

**A GENETIC ALGORITHM BASED MULTI-OBJECTIVE  
APPROACH FOR FUZZY ACTIVITY TIME-COST  
OPTIMIZATION WITH RESOURCES CONSIDERATION**

**SURAJIT KUMAR SAHA**



**DEPARTMENT OF INDUSTRIAL AND PRODUCTION ENGINEERING  
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY**

**Dhaka-1000, Bangladesh**

**October, 2011**

**A GENETIC ALGORITHM BASED MULTI-OBJECTIVE  
APPROACH FOR FUZZY ACTIVITY TIME-COST  
OPTIMIZATION WITH RESOURCES CONSIDERATION**

BY

**SURAJIT KUMAR SAHA**

A thesis submitted to the Department of Industrial and Production Engineering,  
Bangladesh University of Engineering and Technology, in partial fulfillment of the  
requirements for the degree of

**MASTER OF SCIENCE IN INDUSTRIAL AND PRODUCTION ENGINEERING**



**DEPARTMENT OF INDUSTRIAL AND PRODUCTION ENGINEERING  
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY**

**Dhaka-1000, Bangladesh**

**October, 2011**

## CERTIFICATE OF APPROVAL

The thesis titled “**A genetic algorithm based multi-objective approach for fuzzy activity time-cost optimization with resources consideration**” submitted by **Surajit Kumar Saha, Roll No: 0409082009F** has been accepted as satisfactory in partial fulfillment of the requirement for the degree of Master of Science in Industrial and Production Engineering on October 12, 2011.

### Board of Examiners

1. \_\_\_\_\_ Chairman  
**Dr. Sultana Parveen** (Supervisor)  
Associate Professor  
Department of Industrial and Production Engineering  
Bangladesh University of Engineering and Technology (BUET), Dhaka
  
2. \_\_\_\_\_ Member  
**Dr. A.K.M. Masud** (Ex-Officio)  
Professor and Head  
Department of Industrial and Production Engineering  
Bangladesh University of Engineering and Technology (BUET), Dhaka
  
3. \_\_\_\_\_ Member  
**Dr. Md. Ahsan Akhtar Hasin**  
Professor  
Department of Industrial and Production Engineering  
Bangladesh University of Engineering and Technology (BUET), Dhaka
  
4. \_\_\_\_\_ Member  
**Dr. Md. Nazrul Islam** (External)  
Director  
National Productivity Organisation (NPO)  
Govt. of the People’s Republic of Bangladesh  
Motijheel, Dhaka-1000

## **CANDIDATE'S DECLARATION**

---

It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the awarded of any degree or diploma.

---

**Surajit Kumar Saha**

*To the Almighty*

*To my family*

## ACKNOWLEDGEMENT

---

All credits go to God, the most benevolent and the Almighty for guiding, providing and sustaining me through all my life and specially, successful completion of this thesis.

I would like to take opportunity of expressing my sincere respect and heartiest gratitude to my thesis supervisor Dr. Sultana Parveen, Associate Professor, Department of Industrial and Production Engineering, Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh, for her careful supervision, thoughtful suggestions, constant guidance and encouragement throughout the progress of this research work.

I also expressed my sincere gratitude to Dr. A. K. M. Masud, Professor and Head, Department of Industrial and Production Engineering (IPE), BUET, Dr. Md. Ahsan Akhtar Hasin, Professor, Department of Industrial and Production Engineering (IPE), BUET and Dr. Md. Nazrul Islam, Director, National Productivity Organization (NPO), Govt. of the People's Republic of Bangladesh, for their constructive remarks and for kindly evaluating this research.

I am grateful to Mr. Shaunok Datta, undergraduate student of Chemical Engineering, BUET and Mr. Md. Imran Khan, graduate student of Mechanical Engineering in Concordia University, Canada, for helping me to develop computer coding and continue the research.

Finally, I would like to extend my sincere thanks to my parents and brother for their continuous inspiration, sacrifice and support that encourage me to complete the research successfully.

## ABSTRACT

---

The project managers' major concern is to complete project cost effectively and timely. But, in real life, the project completion time and cost may vary significantly as an effect of uncertainty such as weather, political and social disturbances, inflation, site congestion, human factors, productivity level, etc. Minimization of project time requires maximum supply of resources. Moreover, any delay in project completion time adds extra cost. Since, the relationship between time and resources of the activity is no longer a monotonously decreasing or increasing curve, but rather the fuzzy nature. Therefore, it is needed to minimize both project time and cost under uncertainty. In this study, fuzzy set theory is applied to model the managers' behavior in predicting time and resources of an activity and  $\alpha$ -cut method, in fuzzy logic theory, is used as a measure of accepted risk level. Because of NP-hard nature of time-cost tradeoff problem, genetic algorithms are used as a searching mechanism to establish the Pareto optimal solutions under different risk levels incorporating multi-objective approach. In addition, the activities of project are crashed and penalty is induced for delay to make such problem realistic. The proposed model leads the managers to choose the optimal time-cost solutions under different risk levels as well as their associated degree of belief in a more flexible and realistic manner. Finally this proposed model is used to solve a real problem.

# TABLE OF CONTENTS

<b>Title</b>	<b>Page</b>
<b>ACKNOWLEDGEMENT.....</b>	<b>v</b>
<b>ABSTRACT.....</b>	<b>vi</b>
<b>TABLE OF CONTENTS.....</b>	<b>vii</b>
<b>LIST OF TABLES.....</b>	<b>xi</b>
<b>LIST OF FIGURES.....</b>	<b>xii</b>
<b>NOMENCLATURE.....</b>	<b>xiv</b>
<b>LIST OF ABBREVIATIONS.....</b>	<b>xv</b>
<b>CHAPTER 1 INTRODUCTION.....</b>	<b>1</b>
1.1 Rational of the Study.....	1
1.2 Objectives with Specific Aims and Possible Outcome.....	3
1.3 Outline of Methodology.....	3
<b>CHAPTER 2 LITERATURE REVIEW.....</b>	<b>4</b>
2.1 Introduction.....	4
2.2 Project Scheduling with Time-Cost Optimization.....	5
2.2.1 Deterministic Scheduling Problems.....	6
2.2.1.1 Mathematical Models.....	6
2.2.1.2 Heuristic Models.....	7
2.2.1.3 Multi-Objective Optimization.....	7
2.2.2 Non-Deterministic (Uncertainty Based) Scheduling Problems.....	8
2.2.2.1 Probability Theory.....	8
2.2.2.2 Fuzzy Set Theory.....	9
2.2.2.3 Heuristic Method.....	10



2.2.2.4 Multi-Objective Approach for Fuzzy Time- Cost Tradeoff.....	11
--	----

<b>CHAPTER 3 THEORETICAL BACKGROUND.....</b>	<b>13</b>
3.1 Introduction.....	13
3.2 Fuzzy Set Theory.....	13
3.2.1 Definition.....	13
3.2.2 Difference between Crisp Set and Fuzzy Set.....	14
3.2.3 Membership Function of Fuzzy Set .....	15
3.2.4 Concept of Alpha ( $\alpha$ )-cut.....	17
3.2.5 Arithmetic Operations on Fuzzy Numbers.....	18
3.3 Concept of Multi-Objective Optimization.....	19
3.3.1 Pareto Optimum.....	19
3.3.2 Adaptive Weight Approach (AWA).....	19
3.3.3 Modified Adaptive Weight Approach (MAWA).....	21
3.4 Genetic Algorithms.....	23
3.4.1 Biological Background.....	24
3.4.1.1 Chromosomes.....	24
3.4.1.2 Reproduction.....	24
3.4.1.3 Fitness.....	25
3.4.1.4 Evolution.....	25
3.4.2 Basic Principal.....	26
3.4.3 Operators of GA.....	26
3.4.3.1 Encoding of Chromosome.....	26
3.4.3.2 Selection.....	28
3.4.3.3 Crossover.....	30
3.4.3.4 Mutation.....	31
3.4.4 Fitness Function.....	32
3.4.5 Termination.....	32
3.4.6 Genetic Algorithm Process.....	33

<b>CHAPTER 4</b>	<b>PROBLEM FORMULATION.....</b>	<b>34</b>
4.1	Introduction.....	34
4.2	Mathematical Model.....	35
4.3	Assumptions of the Model.....	36
<b>CHAPTER 5</b>	<b>SOLUTION APPROACH .....</b>	<b>37</b>
5.1	Problem Solving Process.....	37
5.2	Solving with Genetic Algorithm.....	41
5.2.1	Algorithm Structure.....	41
5.2.2	Encoding Technique.....	43
5.2.3	Initial Population.....	43
5.2.4	Selection Procedure.....	43
5.2.5	Crossover Operation.....	43
5.2.6	Mutation Operation.....	44
5.2.7	Fitness Function.....	44
5.2.8	Stopping Condition.....	44
<b>CHAPTER 6</b>	<b>RESULT ANALYSIS.....</b>	<b>45</b>
6.1	Introduction.....	45
6.2	Case Study.....	45
6.3	Solving Hypothetical Test Problem with Computer Program.....	48
6.3.1	Considered Required Resources are Available in Store.....	49
6.3.1.1	Result Analysis.....	49
6.3.1.2	Convergence Analysis.....	52
6.3.1.3	Effect of Indirect Cost.....	53
6.3.2	Considered Required Resources are short in Store.....	54
6.3.2.1	Result Analysis.....	55
6.4	Performance of the Proposed Model.....	56
6.4.1	Solving the Test Problem.....	57
6.5	Real Problem.....	58
6.5.1	Solving Real Test Problem with New Model.....	64

6.5.1.1	Considered Resources are Available in Store.....	65
6.5.1.2	Considered Required Resources are Short.....	67
<b>CHAPTER 7</b>	<b>CONCLUSIONS AND RECOMMENDATIONS.....</b>	<b>69</b>
7.1	Conclusions.....	69
7.2	Recommendations.....	70
<b>REFERENCES.....</b>		<b>71</b>
<b>APPENDICES .....</b>		<b>78</b>
	Appendix A: Details results with Hypothetical Case (Resources are available).....	79
	Appendix B: Program Code for GA solver in MATLAB.....	84

## LIST OF TABLES

<b>Table No.</b>	<b>Title</b>	<b>Page</b>
6.1	Detail data of a hypothetical case.....	46
6.2	Optimistic and pessimistic project duration margin.....	48
6.3	Pareto front solutions for $\alpha = 0.3$ (resources are available).....	49
6.4	Effect of $\alpha$ -cut level on the Pareto optimal solutions.....	50
6.5	Effects of generation number on the optimal solutions.....	53
6.6	Effect of indirect costs on the Pareto front solutions.....	54
6.7	Pareto front solutions for $\alpha = 0.3$ (Resources are short in store).....	55
6.8	Test case in detail (Zheng and Ng [66]).....	56
6.9	Parameters for the three models in comparison.....	57
6.10	Results of three models at $\alpha = 1$ .....	57
6.11	Activities and resources of 200 KVA Transformer.....	59
6.12	Detail data of real problem (200 KVA transformer making).....	61
6.13	Optimistic and pessimistic project duration margin (Real problem).....	64
6.14	Pareto front solution for real problem at $\alpha = 0.2$ (Resources are available).....	65
6.15	Effect of $\alpha$ -cut value on the optimum solution in real problem (resources are available).....	66
6.16	Pareto front solution for real problem at $\alpha = 0.2$ (Required resources are short in store).....	68
A1	Results with Different $P_c$ and $P_m$ .....	79
A2	Data table for convergence analysis.....	82
A3	Results with different population size.....	83

## LIST OF FIGURES

Figure No.	Title	Page
2.1	General class of time-cost tradeoff scheduling problems.....	6
3.1	Difference between crisp and fuzzy sets.....	14
3.2	Different membership functions.....	15
3.3 (a)	Triangular membership function of fuzzy set	16
3.3 (b)	$A_{\alpha}$ .....	16
3.4	Trapezoidal membership function of fuzzy set	17
3.5	$A_{\alpha}$ .....	21
	Example of $\alpha$ -cut level.....	
3.6	Concept of adaptive weight approach adapted from Gen and Cheng	25
3.7	[36].....	28
3.8 (a)	Evolution through natural selection.....	29
3.8 (b)	Roulette Wheel.....	29
3.9	Situation before rank selection.....	33
5.1	Situation after rank selection.....	37
	Simple GA process.....	
5.2 (a)	Typical deterministic relationship between resources and time of an	39
5.2 (b)	activity.....	39
5.2 (c)	Crash and Normal activity fuzzy time.....	39
5.3	Crash and Normal activity fuzzy resources.....	40
5.4	Fuzzy relationship between time and resources.....	41
6.1	Problem solving approach.....	46
6.2	Problem solving process with GA.....	48
6.3	CPM network (AON diagram) of 7-activity hypothetical test problem	50
6.4	Possible project duration spectrum for Hypothetical problem.....	51
6.5	At $\alpha = 0.3$ , Pareto front for hypothetical test case.....	52
6.6	Effect of $\alpha$ -cut level on the Pareto optimal solutions.....	52
6.7	Impact of $\alpha$ -cut level on project cost.....	58

6.8	Impact of $\alpha$ -cut level on project duration.....	
	Experimental results from three models.....	60
6.9	CPM network of 8-activity real problem (200 KVA Transformer	64
6.10	making project).....	66
	Project duration spectrum for real test problem.....	
	At $\alpha = 0.2$ , Pareto front solution for Real problem.....	

## NOMENCLATURE

---

$x \in A$	Element $x$ belongs to crisp set $A$
$\{x \notin A\}$	Element $x$ does not belong to crisp set $A$
$\mu_A$	Characteristic function
$\mu_{A_\alpha}(x)$	Membership grade of $x$ in fuzzy set $A_\alpha$
$A_\alpha$	Alpha-cut of fuzzy set $A$
$A \cup B$	Set union
$A \cap B$	Set intersection
$F$	Fitness function
$\forall$	For all
$\vee$	Symbolize maximum
$\wedge$	Symbolize minimum
$\sum$	Summation
$p_i$	Probability
$q_i$	Cumulative probability
$K$	Number of crossover point
$P_c$	Crossover rate
$P_m$	Mutation rate
$G$	Generation number
$l$	Length of chromosome

## LIST OF ABBREVIATIONS

---

TRAPMF	Trapezoidal Membership Function
GBELLMF	Generalized Bell-shaped Membership Function
TRIMF	Triangular Membership Function
GAUSSMF	Gaussian Membership Function
SMF	S-shaped built-in Membership Function
ZMF	Z-shaped built-in Membership Function
PSIGMF	Built-In Membership Function Composed of Product of Two Sigmoid Shaped Membership Functions
DSIGMF	Built-in Membership Function Composed of Difference between Two Sigmoid Membership Functions
PIMF	• -shaped built-in Membership Function
SIGMF	Sigmoid shaped built-in membership function
CPM	Critical Path Method
PERT	Project Evaluation and Review Technique
GA	Genetic Algorithm
NP	Nondeterministic Polynomial
IP	Integer programming
TCTP	Time-Cost Trade-Off Problem
TCO	Time-Cost Optimization
LP	Linear Programming
NSGA	Non-Dominated Sorting Genetic Algorithm
AOA	Activity On Arrows
AON	Activity On Nodes
MAWA	Modified Adaptive Weight Approach
MOO	Multi-Objective Optimization
MF	Membership Function
SAM	Siemens Approximation Method
TSP	Travelling Sales Man Problem
FNET	Fuzzy Network Scheduling





# CHAPTER 1

## Introduction

A project is a group of activities. To complete the project, project planner has to plan the execution order according to precedence relationship of the activities. Here, all activities are arranged in network in their logical sequence and represented by arcs and nodes. This defines paths and gives solutions for project scheduling. Among the paths, a critical path is the sequence of project network activities which add up to the longest overall duration analytically. That determines the shortest time possible to complete the project. Two analytical methods, Critical path method (CPM) and Project Evaluation and Review Technique (PERT) are the most commonly used management techniques for project scheduling. The both techniques deal with deterministic time duration of activities to schedule the project within budget and time limits.

### 1.1 Rational of the Study

Project Scheduling is critical to a project management plan. It is an activity plan with time axis where activities are specified by duration, start and finish times. Proper plans and estimates are needed to achieve the schedule by accelerating some efforts or modifying approaches to meet required deadlines. Critical path Method (CPM) is one of project scheduling tool in project management where duration of project is found. After the project scheduling, the project manager is highly concerned about the project completion time within the deadline and budget. But it can be complicated and caused delay by failure of execution of activities. Failure to meet schedule goals is most often due to unrealistic deadlines, passive project execution, unforeseen problems, or things overlooked in the plan or uncertainties. In that case, the activities should be crashed to accelerate the project to meet deadline. Crashing the schedule can be done by adding workers, equipments, extending working hours, removing delay time and taking innovation approaches etc. Uncertain resources of talent, equipment, or data will likely result

in extending the project. Duration of the activity depends on resource and cost. So, optimum combination should be found to complete the project.

To estimate the duration of activity in CPM network is a probabilistic problem in real life due to uncertainty. Uncertainties such as environmental causes, political influences, execution errors, design errors, accidents, social disturbances etc can delay the activity and extend the project. Project planner can not ensure the completion time of activities. If he is expert, can predict and forecast the situation depends on his experience, analysis of information and own judgment. It is similar to weather forecasting. In this situation, probabilistic approaches are used to address the uncertainty in project where variability in the duration of each activity is considered. It is naturally more risky to define a definite value than define a group or range of possible value. So, expert use linguistic term such as “about”, “more than” or “less than”, “approximate”, “near to”, “lies between”, “range” etc to allocate uncertainty in forecast.

In real life, project information is not saved properly. Expert has to deal with the incomplete data and unavailable data. The lack of statistical data, probability distributions for some activity duration times may be unknown or just partially known. In this case, probability theory may be replaced by fuzzy set theory. Fuzzy set theory is used to tackle vagueness problem [1]. It deals effectively with the ambiguities involved in the process of linguistic estimate times. So, trapezoidal fuzzy numbers is more suitable to make the fuzzy measures of activity time and its resources characterized by linguistic values.

Considering different amount of resources, project planner gets different alternatives of possible project durations and costs. Finding Pareto optimum solutions, multi-objective time-cost trade-off is a NP-hard (Non-deterministic polynomial time) problem [2]. So, it is difficult to find the best solution among possible solutions. Therefore, heuristic models are appropriate to overcome this problem.

This research has been aimed at development of an efficient GA based multi-objective time-cost optimization model with resource consideration incorporating the vagueness or fuzziness of the dynamic conditions of a real world.

## **1.2 Objectives with Specific Aims and Possible Outcome**

The specific objectives of the present research work are as follows:

- (a) To address the uncertainty in CPM network by characterizing activity time and resources by the fuzzy trapezoidal number.
- (b) To develop a mathematical model to calculate the total project cost.
- (c) To develop a genetic algorithm based heuristic algorithm for multi-objective time-cost optimization at different risk level.
- (d) To analyze available profile of variable resources.

The possible outcome of the work is development of a genetic algorithm based heuristic algorithm that uses the information of the fuzzy activity duration, fuzzy resources, indirect cost rate, resources cost rate at available and unavailable states, premium cost rate of beyond the desired project time and project networks as input to solve fuzzy critical path and show the optimal solutions of multi-objective time-cost optimization problem.

## **1.3 Outline of Methodology**

The following step-by-step methodology will be applied to this thesis paper:

- (a) Collection of real production activity data from a company.
- (b) Analysis of precedence of activities and feasible alternatives.
- (c) Characterizing activity duration and resources by fuzzy trapezoidal number.
- (d) Establishing objective function to measure total product cost.
- (e) Establishing fitness function for multi-objective time-cost optimization.
- (f) Development of a Genetic algorithm based heuristic algorithm for optimal solution.
- (g) Making a computer program in MATLAB to solve the problem.
- (h) Analysis of the performance of the developed algorithm.

# CHAPTER 2

## Literature Review

---

### 2.1 Introduction

Project scheduling is the analytical tool of management to schedule the activities and allocation of resources in project. CPM or PERT are used as scheduling technique to calculate project duration by scheduling the activity according to precedence constrains. In real life, project planner's major concern is to minimize project cost or time for the rapid production or to minimize both time and cost for project optimization considering different parameters. However, the construction durations are normally predetermined and stipulated in the tender documents (time being a constraint set by the client), tenderers would usually focus on a single objective only, i.e., to minimize project cost in order to underbid their competitors [3]. Their successes depend on good management of project which makes profit. Therefore, good planning, scheduling and control make the project successful. So, to maximum utilization of resources and minimization of both project cost and duration, project management becomes essential and famous subject in the world. Many researchers show interest on it in recent years. Since, project managers are sincere to practices project management applications.

Management of the project is not an easy task. It becomes more complicated when the interrelated activity number becomes large and project is performed in fuzzy environment. Moreover, in multi-objective time-cost optimization problem, it is difficult to find the best solution among possible solutions due to its NP-hard nature [2].

In CPM and PERT techniques with uncertainty where activity times in a project are approximately known and are more suitably represented by fuzzy sets rather than crisp numbers [3]. That is why, in recent years, many researchers have worked with fuzzy in CPM to represent uncertain activity duration.

## **2.2 Project Scheduling with Time-Cost Optimization**

Time-cost optimization is an important issue in project scheduling. Because, in real life project manager's one of the main concerns is to finish project within the deadline to avoid delay. So, it needs good balancing of project duration, cost and resources.

In a project, there are three important elements, such as project duration or completion time, project cost and resources. These are not independent. Project cost and project duration directly depend on supply of resources. However, in real project, project activities must be scheduled under available resources, such as crew sizes, equipment and materials. The activity duration can be looked upon as a function of resources availability [4]. On the other hand, all combinations of resources demand cost. However, many researchers have pointed out that project time and cost are intricately related [5-8]. It means, if the project is duration compressed then there is leading to an increase in labor and plant costs (i.e., direct costs) [9]. Besides, project overhead (i.e., indirect costs) increasing with the project duration. So, different combinations of resources in a project refer different possible solutions. Thus, project manager has to find best solution among the possible solutions. Ultimately, the project scheduling needs to take account the twin objectives, minimization of both time and cost considering supply of resources.

Resources limitation is not considered in CPM [10]. Evaluating each alternative requires recalculation of schedule using the CPM and assessment of total project. However, TCO problem is NP-hard problems; the required time to solve this problem sharply increases with activity number. Thus, TCO problems can be solved by mathematical or analytical model programming and heuristic methods [11]. Determining the best sets is the goal of TCO.

Generally TCO problems are divided into two types, such as deterministic and non-deterministic scheduling. Moreover, non-deterministic scheduling models are also categorized into two types: fuzzy set models and probabilistic models.

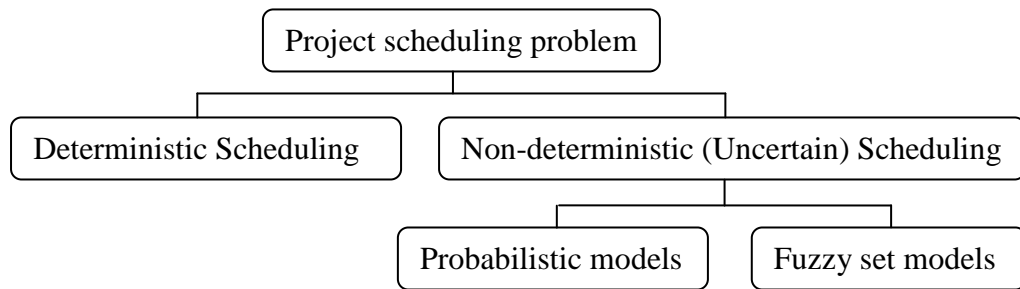


Figure 2.1: General class of time-cost tradeoff scheduling problems.

