DIRECT SYNTHESIS OF BANDPASS FILTERS WITH CAPACITOR COUPLED RESONATORS

BY

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A THESIS

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C E R T I F I C A T E

This is to certify that this work has done by me and it has not been submitted elsewhere for the award of any degree or diploma or for publication.

Countersigned.

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ABSTRACT

In this study, a rational function is determined by approximating a bandpass response directly and this rational function is synthesized into a band pass filter configuration consisting of shunt resonators coupled by capacitors, which is the common bandpass filter network.

The transfer impedance and the transmission function of a common bandpass filter network consisting of shunt resonators coupled by capacitors, have been found out by analysis. It is observed that all the transmission zeroes except one are at the origin and the remaining one is at infinity and the transmission function contains single term at the numerator. To be realizable, the absolute magnitude of the transmission function should be between 0 and 1.

For the rational function approximation of the band pass response; the independent variable, w, is converted into a new variable A such that the aperiodic function of w becomes a periodic function of A and the approximation can be done by Fourier method. Such a transformation of w into A has been obtained by the relation $A = 2\tan^{-1} w$, Because of the fact that the transmission function contains one single term in the numerator, its denominator has been approximated for getting the realizable rational transmission function. After approximation by Fourier method, the denominator of the transmission function has been obtained in the form of a cosine series, which is again converted to a polynomial in w^2 by using Chebyshev polynomials. The Chebyshev polynomial converts cosine of a multiple angle into a polynomial in cosine of the fundamental angle and the previous relation between A and w can be written as $w^2 = \frac{1-\cos A}{1+\cos A}$,

so that the cosine series can be converted to a polynomial in w^2 and thus we obtain the realizable rational function approximation of the band pass response.

The approximation has been done by two methods, one by point matching technique, assuming the values of transmission function for different values of w and the other by assuming a fixed curve for the denominator of the transmission function.

The reflection coefficient and the input impedance are then calculated from the realizable rational transmission function. Since all but one transmission zeroes are at the origin andthe remaining one at infinity, the realization has been done by ladder development of the input impedance realizing a shunt inductance and a series capacitance each time. After realizing all the transmission zeroes at the origin, the one at infinity is realized by a shunt capacitance.

Finally the capacitance matrix transformation of each section of the filter has been used to get the filter realized in the usual form of shunt resonators coupled by capacitors.

Several filters are designed utilizing this procedure. The response curves for the final networks have been observed to be satisfactory compared to Butterworth and Chebyshev filters.

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CHAPTER-1

INTRODUCTION

Filters are selective networks used for frequency discrimination that means for the rejection of unwanted signal frequencies while permitting good transmission of wanted frequencies. The most common filters are designed for lowpass, high-pass, band-pass or band-stop attenuation characteristics.

The lowpass filter passes the package of wave energy from zero frequency upto a determined cut-off frequency and rejects all energy beyond that limits. The highpass filter prevents the transmission of frequencies below a determined point and appears to be electrically transparent to frequencies beyond that point. The band pass filter passes the package of waves from certain lower to upper frequency limits and stops all energy outside these two limits. Band pass filters are the most important and most commonly used in electronic equipments. The band stop filter is used in electronic equipment when a certain unwanted frequency or band of frequencies has to be rejected. Outside the stopband or rejection band all frequencies will pass without appreciable attenuation.

From the frequency domain point of view, an ideal filter is one that passes, without attenuation, all frequencies inside certain frequency limits (called pass band) while providing infinite attenuation for all other frequencies (called stop band).

Since the discovery of the electric wave filter by Cambell and Wagner in 1915, filter theory has evolved essentially along two different points of view. These have been distinguished by the names classical filter theory and modern filter theory.

The classical filter theory originated in the 1920S mainly through the efforts of Zobel[1]. This theory is an application of image parameter equations to the design of filters and, as such, is commonly known as the image parameter theory of filter design. When designing a filter by this method, it is assumed that the filter's load impedance is matched to its image impedance. But in practice this condition is difficult to satisfy, because most loads are constant value resistances and the image impedance is frequency dependent. As a result, design on this basis involves a cut-and-try procedure and, often, final adjustments must be made experimentally in order to meet design requirements. Even so, the classical theory yields good results with speed and a minimum of effort, and there is a wealth of published design information available in this field.

The modern filter theory was developed in 1930S through the efforts of a number of individuals among whom the names of Norton[(2)] Foster[(3], Cauer[(4)], Bode[(5)], Brune[(6)], Guillemin[(7)] and Darlington[(8-9)] need special mention. Essentially this theory involves the approximations of given specifications with a rational transfer function and the realization of this function through the use of different synthesis techniques. Since synthesis procedures involving approximation by polynomial are analytical and exact, designing the filters from its transfer function involves no trial and error. For this reason this method is also called the exact method or the polynomial method. An important feature of this theory is that the approximation part and the realization part are separable.

It is not always that we are given a realizable rational function for which we have to design a filter network. Sometimes a given characteristic is given graphically as a function of frequency.

Sometimes some discrete values are given. It is then necessary to solve the approximation problem: a system function must be found that on the one hand, approximates the given curve or the descrete values with the specified tolerances and, on the other hand, is realizable by a network of the desired form. Stated in other terms, what we need to do is to fit a realizable rational function to the specified data, that is, to determine the coefficients of two polynomials or equivalently to determine the zeroes, poles and constant multipliers of the rational function. We also desire the function to be of the lowest possible order so that a small number of elements will be required for its realization.

The problem of approximation may be solved very easily by the use of Butterworth or Chebyshev functions. From the approximation transmission function obtained by Butterworth[10] or Chebyshev polynomial low pass filter can be synthesized by conventional synthesis procedure. High pass, band pass and band stop filters can then be designed from this low pass model by frequency transformation[7].

After the frequency transformation from low pass to bandpass, the network configuration for band pass consists of parallel resonators and series resonators connected as shunt and series branches respectively. The direct conventional low pass to band pass transformation, although theoretically correct, is not always attainable practically. The element values may be too small or too large. The parasitic capacitance to ground can not be taken into account and therefore may distort the response. The node between a capacitor and a coil in a series arm becomes very sensitive to stray capacitance at some frequencies and the quality of the series arm has to be very high in order to produce a low level insertion loss in the passband.

It is therefore desirable to simplify the network realization in order to remove theselectivity from the series arm and to substitute added selectivity in the parallel arms.

The impedance and the admittance transformation properties of J and K inverters which are theoretically valid at a single frequency are sometimes used to avoid these difficulties.[11] By utilising the concept of coupling introduced by Miltion Dishal[12], the normalised low pass element values, L and C can be converted to new normalised values K and Q, the coupling coefficient and the quality factor respectively. From such a low pass prototype, the band pass filter can be designed so that the network configuration will consist of shunt resonators coupled by capacitances or inductances.

The resulting network obtained by both the above procedures consisting of shunt resonators coupled by capacitors does not have a low pass equivalent andits response exactly equals the response of the low pass prototype after frequency conversion at the band centre. The difference of the two responses increases, when the test frequency moves away from the band centre.

The objective of this study is to determine a rational function approximation directly from the given bandpass response so that it can be synthesized into a configuration consisting of shunt resonators coupled by capacitors. The response of the network thus realized will exactly match the rational function at all frequencies.

A general band pass network configuration consisting of shunt resonators with capacitor coupling between them will be analysed and the transmission function will be determined. The transmission function will be a rational function of polynomials is w^2.

The specified band pass transmission characteristics will then the approximated with the help of Fourier series expansion and Chebyshev polynomial so that the approximate function will be a rational function of type found for the transmission function of a band pass filter consisting of shunt resonators coupled by capacitors.

This rational function will then be synthesized in a conventional method. The network thus obtained should be potentially equivalent to the band pass filter network consisting of shunt resonators with capacitor coupling between them. A network transformation procedure will be illustrated so as to transform this configuration into the general band pass filter configuration with shunt resonators coupled by capacitors.

CHAPTER-2

THEORETICAL FORMULATION OF BAND PASS FILTER SYNTHESIS

2.1 PRELIMINARIES:

In this chapter, the conventional method of Band Pass Filter Synthesis from low pass prototype is discussed in brief. The frequency transformation is treated first. The reactance transformation of the frequency variable makes it possible to get the band pass filters from low pass prototype. A reactance function; having two poles one at origin and the other at infinity with a zero at some frequency w_o, is assumed to be equal to the low pass frequency range from $-\infty$ to ∞. So that low pass frequency zero corresponds to w_o of the band pass, which is the band centre and the low pass response may transformed to be a band pass response. Cutoff frequency for the band pass is calculated. The centre frequency is the geometric mean of two cutoff frequencies.

In article 2.3, J and K inverters are explained for the transformation of the band pass network so that the elements values becomes approximately similar.

Band pass filters may be designed without network transformation from low pass element values where the elements are not capacitances or inductances but they are coupling coefficiemts and quality factors, of the elements as defined by Milton Dishal. This is also discussed in brief.

A different approach of approximation is introduced in this study. The reason for this is explained in article 2.5.

2.2. FREQUENCY TRANSFORMATION:(13)

Band pass filters are generally disigned from low pass filters by reactance transformations of the frequency variable. A reactance transformation is one in which the frequency variable is set equal to a reactance function in a new frequency variable. A reactance function has simple poles and zeroes which alternate on the jw axis. Hence if W_k and W_{k+2} (where $0 < W_k < W_{k+2}$) are two consecutive poles of reactance function X(W) [there is, of course, a zero of X (W) between W_k and W_{k+2}], then on the frequency interval $W_k < W < W_{k+2}$, the frequency $\Omega = X(W)$ assumes all values from $-\infty$ to ∞ (i.e. sweeps the entire axis). Thus the entire $j\Omega$ axis maps into each segment on the jw axis containing two consecutive poles of X(W). It is this phenomenon that allows a low pass filter characteristics to be transformed into other types.

For low pass to band pass transformation we have to take the low pass frequency equal to a reactance function having two poles one at origin and the other at infinity with a zero at some frequency W_o of the transformed band pass frequency. This means that if we take S the low pass independent variable equal to $U_{Lc}(s)$ a reactance function of band pass frequency s, where

$$S = U_{Lc}(s) = \frac{s^2 + W_o^2}{sB}$$

Then we get band pass transformed response as a function of bandpass frequency s.

$$S = \Sigma + j\Omega = U_{LC}(s)$$

$$= \frac{W_o}{B}\left(\frac{s}{W_o} + \frac{W_o}{s}\right)$$

$$= \frac{s^2 + W_o^2}{sB} \qquad \text{....................(2.1)}$$

For $s = jw$

$$U_{LC}(jw) = \frac{-w^2 + w_o^2}{j\, w\, B}$$

$$= j\, \Omega \quad (\Sigma = 0)$$

$$\therefore \quad X(w) = \Omega$$

$$= \frac{1}{j} U_{LC}(jw)$$

$$= \frac{w^2 - w_o^2}{w\, B} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

In Fig.2.1, transmission function T (Ω) is plotted for low pass frequency Ω i.e. X(w), X(w) is plotted for band pass frequency w. The range for Ω from $-\infty$ to ∞ becomes the range for w from 0 to ∞ and the transmission function T (w) (a function of band pass frequency w) now becomes a band pass response having centre frequency w_o and a band width $w_u - w_L$ corresponding to low pass frequency $\Omega = \pm 1$

For $S = j\Omega = -j\,1, \quad s = j\, w_L$

From equation (2.2)

$$-j1 = \frac{-w_L^2 + w_o^2}{j\; w_L\; B}$$

$$w_L^2 + w_L B - w_o^2 = 0$$

$$w_L = -\frac{B}{2} \pm \sqrt{\frac{B^2}{4} + w_o^2}$$

w_L should be greater than 1.

$$w_L = -\frac{B}{2} + \sqrt{\frac{B^2}{4} + w_o^2} \qquad \ldots\ldots\ldots\ldots\ldots (2.3)$$

Similarly, $w_u = \frac{B}{2} + \sqrt{\frac{B^2}{4} + w_o^2}$ $\qquad \ldots\ldots\ldots\ldots\ldots (2-4)$

$w_u - w_L = B$ = Band width for the band pass .

From eqn.(2-3) and (2-4)

$$w_L \cdot w_u = \left(\frac{B^2}{4} + w_o^2\right) - \frac{B^2}{4} = w_o^2, \qquad w_o = \sqrt{w_L\, w_u}$$

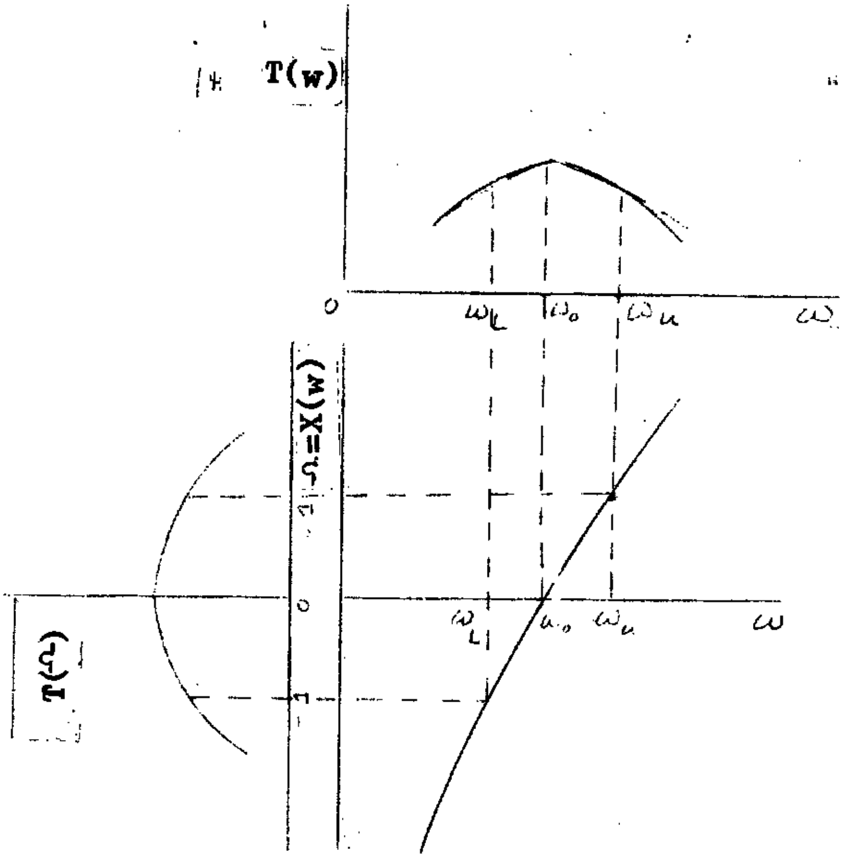


Fig.2.1

Low pass to band pass transformation

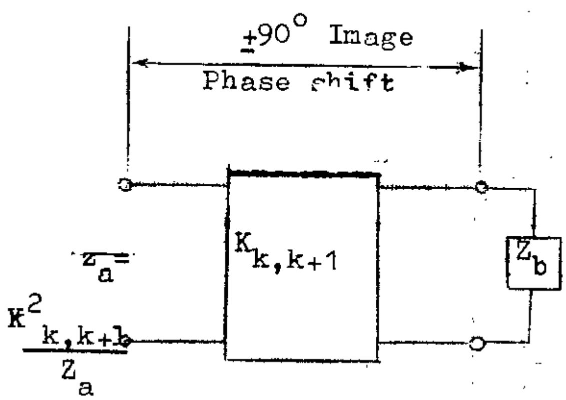


Fig.2.2a

Impedance Inverter

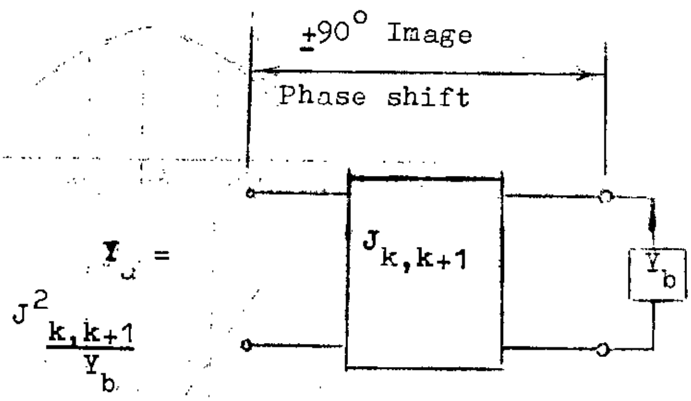


Fig.2.2(b)

Admittance Inverter.

(14)

2.3. NETWORKS TRANSFORMATION BY IMPEDANCE AND ADMITTANCE INVERTERS:

Networks for band pass filters obtained by frequency transformation from low pass prototype have generally elements practically difficult to construct for higher frequencies. Use of impedance and admittance inverter transforms these networks having resonators coupled by inductances or capacitances which can be constructed physically. Sey mour B. Cohn(4) described the process of transformation of band pass filters obtained from low pass prototype to a direct coupled resonator filters using impedance and admittance inverters usually known as K and J inverters respectively.

An idealised impedance inverter operates like a quarter wavelength line of characteristic impedance K at all frequencies. Therefore if it is terminated in an impedance Z_b at one end, the impedance Z_a seen looking in at the other end is $Z_a = \frac{k^2}{Z_b}$ (Fig. 2.2 a)

An idealised admittance inverter operates like a quarter wavelength line of characteristic admittance J at all frequencies. Thus if an admittance Y_b is attached at one end, the admittance Y_a seen looking in the other end is $Y_a = \frac{J^2}{Y_b}$ (Fig. 2.2 (b))

Figure 2.3 shows a typical low pass prototype design and Fig. 2.4 shows the corresponding band pass filter design, which can be obtained directly from the prototype by a low pass to band pass transformation. Fig. 2.5 shows a generalised circuit for a band pass filter having impedance inverter and series type resonator and Fig. 2.6 shows a generalised circuit for the same filter having admittance inverter and shunt type resonators.

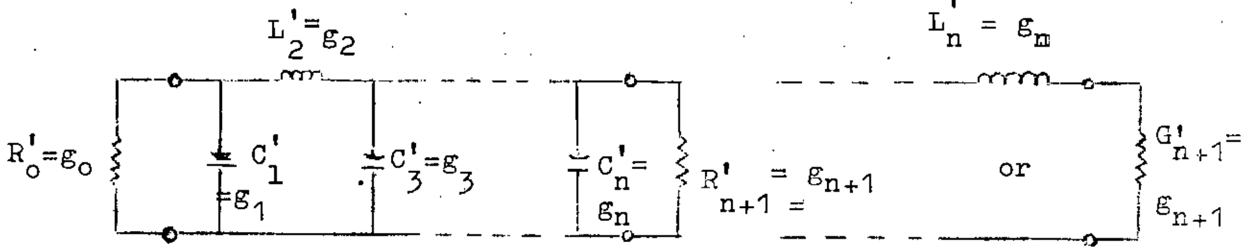
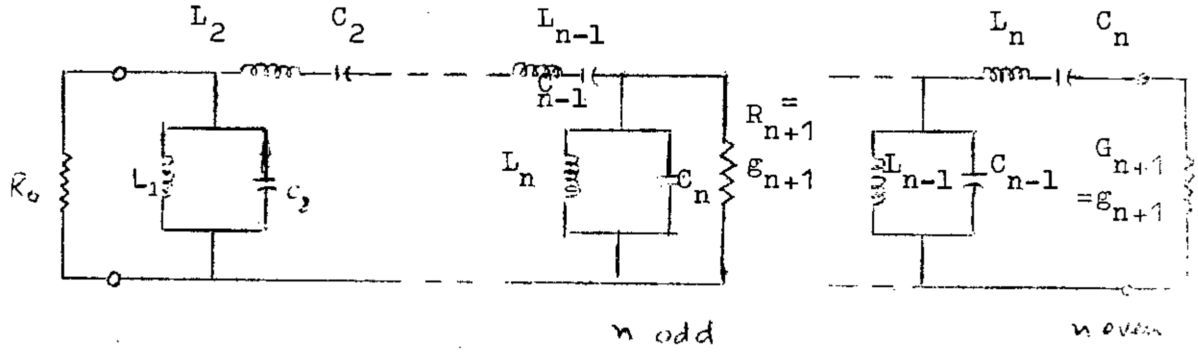


Fig.2.3

A low pass prototype filter.



