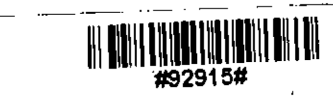
Excitation Mechanism of Erbium in Silicon

A

Thesis submitted to the Department of Electrical and Electronic Engineering, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh in partial fulfillment of the requirements for the Degree of

Master of Science in Electrical and Electronic Engineering



Md. Saiful Anam Siddiqui

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The thesis titled, " Excitation Mechanism of Erbium in Silicon", submitted by Md. Saiful Anam Siddiqui, Roll No.:- 921352P, Session:-1990 - 91 - 92 Registration No.:- 87208 of M. Sc. in Engineering has been accepted as satisfactory in partial fulfillment of the requirements for the Degree of

Master of Science in Electrical and Electronic Engineering

Board of Examiners :

1. ______ 9-2-99 Chairman (Supervisor)
   Dr. Md. Quamrul Huda,
   Assistant Professor
   Department of Electrical and Electronic Engineering
   BUET, Dhaka - 1000.

2. ______ 9-2-99 Member (Ex-Officio)
   Dr. Enamul Basher
   Professor and Head
   Department of Electrical and Electronic Engineering
   BUET, Dhaka - 1000.

3. ______ Member (Internal)
   Dr. M. Rezwan Khan
   Professor
   Department of Electrical and Electronic Engineering
   BUET, Dhaka - 1000.

4. ______ Member (External)
   Dr. Jalalur Rahman
   Professor
   Department of Applied Physics & Electronics
   Dhaka University
   Dhaka – 1000.

Declaration

This is to certify that the work presented in this dissertation has been done by me under direct supervision of Dr. Md. Quamrul Huda, Assistant Professor, Department of Electrical and Electronic Engineering, Bangladesh University of Engineering and Technology , Dhaka, Bangladesh and it has not been submitted else where for the award of any degree or diploma.

Counter Signed

Dr. Md. Quamrul Huda

Signature of the Student

Md. Saiful Anam Siddiqui

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List of Figures

Figure No.	Description	Page No.
Figure 2.1	E – k diagram of (a) direct band gap and (b) indirect band gap material.	6
Figure 2.2	Energy versus wave vector diagram for silicon.	7
Figure 2.3	Electron – hole recombination between (a) conduction band and valence band (b) donor level and valence band (c) conduction band and acceptor level and (d) donor level and acceptor level.	8
Figure 2.4	The energy band diagram for the Shockley – Hall – Read (SHR) generation - recombination kinetics: (1) electron capture (2) electron emission(3) hole capture and (4) hole emission	11
Figure 3.1	Schematic energy level diagram of a free Er^{3+} ion (left-hand side) and Er^{3+} in crystal field (right hand side) (after Xie *et. al.*)[16]	14
Figure 3.2	Time evolution of Er^{3+} luminescence intensity during and after a 30 μs pulse observed by (a) Shin *et. al.* at 9 °K (b) Przybylinska *et. al.* at 77 °K.	18
Figure 5.1	Variation of photoluminescence intensity against erbium lifetime for different values of carrier lifetime.	40
Figure 5.2	Effect of carrier lifetime and carrier density on Photoluminescence intensity for different values of erbium lifetime.	41
Figure 5.3	Temperature dependence of photoluminescence intensity.	42

Figure No.	Description	Page No.
Figure 5.4	(a) Electron occupied erbium atoms as a function of time after providing the optical (laser) excitation and (b) the corresponding erbium photoluminescence.	44
Figure 5.5	Photoluminescence intensity versus time for two different temperatures of 40 °K and 140 °K.	45
Figure 5.6	Photoluminescence decay with respect to time for different values of carrier lifetime with constant erbium lifetime.	46
Figure 5.7	Time decay of photoluminescence intensity for different values of erbium lifetime with constant carrier lifetime.	48
Figure 5.8	Effect of carrier lifetime on photoluminescence intensity for the same value of erbium lifetime.	49
Figure 5.9	Rising and decaying of both (a) electron occupied erbium state and (b) the corresponding photoluminescence intensity.	51
Figure 5.10	Time evolution of photoluminescence intensity during and after a 30 μs laser pulse.	52

ABSTRACT

Excitation mechanism of erbium in silicon and the corresponding luminescence has been studied. The erbium site in silicon has been considered to act as a deep level recombination centre. Shockley - Hall - Read recombination kinetics have been used to explain the energy transfer mechanism from silicon to the erbium atom. Assigning particular values of capture cross - section and emission coefficients to the erbium related level, the rate of capture and emission of electrons has been equated. The steady state density of electron occupied erbium defect sites has been calculated. The rate of excitation of erbium atoms from the ground state ($^4I_{15/2}$) to the first excited state($^4I_{13/2}$) has been calculated from recombination of trapped electrons with holes.

In case of switching condition, expression for gradual increase of the number of erbium sites trapping electrons has been calculated. Then the corresponding increase of erbium luminescence to its steady value as a function of time has been determined. The effect of carrier lifetime, erbium lifetime and temperature on erbium luminescence has been discussed. Effect of a short duration of excitation pulse on photoluminescence intensity has also been analyzed. Excellent agreement with the experimental results on erbium luminescence in the literature has been observed.

Excitation Mechanism of Erbium in Silicon

Approval	ii
Declaration	iii
Acknowledgment	iv
List of figures	v
Abstract	vii

CONTENTS

Chapter One – INTRODUCTION

1.1	Introduction	1
1.2	Objective of the work	2
1.3	Organization of the thesis	3

Chapter Two - LUMINESCENCE FROM SILICON

2.1	Introduction	5
2.2	Direct and indirect band structure	5
2.3	Radiative and non-radiative process	7
2.4	Impurity enhanced luminescence	9
2.5	Shockley – Hall Read (SHR) generation – recombination kinetics	10

Chapter Three – LUMINESCENCE INVOLVING ERBIUM

3.1	Introduction	13
3.2	Optical properties of Er^{3+} ion	13
3.3	Erbium luminescence in silicon	14
3.4	Excitation mechanism of erbium in silicon	16
3.5	The luminescence rise time	18

Chapter Four - MATHEMATICAL MODEL FOR ERBIUM LUMINESCENCE

4.1 Introduction 20
4.2 Steady state excitation mechanism and the related luminescence 20
4.3 Rise time of erbium luminescence 23
4.3.1 Electron occupied erbium state 23
4.3.2 Time dependent erbium luminescence 24
4.4 Decay of erbium luminescence 28
4.4.1 Decay of electron occupied erbium state 28
4.4.2 The luminescence decay 30
4.5 Luminescence corresponding to laser pulse of short duration 32

Chapter Five - RESULTS AND DISCUSSION

5.1 Introduction 36
5.2 Electronic property of erbium related level 36
5.3 The erbium luminescence 39
5.4 Rise time of erbium luminescence 43
5.5 Photoluminescence decay profile 43
5.6 Erbium luminescence under short excitation pulse 50

Chapter Six - CONCLUSION

6.1 Conclusion 53
6.2 Recommendation for future work 54

REFERENCES 55

APPENDIX - A: Details derivation of excited erbium 58
APPENDIX - B: Details derivation of decay equation of erbium filled by electron 62
APPENDIX - C: Details derivation of erbium luminescence decay 66

CHAPTER ONE

INTRODUCTION

1.1. Introduction

Since the advent of solid state electronics, integrated circuit technology has been growing rapidly. Operating speed of IC chip is increasing with simultaneous increase of the word size. High speed data transmission and processing necessarily requires very large bandwidth data buses. With the increasing demand of high speed information transmission, recent interests of research are focused on this advancing field.

Whenever device speed and chip complexity increase, problem related to interconnection become more and more acute. At present, interconnection density (Aluminum interconnector), interconnected driver limited power dissipation and the system bandwidth are the limiting factors to achieve high speed transmission. These limiting factors also largely affect the increased functionality of integrated circuits. Complete integration of optoelectronic on IC chips can be a dependable solution against these present limitations. Under such scheme, data/clock signals will be converted to optical signals before being transmitted through some waveguiding mechanism inside the chip. The optical signal will then be decoded by detectors in the receiving end. A huge increase in system bandwidth is possible with optical signals. Optical transmission is free from any ambient electrical noise, electromagnetic interference, echoes or cross - talk and does not generate any of its own electrical noise. Optical interconnection offers a high degree of flexibility and versatility.

Silicon is the key element in today's semiconductor technology, as it constitutes most of the semiconductor devices that influence our everyday life. If silicon can be realized with optoelectronic components within the single silicon chip, it would extend the functionality of silicon technology from microelectronics to optoelectronic. Several optical functions, such as wave guiding, modulation, detection and optical switching have already been shown [1] to be achievable in silicon. Unfortunately, however, silicon

is not an efficient light emitter due to its indirect bandgap structure. An efficient light emitter in silicon thus seems to be the only missing part towards the optoelectronic functionality in silicon. A considerable efforts have been made to develop a fully silicon based efficient light emitting structure or light source (LED) in recent years. This light source in silicon, in the form of LED should preferably be operated at wavelength around 1.3 μm or 1.5 μm which falls in the absorption minimum of optical fiber communication. Again, silicon band gap energy is much greater than photon energy at this wavelength so that there is no cross-link effect on device performances.

The issue of silicon light source is still at an early stage and several different approaches, such as band gap engineering or the addition of isovalent impurities or structural defects, are being tried to circumvent the inefficient light emission from silicon. The introduction of rare earth element erbium has been demonstrated [2] to be highly potential in efficient light emission. In particular, erbium impurities in silicon exhibit sharp luminescence at ~ 1.54 μm due to its internal 4f - shell transition from the $^4I_{13/2}$ state to the ground ($^4I_{15/2}$) state. This 1.54 μm luminescence is of great interest in telecommunication engineering as losses in optical fiber is minimum at this wavelength.

Despite these promising results, due to the lack of room temperature photoluminescence and electroluminescence at early stages of research, this field of study remained silent for several years. But very recently the observation of photoluminescence [3-7] and electroluminescence [8-10] at room temperature has grown a new interest in erbium doped silicon. Thus the rare earth element erbium might play an important role in the development of silicon based optoelectronic integrated circuits capable of operating at GHz level.

1.2 Objective of the work

It is well known that the rare earth atom erbium when incorporated in silicon, provides a characteristics luminescence at 1.54μm due to the $^4I_{13/2} \rightarrow {}^4I_{15/2}$ emission. But the exact mechanism of excitation of erbium atom in silicon is however yet to be known. While it is clear that erbium atoms take the *electron - hole* recombination energy to reach the

first excited state of $^4I_{13/2}$, there is a dispute over the process of this energy transmission. Few excitation models have so far been developed. But some of the experimentally observed characteristics, especially the temperature and power dependence of erbium luminescence, the rather long rise and decay time of erbium luminescence, etc. are not clearly explained by the models in the literature.

The objective of this research is to develop a mathematical model based on Shockley - Hall - Read (SHR) recombination kinetics for the excitation mechanism of erbium in silicon. Density of excess carriers, life time and the deep state properties will be taken into consideration for estimation of the emitted luminescence power. Effect of different parameters on erbium luminescence will be studied. Behavior of erbium luminescence under short switching condition will also be studied.

1.3 Organization of the thesis

We know that silicon has a very low luminescence efficiency due to its indirect band gap efficiency. Necessities on development of optoelectronic devices, silicon's importance in electronic world, its inability to emit efficient light, and chronological achievement in the development of fully silicon based light emitting diode (LED) has been described in Chapter One.

Band gap, radiative emission, impurity enhanced luminescence in silicon, Shokley – Hall - Read (SHR) recombination kinetics, etc. have been given in details in Chapter Two.

Chapter Three describes the present technological state of erbium luminescence in silicon. The characteristics of erbium luminescence and the published excitation mechanism of erbium in silicon are also given in Chapter Three.

The mathematical model for excitation mechanism of erbium in silicon has been developed in Chapter Four. The erbium luminescence, luminescence rise time and decay time, luminescence under short excitation pulse have also been analyzed mathematically in this Chapter.

Chapter Five gives the results described in chapter 4. Effect of different parameters like the capture and emission coefficient, lifetime, etc. on the erbium luminescence has been shown. The erbium luminescence under short laser excitation pulse is also studied in Chapter Five.

Conclusion on these results and recommendation for future work have been given in Chapter Six.

CHAPTER TWO

LUMINESCENCE FROM SILICON

2.1 Introduction

Whenever an electron in the conduction band recombines with a hole in the valence band, the recombination energy is given up, either as a photon or in the form of a non – radiative process. If this recombination occurs directly, bandgap light is given off and the process is called a radiative process. If recombination occurs through a recombination level within the bandgap, or involves a third particle like a charge carrier or phonon, the recombination energy is dissipated in the crystal lattice and the process is called a non – radiative process.

There are many ways to generate *electron – hole* pairs in silicon. But efficient light emission from silicon depends on complex interplay between the numerous recombination processes of these excited carriers. Light emission from silicon is greatly reduced due to its indirect bandgap structure. So intrinsic radiative efficiency of silicon is very low. In this chapter, direct and indirect band structure, radiative and non-radiative process, and the possible ways to enhance radiative emission from silicon, Shokley – Hall – Read (SHR) generation-recombination kinetics have been described.

2.2 Direct and indirect band structure

The band structure of crystalline solid, i.e. the energy - momentum (E-k) relationship is usually obtained by solving the Schrodinger wave equation assuming a single electron travelling through a perfectly periodic lattice. The wave function of electron is assumed to be in the form of a plane wave moving, for example, in the x - direction with propagation constant k, (called a wave vector). The space-dependent wave function for

the electron is given by

$$\psi_k(x) = U(k_x, x) \; e^{jk_x x} \tag{2.2.1}$$

where the function $U(k_x, x)$ modulates the wave function according to the periodicity of the lattice.

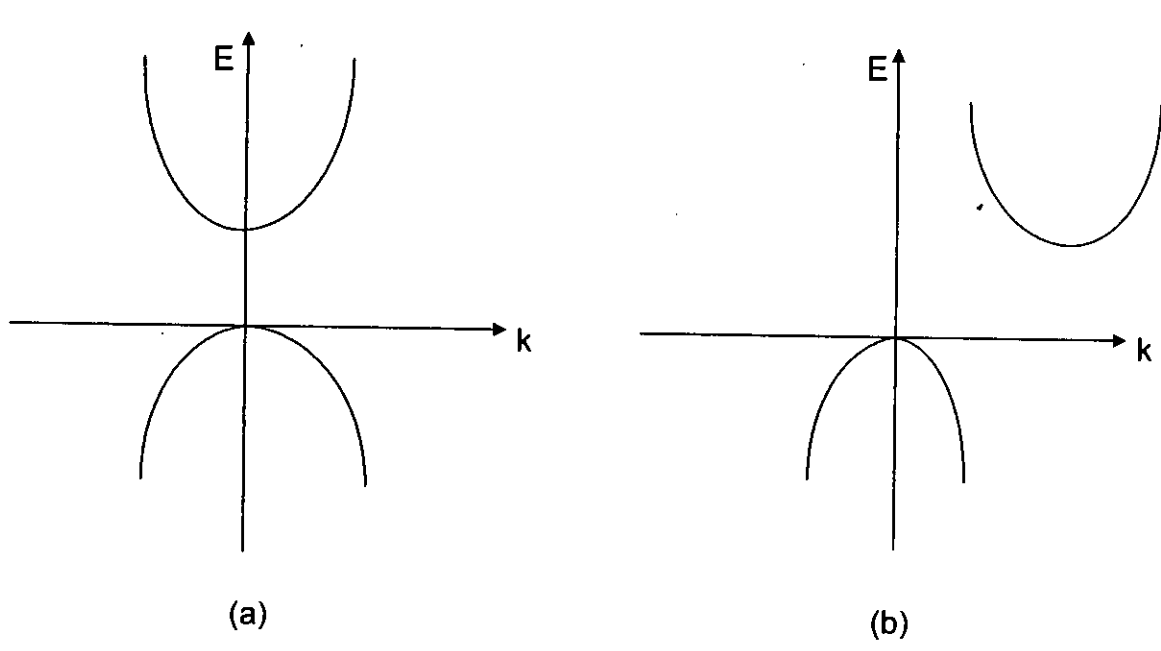

Figure 2.1: E -k diagram of (a) direct bandgap and (b) indirect bandgap material

Energy vs. propagation constant curve (E - k) for two typical cases is shown in figure 2.1. For figure (a) the conduction band minimum and the valence band maximum has the same k value (k = 0). But in fig. (b) k value for the minimum of the conduction band varies from the maximum of the valence band. In the first case, electron can fall directly from conduction band to valence band without changing the value of k. But an electron transition from conduction band minimum to the valence band maximum requires some change in k in the second case. Thus the semiconductor energy bands are classified into two categories:

(a) direct bandgap; and (b) indirect bandgap.

Whenever the conduction band minimum and the valence band maximum within the bandgap have same value of k, the bandgap is called direct bandgap and the material is called direct bandgap material, such as, GaAs, $Al_xGa_{1-x}As$ etc..

Again, if the k value for conduction band minimum varies from the valence band maximum, the bandgap is called indirect bandgap and the material is called indirect bandgap material. Si, Ge, $Si_{1-x}Ge_x$ are the examples of indirect bandgap material.

2.3 Radiative and non - radiative process

Band structure of silicon is shown in figure 2.2. . The following figure shows the E - k relation along the <100> and <111> directions. Since lattice properties are different along different crystalline directions, the energy – momentum (E-k) curve in silicon is a complex picture, as shown in figure 2.2.

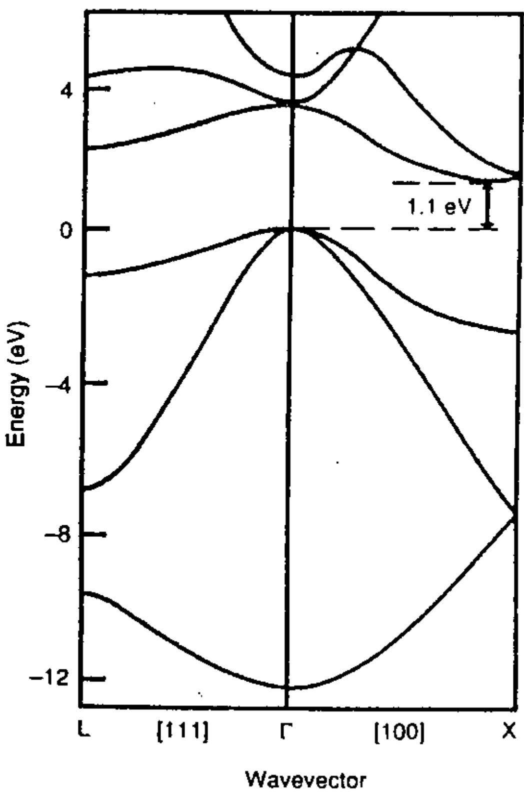

Figure 2.2 : Energy versus wave vector diagram for silicon.
The arrow shows the indirect bandgap of 1.1 eV.
at 300 °K .

It is seen that the probability for intraband smallest energy transition of an electron is extremely low. So an appropriate phonon must participate in the process in order to conserve momentum. The process involving three particle for band to band transition of *electron - hole* pairs in silicon is thus very weak .

