Economic Design of Exponentially Weighted Moving Average (EWMA) Chart with Variable Sampling Interval at Fixed Times (VSIFT) Scheme incorporating Taguchi Loss Function

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BY
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A thesis submitted to the Department of Industrial & Production Engineering, Bangladesh University of Engineering & technology, in partial fulfillment of the requirements for the degree of Master of Science in Industrial & Production Engineering

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CERTIFICATE OF APPROVAL

The thesis titled "**Economic Design of Exponentially Weighted Moving Average (EWMA) Chart with Variable Sampling Interval at Fixed Times (VSIFT) Scheme incorporating Taguchi Loss Function"** submitted by Ineen Sultana**,** Student no: 0412082009F has been accepted as satisfactory in partial fulfillment of the requirements for the degree of Master of Science in Industrial & Production Engineering on April 27, 2014.

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It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma

Ineen Sultana

To the Almighty

To my family

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All credits go to Allah, the most benevolent and the Almighty, for his boundless grace in successful completion of this thesis.

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ABSTRACT

Control charting is a graphical expression and operation of statistical hypothesis testing. Among various types of control chart Exponentially Weighted Moving Average (EWMA) chart is an widely used one and highly useful for not only detecting small changes in process parameters, but also in individual observation The objective of this thesis is to develop an economic statistical design of the exponentially weighted moving average (EWMA) chart using variable sampling intervals with sampling at fixed times (VSIFT) control scheme considering preventive maintenance and Taguchi Loss function to determine the values of the seven test parameters of the chart (i.e., the sample size, the fixed sampling interval, the number of subintervals between two consecutive sampling times, the warning limit coefficient, the control limit coefficient, and exponential weight constant and the time interval of preventive maintenance) such that the expected total cost per hour is minimized. It is because the performance of a production system depends on the breakdown-free operation of equipment and processes. Maintenance and quality control play an important role in achieving this goal. In addition to deteriorating with time, equipment may experience a quality shift (process moves to out-of-control state), which is characterized by a higher rejection rate and a higher tendency to fail the target value. That's why another prime concern of the thesis is to consider an integrated influence of joint optimization of preventive maintenance interval and control parameters incorporating the Taguchi Loss function in the design of EWMA control chart with VSIFT scheme. A mathematical model is given to analyze the cost of the integrated model before the optimization algorithm approach is used to find the optimal values of seven variables (n, h, w, k, η, L *and* t_{pm}) that minimize the hourly cost. Nelder-Mead downhill simplex method and Genetic algorithm approaches are applied to search for the optimal values of the seven test parameters for the economic statistical design of VSIFT EWMA chart, and then a hypothetical example and its solution are provided to have a better understanding about the demonstration of the proposed model.

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NOMENCLATURE

ARL_1 =average run length during in-control period

$C_{lp=}$ cost of lost production (Rs/job)

$C_{resetting=}$ cost of resetting

$T_{eval=}$ evaluation period

E [C_{CM}] FM1$_{=}$expected cost of corrective maintenance (CM) owing to failure mode1

E [$C_{PM]}$ $_{=}$expected cost of preventive maintenance (PM)

E [T_{Cycle}] = expected cycle length

E [T_1] =expected in-control period

t_0 =expected time spent searching for a false alarm

$t_{1=}$expected time to determine occurrence of assignable cause

E [$T_{restore]=}$ expected time to restore the process which may be moved out-of-control owing to machine degradation or external causes

E [TCQ]$_{process\text{-}failure}$ = expected total cost of quality owing to process failure

[ETCPUT] $_{=}$ expected total cost per unit time of integrated maintenance and quality policy

λ_1 = failure owing to external causes

λ_2 = failure owing to machine degradation

$C_{FCPCM\ =}$ fixed cost per CM (Rs/component)

C_{FCPPM} =fixed cost per PM (Rs/preventing component)

LC = maintenance personnel cost (Rs/h of preventing machine)

τ = mean elapse time from the last sample before the assignable cause to the occurrence of assignable cause when the maintenance and quality policies are integrated

$MTTR_{CM\ =}$ mean time required for corrective repair (h)

$MTTR_{PM\ =}$ mean time required for preventive repair (h)

$N_{f\ =}$ number of failures

$t_{PM\ =}$ preventive maintenance interval

P_{FM1}= probability of occurrence of failure owing to failure mode 1

P_{FM2} = probability of occurrence of failure owing to failure mode 2

λ = process failure rate

PR = production rate (job/h)

E [(C_{repair}) $_{FM2}$] =the expected cost of corrective maintenance owing to failure mode 2

$T_{resetting\ =}$ time to perform the resetting of the process which moves out-of-control owing to external reason

α = type I error probability

n =sample size

h = sample frequency.

k = control limit coefficient

g = the time required to take a sample and interpret

L= exponential weight constant

ABBREVIATIONS

GA	Genetic algorithm
FM	Failure Mode
PM	Preventive Maintenance
CM	Corrective Maintenance
MTTR	Mean Time to Repair
VSIFT	Variable sampling interval at Fixed Times
ETCPUT	Expected Total Cost Per Unit Time
EWMA	Exponentially weighted moving average
CTQ	Critical to Quality
TCQ	Total Cost of Quality
ATS	Average Time to Signal
SPC	Statistical process control
ARL	Average run length
USL	Upper Specification Limit
LSL	Lower Specification Limit
UCL	Upper Control Limit
FCPPM	Fixed Cost per Preventive Maintenance
FCPCM	Fixed Cost per Corrective Maintenance

APPENDIX

[L in control] determination

$$L_{\text{in control}} = PR* \frac{A}{d'^2}\int_{\mu-k\sigma\sqrt{\frac{L}{(2-L)n}}}^{\mu+k\sigma\sqrt{\frac{L}{(2-L)n}}}(x-\mu)^2 f(x)dx$$

$$= PR* \frac{A}{d'^2}\left(\frac{1}{\sqrt{2\pi}}*\frac{1}{\sigma\sqrt{\frac{L}{(2-L)n}}}\int_{\mu-k\sigma\sqrt{\frac{L}{(2-L)n}}}^{\mu+k\sigma\sqrt{\frac{L}{(2-L)n}}}(x-\mu)^2 e^{\frac{-(x-\mu)^2}{2*\sigma\sqrt{\frac{L}{(2-L)n}}^2}}\right)dx$$

Let, $\frac{x-\mu}{\sigma\sqrt{\frac{L}{(2-L)n}}} = z$

Therefore,

$$L_{\text{in control}} = PR* \frac{A}{d'^2}*\frac{\sigma^2 * L}{\sqrt{2\pi}*(2-L)n}\int_{-k}^{+k} z^2 e^{\frac{-z^2}{2}})dz$$

$$= PR* \frac{A}{d'^2}*\frac{\sigma^2 * L}{\sqrt{2\pi}*(2-L)n}\left\{-2ke^{\frac{-k^2}{2}} + \int_{-k}^{+k} e^{\frac{-z^2}{2}}dz\right\}$$

$$= PR* \frac{A}{d'^2}*\left\{\frac{\sigma^2 * L}{\sqrt{2\pi}*(2-L)n}*(-2ke^{\frac{-k^2}{2}}) + \frac{\sigma^2 * L}{(2-L)n}*(1-2\phi(-k))\right\}$$

$$[L_{\text{in control}}] = PR* \frac{A}{d'^2}*\frac{L*\sigma^2}{(2-L)n}\left[1-2\phi(-k) - \frac{2k}{\sqrt{2\pi}}e^{-\frac{k^2}{2}}\right]$$

[L out of control] determination

$$L_{\text{out of control}} = PR* \frac{A}{d'^2}\int_{-\infty}^{\infty}(x'-\mu)^2 f(x')dx' - \int_{\mu-k\sigma\sqrt{\frac{L}{(2-L)n}}}^{\mu+k\sigma\sqrt{\frac{L}{(2-L)n}}}(x'-\mu)^2 f(x')dx'\}$$

$$= A' - B'$$

$$A' = PR* \frac{A}{d'^2}\left(\frac{1}{\sqrt{2\pi}}*\frac{1}{\sigma\sqrt{\frac{L}{(2-L)n}}}\int_{-\infty}^{\infty}(x-\mu)^2 e^{\frac{-(x-\mu-\delta\sigma)^2}{2*\sigma\sqrt{\frac{L}{(2-L)n}}^2}}\right)dx$$

Let, $\frac{x-\mu-\delta\sigma}{\sigma\sqrt{\frac{L}{(2-L)n}}} = z$

Therefore,

$$A' = PR* \frac{A}{d'^2}*\frac{\sigma^2 * L}{\sqrt{2\pi}*(2-L)n}\int_{-k-\delta\sqrt{\frac{(2-L)n}{L}}}^{k-\delta\sqrt{\frac{(2-L)n}{L}}}(\delta\sigma + \sigma\sqrt{\frac{L}{(2-L)n}})^2 e^{\frac{-z^2}{2}})dz$$

$= \text{PR}* \frac{\text{A}}{\text{d}'^2}*\int_{-k-\delta\sqrt{\frac{(2-L)n}{L}}}^{k-\delta\sqrt{\frac{(2-L)n}{L}}}(\delta^2\sigma^2 + \frac{\sigma^2*L}{(2-L)n}^2 z^2 + 2\delta\sigma^2 * \sqrt{\frac{L}{(2-L)n}} * z)e^{\frac{-z^2}{2}}\,dz\}$

$= \text{PR}* \frac{\text{A}}{\text{d}'^2} * \{\delta^2\sigma^2 \int_{-k-\delta\sqrt{\frac{(2-L)n}{L}}}^{k-\delta\sqrt{\frac{(2-L)n}{L}}} e^{\frac{-z^2}{2}}dz + \frac{\sigma^2*L}{(2-L)n}\int_{-k-\delta\sqrt{\frac{(2-L)n}{L}}}^{k-\delta\sqrt{\frac{(2-L)n}{L}}} z^2 e^{\frac{-z^2}{2}}dz + 2\delta\sigma^2 *$
$\sqrt{\frac{L}{(2-L)n}} * \int_{-k-\delta\sqrt{\frac{(2-L)n}{L}}}^{k-\delta\sqrt{\frac{(2-L)n}{L}}} z e^{\frac{-z^2}{2}}dz\}$

$= \text{PR}* \frac{\text{A}}{\text{d}'^2}*(\delta^2\sigma^2 + \frac{\sigma^2*L}{(2-L)n})$

Similar algebraic manipulation shows that,

$\text{B}' = PR * \frac{\text{A}}{\text{d}'^2} * [(\delta^2\sigma^2 + \frac{\sigma^2*L}{(2-L)n})*\{\phi(k-\delta\sqrt{\frac{(2-L)n}{L}}) - \phi(-k-\delta\sqrt{\frac{(2-L)n}{L}})\}+$
$\frac{1}{\sqrt{2\pi}}[e^{\frac{-(k-\delta\sqrt{\frac{(2-L)n}{L}})^2}{2}}\{\frac{\sigma^2*L}{(2-L)n} * (-k+\delta\sqrt{\frac{(2-L)n}{L}}) + 2\delta\sigma^2\sqrt{\frac{L}{(2-L)n}}\}] -$
$\frac{1}{\sqrt{2\pi}}[e^{\frac{-(k+\delta\sqrt{\frac{(2-L)n}{L}})^2}{2}}\{\frac{-\sigma^2*L}{(2-L)n} * (k+\delta\sqrt{\frac{(2-L)n}{L}}) - 2\delta\sigma^2\sqrt{\frac{L}{(2-L)n}}\}]]$

Therefore,

$\text{A}' - \text{B}' = PR * \frac{\text{A}}{\text{d}'^2}\ [(\delta^2\sigma^2 + \frac{\sigma^2*L}{(2-L)n})*\{1 - \phi(k-\delta\sqrt{\frac{(2-L)n}{L}}) + \phi(-k-$
$\delta\sqrt{\frac{(2-L)n}{L}})\} + \frac{\sigma^2*L}{\sqrt{2\pi}(2-L)n}\{(k+\delta\sqrt{\frac{(2-L)n}{L}}) * e^{\frac{-(k-\delta\sqrt{\frac{(2-L)n}{L}})^2}{2}} + (k-\delta\sqrt{\frac{(2-L)n}{L}}) *$
$e^{\frac{-(k+\delta\sqrt{\frac{(2-L)n}{L}})^2}{2}}\}]$

$L_{\text{out of control}} = PR * \frac{\text{A}}{\text{d}'^2} * \frac{\sigma^2*L}{(2-L)*n}[\left(1+\delta^2 * \frac{(2-L)n}{L}\right) * \{1 - \phi(\text{k} - \delta * \sqrt{\frac{(2-L)n}{L}})$
$+ \phi(-\text{k} - \delta * \sqrt{\frac{(2-L)n}{L}})\} + (\frac{k+\delta*\sqrt{\frac{(2-L)n}{L}}}{\sqrt{2\pi}})*e^{\frac{-(k-\delta*\sqrt{\frac{(2-L)n}{L}})^2}{2}} + \frac{k-\delta*\sqrt{\frac{(2-L)n}{L}}}{\sqrt{2\pi}})*e^{\frac{-(k+\delta*\sqrt{\frac{(2-L)n}{L}})^2}{2}}$

CHAPTER I

INTRODUCTION

The close relationship between quality and maintenance of manufacturing systems has contributed to the development of integrated models, which use the concept of statistical process control (SPC) and maintenance. Such models not only help to improve quality of products but also lead to lower maintenance cost. In today's competitive arena, quality plays an important role in the business life of an organization. It is obvious that continuous reduction in variation of products and services is necessary for achieving and maintaining a desired level of quality. Since reduction in equipment performance leads to reduction in product quality, a suitable maintenance policy could reduce process variation and help to increase product quality. The close relationship between quality and maintenance has led researchers to develop integrated models, which are more realistic in practice. These models are developed with the aim of reducing total cost of quality.

A control chart is one of the most important techniques of statistical process control. The sample means of measurements of a quality characteristic in samples taken from the production process are plot on this chart. It can be applied to reduce the variability of a process and also used to maintain a process in a state of statistical control. It is a very useful process monitoring technique. Traditionally, control charts are designed with respect to statistical criteria only. Its performance may be unsatisfactory from the economic viewpoint. The economic design of control charts is used to determine various design parameters that minimize total economic costs. The effect of production lot size on the quality of the product may also be significant. If the production process shifts to an out-of-control state at the beginning of the production run, the entire lot will contain more defective items. Hence, it is wiser to reduce the production cycle to decrease the fraction of defective items and, thus, improve output quality. On the other hand, reduction of the production cycle may result in an increase in costs due to frequent setups. A balance must be maintained so that the total cost is minimized. The design of a control chart has economic consequences. The costs of sampling and testing, costs associated with investigating out of control signals and possibly correcting assignable causes and costs of allowing nonconforming units to reach the consumer are all affected by the choice of control chart parameters. Therefore, it is logical to consider the design of control chart from an economic view point. Besides, the performance of a production system strongly depends on the breakdown-free operation of equipment and processes. The performance can be improved if these breakdowns can be minimized in a cost-effective manner. Maintenance and quality control play important roles in achieving this goal. An appropriate Preventive Maintenance (PM) policy not only reduces the probability of machine failure but also improves the performance of the machine in terms of lower production costs and higher product quality. Similarly, an appropriately designed quality control chart may help in identifying any abnormal behavior of the process, thereby helping to initiate a restoration action. However, both PM and quality control add costs in terms of down time, repair/replacement, sampling, inspection, etc. Traditionally, these two activities have been optimized independently. However, researchers have shown that a relationship exists between equipment maintenance and process quality (Pandey et al.2010), Hence joint consideration of these two shop-floor policies has become more cost-effective in improving the performance of the production system.

1.1 Rationale of the Study

In any industry it is very important to control the quality of the product so control chart is a must in this case. But if the control chart is designed without economic viewpoint industries could suffer from huge costs. To overcome this problem many researchers have studied economic design of control chart for many years. It has been observed that an extensive research has already been conducted both in the area of process quality control/economic design of control chart and equipment maintenance/maintenance management separately. The most interesting fact is that, though these two areas have already been proven to be highly correlated (Pandey et al.2010), very few models have investigated the joint consideration of equipment maintenance and process quality. Some very recent literature indicates that such joint consideration has started receiving attention from the research community. Though the integrated models stated above focus on the integration of process quality and preventive maintenance action, most of them ignored the possibility of an equipment failure in terms of machine breakdown or improper functioning of the equipment which results in poor product quality and call for maintenance action. Rather they focused on determination of a warning limit or fixed time interval beyond which planned maintenance will be carried out. The linkage between declining performance of a machine and process is seldom studied. Again except very few, Exponentially Weighted Moving Average (EWMA) chart is not considered for these integrated models which is very effective in determining the small shifts. Another gap of the past researches is that most of the control chart model used to calculate their in control and out control cost based on traditional goal post view. In case of sampling interval policy, most promising one VSIFT (Variable Sampling Interval at Fixed Times) action has not yet been explored by many researchers. Considering the importance of EWMA control chart for detecting small shifts and the significance of integration of process quality along with preventive maintenance and corrective maintenance, the effectiveness of VSIFT sampling interval policy and Taguchi Loss function to calculate the in control and out of control cost an integrated general mathematical cost model is developed to optimize the design parameter and preventive maintenance schedule.

1.2 Objectives of the study

The objective of this thesis is to present an integrated model that can be used to minimize the expected total cost of process failures, inspection, sampling, and corrective maintenance/preventive maintenance (CM/PM) action by jointly optimizing maintenance and EWMA quality control chart parameters. The specific objectives of this research are

- To develop an integrated model of process quality control and different maintenance actions (corrective and preventive) for joint optimization of preventive maintenance interval and EWMA control chart parameters
- To apply Variable Sampling Interval at Fixed Times (VSIFT) policy in case of sampling interval of EWMA chart
- To introduce Taguchi loss function in calculating the in-control and out of control cost of EWMA chart
- To optimize the joint model of preventive maintenance and process quality with a suitable non linear optimization technique

- To design and develop a numerical hypothetical example for clear understanding of the model

This thesis however, presents possible clues in the development of an integrated mathematical model to design EWMA control chart for process control maintaining preventive maintenance schedule by providing mathematical results to help on understanding formulation and analysis of such mathematical model.

1.3 Outline of Methodology

The research work is theoretical in nature. A mathematical cost model is developed that can be used to minimize the expected total cost of process failures, inspection ,sampling and corrective maintenance/Preventive maintenance action by jointly optimizing maintenance and quality control chart parameter. The cost model is composed of some mathematical equations which are used to determine the numerical values of different control chart design parameters and preventive maintenance schedule (sample size, fixed sampling interval, control limit coefficient, warning limit coefficient, number of subintervals between two consecutive sampling times and preventive maintenance interval) The proposed research methodology is outlined below:

- Machine and process failure is categorized such that whenever a complete machine failure occurs, a corrective maintenance action will restore the machine. In parallel process is monitored through a EWMA control chart. Whenever a shift is detected due to machine degradation a corrective action will be carried out, if the shift is due to external reasons, resetting of the process is done. Preventive maintenance is also carried out at some fixed interval which is also a decision variable along with control chart parameters.
- The cost for carrying out both corrective and preventive maintenance will be determined
- Machine failure will be assumed to follow two parameter weibull distribution whereas process failure will be assumed to follow exponential distribution
- EWMA chart with VSIFT policy will be considered to detect the process shift
- Lorenzen-Vance general cost model will be adopted to determine the cost of process failure which in addition with the cost of preventive and corrective

maintenance constitutes the overall cost of the model which is to be minimized.

- ARL value of EWMA chart will be determined by simulation approach proposed by Lucas and Sacucci.
- In control and out of control cost for EWMA chart will be developed from the concept of Taguchi loss function.
- A numerical hypothetical example problem is considered to illustrate and explain the proposed model

CHAPTER II

LITERATURE REVIEW

The performance of a production system strongly depends on the breakdown-free operation of equipment and processes. The performance can be improved if these breakdowns can be minimized in a cost-effective manner. Maintenance and quality control play important roles in achieving this goal. An appropriate Preventive Maintenance (PM) policy not only reduces the probability of machine failure but also improves the performance of the machine in terms of lower production costs and higher product quality. Similarly, an appropriately designed quality control chart may help in identifying any abnormal behavior of the process, thereby helping to initiate a restoration action. However, both PM and quality control add costs in terms of down time, repair/replacement, sampling, inspection, etc. Traditionally, these two activities have been optimized independently. However, researchers have shown that a relationship exists between equipment maintenance and process quality (Pandey et al. 2010), and joint consideration of these two shop-floor policies may be more cost-effective in improving the performance of the production system.