For Shunt Resonators:

$$b_j = \omega_0 C_j = \frac{1}{\omega_0 L_j} = \frac{\omega_1' g_j}{\Delta\omega}$$

For Series Resonators:

$$x_k = \omega_0 L_k = \frac{1}{\omega_0 C_k} = \frac{\omega_1' g_k}{\Delta\omega}$$

ω_1' = Low pass cutoff freq.

ω_1, ω_2 = Band pass cutoff frequencies

$$\Delta\omega = \frac{\omega_2 - \omega_1}{\omega_0}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

Fig.2.4 Band-pass filters and their relation to low pass-prototypes.

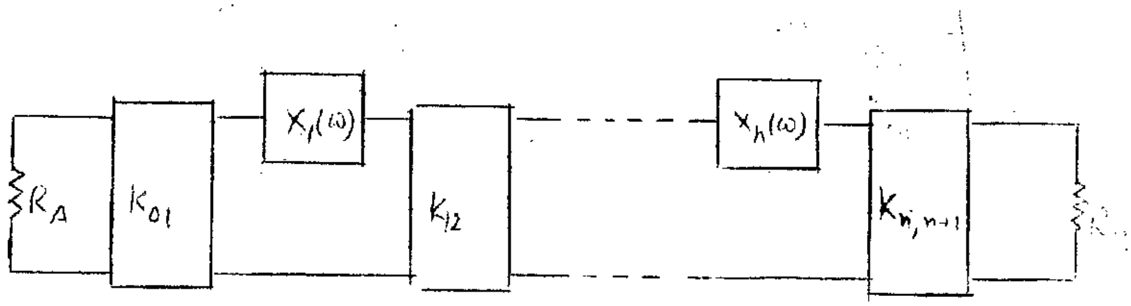


Fig.2.5

The Band-pass filter in Fig.2.4 Converted to use only. series resonators and impedance inverters.

$$x_j = \omega_0 L_j = \frac{1}{\omega_0 C_j} = \omega_1' g_k = \frac{\omega_0}{2} \frac{d X_j(\omega)}{d\omega}\Big|_{\omega=\omega_0}$$

$$k_{01} = \sqrt{\frac{R_A x_1 \Delta\omega}{g_0 g_1 \omega_1'}}, \quad K_{j,j+1} = \frac{\Delta\omega}{\omega_1'} \sqrt{\frac{x_j x_{j+1}}{g_j g_{j+1}}}$$

$$K_{n,n+1} = \sqrt{\frac{R_B x_n \Delta\omega}{\omega_1' g_n g_{n+1}}}, \quad \Delta\omega = \frac{\omega_2 - \omega_1}{\omega_0}$$

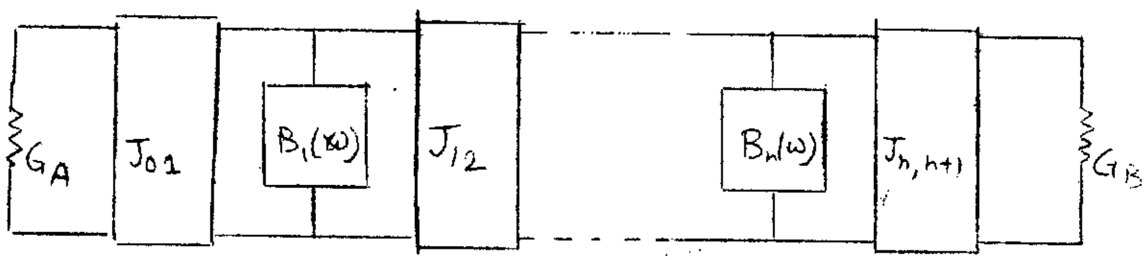


Fig.2.6

The Bandpass filter in fig.2.4 converted to use only shunt regonators and admittance inverters.

$$b_j = \frac{\omega_0}{2} \frac{d B_j(\omega)}{d\omega}\Big|_{\omega=\omega_0} = \omega_0 C_j = \frac{1}{\omega_0 L_j}$$

$$J_{01} = \sqrt{\frac{G_A b_1 \Delta\omega}{g_0 g_1 \Delta\omega}}, \quad J_{j,j+1} = \frac{\Delta\omega}{\omega_1'} \sqrt{\frac{b_j b_{j+1}}{g_j g_{j+1}}}$$

$$J_{n,n+1} = \sqrt{\frac{G_B b_n \Delta\omega}{\omega_1' g_n g_{n+1}}}, \quad \Delta\omega = \frac{\omega_2 - \omega_1}{\omega_0}$$

One of the simplest forms of inverters is a quarter wavelength of transmission line. For an impedance inverter it has an inverter parameter $K = Z_o$, where Z_o is the characteristic impedance of the line. For an admittance inverter it has an inverter parameter $J = Y_o$, where Y_o is the characteristic admittance of the line. Besides this, there are numerous other circuits which operates as inverters. Fig. 2.7(a) shows one of them.

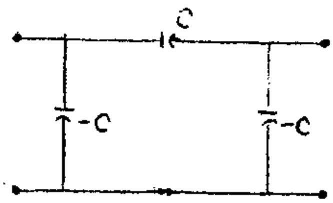


Fig 2.7(a)

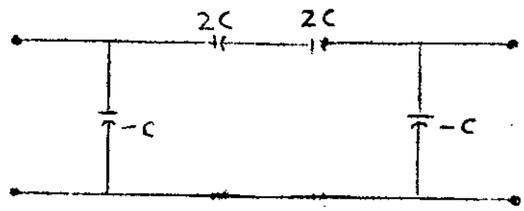


Fig 2.7(b)

Short Circuit and open circuit input impedance of half network Fig. 2.7(b)

$$Z_{sc} = \frac{1}{j\ wc} \quad , \quad Z_{oc} = - \frac{1}{j\ wc}$$

Characteristic impedance

$$= \sqrt{Z_{sc}\ Z_{oc}} = \sqrt{\frac{1}{j\ wc} \times \frac{1}{-jwc}} = \sqrt{\frac{1}{w^2c^2}} = \frac{1}{wc}$$

Image phase shaft

$$B = 2 \tan^{-1} \pm \left(\frac{-Z_{sc}}{+Z_{oc}} \right)$$

$$= 2 \tan^{-1}(\pm 1) = \pm 90^{\circ},$$

Thus B is frequency indipendent.

Thus the circuit of Fig. 2.7 can be used as an impedance or admittance inverter. When used as impedance inverter. The value of K is $\frac{1}{wc}$ and when used as admittance inverter the value of J is wc.

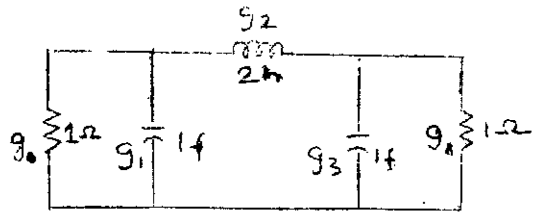
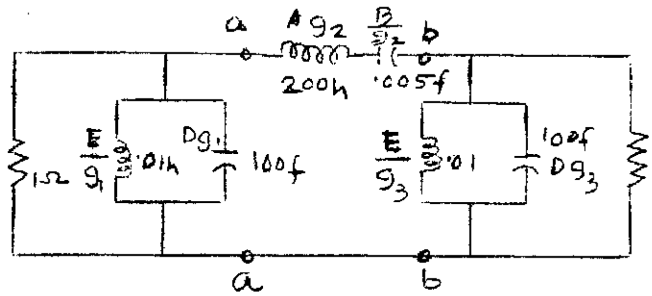


Fig.2.8 (a)
Lowpass Prototype

$g_0=1$
$g_1=1$
$g_2=2$
$g_3=1$
$g_4=1$



$w_m = 1$ rad./sec., w=.01 rad./sec
$R_1 = 1$., $R_2=1$, $\bar{w} = w/w_m = .01$
$w_1' = 1$
A =100, B = .01, D = 100,
E = .01.

Fig.2.8(b)

Band pass transformed by frequency transformation.

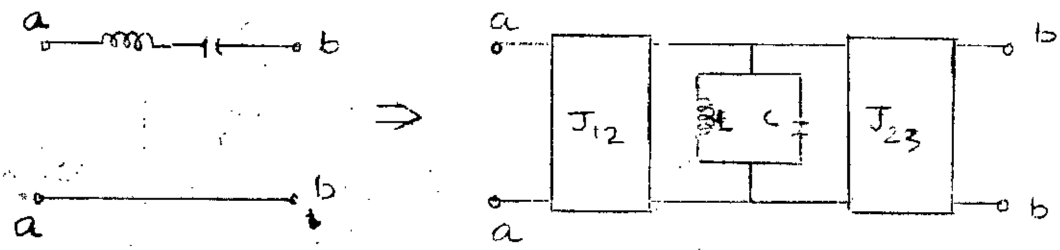


Fig.2.9(a) Series resonator transformed by admittance inverter.

$$b_1 = \omega_0 C_1 = \frac{1}{\omega_0 L_1} = 100$$

$$b_2 = C = \frac{1}{L}$$

$$J_{12} = \frac{\bar{\omega}}{\omega_1'}\sqrt{\frac{b_1 b_2}{g_1 g_2}} = \frac{.01}{1}\sqrt{\frac{100C}{2}} = 0.1\sqrt{C/2}$$

$$J_{23} = \frac{\bar{\omega}}{\omega_1'}\sqrt{\frac{b_2 \cdot b_3}{g_2 g_3}} = \frac{.01}{1}\sqrt{\frac{100C}{2}} = 0.1\sqrt{C/2}$$

Let $L = .01$, $C = 100$

$$J_{12} = \frac{1}{\sqrt{2}} = 0.707, \quad J_{23} = \frac{1}{\sqrt{2}} = 0.707$$

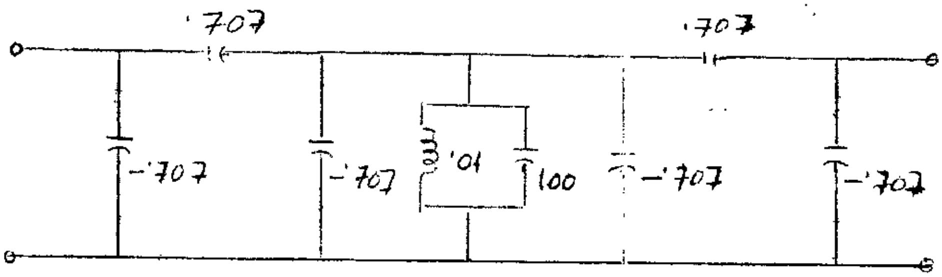


Fig.2.9(b)

Circuit of Fig.2.9(a) with the values of the inverter

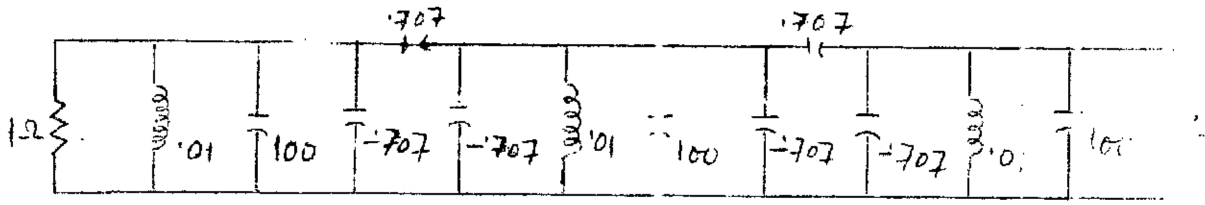


Fig.2.9(c)

Equivalent to circuit of Fig.2.8(b).

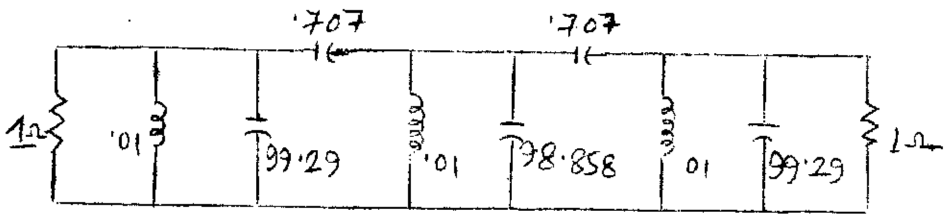


Fig.2.9(d)

The final circuit

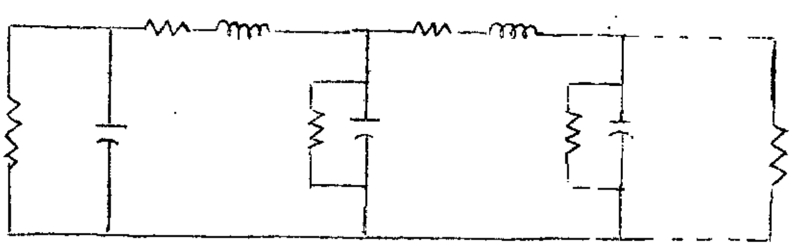


Fig.2.10

Definition of quality factor and coupling coefficient.

As an example let us design a band pass filter consisting of 3 resonators and transform the circuit by admittance inverter of Fig. 2.7(a). The design is shown in Fig. 2.8 and 2.9.

Thus we obtain a network consistings of resonators coupled by capacitors for the normalised band pass filter having center frequency 1 radian and a band width of 10%. After impedance and frequency scaling this network will be transformed into a practically constructable network.

2.4 DESIGN OF BAND PASS FILTER BY NORMALISED K|Q VALUES.(15)

In general the filter designed consists of normalised elements values and normalised ffequency. Elements values are normalised so that they are related to a arbitrary terminating normalizing resistance R_r, the reduced impedance level resulted in simplified calculations. Another normalizing parameter is frequency; the frequency of the 3 db down point is normalised to 1 rad/sec.

Another form of normalisation results when the reactive component of each element is related to the reactive part of the immediately preceding element. The frequency normalization is same as the above procedure. By this normalization, the filter is designed in terms of coefficient of coupling K as defined by Milton Dishal and the normalised quality factor q in place of normalised element values L and C.

The coefficient of coupling K_{ij} is defined by Fig. 2.10

$$K_{12} = \frac{\Omega_{12}}{\Omega_{3dB}}$$

$$K_{23} = \frac{\Omega_{23}}{\Omega_{3dB}}$$

$$\Omega_{12} = (\sqrt{C_1 L_2})^{-1}, \quad \Omega_{23} = (\sqrt{L_2 C_3})^{-1} \text{ ---}$$

Ω_{3dB} = The overall 3dB down frequency of the filter .

The expressions for normalised quality factor of the circuit are

$$\frac{1}{q_1} = \frac{G_1/C_1}{\Omega_{3dB}}$$

$$q_2 = \frac{\Omega_{3dB} L_2}{R_2}$$

$$\vdots$$

From the low pass model of this type of normalisation, the band pass filter can be designed such that network transformation is not required as described in the previous article. The losses in the reactive components can be taken into account by this procedure. Moreover the values of the shunt resonator inductances may be taken to be the same and the capacitances may be corrected including parasitic capacitances.

2.5 RELEVANCE OF THE PRESENT WORK:

Band pass networks are generally designed by the two procedures described in this chapter. Both the procedures are based on the frequency transformation technique from low pass to band pass. For such a band pass filter design, the approximation problem is solved for the low pass prototype.

After frequency transformation, the band pass response does not have arithmatic symmetry. The response at frequencies lower than the center frequency decreases rapidly than the response at frequencies higher than the centre frequency.

Moreover for the final network, the transformation used exactly corresponds at the centre frequency, so that the difference of the two responses (the responses of networks before and after transformation) increases when the test frequency moves away from the band centre.

For both the reasons, low pass response will not exactly corresponds to the band pass response for frequencies other then the centre frequency.

In our study polynomial approximations have been done directly from the band pass response. After getting the rational function approximation we designed the network by exact method. No further approximation is required for getting the final network, so that the response of the final network can be predicted during the time of approximation.

CHAPTER-3

ANALYSIS OF THE BAND PASS FILTER CIRCUIT

3.1. PRELIMINARIES:

In this chapter the practical band pass filter network is analysed. The network consists of parallel resonators with capacitor coupling between then terminated by a resistive load. The input is taken as current source with a resistance parallel to the source.

At first the transfer impedance, $Z_{12}(S)$, is calculated. Transfer impedance contain a single term S^{2n-1} at the numerator, n being the order of the filter i.e. number of resonators. S^{2n-1} term in the numerator indicates that all but one transmission zero (highest order of denominator polynomial is 2n) is at origin and the remaining one is at infinity.

Then the transmission function $|T(j\omega)|^2$, defined by the ratio of power available at the load to the maximum power deliverable by the source, is calculated from this transfer impedance. The maximum value of such a transmission function is unity. Because power available can not be greater than power supplied. Moreover the transmission function can not be negative i.e. load, which is source free, can not supply energy to the source.

Input impedance and the reflection coefficient is than calculated. It was shown that the p.r. condition of input impedance is same as the condition $0 < |T(j\omega)|^2 < 1$ for $|T(j\omega)|^2$. Thus the realizability condition of the transmission function is $0 < |T(j\omega)|^2 < 1$

3.2. THE TRANSFER IMPEDANCE:

The band pass filter networks with shunt resonators coupled by capacitors obtained from the low pass prototype by freqµency transformation and impedance conversion is shown in Fig.3.1.

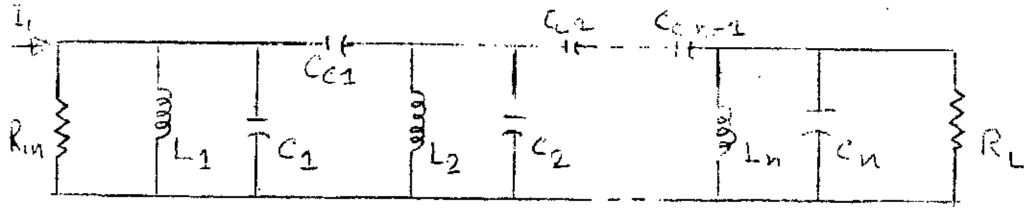


Fig.3.1.

For n =2, i.e. for two resonators the circuit is shown in Fig.3.2.

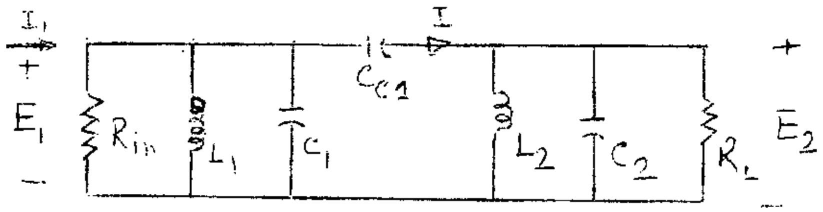


Fig.3.2

We shall first calculate $Z_{12} = \frac{E_2}{I_1}$ for the circuit for Fig.3.2.