## 2.2.1 Deterministic Scheduling Problems

Traditional TCT models mostly focus on deterministic situations. In deterministic project scheduling problem, it is assumed that all data are available. Considering that, many researchers have proposed various mathematical and heuristic models to solve TCT problems.

### 2.2.1.1 Mathematical Models

Several mathematical models such as linear programming, integer programming, or dynamic programming are used to solve TCO problems. Hendrickson and Au [12], Pagnoni [13] and Liu et al. [14] used linear programming to solve them. Burns et al. [15] also proposed a LP/IP hybrid models to solve time-cost trade-off problems. Besides, Kelly [16] formulated a TCT problem considering linear time-cost relationship. On the other hand, some other approaches were proposed to solve time-cost problems considering both linear and discrete time-cost relationships such as Meyer and Shaffer [17] and Patterson and Huber [18] used mixed integer programming. However, for large number of activity in network and complex problem, integer programming needs a lot of computation effort [19]. Besides, Butcher [20], Robinson [21], Elmaghraby [22] and De et al. [23] used dynamic programming to solve the TCT problem. Some researchers, to solve resources constrained scheduling problems used integer and dynamic programming, such as David [24], Elmaghraby [25] and Talbot [26]. In these methods, the relationship between activity duration and cost were generally assumed as: linear or nonlinear; concave, convex or not fixed; discrete or continuous; or hybrid [12-18, 20-26].

Thus, Mathematic model are difficult to create and required great computational effort. Moreover, these approaches could not ensure optimum solution for large and complex problem. Since, these are suitable for small project [27].

### ***2.2.1.2 Heuristic Models***

Due to problem of “combinatory explosion”, some researchers used heuristic approaches to solve the TCO problems. These methods have been widely adopted due to their simplicity and ease of application.

Heuristic approaches rely on the rules of thumb of the decision maker. Fondahl’s method [28], effective cost slope model of Siemens [29] and structural stiffness model of Moselhi [30] are the example of heuristic methods. Recently, Šeda [31] proposed a model for limited resources project scheduling. There, an efficient routine is proposed that, instead of shifting the activities, extends their duration. However, since these methods could only optimize one objective at a time, the solution is not guaranteed to be the “global optimal.”

Some researchers adopted computational optimization technique, such as genetic algorithms and simulated annealing, to solve TCT problem. Li and Love [32], Hegazy [33] and Chua et al. [34] used GA to solve deterministic TCT problems. Li and Love [32] and Hegazy [33] introduced some modification to basic GA, but yet remained as a single-objective optimization tool. Recently, some researchers have worked with GA based resource constrained [11] and multi-resource constrained scheduling [10, 35]. However, the main drawback of the GA-based application is that they required large computational time for the search.

### ***2.2.1.3 Multi-Objective Optimization***

The researches mentioned before are all single objective TCT model. Besides, some researchers worked on multi-objective TCT problem. Multi-objective optimization (MOO) is the process of optimizing systematically and simultaneously a collection of objective functions. It is used to find optimum solution within the multi-objective space. Feng et al. [19] proposed models using Pareto front (multi-objective) approach to solve TCO problem. That model was implemented on a spreadsheet and considered each task’s construction options to



generate the shape of the optimum trade-off curve for direct cost. Indirect cost was then added to determine the optimum TCT strategy. The model, however, is limited to simple networks with finish-to-start relationships and is not capable of dealing with limited resources.

Besides, Gen and Cheng [36] proposed a GA-based multi-objective model, based on an adaptive weight approach for construction of TCO problem. That approach overcomes the weakness of conventional aggregation approaches, in which the weights are the constants determined through a priori knowledge about the problem. However, several deficiencies might still exist in this approach. Zheng et al. [37] modified the adaptive weight approach of Gen and Cheng [36]. Those models induced the adaptive weights derived from previous generations for each objective to exert a search pressure toward an optimum solution. They did not propose any Pareto front to solve TCO problem.

Xiong and Kuang [38] and Ng and Zhang [39] applied ant colony optimization (ACO) algorithm for multi-objective TCT problem combining with modified adaptive weight approach (MAWA). Those algorithms showed optimal (best found) solutions and defined the Pareto front as well.

Geem [40] proposed another algorithm for multi-objective optimization of TCT using harmony search. This study employed a phenomenon-mimicking algorithm. It was devised based upon the music improvisation and the optimization process. He also showed the Pareto front and optimum solutions. However, the above mentioned TCT models mainly focus on deterministic environment.

### **2.2.2 Non-Deterministic (Uncertainty Based) Scheduling Problems**

In real life, project duration is not deterministic, it is probabilistic. Uncertainties effect on the project duration and cost. So, to handle uncertain scheduling a lot of techniques were developed in recent decades.

#### **2.2.2.1 Probability Theory**

Traditionally, uncertain activity duration times in project scheduling was handled by stochastic approaches using probability theory [41]. Classical non-deterministic scheduling models are PERT and Monte Carlo simulation. These are

the most widely used in practice [42]. Ang [43], Ahuja and Arunachalam [44], Padilla and Carr [45], Gong [46] and several others also developed probability theory based scheduling models under uncertainties. Here, uncertainty is associated with randomness. However, in many situations, it is impossible to get the probability distribution of activity duration due to lack of information. Moreover, many projects may never be carried out previously. So, it is hard to model uncertainty using probabilistic approaches.

#### ***2.2.2.2 Fuzzy Set Theory***

Fuzzy set theory has proven to be an effective way of handling vague information in project [47]. Since, some other researchers also have claimed that, it is appropriate for modeling uncertainty that is associated with time elements in project networks [48, 49]. Prade [50] first induced the concept of fuzzy sets into PERT in 1979. Ayyub and Haldar [51], Hadipriono and Sun [52], Wang et al. [53], Wu and Hadipriono [54], Chen and Huang [55] and other researchers developed fuzzy set theory based model to tackle non-deterministic project scheduling problems. Chen and Huang [55] proposed an analytical method that combines fuzzy set theory with PERT technique for measuring criticality in project network. In that model, triangular fuzzy numbers is used to define the activity times. That model also permits the sensitivity analysis by varying the durations of activities on the critical path.

Fuzzy set theory with critical path method can play a significant role in this kind of decision making environment. Several studies have investigated the case where activity times in a project are approximately known and are more suitably represented by fuzzy sets rather than crisp number [56]. Nasution [57] proposed a fuzzy critical path method by considering the interactive fuzzy subtraction and by observing that only the nonnegative part of the fuzzy numbers can have physical interpretation. Lorterapong and Moselhi [58] presented fuzzy network scheduling (FNET) which overcomes the limitations of backward pass calculations to compute project completion time with uncertainties. However, that approach also can handle the situation where multiple critical paths prevail and determine the existence of multiple near-critical paths. Yao and Lin [59] used signed distance ranking of fuzzy numbers to critical path in a fuzzy project network. In 2001,

Chen and Chang [60] used defuzzification method to find possible critical paths in the project network. Chanas and Zielinski [61] proposed a method to make critical path analysis in the network with fuzzy activities times (interval activity times, fuzzy numbers for L-R type) by directly applying the extension principal network of Zadeh [62]. Slyeptosov and Tyshchuk [56] presented an efficient method of computation of fuzzy time windows for late start and finish times of operations in the problem of fuzzy network. To effectively deal with the ambiguities involved in the process of linguistic estimate times, Liang and Han [63] used the trapezoidal fuzzy numbers in the proposed algorithm to perform critical path analysis in a fuzzy environment. In that model, decision-maker's risk attitude was also incorporated into the problem of fuzzy network. Shankar et al. [64] proposed a model using metric distance ranking of fuzzy numbers to solve critical path in fuzzy project network more efficiently. Recently, in [65], Shankar et al. proposed an analytic method for finding critical path in a fuzzy project network. Here, they used a new defuzzification formula for trapezoidal fuzzy number and applied to the float time (slack time) for each activity in fuzzy project network to find critical path. The defuzzification formula which is used for critical path can not be applied to the trapezoidal fuzzy number having equal elements. Because of that trapezoidal fuzzy number is a crisp number. That method can overcome the drawback of the existing fuzzy CPM method [60].

### ***2.2.2.3 Heuristic Method***

Heuristic approach to solve TCT problem under uncertainties is new and challenging field with only a limited number of published papers. Leu et al. [4] proposed a GA-based fuzzy optimal model for construction time-cost trade-off which is more efficient than the traditional (crisp) TCT approach. They combined fuzzy set theory and genetic algorithm to tackle TCT problem under uncertainty. Triangular fuzzy number was used to characterize the activity duration. An acceptable risk level (i.e.,  $\alpha$ -cut level) was defined as the minimum condition that can be acceptable. GA was used as search approach to find optimal solution within the acceptable margin under different risk level. However, activities of project were crashed and costs of crash and normal mode are known. In that research, a new relationship between time-cost of activity under fuzziness was presented. However, since the duration of an activity is associated with a specific

construction method, it would be difficult but not impossible to assign a uniform cost for every possible combination. In addition, the model did not consider how uncertain the estimate is or the extent of risk to which a project is going to face under the proposed estimate [66]. Moreover, that model didn't show any relationship among time, cost and resources.

Lorterapong [67] proposed FNET model in which fuzzy heuristic method was developed to solve resources-constrained scheduling under uncertainty. Hapke and Slowinski [68] developed fuzzy priority heuristic rules to tackle resources constrained scheduling. Pan et al. [69] also used heuristic based approach to solve resources constrained scheduling problem when activity durations were expressed by fuzzy numbers. Leu et al. [70] also developed project scheduling model with resources consideration under fuzzy environment. That model assumed that all activities have crisp resources demand and resources are limited during the project. To obtain good performance using GA search, all GA parameters such as crossover rate, mutation rate were varied for different risk level. Finally, that model was compared with Lorterapong's [67] FNET model. However, the above mentioned models mainly focus on single objective TCT under uncertainties.

#### ***2.2.2.4 Multi-Objective Approach for Fuzzy Time-Cost Tradeoff***

Some researchers considered uncertainty in multi-objective time-cost tradeoff. Zheng and Ng [66] proposed a so-called stochastic model for time-cost trade-off problem incorporating fuzzy set theory and non-replaceable front. The non-replaceable front concept was proposed to assist managers in recognizing promising solutions from numerous candidates on the Pareto front. Utility theory and opportunity cost, were integrated into the model to mimic the decision making process.

Eshtehardian et al. [71] proposed another model for fuzzy multi-objective time-cost optimization. In that paper, accepted risk level of the project manager was defined through  $\alpha$ -cut approach for which a separate Pareto front with set of non-dominated solutions were developed. Therefore the entire range of cost defined by the assigned fuzzy number was considered in that model. Recently, Abbasnia et al. [72] proposed fuzzy logic based approach named Non-dominated Sorting Genetic Algorithm (NSGA) for TCTP under uncertainty. Different Pareto

solutions on different risk level were measured using that model. But, all of above mentioned models did not consider fuzzy time-resources relationship to solve TCO problem.

# CHAPTER 3

## Theoretical Background

### 3.1 Introduction

Fuzzy set theory is used in this research work to characterize project activity time and resources. Multi-objective approach is used to sum up the searching criterions or objectives for time-cost optimization. A heuristic approach, Genetic algorithm is used to solve the problem. In this chapter, all important theory such as fuzzy set theory, Multi-objective approach and Genetic algorithm in this research work are briefly discussed step by step to get clear background.

### 3.2 Fuzzy Set Theory

Fuzzy set theory is developed and is used significantly to deal uncertainties (imprecision, nonspecificity, vagueness, inconsistency etc). It provides not only with a meaningful and powerful representation of measurement uncertainties, but also with a meaningful representation of vague concepts expressed in natural language [73]. Zadeh [62] developed fuzzy sets-are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of a degree. The membership value of each element of a set lies between 0 and 1.

#### 3.2.1 Definition

Fuzzy set is a set whose elements have degrees of membership. If a universal set  $X$  is a collection of objects denoted generally by  $x$ , a fuzzy set  $A$  in  $X$  is defined as follows [74]:  $A = \{(x, \mu_A(x)) \mid x \in X\}$ , Where,  $\mu_A(x)$  is membership function for fuzzy set  $A$  which associates with elements  $x$  in  $X$ . These membership grades are very often represented by real number values ranging in

the closed interval between 0 and 1. The larger  $\mu_{A^{\square}}(x)$  represents the stronger the degree of belongingness for  $x$  in  $A^{\square}$ . Thus, it can be said that, fuzzy set is a set of those elements whose membership values lie between 0 and 1.

### **3.2.2 Difference between Crisp Set and Fuzzy Set**

The characteristic function of a crisp set assigns a value of either 1 or 0 to each individual in the universal set, thereby discriminating between members and nonmembers of the crisp set under consideration. It means that the element whose membership value is only 1 is described as a member of crisp set; value is 0 or except 1 means that element is not member of crisp set. If crisp set  $A$  is defined by its characteristic function  $\bullet_A$  as follows:

membership, the elements between 3 to 6 and 6 to 9 have partial membership and other elements outside the range have zero membership.

### 3.2.3 Membership Function of Fuzzy Set

The membership function is a graphical representation that maps elements of a given universal set  $X$ , which is always a crisp set, into real numbers in  $[0,1]$ . It defines the fuzzy set for the possible elements underneath of it on the horizontal axis. The vertical axis, on a scale of 0 to 1, provides the membership value of the height in the fuzzy set. In literature, the membership function of a fuzzy set  $A \subseteq X$  is most commonly denoted by  $\mu_{A \subseteq X}$ ; thus,

$$\mu_{A \subseteq X}: X \rightarrow [0, 1]$$

A membership function must be piece wise continuous, convex and normalized. Membership functions for fuzzy sets can be defined in any number of ways as long as they follow the rules of the definition of a fuzzy set. The Shape of the membership function defines the fuzzy set and so the decision on which type to use is dependant on the purpose. Different types of fuzzy membership functions are shown Figure 3.2.

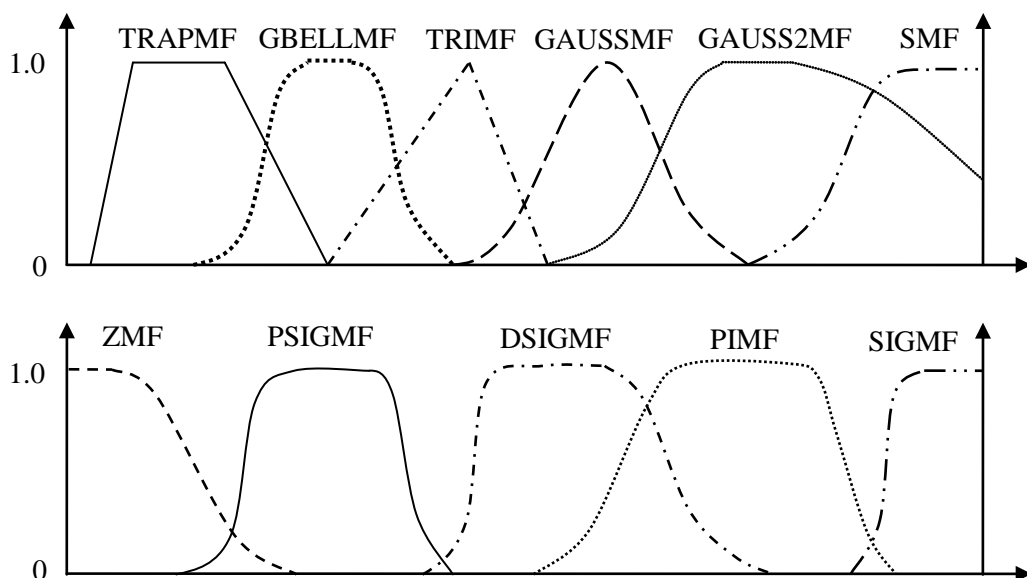


Figure 3.2: Different membership functions.

But, triangular and trapezoidal fuzzy set membership functions are most commonly used.



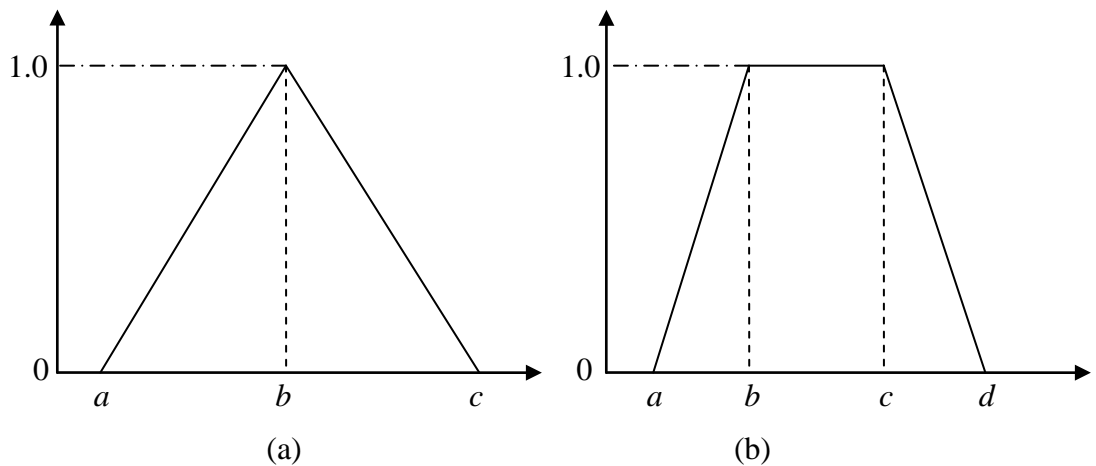


Figure 3.3: Triangular and trapezoidal membership function of fuzzy set  $A$ .

A triangular fuzzy number  $A = (a, b, c)$  shown in Figure 3.3 (a), where  $a$  and  $c$  are lower and upper bounds of the support of the fuzzy number  $A$ , while  $b$  is the modal value. The membership function of triangular fuzzy number  $A$  in the form

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x < b \\ \frac{c-x}{c-b} & b \leq x < c \\ 0 & \text{otherwise} \end{cases}$$

Then centroid of triangular fuzzy number  $A = (a, b, c)$  is defined by:

$$x_c = \frac{a + b + c}{3}$$

The trapezoidal fuzzy set  $A = (a, b, c, d)$  shown in Figure 3.3 (b), where  $a$  and  $d$  are lower and upper bounds of the support of the fuzzy number  $A$ , while  $b$  and  $c$  are the modal values. The membership function in the form

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x < b \\ \frac{c-x}{c-b} & b \leq x < c \\ 0 & \text{otherwise} \end{cases}$$

Then centroid of trapezoidal fuzzy number  $A = (a, b, c, d)$  is defined by:

Fhqr lg#D ,#@<sup>f</sup>

on his judgment. If  $\alpha < 1$ , it means there is uncertainty. Finally,  $\alpha = 0$  indicates maximum uncertainty; thus decision has no confidence on his judgment. So, it can be expressed as, value of  $\alpha$ -cut is inversely proportion to uncertainty. When, decision maker has no confidence or there is maximum uncertainty, then he takes maximum possible range (elements of  $\alpha$ -cut A) between of optimistic and pessimistic margins to complete the project. Maximum and minimum range (elements of  $\alpha$ -cut A) insures minimum and maximum risk respectively. Since, that range is proportion to uncertainty and inversely proportion to  $\alpha$  and degree of risk (Figure 3.4). In conclusion, depending on decision maker's ability to take risk, a  $\alpha$ -cut level is chosen to complete the project within the range.

### **3.2.5 Arithmetic Operations on Fuzzy Numbers**

Fuzzy arithmetic is based on two properties of fuzzy numbers [73]: (1) each fuzzy set and thus also each fuzzy number, can fully and uniquely be represented by its  $\alpha$ -cuts and (2)  $\alpha$ -cuts of each fuzzy number are closed intervals of real numbers for all

### **3.3 Concept of Multi-Objective Optimization**

Multi-objective optimization (MOO) also known as multi-criteria or multi-attribute optimization is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints [75]. These techniques are often used to solve real world problems. Multi-optimization problems have several types of objectives which usually exhibit trade-off relationships. Here, some multi-objective techniques are described.

#### **3.3.1 Pareto Optimum**

An Italian economist, Vilfredo Pareto in nineteenth century developed a concept named Pareto optimum, in his studies of economic efficiency and income distribution. This concept is now broadly used in game theory, production engineering and social sciences.

Pareto optimum technique compares two solutions in multiobjective optimization that has no unified criterion with respect to optima. Such solutions (normally referred to as non-dominated or Pareto optimal solutions) do necessitate improvement in any objective function without sacrificing at least one of the other objective functions [36]. A solution ( $x^*$ ) is non-dominated by another solution ( $x$ ) if and only if it is at least as good as ( $x$ ) in all measures, and better in at least one measure. The region defined by Pareto optimal solutions is called the Pareto front, and the objective of multiobjective optimization is to establish the entire Pareto front for the problem instead of a single best solution [37].

#### **3.3.2 Adaptive Weight Approach (AWA)**

More recently, Gen and Cheng [36] adopted the adaptive weight approach (AWA) in construction TCO (also referred to as the GC approach hereafter). This approach utilizes some useful information from the current population to generate an adaptive weight for each objective, and thereby exerts a search pressure towards the ideal point.

TCO is in fact a bi-objective optimization problem and the twin objective is to concurrently minimize the time and total cost of a project. Two extreme points, namely the maximum extreme point ( $Z^+$ ) and the minimum extreme point ( $Z^-$ ),



cost and time derived from the last generation. However, the above formula cannot hold the test when the adaptive weight is invalid once  $f_w^{pd} = f_w^{pl}$  or  $f_w^{pd} = f_w^{pl}$  [37].

Figure 3.5: Concept of adaptive weight approach adapted from Gen and Cheng [36].

### 3.3.3 Modified Adaptive Weight Approach (MAWA)

Zheng et al. [37] modified adaptive weight approach (MAWA) of Gen and Cheng [36]. That MAWA approach better reflects the practical meaning of adaptive weights and combines the advantages of both the conventional aggregation approach and the GC approach. Under the MAWA, the adaptive weights are formulated through the following four conditions [37]:

1. For  $f_w^{pd} \neq f_w^{pl}$  and  $f_w^{pd} \neq f_w^{pl}$ ;

$$w_c = \bullet_c / \bullet \quad 3.10$$

$$w_t = \bullet_t / \bullet \quad 3.11$$

$$w_c + w_t = 1 \quad 3.12$$

$$2. \text{ For } ]_f^p d\{ = ]_f^p lq \text{ and } ]_w^p d\{ = ]_w^p lq; \quad 3.13$$

$$w_c = w_t = 0.5$$

$$3. \text{ For } ]_w^p d\{ = ]_w^p lq \text{ and } ]_f^p d\{ \neq ]_f^p lq \quad 3.14$$

$$w_c = 0.1, w_t = 0.9$$

$$4. \text{ For } ]_w^p d\{ \neq ]_w^p lq \text{ and } ]_f^p d\{ = ]_f^p lq \quad 3.15$$

$$w_c = 0.9, w_t = 0.1$$

Where  $\bullet_c$  = value for the criterion of cost;  $\bullet_t$  = value for the criterion of time;  $\bullet$  = value for the project;  $w_c$  = adaptive weight for the criterion of cost; and  $w_t$  = adaptive weight for the criterion of time.

By using MAWA, the deficiencies of the GC approach could be overcome as [37]:

1. The adaptive weights in the MAWA represent the relative importance of each criterion. For example, in the current generation, if the range of one criterion is not as wide as another criterion, it will receive a higher weight in order to guide the GA for widening the search scope on this criterion. As their sum is always equal to 1, they can be regarded as weights in theory.
2. The problem is clearly described in the four conditions, while a proper approach to each possible condition is clearly indicated.
3. By introducing the absolute value ( $\bullet_c, \bullet_t$ ) to represent the ratio between the minimum value and deviation between two extreme points, the problem of units is solved. As a result, the adaptive weights are generated independently by comparison with the total absolute value.