Doped impurities introduce additional extrinsic recombination pathways. Acceptor and donor impurity can provide radiative transition to conductiôn band and valence band respectively. Whenever the acceptor impurities and donor impurities are physically close to each other, radiative transition is also possible between them. Impurities may also act as non - radiative recombination centres which is mostly the case in silicon. Figure 2.3 shows some of the radiative process in silicon involving impurities.

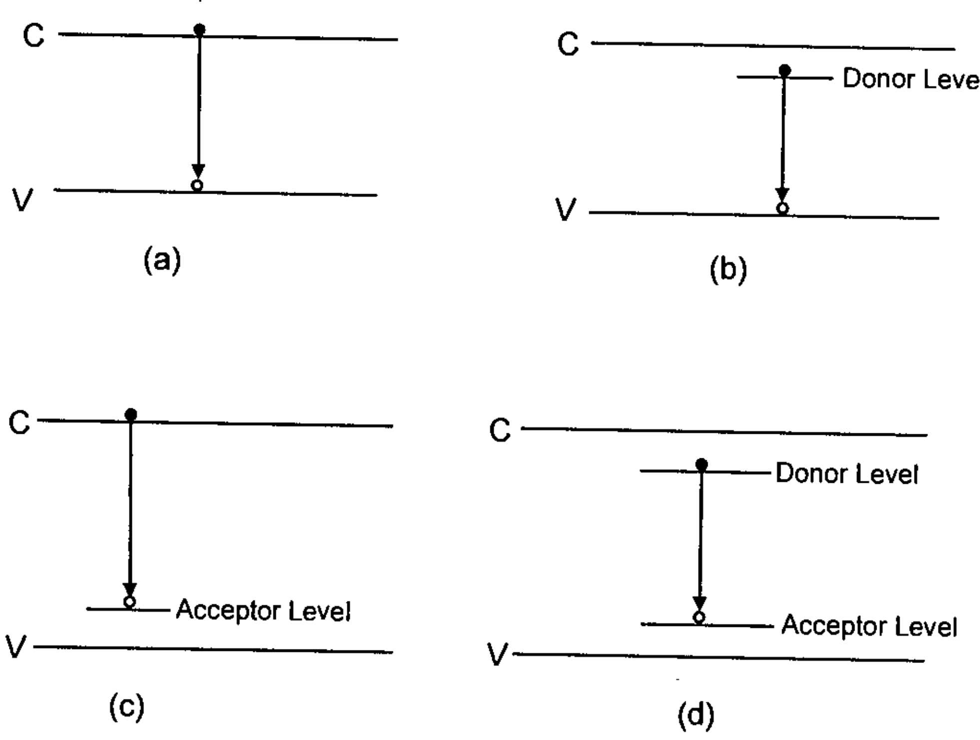

Figure 2.3. : Electron - hole pair recombination between (a) conduction band and valence band, (b) donor level and valence band, (c) conduction band and acceptor level and (d) donor level and acceptor level.

Electron - hole pairs can form bound state called exciton due to coulomb attraction between them. This process may provide another source of radiative emission in silicon. Exciton can move freely through the crystal. Impurity atom attracts the exciton to form bound exciton in extrinsic semiconductor. The annihilation (radiative collapse) of exciton yields a photon.

On the other hand, non - radiative processes are found to be very efficient in silicon. In Auger recombination process, excess energy released by the recombination of *electron - hole* pairs is transferred to a third particle (either an electron or a hole) thereby satisfying energy and momentum conservation law. The electron or hole thermalizes rapidly by phonon emission. Non - radiative recombination through extended defect, trapping centres, deep level impurity sites is also possible.

2.4 Impurity enhanced luminescence

Optically active centres in silicon can enhance radiative efficiency of silicon. Isovalent impurities, impurity complexes or defects induced by radiation damage or thermal processes can serve as optically active centres. Mobile excited carriers are highly localized at these centres and their momentum are diffused. Thus momentum is conserved and the probability of direct transition is enhanced. Isovalent impurities and rare earth elements are the two types of defect centres in silicon which provide radiative transition.

An isovalent impurity is an impurity which has the same outer electronic structure as the atom of the host crystal. But the electronic core-structure of these atoms are different. Carbon, Germanium, Tin, etc., are examples of isovalent impurities in silicon. Because of the different electronic core structure from silicon, they act as trap levels that are used as recombination centres. As the isovalent trap centres are electrically neutral, high concentration of *electron - hole* pairs can be incorporated without promoting non - radiative Auger process.

The rare earth (RE) elements show characteristic luminescence from intra 4f - shell transitions of RE^{3+} ions . The Coulombic interaction and spin - orbit coupling of the 4f electrons result in different energy levels for RE^{3+} ions in silicon. The recombination energy of the *electron* - *hole* pairs at the rare earth impurity centres excite the RE atoms to their higher energy state, which then release the energy by radiative or non - radiative decay to the ground state [11]. The RE^{3+} ions possess partially filled 4f - shells which are shielded by the completely filled external 5s and 5p shells . Due to this shielding, they display atomically sharp luminescence when incorporated in other materials, with wavelengths that are largely independent of surrounding host material and of the operating temperature [12].

2.5 Shockley – Hall – Read (SHR) generation – recombination kinetics

According to Shockley – Hall – Read (SHR) recombination kinetics, there are basically four processes involving generation – recombination of *electron* – *hole* pairs [13]. These processes are as follows:

(1) Electron capture: If the concentration of traps per unit volume is N_t, then the recombination for the electron is given by

$$R_n = c_n n N_t (1 - f_t) \tag{2.5.1}$$

where c_n is the capture coefficient for the electrons and f_t is the occupation probability of the trap.

(2) Electron emission: The rate for the electron emitted from the trap into the conduction band is proportional to the total number of traps times the probability that the trap is filled. Thus the generation rate of the electrons due to this process is

$$G_n = e_n N_t f_t \tag{2.5.2}$$

where e_n is the emission rate and $n_t = N_t f_t$ = the density of the electron occupied traps.

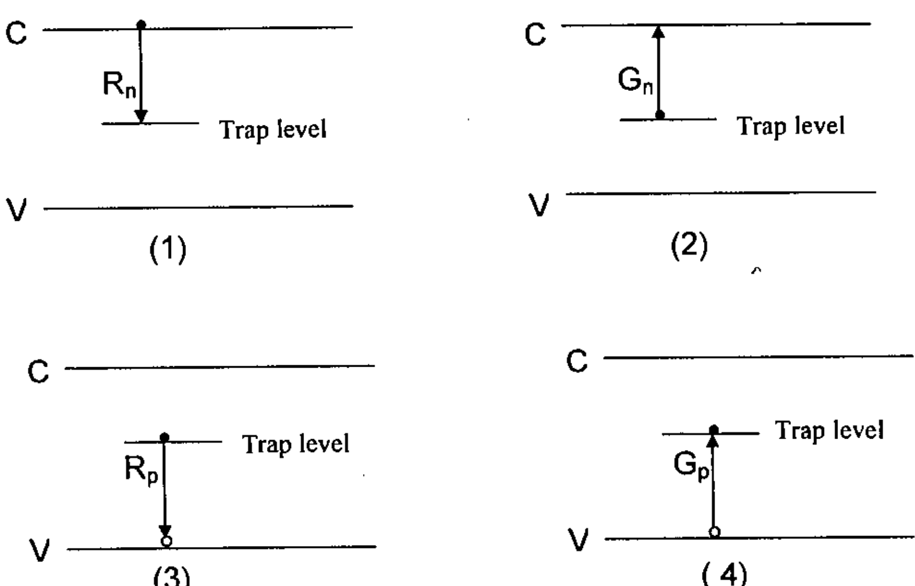

Figure 2.4 : The energy band diagrams for the Shockley – Hall – Read generation – recombination kinetics : (1) electron capture (2) electron emission (3) hole capture and (4) hole emission

(3) Hole capture: The recombination rate of the holes is given by holes capture coefficient, hole density and electron occupied traps as:

$$R_p = c_p p n_t \tag{2.5.3}$$

where, c_p is the capture coefficient for holes.

(4) Hole emission: The generation rate for holes due to this process is proportional to the density of the empty traps

$$G_p = e_p N_t (1 - f_t) \tag{2.5.4}$$

Where, e_p is the emission rate for the holes.

To obtain a relation between e_p and c_p, e_n and c_n we use thermal equilibrium condition. At thermal equilibrium the net generation – recombination of electrons and holes is zero. So

$$R_n - G_n = c_n n_0 N_t (1 - f_{t0}) - e_n N_t f_{t0} = 0 \quad (2.5.5)$$

$$R_p - G_p = c_p p_0 N_t f_{t0} - e_p N_t (1 - f_{t0}) = 0 \quad (2.5.6)$$

where the subscript 0 denotes the steady – state value. Solving the above equation (2.5.5) and (2.5.6) we get

$$e_n = c_n n_0 \frac{(1 - f_{t0})}{f_{t0}} = c_n n_1 \text{ (let)} \quad (2.5.7)$$

$$e_p = c_p p_0 \frac{f_{t0}}{(1 - f_{t0})} = c_p p_1 \text{ (let)} \quad (2.5.8)$$

We know that processes (2) and (4) create an electron – hole pair while processes (1) and (3) destroy an electron – hole pair. At equilibrium two rates are balanced. So, the fraction of the occupied traps f_t is given by

$$f_t = \frac{c_p p_1 + c_n n}{c_n (n + n_1) + c_p (p + p_1)} \quad (2.5.9)$$

At steady state condition, the net rate of electron capture must be equal to that of holes, so we get the net recombination rate

$$R_n - G_n = R_p - G_p = \frac{c_n c_p (np - n_1 p_1)}{c_n (n + n_1) + c_p (p + p_1)} N_t \quad (2.5.10)$$

$$= \frac{(np - n_1 p_1)}{\tau_p (n + n_1) + \tau_n (p + p_1)} \quad (2.5.11)$$

Where $\tau_p = \frac{1}{c_p N_t}$ = hole life time and $\tau_n = \frac{1}{c_n N_t}$ = electron lifetime

CHAPTER THREE

LUMINESCENCE INVOLVING ERBIUM

3.1 Introduction

Rare earth - doped semiconductor has begun to attract considerable attention since they emit sharp and temperature stable luminescence due to the intra 4f shell transition. The optical properties have been widely investigated as RE - doped semiconductor can potentially be used to fabricate optical devices that are insensitive to the surrounding host environment. The use of RE lasers and amplifiers in optical fibre is well known [14].

Since silicon is the most important semiconductor in today's world, a large effort has been devoted to the realization of silicon based optoelectronic devices. Silicon based optoelectronic technology requires efficient light emission from silicon. But unfortunately the band - to - band recombination efficiency in silicon is too small for practical uses and many alternatives are recently investigated [15]. One viable approach suggested by Ennen *et. al* [2] is the doping of silicon with erbium.

Erbium is one of the most widely used rare earth elements. Erbium doped semiconductor are of great interest in integrated optoelectronic technology due to the emission at 1.54 μm, a standard telecommunication wavelength. Erbium doped silicon can be used to integrate light emitting diode (LED) that can be fabricated with other devices on the same silicon chip. This LED may find application in silicon based optoelectronic circuits. This chapter describes the optical properties of Er^{3+} ion, erbium luminescence in silicon, excitation mechanism of erbium in silicon, the luminescence rise time.

3.2 Optical properties of Er^{3+}

It has been well established that the rare earth (RE) element erbium has strong tendency of being in 3^{+} trivalent electronic state when incorporated in silicon. Figure 3.1 shows the schematic energy level diagram of the RE erbium in its trivalent state.

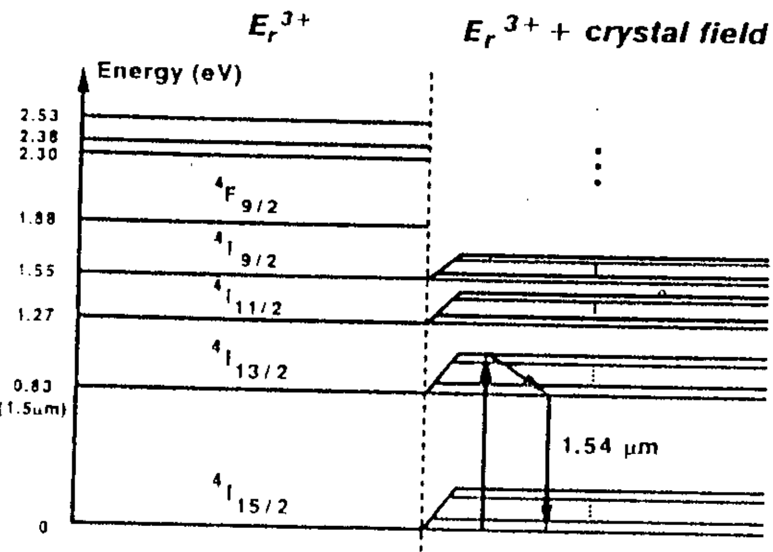

Figure 3.1. Schematic energy level diagram of a free Er^{3+} ion (left-hand side) and Er^{3+} in crystal field (right–hand side)(after Xie *et. al.*[16]).

The crystal field of silicon splits each single state of a free Er^{+3} ion into a manifold of sublevels with slightly different energies. The nature of splitting depends on the erbium site symmetry [17]. The crystal field also intermixes the states of opposite parities [18]. The transition between the intra 4f manifold [$^4I_{13/2} \rightarrow {}^4I_{15/2}$] is forbidden in the case of free ions by the parity selection rule. The transition corresponding to photon energy of 0.8 ev is however made possible by the crystal field in the case of Er^{3+} in silicon. The energy difference between the levels is less than the silicon band gap, making luminescence from erbium feasible in silicon based optoelectronic.

3.3 Erbium luminescence in silicon

Optical properties of various compound III - V semiconductor doped with RE ions have been widely investigated in the last few decades. Sharp intra 4f - luminescence spectra from RE elements Ytterbium (Yb) and Paseodymium (Pm) doped in III - V semiconductor was first observed by Kasatkin *et. al.* [19][20]. Ennen *et. al.* [21] observed Yb - 4f photoluminescence from Yb - doped GaAs in 1983. But this luminescence

intensity was very weak. They also pointed out the potential application of RE elements erbium in semiconductor materials, particularly in silicon for the development of LED and LASER [2]. They successfully showed that the optical transition of erbium in silicon took place between the crystal field split - orbit levels $^4I_{13/2}$ and $^4I_{15/2}$. This transition has wavelength around 1.54 μm coinciding with the low loss window of standard optical communication. This phenomenon makes the Si : Er system very attractive for silicon based LED. Klein *et. al.*[22] suggested erbium to be the best suited RE dopant for device application due to its greater lifetime in comparison to other RE elements.

Ennen *et. al.* [17] and Tang *et. al.* [23] have carried out photo emission analysis from erbium in silicon. They demonstrated that most of the optically active Er^{3+} ions have tetrahedral or lower site symmetries on interstitial lattice sites in silicon. Favannec *et. al.* [24] showed the possibility to enhance the luminescence in Er - doped silicon by the simultaneous presence of impurities such as O, C, N, F or Br. Michel *et. al.*[3] also found a large enhancement in the luminescence intensity by introducing impurities. By using Extended X - ray Absorption Fine Spectroscopy (EXAFS) analysis, Adler *et. al.* [25] demonstrated that the impurities modify the chemical surrounding of erbium in silicon. Since oxygen activates erbium optically, researchers are very much interested in Si : Er system with oxygen co - dopant.

In electrical characterization of erbium in silicon, Er has been demonstrated to present donor behavior in silicon which was suggested by Benton *et. al.* [26]. But Efeoglu *et. al.* [27] observed no shallow erbium donor or acceptor levels in the Molecular Beam Epitaxy (MBE) grown sample. Priolo *et. al.* [28] pointed out that the oxygen co - implant enhances the erbium donor activities. The erbium donor concentration in Czocralsky (CZ) Si increases with increasing dose up to a maximum value of $\sim 5\times10^{16}$ donors $/cm^3$ at an erbium concentration of $\sim 4\times10^{17}/cm^3$ [26] coinciding with the value for maximum photoluminescence. They suggested that this donor state may be the result of the formation of Er - O complexes in Er doped CZ silicon. Based on these results they suggested that electrical and optical activation of erbium in silicon are correlated i.e. the Er - donor behavior might be associated with the optically active Er^{3+} state. Ennen *et.*

al. [29] first reported electroluminescence from erbium doped silicon at 77 °K. Erbium doped epilayer on an n - type silicon was grown by MBE at an erbium concentration of $5.6\times10^{18}/cm^3$.

Sharp fall of luminescence intensity with increasing temperature is the major limitation of Si : Er luminescence. This phenomenon is called temperature quenching. Coffa *et. al* [6] demonstrated that the temperature dependence of 4f – shell luminescence intensity by assuming that emission of an electron from the erbium related level to the conduction band is responsible for thermal quenching. Priolo *et. al* [30] and A. Polman [31] demonstrated that the dissociation of a bound exciton (due to the release of the trapped electron from erbium site) results temperature quenching. Kik. *et. al.* [32] performed the luminescence decay measurement within the temperature range 12 – 170 °K. They observed that the luminescence intensity decreases by three order of magnitude as temperature rises from 12 to 150 °K. They also found nearly a single exponential decay at lower temperature. And strongly non - exponential decay at higher temperature. They pointed out that decrease of lifetime with temperature results temperature quenching.

Despite the effect of temperature quenching, Ren *et. al.* [33] were first able to fabricate Si : Er LED with peak erbium concentration of $10^{18}/cm^3$. Another p^+n^+ LED structure has been reported by Franzo *et. al.* [34] where erbium was incorporated at a concentration of $10^{19}/cm^3$ along with $10^{20}/cm^3$ of oxygen.

3.4 Excitation mechanism of erbium in silicon

Several groups [9,30,32,35 -38] have proposed several luminescence mechanism for Si:Er system. It is accepted that erbium ions in silicon are excited to the $^4I_{13/2}$ state by the recombination energy of electron - hole pairs (generated either electrically or by optical pumping) near an erbium related level in the band gap. Radiative decay of these ions to the ground state, $^4I_{15/2}$, provides photon at 1.54 μm. Thus the excitation occurs likely through a Auger Process. Coffa *et. al.* [36] proposed that the excitation mechanism of

Er atoms in silicon occurs through Auger process, where the recombination of excitons bound to Er - donor atoms results the excitation of 4f - shell electron. Under this assumption, the steady state population of excitons controls the luminescence. Shin *et. al.* [37], De Dood *et. al.* [39] supported the same excitation mechanism as described by Coffa *et. al.*[36]. Priolo *et. al.* [30] proposed temperature dependence of excitation mechanism and Er - 4f shell luminescence intensity. They pointed out that electron emission from the erbium related level to the conduction band results thermal quenching. They assumed that the level is a donor and becomes neutral by capturing an electron. By solving rate equation they proposed an expression for Er - 4f shell luminescence intensity with respect to temperature given by

$$f(T) = \frac{f_0}{1 + B\,e^{-\frac{E}{kT}}} \qquad (3.4.1)$$

where f_0 = intensity at very low temperature

T = temperature °K

E = activation energy, ev

k = Boltzmann constant, ev/°K

B = fitting parameter

They used activation energy about 0.15 ev, which was observed from Deep Level Transient Spectroscopy (DLTS) [25] measurements.