From the figure,

$$I = E_2 \left[\frac{1}{L_2 s} + C_2 s + \frac{1}{R_L}\right] = E_2 \; \frac{R_L + L_2 s + L_2 C_2 R_L s^2}{R_L L_2 s}$$

$$E_1 = I \cdot \frac{1}{C_{c_1} s} + E_2$$

$$= E_2 \cdot \frac{R_L + L_2 s + L_2 C_2 R_L s^2}{R_L L_2 C_{c1} s^2} + E_2$$

$$= E_2 \cdot \frac{R_L + L_2 s + L_2 C_2 R_L s^2 + R_L C_{c1} L_2 s^2}{R_L L_2 C_{c1} s^2}$$

$$= E_2 \frac{R_L + L_2 s + (R_L L_2 C_2 + R_L L_2 C_c) s^2}{R_L L_2 C_{c1} s^2}$$

$$I_1 = E_1 \left[\frac{1}{L_1 s} + C_1 s + \frac{1}{R_{in}} \right] + I$$

$$= \frac{E_2 \quad R_L + L_2 s + (R_L L_2 C_2 + R_L L_2 C_{c1}) s^2}{R_L L_2 C_c s^2} \times \frac{R_{in} + L_1 s + L_1 C_1 R_{in} s^2}{R_{in} L_1 s}$$

$$+ E_2 \frac{R_L + L_2 s + L_2 C_2 R_L s^2}{R_L L_2 s}$$

$$= E_2 \frac{[R_L + L_2 s + (R_L L_2 C_2 + R_L L_2 C_{c1}) s^2][R_{in} + L_1 s + L_1 C_1 R_{in} s^2]}{R_L L_2 C_{c1} s^2 \quad R_{in} L_1 s}$$

$$+ E_2 \frac{R_L + L_2 s + L_2 C_2 R_L s^2}{R_L L_2 s}$$

$$= E_2 \{ R_L R_{in} + (R_L L_1 + R_{in} L_2) s + [R_L L_1 C_1 R_{in} + L_1 L_2 + R_{in} (R_L L_2 C_2$$
$$+ R_L L_2 C_{c1})] s^2 + [L_2 L_1 C_1 R_{in} + L_1 (R_L L_2 C_2 + R_L L_2 C_{c1})] s^3$$
$$+ [R_1 C_1 R_{in} (R_L L_2 C_2 + R_L L_2 C_{c1})] s^4 \} / R_L L_2 C_{c1} L_1 R_{in} s^3$$
$$+ E_2 \{ C_{c1} L_1 R_{in} R_L s^2 + C_{c1} L_1 R_{in} L_2 s^3 + C_{c1} L_1 R_{in} L_2 C_2 R_L s^4 \}$$
$$(R_2 L_2 C_1 L_1 R_{in} s^3 \}$$

$$= E_2 \{ R_L R_{in} + (R_L L_1 + R_{in} L_2) s + [R_L L_1 C_1 R_{in} + L_1 L_2 + R_{in} (R_L L_2 C_2 +$$
$$R_L L_2 C_{c1}) + C_{c1} L_1 R_{in} R_L] s^2 + [L_2 L_1 C_1 R_{in} + L_1 (R_L L_2 C_2 +$$
$$R_L L_2 C_{c1}) + C_{c1} L_1 R_{in}] s^3 + [L_1 C_1 R_{in} (R_2 L_2 C_2 + R_2 L_2 C_{c1}) +$$
$$C_{c1} L_1 R_{in} L_2 C_2 R_2] s^4 \} / R_L L_2 C_{c1} L_1 R_{in} s^3$$

$$\therefore \frac{E_2}{I_1} = \frac{b_3 s^3}{d_0 + d_1 s + d_2 s^2 + d_3 s^3 + d_4 s^4}$$

$$= Z_{12}$$

Thus the filter with two resonators the transfer impedance Z_{12} have the form

$$Z_{12}(S) = \frac{b_3 S^3}{d_0 + d_1 S + d_2 S^2 + d_3 S^3 + d_4 S^4} \quad \ldots\ldots\ldots\ldots\ldots(3.1)$$

Analysis in a similar manner will show that for n =3

$$Z_{12}(S) = \frac{b_5 S^5}{d_0 + d_1 S + d_2 S^2 + d_3 S^3 + d_4 S^4 + d_5 S^5 + d_6 S^6} \quad \ldots\ldots\ldots\ldots\ldots(3.2)$$

In general for n resonators, the impedance

$$= \frac{b S^{2n-1}}{d_0 + d_1 S + \cdots + d_{2n} S^{2n}} \quad \ldots\ldots\ldots\ldots\ldots(3.3)$$

For an increasing order, three elements are added in a circuit as shown figure 3.3.

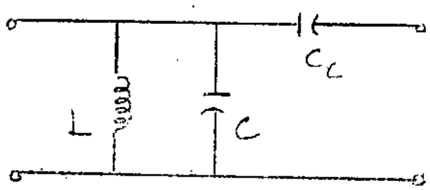


Fig.3.3.

But $Z_{12}(S)$ has two more transmission zeroes at the origin. In the Fig.3.3 L and C_c will cause the transmission zeroes, C can not produce **any** independent transmission zero. The capacitances C_1, C_c and C_2 are in π form as shown in fig. 3.5(a) combination of three capacitances are equivalent to the combination of two capacitances as shown in Fig. 3.5 (b). Again for the next stage, C_{c2}, C_3 and C'_2 form the π circuit of Fig.3.6(a) and is equivalent to circuit of Fig.3.6(b). The network of Fig.3.4 will then be equivalent to the network of Fig.3.7, where the transmission zero at infinity is for C'_3. The network of 3.4 and Fig.3.7 are called potentially equivalent. Obtaining one of them, the other can be found out easily by changing the internal capacitance matrices.

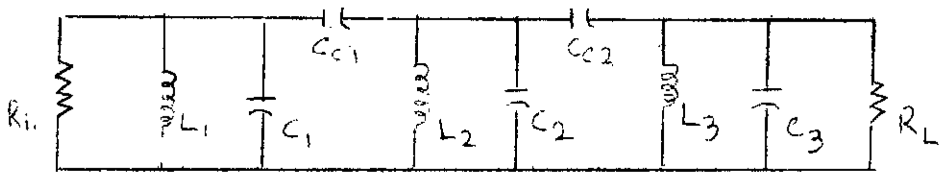
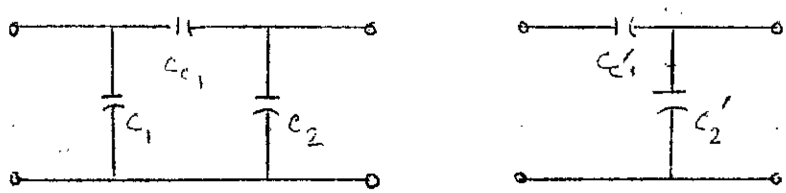


Fig.3.4



(a) Fig.3.5 (b)

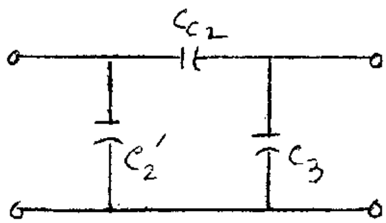


Fig.3.6(a)

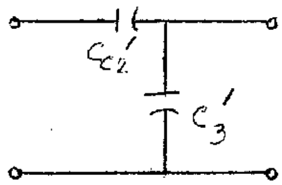


Fig.3.6(b)

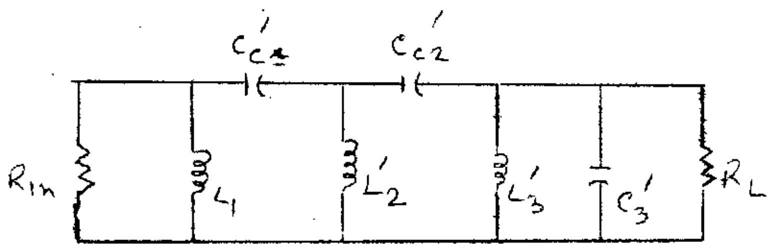


Fig.3.7

Fig.3.4 & 3.7 are potentially equivalent network.

Knowing one, the other can be obtained by the transformation of figure 3.5 & 3.6

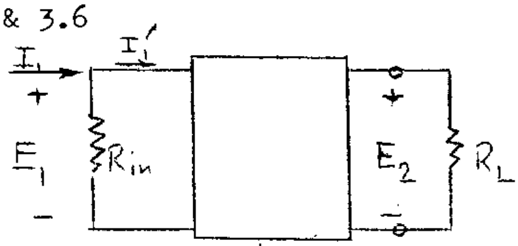


Fig.3.8

Definition of Transmission function.

3.3. THE TRANSMISSION FUNCTION:

The transmission function is defined as the ratio of the power available at the load to the maximum power deliverable by the source..

$$\left|t(j\omega)\right|^2 = \frac{\text{Power available at the load}}{\text{Maximum power deliverable by the source}} \quad \ldots\ldots\ldots(3.5)$$

$$= \frac{|E_2|^2/R_L}{|I_1|^2 \frac{R_{in}}{4}}$$

$$= \frac{4}{R_{in}R_L} \left|\frac{E_2}{I_1}\right|^2$$

$$= \frac{4}{R_{in}R_L} \left|Z_{12}(j\omega)\right|^2 \quad \ldots\ldots\ldots(3.6)$$

for $n = 2$,

$$Z_{12}(s) = \frac{c_3 s^3}{d_0 + d_1 s + d_2 s^2 + d_3 s^3 + d_4 s^4}$$

$$|Z_{12}(j\omega)|^2 = Z_{12}(s)\,Z_{12}(-s)\Big|_{s=j\omega}$$

$$= \frac{c_3 \omega^6}{A_0 + A_1\omega^2 + A_2\omega^4 + A_3\omega^6 + A_4\omega^8} \quad \ldots\ldots(3.7)$$

Therefore for a filter of order 2

$$|t(j\omega)|^2 = \frac{4\,|Z_{12}(j\omega)|^2}{R_{in}R_L}$$

$$= \frac{\omega^6}{A_0 + A_1\omega^2 + A_2\omega^4 + A_3\omega^6 + A_4\omega^8} \quad \ldots\ldots(3.8)$$

In a similar way it can be shown that for a filter of order n

$$Z_{12}(s) = \frac{b_{2n-1}\, s^{2n-1}}{c_0 + c_1 s + c_2 s^2 + \cdots + c_{2n} s^{2n}} \quad \ldots\ldots(3.9)$$

Therefore the transmission function $|t(j\omega)|^2$ have the form

$$|t(j\omega)|^2 = \frac{\omega^{2(2n-1)}}{A_0 + A_1\omega^2 + A_2\omega^4 + \cdots + A_{2n}\omega^{4n}} \quad \ldots\ldots(3.9)$$

3.4 REALIZABILITY CONDITION OF THE TRANSMISSION FUNCTION:

The power taken by the load can not be negative or greater than maximum power deliverable by the source. So that from Article 3.1, it can be concluded that the transmission function $|t(jw)|^2$ must be between 0 to 1. This condition is necessary and sufficient for the p.r. property of the input impedance at the driving point.

To explain this, we shall define reflection coefficient, $\rho(s)$, by the equation

$$|\rho(j\omega)|^2 = 1 - |T(j\omega)|^2 \quad --- (3.10)$$

where $|\rho(jw)|^2$ is the square magnitude of $\rho(s)$ at $S = jw$.

Let the driving point impedance Z_1 be $\frac{E_1}{I_1}$ as shown in Fig.3.8. Then

$$\frac{I_1'}{I_1} = \frac{R_{in}}{R_{in} + Z_1} \quad ---- (3.11)$$

Let us take $R_{in} = R_L = 1$ ohm.

$$\frac{E_2}{I_1} = \frac{E_2}{I_1'} \times \frac{I_1'}{I_1} = Z_{12}' \times \frac{1}{1+Z_1} \quad \cdots (3.12) \quad \left[Z_{12}' = \frac{E_2}{I_1'}\right]$$

Let $Z_1(s) = \frac{E_1}{I_1'} = \frac{m_1 + n_1}{m_2 + n_2} = \frac{n_1}{m_2} \cdot \frac{(m_1/n_1)+1}{(n_2/m_2)+1} \quad \cdots\cdots (3.13)$

where m_1 and m_L are even and n_1 and n_2 are add functions of S. Then according to the Darlington's Synthesis procedure it can be shown that

$$z_{11} = \frac{n_1}{m_2} \qquad z_{22} = \frac{n_2}{m_2} \qquad z_{12} = \frac{\sqrt{n_1 n_2 - m_1 m_2}}{m_2}$$

For this case, z_{12}, having all the transmission zero of Z_{12}, must have (b s^{n-1}) in the numerator so that z_{11}, z_{22} and z_{12} must contain even function s at the denominator which is m_2.

From Fig.3.9, it can be shown that

$$Z_{12}' = \frac{z_{12}}{1 + z_{22}}$$

So that

$$Z_{12}' = \frac{(\sqrt{n_1 n_2 - m_1 m_2})/m_2}{1 + n_2/m_2}$$

$$= \frac{\sqrt{n_1 n_2 - m_1 m_2}}{m_2 + n_2} \qquad \cdots\cdots\cdots (3\cdot15)$$

$$z_{12} = \frac{E_2}{I_1} = Z_{12}' \times \frac{1}{1+Z_1}$$

$$= \frac{\sqrt{n_1 n_2 - m_1 m_2}}{m_2 + n_2} \times \frac{1}{1 + \frac{m_1+n_1}{m_2+n_2}}$$

$$= \frac{\sqrt{n_1 n_2 + m_1 m_2}}{m_1 + m_2 + n_1 + n_2}$$

$$|z_{12}(j\omega)|^2 = -\frac{n_1 n_2 - m_1 m_2}{(m_1+m_2)^2 - (n_1+n_2)^2}\Bigg|_{s=j\omega}$$

$$= \frac{m_1 m_2 - n_1 n_2}{(m_1+m_2)^2 - (n_1+n_2)^2}\Bigg|_{s=j\omega}$$

$$\left|\frac{1-z_1}{1+z_1}\right|^2_{s=j\omega} = \frac{m_2+n_2-(m_1+n_1)}{(m_2+m_1)^2-(n_2+n_1)^2}\Bigg|_{s=j\omega}$$

$$1 - \left|\frac{1-z_1}{1+z_1}\right|_{s=j\omega} = \frac{4(m_1 m_2 - n_1 n_2)}{(m_2+m_1)^2-(n_2+n_1)^2}\Bigg|_{s=j\omega}$$

$$\therefore \; 4|z_{12}(j\omega)|^2 = \left[1 - \left|\frac{1-z_1}{1+z_1}\right|^2_{s=j\omega}\right]$$

$\therefore$ According to the definition (3.1)

$$|t(j\omega)|^2 = \frac{4}{R_{in} R_2}|Z_{12}(j\omega)|^2$$

$$= 4|z_{12}(j\omega)|^2$$

$$= 1 - \left|\frac{1-z_1}{1+z_1}\right|^2 \qquad \cdots\cdots\cdots (3\cdot17)$$

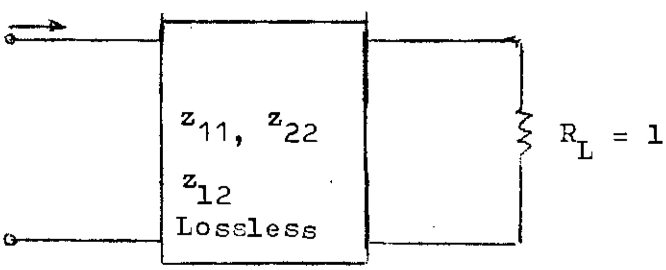


Fig.3.9

Now we consider the case with source resistance R_{in} and Load resistances R_L.

Input impedance to the lossless network can be written as

$$\frac{E_1}{I_1} = Z_1(jw) = R_{11} + j\, X_{11} .$$

Power entering and leaving lossless the network will be equal so that

$$|I_1'|^2 R_{11} = |E_2|^2 / R_L$$

For this case

$$\frac{I_1'}{I_1} = \frac{R_{in}}{R_{in} + Z_1}$$

$$\therefore \quad |I_1'|^2 = |I_1|^2 R_{in}^2 / |R_{in} + Z_1|^2$$

$$\therefore \quad \frac{E_2^2}{R_L R_{in} |I_1|^2} = \frac{|I_1|^2 R_{11}}{R_{in}} \times \frac{R_{in}^2}{|I_1'|^2 |R_{in} + Z_1|^2}$$

$$= \frac{R_{11} R_{in}}{|R_{in} + Z_1|^2}$$

$$|R_{in} - Z_1|^2 = (R_{in} - R_{11})^2 + X_{11}^2$$

$$|R_{in} + Z_1|^2 = (R_{in} + R_{11})^2 + X_{11}^2$$

$$1 - \left|\frac{R_{in} - Z_1}{R_{in} + Z_1}\right|^2 = 1 - \frac{(R_{in} - R_{11})^2 + X_{11}^2}{(R_{in} + R_{11})^2 + X_{11}^2}$$

$$= \frac{R_{in}^2 + R_{11}^2 + 2R_{in}R_{11} + X_{11}^2 - R_{in}^2 - R_{11}^2 + 2R_{in}R_{11} - X_{11}^2}{(R_{in} + R_{11})^2 + X_{11}^2}$$

$$= \frac{4R_{in}R_{11}}{(R_{in} + R_{11})^2 + X_{11}^2}$$

$$= \frac{4R_{in}R_{11}}{|R_1 + Z_1|^2}$$

$$\therefore \frac{|E_2|^2}{R_1R_2|I_1|^2} = \frac{R_{in}R_{11}}{|R_{in} + Z_1|^2}$$

$$= \frac{1}{4}\left(1 - \left|\frac{R_{in} - Z_1}{R_{in} + Z_1}\right|^2\right)$$

$$\therefore |t(j\omega)|^2 = \frac{4}{R_{in}R_L}|Z_{12}(j\omega)|^2$$

$$= 1 - \left|\frac{R_{in} - Z_1}{R_{in} + Z_1}\right|^2 \quad \cdots\cdots (3.18)$$

Thus the transmission coefficient i.e. the ratio of the power available at the load to the maximum deliverable power $|t(j\omega)|^2$ is given by

$$|t(j\omega)|^2 = 1 - \left| \frac{R_{in} - Z_1(j\omega)}{R_{in} + Z_1(j\omega)} \right|^2$$

$$= 1 - \left| \frac{1 - Z_1(j\omega)}{1 + Z_1(j\omega)} \right|^2 \quad \ldots (3.20) \quad \text{for } R_{in} = 1\Omega$$

$|\rho(j\omega)|^2$ the reflection function is defined by,

$$|\rho(j\omega)|^2 = 1 - |t(j\omega)|^2$$

$$\therefore |\rho(j\omega)|^2 = \left| \frac{R_{in} - Z_1(j\omega)}{R_{in} + Z_1(j\omega)} \right|^2 \quad \ldots\ldots (3.21)$$

$\therefore \rho(s)$ the reflection coefficient can be written as

$$\rho(s) = \frac{R_{in} - Z_1(s)}{R_{in} + Z_1(s)}$$

$$R_1 \rho(s) + Z_1(s)\rho(s) = R_{in} - Z_1(s)$$

$$Z_1(s)\left[1 + \rho(s)\right] = R_{in}\left[1 - \rho(s)\right]$$

$$Z_1(s) = R_{in} \frac{1 - \rho(s)}{1 + \rho(s)} \quad \ldots\ldots\ldots (3.23)$$

$$\frac{Z_1(s)}{R_{in}} = \frac{1 - \rho(s)}{1 + \rho(s)} \quad \ldots\ldots\ldots \quad (3.24)$$

The equation (3.24) maps the right half of the $Z_1(s)$ plan upon the interior of the unit circle of the $\rho(s)$ plane and vice-versa. Therefore if $Z_1(s)$ in p.r. then $Re[Z_1(s)] \geq 0$ for $Re(s) \geq 0$ According to the mapping property of equation (3.24), it then follows $|\rho(s)| \leq 1$ for $Re(s) \geq 0$. Conversely, if $|\rho(s)| \leq 1$ for $Re(s) \geq 0$, then $Z_1(s)$ must be p.r..