Zheng et al. [37] proposes a fitness formula in accordance with the proposed adaptive weight:

$$i\#(q)\#G\# w\# ]_w^p d\{ \# w$$

### 3.4 Genetic Algorithms

Genetic algorithms are search and optimization tools that enable the fittest candidate among strings to survive and reproduce based on random information search and exchange imitating the natural biological selection [76]. Inspired by Darwin's theory of evolution, Genetic Algorithms (GAs) are search algorithms based on the mechanics of natural selection and natural genetics.

In the middle of the twentieth century some computer scientists worked on evolutionary systems with the notion that this will yield to an optimization mechanism for an array of engineering queries [77]. GAs were invented and developed by John Holland, his students and his colleagues at the University of Michigan. Holland's book *Adaptation in Natural and Artificial Systems* [78] set forth the lexicon from which all further dialogue concerning GAs would be developed. In essence, his theoretical framework provided the point of reference for all work on genetic algorithms up until recently whereupon it has taken on a new direction, given new technology [76].

GA is a search method that can be used for both solving problems and modeling evolutionary systems. The production of new strings in the following generations depend on the initial "parent" strings, the offspring "child strings" are created using parts and portions of the parent strings; the fittest candidate, preserving the best biological features and thus improving the search process. The whole search process is not completely a random process; genetic algorithms utilize the chronological information about old strings in order to be able to produce new and enhanced ones [79].

GAs procedure begins by generating an initial collection (referred to as population) of random solution that are encoded in the form of strings called chromosomes or the genotype or the genome. The fitness of each individual chromosome is determined by evaluating its performance with respect to an objective function. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are selected (natural survival of the fittest process) and best chromosomes exchange information to produce offspring genes that are evaluated and can replace less fit member in the population. This process is continuing until the terminating condition is met. Thus, the optimum solution is found at the end of the GA process.



Advantage of GA:

Robust search algorithm.

Require no knowledge or gradient information about the response surface.

Discontinuities present on the response surface have little effect on overall optimization performance.

Has resistance to becoming trapped in local optima.

Applicable to large problems.

Applicable for a wide variety of optimization problems.

Disadvantage:

Random search is time consuming

Cannot tell when or if an optimum solution is solution is obtained.

Configuration is not straightforward.

### **3.4.1 Biological Background**

#### ***3.4.1.1 Chromosomes***

All living organism consist of cells. In each cell there is the same set of chromosomes. All the genetic information gets stored in the chromosomes. Each chromosome is build of Dioxy Ribo Nucleic Acid (DNA) and serves as a model for the whole organism. A chromosome consists of genes, blocks of DNA. Each gene encodes a particular protein. Basically, each gene encodes a trait, for example eye color. Possible setting for a trait is called alleles. The set of all the genes of a specific species is called genome. Each and every gene has a unique position on the genome called locus. A particular set of genes in genome is called genotype.

#### ***3.4.1.2 Reproduction***

Reproduction is a process where genetic information is shared between the parents in order to create new offspring. It is expected that all good qualities of parents will be passed into child. In reproduction, individual strings are copied according to their fitness value means that strings with a higher value have a higher probability of contributing one or more offspring in the next generation. The

reproduction operator may be implemented in algorithmic form in a number of ways. In biology, the most common form of reproduction is crossover, two chromosomes are cut at one point and the halves are spliced to create new chromosomes. The newly created offspring (chromosome) are modified using mutation operator. Mutation consists in changing the value of genes in chromosome. A few random changes by mutation operator can be a good way of exploring the search space quickly.

### 3.4.1.3 Fitness

Fitness describes the ability to both survive and reproduce, and is equal to the average contribution to the gene pool of the next generation that is made by an average individual of the specified genotype (string).

### 3.4.1.4 Evolution

Evolution is the process of change in all forms of life over generations. In evolution, each species faces the problems of searching for a beneficial adaptation to adapt to the rapidly changing environment around them.

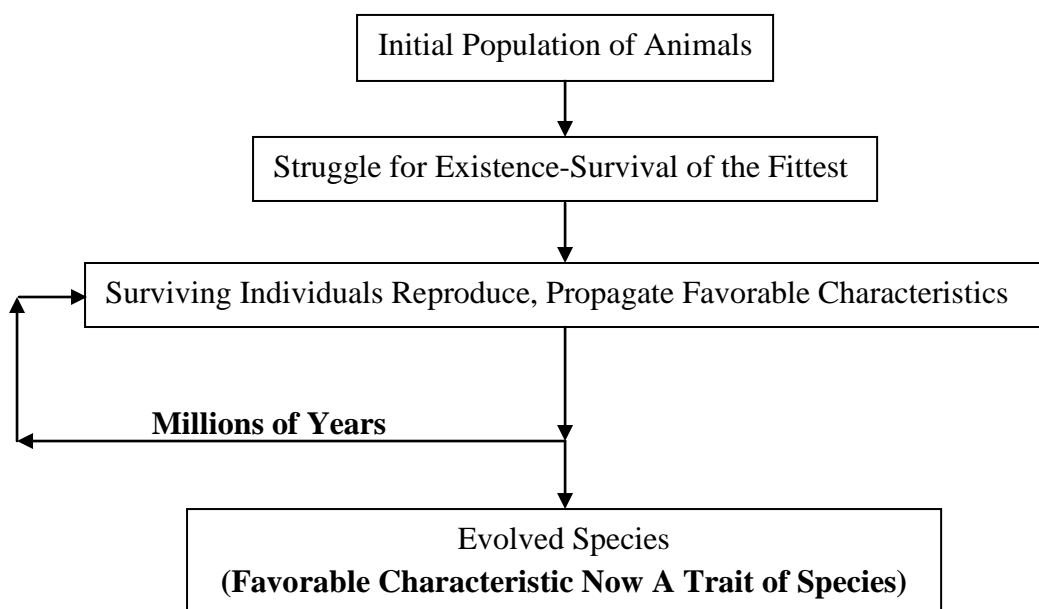


Figure 3.6: Evolution through natural selection.

### 3.4.2 Basic Principal

Genetic algorithm starts their process with a set of possible solutions that is called initial population. Solutions from one population are taken and used to form a new population. This is motivated by a hope that the new population will be better than the old one. New solutions (offspring) are selected according to their fitness-the more suitable they are the more chances they have to reproduce by mating (crossover). Repeat the cycle until some conditions are satisfied.

The outline of the genetic algorithm

1. Choose the population size  $n$ , length of chromosome, crossover rate, mutation rate and stopping condition for problem.
2. Randomly generate population of  $n$  chromosomes and evaluate the fitness  $f(x)$  of each chromosome  $x$  in the population.
3. Select the best chromosomes according to their fitness to create new population of size  $n$  using selection operator.
4. Randomly select parent chromosomes with a crossover rate, crossover the parents randomly at randomly chosen points. If no crossover is performed, offspring are the exact copy of parents. Replace the old generation with the new crossed population of size  $n$ .
5. Apply the mutation operator with a mutation probability; randomly mutate bits (locus) in chromosomes to create final new population of size  $n$ .
6. Test the new population. If the end condition is satisfied, stop, and return the best solution in current population. Otherwise go to step 3.

### 3.4.3 Operators of GA

#### 3.4.3.1 *Encoding of Chromosome*

Encoding is a process by which the possible solution is represented in the form of string. Length of chromosome  $L$  represents the number of bits in the string. Each bit in the string represents a characteristic of a solution.

##### i. Binary Encoding

Most common method of encoding is binary. The chromosome could look like this:

Chromosome 1	1	1	0	1	0	0	1	1	0
Chromosome 2	1	0	1	1	0	1	0	1	1

Information of solution is coded in the string by binary digit 0 and 1. So, chromosomes are strings of 1s and 0s.

#### ii. Permutation Encoding

Permutation encoding can be used in ordering problems, such as traveling sales man problem (TSP) or jobs scheduling in machines etc. Every chromosome is a string of number which represents number in a sequence. In the jobs scheduling in machine problem each number would represent a machine to be worked.

Chromosome 1	1	3	5	2	6	7	4	8	9
Chromosome 2	9	4	1	3	2	6	7	5	8

#### iii. Value Encoding

Value Encoding is used in problems where complicated values, such as real numbers, are used and where binary encoding would not suffice. It is good for some problems, but it is often necessary to develop some specific crossover and mutation techniques for these chromosomes. When the string is stored as arrays of real numbers, then the coding is called real number encoding.

Chromosome 1	1.35	3.25	5.36	2.36	6.23	7.69	4.36	8.39	9.25
Chromosome 2	E	S	N	W	S	N	W	E	S

In chromosome 1 above, 1.35 could represent a particular distance of task 1, 3.25 represent distances of task 2, etc. For the chromosome 2, E, S, N, etc are the direction of the task.

#### iv. Tree Encoding

Tree encoding is used to actually have programs or expressions evolve. In tree encoding every chromosome is a tree of some objects, such as functions or commands in the programming language. LISP is often used for this because

programs in LISP can be represented in this form and then be easily parsed as a tree.

### 3.4.3.2 Selection

Selection is operator of GA by which the best solution or a few best solutions (chromosomes) are chosen for new population from the current population.

There are many methods for selecting the best chromosomes.

#### i. Roulette Wheel

Simple reproduction allocates offspring strings using a roulette wheel with slots sized according to their fitness (shown in figure 3.7).

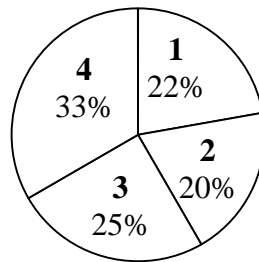


Figure 3.7: Roulette Wheel.

The chromosomes with bigger slots are, the more chances to be selected they have. This is a way of choosing members from the population of chromosomes in a way that is proportional to their fitness. Better the chromosomes will be selected more times; however it is not guaranteed that the fittest member goes to the next generation.

#### ii. Rank Selection

The roulette wheel method has some limitation when the fitness of chromosomes differs greatly. For example, if the best chromosome's fitness is 90% of the entire roulette wheel then the other chromosomes with very slim slots will have very small chances to be selected. To overcome the problem, rank selection method ranks the chromosomes according to their fitness and then every chromosome receives order number from this ranking. The worst will have fitness 1, second

worst will have 2, etc and the best will have fitness  $n$  (number of chromosome in population).

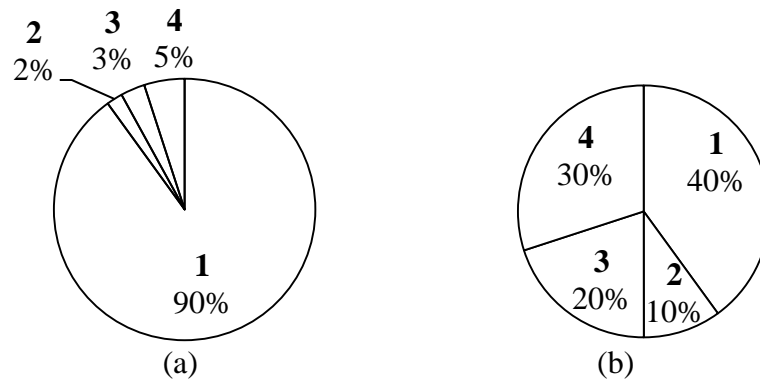


Figure 3.8: (a) situation before rank selection and (b) situation after rank selection.

### iii. Tournament

In general tournament selection  $n$  individuals are selected at random and fittest is selected. The most common type of tournament selection is binary tournament selection, where just two individuals are selected.

### iv. Elitism

The best chromosome (or a few best chromosomes) is copied to the population in the next generation. The rest are chosen in classical way. Elitism can very rapidly increase performance of GA, because it prevents losing the best found solution to date. A variation is to eliminate an equal number of the worst solutions, i.e. for each “best chromosome” carried over a “worst chromosome” is deleted.

### v. Steady-State selection

Steady-State selection is not particular method of selecting parents. Main idea of this selection is that big part of chromosomes should survive to next generation. In every generation, a few (good - with high fitness) chromosomes are selected for creating a new offspring strings. Then some (bad - with low fitness) chromosomes are removed and the new offspring strings are placed in their place. The rest of population survives to new generation.

### 3.4.3.3 Crossover

Crossover is a process in which two chromosomes exchange their genetic materials to produce new offsprings which possess both their characteristics. It is used to vary the programming of a chromosome or chromosomes from one generation to next. The concept of reproduction and crossover in GA are based on the biological evolution concept. Crossover between two good solutions (chromosomes) may not always yield a better or as good a solution. Since parents are good, probability of the child being good is high. There are many crossover techniques for organisms which use different data structures to store themselves.

#### i. Single point crossover

A random point is chosen on the individual mating chromosomes (strings) and the genetic material is exchanged at this point. For example, two mating chromosomes (parents) below are selected randomly for crossover. The crossover point is chosen 4 randomly.

Parent 1	1	3	5	2	6	7	4	8	9
Parent 2	9	4	1	3	2	6	7	5	8

Offspring 1	1	3	5	2	2	6	7	5	8
Offspring 2	9	4	1	3	6	7	4	8	9

After the crossover, it is found that, offspring 1 copied (a) every thing till this crossover point 4 from 1<sup>st</sup> parent and (b) every thing from 2<sup>nd</sup> parent after this crossover point. For offspring 2, the process is reversed.

#### ii. Two-point Crossover

Two-point crossover is similar to single point crossover except this time two crossover points are randomly chosen. Here the crossover points are 3 and 8.

Parent 1	1	3	5	2	6	7	4	8	9
Parent 2	9	4	1	3	2	6	7	5	8

Offspring 1	1	3	5	3	2	6	7	8	9
Offspring 2	9	4	1	2	6	7	4	5	8

iii. Uniform Crossover

In uniform crossover each gene (bit) is selected randomly from one of the corresponding genes of the parent chromosomes.

Parent 1	<b>1</b>	<b>3</b>	<b>5</b>	<b>2</b>	<b>6</b>	<b>7</b>	<b>4</b>	<b>8</b>	<b>9</b>
Parent 2	9	4	1	3	2	6	7	5	8

Offspring 1	9	<b>3</b>	1	<b>2</b>	2	<b>7</b>	<b>4</b>	<b>8</b>	8
Offspring 2	<b>1</b>	4	<b>5</b>	3	<b>6</b>	6	7	5	<b>9</b>

**3.4.3.4 Mutation**

Mutation is a process by which the genes (bits) are changed with a certain probability to form a new set of chromosome. It is done to maintain diversity in the population set. Mutation decreases the probability of solution being trapped in local optimum. Therefore the chromosomes near the local optimum will be chosen to crossover because they will have the better fitness and there will be very little chance of finding the global optimum. So mutation is a completely random way of getting to possible solutions that would otherwise not be found.

Mutation is performed after crossover by randomly choosing bits in the new generation to mutate. A mutation rate is used to change the value. In case of binary encoding randomly selected bits are inverted. In case of other types of encoding different mutation processes are used. For example, some mutation processes are shown below.

i. Mutation of a binary encoding

Offspring	1	<b>1</b>	0	1	<b>0</b>	1	0	0	<b>1</b>
Mutated Offspring	1	<b>0</b>	0	1	<b>1</b>	1	0	0	<b>0</b>

ii. Mutation of permutation encoding

Two numbers are selected randomly and exchanged.

Offspring	<b>1</b>	3	5	2	6	<b>7</b>	4	8	9
Mutated Offspring	<b>7</b>	3	5	2	6	<b>1</b>	4	8	9



### iii. Mutation of real value encoding

Values of randomly selected bits are added (or subtracted) small values.

Offspring	1.35	3.25	<b>5.36</b>	2.36	6.23	7.69	4.36	<b>8.39</b>	9.25
Mutated Offspring	1.35	3.25	<b>5.67</b>	2.36	6.23	7.69	4.36	<b>8.03</b>	9.25

For mutation of value encoding, values of randomly selected bits are mutated within their range. Here every bit has value between 1 and 9.

Offspring	1	3	<b>5</b>	3	2	6	7	8	<b>9</b>
Mutated Offspring	1	3	<b>6</b>	3	2	6	7	8	<b>2</b>

### 3.4.4 Fitness Function

Fitness function is a type of objective function that qualifies the optimality of a solution (that is, a chromosome) in a genetic algorithm so that particular chromosome may be ranked against all the other chromosomes. It generates fitness value that is assigned to each solution depending on how close it actually is to solving the problem. Optimal chromosomes, or at least chromosomes which are more optimal, are allowed to breed and mix their datasets by any of several techniques, producing a new generation that will (hopefully) be even better. An ideal fitness function correlates closely with the algorithm's goal, and yet may be computed quickly. Another way of looking at fitness functions is in the terms of a fitness landscape, which shows the fitness for each possible chromosome.

Definition of the fitness function is not straightforward in many cases and often is performed iteratively if the fittest solutions produced by GA are not what are desired. In some cases, it is very hard or impossible to come up even with a guess of what fitness function definition might be. Interactive genetic algorithms address this difficulty by outsourcing evaluation to external agents (normally humans).

### 3.4.5 Termination

This generational process is repeated until a termination (stopping) condition has been reached. Common terminating conditions are:

A solution is found that satisfies minimum criteria

Fixed number of generations reached

Allocated budget (computation time/money) reached

The highest ranking solution's fitness is reaching or has reached a plateau such that successive iterations no longer produce better results

Manual inspection

Combinations of the above.

### 3.4.6 Genetic Algorithm Process

The complete GA process is shown in figure 3.9.

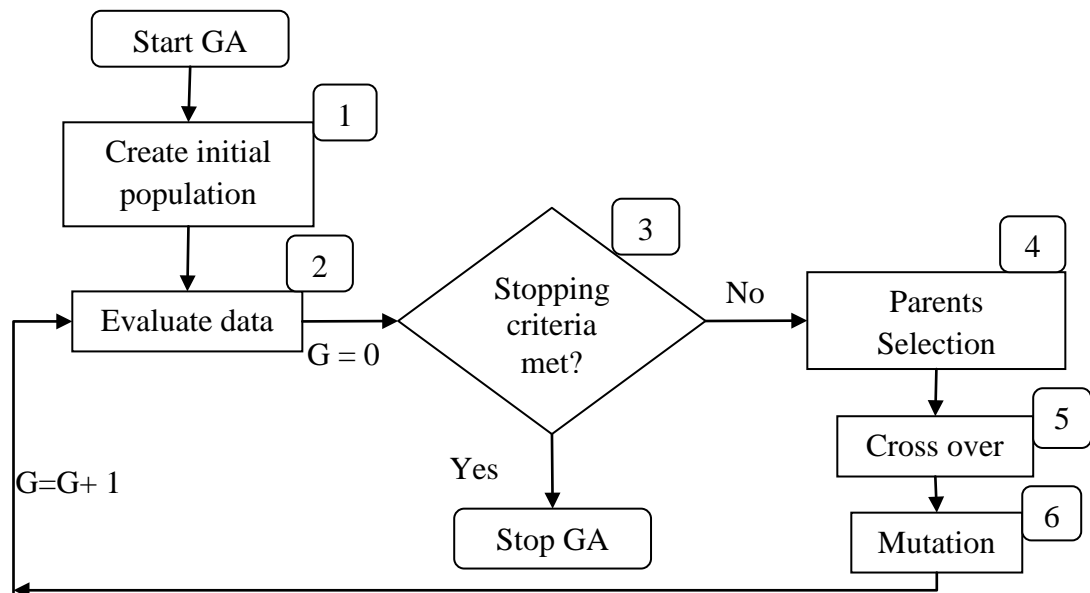


Figure 3.9: Simple GA process.

1 Create  $n$  different strings by randomly changing the bits. A bit can be presented as duration of duration of an activity.

2 Evaluate the fitness value for each chromosome. Measure  $p_i$  and  $q_i$ ,  $i =$  number of chromosome string.

3 Check whether stopping criteria is met or not. If “yes” go to STOP GA. Stopping criteria may be: total number of generations. Otherwise go to step 4.

4 Select parents using selection mechanism. These parents will undergo genetic operations.

5 Crossover is done randomly with a rate  $P_c$ .

6 Mutation is done at rate  $P_m$ . Go to step 2.

# CHAPTER 4

## Problem Formulation

### 4.1 Introduction

The management of a large project requires analytical tools for scheduling activities and allocating resources. A schedule consists of a list of a project's terminal elements with intended start and finish dates. Terminal element is the lowest element in a schedule, which is not further subdivided. Those items are often estimated in terms of resource requirements, budget and duration, linked by dependencies and scheduled.

The information of project activities may also be presented by a network diagram in which nodes or arrows signify activities. The relationships between activities are represented by arrows between the nodes. All arrows to a node begin at the node's immediate predecessors, indicating that the activity cannot be started until all activities prior to that node in the network are completed.

In time-cost trade optimization problem, a relationship between project duration and total project cost with respect to resources is very important. Generally, project total cost is a summation of indirect cost and direct cost. Typically direct cost decreases and indirect cost increases with duration of activity. Managers try to finish their project within targeted completion time with least costs. Here, penalty is added for delay beyond desired project completion time. But, in a competitive environment reducing both project cost and time is critical. Two types of time-cost trade-off are observed in scheduling problems such as i) single objective and ii) multi-objective time-cost tradeoff. More over, resources allocation is important factor in scheduling problem. Deterministic and uncertain scheduling are also two categories. Uncertain scheduling is more realistic. The following mathematical model considers resources unconstrained scheduling involving uncertainties in project for TCT.

## 4.2 Mathematical Model

The objective of this model is to find the optimum combination of activity durations and project cost at different risk levels so that the project hopefully completely within the desired duration selected from feasible project time spectrum. If the project duration is more than the desired duration within feasible time spectrum, then delay fine will be added with total project cost. Moreover, premium costs for extra resources (required resources beyond the available limit which are hired or bought to complete project early) are also added in direct cost of each activity. The durations of each activity and their corresponding resources are used to calculate direct costs in present project cost function.

$$F = \sum_{l=1}^L \sum_{r=1}^u F_{g_{lr}} + F_{lg} + F_{lm} \quad (14)$$

$$F_{g_{lr}} = U_{lm}^u$$

$t_i$  = Time of activity i at a specific  $\alpha$ -cut level,

$EST_i$  = Earliest starting time of activity i at a specific  $\alpha$ -cut level,

$C_{dij}$  = Direct cost of resource type j of activity i,

$R_{ij}^r$  = Required amount of resource type j of activity i at a duration  $t_i$ ,

$R_{ij}^n$  = Available amount of resource type j of activity i at a duration  $t_i$ ,

$M_{ij}$  = Cost rate of available resource type j of activity i,

$V_{ij}$  = Cost rate of resource type j of activity i beyond available limit,

$P_{ij}$  = Premium rate of resource type j of activity i for extra resources,

$C_{id}$  = Total indirect cost of project at a duration T,

$C_L$  = Indirect cost rate of project,

$C_p$  = premium cost (delay fine) rate of above desired project completion time.

$J_w^{plq}$  = Minimum limit of possible project duration at a specific  $\alpha$ -cut level,

$J_w^{pdl}$  = Maximum limit of possible project duration at a specific  $\alpha$ -cut level,

$W_{plq}$

# CHAPTER 5

## Solution Approach

### 5.1 Problem Solving Process

To solve this problem, it is necessary to define network of the project, fuzzy crash and normal durations of each activity, fuzzy resources of each activity, •-cut levels, cost rate of each resource, desired project completion time, penalty for delay etc as basic input information. Then using GA, Pareto optimum solutions have to be generated.

