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DETERMINATION OF POWER SYSTEM STABILITY BY LIAPUNOV METHOD

A THESIS

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ABSTRACT

This thesis investigates stability of a typical power system problem-Single machine infinite bus system by second method of Liapunov which provides information on stability without solving the system differential equations.

The equations of a single machine infinite bus system equipped with voltage regulator and excitation system having the provision of auxiliary stabilizing signal are expressed in a state space form.The system equations were linearized about an equilibrium point and the stability of the equilibrium point was investigated through the second method of Liapunov. To investigate into the stability of the nonlinear system a variable gradient method of searching was employed. Stability of different operating conditions were studied considering (i) the synchronous machine model with no exciter dynamics(ii) with exciter dynamics (iii) with exciter and an additional stabilizing signal derived from rotor velocity. The effect of variation of exciter and regulator gain was also investigated through the second method. It was observed that the system could be operated with significantly large gain of the exciter provided it is equipped with additional stabilizing signals.

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NOTATION

E_{fd}	Generator open circuit field voltage.
E_o	Operating value of E_{fd} (E_{fdo})
e_d	Direct Axis component of armature voltage
e_q	Quadrature axis component of armature voltage
e_t	Generator terminal voltage.
e_{tr}	Reference terminal voltage
i_d	Direct axis armature current
i_q	Quadrature axis armature current
i_{fd}	Field current
K_r	Regulator gain
M	Generator inertia constant
n	Speed deviation
P_{in}	Net input power
P_o	Power Output (P_{out})
P	Differential operator
R	Armature resistance
R_e	Equivalent resistance from generator terminal to the busbar
r_{fd}	Field Winding resistance.
T_{in}	Net input torque
T_m	Inertia constant (2H) seconds
U(t)	Generator Input voltage (E_{fd})
$U_s(t)$	Stabilizing signal
V	Busbar voltage

V_{fd}	Applied Field Voltage
X_{afd}	Mutual reactance between field and armature.
X_e	Equivalent reactance from machine terminal to the busbar.
X_{ffd}	Self reactance of generator field winding.
X_d	Direct axis armature reactance
X_q	Quadrature axis armature reactance
δ	Rotor angular position, radian
Ψ_d	Direct axis armature flux linkage
Ψ_{fd}	Field flux linkage.
Ψ_q	Quadrature axis armature flux linkage
w	Angular frequency, rad/sec
τ_r	Regulator time constant, seconds.

All quantities normalized except as indicated.

CHAPTER- 1

INTRODUCTION

1.1 A Power System.

An electric power system consists of three principal components: the generating stations, the transmission lines, and the distribution systems. The transmission lines are the connecting links between all the generating stations and the distribution systems. A distribution system connects all the individual loads in a given area to the transmission lines. A well developed power system integrates a large number of generating stations so that their combined output is readily available throughout the region served.

Interconnection of systems brought many new problems, most of which have been solved satisfactory. Interconnection increases the amount of current which flows when a short circuit occurs on a system and requires the installation of breakers able to interrupt a large current. Disturbance on such a system may propagate throughout the rest of the system, resulting in major failure of power supply in absence of adequate safeguards. Not only must the interconnected systems have the same nominal frequency, but also the synchronous machines of one system must remain in step with the synchronous machines of interconnected systems. For reliable production and transmission of electrical energy, it is essential that the power system be stable in the sense described in the following sections. Before elaborating on the concept of stability of a power system let it be considered from system view point.

1.2 Stability of Continous Systems:

The stability of an undriven physical system (i.e. system having no driving force or input) is determined by its behaviour when subjected to external purterbation which displace the system from its original rest position. Here a system is said to be stable if it remains close to its original rest position (or operating point) for all times after the disturbance has ceased; and the system will be asymptotically stable if it, in time, returns to its original rest position after the disturbance has ceased. In many cases it happens that a system exhibits its behaviour only within a limited region around the original rest point. This region is then called region of attraction or the stability region of the given system.The exact location of the boundary of the region is in most cases very difficult to determine. If no such boundary exists the system is called globally or absolutely stable. Also a system may have more than one stable position and may come to rest at a point other than original position if the applied disturbance is large enough. If a system does not exhibit any of the characteristics just mentioned it is said to be unstable. The stability of a system in the absence of a driving force which has just been discussed is also termed as Liapunov stability. The concepts are further elaborated in the subsequent chapter.

In considering the stability of the driven system (i.e. system with driving force or forcing function) a different concept is used. The stability of the equilibrium point in the presence of a driving force or input gives rise to the term "bounded input and bounded output stability" which means that if the input belongs to a set which is bounded then the resulting response should also be bounded for stable operation.

1.3 Power System Stability:

Since power system is also a continuous system the concept described in the previous section is also applicable to it. One of the major problems of power system operation is resynchronisation of some or all of the machines of a system following a fault on the system. Maintenance of stable operation at a steady operating point in the presence of small perturbations also sometimes posses a problem. Depending on the magniture and type of disturbance, power system stability may be defined [5] in more explicit ways. These are described below.

Steady State Stability is the ability of the power y system to maintain power transfer over the system without loss of stability or synchronism when the magnitude of power transfer is increased gradually. It is assumed that the increase in power level upto this limit occurs slowly enough to allow regulating devices to respond with their steady state characteristics, and that inertia effects are negligible.

Transient Stability is the ability of the power system to maintain stability in the presence of a sudden large change in load occasioned by system switching or by a fault.

Dynamic Stability is the ability of the power system to maintain stability for small disturbance and to prevent growth of oscillation. Dynamic instability generally occurs due to lack of damping torque. Due to the small disturbance assumption, this type of stability is usually investigted through the use of linearized models.

As is clear from the definition, the steady state stability and dynamic stability are of small perturbation stability in nature . Though they are also to be taken care of, the system engineers are often confronted with the second type of stability problems that is, transient stability or sometimes referred to as first swing stability.

1.4 Methods od Determining Stability:

There are very many ways of evaluating the performance and stability of a power system. There are again various methods of determining stability of a system under different conditions of operation. As for example, the load flow study is widely used to determine the steady state stable operating conditions. This method involves iterative solution of algebraic equations of voltage, current and power of the various machines in the network under steady conditions. Load flow of a system may also be studied with the help of a.c. calculating boards, analog and digital computers. Since the dynamic equations of the system are not taken into account, this method is limited only to determining the steady state behaviour of the system.

In order to study the transient stability characteristics of a system the dynamic model has to be considered. The easiest representation of a synchronous ~~ma~~ generator supplying power to a power network is given by

$$M \frac{d^2\delta}{dt^2} = P_{in} - \frac{E_1 E_2}{X} \sin\delta \qquad (1.1)$$

where E_1 and E_2 are machine internal voltages and X is transfer (or mutual) reactance of the system, neglecting damping, exciter and governor dynamics. This equation is also known as "Swing equation". The analytical solution of this equation is not that easy. Assuming all quantities like P_{in}, E_1, E_2 and X are constants. This involves an elliptical integral[13]. In general, allthese variables, are not constant and the system reactance X depends on the type of fault or disturbance appearing on it.

The swing equation with $P_{in} \neq 0$ has been solved by a calculating machine called integraph or differential analyzer[14]. Many numerical techniques has also been developed to solve this equation. The point by point method where the acrelating power is assumed constant over a time step is one of the very widely used methods. Moreover, this equation can be programmed on an analog or a digital machine for efficient solution.

The equal area criterion, a graphical method, is applicable for stability study between a pair or machines on a power system. The usual assumption of no damping , no exciter and governor dinamics are made. For stable operation, it demands, the area between input power line and above the fault-on curve must be equal to the area between input power line and below the post-fault curve for the specified time.

A relatively detailed representation of a synchronous machine feeding a power system involves differential equations in terms of different flux linkage (or currents), excitation and governor dynamics in addition to the swing equation. Then the system may be represented by the equations

$$\dot{\underline{X}} = \underline{f} (\underline{X} , \underline{u}) \qquad (1.2)$$

where $\underline{X}$ is a stable vector of system variables stated and $\underline{u}$ is a vector of forcing functions which may be prime mover input, excitation input or any auxiliary signal. The set of differential equations (1.2) may be solved on a digital computer and stability of the system can be investigated for any fault on the system.

In order to determine the transient stability of a system, the system equations (1.1) and (1.2) has to be solved on a digital computer for a significant period of real time. This is costly and time consuming. Moreover, it may not be feasible to study the stability behavior of the system for any and all types of faults at every possible location.

In contrast to all these conventional methods discussed, there is a relatively new approach of solving stability problems known as Liapunov method. The details of this method has been provided in a seperate chapter. The stability of an operating point can be determined from a fictitáus energy function called "Liapunov Function" without actually solving the set of system differential equations. The Liapunov function and its derivative are expressed as functions of system states.

This method can be used as a on-line method for finding power system stability. A small analog or digital computer may be used for this purpose. By the online computing devices the stability of the system can be determined by testing Liapunov function and its derivative for a particular value of states which are often measurable. For unstable condition proper action can be taken by this device to remove the machine from the rest of the system or any other supervisory control may be introduced to stabilize it.

1.5 Scope of thesis

The stability domain of a single machine infinite bus power system model has been evaluated for particular operating conditions. Both linear and nonlinear system model with and without voltage regulator and stabilizing signals have been considered. The effect of exciter gain or stability region of a power system has also been investigated in this thesis.

CHAPTER 2

LIAPUNOV STABILITY

This chapter expands on the definination and concept of Liapunov stability. Various method of generating and testing of a Liapunov function have been discussed. The content of this thesis are collected from different references[2,6] and are reproduced here only for the sake of completeness of the thesis.

2.1. Equilibrium State- Definition and Stability

Consider a system defined by

$$\dot{X} = f(x, t) \tag{2.1}$$

Where X is a state vector (n vector) and f(x, t) is an n vector whose elements are functions of $X_1, X_2, \ldots X_n$ and t. The solution of the equation (2.1) is $\Phi(t, x_o, t_o)$ where $x = x_o$ at $t = t_o$ and t is the observed time. Thus

$$\Phi(t_o, x_o, t_o) = X_o$$

For this system a state x_e where

$$f(X_e, t) = 0 \text{ for all } t \tag{2.2}$$

is called an equilibrium state' of the system. The equilibrium state X_e is said to be stable if for each real number $\in > 0$ there is a real number $\delta(\in, t_o) > 0$ such that the inequality $\|X_o - X_e\| \leqslant \delta$ implies

$$\|\Phi(t ; x_o, t_o) - x_e\| \leqslant \in \quad \text{for all } t > t_o$$

The real number δ depends on $\in$ and, in general, also depends on t_o. If δ does not depend on t_o, the equilibrium state is said to be "uniformly stable". Figure (2.1) and (2.2) show a stable equilibrium state X_e of a second order system and the representative trajectory starting from x_o. $S(\in)$ and $S(\delta)$ are circles of radius $\in > 0$ and $\delta > 0$ about the equilibrium state X_e. The solution of equation (2.1) is said to be "bounded" if there exists for a given $\delta > 0$ a constant $\in(\delta, t_o)$ such that $\|x_o - x_e\| \leqslant \delta$ implies.

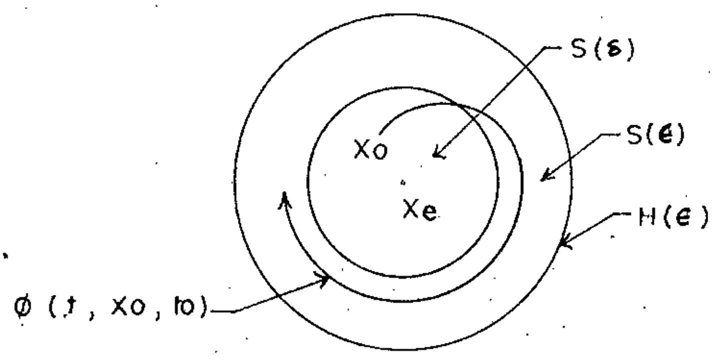


FIGURE 2·1

STABLE EQUILIBRIUM STATE AND REPRESENTATIVE TRAJECTORY

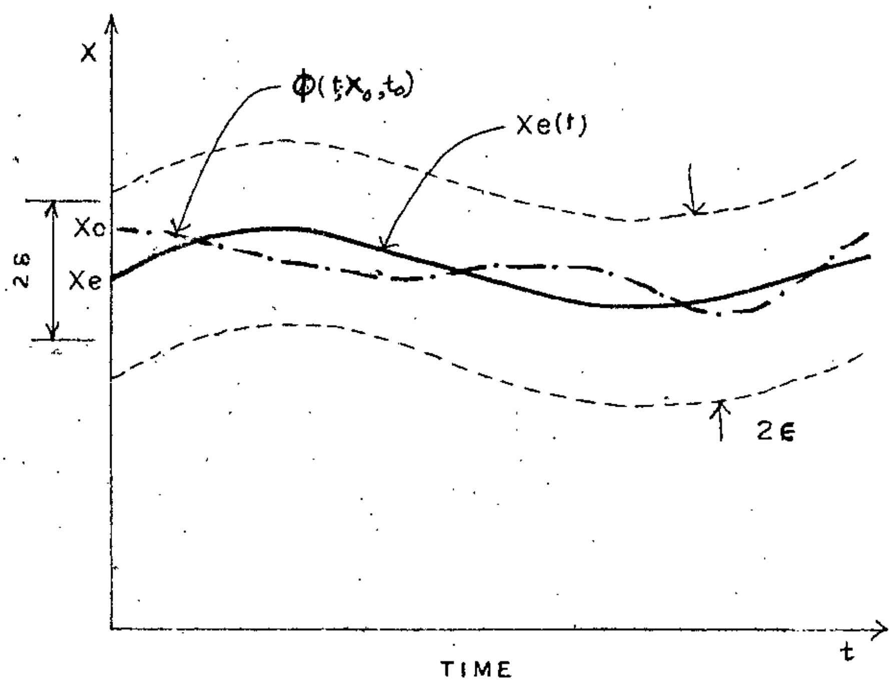


FIGURE 2·2

ILLUSTRATION OF LIAPUNOV STABILITY

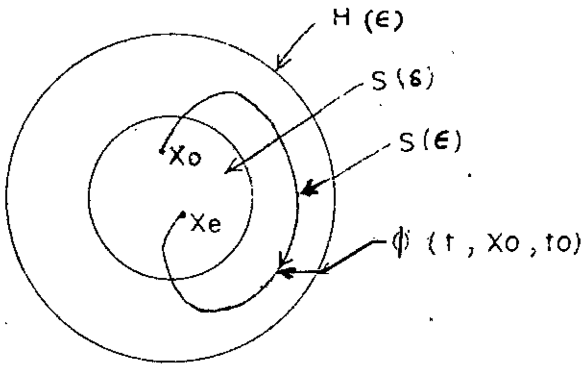


FIGURE 2·3

ASYMPTOTICALLY STABLE EQUILIBRIUM STATE Xe AND REPRESENTATIVE TRAJECTORY STARTING FROM Xo.

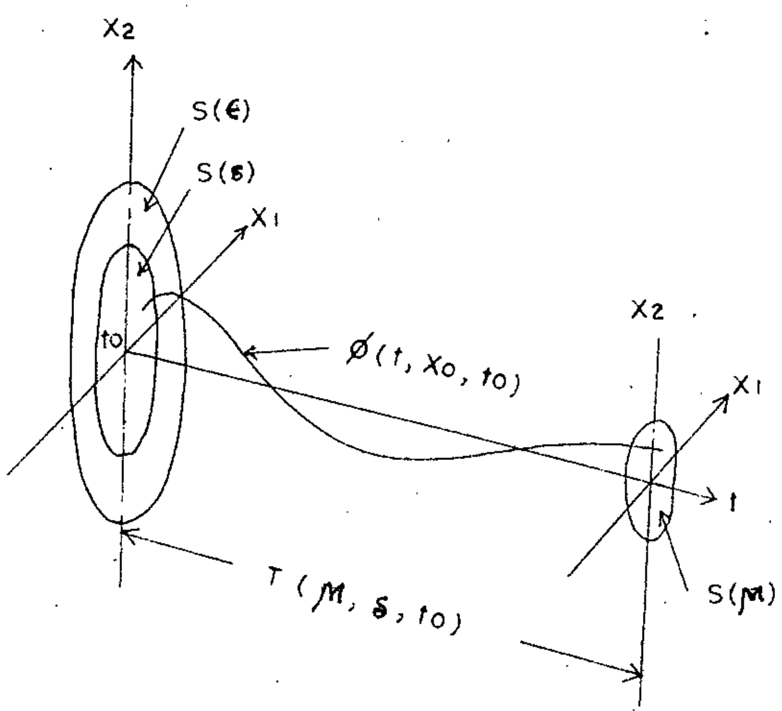


FIGURE 2·4

PLOT OF REPRESENTATIVE TRANJECTORY AS TIME ELAPSES

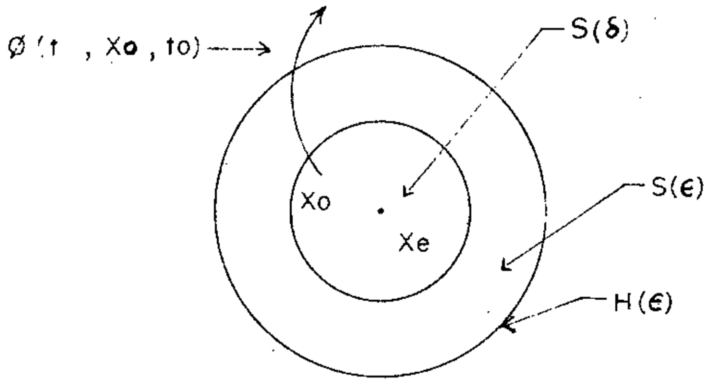


FIGURE 2 5

UNSTABLE EQUILIBRIUM STATE Xe AND REPRESENTATIVE TRAJECTORY STARTING FROM Xo.

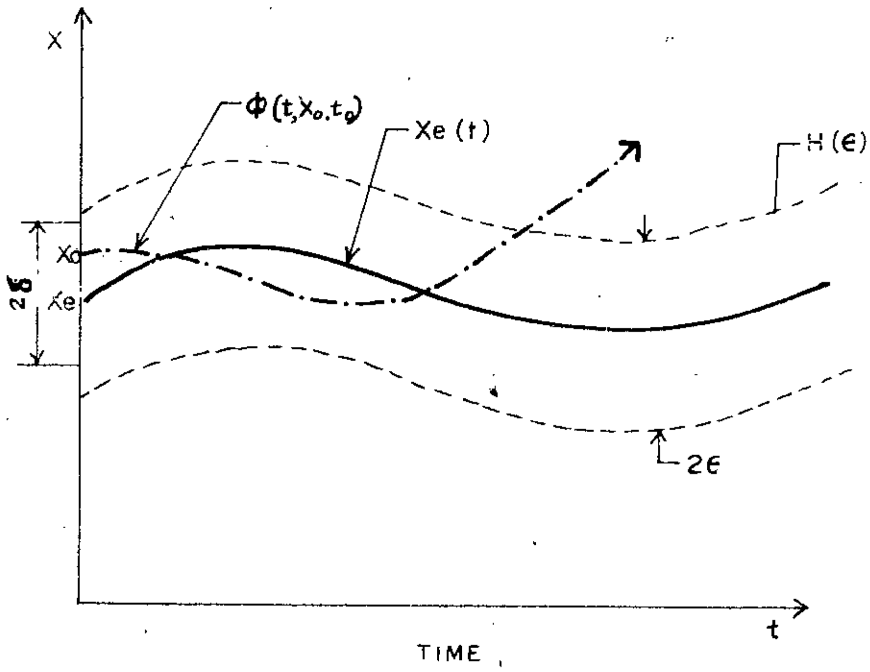


FIGURE 2 · 6

PLOT OF REPRESENTATIVE TRAJECTORY AS TIME ELAPSES

$$\|\phi(t\ ;\ x_o,\ t_o) - x_e\| \leqslant \epsilon(\delta\ ,\ t_o) \text{ for all } t \geqslant t_o$$

If ϵ does not depend on t_o, the solution is said to be "uniformly bounded".

An equilibrium state X_e of the system considered is said to be "asymptotically stable" if it is stable and if every solution starting at a state X_o sufficiently near X_e converges to X_e as t increases indefinitely. Namely, given two real numbers $\delta > 0$ and $\mu > 0$ there are real numbers $\epsilon > 0$ and $T(\mu, \delta, t_o)$ such that $\| X_o - X_e \| \leqslant \delta$ implies

$$\|\phi(t\ ;\ x_o,\ t_o) - x_e\| \leqslant \epsilon \text{ for all } t \geqslant t_o$$

and

$$(t\ ;\ x_o,\ t_o) - x_e\| \leqslant \mu \text{ for all } t \geqslant t_o + T\ (\mu,\ \delta,\ t_o)$$

Figure 2.3 and 2.4 shows an asymptotically stable state X_e of a second order system and the representative trajectory starting from x_o. $S(\epsilon)$, $S(\delta)$ and $S(\mu)$ are respectively, circular regions of radius $\epsilon > 0$, $\delta > 0$ and $\mu > 0$ about X_e. If the equilibrium state X_e is asymptotically stable, then every motion starting at a state x_o in $S(\delta)$ converges, without leaving $S(\epsilon)$, to the origin as time increases indefinitely. Asymptotic stability is more important than mere stability. Since asymptotic stability is a local concept, merely to have established asymptotic stability may not mean that the system will operate properly. The largest regions of asymptotic is called the "domain of attraction". If asymptotic stability holds for all states (all points in the state space) from which motions originate, the equilibrium state as is said to be "asymptotically stable in the large".

An equilibrium state is said to be unstable if it is neither stable nor asymptotically stable. Figure 2.5 and 2.6 shows an unstable equilibrium state for some real number $\epsilon > 0$ and any real number $\delta > 0$, no matter how small, there is always in the circular region $S(\delta)$ a state x_0 such that the motion starting from this state reaches the boundary circule $H(\epsilon)$ of $S(\epsilon)$.

2.2 First Method of Liapunov

The first method of Liapunov consists of all procedures in which the explicit form of the solution is used for stability analysis. In this method each equilibrium state, if there is more than one is investigated separately. Consider a nonlinear system

$$\dot{x} = f(x) \qquad (2.3)$$

where x is a state vector (n vector), and f (x) is an n vector and is continuously differentiable in $x_1, x_2, \text{------}, x_n$. Let us expand the nonlinear vector function f (x) in Taylor series about the equilibrium state X_e in question. Introducing a new vector, $y = x - x_e$, the equilibrium state can be shifted to the origin. By expanding f(x) in Taylor series about $x = x_e$, Equation (2.3) becomes -

$$\dot{y} = Ay + G(y)\,y$$

where A is the n x n "Jacobian matrix given by.

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \text{------} & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \text{------} & \frac{\partial f_2}{\partial x_n} \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \text{------} & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

where f_1, f_2, ———————, f_n are the n components of f(x). All the partial derivaties appearing in the Jacobian matrix A are evaluated at the equilibrium state $x = x_e$, or $y = 0$. The n x n matrix $G(y)$ contains arising from the higher-order derivatives in the Taylor series expansion. The elements of G(y) vanish at the equilibrium state. Equation (2.3) can thus be linearized near the origin as follows.

$$\dot{y} = Ay \tag{2.4}$$

Liapunov showed that if all the eigenvalues of the constant matrix A in equation (2.4) have non-zero real parts, then the stability of the equilibrium state X_e of the original nonlinear equation (2.3), is the same as that of the equilibrium state y= 0 of the linearized equation (2.4). Hence if the eigenvalues of A have negative real parts, then the equilibrium state X_e is asymptotically stable and all solutions of the system with initial state x(o) sufficiently close to x_e, approach x_e as $t \rightarrow \infty$ and if at least one of the eigenvalues of A has a positive real part; then the equilibrium state X_e is unstable. If however, at least one of the eigenvalues of A has a zero real part, the local stability behavior of the equilibrium state $x = x_e$ of the system, equation (2.3), can not be determined by equation (2.4). This is a critical case. The first method of Liapunov concerns only stability in the small, that is wheather a motion originating from a neighbourhood of an equilibrium state X_e will approach X_e as t increased indifinitely.

2.3 Second Method of Liapunov- Theorems.

The second method of Liapunov is based on a generalization of the idea that if the system has an asymptotically stable equilibrium state, then the stored energy of the system displaced within the domain of attraction decays with increasing time until finally assumes its minimum value at the equilibrium state. The second method of Liapunov consists of determination of a fictious "energy" function called "Liapunov Function". The idea of the Liapunov function is more general than that of energy and is more widely applicble. Liapunov functions are function of x_1, x_2, ———————————, x_n, and t. The Liapunov function is usually denoted as V (x, t). If Liapunov function do not include t explicitly, then we denote them as V(x). In the second method of Liapunov the sign behaviours of V(x, t) and its time derivative $\dot{V}$ (x, t) gives information on stability, asymptonic stability or instability of the equilibrium state under consideration without directly solving for the solution for both linear and nonlinear systems. The essence of the "second method of Liapunov" is given below.