Since temperature dependence of processes like electron emission, the hole capture, etc. are not known, they considered B as a fitting parameter. Although the calculated curve and experimental results show good agreement but this model is inconsistent with the experimentally observed fact that the lifetime of erbium luminescence decreases with increasing temperature.

Kik *et. al.* [32] explained temperature dependence of luminescence mechanism by another process which they called energy back transfer (EBT) process. They pointed out that this is a reverse process of the excitation process. In this process, the relaxation energy of intra 4f - shell is transferred back to the host, forming an electron - hole pair at the erbium related level. The pair thermally dissociates at elevated

temperature resulting in quenching the 4f - shell luminescence. Taguchi *et. al.*[38] demonstrated the multi-phonon energy transfer excitation mechanism.

3.5 The Luminescence rise time

Shin *et. al.*[37] observed a longer rise time for the Er luminescence intensity in Er doped silicon. The same observation was also found by Przbylinska *et. al.* [40]. They observed the luminescence to increase for a certain duration after the termination of the excitation pulse. The corresponding time evolution of the PL intensity during and after 30 μs pulse is shown in figure 3.2.

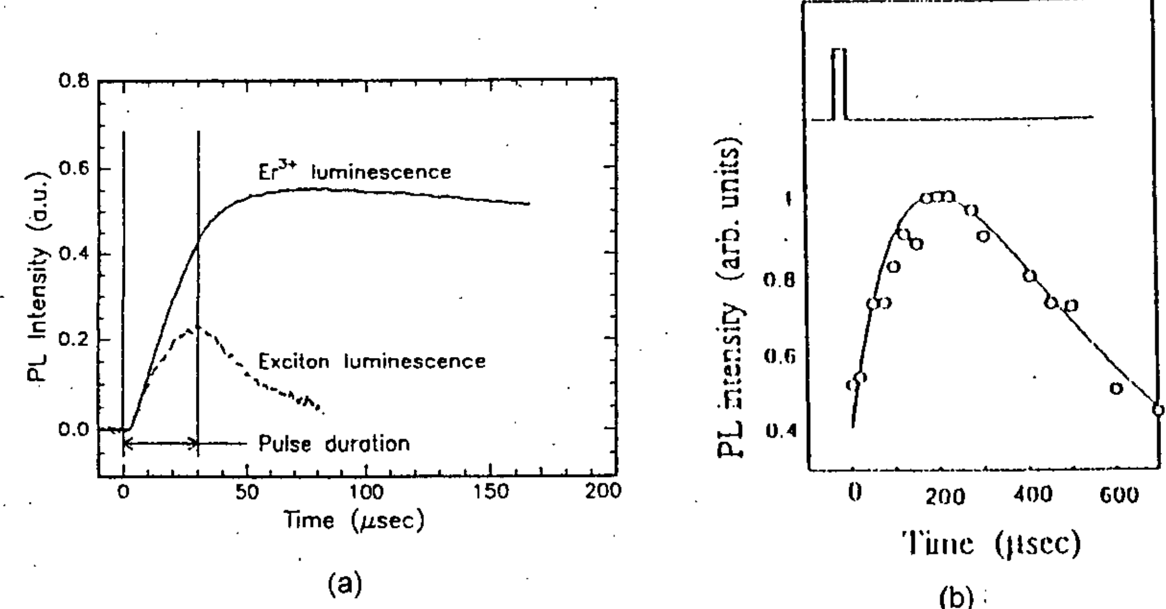

Figure 3.2 Time evolution of Er^{3+} luminescence intensity during and after a 30 μs excitation pulse observed by (a) Shin *et. al.* at 9 °K and (b) Przbylinska *et. al.* at 77 °K.

They demonstrated that Er^{3+} ions will remain excited at a considerable rate up to 50→ 100 μs after the excitation pulse is switched off. They observed this phenomenon only. But they did not give any explanation or mathematical solution in their model. They proposed that the recombination of exciton bound to an erbium related trap level within the silicon bandgap is the possible way to excite erbium in silicon. However, the coulombic attraction between e – h in an exciton is very weak. So they can dissociate at

dissociate at an elevated temperature. Furthermore, binding energy of an exciton to a deep level is also very low (typically within the range of 10 meV). As a result, excitation of erbium through bound exciton is an unlikely process at temperature close to the room temperature. Apart from that, the excitation mechanisms described above remain ambiguous due to taking several critical assumptions for the estimation of many parameters. A new approach on the process of erbium excitation mechanism, based on Shockley – Hall – Read (SHR) recombination kinetics will be developed in the next chapter.

CHAPTER FOUR

MATHEMATICAL MODEL FOR ERBIUM LUMINESCENCE

4.1 Introduction

It is believed that, erbium luminescence in silicon is *electron – hole* mediated process. In the present work, it is proposed that the process occurs though recombination of electron (trapped at erbium level) with hole. The recombination energy provides energy to excite erbium ions from the ground state, $^4I_{15/2}$, to the first excited state, $^4I_{13/2}$. The first excited state then relaxes radiatively to the ground state. In this chapter, excitation mechanism of erbium, electron occupied erbium states, time dependent erbium luminescence, decay of erbium luminescence, effect of short duration of excitation pulse on erbium luminescence, etc. have been described and their mathematical solution have been developed.

4.2 Steady state excitation mechanism and the related luminescence

As described in the previous chapter, the recombination of *electron - hole* pair at erbium related level excites erbium in silicon. The gateway for erbium excitation can be a donor level, a hole trap, an acceptor level or an electron trap. Since past studies [26, 28] showed the donor activity of erbium in silicon and erbium has deep states [30,41] in the upper half of the band gap, the energy transfer to the erbium atom can be considered to be through a donor level or a defect state situated in the upper half of silicon band gap. Whatever might be the erbium state, due to its presence in the close vicinity of Er^{3+} ions, it will capture free electron from the conduction band by the coulomb attraction. If the occupied trap can capture a hole before the electron is thermally dissociated, the corresponding Er^{3+} ion takes the recombination energy of the *electron - hole* pair and thus reaches its first excited state $^4I_{13/2}$.

Let,

N_{er} = total number of active erbium sites per unit volume

n_{er} = total number of active erbium filled by electron at a steady state of optical excitation per unit volume

$N_{er} - n_{er}$= the number of available free Er sites for electron capture

e_n = electron emission rate, s^{-1}

e_p = hole emission rate, s^{-1}

c_n = electron capture coefficient , $cm^3\ s^{-1}$

c_p = hole capture coefficient , $cm^3\ s^{-1}$

Therefore, electron emission rate = $(e_n + c_p p)n_{er}$ (4.2.1)

electron capture rate = $(e_p + c_n n)(N_{er} - n_{er})$ (4.2.2)

At steady state, Electron emission rate = Electron capture rate

$$(e_n + c_p p)n_{er} = (e_p + c_n n)(N_{er} - n_{er}) \tag{4.2.3}$$

Thus the fraction of erbium sites occupied by electron is given by

$$f_t = \frac{n_{er}}{N_{er}} = \frac{e_p + c_n n}{(e_n + e_p + c_n n + c_p p)} \tag{4.2.4}$$

whenever a trapped electron (in erbium level) recombines with a hole before the electron gets dissipated thermally back to the conduction band, the recombination energy excites the corresponding erbium atom. But, if an erbium atom is already in the excited state due to the previous recombination of *electron – hole* pair and

recombination takes place before the corresponding excited erbium atom comes to its ground state, the recombination energy of the later will be wasted away.

So, let

n_{er}^{*} = total number of excited Er atoms at steady state per unit volume

$N_{er} - n_{er}^{*}$ = the number of available Er sites for excitation at any time

So, the number of erbium sites which are filled with electron and are available for excitation = $f_t(N_{er} - n_{er}^{*})$ (4.2.5)

Thus the excitation rate = $f_t c_p p(N_{er} - n_{er}^{*})$ (4.2.6)

Assuming no change in electronic property of the corresponding defect levels of excited erbium ions, the rate of erbium excitation at steady state is given by

$$f_t c_p p(N_{er} - n_{er}^{*}) = \frac{n_{er}^{*}}{\tau} \quad (4.2.7)$$

where τ is the decay time of of $^4I_{13/2}$ state of erbium to the ground state. τ includes both radiative and non–radiative lifetime. So, at equilibrium, the number of excited erbium ions is given by

$$n_{er}^{*} = \frac{f_t c_p p N_{er}}{f_t c_p p + \frac{1}{\tau}} \quad (4.2.8)$$

The luminescence strength is then given by

$$I \;\alpha\; \frac{n_{er}^{*}}{\tau_{er}} \quad (4.2.9)$$

$$\alpha\; \frac{f_t N_{er}}{\tau_{er}} \frac{c_p p}{f_t c_p p + \frac{1}{\tau}} \quad (4.2.10)$$

Where τ_{er} radiative lifetime of erbium.

4.3 Rise time of erbium luminescence

Rise time of erbium luminescence to the steady state (after providing a laser excitation power) can be obtained by solving time equation for electron occupied erbium states. The corresponding time dependent erbium luminescence can then be determined by solving the rate equation of excited erbium atoms. Before the application of laser power, the erbium states are free of electron and so free erbium ions. The emission rate is less than the capture rate. The erbium atoms capture electron at faster rate just after the excitation. After sometimes, capture rate becomes equal with the emission rate. Finally erbium luminescence gets to steady value during the external excitation.

4.3.1 Electron occupied erbium states

When a light pulse is applied to a semiconductor, *electron – hole* pairs are generated during photon absorption. The generation of electrons and holes can be considered as instant. But capture of electron or hole to a trap level is limited by the corresponding capture coefficient. Capture cross-section, thermal velocity of electron or hole largely affect the capture rate. Capture rate is equal to the capture coefficient multiplied by the density of electron or hole. The time dependent electron occupied erbium states has been determined by solving the rate equation.

We know from equations (4.2.1) and (4.2.2) that

$$\text{electron emission rate} = (e_n + c_p p)n_{er}(t)$$

$$\text{electron capture rate} = (e_p + c_n n)(N_{er} - n_{er}(t))$$

Thus the rate of change of electron occupied trap density at time t is given by,

$$\frac{dn_{er}(t)}{dt} = (e_p + c_n n)(N_{er} - n_{er}(t)) - (e_n + c_p p)n_{er}(t) \qquad (4.3.1.1)$$

$$\Rightarrow \qquad \frac{dn_{er}(t)}{dt} + an_{er}(t) = bN_{er} \qquad (4.3.1.2)$$

where,

$$a = (e_p + e_n + c_n n + c_p p) \quad (4.3.1.3)$$

$$b = (e_p + c_n n) \quad (4.3.1.4)$$

Integration factor = $e^{\int a\,dt} = e^{at}$

So the solution of the above differential equation is given by

$$n_{er}(t) = \frac{b}{a} N_{er} + Ce^{-at} \quad (4.3.1.5)$$

where, C is an integration constant

putting $n_{er}(0) = 0$, we get

$$C = -\frac{b}{a} N_{er} \quad (4.3.1.6)$$

$$\Rightarrow \quad n_{er}(t) = \frac{b}{a} N_{er}(1 - e^{-at}) \quad (4.3.1.7)$$

This is the time varying equation of electron occupied erbium states.

4.3.2 Time dependent erbium luminescence

The time dependent erbium luminescence can be obtained by balancing the useful rate of electron – hole recombination with the decay rate. We know from equations (4.2.5) and (4.2.6) in section 4.2 that, the number of electron occupied erbium sites available for excitation is $f_t(N_{er} - n^*_{er})$ and the excitation rate = $f_t c_p p(N_{er} - n^*_{er})$. All of the excited erbium ions do not relax radiatively. Some of them decay through non-radiative channel. So we do not get luminescence from these excited erbium ions. They are wasted.

$$\text{So de-excitation rate} = \frac{N^*_{er}}{\tau}, \quad (4.3.2.1)$$

Where, τ is the lifetime of erbium including radiative and non – radiative lifetime,

Time rate of change of excited erbium = excitation rate – de-excitation rate

So the rate of change of excited erbium is given by

$$\frac{dn_{er}^{*}(t)}{dt} = f_t(t)\left[N_{er} - n_{er}^{*}(t)\right]c_p p - \frac{n_{er}^{*}(t)}{\tau} \qquad (4.3.2.2)$$

putting the value of $n_{er}(t)$ from equation (4.3.1.7) and $f_t(t)$, we get

$$\Rightarrow \frac{dn_{er}^{*}(t)}{dt} + \left[\frac{1}{\tau} + \frac{c_p p}{N_{er}}\left(\frac{b}{a}N_{er}(1-e^{-at})\right)\right]n_{er}^{*}(t) = \frac{N_{er}c_p p}{N_{er}}\left(\frac{b}{a}N_{er}(1-e^{-at})\right) \qquad (4.3.2.3)$$

$$\Rightarrow \frac{dn_{er}^{*}(t)}{dt} + \left[\frac{1}{\tau} + \frac{b}{a}c_p p - \frac{b}{a}c_p pe^{-at}\right]n_{er}^{*}(t) = \frac{b}{a}c_p pN_{er}(1-e^{-at}) \qquad (4.3.2.4)$$

$$\Rightarrow \frac{dn_{er}^{*}(t)}{dt} + \left[D_1 - D_2 e^{-at}\right]n_{er}^{*}(t) = D_2 N_{er}(1-e^{-at}) \qquad (4.3.2.5)$$

where, $$D_1 = \frac{1}{\tau} + \frac{b}{a}c_p p \qquad (4.3.2.6)$$

$$D_2 = \frac{b}{a}c_p p \qquad (4.3.2.7)$$

$$\text{Integration factor} = e^{\int (D_1 - D_2 e^{-at})dt} = e^{D_1 t + \frac{D_2}{a}e^{-at}} \qquad (4.3.2.8)$$

The solution (details derivation is given in APPENDIX – A) of the above differential equation is given by

$$\Rightarrow n_{er}^{*}(t) = \frac{D_2 N_{er}\int e^{D_1 t} e^{\frac{D_2}{a}e^{-at}}dt - D_2 N_{er}\int e^{(D_1 - a)t + \frac{D_2}{a}e^{-at}}dt}{e^{D_1 t + \frac{D_2}{a}e^{-at}}} + Ce^{-(D_1 t + \frac{D_2}{a}e^{-at})}$$

(4.3.2.9)

where, C is an integration constant

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2N_{er}[e^{\frac{D_2}{a}e^{-at}}\int e^{D_1t}dt-\int\{\frac{d}{dt}(e^{\frac{D_2}{a}e^{-at}})\int e^{D_1t}dt\}dt]}{e^{D_1t+\frac{D_2}{a}e^{-at}}}$$

$$-\frac{D_2N_{er}\int e^{(D_1-a)t+\frac{D_2}{a}e^{-at}}dt}{e^{D_1t+\frac{D_2}{a}e^{-at}}}+Ce^{-(D_1t+\frac{D_2}{a}e^{-at})} \qquad (4.3.2.10)$$

In this way we get

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2N_{er}}{D_1}-\frac{D_2N_{er}e^{-at}}{\tau D_1(D_1-a)}-\frac{D_2{}^2N_{er}e^{-2at}}{\tau D_1(D_1-a)(D_1-2a)}$$

$$-\frac{D_2{}^3N_{er}e^{-3at}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)}-\frac{D_2{}^4N_{er}e^{-4at}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)(D_1-4a)}$$

$$--\frac{D_2{}^rN_{er}}{\tau D_1(D_1-a)(D_1-2a)...(D_1-ra+a)IF}\int e^{(D_1-ra)t+\frac{D_2}{a}e^{-at}}dt$$

$$+Ce^{-(D_1t+\frac{D_2}{a}e^{-at})} \qquad (4.3.2.11)$$

Letting $a > b$, $a > D_2$, and $D_1 > a$ so, $D_1 > D_2$ and $D_1 > b$

So we only take the first three terms

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2N_{er}}{D_1}-\frac{D_2N_{er}e^{-at}}{\tau D_1(D_1-a)}-\frac{D_2{}^2N_{er}e^{-2at}}{\tau D_1(D_1-a)(D_1-2a)}-\frac{D_2{}^3N_{er}e^{-3at}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)}$$

$$+Ce^{-(D_1t+\frac{D_2}{a}e^{-at})} \qquad (4.3.2.12)$$

at t = 0, $n_{er}^{*}(0) = 0$

$$0 = \frac{D_2 N_{er}}{D_1} - \frac{D_2 N_{er}}{\tau D_1(D_1 - a)} - \frac{D_2^2 N_{er}}{\tau D_1(D_1 - a)(D_1 - 2a)}$$

$$- \frac{D_2^3 N_{er}}{\tau D_1(D_1 - a)(D_1 - 2a)(D_1 - 3a)} + Ce^{-\frac{D_2}{a}} \qquad (4.3.2.13)$$

$$\Rightarrow C = [\frac{D_2 N_{er}}{\tau D_1(D_1 - a)} + \frac{D_2^2 N_{er}}{\tau D_1(D_1 - a)(D_1 - 2a)} + \frac{D_2 3 N_{er}}{\tau D_1(D_1 - a)(D_1 - 2a)(D_1 - 3a)} - \frac{D_2 N_{er}}{D_1}]e^{\frac{D_2}{a}} \qquad (4.3.2.14)$$

Putting the value of C in equation (4.3.2.12), we get

$$\Rightarrow n_{er}^{*}(t) = \frac{D_2 N_{er}}{D_1}[1 - e^{\frac{D_2}{a}} e^{-(D_1 t + \frac{D_2}{a} e^{-at})}]$$

$$+ \frac{D_2 N_{er}}{\tau D_1(D_1 - a)}[e^{\frac{D_2}{a}} e^{-(D_1 t + \frac{D_2}{a} e^{-at})} - e^{-at}]$$

$$+ \frac{D_2^2 N_{er}}{\tau D_1(D_1 - a)(D_1 - 2a)}[e^{\frac{D_2}{a}} e^{-(D_1 t + \frac{D_2}{a} e^{-at})} - e^{-2at}]$$

$$+ \frac{D_2^3 N_{er}}{\tau D_1(D_1 - a)(D_1 - 2a)(D_1 - 3a)}[e^{\frac{D_2}{a}} e^{-(D_1 t + \frac{D_2}{a} e^{-at})} - e^{-3at}] \qquad (4.3.2.15)$$

The luminescence intensity is proportional to the density of excited erbium atoms and inversely proportional to the radiative lifetime of excited erbium. So

$$I(t)\, \alpha\, \frac{n_{er}^{*}(t)}{\tau_{er}} \qquad (4.3.2.16)$$

It is the time varying equation of luminescence intensity. It is seen from the equation that luminescence intensity rising gradually to get steady state.

4.4 Decay of erbium luminescence

When the external excitation pulse on a erbium doped specimen is switched off, the number of erbium states occupied by electron starts decreasing from the steady state. The strength of erbium luminescence, as a consequence gradually decays to zero. In the following two sections, decay equation of electron occupied erbium states and the corresponding erbium luminescence have been developed.