Figure 5.1 shows the typical activity time-resources relation which was originally developed by Chua et al. [34]. At crash mode, maximum amount of resources (normal + additional) expended to complete activity in shorter time.

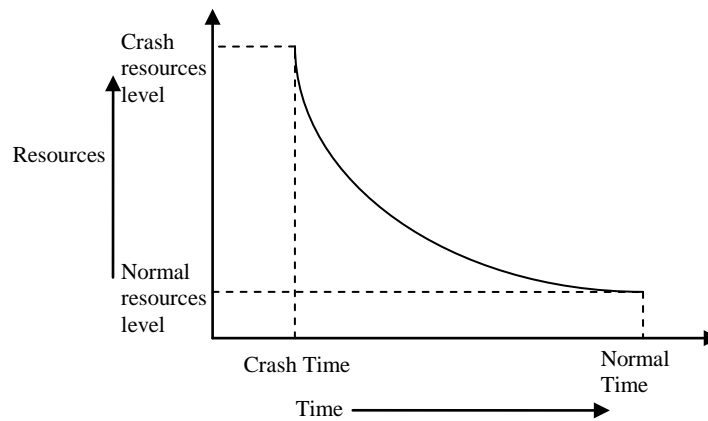


Figure 5.1: Typical deterministic relationship between resources and time of an activity.

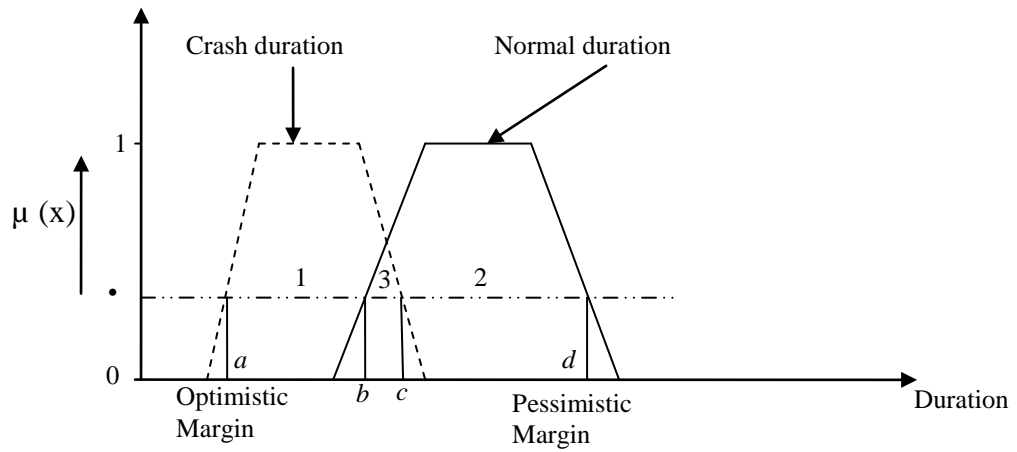
But, in real life, the activity resources-time relationship under uncertainty is no longer a monotonously decreasing curve. Because of uncertainties such as environmental variables, space congestion, work errors, accidents, productivity level, etc dynamically affect on activity duration and corresponding resources during project implementation.

Fuzzy relationship between the time and resources of an activity is shown in figure 5.2. For simplicity, only the case with one fuzzy crash duration membership function and fuzzy normal duration membership function of an activity is explained and expressed in figure 5.2. The activity duration is characterized by a fuzzy trapezoidal number due to uncertainty in process. An acceptable risk level ( $\alpha$ -cut level) is defined as the minimum condition that can be accepted in the proposed algorithm. Here, the activity duration divided into three regions- crash time (region 1), normal time (region 2) and overlapping time (region 3) as in figure 5.2 (a). Within the region 1, the activity needs to be performed in a crash mode, i.e., it needs maximum effort to reduce the time. Within the region 2, the activity can be finished in normal mode. The cost in a crash mode is generally higher than that in a normal mode since more resources are needed to invest in the activity so as to finish as early as possible. When the activity time falls within the region 3, it can be performed in either normal mode or crash mode. But it is reasonable to finish the activity in normal mode as it will be cost effective.

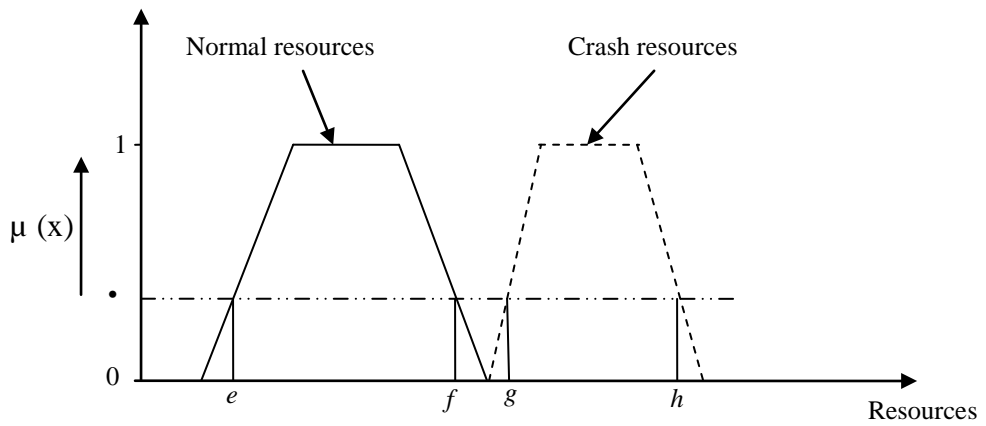
At a specific  $\alpha$ -cut level, point  $h$ ,  $g$ ,  $f$  and  $e$  in figure 5.2 (b) represent corresponding resources of points  $c$ ,  $a$ ,  $d$  and  $b$  in figure 5.1 (a) respectively. The amount of resources, resources cost rate and indirect cost per unit time of each activity which are the inputs of the next steps.

After developing the objective function, the total project duration is determined by the critical path method and the total project cost of the project can be calculated by cost equation at a specific  $\alpha$ -cut level. The multi-objective optimum solution would be the solution which has the maximum fitness value obtained from the fitness function.

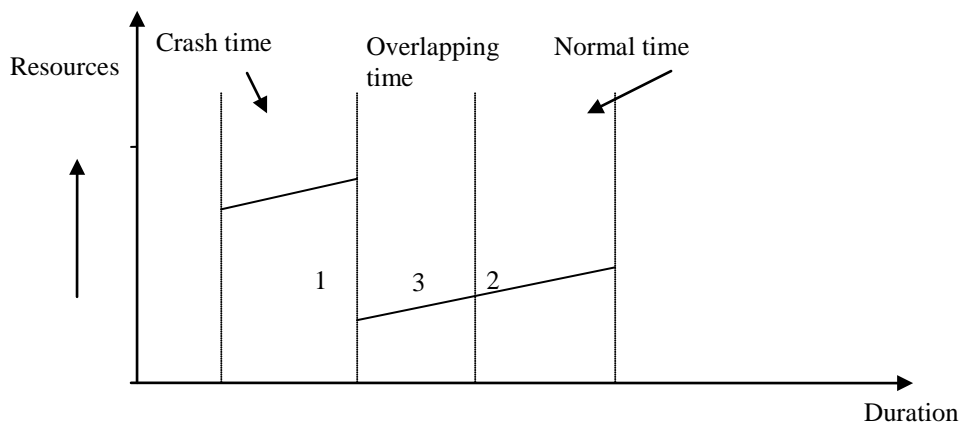
Solving with the GA requires several parameters such as encoding type, population size, selection method, reproduction operations, termination condition, etc. In GA process fittest solutions in each generation are stored in Fittest Table and further their fitness are calculated again to get best fittest solution and least cost solution. This additional step makes the algorithm more efficient and feasible. Moreover, all fittest solutions in each generation are plotted in graph to locate the Pareto front. Thus, using GA as a solving procedure a result is obtained.



(a) Crash and Normal activity fuzzy time



(b) Crash and Normal activity fuzzy resources



(c) Fuzzy relationship between time and resources

Figure 5.2: Concept of fuzzy activity duration and resources.



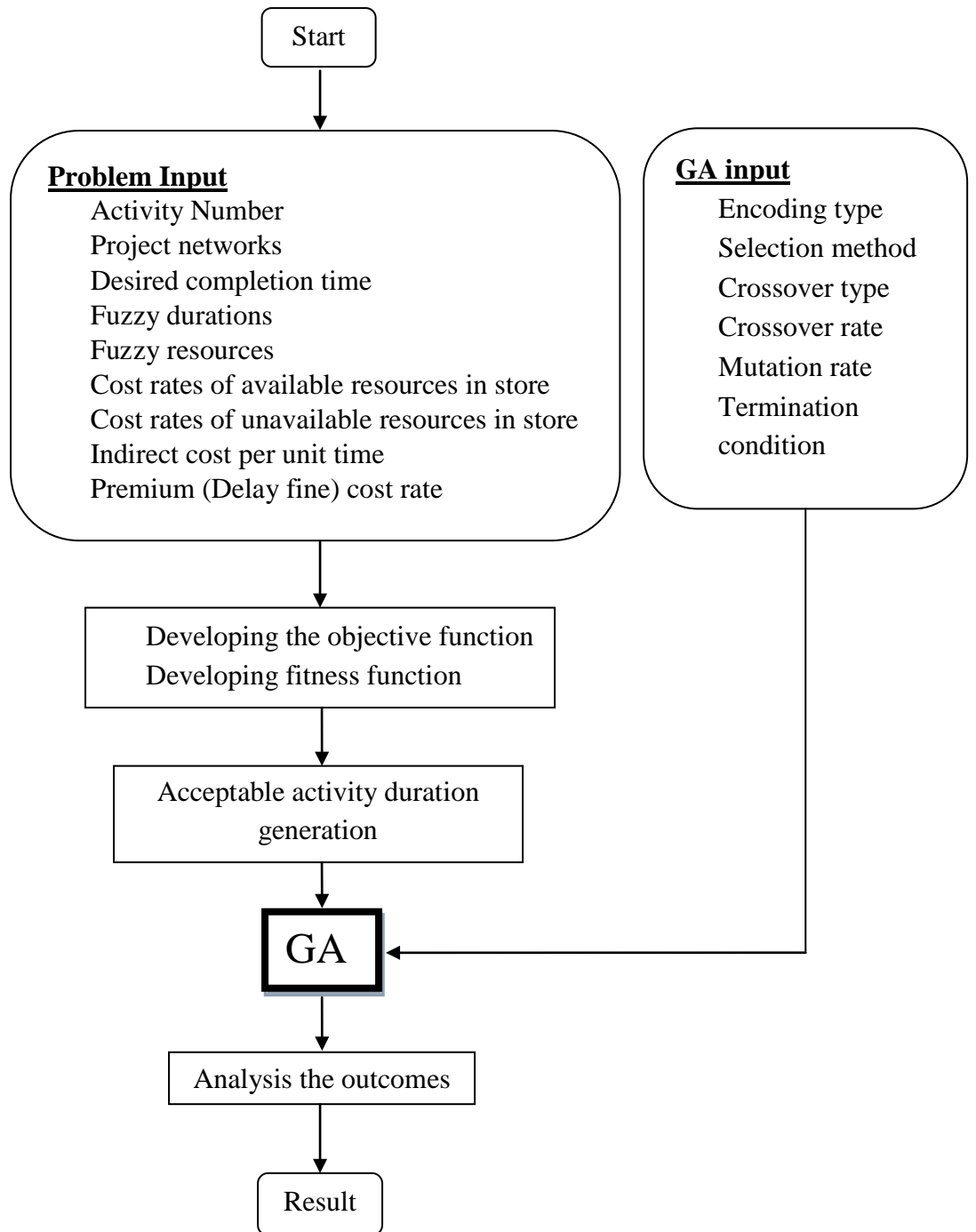


Figure 5.3: Problem solving approach.

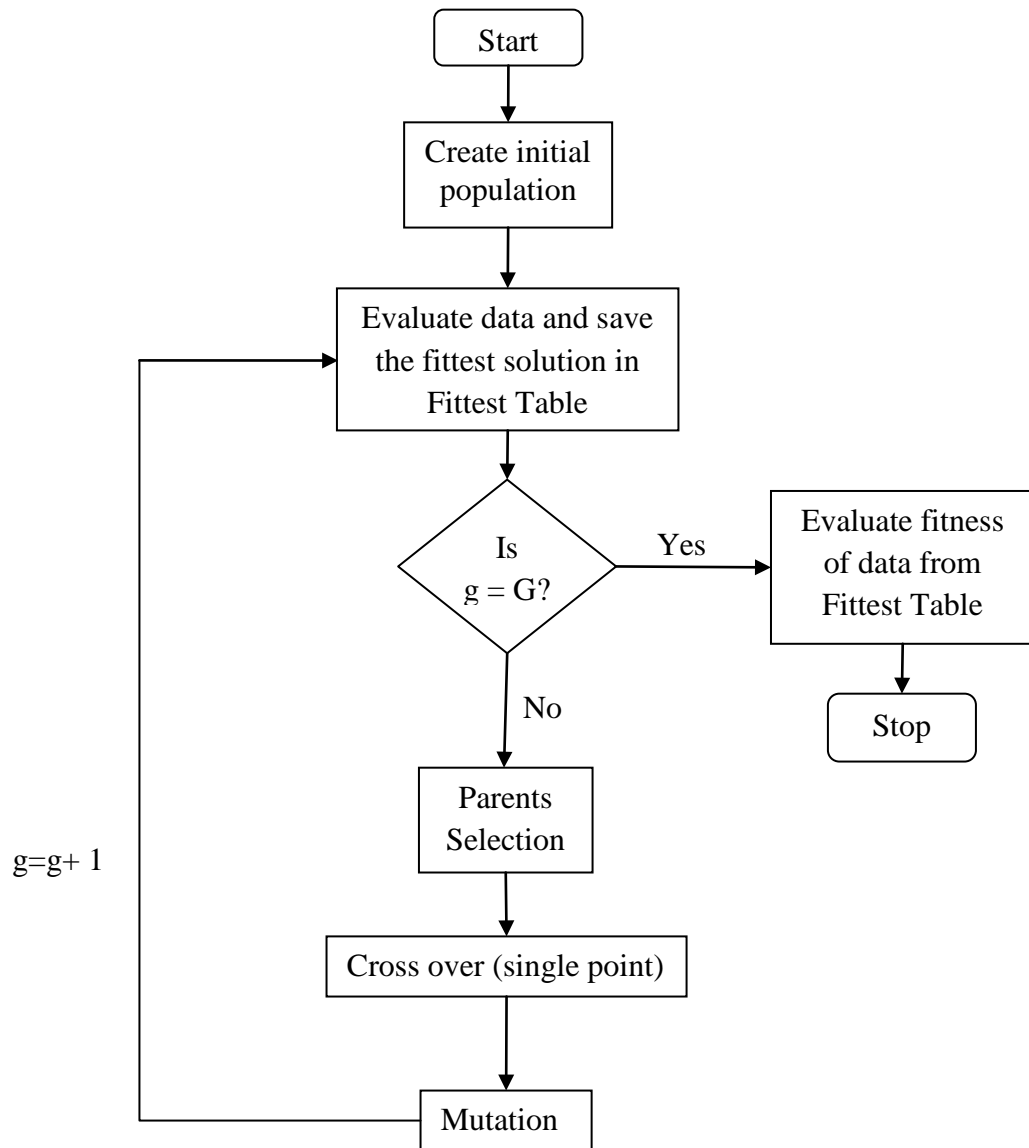


Figure 5.4: Problem solving process with GA.

## 5.2 Solving with Genetic Algorithm

### 5.2.1 Algorithm Structure

Step 1: Take a value of  $\alpha$ -cut level as a computer input which range from 0 to 1 with 0.1 increments.

Step 2: Specify an expected project completion time (Select from project completion time spectrum corresponding to a specific  $\alpha$ -cut level). Above that time the penalty is added for delay time.

Step 3: Create  $n$  no. of chromosomes (project schedule) for an initial population pool. The initial population number is taken as computer input for program. Check the chromosome strings whether they are valid or not.

Step 4: Evaluate the fitness function.

Step 5: Calculate the probability  $p_i$  and cumulative probability  $q_i$  for  $i$  chromosomes (e.g.,  $i = 100$ ).

Step 6: Select the chromosomes with some selection mechanism. Roulette wheel mechanism is used.

Step 7: Select the chromosomes as parents which will undergo breeding to create next generation.

Step 8: Take crossover rate  $P_c$  and mutation rate  $P_m$  as computer program input. Crossover rate indicates how many chromosomes will undergo the crossover operation. Mutation rate indicates how many bits will undergo through mutation.

Step 9: Select randomly the chromosomes for crossover. Apply single point crossover. No. of crossed chromosomes =  $P_c \times$  population size,  $n$ .

Cross point,  $K = [1, l - 1]$ , where,  $l =$  number of activities. If project has 7 activities then there are 1 to 6 points where crossover can take place.

Step 10: Select randomly the bits in chromosomes which undergo the mutation process.

No. of bits for mutation =  $P_m \times$  population size,  $n \times$  no. of activity,  $l$ .

The selected bits will be swapped with the randomly selected activity duration within its range.

Step 11: A generation is completed and a new set of population (offspring) has been created.

Step 12: Evaluate the fitness function value for the new population and save the fittest value in each generation in Fittest Table.

Step 13: Go to step 4 until the stopping condition is met.

Stopping condition may be: predefined number of generation.

Step 14: Evaluate the fitness value of data in Fittest Table. Save the best fittest solution and least cost solution. Finally, Plot the cost and time of fittest solution of each generation in T-C graph to locate the Pareto front.

### **5.2.2 Encoding Technique**

Direct value encoding approach is followed to encoding the project scheduling with multi-objective time-cost optimization problem. In the chromosome, value in each box represents duration of activity. Values are taken from the durations of corresponding activities, which are restricted within the pessimistic and optimistic values, based upon the  $\alpha$ -cut level. For seven activity problem, first box represents the duration of first activity and the other boxes represent the other activities by order.

### **5.2.3 Initial Population**

Either heuristic procedures or random criteria can be used to generate feasible strings that form the initial population. In this research, a random generation is allowed to create the initial population pool by taking durations from the time spectrum at a specific  $\alpha$ -cut level in a chromosome string. The size of the initial population depends on the problem's size. A number is taken as input to generate initial population randomly. It is necessary to check the feasibility of the chromosomes create randomly.

### **5.2.4 Selection Procedure**

There are several common techniques for the chromosomes from the initial population pool. Most common techniques are tournament selection, roulette wheel selection, rank selection, etc. In this algorithm, roulette wheel selection method is used. This strategy is rather elitism and makes it hard for low fitness chromosomes to survive when there are big fitness differences among the chromosomes.

### **5.2.5 Crossover Operation**

In crossover operation the pairs of chromosomes are crossed in one or more points. In case of bit representation it is easy. In case of value encoding if the point is selected randomly then there is chance of creating a repetition of

operations in the new generation. Partially matched crossover technique solves the problem but the process is complicated.

Crossover rate  $P_c$  is used to control the number of chromosomes for crossover. Crossover rate is the percentage of the total chromosomes that should undergo crossover. A random number is generated between the 1 and  $l-1$  to locate the crossover points for each pairs.

### 5.2.6 Mutation Operation

In the proposed algorithm, a bit is selected randomly and then replaced with the randomly selected value within its accepted region at the specific  $\bullet$ -cut level. A mutation rate  $P_m$  is used to control the percentage of bits on which mutation is applied.

### 5.2.7 Fitness Function

In GA the best fit solutions survive over generations. Fitness of a solution hence should reflect the quality of the schedules generated in different generations applying the GA operators to regarding the objectives. In this problem, two objectives are minimization of project cost and duration for TCO. So, multi-objective function is considered to a fitness function for GA (section 3.3.3). The fitness function is defined as follows,

$$f_i(q) = \sum_{j=1}^n w_j^p d_j^q$$

# CHAPTER 6

## Result Analysis

### 6.1 Introduction

The proposed multi-objective time-cost optimization problem has been solved with genetic algorithm technique which is coded in MATLAB 7.7.0.471 (R2008b) and run on personal computer having Intel (R) Pentium (R) Dual CPU 2.80 GHz and 512 MB of RAM. One folder is created where input are given into text files. For each activity, one text file is used for time schedules and their corresponding resources and another text file is used for resource cost rates at available and crash condition and resources limit at available condition. All GA parameters are inputted through dialogs.

### 6.2 Case Study

A hypothetical test and a real case are presented illustrating project scheduling with multi-objective time-cost optimization problem. To determine the effectiveness of the proposed model, a test case is also adopted from Zheng and Ng [66]. As shown in Figure 6.1, the hypothetical project case consists of a seven activities CPM network (AON diagram). The duration and resources of each activity are assumed to be trapezoidal fuzzy number. The durations, resources, resource cost rates for both available and unavailable resources of each activity are shown in Table 1. In Table 2 and Figure 2, the optimistic and pessimistic project duration margins at different alpha-cut levels are defined. Only integer activity duration and resources are considered. Each activity is performed at two different modes, crash and normal mode. Moreover, if the required number of resource exceeds the available limit of resource; extra cost is needed to get extra resources. In addition, delay fine per day will be considered for extensions beyond the desired project duration.

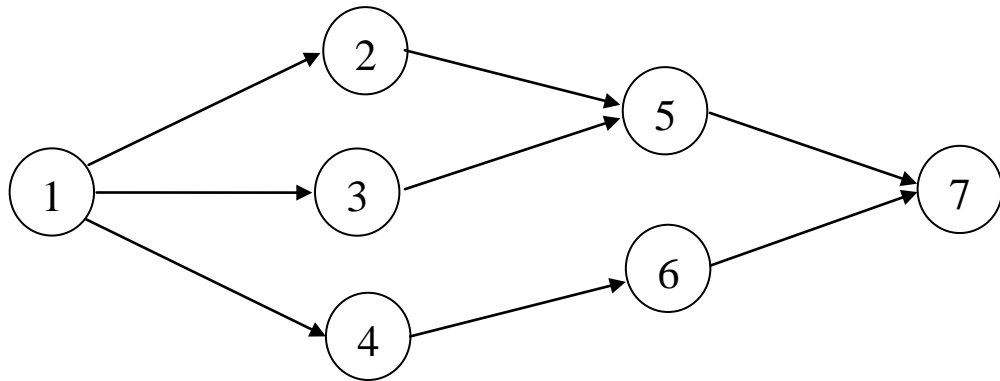


Figure 6.1: CPM network (AON diagram) of 7-activity hypothetical test problem.

Table 6.1: Detail data of a hypothetical test case.