Consider the system defined by the equation (2.1) and

$$f(0, t) = 0 \quad \text{for all } t$$

Suppose that there exists a scalar function V (x, t) which has continuous first partial derivatives. If V (x, t) satisfies the following conditions.

1. V (x, t) is positive definite, namely V (o, t) = 0 and
2. $V(x, t) \geqslant \alpha(\|x\|) > 0$ for all $x \neq 0$ and all t where α is a continuous, nondecreasing scalar function such that $\alpha(o) = 0$.
3. The total derivative $\dot{V}$ is negative for all $x \neq 0$ and all t, or $\dot{v}(x, t) \leqslant -\gamma(\|x\|) < o$ for all $x \neq 0$ and all t where γ is a continuous nondecreasing scalar function such that $\gamma(0) = 0$.

4. There exist a continuous, nondecreasing scalar function such that $\beta(o) = 0$ and for all t, $V(x, t) \leqslant \beta(\|x\|)$.

5. $\alpha(\|x\|)$ approaches infinity as $\|x\|$ increases indefinitely,

or $\alpha(\|x\|) \rightarrow \infty$ as $\|x\| \rightarrow \infty$

then the origin of the system, $x = 0$, is "uniformly asymptotically stable in the large".

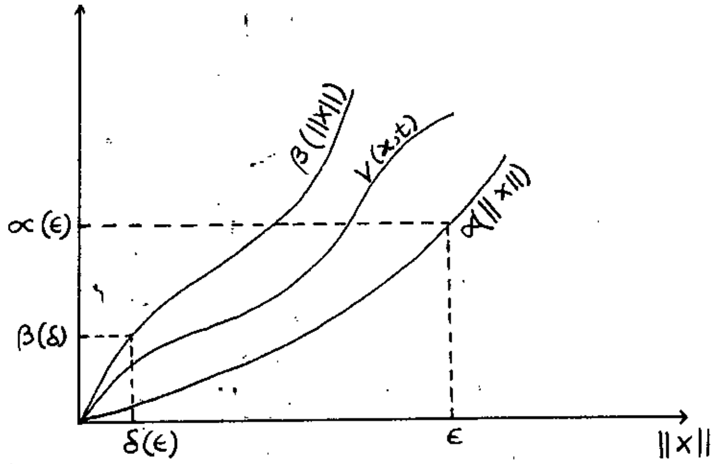


FIGURE 2.7 CURVES $\alpha(\|x\|)$, $\beta(\|x\|)$ AND $V(x,t)$.

If there exist a scaler function $V(x, t)$, with continuous first partial derivatives, satisfying the following conditions:

a) $V(x,t) > 0$ for all $x \neq 0$ in the region Ω (can be entire state space) which includes the origin and all t

$V(0, t) = 0$ for all t

b) $\dot{V}(x,t) \leq 0$ for all $x \neq 0$ in Ω and all t

$\dot{V}(0,t) = 0$ for all t

then the origin of the system is "uniformly asymptotically stable".

If there exist a scalar function $V(x, t)$, with continuous first partial derivatives, satisfying the following conditions:

a) $V(x,t) > 0$ for all $x \neq 0$ in Ω and all t

$V(0, t) = 0$ for all t

(b) $\dot{V}(x, t) \leq 0$ for all $x \neq 0$ in Ω and all t

$\dot{V}(0,t) = 0$ for all t

then the origin of the system is "uniformly stable"

If there exist a scalar function $V(x,t)$, with continuous first partial derivatives, satisfying the following conditions

(a) $V(x, t) > 0$ for all $x \neq 0$ and all t

$V(0, t) = 0$ for all t

(b) $\mathring{V}(x,t) \leq 0$ for all $x \neq 0$ and all t

$\mathring{V}(0, t) = 0$ for all t

(c) $V(\phi(t; x_o, t_o), t)$ does not vanish identically in $t \geq t_o$ for any t_o and any $x_o \neq 0$, where $\phi(t; x_o, t_o)$ denotes the solution starting from X_o at t_o

then the origin of the system is "uniformly asymptically stable in the large".

Finally if there exists a scalar function $V(x,t)$, with continuous first partial derivatives, satisfying the following condition:

(a) $V(x, t) > 0$ for all $x \neq 0$ in Ω and all t

$V(0, t) = 0$ for all t

(b) $\mathring{V}(x, t) > 0$ for all $x \neq 0$ in Ω and all t

$\mathring{V}(0, t) = 0$ for all t

or (c) $V(x, t) < 0$ for all $x \neq 0$ in Ω and all t

$V(0, t) = 0$ for all t

(d) $\mathring{V}(x, t) < 0$ for all $x \neq 0$ in and all t

$\mathring{V}(0, t) = 0$ for all t

then the origin of the system is unstable".

2.4. Second Method for Linear Time Invariant systems

Consider the following system

$$\mathring{X} = A x \qquad (2.5).$$

where x is a sate vector (n vector) and A is an n x n constant matrix. We assume henceforth that A is nonsingular. The stability of the equilibrium state of the linear time- invariant system can be investigated easily by use of second method of Liapunov.

The following is a basic theorem of stability analysis of linear time invariant systems by means of second method of Liapunov.

The equilibrium state X = 0 of the system given by equation (2.5) is asymptotically stable if and only if given any positive definite Hermilian matrix Q (or positive definite real symmetric matrix Q), there exist a positive definite Hermilian matrix p (or positive definite real symmetric matrix p) such that

$$A^{*} P + PA = -Q \qquad (2.6)$$

The scalar function X^{*} PX is a Liapunov function for the system of equation (2.5).

In other words a necessary and sufficient condition for X = 0 to be an asymptotically stable solution of (2.5) is that there exist a positive definite real symmetric matrix P (or positive definite Hermilien matrix p) satisfying the equation

$$A^{T}P + PA = -I \qquad \text{------------------------------}(2.7)$$

where I is the identify ~~mirix~~ matrix and X^{T} PX is the liapunov function.

"Sylvester's criteres " may be applied for the positive definiteness test of the P matrix. A necessary and sufficient condition in order that the n x n real symmetric P matrix, be positive definite is that the determinant of P be positive definite and the successive principal menors of the determinant of p be positive, or

$$D_{11} = P_{11} > 0 \qquad D_{22} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} > 0 \qquad D_{33} = \begin{vmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{vmatrix} > 0$$

$$D_{nn} = \det P = \begin{vmatrix} P_{11} & P_{12} & \text{———} & P_{1n} \\ P_{21} & P_{22} & \text{———} & P_{2n} \\ & & & \\ P_{n1} & P_{n2} & \text{———} & P_{nn} \end{vmatrix} > 0 \qquad (P_{ij} = P_{ji}) \qquad (2.76)$$

This test is executed by the computer program no.3 of appendix 2.

2.5 Liapunov Stability of Nonlinear Systems.

In a linear free dynamic system if the equilibirum state is locally asymptotically stble, then it is asymptotically stable in the large. In a non-linear free dynamic system, however, an equilibrium state can be locally asymptotically stble without being asymptotically stable in the large. Hence implication of asymptotic stability of equilibrium states of linear systems and those of nonlinear systems are quite different.

For asymptotic stability of equilibrium states of nonlincar systems, stability analysis of linearized models of nonlinear systems is completely inadequate. For investigation of nonlinear system without linearization, Several methods based on the second method of Liapunov are available. ~~for this purpose.~~ Krasovskii's method for testing sufficient condition for asymptotic stability of nonlinear systems.

Schultz- Gibson's variable gradient method for generating Liapunov functions of nonlinear systems, Lure's method applicable to stability analysis of certain nonlinear control systems, Zubov's method for constructing domain of attraction, and others. Of these "variable gradient method of generating Liapunov function" is given below.

2.6 Schulz and Gibson method of generating:

One useful systematic approach is the "variable gradient method of Schultz and Gibson". This method is based on the fact that if a particular Liapunov function exists which is capable of proving asymptotic stability of a given system, then a unique gradient of this Liapunov function also exists.

Consider the system described by the following equation:

$$\dot{X} = f(x, t)$$

where x is a state vector (n vector) and f (x, t) is an n vector, elements of which are functions of x_1, x_2 ———— x_n and t. We assume the equilibrium state under consideration to be the origin of the state space. Let us denote a tentative Liapunov function as V. In this tentative Liapunov function we assume, for convenience, that V is an explicit function of x_1, x_2 .... x_n, but not an explicit function of t. Then

$$\dot{V} = \frac{\partial V}{\partial x_1}\dot{x}_1 + \frac{\partial V}{\partial x_2}\dot{x}_2 + \text{———} + \frac{\partial V}{\partial x_n}\dot{x}_n$$

$\dot{V}$ can be determined from ∇V, the gradient of V, as follows :

$$\dot{V} = (\nabla V)' \dot{x} \qquad (2.8a)$$

where $(\nabla V)'$ is the transpose of (∇V), ∇V is given by

$$\nabla V = \begin{bmatrix} \frac{\partial V}{\partial x_1} \\ \vdots \\ \frac{\partial V}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla V_1 \\ \vdots \\ \nabla V_n \end{bmatrix}$$

V is obtained as line integral of ∇V is

$$V = \int_0^{x} (\nabla V) \cdot dx \qquad (2.8)$$

The upper limit here is not meant to imply that V is a vector quantity, but rather that the integral is a line integral to an arbitrary point in the state space located at $(x_1, x_2 \text{ ——— } x_n)$. This integral can be made independent of the path of integration. The simplest path of integration is indicated by the expanded form of equation (2-8) to be

$$V = \int_0^{x_1 \; (x_2 = x_3 \text{ ——— } n_n = 0)} \nabla V_1 \, dx_1$$

$$+ \int_0^{x_2 \; (x_1 = x_1, \; x_3 = x_4 \text{ ——— } x_n = 0)} \nabla V_2 \, dx_2$$

$$+ \text{ ——————— }$$

$$+ \int_0^{x_n \; (x_1 = x_1, \; x_2 = x_2 \; \ldots\ldots\ldots\ldots \; x_n - 1 = x_{n-1})} \nabla V_n \, dx_n \qquad (2\cdot 9)$$

were ∇Vi is the component of ∇V in the x_i direction.

For a scaler function of V to be obtained uniquely from a line integral of a vector function ∇V, the following matrix F formed by $\frac{\partial \nabla V_i}{\partial x_j}$

$$F = \begin{bmatrix} \frac{\partial \nabla V_1}{\partial x_1} & \frac{\partial \nabla V_1}{\partial x_2} & \text{———} & \frac{\partial \nabla V_1}{\partial x_n} \\ \frac{\partial \nabla V_2}{\partial x_1} & \frac{\partial \nabla V_2}{\partial x_2} & \text{———} & \frac{\partial \nabla V_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial \nabla V_n}{\partial x_1} & \frac{\partial \nabla V_n}{\partial x_2} & \text{———} & \frac{\partial \nabla V_n}{\partial x_n} \end{bmatrix}$$

must be symmetric. The condition on the matrix F is thus a generalized curl requirement for the n-dimensional case. The problem of determining a V function which satisfies Liapunov theorem is then transformed into the problem of finding a ∇V such that the n-dimensional curl of ∇V equals zero. Further, the V and $\dot{V}$ determined from ∇V must be a sufficient to prove stability, that is, they must satisfy Liapunov's theorem. We first set ∇V is equal to an arbitrary column vector.

$$\nabla V = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots\cdots + a_{2n}x_n \\ \vdots \qquad \vdots \qquad \qquad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots\cdots + a_{nn}x_n \end{bmatrix} \tag{2.10}$$

The a_{ij} are completely undetermined quantities. The a_{ij} may be constant or function of time t and/or functions of state variables. It is convenient, however, to choose a_{nn} as a constant or as a function of time t. Several of the a_{ij} may be chosen to be zero or they are obvious from constraints on $\dot{V}$ imposed by the investigator, or they are determined from the curl equations.

If the equilibrium state X = 0 of a nonlinear system is asymptotically stable, then we may be able to obtain a Liapunov function by the method just described by following the following procedures:

1. Assume ∇V of the form of equation (2.10)
2. $\dot{V}$ is determined from ∇V
3. Constrains are applied to make $\dot{V}$ to be negative definite or atleast negative semidefinite.

4. By using the n (n-1) /2 curl equations implied by the statement that F must be symmetric, the remaining unknown coefficients in ∇V are determined.

5. $\mathring{V}$ is rechecked as the addition of terms required as a result of step 4 may alter $\mathring{V}$.

6. V is determined by equation (3.9).

7. Region of assymptofic stability is then examined.

CHAPTER- 3

MATHEMATICAL MODEL OF A POWER SYSTEM

3.1 Single Machine Infinite Bus System

The system considered for the purpose of investigation is a relatively simple one. A synchronous generator feeding an infinite busbar through a transformer and a double circuit transmission line has been considered here. The system considered is shown in figure 3.1.

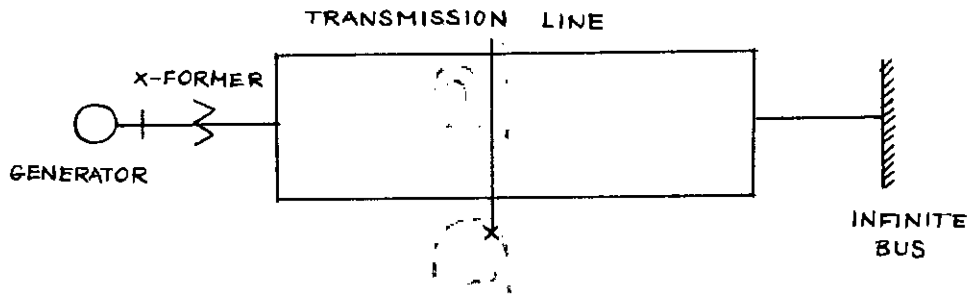


FIGURE - 3.1 SYSTEM CONFIGURATION FOR SINGLE MACHINE CASE.

In stability studies, the interval of time of interest is relatively small compared to the time constants of the governor and the prime mover. While Governor and prime mover responses are important, they are less effective in the early part of the machine swing and may, if considered , contribute to stability. For simplicity, the torque variations due to governor control are neglected. The amortissor windings are known to aid stability, so these are also not considered.

The equations for the machine as developed by G.Shackshaft[15] and J.M. Undrill[10] are given below. (The list of symbols is given in page VIII).

(a) Voltage and flux linkage equations for the armature.

$$e_d = -\frac{w}{w_o}\psi_q - R\,i_d + \frac{1}{w_o}\,p\,\psi_d \qquad (3.1)$$

$$e_q = \frac{w}{w_o}\psi_d - R\,i_q + \frac{1}{w_o}\,\psi_q \qquad (3.2)$$

$$\psi_d = X_{afd}\,i_{fd} - X_d\,i_d \qquad (3.3)$$

$$\psi_q = -X_q\,i_q \qquad (3.4)$$

(b) Voltage and flux linkage relations for the rotor

$$E_{fd} = \frac{X_{afd}}{W_o\, r_{fd}} \; p\,\psi_{fd} + X_{afd} \; i_{fd} \qquad (3.5)$$

$$V_{fd} = r_{fd} \; i_{fd} + \frac{1}{w_o} \; p\,\psi_{fd} \qquad (3.6)$$

As defined by C.Concordia[7]

$$E_{fd} = \frac{X_{afd}}{r_{fd}} \; V_{fd} \qquad (3.7)$$

$$\psi_{fd} = X_{ffd} \; i_{fd} - X_{afd} \; i_d \qquad (3.8)$$

(c) The equation for the terminal voltage of the machine is

$$e_t^2 = e_d^2 + e_q^2 \qquad (3.9)$$

(d) The system connection relationships for the machine with the bus bar are

$$e_d = R_e i_d - \frac{w}{w_o} X_e i_q + \frac{X_e}{w_o} \; p i_d + V_d \qquad (3.10)$$

$$e_q = \frac{w}{w_o} X_e i_d + R_e i_q + \frac{X_e}{w_o} \; p i_q + V_q \qquad (3.11)$$

Where $V_d = V \sin\delta$ ——————— (3.12)

$V_q = V \cos\delta$ ——————— (3.13)

(e) The torque equations for the machine

$$p\,\delta = W_o \; n \qquad (3.14)$$

$$T_m\, p_n = T_{in} - (\psi_d \; i_q - \psi_q i_d) \qquad (3.15)$$

(f) The voltage regulator equation[11] is written as

$$E_{fd} = \frac{K_r}{1 + \tau_r \; p} \left(e_t - e_{tr} - U_s(t) \right) \qquad (3.16)$$

Where K_r is less than zero.

Equating (3.24) & (3.25)

$$-(X_q + X_e)\, pi_q = -WX_{afd}\, i_{fd} + W(X_d + X_e)\, id + w_o(R_e + R)\, i_q + V\cos\delta \quad \text{------}(2.25)$$

The simultaneous equations (3.19), (3.22) & (3.26) may be written as

$$\begin{bmatrix} X_{ffd} & -X_{afd} & 0 \\ X_{afd} & -(x_d + x_e) & 0 \\ 0 & 0 & -(x_q + x_e) \end{bmatrix} \begin{bmatrix} pi_{fd} \\ pid \\ pi_q \end{bmatrix} = \begin{bmatrix} -w_o r_{fd} & 0 & 0 \\ 0 & w_o(R_e+R) & -w(x_e + x_q) \\ -wx_{afd} & w(x_e + x_d) & w_o(Re+R) \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{w_o\, rfd}{X_{afd}} E_{fd} \\ w_o V \sin \\ w_o V c \end{bmatrix}$$

$$\text{------------}(3.27)$$

which in turn gives the following non liner differential equations.

Solving (3-19) and (3-22) for pi_{fd} and id we have

$$pi_{fd} = \frac{r_{fd}(x_d + x_e)\, ifd}{\Delta d} \quad \frac{X_{afd}(Re + R)}{\Delta d} i_d \quad \frac{X_{afd}(X_2 + X_e)}{\Delta d}(1+n)\, iq$$

$$+ \frac{x_{afd}\, V}{\Delta d} \sin\delta - \frac{r_{fd}(x_d + x_e)}{X_{afd}\,\Delta d} E_{fd} \quad \text{----------}(3.28)$$

$$pi_d = \frac{r_{fd}\, x_{afd}}{\Delta d} i_{fd} + \frac{x_{ffd}(Re + R)}{\Delta d} id - \frac{X_{ffd}(X_q + x_e)}{\Delta d}(1+n)\, i_q$$

$$+ \frac{X_{ffd}\, V}{\Delta d} \sin\delta - \frac{r_{fd}}{\Delta d} E_{fd} \quad \text{------------------}(3.29)$$

From equation (3.26)

$$pi_q = \frac{X_{afd}}{\Delta q}(1+n)\, ifd - \frac{(X_d + X_e)}{\Delta q}(1+n)\, id - \frac{(R_e + R)}{\Delta q} iq - \frac{V}{\Delta q}\cos\delta \quad \text{----}(3.30)$$

Where $\Delta d = \dfrac{x^2_{afd} - X_{ffd}(X_d + x_e)}{w_o}$ ------------ (3.31)

$$\Delta q = \frac{X_q + X_e}{W_o} \quad \text{----------------}(3.32)$$

$$(1 + n) = \frac{w}{w_o} \qquad \text{------------(3.33)}$$

Substituting equations (3.29) and (3.30) in equations (3.10) and (3.11) gives

$$e_d = \frac{X_e \; r_{fd} \; X_{afd}}{w_o \; \Delta d} + \left[\frac{X_e \; X_{ffd} \; (R_e + R)}{w_o \; \Delta d} + R_e \right] i_d$$

$$- \left[\frac{X_e \; X_{ffd} \; (X_q + X_e)}{w_o \; \Delta d} + X_e \right] (1+n) \, iq - \frac{X_e \; r_{fd}}{w_o \; \Delta d} E_{fd}$$

$$+ \left[V + \frac{X_{ffd} \; X_e \; V}{w_o \; \Delta d} \right] \sin \delta \qquad (3.34)$$

$$e_q = \frac{X_e \; X_{afd}}{\omega_o \Delta_q} (1 + n) \, i_{fd} + \left[X_e - \frac{X_e (X_d + X_e)}{w_o \; \Delta_q} \right] (1 + b) \, id$$

$$+ \left[R_e - \frac{R \; X_e (R_e + R)}{w_o \; \Delta q} \right] i_q + \left[V - \frac{X_e \; V}{w_o \; \Delta q} \right] \cos \delta \quad (3.35)$$

Substituting equations (3.3) and (3.4) in equations (3.15) gives

$$pn = \frac{T_{in}}{T_m} - \frac{X_{afd}}{T_m} \, i_{fd} \, i_q + \frac{X_d - X_q}{T_m} \, i_d i_q \qquad (3.36)$$

$$pr_{fd} = \frac{K_r}{\tau_r} \; e_t - \frac{1}{\tau_r} \; E_{fd} + \frac{(E_o - Kr \, e_{tr})}{\tau_r} - \frac{kr \; U_s(t)}{\tau_r} \qquad (3.37)$$

where

$$e_t = (e_d^2 + e_q^2)^{\frac{1}{2}}$$

3.2 Nonlinear model of single machine infinite bus system.

Equations (3.14),(3.28) to (3.30) and (3.36) and (3.37) can be grouped to give the equations of the nonlinear model in terms of different states.

$$p i_{fd} = a_{11} i_{fd} + a_{12} i_d + a_{13} (1+n) i_q + a_{15} \sin\delta + d_1 E_{fd}$$

$$p i_{d} = a_{21} i_{fd} + a_{22} i_d + a_{23} (1+n) i_q + a_{25} \sin\delta + d_2 E_{fd}$$

$$p i_q = a_{31} (1+n) i_{fd} + a_{32} (1+n) i_d + a_{33} i_q + a_{35} \cos\delta$$

$$p n = a_{41} i_{fd} i_q + a_{42} i_d i_q + k$$

$$p \delta = w_o n$$

$$p E_{fd} = \frac{K_r}{T_r} e_t - \frac{k_r}{r} U_e(t) + \frac{1}{T_r} (E_o - K_r e_t) - \frac{1}{T_r} E_{fd} \quad (3.38)$$

where

$$a_{11} = \frac{r_{fd} (X_d + X_e)}{\Delta d} \quad (3.39)$$

$$a_{12} = \frac{X_{afd} (R_e + R)}{\Delta d} \quad (3.40)$$

$$a_{13} = -\frac{X_{afd} (X_q + X_e)}{\Delta d} \quad (3.41)$$

$$a_{15} = \frac{X_{afd}}{\Delta d} \quad (3.42)$$

$$d_1 = \frac{r_{fd} (X_d + X_e)}{X_{afd} \quad \Delta d} \quad (3.43)$$

$$a_{22} = \frac{X_{ffd} (R_e + R)}{\Delta d} \quad (3.45)$$

$$a_{21} = \frac{r_{fd} \quad X_{afd}}{\Delta d} \quad (3.44)$$

$$a_{23} = -\frac{X_{ffd} (X_q + X_e)}{\Delta d} \quad (3.46)$$

$$a_{25} = \frac{X_{ffd} \quad V}{\Delta d} \tag{3.47}$$

$$d_2 = -\frac{X_{fd}}{\Delta d} \tag{3.48}$$

$$a_{31} = \frac{X_{afd}}{\Delta q} \tag{3.49}$$

$$a_{32} = -\frac{X_d + X_e}{\Delta q} \tag{3.50}$$

$$a_{33} = -\frac{R_e + R}{\Delta q} \tag{3.51}$$

$$a_{35} = -\frac{V}{\Delta q} \tag{3.52}$$

$$a_{41} = -\frac{X_{afd}}{T_m} \tag{3.53}$$

$$a_{42} = \frac{X_d - X_q}{T_m} \tag{3.54}$$

$$K = -a_{41} \, i_{fdo} \quad i_{qo} - a_{42} \quad i_{do} \quad i_{qo} \tag{3.55}$$

3.3 Linearized model of single machine infinite bus system.

For small perturbations, the system equations can be linearized about an operating point. In such cases, the change in speed deviation is negligibly small, so that $w/w_o = 1$. To include the voltage regulator action in the general linearized model, it is necessary to obtain an expression for perturbation of terminal voltage in terms of the states chosen.