4.4.1 Decay of electron occupied erbium state

We know from equations (4.2.1) and (4.2.2) that

the rate of electron emission from erbium level $= (e_n + c_p p) n_{er}(t)$

the rate of electron capture by erbium level = $(e_p + c_n n)(N_{er} - n_{er}(t))$

We also know from equation (4.3.1.1) that the rate of change of electron occupied trap density at time, t, is given by,

$$\frac{dn_{er}(t)}{dt} = (e_p + c_n n)(N_{er} - n_{er}(t)) - (e_n + c_p p) n_{er}(t)$$

In this case the electron emission rate will be higher than the capture rate of trap level. So the time rate of change of the electron occupied trap density will be negative.

$$\Rightarrow \quad \frac{dn_{er}(t)}{dt} = (e_p + c_n n) N_{er} - (e_p + e_n + c_n n + c_p p) n_{er}(t) \qquad (4.4.1.1)$$

$$\Rightarrow \quad \frac{dn_{er}(t)}{dt} + (e_p + e_n + c_n n + c_p p) n_{er}(t) = (e_p + c_n n) N_{er} \qquad (4.4.1.2)$$

Let us assume steady state value of electron and holes as N and P respectively. Whenever the laser pulse is off, the excess carrier concentration starts to decrease and this decay nature will be exponential. Assuming electron and hole decay as

$$n = Ne^{-\frac{t}{\tau'}}, \quad p = Pe^{-\frac{t}{\tau'}} \text{ and } N \approx P$$

Where, τ' = life time of carriers

Putting the value of n and p, We get

$$\frac{dn_{er}(t)}{dt}+\{(e_p+e_n)+(c_n+c_p)Ne^{-\frac{t}{\tau'}}\}n_{er}(t)=(e_p+c_nNe^{-\frac{t}{\tau'}})N_{er} \qquad (4.4.1.3)$$

$$\Rightarrow \frac{dn_{er}(t)}{dt}+\{E+De^{-\frac{t}{\tau'}}\}n_{er}(t)=(e_p+c_nNe^{-\frac{t}{\tau'}})N_{er} \qquad (4.4.1.4)$$

Where, $E=e_p+e_n$ and $D=(c_n+c_p)N$

$$\text{Integration factor} = e^{\int(E+De^{-\frac{t}{\tau'}})dt} = e^{(Et-D\tau'e^{-\frac{t}{\tau'}})} \qquad (4.4.1.5)$$

The solution of the above equation (details derivation is shown in APPENDIX – B) is given by

$$n_{er}(t)=\frac{e_pN_{er}\int e^{Et}e^{-D\tau'e^{-\frac{t}{\tau'}}}dt+c_nN_{er}N\int e^{(E-\frac{1}{\tau'})t-D\tau'e^{-\frac{t}{\tau'}}}dt}{e^{Et-D\tau'e^{-\frac{t}{\tau'}}}}+Ce^{-Et+D\tau'e^{-\frac{t}{\tau'}}} \qquad (4.4.1.6)$$

where, C is an integration constant

$$\Rightarrow n_{er}(t)=\frac{e_pN_{er}}{E}+(c_nN_{er}N-\frac{e_pN_{er}D}{E})\frac{1}{IF}\int e^{(E-\frac{1}{\tau'})t-D\tau'e^{-\frac{t}{\tau'}}}dt+Ce^{-Et+D\tau'e^{-\frac{t}{\tau'}}} \qquad (4.4.1.7)$$

As E << 1/τ' neglecting E

$$\Rightarrow n_{er}(t)=\frac{e_pN_{er}}{E}+(c_nN_{er}N-\frac{e_pN_{er}(c_n+c_p)N}{e_p+e_n})\frac{1}{IF}\int e^{-\frac{t}{\tau'}-D\tau'e^{-\frac{t}{\tau'}}}dt+Ce^{-Et+D\tau'e^{-\frac{t}{\tau'}}}$$

$$(4.4.1.8)$$

$$\Rightarrow n_{er}(t)=\frac{e_pN_{er}}{E}+\frac{(e_nc_n-e_pc_p)N_{er}}{(e_p+e_n)(c_n+c_p)}e^{-Et}+Ce^{-Et+D\tau'e^{-\frac{t}{\tau'}}} \qquad (4.4.1.9)$$

neglecting first term (as $\frac{e_p N_{er}}{E} \approx 0$) We get

$$n_{er}(t) = \frac{(e_n c_n - e_p c_p)N_{er}}{(e_p + e_n)(c_n + c_p)} e^{-Et} + Ce^{-Et + D\tau' e^{-\frac{t}{\tau'}}} \qquad (4.4.1.10)$$

at $t = 0,\ n_{er}(0) = f_{t0} N_{er}$ We get

$$\Rightarrow \quad C = e^{-D\tau'}\left[f_{t0} N_{er} - \frac{(e_n c_n - e_p c_p)N_{er}}{(e_p + e_n)(c_n + c_p)} \right] \qquad (4.4.1.11)$$

Putting the value of C in equation (4.4.1.10), we get

$$n_{er}(t) = A_1 e^{-Et} + B_1 e^{-Et + D\tau' e^{-\frac{t}{\tau'}}} \qquad (4.4.1.12)$$

where, $$A_1 = \frac{(e_n c_n - e_p c_p)N_{er}}{(e_p + e_n)(c_n + c_p)} \qquad (4.4.1.13)$$

$$B_1 = e^{-D\tau'}\left[f_{t0} N_{er} - \frac{(e_n c_n - e_p c_p)N_{er}}{(e_p + e_n)(c_n + c_p)} \right] \qquad (4.4.1.14)$$

This is the time varying decay equation of erbium state filled by electron.

4.4.2 The luminescence decay

After the termination of laser pulse, the erbium luminescence starts to decrease and finally falls down to zero. We know from equations (4.2.5) and (4.2.6) in section 4.2, the number of electron occupied erbium sites available for excitation is $f_t(N_{er} - n_{er}^*)$ and the rate of excitation = $f_t c_p p(N_{er} - n_{er}^*)$.

We also know from equation (4.3.2.2) in section 4.3.2 that the rate of change of excited erbium is given by

$$\frac{dn_{er}^*(t)}{dt} = f_t(t)\left[N_{er} - n_{er}^*(t) \right] c_p p - \frac{n_{er}^*(t)}{\tau}$$

In this case, decay rate of excited erbium is greater than the de-excitation rate, so that the rate of change of excited erbium will be negative.

$$\Rightarrow \quad \frac{d\overset{*}{n}_{er}(t)}{dt}+[\frac{1}{\tau}+\frac{c_p p}{N_{er}}n_{er}(t)]\overset{*}{n}_{er}(t)=c_p p n_{er}(t)$$

Putting the value of $n_{er}(t)$ from equation (4.4.1.12) and p, (details derivation is given in APPENDIX – C) we get

$$\Rightarrow \quad \frac{d\overset{*}{n}_{er}(t)}{dt}+[\frac{1}{\tau}+\frac{c_p N}{N_{er}}\{A_1 e^{-(E+\frac{1}{\tau'})t}+B_1 e^{-(E+\frac{1}{\tau'})t+D\tau' e^{-\frac{t}{\tau'}}}\}]\overset{*}{n}_{er}(t)=$$

$$c_p N\{A_1 e^{-(E+\frac{1}{\tau'})t}+B_1 e^{-(E+\frac{1}{\tau'})t+D\tau' e^{-\frac{t}{\tau'}}}\} \qquad (4.4.2.1)$$

neglecting E as $E<<\frac{1}{\tau'}$

$$\Rightarrow \quad \frac{d\overset{*}{n}_{er}(t)}{dt}+[\frac{1}{\tau}+A_2 e^{-\frac{t}{\tau'}}+B_2 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}}]\overset{*}{n}_{er}(t)=A_3 e^{-\frac{t}{\tau'}}+B_3 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}} \qquad (4.4.2.2)$$

Where, $A_2=\frac{c_p A_1 N}{N_{er}}$, $B_2=\frac{c_p B_1 N}{N_{er}}$, $A_3=c_p A_1 N$, $B_3=c_p B_1 N$

$$\text{Integration factor} \quad =\exp[\int(\frac{1}{\tau}+A_2 e^{-\frac{t}{\tau'}}+B_2 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}})dt] \qquad (4.4.2.3)$$

$$=\exp[\frac{t}{\tau}-A_2\tau' e^{-\frac{t}{\tau'}}-\frac{B_2}{D}e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}}] \qquad (4.4.2.4)$$

The solution of the equation (4.4.2.2) is given by

$$\overset{*}{n}_{er}(t)=\frac{\int(A_3 e^{-\frac{t}{\tau'}}+B_3 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}})\exp(\frac{t}{\tau}-A_2\tau' e^{-\frac{t}{\tau'}}-\frac{B_2}{D}e^{D\tau' e^{-\frac{t}{\tau'}}})dt}{\exp(\frac{t}{\tau}-A_2\tau' e^{-\frac{t}{\tau'}}-\frac{B_2}{D}e^{D\tau' e^{-\frac{t}{\tau'}}})}$$

$$+C\exp[-\frac{t}{\tau}+A_2\tau' e^{-\frac{t}{\tau'}}+\frac{B_2}{D}e^{D\tau' e^{-\frac{t}{\tau'}}}] \qquad (4.4.2.5)$$

where C is an integration constant.

to calculate the value of C, at t = 0, the value inside the integration sign will be zero due to the same upper and lower limit.

$$\text{At } t=0,\ n_{er}^{*}(0)=\frac{D_2 N_{er}}{D_1} \tag{4.4.2.6}$$

$$C=\frac{D_2 N_{er}}{D_1}e^{-(A_2\tau'+\frac{B_2}{D}e^{D\tau'})} \tag{4.4.2.7}$$

Putting the value of integration constant, C, we get

$$\Rightarrow \quad n_{er}^{*}(t)=\frac{\int_0^t (A_3 e^{-\frac{t}{\tau'}}+B_3 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}})\exp(\frac{t}{\tau}-A_2\tau' e^{-\frac{t}{\tau'}}-\frac{B_2}{D}e^{D\tau' e^{-\frac{t}{\tau'}}})dt}{\exp(\frac{t}{\tau}-A_2\tau' e^{-\frac{t}{\tau'}}-\frac{B_2}{D}e^{D\tau' e^{-\frac{t}{\tau'}}})}$$

$$+\frac{D_2 N_{er}}{D_1}e^{-(A_2\tau'+\frac{B_2}{D}e^{D\tau'})}\exp[-\frac{t}{\tau}+A_2\tau' e^{-\frac{t}{\tau'}}+\frac{B_2}{D}e^{D\tau' e^{-\frac{t}{\tau'}}}] \tag{4.4.2.8}$$

where $D_1=\frac{b}{a}c_p p+\frac{1}{\tau}$, $D_2=\frac{b}{a}c_p p$

$a=e_n+e_p+c_n n+c_p p$ and $b=e_p+c_n n$

This is the time varying equation of luminescence decay.

4.5 Luminescence corresponding to laser pulse of short duration

Now, we want to see what might happen to the luminescence intensity if a short pulse (say, time = t_1) is applied to the system. From equation (4.4.1.10), the time dependent decay of electron occupied erbium states is given by

$$n_{er}(t)=\frac{(e_n c_n-e_p c_p)N_{er}}{(e_p+e_n)(c_n+c_p)}e^{-Et}+Ce^{-Et+D\tau' e^{-\frac{t}{\tau'}}}$$

The value of integration constant will be different. Under this condition at $t=t_1$,

$$n_{er}(t_1)=\frac{b}{a}N_{er}(1-e^{-at_1})=\text{Q (let)} \tag{4.5.1}$$

The value at $t = t_1$ in equation (4.4.1.10) will be considered as the initial value i.e. t = 0 in this case, $n_{er}(0) = Q$

$$\therefore \quad Ce^{D\tau'} = Q - \frac{(e_n c_n - e_p c_p)N_{er}}{(e_p + e_n)(c_n + c_p)} \tag{4.5.2}$$

$$\Rightarrow \quad C = e^{-D\tau'}\left[Q - \frac{(e_n c_n - e_p c_p)N_{er}}{(e_p + e_n)(c_n + c_p)}\right] = C_1 \tag{4.5.3}$$

So we get

$$n_{er}(t) = \frac{(e_n c_n - e_p c_p)N_{er}}{(e_p + e_n)(c_n + c_p)} e^{-Et} + e^{-D\tau'}\left[Q - \frac{(e_n c_n - e_p c_p)N_{er}}{(e_p + e_n)(c_n + c_p)}\right] e^{-Et + D\tau' e^{-\frac{t}{\tau'}}} \tag{4.5.4}$$

$$\therefore \quad n_{er}(t) = A_1 e^{-Et} + C_1 e^{-Et + D\tau' e^{-\frac{t}{\tau'}}} \tag{4.5.5}$$

The decay equation of excited erbium under this condition is calculated in the following way:

From equation (4.3.2.2)

$$\frac{d\overset{*}{n}_{er}(t)}{dt} + [\frac{1}{\tau} + \frac{c_p p}{N_{er}} n_{er}(t)]\overset{*}{n}_{er}(t) = c_p p n_{er}(t)$$

putting the new value of $n_{er}(t)$ from equation (4.5.5)

$$\frac{d\overset{*}{n}_{er}(t)}{dt} + [\frac{1}{\tau} + \frac{c_p p}{N_{er}}(A_1 e^{-Et} + C_1 e^{-Et + D\tau' e^{-\frac{t}{\tau'}}})]\overset{*}{n}_{er}(t) = c_p p[A_1 e^{-Et} + C_1 e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}] \tag{4.5.6}$$

Putting the value of $n = Ne^{-\frac{t}{\tau'}}$, $p = Pe^{-\frac{t}{\tau'}}$ and $N \approx P$ and neglecting E as $E << \frac{1}{\tau'}$, we get

$$\Rightarrow \quad \frac{d\overset{*}{n}_{er}(t)}{dt} + [\frac{1}{\tau} + A_2 e^{-\frac{t}{\tau'}} + B_4 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}}]\overset{*}{n}_{er}(t) = A_3 e^{-\frac{t}{\tau'}} + B_5 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}} \tag{4.5.7}$$

where, $A_2 = \frac{c_p A_1 N}{N_{er}}$, $B_4 = \frac{c_p C_1 N}{N_{er}}$, $A_3 = c_p A_1 N$, $B_5 = c_p C_1 N$

Integration factor $= \exp[\int(\frac{1}{\tau} + A_2 e^{-\frac{t}{\tau'}} + B_4 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}})dt]$ (4.5.8)

$$= \exp[\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_4}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}] \quad (4.5.9)$$

The solution of this equation under the condition is given by

$$n^*_{er}(t) = \frac{\int_0^t (A_3 e^{-\frac{t}{\tau'}} + B_5 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}}) \exp(\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_4}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}) dt}{\exp(\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_4}{D} e^{D\tau' e^{-\frac{t}{\tau'}}})}$$

$$+ C \exp[-\frac{t}{\tau} + A_2 \tau' e^{-\frac{t}{\tau'}} + \frac{B_4}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}] \quad (4.5.10)$$

where C is an integration constant.

to calculate the value of C, at t = 0, the value inside the integration sign will be zero due to the same upper and lower limit. The value of integration constant will be different from equation (4.4.2.5). Under this condition at $t = t_1$,

$$n^*_{er}(t_1) = \frac{D_2 N_{er}}{D_1}[1 - e^{\frac{D_2}{a}} e^{-(D_1 t_1 + \frac{D_2}{a} e^{-a t_1})}]$$

$$+ \frac{D_2 N_{er}}{\tau D_1 (D_1 - a)}[e^{\frac{D_2}{a}} e^{-(D_1 t_1 + \frac{D_2}{a} e^{-a t_1})} - e^{-a t_1}]$$

$$+ \frac{D_2^2 N_{er}}{\tau D_1 (D_1 - a)(D_1 - 2a)}[e^{\frac{D_2}{a}} e^{-(D_1 t_1 + \frac{D_2}{a} e^{-a t_1})} - e^{-2a t_1}]$$

$$+ \frac{D_2^3 N_{er}}{\tau D_1 (D_1 - a)(D_1 - 2a)(D_1 - 3a)}[e^{\frac{D_2}{a}} e^{-(D_1 t_1 + \frac{D_2}{a} e^{-a t_1})} - e^{-3a t_1}]$$

$$= n^*_{er}(0) \text{ (let)} \quad (4.5.11)$$

The value at $t = t_1$ in equation (4.3.2.15) will be considered as the initial value i.e. t = 0 for equation (4.5.10)

$$n_{er}^{*}(t) = \frac{\int_0^t (A_3 e^{-\frac{t}{\tau'}} + B_5 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}}) \exp(\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_4}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}) dt}{\exp(\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_4}{D} e^{D\tau' e^{-\frac{t}{\tau'}}})} s$$

$$+ n_{er}^{*}(0) \frac{D_2 N_{er}}{D_1} e^{-(A_2 \tau' + \frac{B_4}{D} e^{D\tau'})} \exp[-\frac{t}{\tau} + A_2 \tau' e^{-\frac{t}{\tau'}} + \frac{B_4}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}] \qquad (4.5.12)$$

This is the equation of excited erbium decay whenever a short pulse is applied.

CHAPTER FIVE

RESULTS AND DISCUSSION

5.1 Introduction

Electronic behavior of erbium in silicon has been analyzed in details in the previous chapter. Shokley - Hall - Read (SHR) recombination kinetics was used to find out the luminescence strength of erbium emission. Mathematical expression for the rise time of the luminescence and the luminescence decay has also been developed. Results of these mathematical analysis, their significance and the effect of different parameters on erbium luminescence have been described in this chapter.

5.2 Electronic property of erbium level

We know that whenever silicon is doped with an impurity, the impurity forms trap level within the silicon bandgap. Past studies showed [30,41] that erbium doped in silicon forms trap levels in the upper half of the silicon bandgap. Few reports [30,37,41] have demonstrated the formation of erbium impurity level at around 0.15 eV from the silicon conduction band. So we assume 0.15 eV as activation energy. The value of electron emission rate and hole emission rate have been calculated from the figure 2 of the paper of S. Libertino *et. al.*[41]. The electron emission rate and hole emission rate are given as follows :

$$e_n = Ae^{-\frac{E_a}{kT}} \tag{5.2.1}$$

$$e_p = Ae^{-\frac{(E_g-E_a)}{kT}} \tag{5.2.2}$$

Where,

$E_g =$ Silicon bandgap energy, 1.1 eV

$E_a =$ activation energy, 0.15 eV

$k =$ Boltzmann constant, $eV/°K$

$T =$ Temperature, $°K$

$A =$ constant depends on the property of the trap semiconductor, 7.43×10^{10}/s.

The electron capture coefficient, $c_n = \sigma_n V_{th}$ (5.2.3)

The hole capture coefficient, $c_p = \sigma_p V_{th}$ (5.2.4)

where,

$V_{th} = \sqrt{\frac{3kT}{m^*}}$ = thermal velocity

σ_n = electron capture cross section area, cm^2

σ_p = hole capture cross-section area, cm^2

k = Boltzmann constant, $eV/°K$

T = Temperature, $°K$

m^* = effective mass of electron or hole

The electron capture cross – section area (σ_n) and the hole cross – section area (σ_p) have been taken as 10^{-20} cm^2 [32]. Electron and hole effective mass are taken as $m_e^* = 1.1\ m_0$ and $m_h^* = 0.56\ m_0$, where m_0 is the free electron mass, 9.1095×10^{-31} Kg [13].