Activity no. (i)	Mode of activity	Activity duration and corresponding required resources				Cost rates in taka/unit for available and unavailable resources and available limit of resources in store
		$t_i$	$R_{11}^r$	$R_{12}^r$	$R_{13}^r$	
1	Crash	14	462	470	434	$M_{11}=17/\text{unit}, M_{12}=13/\text{unit},$ $M_{13}=18/\text{unit}, V_{11}=30/\text{unit},$ $V_{12}=30/\text{unit}, V_{13}=32/\text{unit},$ $R_{11}^n=602, R_{12}^n=511,$ $R_{13}^n=556.$
		16	528	486	496	
		17	561	498	527	
		19	602	511	556	
	Normal	18	252	378	270	
		20	280	420	300	
		22	308	463	330	
		24	336	504	360	
2	Crash	15	1485	360	$M_{21}=1/\text{unit}, M_{22}=4/\text{unit},$ $V_{21}=2/\text{unit}, V_{22}=8/\text{unit},$ $R_{21}^n=2016, R_{22}^n=512.$	
		17	1683	408		
		19	1881	456		
		21	2016	512		
	Normal	20	860	180		
		22	946	198		
		23	989	207		
		25	1075	225		
3	Crash	15	1125	225	$M_{31}=2/\text{unit}, M_{32}=10/\text{unit},$ $V_{31}=4/\text{unit}, V_{32}=16/\text{unit},$ $R_{31}^n=1802, R_{32}^n=384.$	
		18	1332	288		
		21	1554	336		
		24	1802	384		
	Normal	25	725	150		
		27	783	162		
		30	870	180		
		33	957	198		

4		$t_4$	$R^r_{41}$	$R^r_{42}$	$R^r_{43}$	$R^r_{44}$	$M_{41}=19/\text{unit}, M_{42}=62/\text{unit},$ $M_{43}=34/\text{unit}, M_{44}=25/\text{unit},$ $V_{41}=30/\text{unit}, V_{42}=100/\text{unit},$ $V_{43}=55/\text{unit}, V_{44}=41/\text{unit},$ $R^n_{41}=712, R^n_{42}=251,$ $R^n_{43}=395, R^n_{44}=569.$
	crash	12	588	204	312	456	
		13	637	221	338	494	
		14	686	238	364	532	
		15	712	251	395	569	
	Normal	15	345	120	210	270	
		17	391	136	238	306	
18		414	144	252	324		
		20	460	161	284	360	
5	22-30	$t_5$	$R^r_{51}$	$R^r_{52}$	$R^r_{53}$	$M_{51}=10/\text{unit}, M_{52}=12/\text{unit},$ $M_{53}=23/\text{unit}, V_{51}=17/\text{unit},$ $V_{52}=20/\text{unit}, V_{53}=39/\text{unit},$ $R^n_{51}=812, R^n_{52}=690,$ $R^n_{53}=346.$	
	Crash	22	682	572	286		
		24	744	624	312		
		25	788	658	326		
		26	812	690	346		
	Normal	25	350	325	150		
		27	378	351	162		
		28	392	364	168		
30		419	388	180			
6		$t_6$	$R^r_{61}$	$R^r_{62}$	$R^r_{63}$	$M_{61}=7/\text{unit}, M_{62}=12.5/\text{unit},$ $M_{63}=36/\text{unit}, V_{61}=12/\text{unit},$ $V_{62}=21/\text{unit}, V_{63}=60/\text{unit},$ $R^n_{61}=2350, R^n_{62}=1408,$ $R^n_{63}=473.$	
	Crash	14	1904	1064	364		
		16	2176	1216	416		
		18	2350	1408	473		
	Normal	18	1278	756	252		
		20	1420	840	280		
		22	1562	924	308		
24		1704	1008	336			
7		$t_7$	$R^r_{71}$	$R^r_{72}$	$R^r_{73}$	$M_{71}=13/\text{unit}, M_{72}=20/\text{unit},$ $M_{73}=23/\text{unit}, V_{71}=24/\text{unit},$ $V_{72}=32/\text{unit}, V_{73}=40/\text{unit},$ $R^n_{71}=1296, R^n_{72}=835,$ $R^n_{73}=714.$	
	Crash	9	774	504	432		
		11	946	616	528		
		13	1108	721	619		
		15	1296	835	714		
	Normal	13	546	377	299		
		15	630	435	345		
		17	708	490	387		
18		756	522	414			



Table 6.2: Optimistic and pessimistic project duration margin

•	Crash duration		Normal duration	
	Minimum	Maximum	Minimum	Maximum
0	<b>60</b>	84	81	<b>105</b>
0.1	<b>64</b>	80	85	<b>101</b>
0.2	<b>64</b>	80	85	<b>101</b>
0.3	<b>64</b>	80	85	<b>101</b>
0.4	<b>67</b>	78	88	<b>99</b>
0.5	<b>67</b>	78	88	<b>99</b>
0.6	<b>68</b>	77	89	<b>98</b>
0.7	<b>68</b>	77	89	<b>98</b>
0.8	<b>69</b>	76	89	<b>97</b>
0.9	<b>69</b>	76	89	<b>97</b>
1	<b>69</b>	76	89	<b>97</b>

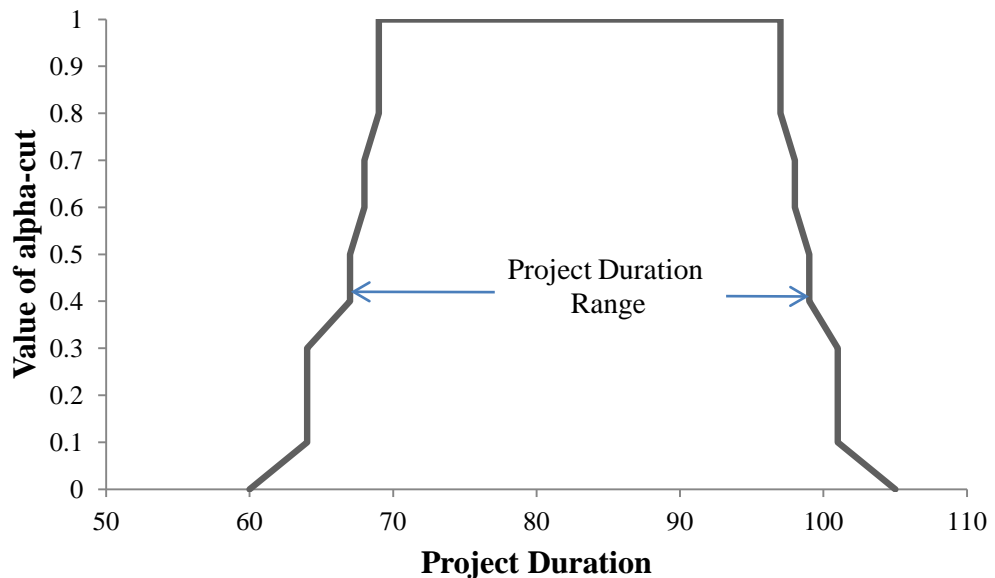


Figure 6.2: Possible project duration spectrum for Hypothetical test problem.

### 6.3 Solving Hypothetical Test Problem with Computer Program

This hypothetical test case was solved by proposed computer program. The test case was tested several times while various combinations of parameters such as generation number, population number, crossover rate, mutation rate and indirect cost rate were changed. The complete result is shown in appendix A1.

### 6.3.1 Considered Required Resources are Available in Store

It was assumed that required resources were available. It means that, available number of each resource,  $(R_{ij}^n)$  was greater than or equal to required number of resource,  $(R_{ij}^r)$ ; so that extra cost was not needed for any resource.

#### 6.3.1.1 Result Analysis

Best result was obtained using the following parameters for this hypothetical test case using proposed computer program.

Value of  $\alpha$ -cut level: 0.3

Generation number: 10000

Number of initial population: 200

Activity number: 7

Probability of mutation: 0.6

Probability of crossover: 0.4

Indirect cost rate: 1500

Premium cost: 3000

Desired completion time at a specific level: 83

Project networks: 1-3-5-7; 1-2-5-7; 1-4-6-7

Best optimum solution with respect to fitness:

Duration array: 15, 16, 16, 15, 25, 18, 10.

Related Cost: 229627; Related Time: 66.

Best optimum solution with respect to minimum cost:

Duration array: 18, 16, 16, 15, 25, 18, 13.

Related Cost: 217824; Related Time: 72.

Table 6.3: Pareto front solutions for  $\alpha = 0.3$  (resources are available).

Project Time	Project Cost	Duration of activities						
64	236960	15	16	16	15	23	18	10
66	229627	15	16	16	15	25	18	10
69	222222	15	16	16	15	25	18	13
72	217824	18	16	16	15	25	18	13

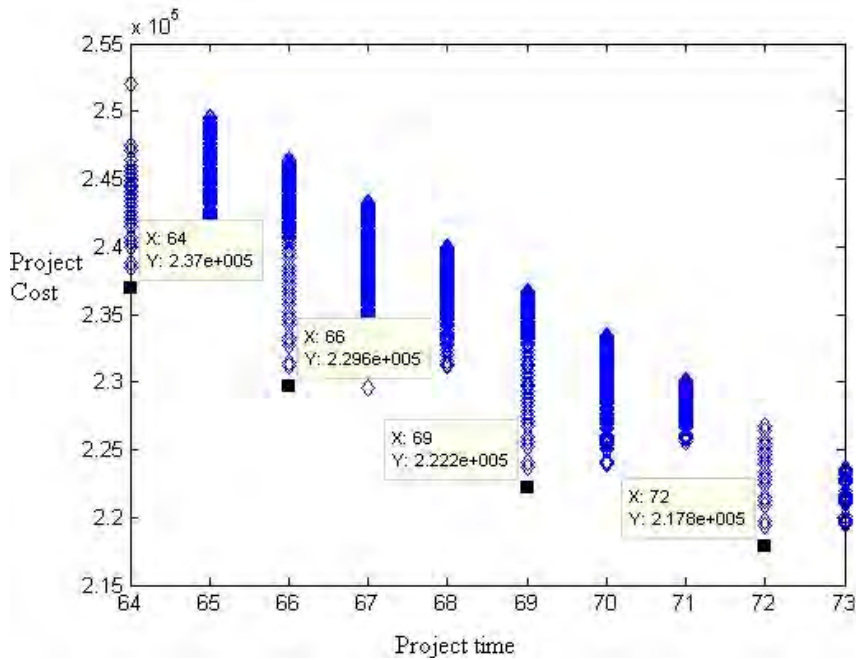


Figure 6.3: At  $\bullet = 0.3$ , Pareto front for hypothetical test case.

The Table 6.3 and Figure 6.3 show Pareto optimal solutions of the hypothetical TCO problem. Moreover, with desired project completion time 83 days, optimum solutions at different risk levels ( $\bullet$ -cut level) are shown in Table 6.4, Figure 6.4, 6.5 and 6.6. An analysis is found that as  $\bullet$ -cut value increases, the project durations and costs of Pareto optimal solutions also increases. It is because; the space or times spectrum of each activity becomes narrow for the selection of optimal solutions. Alternatively, it can be said that total cost and duration of optimal solutions of a project increase with risk level. At  $\bullet = 0.6$  and above, best optimum solution with respect to fitness (OBF) and best optimum solution with respect to minimum cost (OMC) were found same (shown in Figure 6.5 and 6.6).

Table 6.4: Effect of  $\bullet$ -cut level on the Pareto optimal solutions.

Value of $\bullet$ -cut level	Optimum Solutions								
	Time	Cost	Duration of activities						
0	<b>60</b>	<b>224894</b>	<b>14</b>	<b>15</b>	<b>15</b>	<b>15</b>	<b>22</b>	<b>18</b>	<b>9</b>
	63	221508	14	15	15	15	25	19	9
	67	217419	14	15	15	15	25	18	13
	<b>71</b>	<b>215701</b>	<b>18</b>	<b>15</b>	<b>15</b>	<b>15</b>	<b>25</b>	<b>18</b>	<b>13</b>
0.2	64	236960	15	16	16	15	23	18	10
	<b>66</b>	<b>229627</b>	<b>15</b>	<b>16</b>	<b>16</b>	<b>15</b>	<b>25</b>	<b>18</b>	<b>10</b>
	69	222222	15	16	16	15	25	18	13
	<b>72</b>	<b>217824</b>	<b>18</b>	<b>16</b>	<b>16</b>	<b>15</b>	<b>25</b>	<b>18</b>	<b>13</b>

0.4	67	246226	15	17	17	15	24	18	11
	68	236472	15	17	17	15	25	18	11
	69	233979	15	17	17	15	24	18	13
	<b>70</b>	<b>224225</b>	<b>15</b>	<b>17</b>	<b>17</b>	<b>15</b>	<b>25</b>	<b>18</b>	<b>13</b>
	<b>73</b>	<b>219827</b>	<b>18</b>	<b>17</b>	<b>17</b>	<b>15</b>	<b>25</b>	<b>18</b>	<b>13</b>
0.6	68	249023	16	17	17	15	24	18	11
	69	239238	16	17	17	15	25	18	11
	70	236745	16	17	17	15	24	18	13
	71	226991	16	17	17	15	25	18	13
	<b>73</b>	<b>219827</b>	<b>18</b>	<b>17</b>	<b>17</b>	<b>15</b>	<b>25</b>	<b>18</b>	<b>13</b>
0.8	69	250800	16	17	18	15	25	18	11
	70	241241	16	18	18	15	25	18	11
	71	238553	16	17	18	15	24	18	13
	72	228799	16	17	18	15	25	18	13
	<b>74</b>	<b>221635</b>	<b>18</b>	<b>17</b>	<b>18</b>	<b>15</b>	<b>25</b>	<b>18</b>	<b>13</b>
1	69	250800	16	17	18	15	25	18	11
	70	241241	16	18	18	15	25	18	11
	71	238553	16	17	18	15	24	18	13
	72	228799	16	17	18	15	25	18	13
	<b>74</b>	<b>221635</b>	<b>18</b>	<b>17</b>	<b>18</b>	<b>15</b>	<b>25</b>	<b>18</b>	<b>13</b>

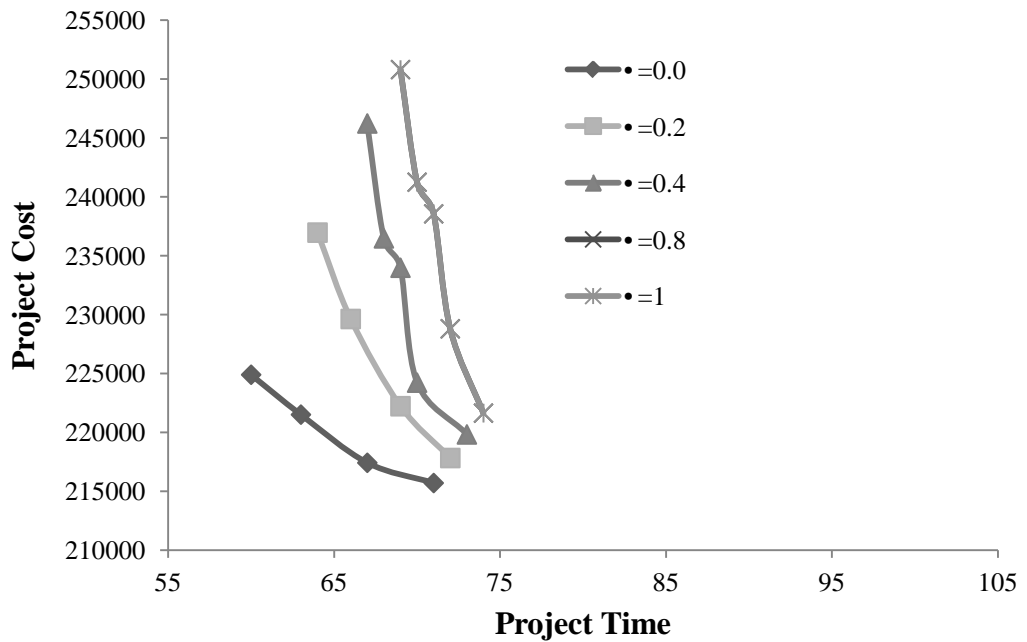


Figure 6.4: Effect of  $\alpha$ -cut level on the Pareto optimal solutions.

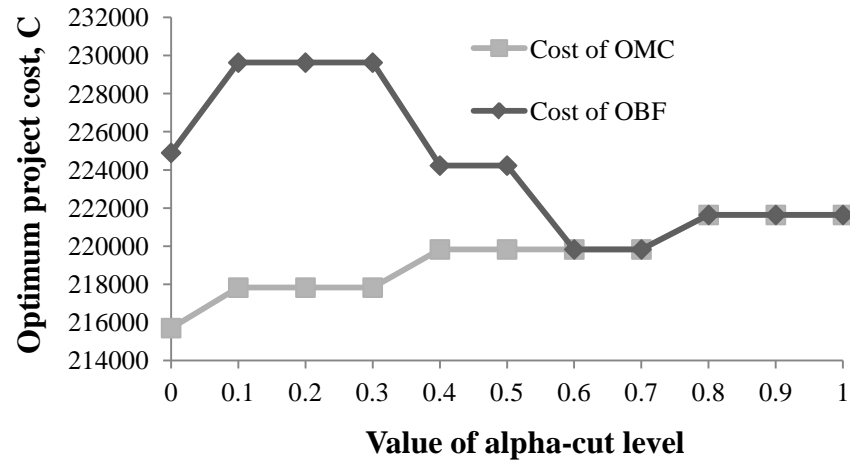


Figure 6.5: Impact of  $\alpha$ -cut level on project cost.

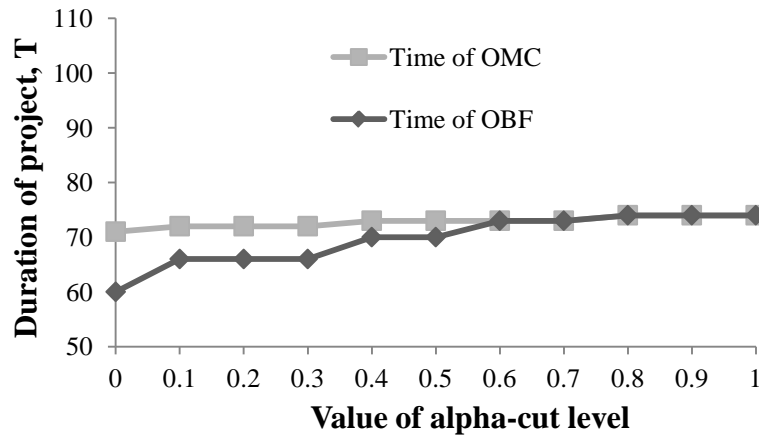


Figure 6.6: Impact of  $\alpha$ -cut level on project duration.

### 6.3.1.2 Convergence Analysis

Table 6.5 shows the convergence of hypothetical test case solving by GA based proposed computer program. The generation number was taken for convergence analysis to find optimal solutions of TCO problem. The parameters such as value of alpha-cut level = 0.3, desired project completion time = 83, indirect cost = 1500/day, premium cost = 3000/day, number of population = 200, crossover rate = 0.4 and mutation rate = 0.6 were used in this analysis. It is observed from Table 6.5, optimum solutions are closing with generation number. At 1000 generation the Pareto optimum solutions were found. The details results are shown in Table A2 of appendix A.

Table 6.5: Effects of generation number on the optimal solutions.

No. of Generation	Optimal solution		No. of Generation	Optimal solution		
	Project Duration	Project Cost		Project Duration	Project Cost	
2	67	248919	600	64	236960	
	73	238984		66	229627	
4	69	250292		69	222222	
	72	232426		72	219683	
6	69	244391		73	219519	
	71	232941		800	64	236960
	72	230519	66		229627	
10	66	231486	69		225229	
	71	226159	70		223917	
50	64	242204	72		219683	
	65	239726	1000		64	236960
	67	231435		66	229627	
	69	222222		69	222222	
100	64	254900		72	217824	
	65	240181		2000	64	236960
	66	234205			66	229627
	69	222222	69		222222	
	74	221214	72		217824	
200	64	244396	4000		64	236960
	65	242040		66	229627	
	66	233012		69	222222	
	67	231081		72	217824	
	70	227610	6000	64	236960	
	71	226159		66	229627	
	73	221045		69	222222	
400	64	244590		72	217824	
	65	238768		8000	64	236960
	67	232562	66		229627	
	68	231250	69		222222	
	69	225940	72		217824	
	70	224156	10000	64	236960	
	73	219519		66	229627	
		69		222222		
		72		217824		

### 6.3.1.3 Effect of Indirect Cost

Table 6.6 shows the effect of indirect cost/day on optimum solutions. The parameters were same as convergence analysis. It is observed that as the indirect

cost rate increases, the number of optimum solutions decreases. It means that there is greater incentive to reduce indirect costs by expending the project. Alternatively it can be said that the pressure to complete the project earlier increases with indirect cost.

Table 6.6: Effect of indirect costs on the Pareto front solutions.

Indirect cost	Optimum Solutions		Indirect cost	Optimum Solutions	
	Project Time	Project Cost ( $10^5$ )		Project Time	Project Cost ( $10^5$ )
50	64	1.46	1000	64	2.05
	66	1.339		66	1.966
	67	1.324		67	1.961
	69	1.222		69	1.877
	72	1.134		72	1.818
	76	1.121			
	81	1.104			
100	64	1.489	1250	64	2.21
	66	1.388		66	2.131
	67	1.358		67	2.128
	69	1.256		69	2.05
	72	1.170		72	1.998
	76	1.159			
	81	1.144			
250	64	1.57	1500	64	2.37
	66	1.471		66	2.296
	67	1.458		69	2.222
	69	1.36		72	2.178
	72	1.278			
	81	1.266			
	500	64		1.73	1750
66		1.636	66	2.461	
67		1.626	69	2.395	
69		1.532	72	2.358	
73		1.458			
750	64	1.89	2000	64	2.69
	66	1.801		66	2.626
	67	1.793		69	2.567
	69	1.705		72	2.538
	72	1.638			

### 6.3.2 Considered Required Resources are short in Store

It was considered that required number of resource was greater than the available number of resources of each activity. So, extra costs were added for extra

resources beyond the available limit of resources. Cost rates of each resource are given in Table 6.1. But, available limit of each resource were redefined for that state namely  $R_{11}^n=300$ ,  $R_{12}^n=450$ ,  $R_{13}^n=345$ ,  $R_{21}^n=1032$ ,  $R_{22}^n=216$ ,  $R_{31}^n=928$ ,  $R_{32}^n=192$ ,  $R_{41}^n=400$ ,  $R_{42}^n=150$ ,  $R_{43}^n=250$ ,  $R_{44}^n=342$ ,  $R_{51}^n=400$ ,  $R_{52}^n=350$ ,  $R_{53}^n=174$ ,  $R_{61}^n=1600$ ,  $R_{62}^n=900$ ,  $R_{63}^n=300$ ,  $R_{71}^n=700$ ,  $R_{72}^n=400$ ,  $R_{73}^n=350$ . Other parameters were same as previous state.

### 6.3.2.1 Result Analysis

Value of  $\alpha$ -cut level: 0.3

Generation number: 10000

Number of initial population: 200

Activity number: 7

Probability of mutation: 0.6

Probability of crossover: 0.4

Indirect cost rate: 1500

Premium cost: 3000

Desired completion time at a specific level: 83

Project networks: 1-3-5-7; 1-2-5-7; 1-4-6-7

Best optimum solution with respect to fitness:

Duration array: 15, 16, 16, 15, 25, 18, 13.

Related Cost: 228986; Related Time: 69.

Best optimum solution with respect to minimum cost:

Duration array: 18, 16, 16, 15, 25, 18, 13.

Related Cost: 219944; Related Time: 72.

Table 6.7: Pareto front solutions for  $\alpha = 0.3$  (Resources are short in store).

Project Time	Project Cost	Duration of activities						
64	255800	15	16	16	15	23	18	10
66	242281	15	16	16	15	25	18	10
69	228986	15	16	16	15	25	18	13
72	219944	18	16	16	15	25	18	13



This analysis shows that, extra costs for extra resources beyond the limit of resources availability are induced in project total cost. So that, best optimum project duration with respect to fitness is increased (from 66 to 69 days) to reduce the project total cost.

#### 6.4 Performance of the Proposed Model

To illustrate the concept and determine the effectiveness of the proposed model, a test case adapted from Zheng and Ng [66] was fitted into the proposed model. The robustness of the new proposed model in the deterministic situation was compared with two other previous models: (1) the multiobjective modified adaptive weighting approach (MAWA) model (Zheng et al. [37]) and (2) a stochastic time–cost optimization model incorporating fuzzy sets theory and non-replaceable front (Zheng and Ng [66]).