From equation (3.9)

$$\Delta e_t = \frac{e_{qo}}{e_{to}} \Delta e_d + \frac{e_{qo}}{e_{to}} \Delta e_q \tag{3.56}$$

The voltage regulator equation in the linearized form is

$$p \; \Delta \; E_{fd} = \frac{k_r}{\tau_r} \Delta e_t - \frac{1}{\tau_r} \Delta E_{fd} - \frac{k_r}{\tau_r} U_e(t) \qquad (3.57)$$

Substituting the linearized forms of e_d and e_q in (3-56) one gets

$$p\Delta E_{fd} = c_{61} \; \Delta i_{fd} + C_{62} \; \Delta i_d + C_{63} \; \Delta i_q + C_{65} \Delta \delta + C_{66} \; \Delta E_{fd}$$

$$- \frac{K_r}{\tau_r} \; U_e(t) \qquad (3.58)$$

Where

$$C_{61} = \frac{K_r \; e_{do}}{\tau_r \; e_{to}} \; \frac{r_{fd} \; X_{afd} \; X_e}{w_o \; \Delta d} + \frac{K_r \; e_{qo}}{\tau_r \; e_{to}} \; \frac{X_e \; X_{afd}}{w_o \; \Delta q} \qquad (3.59)$$

$$C_{62} = \frac{K_r \; e_{do}}{\tau_r \; e_{to}} \left[R_e + \frac{X_e \; X_{ffd} \; (R_e + R)}{w_o \; \Delta d} \right] + \frac{K_r \; e_{qo}}{\tau_r \; e_{to}} \left[X_e \; \frac{X_e (X_d + X_e)}{w_o \; \Delta q} \right]$$

$$(3.60)$$

$$C_{63} = - \frac{K_r \; e_{do}}{\tau_r e_t} \left[X_e + \frac{X_e \; X_{ffd} \; (X_q + X_e)}{\tau_r \; e_{to}} \right] + \frac{K_r \; E_{qo}}{\tau_r \; e_{to}} \left[R_e - \frac{X_e (R_e + R)}{w_o \; \Delta q} \right]$$

$$(3.61)$$

$$C_{65} = - \frac{K_r \; e_{qo}}{\tau_r \; e_{to}} \; \sin \delta_o \left[V - \frac{X_e V}{w_o \Delta q} \right] + \frac{K_r \; e_{do}}{\tau_r \; e_{to}} \cos \delta_o \left[V + \frac{X_e \; X_{ffd} \; V}{w_o \; \Delta d} \right] \qquad (3.62)$$

$$C_{66} = - \frac{k_r \; e_{do} \; r_{fd} \; X_e}{\tau_r \; e_{to} \; w_o \; \Delta d} - \frac{1}{\tau_r} \qquad (3.63)$$

The linearized equations are given below:-

$$p\,\Delta i_{fd} = C_{11}\,\Delta i_{fd} + C_{12}\,\Delta i_d + C_{13}\,\Delta i_q + C_{15}\,\Delta\delta + d\,\Delta E_{fd}$$

$$p\,\Delta i_d = C_{21}\Delta i_{fd} + C_{22}\,\Delta i_d + C_{23}\,\Delta i_q + C_{25}\,\Delta\delta + d_2\,\Delta E_{fd}$$

$$p\,\Delta i_q = C_{31}\Delta i_{fd} + C_{32}\,\Delta i_d + C_{33}\,\Delta i_q + C_{35}\,\Delta\delta \; .$$

$$p\,n = C_{41}\Delta i_{fd} + C_{42}\,\Delta i_d + C_{43}\,\Delta i_q$$

$$p\,\delta = W_o\, n$$

$$p\,\Delta E_{fd} = C_{61}\,\Delta i_{fd} + C_{62}\,\Delta i_d + C_{63}\,\Delta i_q + C_{65}\,\Delta\delta + C_{66}\,\Delta E_{fd} - \frac{K_r}{T_r}\,U_s(t) \qquad (3.64)$$

Where = a_{ij} ; j = 1, 2, 3 j = 1,2,3

$$C_{15} = a_{15}\cos\delta_o$$

$$C_{25} = a_{25}\cos\delta_o$$

$$C_{35} = -\,a_{35}\sin\delta_o$$

$$C_{41} = a_{41}\; i_{qo}$$

$$C_{42} = a_{42}\; i_{qo}$$

$$C_{43} = a_{41}\, i_{fdo} + a_{42}\; i_{do} \qquad (3.65)$$

CHAPTER- 4

STABILITY STUDY OF SINGLE MACHINE INFINITE BUS SYSTEM WITHOUT VOLTAGE REGULATOR

4.1. Introduction

A synchronous generator feeding an infinite bus through a step up transformer and a double circuit transmission line which has been shown in fig. 3.1 is considered. As has been shown in previous chapter, the more exact representation of the system without voltage regulator action is a set of five first order nonlinear differential equation in terms of field, direct and quadrature axis armature currents, speed and rotor angular position of the generator. Or mathematically,

$$\overset{\circ}{\underline{X}} = \underline{f}(\underline{X}, U) \qquad (4.1)$$

The control U is the excitation to the synchronous generator . When linearized about an operating point, the procedure of which has been shown in section 3.3, the system of equation (4.1) is transformed to

$$\overset{\circ}{\underline{X}} = \underline{\underline{A}}\underline{X} + \underline{B}\, U \qquad (4.2)$$

Where the state variables X or now perturbations from the original state variables in (4.1) from the operating equilibrium states. The matrices (vectors) A and B are constants depending on system parameters and operating points. It should be noted that representation (4.2) is only approximate and this approximation is fair only if the perturbations or disturbances considered are small. Because of obvious simplicity of the linearized system, this is considered first and stability problem of the nonlinear system is considered in latter sections.

4.2 Linearized Model

Equation (4.2) reproduced is

$$\overset{\circ}{\underline{X}} = \underline{A}\ \underline{X} + \underline{B}\, U \qquad (4.3)$$

Liapunov stability is the stability of the undriven system or the natural dynamic part, that is

$$\overset{\circ}{\underline{X}} = \underline{A}\ \underline{X} \qquad (4.4)$$

Determination of stability domain by second method of Liapunov for this system has been discussed in section 2.4. A necessary and sufficient condition for the equilibrium state X = 0 to be an assymptotically stable solution of equation(4.4) is that there exist a positive definite real symmetric matrix P satisfying the equation

$$A^T P + PA = -I \qquad (4.5)$$

Where I is the identity matrix and a possible Liapunov function is

$$V = X^T PX \qquad (4.6)$$

and its derivative $\dot{V}$ is

$$\dot{V} = -X^T IX \qquad (4.7)$$

The matrix P is assumed to be real symmetric i.e. of the form

$$P = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 \\ p_2 & p_6 & p_7 & p_8 & p_9 \\ p_3 & p_7 & p_{10} & p_{11} & p_{12} \\ p_4 & p_8 & p_{11} & p_{13} & p_{14} \\ p_5 & p_9 & p_{12} & p_{14} & p_{15} \end{bmatrix}$$

Equation (4.5) is an algebraic equation with 15 variables p_1, p_2, ---------- p_{15}. These 15 simultaneous equations are solved by Gauss- Jordan method with normalization. The matrix P is then tested for positive definiteness by "Sylvesters criterion" as described in the section 2.4.

The example considered is : The synchronous generator is supplying 66% of of the rated load at an operating power angle of 60^{o} and is operating at an excitation below normal. The different operating conditions are given in Appendix 1. The A matrix for this particular operating point is given as

$$A = \begin{bmatrix} -0.40013 & -22.233 & 369.7 & 0.0 & -149.52 \\ -0.15191 & -27.376 & 455.22 & 0.0 & -184.155 \\ 171.\ 3775 & -451.406 & -22.672 & 0.0 & 263.8 \\ -0.0687 & 0.02988 & -0.211 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 377.0 & 0.0 \end{bmatrix}$$

The choice of this particular operating point was made because the stability margin is low and any small perturbation from the operating point my tend to make the system unstable. The 15 simultaneous equations are,

$-0.80026\ p_1 - 0.30382\ P_2 + 242.755\ P_3 - 0.1374\ P_4 = -1.0$

$-22.233\ P_1 - 27.77613\ P_2 - 451.406\ P_3 + 0.02988\ P_4 - 0.15191\ P_6$

$+ 171.\ 3775\ P_7 - 0.0687\ P_3 +$ ~~$0.02988\ P_4$~~ $-$ ~~$0.15191\ P_6$~~ $= 0.0$

$369.7\ p_1 + 455.22\ p_2 - 23.07213\ p_3 - 0.211\ p_4 - 0.15191\ p_7$

$+ 171.3775\ p_{10} - 0.0687\ p_{11} = 0.0$

$-0.40013\ P_4 + 377.0\ P_5 - 0.15191\ p_8 + 171.3775\ p_{11} - 0.0687\ P_{13} = 0.0$

$-149.52 p_1 - 184.155\ p_2 + 263.8\ p_3 - 0.40013\ p_5 - 0.15191\ p_9$

$+ 171.3775\ p_{12} - 0.0687\ p_{14} = 0.0$

$-44.466\ p_2 - 54.752\ p_6 - 902.812\ p_7 + 0.05976\ p_8 = -1.0$

$369.7\ p_2 - 22.233\ p_3 + 455.22\ p_6 - 50.048\ p_7 - 0.211\ p_8$

$-451.406\ p_{10} + 0.02988\ p_{11} = 0.0.$

$$-22.233\, p_4 - 27.376\, p_8 + 377.0\, p_9 - 451.406\, p_{11} + 0.02988\, p_{13} = 0.0$$

$$-149.52\, p_2 - 22.233\, p_5 - 184.155\, p_6 + 263.8\, p_7 - 27.376\, p_9$$

$$-451.406\, p_{12} + 0.02988\, p_{14} = 0.0$$

$$739.4\, p_3 + 910.44\, p_7 - 45.344\, p_{10} - 0.422\, p_{11} = -1.0$$

$$369.7\, p_4 + 455.22\, p_8 - 22.672\, p_{11} + 377.0\, p_{12} - 0.211\, p_{13} = 0.0$$

$$-149.52\, p_3 + 369.7\, p_5 - 184.155\, p_7 + 455.22\, p_9 + 263.8\, p_{10}$$

$$754.0\, p_{14} = -1.0$$

$$-149.52\, p_4 - 184.155\, p_8 + 263.8\, p_{11} + 377\, p_{15} = 0.0$$

$$-299.04\, p_5 - 368.31\, p_9 + 527.6\, p_{12} = -1.0 \qquad (4.8)$$

The Gauss- Jordan method solving simultaneous algebric equations gave the 'P' matrix given below.

$$P = \begin{bmatrix} 5.69625 & -4.64682 & -0.00015 & -15.99779 & 3.85916 \\ -4.64682 & 3.81758 & -0.00019 & 20.41740 & -3.15511 \\ -0.00015 & -0.00019 & 0.03445 & -2.01270 & -0.01708 \\ -15.99779 & 20.41740 & -2.01270 & 16204.86718 & -0.00132 \\ 3.85916 & -3.15511 & -0.01708 & -0.00132 & 5.03694 \end{bmatrix}$$

As a recheck, the matrix P obtained was fed into the equation TP + PA and was found equal to the identity matrix.

$$I = \begin{bmatrix} -1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -1.0 \end{bmatrix}$$

The P matrix was then tested for positive definiteness by sylvesters criterion and the values of the successive principal minors of the 'P' matrix are

$$D_{11} = p_1 = 5.69625 > 0$$

$$D_{22} = \begin{vmatrix} 5.69625 & -4.64682 \\ -4.64682 & 3.81758 \end{vmatrix} = 0.15294 > 0$$

$$D_{33} = \begin{vmatrix} 5.69625 & -4.64682 & -0.00015 \\ -4.64682 & 3.81758 & -0.00019 \\ -0.00015 & -0.00019 & 0.03445 \end{vmatrix} = 0.005269 > 0$$

$$D_{44} = \begin{vmatrix} 5.69625 & -4.6482 & -0.00015 & -15.99779 \\ -4.64682 & 3.81758 & -0.00019 & 20.41740 \\ -0.00015 & -0.00019 & 0.03445 & -2.01270 \\ -15.99779 & 20.4174 & -2.0127 & 16204.86718 \end{vmatrix} = 73.9387 > 0$$

$$D_{55} = \text{DET. P} = 177.61966 > 0$$

Therefore the 'P' matrix is positive definite real symmetric so the system considered is stable. The computer programs used for finding p matrix and checking positive definiteness are given in appendix 2.

The Liapunov function is

$$V = 5.69625 (\Delta i_{fd})^2 + 3.81758 (\Delta id)^2 + 0.03445 (\Delta i_q)^2$$

$$+ 16204.68718 (n)^2 + 5.03694 (\Delta\delta)^2 - 9.294 \Delta i_{fd} \Delta id$$

$$0.0003 \Delta i_{fd} \Delta iq - 31.994 \Delta i_{fd} n + 7.718 \Delta i_{fd} \Delta\delta$$

$$-0.00038 \Delta id \Delta iq + 40.834 \Delta id\, n - 6.31 \Delta id \Delta\delta$$

$$-4.024 \Delta iq\, n - 0.034 \Delta iq \Delta\delta - 0.0026\, n \Delta\delta$$

Which is positive for all values of states not equal to zero

$$V = -\left((\Delta i_{fd})^2 + \Delta id^2 + \Delta iq^2 + \Delta n^2 + \Delta \delta^2 \right)$$

which is negative for all values of states.

Thr programme -8 of the appendix 3 is executed to test the liapunov function, V

The equilibrium point or the operating point is found to be stable for normal power transfer with normal excitation. This analysis is of course valid for small perturbation from equilibrium states.

4.3 Nonlinear Model

As discussed, a more exact representation of the synchronous machine involves nonlinear differential equations of the form

$$\overset{\circ}{X} = f(x) \qquad (4\cdot 9)$$

The dependence of f on u has not been considered here, because only the free dynamic part is of ~~merex~~ importance for determining the Liapunov stability.

The variable gradient method of Schultz and Gibson has been used here to generate a Liapunov function. The procedure, which has been discussed in detail in section 2.6, in brief, involves assumption of the gradient of V interms of some unknown coefficients. The gradient, in turn, has to satisfy few curl requirements. A good measure of guess can be made about some of the unknown co-efficients from the curl equations. The other co-efficients are generated by trial and error methods. The success of the scheme depends on as to how close a trial can be made .

The operating point of the synchronous generator was considered at a point quite high on the power angle curve. The machine deliver a power of 91.6% of the rated load under normal excitation at an angle of 60°. Any large disturbance at

~~max angle of 60°~~
at this operating point would be quite severe from stability view point.

Substituting the values of the parameter in the nonlinear system equations of section 3.2 yields .

$$p\, i_{fd} = -0.400013\, i_{fd} - 22.233\, id + 369.0\, i_q - 298.44 \sin\delta + 369\text{-}7\, iq\, n$$

$$p_{id} = -0.15191\, i_{fd} - 27.37\, id + 455.22\, i_q - 360 \sin\delta + 455.22\, iq.n$$

$$p i_q = 171.3775\, i_{fd} - 451.\ 406\, id - 22.672\, iq - 305\,\delta$$
$$+ 171.3775\, i_{fd}\, n - 451.406\, I_d\, n$$

$$pn = -0.0936\, i_{fd}\, iq + 0.0406\, i_d\, i_q + 0.1532$$

$$p\,\delta = 377.0\, n \qquad (4.10)$$

$$\nabla V = \begin{bmatrix} a_{11}\, i_{fd} + a_{12}\, id + a_{13}\, iq + a_{14}\, n + a_{15} \\ a_{21}\, i_{fd} + a_{22}\, id + a_{23}\, iq + a_{24}\, n + a_{25} \\ a_{31}\, i_{fd} + a_{32}\, id + a_{33}\, iq + a_{34}\, n + a_{35} \\ a_{41}\, i_{fd} + a_{42}\, id + a_{43}\, iq + a_{44}\, n + a_{45} \\ a_{51}\, i_{fd} + a_{52}\, id + a_{53}\, iq + a_{54}\, n + a_{55} \end{bmatrix} = \begin{bmatrix} \nabla V_1 \\ \nabla V_2 \\ \nabla V_2 \\ \nabla V_3 \\ \nabla V_4 \end{bmatrix} \qquad (4..11)$$

The aij's are completely undetermined quantities. These are determined from various constraints on V and the curl e uations.

The derivative of Liapunov function is

$$\dot{V} = \nabla V_1\ p i_{fd} + \nabla V_2\ p i_d + \nabla V_3\ p i_q + \nabla V_4\ pn + \nabla V_5\ p\delta \qquad (4.12)$$

There are 5(5-1) /2 = 10 curl equation for this problem, which

1. $\dfrac{\partial \nabla V_1}{\partial i_d} = \dfrac{\partial \nabla V_2}{\partial i_{fd}}$

2. $\dfrac{\partial \nabla V_1}{\partial iq} = \dfrac{\partial \nabla V_3}{\partial i_{fd}}$

3. $\dfrac{\partial \nabla V_1}{\partial n} = \dfrac{\partial \nabla V_4}{\partial i_{fd}}$

4. $\frac{\partial \nabla V_1}{\partial \delta} = \frac{\partial \nabla V_5}{\partial i_{fd}}$

5. $\frac{\partial \nabla V_2}{\partial i_q} = \frac{\partial \nabla V_3}{\partial i_d}$

6. $\frac{\partial \nabla V_2}{\partial n} = \frac{\partial \nabla V_4}{\partial i_d}$

7. $\frac{\partial \nabla V_2}{\partial \delta} = \frac{\delta \nabla V_5}{\delta i_d}$

8. $\frac{\partial \nabla V_3}{\partial n} = \frac{\partial \nabla V_4}{\partial i_q}$

9. $\frac{\partial \nabla V_3}{\partial \delta} = \frac{\partial \nabla V_5}{\partial i_q}$

10. $\frac{\partial \nabla V_4}{\partial \delta} = \frac{\partial \nabla V_5}{\partial n}$

The Liapunov function for the system is then written as

$$V = \int_0^{\dot{i}_{fd}} a_{11}\ i_{fd}\ di_{fd} + \int_0^{\dot{i}_d} (a_{21}\ i_{fd} + a_{22}\ id)\ d\ id + \int_0^{\dot{i}_q} (a_{31}\ ifd + a_{32}\ id + a_{33}\ iq)\ d\ iq$$

$$+ \int_0^{n} (a_{41}\ i_{fd}\ a_{42}\ id + a_{43}\ iq + a_{44}\ n)\ dn$$

$$+ \int_0^{\delta} (a_{51}\ i_{fd} + a_{52}\ id + a_{53}\ iq + a_{54}\ n + a_{55})\ d\delta \qquad (4.14)$$

A trial and error method was developed to find the values of aij satisfying all the constraints viz.(i) curl equations (ii) negative definiteness of $\overset{\circ}{V}$

(iii) Positive definiteness of V. To make this method efficient and fast the following assumptions were made.

(1) To satisfy the curl equations, the values of aij were assumed as

$$a_{ij} = a_{ji} \qquad \begin{matrix} i = 1, 2 \quad 5 \\ j = 1, 2 \quad 5 \end{matrix}$$

(2) The values of aij for i = j and i = 1,2 ....5 were assumed positive except a_{55} which may be negative (3) a_{11} > (a_{2j} ; j = 1,2) > (a_{3j} ; j = 1,2,3) > (a_{4j} ; j = 1,2,4) ≥ (a_{5j} ; j = 1,2,5). Assumptions (2) and (3) were made to make V positive definite. Substituting these values of aij, $\mathring{V}$ is tested for negative definiteness.

$$\mathring{V} = \nabla V_1 \; p i_{fd} + \nabla v_2 \; p i_{1d} + \nabla v_3 \; p i_q + \nabla v_4 \; pn + \nabla v_5 \; p\delta \quad (4.15)$$

The equilibrium point is assumed as the origin of the system. Expressions (4.10) and (4.11) were substituted in expression (4.15) for a set of trial values of a_{ij} satisfying all other constraints. Considering the different states ad different co-ordinates in the state space, the derivative of the Liapunov function $\mathring{V}$, was calculated in steps for all possible values along the co -ordinate i_{fd} (say) keeping i_d, i_q ..... δ constant. The Limiting value of the states value of the states/i_{fd} was found where V becomes just positive. The procedure was continued, taking variations in the states along all the co-ordinates for a particular set of aij's . Thus the bounardy states of the stability domain are obtained outside which the system becomes unstable. The computer program for testing positive definiteness and negative definiteness are given in appendix 3.

The values of aij's are

5.872	-4.025	-1.28	-1.0	-1.0
-4.025	4.025	1.28	1.0	1.0
-1.28	1.28	1.28	1.0	1.0
- 1.0	1.0	1.0	1.0	1.0
- 1.0	1.0	1.0	1.0	-0.202

The boundary states as obtained by program 6 of appendix 3 are

$$
\begin{aligned}
i_{fd} &= 5.0024 \\
i_{d} &= 1.6143 \\
i_{q} &= 1.003 \\
n &= 0.047 \\
\delta &= 2.012
\end{aligned}
\qquad (4.16)
$$

The Liapunov function is

$$V = 0.9235 \; i^2_{fd} + 1.372 \; (- i_{fd} + i_d)^2 + 0.14 \; (-i_{fd} + id + iq)^2$$

$$+ 0.5 \; (- i_{fd} + i_d + i_q + n - 0.202)^2$$

The derivative of V is

$$\dot{V} = (5.872 \, i_{fd} - 4.025 \, id - 1.28 \, i_q - n - \delta_{fd})$$

$$(-0.40013 \, i_{fd} - 22.233 \, id + 369.7 \, iq - 298.44 \sin \delta + 369.7 \, i_q \, n)$$

$$+ (-4.025 \, i_{fd} + 4.025 \, id + 1.28 \, iq + n + \delta_{fd})$$

$$(-0.15191 \, i_{fd} - 27.376 \, id + 455.22 \, iq - 368.0 \sin \delta + 455.22 \, iq \, n)$$

$$+ (-1.28 \, i_{fd} + 1.28 \, id + 1.28 \, iq + n + \delta_{fd})$$

$$.(171.3775 \, i_{fd} - 451.406 \, id - 22.672 \, iq - 305.0 \cos \delta + 171.3775 \, i_{fd} n - 451.406 \, idn)$$

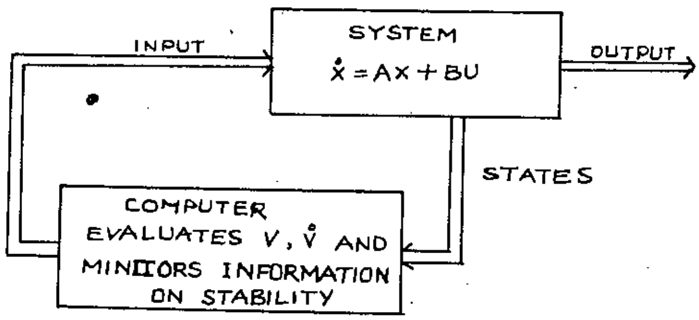
$$+ (-i_{fd} + id + iq + n + \delta \quad \delta_{fd})$$

$$(-0.0936 \, i_{fd} \, iq - 0.0406 \, id \, iq + 0.1532)$$

$$+ (-i_{fd} + id + iq + n + \qquad - 0.202 \, \delta_{fd}) \; (377.0 \, n)$$

It can be easily seen that the V function is positive definite for all values of the states, but the V function is negative definite only in a region in the neighbourhood of the equilibrium states. The boundary states for stable operation is shown in (4.16) . This is quite expected, if the disturbance or fault is severe to push the machine states beyond the critical values, the system will definitely be unstable. An idea about the range of stable operation can be obtained from the V function.

If a disturbance appears on the system, the states will be perturbed from the equilibrium point and the amount of perturbation will depend on the magnitude, duration and type of fault. However these informations, on disturbance are not necessary for determining stability. The V function (Liapunov function) has been expressed as a function of system states which may be measured and metered directly or indirectly. So a scheme may be developed using small computing devices (analog or digital) to evaluate the energy function and its derivative which is also a function of the states. So long as the V function is positive and its derivative is negative , the system is stable. Any violation of this condition will make the system unstable and proper action should be taken to remove the machine from the rest of the system or any other supervisory control to stabilize it should be attempted. The procedure suggested can be illustrated by a figure.



BLOCK DIAGRAM OF AN ONLINE AUTOMATIC CONTROL DEVICE FOR A POWER SYSTEM

FIGURE 4.1.