Thus p.r. property of $Z_1(s)$ can be assumed by the relation

$$|\rho(s)| \leq 1 \quad \text{-----------------------}(3.25)$$

i.e. $$|\rho(j\omega)|^2 \leq 1 \quad \text{- - - - - - - - - - - - - - -}(3.26)$$

But $|\rho(j\omega)|^2 = 1 - |T(j\omega)|^2$

So that if $|\rho(j\omega)|^2 \leq 1$

$$|T(j\omega)|^2 \leq 1 \quad \text{. . . . .} \; (3.27)$$

So that the condition of realizability of $|T(j\omega)|^2$ as a transmission function is

$$|T(j\omega)|^2 \leq 1 \quad \text{. . . . .} \; (3.27)$$

CHAPTER-4

THE METHOD OF APPROXIMATION

4.1 PRELIMINARIES:

This chapter describes in details the method of approximation.

In article 4.2 we describe in brief the Butterworth and Chebyshev methods of approximation of Low pass filter and the frequency transformation for band pass circuit. After network transformation of series resonator in a parallel resonator and a capacitor coupling, the transmission zero at infinity is changed to be transmission zero at origin. Thus the final band pass circuit by this method has all but one transmission zero at the origin and one at infinity.

In article 4.3 a general describtion of the method is given.

In article 4.4 band pass response curve is approximated by Fourier method. The value of the approximate response is exactly same at the chosen points. However it may be distorted at any other point between the specific points.

In article 4.5 method of obtaining the polynomial in w^2 is described . This is done with the help of Chebyshev polynomial. By Chebyshev polynomial cosine terms of Fourier series is converted into a polynomial of fundamental component Cos A, which is assumed to be equal to $\frac{1-w^2}{1+w^2}$ so that all the terms of Fourier expansion becomes polynomial of in w^2. Thus the polynomial is obtained. From this polynomial the network can be synthesized.

4.2 APPROXIMATION BY USE OF BUTTERWORTH AND CHEBYSHEV FUNCTIONS:

The ideal transmission function for filters which has the magnitude units for pass band and zero for stop band is not practically realizable. To be realizable $|t'(j\omega)|^2$ is to be expressed as a

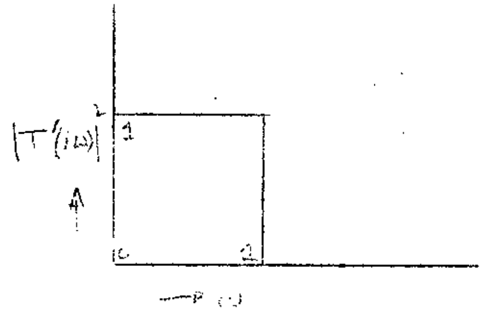


Fig.4.1

rational polynomial in w^2. Such a rational polynomial can not be obtained for the ideal transmission function shown in Fig.4.1 for low pass. Realizable rational function will be approximately equal to the ideal function. The difference between the ideal and the approximate (practically realizable) function will depend on the method of approximation. In general better approximation can be obtained by using higher order rational functions.

As a specific example of the approximation problem, let us consider the magnitude characteristic of the ideal low pass filter shown in Fig.4.1. There exists two polynomial approximations to it which are of great importance in both theory and application. One of these is named the Butterworth or maximally flat response and the other is named the Chebyshev or equal ripple response.

Assume that $|T'(j\omega)|^2$ is the squared magnitude of the normalised i.e. units bandwidth and units magnitude ideal low pass filter. Then a possible expression for

$$|T'(jw)|^2 = \frac{1}{1+F(w^2)} \qquad \ldots(4.1)$$

$$\text{where } F(w^2) = \begin{cases} 0 & \text{if } 0 \leqslant w \leqslant 1 \\ \infty & \text{if } w \quad 1 > 1 \end{cases}$$

Evidently a possible $F(w^2)$ is

$$F(w^2) = \lim_{n \to \infty} w^{2n} \qquad \ldots\ldots\ldots\ldots\ldots(4.2)$$

Which suggests an approximation for $F(w^2)$ as

$$F(w^2) \approx w^{2n} \quad (n \text{ finite}) \qquad ....(4.3)$$

Then

$$|T'(jw)| \approx \frac{1}{1 + w^{2n}} = |T(jw)|^2 \qquad .....(4.4)$$

Thus $|T(jw)|^2$ approximates the ideal function $|T'(jw)|^2$.

This approximation is named Butterworth approximation, and the filter is known as Butterworth filter. The normalised Butterworth magnitude function is given by

$$|T(jw)| = \frac{1}{\sqrt{1 + w^{2n}}} \qquad ....(4.5)$$

Another possible expression for $F(w^2)$ of Eq 4.1 is

$$F(w^2) = \lim_{n \to \infty} \epsilon^2 P_m^2(w) \qquad ....(4.6)$$

where p_n is an nth degree polynomial. If $p_n(w) = T_n(w)$ = nth order Chebyshev polynomial, and $0 \leq \epsilon \leq 1$, then similar to equation 2.3.

$$|T'(jw)| \approx \frac{1}{\sqrt{1 + \epsilon^2 T_n^2(w)}} = |T(jw)| \qquad ....(4.7)$$

$|T(jw)|$ is the normalised Chebyshev approximation to $|T'(jw)|$. The filter realizing $|T(jw)|$ is known as the Chebyshev filter. The trigonometric form for the polynomials $T_n(w)$ is given by

$$T_n(w) = \text{Cos}\,(n\ \text{Cos}^{-1} w), \quad w \leq 1$$
$$= \text{Cosh}\,(n\ \cosh^{-1} w) \quad w > 1 \qquad (4.8)$$

A recursive relation develops from (4.8) and yields polynomials

$$T_1 = w, \qquad T_2 = 2w^2 - 1 \qquad -------(4.9)$$
$$T_n = 4w^3 - 3w, \qquad T_{n+1} = 2wT_n - T_{n-1}$$

Plots of eqs. 4.5 and 4.7 will show that for the Butterworth function, pass band characteristic becomes flatter as n increase while for the Chebyshev function the pass band characteristic has n number of ripple peaks and valleys. However for both the cases, the cut off becomes shaper as n increases. For Butterworth filter due to its maximally flat character, closely approximadtes the ideal filter characteristics for low frequencies, however the error becomes large as frequency increases. On the other hand the deviation between the ideal characteristic and the Chebyshev response is spread out from w = 0 to w = 1 as a series of equal ripples.

Band pass response can be obtained by frequency transformation of the low pass response. The element values will also be changed for the transformed band pass filter. This is explained previously in Chapter 2.

For the band frequency $\bar{w}$, $|T(j\bar{w})|^2$ can be calculated for Butterworth response.

$$w = \frac{\bar{w}^2 - w_o^2}{\bar{w} B} \qquad \begin{array}{l} w = \text{low pass frequency} \\ \bar{w} = \text{Band pass frequency.} \end{array}$$

$$|T(jw)|^2 = \frac{1}{1+w^{2n}}$$

$$|T(j\bar{w})|^2 = \frac{1}{1 + \left(\frac{\bar{w}^2 - w_o^2}{\bar{w} B} \right)^{2n}}$$

$$= \frac{\bar{w}^{2n} \; B^{2n}}{(\bar{w} B)^{2n} + (\bar{w}^2 - w_o^2)^{2n}}$$

$$= \frac{\bar{w}^{2n}}{A_o + A_1 \bar{w}^2 + \ldots\ldots\ldots + A_{2n} \bar{w}^{4n}} \qquad \ldots\ldots\ldots(4.10)$$

Similar expression can be obtained for Chebyshev function also.

Equation (4.10) is similar to Eqn. 3.9 except the difference in the numerator. The numerator obtained from the circuit analysis is $w^{2(2n-1)}$, whereas in this case it is w^{2n}. This is due to the fact that the transformed band pass filter, being practically difficult to construct, is to be modified and the series resonators are converted to the parallel resonators with additional capacitors between the resonator as shown in Fig.4.2.

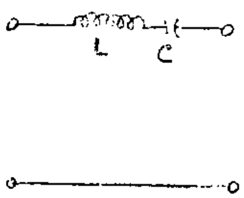


Fig.4.2(a)

Band pass series branch Before modification.

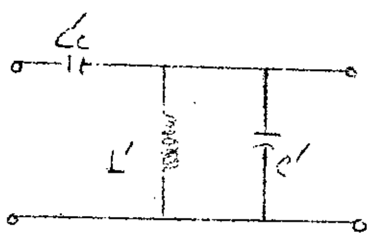


Fig.4.2(b)

Band pass parallel resonator corresponding to series resonator of 4.2(a) after modification.

The circuit of Fig.4.2(a) has a transmission zero at origin due to capacitance and a transmission zero at infinity due to inductance. The circuit of Fig.4.2(b) has a transmission zero at origin due to series capacitance and a transmission zero at origin due to shunt inductance. The shunt capacitance can not have any independent transmission zero. All such capacitances of all the branches will contribute one transmission zero at infinity. Because at that time the shunt inductances will become open (infinite impedance) and the series capacitances will become shorted (zero impedance). So that all the shunt capacitances will contribute one transmission zero at $s=\infty$.

For this reason all the transmission zero of the modified circuit will be at the origin except only one which is at infinity. For this reason equation 3.5, has $w^{2(2n-1)}$ tern as numerator indicating all the transmission zeroes except one at the origin while the remaining one at infinity whereas equation 4.10contains w^{2n} as the numerator indicating equal number of transmission zeros at the origin and at infinity.

For this reason, the response of the final network obtained by Butterworth or Chebyshev function and after being transformed for band pass after modification to be easily constructable becomes smaller at frequencies near origin than that at frequencies near infinity, that means for frequency lower than the centre frequency the value of $|T\ (jw)|^2$ is smaller than that at the frequency higher than the centre frequency by the same amount. Moreover this difference will increase for higher order filters.

4.3. GENERAL DESCRIPTION OF THE FOURIER METHOD OF APPROXIMATION

The Fourier series expansion and the Chebyshev polynomial may be simultaneously used for a rational function approximation of a given band pass response so that the rational function, $|T\ (jw)|^2$, thus obtained can be synthesized in a network configuration consisting of shunt resonators coupled by capacitors. In article 3.2 we have shown that for such a network configuration $|T\ (jw)|^2$, the transmission function, will have the form

$$|T(j\omega)|^2 = \frac{\omega^{2(2m-1)}}{AR_1 + AR_2\omega^2 + AR_3\omega^4 + \cdots + AR_{2m+1}\omega^{4m}} \qquad \cdots (4.11)$$

(where m is the number of resonators).

Equation (4.11) can be written as

$$|T(j\omega)|^2 = \frac{\dfrac{\omega^{2(2m-1)}}{(1+\omega^2)^{2m}}}{\dfrac{AR_1 + AR_2\omega^2 + AR_3\omega^4 + \cdots + AR_{2m+1}\omega^{4m}}{(1+\omega^2)^{2m}}} \quad --(4.12)$$

$$= \frac{Y(\omega^2)}{XX(\omega^2)}$$

where
$$Y(\omega^2) = \frac{\omega^{2(2m-1)}}{(1+\omega^2)^{2m}} \quad -------(4.13)$$

$$XX(\omega^2) = \frac{AR_1 + AR_2\omega^2 + AR_3\omega^4 + \cdots + AR_{2m+1}\omega^{4m}}{(1+\omega^2)^{2m}} \quad --(4.14)$$

By conversion of the frequency variable, w, in to a new variable, A, so that the range of w from 0 to ∞ will be changed from 0 to π for A, $XX(w^2)$ can be converted into F(A) such that F (A) can be expanded in a trigometric series (4.15) by Fourier series expansion.

$$F(A) = A_1 + A_2 \cos A + A_3 \cos 2A + \cdots\cdots + A_n \cos(n-1)A \quad \ldots\ldots(4.15).$$

The value of F(A) at some A is same as the value of $XX(w^2)$ at corresponding w. The values of $A_1, A_2 \cdots A_n$ can be found out from n known values of F(A) corresp8nding to n values of A. Thus if $|T(jw)|^2$ be given for n values w, then $XX(w^2)$ and A can be calculated corresponding to these n values of $|T(jw)|^2$ and w. We thus obtain n values of F(A) corresponding to n values of A. We then obtain n number of equations from (4.15) involving n number of unknown A_i's so that A_i (i = 1 to n) can be obtained from these n equations.

Knowing the values of A_1, A_2 ....A_n for equation (4.15), equation (4.14) can be obtained with the help of Chebyshev polynomial by converting A into w. which isshown below.

The Chebyshev polynomial of order n is defined by

$$T_n(x) = \text{Cos}\ (n \cos^{-1} x) \quad \ldots\ldots(4.16).$$

Let us assume,

$$x = \text{Cos}\ A, \qquad A = \text{Cos}^{-1} x \quad \ldots\ldots(4.17)$$

Then (4.16) becomes

$$T_n(x) = \cos n A$$

$$\left.\begin{aligned} &= \text{A polynomial in } \cos A \\ &= \text{A polynomial in } x \end{aligned}\right) \quad \ldots\ldots(4.1o,$$

With the help of equation (4.18) , the trigonometric series (4.15) can be converted into an equivalent series, G(x), a polynomial in x where

$$G\ (x) = B_1 + B_2 \quad B_3\ x^2 + \ldots\ldots\ldots\ldots + B_n\ x^{n-1} \ldots\ldots(4.19).$$

The coefficients B_1, B_2.........B_n in the polynomial (4.19) ... be computed by substituting for each cosine term of series (4.15) by its equivalent polynomial in x according to (4 .18) and collecting the coefficients of like powers of x.

The transformation of w into A can be obtained by the equation

$$A = 2 \tan^{-1} w, \qquad w = \tan \frac{A}{2} \quad \ldots\ldots(4.20)$$

Plot of w and A is given in figure 4.3 From the figure, it is clear that the change of the variable w into A transforms w from - ∞ to ∞ into A from $-\pi$ to π so that $XX(w^2)$, an aperiodic function **of w will be transformed into F(A), a periodic function** of A, which can be expanded in Fourier series.

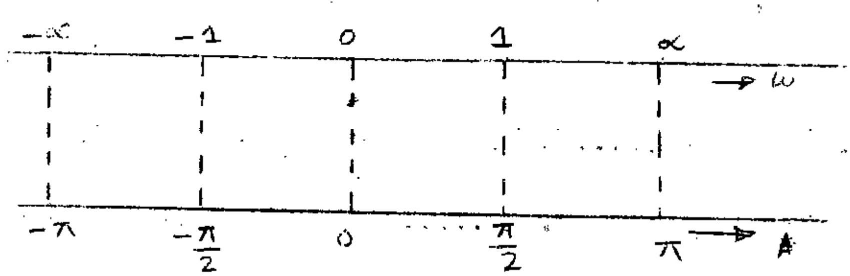


Fig 4.3

From the equation (4.20)

$$\omega^2 = \tan^2 \frac{A}{2} = \frac{\sin^2 A/2}{\cos^2 A/2} = \frac{1-\cos^2 A/2}{\cos^2 A/2}$$

$$= \frac{2-2\cos^2 A/2}{2\cos^2 A/2} = \frac{2-(\cos A+1)}{\cos A+1}$$

$$= \frac{1-\cos A}{1+\cos A} = \frac{1-x}{1+x}$$

i.e.

$$\omega^2 = \frac{1-x}{1+x} \quad \text{-------------------} (4.21)$$

$$x = \frac{1-\omega^2}{1+\omega^2} \quad \text{-------------------} (4.22)$$

Substituting the value of x in equation (4.20)

$$G(x) = B_1 + B_2 x + B_3 x^3 + \cdots\cdots + B_n x^{n-1} \quad \cdots (4.20)$$

$$XX(\omega) = B_1 + B_2 \frac{1-\omega^2}{1+\omega^2} + B_3\left(\frac{1-\omega^2}{1+\omega^2}\right)^2 + \cdots\cdots + B_n\left(\frac{1-\omega^2}{1+\omega^2}\right)^{n-1} \quad (4.23)$$

$$= \frac{AR_1 + AR_2\omega^2 + AR_3\omega^4 + \cdots\cdots + AR_n\omega^{2(n-1)}}{(1+\omega^2)^{n-1}}$$

$$\cdots (4.24)$$

AR_1, AR_2 ....AR_n can be obtained from the known values of B_1, B_2 --------B_n and the expansion of (4.23)

Equations (4.24) and (4.14) are same for

$$n-1 = 2m$$

$$\text{i.e. } n = 2m+1$$

Here m is the number of resonators and n is the number of required known values of $XX(w^2)$ i.e. $|T(jw)|^2$. This means that for a filter with m resonators, (2m +1) values of Transmission function are to be taken to get the polynomial approximation .

4.4 EXPANSION IN FOURIER SERIES OF COSINE TERMS

In equation (4.24), the denominator $XX(w^2)$ of transmission function $|T(jw)|^2$ is expressed as a rational polynomial of w^2. The values of coefficients of expansion, AR_i's can be calculated by changing the w by A and expressing XX as a Fourier series of cosine terms.

$$XX(A) = R_1 + R_2 \cos A + R_3 \cos 2A + \ldots\ldots\ldots + R_n \cos(n-1) \ldots \qquad \ldots\ldots(4.25)$$

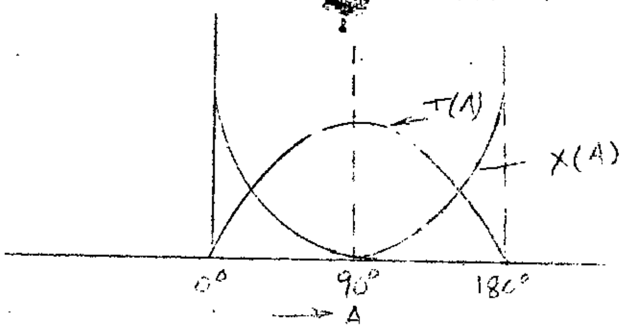


Fig 4.4

The range of w from 0 to ∞ has been taken equivalent to the range of A from 0 to 180°. For the range -180° to 0°, symmetry may be assumed very easily as shown in fig. 4.4. Then the Fourier series expansion will involve cosine terms only. The approximation problem may be solved by taking n different value of $|T(jw)|^2$ corresponding to n values of w^2 and then calculating corresponding n values of XX (w^2) from $|T(jw)|^2$ and Y (w^2) and n values of A from w. Putting these values, equation (4-25) becomes n number of equations for n number of unknown values R, so that these values of R can be calculated by solving the equations (4-26)

$$R_1 + R_2 \cos A_1 + R_3 \cos 2A_1 + \ldots\ldots\ldots\ldots + R_n \cos(n-1)A_1 = XX_1$$

$$\vdots$$

$$R_1 + R_2 \cos A_n + R_3 \cos 2A_n + \ldots\ldots\ldots\ldots + R_n \cos(n-1)A_n = XX_n \qquad ---(4\text{-}26)$$

In matrix form

$$\begin{bmatrix} 1 & \cos A_1 & \cos 2A_1 & \cdots & \cos (n-1)A_1 \\ \vdots & & & & \\ 1 & \cos A_n & \cos 2A_n & \cdots & \cos(n-1)A_n \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} = \begin{bmatrix} XX_1 \\ \vdots \\ XX_n \end{bmatrix} \qquad \ldots\ldots(4.27)$$

$$[A]\ [R] = [XX] \qquad \ldots\ldots\ldots(4.28)$$

$$[R] = [A^{-1}]\ [XX] \qquad \ldots\ldots\ldots(4\text{-}28\ a)$$

R is a column matrix.