Table 6.7: Test case in detail (Zheng and Ng [66]).

Activity description	Activity Number	Precedent activity	Option	Duration (days)		Direct cost (\$)	
				• = 1	• = 0	• = 1	• = 0
Site preparation	1		1	14	±2	23,000	±4600
			2	20	±3	18,000	±3000
			3	24	±5	12,000	±2000
Forms and rebar	2	1	1	15	±3	3,000	±100
			2	18	±5	2,400	±400
			3	20	±5	1,800	±60
			4	23	±7	1,500	±360
			5	25	±6	1,000	±120
Excavation	3	1	1	15	±5	4,500	±750
			2	22	±2	4,000	±533
			3	33	±5	3,200	±853
Precast concrete girder	4	1	1	12	±2	45,000	±1500
			2	16	±3	35,000	±10500
			3	20	±4	30,000	±7000
Pour foundation and piers	5	2,3	1	22	±6	20,000	±6000
			2	24	±4	17,500	±1750
			3	28	±5	15,000	±1500
			4	30	±5	10,000	±1000
Deliver PC girders	6	4	1	14	±2	40,000	±4000
			2	18	±3	32,000	±2133
			3	24	±4	18,000	±2400
Erect girders	7	5,6	1	9	±2	30,000	±2000
			2	15	±2	24,000	±7200
			3	18	±3	22,000	±4400

### 6.4.1 Solving the Test Problem

To solve the test problem, direct cost of each activity was inputted as resource in the proposed model. Because of, the proposed model was designed with resources consideration for TCO problem. The parameters are shown in Table 6.9.

Table 6.9: Parameters for the three models in comparison.

Models	Indirect cost (\$/day)	Opportunity cost (\$/day)	Penalty for delay (\$/day)	Planned /desired time (day)	Planned Cost (\$)	$P_m$	$P_c$	Population size	Generation number
MAWA model [37]	500	×	×	×	×	0.6	0.4	50	100
Zheng and Ng [66]	500	500	×	75	155000	0.6	0.4	50	100
New proposed model	500	×	1500	75	×	0.6	0.4	50	100

Table 6.10: Results of three models at  $\bullet = 1$

MAWA model [37]		Zheng and Ng [66]		New proposed model	
Time (day)	Cost (\$)	Time (day)	Cost (\$)	Time (day)	Cost (\$)
61	173000	68	152500	60	173500
62	172000	74	149500	61	173000
63	162500			62	171000
66	161500			63	162500
67	157000			66	161500
68	152500			67	157000
74	149500			68	152500
77	149000			74	149500
78	146500				
84	143500				
87	143000				

An analysis is found in Table 6.10 and Figure 6.7 that, the new proposed model is able to find out the optimal solutions and define the Pareto front as well. However, when comparing the results with that produced by the new proposed model, at least one outcome (time = 60 days and cost=\$173500) was better than the best

solutions generated by MAWA model [37]. Moreover, one of obtained solutions of MAWA model [37] (time = 62 days and cost=\$172000) was not Pareto solution. On the other hand, Zheng and Ng's model [66] found optimal solutions within their planned cost and time. The proposed model also found optimal solutions within the desired time. Here, penalty was added for delay beyond the desired project completion time. So, the managers get more options for choosing suitable conditions. Finally, it can be said that the proposed model is better than MAWA model [37] and more realistic than Zheng and Ng's model [66].

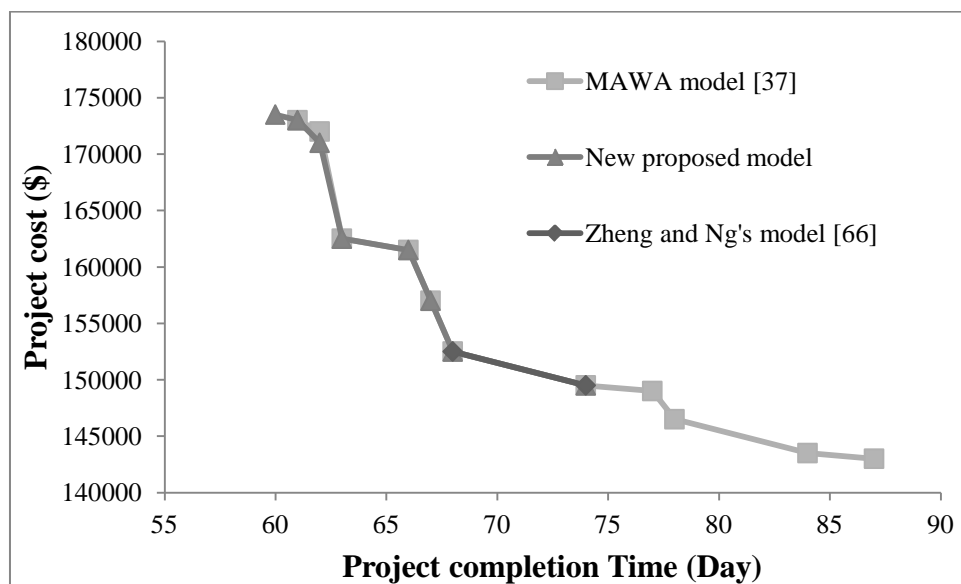


Figure 6.7: Experimental results from three models.

## 6.5 Real Problem

Transformer making project is taken as real problem. So that, the data is taken from an electric company namely AHK Electric Company Ltd, Nondipara, Sabujbag, Dhaka. It is a small electric company and its products are different types of transformer, LT and HT switch gears. This company also sales generator and other electrical goods.

Transformer is electrical device that transfers electrical energy from one circuit to another through inductively coupled conductors—the transformer's coils. Making a 200 KVA transformer a lot of resources and many activities are required, shown in Table 6.11. Several times, I saw the transformer manufacturing process. In

manufacturing process, some resources are directly assembled and some are manufactured. Moreover, some resources (e.g. technicians, labors, machineries, equipments etc) are used in manufacturing process; these are important or responsible for activity duration. Much resources of that type are required to complete project in shortest time. So, in this proposed paper, these resources are considered for multi-objective time-cost optimization as time variable resources. Other resources which are assembled are not considered in optimization and their related costs are added as fixed cost.

The transformer making project with an 8-activity CPM network (AON diagram) is presented illustrating project scheduling with multi-objective time-cost optimization problem. The precedence relationships of network are depicted in Figure 6.8. The activity durations and resources of each activity are assumed to be trapezoidal fuzzy number. The durations, their related resources which are time variable, resources cost rate for both normal and crash modes of each activity are shown in Table 6.12. All cost rates of resources are given at current market price.

In Table 6.13 and Figure 6.9, the optimistic and pessimistic project duration is defined for different alpha-cut level. Only integer activity duration and resources are considered. Each activity is performed at two different modes, crash and normal mode. Moreover, if the required number of resource exceeds the available limit of resource; extra cost will be needed to get extra resources. In addition, delay fine per day will be considered for extensions beyond the desired project duration.

Table 6.11: Activities and resources of 200 KVA Transformer.

Activity	Resources
1. Core making	Technician and labor, Silicon core
2. Winding (H.V. and L.V.)	Technician and labor, Copper strip, Super enamel wire
3. Coil assembly and connection	Technician and labor, D.B.C paper, Aika (additives), Insulator paper
4. Baking	Heat chamber of 8*1500 W
5. Tank fabrication	M.S. Sheet, Angle, Flat bar, Flange, M.S.

	rod/tread rods and nuts, G.I. pipe/M.S. Pipe, Elbow, Welding rod (Ferro graph), Welding rod (botic)
6. Box-up	Thermometer pocket, Crane Hooke, Wiring cable, Earthling bush, Earthling horn, Silicon pot, Silica jell, Chaka, H.T. connector, L.T. connector, H.T. rubber, L.T. rubber, H.T. Got ka, H.T. Got Ka, Channel Rubber, Core channel, Bottom channel, Oil key, Ring set, Cotton tape, Varnish, Copper sheet (1mm), Bus bar, Press board (1mm), Press board (2mm), Tap changer, Cotton tube, H.T. insulator, H.T. insulator, Wood, Nut bolt with washer, Super glow, Thermometer, Name plate, Coke sheet/rubber sheet, Back light sheet/white, Coolant oil, Technician and labor
7. Testing	Technician and labor
8. Painting and finishing	Labor, Paint

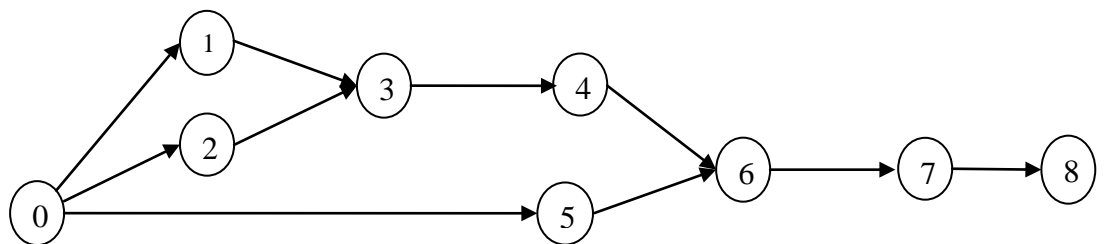


Figure 6.8: CPM network of 8-activity real problem (200 KVA Transformer making project).









Table 6.13: Optimistic and pessimistic project duration margin (Real problem).

•	Crash duration		Normal mode	
	Minimum	Maximum	Minimum	Maximum
0	<b>55.5</b>	94.5	66.5	<b>108.5</b>
0.1	<b>62</b>	89	72	<b>102</b>
0.2	<b>62</b>	89	72	<b>102</b>
0.3	<b>62</b>	89	72	<b>102</b>
0.4	<b>65</b>	86	76	<b>98</b>
0.5	<b>65</b>	86	76	<b>98</b>
0.6	<b>67</b>	83	80	<b>97</b>
0.7	<b>67</b>	83	80	<b>97</b>
0.8	<b>69</b>	81	82	<b>96</b>
0.9	<b>69</b>	81	82	<b>96</b>
1	<b>69</b>	81	82	<b>96</b>

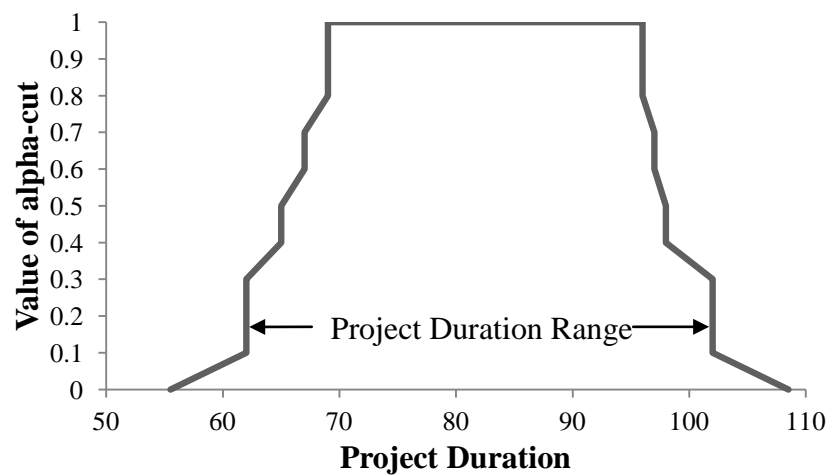


Figure 6.9: Project duration spectrum for real test problem.

### 6.5.1 Solving Real Test Problem with New Model

This real test case was solved by computer program. Parameters of this real problem such as generation number = 10000, population number = 200, crossover rate = 0.4, mutation rate = 0.6 and indirect cost rate = Tk. 300/hour were considered. Premium (delay fine) cost was also considered for extra project duration beyond the desired project duration. Since fixed cost of transformer making was Tk. 75000 which should be added with cost of optimal solutions to find total cost of transformer.

**6.5.1.1 Considered Resources are Available in Store**

Similar to hypothetical data, it was assumed that all required resources were available.

Best result was obtained using the following parameters for this real test case using proposed computer program.

Value of  $\alpha$ -cut level: 0.2

Generation number: 10000

Number of initial population: 200

Activity number: 8

Probability of mutation: 0.6

Probability of crossover: 0.4

Indirect cost rate: 300

Premium cost: 500

Desired completion time at a specific level: 70

Project networks: 0-2-3-4-6-7-8, 0-1-3-4-6-7-8, 0-5-6-7-8

Best optimum solution with respect to fitness:

Duration array: 6, 18, 9, 26, 24, 11, 1, 3.

Related Cost: Tk.  $2.3129 \times 10^5$ ; Related Time: 68 hrs.

Best optimum solution with respect to minimum cost:

Duration array: 6, 18, 10, 26, 24, 12, 1, 3.

Related Cost: Tk.  $2.2821 \times 10^5$ ; Related Time: 70 hrs.

Table 6.14: Pareto front solutions for real problem at  $\alpha = 0.2$  (Resources are available).

Optimum Solutions (Pareto front solutions)									
Time (Hr.)	Cost (Tk.) ( $10^5$ )	Duration of activities (hr.)							
62	2.638	6	12	9	26	24	11	1	3
63	2.611	6	12	10	26	24	11	1	3
<b>68</b>	<b>2.3129</b>	<b>6</b>	<b>18</b>	<b>9</b>	<b>26</b>	<b>24</b>	<b>11</b>	<b>1</b>	<b>3</b>
<b>70</b>	<b>2.2821</b>	<b>6</b>	<b>18</b>	<b>10</b>	<b>26</b>	<b>24</b>	<b>12</b>	<b>1</b>	<b>3</b>

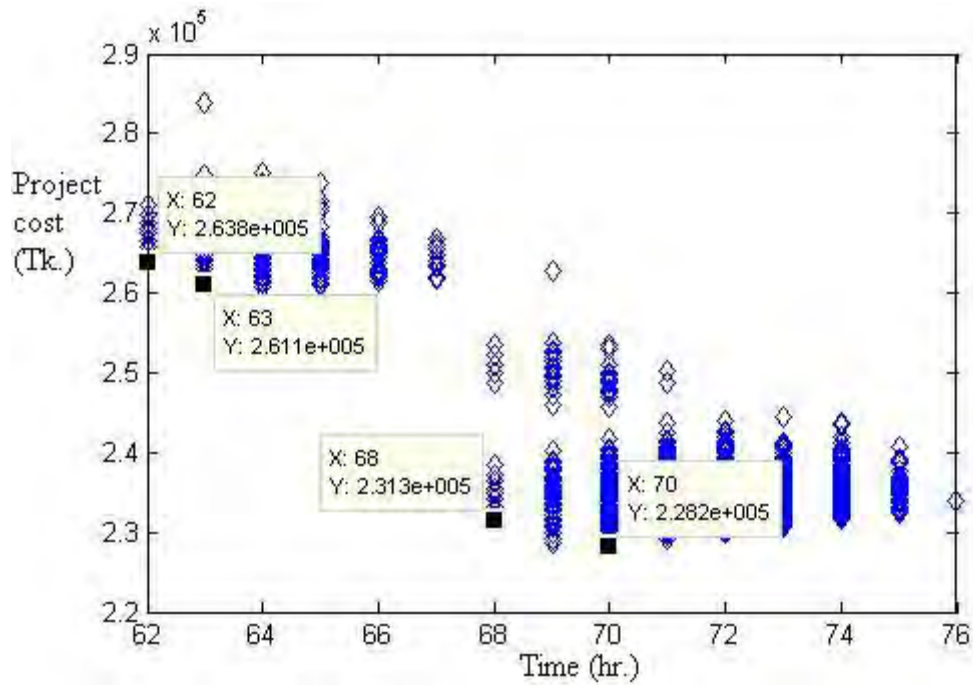


Figure 6.10: At  $\bullet = 0.2$ , Pareto front solution for Real problem.

Table 6.15: Effect of  $\bullet$ -cut value on the optimum solution in real problem (resources are available).

Value of $\bullet$ -cut level	Optimum Solutions									
	Time (Hr.)	Cost (Tk.) ( $10^5$ )	Duration of activities							
0	<b>55.5</b>	<b>2.553</b>	6	11	8	24	24	10	0.5	2
	<b>64.5</b>	<b>2.137</b>	6	18	10	24	24	10	0.5	2
0.2	62	2.638	6	12	9	26	24	11	1	3
	63	2.611	6	12	10	26	24	11	1	3
	<b>68</b>	<b>2.3129</b>	6	18	9	26	24	11	1	3
	<b>70</b>	<b>2.2821</b>	6	18	10	26	24	12	1	3
0.4	65	2.70	6	13	9	28	25	11	1	3
	66	2.675	6	13	10	28	25	11	1	3
	<b>70</b>	<b>2.32</b>	6	18	9	28	24	11	1	3
	<b>71</b>	<b>2.298</b>	6	18	10	28	24	11	1	3
0.6	67	2.70	6	13	9	30	24	11	1	3
	68	2.675	6	13	10	30	24	11	1	3
	<b>72</b>	<b>2.338</b>	6	18	9	30	24	11	1	3
	<b>73</b>	<b>2.316</b>	6	18	10	30	24	11	1	3
0.8	69	2.711	6	13	9	32	24	11	1	3
	<b>74</b>	<b>2.355</b>	6	18	9	32	24	11	1	3
	<b>75</b>	<b>2.33</b>	6	18	10	32	24	11	1	3
1	69	2.711	6	13	9	32	24	11	1	3
	<b>74</b>	<b>2.355</b>	6	18	9	32	24	11	1	3
	<b>75</b>	<b>2.33</b>	6	18	10	32	24	11	1	3

### 6.5.1.2. Considered Required Resources are short in store

It was considered that required number of resource was greater than the available number of resources of each activity. So, extra costs were added for extra resources beyond the available limit of resources. Cost rates of each resource are given in Table 6.12. But, available limit of each resource were redefined for that state namely  $R_{11}^n = 1$  man/hr,  $R_{12}^n = 2$  man/hr,  $R_{13}^n = 320$  kg,  $R_{21}^n = 1$ /hr,  $R_{22}^n = 2$ /hr,  $R_{23}^n = 73$  kg,  $R_{24}^n = 92$  kg,  $R_{31}^n = 1$ /hr,  $R_{32}^n = 2$ /hr,  $R_{33}^n = 11$  kg,  $R_{41}^n = 600$  KW-Hr,  $R_{51}^n = 1$ /hr,  $R_{52}^n = 3$ /hr,  $R_{53}^n = 76$  kg,  $R_{54}^n = 4$  pcs,  $R_{55}^n = 5$  pcs,  $R_{61}^n = 1$ /hr,  $R_{62}^n = 2$ /hr,  $R_{71}^n = 1$ /hr,  $R_{72}^n = 1$ /hr,  $R_{81}^n = 2$ /hr.

Best result was obtained using the following parameters for this real test case using proposed computer program.

Value of • -cut level: 0.2

Generation number: 10000

Number of initial population: 200

Activity number: 8

Probability of mutation: 0.6

Probability of crossover: 0.4

Indirect cost rate: 300

Premium cost: 500

Desired completion time at a specific level: 70

Project networks: 0-2-3-4-6-7-8, 0-1-3-4-6-7-8, 0-5-6-7-8

Best optimum solution with respect to fitness:

Duration array: 6, 18, 10, 26, 24, 11, 1, 3.

Related Cost: Tk.  $2.291 \times 10^5$ ; Related Time: 69 hrs.

Best optimum solution with respect to minimum cost:

Duration array: 6, 18, 10, 26, 24, 12, 1, 3.

Related Cost: Tk.  $2.282 \times 10^5$ ; Related Time: 70 hrs.

Table 6.16: Pareto front solution for real problem at  $\bullet = 0.2$  (Required resources are short in store).

Optimum Solutions (Pareto front)									
Time (Hr.)	Cost (Tk.) ( $10^5$ )	Duration of activities (hr.)							
62	2.766	6	12	9	26	24	11	1	3
63	2.734	6	12	10	26	25	11	1	3
64	2.719	6	12	10	26	24	12	1	3
68	2.329	6	18	9	26	24	11	1	3
<b>69</b>	<b>2.291</b>	<b>6</b>	<b>18</b>	<b>10</b>	<b>26</b>	<b>24</b>	<b>11</b>	<b>1</b>	<b>3</b>
<b>70</b>	<b>2.2821</b>	<b>6</b>	<b>18</b>	<b>10</b>	<b>26</b>	<b>24</b>	<b>12</b>	<b>1</b>	<b>3</b>

This analysis shows that, extra costs for extra resources above the normal resources availability are induced in project total cost. So that, best optimum project duration with respect to fitness (from 68 to 69) increases to reduce the project total cost.

# CHAPTER 7

## Conclusions and Recommendations

---

### 7.1 Conclusion

This research work has developed a practical model of GA based multi-objective time-cost optimization in a fuzzy environment with resources consideration. It provides an efficient technique for TCO problems incorporating multi-objective approach MAWA and critical path analysis under uncertainty. This proposed model tries to find the relationship between fuzzy activity time and its resources at crash and normal mode. Resources are varied with activity time to get time-cost optimum solutions. At different risk level ( $\alpha$ -cut level), to find the Pareto optimal solutions and best optimal solutions with respect to fitness and minimum project cost, a computer code of GA based solver is used.

A hypothetical test case is developed based on proposed model. The proposed model is used to solve the test case to show its robustness at different risk level changing its parameters. Moreover, a simple test case adapted from Zheng and Ng is used to test its performance and effectiveness. The results that produced by proposed model is also used to compare with MAWA model and Zheng and Ng's model. Thus, it is found that this proposed model can find the Pareto optimum solutions properly. Finally a real case is collected and it is solved by this proposed model.

Though a number of researchers have been carried out to solve project TCO problem, none of them considered both time and resources as fuzzy input at a time. This research work has also been proposed GA based multi-objective fuzzy TCO model whereas most of the previous works were GA based single objective fuzzy TCT model.

## **7.2 Recommendations**

The performance of the presented model can be further analyzed in terms of number of iteration by comparing it with other best known algorithms for multi-objective TCT project scheduling. Here, Roulette wheel selection method and single point crossover have been used. Other selection and crossover methods can be used and compared with the proposed algorithm to observe the result.

This presented model is solved by GA. It can be compared with other searching algorithms.

The algorithm can also be extended by considering limited resources time-cost trade-off with other different types of fuzzy numbers.

# References

- [1] Han, T.C., Chung, C.C., and Liang, G.S., “Application of fuzzy critical path method to airport’s cargo ground operation systems”, *Journal of Marine Science and Technology*, Vol. 14, No. 3, pp. 139-146, 2006.
- [2] Iranmanesh, H., Skandari, M. R. and Allahverdiloo, M., “Finding pareto optimal front for the multi-mode time, cost quality trade-off in project scheduling”, *World Academy of Science, Engineering and Technology*, Vol. 40, 2008.
- [3] Park, W. R., and Chapin, W. B., “Construction bidding: Strategic pricing for profit”, Wiley, New York, 1992.
- [3] Shankar, N.R., Sireesha, V., Rao, K.S. and Vani, N., “Fuzzy critical path method based on metric distance ranking of fuzzy numbers”, *Int. Journal of Math. Analysis*, Vol. 4, No. 20, pp. 995-1006, 2010.
- [4] Leu, S.S., Chen, A.T. and Yang, C.H., “A GA-based fuzzy optimal model for construction time-cost trade-off”, *International Journal of Project Management*, Vol. 19, pp. 47-58, 2001.
- [5] Kasprowicz, T., “Multi-objective optimization of construction schedules”, *Proc., 1st Congress on Computing in Civil Engineering*, Part 1, Washington, D.C., pp. 185–190, 1994.
- [6] Laptali, E., Bouchlagham, N., and Wild, S., “Planning and estimating in practice and the use of integrated computer models.” *Autom. Constr.*, Vol. 7, No. 1, pp. 71–76, 1997.
- [7] Nkasu, M. M., and Leung, K. H., “Resources scheduling decision support system for concurrent project management.” *Int. J. Prod. Res.*, Vol. 35, No. 11, pp. 3107–3132, 1997.
- [8] Wang, C. H., and Huang, Y. C., “Optimization model for construction project durations using a multistage decision process.” *Eng. Optimiz.*, Vol. 30, No. 2, pp. 155–173, 1998.
- [9] Adrian, J. J., “Construction accounting: Financial, managerial, auditing and tax”, Prentice-Hall, Englewood Cliffs, N.J, 1979.