CHAPTER- 5

STABILITY STUDY OF SINGLE MACHINE INFINITE BUS SYSTEM WITH VOLTAGE REGULATOR AND STABILIZING-SIGNAL

5.1 Introduction

In the previous chapter, the stability of the synchronous generator infinite bus system was considered without taking the voltage regulator action into consideration. In practice, a synchronous generator is equipped with automatic voltage regulators having terminal voltage feedback. Sometimes, additional internal feedback loops in the regulator and exciter are provided for tight loop stabilizstion. The modern ternd is to have high gain fast acting solid state excitation systems for power system. This though is helpful in regulation worsens stability of the system because of introduction of high gain in the forward loop. It has been found that introduction of additional stabilizing signals proportional to speed deviation and acceleration of the system with fast acting regulating devices can improve stability substantially [12, 16].

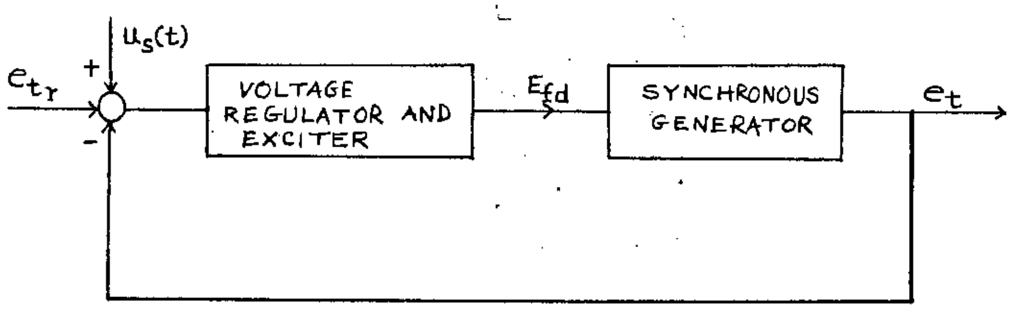


FIGURE 5.1 SYNCHRONOUS GENERATOR CONFIGURATION WITH VOLTAGE REGULATOR

The dynamic equation of the synchronous generator with a voltage regulator has been given in section 3. Considering a single time constant for the voltage regulator and excitation system. These equations can be rewritten as

$$\dot{\underline{X}}(t) = \underline{f}\,[\,x(t),\quad u_s(t)\,] \qquad (5.1)$$

where the states are the different currents, speed deviation and angular position of the generator and the excitation of the generator. U_s (t) is additional stabilizing signal, usually functions of rotor velocity acceleration etc. These when linearized about steady operating point can be expressed as

$$\underline{\overset{o}{X}}(t) = \underline{A} \; \underline{X}(t) + B . U_s(t) \qquad (5.2)$$

when the new states X(t) are the deviation of the original states of equation (5.1) from the equilibrium point.

5.2 Linearized model without stabilizing signal

The equation of the linearized synchronous machine with excitation scheme in absence of the stabilizing auxiliary signal.

$$\overset{o}{X}(t) = A \; x(t) \qquad (5.3)$$

This is the free dynamic part of equation(5.2). As has been discussed, since the A matrix contains a high gain term of the excitation system, the system is inherently has very little stability limit. The range of the exciter gain K_r, for stable operation of the system, is under investigation in this section.

The A matrix for the under excited operating condition of the synchronous machine given in appendix 2, is

$$A = \begin{bmatrix} -0.40013 & -22.233 & 369.7 & 0.0 & -144.52 & 0.711 \\ -0.15191 & -27.376 & 455.22 & 0.0 & -184.155 & 0.27 \\ 171.3775 & -451.406 & -22.672 & 0.0 & 263.8 & 0.0 \\ -0.0607 & 0.02980 & -0.211 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 377.0 & 0.0 & 0.0 \\ 27.7983\, k_r & -12.2572\, kr & 2.521\, kr. & 0.0 & -22.53\, kr & 0.003\, kr - 80.0 \end{bmatrix}$$

The procedure for finding the Liapunov function is similar to that of the linearized system in section 4. If the matrix equation

$$A^T P + PA = -I \qquad (5.5)$$

can be solved for a positive definite real symmetric matrix P, then the system is stable. The P matrix is assumed to be of the form

$$P = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \\ p_2 & p_7 & p_8 & p_9 & p_{10} & p_{11} \\ p_3 & p_8 & p_{12} & p_{13} & p_{14} & p_{15} \\ p_4 & p_9 & p_{13} & p_{16} & p_{17} & p_{18} \\ p_5 & p_{10} & p_{14} & p_{17} & p_{19} & p_{20} \\ p_6 & p_{11} & p_{15} & p_{18} & p_{20} & p_{21} \end{bmatrix}$$

The matrix equation 5.5 is arranged as a set of 21 simultaneous equations. It can be seen that all these equations are function of the gain of the excitation system, K_r since the A matrix is a function of k_r.

The simultaneous equations are

$$-0.80026\, p_1 - 0.30382\, p_2 + 342.755\, p_3 - 0.1374\, p_4 + 55.59662\, k_r\, p_6 = -1.0$$

$$-22.233\, p_1 - 27.77613\, p_2 - 451.406 p_3 + 0.02980\, p_4 - 12.2572 p_6\, k_r$$

$$-0.15191 p_7 + 171.3775 p_8 - 0.0687\, p_9 + 27.7983 p_{11} k_r = 0.0$$

$$369.7 p_1 + 455.22\, p_2 - 23.07213 p_3 - 0.211 p_4 + 2.521\, p_6 kr - .15191 p_8$$

$$+ 171.3775 p_{12} - 0.0687 p_{13} + 27.7983 p_{15} kr = 0.0$$

$$-0.40013 p_4 + 377.0 p_5 - 3.15191 p_9 + 171.3775 p_{13} - 0.0687 p_{16} + 27.7983\, k_r p_{18} = 0.0$$

$$-149.52 p_1 - 184.155 p_2 + 2638 p_3 - 0.40013 p_5 - 22.53 p_6 k_r - 0.15191\, p_{10}$$

$$+ 171.3775 p_{14} - 0.0687 p_{17} + 27.7983\, p_{20} k_r = 0.0\ .$$

$0.711p_1 + 0.27p_2 + (0.003\ kr - 80.4)\ p_6 - 0.15191\ p_{11} + 171.3775\ p_{15}$

$-0.06387p_{18} + 27.7983\ kr\ p_{21} = 0.0$

$-44.466p_2 - 54.752p_7 \quad -902.812p_8 + 0.05976p_9 \quad -24.5144p_{11}\ k_r = -1.0$

$369.7\ p_2 - 22.233p_3 + 455.22p_7 - 50.048p_8 - 0.211p_9 + 2.521\ kr\ p_{11}$

$-451.406p_{12} + 0.02988p_{13} + 12.2572p_{15} \quad = 0.0$

$-22.233p_4 \quad -27.376p_9 + 377.0p_{10} - 451.406\ p_{13} + 0.02988p_{16}$

$+ 12.2572p_{18}kr = 0.0$

$-149.52p_2 \quad -22.233p_5 \quad -184.155p_7 + 263.8p_8 \quad -27.376\ p_{10}$

$-22.53krp_{11} \quad -451.406p_{14} + 0.02988\ p_{17} \quad -12.2572\ kr\ p_{20} = 0.0$

$0.711p_2 \quad -22.233p_6 + 0.27p_7 + (0.003\ kr - 107.376)\ p_{11}$

$-451.406\ p_{15} + 0.02988p_{18} + 12.2592\ p_{21}kr = 0.0$

$139.4p_3 + 910.44p_8 - 45.344p_{12} - 0.422p_{13} + 5.042p_{15}kr = 1.0$

$369.7\ p_4 + 455.22\ p_9 - 22.672p_{13} + 377.0p_{14} \quad -0.211p_{16}$

$+ 2.521\ kr\ p_{18} = 0.0$

$-149.52p_3 + 369.7\ p_5 - 184.155p_8 + 455.22p_{10} + 263.8\ p_{12}$

$-22.672p_{14} - 22.53kr\ p_{15} \quad -0.211p_{17} + 2.521\ kr\ p_{20} = 0.0$

$0.711p_3 + 369.7p_6 + 0.27p_8 + 455.22\ p_{11} + (0.003\ kr - 102.682)p_{15}$

$-0.211\ p_{18} + 2.521\ kr\ p_{21} = 0.0$

$754p_{17} = -1.0$

$-149.52p_4 - 184.155p_9 + 263.8\ p_{13} - 22.53\ kr\ p_{18} + 377.0\ p_{19} = 0.0$

$0.711p_4 + 0.27p_9 + (0.003\ kr - 80.0)\ p_{18} + 377.0p_{20} = 0.0$

$-299.04p_5 \quad -368.31\ p_{10} + 527.6\ p_{14} \quad -45.06\ kr\ p_{20} = -1.0$

$0.711p_5 \quad -149.52p_6 + 0.27p_{10} - 184.155p_{11} + 263.8\ p_{15}$

$+ (0.003\ kr \quad -80.0)\ p_{20} \quad -22.53\ kr\ p_{21} = 0.0$

$$1.422\ p_6 + 0.54p_{11} + (0.006\ kr - 160.0)\ p_{21} = -1.0 \qquad (5.6)$$

case 1 Exciter gain (kr) = -2.0

The A matrix is

$$A = \begin{bmatrix} -0.40013 & -22.233 & 369.7 & 0.0 & -149.52 & 0.711 \\ -0.15191 & -27.376 & 455.22 & 0.0 & -104.155 & 0.27 \\ 171.3775 & -451.4058 & -22.672 & 0.0 & 263.7998 & 0.0 \\ -0.0687 & 0.0299 & -0.211 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 377.0 & 0.0 & 0.0 \\ -55.5966 & 24.5144 & -5.042 & 0.0 & 45.06 & -80.006 \end{bmatrix}$$

The 'P' matrix is

$$P = \begin{bmatrix} 8.366 & -6.838 & -0.0014 & -66.7888 & 8.616 & 0.0456 \\ -6.838 & 5.621 & -0.0003 & 70.937 & -7.0194 & -0.0371 \\ -0.0014 & -0.0003 & 0.0348 & -4.4654 & -0.0276 & -0.0005 \\ -66.78 & 70.93 & -4.465 & 36453.87 & -0.0013 & -0.094 \\ 8.616 & -7.019 & -0.0276 & -0.0013 & 11.298 & 0.0552 \\ 0.0456 & -0.0371 & -0.0005 & -0.094 & 0.0552 & 0.0065 \end{bmatrix}$$

The value of the successive principal minors of the 'p' matrix are given below.

$D_{11} = 8.366 > 0$

$D_{22} = 0.259 > 0$

$D_{33} = 0.009 > 0$

$D_{44} = 242.35 > 0$

$D_{55} = 552.76 > 0$

$D_{66} = 3.45 > 0$

The 'p' matrix is positive definite real symmetric so the system with $K_r = -2.0$ is stable.

The Liapunov function is

$$V = 8.366\,(\Delta i_{fd})^2 + 5.621\,(\Delta id)^2 + 0.0348\,(\Delta iq)^2 + 36453.89\,(n)^2$$

$$+ 11.298\,(\Delta\delta)^2 + 0.0065\,(\Delta E_{fd})^2 - 13.676\,\Delta i_{fd}\,\Delta id$$

$$-0.0028\,\Delta i_{fd}\Delta i_q - 133.5776\,\Delta i_{fd}\,n + 17.232\,\Delta i_{fd}\Delta\delta + 0.0912\,\Delta i_{fd}\Delta E_{fd}$$

$$-0.006\,\Delta id\Delta iq + 141.874\,\Delta id\,n - 14.0388\,\Delta id\Delta\delta - 0.0742\,\Delta i_d\Delta E_{fd}$$

$$-8.9308\,\Delta i_q\,n - 0.0552\,\Delta iq\,\Delta\delta - 0.001\Delta i_q\,\Delta E_{fd}$$

$$-0.0026\,n\,\Delta\delta - 0.188\,n\,\Delta E_{fd} + 0.1104\,\Delta\delta\,\Delta E_{fd}.$$

which is positive for all values of states not equal to zero.

$$\dot{V} = -\left(\Delta i_{fd}^{\,2} + \Delta i_d^{\,2} + \Delta i_q^{\,2} + n^2 + \Delta\delta^2 + \Delta E_{fd}^{\,2}\right)$$

which is negative for all values of states.

Case -2 Exciter gian (kr) = -3.8

The A matrix is

$$A = \begin{bmatrix} -0.40013 & -22.233 & 369.7 & 0.0 & -149.52 & 0.711 \\ -0.1519 & -27.376 & 455.22 & 0.0 & -184.155 & 0.27 \\ 171.3775 & -451.4058 & -22.672 & 0.0 & 263.7998 & 0.0 \\ -0.0627 & 0.0299 & -0.211 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 377.0 & 0.0 & 0.0 \\ -105.6335 & 46.5773 & -9.5798 & 0.0 & 85.614 & -80.0114 \end{bmatrix}$$

The P matrix is

$$P = \begin{bmatrix} -297.35 & 243.4822 & 0.1246 & 4302.99 & -383.36 & -1.815 \\ 243.48 & -199.67 & -0.01 & -4247.6 & 311.36 & 1.48 \\ 0.1246 & -0.01 & 0.0091 & 198.06 & 0.84 & 0.0017 \\ 4302.99 & -4247.6 & 198.06 & 1644904.0 & -0.013 & -2.3704 \\ -383.36 & 311.36 & 0.84 & -0.0013 & -509.74 & -2.3704 \\ -1.815 & 1.4203 & 0.0017 & 12.7351 & -2.3704 & -0.0049 \end{bmatrix}$$

The value of the successive principal minors of this 'P' matrix is

$$D_{11} = -297.35 < 0$$

$$D_{22} = 91.44 > 0$$

$$D_{33} = 3.353 > 0$$

$$D_{44} = 387381.56 > 0$$

$$D_{55} = 1520427 > 0$$

$$D_{55} = 9454.16 > 0$$

'P' matrix is not positive definite real symmetric so this system with $k_r = -3.8$ is unstable.

Case – 3 Exciter gain (kr) = -100.0

The A matrix is

$$\begin{bmatrix} -0.40018 & -22.233 & 369.7 & 0.0 & -149.52 & 0.711 \\ -0.1519 & -27.3760 & 455.22 & 0.0 & -184.155 & 0.27 \\ 171.3775 & -451.4058 & -22.672 & 0.0 & 263.7998 & 0.0 \\ -0.0687 & 0.0299 & -0.211 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 377.0 & 0.0 & 0.0 \\ -2779.8308 & 1225.72 & -252.1 & 0.0 & 2252.99 & -80.3 \end{bmatrix}$$

The 'P' matrix is

$$P = \begin{bmatrix} 41.1419 & -33.55 & 0.014 & -144.67 & -17.015 & -0.0066 \\ -33.55 & 27.435 & -0.0088 & -162.97 & 13.7998 & 0.0045 \\ 0.014 & -0.0088 & 0.2095 & 9.6948 & -0.0495 & -0.0204 \\ 144.6765 & -162.976 & 9.6984 & -100150.19 & -0.0013 & 0.7532 \\ -17.05 & 13.7998 & -0.0495 & -0.0013 & -33.5155 & 0.0043 \\ -0.0066 & 0.0045 & -0.0204 & 0.7532 & 0.0043 & 0.0062 \end{bmatrix}$$

The value of the successive principal minors of the 'P' matrix according to the equation (2.76) are

$$D_{11} = 41.1419 > 0$$
$$D_{22} = 3.1426 > 0$$
$$D_{33} = 0.6579 > 0$$
$$D_{44} = -84073.25 < 0$$
$$D_{55} = 3416282 > 0$$
$$D_{55} = 14394.695 > 0$$

'P' matrix is not positive definite real symmetric so this system with k_r = -100.0 is unstbble.

5.3 Linearized model with stabilizing signal.

The equation of the lenearized system considering the auxiliary stabilizing signal is

$$\dot{X}(t) = A\ X(t) + B\ U_s(t) \qquad (5.7)$$

Liapunov stability, as such, is the stability of the free dynamic system. The effect of the input or control term can not be taken into consideration.

Stability of a driving system gives rise to the concept of bounded input bounded output stability which means that if the input to a system belongs to a bounded set, then for stability, the response will also be bounded. This in other word is ; for stable operation, if

$$| U_s (t) | \leq M$$

Then $\| X (t) \| \leq N$

Where M and N belongs to a set of bounded numbers. This type of system stability is outside the scope of this thesis.

A special type of the auxiliary stabilizing signal; viz a feedback derived from the rotor velocity of the generator, has found importance in excitation system of modern power system. Most of the time, the rotor velocity is not directly fed back. A particular study related. U_s to be 20 times per unit frequency deviation for satisfactory system operation.(8) Choice of this particular feedback converts the bounded input bounded output problem given by equation (5.7) to a Liapunov problem with a different A matrix i.e.

$$\overset{\circ}{X}(t) = A_1 \; X (t) \qquad (5.8)$$

The A_1 matrix as a function of exciter gain, Kr

A_1 =

-0.40013	-22.233	369.7	0.0	-149.52	0.711
-0.15191	-27.376	455.22	0.0	-184.155	0.27
171.3775	-451.406	-22.672	0.0	263.8	0.0
-0.0687	0.02408	-0.211	0.0	0.0	0.0
0.0	0.0	0.0	377.0	0.0	0.0
27.7983 kr	-12.272 kr	2521 kr	-1600 kr	-22.53 kr	(0.003-80.0)

For positive definite real symmetric matrix P the equation

$$A_1^T \, P + PA_1 = -I$$

is solved. The rest of the procedure is same as before.

Case-1 Exciter gain (K_r) = -2.0

A_1 matrix is

$$
A_1 = \begin{bmatrix}
-11^{\circ}40013 & -22.233 & 369.7 & 0.0 & -149.52 & 0.11 \\
-0.1519 & -27.376 & 455.22 & 0.0 & -184.155 & 0.27 \\
171.3775 & -451. & -22.672 & 0.0 & 263.7998 & \\
-0.0687 & 0.0299 & -0.211 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 377.0 & 0.0 & 0.0 \\
-55.5966 & 24.5144 & -5.042 & 3200.0 & 45.06 & -00.000
\end{bmatrix}
$$

The 'P' matrix is

$$
P = \begin{bmatrix}
8.8481 & -7.2327 & -0.0019 & -71.9314 & 9.2438 & 0.0482 \\
-7.2327 & 5.9441 & -0.0002 & 77.0328 & -7.5295 & -0.0391 \\
-0.0019 & -0.0002 & 0.0349 & -4.9946 & -0.0288 & -0.0005 \\
-71.9314 & 77.0328 & -4.9946 & 40633.77 & -1.3474 & 0.1586 \\
9.2438 & -7.5295 & -0.0288 & -1.3474 & 12.0789 & 0.0586 \\
0.0482 & -0.0391 & -0.0005 & 0.1586 & 0.0586 & 0.0065
\end{bmatrix}
$$

The value of the successive principal minors of this 'P' matrix are

$D_{11} = 8.8481 > 0$

$D_{22} = 0.2817 > 0$

$D_{33} = 0.0098 > 0$

$D_{44} = 285.89 > 0$

$D_{55} = 650.36 > 0$

$D_{66} = 4.0629 > 0$

The Liapunov function is

$$V = 0.8481\ (\Delta i_{fd})^2 + 5.9441\ (\Delta id)^2 + 0.0349\ (\Delta i_q)^2$$

$$+ 40633.77\ n^2 + 12.0789\ (\Delta\delta)^2 + 0.0065\ (\Delta E_{fd})^2$$

$$- 14.4654\ \Delta i_{fd}\ \Delta i_d - 0.0038\ \Delta i_d\ \Delta i_q\ -143.0628\ \Delta i_{fd}\ n$$

$$+ 10.4876\ \Delta i_{fd}\ \Delta\delta + .0964\ \Delta i_{fd}\ \Delta E_{fd}\ -0.0004\ \Delta i_d\ \Delta i_q$$

$$+ 154.06\ \Delta id\ n - 15.059\ \Delta_{id}\Delta\delta - 0.0882\ \Delta i_d\ \Delta E_{fd}$$

$$-9.9892\ \Delta i_q n - 0.0576\ \Delta i_q \Delta\delta - 0.001\Delta i_q\ \Delta E_{fd}$$

$$-2.6948\ n\ \Delta\delta + 0.3172\ n\ \Delta E_{fd} + 0.1172\ \Delta\delta\ \Delta E_{fd}$$

which is positive for all value of states not equal to zero.

$$\dot{V} = -(\ \Delta i_{fd}^{\ 2} + \Delta i_d^{\ 2} + \Delta i_q^{\ 2} + n^2 + \Delta\delta^2 + \Delta\ E_{fd}^{\ 2}\ )$$

which is negative for all values of states.

Case 2 Exciter gain (K_x) = -5.0

A_f matrix is

A_f =

-0.40013	-22.233	369.7	0.0	-149.52	0.711
-0.1519	-27.376	455.22	0.0	-184.155	0.27
~~+0.1519~~ 171.3775	-451.4050	-22.672	0.0	263.799	0.0
-0.0687	0.0299	-0.211	0.0	0.0	0.0
0.0	0.0	0.0	377.0	0.0	0.0
-130.9915	61.2860	-12.605	8000.0	112.65	-80.015

The P matrix is

$$P = \begin{bmatrix} 101.1187 & -82.9212 & -0.0340 & -1695.97 & 125.1687 & 0.5495 \\ -82.9212 & 68.1405 & -0.0044 & 1649.65 & -101.6398 & -0.4892 \\ -0.0340 & -0.0044 & 0.0439 & -71.3308 & -0.3449 & -0.0019 \\ -1695.9771 & 1649.65 & -71.33 & 594730.43 & 102.307 & -4.8213 \\ 125.16 & 101.63 & -0.3449 & 102.30 & 167.91 & 0.7832 \\ 0.5995 & -0.4892 & -0.0019 & -4.8213 & 0.7832 & 0.0099 \end{bmatrix}$$

From equation (2.76), the values of the successive principal minors of this 'P' matrix are

$$D_{11} = 101.1187 > 0$$

$$D_{22} = 14.36 > 0$$

$$D_{33} = 0.5243 > 0$$

$$D_{44} = 47089.84 > 0$$

$$D_{55} = 207379.69 > 0$$

$$D_{66} = 1285.72 > 0$$

therefore the P matrix is positive definite real symmetric so the system is stable.

The Liapunov function is

$$\begin{aligned} V = {} & 101.1187 (\Delta i_{fd})^2 + 68.1405 (\Delta i_d)^2 + 0.0439 (\Delta i_q)^2 \\ & + 594730.43\, n^2 + 167.91\, \Delta\delta^2 + 0.0099 (\Delta E_{fd})^2 \\ & -165.8424\, \Delta i_{fd} \Delta i_d - 0.068\, \Delta i_{fd} \Delta i_q - 3391.94\, \Delta i_{fd}\, n \\ & +250.3374\, \Delta i_{fd} \Delta\delta + 1.199\, \Delta i_{fd} \Delta E_{fd} - 0.0088\, \Delta i_d \Delta i_q \\ & + 3299.3\, \Delta i_d\, n - 203.2796\, \Delta i_d \Delta\delta - 0.9784\, \Delta i_d \Delta E_{fd} \\ & -142.6616\, \Delta i_q\, n - 0.6898\, \Delta i_q \Delta\delta - 0.0038\, \Delta i_q \Delta E_{fd} \\ & + 204.614\, n \Delta\delta - 9.6426\, n \Delta E_{fd} + 1.5664\, \Delta\delta \Delta E_{fd} \end{aligned}$$

which is positive for all values of states not equal to zero

$$\dot{V} = -(\Delta i_{fd}^2 + \Delta i_d^2 + \Delta i_q^2 + n^2 + \Delta \delta^2 + \Delta E_{fd}^2)$$

which is negative for all values of states.

Case-3 Exciter gain (k_r) = -100.0

A_1 matrix is

A_1 =

-0.40013	-22.233	369.7	0.0	-149.52	0.711
-0.1519	-27.376	455.22	0.0	-184.155	0.27
171.3775	-451.4058	-22.672	0.0	263.799	0.0
-0.0687	0.0299	-0.211	0.0	0.0	0.0
0.0	0.0	0.0	~~0.0~~ 377.0	0.0	0.0
-2779.83	1225.719	-252.1	160000.0	2252.9	-80.3

The 'P' matrix is

P =

56.52	-47.31	0.087	-2630.44	-3.06	0.065
-47.31	39.72	-0.0805	2231.53	2.90	-0.0554
0.0878	-0.0805	0.2415	-16.129	-0.555	-0.0229
-2630.44	2231.53	-16.12	19142.12	908.07	-2.139
-3.0607	2,9039	-0.555	908.07	28.85	0.099
0.065	-0.0554	-0.0229	-2.1396	0.099	0.0066

The values of the successive principal minors of this P matrix are

$D_{11} = 56.52 > 0$

$D_{22} = 6.742 > 0$

$D_{33} = 1.6247 > 0$

$D_{44} = 99524.31 > 0$

$D_{55} = 1937361 > 0$

$D_{66} = 809608.59 > 0$

Therefore the 'P' matrix is positive definite real symmetric so the system is stable.