Thus from n known values of $|T(jw)|^2$, we get $R_1 \ldots\ldots R_n$. Then we get the approximate continuous function XX(A) according to equation (4.25). This continuous function XX(A) will have the exactly equal values for the specified points. The intermediate point may have values not permitted to the specification if the values of n fixed points are not chosen properly. Out of these n fixed points 3 points one at centre frequency $w_o = 1$ rad for normalised frequency scale, and two at the cutt off frequencies will be fixed. The value of $|T(jw)|^2$ at the center frequency is unity, at the two cut off frequencies is 0.5. The cutt off frequency points will be fixed (values of w) by the specification of the band width. The stopband attenuation will be specified. The value of $|T(jw)|^2$ is at any other frequency which is at stop band range will be lower than the specified. (obtained from the specified stopband attenuation). For each point, XX(A) will be found for $|T(jw)|^2$ and $Y(w^2)$ and value of A will be found from corresponding value of w. Thus knowing n values of XX corresponding to n values of A, we can calculate n values of R by (4.28) and have the continuous expression XX(A) from eq.(4.25).

Assuming a fixed curve, $XX(w^2)$ can be approximated by a Fourier series expansion taking n number of terms. The fixed curve should have symmetry so that the expansion consist of cosine terms only. One such assumed curve is shown in 5.1. Fourier series expansion of this curve will consist of cosine terms only assuming the symmetry described previously.

4.5 EXPRESSING IN POLYNOMIAL OF w^2

In the previous article we explain the procedure to transmission function as a cosine series of n terms . This cosine seri... can be expressed as a polynomial in w^2 by expanding cos n A in te... of cos A according the the Chebyshev polynomial:

$$XX(A) = R_1 + R_2 \cos A + R_3 \cos 2A + \ldots\ldots + R_n \cos(n-1)\ldots$$

$$= R_1 + R_2 T_1(x) + R_3 T_2(x) + \ldots\ldots + R_n T_{n-1}(x)$$

$$= \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix} \begin{bmatrix} 1 \\ T_1 \\ T_2 \\ \vdots \\ T_{n-1} \end{bmatrix} \ldots\ldots\ldots \quad \textbf{(4.29)}$$

By use of eq. (4.18) each term of the column matrix $\begin{bmatrix} 1 & T_1 & T_2 \ldots T_{n-1} \end{bmatrix}$ in the right side of eq. (4.26) is a polynomial in x, so that this can be written as

$$\begin{bmatrix} 1 \\ T_1 \\ T_2 \\ \vdots \\ \vdots \\ T_{n-1} \end{bmatrix} = \begin{bmatrix} CP_{11} & CP_{12} & \ldots & CP_{1n} \\ & & & \\ \vdots & & & \vdots \\ CP_{N1} & CP_{n2} & \ldots & CP_{nn} \end{bmatrix} \begin{bmatrix} 1 \\ x \\ x^2 \\ \vdots \\ \vdots \\ x^{n-1} \end{bmatrix}$$

$$\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} CP \end{bmatrix} \begin{bmatrix} X \end{bmatrix} \ldots\ldots\ldots \quad \textbf{(4.30)}$$

The **rows** of CP are the coefficients of the Chebyshev polynomial, **row** 1 for T_0, **row** 2 for T_1 and so on, the last **row** is for T_{n-1} .

Again the column matrix $[(X)]$, can be expressed as a functi[illegible]r of w^2 by the relation (4.12) i.e.

$$x = \frac{1 - w^2}{1 + w^2} \qquad \text{.......... (4.[illegible])}$$

$$\begin{bmatrix} 1 \\ x \\ x^2 \\ \vdots \\ x^{n-1} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1-\omega^2}{1+\omega^2} \\ \vdots \\ \left(\frac{1-\omega^2}{1+\omega^2}\right)^{n-1} \end{bmatrix} = \frac{1}{(1+\omega^2)^{n-1}} \begin{bmatrix} (1+\omega^2)^{n-1} & 1 \\ (1+\omega^2)^{n-2} & (1-\omega^2) \\ \vdots & \\ 1 & (1-\omega^2)^{n-1} \end{bmatrix} \qquad \text{--- (4.31)}$$

Each term of the matrix on the right side of eqn.(4.[illegible]) can again be expressed as a polynomial in w^2.

$$\begin{bmatrix} (1+\omega^2)^{n-1} & \\ (1+\omega^2)^{n-2} & (1-\omega^2) \\ (1+\omega^2)^{n-3} & (1-\omega^2)^2 \\ \vdots & \\ 1 & (1-\omega^2)^{n-1} \end{bmatrix} =$$

$$\begin{bmatrix} (AW_{n1} + AW_{n2}\omega^2 + \cdots + AW_{nn}\omega^{2(n-1)})(BW_{11} + BW_{12}\omega^2 + \cdots + BW_{1n}\omega^{2(n-1)}) \\ \vdots \\ (AW_{11} + AW_{12}\omega^2 + \cdots + AW_{1n}\omega^{2(n-1)})(BW_{n1} + BW_{n2}\omega^2 + \cdots + BW_{nn}\omega^{2(n-1)}) \end{bmatrix} \qquad \cdots \text{(4.32)}$$

AW_{n1}, AW_{n2} .... AW_{nn} are the coefficients of the binomial expansion $(1+w^2)^{n-1}$, BW_{n1}, BW_{n2}.......... BW_{nn} are the coefficients of binomial expansion $(1-w^2)^{n-1}$. BW_{12}, B_{13}........B_{1n} are equal to zero, so that the highest order of the polynomial of 1st column is $w^{2(n-1)}$. Similarly BW_{23}, BW_{24} ....BW_{2n} are equal to zero and AW_{2n} is also equal to zero so that the highest order of the polynomial of the second column is also $w^{2(n-1)}$. Thus it can be explained that the highest order of the polynomial of each column of matrix is $w^{2(n-1)}$. So that this can be written as

$$\begin{bmatrix} 1 \\ \frac{1-\omega^2}{1+\omega^2} \\ \vdots \\ \left(\frac{1-\omega^2}{1+\omega^2}\right)^{n-1} \end{bmatrix} = \frac{1}{(1+\omega^2)^{n-1}} \begin{bmatrix} CW_{11} & CW_{12} & \cdots & CW_{1n} \\ \vdots & & & \\ CW_{n1} & CW_{n2} & \cdots & CW_{nn} \end{bmatrix} \begin{bmatrix} \omega^0 \\ \vdots \\ \omega^{2(n-1)} \end{bmatrix} \quad (4.33)$$

Now the equations, may be written in matrix form

$XX(A) = [R]\,[T]$ .....**(4.34)** R is row matrix
T is colum matrix

$[T] = [CP]\,[X]$ ....**(4.35)** CP is square matrix.
X is column matrix.

$[X] = \frac{1}{(1+w^2)^{n-1}}\,[CW]\,[W]$ ....**(4.36)** CW is square matrix.
W is a column matrix.

From these equation (4.31),

$$XX(w^2) = [R]\,[T]$$

$$= [R]\,[CP]\,[X]$$

$$= \frac{1}{(1+\omega^2)^{n-1}} [R] [CP] [CW] [W]$$

$$= \frac{[R][CP][CW] \cdot [W]}{(1+\omega^2)^{n-1}} \quad \text{- - - -} \quad (4.37)$$

$[R] [CP] [CW] [W]$

$$= [R] [CPW] [W]$$

$$= [W]^T [CPW]^T [R]^T \quad \ldots\ldots\ldots \quad (4.38)$$

$[R]^T$ is a column matrix which equal to the matri[x] $[R]$ of $[R]$ of equation (4.18). By equation (4.18)

$$[R]^T = A^{-1} [XX] \quad \ldots\ldots\ldots \quad (4.39)$$

From (4.32), (4.33) and (4.34)

$$XX(\omega^2) = \frac{[W]^T [CPW]^T A^{-1} [XX]}{[1+\omega^2]^{n-1}}$$

$$= \frac{[1 \quad \omega^2 \ldots\ldots\ldots \omega^{2(n-1)}] [AC] \begin{bmatrix} XX_1 \\ \vdots \\ XX_n \end{bmatrix}}{(1+\omega^2)^{n-1}}$$

$$= [W]^T [AR] / (1+\omega^2)^{n-1}$$

$$= \frac{AR_1 + AR_2 \omega^2 + \ldots\ldots\ldots + AR_n \omega^{2(n-1)}}{(1+\omega^2)^{n-1}}$$

AR is a column matrix given by

$$AR = [AC] \begin{bmatrix} XX_1 \\ \vdots \\ XX_n \end{bmatrix}$$

$$= [CPW]^T [A^{-1}] [XX]$$

$$= [CW]^T [CP]^T [A^{-1}] [XX]$$

$$= [CR] [A^{-1}] [XX]$$

where

$$[CR] = [CW]^T [CP]^T$$

Thus we get $AR_1, AR_2 \ldots\ldots AR_n$ and from eq (4.2.1) we get $XX(\omega^2)$ and from eq. (4.12) we get $|T(j\omega)|^2$

From eq. (4.12)

$$|T(j\omega)|^2 = \frac{\omega^{2(2m-1)} / (1+\omega^2)^{2m}}{\{AR_1 + AR_2\omega^2 + AR_3\omega^4 + \cdots + AR_{2m+1}\omega^{4m}\}/(1+\omega^2)^{2m}}$$

$$= \frac{\omega^{2(2m-1)}}{AR_1 + AR_2\omega^2 + AR_3\omega^4 + \cdots\cdots + AR_{2m+1}\omega^{4m}}$$

Here $2m = n-1$

$$|T(j\omega)|^2 = \frac{\omega^{2(n-1)}}{AR_1 + AR_2\omega^2 + AR_3\omega^4 + \cdots + AR_n\omega^{2(n-1)}}$$

for $m = 2$, $n = 5$

$$|T(j\omega)|^2 = \frac{\omega^6}{AR_1 + AR_2\omega^2 + AR_3\omega^4 + AR_4\omega^6 + AR_5\omega^8}$$

For $m = 3$, $n = 7$

$$|T(j\omega)|^2 = \frac{\omega^{10}}{AR_1 + AR_2\omega^2 + AR_3\omega^4 + AR_4\omega^6 + AR_5\omega^8 + AR_6\omega^{10} + AR_7\omega^{12}}$$

and so on.

CHAPTER-5

SYNTHESIS OF THE BANDPASS FILTERS

5.1. PRELIMINARIES:

The synthesis of band pass filter by this method essentially consists of three parts, the first is the approximation, the second is the realization of the network and the third is the transformation of the network.

In this chapter we discuss these three parts separately.

Approximation is done by two methods, one by approximating denominator of $|T(jw)|^2$ assuming n number of values and expanding in Fourier series of cosine terms and the second by assuming a fix curve. Since all but one transmission zeroes are at origin and the rest is at infinity, the realization can be done by Ladder development of the input impedance realizing shaunt inductance and series capacitance each time. After realizing all the transmission zero at origin, the one at infinity is realized by a shunt capacitance.

Capacitance matrix transmission of each section of the filter may be used to get the filter realized in the usual form of parallel resonators coupled by capacitances.

5.2 APPROXIMATION PROBLEM:

In the analysis of the band pass filters consisting of resonators coupled by capacitors we have shown (art.3.1) that the transmission function of such a filter has the form

$$|T(j\omega)|^2 = \frac{\omega^{2(n-2)}}{A_1 + A_2\omega^2 + \cdots + A_n\omega^{2(n-1)}} \quad \text{---(5.1)}$$

Where, $2(n-1) = 4m$, m being the order of the filter

i.e. $n = 2m + 1$ ....(5.2).

$|T(jw)|^2$ may be obtained from n different values of $|T(jw)|^2$ corresponding to n values of w, the frequency variable.

In article 3.2 we have shown that to realize the network, the value of $|T(jw)|^2$ must be such that

$$0 \leq T(jw)^2 \leq 1 \quad \text{.........................(5.3).}$$

This condition is also sufficient for $|T(jw)|^2$ to be realize[illegible]

Therefore we have to obtain the expression 5.1 for $|T(jw)|^2$ so that it satisfies 5.3. For getting the continuous function $|T(jw)|^2$ in polynomial form we have to take n values of $|T(jw)|^2$ correspon-ding n different values of frequency variable, w. After getting the continuous expression of $|T(jw)|^2$, it may exceed the range 5.3 for other values of w. This is the main problem of approximation.

The required specifications, generally given, will fix up 5 such points, one for band centre, two for band edges and two for required attenuation at some different values of w. Approximation and network obtained by these five points will give a filter of order 2 which is easily seen from equation 5.2 (n=5, m=2). For such a filter of order 2 has a maximum limit of attenuation. Beyond this limit, the continuous function $|T(jw)|^2$, exceed the limit (5.3). So that it can not be realized.

Higher attenuation will be obtained if we increase the order of the filter from 2. For order 3, (n= 2 m+1 = 2.3 +1 = 7) 7 points will be required. Five given points and two assumed points will then be required to solve the problem of satisfying equation 5.3. The two assumed $|T(jw)|^2$ values for two w values is to be within the specified tolerance. Suppose at $w = 2w_c$, attenuation A, is specified. Then the assumed point may be at $w = 3 W_c$ and the value of $|T($[illegible] may be taken more than A. By changing this value we can have the approximate function satisfying the range (5.3). If still higher attenuation is required than two more points will be taken zand the specification can be satisfied along with the condition 5.3, If $|T(jw)|^2$ becomes greater than 1 by a small value, then by dividing $|T(jw)|^2$ its maximum value, the range 5.3 may be satisfied. Then the realized net work will have response of new $|T(jw)|^2$ after division and the bandwidth will be changed. But if $|T(jw)|^2$ becomes smaller than zero, i.e. negative then the change of values of $|T(jw)|^2$ at the chosen w will be required.

From equation (4.12)

$$|T(j\omega)|^2 = \frac{Y(\omega^2)}{XX(\omega^2)}, \quad where \quad Y(\omega^2) = \frac{\omega^{2(n-2)}}{(1+\omega^2)^{n-1}}$$

Thus $Y(w^2)$ is always positive. Moreover when n is fixed, the value of $Y(w^2)$ at any w can be easily calculated. The approximation is done with the value of $XX(w^2)$ which is given by

$$XX(w^2) = \frac{Y(w^2)}{|T(jw)|^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5.4)$$

For the values of A from 0 to 180° (i.e. for w from 0 to ∞), the value of XX(A) is shown in Figure 1. The values of $XX(w^2)$ is calculated for specific values of $|T(jw)|^2$ at $A = 90^\circ$ and higher. Then symmetry is assumed for XX(A) at left and right side of 90°. So that the values of XX(A) at lower points is assumed to be same as corresponding higher points. Value at 80° equal to the value at 100°, value at 75° equal to the value at 105°, and so on. By this assumption odd harmonics will become zero. The value of XX(A) is given by

$$XX(A) = R_1 + R_3 \cos 2A + R_5 \cos 4A.$$

and the solution of the problem will become easier.

For the approximation assuming a fixed curve, the curve is taken to be as shown in fig. 5.1.

The approximate values of the curve obtained by Fourier analysis is

$$XX(A) = R_o + R_1 \cos A + R_2 \cos 2A + \text{------} + R_n \cos nA \qquad \ldots\ldots 5.5(a)$$

Where R_n is given by the relation

$$R_n = \frac{2XX_m}{\pi n^2}\left[\frac{1}{x_1 - x_2}\right]\left[\cos nx_2 - \cos nx_1\right] + \frac{2XX_m}{\pi n^2}\left[\frac{1}{x_3 - \frac{\pi}{2} + x_2}\right]\left[\cos nx_3 - \cos nx_2\right] \text{----} 5.5(b)$$

R_o has been calculated from the values of R_1, R_2, R_3 so that at $A=90^o$, the value of XX(A) becomes equal to the value Y(A). Because at $=90^o$, the value of the transmission function $T(jw)$ ^{2} should be equal to unity.

By this process, attenuation may be increased by increasing the value of XX_m. But at the same time the band width of the filter will be decreased. Band width can be increased by decreasing the value of x_2. By increasing the values of x_1, the attenuation may be increased, but for this case also the band width of the filter will be decreased.

Y(A) which is fixed, increases if A increases from 90^o for a specific range. So that in this range, if we can increase XX(A). he response will become uniform. This can be done by decreasing the value of X_3.

Assuming X_3 to be equal to $(180^o - X_2)$ the value of R_1, R_3......... from equation 5.5(b) becomes zero. So the odd harmonics will becomes zero. The solution of the problem will become easier. The values of

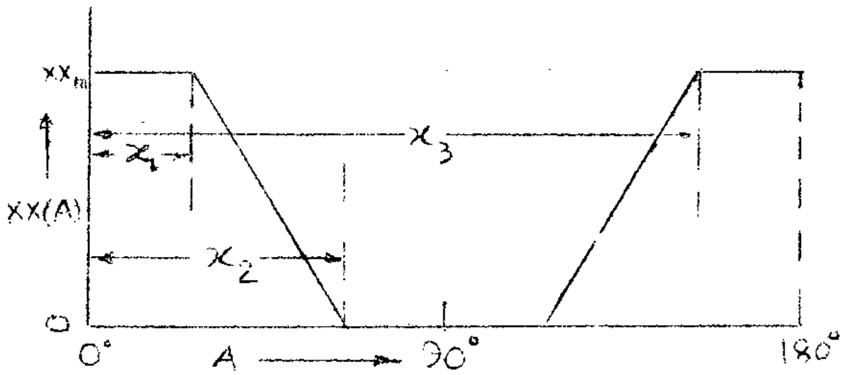
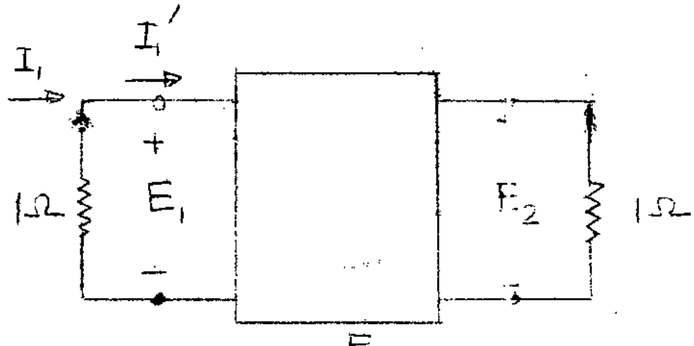


Fig. 5.1

Assumed curve XX(A)



$Z_1 = \frac{E_1}{I_1'}$

Fig. 5.2

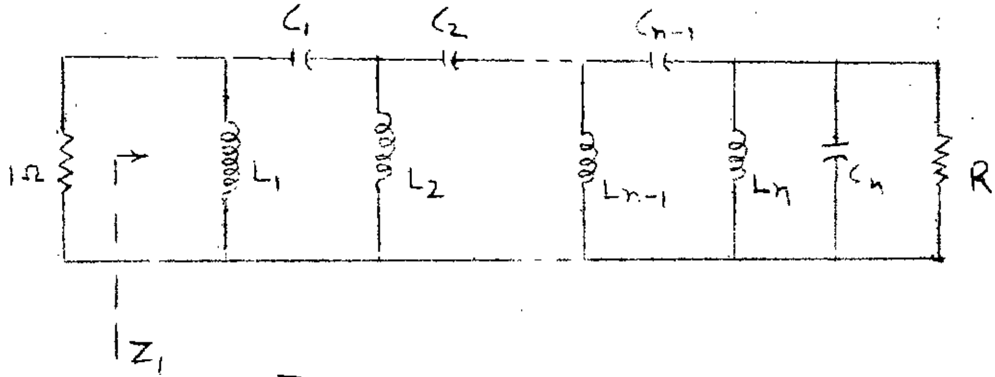


Fig. 5.3

Realization of input impedance Z_1 of Fig 5.2 in Ladder form

XX(A) at stop band range is higher, at 90° it is very small as compared to the values at stop band. So that the odd harmonics effects XX(A) very much near 90° and it may be negative at frequencies near 90°. For this reason this assumption of symmetry about 90°, makes the approximation problem easier.