Though recent literature indicates that such joint consideration has started receiving attention from the research community but it all started when at first Tagaras (1988) developed an integrated cost model for the joint optimization of process control and maintenance. Following him, Rahim (1993) jointly determined the optimal design parameters of an X- bar control chart and preventive maintenance time for a production system with an increasing failure rate. They generalized the model for the economic design of X- bar control charts of Duncan (1956), starting from the more recent papers of Lorenzen and Vance (1986) and Banerjee and Rahim (1988). The classical model of Duncan (1956) and its several extensions including the unified model of Lorenzen and Vance (1986) assumed exponentially distributed in-control periods and provided uniform sampling schemes. Banerjee and Rahim (1988), however, assumed a Weibull-distributed in-control period having an increasing failure rate and used variable sampling intervals. This article was an extension of the work of Banerjee and Rahim (1988), where a general distribution of in-control periods having an increasing failure rate was assumed and the possibility of age-dependent repair before failure was considered. A general distribution of the in-control period was considered and the salvage value of the equipment was introduced. The model allowed the possibility of age-dependent replacement before failure. A replacement before failure was meaningful only when such a replacement yields economic benefits. Intuitively, the residual life beyond a certain age for systems involving increasing hazard rate shock models would be rather short. Consequently, more frequent sampling could be necessary after the system attains a certain age. This, in turn, might increase the operational cost as a result of frequent sampling. Therefore, they argued that, it is conceivable that terminating a production cycle at some time beyond this age might yield additional economies. A truncated production cycle was defined to be a production cycle which terminates after the detection of a failure or at a certain pre specified age, whichever comes first. The question of replacement before failure did not arise for the Markovian shock model because of its memory less property. Following an approach similar to that of Banerjee and Rahim (1988), the focus of their study was to propose a manner by which the frequency of sampling was to be regulated, while taking into account the underlying probability distribution of the in-control duration. The criterion for choosing the sampling plan was that the expected cost per hour of operation should be minimized when the lengths of the sampling intervals were chosen in such a way as to maintain a constant integrated hazard over each sampling interval. Several different truncated and non truncated probability models were chosen. It was proposed that economic benefits could be achieved by adopting a non uniform inspection scheme and by truncating a production cycle when it attains a certain age. Numerical examples were presented to support this proposition. Finally, the effect of model specification in the choice of failure mechanism was investigated.

Rahim (1994) presented a model for jointly determining an economic production quantity, inspection schedule and control chart design of an imperfect production process. The process was subjected to the occurrence of a non- Markovian shock having an increasing failure rate. The product quality of the process was monitored under the surveillance of an X-bar control chart. The objective was to determine the optimal control chart design parameters and production quantity so as to minimize the expected total cost (the quality control cost and inventory control cost) per unit time. Both uniform and non- uniform inspection schemes were considered. For non-uniform inspection schemes, the lengths of the inspection intervals were regulated to maintain a constant integrated hazard rate over each inspection interval. Examples of weibull shock models having increasing failure rate were provided. It was shown that the non-uniform and decreasing process inspection intervals scheme resulted in a lower cost than the uniform inspection scheme.

Ben-Daya and Duffuaa (1995) discussed the relationship and interaction between maintenance, production and quality. Models relating these three important components of any manufacturing system had been briefly reviewed. Two approaches had been proposed for linking and modeling the relationship between maintenance and quality so that they could be jointly optimized. One approach was based on the idea that maintenance affects the failure pattern of the equipment and that it should be modeled using the concept of imperfect maintenance. The second approach was based on Taguchi's approach to quality. Chiu and Huang (1996) studied the economic design of X-bar control charts in a situation in which the duration time that the process remains in the in-control state follows a general distribution which has an increasing hazard rate. In this situation, the active and persistent action for quality control was to design a process in which a preventive maintenance procedure was performed periodically. They addressed first the relationship between preventive maintenance and x-bar control charts. A cost function which was opposed to those given by Banerjee and Rahim (1993) and by Hu (1994) is derived. The computational results indicated that the proposed model under a preventive maintenance policy had a lower expected total cost per hour than have those of Banerjee and Rahim's and Hu's Weibull shock models. Numerical examples also demonstrated that the model has great flexibility when applied in the situation previously mentioned. They also presented the advantages of the combination of a preventive maintenance policy and x-bar control charts and concluded that a preventive maintenance policy performed under a certain condition could be particularly instrumental in reducing the expected total cost per hour.

Ben-Daya (1999) argued that production quantity, product quality, and the maintenance of the production process are interrelated problems. Traditionally, these three problems used to be dealt with separately. There is a need to develop models that capture the interdependence between these three main components of any modern production process, and allow their joint optimization. He developed an integrated model for the joint optimization of the economic production quantity, the economic design of X- bar control chart and the optimal maintenance level. This was done for a deteriorating process where the in-control period follows a general probability distribution with increasing hazard rate. The proposed model consist of the four different costs: the production setup cost; the inventory holding cost; the quality control cost; and the preventive maintenance cost. In the proposed model, the Preventive Maintenance (PM) actions were supposed to change the failure pattern of the equipment. It was assumed that, after PM, the age of the system would be reduced proportional to the PM level. This reduction in the age of the equipment would affect the time to shift distribution and consequently the design of the control chart. It would also affect the length of the production run and consequently the EPQ. Thus the modeling of the effect of maintenance on the time shift distribution provided the underlying link and allowed the integration of EPQ, economic design of control chart, and the optimization of the preventive maintenance effort. Compared to the case with no PM, the extra cost of maintenance results in lower quality control cost which would lead to lower overall expected cost. These issues were illustrated using an example of a Weibull shock model with an increasing hazard rate.

Cassady et al. (2000) argued that the relationship between equipment maintenance and product quality is an important one that has yet to be sufficiently explored and exploited. Therefore, they set their objective as to find methods of utilizing the maintenance-quality relationship to achieve productivity gains. With that objective in mind, the paper addressed a preliminary study into how existing preventive maintenance and statistical process control technologies could be combined to achieve greater productivity gains than could be achieved using neither or one. A combined control chart-preventive maintenance strategy was defined for a process which shifts to an out of control condition due to a manufacturing equipment failure. An X bar chart was used in conjunction with an age-replacement preventive maintenance policy to achieve a reduction in operating costs that was superior to the reduction achieved by using only the control chart or the preventive maintenance policy. This superior cost performance was demonstrated using a simulation-optimization approach.

Ben- Daya and Rahim (2000) developed an integrated model for the joint optimization of the maintenance level and the economic design of X-bar chart. The paper provided a model for incorporating the effects of preventive maintenance on quality control charts. The model allowed the joint optimization of quality control charts (number of inspections, sample size, sampling intervals and control limit) and preventive maintenance level to minimize total expected cost. The model assumed imperfect maintenance, meaning that maintenance reduce failure rates but does not reduce it to that of a new system. It was assumed that the reduction in age of the system is proportional to the PM level used. This change in the shift distribution affected directly quality control costs thus providing the underlying link between maintenance and quality. The proposed model had been developed for a process having a general shift distribution with increasing hazard rates. The PM schedule was coordination with that of the quality control inspections. The lengths of the sampling intervals were chosen such that the integrated hazard over each interval should be equal. In the proposed model, sampling and PM are performed concurrently, which was varied from general policies where sampling and PM are done at different intervals. These restrictions were imposed for mathematical simplicity and operational convenience (i.e., one operator does two jobs at the same time). Conducting PM together with monitoring inspection might not be the best policy. Other policies were possible and were being investigated. An example was used to show that the proposed model captured the interdependence between quality control parameters and maintenance level. In particular, the example showed that higher PM levels lead to more reduction in quality control costs. If the savings in these costs compensate for the added maintenance cost, the overall cost would be reduced compared to the no PM case. Comparison between uniform and non-uniform schemes with and without PM and effects of different shift distribution parameters had also been investigated. Finally they argued that maintenance and quality control were interrelated but have been treated separately in the past and there were few attempts to integrate them in one model. So their purpose was to provide a framework for capturing their interactions leading to models for their joint optimization. Lee and Rahim (2001) also investigated the joint design of maintenance and SPC and suggested that the integrated model could result in significant cost savings.

Lindeman et al. (2005) demonstrated the value of integrating Statistical Process Control and maintenance by jointly optimizing their policies to minimize the total costs associated with quality, maintenance, and inspection. In their model, Statistical Process Control monitored the equipment and provided signals indicating equipment deterioration, while Planned Maintenance was scheduled at regular intervals to preempt equipment failure. The determination of an unstable process, via Statistical Process Control, resulted in an early Reactive Maintenance to restore the equipment. Otherwise a Planned Maintenance was supposed to occur after a specified period of operation. In this sense, they propose an ‘‘adaptive’’ maintenance policy, where the maintenance schedule adapted to the stability of the process. The completion of maintenance was supposed to return the process to its original operating condition and resulted in a process renewal. The model derived and optimal policy to minimize the cost per unit time. Finally, a sensitivity analysis was conducted to develop insights into the economic and process variables that influence the integration efforts.

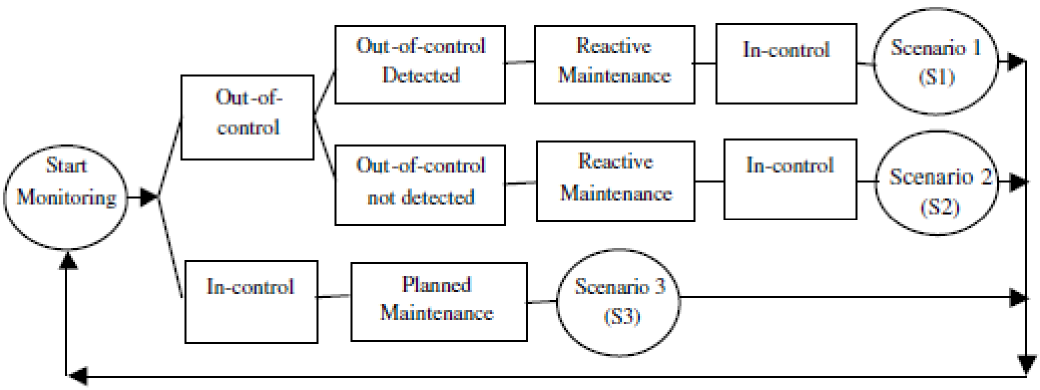


Fig.2.1 Three scenarios proposed by Lindeman et al. (2005)

Three scenarios were considered in their paper as showed in Fig.2.1. They assumed that the process begins in-control and inspections occur after h hours of production to determine whether the process has shifted from an in-control to an out-of-control state. Sometime between the jth and (j+1)th sampling interval an assignable cause occurs and the process shifts to an out-of-control state. The process continue to operate; however, the control chart does not detect an out-of-control condition until the (j +i)th sample. A time lag is associated with collecting the data and plotting the results on the chart. The control chart then signals an out-of-control condition and a search for an assignable cause takes place to validate the signal. A valid control-chart signal then results in Reactive Maintenance that restores the equipment to a ‘‘good-as new’’ condition (a renewal) this is scenario 1. In Scenario 2 in Fig.2.1, the process shifts to an out-of-control state, but the control chart does not signal an out-of-control condition before the Planned Maintenance. As in Scenario 1, the process begins in an in-control state and sometime between the jth and (j +1)th sampling interval, the process shifts to an out-of-control state. However, the process continues to operate because the control chart does not detect an out-of-control condition. At the (k +1)th sampling interval, maintenance begins, and the out-of-control state is identified. They considered that to be Reactive Maintenance because the out-of-control condition occurred before the scheduled maintenance and additional time and expense would be incurred to identify and resolve the equipment problem. Completing the maintenance causes the process to renew and return to the in-control state. In Scenario 3 in Fig.2.1, the process still remains in an in-control state at the time of the Planned Maintenance. The Planned Maintenance would take place at the (k + 1)th sampling interval to preempt a process failure. Typically, the activities associated with planned maintenance are less costly than those associated with Reactive Maintenance because preparation activities can be conducted off-line before the Planned Maintenance. Finally, after maintenance completion, the process renews itself. The approach assumed that the process monitoring and maintenance schedule followed a rolling schedule.

Kuo (2006) studied the joint machine maintenance and product quality control problem of a finite horizon discrete time Markovian deteriorating, state unobservable batch production system. In the paper, he used dynamic programming (DP) to find the optimal machine maintenance and product quality control policy for a finite horizon discrete time Markovian deteriorating, state unobservable batch production system. Unlike previous studies, both the timing of the sampling action and the sample size were directly included in the action space of dynamic programming model of the system. Using the posterior probability of the machine being in the out-of-control state as our DP modeling state, he formulated the system as a partially observable Markov decision process and derived some properties of the optimal value function which enabled him to efficiently search the optimal maintenance and quality control policy to minimize the expected total discounted system cost.

Al-Ghazi et al. (2007) incorporated Taguchi's quadratic loss function in the economic design of the control chart. This was done by redefining the in-control and out-of-control costs using Taguchi's loss function in the general model for the economic design of *x- bar*-control charts developed by Banerjee and Rahim (1993). Both cases of increasing hazard rate (Weibull failure rate) and constant hazard rate (exponential distribution) was presented and followed by numerical examples. Finally, sensitivity analysis was conducted to study the effect of important parameters on the cost. Panagiotidou and Tagaras (2007) presented an economic model for the optimization of preventive maintenance in a production process with two quality states. The equipment starts its operation in the in-control state but it might shift to the out-of-control state before failure or scheduled preventive maintenance. The time of shift and the time of failure were generally distributed random variables. The two states were characterized by different failure rates and revenues. They first derived the structure of the optimal maintenance policy, which was defined by two critical values of the equipment age that determine when to perform preventive maintenance depending on the actual (observable) state of the process. They then provided properties of the optimal solution and showed how to determine the optimal values of the two critical maintenance times accurately and efficiently. The proposed model and, in particular, the behavior of the optimal solution as the model parameters and the shift and failure time distributions change were illustrated through numerical examples.

Zhou and Zhu et al. (2008) developed an integrated model of control chart and maintenance management with reference to the integrated model proposed by Linderman et al. (2005). In their model, control chart was used to monitor the equipment and to provide signals that indicate equipment deterioration, while Planned Maintenance was scheduled at regular intervals to pre-empt equipment failure. Based on Alexander's cost model, they investigated the economic behavior of the integrated model, and gave an optimal design for determining the four policy variables (the sample size (n), the sampling interval (h), scheduled sampling times (k), control limit, (L)) that minimize hourly cost. The integrated model espoused the framework as shown in Fig. 2.2.

The process begins with an in-control state with a Process Failure Mechanism that follows a Weibull distribution. They assumed that process inspections with control chart occur after h hours of production to determine whether the process has shifted from an in-control to an out-of-control state. The quality characteristic was measured and plotted on a control chart to assess the state of the process. If the control chart does not signal an out-of-control condition after k inspection intervals, then scheduled or Planned Maintenance occurs at the (k + 1)th sampling interval. However, if the control chart signals an out-of-control condition at any of the j inspections, a search for an assignable cause takes place to validate the signal. A valid control-chart signal then results in Reactive Maintenance and a false signal results in Compensatory Maintenance. They assumed that the completion of Planned, Reactive and Compensatory maintenance restores the equipment to a “good-as-new” condition (a renewal). As shown in Fig. 2.2., the integrated model might result in four different scenarios. In S1, the process begins with an “in-control” state and inspections occur after h hours of monitoring as to whether the process has shifted from an “in-control” to an “out-of-control” state. And there is an alert signal in the control chart before the scheduled time when maintenance should be performed. But if the signal is false, that is to say, the process is still “in-control”. Since searching and determining false signal take time and incur cost, Compensatory Maintenance would be performed. Similar to S1, there is also a signal in S2. While the signal is valid and the process shifts to an “out-of-control” state, it results in Reactive Maintenance. In S3 and S4, no signal occurs in the control chart before the scheduled time. Then at the (k + 1) th sampling interval, appropriate maintenance should be arranged. In S3, the process is always “in-control”, and Planned Maintenance was suggested to be performed. When the process shifts to an “out-of-control” state in S4, Reactive Maintenance would take place because the “out-of-control” condition occurred before the scheduled time and additional time and expense would be incurred to identify and solve the equipment problem.

Panagiotidou and Tagaras (2008) developed an economic model for the optimization of maintenance procedures in a production process with two quality states. In addition to deteriorating with age, the equipment might experience a jump to an out-of-control state (quality shift), which was characterized by lower production revenues and higher tendency to failure.

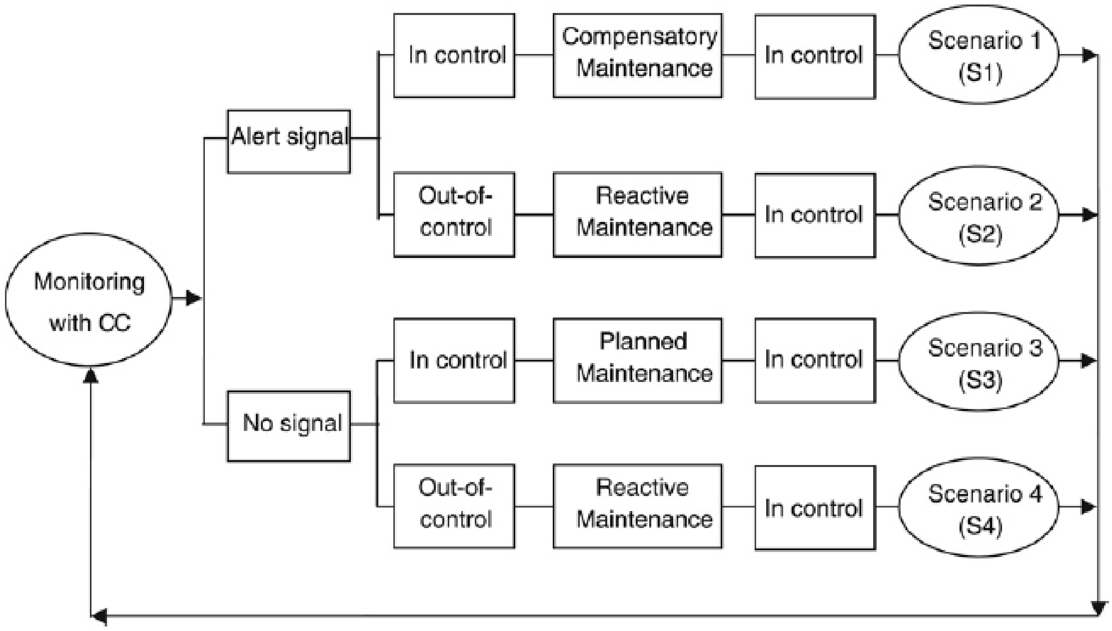


Fig.2.2 Four scenarios proposed by Zhou and Zhu et al. (2008)

The times to quality shift and failure were allowed to be generally distributed random variables. They considered two types of maintenance: minimal maintenance (MM) that upgraded the quality state of the equipment without affecting its age and perfect preventive maintenance (PM) that fully upgraded the equipment to the as-good-as-new condition. They derived the expression for the expected profit per time unit and they investigated, through a large number of numerical examples, the type of the optimal solution. It was concluded that in practically every case the optimal maintenance policy was an extreme one: it either calls for immediate MM as soon as a quality shift occurs (active policy) or it allows operation in the out-of-control state until the time of a scheduled PM action (passive policy).

Yeung et al. (2008) extended the initial preliminary investigation of this idea of using an X-bar chart in conjunction with an age-replacement preventive maintenance policy. They formulated a partially observable, discrete-time Markov decision process in order to obtain the near-optimal combined preventive maintenance/statistical process control policy that minimized the costs associated with maintenance, sampling, and poor quality. They developed transition probabilities for the various states of the infinite horizon problem and a solution algorithm for finding the best policy in polynomial time by finding a control limit on the sampling policy. They also performed sensitivity analysis on the decision variables for each of the various input parameters. It was shown that in every case a combined PM and SPC policy was the most cost efficient.

Panagiotidou and Nenes (2009) considered for the first time the possibility of combining, and optimizing economically, PM actions and quality control decisions using an adaptive sampling scheme. The paper proposed a model for the economic design of a variable-parameter (Vp) Shewhart control chart used to monitor the mean in a process, where, apart from quality shifts, failures might also occur. Quality shifts result in poorer quality outcome, higher operational cost and higher failure rate. Thus, removal of such quality shifts, besides improving the quality of the outcome and reducing the quality cost, is also a preventive maintenance (PM) action since it reduces the probability of a failure and improves the equipment reliability. The proposed model allowed the determination of the scheme parameters that minimize the total expected quality and maintenance cost of the procedure. The monitoring mechanism of the process employed an adaptive Vp-Shewhart control chart. To evaluate the effectiveness of the proposed model, its optimal expected cost was compared against the optimum cost of a fixed-parameter (Fp) chart. Mehrafrooz et al. (2011) presented an integrated model which considered complete failure and planned maintenance simultaneously. This model leads to six different scenarios as shown in Fig.2.3. A new procedure for calculating average cost per time unit was also presented. Through a numerical example, effects of the model parameters on the average cost were studied and it is shown that integrated model inquired lower cost than the planned maintenance model.

Chen et al. (2011) presented an integrated model for combining the preventive maintenance and the economic design of *x*-bar control charts using the Taguchi loss function. They argued that in a traditional system, a product would be accepted if the product measurement meets the specification requirement and there are no quality losses. The quality losses tend to be constant if the product measurements beyond the quality specification limit. But they referred to Taguchi who suggested that any deviation from characteristic's target value results in a loss. If a characteristic measurement is the same as the target value, the loss is zero.