The Liaponov function is

$$V = 58.52\,(\Delta i_{fd})^2 + 39.72\,(\Delta i_d)^2 + 0.2415\,(\Delta i_q)^2$$
$$+ 191429.12\,n^2 + 28.85\,(\Delta\delta)^2 + 0.0066\,(\Delta E_{fd})^2$$
$$-94.62\,\Delta i_{fd}\,\Delta i_d + 0.174\,\Delta i_{fd}\,\Delta i_q - 5260.80\,\Delta i_{fd}\,\eta$$
$$-6.12\,\Delta i_{fd}\,\Delta\delta + 0.13\,\Delta i_{fd}\,\Delta E_{fd} - 0.161\,\Delta i_d\,\Delta i_q$$
$$+ 4463.06\,\Delta i_d\,n + 5.8\,\Delta i_d\,\Delta\delta - 0.1108\,\Delta i_d\,\Delta E_{fd}$$
$$-32.258\,\Delta i_q\,n - 1.110\,\Delta i_q\,\Delta\delta - 0.0458\,\Delta i_q\,\Delta E_{fd}$$
$$+ 1816.14\,n\,\Delta\delta - 4.278\,\eta\,\Delta E_{fd} + 0.198\,\Delta\delta\,\Delta E_{fd}$$

which is positive for all values of states not equal to zero

$$\dot{V} = -\left(\Delta i_{fd}^2 + \Delta i_d^2 + \Delta i_q^2 + n^2 + \Delta\delta^2 + \Delta E_{fd}^2\right)$$

which is negative for all values of states.

Case-4 Exciter gain (K_r) = -800.0

A_1 matrix is

A_1 =

-0.40013	-22.233	369.7	0.0	-149.52	0.711
-0.1519	-27.376	455.22	0.0	-184.155	0.27
171.3775	-451.4058	-22.672	0.0	263.7998	0.0
-0.060.7	0.0299	-0.211	0.0	0.0	0.0
0.0	0.0	0.0	377.0	0.0	0.0
-22238	9805.75	-2016	1280000.0	18023.99	82.4

The 'P' matrix is

$$P = \begin{bmatrix} 301.1812 & -332.44 & 2.4734 & -1764.86 & -110.71 & 0.069 \\ -332.44 & 295.77 & -2.189 & 15260.87 & 91.71 & -0.075 \\ 2.47 & -2.189 & 15.1303 & -174.03 & -10.73 & -0.1925 \\ -17643.867 & 15260.87 & -174.03 & 875613.5 & 5902.11 & -1.7384 \\ -110.71 & 91.71 & -10.73 & 5902.11 & 69.1 & 0.1757 \\ 0.069 & -0.075 & -0.1925 & -1.7384 & 0.1757 & 0.0064 \end{bmatrix}$$

The values of the successive principal minors of this 'P' matrix are

$D_{11} = 381.18 > 0$

$D_{22} = 2225.51 > 0$

$D_{33} = 33630.5 > 0$

$D_{44} = 0.129194 \text{ E } 10 > 0$

$D_{55} = 0.35923 \text{ E } 11 > 0$

$D_{66} = 0.13512 \text{ E } 09 > 0$

Therefore the 'P' matrix is positive definite real symmetric so the system is stable.

The Liapunov function is

$$V = 381.1812\ (\Delta i_{fd})^2 + 295.77\ (\Delta id)^2 + 15.1303\ (\Delta i_q)^2$$
$$875613.5\ n^2 + 69.1\ (\Delta\delta)^2 + 0.0064\ (\Delta E_{fd})2$$
$$-664.08\ \Delta i_{fd}\,\Delta id + 4.9468\ \Delta i_{fd}\,\Delta iq - 35287.72\ \Delta i_{fd}\ n$$
$$-221.42\ \Delta i_{fd}\,\Delta\delta + 0.138\ \Delta i_{fd}\ \Delta E_{fd} - 4.378\ \Delta id\ \Delta iq$$
$$+ 30537.74\ \Delta id\, n + 183.42\ \Delta i_d\ \Delta\delta - 0.15\,\Delta id\ \Delta E_{fd}$$
$$- 348.06\ \Delta i_q\, n - 21.46\ \Delta i_q\,\Delta\delta - 0.385\ \Delta iq\ \Delta E_{fd}$$
$$+ 11804.22\ n\,\Delta\delta - 3.4768\ n\ \Delta E_{fd} + 0.3514\ \Delta\delta\ \Delta E_{fd}$$

Which is positive for all values of states not equal to zero.

$$V = -(\Delta i_{fd}^2 + \Delta i_d^2 + \Delta i_q^2 + n^2 + \Delta \delta^2 + \Delta E_{fd}^2)$$

Which is negative for all values of states.

5.4 Nonlinear model without Stabilizing Signal

The equation of the nonlinear model with voltage regulator and exciter has been presented in section 3. The stabilizing signal $U_e(t)$ has been assumed absent here. The operating conditions are those of the normally excited machine given in appendix 1. The equations are rewritten as

$$p i_{fd} = 0.40013\, i_{fd} - 22.233\, id + 369.7\, i_q - 290.44 \sin\delta + 369.7\, i_q n + 0.712\, E_{fd}$$

$$p i_d = -0.15191\, i_{fd} - 27.376\, id + 455.22\, i_q - 368.0 \sin\delta + 455.22\, i_q n + 0.271\, E_{fd}$$

$$p i_q = 171.3775\, i_{fd} - 451.406\, id - 22.672\, iq - 305.0 \cos\delta + 171.3775\, i_{fd}\, n - 451.406\, id\, n$$

$$pn = -0.0936\, i_{fd} + 0.0406\, id\, iq + 0.1532$$

$$p\delta = 377.0\, n$$

$$pE_{fd} = -80.0\, E_{fd} - 84.2\, k_r + 80.0\, e_t k_r + 111.2$$

Where

$$e_t^2 = e_d^2 + e_q^2$$

$$e_d = -0.0003225\, i_{fd} + 0.0116\, i_d - 0.17\,(1+n)\, iq + 0.00057\, E_{fd} + 0.187 \qquad (5.9)$$

$$e_q = 0.365(1+n)\, i_{fd} - 0.164\,(1+n) + 0.0216\, iq + 0.175$$

The procedure of finding the Liapunov function is similar to that for the nonlinear system presented in section 4.2. The expression for the gradient is assumed as

$$\nabla V = \begin{bmatrix} a_{11}\, i_{fd} + a_{12}\, id + a_{13}\, iq + a_{14}\, n + a_{15}\,\delta + a_{16}\, E_{fd} \\ a_{12}\, i_{fd} + a_{22}\, id + a_{23}\, iq + a_{24}\, n + a_{25}\,\delta + a_{26}\, E_{fd} \\ a_{13}\, i_{fd} + a_{23}\, id + a_{33}\, iq + a_{34}\, n + a_{35}\,\delta + a_{36}\, E_{fd} \\ a_{14}\, i_{fd} + a_{24}\, i_{q} + a_{34}\, iq + a_{44}\, n + a_{45} + a_{46}\,\delta\, E_{fd} \\ a_{15}\, i_{fd} + a_{25}\, id + a_{35}\, iq + a_{45}\, n + a_{55}\,\delta + a_{56}\, E_{fd} \\ a_{16}\, i_{fd} + a_{26}\, id + a_{36}\, iq + a_{46}\, n + a_{56}\,\delta + a_{66}\,\delta\, E_{fd} \end{bmatrix} = \begin{bmatrix} \nabla V_1 \\ \nabla V_2 \\ \nabla V_3 \\ \nabla V_4 \\ \nabla V_5 \\ \nabla V_6 \end{bmatrix} \qquad (5\text{-}10)$$

The 15 curl equations

1. $\dfrac{\partial \nabla V_1}{\partial i_d} = \dfrac{\partial \nabla V_2}{\partial i_{fd}} = a_{12}$

2. $\dfrac{\partial \nabla V_1}{\partial i_q} \quad \dfrac{\partial \nabla V_3}{\partial i_{fd}} = a_{13}$

3. $\dfrac{\partial \nabla V_1}{\partial n} = \dfrac{\partial \nabla V_4}{\partial i_{fd}} = a_{14}$

4. $\dfrac{\partial \nabla V_1}{\partial \delta} = \dfrac{\partial \nabla V_5}{\partial i_{fd}} = a_{15}$

5. $\dfrac{\partial \nabla V_1}{\partial E_{fd}} = \dfrac{\partial \nabla V_6}{\partial i_{fd}} = a_{16}$

6. $\dfrac{\partial \nabla V_2}{\partial i_q} = \dfrac{\partial \nabla V_3}{\partial id} = a_{23}$

7. $\dfrac{\partial \nabla V_2}{\partial n} = \dfrac{\partial \nabla V_4}{\partial id} = a_{24}$

$$8.\ \frac{\partial \nabla V_2}{\partial \delta} = \frac{\partial \nabla V_5}{\partial\, id} = a_{25}$$

$$9.\ \frac{\partial \nabla V_2}{\partial E_{fd}} = \frac{\partial \nabla V_6}{\partial\, id} = a_{26}$$

$$10.\ \frac{\partial \nabla V_3}{\partial n} = \frac{\partial \nabla V_4}{\partial\, iq} = a_{34}$$

$$11.\ \frac{\partial \nabla V_3}{\partial \delta} = \frac{\partial \nabla V_5}{\partial\, iq} = a_{35}$$

$$12.\ \frac{\partial \nabla V_3}{\partial E_{fd}} = \frac{\partial \nabla V_6}{\partial\, iq} = a_{36}$$

$$13.\ \frac{\partial \nabla V_4}{\partial \delta} = \frac{\partial \nabla V_5}{\partial n} = a_{45}$$

$$14.\ \frac{\partial \nabla V_4}{\partial E_{fd}} = \frac{\partial \nabla V_6}{\partial n} = a_{46}$$

$$15.\ \frac{\partial \nabla V_5}{\partial E_{fd}} = \frac{\partial \nabla V_6}{\partial \delta} = a_{56} \qquad (5.11)$$

are satisfied.

From equation (2.8a)

$$\mathring{V} = \nabla V_1\ p\, i_{fd} + \nabla V_2\, p i_d + \nabla V_3\, p i_q + \nabla V_4\, p n + \nabla V_5\, p\delta + \nabla V_6\, pE_{fd} \qquad (5.12)$$

The Liapunov function is

$$V = \int_0^{i_{fd}} a_{11}\, i_{fd}\, di_{fd} + \int_0^{i_d} (a_{21}\, i_{fd} + a_{22} id)\, d\, id + \int_0^{i_q} (a_{31}\, i_{fd} + a_{32}\, id + a_{33}\, iq)\, di_q$$

$$+ \int_0^{n} (a_{41}\, i_{fd} + a_{42}\, id + a_{43}\, iq + a_{44}\, n)\; dn$$

$$+ \int_0^{\delta} (a_{51}\, i_{fd} + a_{52}\, id + a_{53}\, iq + a_{54} n + a_{55} \quad d\delta$$

$$+ \int_0^{E_{fd}} E_{fd} \; (a_{61}\, i_{fd} + a_{62}\, id + a_{63}\, iq + a_{64} n + a_{65} + a_{66}\, E_{fd}) \; d\, E_{fd} \qquad (5.13)$$

In order to evaluate the unknown co-efficient a_{if}, computer program 7 in appendix has been used. The equilibrium point or operating point as given in appendix 1 are

$i_{fd} = 2.473309$

$i_d = 0.564341$

$i_q = 0.734433$

$n = 0.0$

$\delta = 1.04719$

$E_{fd} = 1.39$

Case -1 Exciter gain (Kr) = - 2.0

The values of a_{ij}'s are searched so that the derivative of the Liapunov function V is negative definite at the same time satisfying all the curl equations in the same way as in the previous section 4.

The values of a_{ij}'s are

16.4399	-11.72	-3.02	-2.92	-1.0	-1.0
-11.72	11.72	3.02	2.92	1.0	1.0
-3.02	3.02	3.02	2.92	1.0	1.0
-2.92	2.92	2.92	2.92	1.0	1.0
-1.0	1.0	1.0	1.0	1.0	1.0
-1.0	1.0	1.0	1.0	1.0	-5.66

The boundary states are

$i_{fd} = 5.1213$

$i_d = 1.6102$

$i_q = 1.0321$

$n = 0.048$

$\delta = 2.14$

$E_{fd} = 3.22$

From equation (4.9) the Liapunov function is determined. The Liapunov function is

$$V = 2.3599\, i_{fd}^2 + 4.35\,(-i_{fd} + id)^2 + 0.05\,(-i_{fd} + id + i_q)^2$$
$$+ 0.96\,(-i_{fd} + id + iq + n)^2 + 0.5\,(-i_{fd} + id + iq + n + \delta - 5.66\,E_{fd})^2$$

The derivative of V is

$$\mathring{V} = (16.4399\, i_{fd} - 11.72\, id - 3.02\, i_q - 2.92\, n - \delta - E_{fd})$$
$$(-0.40013\, i_{fd} - 22.233\, id + 369.7\, i_q - 298.44\, \sin\delta + 369 + iq\, n + 0.711\, E_{fd})$$
$$+ (-11.72\, i_{fd} + 11.72\, i_d + 3.02\, i_q + 2.92\, n + \delta + E_{fd})$$
$$(-0.1519\, i_{fd} - 27.376\, i_d - 368.0\, \sin\delta + 455.22\, iq\, n + 0.27\, E_{fd})$$
$$+ (-3.02\, i_{fd} + 3.02\, id + 3.02\, iq + 2.92\, n + \delta + E_{fd})$$
$$(171.3775\, i_{fd} - 451.406\, id - 22.672\, iq - 305.0\, \cos\delta + 171.3775\, i_{fd} n - 451.406\, i_d n)$$
$$+ (-2.92\, i_{fd} + 2.92\, id + 2.92\, i_q + 2.92\, b + \delta + E_{fd})$$
$$(-0.0936\, i_{fd}\, i_q + 0.0406\, id\, iq + 0.1532)$$
$$+ (-i_{fd} + id + iq + n + \delta + E_{fd})\,(377.0\, n)$$
$$+ (-i_{fd} + id + iq + n + \delta - 5.66\, E_{fd})\,(-80.0\, E_{fd} - 84.2\, k_r + 80.0\, e_t\, k_r$$
$$k_r + 111.2)\,.$$

It is clear that V is positive definite for all values of the states. The $\overset{\circ}{V}$ function is negative definite only in a small which is the stable region of operation of the machine for this particular gain. Test on V is done by the computer program 8 in appendix 3.

Case -2 Exciter gain (K_r) = -4.0

It was not possible to find a set of a_{ij} which would make V negative definite satisfying the Curl equations at the same time. From the nonlinear model it is not possible to affirm the instability of the system because the V function gives only a sufficient condition. Failure to find a positive definite V with negative definite $\overset{\circ}{V}$ does not necessarily mean instability of the system. However, the linear analysis gives a necessary and sufficient condition on stability, which showed that for $K_r > -3.8$ the system is unstable. So there is every reason to believe that, in this case, failure to find a positive definite V was due to the instability of the system. Any other gain higher than this value also gives value also give similar result.

5.3 <u>Nonlinear model with Voltage regulator and Stabilizing signal</u>

The equation of the synchronous generator infinite bus system provided with an excitation system which has an auxiliary stabilizing signal derived from the rotor velocity is

$$pi_{fd} = -0.40013\ i_{fd} - 22.233\ i_d + 369.7\ iq \quad -298.44\ \sin\delta$$
$$+ 369.7\ iqn + 0.712\ E_{fd}$$

$$pi_d = -0.15191\ i_{fd} \quad -27.37\ id + 455.22\ iq \quad -360.0\ \sin\delta$$
$$+ 455.22\ iq\ n + 0.271\ E_{fd}$$

$$p i_q = 171.3775\, i_{fd} - 451.406\, id \; -22.672\, iq \; - 305.0 \cos\delta$$

$$+\ 171.3775\, i_{fd}$$ [4A] ~~+ 0.0406 id iq + 0.1532~~

$$pn = 0.0936\, i_{fd}\, iq + 0.0406\, id\, iq + 0.1532$$

$$p\,\delta = 377.0\ n$$

$$p_{Efd} = 1600\ n\ kr - 80.\ E_{fd} - 84.2\ k_r + 80.0\ e_t\, k_r + 111.2$$

Where

$$e_t^{\,2} = e_d^{\,2} + e_q^{\,2}$$

$$e_d = 0.0003225\, i_{fd} + 0.0116\, id - 0.17\ (1 + n)\, i_q + 0.00057\, E_{fd} + 0.187$$

$$e_q = 0.365\ (1+n)\, i_{fd} - 0.164\ (1 + n)\, i_d + 0.0216\, iq + 0.175\ . \tag{5.14}$$

The stabilizing signal $U_s(t)$ [8] is taken as 20 times the per unit speed deviation.

The procedure for generating and testing the Liapunov function is similar to that of the previous section.

The value of a_{ij}

6.721	-4.729	-3.02	-2.82	-1.0	-1.0
-4.729	4.729	3.02	2.82	1.0	1.0
-3.02	3.02	3.02	2.82	1.0	1.0
-2.82	2.02	2.82	2.82	1.0	1.0
-1.0	1.0	1.0	1.0	1.0	1.0
-1.0	1.0	1.0	1.0	1.0	0.029

Case 1 Exciter gain (Kr) = -2.0

The boundary states are

$$i_{fd} = 6.2232$$

$$i_d = 1.7221$$

$$i_q = 0.9847$$

$$n = 0.044$$

$$\delta = 2.102$$

$$E_{fd} = 3.32$$

The Liapunov function is

$$V = 0.995\, i_{fd}^2 + 0.854\,(-i_{fd} + i_d)^2 + 0.096\,(-i_{fd} + id + i_q)^2$$

$$+ 0.914\,(-i_{fd} + id + i_q + n)^2 + 0.5\,(-i_{fd} + id + iq + n + \delta + 0.029\, E_{fd})^2$$

which is positive definite for all values of states.

The derivative of V function is

$$\dot{V} = (6.721\, i_{fd} - 4.729\, id - 3.02\, i_q - 2.82\, b - \delta - E_{fd})$$

$$(-0.40013\, i_{fd} - 22.233\, id + 369.7\, i_q - 298.44\, \sin\delta + 369.7\, iq\, n + 0.711\, E_{fd})$$

$$+ (-4.4.729\, i_{fd} + 4.729\, id + 3.02\, i_q + 2.82\, n + \delta + E_{fd})$$

$$(-0.1519\, i_{fd} - 27.376\, id + 455.22\, iq - 368.0\, \sin\delta + 455.22\, i_q n + 0.27\, E_{fd})$$

$$+ (-3.02\, i_{fd} + 3.02\, id + 3.02\, iq + 2.82\, b + \delta + E_{fd})$$

$$(171.3775\, i_{fd} - 451.406\, id - 22.672\, iq - 305.0\, \cos\delta + 171.3775\, i_{fd} n - 451.4\, i_d n)$$

$$+ (-2.82\, i_{fd} + 2.82\, id + 2.82\, iq + 2.82\, n + \delta + E_{fd})$$

$$(-0.0936\, i_{fd}\, iq + 0.0406\, i_d i_q + 0.1532)$$

$$+ (-i_{fd} + i_d + iq + n + \delta + E_{fd})(377.0\, n) + (-i_{fd} + i_d + i_q + n + \delta + 0.029\, E_{fd})$$

$$(-1600.0\, n\, kr - 80.0\, E_{fd} - 84.2\, kr + 80.0\, e\, kr. + 111.2)$$

Which is negative definite within the region specified. So the system considered with $k_r = 2.0$ is stable within this region.

Case-2 Exciter gain (kr) = 20.0

The boundary states are

$$i_{fd} = 5.7321$$

$$i_d = 1.6281$$

$$i_q = 1.0021$$

$$n = 0.042$$

$$\delta = 2.028$$

$$E_{fd} = 4.21$$

The Liapunov function is

$$\bar{V} = 0.995\, i_{fd}^2 + 0.854\,(-i_{fd} + i_d)^2 + 0.096\,(i_{fd} + id + i_q)^2$$

$$+ 0.914\,(-i_{fd} + i_d + i_q + n)^2\; 0.5\,(-i_{fd} + id + iq + n + \delta + 0.029\, E_{fd})^2$$

which is positive definite for all values of states.

The derivative of V is

$$\mathring{V} = (6.721\, i_{fd} - 4.729\, id - 3.02\, iq - 2.82\, n, - \delta - E_{fd})$$

$$(-0.40013\, i_{fd} - 22.233\, id + 369.7\, iq - 298.44 \sin\delta + 369.7\, i_q n + 0.711\, E_{fd})$$

$$+ (-4.729\, i_{fd} + 4.729\, id + 3.02\, iq + 2.82\, n + \delta + E_{fd})$$

$$(-0.15191\, i_{fd} - 27.376\, id + 455.22\, iq - 368.0 \sin\delta + 455.22\, i_q n + 0.27\, E_{fd})$$

$$+ (-3.02\, i_{fd} + 3.02\, id + 3.02\, iq + 2.82\, n + \delta + E_{fd})$$

$$(171.3775\, i_{fd} - 451.406\, id - 22.672\, iq - 305.0 \cos\delta + 171.3775\, i_{fd}\, n - 451.406\, i_d\, n)$$

$$+ (-2.82\, i_{fd} + 2.82\, id + 2.82\, i_q + 2.82\, n + \delta + E_{fd})$$

$(-0.0936\ i_{fd}\ i_q + 0.0406\ id\ iq + 0.1532)$

$+ (-i_{fd} + id + iq + n + \delta + E_{fd})\ (377.0\ n)$

$+ (k - i_{fd} + id + iq + n + \delta + 0.029\ E_{fd})$

$(-1600.0\ n\ kr - 80.0\ E_{fd} - 84.2\ kr + 80.0\ e_t + 111.2)$

Which is negative definite within the region specified. So this system considered with kr = -20.0 is stable within this region.

Case -3 Exciter gain (kr) = -200.0

The boundary states are

$i_{fd} = 5.0214$

$i_d = 1.7428$

$i_q = 0.8732$

$n = 0.04$

$\delta = 2.001$

$E_{fd} = 3.74$

The Liapunov function is

$V = 0.995\ i_{fd}^2 + 0.854\ (-i_{fd} + id)^2 + 0.096\ (-i_{fd} + id + iq)^2$

$+ 0.914\ (-i_{fd} + id + iq + n)^2 + 0.5\ (-i_{fd} + id + iq + n + \delta + 0.029\ E_{fd})^2$

Which is positive definite for all values of states.

The derivative of V function is

$$\dot{V} = (6.721\, i_{fd} - 4.729\, id - 3.02\, iq - 2.82\, n - \delta - E_{fd})$$

$$(-0.40013\, i_{fd} - 22.233\, id + 369.7\, iq - 290.44 \sin\delta + 369.7\, iq\, n + 0.711\, E_{fd})$$

$$+ (-4.729\, i_{fd} + 4.729\, id + 3.02\, i_q + 2.82\, n + \delta + E_{fd})$$

$$(-0.15191\, i_{fd} - 27.376\, id + 455.22\, iq - 368.0 \sin\delta + 455.22\, i_q n + 0.27\, E_{fd})$$

$$+ (-3.02\, i_{fd} + 3.02\, id + 3.02\, iq + 3.82\, n + \delta + E_{fd})$$

$$(171.3775\, i_{fd} - 451.406\, id - 22.672\, iq - 305.0 \cos\delta + 171.3775\, i_{fd}\, n - 451.4\, i_d n)$$

$$+ (-2.82\, i_{fd} + 2.82\, id + 2.82\, iq + 2.82\, n + \delta + E_{fd})$$

$$(-0.0939\, i_{fd} iq + 0.0406\, id\, iq + 0.1532)$$

$$+ (-i_{fd} + id + iq + n + \delta + E_{fd})(377.0\, n)$$

$$+ (-i_{fd} + id + i_q + n + \delta + 0.029\, E_{fd})$$

$$(-1600.0\, n\, kr - 80.0\, E_{fd} - 84.2\, kr + 80.\, e_t\, kr + 111.2)$$

Which is negative definite with the region specified. So the system considered with the Kr = -200.0 is stable within this region.

5.6 Summary of Results

One factor of interest in stability study is the effect of voltage regulator gain on system stability. The regulator gain was varied from -1.0 to 1000.0 for both linearized and nonlinear model with voltage regulator and stabilizing signal. The results are summarized in the following tables.