- [10] Weng, H.J., and Chen, P. H., “Smart GA-based cost scheduling model for resources- constrained projects”, *22<sup>nd</sup> International Symposium on Automation and Robotics in Construction*, 2005.
- [11] Chen, P. H., and Weng, H., “A two phase GA model for resources-constrained project scheduling”, *Automation of Construction*, Vol. 18, pp. 485-498, 2009.
- [12] Hendrickson, C., and Au,T., “ Project management for construction”, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1989.
- [13] Pagnoni, A., “Project engineering: Computer oriented planning and operational decision making, Springer, Berlin, 1990.
- [14] Liu, L., Burns,S.A., and Feng, C., “ Construction time-cost trade-off analysis using LP/IP”, *Journal of Construction Engineering and Management*, ASCE, Vol. 121 (4), pp. 446-454, 1995.
- [15] Burns, S.A., Liu, L., and Feng, C.W., “The LP/IP hybrid method for construction time-cost trade-off analysis”, *Constr. Manag. Econ.*, Vol. 14, No. 3, pp. 265-276, 1996.
- [16] Kelly, James, E. Jr., “Critical path planning and scheduling: mathematical basis”, *Operational Research*, Vol. 9 no. 3, pp. 167-179, 1961.
- [17] Mayer, W. L., and Shaffer, L.R., “Extension of the critical path method through the application of integer programming”, *Civ. Engrg. Constr. Res. Ser. 2, Univ. of Illinois, Urbana, III*, 1963.
- [18] Patterson, J. H., and Huber, D., “A horizon-varying, zero-one approach to project scheduling”, *Mgmt. Sci.*, Vol. 20, No. 6, pp. 990-998, 1974.
- [19] Feng, C.W., Liu, L., and Burns, S.A., “Using genetic algorithms to solve construction time-cost trade-off problems”, *Journal of Computing in Civil Engineering*, Vol. 11, No. 3, pp. 184-189, 1997.
- [20] Butcher, W.S., “Dynamic programming for project cost-time curve”, *Journal of Construction Division, ASCE*, Vol. 93 (C01), pp. 59-73, 1967.
- [21] Robinson, D. R., “A dynamic programming solution to cost-time tradeoff for CPM”, *Mgmt. Sci.*, Vol. 22, No. 2, pp. 158-166, 1975.
- [22] Elmaghraby, S. E., “Resource allocation via dynamic programming in activity networks”, *Eur. J. Operational Res.*, Vol. 64, pp. 199-215, 1993.
- [23] De, P., Dunne, E. J., and Wells, C. E., “The discrete time-cost trade-off problem revisited”, *Eur. J. Operational Res.*, Vol. 81, pp. 225-238, 1995.

- [24] David, E. W., "Project scheduling under resources constraints-historical review and categorization of procedures", *AIEE Trans.*, Vol. 5, No. 4, pp. 297-312, 1973.
- [25] Elmaghraby, S. E., "Activity networks", Wiley, New York, 1977.
- [26] Talbot, F. B., "Resources-constrained project scheduling with time-resources tradeoffs: the non-preemptive case", *Management Science*, Vol. 28, pp. 1197-1210, 1982.
- [27] Panagiotakopoulos, D., "Cost-time model for large CPM project networks", *Journal of Construction Engineering and Management, ASCE*, Vol. 103 (C02), pp. 201-211, 1977.
- [28] Fondahl, J. W., "A non-computer approach to the critical path method for the construction industry", Technical Report No. 9, The Construction Institute, Department of Civil Engineering, Stamford University, 1961.
- [29] Siemens N., "A simple CPM time-cost trade-off algorithm", *Management Science*, Vol. 17, No. 6, pp. B-354-OB-363, 1971.
- [30] Moselhi, O., "Schedule compression using the direct stiffness method", *Canadian Journal of Civil Engineering*, Vol. 20, No. 1, pp. 65-72, 1993.
- [31] Šeda, M., "Flexible heuristics for project scheduling with limited resources", *World Academy of Science, Engineering and Technology*, Vol. 30, pp. 170-174, 2007.
- [32] Li, H., and Love, P., "Using improved genetic algorithms to facilitate time-cost optimization", *Journal of Construction Engineering and Management, ASCE*, Vol. 123, No. 3, pp. 233-237, 1997.
- [33] Hegazy, T., "Optimization of construction time-cost trade-off analysis using genetic algorithms", *Canadian Journal of Civil Engineering*, Vol. 26, No. 6, pp. 685-697, 1999.
- [34] Chua, D.K.H., Chan, W.T., and Govinda, K., "A time-cost trade-off model with resource consideration using genetic algorithm", *Civil. Eng. Syst.*, Vol. 14, pp. 291-311, 1997.
- [35] Fathi, H. and Afshar, A., "Multiple resource constraint time-cost-resource optimization using genetic algorithm", *Advancing and Integrating Education, Research & Practice, First International Conference on Construction in Developing Countries (ICCIDC-I)*, Pakistan, pp. 42-50, 2008.

- [36] Gen, M., and Cheng, R., “Genetic algorithms & engineering optimization”, Wiley-Interscience, New York, 2000.
- [37] Zheng, D. X.M., Ng, S.T., and Kumaraswamy, M.M., “Applying a genetic algorithm-based multiobjective approach for time-cost optimization”, *Journal of Construction Engineering and Management*, Vol. 130, No. 2, pp. 168-176, 2004.
- [38] Xiong, Y. and Kuang, Y., “Applying an ant colony optimization algorithm-based multiobjective approach for time–cost trade-off”, *Journal of Construction Engineering and Management*, ASCE, Vol. 134, No. 2, pp. 153- 156, 2008.
- [39] Ng, S. T. and Zhang, Y., “Optimizing construction time and cost using ant colony optimization approach”, *Journal of Construction Engineering and Management*, ASCE, Vol. 134, No. 9, pp. 721-728, 2008.
- [40] Geem, Z. W., “Multiobjective optimization of time-cost trade-off using harmony search”, *Journal of Construction Engineering and Management*, ASCE, Vol. 136, No. 6, pp. 711- 716, 2010.
- [41] Malcolm, D.G., Roseboom, J.H., Clark, C.E. and Fazar, W., “Applications of a technique for research and development program evaluation”, *Operation Research*, Vol. 7, pp. 646-669, 1959.
- [42] Diaz, C.F. and Hadipriono, F.C., “Nondeterministic networking methods”, *Journal of Construction Engineering and Management*, ASCE, Vol. 119, No. 1, pp. 40-57, 1993.
- [43] Ang, A., “Analysis of activity network under uncertainty”, *Journal of Engineering Mechanics*, ASCE, Vol. 101, No. 4, pp. 373-387, 1975.
- [44] Ahuja, H.M. and Arunachalam, V., “Risk evaluation in resources allocation”, *Journal of Construction Engineering and Management*, ASCE, Vol. 110, No. 4, pp. 324-336, 1984.
- [45] Padilla, E.M. and Carr, R.I., “Resources strategies for dynamic project management”, *Journal of Construction Engineering and Management*, ASCE, Vol. 117, No. 2, pp. 279-293, 1991.
- [46] Gong, D., “Risk analysis oriented network scheduling backward pass calculation and time disturbance analysis”, PhD thesis, Department of Civil Engineering, Norwegian Institute of Technology, Norway, 1993.
- [47] Carlsson, C, “On the relevance of fuzzy sets in management science methodology”, *In TIMES/studies in the Management Sciences*, Eds H.J.

- Zimmermann, L. A. Zadeh & B.G. Gaines, Elsevier, Netherlands, Vol. 20, pp. 11-28, 1984.
- [48] Chanas, S. and Kamburowski, J., "The use of fuzzy variables in PERT", *Fuzzy Sets and Systems*, Vol. 5, pp. 11-19, 1981.
- [49] McCahon, C.H., "Using PERT as an approximation of fuzzy project network analysis", *IEEE Transactions on Engineering Management*, Vol. 40, No. 2, pp. 146-153, 1993.
- [50] Prade, H., "Using fuzzy set theory in a scheduling problem: a case study", *Fuzzy Sets and Systems*, Vol. 2, pp. 153-165, 1979.
- [51] Ayyub, B.M. and Haldar, A., "Project scheduling using fuzzy set concepts", *Journal of Construction Engineering and Management, ASCE*, Vol. 110, pp. 189-203, 1984.
- [52] Hadipriono, C. and Sun, K., "Angular fuzzy set models for linguistic values", *Civil Engineering Systems*, Vol. 7, pp. 148-156, 1990.
- [53] Wang, K.H., Chi, J.H. and Wan, E.H., "Decision making of project under fuzzy information", *Journal of the Chinese Institute of Engineers*, Vol. 16, pp. 533-541, 1993.
- [54] Wu, R.M.K. and Hadipriono, F.C., "Fuzzy modus ponens deduction technique for construction scheduling", *Journal of Construction Engineering and Management, ASCE*, Vol. 120, pp. 162-179, 1994.
- [55] Chen, C.T. and Huang, S.F., "Applying fuzzy method for measuring criticality in project network", *Information Science*, Vol. 177, pp. 2448-2458, 2007.
- [56] Slyeptosov, A.I., and Tyshchuk, T.A., "Fuzzy temporal characteristics of operations for project management on the network models basis", *European Journal of operations Research*, Vol. 147, pp. 253-265, 2003.
- [57] Nasution, S.H., "Fuzzy critical path method", *IEEE Transactions on Systems, man and cybernetics*, Vol. 24, pp. 48-57, 1994.
- [58] Lorterapong, P. and Moselhi, O., "Project-network analysis using fuzzy sets theory", *Journal of Construction Engineering and Management*, Vol. 122, No. 4, pp. 308-318, 1996.
- [59] Yao, J.S. and Lin, F.T., "Fuzzy critical path method based on signed distance ranking of fuzzy numbers", *IEEE Transactions on Systems, man and cybernetics-part A: Systems and humans*, Vol. 30, No. 1, 2000.

- [60] Chen, S.M. and Chang, T.H., “Finding multiple possible critical paths using fuzzy PERT”, *IEEE Transactions on Systems, man and cybernetics-part A: Systems and humans*, Vol. 31, No. 6, 2001.
- [61] Chanas, S. and Zielinski, P., “Critical path analysis in the network with fuzzy activity times”, *Fuzzy Sets and Systems*, Vol. 122, pp. 195-204, 2001.
- [62] Zadeh, L.A., “Fuzzy sets”, *Information and Control*, Vol. 8, pp. 138-353, 1965.
- [63] Liang, G.S. and Han, T.C., “Fuzzy critical path for project network”, *Information and Management Sciences*, Vol. 15, No. 4, pp. 29-40, 2004.
- [64] Shankar, N. R., Sireesha, V. and Rao, P.P.B., “Fuzzy critical path method based on metric distance ranking of fuzzy numbers”, *International Journal of Math. Analysis*, Vol. 4, No. 20, pp. 995-1006, 2010.
- [65] Shankar, N. R., Sireesha, V. and Rao, P.P.B., “An analytical method for finding critical path in a fuzzy project network”, *International Journal of Contemp. Math. Sciences*, Vol. 5, No. 20, pp. 953-962, 2010.
- [66] Zheng, D. X. M. and Ng, S. T., “Stochastic time–cost optimization model incorporating fuzzy sets theory and nonreplaceable front”, *Journal of Construction Engineering and Management, ASCE*, Vol. 131, No. 2, pp. 176- 186, 2005.
- [67] Lorterapong, P., “A fuzzy heuristic method for resources-constrained project scheduling”, *Project management Journal*, Vol. 25, pp. 12-18, 1994.
- [68] Hapke, M. and Slowinski, R., “Fuzzy priority heuristics for project scheduling”, *Fuzzy Sets and Systems*, Vol. 83, pp. 291-299, 1996.
- [69] Pan, H., Willis, R. and Yeh, C.H., “Resources-constrained project scheduling with fuzziness”, In Mastorakis (Eds.), *Advances in Fuzzy Systems and Evolutionary Computation*, Danvers: WSES Press, pp. 173-179, 2001.
- [70] Leu, S.S., Chen, A.T. and Yang, C.H., “Fuzzy optimal model for resource-constrained construction scheduling”, *Journal of Computing in Civil Engineering, ASCE*, Vol. 13, No. 3, pp. 207-216, 1999.
- [71] Eshtehardian, E., Afshar, A. and Abbasnia, R., “Time–cost optimization: using GA and fuzzy sets theory for uncertainties in cost”, *Construction Management and Economics*, Vol. 26, pp. 679 – 691, 2008.

- [72] Abbasnia, R., Afshar, A. and Eshtehardian, E., “Time-cost trade-off problem in construction project management, based on fuzzy logic”, *Journal of Applied Sciences*, Vol. 8, No. 22, pp. 4159-4165, 2008.
- [73] Klir, G.J. and Yuan, B., “Fuzzy sets and fuzzy logic: theory and applications”, Prentice-Hall of India private Ltd, 2006.
- [74] Zimmermann, H.J., “Fuzzy Set Theory”, Boston: Kluwer, 1996.
- [75] Sawaragi, Y., Nakayama, H. and Tanino, T., “Theory of Multiobjective Optimization”, *Mathematics in Science and Engineering, Orlando, FL: Academic Press Inc*, Vol. 176, 1985.
- [76] Said, Y. H., “On genetic algorithms and their applications”, *Handbook of Statistics*, Elsevier B.V., Vol. 24, pp. 361-392, 2005.
- [77] Mitchell, M., “An Introduction to Genetic Algorithms”, MIT Press, Cambridge, MA, 1998.
- [78] Holland, J.H., “Adaptation in Natural and Artificial Systems”, University of Michigan Press, Ann Arbor, 1975.
- [79] Goldberg, D.E., “Genetic Algorithms in Search, Optimization and Machine Learning”, Addison–Wesley, Boston, 1989.

# Appendices

---

Table 6.12: Detail data of real problem (200 KVA transformer making).

Activity (i)	Mode	Activity duration and it's corresponding required resources				Cost rate of available and unavailable resources respectively in Tk./unit and available limit for each resource type in store	
1. Core making	Mode	t <sub>1</sub> , (hr)	Technicians/hr*, R <sup>r</sup> <sub>11</sub>	Labor/hr*, R <sup>r</sup> <sub>12</sub>	Silicon core, R <sup>r</sup> <sub>13</sub> (Kg)		M <sub>11</sub> =26/man/hr, M <sub>12</sub> =13/man/hr, M <sub>13</sub> =260/kg, V <sub>11</sub> =45/man/hr, V <sub>12</sub> =30/man/hr, V <sub>13</sub> =360/kg, R <sup>n</sup> <sub>11</sub> =3 man/hr, R <sup>n</sup> <sub>12</sub> =5man/hr, R <sup>n</sup> <sub>13</sub> =546 kg.
	<b>Crash</b> (approximately 5 hrs)	4	3	5	410		
		5			455		
		5			455		
		6			546		
	<b>Normal</b> (approximately 7 hrs)	6	1	2	315		
		7			320		
		7			320		
		8			330		
	2. Winding (H.V. and L.V.)	Mode	t <sub>2</sub> , (hr)	Technician s/hr*, R <sup>r</sup> <sub>21</sub>	Labor/hr*, R <sup>r</sup> <sub>22</sub>	Copper strip, R <sup>r</sup> <sub>23</sub> (Kg)	
<b>Crash</b> (approximately between 13 & 15 hrs)		11	3	6	98	105	
		13			109	112	
		15			119	119	
		18			138	128	
<b>Normal</b> (approximately between 20 & 22 hrs)		18	1	2	68	87	
		20			70	89	
		22			72	91	
		24			74	93	
3. Coil assembly and connection		Mode	t <sub>3</sub> , (hr)	Technicians/hr*, R <sup>r</sup> <sub>31</sub>	Labors/hr*, R <sup>r</sup> <sub>32</sub>	Insulation paper, R <sup>r</sup> <sub>33</sub> (Kg)	
	<b>Crash</b> (approximately between 9 & 10 hrs)	8	2	4	18		
		9			19		
		10			20		
		11			21		



	<b>Normal</b> (approximately between 12 & 13 hrs)	10 12 13 14	1	2 2 2 2	9 10 11 12					
4. Baking	Mode	t <sub>4</sub> , (hr)	Power required for heating, R <sub>41</sub> <sup>r</sup> (KW-Hr)				M <sub>41</sub> =5.58/ (KW-Hr), V <sub>41</sub> =5.58/ (KW-Hr), R <sub>41</sub> <sup>n</sup> =600 KW-Hr.			
	<b>Normal</b> (approximately between 32 & 42 hrs)	24	300							
		32	400							
		42	525							
		48	600							
5. Tank fabrication	Mode	t <sub>5</sub> , (hr)	Technicians/hr*, R <sub>51</sub> <sup>r</sup>	Labor/hr*, R <sub>52</sub> <sup>r</sup>	M.S. Sheet, R <sub>53</sub> <sup>r</sup> (kg)	Welding rod (Bo tic) R <sub>54</sub> <sup>r</sup> (pcs)	Welding rod (Ferro graph) R <sub>55</sub> <sup>r</sup> (pcs)	M <sub>51</sub> =26/man/hr, M <sub>52</sub> =13/man/hr, M <sub>53</sub> = 273/kg, M <sub>54</sub> = 900/pcs, M <sub>55</sub> = 700/pcs, V <sub>51</sub> =45/man/hr, V <sub>52</sub> = 30/man/hr, V <sub>53</sub> =330/kg, V <sub>54</sub> =1350/pcs, V <sub>55</sub> =950/pcs, R <sub>51</sub> <sup>n</sup> =2/hr, R <sub>52</sub> <sup>n</sup> =5/hr, R <sub>53</sub> <sup>n</sup> =122 kg, R <sub>54</sub> <sup>n</sup> =8 pcs, R <sub>55</sub> <sup>n</sup> =10 pcs		
	<b>Crash</b> (approximately between 22 & 24 hrs)	20	2	5	102	6	7			
		22	2	5	106	7	8			
		24	2	5	114	7	9			
		26	2	5	122	8	10			
	<b>Normal</b> (approximately between 26 & 28 hrs)	24	1	3	67	2	3			
		26	1	3	70	3	4			
		28	1	3	76	4	5			
		29	1	3	80	4	5			
	6. Box-up	Mode	t <sub>6</sub> , (hr)	Technicians/hr*, R <sub>61</sub> <sup>r</sup>		Labor/hr*, R <sub>61</sub> <sup>r</sup>			M <sub>61</sub> =26/man/hr, M <sub>62</sub> =13/man/hr, V <sub>61</sub> =45/man/hr, V <sub>62</sub> =30/man/hr, R <sub>61</sub> <sup>n</sup> =2/hr, R <sub>62</sub> <sup>n</sup> =5/hr.	
		<b>Crash</b> (approximately 11 hrs)	10	2		5				
			11	2		5				
11			2		5					
12			2		5					
<b>Normal</b> (approximately between 14 & 15 hrs)		12	1		2					
		14	1		2					
		15	1		2					
		17	1		2					

7. Testing	Mode	t <sub>7</sub> , (hr)	Technicians/hr*, R <sub>71</sub> <sup>r</sup>	Labor/hr*, R <sub>72</sub> <sup>r</sup>	M <sub>71</sub> =26/man/hr, M <sub>72</sub> =13/man/hr, V <sub>71</sub> =45/man/hr, V <sub>72</sub> =30/man/hr, R <sub>71</sub> <sup>n</sup> =1/hr, R <sub>72</sub> <sup>n</sup> =1/hr
	<b>Normal</b> (approximately 1 hrs)	0.5	1	1	
		1	1	1	
		1	1	1	
		1.5	1	1	
8. Painting & finishing	Mode	t <sub>8</sub> , (hr)	Labor/hr*, R <sub>81</sub> <sup>r</sup>		M <sub>81</sub> =13/man/hr, V <sub>81</sub> =30/man/hr, R <sub>81</sub> <sup>n</sup> =2/hr
	<b>Normal</b> (approximately 3 hrs)	2	2		
		3	2		
		3	2		
		4	2		

\*for technician and labor, wages is given like Taka/man/hour. So, for these resources, it is inputted (man × duration) as resources of activity in input text of activity durations and their related resources.

**Appendix A: Details results with Hypothetical Case (Resources are available)**

Value of Alpha-Cut = 0.3, Desired Project Completion Time = 83, Number of Population = 200, Number of Generation = 200, Indirect Cost= 1500/day,  
Premium Cost = 3000/day

Table A1: Results with different  $P_c$  and  $P_m$ .