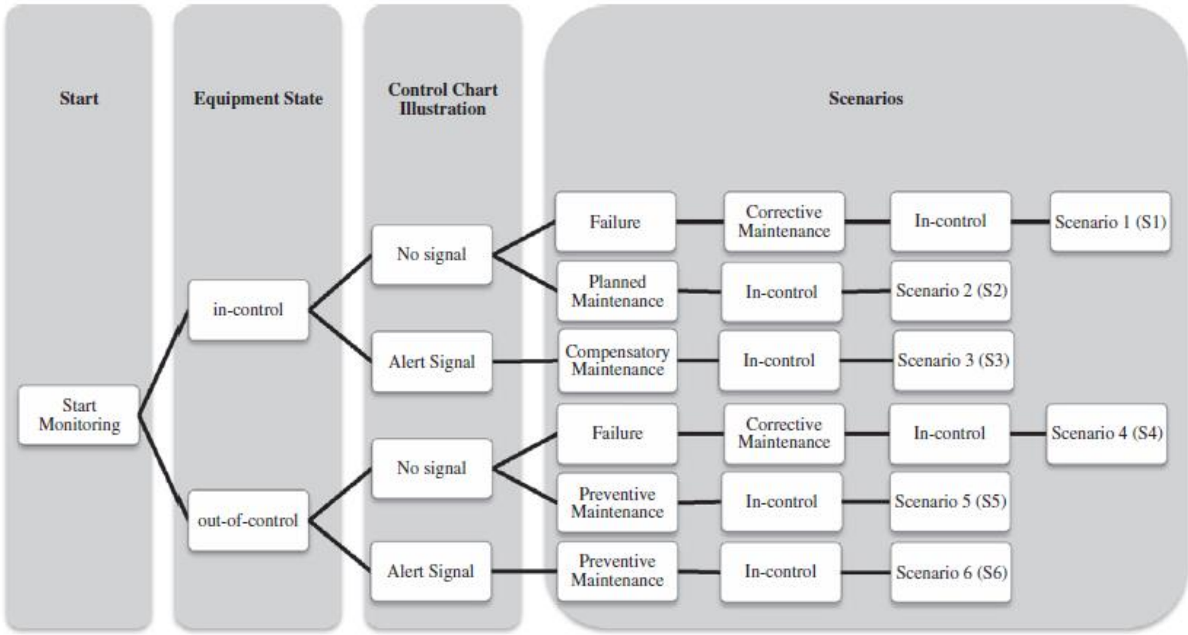


Fig. 2.3 Scenarios proposed by Mehrafrooz et al. (2011)

This concept of the Taguchi's loss function was employed in their research to construct the quality losses for economic design of *X-bar* control chart and PM consideration. The maintenance activities were coordinated with the statistical characteristics of the sampling results. The model assumed that preventive maintenance was performed on the basis of sampling characteristic. Finally, a numerical experiment was conducted to investigate the model's working underlying the effect of preventive maintenance on the quality control costs.

Pandey et al. (2012) introduced a new integrated approach for joint optimization of maintenance (preventive maintenance interval) and process control policy (control chart parameters) using "Taguchi Loss Function" and a way to categorize the machine and process failure. Two failure modes were considered (showed in Fig.2.4.), failure mode 1 leads to immediate breakdown of the machine. This machine failure was assumed to follow a two parameter weibull distribution. Failure mode 2 could be caused either from external reasons or malfunction of the machine to some extent which immediately affects the process mean. To monitor these types of shift an X-bar control chart was suggested to use.

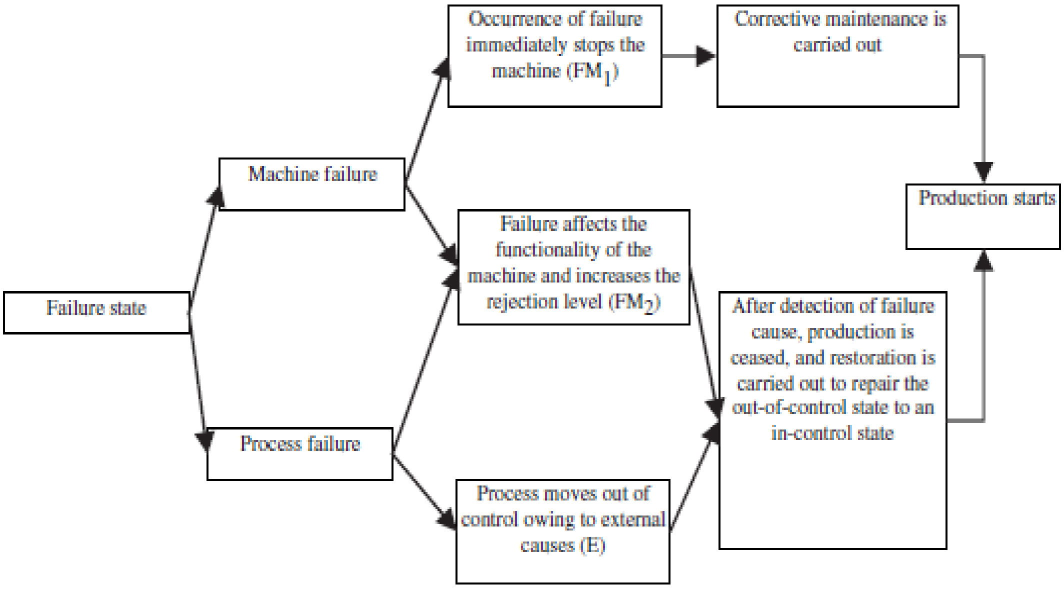


Fig.2.4 Failure modes and actions proposed by Pandey et al. (2012)

Whenever a complete failure occurs, corrective maintenance action, which restores the machine, would be implemented immediately. In parallel, the process would be monitored through a control chart in order to identify the actual operating state. Whenever a quality shift would be detected owing to machine degradation (partial failure), a corrective maintenance action was suggested to be performed to restore an in-control state through a repair action. Whenever quality shift would be detected owing to external reasons, a resetting of the process was suggested to be performed to restore the process to its in-control state. Thus, this type of maintenance action was expected to result in twin benefits, i.e. eliminating the quality cost related to out-of-control operation owing to external reasons and machine degradation and also improving the machine's reliability by protecting it against failures. Two types of maintenance policies were considered: minimal corrective maintenance that maintains the state of the equipment without affecting the age and imperfect preventive maintenance that upgrades the equipment in between 'as good as new' and 'as bad as old' condition. The proposed model enabled the determination of the optimal value of each of the four decision variables, i.e. sample size (n), sample frequency (h), control limit coefficient (k), and preventive maintenance interval (t_{PM}) that minimizes the expected total cost of the integration per unit time. A numerical example was presented to demonstrate the effect of the cost parameters on the joint economic design of preventive maintenance and process quality control policy. The sensitivity of the various parameters was also examined.

Safaei et al. (2012) proposed a multi objective model of the economic-statistical design of the X-bar control chart by incorporating the Taguchi loss function and intangible external costs. The model minimized the mean hourly loss-cost while minimizing out-of-control average run length and maintaining reasonable in-control average run length. A multi objective evolutionary algorithm, namely NSGA-II, was then developed and used to obtain the Pareto optimal solution of the model. Some sensitivity analyses were next performed to investigate the effect of parameter estimation on the chart performances. Finally, a comparison study with a traditional economic design model revealed that the proposed multiple objective design of the X-bar control chart presented a better approach for quality engineers to improve the processes.

Morales (2013) extended on the application of SPC with PM as some points were not completely covered by previous studies. First, most SPC was performed with the X-bar control chart which did not consider the variability of the production process. Second, many studies of design of control charts considered just the economic aspect while statistical restrictions must be considered to achieve charts with low probabilities of false detection of failures. Third, the effect of PM on processes with different failure probability distributions had not been explored as most of the studies consider one distribution Hence, he presented the Economic Statistical Design (ESD) of joint X- bar S control charts to monitor mean and variability in a production process. In addition, the cost model integrated PM with general failure distribution (cases with Exponential, Gamma, and Weibull distributions are presented) and constant and variable sampling intervals. Experiments showed that PM decreases costs for processes with high failure rates and reduces the sampling frequency of units for testing under SPC. From the results it was observed that, when the failure rates were small, there was little or no cost benefit in performing PM with different failure probability distributions. This could be attributed to the concept that a ‘‘good’’ process does not need much maintenance as a ‘‘bad’’ process (with higher failure rates) would require. In the case of high failure rates it would be convenient to perform PM with significant cost benefits, either with or without interruption of the process. A result of the effect of PM on the reliability of the process would be the increase in the length of the sampling intervals (constant or variable), which means a reduction in the sampling frequency.

Wang (2011) developed models for the maintenance of a system based on *np* control charts with respect to the sampling interval. At any given time, the system was assumed to be in one of the three possible states; in-control, out-of-control and failure. If the control charts signals, suggesting the possibility of an out-of-control state, an investigation would be carried out. He assumed that this investigation was perfect in that it revealed the true state of the system. If an assignable cause was confirmed by the investigation, a minor repair would be carried out to remove the cause. If the assignable cause was not attended to, it would gradually develop into a failure. When a failure occurs, the system could not operate and a major repair was needed. He discussed three models depending on the assumptions related to the renewal mechanism, the occurrence of failures, and the time between minor repairs. The paper optimized the performance of such a system in terms of the sampling interval. Geometric processes were utilized for modeling the lifetimes between minor repairs if the minor repair could not bring the system back to an as good as new condition. The expected cost per unit time for maintaining the systems with respect to the sampling interval of the control chart was obtained. Numerical examples were conducted to demonstrate the applicability of the methodology derived.

The interesting fact is almost all of the above researches proposed integration of SPC and maintenance and developed their model using X-bar control chart which has the disadvantage of not detecting small changes in mean efficiently. So very recently researchers are paying attention to other types of control charts for the purpose of developing an integrated model. No doubt, being able to detect small changes in a quick manner (The exponentially weighted moving average (EWMA) chart, is leading the journey. EWMA chart was introduced in the quality control literature by Roberts (1959), has recently received further attention as a process monitoring and control tool. Papers by Robinson and Ho (1978), Crowder (1987 A), and Lucas and Saccucci (1987) gave numerical procedures which made the properties of EWMA schemes easy to investigate. Roberts (1959) and Chantraine (1987) presented graphical methods which made the use of the EWMA nearly as simple as the traditional

Shewhart control chart as a factory floor technique. Hunter (1986) promoted the EWMA as a forecasting tool. In using a technique such as the EWMA, one should distinguish between two of its possible uses, that of process monitoring for detecting shifts and that of process forecasting. The nature of the process suggests which of these uses of the EWMA, if either, is appropriate. For processes which are essentially white noise (random variation) with periodic shifts in mean level, the EWMA is useful for monitoring the process and alerting the user that a shift was occurred. Lucas and Saccucci (1987) have shown that in this situation, the EWMA is as powerful as the CUSUM for purposes of detecting the shift. For processes with gradual drift, the EWMA is useful for forecasting. Crowder (1989) reviewed a simple procedure for designing a EWMA scheme for purposes of process monitoring and detection of shifts. The usefulness of the EWMA for this problem will be illustrated graphically. He distinguished between two different uses of the EWMA that of forecasting future observations from processes with drift and that of monitoring processes subject to occasional shifts in mean level. For the latter case, he motivated the use of the EWMA graphically, illustrating how the EWMA provided a clearer picture of process shifts and produced smaller ARLs than the traditional X-bar chart. Aparisi and Diaz (2004) presented a software program developed in Windows environment for the optimal design of the EWMA and MEWMA chart parameters, to protect the process in the case of shifts of given size. Optimization had been done using genetic algorithms. Throughout this decade various researchers contributed to the EWMA chart.

Park et al. (2004) compared the Variable Sampling Rate (VSR) approach with Fixed Sampling Rate (FSR) approach in case of a EWMA chart. A VSR EWMA chart is a EWMA chart with the VSR sampling scheme. The properties of the VSR EWMA chart were obtained by using a Markov chain approach. The model contained cost parameters which allow the specification of the costs associated with sampling, false alarms and operating off target as well as search and repair. Control charts that vary the sample size are called Variable Sample Size (VSS) charts. A large sample size is used when there is some indication of a problem and a small sample size is used when there is no indication of a problem. Variable Sampling Rate (VSR) charts allow both the sample size and the sampling interval to vary depending on the previous value of the control statistic. The idea of the VSR chart was to combine the VSI and VSS features. The economic design model of Park and Reynolds (1999) had been applied to evaluate the expected cost per hour associated with the operation of VSR and FSR EWMA charts in the single and double occurrence models. The optimal parameters of the VSR and FSR EWMA charts were obtained in this economic model for some given sets of values of the process and cost parameters. For the parameter combinations considered here, it was shown that the optimal control limit of the VSR chart was considerably higher than that of the FSR chart. This results in a much lower false alarm rate for the VSR chart. The percent reductions in cost presented in the tables showed that applying the VSR scheme in place of the FSR scheme in the EWMA chart could result in substantial cost savings. A double occurrence model was studied for a possible situation in which, given that a special cause has occurred, a second special cause may arrive before a signal is given. It was shown that such a model change has little effect on determining the optimal chart parameters.

Serel and Moskowitz (2008) used control charts with exponentially weighted moving average (EWMA) statistics (mean and variance) to jointly monitor the mean and variance of a process. A EWMA cost minimization model was presented to design the joint control scheme based on pure economic or both economic and statistical performance criteria. The pure economic model was extended to the economic-statistical design by adding constraints associated with in-control and out-of-control average run lengths. The quality related production costs were calculated using Taguchi's quadratic loss function. The optimal values of smoothing constants, sampling interval, sample size, and control chart limits were determined by using a numerical search method. The average run length of the control scheme was computed by using the Markov chain approach. Computational study indicated that optimal sample sizes decrease as the magnitudes of shifts in mean and/or variance increase, and higher values of quality loss coefficient lead to shorter sampling intervals. The sensitivity analysis results regarding the effects of various inputs on the chart parameters provided useful guidelines for designing an EWMA-based process control scheme when there exists an assignable cause generating concurrent changes in process mean and variance.

Chou et al. (2008) developed the economic design of the exponentially weighted moving average (EWMA) chart using variable sampling intervals with sampling at fixed times (VSIFT) control scheme to determine the values of the six test parameters of the chart (i.e., the sample size, the fixed sampling interval, the number of subintervals between two consecutive sampling times, the warning limit coefficient, the control limit coefficient, and exponential weight constant) such that the expected total cost per hour is minimized. The sampling scheme of a VSIFT chart was to use the sampling interval (denoted as h) between fixed time points as long as the sample point is close to the target so that there is no indication of process change. However, if the sample point is far away from the target, but still within the control limits, so that there is some indication of process shift, then additional samples were allowed between the two fixed sampling time points. The control charts using variable sampling intervals with sampling at fixed times (VSIFT) policy had been shown to give substantially faster detection of most process shifts than the conventional control charts. The genetic algorithms (GA) were applied to search for the optimal values of the six test parameters of the VSIFT EWMA chart, and then an example and its solution were provided. Finally, a sensitivity analysis was carried out to investigate the effect of model parameters on the solution of the economic design.

Serel (2009) studied the economic design of EWMA-based mean and dispersion charts where a linear, quadratic, or exponential loss function was used for computing the costs arising from poor quality. The chart parameters (sample size, sampling interval, control limits and smoothing constant) minimizing the overall cost of the control scheme are determined via computational methods. Using numerical examples, he compared the performances of the EWMA charts with Shewhart X-bar and S charts, and investigated the sensitivity of the chart parameters to changes in process parameters and loss functions. His computational study suggested that using a different type of quality loss function (linear versus quadratic) leads to a significant change in sampling interval while affecting the sample size and control limits very little. It was also observed that the overall costs are insensitive to the choice of Shewhart or EWMA charts. Numerical results implied that rather than sample size or control limits, the users need to adjust the sampling interval in response to changes in the cost of poor quality.

Xue et al. (2010) investigated the economic-statistical design of EWMA charts with variable sampling intervals (VSIs) under non-normality to reduce the process production cycle cost and improve the statistical performance of control charts. The objective was to minimize the cost function by adjusting the control chart parameters which suffice for the statistical restriction. First, using the Burr distribution to approximate various non-normal distributions, the economic-statistical model of the VSI EWMA charts under non-normality was developed. Further, the genetic algorithms were used to search for the optimal values of parameters of the VSI EWMA charts under non-normality. Finally, a sensitivity analysis was carried out to investigate the effect of model parameters and statistical restriction on the solution of the economic-statistical design. The result of sensitivity analysis showed that a large lower bound of average time to signal when the process is in control increases the control limit coefficient, no model parameter significantly affects the short sampling intervals, and so on. The economic-statistical design method proposed in this paper could improve the statistical performance of economic design of control charts and the general idea could be applied to other VSI control charts.

Charongrattanasakul and Pongpullponsak (2009) studied the joint effects of exponentially weighted moving average (EWMA) control chart of SPC and planned maintenance, as well comparing the optimal values of an X-bar control chart and a EWMA control chart. A mathematical model was developed to analyze the cost computed from the EWMA control chart. They developed an integrated model of a control chart and planned maintenance with reference to the integration of three scenarios first proposed by Linderman (2005), Kathleen E. McKone-Sweet (1986) and four scenarios by Zhou and Zhu (2008). A control chart was used to monitor the equipment and to provide signals indicating equipment deterioration, while planned maintenance was scheduled at regular intervals to pre-empt equipment failure. Based on Alexander's (1995) cost model, the economic behavior of the integrated model was investigated and an optimal design was developed to determine the four policy variables (n,h,L,k) that minimize hourly cost. The result of this case study showed that the integrated model managed with the inclusion of a EWMA control chart was better than the integrated model with X-bar control chart models.

Charongrattanasakul and Pongpullponsak (2011) studied integrated systems approach to Statistical Process Control (SPC) and Maintenance Management (MM). Previously, only four policies which were in control alert signal, out of control alert signal, in control no signal, and out of control no signal, were used in the consideration (Zhou & Zhu, 2008). The objectives of their research were to develop an integrated model between Statistical Process Control and Planned Maintenance of the EWMA control chart. To do this, warning limit was considered to increase the policy from four to six such as warning limit alert signal and warning limit no signal. As shown in Fig.2.5, the framework of the integrated model illustrated six different scenarios. Scenario 1 to 4 was just similar to the (Zhou & Zhu, 2008). When the process shifts to an ‘‘out-of-control’’ state in Scenario 5, Reactive Maintenance would take place because the ‘‘out-of-control’’ condition occurred before the scheduled time, and additional time and expense will be incurred to identify and solve the equipment problem. In Scenario 6, the process begins with an ‘‘in-control’’ state and no signal occurs in the control chart before the scheduled time. Then at the (k + 1)th sampling interval, appropriate maintenance should be arranged.

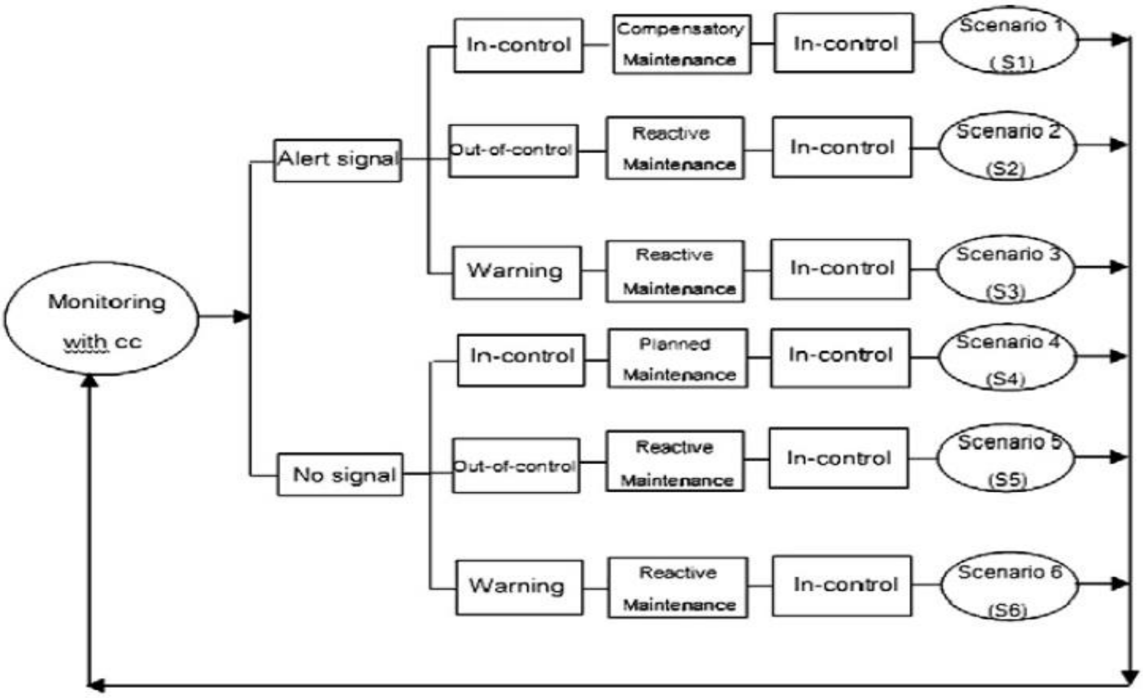


Fig. 2.5 Six scenarios proposed by Charongrattanasakul and Pongpullponsak (2011)

And in Scenario 6, the process is always ‘‘in-control’’, so Planned Maintenance was proposed to be performed. A mathematical model was given to analyze the cost of the integrated model before the genetic algorithm approach was used to find the optimal values of six variables (i.e., the sample size (n), the sampling interval (h), the number of subintervals between two consecutive sampling times (g), the warning limit coefficient (w), the number of samples taken before Planned Maintenance, (k) and the exponential weight constant (r)), that minimize the hourly cost. A comparison between four-policy and six-policy models showed that the six policy model contained the hourly cost higher than that of the four policy model, it was because the addition of the warning limit in the model leads into increased ability of defective product detection. This consequently results to the increase of repairing and maintenance of machines; therefore the hourly cost was higher. Finally, multiple regressions were employed to demonstrate the effect of cost parameters.

CHAPTER III

COMPUTATIONAL OPTIMIZATION PROCEDURE

3.1 Nelder Mead Downhill Simplex Algorithm

The Nelder-Mead algorithm was originally published in 1965 is one of the best known algorithms for multidimensional unconstrained optimization without derivatives. The algorithm is stated using the term simplex (a generalized triangle in N dimensions) and finds the minimum of a function of N variables. It is effective and computationally compact. Since, this method does not require any derivative information; therefore, it is quite suitable for problems with non-smooth functions. It is widely used to solve parameter estimation and similar statistical problems, where the function values are uncertain or subject to noise and that is a particular cause of choosing this particular algorithm for my analysis.

For two variables, a simplex is a triangle, and the method is a pattern search that compares function values at the three vertices of a triangle. The worst vertex, where f(x,y) is largest, is rejected and replaced with a new vertex. A new triangle is formed and the search is continued. The process generates a sequence of triangles, for which the function values at the vertices get smaller and smaller. Eventually, the size of the triangles is reduced and the coordinates of the minimum point are found.