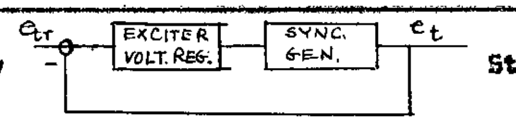
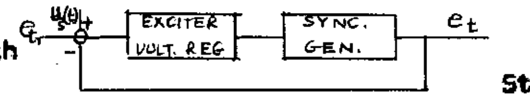
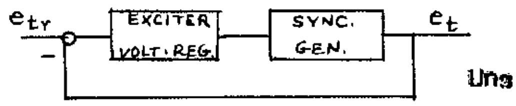
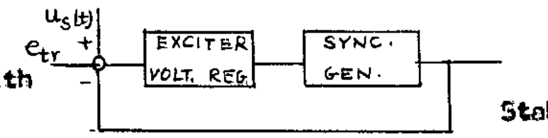
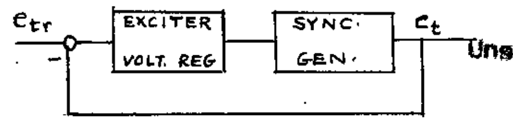
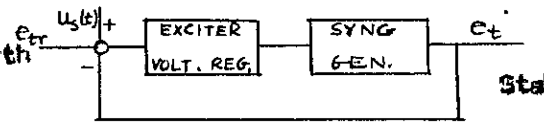
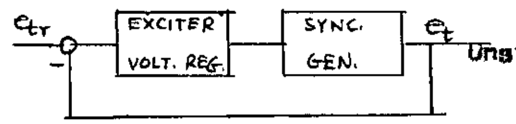
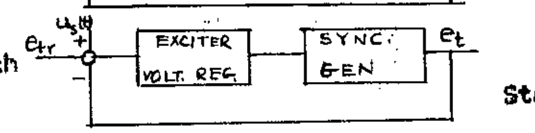
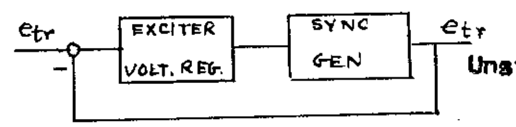
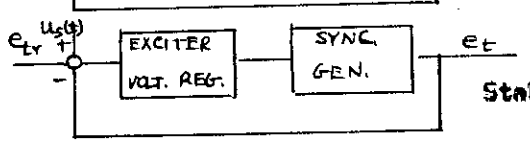
CASE	REGULATOR GAIN	CONTROL SIGNAL	SYSTEM CONFIGURATION	STABILITY
1	-2.0	Voltage Regulator only		Stable
2	-2.0	Voltage Regulator with stabilizing signal		Stable
3	-3.8	Voltage regulator only		Unstable
4	-3.8	Voltage regulator with stabilizing signal		Stable
5	-10.0	Voltage regulator only		Unstable
6	-10.0	Voltage regulator with stabilizing signal		Stable
7	-100.0	Voltage regulator only		Unstable
8	-100.0	Voltage regulator with stabilizing signal		Stable
9	-1000.0	Voltage regulator only		Unstable
10	-1000.0	Voltage regulator with stabilizing signal		Stable

Table 5.1 Linearized Model.

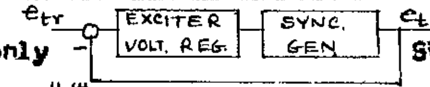
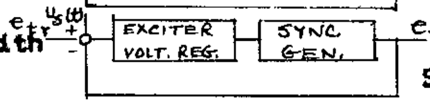
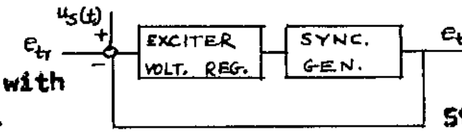
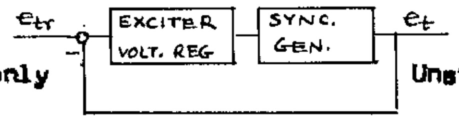
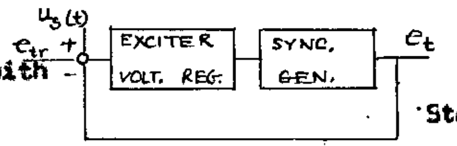
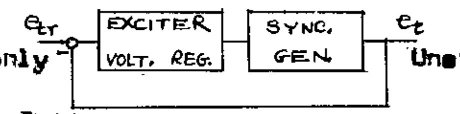
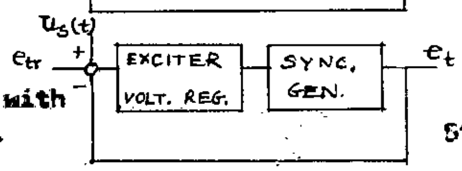
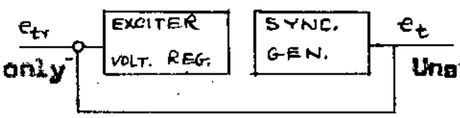
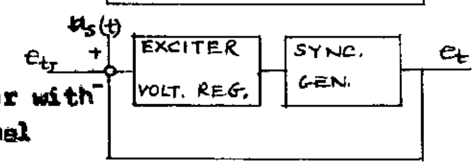
CASE	REGULATOR GAIN	CONTROL SIGNAL	SYSTEM CONFIGURATION	STABILITY
1	-2.0	Voltage regulator only		Stable
2	-2.0	Voltage regulator with Stabilizing signal		Stable
3	-4.0	Voltage regulator only		Unstable
4	-4.0	Voltage regulator with stabilizing signal		Stable
5	-10.0	Voltage regulator only		Unstable
6	-10.0	Voltage regulator with stabilizing signal		Stable
7	-200.0	Voltage regulator only		Unstable
8	-200.0	Voltage regulator with stabilizing signal		Stable
9	-1000.0	Voltage regulator only		Unstable
10	-1000.0	Voltage regulator with stabilizing signal		Stable

Table 5.2 Nonlinear Model

CHAPTER -6

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

6.1 Conclusions

The second method of a Liapunov was used to find the stability of a synchronous generator infinite bus power system equipped with solid state excitation system and having provision for auxiliary stabilizing signal. Both nonlinear and linearized model of the original system equations were considered. The linearized system is valid for only small perturbation about the equilibrium state. Though the existance of a positive definite P matrix gives necessary and sufficient condition for the stability of the equilibrium point, the linearized system gives only stability in the small because of the small perturbation condition.

A variable gradient method originally suggested by Schultz and Gibson was used to find a Liapunov function for the nonlinear system. This is an iterative method, the number of searchs for the parameters depending very much on guess work satisfying , ~~of course,~~ certain constraints. A set of computer routines were developed to give an efficient convergence.

Use of high gain static excitation system, through very much desirable from performance viewpoint, is detrimental to stability. This is taken care by using an additional stabilizing signal. This stabilizing signal though often a function of rotor velocity and acceleration, only that derived from the rotor velocity was utilized in this work, because that can be easily accomodated in the Liapunov problem.

A factor of interest in this study was the effect of the regulator gain on system stability. It was observed with both linear and nonlinear models that the system was stable for a gain (magnitude) of upto 4. Any further increase resulted in an unstable situation. Use of the stabilizing signal, however, increased the domain of attraction keeping the system stable for significantly large value of exciter gain.

The second method can be used as an on-line method for finding stability of a system. The fictious energy function V and its derivative are expressed directly as functions of system states which are often measurable. A small analog or digital computer may be quite sufficient for this purpose. The stability of the system can be determined by testing V function and its derivative for a particular value of states by this online computing device. For unstable condition this proper action can be taken by this device to remove the machine from the rest of the system or any other supervisory control may be introduced to stabilize it.

6.2 Suggestion for further research.

In this thesis only single machine infinite bus system was considered. The method can be extended to a two machine and even multimachine system including governer and primemover dynamics and effect of the damper windings.

A stabilizing signal derived from rotor velocity, acceleration and other quantities may improve domain of attraction. This may give rise to the bounded input and bounded output stability problems. The effect of such stabilizing signal on system stability could be investigated.

Further investigation to develop more efficient method of generating the Liapunov function for the nonlinear model should be made. Better ways of finding Liapunov function which will provide larger or even the largest domain of attraction also has to be considered. The generation of the Zubov function to give the largest stability region may also be considered.

APPENDIX -1

The date for the machine was taken from Roy[12]. The system configuration is given in figure 2.1.

Machine rating 46 MVA. 13.8 KV.

X_{efd}	.562		
X_d	.6803	X_{tie}	= .7/Section.
X_q	.4363	r_{tie}	= .07/section.
r_{fd}	.000508	X_{transf}	= .1
X_{ffd}	.692	X_e	= .8
H	3	R_e	= .07
R	.00435	r	= .8
		k_r	= 4.5 to 1000

The operating point for the linear model (under excited) are

i_{fdo}	= 1.77935	δ_o	= 60^o
i_{do}	= .30167	V	= 1
i_{qo}	= .718637	P_o	= .662
e_{to}	= .85077	Q_o	= .0147
E_{fd}	= 1		

The operating points for nonlinear model (normally excited) are

i_{fdo}	= 2.473309	δ_o	= 60^o
i_{do}	= .564341	V	= 1
i_{qo}	= .734433	P_o	= .916
e_{to}	= 1.05208	Q_o	= .3324
E_{fd}	= 1.39		

APPENDIX - 2

1. Program for P(I, J), Linearized model without the voltage regulator.
2. Program for P(I,J), Linearized model with the voltage gregulator.
3. Program for Sylvester's Criterion for positive defineteness.
4. Generalized program for linearized model without voltage regulator.

5a. Generalized program for linearized model with stabilizing signal.