Erbium lifetime (τ) has been obtained by the following equation [32]:

$$\tau = \frac{1}{W_0 + W_{BT} e^{-\frac{\Delta E}{kT}}} \quad (5.2.5)$$

where, $\frac{1}{W_0}$ = life time at low temperature, 500 μs

W_{BT} = fitting constant, $10^8 \sim 10^{10}$/sec

ΔE = Activation energy, 135 ± 5 mev

k = Boltzmann constant, $eV/°K$

T = Temperature, $°K$

Carrier lifetime has been taken in the range $5\,\mu s \rightarrow 400\,\mu s$ [42]. In developing the rising equation of electron occupied erbium state and the corresponding erbium luminescence, we assume the carrier concentrations as $n = n_0 + \Delta n$ and $p = p_0 + \Delta p$ for electron and hole respectively. Where n_0 and p_0 are the electron and hole concentration at thermal equilibrium respectively. Δn is the optically generated electron density and Δp is the optically generated hole concentration. Since the equilibrium value of electron (n_0) and hole (p_0) is very low ($\approx 10^{10}$ /cm^3 at 300 °K) as compared to the typically optically generated excess carrier concentration ($\approx 10^{17}$/cm^3), we neglect the thermal equilibrium value. So, $n \approx \Delta n$ and $p \approx \Delta p$. Again the excess carrier concentration are equal ($\Delta n = \Delta p$) for the optical generation process, since we assume that electrons and holes are created in pairs.

Excess electron concentration generated by laser irradiation, assuming 100% absorption efficiency, can be obtained in the following way:

Optical generation rate, $$g_{op} = \frac{\frac{P}{A.d}}{E \times 1.6 \times 10^{-19}} \ / \text{cm}^3 - \text{sec} \qquad (5.2.5)$$

Thus the carrier concentration, $$N = g_{op} \times \tau' \ / \text{cm}^3 \qquad (5.2.6)$$

Where,

$A =$ cross – section area of the laser spot, m^2

$d =$ depth of the penetration of the laser light ,m

$P =$ optical power, watt

$E = \dfrac{1.2398}{\lambda} =$ photon energy, eV

$\lambda =$ wave length of photon in µm

$\tau' =$ life time of carrier

For a laser power of 28 mw with a spot diameter of 3mm and penetration depth of 1 µm and the carrier lifetime of 100 µs, the minority carrier concentration is about 1.0275×10^{18}/cm^3.All the curves have been plotted using the above mentioned value.

5.3 The erbium luminescence

Equation (4.3.1.7) gives number of erbium states occupied by electron and equation (4.3.2.16) gives the value of corresponding photoluminescence intensity. The variation of photoluminescence with erbium lifetime (τ) is shown in figure 5.1 for different values of silicon lifetime (τ'). It is believed that radiative lifetime of erbium decay remains more or less constant to a value around 1 ms, whereas variation of the non - radiative component changes the effective lifetime [24]. As a result, increase of erbium lifetime τ corresponds to an increased quantum efficiency of erbium emission and hence the luminescence strength.

Figure 5.2 shows the effect of carrier lifetime and carrier concentration on erbium luminescence for different values of erbium lifetime at 77 °K. It can be seen from the figure that luminescence intensity increases with the increase of carrier lifetime and carrier concentration. This is quite reasonable. As τ' increases, the carrier concentration increases. More carrier concentration excites more Er^{3+} ions resulting higher photoluminescence intensity. Because we know that the carrier concentration is equal to the optical generation rate (g_{op}) multiplied by the carrier lifetime (τ'). Optical generation rate remains the same for the same excitation power as we know from the equation (5.2.5).

The variation of photoluminescence intensity with increasing temperature is shown in figure 5.3. In this figure, the PL intensity is plotted as a function of reciprocal temperature. As can be seen that the PL intensity changes with increasing temperature. This change is low at temperature up to 100 °K. Beyond 100 °K, PL intensity falls sharply with temperature. Finally at room temperature virtually no measurable signal is left. This phenomenon is called temperature quenching. At temperature below 100 °K, electron trapped at the trap level recombine with hole before being thermalized and this recombination energy excites the corresponding Er atom. But at higher temperature (beyond 100 °K), electrons emission rate increases exponentially in comparison to the recombination rate. So the electron emission from the Er level is higher than that of

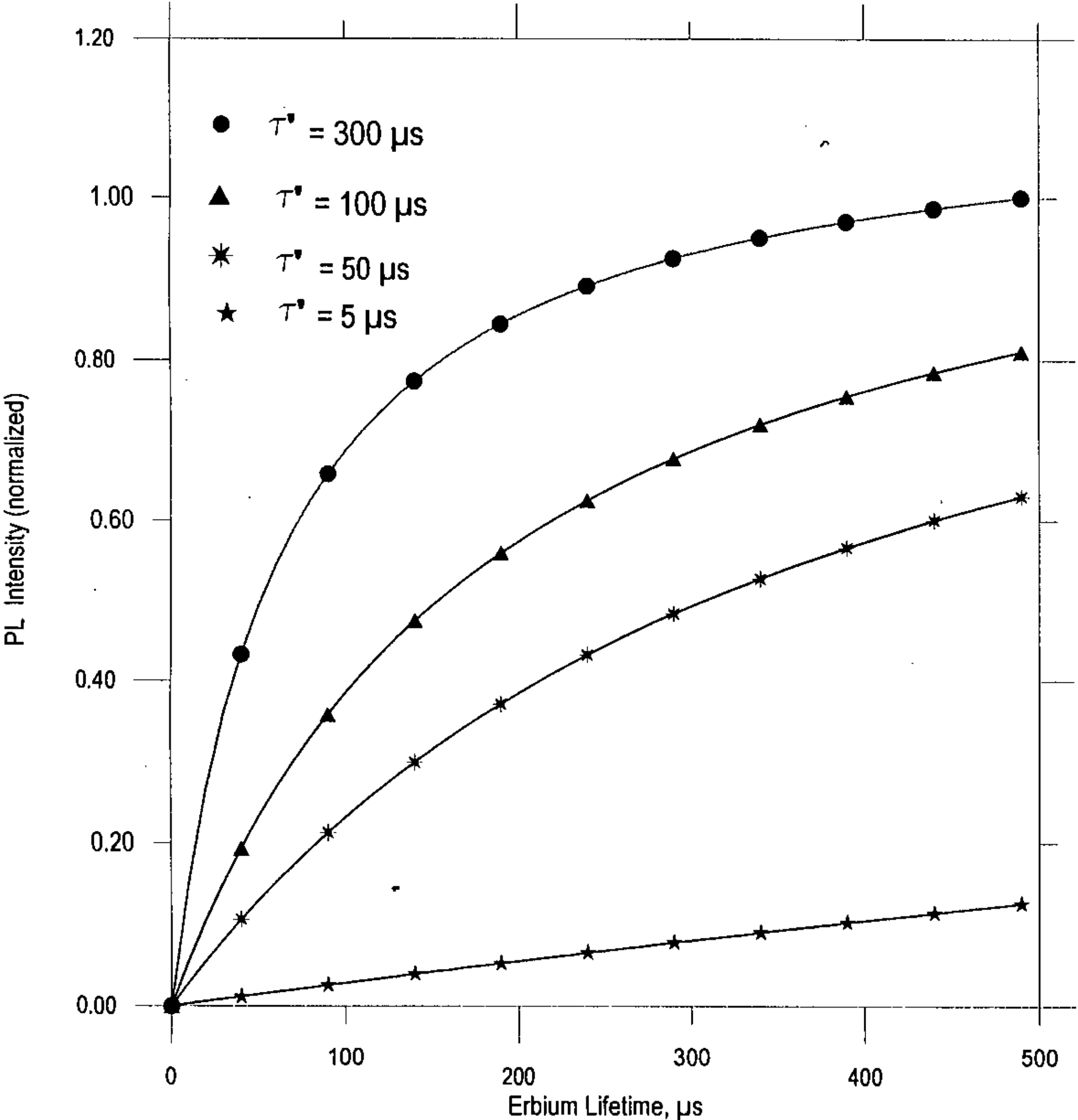

Figure 5.1 : Variation of Photoluminescence Intensity against erbium lifetime for different values of carrier lifetime.

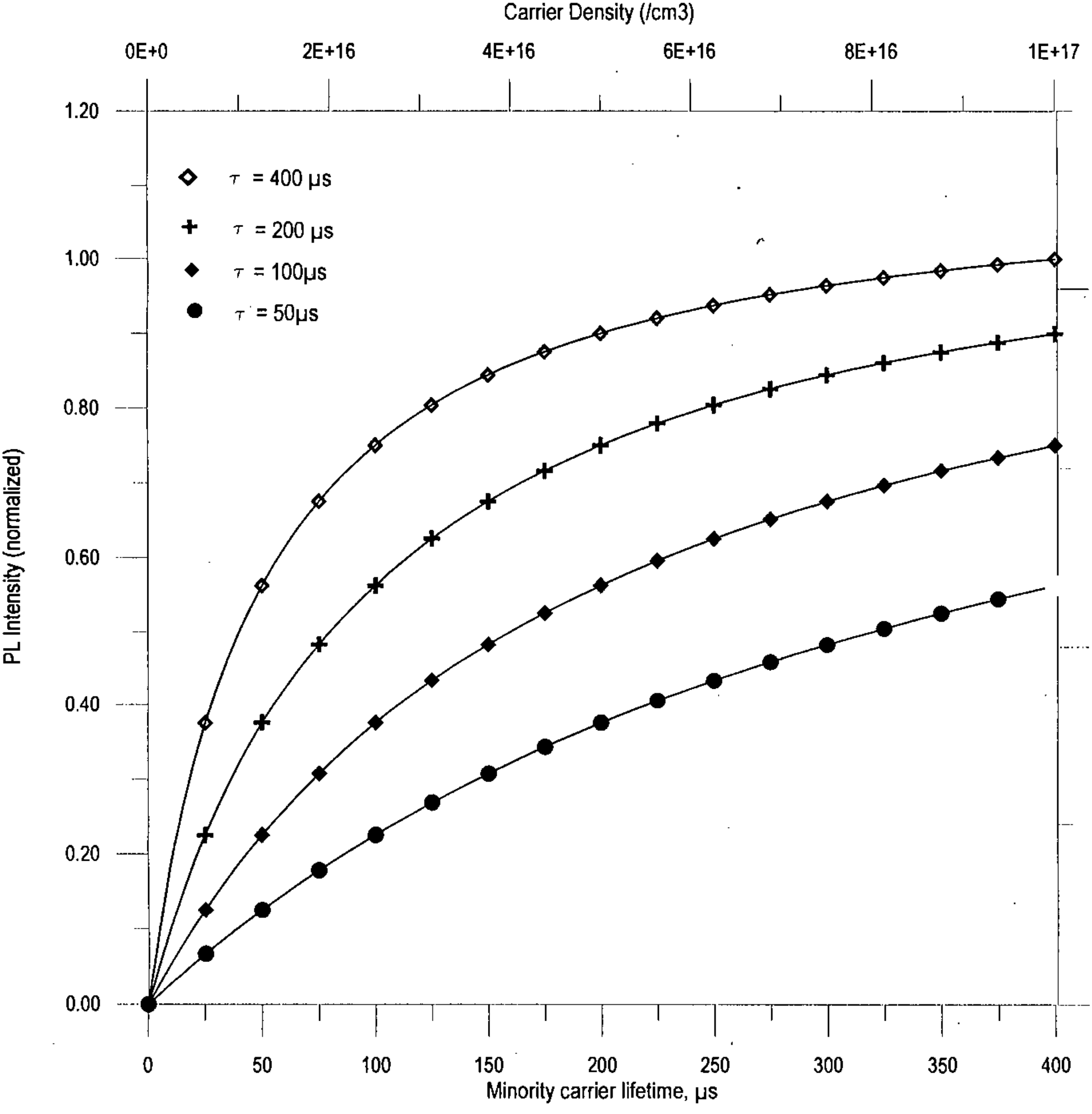

Figure 5.2 : Effect of carrier lifetime and carrier density on photoluminescence intensity for different values of erbium lifetime.

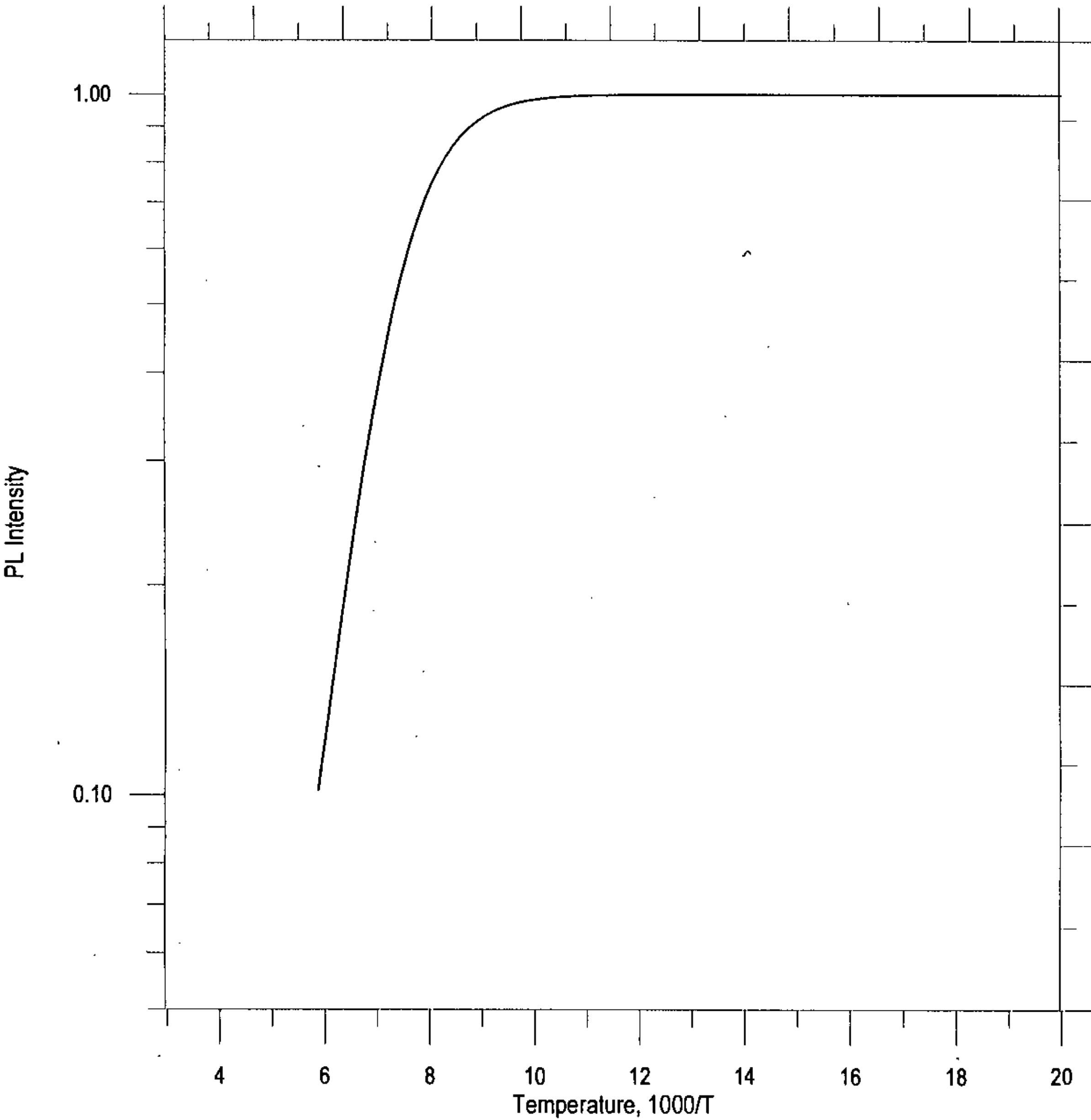

Figure 5.3 : Temperature dependence of photoluminescence intensity

recombination with holes. As a result, electrons trapped at Er level thermalize to the conduction band before being recombined. Since the electron thermalization occurs before the energy transfer takes place, the effective excitation through this Er - level reduces resulting temperature quenching. Another reason for temperature quenching is the rapid reduction of Er lifetime (ι) with temperature [32]. This reduction of Er lifetime actually corresponds to an enhanced non - radiative component of Er decay, thereby reducing the radiative emission.

5.4 Rise time of erbium luminescence

Whenever a laser excitation pulse is applied, excess *electron – hole* pairs are generated. The time dependent expression for electron occupied erbium states has been given in equation (4.3.1.7). The Er luminescence reaches to its steady value as given in equation (4.3.2.16). The corresponding results are shown in figure 5.4. It is seen from the figure that electron occupied erbium states attain the final steady state faster when compared to Er luminescence. Figure 5.5 shows the rising curves of PL intensity at two different temperature (40°K and 140°K). It is seen from the figure that as temperature rises, the PL intensity decreases. Up to temperature 100°K, one curve falls on another curve (not shown), i.e. there is no significant change in PL intensity. But beyond 100°K, there is significant change in PL intensity. Such change in PL is shown at 140°K as for example.

5.5 Photoluminescence decay profile

The time dependent photoluminescence decay profile at 77 °K for different values of minority carrier lifetime (τ') with constant Er lifetime(τ) is shown in figure 5.6. Er lifetime and carrier lifetime shows significant effect on photoluminescence intensity.