3.1.1 Initial Triangle

Let f (x, y) be the function that is to be minimized. At the beginning, three vertices of a triangle are considered such that, $V_k=(x_k, y_k)$ where, k=1, 2, 3. For simplicity let's assume the points as $B=(x_1, y_1)$, $G=(x_2, y_2)$, and $W=(x_3, y_3)$. The function f (x, y) is then evaluated at each of the three points i.e. $z_k=f(x_k, y_k)$ for k=1, 2, 3. The subscripts are then reordered so that $z_1 \leq z_2 \leq z_3$.

3.1.2 Midpoint of the Good Side

The construction process uses the midpoint of the line segment joining B and G.

$$M = \frac{B+G}{2} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \qquad \text{……..…….(3.1)}$$

3.1.3 Reflection using the Point R

The function decreases as the points move along the side of the triangle from W to B, and also along the side from W to G. Hence it is feasible that f (x, y) takes on smaller values at points that lie away from W on the opposite side of the line between B and G. So, a test point R is chosen, that is obtained by "reflecting" the triangle through the side BG.

$$R = M + (M - W) = 2M - W \qquad \text{……..…….(3.2)}$$

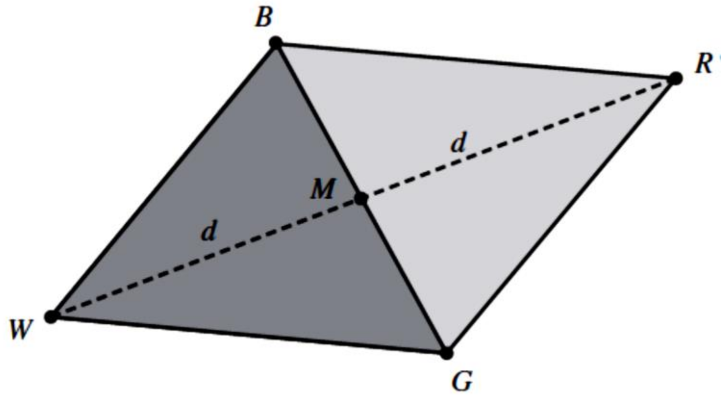


Fig.3.1 Initial triangle BGW, midpoint (M) and reflection point (R) in Nelder Mead method

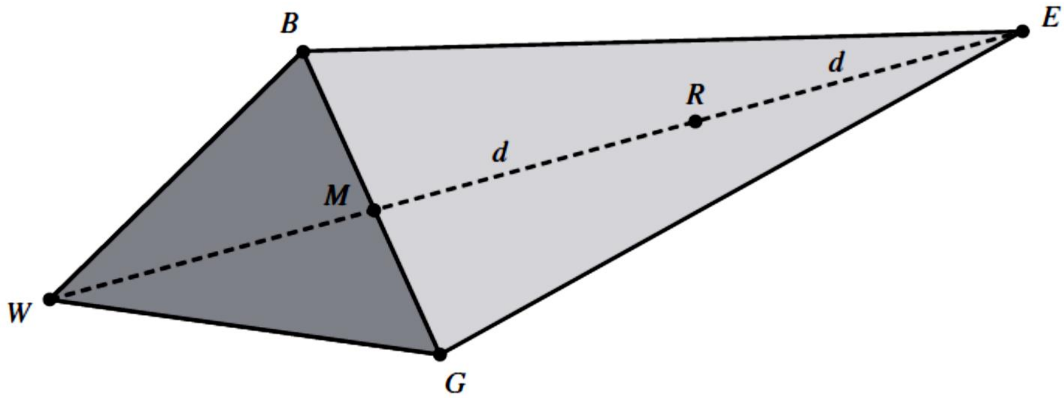


Fig.3.2 Expanded triangle BGE, reflection point (R) and extended point (E) in Nelder Mead method

3.1.4 Expansion using the Point E

If the function value at R is smaller than the function value at W, then the point has moved in the correct direction toward the minimum. Perhaps the minimum is just a bit farther than the point R. So in the next step, the line segment through W, M and R is extended to the point E. This forms an expanded triangle BGE. The point E is found by moving an additional distance d along the line joining M and R. If the function value at E is less than the function value at R, then, E is the new better vertex.

$$E = R + (R - M) = 2R - M \qquad \text{……..…….(3.3)}$$

3.1.5 Contraction using the Point C

If the function values at R and W are the same, another point must be tested. Perhaps the function is smaller at M, but W cannot be replaced with M because the points must form a simplex, in this case a triangle. So, the two midpoints C_1 and C_2 of the line segments WM and MR is considered, respectively. The point with the smaller function value is called C, and the new triangle is BGC.

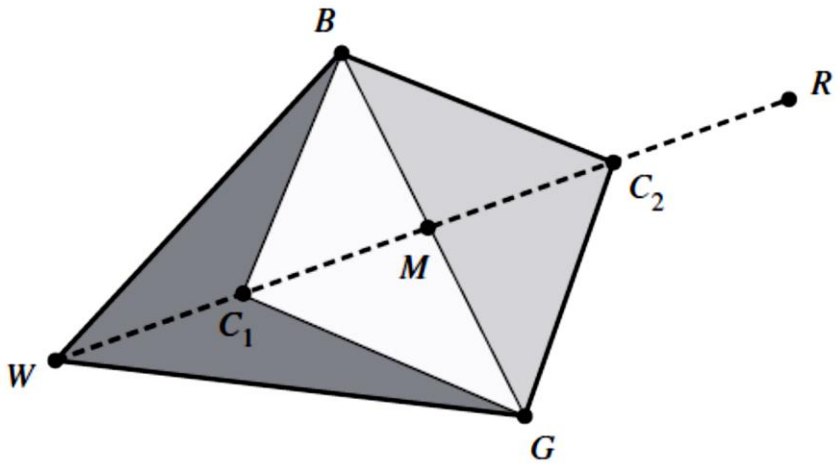


Fig.3.3 Contraction point C_1 and C_2 in Nelder Mead method

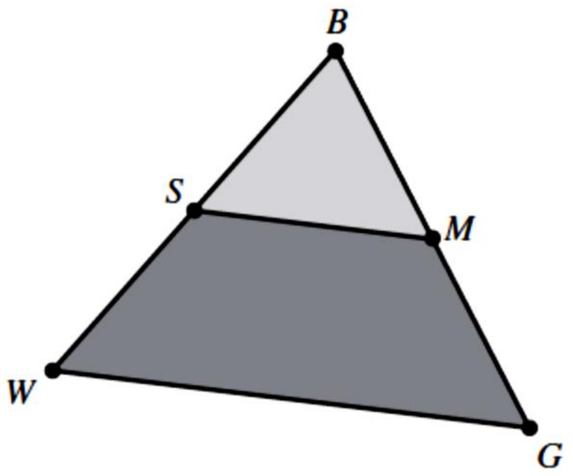


Fig.3.4 Shrinking of simplex (triangle) towards B

3.1.6 Shrink Towards B

If the function value at C is not less than the value at W, the points G and W must be shrunk towards B. The point G is replaced with M, and W is replaced with S, which is the midpoint of the line segment joining B with W.

3.1.7 Termination Tests

A practical implementation of the Nelder-Mead method must include a test that ensures termination in a finite amount of time. The termination test is often composed of three different parts i.e.

1. term_x ,
2. term_f and
3. fail.

'term_x' is the domain convergence or termination test. It becomes true when the working simplex S is sufficiently small in some sense (some or all vertices xj are close enough). 'term_f' is the function-value convergence test. It becomes true when (some or all) function values f_j are close enough in some sense. 'fail' is the no-convergence test. It becomes true if the number of iterations or function evaluations exceeds some prescribed maximum allowed value. The algorithm terminates as soon as at least one of these tests becomes true. The flow chart of the working procedure of the algorithm is as follows:

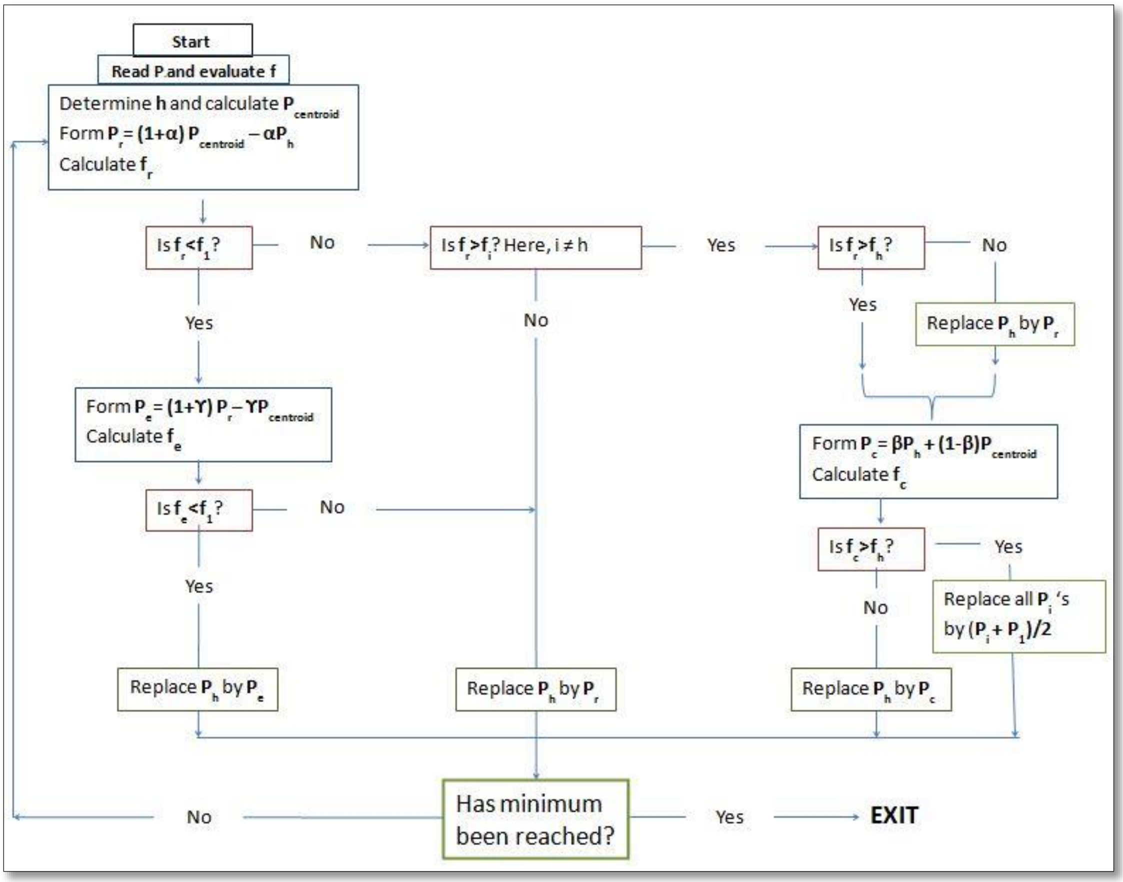


Fig.3.5 Flow chart of Nelder Mead algorithm's working procedure

3.1.8 Convergence of Nelder Mead Method

Rigorous analysis of the Nelder-Mead method seems to be a very hard mathematical problem. Known convergence results for direct search methods in simplex terms rely on one or both of the following properties:

(a) The angles between adjacent edges of the working simplex are uniformly bounded away from 0 and π throughout the iterations, i.e., the simplex remains uniformly non-degenerate.

(b) Some form of "sufficient" descent condition for function values at the vertices is required at each iteration.

In general, the original Nelder-Mead method does not satisfy either of these properties. By design, the shape of the working simplex can almost *degenerate* while “adapting itself to the local landscape”, and the method uses only *simple* decrease of function values at the vertices to transform the simplex. Hence, very little is known about the convergence properties of the method

3.1.9 Advantages and Disadvantages

In many practical problems, like parameter estimation and process control, the function values are uncertain or subject to noise. Therefore, a highly accurate solution is not necessary, and may be impossible to compute. All that is desired is an improvement in function value, rather than full optimization.

The Nelder-Mead method frequently gives significant improvements in first few iterations and quickly produces quite satisfactory results. Also, the method typically requires only one or two function evaluations per iteration, except in shrink transformations, which are extremely rare in practice. This is very important in applications where each function evaluation is very expensive or time-consuming. For such problems, the method is often faster than other methods, especially those that require at least n function evaluations per iteration. In many numerical tests, the Nelder-Mead method succeeds in obtaining a good reduction in the function value using a relatively small number of function evaluations.

Apart from being simple to understand and use, this is the main reason for its popularity in practice.

On the other hand, the lack of convergence theory is often reflected in practice as a numerical breakdown of the algorithm, even for smooth and well-behaved functions.

The method can take an enormous amount of iterations with negligible improvement in function value, despite being nowhere near to a minimum. This usually results in premature termination of iterations. A heuristic approach to deal with such cases is to restart the algorithm several times, with reasonably small number of allowed iterations per each run.

3.2 Genetic Algorithm

GA is a search algorithm developed by Holland (1975) which is based on the mechanics of natural selection and genetics to search through decision space for optimal solutions. The metaphor underlying GAs is natural selection. In evolution, the problem that each species faces is to search for beneficial adaptations to the complicated and changing environment. In other words, each species has to change its chromosome combination to survive in the living world. In GA, a string represents a set of decisions (chromosome combination), that is a potential solution to a problem. Each string is evaluated on its performance with respect to the fitness function (objective function). The ones with better performance (fitness value) are more likely to survive than the ones with worse performance. Then the genetic information is exchanged between strings by crossover and perturbed by mutation. The result is a new generation with (usually) better survival abilities. This process is repeated until the strings in the new generation are identical, or certain termination conditions are met. A generic flow of GA is given in Fig.3.6 This algorithm is continued since the stopping criterion is reached. GAs is used in forming models to solve optimization problems. Readers can find more details of GAs in Gen and Cheng (2000), Kaya (2008).GAs are different from other search procedures in the following ways (Chen, 2004): (1) GAs consider many points in the search space simultaneously, rather than a single point; (2) GAs work directly with strings of characters representing the parameter set, not the parameters themselves; (3) GAs use probabilistic rules to guide their search, not deterministic rules. Because GAs considers many points in the search space simultaneously there is a reduced chance of converging to local optima. In a conventional search, based on a decision rule, a single point is considered and that is unreliable in multimodal space. GAs consists of four main sections. They are as follows: Encoding, Selection, Reproduction, and Termination (Gen & Cheng, 2000; Mitchell, 1996).

3.2.1 Outline of the Basic Genetic Algorithm

[Start] Generate random population of *n* chromosomes (suitable solutions for the problem)

[Fitness] Evaluate the fitness *f(x)* of each chromosome *x* in the population

[New population] Create a new population by repeating following steps until the new population is complete

[Selection] Select two parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to be selected)

[Crossover] With a crossover probability crossover the parents to form a new offspring (children). If no crossover was performed, offspring is an exact copy of parents.

[Mutation] With a mutation probability mutate new offspring at each locus (position in chromosome).

[Accepting] Place new offspring in a new population

[Replace] Use new generated population for a further run of algorithm

[Test] If the end condition is satisfied, stop, and return the best solution in current population

[Loop] Go to step 2

The GA, based on the concept of natural genetics, is directed toward a random optimization search technique. The GA solves problems using the approach inspired by the process of Darwinian evolution. The current GA in science and engineering refers to the models introduced and investigated by Holland (1975). In the GA, the solution of a problem is called a ‘‘chromosome’’. A chromosome is composed of genes (i.e., features or characters). Although there are several kinds of numerical optimization methods, such as neural network, gradient-based search, GA, etc., the GA has advantages in the following aspects:

1. The operation of GA uses the fitness function values and the stochastic way (not deterministic rule) to guide the search direction of finding the optimal solution. Therefore the GA can be applied for many kinds of optimization problems.

2. The GA can lead to a global optimum by mutation and crossover technique to avoid trapping in the local optimum.

3. The GA is able to search for many possible solutions (or chromosomes) at the same time. Hence, it can obtain the global optimal solution efficiently.

Based on these points, GA is considered as an appropriate technique for solving the problems of combinatorial optimization and has been successfully applied in many areas to solve optimization problems (e.g., Chou, Wu, & Chen, 2006).

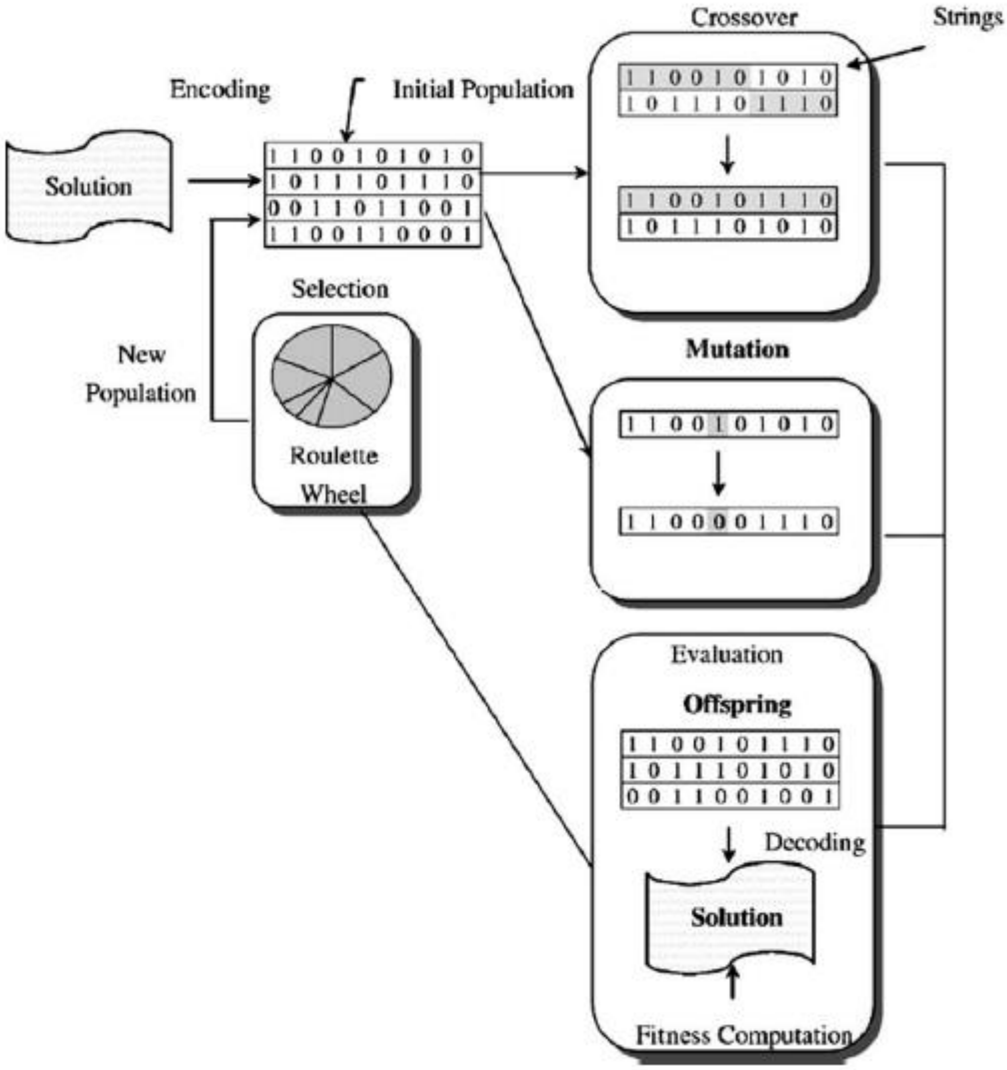


Fig. 3.6 Flow chart of Genetic Algorithm's working procedure

3.2.2 Parameters of GA

Crossover probability says how often will be crossover performed. If there is no crossover, offspring is exact copy of parents. If there is a crossover, offspring is made from parts of parents' chromosome. If crossover probability is 100%, then all offspring is made by crossover. If it is 0%, whole new generation is made from exact copies of chromosomes from old population (but this does not mean that the new generation is the same!).

Mutation probability says how often will be parts of chromosome mutated. If there is no mutation, offspring is taken after crossover (or copy) without any change. If mutation is performed, part of chromosome is changed. If mutation probability is 100%, whole chromosome is changed, fit is 0%, nothing is changed.

Population size says how many chromosomes are in population (in one generation). If there are too few chromosomes, GA has a few possibilities to perform crossover and only a small part of search space is explored. On the other hand, if there are too many chromosomes, GA slow down.

CHAPTER IV

MODEL DEVELOPMENT

4.1 Problem Identification

There are two key tools for the control of production processes — Statistical Process Control (SPC) and Maintenance Management (MM), which are traditionally separated (both in science and in business practice), even though their goals overlap a great deal. Their common goal is to achieve optimal product quality, little downtime and cost reduction by controlling variances of the process. Since single or separated parallel applications may not be fully effective, the integration of statistical process control and maintenance has already drawn the attention of the researchers. But the following gaps are observed in the literature on this subject:

1. Most of the integrated general cost models integrate maintenance management policy and process quality control considering $\overline{X}$ control chart but EWMA control chart is a very effective one for detecting small shifts of the process parameter and for individual measurement. In real life, detecting small change in process parameter is a vital one and in process manufacturing industry individual measurement or measurement when sample size is 1 is necessary. So integrated cost model of EWMA chart with maintenance management becomes a required one.
2. Most of the integrated models focus on process quality problems and completely ignore the possibility of an equipment failure in terms of machine breakdown or improper functioning of the equipment that result in poor product quality and call for maintenance action.
3. Most of the integrated models focus on the investigation of preventive replacement or perfect PM policy that restores the equipment to an 'as-good-as-new' state. Relatively very few papers have appeared that incorporate the aspect of imperfect PM. But in real life aspect imperfect PM has attained more acceptances.
4. Most of the integrated models consider the conventional goal-post approach for the economic design of control chart; however any deviation of quality characteristics from the target value is an indirect loss to the customers. Thus,

incorporating the quadratic loss function approach with the economic design of maintenance and process quality control chart policy should give better results.