5b. Generalized program for linearized model with voltage regulator.

```
C          JOB    NO UET91812
C          COMPUTER USED  IBM 360
C          BY MD. MIZANUR RAHMAN
C          PROGRAM No.1
C          PROGRAM FOR P(I,J)
C          LINEARIZED MODEL EXCLUDING VOLTAGE REGULATOR ACTION
           ACTION DIMENSION C(30,30),D(30,30),P(15,15)
    101    READ(1,5) N,M,L
           IF(N) 99,99,100
    100    WRITE (3,501)
           WRITE (3,302) N,M,L
           READ ( 1,509)(D(I,J), J=1,M),I=1,N)
           READ (1,502)((C(I,J), J=1,N),I=1,N)
           WRITE ( 3,501 )
           WRITE (3,503)((C(I,J),J=1,N),I=1,N)
           WRITE (3,504)
           WRITE (3,510)((D(I,J),J=1,M),I=1,N)
           CALL SOBLE ( C,N,D,M,DT )
           WRITE (3,505)
           WRITE(3,511)((D(I,J),J=1,M),I=1,N)
           P(1,1) = D (1,1)
           P(1,2) = D (2,1)
           P(1,3) = D (3,1)
           P(1,4) = D (4,1)
           P(1,5) = D (5,1)
           P(2,2) = D (6,1)
           P(2,3) = D (7,1)
           P(2,4) = D (8,1)
           P(2,5) = D (9,1)
           P(3,3) = D (10,1)
           P(3,4) = D (11,1)
           P(3,5) = D (12,1)
           P(4,4) = D (13,1)
           P(4,5) = D (14,1)
           P(5,5) = D (15,1)
           P(2,1) = P (1,2)
           P(3,1) = P(1,3)
           P(4,1) = P(1,4)
           P(5,1) = P(1,5)
           P(3,2) = P(2,3)
           P(4,2) = P(2,4)
           P(5,2) = P(2,5)
           P(4,3) = P(3,4)
           P(5,3) = P(3,5)
           P(5,4) = P(4,5)
           WRITE (3,15)
           WRITE(3,8)((P(I,J),J=1,L),I = 1,L)
           GO TO 101
      5 FORMA T (3I2)
      8 FORMA T (4x,5F20.6)
     15 FORMA T (1H0,14HMATRI x P(I,J)//)
    301 FORMA T (1H0,35H VALUES OF N, M & L RESPECT IVELY//)
    302 FORMA T (3I4)
    509 FORMA T (15F 4.1)
```

```
501   FORMA T (1H0,16HGIVEN MATRIX  C//)
502   FORMA T (5F10.5)
503   FORMA T (4X,5F15.5)
504   FORMA T(1H0,16HGIVEN MATRIX D//)
510   FORMA T (2X,15F6.1)
505   FORMA T (1H0,30H SOLUTION OF GIVEN EQUATIONS //)
511   FORMA T (55X, E14.8)
 99   STOP
      END
```

```
C         JOB NO UB T 91812
C         COMPUTER USED IBM 360
C         BY MD. MIZANUR RAHMAN
C         PROGRAM NO 2
C         PROGRAM FOR P(I,J)
C         LINEARIZED MODEL INCLUDING VOLTAGE REGULATOR ACTION
          DIMENSION C(30,30),D(30,30),RK(15),Q(30,30)R(30,30),
          P(30,30)
          READ (1,5)N,M,L
          WRITE (3,501)
          WRITE (3,502)N,M,L
          READ(1,602)((C(I,J),J=1,N),I=1,N)
          WRITE (3,501)
          WRITE (3,503)((C(I,J),J=1,N),I=1,N)
          READ (1,502)(RK(K),K =1,5)
          WRITE (3,520)
          WRITE (3,502)(RK(K),K=1,5)
          READ (1,509)((D(I,J),J=1,M)I=1,N)
          WRITE (3,504)
          WRITE (3,510)((D(I,J),J=1,M),I=1,N)
          DO  800 I = 1,N
          DO  800 J = 1,N
      800 Q (I,J)=C(I,J)
          DO  802 I = 1,N
          DO 802  J = 1, M
      802 R(I,J) = D(I,J )
          DO  * 99K = 1,5
          DO 801 I =1,N
          DO 801 J = 1,N
      801 C(I,J)=Q(I,J)
          DO 803 I=1, N
          DO 803 J = 1,M
      803 D(I,J)=R(I,J)
          C(1,6)=55.5966*RK(K)
          C(2,6)=-24.5149*RK(K)
          C(2,11)=27.7983*RK(K)
          C(3,6)=2.52*RK(K)
          C(3,15)=27.7983*RK(K)
          C(4,6)=-1600.0*RK(K)
          C(4,18)= 27.7983*RK(K)
          C(5,6)=-22.63*RK(K)
          C(5,20)=27.7983*RK(K)
          C(6,6)=-80.40013 +.003*RK(K)
          C(6,21)=27.7983)*RK(K)
          C(7,11)=-24.5149*RK(K)
          C(8,11)=2.52*RK(K)
          C(8,15)=-12.2572*RK(K)
          C(9,11)=1600.0*RK(K)
          C(9,18)=-12.2572*RK(K)
          C(10,11)=-22.63*RK(K)
          C(10,20)=-12.2572*RK(K)
          C(11,11)=-107.376+.003*RK(K)
          C(11,21)=-12.2572*RK(K)
          C(12,15)=5.042*RK(K)
```

```
C(13,15) = -1600.0*RK(K)
C(13,18)=2. 52*RK(K)
C(14,15)= -22.63*RK(K)
C(14,20)=2. 52*RK(K)
C(15,15) = -102.672 + .003*RK(K)
C(15,21) =2.52*RK(K)
C(16,18) = -3200.0*RK(K)
C(17,18) = -22.63*RK(K) C(17,20) = -1600.0*RK(K)
C(18,18) =- 80.0 + .003*RK(K)
C(18,21) =-1600.0*RK(K)
C(19,20) = -45.06 *RK(K)
C(20,20) = -80.0+.003*RK(K)
C(20,21) = -22.63*RK(K)
C(21,21) = -160.0+.006*RK(K)
WRI TE(3,521)
WR I TE(3,503)((C  (I,J), J=1, N), I=1,N)
WRI TE(3, 504)
WR I TE(3,510) (I,J), J=1,M),I =I,N)
CALL COSLE(C,N,D,M,DT)
WR I TE (3, 5051)
WR I TE (3, 511) (D (I,J), J=1, M), I=1,N)
P(1,1) =D(1,1)
P(1,2) =D (2,1)
P(1,3) =D(3,1)
P(1,4) =D(4,1)
P(1,5)=D(5,1)
P(1,6) =D(6,1)
P(2,2)=D(7,1)
P(2,3) =D(8,1))
P(2,4)=D(9,1)
P(2,5)=D(10,1)
P(2,6)=D(11,1)
P(3,3)=D(12,1)
P(3,4)=(13,1)
P(3,5)=D(14,1)
P(3,6)=D(15,1)
P(4,4)=D(16,1)
P(4,5)=D(17,1)
P(4,6)=D(18,1)
P(5,5)=D(19,1)
P(5,6)=D(20,1)
P(6,6)=D(21,1)
P(2,1) =P(1,2)
P(3,1)=P(1,3)
P(4,1)=P(1,4)
P(5,1)=P(1,5)
P(6,1)=P(1,6)
P(3,2) =P(2,3)
P(4,2)=P(2,4)
P(5,2)=P(2,5)
P(6,2)=P(2,6)
P(4,3)=P(3,4)
P(5,3) =P (3,5)
```

```
      P(6,3)=P(3,6)
      P(5,4) = P(4,5)
      P(6,4) = P(4,6)
      P(6,5) = P(5,6)
      WRITE (3,15)
      WRITE (3,8)((P(I,J),J = 1,L),I=1,L)
   99 CONTINUE
    5 FORMA T (3I2)
    8 FORMA T (4x,6F20.12)
   15 FORMA T (1HO,14HMATRIX P(I,J)//)
  301 FORMAT (1HO,35H VALUES OF N,M & L RESPECTIVELY//)
  302 FORMA T (3I4)
  501 FORMA T ( HO,16H GIVEN MATRIX C//)
  502 FORMA T (5F10.5)
  503 FORMA T (4x,7F 15.4)
  504 FORMA T (1HO,16HGIVEN MATRIX D//)
  505 FORMA T (1HO,30H SOLUTION OF GIVEN EQUATIONS//)
  509 FORMA T (15F 4.1 )
  510 FORMA T (2x, 21F6.1 )
  511 FORMA T (35x, E14.8)
  520 FORMA T (1HO, 36H GAIN OF THE REGULATOR CONSIDERED //)
  521 FORMA T (1HO, 40HGIVEN MATRIX C WITH GAIN CONSIDERED//)
  602 FORMA T (7F10.4)
      STOP
      END
```

```
C         JOB NO UE T 91812
C         COMPUTER  USED IBM 360
C         BY MD. MIZANUR RAHMAN
C         PROGRAM NO2 3
C         PROGRAM FOR SYLVESTERS CRITERION FOR POSITIVE DEFINITENESS
          DIMENSION C(30,30)D(30,30),Q(30,30),DTT(15)
          COMMON IPVOT,INDEX,PIVOT
    602   READ (1,5)(N,M)
          IF (N) 600,600,601
    601   WRITE (3,301)
          WRITE (3,302)(N,M)
          READ (1,502)((C(I,J),J=1,N),I=1,N)
          WRITE (3,501)
          WRITE (3,503)((C(I,J),J=1,N),I=1,N)
          DO 610 I = 1, N
          DO 610 J = 1,N
    610   Q(I,J)=C(I,J)
          DO 612 N = 2,6
          DO 613 I = 1, N
          DO 613 J =I, N
    613   C(I,J) Q(I,J )
          DO 614 I = 1, N
          DO 614 J = 1, M
    614   D(I,J)=1.0
          WRITE (3,701)
          WRITE (3,503)((C(I,J),J=1,N),I=1,N)
          WRITE (3,700)N
          CALL SOSLE (C,N,D, M,DET)
          DET(N) = DET
          WRITE (3,507)DET
    612   CONTINUE
          IF(N-6) 12,80,600
     80   IF(DTT(6))11,11,12
     12   IF(DTT(5)) 11,11,13
     13   IF(DTT(4)) 11,11,14
     14   IF(DTT(3)) 11,11,15
     15   IF(DTT(2)) 11,11,16
     16   IF (C(I,1)) 11,11,10
     10   WRITE (3,34)
          GO TO 602
     11   WRITE (3,35)
          GO TO 602
      5   FORMAT (2I2 )
     34   FORMAT(1H0, 93HMATRIX IS PROSITIVE DEFINITE REAL
          SYMMETRIC ISO THE SYSTEM CONSIDERRD IS STABLE/////)
     35   FORMAT(1H0, 98H MATRIX P IS NOT POSITIVE DEFINITE REAL
          SYMMETRIC  //// )
    301 FORMAT (35H VALUES OF N AND M RESPECTIVELY ///)
    302 FORMAT 2I4
    501 FORMAT XH (1H0, 16H GIVEN MATRIX C// )
    502 FORMAT (5E15.8)
    503 FORMAT (4X, 5E20.8)
```

```
510  FORMAT (4x, 7F10.5)
507  FORMAT (1HO 22H VALUE OF DETERMINATNT, E20 .8)
7008 FORMAT (1HO 27 HORDER OF THE DETERMINENT,I4)
701  FORMAT (1HO 20 H THE DETERMINENT IS ///)
600  STOP
     END
```

```
C       JOB NO UET 91812
C       COMPUTER USED IBM 360
C       BY MD. MIZANUR RAHMAN
C       PROGRAM NO 4
C       GENERALIZED PROGRAM
C       LINEARIZED MODEL EXCLUDING VOLTAGE REGULATOR ACTION
C       TO FIND THE MATRIX P(I,J)
        DIMENSION C(# 40,40), D(40,40),A(15,15) ,F(15,15),
        G(15,15),H(15,15),1AT(15,15),P(15,15),DTT(15)
        READ (1,1)N,M,L
        READ (1,2)((C(I,J),J=1,N) I=1,N)
        READ(1,3)((D(I,J),J=1,M),I=1,N)
        READ (1,2( (( A(I,J),J = 1, L),I =1,L )
        DO 4 I = 1,L
        DO 4 J = 1,L
      4 AT (J,I)=A (I,J)
        WRITE (3,5)
        WRITE (3,6)N,M,L
        WRITE (3,7)
        WRITE (3,8)((C(I,J)vJ = 1,N),I=1,N )
        WRITE (3,9)
        WRITE (3,10)((D(I,J),J=1,M),I=1,N)
        WRITE(3,50)
        WRITE (3,8)((A(I,J), J=1,L),I=1,L )
        WRITE (3,11)
        WRITE (3,8)((AT(I,J),J =1,L)I=1,L)
        CALL SOSLE(C,N,D,M,DT)
        WRITE (3,12)
        WRITE (3,14)((D(I,J),J = 1,M),I=1,N)
        P(I,1)=D(1,1)
        P(1,2)=D(2,1)
        P(1,3)=D(3,1)
        P(1,4)=D(4,1)
        P(1,5)=D(5,1)
        P(2,2)=D(6.1)
        P(2,3(=D(7,1 )
        P(2,4)=D(8v1)
        P(2 5)=D(9,1)
        P(3;3)=D(10,1)
        P(3,4)=D(11,1)
        P(3,5)=D(12,1)
        P(4,4)=D(13,1)
        P(4,5)=D(14,1)
        P(5,5)=D(15,1 )
        P(2,1)=P(1,2=)
        P(3,1)=P(1,3 )
        P(4,1)=P(1,4)
        P(5,1)=P(1,5)
        P(3,2)=P(2,3)
        P(4,2)=P(2,4)
        P(5,2)=P2,5)
        P(4,3)=P(3,4)
        P(5,3)=P(3,5)
        P(5,4)=P(4,5)
```

```
      WRITE (3,15)
      WRITE (3,8)((P(I,J),J = 1,L),I=1,L)
C     TO FIND AT*P+P*A
      DO 16 I = 1, L
      DO 16 J = 1, L
      F(I,J)= 0.0
      DO 16 I1 = 1,L
   16 F(I,J)=F(I,J) + AT(I,I1)*P(I1,J )
      DO 17 I2 = 1,L
      DO 17 J2 = 1, L
      G (I2, J2)= 0.0
      DO 17 I3= 1,L
   17 G(I2,J2)=G(I2,J2)+P(I2,I3)*A(I3,J2)
      DO 18 I3 = I,L
      DO 18 J3 = 1,L
   18 H(I3,J3)=F(I3,J3)+ G(I3,J3)
      WRITE (3,19)
      WRITE (3,20)((H(I,J),J=1,L),I = 1,L )
C     TEST OF MATRIX P(I,J )
   26 DO 21 I =1,L
      DO 21 J = 1,L
   21 C(I,J )=P(I,J )
      DO 22 I = 1, L
      DO 22 J =1, L
   22 D( I J)= 0.0
      WRITE (3,7)
      WRITE (3,8)((C ( I, J ),J = 1, L ) ,I = 1,L )
      CALL SOSLE (C,L,D,M,DT)
      WRITE (3,23)L
      DTT (L) = DT
      WRITE (3, 24) DT
      L = L - 1
      IF(R L -1) 85,85,26
   85 IF(DTT(5)) 40,40,81
   81 IF(DTT(4)) 40,40,82
   82 IF(DTT(3)) 40,40,83
   83 IF(DTT(2)) 40, 40,42
   42 IF(P(1,1)) 40,40,43
   43 WRITE (3,34)
      GO TO 60
   40 WRITE (3,35)
    1 FORMAT (22x 3I2)
    2 FORMAT (5F10.5)
    3 FORMAT (15F 4.1)
    5 FORMAT (1H0, 17H VALUES OF N,M,L//)
    6 FORMAT (4 x , 5I4)
    7 FORMAT (1H0, 14H MATRIX C (1,J)//)
    8 FORMAT (4 x, 5F20.6)
    9 FORMAT (1H0, 14H MATRIX  D(I,J)//)
   10 FORMAT (4x, 15F8.1)
   11 FORMAT (1 H0, 15H MATRIX AT (I,J)//)
   12 FORMAT (1H0,30H SOLUTION OF GIVEN EQUATIONS//)
```

```
13 FORMAT (1H0,22H REGULATOR GAIN RK(K)//)
14 FORMAT(35x, E14.8/)
15 FORMAT(1H0 14H MATRIX P(I,J)//)
19 FORMAT (1H0,28HRESULT OF MATRIX AT*P+P*A//)
20 FORMAT (4x, 5F10.1)
23 FORMAT ( 1H0, 31H ORDER OF THE DETERMINENT IS,I3)
24 FORMAT(1H0, 31H VALUE OF THE DETERMINENT IS E15.8)
34 FORMAT (1H0 91H MATRIX P IS POSITIVE DEFINITE REAL
   SYMMETRIC ISO THE SYSTEM CONSIDERED IS STABLE//////)
35 FORMAT(1H0,100H MATRIX P IS NOT POSITIVE DEFINITE
   REAL SYMMETRIC SO THE SYSTEM CONSIDERED IS UNSTABLE
   ////// )
50 FORMAT (1H0,14H MATRIX A(I,J)//)
60 STOP
   END
```

```
C      JOB NO UBT 91812
C      COMPUTER USED IBM 360
C      BY MD. MIZANUR RAHMAN
C      PROGRAM NO 5(a)
C      GENERALIZED PROGRAM
C      LINEARIZED MODEL INCLUDING VOLTAGE REGULATOR AND
       STABILIZING SIGNAL
C      TO FIND THE MATRIX P(I,J)
       DIMENSION C(40,40),D(40,40),A(15,15),F(15,15),
       G(15,15), H(15,15),1 AT(15,15), P(15,15),RK(15),
       Q(30,30), R(30,30),DTT (15)
       READ (1,1)N,M,L
       XL=L
       READ (1,502)(RK(K),K =1,10)
       WRITE (3,520)
       WRITE (3,502)(RK(K),K =1,10)
       READ (1,509)((D(I,J),J = 1,M),I = 1,N)
       WRITE (3,504)
       WRITE (3,510)((D(I,J),J=1,M),I = 1, N )
       READ(1,2)((A(1,J),J = 1, L), 1=1, L )
       DO 4 I = 1, L
       DO 4 J = 1, L
    4  AT(J,I )= A(1,J)
       WRITE (3,5)
       WRITE (3,6)N,M,L
       WRITE (3,50)
       WRITE (3,8)((A(I,J),J = 1,I=1,L)
       WRITE (3,11)
       WRITE (3,8)(& AT(I,J),J = 1,L)yI=1,L)
       DO 900 I = 1,N
       DO 900 J = 1,N
  900  C(I,J)=0.0
       C(I,1)=2.0 *A(1,1)
       C(1,2)=2.0*A(2,1)
       C(1,3)=2.0*A(3,1)
       C(1,4)=2.0*A(4,1)
       C(2,1)=A(1,2)
       C(2,2)=A(1,1)+ A(2,2)
       C(2,3)=A(3,2)
       C(2,4)=A(4,2)
       C(2,7)=A(2,1)
       C(2,8)=A(3,1)
       C(2,9)=A(4,1)
       C(3,1)=A(1,3)
       C(3,2)=A(2,3)
       C(3,3)=A(1,1)+A(3,3)
       C(3,4)=A(4,3)
       C(3,8)=A(2,1)
       C$3,12)=A(3,1)
       C(3,13)=A(4,1)
       C(4.4)=A(1,1)
       C(4,5)=A(5.4)
       C(4.9)=A(2,1)
       C(4,13)=A(3,1)
       C(4,16)A(4,1)
       C(5,1)=A (1,5)
```

```
C(5,2) = A(2,5)
C(5,3) = A(5,3)
C(5,3) = A(3,5)
C(5,5) = A(1,1)
C(5,10)= A(2,1)
C(5,14)= A(3,1)
C(5,17)= A(4,1)
C(6,1) = A(1,6)
C(6,2) = A(2,6)
C(6,11)= A(2,1)
C(6,15)= A(3,1)
C(6,18)= A(4,1)
C(7,2) = * 2.0*A(1,2)
C(7,7) = 2.0*A(2,2)
C(7,8) = 2.0*A(3,2)
C(7,9) = 2.0*A(4,2)
C(8,2) = A ( 1,3)
C(8,3) = A(* 1,2)
C(8,7) = A(2,3)
C(8,8) = A(2,2) + A(3,3 )
C(8,9) = A(4,3)
C(8,12)= A(3,2)
C(8,15)= A (4,2)
C(9,4) = A(1,2)
C(9,9 )= A(2,2)
C(9,10)=A (5,4)
C(9,13)= A(3,2)
C(9,16)= A(4,2)
C(10,2)= A(1,5)
C(10,5)= A(1,2 )
C(10,7)= A(2,5)
C(10,8)= A 3,5)
C(10,10)=A(2,2)
C(10,14)=A(3,2)
C(10,17)=A(4,2)
C(11,2) =A(1,6)
C(11,6) =A(1,2)
C(11,7) =A(2,6)
C(11,15)=A(3,2)
C(11,18) =A(4,2)
C(12,3) =2.0*A(1,3)
C(12,8) =2.0*A(2,3)
C(12,12)=2.0*A(3,3)
C(12,13)=2.0*A(4,3)
C(13,4) =A(1,3)
C(13,9) = A(2,3)
C(13,13))=A(3,3)
C(13,14)=A(5,4)
C(13,16) = A(4,3)
C(14,3) = A(1,5)
C(14,5)=(A(1,3)
C(14,8)=A(2,5)
C(14,10)=A(2,3)
C(14,12)= A(3,* 5)
C(14,14) =A(3,3)
```

```
      C(14,17) = A(4,3)
      C(15,3)  = A (1,6)
      C(15,6)  = A(1,3)
      C(15,8)  = A(2,6)
      C(15,11) = A(2,3)
      C(15,18) = A(4,3)
      C(16,17) = 2.0*A(5,4)
      C(17,4)  = A(1,5)
      C(17,9)  = A(2,5)
      C(17,13) = A(3,5)
      C(17,19) = A(5,4)
      C(18,4)  = A(1,6)
      C(18,9)  = A(2,6)
      C(18,20) = A(5,4)
      C(19,5)  = 2.0*A(1,5)
      C(19,10)=2.0*A(2,5)
      C( 19,14)= 2.0*A(3,5)
      C(20,5) = A ( 1,6)
      C(20,6) = A(1,5)
      C(20,10)= A(2,6)
      C(20,11)= A(2,5)
      C(20,15)= A(3,5)
      C(21,6) = 2.0*A(1,6)
      C(21,11)= 2.0*A(2,6)
      WRITE (3, 501)
      WRITE (3,503)((C(I,J), J=1,N),I=1, N)
      DO 800 I = 1, N
      DO 800 J = 1, N
  800 Q(I,J) = C(I,J )
      DO 802 I = 1, N
      DO 802 J = I, M
  802 R(I, J)= D (I, J )
      DO 99 K = 1,10
      L = IL
      DO 801 I = 1, N
      DO 801 J = 1, N
  801 C(I,J) = Q (I,J )
      DO 803 I = 1, N
      DO 803 J = 1, M
  803 D(I, J)= R (I,J )
      WRITE (3, 727 ) RK(K)
      A(6,1)=27.79831 *RK(K)
      A(6,A)=27.79831*RK(K)
      A(6,2)=-212.2572*RK(K)
      A(6,3)=2.521 *RK(K)
      A(6,5) = -22.53*RK(K)
      A(6,6) = 0.003*RK(K) -80.0
      WRITE (3,725)
      WRITE (3,8)((A(I,J),J=1,L), I = 1, L)
      DO 730 I = 1, L
  730 AT(I,6) = A(6,I )
      WRITE (3,726)
      WRITE (3,8)((AT(I,J),J=I,L),I=1, L)
      C(1,6) =2.0*A(6,1)
      C(2,6) =A(6,2)
```

```
C(2,11) = A (6,1)
C(3,6)  = A(6,3)
C(3,15) = A(6,1)
C(4,6)  = A(6,4)
C(4,10) = A(6,1)
C(5,6)  = A(6,5)
C(5,20) = A(6,1)
C(6,6)  = A(6,6)+ A(1,1)
C(6,21) = A(6,1)
C(7,11) = 2.0*A(6,21)
C(8,11) = A(6,3)
C(8,15 ) = A(6,2)
C(9,11)  = A(6,4)
C(9,18)  = A(6,2)
C(10,11)= A(6,5)
C(10,20) = A(6,2)
C(11,11) = A(6,6) + A( 2,2 )
C(11,21) = A(6,2 )
C(12,15) = 2.0*A(6, 3 )
C(13,15 )= A (6,4)
C(13,18) = A(6,3 )
C(14,15) = A(6,5)
C14,20 ) = A (6,3)
C(15,15) = A(6,6) + A (3,3 )
C(15,21) = A ( 6,3 )
C(16,18) = 2.0*A(6,4 )
C(17,18) = A(6,5)
C(17,20) = A (6,4)
C(18,18) = A(6,6)
C(18,21) = A (6,4 )
C(19,20)  = 2.0*A(6,5 )
C20,20)   = A (6,6)
C(20,21)  = A (6,5 )
C(21,21 ) = 2.0*A(6,6)
WRITE (3, 521 )
WRITE (3,503)((C(I,J), J = 1,N), I = 1,N )
WRITE (3, 504 )
WRITE (3, 510)((D(I,J), J = 1,M), I=1, N )
CALL SOSLE (C,N,D,M,DT )
WRITE (3,12 )
WRITE (3,14) ((D(I,J ), J = 1, M),I=1,N)
P(1,1 ) =  D(1,1)
P(1,2 ) = D (2,1 )
P(1,3 ) = D (3,1 )
P(1,4)  = D ( 4,1 )
P(1,5 ) = D ( 5,1)
P(1,6 ) = D ( 6,1 )
P(2,2 ) = D (7,1 )
P(2,3 ) = D 8,1 )
P(2,4 ) = D 9,1)
P(2,5)  = D (10,1 )
P(2,6)  = D(11,1 )
P(3,3 ) = D (12,1 )
P(3,4 ) = D(13,1 )
```

```
      P(3,5) = D (14,1 )
      P(3,6) = D (15,1 )
      P(4,4) = D (16,1 )
      P(4,5) = D (17,1 )
      P(4,6) = D (18,1 )
      P(5,5) = D (19,1 )
      P(5,6) = D (20,1))
      P(6,6) = D (21,1 )
      P(2,1) = P(1,2  )
      P(3,1) = P (1,3  )
      P(4,1 )= P (1,4)
      P(5,1 )= P( 1,5)
      P(6,1) = P(1,6 )
      P(3,2 )= P(2,3 )
      P(4,2) = P(2,4)
      P(5,2) = P(2,5)
      P(6,2) = P(2,6)
      P(4,3) = P(3,4)
      P(5,3) = P(3,5)
      P(6,3) = P(3,6)
      P(5,4)  = P(4,5 )
      P(6,4 )= P(4,6)
      P(6,5) = P( 5,6)
      WRITE (3,15)
      WRITE (3,8)((P(I, J), J=1, L) I=1, L)
      TO FIND AT*P+P*A
      DO 16 I = 1, L
      DO 16 J = 1, L
      F(I, J= = 0.0
      DO 16 I1=1,L
   16 F(I,J) = F(I,J)+AI(I,I1)*A(I1,J)
      DO 17 I2 = 1, L
      DO 17 J2=1, L
      G(I2, J2) = 0.0
      DO 17 I3=1, L
   17 G(I2, J2)=G(I2,J2) + P(I2,I3 ) *A(I3,J2 )
      DO 18 I3=1, L
      DO 18 J3=1, L
   18 H(I3, J3)=F(I3,J3) + G(I3,J3 )
      WRITE (3,19 )
      WRITE (3,20) ((H(I,J), J = 1, L),I=1, L)
C     TEST OF MATRIX P(I,J )
   26 DO 21 I=1,L
      DO 21 J =1,L
   21 C(I,J) = P(I,J)
      DO 22 I = 1,L
      DO 22 J = 1, L
   22 D(I,J) = 0.0
      WRITE (3, 901)
      WRITE (3,8)((C(I,J), J = 1, L), I = 1, L)
      CALL SOSLE (C,L,D,M,DT )
      WRITE (3, 23)L
      DTT (L) = DT
      WRITE (3, 24)DT
```

```
      L = L - 1
      IF (L -1) 85, 85, 86
 85   IF(DT(6)) 40,40, 80
 80   IF(DTT (5))40,40,81
 81   IF(DTT(4)) 40,40,82
 82   IF(DTT(3)) 40,40,83
 83   IF(DTT(2)) 40,40,42
 42   IF(P(1,1))40,40,43
 43   WRITE (3,54 )
      GO TO 99
 40   WRITE (3,35)
 99   CONTINUE
  1   FORMAT (3I2)
  2   FORMAT (6F10.4)
  5   FORMAT(1HO,17H VALUES OF N,M,L//)
  6   FORMAT (4x, 3I4)
  7   FORMAT (1HO, 14HMATRIX C(I,J)//)
  8   FORMAT(4x, 6F11.4)
  9   FORMAT (1HO,14H MATRIX D(I,J)//)
 10   FORMAT (4x, 15F8 8.1 )
 11   FORMAT (1HO, 15H MATRIX AT (I,J)//)
 12   FORMAT (1HO, 30H SOLUTION OF GIVEN EQUATIONS//)
 14   FORMAT (55x, E14.8/)
 15   FORMAT (1HO, 14H MATRIX P(1,J )//)
 19   FORMAT (1HO,28HRESULT OF MATRIX AT*P+P*A//)
 20   FORMAT (4x, 6F8.1)
 23   FORMAT(1HO, 31HORDER OF THE DETERMINENT IS, I5 )
 24   FORMAT (1HO,31HVALUE OF THE DETERMINENT IS,E15.8)
 34   FORMAT (1HO,93HMATRIX P IS POSITIVE DEFINITE REAL SYMMETRIC
     1SO xxxxx SO THE SYSTEM CONSIDERED IS STABLE//////)
 35   FORMAT (1HO,100H MATRIX P IS NOT POSITIVE DEFINITE REAL
      SYMMETRIC SO THE SYSTEM CONSIDERED IS UNSTABLE///////)
 50   FORMAT(1HO,14HMATRIX A(I,J)//)
501   FORMAT (1HO, 16HGIVEN MATRIX C//)
502   FORMAT(5F10.5)
503   FORMAT(4x, 7F15.4)
504   FORMAT(1HO,16HGIVEN MATRIX D//)
509   FORMAT(15F 4.1)
510   FORMAT(2X,21F6.1)
520   FORMAT(1HO, 36HGAIN OF THE REGULATOR CONSIDERED//)
521   FORMAT (1HO,40H GIVEN MATRIX C WITH GAIN CONSIDERED//)
602   FORMAT(7F10.4)
725   FORMAT (1HO, 38H MATRIX A(I,J ) WITH GAIN CONSIDERED//)
726   FORMAT(1HO, 39H MATRIX AT(I,J)WITH GAIN CONSIDERED///)
727   FORMAT(1HO,19 H REGULATOR GAINIS,F10.5)
901   FORMAT (1HO, 28H THE DETERMINENT CONSIDERED//)
 60   STOP
      END
```

```
C         JOB NO UBT 91812
C         COMPUTER USED IBM 360
C         BY MD. MIZANUR RAHMAN
C         PROGRAM NO 3(b)
C         GENERALIZED PROGRAM
C         LINEARIZED MODEL INCLUDING VOLTAGE REGULATOR ACTION
          ONLY
C         TO FIND THE MATRIX P(I,J)
          DIMENSION C(40,40),D(40,40),A(15,15),F(15,15),G(15,15),
          H(15,15),ZAT(15,15),P(15,15),RK(15),Q(30,30),R(30,30),
          DTT(15)
          READ (1,1)N,M,L
          IL = L
          READ(1,502)(RK(K),K = 1,10)
          WRITE (3,520)
          WRITE (3,502)(RK(K),K = 1,10)
          READ (1, 509)((D(I,J),J = 1,M),I =1,N )
          WRITE (3,504 )
          WRITE (3,510)((D(I,J),J =1,M),I =1,N)
          READ(1,2)((A(I,J),J =1,L),I=1,L )
          DO 4 I =1, L
          DO 4 J =1, L
        4 AT(J,I)=A(I,J)
          WRITE(3,5)
          WRITE (3,6,)N,M,L
          WRITE (3,50)
          WRITE (3,8)((A(I,J),J =1,L),I=1,L)
          WRITE (3,11)
          WRITE (3,8)((AT(I,J),J =1, L)I=1,L)
          DO 900 I =1, N
          DO 900 J =1,N
      900 C(I,J ) =0.0
          C(1,1)=2.0*A(1,1)
          C(1,2)=2.0*A(2,1)x
          C(1,3)=2.0*A(3,1)
          C(1,4)=2.0*A(4,1)
          C(2,1)=A(1,2 )
          C(2,2)=A(1,1)+A(2,2)
          C(2,3)=A(3,2)
          C(2,4)=A(4,2)
          C(2,7)=A(2,1)
          C(2,8)=A3,1)
          C(2,9)=A(4,1)
          C(3,1)=A(1,3)
          C(3,2)=A(2,3)
          C(3,3)=A(1,1)+A(3,3)
          C(3,4)=A(A(4,3)
          C(3,8)=A(2,1)
          C(3,12)=A(3,1)
          C(3,13)=A(x 4,1 )
          C(4,4)= A(1,1)
          C(4,5)= A(x 5,4 )
          C(4,9)=A (2,1 )
          C(4,13)=A(3,1)
          CCCCCxx
          C(4,16)= A(4, 1 )
          C(5,1) = A(1, 5)
```

```
C (5,2) = A (2,5)
C (5,3) = A (3,5)
C (5,5) = A (1,1)
C (5,10)= A (2,1)
C (5,14)= A(3,1)
C(5,17) = A(4,1 )
C(6,1)  = A(1,6)
C(6,2) = A (2,6)
C(6,11)= A (2,1 )
C(6,15)= A(3,1 )
C(6,18)= A(4,1 )
C(7,2) = * 2.0*A(1,2)
C(7,7) = 2.0*A(2,2)
C(7,8) = 2.0*A(3,2)
C(7,9) = 2.0*A(4,2)
C(8,2) = A(1,3 )
C(8,3) = A(1,2 )
C(8,7) = A(2,3)
C(8,8) = A(2,2) + A(3,3 )
C(8,9) = A (4,3 )
C(8,12)= A(3,2 )
C(8,13)= A(4,2)
C(9,4) = A(1,2 )
C(9,9) = A(2,2 )
C(9,10) = A(5,4 )
C(9,13) = A(3,2 )
C(9,16) = A(4,2 )
C(10,2) = A(1,5 )
C(10,5) = A(1,2 )
C(10,7,) =A(2,5)
C(10,8) )= A(3,5)
C(10,10) = A(2,2)
C(10,14) = A(3,2)
C(10,17) = A(4,2)
C(11,2) = A (1,6)
C(11,6) = A(1,2)
C(11,7* = A(2,6)
C(11,15) = A(3,2)
C(11,18) = A(42(2)
C(12,3 ) = 2.0*A(1,3 )
C(12,8 ) = 2.0*A(2,3)
C(12,12)= 2.0*A(3,3 )
C(12,13)= 2.0*A(4,3)
C(13,4)=A(1,* 3 )
C(13,9) = A(* 2,3 )
C(13,13) = A(3,3 )
C(13,14)= A(5,4)
C(13,16) = A(4,3 )
C(14,3) = A(1,5)
C14,5)  = A(1,5 )
C(14,8* = A(2,5 )
C(14,10)= A(2,3)
C(14,12)= A(3,5)
C(14,14) = A(3,3 )
```

```
      C(14,17) = A(4,3)
      C(15,5)  = A(1,6)
      C(15,6)  = A(1,5)
      C(15,8)  = A(2,6)
      C(15,11) = A(2,5)
      C(15,18) = A(4,3)
      C(16,17) = 2.0*A(5,4 )
      C(17,4)  = A(1,5)
      C(17,9)  = A(2,5)
      C(17,13) = A(5,5 )
      C(17,19) = A(5,4 )
      C(18,4 ) = A(1,6)
      C(18,9 ) = A(2,6)
      C(18,20) = A(5,4 )
      C(19,5)  = 2.0*A(1,5)
      C(19,10) = 2.0*A(2,5)
      C(19,14) = 2.0*A(5, 5)
      C(20,5)  = A(1,6)
      C(20,6)  = A(1,5)
      C(20,10) = A(2,6)
      C(20,11) = A(2,5 )
      C(20,15) = A(5,5)
      C(21,6)  = 2.0*A(1,6)
      C(21,11)=2.0*A(2,6)
      WRITE(3,501 )
      WRITE (3,503)((C(I,J),J=1, N),I=1,N)
      DO 800 I =1, N
      DO 800 J = 1, N
  800 Q(I, J) = C(I,J )
      DO 802 I =1, N
      DO 802 J =1, M
  802 R(I, J )= D(I,J)
      DO 99 K = 1, 10
      L = IL
      DO 801 I = 1, N
      DO 801 J = 1, N
  801 C(I,J)=Q(I,J )
      DO 803 I = 1, N
      DO 803 J =1, M
  803 D(I,J)=R(I, J)
      WRITE(3,727) RK(K)
      A(6,1)= 27.79831*RK(K)
      A(6,2)=-12.2572*RK(K)
      A(6,3)=2.521*RK(K)
      A(6,4)= -1600.0*RK(K)
      A(6,5)=-22.53*RK(K)
      A(6,6)=0.003*RK(K)-80.0
      WRITE (3,725)
      WRITE (3,8)((A(I,J), J=1, L), I =1, L)
      DO 730 I=1, L
  730 AT(I, 6)=A(6,1)
      WRITE(3,726)
      WRITE(3,8)((AT(I,J),J=1,L),I=1, L)
      C(1,6) = 2.0*A(6.1 )
```

```
C(2,6) = A(6,2 )
C(2,11)= A6, 1 )
C(3,6) = A(6,3)
C(3,15)= A(6,1)
C(4,6) = A(6,4)
C(4,18)= A(6,1)
C(5,6) = A(6,5)
C(5,20)= A(6,1)
C(6,6,)= A(6,6) + A(1,1)
C(6,21)= A(6,1)
C(8,11)= A(6,3)
C(8,15)= A(6,2)
C(9,11)= A(6,4)
C(9,18)= A(6,2)
C(10,11)=A(6,5)
C(10,20)=A(6,2)
C( 11,11)=A(6,6) + (2,2)
C(11,21)= A(6,2 )
C(12,15)= 2.0*A(6,3)
C(13,15)= A(6,4)
C(13,18)= A(6,3)
C(14,15)= A(6,5)
C(14,20)= A(6,3)
C(15,15)= A(6,6) + A(3,3)
C(15, 21)= A(6,3)
C(16,18)= 2.0*A(6,4)
C(17,18)= A(6,5)
C(17,20)= A(6,4)
C(18,18)= A(6,6)
C(18,21)= A(6,4)
C(19,20) = 2.0*A(6,5 )
C(20,20) = A(6,6)
C(20,21) = A(6,5)
C(21,21) = 2.0*A(6,6)
WRITE(3,521)
WRITE(3,503)((C(I,J),J=1,N),I=1,N)
WRITE (3,504)
WRITE (3,510)((D(I,J),J=1, I=1, N)
CALL SOSLE (C, N, D, M, DT)
W RITE(3, 12)
WRITE (3, 14)((D(I, J), J=1, M), I=1, N)
P(I, 1)= D(1,1)
P(1,2) = D(2,1)
P(1,3) = D(3,1)
P(1,4) = D(4,1)
P(1,5) = D(5,1)
P(1,6) = D(6,1)
P(2,2) = D(7,1)
P(2,3) = D(8,1)
P(2,4) = D(9,1)
R(2,5) = D(10,1)
P(2,6) = D(11,1)
P(3,3) = D(12,1)
```

```
      P(3,4) = D(13,1)
      P(3,5) = D(14,1)
      P(3,6) = D(15,1)
      P(4,4) = D(16,1 )
      P(4,5) = D(17,1)
      P(4,6) = D(18,1)
      P(5,5) = D(19,1)
      P(5,6) = D(20,1)
      P(6,6) = D(21,1)
      P(2,1) =PR(1,2)
      P(3,1) =PR(R 1,3 )
      P(4,1) = P(1,4)
      P(5,1) = P(1,5)
      P(6,1) = P(1,6)
      P(3,2) = P(2,3)
      P(4,2) = P(2,4)
      P(5,2) = P(2,5 )
      P(6,2) = P(2,6)
      P(4,3) = P(3,4)
      P(5,3) = P(3,5)
      P(6,3) = P(3,6)
      P(5,4) = P(4,5)
      P(6,4) = P(4,6)
      P(6,5) = P(5,6)
      WRITE (3,15)
      WRITE (3,8)((P(I,J),J=1, L),I=1,L)
C     TO FIND AT *P+P*A
      DO 16 I = 1,L
      DO 16 J= 1, L
      F(I,J)=0.0
      DO 16 I1=1,L
   16 F(I,J=F(I,J)+AT(I,I1)*P(I1,J)
      DO 17 I2=1,L
      DO 17 J2=1, L
      G(I2,J2)=0.0)
      DO 17 I3=1, L
   17 G(I2,J2)=G(I2,J2)+P(I2,I3)*A(I3,J2)
      DO 18 I3=1,L
      DO 18 J3=1,L
   18 H(I3,J3)=F(S I3,J3)+ G(I3,J3)
C     TEST OF MATRIX P(I,J)
   26 DO 21 I=1, L
      DO 21 J=1, L
   21 C(I,J) = P(I,J)
      DO 22 I =1, L
      DO 22 J =1,L
   22 D(I,J)=0.0
      WRITE (3,7)
      WRITE (3,8((C(I,J),J=1,L), I=1,L)
      CALL SOSLE (C,L,D,M, DT)
      WRITE (3, 23)L
      DTT(L) = DT
      WRITE (3,24)DT
      L=L-1
```

```
   IF(L-1) 85,85,26
 85IF(DTT(6)) 40,40, 80
80 IF (DTT(5))40,40, 81
81 IF(DTT(4)) 40,40,82
82 IF (DTT(* 3)) 40,40,83
83 IF (DTT(2)) 40,40,42
42 IF(P(1,1)) 40,40, 43
43 WRITE (3, 34 )
   TO TO 99
40 WRITE (3, 35)
99 CONTINUE
 1 FORMAT (3I2 )
 2 FORMAT(6F10.4)
 5 FORMAT(1HO,17H VALUES OF N,M,L//)
 6 FORMAT (4x, 3I4)
 7 FORMAT(1HO, 14HMATRIX C(I, J)//)
 8 FORMAT(4x, 6F11.4)
 9 FORMAT (1HO, 14HMATRIX D(I,J)//)
10 FORMAT (4x, 15F 8.1)
11 FORMAT(1HO, 15HMATRIX AT(I,J)//)
12 FORMAT (1HO, 50H SOLUTION OF GIVEN EQUATIONS//)
14 FORMAT(35x, E14.8/)
15 FORMAT (1HO, 14HMATRIX P(I,J)//)
16                    RESULT OF MATRIX AT*P+P*A//)
19 FORMAT(1HO, 26HXXXXXXXXXXXXXXXXXXXX
20 FORMAT (4x, 6F8.1)
23 FORMAT (1HO, 31H ORDER OF THE DETERMINENT IS, I3)
24 FORMAT(1HO, 31H VALUE OF THE DETERMINENT IS E15,8)
34 FORMAT(1HO, 93HMATRIX P IS POSITIVE DEFINITE REAL
   SYMMETRIC 1SO THE SYSTEM CONSIDERED IS STABLE//////)
35 FORMAT(1HO, 100HMATRIX P IS NOT POSITIVE DEFINITE REAL
   SYMMETRIC SO THE SYSTEM CONSIDERED IS UNSTABLE///////)
50 FORMAT(1HO,14H MATRIX A(I,J)//)
501 FORMAT (1HO, 16H GIVEN MATRIX C//)
502 FORMAT (15F10.5)
503 FORMAT (4x, 7F15.4)
504 FORMAT (1HO, 16H GIVEN MATRIX D//)
509 FORMAT (15F4.1)
510 FORMAT (2x, 21F6,1)
520 FORMAT(1HO, 36H GAIN OF THE REGULATOR CONSIDERED//)
521 FORMAT(XXXXXX (1HO,40HGIVEN MATRIX C WITH GAIN
    CONSIDERED//)
602 FORMAT (7F10.4)
725 FORMAT (1HO, 38HMATRIX A(I,J)WITH GAIN CONSIDERED)
726 FORMAT (1HO,39H MATRIX AT (I,J) WITH GAIN CONSIDRED///)
727 FORMAT(1HO, 19H REGULATOR GAIN IS F10.5)
 60 STOP
    END
```

```
      SUBROUTINE SOSLE(E,N,B,M,DT)
      FIMENSION E(40,40),B(40,40),MR(40)),KN(40,2),PVT(40)
500   DT = 1.0
      DO = 501 J = 1, N
501   MR(J) = 0
      DO 502 I =1, N
      I = 0.0
      DO 503 J =1,N
      IF(MR(J)-1) 504, 503,504
504   DO 505 K =1,N
      IF(MR(K)-1) 506, 505, 507
506   IF (ABS(T)-ABS(E(J,K))) 508,505,505
508   IR=J
      IC=K
      T=E (J,K)
505   CONTINUE
503   CONTINUE
      MR(IC)=MR(IC)+1
      IF(IR-IC) 509, 510, 509
509   DT=-DT
      DO 511 L =1, N
      T=E(IR,L)
      E(IR,L)=E(IC,L)
511   E(IC,L)=T
      IF(M) 510, 510, 512
512   DO 513 L=1, M
      T=B(IR,L)
      B(IR,L)=B(IC,L)
513   B(IC,L)=T
510   KN(I,1)=IR
      KN(I,2)=IC
      PVT(I)=E(IC,IC)
      DT=DT*PVT(I)
      E(IC,IC)=1.0
      DO 514 L =1, N
514   E(IC,L)=E(IC,L)/PVT(I)
      IF(M) 515,515,516
516   DO 517 L= 1, M
517   B(IC,L)=B(IC,L)/PVT(T)
515   DO 502 LI =1,N
      IF(LI-IC)518,502,518)
518   T=E(LI,IC)
      E(LI,IC)=0.0
      DO 519 L=1,N
519   E(LI,L)=E(LI,L)-E(IC,L)*T
     IF(M) 502, 502, 520
520   DO 521 L=1, M
521   B(LI,L)=B(LI,L)-B(IC,L)*T
502   CONTINUE
522   DO 523 I =1,N
      L=N-I+1
      IF(KN(L,1)-KN(L,2)) 524,523, 524
524   JR=KN (L,1)
      JC=KN(L,2)
```

```
      DO 525 K = 1, N
      T = B(K, JR )
      B(K, JR) = B(K,JC)
      B(K,JC) = T
  525 CONTINUE
  523 CONTINUE
  507 RETURN
      END
```