It is seen that the Er luminescence decays exponentially as observed experimentally [6,30,43]. PL intensity has larger values and decreases at slower rate for higher values of carrier lifetime. When excitation pulse is applied, excess carriers provide excitation to erbium atom. If *electron - hole* pairs recombine within a very short time (τ' low), there is little possibility to excite Er^{3+} ions. The recombination energy of the short lived

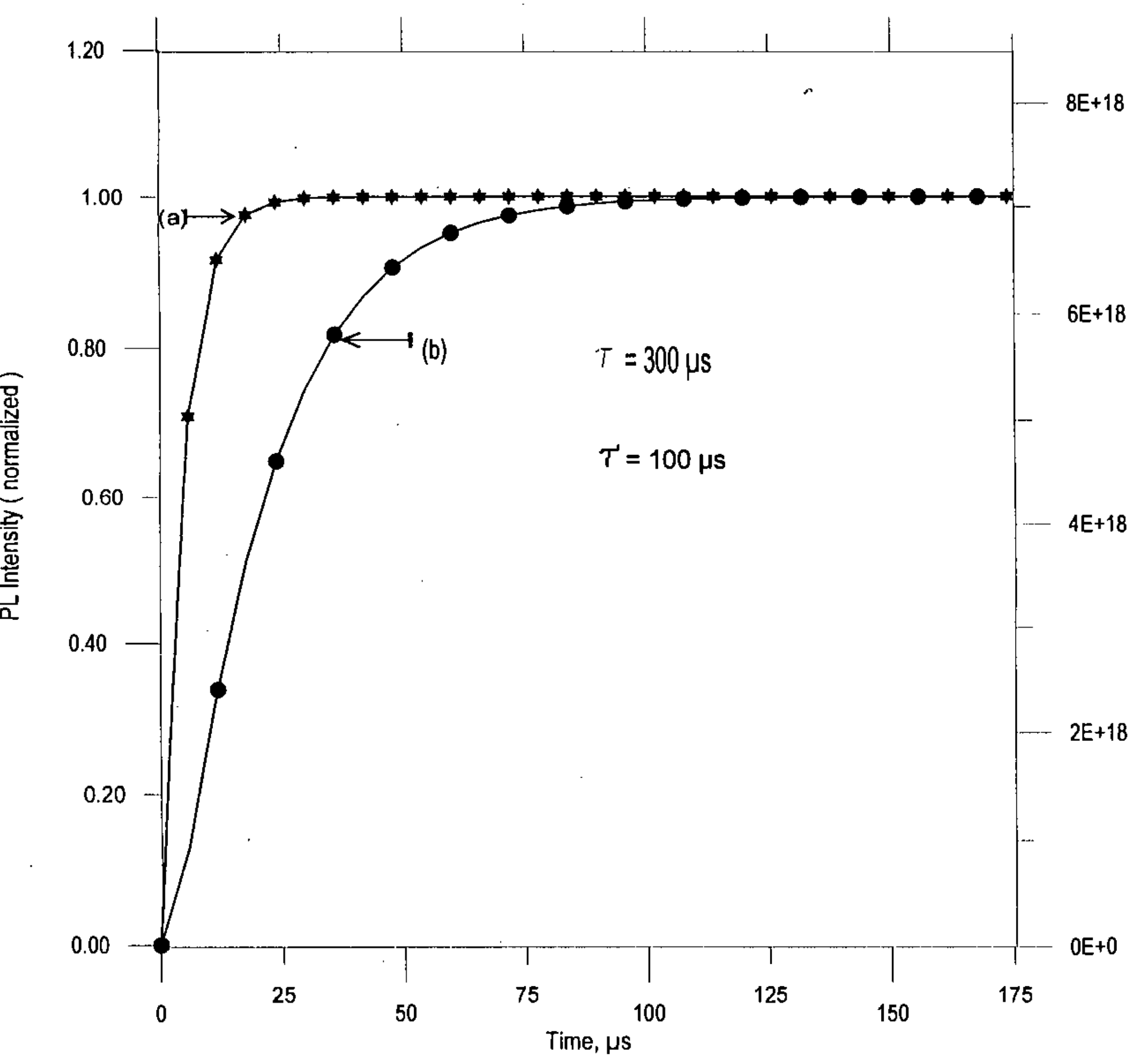

Figure 5.4 : (a) Electron occupied erbium atoms as a function of time after providing the optical (laser) excitation and (b) the corresponding erbium photoluminescence.

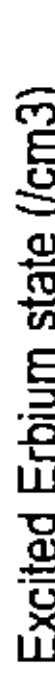

Figure 5.5 : Photoluminescence intensity versus time for two different temperatures of 40 °K and 140 °K.

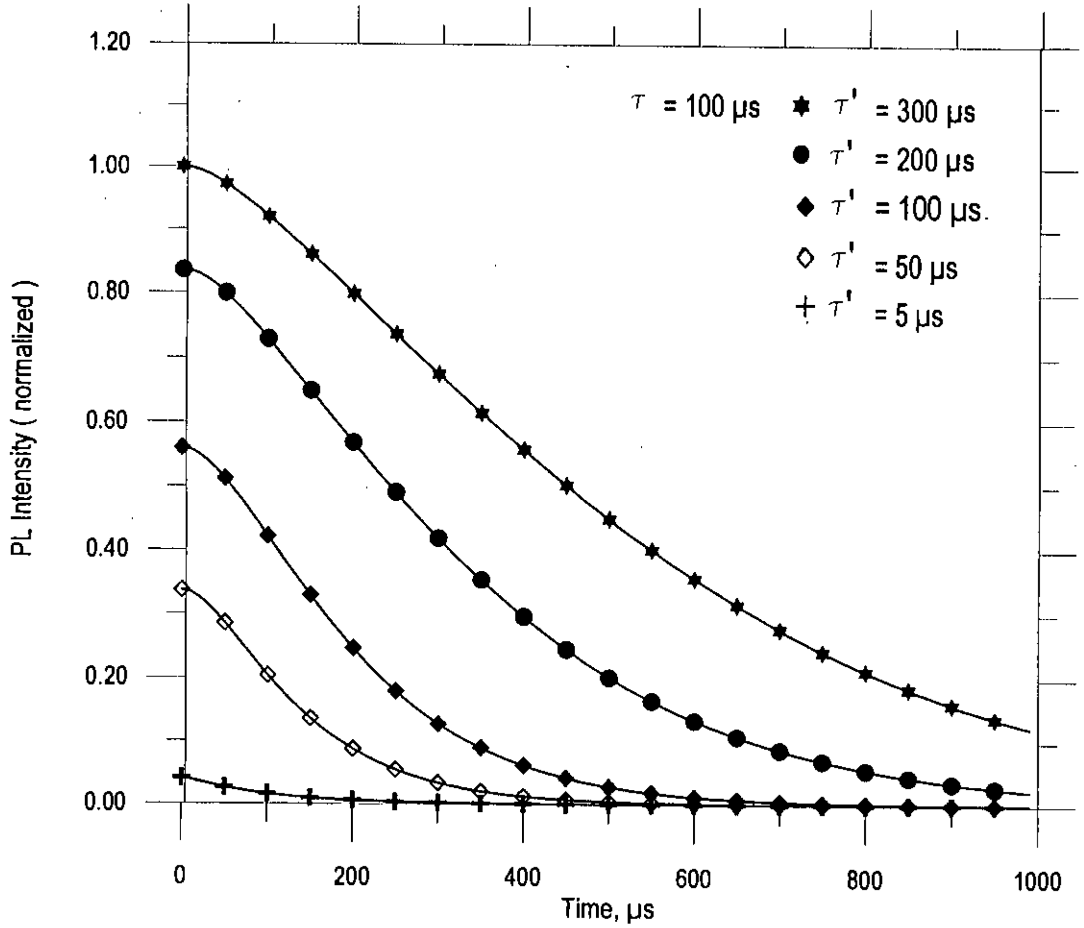

Figure 5.6 : Photoluminescence decay with respect to time for different values of carrier lifetime with constant erbium lifetime.

electron - *hole* pair at the vicinity of the excited Er^{3+} ions can not be used until the excited Er^{3+} relaxes to the ground state. In case of longer carrier lifetime, excess carrier decay slowly after the laser excitation is being switched off. This corresponds to a slower decay of Er luminescence.

Luminescence decay of Er for different values of Er lifetime(τ) with constant carrier lifetime(τ') is shown in the figure 5.7. The points at Y – axis represent the steady state PL intensity. As seen from rising curve (Fig. 5.4) that after certain time PL intensity reaches to steady state value during the excitation pulse. After turning off the excitation, Er luminescence starts to decrease from this steady value. Figure shows that PL intensity has higher value as we go from Er lifetime 50 μs to 500 μs and then decays gradually with time. We know that excited erbium relaxes in two ways: radiatively and non - radiatively. Radiative lifetime remains almost same for all conditions. At lower lifetime, more efficient non - radiative decay is likely to occur than radiative decay. So the lower PL intensity is observed. At longer lifetime, radiative lifetime dominates the decaying process and higher PL intensity is observed. Longer lifetime corresponds to slower rate of decay of the luminescence. This is a result of the stronger quantum efficiency of Er luminescence.

Figure 5.8 shows that the photoluminescence decay profile of erbium for two different values of carrier lifetime (τ' = 5, 300 μs) taking constant erbium lifetime (τ = 100 μs). In the first case PL intensity exhibits a faster exponential decay characterized by a lifetime of 100 μs. Excess carrier decays at faster rate due to lower lifetime in this case. Only the natural exponential decay of excited erbium has been observed. But in the second case, deviation from exponential decay is observed for a short period and then the luminescence decays in the same way as seen in the first case. The carriers stay longer period compared to excited erbium, i.e. excited erbium decays before the carrier decay. This deviation of decay from the exponential nature for a short period is due to the excitation by the excess carrier.

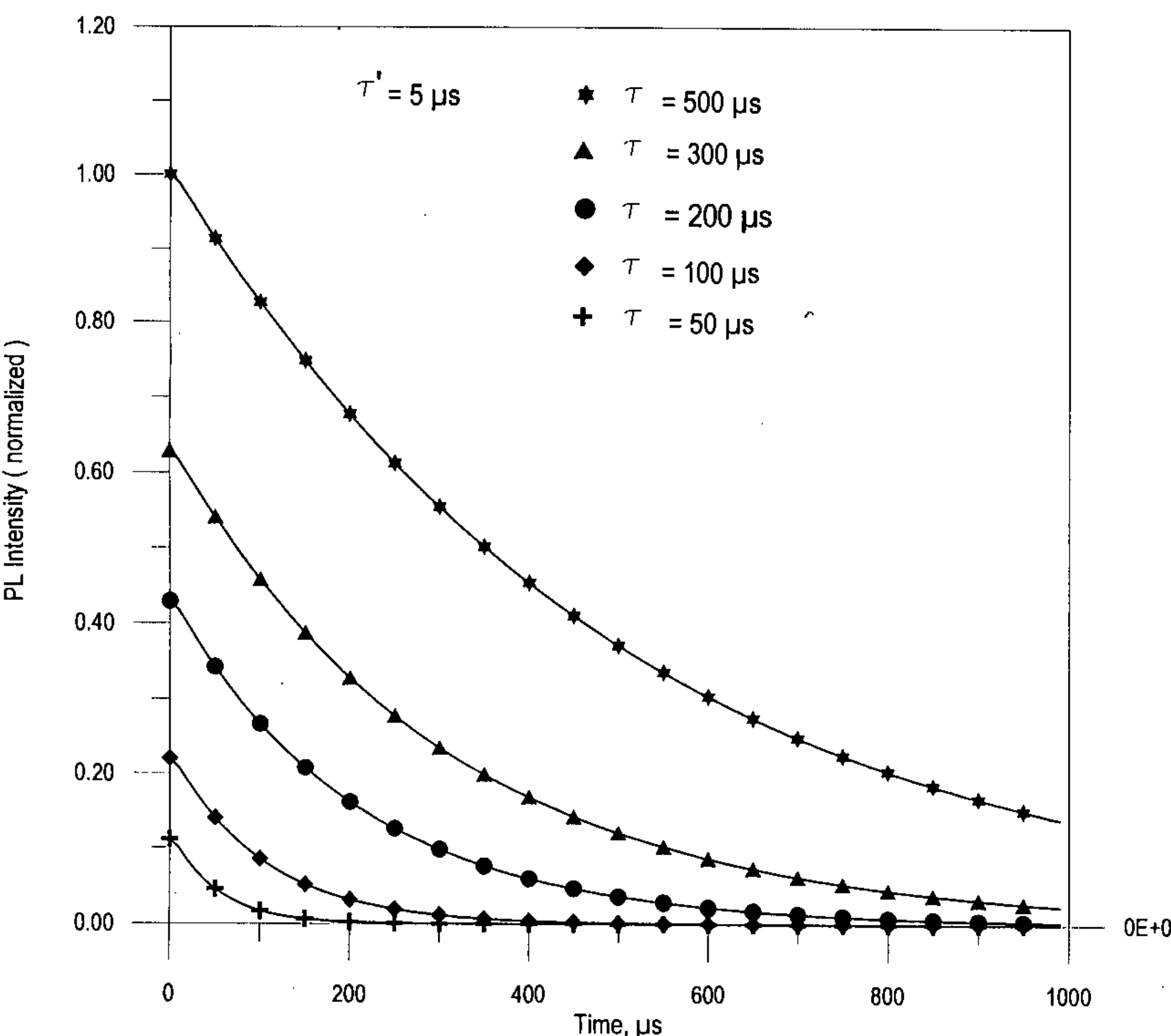

Figure 5.7 : Time decay of photoluminescence intensity for different values of erbium lifetime with constant carrier lifetime.

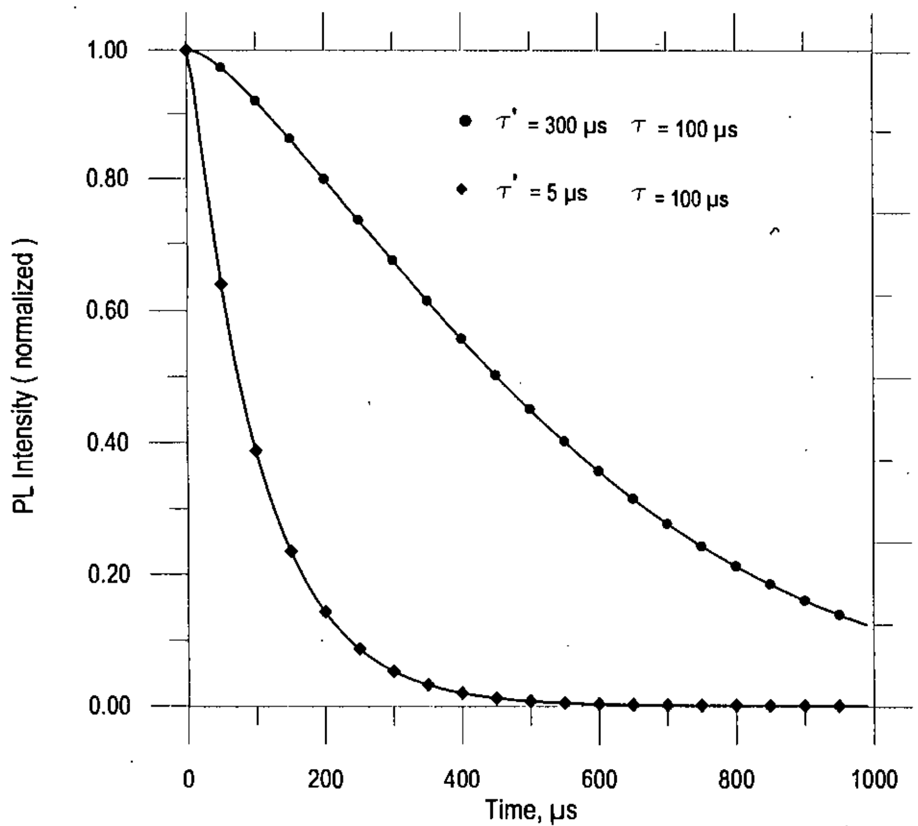

Figure 5.8 : Effect of carrier lifetime on photoluminescence intensity for the same value of erbium lifetime.

5.6 Erbium luminescence under short excitation pulse

Figure 5.9 shows the combination of rising and decaying profile of concentration of electron occupied erbium levels. The corresponding PL variation as a function of time is also given in the figure. It is seen from the figure that as long as the excitation pulse is applied, both the density of electron occupied erbium states and luminescence level remain at steady values. In figure 5.9, we see that the density of electron occupied Er states (N_{er}) reaches the steady state value at around $\approx$ 50 μs, whereas the corresponding PL intensity reaches to the steady value at $\approx$ 450 μs. If the excitation is switched off during an interval, say at 350 μs, density of electron occupied erbium levels starts to decay because it already attains its steady state value. But PL intensity will rise for a short time before decaying. This is because the Er^{3+} ions are still being excited after turning off the external optical excitation. The excess *electron - hole* pairs generated do not die down instantly. These excess *electron – hole* pairs can provide a reasonable pumping action of Er atoms to their excited state.

Mathematical expression for electron occupied Er density after providing a short excitation pulse (say 30 μs) as a function of time has been developed in equation (4.5.9). The expression for corresponding PL intensity is given in equation (4.5.12). Figure 5.10 shows the time evolution of the photoluminescence intensity during and after 30 μs at 77°K for erbium lifetime of 100 μs with carrier lifetime of 50 μs. It can be seen that the luminescence intensity continues to increase even after the termination of the excitation pulse. This increase is considerable and constitutes about 75% of the maximum observed intensity. The rise time is found to be at the range 50 μs to 70 μs. After that the luminescence intensity starts to decrease as found in the decay profile. This figure implies that the Er^{3+} ions are still being excited after the excitation pulse is turned off. This same characteristics was observed from time -resolved spectroscopy by Shin *et. al* [37] and Przybylinska *et. al* [40] for erbium doped silicon at 9 °K and 77 °K respectively. Neither they explained theoretically nor analyzed mathematically. The model developed in this thesis however shows good agreement with the experimental result.

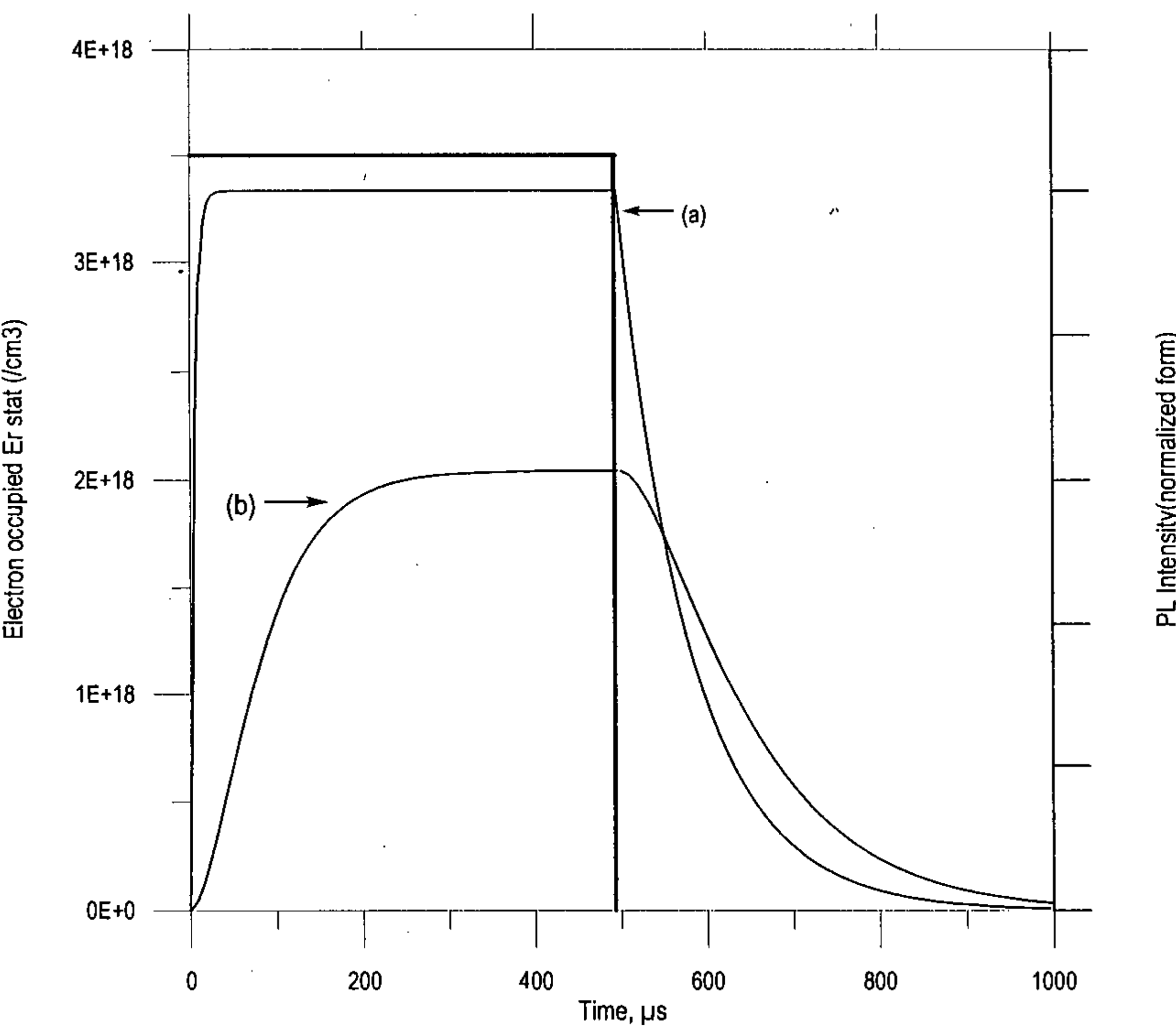

Figure 5.9 : Rising and decaying of both (a) electron occupied erbium state and (b) the corresponding photoluminescence intensity.