5. Besides, in case of sampling policy, variable sampling interval has received more attention than fixed sampling interval and has become more realistic one. Variable Sampling Interval at Fixed Times (VSIFT) is one of the latest sampling technique which has been used in a very few study and recently it has already been proven as the most promising technique of sampling policy. But no integrated model has ever been developed with VSIFT approach.

The above-mentioned gaps are addressed in this thesis.

4.2 Model Formulation

It is always required to consider the design of a control chart from an economical point of view. Because the choice of control chart parameters such as sample size (**n**), sampling frequency or interval between the samples (**h**), and width of the control limits (**k**) etc affect the whole cost. Selection of these control chart parameters is usually called the "Design of the Control Chart". Economic models are generally formulated using a total cost function, which expresses the relationships between the control chart design parameters and associated costs of process control. The production, monitoring and adjustment process are a series of independent cycles over time. Each cycle begins with the production process in the in-control state and continues until process monitoring via the control chart results in an out-of-control signal. After adjustment it returned to the in-control state to begin another cycle. Then the expected cost per unit time is,

$$E(A) = \frac{E(C)}{E(T)}$$

where **C, T** are dependent random variables.

The above equation is optimized to design economically optimal control chart. The sequence of production-monitoring-adjustment, with accumulation of costs over the cycle, can be represented by a particular type of stochastic process called Renewal Reward Process. Where time cost is given by the ratio of the expected reward per cycle to the expected cycle length.

4.2.1 The VSIFT EWMA chart

The sampling scheme of a VSIFT chart is to use the sampling interval (denoted as h) between fixed time points as long as the sample point is close to the target so that there is no indication of process change. However, if the sample point is far away from the target, but still within the control limits, so that there is some indication of process shift, then additional samples are allowed between the two fixed sampling time points. Suppose that the interval h between two fixed times is divided into η subintervals of length d, where d = h/η. For example, if h = 1 h and η = 6 (i.e., d = 10 min), then samples would always be taken every hour. But if the sample point indicates a problem, then the next sample would be taken 10 min later. The ith sample statistic of an EWMA chart is

$$X_i = \mathrm{L}Y_i + (1-\mathrm{L})\, X_{i-1} \qquad \text{for } 0 < \mathrm{L} \leq 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.1)}$$

Where X_0 is the starting value and is often taken to be the process target value, the sequentially recorded observations Y_i can either be individually observed values from the process or sample averages obtained from rational subgroups and L is the exponential weight constant. The upper and lower control limits (denoted by UCL and LCL, respectively) for the EWMA chart are

$$\mathrm{UCL} = \mu_0 + k\sigma_x \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.2)}$$

$$\mathrm{LCL} = \mu_0 - k\sigma_x \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.3)}$$

Where μ_0 is the in-control value of the process mean and is usually identical to X_0 , k is the control limit coefficient of the EWMA chart that determines the size of the critical region of the chart, and σ_x is the asymptotic standard deviation of the sample statistic in Eq. (1) and is equal to

$$\sigma_x = \sigma\sqrt{\frac{L}{(2-L)n}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(4.4)}$$

where σ is the standard deviation of the process characteristic and n is the sample size. The upper and lower warning limits (denoted by UWL and LWL, respectively) for the EWMA chart are

$$\mathrm{UWL} = \mu_0 + w\sigma_x \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.5)}$$

LWL = μ_0 - $w\sigma_x$ ……………………….(4.6)

where w is the warning limit coefficient of the EWMA chart. If the last sample point falls in the safe region (i.e., LWL ≤ X_i ≤ UWL), then take the next sample at the next fixed sampling time point. If the last sample point falls in the warning region (i.e., UWL < X_i ≤ UCL or LCL ≤ X_i < LWL), then take the next sample using the sampling interval *d*. A search for the assignable cause is under taken when the sample point falls outside the control limits. Thus, when the VSIFT EWMA chart is applied to maintain current control of a process, six test parameters (i.e., *n, h, g, k, w,* and *k*) should be determined. The economic design of VSIFT EWMA chart is to determine the optimal values of the six test parameters such that the average total cost per hour associated with test procedure may be minimized.

4.2.2 *Process assumptions for VSIFT EWMA control chart*

To simplify the mathematical manipulation and analysis, the following assumptions are made:

1. The process characteristic monitored by the VSIFT EWMA chart follows a normal distribution with mean μ and standard deviation σ.
2. In the start of the process, the process is assumed to be in the safe state; that is, $\mu = \mu_0$
3. The process mean may be shifted to the out-of-control region; that is, $\mu = \mu_0 + \delta\, \sigma$.
4. The process standard deviation σ remains unchanged.
5. The time between occurrences of the assignable cause is exponential with a mean of λ occurrences per hour.
6. When the process goes out of control, it stays out of control until detected and corrected.
7. During each sampling interval, there exists at most one assignable cause which makes the process out of control. The assignable cause will not occur at sampling time.
8. The measurement error is assumed to be zero.

4.2.3 Problem Statement and Assumptions

According to Pandey et al. (2012) in the problem statement, a production system is considered consisting of a single machine producing products of the same type with a constant Production Rate (PR, items per hour) on a continuous basis (three shifts of seven hours each, six days a week). Further, a single component operating as a part of machine is considered with time-to-failures following a two-parameter Weibull distribution. The failures of most components in a mechanical system like production machines can be modeled using a two-parameter Weibull distribution. However, the approach proposed in this thesis is generic and can be used with other distributions also.

Machine failures are divided into two failure modes:

(1) Failure mode 1 (FM_1): leads to reduction in process quality owing to shifting of the process mean.

(2) Failure mode 2 (FM_2): leads to immediate breakdown of the machine.

Similar classifications are also used by Lad and Kulkarni (2008). They have defined the failure of a machine tool as any event that either brings the machine down or leads to the machine still running but producing higher rejections. This means that if a failure occurs, it is not necessary that it be detected immediately, and the machine is stopped, but it may also affect the quality of products being manufactured on the machine. For example, in the case of a grinding machine, if the work-head belt is broken, it will stop the machine completely, while if a loosening of the ball screw chuck nut occurs, the machine may continue to run but may result in oval components being produced.

It is therefore necessary to consider these types of failures, and the corresponding failure costs, which may be situation-specific, in maintenance-planning decisions. The failures belonging to the second type of failure mode can also be considered as partial failures and are defined by Black and Mejabi (1995) as degradation in the machine performance without complete failure. Thus, the problem is to jointly optimize design parameters for the preventive maintenance and process quality policy.

Let us consider the following assumptions:

1. Corrective maintenance is minimal in nature, i.e. after corrective action, the equipment has the same age as it did at the time of failure.
2. Preventive maintenance is imperfect in nature.
3. A single part gets manufactured on the machine with a single critical to quality (CTQ) characteristic.
4. In this model it is assumed that failure modes FM_1 and FM_2 are independent and result in failures with an appropriate time to failure distribution. For a given failure, let the probability that it is due to FM be P_{FM1} and P_{FM2} respectively. Since it is assumed that these are the only failure mode types, $P_{FM1} + P_{FM2}=1$.These probabilities can be obtained from the failure reports maintained by maintenance personnel from production lines. The failure reports mainly cover the following information: Component ID, Time to failure, Time of repair, Failure mode, Action taken, Failure cost.
5. The process is monitored by a VSIFT EWMA control chart.

4.2.3 Problem Description

If FM_1 occurs, it immediately stops the machine. Corrective actions are taken to repair the machine to its operating condition. Thus, the expected cost of corrective maintenance $E[C_{CM}]_{FM1}$ includes the cost of down time, and the cost of repair/restore action. FM_2 affects the functionality of the machine and in turn increases the rejection level. In other words, FM_2 affects the process rejection rate. It is assumed that whenever FM_2 is detected, the process is stopped immediately, and corrective actions are taken to repair the process back to the normal condition. Apart from failures owing to FM_2, the process may also deteriorate owing to external causes (E) such as environmental effects, operators' mistakes, use of wrong tool, etc. The process is reset to the in-control state if an external event 'E' is detected. The time-to-failure of the process is assumed to follow an exponential distribution (Ben-Daya 1999). The detection of FM_2 or an external cause is achieved by monitoring the process. In this thesis a VSIFT EWMA control chart mechanism is considered for process monitoring. Let the design parameters of the control chart be sample size n, length of the fixed sampling interval h, the control limit coefficient k ,warning limit coefficient w, number of subinterval η and exponential wightage constant L that determine the distance between the centre line and the control limits. Thus, the expected total cost of process failure E $[TCQ]_{process\text{-}failure}$ owing to FM and external events considering the cost of down time, cost of rejections owing to process shifts, cost of repair/resetting, cost of sampling/inspection, cost of investigation of false/valid alarm, and cost of deviation from the target value of the CTQ. Apart from the above corrective actions, the machine can undergo preventive maintenance t_{PM} to minimize the unplanned downtime losses. In this thesis, imperfect preventive maintenance has been considered. This means that the PM upgrades the equipment to a state between the as-good-as-new and as-bad-as-old conditions. The frequency of failures can be significantly decreased through PM. Reduction in FM_2 reduces the quality costs related to the out-of-control operation. However, PM also consumes some resources and productive machine time that could otherwise be used for production. The expected cost of PM (E $[C_{PM}]$) comprises the cost of downtime and cost of performing preventive maintenance actions.

4.3 Mathematical Model

4.3.1 Development of cost function

The cost function, which is the objective function of the economic design of the VSIFT EWMA chart integrating maintenance management, is developed based on the economic model given in Lorenzen and Vance (1986).

Expected Cycle Length Determination

The cycle length is defined as the total time from which the process starts in control, shifts to an out-of-control condition, has the out-of-control condition detected, and results in the assignable cause being identified and the process being corrected. Four time intervals in an expected cycle length are respectively

1. The interval the process is in control, denoted by T_1,

2. The interval the process is out of control before the final sample of the detecting subgroup is taken, denoted by T_2,

3. The interval to sample, inspect, evaluate and plot the subgroup result, denoted by T_3, and

4. The interval to search for the assignable cause and correct the process, denoted by T_4.

When the expected cycle length is determined, the cost components can be converted to a ‘‘per hour of operation’’ basis.

The average time interval that the process is in control can be expressed as

$$E\ (T_1) = \frac{1}{\lambda} + (1 - \gamma_1) * t_0\ * \frac{s}{ARL1} \qquad \text{………………. (4.7)}$$

where t_0 is expected time of searching for an assignable cause under a false alarm, s is the expected sampling frequency while in control and

$\gamma_1 =$ 1; if production continues during searches

0; if production ceases during searches

Calculation of the expected sampling frequency while in control

The first sampling occurs at time point h (i.e., this is the only one sampling during the first interval [0, h]). When the process is in control, except for the first interval, the possible values of the sampling frequency for any interval with fixed length h (i.e., the intervals [h, 2h], [2h, 3h], [3h, 4h] ...) are 1, 2, ..., η. For any interval between two consecutive fixed sampling time points (except for the first interval), define A_i as the event that the process is in control at the ith sampling given that the (i -1)st sample point falls in the warning region, for i =1,2,..., η. Then the expected sampling frequency for any interval with fixed length h (excluding the first interval), denoted by v, is

$$v = \sum_{i=1}^{\eta} iP(A_i)$$

$$= (1-\rho)+2\rho\ (1- \rho)+3\rho^2\ (1- \rho)+\ldots\ldots+(\eta-1)\ \rho^{\eta-2}(1- \rho) + \eta\rho^{\eta-1}$$

$$= \frac{1-\rho^{\eta-1}}{1-\rho} + (2\eta - 1)\rho^{\eta-1} \qquad \text{………………… (4.8)}$$

Let B_j be the event that the process is in control at the start of the jth interval with fixed length h, for j =2,3,..., ∞. Then, based on the model assumptions, we have

$P(B_j) = 1 - \int_0^{(j-1)h} \lambda e^{-\lambda t}\, dt$

$= e^{-(j-1)\lambda h}$ ……………………(4.9)

for j =2,3,..., ∞.

From Eqs. (4.8) and (4.9),the expected sampling frequency while in control is,

$s = 1 + \left(\frac{1-\rho e^{\eta-1}}{1-\rho} + (2\eta - 1)\rho^{\eta-1}\right) * \sum_{j=2}^{\infty} e^{-(j-1)\lambda h}$

$=1 + \left(\frac{1-\rho e^{\eta-1}}{1-\rho} + (2\eta - 1)\rho^{\eta-1}\right) * \left(\frac{e^{-\lambda h}}{1-e^{-\lambda h}}\right)$ ……………………(4.10)

$\eta = \frac{h}{d}$ ……………………(4.11)

where ρ is the conditional probability that the sample point is plotted in the warning region given that the process is in control and is equal to

$\rho = \frac{2[\varphi(k) - \varphi(\omega)]}{\varphi(k) - \varphi(-k)}$ ……………………(4.12)

where $\varphi(.)$ is the cumulative function of the standard normal random variable,

α is the probability of false alarm and for EWMA chart it can be calculated by

$\alpha = 1/ARL_1$ ……………………(4.13)

Here, ARL_1 denotes Average Run Length when process is in control

The average time interval that the process is out of control before the final sample of the detecting subgroup is taken may be written as

$E(T_2)=ATS_2 - \xi$ ………………...(4.14)

$ATS_2 = (ATS_2)_{m/c} + (ATS_2)_{external}$ ………………...(4.15)

where ATS_2 is defined as the average time from the occurrence of an assignable cause to the time when the chart indicates an out-of-control signal and in this study the value of ATS_2 is obtained calculating the in control and out of control ARL for EWMA chart using Markov Chain approach as proposed by Lucas and Saccucci

(1990) and ξ is average time lag between the sampling time point, which is just prior to the occurrence of the assignable cause, and the time point that the assignable cause occurs and it can be shown that

$$\xi = \sum_{j=0}^{\eta-1} \left(\rho_{1j}\,\tau_{1j} + \rho_2\,\tau_2\right) \qquad \text{..................(4.16)}$$

where p_{1j} is the ratio of the sampling interval h- jd to the average sampling interval and is equal to

$$p_{1j} = \frac{\rho^{j}(1-\rho)(h-jd)}{\rho\sum_{i=0}^{\eta-2}\rho^{i}d+(1-\rho)\sum_{i=0}^{\eta-1}\rho^{i}(h-id)} \qquad \text{..................(4.17)}$$

p_2 is the ratio of the sampling interval *d* to the average sampling interval and is equal to

$$p_2 = \frac{\rho\sum_{i=0}^{\eta-2}\rho^{i}d}{\rho\sum_{i=0}^{\eta-2}\rho^{i}d+(1-\rho)\sum_{i=o}^{\eta-1}\rho^{i}(h-id)} \qquad \text{..................(4.17)}$$

τ_{1j} is defined as, given that the assignable cause occurs between the sampling time points *ih + jd* and (*i +1*)*h*, the expected in-control time interval during this period and, from the definition, it may be shown that

$$\tau_{10} = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda\,(1-e^{-\lambda d})} \qquad \text{..................(4.18)}$$

$$\tau_{1j} = \frac{1}{\lambda} - \frac{(\eta-j)de^{-\lambda(\eta-j)h}}{1-e^{-\lambda(\eta-j)h}} \qquad \text{..................(4.19)}$$

For j = 1, 2…..η-1

and τ_2 is defined as, given that the assignable cause occurs between the *i*th and *(i +* 1)st sampling time points with sampling interval d, the expected in-control time interval during this period and is equal to

$$\tau_2 = \frac{1-(1+\lambda d)e^{-\lambda d}}{\lambda(1-e^{-\lambda d})} \qquad \text{..................(4.20)}$$

The average time interval to sample, inspect, evaluate and plot the subgroup result is equal to

$E(T_3)=n*g$ ………………..(4.21)

where g is the average sampling, inspecting, evaluating and plotting time for each sample. The average time interval to search for the assignable cause and correct the process is

$E\ (T_4) = t_1 + E\ (T_{restore})$

$$E\ (T4) = t_1 + \ (T_{resetting} * \frac{\lambda 1}{\lambda} + MTTR_{Cr} * \frac{\lambda 2}{\lambda}) \quad \text{……………………(4.22)}$$

where t_1 is the average time to search for the assignable cause and $E(T_{restore})$is the expected time to repair or reset the process. Therefore, from Eqs. (4.7), (4.14), (4.21) and (4.22), the expected cycle length is

$E\ (T_{cycle}) = E\ (T_1) + E(T_2) + E(T_3) + E(T_4)$ ……………………(4.23)

Expected Cost Calculation

To estimate the expected cost of corrective maintenance owing to FM_1 and preventive maintenance, the analyst must have the following information:

1. The amount of time that the equipment is expected to be down each time CM/PM is required. This can include the time to perform the maintenance as well as Reliasoft (2009) any logistical delays (i.e. waiting for labor and/or materials required).
2. The cost of CM/PM including the downtime, labor, materials, and other costs.
3. The degree to which the equipment will be restored by CM/PM (e.g. 'as good as new,' 'as bad as old,' 'Imperfect'). This is quantified in terms of a restoration factor. The restoration factor can be determined empirically or based on expert judgment as calculated in) and Lad and Kulkarni (2010) respectively.
4. The probability that the equipment will fail owing to a particular failure mode.

The expected cost of minimal corrective maintenance owing to FM_1 is given as:

$E[C_{CM}]_{FM1}=\{MTTR_{CM}[PR.C_{lP}+LC]+C_{FCPCM}\}*P_{FM1}*N_f$ ………………………(4.24)

$MTTR_{CM}$ [PR. C_{lP} +LC] is the down time cost owing to corrective maintenance

The expected total cost of preventive maintenance action of component will be

$$E[C_{PM}]=\{MTTR_{PM}[PR.C_{lP}+LC]+C_{FCPPM}\}*\frac{T_{eval}}{t_{PM}} \quad \text{……………….(4.25)}$$

where $MTTR_{PM}$ [PR. C_{lP} +LC] is the down time cost owing to Preventive maintenance

Let the cost per unit time for investigating a false alarm is $C_{false.}$ This includes the cost of searching and testing for the cause. Then the expected cost of false alarm

$$E[C_{false}] = C_{false} * \frac{s}{ARL_1} * t_0 \quad \text{……………….(4.26)}$$

Let 'a' be the fixed cost per sample of sampling and 'b' be the variable cost per unit sampled. Thus, the expected cost per cycle for sampling is the sum of the fixed cost per sample and variable cost per unit sampled, and is given as:

$$E\ [\text{Cost of sampling}] = \frac{(a+bn)\{\frac{1}{\lambda} + ATS_2 - \xi + \eta g + \gamma_1 t_1 + \gamma_2\ E(T_{restore})}{\rho \sum_{i=0}^{\eta-2} \rho^i d + (1-\rho) \sum_{i=0}^{\eta-1} \rho^i (h - id)} \quad \text{……………….(4.27)}$$

γ_2 = 1; if production continues during process correction

0; if production ceases during process correction

Let $C_{resetting}$ be the cost for finding and resetting the assignable cause owing to external reasons, downtime if process ceases functioning, and for finding and resetting the process. The expected value of $C_{resetting}$ can be calculated as:

$$E[C_{resetting}] = [C_{resetting} * T_{resetting}] * \frac{\lambda_1}{\lambda} \quad \text{…………..(4.28)}$$

The expected cost of corrective maintenance action owing to failure mode FM_2 of the component and for finding and repairing the assignable cause owing to machine failure is given by:

$$E[(C_{repair})FM_2] = \{MTTR_{CM}[PR.C_{lP} + LC] + C_{FCPPM}\} * \frac{\lambda_2}{\lambda} \quad \text{……………….(4.29)}$$

Consideration of Taguchi Loss

To calculate the cost of quality loss incurred owing to defectives produced while the process is in control and out of control a Taguchi loss function has been used.

Consider a Critical to Quality (CTQ) with bilateral tolerances of equal value (d'^2). The cost to society for manufacturing a product out of specification is A Rs/unit, and uniform rejection cost is incurred beyond the control limits.

[$L_{in\ control}$] determination

The quality loss per unit time incurred while process is in control state is given as

$$L_{in\ control} = PR* \frac{A}{d'^2} \int_{\mu-k\sigma\sqrt{\frac{L}{(2-L)n}}}^{\mu+k\sigma\sqrt{\frac{L}{(2-L)n}}} (x-\mu)^2 f(x)dx \qquad \ldots\ldots\ldots\ldots\ldots(4.30)$$

where x is a random variable denoting sample means of the quality characteristic and $f(x)$is its normal density function with mean μ and standard deviation $\sigma\sqrt{\frac{L}{(2-L)n}}$ .Now any deviation from the target value will incur a loss. This was not the case under the classical SPC approach.

From algebraic manipulations,

$$[L_{in\ control}] = PR* \frac{A}{d'^2} * \frac{L*\sigma^2}{(2-L)n} [1\text{-}2^{\varphi(-k)} - \frac{2k}{\sqrt{2\pi}} e^{-\frac{k^2}{2}}] \qquad \ldots\ldots\ldots\ldots\ldots(4.31)$$

[$L_{out\ of\ control}$] determination

$$L_{out\ of\ control} = PR* \frac{A}{d'^2} \int_{-\infty}^{\infty} (x'-\mu)^2 f(x')dx' - \int_{\mu-k\sigma\sqrt{\frac{L}{(2-L)n}}}^{\mu+k\sigma\sqrt{\frac{L}{(2-L)n}}} (x'-\mu)^2 f(x')dx'\}$$

where $x'$is a random variable denoting sample means and $f(x')$ is its normal density function with mean $\mu+\delta\sigma$ and standard deviation $\sigma\sqrt{\frac{L}{(2-L)n}}$.