Crossover rate, $P_c$	Mutation rate, $P_m$	Duration of the activities (Best choice with respect to fitness)							Project Duration (Best choice with respect to fitness)	Cost (Best choice with respect to fitness)	Duration of the activities (Best choice with respect to least cost)							Project Duration (Best choice with respect to least cost)	Cost (Best choice with respect to least cost)
		1	2	3	4	5	6	7			1	2	3	4	5	6	7		
0.1	0.1	15	16	16	15	23	18	10	64	236960	18	17	17	15	25	19	13	73	221353
0.1	0.2	15	16	16	15	23	18	10	64	236960	18	16	17	15	25	18	13	73	219632
0.1	0.3	15	16	16	16	25	20	10	66	234538	18	16	17	15	25	18	13	73	219632
0.1	0.4	15	16	16	18	23	18	10	64	242537	18	16	16	15	25	18	13	72	<b>217824</b>
0.1	0.5	15	16	16	15	23	19	10	64	238486	19	18	16	15	25	18	13	75	223495
0.1	0.6	15	16	16	16	25	19	10	66	233012	18	17	16	15	25	19	14	74	224200
0.1	0.7	16	16	16	15	25	18	10	67	232393	18	16	17	15	25	19	13	73	221158
0.1	0.8	15	16	16	15	23	18	10	64	236960	19	16	16	15	25	19	13	73	221631
0.1	0.9	15	16	16	16	25	18	10	66	231486	18	16	16	17	25	18	13	72	221542
0.2	0.1	15	16	16	15	23	18	10	64	236960	15	16	16	15	25	19	13	69	223748
0.2	0.2	15	16	16	15	23	18	10	64	236960	18	16	16	15	25	18	13	72	217824
0.2	0.3	15	16	16	16	23	18	10	64	238819	18	16	16	17	26	19	13	73	225002
0.2	0.4	15	16	16	16	25	19	10	66	233012	18	16	16	17	25	18	13	72	221542
0.2	0.5	15	16	16	15	23	20	10	64	240012	18	16	16	15	25	19	14	73	222505
0.2	0.6	15	16	16	15	25	19	10	66	231153	18	17	16	15	25	20	13	73	222571
0.2	0.7	15	16	16	15	25	18	10	66	<b>229627</b>	18	18	17	15	25	18	13	74	221522
0.2	0.8	18	16	16	15	25	18	13	72	217824	18	16	16	15	25	18	13	72	<b>217824</b>
0.2	0.9	15	16	16	17	23	21	10	74	245256	18	16	17	15	25	19	13	73	221158
0.3	0.1	15	16	16	15	23	18	10	64	236960	19	16	16	15	25	19	10	70	229036
0.3	0.2	15	16	16	15	23	18	10	64	236960	15	16	16	15	25	18	13	69	222222
0.3	0.3	15	16	16	15	23	18	10	64	236960	18	16	16	15	26	20	13	73	222810

Table A1 (continue)

0.3	0.4	15	16	16	16	23	18	10	64	238819	19	16	16	16	25	18	13	73	221964
0.3	0.5	15	16	16	16	23	19	10	64	240345	18	17	16	16	25	18	13	73	221378
0.3	0.6	15	17	17	15	23	18	10	65	238963	18	18	16	16	25	18	13	74	223073
0.3	0.7	15	16	16	16	23	19	10	64	240345	18	17	17	16	25	18	14	74	224841
0.3	0.8	15	16	16	15	25	18	10	66	<b>229627</b>	18	20	19	15	25	18	13	74	223208
0.3	0.9	15	17	16	15	23	18	10	65	238655	15	20	17	15	25	18	13	73	226990
0.4	0.1	15	16	16	15	25	18	10	66	<b>229627</b>	15	16	16	17	25	18	13	69	225940
0.4	0.2	15	16	16	15	25	18	10	66	<b>229627</b>	18	17	16	15	25	18	13	73	219519
0.4	0.3	15	16	16	15	23	22	10	64	243064	18	16	17	16	25	20	13	73	224543
0.4	0.4	15	17	16	15	25	19	10	67	232848	18	16	17	15	25	18	13	73	219632
0.4	0.5	15	16	16	15	25	18	13	69	222222	15	16	16	15	25	18	13	69	222222
<b>0.4</b>	<b>0.6</b>	15	16	16	15	25	18	10	<b>66</b>	<b>229627</b>	18	16	16	15	25	18	13	<b>72</b>	<b>217824</b>
0.4	0.7	15	16	16	16	23	18	13	67	231414	19	16	16	15	26	18	13	74	222039
0.4	0.8	15	16	16	15	23	20	10	64	240012	18	16	16	16	26	18	13	73	221617
0.4	0.9	15	16	16	16	23	19	13	67	232940	18	16	16	16	25	19	13	72	221209
0.5	0.1	15	16	16	15	25	18	10	66	<b>229627</b>	15	17	17	15	25	18	14	71	227380
<b>0.5</b>	<b>0.2</b>	15	16	16	15	25	18	10	66	<b>229627</b>	18	16	16	15	25	18	13	72	<b>217824</b>
0.5	0.3	15	16	16	15	23	20	10	64	240012	18	16	16	16	25	21	13	72	224261
0.5	0.4	15	16	16	15	23	19	10	64	238486	18	17	16	15	25	19	13	73	221045
0.5	0.5	15	16	16	16	23	18	10	64	238819	19	18	18	15	25	18	13	75	224111
0.5	0.6	15	16	16	15	23	20	10	64	240012	18	16	16	16	25	21	13	72	224261
0.5	0.7	15	16	16	15	23	18	10	64	236960	18	16	17	16	25	19	13	73	223017
0.5	0.8	15	16	16	15	23	19	10	64	238486	19	16	16	15	25	19	13	73	221631
0.5	0.9	15	17	17	15	25	19	10	67	233156	18	17	17	15	25	18	14	74	222982
0.6	0.1	15	16	16	15	25	18	10	66	<b>229627</b>	18	16	16	15	25	19	13	72	219350
0.6	0.2	15	16	16	15	23	18	10	64	236960	18	16	18	15	26	18	13	75	223374
0.6	0.3	15	16	16	15	25	19	10	66	231153	15	16	16	15	26	18	13	70	224156
0.6	0.4	15	16	16	16	23	18	10	64	238819	18	16	16	16	25	18	13	72	219683
0.6	0.5	15	16	16	15	25	19	10	66	231153	18	16	16	15	25	19	13	72	219350
0.6	0.6	15	16	16	16	23	18	13	67	231414	18	16	16	15	27	18	13	74	221692
0.6	0.7	15	16	17	15	25	19	10	67	232961	18	16	17	15	25	18	13	73	219632

A1 (Continue)

0.6	0.8	15	16	16	15	25	19	13	69	223748	18	17	16	15	25	19	14	74	222674
0.6	0.9	15	16	16	15	25	20	10	66	232679	18	16	16	15	27	18	13	74	221692
0.7	0.1	15	16	16	15	23	18	10	64	236960	15	17	17	15	25	19	13	70	225751
0.7	0.2	15	16	16	15	25	19	10	66	231153	18	16	16	15	25	19	13	72	219350
0.7	0.3	15	16	16	15	25	18	13	69	222222	18	16	16	15	25	18	13	72	217824
0.7	0.4	15	16	16	16	25	18	13	69	224081	18	16	16	16	25	18	13	72	219683
0.7	0.5	15	16	16	15	25	19	13	69	223748	18	16	17	16	25	19	13	73	223017
0.7	0.6	15	16	16	18	23	19	10	64	244063	18	17	17	15	25	19	13	73	221353
0.7	0.7	15	17	16	16	25	18	10	67	233181	19	16	16	15	25	18	13	73	220105
0.7	0.8	15	16	16	15	23	23	10	64	244590	19	17	17	15	25	18	13	74	222108
0.7	0.9	15	16	16	18	25	19	10	66	236730	18	16	16	18	25	18	13	72	223401
0.8	0.1	15	16	16	15	23	18	10	64	236960	18	16	17	15	25	19	13	73	221158
0.8	0.2	15	16	16	15	25	18	10	66	<b>229627</b>	18	16	16	16	25	18	13	72	219683
0.8	0.3	15	16	16	15	23	18	10	64	236960	15	16	16	15	25	18	13	69	222222
0.8	0.4	15	16	16	15	23	18	10	64	236960	20	16	16	15	25	18	13	74	222386
0.8	0.5	15	16	16	16	25	19	10	66	233012	18	18	17	17	25	18	13	74	225240
0.8	0.6	15	16	16	16	23	18	13	67	231414	18	16	17	16	25	18	14	74	224646
0.8	0.7	15	16	16	18	23	19	10	64	244063	19	18	16	15	25	18	13	75	223495
0.8	0.8	15	16	16	15	25	22	10	66	235731	18	16	17	15	25	18	14	74	222787
0.8	0.9	15	16	16	17	23	18	10	64	240678	18	16	17	15	25	20	13	73	222684
0.9	0.1	15	16	16	15	23	18	10	64	236960	18	16	17	15	25	19	13	73	221158
0.9	0.2	15	16	16	15	23	18	10	64	236960	18	16	17	15	25	18	13	73	219632
0.9	0.3	15	16	16	15	25	19	10	66	231153	18	16	16	15	25	20	13	72	220876
0.9	0.4	15	16	16	15	25	18	10	66	<b>229627</b>	18	16	16	15	25	19	13	72	219350
0.9	0.5	15	16	16	15	23	18	10	64	236960	18	17	16	15	25	18	13	73	219519
0.9	0.6	15	16	16	15	25	18	10	66	<b>229627</b>	19	16	16	15	25	18	13	73	220105
0.9	0.7	18	16	16	15	23	18	10	67	232562	18	16	17	15	25	20	13	73	222684
0.9	0.8	15	16	16	15	23	19	13	67	231081	18	16	17	15	27	18	13	75	223500
0.9	0.9	15	16	16	15	23	19	13	67	231081	18	18	16	15	25	19	13	74	222740

Value of Alpha-Cut = 0.3, Desire Completion Time = 83, Number of Population = 200,  $P_c = 0.4$ ,  $P_m = 0.6$ , Indirect Cost= 1500/day, Premium Cost =3000/day

Table A2: Data table for convergence analysis

No. of Generation	Project Duration	Project Cost	Duration of the Activities							No. of Generation	Project Duration	Project Cost	Duration of the Activities							
			16	16	18	18	23	18	10				15	16	16	15	23	18	10	
2	67	248919	16	16	18	18	23	18	10	600	64	236960	15	16	16	15	23	18	10	
	73	238984	20	16	18	18	25	18	10		66	229627	15	16	16	15	25	18	10	
4	69	250292	15	17	16	15	27	15	10		69	222222	15	16	16	15	25	18	13	
	72	232426	19	18	16	15	25	19	10		72	219683	18	16	16	16	25	18	13	
6	69	244391	15	17	16	16	25	19	12		800	73	219519	18	17	16	15	25	18	13
	71	232941	15	16	18	18	25	19	13			64	236960	15	16	16	15	23	18	10
	72	230519	15	16	18	15	25	19	14	66		229627	15	16	16	15	25	18	10	
10	66	231486	15	16	16	16	25	18	10	69		225229	18	16	16	15	25	18	10	
	71	226159	15	17	17	15	26	18	13	70		223917	15	17	16	15	25	18	13	
50	64	242204	15	16	16	17	23	19	10	1000		72	219683	18	16	16	16	25	18	13
	65	239726	16	16	16	15	23	18	10		64	236960	15	16	16	15	23	18	10	
	67	231435	15	16	17	15	25	18	10		66	229627	15	16	16	15	25	18	10	
	69	222222	15	16	16	15	25	18	13		69	222222	15	16	16	15	25	18	13	
100	64	254900	15	16	16	15	23	16	10		2000	72	217824	18	16	16	15	25	18	13
	65	240181	15	17	16	15	23	19	10			64	236960	15	16	16	15	23	18	10
	66	234205	15	16	16	15	25	21	10	66		229627	15	16	16	15	25	18	10	
	69	222222	15	16	16	15	25	18	13	69		222222	15	16	16	15	25	18	13	
	74	221214	18	18	16	15	25	18	13	72		217824	18	16	16	15	25	18	13	
200	64	244396	15	16	16	19	23	18	10	4000		64	236960	15	16	16	15	23	18	10
	65	242040	15	17	16	16	23	19	10		66	229627	15	16	16	15	25	18	10	
	66	233012	15	16	16	16	25	19	10		69	222222	15	16	16	15	25	18	13	
	67	231081	15	16	16	15	23	19	13		72	217824	18	16	16	15	25	18	13	
	70	227610	15	17	17	16	25	19	13		6000	64	236960	15	16	16	15	23	18	10
	71	226159	15	17	17	15	26	18	13			66	229627	15	16	16	15	25	18	10
	73	221045	18	17	16	15	25	19	13	69		222222	15	16	16	15	25	18	13	
	400	64	244590	15	16	16	15	23	23	10		8000	72	217824	18	16	16	15	25	18
65		238768	15	16	17	15	23	18	10	64			236960	15	16	16	15	23	18	10
67		232562	18	16	16	15	23	18	10	66			229627	15	16	16	15	25	18	10
68		231250	15	17	16	15	23	18	13	69	222222		15	16	16	15	25	18	13	
69		225940	15	16	16	17	25	18	13	72	217824		18	16	16	15	25	18	13	
70		224156	15	16	16	15	26	18	13	10000	64		236960	15	16	16	15	23	18	10
73		219519	18	17	16	15	25	18	13		66	229627	15	16	16	15	25	18	10	
									69		222222	15	16	16	15	25	18	13		
									72		217824	18	16	16	15	25	18	13		

Value of alpha-cut = 0.3, Desire completion time = 83, Number of Generation = 200, Pc = 0.4, Pm = 0.6, indirect cost= 1500/day, premium cost =3000/day.  
 Table A3: Results with different population size.

No. of population	Project Duration (Best choice with respect to fitness)	Cost (Best choice with respect to fitness)	Project Duration (Best choice with respect to least cost)	Cost (Best choice with respect to least cost)
20	65	243661	77	228516
40	66	243255	75	225901
60	67	231322	73	219519
100	64	238486	74	223356
200	66	<b>229627</b>	72	<b>217824</b>

## **Appendix B: Program Code for GA solver in MATLAB**

(This GA based program was coded for 7-activity problem)

```
clear
clc
%accepting input from user
g=input('Enter generation number:');
n=input('Enter number of initial population:');
l=input('Enter activity number:');
pm=input('Enter probability of mutation:');
pc=input('Enter probability of crossover:');
cl=input('Enter indirect cost rate:');
cp=input('Enter premium cost:');
ta=input('Enter desired completion time at a specific level:');
N=input('Enter project networks:');
%loading input from text files
load data1.txt
load data2.txt
load data3.txt
load data4.txt
load data5.txt
load data6.txt
load data7.txt
load mvr1.txt
load mvr2.txt
load mvr3.txt
load mvr4.txt
load mvr5.txt
load mvr6.txt
load mvr7.txt
table1=data1;
table2=data2;
table3=data3;
table4=data4;
table5=data5;
table6=data6;
table7=data7;
%initiating formation of random table
%IF MORE TABLE ADDED CHANGE HERE>>>>>
for i=1:n
    %forming a table using random method to
    %get value from every data tables randomly in a new matrix
    %"oldstat"(n*1)
    oldstat(i,1)=table1(randi(length(table1(:,1)),1),1);
    oldstat(i,2)=table2(randi(length(table2(:,1)),1),1);
    oldstat(i,3)=table3(randi(length(table3(:,1)),1),1);
    oldstat(i,4)=table4(randi(length(table4(:,1)),1),1);
    oldstat(i,5)=table5(randi(length(table5(:,1)),1),1);
    oldstat(i,6)=table6(randi(length(table6(:,1)),1),1);
    oldstat(i,7)=table7(randi(length(table7(:,1)),1),1);
end
for k=1:g
for j=1:n
    T(j)=0;
    C(j)=0;
    %starting objective analysis of the random system...
```



```

%after this, fitness will be tasted and crossover
%will be considered using Roulette Wheel and random point
selection
tim=zeros(1,length(N(:,1)));
for p=1:length(N(:,1))
    %initializing consideration of network setup
    for o=1:length(N(1,:))
        %counting total time for desired network basis
        if N(p,o)~=0
            tim(p)=tim(p)+oldstat(j,N(p,o));
        end
    end
end
end
%as the maximum value of the network will be considered.
T(j)=max(tim);

c=zeros(1,1);
%IF MORE TABLE ADDED CHANGE HERE>>>>>

for lr=1:length(table1(:,1))
    if oldstat(j,1)==table1(lr,1)
        for ir=1:length(mvr1(1,:))
            if table1(lr,ir+1)>=mvr1(3,ir)
                p(ir)=mvr1(2,ir)-mvr1(1,ir);
            else
                p(ir)=0;
            end
            c(1)=c(1)+table1(lr,ir+1)*mvr1(1,ir)+p(ir)*(table1(lr,ir+1)-
mvr1(3,ir));
        end
    end
end

for lr=1:length(table2(:,1))
    if oldstat(j,2)==table2(lr,1)
        for ir=1:length(mvr2(1,:))
            if table2(lr,ir+1)>=mvr2(3,ir)
                p(ir)=mvr2(2,ir)-mvr2(1,ir);
            else
                p(ir)=0;
            end
            c(2)=c(2)+table2(lr,ir+1)*mvr2(1,ir)+p(ir)*(table2(lr,ir+1)-
mvr2(3,ir));
        end
    end
end

for lr=1:length(table3(:,1))
    if oldstat(j,3)==table3(lr,1)
        for ir=1:length(mvr3(1,:))
            if table3(lr,ir+1)>=mvr3(3,ir)
                p(ir)=mvr3(2,ir)-mvr3(1,ir);
            else
                p(ir)=0;
            end
        end
    end
end

```

```

        end
        c(3)=c(3)+table3(lr,ir+1)*mvr3(1,ir)+p(ir)*(table3(lr,ir+1)-
mvr3(3,ir));
        end
    end
    end

    for lr=1:length(table4(:,1))
    if oldstat(j,4)==table4(lr,1)
        for ir=1:length(mvr4(1,:))
            if table4(lr,ir+1)>=mvr4(3,ir)
                p(ir)=mvr4(2,ir)-mvr4(1,ir);
            else
                p(ir)=0;
            end
        end
        c(4)=c(4)+table4(lr,ir+1)*mvr4(1,ir)+p(ir)*(table4(lr,ir+1)-
mvr4(3,ir));
        end
    end
    end

    for lr=1:length(table5(:,1))
    if oldstat(j,5)==table5(lr,1)
        for ir=1:length(mvr5(1,:))
            if table5(lr,ir+1)>=mvr5(3,ir)
                p(ir)=mvr5(2,ir)-mvr5(1,ir);
            else
                p(ir)=0;
            end
        end
        c(5)=c(5)+table5(lr,ir+1)*mvr5(1,ir)+p(ir)*(table5(lr,ir+1)-
mvr5(3,ir));
        end
    end
    end

    for lr=1:length(table6(:,1))
    if oldstat(j,6)==table6(lr,1)
        for ir=1:length(mvr6(1,:))
            if table6(lr,ir+1)>=mvr6(3,ir)
                p(ir)=mvr6(2,ir)-mvr6(1,ir);
            else
                p(ir)=0;
            end
        end
        c(6)=c(6)+table6(lr,ir+1)*mvr6(1,ir)+p(ir)*(table6(lr,ir+1)-
mvr6(3,ir));
        end
    end
    end

    for lr=1:length(table7(:,1))
    if oldstat(j,7)==table7(lr,1)
        for ir=1:length(mvr7(1,:))
            if table7(lr,ir+1)>=mvr7(3,ir)
                p(ir)=mvr7(2,ir)-mvr7(1,ir);
            else
                p(ir)=0;
            end
        end
    end
end

```

```

        end
        c(7)=c(7)+table7(lr,ir+1)*mvr7(1,ir)+p(ir)*(table7(lr,ir+1)-
mvr7(3,ir));
        end
    end
    end

    for st=1:l
        C(j)=C(j)+c(st);
    end
    if T(j)>ta
        D=cp;
    else
        D=0;
    end
    cid=c1*T(j)+D*(T(j)-ta);
    C(j)=C(j)+cid;
end
Ztmx=max(T);
Ztmn=min(T);
Zcmx=max(C);
Zcmn=min(C);
%initializing fitness check calculaitions
if Ztmx==Ztmn && Zcmx==Zcmn
    Wt=.5;
    Wc=.5;
elseif Ztmx~=Ztmn && Zcmx==Zcmn
    Wt=.1;
    Wc=.9;
elseif Ztmx==Ztmn && Zcmx~=Zcmn
    Wt=.9;
    Wc=.1;
else
    Vc=Zcmn/(Zcmx-Zcmn);
    Vt=Ztmn/(Ztmx-Ztmn);
    V=Vt+Vc;
    Wc=Vt/V;
    Wt=Vc/V;
end
%producing fitness values
for i=1:n
    f(i,1)=((Ztmx-T(i)+1)/(Ztmx-Ztmn+1)*Wt)+((Zcmx-C(i)+1)/(Zcmx-
Zcmn+1)*Wc);
end
%applying Roulette Wheel Selection
fmax=max(f);
fmin=min(f);
fav=mean(f);
newstat=oldstat;

%determining position of maximum and minimum fit setup
for i=1:n
    if f(i)==fmin
        aka=i;
    end
end

```

```

        elseif f(i)==fmax
            ao=i;
        end
    end
end
deter(k,:)=oldstat(ao,:);
Cost(k)=C(ao);
Time(k)=T(ao);
disp('Best fit data of this generation')
disp(oldstat(ao,:))
disp('Time')
disp(T(ao))
disp('Cost')
disp(C(ao))
%Roulette Elimination of least fit setup by best fit setup
for i=1:n
    if f(i)<fav*0.5
        newstat(i,:)=newstat(ao,:);
        T(i)=T(ao);
        C(i)=C(ao);
        f(i)=f(ao);
    end
end
end
numcross=pc*n; %number of possible crossover
numcross=round(numcross);
nummutate=pm*n*1; % number of possible mutation
nummutate=round(nummutate);
%generating random coupling for crossover
r1=randi([1 n/2], numcross/2,1);
r2=randi([(n/2+1) n], numcross/2,1);
crosspoint=randi(6,1);
%performing crossover
for i=1:length(r1)
    temp1=newstat(r1(i),1:crosspoint);
    temp2=newstat(r2(i),1:crosspoint);
    newstat(r1(i),1:crosspoint)=temp2;
    newstat(r2(i),1:crosspoint)=temp1;
end
end
%performing mutation
a=randi(1,1,nummutate);
%IF MORE TABLE ADDED, CHANGE HERE>>>>>>
for i=1:length(a)
    if a(i)==1

newstat(randi(n,1),a(i))=table1(randi(length(table1(:,1)),1),1);
        elseif a(i)==2

newstat(randi(n,1),a(i))=table2(randi(length(table2(:,1)),1),1);
        elseif a(i)==3

newstat(randi(n,1),a(i))=table3(randi(length(table3(:,1)),1),1);
        elseif a(i)==4

newstat(randi(n,1),a(i))=table4(randi(length(table4(:,1)),1),1);
        elseif a(i)==5

newstat(randi(n,1),a(i))=table5(randi(length(table5(:,1)),1),1);
        elseif a(i)==6

```

```

newstat(randi(n,1),a(i))=table6(randi(length(table6(:,1)),1),1);
    elseif a(i)==7

newstat(randi(n,1),a(i))=table7(randi(length(table7(:,1)),1),1);
    end
end

    if k>=2
        if rzcmn<=Zcmn && rztmn<=Ztmn
            oldstat=reservestat;
        end
    end
    rzcmn=Zcmn;
    rztmn=Ztmn;
    reservestat=oldstat;
    oldstat=newstat;
end
%FOR BEST FIT DATA TABLE
for i=1:g
bZtmx=max(Time);
bZtmn=min(Time);
bZcmx=max(Cost);
bZcmn=min(Cost);
%initializing fitness check calculaitions
if bZtmx==bZtmn && bZcmx==bZcmn
    bWt=.5;
    bWc=.5;
elseif bZtmx~=bZtmn && bZcmx==bZcmn
    bWt=.1;
    bWc=.9;
elseif bZtmx==bZtmn && bZcmx~=bZcmn
    bWt=.9;
    bWc=.1;
else
    Vc=bZcmn/(bZcmx-bZcmn);
    Vt=bZtmn/(bZtmx-bZtmn);
    V=Vt+Vc;
    bWc=Vt/V;
    bWt=Vc/V;
end
    fit(i,1)=((bZtmx-Time(i)+1)/(bZtmx-bZtmn+1)*Wt)+((bZcmx-
Cost(i)+1)/(bZcmx-bZcmn+1)*Wc);
end
fmax=max(fit);
fmin=min(fit);
%determining position of maximum fit setup
for i=1:g

    if fit(i)==fmax
        ao=i;
    end
end
disp('Best Operational Choice From Best Fit Table:')
    disp(deter(ao,:));
    disp('Related Cost:')

```

```

        disp(Cost(ao))
        disp('Related Time:')
        disp(Time(ao))
Mincost=min(Cost);
Mintime=min(Time);
for i=1:g
    if Cost(i)==Mincost
        point=i;
    end
end
disp('Best Operational Choice with respect to Cost:')
    disp(deter(point,:));
    disp('Related Cost:')
    disp(Mincost)
    disp('Related Time:')
    disp(Time(point))
for i=1:g
    if Time(i)==Mintime
        point=i;
    end
end
disp('Best Operational Choice with respect to Time:')
    disp(deter(point,:));
    disp('Related Cost:')
    disp(Cost(point))
    disp('Related Time:')
    disp(Mintime)
disp('Population in last generation:')
disp('Duration of activites:')
disp(oldstat)
favg=mean(fit);
lsr=length(deter(:,1));
tip=0
disp('Better Data set within Best Fit table ')
for i=1:lsr
    if fit(i)>=favg
        tip=tip+1;
    end
    disp('Data Set')
    disp(deter(i,:))
    disp('Time')
    disp(Time(i))
    Ctime(tip)=Time(i);
    Ccost(tip)=Cost(i);
    disp('Cost')
    disp(Cost(i))
end
end
plot(Time, Cost, 'bd ')

```