5.3 REALIZATION OF THE FILTER NETWORK

After getting $|T(jw)|^2$, we shall synthesize the filter from the input impedance by ladder development.

For the circuit shown in figure, 5.2

$$Z_1 = \frac{E_1}{I_1'} = \frac{m_1 + n_1}{m_2 + n_2}$$

$|T(jw)|^2$ is obtained by approximation.

The reflection function $|\rho(jw)|^2$ is given by

$$|\rho(j\omega)|^2 = 1 - |T(j\omega)|^2 = 1 - \frac{B(\omega^2)}{A(\omega^2)}$$

$$= \frac{A(\omega^2) - B(\omega^2)}{A(\omega^2)} \quad --- (5.5)$$

$$= \rho(s)\,\rho(-s)\Big|_{s=j\omega}$$

$$\rho(s) = \frac{1 - Z_1(s)}{1 + Z_1(s)} = \frac{(m_2 - m_1) + (n_2 - n_1)}{(m_1 + m_2) + (n_2 + n_1)} \quad --- (5.6)$$

$$Z_1(s) = \frac{1 - \rho(s)}{1 + \rho(s)} \quad ------- (5.7)$$

From the transmission function $|T(jw)|^2$ we can calculate $|\rho(jw)|^2$, the reflection function.

ρ (s) will be obtained from $|\rho(jw)|^2$ in 5.5 by finding out the roots of the equations

$$A(-s^2) = 0, \quad A(-s^2) - B(-s^2) = 0.$$

and collecting the left half zeros of the denomination and the numerator polynomial.

$$A(-s^2) = (s \pm \alpha_1 \pm j\omega_1)(s \pm \alpha_2 \pm j\omega_2)\cdots \quad --(5.9)$$

Then the denominator $G(s)$, of $f(s)$ is given by

$$G(s) = (s + \alpha_1 \pm j\omega_1)(s + \alpha_2 \pm j\omega_2)\cdots \quad --(5.10)$$

Similarly numerator of $f(s)$ can also be obtained. For numerators selecting left half zeros is not necessary. However if we take the left zeros and calculate the numerator polynomial of (s), The resultant network synthesised will have gain×band width to be maximum.

Thus we obtain

$$f(s) = \frac{(m_2 - m_1)(n_2 - n_1)}{(m_2 + m_1)(n_2 + n_1)} = \frac{H(s)}{G(s)} \quad ----(5.11)$$

G(s) and H(s) calculated in this manner are polynomials of S.

From equation 5-11.

$$2m_2 + 2n_2 = G(s) + H(s)$$

$$m_2 + n_2 = \frac{G(s) + H(s)}{2} \quad ----(5\text{-}12)$$

Similarly

$$m_1 + n_1 = \left[G(S) - H(S)\right] / 2 \qquad (5\text{-}13)$$

Thus we get the expression of $Z_1 = \frac{E_1}{I}$

$$Z_1 = \frac{m_1 + n_1}{m_2 + n_2} \quad -------(5\text{-}14)$$

Since all except one transmission zeroes are at origin, we can develop $Z_1(s)$ in a ladder form shown in the fig 5.3, transmission zero at infinity will be synthesised by the last capacitance.

This network is potentially equivalent to the band pass filter consisting of resonators coupled by capacitors. So that changing the internal capacitance matrices such that inputimpedance remain invariant, we can get the required filter configuration.

5.4 NETWORK TRANSFORMATION:

In the synthesis procedure described in article 5.3, we obtained the filter network by ladder development of Z_1, the input impedance. If we change the network, so that Z_1 remains constant, then the changed network will also have the same transmission property. A real nonsingular transformations of the loop currents in the network, keeping the input loop current i_1 to be constant, will lead to networks involving a number of variations in structure and element values, while presenting the same input impedance at the driving point.

By this process we can change the terminating resistance R of the synthesized network to be equatl to 1, the normalised value. The impedance level of the output terminal is to be changed.

For this change an additional capacitance C will be required which is shown below.

We want to change R to be 1 so that remains constant. For this the inductance L_n will have a new value L_n/R. The capacitance matrix for C_{n-1} and C_n for the circuit of Fig. 5.3 can be written as

$$\begin{bmatrix} C_{n-1} & -C_{n-1} \\ -C_{n-1} & C_n + C_{n-1} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.15)$$

A new capacitance matrix written as

$$\begin{bmatrix} C_{n-1} & -C_{n-1} \times \sqrt{R} \\ -C_{n-1} \times \sqrt{R} & R\,(C_n + C_{n-1}) \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.16)$$

will keep Z constant and change the output side so as to include the change in R for the capacitance. For the capacitance matrix (2)

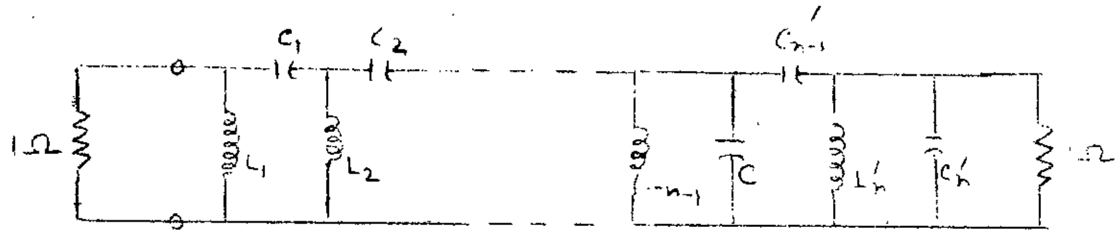


Fig 5.4

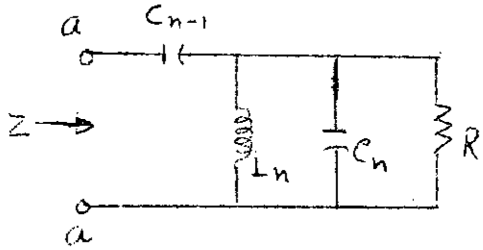


Fig 5.5

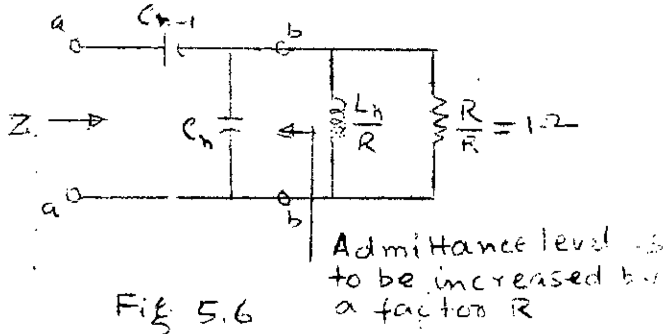


Fig 5.6

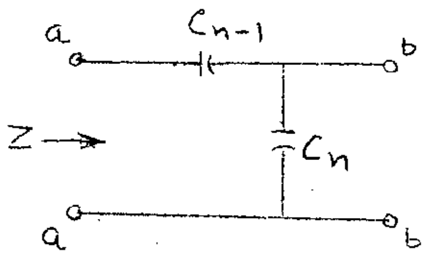


Fig 5.7

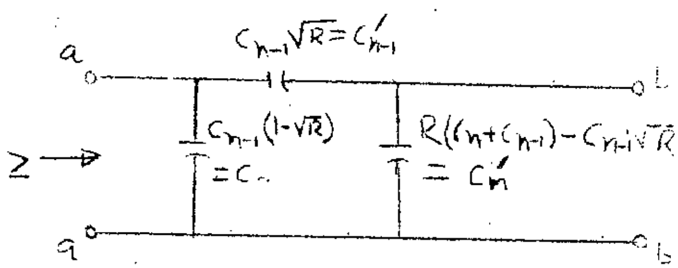


Fig 5.8

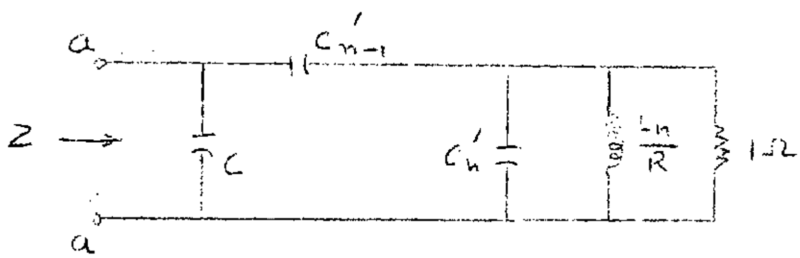
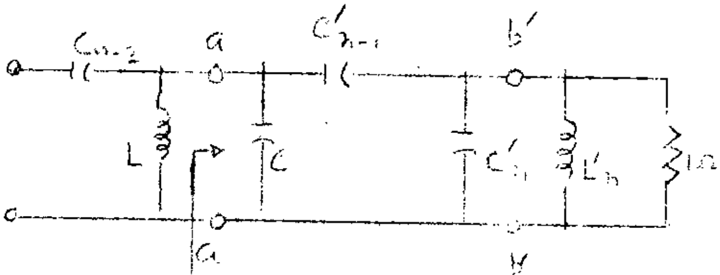


Fig 5.9

Transformation of input impedance so that Load resitance becomes 1Ω



Admittance level to be increased at aa

Fig 5.10

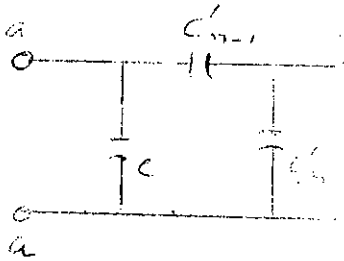
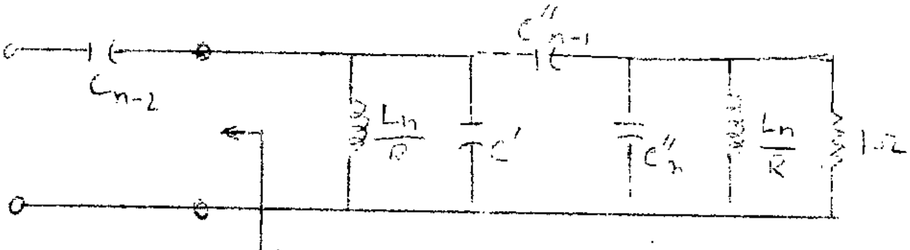
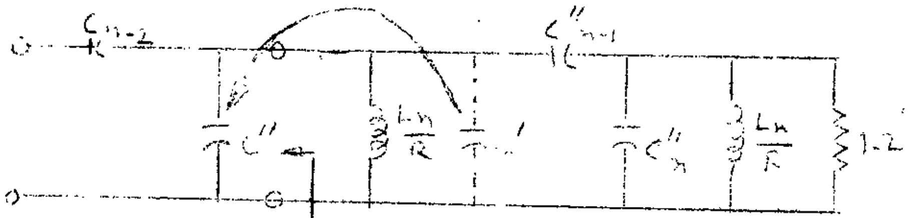


Fig 5.11



Admittance level is to be increased

Fig. 5.12



Admittance level is to be increased.

Fig 5 13

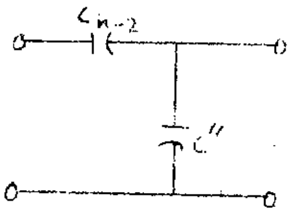


Fig 5.14

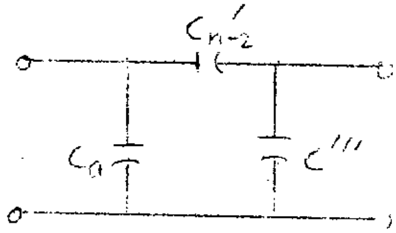


Fig 5.15

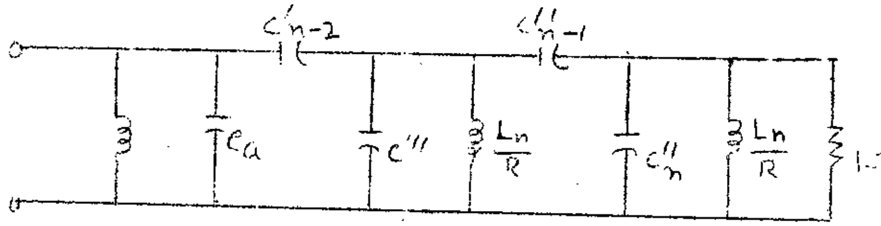


Fig 5.16

Transformation for getting same inductance

The circuit can be written as Fig.5.8.

Fig.5.7 and Fig.5.8 will have the same Z at the terminals aa while the admittance level at terminals bb will be increased by a factor R, i.e. the impedance level is decreased by the factor R, so that we can get the output resistance to be equal to 1. The circuit of the figure 5.5 will then be transformed as that of fig. 5.9 having same impedance at the terminal aa while the output impedance is decreased by a factor R. For this change we require an additional capacitance C. The network of Fig. 5.3 will now have the form of Fig. 5.4.

After getting the network in the form Fig.5.4, the inductances excepting the first one (L_1), can be made equal to the inductance $\frac{L_n}{R}$ by lowering the impedance level in each case by a factor of $(L_n/R) \times (1/L)$, L being the inductance of the respective branch.

For this change the capacitance seen at aa is again to be changed, admittance level increased by a factor $\left(\frac{L_n}{R} \times \frac{1}{L}\right)$. So a new set of capacitances will be obtained. Taking C' to be left side of the inductance and increasing its admittance level, the new circuit becomes .as fig.5.13.

Admittance level of terminal aa of Fig. 5.14 can be increased by a factor $\frac{L_n}{R} \times \frac{1}{L}$, keeping the admittance at bb invariant, involving one more capacitance, C_a, as shown in fig. 5.15 The complete circuit on the right side now becomes

In a similar manner circuit of Fig. 5.3 can be converted to a network configuration cisting of shunt resonator coupled by capacitances.

5.5. COMPUTER PROGRAMME:

Computer programmes have been written for the complete procedure as has been described in previous sections. The main steps are shown in the flow chart of fig.5.17. The values of denominator of transmission function $XX(w^2)$ are very high compared to that of the numerator $Y(w^2)$ at frequencies of stopband. At centre frequency the transmission function is unity so that the value of its denominator and numerator is equal. For this reason, double precision is used for the entire programme.

Values of the approximated transmission function with the valu es of its denominator and numerator and the attenuation of the fil are calculated for different frequency ranges for plotting curves which will be shown in the next chapter.

The programme is a generalised one for any order of filters and is only limited by the storage capacity of the computer.

The operating time required for a filter of order 2 is approximately 15 minutes and for a filter of order 3 is approximately 20 minute for the IBM 360/30 computer which has been used for computer.

The poles and zeroes of the reflection coefficient are calculated with the help of a subroutine written applying Newton-Rapson method. Major time is required for this subroutine. It was observed that about 45 minutes time is required for a filter of order 4 where a polynomial of sixteenth order has to be solved.

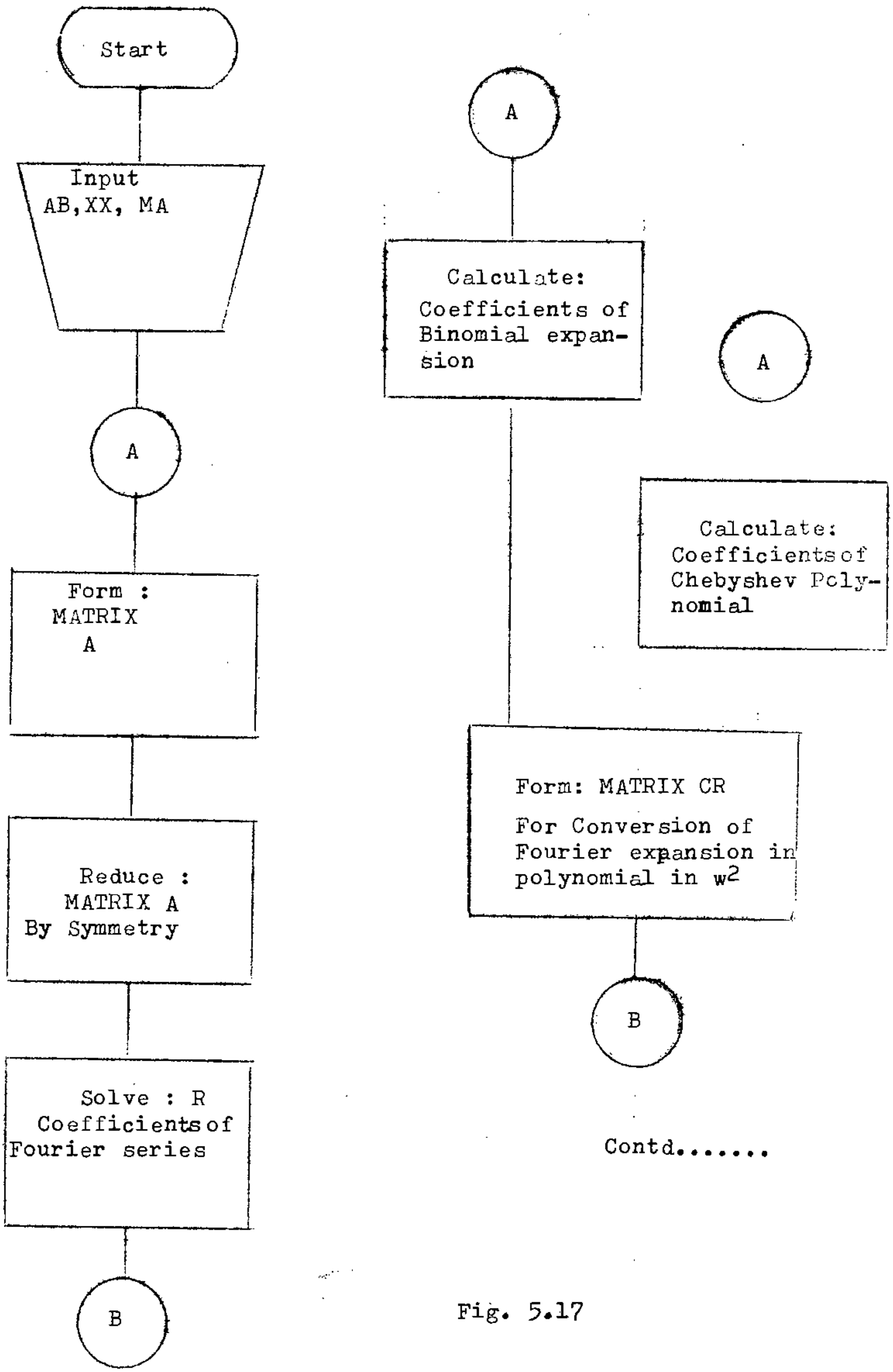
R; the coefficients of the Fourier series are calculated by two method- one by assuming fixed points(point matching technique) and the other by assuming a regular curve. The flow chart of Fig. 15.17 shows the 1st method. The flow chart of the second methos is shown in figure 5.18. The programme for fig.5.18 is same as that of Fig.5.17 except in the calculation of R.

Inputs to the programme of Fig.15.17 are the order of the filter MA. The values of the denominator of transmission function, XX which are assumed depending on the requirement and the angles AB in degrees corresponding to the values of XX.

The outputs are the values of elements of final network, and the values of transmission function, its numerator and denominator, and the attenuation in dB for different frequencies.

Inputs to the programme of Fig.15.18 are the angles AB x, sym in degrees, AFMX, the maximum value of XX, in number and, IORD, the order of the filter, Outputs are same as in the previous case.