From algebraic manipulation,

$$L_{\text{out of control}} = R * \frac{A}{d'^2} * \frac{\sigma^2 * L}{(2-L)*n}\left[\left(1+\delta^2 * \frac{(2-L)n}{L}\right) * \{1 - \varphi(k - \delta * \sqrt{\frac{(2-L)n}{L}}) + \varphi(-k - \delta * \sqrt{\frac{(2-L)n}{L}})\} + \left(\frac{k+\delta*\sqrt{\frac{(2-L)n}{L}}}{\sqrt{2\pi}}\right) * e^{\frac{-(k-\delta*\sqrt{\frac{(2-L)n}{L}})^2}{2}} + \frac{k-\delta*\sqrt{\frac{(2-L)n}{L}}}{\sqrt{2\pi}}) * e^{\frac{-(k+\delta*\sqrt{\frac{(2-L)n}{L}})^2}{2}} \quad \text{..............(4.32)}$$

Thus, the expected process quality loss incurred per cycle in the in-control state is:

$$E[L_{\text{incontrol}}] = [L_{\text{incontrol}}] * \frac{1}{\lambda} \quad \text{..................(4.33)}$$

The quality loss per unit time incurred while the process is operating in out-of-control state owing to machine failure is given as:

$$[L_{\text{out of control}}]_{M/C} = PR * \frac{A}{d'^2} * \frac{\sigma^2 * L}{(2-L)*n}\left[\left(1+{\delta_{M/C}}^2 * \frac{(2-L)n}{L}\right) * \{1 - \varphi(k - \delta_{M/C} * \sqrt{\frac{(2-L)n}{L}}) + \varphi(-k - \delta_{M/C} * \sqrt{\frac{(2-L)n}{L}})\} + \left(\frac{k+\delta_{M/C}*\sqrt{\frac{(2-L)n}{L}}}{\sqrt{2\pi}}\right) * e^{\frac{-(k-\delta_{M/C}*\sqrt{\frac{(2-L)n}{L}})^2}{2}} + \frac{k-\delta_{M/C}*\sqrt{\frac{(2-L)n}{L}}}{\sqrt{2\pi}}) * e^{\frac{-(k+\delta_{M/C}*\sqrt{\frac{(2-L)n}{L}})^2}{2}} \quad \text{..............(4.34)}$$

Thus, the expected quality loss incurred per cycle in the out-of-control state owing to machine failure is:

$$E\left[(\text{cost of } L_{\text{out of control}})_{M/C}\right] = [L_{\text{out of control}}]_{M/C} * \left\{ATS_2 - \xi + n * g + \gamma_1 t_1 + \gamma_2 * E\,(T_{\text{restore}})\right\} * \frac{\lambda_2}{\lambda} \quad \text{.............(4.35)}$$

Similarly, the quality loss per unit time incurred while the process is operating in out-of- control state owing to external reasons is given as:

$$[L_{\text{out of control}}]_{E} = PR * \frac{A}{d'^2} * \frac{\sigma^2 * L}{(2-L)*n}\left[\left(1+{\delta_E}^2 * \frac{(2-L)n}{L}\right) * \{1 - \varphi(k - \delta_E * \sqrt{\frac{(2-L)n}{L}}) + \varphi(-k - \delta_E * \sqrt{\frac{(2-L)n}{L}})\} + \left(\frac{k+\delta_E*\sqrt{\frac{(2-L)n}{L}}}{\sqrt{2\pi}}\right) * e^{\frac{-(k-\delta_E*\sqrt{\frac{(2-L)n}{L}})^2}{2}} + \frac{k-\delta_E*\sqrt{\frac{(2-L)n}{L}}}{\sqrt{2\pi}}) * e^{\frac{-(k+\delta_E*\sqrt{\frac{(2-L)n}{L}})^2}{2}} \quad \text{..............(4.36)}$$

Thus, the expected quality loss cost incurred in out-of-control state owing to external reasons is:

$$E\,[(\text{cost of } L_{\text{out of control}})E\,] = [L_{\text{out of control}}]E * \{ATS_2 - \xi + n * g + \gamma_1 t_1 + \gamma_2 * E\,(T_{\text{restore}})\} * \frac{\lambda_1}{\lambda} \qquad \text{.............(4.37)}$$

Adding Equations (4.26), (4.27), (4.28), (4.29), (4.33), (4.35), and (4.37) gives the expected cost of process failure per cycle as:

$$E[C_{\text{process}}] = E[C_{\text{false}}] + E\,[\text{Cost of sampling}] + E\,[C_{\text{resetting}}] + E\,[(C_{\text{repair}})_{FM2}] + E\,[L_{\text{in control}}] + E\,[(\text{cost of } L_{\text{out of control}})_{M/C}] + E\,[(\text{cost of } L_{\text{out of control}})_{E}\,] \qquad \text{.............(4.38)}$$

The process failure is assumed to be repetitive in nature, i.e. every time when the process moves out-of-control from the in-control state and is again restored, it will take the same expected time (having fixed expected cycle length).If there are M process failure cycles in a given evaluation period, the expected process quality control cost for the evaluation period will be:

$$E\,[TCQ]_{\text{process- failure}} = E[C_{\text{process}}] * M \qquad \text{.............(4.39)}$$

$$\text{where },\quad M = \frac{T_{\text{eval}}}{E[T_{\text{cycle}}]} \qquad \text{.............(4.40)}$$

Let the rate of failure owing to machine degradation be λ_2 and that owing to external causes be λ_1. Thus the process failure rate λ is the sum of failure rates owing to machine degradation and owing to assignable causes. It can be written as

$$\lambda = \lambda_1 + \lambda_2 \qquad \text{.............(4.41)}$$

N_f is the number of machine failures and T_{eval} is the evaluation period and t_{PM} is the preventive maintenance interval. Pandey et al (2012) used a simulation approach to generate a regression model to establish a relationship between N_f and t_{PM}

$$N_f = 0.0437 * (t_{PM})^{0.8703} \qquad \text{.............(4.42)}$$

The process failure rate owing to machine degradation can be calculated as

$$\lambda_2 = \frac{N_f * P_{FM_2}}{T_{\text{eval}}} \qquad \text{.............(4.43)}$$

Optimization Model

The problem is to determine the optimal values of the decision variables (n, h, w, k, η, L *and* t_{pm}) that minimize the expected total cost per unit time of the system ETCPUT. Recall that the age of the equipment after a PM is reduced according to the restoration factor. The expected total cost per unit time of preventive maintenance and control chart policy ETCPUT is the ratio of the sum of the expected total cost of the process quality control (E [TCQ] $_{\text{process-failure}}$), expected total costs of the preventive maintenance E[C_{PM}] and expected total cost of machine failure (E[C_{CM}]$_{FM1)}$ to the evaluation time. Therefore, the expected total cost per unit time for the integrated model is given as:

$$\text{Minimize [ETCPUT]} = \frac{\text{E}[\text{C}_{\text{CM}}]\text{FM1} + \text{E}[\text{C}_{\text{PM}}] + \text{E}[\text{TCQ}]\text{process}-\text{failure}}{\text{Teval}} \qquad \ldots\ldots\ldots\ldots(4.44)$$

where [ETCPUT] = f (n, h, w, k, η, L *and* t_{pm}) and T_{eval} is the Planning period for which the analysis is done.

CHAPTER V

NUMERICAL EXAMPLE

Equation (4.44) indicates that optimizing the seven variables (n, h, w, k, η, L *and* t_{pm}) is not a simple process. In this section, a numerical example (According to Pandey et al. 2012) is presented to illustrate the nature of the solution obtained from the economic design of the proposed integrated model.

To illustrate, consider a single component operating as a part of a machine. Machine failure is assumed to follow a two-parameter Weibull distribution. The machine considered here is expected to operate for three shifts of seven hours each for six days in a week. Time to execute a preventive maintenance action T_{PM} = 7 time units with a restoration factor RF_{PM} = 0.6 (it implies 60% restoration of life and sets the age of the block to 40% of the age of the block at the time of the maintenance action) and time to execute corrective maintenance action T_{CM}=12 time units with restoration factor RF_{CM} = 0 (repair is minimal, i.e. the age of a repaired machine is the same as its age when it failed).The time to failure for the component was obtained through simulation.

A hypothetical example is illustrated in this section to analyze the proposed integrated model. It is assumed that the EWMA control chart is used to monitor a CTQ characteristic. Assuming that the process, in its in-control state, is characterized by a process mean of μ_0 = 24 mm and a process standard deviation of σ= 0.01 and that the magnitude of the shift owing to external reasons is δ_E = 1.5 and owing to machine failure is $\delta_{M/C}$ = 0.6, which occurs at random and results in a shift of process mean from μ_0 to $\mu_0 + \delta\sigma$.

The initial values of necessary parameters are given in Table 5.1

Table 5.1 Initial values of parameters for the hypothetical numerical example

Parameter	Value	Parameter	Value
δ_E	1.5	PR	10
$\delta_{M/C}$	0.6	T_{eval}	1000
g	20	C_{lp}	400
a	100	LC	500
b	50	C_{FCPPM}	1000
C_{false}	1200	$MTTR_{CM}$	12
A	500	$MTTR_{PM}$	7
C_{Frej}	2500	d'	0.05
C_{FCPCM}	10000	μ_0	24
C_{reset}	5000	σ	0.01
t_0	1	λ_1	0.05
t_1	1	P_{FM1}	0.4
$T_{resetting}$	2	P_{FM2}	0.6

CHAPTER VI

RESULTS AND DISCUSSIONS

The thesis work is theoretical in nature. A mathematical hypothetical problem is considered for having better demonstration about the implementation of the proposed model. The proposed model is composed of some mathematical equations which are required to determine the optimal values of seven decision variables i.e. the sample size, the fixed sampling interval, the control limit coefficient, the warning limit coefficient, the number of subintervals between two consecutive sampling times, exponential weight constant and the time interval of preventive maintenance. A computer programming code of the proposed model has been generated in Matlab 7.8.0 (R2009a) and two of the solution methods have been used to get the optimal value.

6.1 Result Discussion

Firstly the Nelder-Mead Downhill simplex method is mainly used in order to find the most economic design for a given set of input parameters. The numerical results obtained from three of the Nelder-Mead Simplex runs are summarized in Table 6.1. Although the Nelder-Mead method does not guarantee convergence to the global optimal solution, but in this case it is apparent that the cost values resulting from implementations with different initial points are close to each other. In the table 6.1 three of such implementations' results are summarized and it is observed that the results are more or less similar for the cost values and for the variables except w and η. The rest of the five variables showed nearly very close result.

Table 6.1 Optimization using Nelder-Mead downhill simplex method

n	h	k	w	η	L	t_{PM}	Cost
10.012	9.9879	5	1.1556	8.0750	0.8	199.4712	708.5763
10	9.9199	5	3.9433	3.1552	0.8	197.4901	709.1733
10.004	9.8631	5	2.6368	3.6783	0.8	199.7448	709.0018

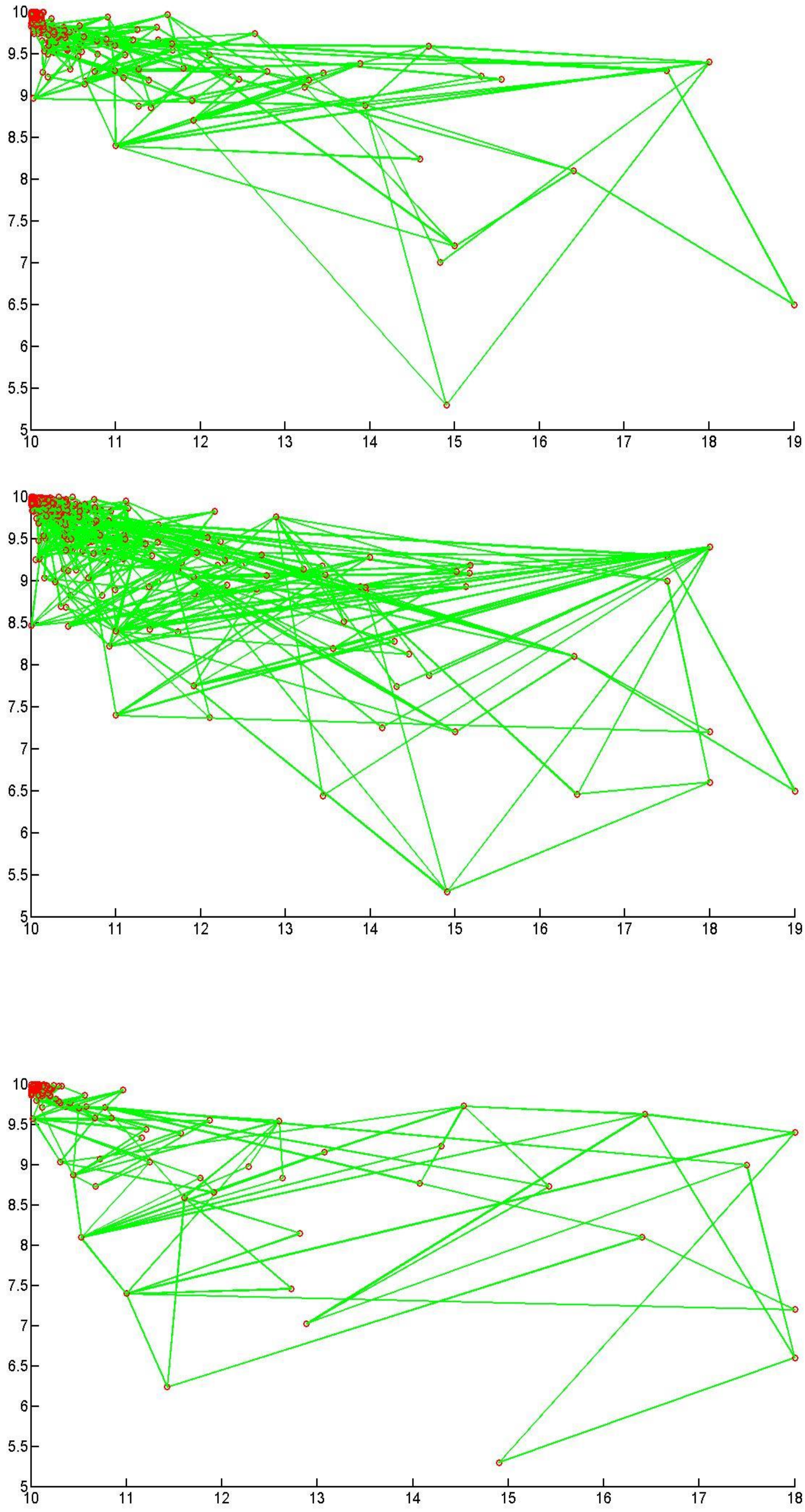


Fig.6.1 Convergence path of points in the domain in Nelder-Mead method

So it can be concluded that as an optimization technique Nelder-Mead algorithm inhibited good convergence. And the Convergence paths of points in the domain in Nelder-Mead method are shown in the Fig 6.1. From Fig 6.1 it can be observed that the initial parameters are set different and therefore the starting simplex of the Nelder-Mead algorithms are different. And it is apparently seen that the transition of the simplex from larger approximation to the optimal solution is different. But the destinations of the paths or the convergence points are same at the upper left corner of the graph. So we can certainly conclude that the solutions found using this method might reach to the global optimal value.

Moreover, to validate the effectiveness of the result obtained in this approach, another technique genetic algorithm approach is also used to have the optimal values of decision variables that minimize the expected total cost of system per unit time ETCPUT. It has been observed that, the results are very close to the results of Nelder-Mead method. There are three results summarized in table 6.2 obtained using GA. Stall generation (G) and mutation rate (m) have been changed to have better view of the result. The Convergence steps with number of generations in GA are shown in the Fig 6.2.From the Fig.6.2 it has been observed that for the first graph when stall generation or iteration was set to 400 the convergence path was decreasing step by step. And the fine or smooth portion was not reached quickly. On the contrary, in the second and third graph when the stall generation was set to 1000, the stepped solution became more stable ensuring an optimized output. By increasing the mutation rate to 0.05, the fine or smoother convergent portion was attained earlier as being seen in the third graph. So it can be concluded that with the increase of stall generation or iteration the chance of having optimal result can be ensured effectively. Besides, in the third graph when mutation rate was increased that mean the variation of variable parameter can form in a large domain then the path or results can drastically fall and go toward the convergent result very soon. So increasing the mutation rate is also good for having the global optimal solution.

Table 6.2 Optimization data using Genetic Algorithm

G	m	n	h	k	W	η	L	t_{PM}	Cost
400	0.01	10.058	9.99	4.994	3.290	2.025	0.8	199.62	710.40
1000	0.01	10.029	9.998	4.996	3.195	2.002	0.796	199.90	710.85
1000	0.05	10.031	9.993	5	3.202	2.018	0.799	199.91	709.12

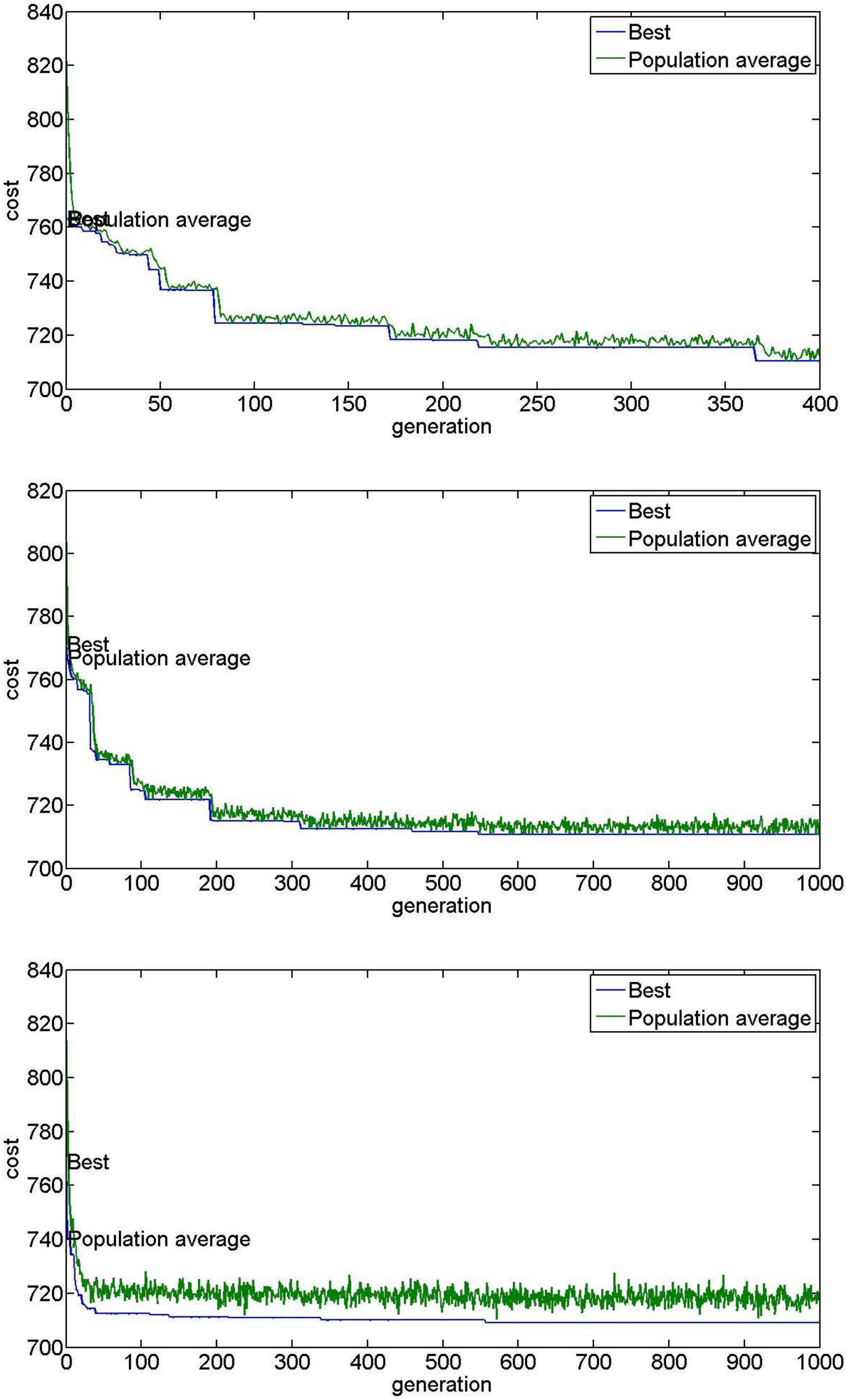


Fig.6.2 Minimization of cost (Best cost and Population average) with number of generations in GA

6.2 Sensitivity Analysis

In order to study the effects of some of the model parameters, a sensitivity analysis is performed with the illustrative example shown in chapter 5. In Table.6.3, level 1 is the basic level which was used to solve the example in Section 6.Levels 2 and 3 represent the values of these parameters at -10% and +10% of the basic level respectively. For sensitivity analysis, ½ fraction factorial Design of Experiment is performed and the estimated effects of each parameter are shown in the Table 6.4.Besides.One way Analysis of Variance or ANOVA is also performed to identify the most sensitive parameter of the proposed model.

The optimum value of the decision variables for level 1 or basic level are kept constant and the corresponding values of the objective function at two different levels (one -10% decreasing and one +10% increasing) of the model parameters are calculated for 16 different combinations for the ½ fraction factorial Design of Experiment. The result of effects of each parameter and the F values and P values found for each of the parameters from the Analysis of Variance are shown in Table 6.4. The following observations are made from the analysis.