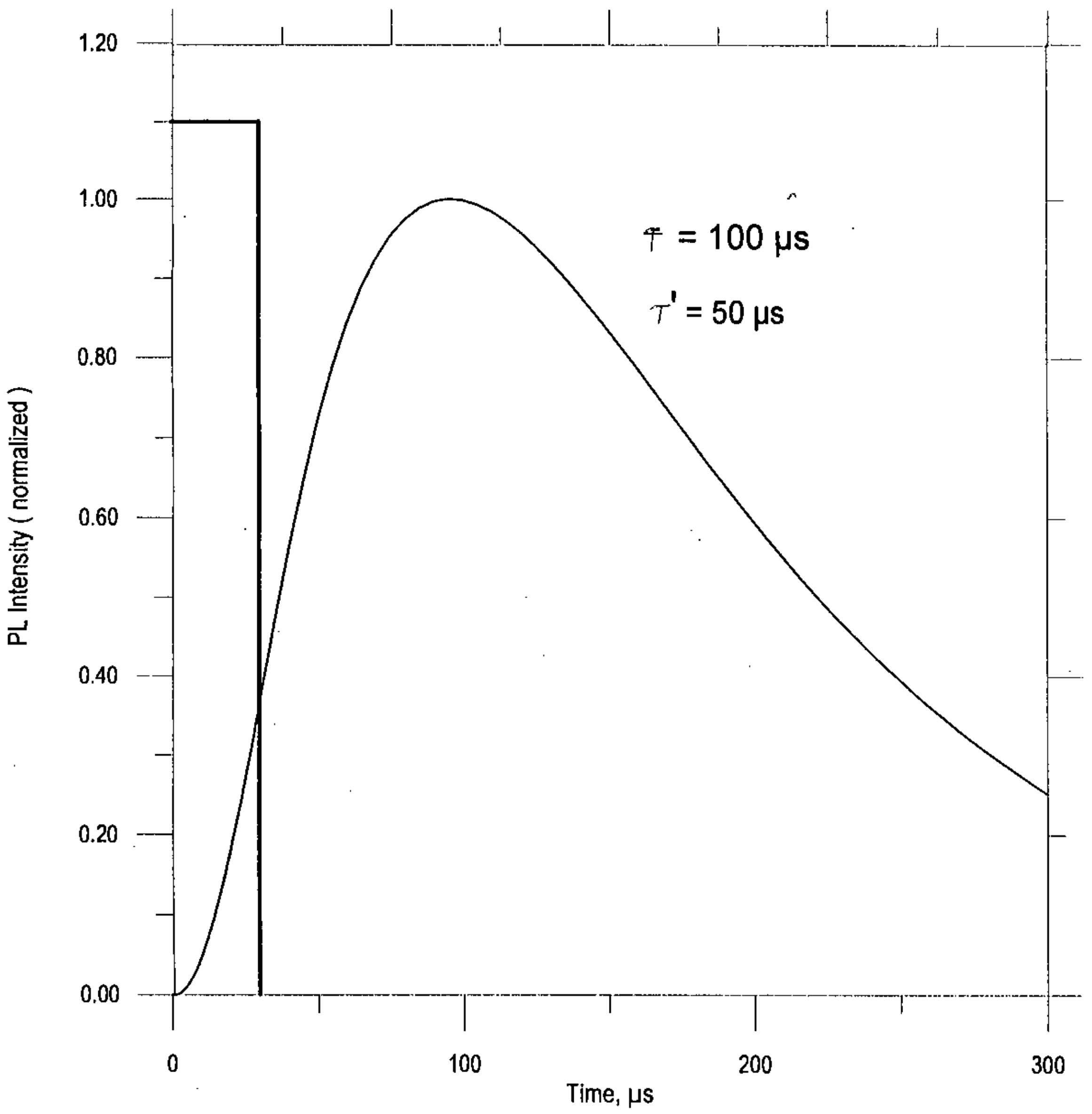

Figure 5.10 : Time evolution of the Photoluminescence intensity during and after 30 μs laser pulse.

CHAPTER SIX

CONCLUSION

6.1 Conclusion

The optical properties of erbium have been discussed. Êrbium in its trivalent charge state emits photon at 1.54 μm, a standard wavelength in the optical communication. Since loss is minimum at this wavelength. Si : Er LED system has been discussed where researchers have focused on how to get efficient light from silicon. Previous models for excitation mechanism of erbium in silicon have been discussed and remarks have been given logically why they are not the actual excitation mechanism.

Based on Shockley - Hall - Read (SHR) recombination kinetics, the mathematical model for excitation mechanism of erbium in silicon has been developed. According to the model, Er atoms form defect level in the upper half of the silicon band. *Electron – hole* recombination in the Er site provides energy for exciting the Er atom to the higher energy state of $^4I_{13/2}$. Radiative transition of the Er atoms from the excited state to ground state ($^4I_{15/2}$) corresponds to luminescence at 1.54 μm.

We have investigated the effect of various parameters and conditions (carrier lifetime, erbium lifetime, temperature, short optical excitation etc.) on the PL intensity and discussed the effect with graphical representations. In each case, the result obtained so far using this model shows good agreement with the published experimental results. We see in figure 5.4 that the density of erbium sites occupied by electron rises to the steady state value with an exponential nature. The luminescence intensity comparatively takes a longer time to set to the final steady value. The same result was observed by Shin *et. al.*[37] and Przybylinska *et.al.*[40]. So attention should be given to this phenomenon in fabricating the optoelectronic devices from erbium doped silicon.

The decay of Er luminescence and unusual effect of short excitation pulse on luminescence intensity which was observed by Shin *et. al.* [37] and Przybylinska *et. al.*[40] have been explained mathematically. Thus the model developed is the model for excitation mechanism of erbium in silicon.

6.2 Recommendation for Future Work

As we know from previous discussion that efficient light emission from silicon is greatly hampered by its indirect band structure. So optical doping of erbium is suggested due to its internal 4f – transition at 1.54 μm. Again we also know that the attainment of photoluminescence and electroluminescence at room temperature is largely affected by the temperature quenching effect. As we see in figure 5.3, the PL intensity falls sharply with the increase of temperature. So it is a great problem to fabricate optoelectronic devices.

Again we see in figure 5.4 that the PL intensity takes comparatively longer time (in the order of 100 μs) to become steady state. This phenomenon is a great hurdle in high frequency optoelectronic operation. Since in high speed information transmission requires a quick responding device.

So it is my opinion that the future research effort in this field should be concentrated on the following two areas:

(i) reduction of rise time in the PL intensity ; and

(ii) the attainment of the room temperature photo and electro - luminescence from erbium incorporated silicon.

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Details derivation of excited erbium:

Let,

N_{er} = total number of erbium atom

$n_{er}(t)$ = the number of erbium traps occupied by electron at time, t

$n^*_{er}(t)$ = the number of excited traps at time, t

$f_t(t)$ = the ratio of occupied traplevels to steady state traps at time t, $\frac{n_{er}(t)}{N_{er}}$

τ = the life time of erbium

The rate of change of excited erbium is given by

$$\frac{dn^*_{er}(t)}{dt}+\frac{n^*_{er}(t)}{\tau}=f_t(t)\left[N_{er}-n^*_{er}(t)\right]c_pp$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\frac{n^*_{er}(t)}{\tau}=f_t(t)N_{er}c_pp-f_t(t)c_ppn^*_{er}(t)$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\frac{n^*_{er}(t)}{\tau}+f_t(t)c_ppn^*_{er}(t)=f_t(t)N_{er}c_pp$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\left[\frac{1}{\tau}+f_t(t)c_pp\right]n^*_{er}(t)=f_t(t)N_{er}c_pp$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\left[\frac{1}{\tau}+c_pp\frac{n_{er}(t)}{N_{er}}\right]n^*_{er}(t)=N_{er}c_pp\frac{n_{er}(t)}{N_{er}}$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\left[\frac{1}{\tau}+\frac{c_pp}{N_{er}}\left(\frac{b}{a}N_{er}(1-e^{-at})\right)\right]n^*_{er}(t)=\frac{N_{er}c_pp}{N_{er}}\left(\frac{b}{a}N_{er}(1-e^{-at})\right)$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\left[\frac{1}{\tau}+\frac{b}{a}c_pp-\frac{b}{a}c_ppe^{-at}\right]n^*_{er}(t)=\frac{b}{a}c_ppN_{er}(1-e^{-at})$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt} + \left[D_1 - D_2 e^{-at}\right] n^*_{er}(t) = D_2 N_{er}(1 - e^{-at})$$

where, $D_1 = \frac{1}{\tau} + \frac{b}{a} c_p p$, $D_2 = \frac{b}{a} c_p p$

Integration factor $= e^{\int (D_1 - D_2 e^{-at})dt} = e^{D_1 t - D_2 \int e^{-at} dt} = e^{D_1 t + \frac{D_2}{a} e^{-at}}$

Multiplying both sides by integration factor we get

$$e^{D_1 t + \frac{D_2}{a} e^{-at}} \frac{dn^*_{er}(t)}{dt} + \left[D_1 - D_2 e^{-at}\right] e^{D_1 t + \frac{D_2}{a} e^{-at}} n^*_{er}(t) = D_2 N_{er}(1 - e^{-at}) e^{D_1 t + \frac{D_2}{a} e^{-at}}$$

$$\Rightarrow e^{D_1 t + \frac{D_2}{a} e^{-at}} dn^*_{er}(t) + \left[D_1 - D_2 e^{-at}\right] e^{D_1 t + \frac{D_2}{a} e^{-at}} n^*_{er}(t)dt = D_2 N_{er} e^{D_1 t + \frac{D_2}{a} e^{-at}} dt - D_2 N_{er} e^{(D_1 - a)t + \frac{D_2}{a} e^{-at}} dt$$

$$\Rightarrow d(e^{D_1 t + \frac{D_2}{a} e^{-at}} n^*_{er}(t)) = D_2 N_{er} e^{D_1 t + \frac{D_2}{a} e^{-at}} dt - D_2 N_{er} e^{(D_1 - a)t + \frac{D_2}{a} e^{-at}} dt$$

$$\Rightarrow \int d(e^{D_1 t + \frac{D_2}{a} e^{-at}} n^*_{er}(t)) = D_2 N_{er} [\int e^{D_1 t + \frac{D_2}{a} e^{-at}} dt - \int e^{(D_1 - a)t + \frac{D_2}{a} e^{-at}} dt]$$

Integrating both sides we get

$$\Rightarrow e^{D_1 t + \frac{D_2}{a} e^{-at}} n^*_{er}(t) = D_2 N_{er} \int e^{D_1 t + \frac{D_2}{a} e^{-at}} dt - D_2 N_{er} \int e^{(D_1 - a)t + \frac{D_2}{a} e^{-at}} dt + C$$

where C is an integration constant

$$\Rightarrow e^{D_1 t + \frac{D_2}{a} e^{-at}} n^*_{er}(t) = D_2 N_{er} \int e^{D_1 t} e^{\frac{D_2}{a} e^{-at}} dt - D_2 N_{er} \int e^{(D_1 - a)t + \frac{D_2}{a} e^{-at}} dt + C$$

Dividing both sides by integration factor we get

$$\Rightarrow n^*_{er}(t) = \frac{D_2 N_{er} \int e^{D_1 t} e^{\frac{D_2}{a} e^{-at}} dt - D_2 N_{er} \int e^{(D_1 - a)t + \frac{D_2}{a} e^{-at}} dt}{e^{D_1 t + \frac{D_2}{a} e^{-at}}} + Ce^{-(D_1 t + \frac{D_2}{a} e^{-at})}$$

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2 N_{er}[e^{\frac{D_2}{a}e^{-at}}\int e^{D_1 t}dt-\int\{\frac{d}{dt}(e^{\frac{D_2}{a}e^{-at}})\int e^{D_1 t}dt\}dt]}{e^{D_1 t+\frac{D_2}{a}e^{-at}}}$$

$$-\frac{D_2 N_{er}\int e^{(D_1-a)t+\frac{D_2}{a}e^{-at}}dt}{e^{D_1 t+\frac{D_2}{a}e^{-at}}}+Ce^{-(D_1 t+\frac{D_2}{a}e^{-at})}$$

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2 N_{er}[e^{\frac{D_2}{a}e^{-at}}\frac{e^{D_1 t}}{D_1}-\int e^{\frac{D_2}{a}e^{-at}}\frac{D_2 e^{D_1 t}(-a)e^{-at}}{aD_1}dt]}{e^{D_1 t+\frac{D_2}{a}e^{-at}}}$$

$$-\frac{D_2 N_{er}\int e^{(D_1-a)t+\frac{D_2}{a}e^{-at}}dt}{e^{D_1 t+\frac{D_2}{a}e^{-at}}}+Ce^{-(D_1 t+\frac{D_2}{a}e^{-at})}$$

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2 N_{er}}{D_1}-\frac{D_2 N_{er}}{\tau D_1.IF}\int e^{(D_1-a)t}e^{\frac{D_2}{a}e^{-at}}dt+Ce^{-(D_1 t+\frac{D_2}{a}e^{-at})}$$

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2 N_{er}}{D_1}-\frac{D_2 N_{er}}{\tau D_1.IF}[e^{\frac{D_2}{a}e^{-at}}\int e^{(D_1-a)t}dt-\int\{\frac{d}{dt}(e^{\frac{D_2}{a}e^{-at}})\int e^{(D_1-a)t}dt\}dt]$$

$$+Ce^{-(D_1 t+\frac{D_2}{a}e^{-at})}$$

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2 N_{er}}{D_1}-\frac{D_2 N_{er}}{\tau D_1.IF}[e^{\frac{D_2}{D_1}e^{-at}}\frac{e^{(D_1-a)t}}{(D_1-a)}-\int e^{\frac{D_2}{a}e^{-at}}\frac{D_2 e^{-at}(-a)e^{(D_1-a)t}}{a(D_1-a)}dt]$$

$$+Ce^{-(D_1 t+\frac{D_2}{a}e^{-at})}$$

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2 N_{er}}{D_1}-\frac{D_2 N_{er}e^{-at}}{\tau D_1.(D_1-a)}-\frac{D_2^2 N_{er}}{\tau D_1.(D_1-a)IF}[\int e^{\frac{D_2}{a}e^{-at}}e^{(D_1-2a)t}dt]+Ce^{-(D_1 t+\frac{D_2}{a}e^{-at})}$$

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2 N_{er}}{D_1}-\frac{D_2 N_{er}e^{-at}}{\tau D_1.(D_1-a)}-\frac{D_2^2 N_{er}}{\tau D_1(D_1-a)IF}[e^{\frac{D_2}{a}e^{-at}}\int e^{(D_1-2a)t}dt$$

$$-\int\{\frac{d}{dt}(e^{\frac{D_2}{a}e^{-at}})\int e^{(D_1-2a)t}dt\}dt\}]+Ce^{-(D_1 t+\frac{D_2}{a}e^{-at})}$$

$$\Rightarrow n_{er}^{*}(t)=\frac{D_2 N_{er}}{D_1}-\frac{D_2 N_{er}e^{-at}}{\tau D_1.(D_1-a)}-\frac{D_2^2 N_{er}}{\tau D_1.(D_1-a)IF}[\frac{e^{\frac{D_2}{a}e^{-at}}e^{(D_1-2a)t}}{(D_1-2a)}$$

$$-\int e^{\frac{D_2}{a}e^{-at}}\frac{D_2 e^{-at}(-a)}{a(D_1-2a)}e^{(D_1-2a)t}dt]+Ce^{-(D_1 t+\frac{D_2}{a}e^{-at})}$$

$$\Rightarrow n^*_{er}(t) = \frac{D_2N_{er}}{D_1} - \frac{D_2N_{er}e^{-at}}{\tau D_1.(D_1-a)} - \frac{D_2^2N_{er}e^{-2at}}{\tau D_1.(D_1-a)(D_1-2a)}$$

$$-\frac{D_2^3N_{er}}{\tau D_1(D_1-a)(D_1-2a)IF}\int e^{\frac{D_2}{a}e^{-at}} e^{(D_1-3a)t}dt + Ce^{-(D_1t+\frac{D_2}{a}e^{-at})}$$

In this way we get

$$\Rightarrow n^*_{er}(t) = \frac{D_2N_{er}}{D_1} - \frac{D_2N_{er}e^{-at}}{\tau D_1.(D_1-a)} - \frac{D_2^2N_{er}e^{-2at}}{\tau D_1.(D_1-a)(D_1-2a)} - \frac{D_2^3N_{er}e^{-3at}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)}$$

$$-\frac{D_2^4N_{er}e^{-4at}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)(D_1-4a)} + Ce^{-(D_1t+\frac{D_2}{a}e^{-at})}$$

$$-\frac{D_2^rN_{er}}{\tau D_1(D_1-a)(D_1-2a)...(D_1-ra+a)IF}\int e^{(D_1-ra)t+\frac{D_2}{a}e^{-at}}dt$$

Letting $a > b$, $a > D_2$, and $D_1 > a$ so, $D_1 > D_2$ and $D_1 > b$

So we only take the first three terms

$$\Rightarrow n^*_{er}(t) = \frac{D_2N_{er}}{D_1} - \frac{D_2N_{er}e^{-at}}{\tau D_1(D_1-a)} - \frac{D_2^2N_{er}e^{-2at}}{\tau D_1(D_1-a)(D_1-2a)} - \frac{D_2^3N_{er}e^{-3at}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)}$$

$$+Ce^{-(D_1t+\frac{D_2}{a}e^{-at})}$$

at $t = 0$, $n^*_{er}(0) = 0$

$$0 = \frac{D_2N_{er}}{D_1} - \frac{D_2N_{er}}{\tau D_1(D_1-a)} - \frac{D_2^2N_{er}}{\tau D_1(D_1-a)(D_1-2a)} - \frac{D_2^3N_{er}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)} + Ce^{-\frac{D_2}{a}}$$

$$\Rightarrow C = [\frac{D_2N_{er}}{\tau D_1(D_1-a)} + \frac{D_2^2N_{er}}{\tau D_1(D_1-a)(D_1-2a)} + \frac{D_2^3N_{er}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)} - \frac{D_2N_{er}}{D_1}]e^{\frac{D_2}{a}}$$

$$n^*_{er}(t) = \frac{D_2N_{er}}{D_1} - \frac{D_2N_{er}e^{-at}}{\tau D_1(D_1-a)} - \frac{D_2^2N_{er}e^{-2at}}{\tau D_1(D_1-a)(D_1-2a)} - \frac{D_2^3N_{er}e^{-3at}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)}$$

$$+[\frac{D_2N_{er}}{\tau D_1(D_1-a)} + \frac{D_2^2N_{er}}{\tau D_1.(D_1-a)(D_1-2a)} + \frac{D_2^3N_{er}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)} - \frac{D_2N_{er}}{D_1}]e^{\frac{D_2}{a}}e^{-(D_1t+\frac{D_2}{a}e^{-at})}$$

$$\Rightarrow n^*_{er}(t) = \frac{D_2N_{er}}{D_1}[1 - e^{\frac{D_2}{a}}e^{-(D_1t+\frac{D_2}{a}e^{-at})}] + \frac{D_2N_{er}}{\tau D_1(D_1-a)}[e^{\frac{D_2}{a}}e^{-(D_1t+\frac{D_2}{a}e^{-at})} - e^{-at}]$$

$$+\frac{D_2^2N_{er}}{\tau D_1(D_1-a)(D_1-2a)}[e^{\frac{D_2}{a}}e^{-(D_1t+\frac{D_2}{a}e^{-at})} - e^{-2at}]$$

$$+\frac{D_2^3N_{er}}{\tau D_1(D_1-a)(D_1-2a)(D_1-3a)}[e^{\frac{D_2}{a}}e^{-(D_1t+\frac{D_2}{a}e^{-at})} - e^{-3at}]$$

It is the time varying equation of excited erbium atom.