APPENDIX - 3

6. Program for nonlinear model without voltage regulator.

7. Program for nonlinear model model with voltage regulator and stabilizing signal.

8. (a) Subprogram -3 for voltage regulator.

   (b) Subprogram 04 for stabilizing signal.

8. Program for the test of a liepanov function.

```
C       JOB NO UET 91812
C       COMPUTER USED IBM 360
C       BY MD. MIZANUR RAHMAN
C       PROGRAM NO. 6
C       PROGRAM FOR NONLINEAR MODEL
C       EXCLUDING VOLTAGE REGULATOR ACTION
        DIMENSION A(15,15),S(15), V(15)
        READ (1,17)(S(I),I=1,5)
   15   READ (1,55)N
        IF(N) 56,56,50
   50   READ (1,17)((A(I,J),J=1,N),I=1,N)
        T=0.0
  100   WRITE (3,51)
        WRITE (3,17)((A(I,J),J=1,N),I=1,N)
        WRITE(3,80)
    2   CALL VOD S(S,A,N.V,VDOT )
        IF(S(1)-7.0)200,200,201)
  200   S(I) =S(1)+.05
        IF (VDOT) 2,2,201
  201   WRITE (3,112)(S(I),K=1,,N),VDOT
        Z1=VDOT
    4   S(I)=2.4753
    8   CALL VOD S(S,A.N,V,VDOT )
        IF(S(2)-2.5) 202,202,203
  202   S(2)= S(2)+.01
        IF (VDOT)8,8,203
  203   WRITE (3,112)(S(I),I=1,N),VDOT
        Z2=KR VDOT
    9   S(2)=0.5643
   10   CALL VOD S(S,A,N,V,VDOT )
        IF (S(n)-2.0) 204,204, 205
  204   S(3)=S(3)+.001
        IF (VDOT) 10,10, 205
  205   WRITE (3, 112)(SI),I=1,N),VDOT
        Z3=VDOT
   11   S(3)=0.7344
   12   CALL VOD S(S,A,N,V,VDOT)
        IF (S(4)-.06) 206, 206, 207
  206   S(4)=S(4=+.001
        IF (VDOT) 1x2 12,12,207
  207   WRITE (3,112)(S(I),I=1,N),VDOT
        Z4= VDOT
   13   S(4)= 0.0
   14   CALL VOD S(S,A,N,V,VDOT)
        IF(S(5)-3.0) 208,208,209
  208   S(5)=S(5)+.05
        IF(VDOT) 14,14,209
  209   208 WRITE (3,112)(S(I),I=1,N),VDOT
        Z5 = VDOT
        IF (ZI) 300,300,301
  301,  IF 'Z2) 300,300,302 302
  302   IF(Z3)  300,300,303
  303   IF (Z4) 1 300,300,303 304
  304   IF (Z5) 300 300, 305
```

```
300 WRITE (3,307)
    GO TO 309
305 WRITE (3,308)
309 X5 = 1.04716
    A (5 5,5) = A(5,5)-3.0
    T = T + 1.0
    IF(T-4.0) 100,100,64
 64 A(5,5)=-6.0
    GO TO 15
  7 FORMAT( 5x, 6E 10.4)
 17 FORMAT (5F10.5)
 51 FORMAT(1H1,5x, 18H VALUES OF A(I,J)//)
 55 FORMAT (I2)
 80 FORMAT(1H0, 10x, 35H FIRST UNSTABLE VALUES OF
    STATES//)
112 FORMAT(1H0, 74H FOR SUCH COMBINATION OF A(I,J)THE
    SYSTEM CONSIDERED IS UNSTABLE////)
308 FORMAT (1H0, 84H CHANGE THE COMBINATION OF A(I,J),
    TO OTHER SUIABLE VALUES FOR STABILITY /////)
 56 STOP
    END
```

```
C         SUBROUTINE VOD S(X,C,N,V,VDOT)
          SUBPROGRAM NO 2
          DIMENSION X (5), C(5,5),V(5)
          B11=C(1,1)*X(1)+C(1,2)*X(2)+C(1,3)*X(3)+C(1,4)*X(4)+C
          (1,5)*X(5)
          B22=C(2,1)*X(1)+C(2,2)*X(2,m)+ C(2,3)*X(3)+C(2,4)*X(4)+
          C(2,5)*X(5)
          B33=C(3,1)*X(1)+C(3,2)*X(2)+C(3,3)*X(3)+C(3,4)*X(4)+C
          (3,5)*X(5)
          B44=C(4,1)*X(1)+C(4,$ 2)*X(2)+C(4,3)*X(3)+C(4,4)*X(4)
          +C(4,5)*X(5)
          B55=C(5,1)*X(1)+C(5,2)*X(2)+C(5,3)*X(3)+C(5,4)*X(4)+C
          (5,5) *X(5)
          XDOT 1= -0.40013*X(1)-22.233*X(2)+369.7*X(3)-298.44*SIN(
          (X(5))1+ 369.7*X(3)*X(4)
          XDOT$ 2=-0.15191*X(1) -27.376*X(2)+455.22*X(3)-368.0*
          SIN(X(5)) 2+ 544.22* X(3)*X(4)
          XDOT 3= 171.3775*X(1)-451.406*X(2)-22.672*X(3)-305.0
          *COS (X(5)) + 3171.3775*X(1)*X(4)-451.406*X(2)*X(4)
          XDOT 4=-.0936*X(1)*X(3)+0.0405*X(2)*X(3)+0.1532
          XDOT 5=377.0*X(4)
          V(1) = B11*XDOT1
          V(2) = B22*XDOT2
          V(3)=  B33*XDOT3
          V(4) = B44*XDOT4
          V(5) = B55*XDOT5
          VDOT = V(1)+V(2)+V(3)+V(4)+V(5)
          RETURN
          END
```

```
C       JOB NO UBT 91812
C       COMPUTER USED IBM 360
C       BY MD. MIZANUR RAHMAN
C       PROGRAM No7
C       PROGRAM FOR NONLINEAR B MODEL
C       INCLUDING VOLTAGE REGULATOR ACTION AND STABILIZING
        SIGNAL
        DIMENSION A(15,15),S(15),& V(15),RK(15)
        READ (1,17)(S(1),I=1,6 6)
        READ (1,17)(RK(K), K=1, 6)
     15 READ (1,55)N
        IF(N) 56,56, 50
     50 READ (1,17)((A(I,J),J=1,N),I-1, N)
        DO 54 K = 1, 2
        T = 0.0
    100 WRITE (3,51)
        WRITE(3,17)((A(I,J)J=1,N),I=1,N)
        WRITE (3,80)
      2 CALL VOD S( RK, S, A, N, V, VDOT )
        IF (S(1)-7.0)200,200,201
    200 S(I)= S(1)+.05
        IF(VODT)2,2,201
    201 WRITE (3,112)(S(I),I=1,N),VDOT
      4 S(1)=2.4733
      8 CALL VOD S(RK,S,A, N, V, VDOT )
        IF (S(2)-2.5) 202,202, 203
    202 S(2)=S(2)+.01
        IF (VDOT)8,8, 203
    203 WRITE (3,112)(S(I),I=1, N)VDOT
      9 S(2) =0.5643
     10 CALL VOD S(RK, S,A,N,V,VDTO)
        IF (S(& 3)-2.0)204,204,205
    204 S(3)=S(3)+.001
        IF(VDOT) 10,10,205)
    205 WRITE (3,112)S(I),I=1,N),VDOT
     11 S(3)=0.7344
     12 CALL VOD S(RK., S,A,N,V,VDOT)
        IF (S($)-.06) 206, 206, x2 207
    206 S(4)= S(4)+.001
        IF (VDOT) 12, 12, 207
    207 WRITE (3, 112,(S(I),I=1,N),VDOT
     13 S(4)=0.0
     14 CALL VOD S(RK, S,A, N, V, VDOT )
        IF (S(5)-3.0) 208, 208, 209
    208 S(5)=S(5)+.05
        IF(VDOT )14,14,209
    209 WRITE (3,112)S(I), I=1, N), VDOT
     40 S(5) = 1.04714
    412 CALL VOD S(RK, S, A, N, V, VDOT)
        IF (S(6) -3.0) 42,xxxxxx 42,44.
     42 S(6)= (S(6)+.05

        IF (VDOT ) 41, 41, 44
     44WRITE (3, 112)IS( 1),I =1, N), VDOT

    309 X6= 1.39
        A (6)=A(6,6)-3.0
```

```
      T=T+1.0
      IF(T-4.0)100,64
   64 A(6,6)=-6.0
   54 CONTINUE
      GO TO 15
    7 FORMAT ((5X, 6E10.4)
   17 FORMAT (16F10.5)
   51 FORMAT(1H1, 5X, 18H VALUES OF A(I,J)//)
   55 FORMAT (I2)
   80 FORMAT (1H0,10X,35H FIRST UNSTABLE VALUES OF
     STATES //)
  112 FORMAT (6F14.5,E20.8/)
  507 FORMAT (1H0, 74H FOR SUCH COMBINATION OF A(I,J)
     THE SYSTEM CONSIDERED IS UNSTABLE ////)
  508 FORMAT (1H0, 84 CHANGE THE COMBINATION OF A(I,J)
     TO OTHER SUITABLE VALUES FOR STABILITY/////)
   56 STOP
      END
```

```
      SUBROUTINE VODS(RK,X,C,N,V,VDOT)
C     SUVPROGRAM NO3
      DIMENSION X (6) ,C(6,6), V(6),RK(15)
      B11=C(1,1)*X(1)+C(1,2)*X(2) +C(1,3)*X(3)+C(1,4)*X(4)+
         C(1,5)*X(5)1+C(1,6)6)*X(6)
      B22=C(2,1)*X(1)+C(2,2)*X(2)+C(2,3)*X(3)+C(2,4)*X(4)+
         C(2,5)*X(5)1+C(2,6)*X(6)
      B33=C(3,1)*X(1)+C(3,2)*X(2)+C(3,3)*X(3)+C(3,4)*X(4)
         +C(3,5)*X(5) 1+ C(3,6)*X(6)
      B44=C(4,1)*X(1)+C(4,2)*X(2)+C(4,3),*X(3)+C(4,4)*X(4)+C
         (4,5)*X(5)1+C(4,6)(*X(6)
      B55=C(5,1)*X(1)*C(5,2)*X(2)+C(5,3)*X(3)+C(5,4)*X(4)+C
         (5,5) *X(5) ++ C(5,6) *X(6)
      B66=C(6,1)*X(1)+C(6,2)*X(2) +C(6,3)*X(3)+C( 6, 4)
         *X(4)+C(6,5)*X( 5)1+C(6,6),*X(6)
      XDOT 1= -0.40013*X(1) -22.235*X(2)+369.7*X(3)- 298.
         44*SIN (X(3)) 1+ 369.7*X(3)*X(4)+.712*X(6)
      XDOT 2= -0.15191 *X(1)-27.376*X(2)+455. 22*X(3)-368.0
         *SIN (X()) 2+ 455.22*X(3)*X(4)+.271*X(6)
      XDOT 3=171.3775 *X(1)-451.406*X(2)-22.672*X(3)-305.0
         *COS (X(3)  3+  171.3775*X(1) *X(4)-451.406
         *X(2)*X(4)
      XDOT 4=-.0936*X(1)*X(3)+0.0406 *X(2)*X(3)+0.1532
      XDOT 5=377.0*X(4)
      ED=-.0003225*X(1)+.0116*X(2)-.17*(1.0+X(4))*X(3)+.000575
         *X(6)+. 187
      EQ=-.365 *(1).0+X(4))*X(1)-.164*(1.0+X(4))*X(2) + .0216*
         X(3)+.175
      ET=(ED**2+EQ**2) **.5
      XDOT 6=1600.*X(4)*RK(K)-80.*X(6)-84.2*RK(K)+80.*ET*RK(K)+
         111.2
      V(1) = B11*DOT1
      V(2) = B22*XDOT2
      V(3) = B33*XDOT3
      V(4) = B44*XDOT4
      V(5) = B55*XDOT 5
      V(6) = B66*XDOT6
      VDOT= V(X)+V(2)+V( 3)+V(4)+ V(5)+(V6)
      RETURN
      END
```

```
SUBROUTINE VOD S ( RK, X,C, N, V, VDCT)
SUBPROGRAM NO.4
DIMENSION X (6), C(6,61, V(6), RK (15)
B11= C(1,1) * x (1) + C (1,2) * x (2) + C1,3,) x X (3) + C (1,4) * X (4) +C(1),5)*X
                                                                              (5).
1+C(1,6)* X(6)
B22= C(2,1) * X (1) + C(2,2) * X (2) + C(2,3) * X(3) + C(2,4)*X(4)+C(2,5)*X(5)
1+C(2,6)* X(6)
B 33=C (3,1)* Y (1)+ C(3,2 )* X(2) +C(3,3) *X(3)+C(3,4)*X(4) + C (3,5)* X (5)
1+C(3,6)* X (6)
B44= C(4,1)* X(1) + C(4,2) * X (2) + C (4,3) * X(3) + C (4,4)*X(4)+C(4,5)* X(5)
1+C(4,6) * X (6)
B55= C(5,1)* X (1) + C (5,2) * X (2) + C(5,3)*X(3) +C(5,4)*X(4) +C(5,5)* X(5)
1+C(5,6) * X (6)
B66= C(6,1) * X (1) + C(6,2) * X(2) +C(6,3)*X(3) +C(6 ,4)*X(4)+C(6,5)* X(5)
1+C(6,6)* X(6)
XDOT 1 = -0.40013* X (1) -22.233 * X(2) + 369.7 *X(3) -298.44*SIN(X(5))
1+369.7* X(3) * X(4) + 0.712 * X(6)
XDOT2 = -0.15191*X(1) -27.376*X(2) + 455.22*X(3) -368.0*SIN (X(5) )
2+455.22*X(3) * X(4) +0.271 * X(6)
XDOT3=171.3775* X(1) -451.406*X(2) -22.672*X(3) -305.0* COS(X(5) )
3+171.3775*X(1) *X(4) -451.406* X(2) *X (4)
XDOT4 = -0.0936*X(1)* X (3) + 0.0406 * X(2) * X(3) + 0.1532
XDOT5 = 377.0 * X (4)
ED = - .0003225 * X(1) + 0.0116*X(2) -0.017* (1.0+ X (4)) *X(3) + 0.000575*X(6)+ .187
EQ=0.365*(1.0+ X(4)) * X(1)-.164* (1.0 + X (4) ) * X(2)+0.0216*X(3) +.175
ET =(ED ** 2 + EQ ** 2) **.5
XDOT6 = -80.*X(6) -84.2* RK (K)+80.0* ET*RK (k) + 111.2
V(1) = B 11* XDOT 1
V(2) = B22*XDOT2
V(3)=B33* X DOT3
V(4)= B44 * X DOT4
V(5) = B 55* XDOT5
V(6) = B 66* XDOT6
VDOT = V(1) + V(2) + V(3) + V(4) + V(5) + V(6)
RETURN
END
```

```
C       PROGRAM NO 8
C       TEST OF V
        HXXXXXXXXX
        DIMENSION A(5,5), X(5)
   13   READ (1,5)((A(I,J),J=1, 5)), I=1, & 5)
        WRITE (3,6)
        WRITE (3,5)((A(I,J),J=1,5),I=1, 5)
        WRITE (3,51)
   14   READ (1,1) K,N,M,H,P,Q
        IF(N) 13,23, 24
   24   READ (1, 5)(X(I),I=1,N)
   12   V1=0.5*A(1,1)*X(1)**2+0.5*(A(2,1)*X(1)+A(2,2)*X(2))*@2
        1-0.5*A(2,1)*X(1)**2+0.5*(A(3,& 1)*X(1)+A(3,2)*X(2)
        +A(3,3)*X(3)) **2
        2-0.5*(A(3,1)*X(1)+A(3,2)*X(2))**2+0.5*(A(4,1)*X(1)
        +A(4,2)*X(2)3+A(4,3)*X(3)+A(4,4)*X(4)**2-0.5*(A(4,1)
        *(1)*X(1) +A(4,2*2 )*X(2)4+A(4),3)*X(3))**2+0.5*(A(5,1)
        *X(1) +A(5,2)*X(2)+A(5,3)*X(3)5+A(5,4)*X(4)+A(5,5)
        *X(4)+A(5,5)*X(5))**2
        V2 = - 0.5*(A(5,1=*X(1)+A(5,2)*X(2)
        1+ A(5,3)*X(3)+A(5) 4)*X(4))**2
        V= V1+ V2
        WRITE (3, & 50)(X(I),I =1,N),V
        Y= 0.4/M
        X(K)= X(K)+Y
        P=P + 1.0
        IF (P-Q)12,14,14
    1   FORMAT (3X2,3F10.5 )
    5   FORMAT (5F10.5)
    6   FORMAT (1H1, 5X, 18H VALUES OF A(I, J)//)
   50   FORMAT (1H0, 5 F15.5,10x,E15.5/)
   51   FORMAT (5X,44HRESULT KG OF V FOR DIFFERENT VALUES OF
        X//)
   23   STOP
        END
```

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