It is apparent from the analysis that, δ_E has the highest impact on the model because the magnitude of effect is the highest (233.226) .On the other hand, g and $\delta_{M/C}$ both has less impact on the model due to less magnitude of effect. This suggests that the economic model is quite robust to g and variation in the magnitude of the shift in the process mean owing to assignable causes attributed to machine failure with failure mode 2 $\delta_{M/C}$.

Moreover, from the ANOVA test it is also found that δ_E has highest F value where P value is 0.00 and in case of g and $\delta_{M/C}$ have highest P value where F value is 0.00.So it also proved that δ_E is significant and g , $\delta_{M/C}$ are least significant. The reason behind this analysis is also justified as in this model, due to maintaining a preventive maintenance change of $\delta_{M/C}$ does not imply that much change to the optimal cost of the model. On the contrary, change of δ_E or the magnitude of the shift in the process mean owing to assignable causes attributed to machine failure with failure mode 2 implies drastic changes on the optimal cost or value of the decision variables of the model. For this reason the test becomes highly significant for δ_E .

Table 6.3 Experimental data set for sensitivity analysis

Parameter	**Basic(1)**	**Level 2 (-10%)**	**Level 3 (+10%)**
δ_E	1.5	1.35	1.65
$\delta_{M/C}$	0.60	0.54	0.66
λ	0.05	0.045	0.055
b	50	45	55
g	20	18	22

Table 6.4 Results from DOE and One way ANOVA

Parameter	**Effect**	**F value**	**P value**
δ_E	233.226	5862.55	0.000
$\delta_{M/C}$	1.057	0.00	0.987
λ	4.548	0.01	0.943
b	9.889	0.03	0.876
g	0.341	0.00	0.996

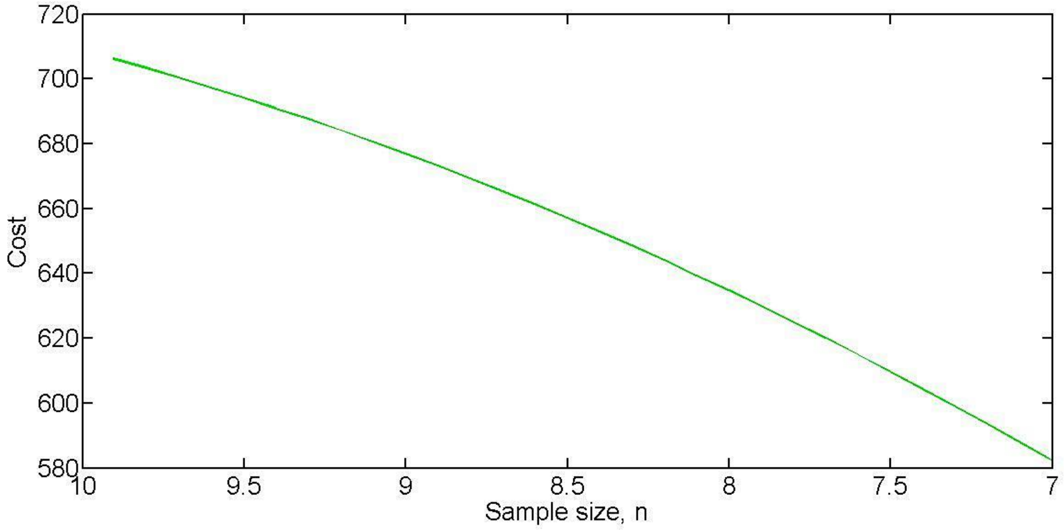
6.3 Effect of changes in variable parameter on cost

To observe the effect of the decision variables on the total cost, an analysis for all the variables was performed. In total 30 data points were used for each of the independent variable keeping all other variables constant. In this data set, 15 points are taken following increasing trend and the rest 15 points are taken following decreasing trend of the respective variable to have the understanding about the relationship or effect of the variable change on the total cost.

Table 6.5 Variation of cost with the change of sample size and fixed sampling interval

Variation of cost with sample size				Variation of cost with fixed sampling interval			
n *(increasing)*	cost	*n* *(decreasing)*	cost	*h* *(increasing)*	cost	*h* *(decreasing)*	cost
10.17	713.42	9.90	706.23	10.03	707.97	9.77	709.59
10.34	717.60	9.80	703.37	10.21	706.97	9.67	710.19
10.50	721.56	9.70	700.42	10.38	706.01	9.58	710.80
10.67	725.30	9.60	697.37	10.55	705.08	9.48	711.43
10.84	728.84	9.50	694.23	10.72	704.18	9.39	712.07
11.00	732.19	9.40	690.99	10.89	703.30	9.29	712.72
11.17	735.35	9.30	687.66	11.06	702.46	9.20	713.39
11.34	738.34	9.20	684.22	11.23	701.64	9.10	714.07
11.50	741.17	9.10	680.68	11.40	700.84	9.00	714.76
11.67	743.83	9.00	677.04	11.58	700.07	8.91	715.47
11.84	746.36	8.90	673.29	11.75	699.32	8.81	716.19
12.00	748.74	8.80	669.44	11.92	698.59	8.72	716.93
12.17	751.00	8.70	665.49	12.09	697.88	8.62	717.69
12.34	753.13	8.60	661.42	12.26	697.20	8.53	718.46
12.50	755.16	8.50	657.25	12.43	696.53	8.43	719.25
12.67	757.08	8.40	652.98	12.60	695.88	8.34	720.06
12.84	758.90	8.30	648.59	12.77	695.24	8.24	720.89
13.00	760.63	8.20	644.10	12.95	694.63	8.15	721.74
13.17	762.28	8.10	639.50	13.12	694.03	8.05	722.60
13.34	763.85	8.00	634.79	13.29	693.44	7.95	723.49
13.50	765.35	7.90	629.98	13.46	692.87	7.86	724.40
13.67	766.78	7.80	625.06	13.63	692.32	7.76	725.33
13.84	768.15	7.70	620.04	13.80	691.78	7.67	726.29
14.00	769.47	7.60	614.92	13.97	691.25	7.57	727.27
14.17	770.73	7.50	609.69	14.14	690.73	7.48	728.27
14.34	771.95	7.40	604.37	14.32	690.23	7.38	729.30
14.50	773.12	7.30	598.95	14.49	689.74	7.29	730.36
14.67	774.26	7.20	593.44	14.66	689.26	7.19	731.44
14.84	775.36	7.10	587.84	14.83	688.79	7.10	732.56
15.00	776.42	7.00	582.15	15.00	688.33	7.00	733.70

The variation of cost with the change of sample size and fixed sampling interval is shown in the Table 6.5 and the patterns of the relationships are shown in the Fig 6.3 and Fig 6.4 respectively. The green curve shows pattern for decreasing the value of the variable and the red curve shows the pattern for increasing the value of the variable.



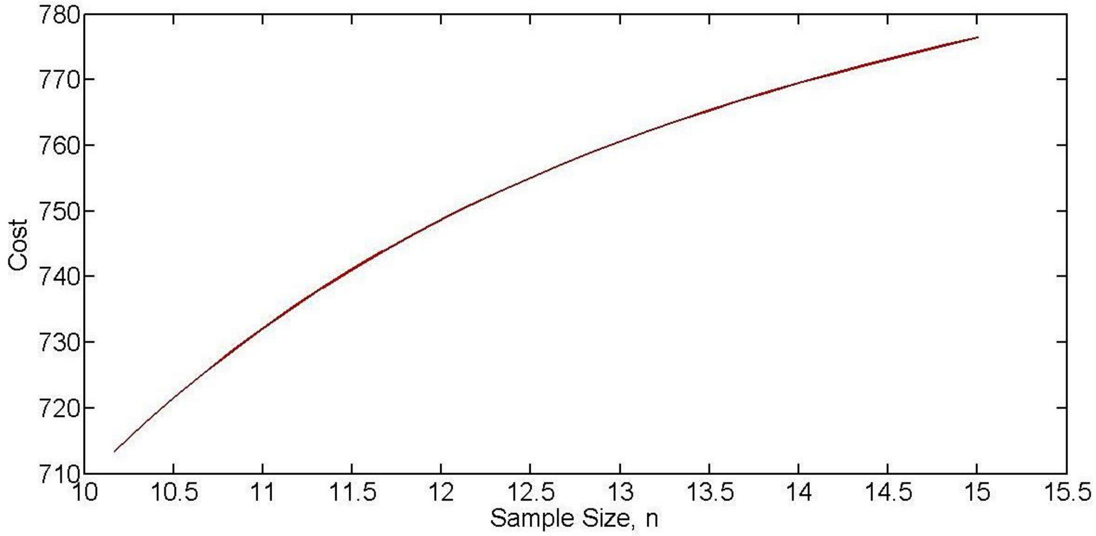
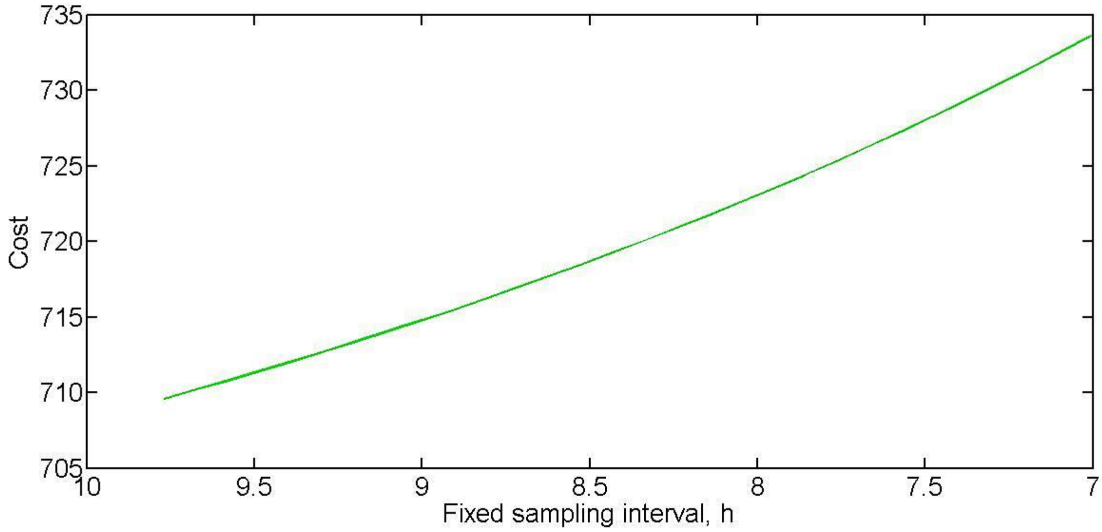


Fig.6.3 Relationship between sample size variability and cost (Decreasing and Increasing respectively)



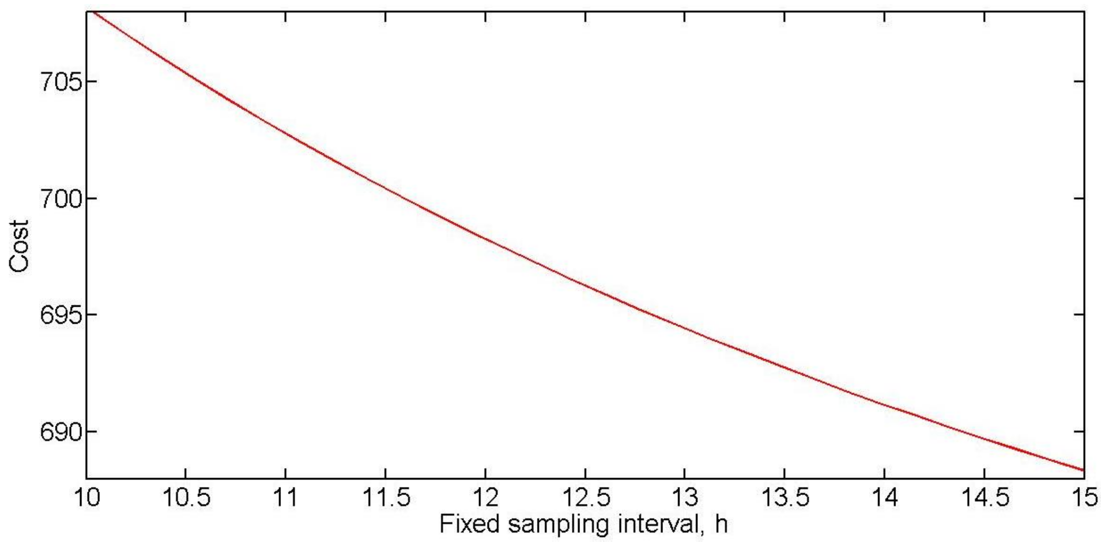


Fig.6.4 Relationship between fixed sampling interval variability and cost (decreasing and increasing respectively)

It can be easily seen that with the increase of sample size the cost is increasing. The relationship is almost linear. It is quite justified as increase in sample size will definitely increase the sampling cost, again it will also influence the in control and out control cost calculated by Taguchi Loss Function and also make them increase. On the other hand from the above figure Fig.6.4 it can be found that with increase of fixed sampling interval the cost is decreasing. The reason is with higher sampling interval sampling frequency decreases which in turn decrease the sampling cost.

Table 6.6 Variation of cost with change of control limit coefficient and warning limit coefficient

Variation of cost with control limit coefficient				Variation of cost with warning limit coefficient			
k (increasing)	cost	*k (decreasing)*	cost	*w (increasing)*	cost	*w (decreasing)*	cost
5.07	702.96	4.90	717.19	2.72	709.00	2.55	709.00
5.13	696.45	4.80	724.37	2.79	709.00	2.46	709.00
5.20	689.46	4.70	730.61	2.87	709.00	2.37	709.00
5.27	681.99	4.60	735.97	2.95	709.00	2.29	709.00
5.33	674.05	4.50	740.54	3.03	709.00	2.20	709.00
5.40	665.65	4.40	744.38	3.11	709.00	2.11	709.00
5.47	656.80	4.30	747.59	3.19	709.00	2.02	709.00
5.53	647.52	4.20	750.24	3.27	709.00	1.93	709.01
5.60	637.84	4.10	752.42	3.35	709.00	1.85	709.01
5.67	627.78	4.00	754.20	3.42	709.00	1.76	709.02
5.73	617.39	3.90	755.64	3.50	709.00	1.67	709.03
5.80	606.70	3.80	756.81	3.58	709.00	1.58	709.04
5.87	595.75	3.70	757.75	3.66	709.00	1.49	709.07
5.93	584.60	3.60	758.52	3.74	709.00	1.41	709.12
6.00	573.28	3.50	759.14	3.82	709.00	1.32	709.19
6.07	561.85	3.40	759.66	3.90	709.00	1.23	709.29
6.13	550.37	3.30	760.09	3.98	709.00	1.14	709.45
6.20	538.89	3.20	760.45	4.05	709.00	1.05	709.69
6.27	527.45	3.10	760.77	4.13	709.00	0.97	710.04
6.33	516.13	3.00	761.05	4.21	709.00	0.88	710.54
6.40	504.95	2.90	761.29	4.29	709.00	0.79	711.25
6.47	493.98	2.80	761.52	4.37	709.00	0.70	712.24
6.53	483.26	2.70	761.72	4.45	709.00	0.62	713.61
6.60	472.84	2.60	761.90	4.53	709.00	0.53	715.50
6.67	462.74	2.50	762.07	4.61	709.00	0.44	718.09
6.73	453.02	2.40	762.22	4.68	709.00	0.35	721.66
6.80	443.69	2.30	762.35	4.76	709.00	0.26	726.62
6.87	434.78	2.20	762.47	4.84	709.00	0.18	733.63
6.93	426.32	2.10	762.57	4.92	709.00	0.09	743.85
7.00	418.31	2.00	762.65	5.00	709.00	0.00	759.45

The variation of cost with the change of control limit coefficient and warning limit coefficient is shown in the Table 6.6 and the patterns of the relationships are shown in the Fig 6.5 and Fig 6.6 respectively. The green curve shows pattern for decreasing the value of the variable and the red curve shows the pattern for increasing the value of the variable.

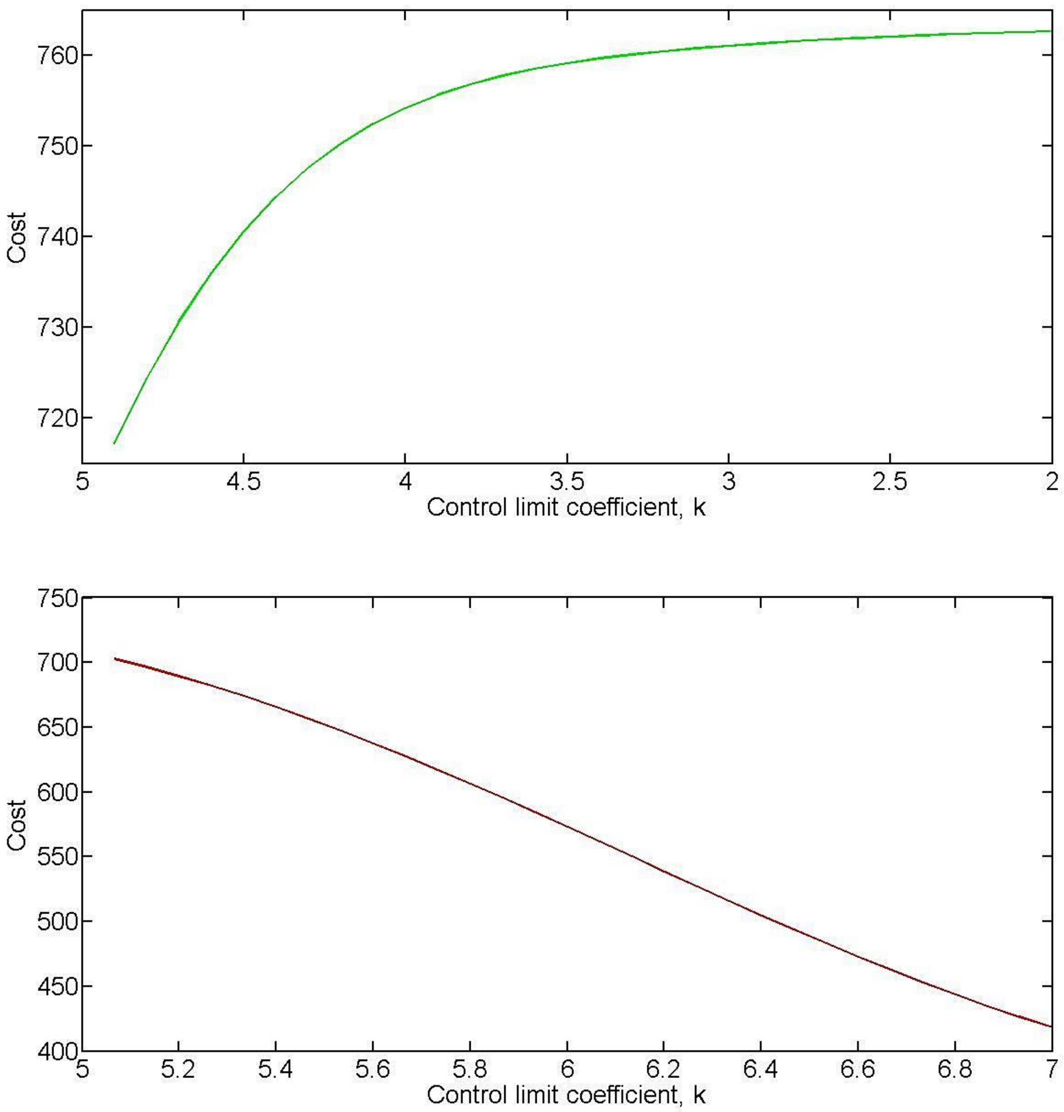


Fig.6.5 Relationship between control limit coefficient variability and cost (Decreasing and Increasing respectively)

It can be found that with the increase of control limit coefficient the cost is decreasing. The relationship is almost linear. But when the coefficient is decreasing the cost increases rapidly at first but almost started to converge from the value of 3.5 and lower. The possible reason is with higher control limit coefficient value more tighter control is achieved which in turn decreases the low quality product and decreases the final rejection cost. In case of warning limit coefficient, with the decrease of the coefficient, cost is constant at first but increase rapidly from a certain value. With the increase of the coefficient cost decreases rapidly at first but becomes constant from a certain value. As low warning limit results in higher no. of sampling and more sampling results in high sampling cost, this result is acceptable.