Details derivation of decay equation of erbium filled by electron:

Let, N_{er} = total number of erbium atom

$n_{er}(t)$ = the number of erbium traps occupied by electron at time, t

$N_{er} - n_{er}(t)$ = the number of empty traps at time, t

τ' = life time of silicon atom

$\therefore$ electron emission rate $= (e_n + c_p p)n_{er}(t)$

$\therefore$ electron capture rate = $(e_p + c_n n)(N_{er} - n_{er}(t))$

$\therefore$ The rate of change of occupied trap density at time t is given by,

$$\frac{dn_{er}(t)}{dt} = (e_p + c_n n)(N_{er} - n_{er}(t)) - (e_n + c_p p)n_{er}(t)$$

$$\Rightarrow \frac{dn_{er}(t)}{dt} = (e_p + c_n n)N_{er} - (e_p + e_n + c_n n + c_p p)n_{er}(t)$$

$$\Rightarrow \frac{dn_{er}(t)}{dt} + (e_p + e_n + c_n n + c_p p)n_{er}(t) = (e_p + c_n n)N_{er}$$

Let $n = Ne^{-\frac{t}{\tau'}}$, $p = Pe^{-\frac{t}{\tau'}}$ and $N \approx P$

We get,

$$\frac{dn_{er}(t)}{dt} + \{(e_p + e_n) + (c_n + c_p)Ne^{-\frac{t}{\tau'}}\}n_{er}(t) = (e_p + c_n Ne^{-\frac{t}{\tau'}})N_{er}$$

$$\Rightarrow \frac{dn_{er}(t)}{dt} + \{(e_p + e_n) + (c_n + c_p)Ne^{-\frac{t}{\tau'}}\}n_{er}(t) = (e_p + c_n Ne^{-\frac{t}{\tau'}})N_{er}$$

$$\Rightarrow \frac{dn_{er}(t)}{dt} + \{E + De^{-\frac{t}{\tau'}}\}n_{er}(t) = (e_p + c_n Ne^{-\frac{t}{\tau'}})N_{er}$$

Where, $E = e_p + e_n$ and $D = (c_n + c_p)N$

Integration factor $= e^{\int (E + De^{-\frac{t}{\tau'}})dt} = e^{(\int Edt + \int De^{-\frac{t}{\tau'}}dt)}$

$= e^{(Et + De^{-\frac{t}{\tau'}}(-\tau'))} = e^{(Et - D\tau' e^{-\frac{t}{\tau'}})}$

Multiplying both sides by integration factor we get

$$e^{Et - D\tau' e^{-\frac{t}{\tau'}}} \frac{dn_{er}(t)}{dt} + e^{Et - D\tau' e^{-\frac{t}{\tau'}}} (E + De^{-\frac{t}{\tau'}}) n_{er}(t) = (e_p + c_n Ne^{-\frac{t}{\tau'}}) N_{er} e^{Et - D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow d\{e^{Et - D\tau' e^{-\frac{t}{\tau'}}} n_{er}(t)\} = e_p N_{er} e^{Et - D\tau' e^{-\frac{t}{\tau'}}} dt + c_n N_{er} N e^{(E - \frac{1}{\tau'})t - D\tau' e^{-\frac{t}{\tau'}}} dt$$

Integrating both sides we get

$$\int d\{e^{Et - D\tau' e^{-\frac{t}{\tau'}}} n_{er}(t)\} = e_p N_{er} \int e^{Et - D\tau' e^{-\frac{t}{\tau'}}} dt + c_n N_{er} N \int e^{(E - \frac{1}{\tau'})t - D\tau' e^{-\frac{t}{\tau'}}} dt + C$$

92915

Where C is an integration constant

$$e^{Et - D\tau' e^{-\frac{t}{\tau'}}} n_{er}(t) = e_p N_{er} \int e^{Et} e^{-D\tau' e^{-\frac{t}{\tau'}}} dt + c_n N_{er} N \int e^{(E - \frac{1}{\tau'})t - D\tau' e^{-\frac{t}{\tau'}}} dt + C$$

dividing both sides by integration factor we get

$$n_{er}(t) = \frac{e_p N_{er} \int e^{Et} e^{-D\tau' e^{-\frac{t}{\tau'}}} dt + c_n N_{er} N \int e^{(E - \frac{1}{\tau'})t - D\tau' e^{-\frac{t}{\tau'}}} dt}{e^{Et - D\tau' e^{-\frac{t}{\tau'}}}} + Ce^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow n_{er}(t) = \frac{e_p N_{er}}{IF} [e^{-D\tau' e^{-\frac{t}{\tau'}}} \int e^{Et} dt - \int \{\frac{d}{dt}(e^{-D\tau' e^{-\frac{t}{\tau'}}}) \int e^{Et} dt\} dt]$$

$$+ \frac{c_n N_{er} N}{IF} \int e^{(E - \frac{1}{\tau'})t - D\tau' e^{-\frac{t}{\tau'}}} dt + Ce^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow n_{er}(t) = \frac{e_p N_{er}}{IF} [e^{-D\tau' e^{-\frac{t}{\tau'}}} \frac{e^{Et}}{E} - \int e^{-D\tau' e^{-\frac{t}{\tau'}}} (-D\tau') e^{-\frac{t}{\tau'}} (\frac{-1}{\tau'}) \frac{e^{Et}}{E} dt$$

$$+ \frac{c_n N_{er} N}{IF} \int e^{(E - \frac{1}{\tau'})t - D\tau' e^{-\frac{t}{\tau'}}} dt + Ce^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow \; n_{er}(t) = \frac{e_p N_{er}}{E} + (c_n N_{er} N - \frac{e_p N_{er} D}{E}) \frac{1}{IF} \int e^{(E - \frac{1}{\tau'})t - D\tau' e^{-\frac{t}{\tau'}}} dt \; + C e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

As E $<< 1/\tau'$ neglecting E

$$\Rightarrow n_{er}(t) = \frac{e_p N_{er}}{E} + (c_n N_{er} N - \frac{e_p N_{er} (c_n + c_p) N}{e_p + e_n}) \frac{1}{IF} \int e^{-\frac{t}{\tau'} - D\tau' e^{-\frac{t}{\tau'}}} dt + C e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow n_{er}(t) = \frac{e_p N_{er}}{E} + N_{er} N \{c_n - \frac{e_p c_n + e_p c_p}{e_p + e_n}\} \frac{1}{IF} \int e^{-\frac{t}{\tau'} - D\tau' e^{-\frac{t}{\tau'}}} dt + C e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow n_{er}(t) = \frac{e_p N_{er}}{E} + N_{er} N \{\frac{e_p c_n + e_n c_n - e_p c_n - e_p c_p}{e_p + e_n}\} \frac{1}{IF} \int e^{-\frac{t}{\tau'} - D\tau' e^{-\frac{t}{\tau'}}} dt$$

$$+ C e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow \; n_{er}(t) = \frac{e_p N_{er}}{E} + \{\frac{e_n c_n - e_p c_p}{e_p + e_n}\} \frac{N_{er} N}{IF} \int e^{-\frac{t}{\tau'} - D\tau' e^{-\frac{t}{\tau'}}} dt + C e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow \; n_{er}(t) = \frac{e_p N_{er}}{E} + \{\frac{e_n c_n - e_p c_p}{e_p + e_n}\} \frac{N_{er} N}{IF} \int e^{-\frac{t}{\tau'}} e^{-D\tau' e^{-\frac{t}{\tau'}}} dt + C e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

Let $\quad D\tau' e^{-\frac{t}{\tau'}} = z \quad \therefore \; D\tau'(-\frac{1}{\tau'}) e^{-\frac{t}{\tau'}} dt = dz \quad \therefore \; e^{-\frac{t}{\tau'}} dt = -\frac{1}{D} dz$

$$\Rightarrow \; n_{er}(t) = \frac{e_p N_{er}}{E} + \{\frac{e_n c_n - e_p c_p}{e_p + e_n}\} \frac{N_{er} N}{IF} \int \frac{1}{-D} e^{-z} dt + C e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow \; n_{er}(t) = \frac{e_p N_{er}}{E} + \frac{(e_n c_n - e_p c_p)}{(e_p + e_n)(c_n + c_p) N} \frac{N_{er} N}{IF} e^{-z} + C e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow \; n_{er}(t) = \frac{e_p N_{er}}{E} + \frac{(e_n c_n - e_p c_p)}{(e_p + e_n)(c_n + c_p)} \frac{N_{er}}{e^{-D\tau' e^{-\frac{t}{\tau'}}} e^{Et}} e^{-D\tau' e^{-\frac{t}{\tau'}}} + Ce^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

$$\Rightarrow \; n_{er}(t) = \frac{e_p N_{er}}{E} + \frac{(e_n c_n - e_p c_p) N_{er}}{(e_p + e_n)(c_n + c_p)} e^{-Et} + Ce^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

neglecting first term (as $\frac{e_p N_{er}}{E} \approx 0$)

$$\Rightarrow \; n_{er}(t) = \frac{(e_n c_n - e_p c_p) N_{er}}{(e_p + e_n)(c_n + c_p)} e^{-Et} + Ce^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

at $t = 0,\; n_{er}(0) = f_{t0} N_{er} = \frac{(e_n c_n - e_p c_p) N_{er}}{(e_p + e_n)(c_n + c_p)} + Ce^{D\tau'}$

$$\Rightarrow Ce^{D\tau'} = f_{t0} N_{er} - \frac{(e_n c_n - e_p c_p) N_{er}}{(e_p + e_n)(c_n + c_p)} \quad \therefore C = e^{-D\tau'} \left[f_{t0} N_{er} - \frac{(e_n c_n - e_p c_p) N_{er}}{(e_p + e_n)(c_n + c_p)} \right]$$

$$\Rightarrow \; n_{er}(t) = A_1 e^{-Et} + B_1 e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}$$

where, $A_1 = \frac{(e_n c_n - e_p c_p) N_{er}}{(e_p + e_n)(c_n + c_p)}$, $B_1 = e^{-D\tau'} \left[f_{t0} N_{er} - \frac{(e_n c_n - e_p c_p) N_{er}}{(e_p + e_n)(c_n + c_p)} \right]$

This is the decay equation of erbium at any time t.

Details derivation of erbium luminescence decay:

Let,

N_{er} = total number of erbium atom

$n_{er}(t)$ = the number of erbium traps occupied by electron at time, t

$n^*_{er}(t)$ = the number of excited traps at time, t

$f_t(t)$ = the ratio of occupied traplevels to steady state traps at time t, $\frac{n_{er}(t)}{N_{er}}$

τ = the life time of erbium

τ' = life time of silicon atom

The rate of change of excited erbium is given by

$$\frac{dn^*_{er}(t)}{dt}+\frac{n^*_{er}(t)}{\tau}=f_t(t)\left[N_{er}-n^*_{er}(t)\right]c_pp$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\frac{n^*_{er}(t)}{\tau}=f_t(t)N_{er}c_pp-f_t(t)c_ppn^*_{er}(t)$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\frac{n^*_{er}(t)}{\tau}+f_t(t)c_ppn^*_{er}(t)=f_t(t)N_{er}c_pp$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\left[\frac{1}{\tau}+f_t(t)c_pp\right]n^*_{er}(t)=f_t(t)N_{er}c_pp$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+\left[\frac{1}{\tau}+c_pp\frac{n_{er}(t)}{N_{er}}\right]n^*_{er}(t)=N_{er}c_pp\frac{n_{er}(t)}{N_{er}}$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+[\frac{1}{\tau}+\frac{c_pp}{N_{er}}n_{er}(t)]n^*_{er}(t)=c_ppn_{er}(t)$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt}+[\frac{1}{\tau}+\frac{c_pp}{N_{er}}\{A_1e^{-Et}+B_1e^{-Et+D\tau'e^{-\frac{t}{\tau'}}}\}]n^*_{er}(t)=c_pp\{A_1e^{-Et}+B_1e^{-Et+D\tau'e^{-\frac{t}{\tau'}}}\}$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt} + [\frac{1}{\tau} + \frac{c_p Ne^{-\frac{t}{\tau'}}}{N_{er}}\{A_1 e^{-Et} + B_1 e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}\}]n^*_{er}(t) =$$

$$c_p Ne^{-\frac{t}{\tau'}}\{A_1 e^{-Et} + B_1 e^{-Et + D\tau' e^{-\frac{t}{\tau'}}}\}$$

$$\Rightarrow \frac{dn^*_{er}(t)}{dt} + [\frac{1}{\tau} + \frac{c_p NA_1}{N_{er}} e^{-(E+\frac{1}{\tau'})t} + \frac{c_p NB_1}{N_{er}} e^{-(E+\frac{1}{\tau'})t + D\tau' e^{-\frac{t}{\tau'}}}]n^*_{er}(t) =$$

$$c_p NA_1 e^{-(E+\frac{1}{\tau'})t} + c_p NB_1 e^{-(E+\frac{1}{\tau'})t + D\tau' e^{-\frac{t}{\tau'}}}$$

neglecting E as $E << \frac{1}{\tau'}$ we get

$$\Rightarrow \frac{dn^*_{er}(t)}{dt} + [\frac{1}{\tau} + A_2 e^{-\frac{t}{\tau'}} + B_2 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}}]n^*_{er}(t) = A_3 e^{-\frac{t}{\tau'}} + B_3 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}}$$

where, $A_2 = \frac{c_p NA_1}{N_{er}}$, $B_2 = \frac{c_p NB_1}{N_{er}}$, $A_3 = c_p NA_1$, $B_3 = c_p NB_1$

Integration factor $= \exp[\int(\frac{1}{\tau} + A2 e^{-\frac{t}{\tau'}} + B2 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}})dt]$

$$= \exp[\int \frac{1}{\tau} dt + A2 \int e^{-\frac{t}{\tau'}} dt + B2 \int e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}} dt]$$

$$= \exp[\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}]$$

Multiplying both sides by integration factor we get

$$\exp[\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}]\{\frac{dn^*_{er}(t)}{dt} + [\frac{1}{\tau} + A_2 e^{-\frac{t}{\tau'}} + B_2 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}}]\}n^*_{er}(t) =$$

$$[A_3 e^{-\frac{t}{\tau'}} + B_3 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}}] \exp[\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}]$$

$$\Rightarrow \exp[\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}]\, dn^*_{er}(t) + [\frac{1}{\tau} + A_2 e^{-\frac{t}{\tau'}} + B_2 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}}]\}n^*_{er}(t)dt =$$

$$[A_3 e^{-\frac{t}{\tau'}} + B_3 e^{-\frac{t}{\tau'} + D\tau' e^{-\frac{t}{\tau'}}}] \exp[\frac{t}{\tau} - A_2 \tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}]dt$$

$$\Rightarrow d\left[n_{er}^{*}(t)\exp[\frac{t}{\tau} - A_2\tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}\right] =$$

$$[A_3 e^{-\frac{t}{\tau'}} + B_3 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}}]\exp[\frac{t}{\tau} - A_2\tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}]dt$$

Integrating both sides we get

$$\int d\left[n_{er}^{*}(t)\exp[\frac{t}{\tau} - A_2\tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}\right] =$$

$$\int (A_3 e^{-\frac{t}{\tau'}} + B_3 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}})\exp[\frac{t}{\tau} - A_2\tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}]dt$$

$$\Rightarrow n_{er}^{*}(t)\exp[\frac{t}{\tau} - A_2\tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}} =$$

$$\int (A_3 e^{-\frac{t}{\tau'}} + B_3 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}})\exp[\frac{t}{\tau} - A_2\tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}]dt + C$$

where C is an integration constant.

Dividing both sides by integration factor we get

$$n_{er}^{*}(t) = \frac{\int (A_3 e^{-\frac{t}{\tau'}} + B_3 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}})\exp(\frac{t}{\tau} - A_2\tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}})dt}{\exp(\frac{t}{\tau} - A_2\tau' e^{-\frac{t}{\tau'}} - \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}})}$$

$$+ C\exp[-\frac{t}{\tau} + A_2\tau' e^{-\frac{t}{\tau'}} + \frac{B_2}{D} e^{D\tau' e^{-\frac{t}{\tau'}}}]$$

to calculate the value of C, at t = 0, the value inside the integration sign will be zero due to the same upper and lower limit.

At $t = 0$, $n_{er}^{*}(0) = \frac{D_2 N_{er}}{D_1} = C e^{A_2\tau' + \frac{B_2}{D} e^{D\tau'}}$ $\therefore$ $C = \frac{D_2 N_{er}}{D_1} e^{-(A_2\tau' + \frac{B_2}{D} e^{D\tau'})}$

Putting the value of C we get

$$\Rightarrow \quad n_{er}^{*}(t)=\frac{\int(A_3 e^{-\frac{t}{\tau'}}+B_3 e^{-\frac{t}{\tau'}+D\tau' e^{-\frac{t}{\tau'}}})\exp(\frac{t}{\tau}-A_2\tau' e^{-\frac{t}{\tau'}}-\frac{B_2}{D}e^{D\tau' e^{-\frac{t}{\tau'}}})dt}{\exp(\frac{t}{\tau}-A_2\tau' e^{-\frac{t}{\tau'}}-\frac{B_2}{D}e^{D\tau' e^{-\frac{t}{\tau'}}})}$$

$$+\frac{D_2 N_{er}}{D_1}e^{-(A_2\tau'+\frac{B_2}{D}e^{D\tau'})}\exp[-\frac{t}{\tau}+A_2\tau' e^{-\frac{t}{\tau'}}+\frac{B_2}{D}e^{D\tau' e^{-\frac{t}{\tau'}}}]$$

where $D_1=\frac{b}{a}c_p p+\frac{1}{\tau}$, $D_2=\frac{b}{a}c_p p$

$a=e_n+e_p+c_n n+c_p p$, $b=e_p+c_n n$