Flow Chart



Contd.......

Fig. 5.17

Flow Chart

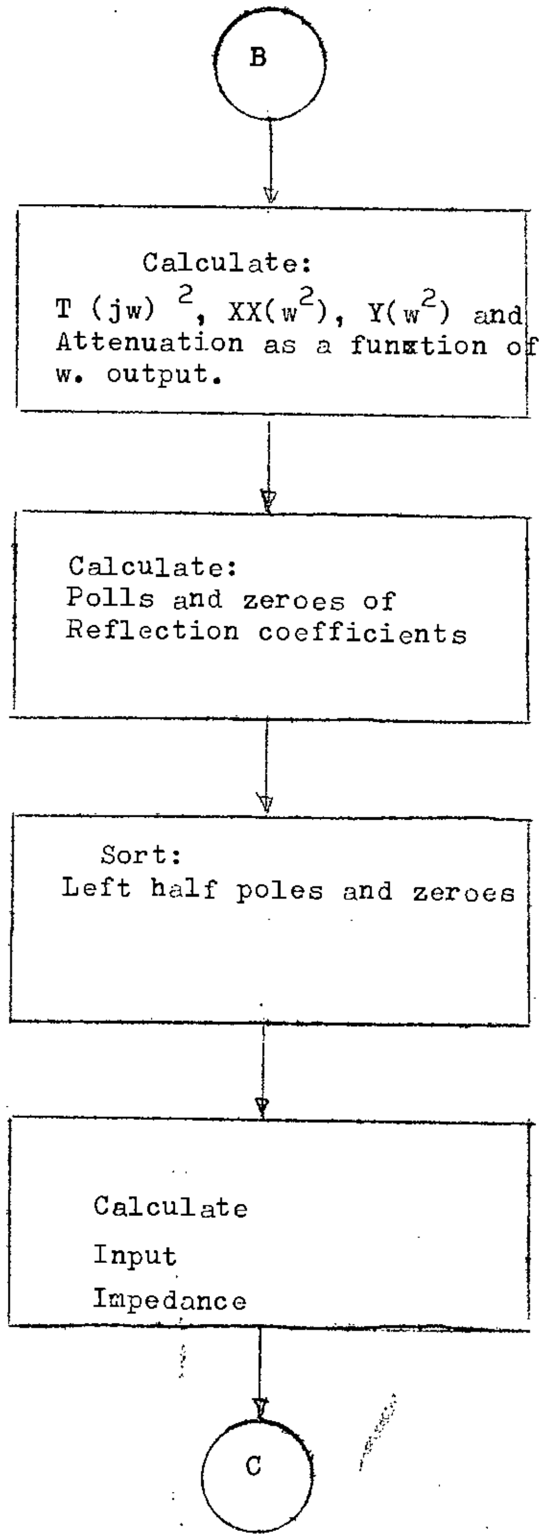


Fig.5.17 (Contd.......)

Flow chart

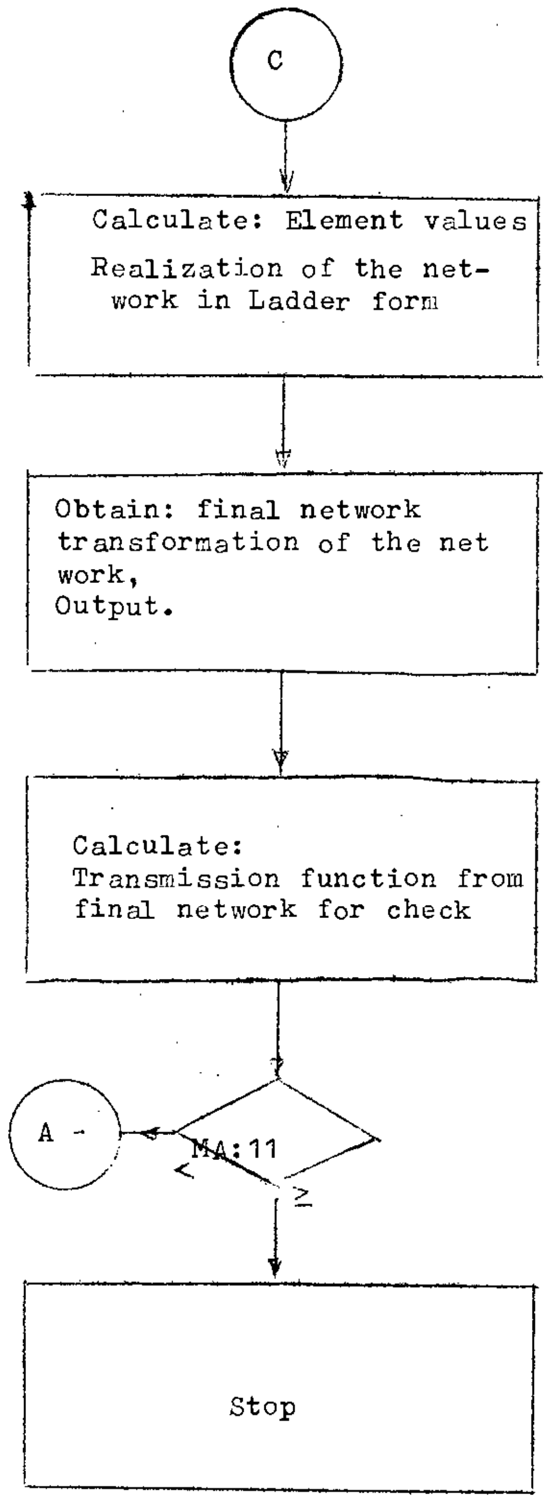


Fig.5.17.

Flow chart

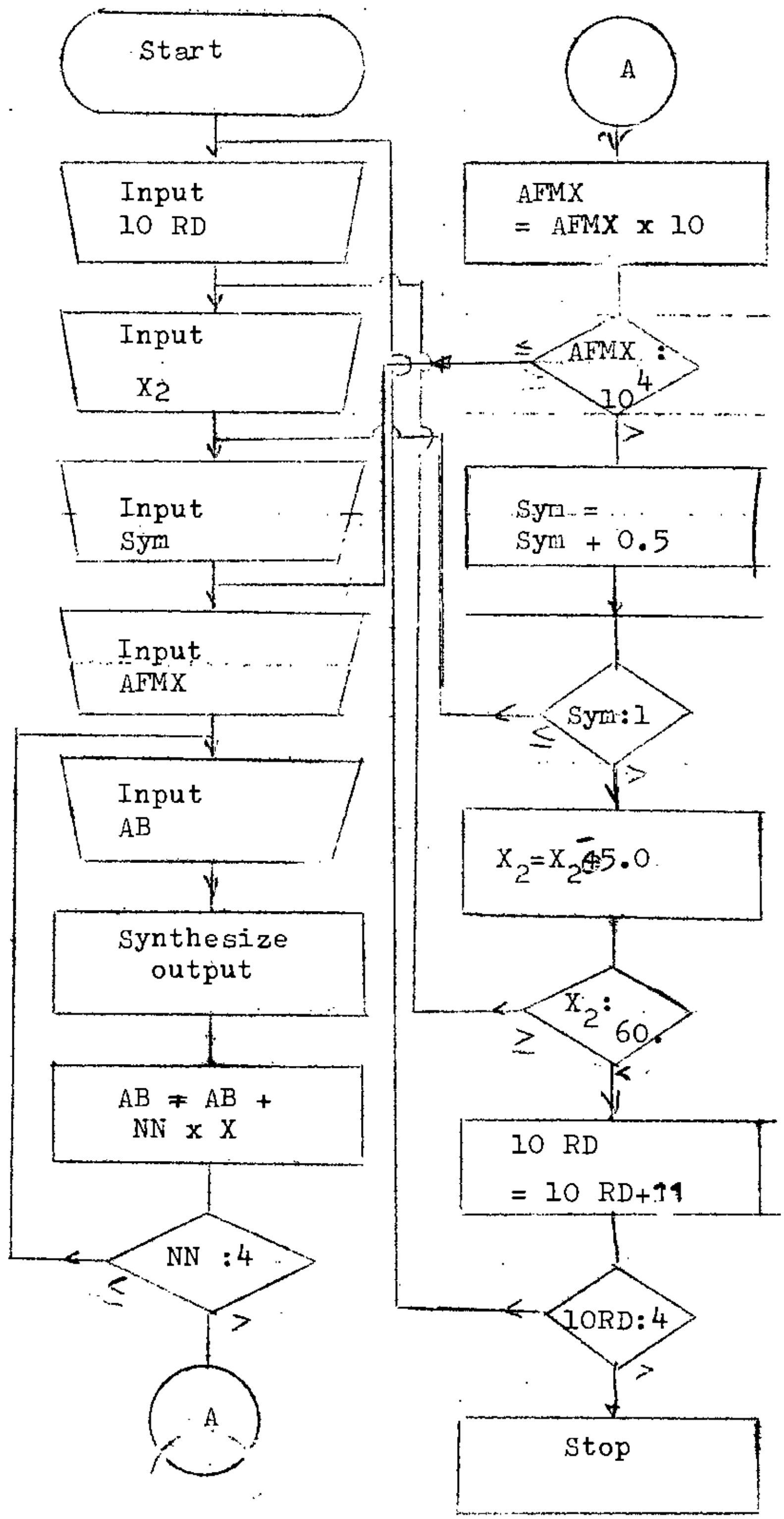


Fig. 5.18

CHAPTER-6

RESULTS AND DISCUSSION

6.1 PRELIMINARIES

In this chapter the results of filters synthesized for different orders are discussed compared with those of Butterworth and Chebyshev filters.

In article 6.2 Butterworth and Chebyshev band pass filters are designed for band width of .07 for different orders.

In article 6.3 second order filters are discussed. The element values are tabulated for Fourier, Chebyshev and Butterworth filters. The attenuation curves are plotted for different types of filters

In article 6.4 third and fourth order filters are discussed.

In article 6.5 discussions have been made in the results obtained using IBM Model 360/30 computer.

6.2. BUTTERWORTH AND CHEBYSHEV FILTER DESIGN

Tabulated Butterworth and Chebyshev low pass, values are taken from Hand Book of Filter Synthesis, by Anatol, I.Zverev [15]. These values are converted for the band pass response. For Fourier approximation by point matching technique we have taken cutoff frequency points corresponding to ~~38°~~ 88° and 92°, because, for the normalised case, centre frequency 1 rad/sec corresponds to 90° when converted to angle according to the equation (4.20) and by the same equation 92° degree corresponds to a frequency 1.035 rad/sec. Assuming these values as cut off frequencies, we may obtain a band width of (.0355 x 2) = .070 rad/sec. Normalised low pass Butterworth and Chebyshev, filters are transformed to band pass filters for this band width. The element values of the Butterworth and Chebyshev filters are then calculated for second third and fourth order filters. These values are tabulated. Calculations are shown in Appendix A-1. A computer programme is prepared for calculation of response curves which are then shown graphically.

6.3 SECOND ORDER FILTERS (FILTERS WITH RESONATOR 2)

For Fourier method of approximation, the value of the angle is taken to be 90° corresponding to w = 1 rad/sec as center point and the cuttoff points are taken to be 92° and 88°. The frequency corresponding to 92° in 1.035, so that band width becomes equal to 2x(1.035 -1.0) = 0.070 radians/sec. approximately. The value of $T(jw)^2$ at points 88°, 90° and 92° are respectively 0.5, 1.0 and 0.5 approximately corresponding values of $Y(w^2)$, the numerator of $T(jw)^2$ is 0.05814, 0.0625, 0.0668, we have considered $XX(w^2)$, the denominator of $T(jw)^2$ to be symmetrical so that the value of $XX(w^2)$ at 92° comes $\frac{Y(w^2)}{T(jw)^2}$ = 0.1337. The value of $XX(w^2)$ at 88° comes to be the same i.e. 0.1337. So that the values of $XX(w^2)$ at 88°, 90° and 92° become 0.1337, .0625 and 0.1337 respectively. The remaining two points required for m =2, are assumed in such a way that the attenuation at stop band becomes very high while at the same time $T(jw)^2$ remains positive. Preliminary testing value of 3,4,5 and 6 have been taken satisfying the foregoing conditions for angles 85° and 95°. For $XX(85^\circ) = XX(95^\circ) = 3$. It is found that there is no ripple. But for $XX(85^\circ) = XX(95^\circ) = 5$, there is a .7 db ripple in the pass band. Increasing the values of $XX(85^\circ)$

and XX (95^o), so as to obtain sharp attenuation it is observed that the ripple becomes larger and at some point with XX(85^o) = XX(95^o) = 7.5, XX (w^2) and T (jw) ^{2} becomes negative for which the network realization is not possible.

For Fourier approximation assuming the specific curve for XX(w^2), the value x_2 is taken to be 60^o and the value of X_1 is taken to be 0. The maximum values of XX(A) have been taken 10^3, 10^4. For fig.5.1, for X_2 to be 60^o, the value of XX (A) is assumed to be zero for obtaining a reasonable band width. After approximation, the band width becomes 0.18. The results are shown in table-1.

Computations have been made for different X2 values also. For X_2 greater than 60^o, though the attenuation increased the band-width becomes smaller and the attenuation is very poor compared to the Chebyshev and Butterworth filter and the value of the capacitance, C_2 becomes negative, after network transformation, which is considerable.

6.4. THIRD ORDER AND FOURTH ORDER FILTERS:

For the design of third order filter, the values XX(A) is assumed at 7 points, the value of A at these points are 80^o, 85^o, 88^o 90^o, 92^o, 95^o, 100^o. The band width is assumed to be .07 as in the case of 2nd order. The values of XX(A) at the cut off points 88^o and 92^o are calculated to be .0358. Symmetry is assumed in this case also so that the value of XX(A) at 85^o is equal to that at 95^o and the value at 80^o is equal to that at 100^o. Different sets of values are taken for these points. On the basis of previous discussion we took the value of XX at 85^o and 95^o to be 14 and that at 80^o and 100^o to be 10^3. The response curve for these values is approximately similar to the Chebyshev, and Butterworth response curve. If we want to increase the attenuation, the value at 85° and 95° is to be increased and the the value at 80° and 100° has to be increased also. If we change only one of these values, ripple occurs at passband. When the change is sufficiently large, ripple becomes so large that the value of XX(A) is negative for which $T(jw)^2$ is also negative and the network realization is not possible.

For fourth order filter the values of A are taken to 75°, 80°, 85°, 88°, 90°. 92°, 100° and 105°. symmetry is assumed in this case also, so that we can assume three values of XX(A) one at 75° and 105°, one at 80° and 100° and the rest at 85° and 95°. The values of XX(A) is assumed to be 10^5, 5×10^3 and 3.4 at 75°, 80° and 85° respectively. The filter has been synthesized with three values. The pass band response of this filter has been found to be quite satisfactory, though considerable attenuation has been obtained at the stop band.

The results are shown in table 2 & 3.

TABLE-1

ORDER OF THE FILTER =2

	BW	L_1	C_1	C_{c1}	L_2	C_2
1. Butterwidth	.07	.0495	19.2	1.00	.0495	19.2
2. Chebyshev (.5 dB)	.07	.0359	26.44	1.41	.0359	26.44
3. Fourier(3)	.07	.0336	28.55	1.10899	.0585	16.06
4. Fourier(4)	.07	.03007	31.82	1.2871	.04620	20.47
5. Fourier(5)	.07	.02608	36.67	1.485	.03889	24.40
6. Fourier (Assumed curve)	.18	.07932	11.28	1.138	.179	4.635

(3) $XX(85^\circ) = XX(95^\circ) = 3$,

(4) $XX(85^\circ) = XX(5^\circ) = 4$

(5) $XX(85^\circ) = XX(95^\circ) = 5$

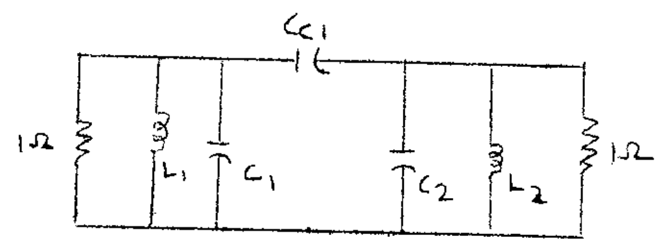


TABLE-2

ORDER OF THE FILTER =3

	BW	L_1	C_1	C_{c1}	L_2	C_2	C_{c2}	L_3	C_3
1. Butterwidth	.07	.07	13.48	.7071	.07	12.77	.7071	.07	13.48
2. Chebyshev	.07	.0376	25.396	1.204	.0376	24.192	1.204	.0376	25.396
3. Fourier(1)	.07	.02559	37.82	1.0504	.05590	15.94	.9634	.0590	17.03
4. [illegible]		.0530[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	14.05

(1) $XX(85^\circ) = XX(95^\circ) = 14$, $XX(80^\circ) = XX(100^\circ) = 1000$

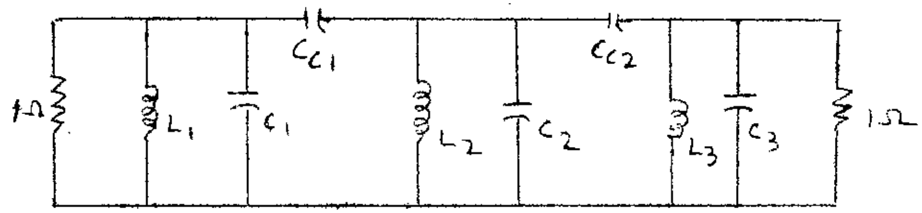


TABLE-3

ORDER OF THE FILTER=4

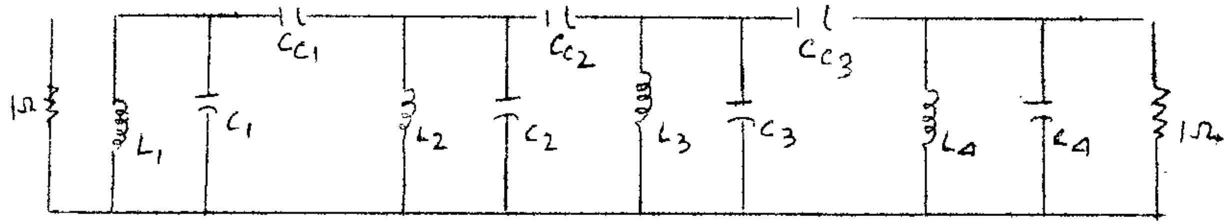
BW = .07, W_m = 1 rad/sec.

	L_1	C_1	C_{c1}	L_2	C_2	C_{c2}	L_3	C_3	C_{c3}	L_4	C_4
1.Butterworth	.0913	9.305	0.645	.0913	8.882	0.421	.0913	8.882	.645	.0913	9.305
2.Chebyshev (.5 dB)	.0384	24,885	1.185	.0384	23.893	.992	.0384	23.893	1.185	.0384	24.885
3.Fourier[1]	.008312	119.14	.733	.2942	2.475	.19	.2942	2.97	.265	.2942	3.154

(1) $XX(85^\circ) = XX(95^\circ) = 34$

$XX(80^\circ) = XX(100^\circ) = 5000$

$XX(75^\circ) = XX(105^\circ) = 100,000$



6.5 DISCUSSION OF THE RESULTS:

Graphs are plotted for Butterworth Chebyshev and Fourier filters to show the different attenuation characteristics. In Fig.6.1 and 6.2 Attenuation Characteristic of Fourier filters of Second order are shown. Attenuation in dB is plotted, as a function of normalised frequency in radian/sec. It is observed that the attenuation at the stop band of filter can be increased by increasing the value of XX(85^{o}) , where XX is the value of the denominator of transmission function.