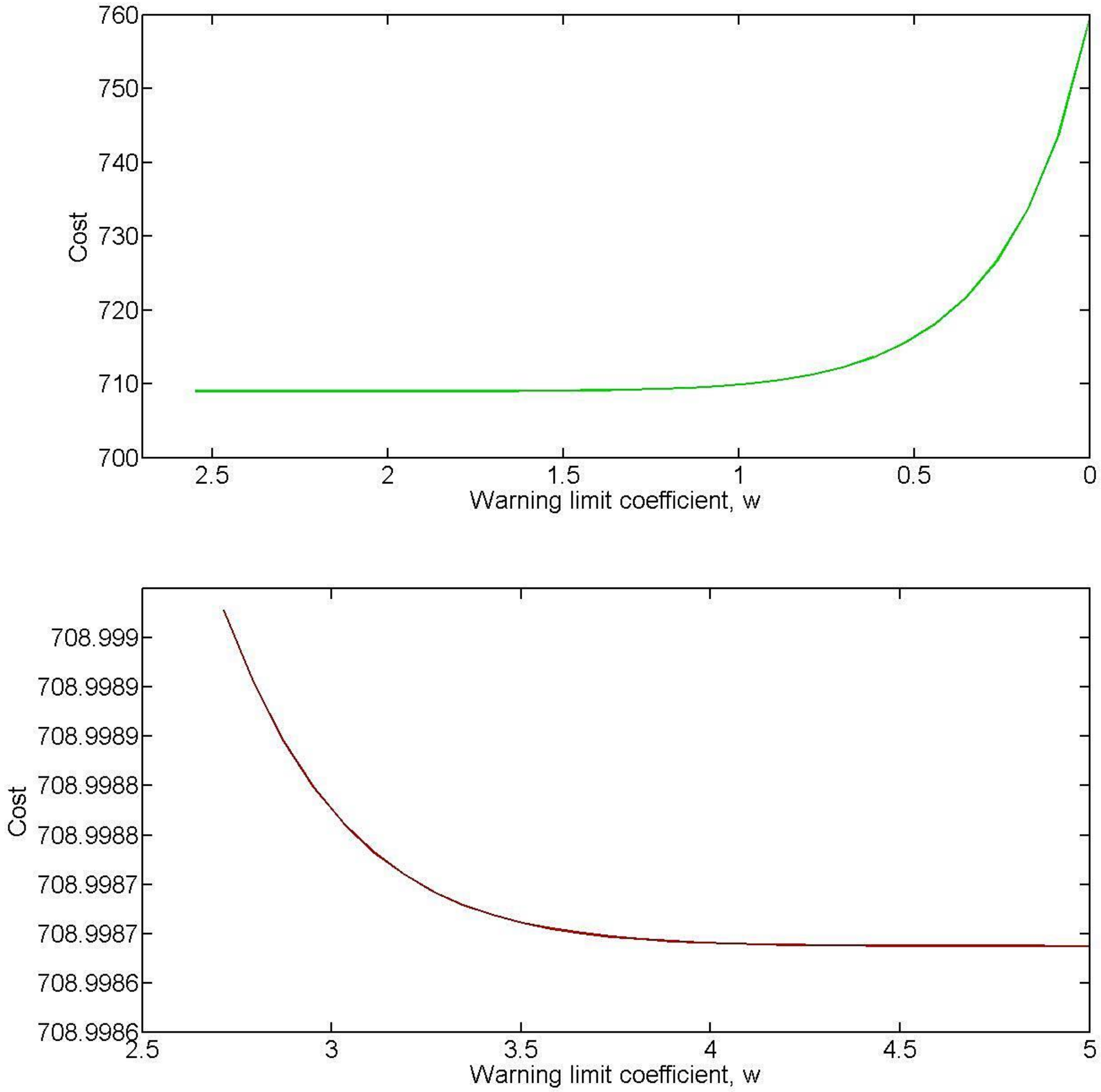


Fig.6.6 Relationship between warning limit coefficient variability and cost (Decreasing and Increasing respectively)

The variation of cost with the change of no of subinterval between two consecutive sampling time sand Exponential weight constant is shown in the Table 6.7 and the patterns of the relationships are shown in the Fig 6.7 and Fig 6.8 respectively. The green curve shows pattern for decreasing the value of the variable and the red curve shows the pattern for increasing the value of the variable.

Table 6.7 Variation of cost with the change of no of subinterval between two consecutive sampling times and Exponential weight constant

Variation of cost with no of subinterval between two consecutive sampling times				Variation of cost with Exponential weight constant			
n (increasing)	cost	*n (decreasing)*	cost	*n (increasing)*	cost	*n (decreasing)*	cost
3.78	709.00	3.62	709.00	0.81	705.81	0.79	712.81
3.89	709.00	3.57	709.00	0.81	702.51	0.78	716.44
3.99	709.00	3.51	709.00	0.82	699.09	0.78	719.88
4.10	709.00	3.45	709.00	0.83	695.56	0.77	723.12
4.21	709.00	3.40	709.00	0.83	691.92	0.76	726.17
4.31	709.00	3.34	709.00	0.84	688.17	0.75	729.03
4.42	709.00	3.29	709.00	0.85	684.32	0.74	731.70
4.52	709.00	3.23	709.00	0.85	680.37	0.73	734.18
4.63	709.00	3.17	709.00	0.86	676.33	0.73	736.47
4.73	709.00	3.12	709.00	0.87	672.19	0.72	738.59
4.84	709.00	3.06	709.00	0.87	667.97	0.71	740.52
4.94	709.00	3.01	709.00	0.88	663.66	0.70	742.29
5.05	709.00	2.95	709.00	0.89	659.27	0.69	743.89
5.15	709.00	2.90	709.00	0.89	654.81	0.68	745.33
5.26	709.00	2.84	709.00	0.90	650.28	0.68	746.62
5.36	709.00	2.78	709.00	0.91	645.69	0.67	747.77
5.47	709.00	2.73	709.00	0.91	641.03	0.66	748.78
5.57	709.00	2.67	709.00	0.92	636.32	0.65	749.67
5.68	709.00	2.62	709.00	0.93	631.57	0.64	750.44
5.79	709.00	2.56	709.00	0.93	626.77	0.63	751.09
5.89	709.00	2.50	709.00	0.94	621.92	0.63	751.65
6.00	709.00	2.45	709.00	0.95	617.05	0.62	752.12
6.10	709.00	2.39	709.00	0.95	612.14	0.61	752.50
6.21	709.00	2.34	709.00	0.96	607.22	0.60	752.81
6.31	709.00	2.28	709.00	0.97	602.27	0.59	753.05
6.42	709.00	2.22	709.00	0.97	597.31	0.58	753.24
6.52	709.00	2.17	709.00	0.98	592.33	0.58	753.37
6.63	709.00	2.11	709.00	0.99	587.36	0.57	753.46
6.73	709.00	2.06	709.00	0.99	582.38	0.56	753.51
6.84	709.00	2.00	709.00	1.00	577.41	0.55	753.52

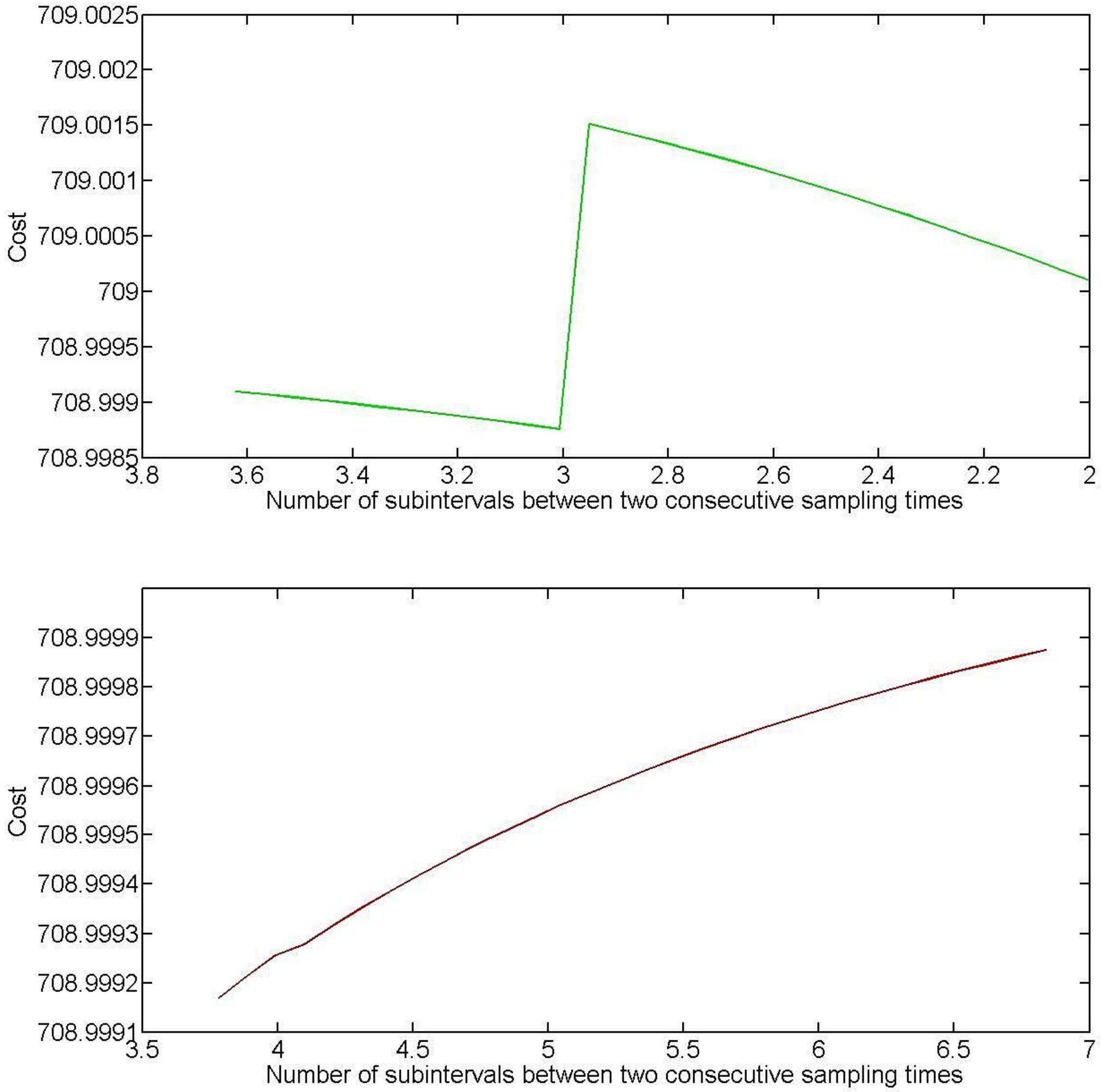


Fig.6.7 Relationship between number of subinterval between two consecutive sampling times variability and cost (Decreasing and Increasing respectively)

It can be found that with the increase of no of subinterval between two consecutive sampling times the cost is increasing slowly and with the decrease of no of subinterval between two consecutive sampling times the cost is decreases at first then increases for a time then again started to decrease. More subintervals mean more sampling, so this is justified. In case of Exponential weight constant with decrease of it the cost increases and with increase of it the cost decreases.

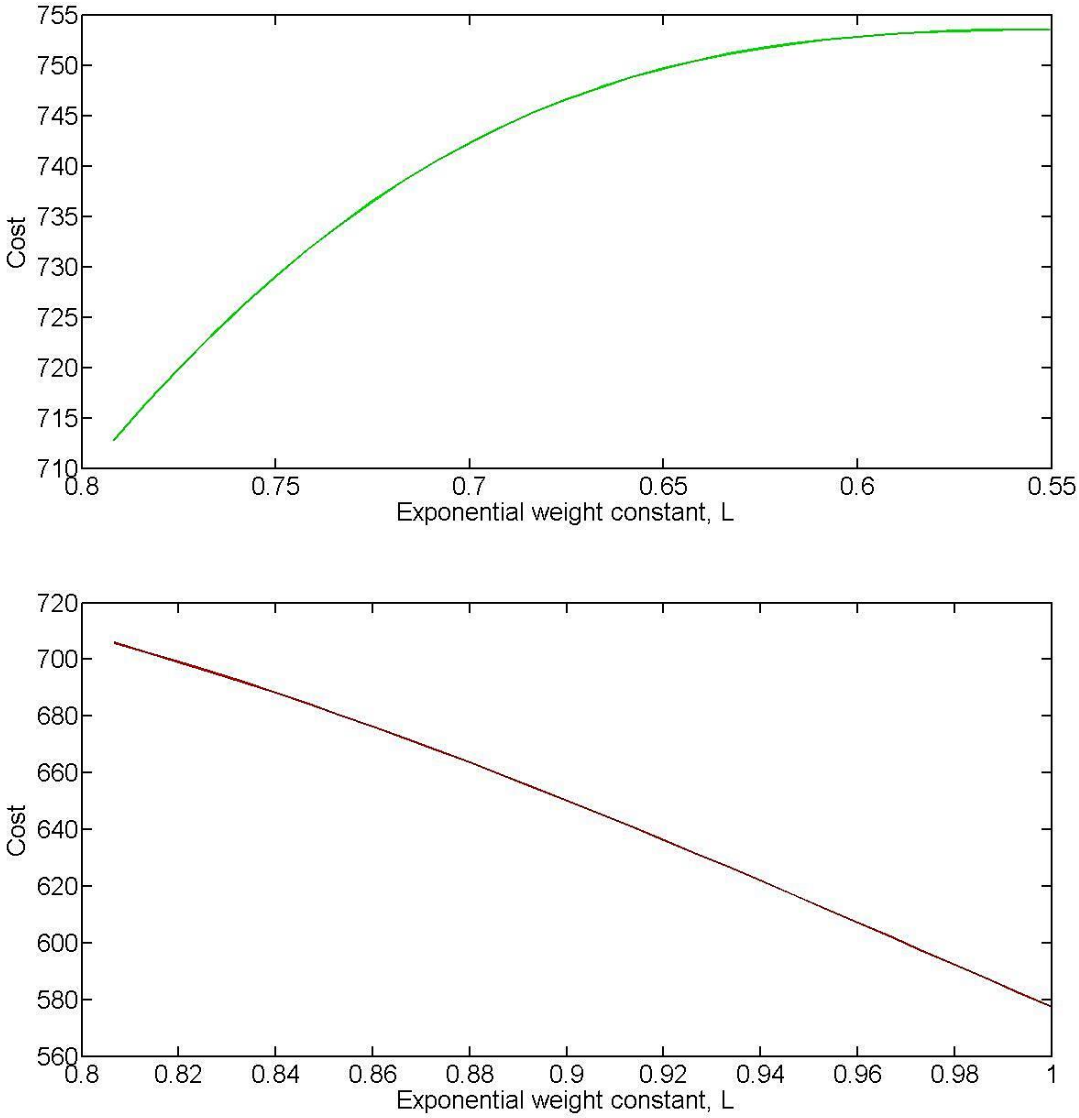


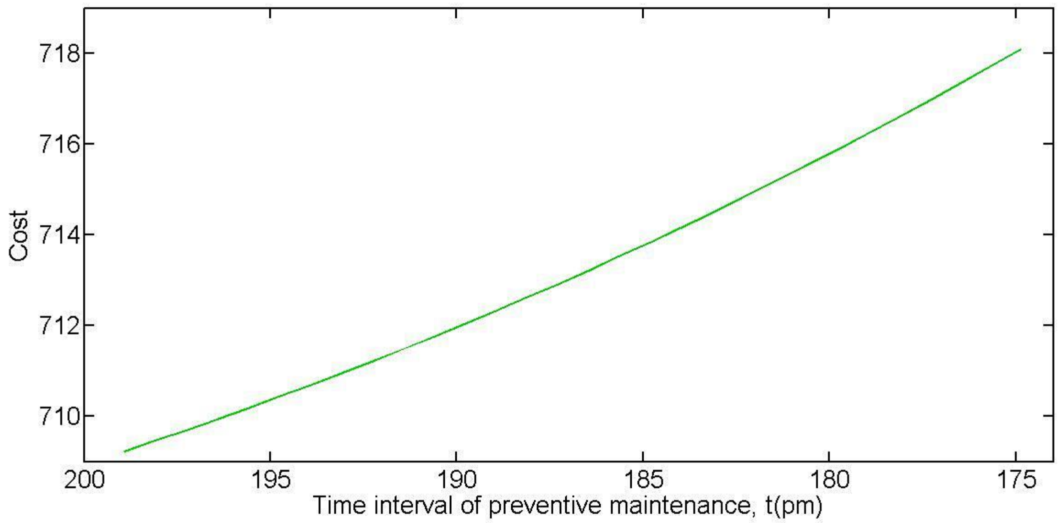
Fig.6.8 Relationship between Exponential weight constant variability and cost (Decreasing and Increasing respectively)

The variation of cost with the change of time interval of preventive maintenance is shown in Table 6.8 and the patterns of the relationships are shown in the Fig 6.9. The green curve shows pattern for decreasing the value of the variable and the red curve shows the pattern for increasing the value of the variable.

Table 6.8 Variation of cost with the change of time interval of preventive maintenance

Variation of cost with time interval of preventive maintenance			
t_{PM} *(increasing)*	cost	t_{PM} *(decreasing)*	cost
200.09	708.91	198.92	709.22
200.43	708.82	198.09	709.45
200.77	708.73	197.26	709.69
201.11	708.64	196.43	709.93
201.45	708.55	195.60	710.17
201.80	708.46	194.77	710.42
202.14	708.37	193.94	710.68
202.48	708.29	193.11	710.94
202.82	708.20	192.28	711.20
203.16	708.12	191.45	711.47
203.51	708.03	190.62	711.75
203.85	707.95	189.80	712.03
204.19	707.87	188.97	712.32
204.53	707.79	188.14	712.61
204.87	707.71	187.31	712.91
205.21	707.63	186.48	713.21
205.56	707.55	185.65	713.52
205.90	707.47	184.82	713.83
206.24	707.39	183.99	714.15
206.58	707.31	183.16	714.48
206.92	707.23	182.33	714.81
207.27	707.16	181.51	715.15
207.61	707.08	180.68	715.50
207.95	707.01	179.85	715.85
208.29	706.94	179.02	716.21
208.63	706.86	178.19	716.57
208.97	706.79	177.36	716.94
209.32	706.72	176.53	717.32
209.66	706.65	175.70	717.70
210.00	706.58	174.87	718.09

It can be found that with the increase of time interval of preventive maintenance the cost is decreasing almost linearly. It is quite expected because a significant amount of cost incurs with every preventive maintenance activity. Though preventive maintenance is good for the machine life but the cost of carrying preventive maintenance activity is too much high compared to the benefit it added.



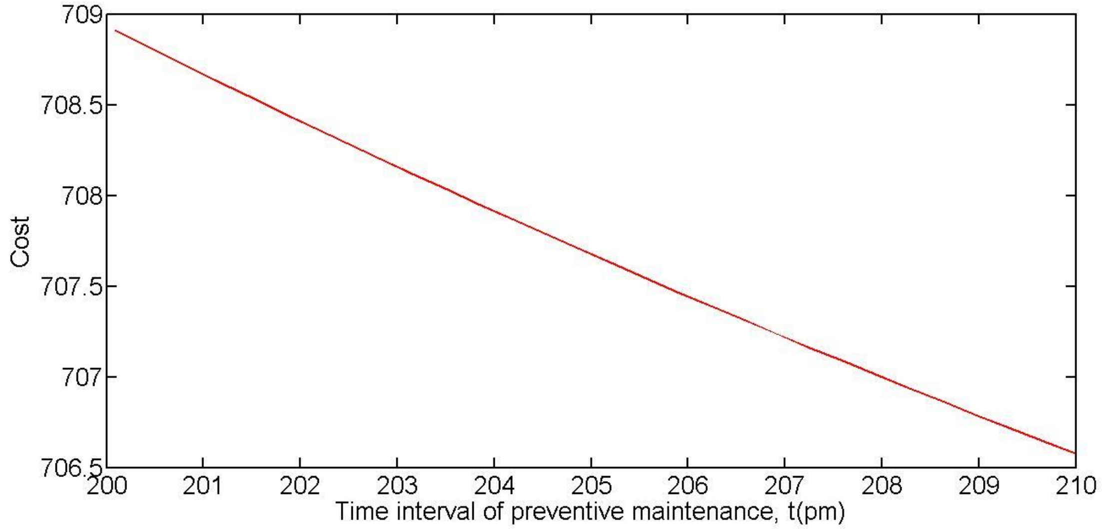


Fig.6.9 Relationship between time interval of preventive maintenance variability and cost (Decreasing and Increasing respectively)

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The EWMA chart is a widely applied control chart for detecting small shifts in process mean. The control charts using variable sampling intervals with sampling at fixed times (VSIFT) policy has been shown to give substantially faster detection of most process shifts than the conventional control charts. In the present thesis, an integrated model which takes advantage of the two traditional but separately used manufacturing process control tools – statistical process control and maintenance management – is proposed to determine the values of the six test parameters of the chart (i.e., the sample size, the fixed sampling interval, the number of subintervals between two consecutive sampling times, the warning limit coefficient, the control limit coefficient, and the exponential weight constant) and one the preventive maintenance interval such that the expected total cost per hour is minimized. Nelder Mead and Genetic Algorithm approach is applied to find the optimal values of control chart parameters and preventive maintenance schedule ($n, h, k, w, \eta, L, t_{PM}$) respectively that minimize the hourly cost. To validate the effectiveness of the solution process performed for optimization of the integrated cost model, two approaches are used and approximately similar result is found.

This research offers a promising conceptual contribution that suggests that maintenance and process control decisions, often considered independent in practice, need to be coordinated. In our model, Statistical Process Control monitors the equipment and provides signals indicating equipment deterioration, while Planned Maintenance is scheduled at regular intervals to preempt equipment failure. Our research demonstrates the benefits from conducting maintenance as an adaptive maintenance policy, where the maintenance schedule adapts to the stability of the process. The analysis demonstrates considerable economic benefit in coordinating process-control and maintenance policies and indicates conditions for which the benefits are most pronounced.

This thesis opens up a promising research area and suggests that maintenance and process quality control decisions, when considered jointly, can lead to better process performance. An integrated model using Taguchi loss function is developed for the joint optimization of preventive maintenance interval and quality control policy of the process subject to machine failures and quality shifts. Thus, the methodology developed in this thesis is a step towards better planning through joint consideration of maintenance and process quality-control policy.

7.2 Recommendations

While the model's effectiveness is demonstrated using a EWMA chart, it is possible to use a variety of other control charts, such as CUSUM control chart, thereby expanding the model's applicability to diverse environments with unique process control approaches. However, further research could be conducted on the subject assuming Weibull or a more general distribution for the time to failure or out-of-control condition. The issue of two or more out-of-control conditions and a multiple equipment manufacturing process is also an issue which requires further research.

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