In fig.6.3 and 6.4 comparison is shown with Butterworth and Chebyshev 5 dB filters. In fig. 6.3 the characteristic of Fourier filter for XX (85^{o}) = XX (95^{o}) = 5.0 is plotted with those Butter worth and Chebyshev filters. The stop band attenuation is highest for the Fourier filter. But at the same time the pass band ripple also becomes highest (approximately 1 dB). In fig. 6.4, Fourier filter characteristics are plotted for XX(95^{o}) = XX (95^{o}) = 3. In this case there is no ripple and the attenuation at stop band is in between Butterworth and Che byshev .5 dB ripple filters.

In Fig.6.5 the pass band response is plotted. The response of the Fourier filter for XX(85^{o}) = 3 in the pass band is observed to be similar to the Butterworth filter.

In Fig.6.6 the numerator of the transmission function is plotted as a function of normalised frequency. For the filters of Fig. 6.1 and 6.2, symmetry has been taken for the denominator, XX. The transmission function can not be symmetrical about centre frequency. But at the frequency lower than the centre frequency the value of the transmission function will be smaller and at the frequency higher than the centre the value will be greater than that at the centre frequency. This is observed in fig.6.5.For symmetrical response the value of XX may be taken so that $|T(jw)|^2$ is same at both sides of centre frequency at equal distance. For this XX at lower frequency should be smaller than that at higher frequency than the centre frequency. The difference **is** values of XX at equal distance apart from the centre frequency, will cause the odd harmonics in the Fourier expansion. Fig.6.7 explains the numerator polynomial as a function of angle A of Fourier series expansion.

In Fig.6.8, the effect of increasing the value of XX at 85° and 95° is explained. It is observed that if the values are increased the ripple occurs at the pass band. Further continuation of this procedure which resulted in negative values for XX (w^2) in the pass band, the realization was not possible.

Fig.6.9 and 6.10 explain the response curve for second order filter designed for the assumed curve shown in fig.6.9.

Fig.6.11, 612 and 6.13 are characteristic curves for the third order filter. It is observed from Fig.6.11 that the response of Fourier filter is satisfactory compared with the Butterworth and Chebyshev filter,**In the sense**,that the Fourier filter has got almost the same sharp cutoff as the Chevyshev filter but without the ripple effect of the latter in the passband. The Butterworth Filter has got a comparable passband response but with a less sharp cut off characteristic. These remarks will be evident from a comparison of curves (a) (b) and (c) of Fig.6.11. Systematic synthesis procedure for Fourier filter can be obtained via the assumed curve shown in Fig.6.9, but the point -

matching technique does not yield good high order filters as shown in Fig.6.14 where a fourth order filter response has been obtained using point-matching technique. The passband ripple magnitude becomes unacceptably large though a satisfactory stopband response can be obtained without much difficulty.

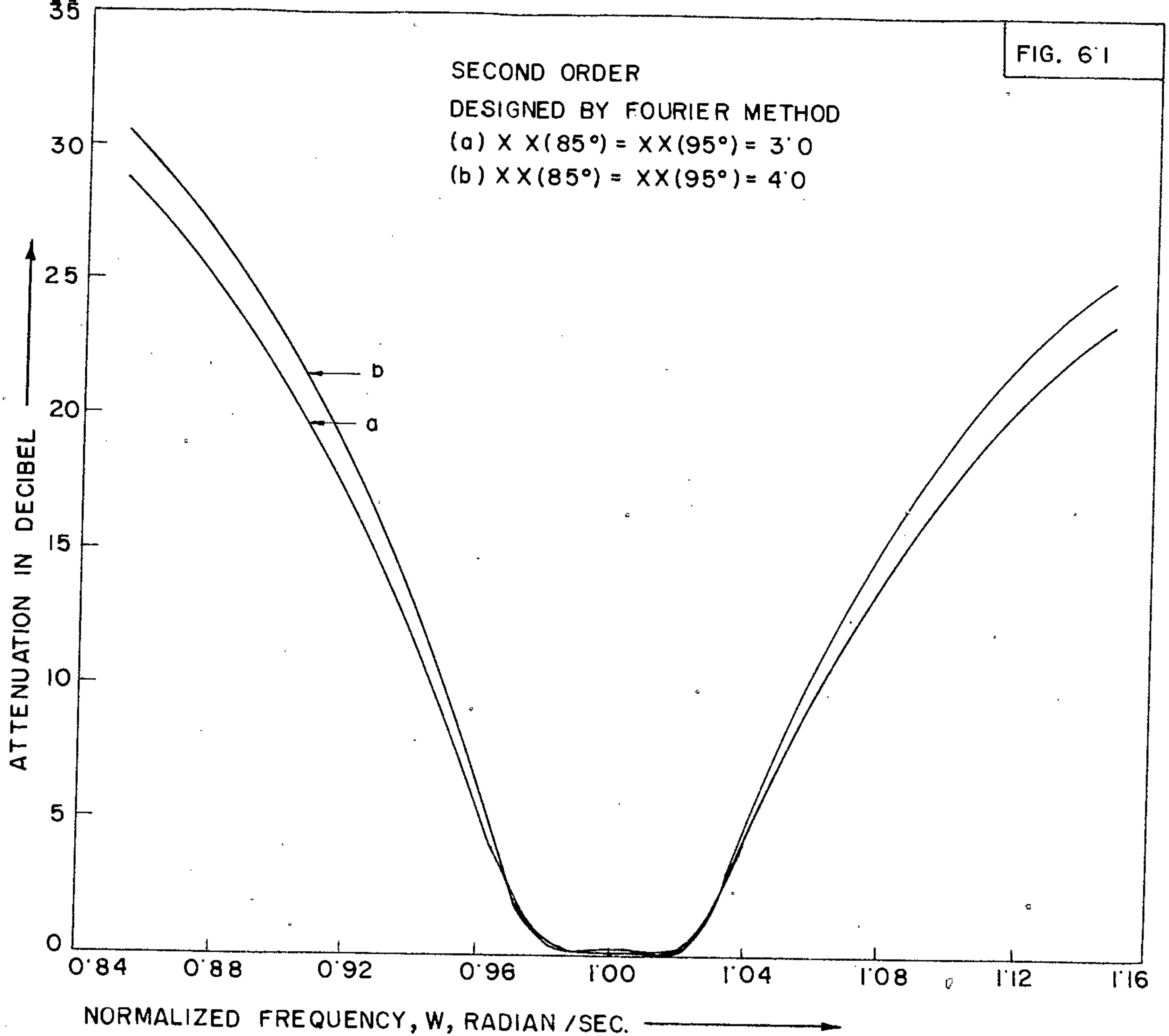
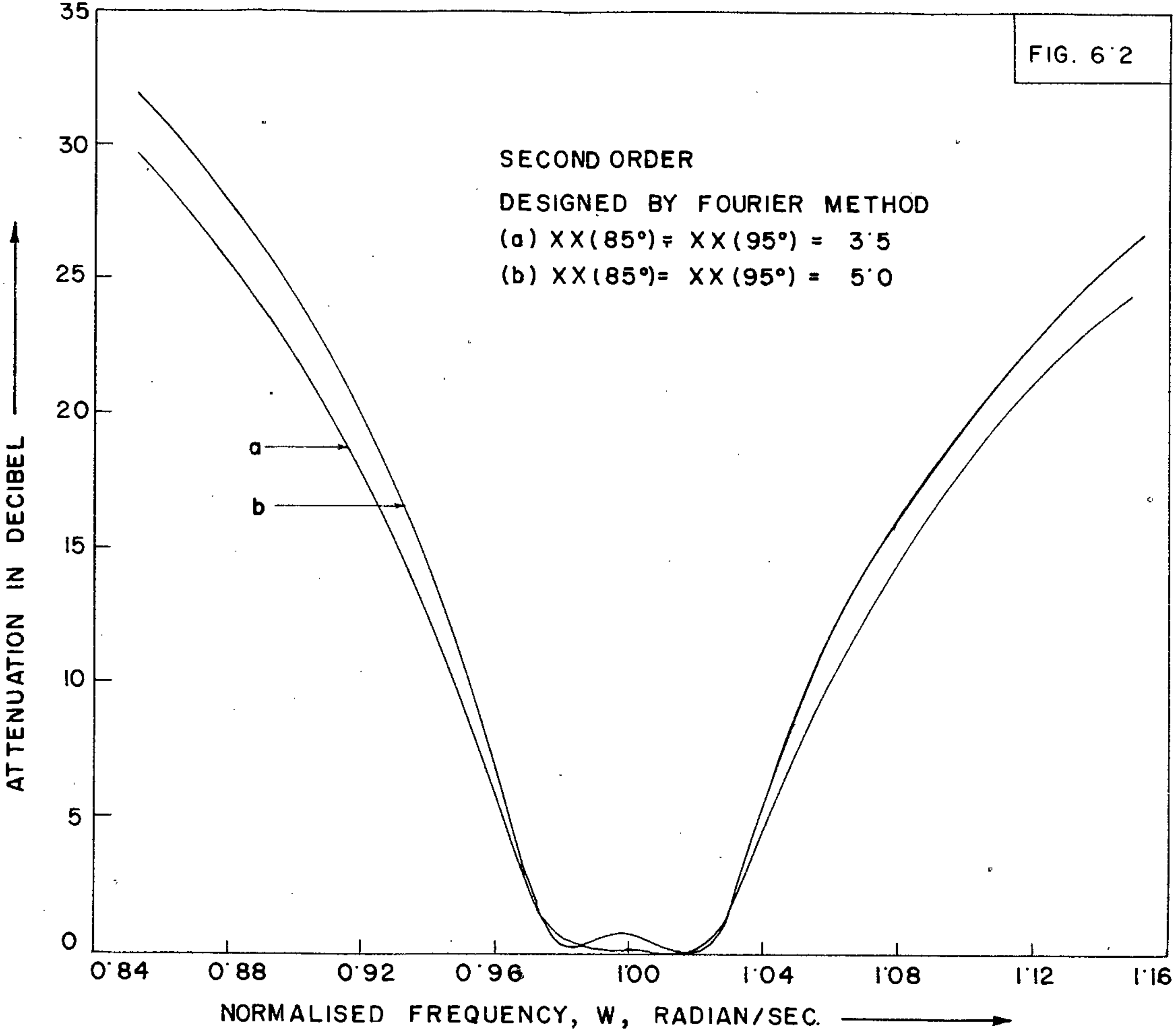

SECOND ORDER
DESIGNED BY FOURIER METHOD
(a) X X(85°) = XX(95°) = 3·0
(b) XX(85°) = XX(95°) = 4·0
b
a
ATTENUATION IN DECIBEL
35
30
25
20
15
10
5
0
0·84
0·88
0·92
0·96
1·00
1·04
1·08
1·12
1·16
NORMALIZED FREQUENCY, W, RADIAN /SEC.

FIG. 6·1


SECOND ORDER
DESIGNED BY FOURIER METHOD
(a) XX(85°) = XX(95°) = 3·5
(b) XX(85°) = XX(95°) = 5·0
a
b
ATTENUATION IN DECIBEL
NORMALISED FREQUENCY, W, RADIAN/SEC.
0
5
10
15
20
25
30
35
0·84
0·88
0·92
0·96
1·00
1·04
1·08
1·12
1·16
FIG. 6·2

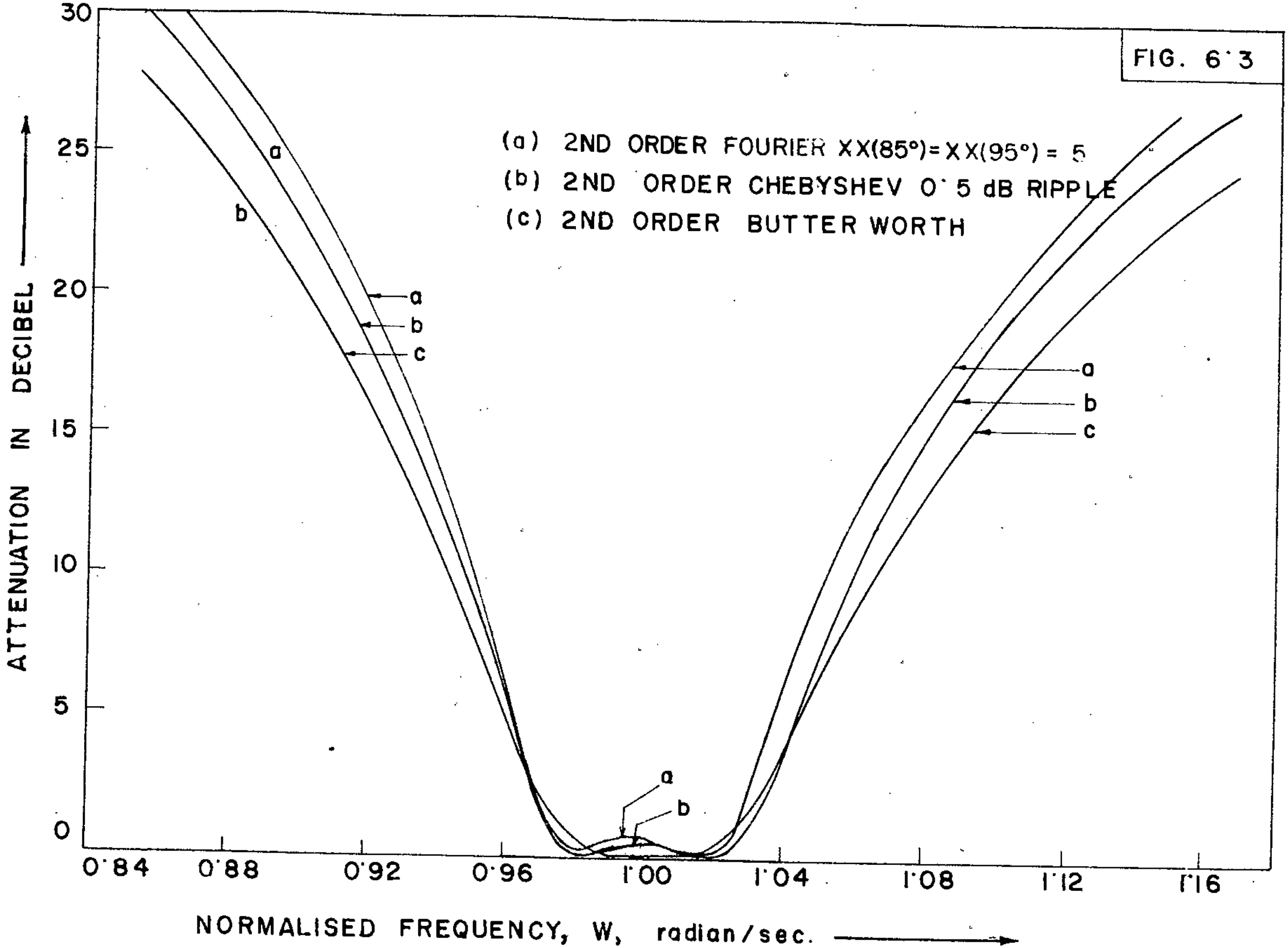


FIG. 6·3
(a) 2ND ORDER FOURIER XX(85°)=XX(95°) = 5
(b) 2ND ORDER CHEBYSHEV 0·5 dB RIPPLE
(c) 2ND ORDER BUTTER WORTH
ATTENUATION IN DECIBEL
NORMALISED FREQUENCY, W, radian/sec.
30
25
20
15
10
5
0
0·84
0·88
0·92
0·96
1·00
1·04
1·08
1·12
1·16
a
b
c

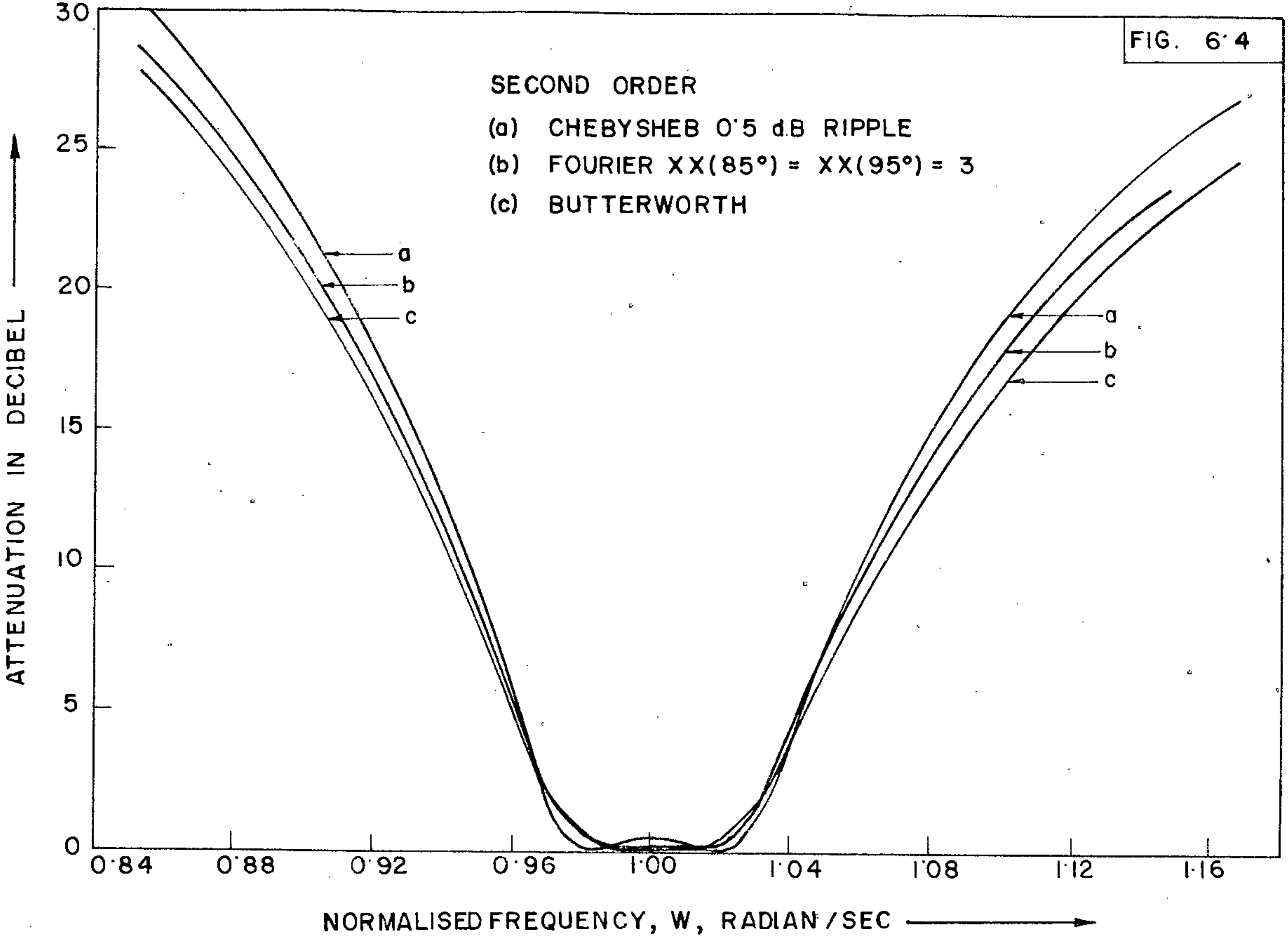

SECOND ORDER
(a) CHEBYSHEB 0·5 d.B RIPPLE
(b) FOURIER XX(85°) = XX(95°) = 3
(c) BUTTERWORTH
ATTENUATION IN DECIBEL
NORMALISED FREQUENCY, W, RADIAN /SEC
0
5
10
15
20
25
30
0·84
0·88
0·92
0·96
1·00
1·04
1·08
1·12
1·16
a
b
c
a
b
c

FIG. 6·4

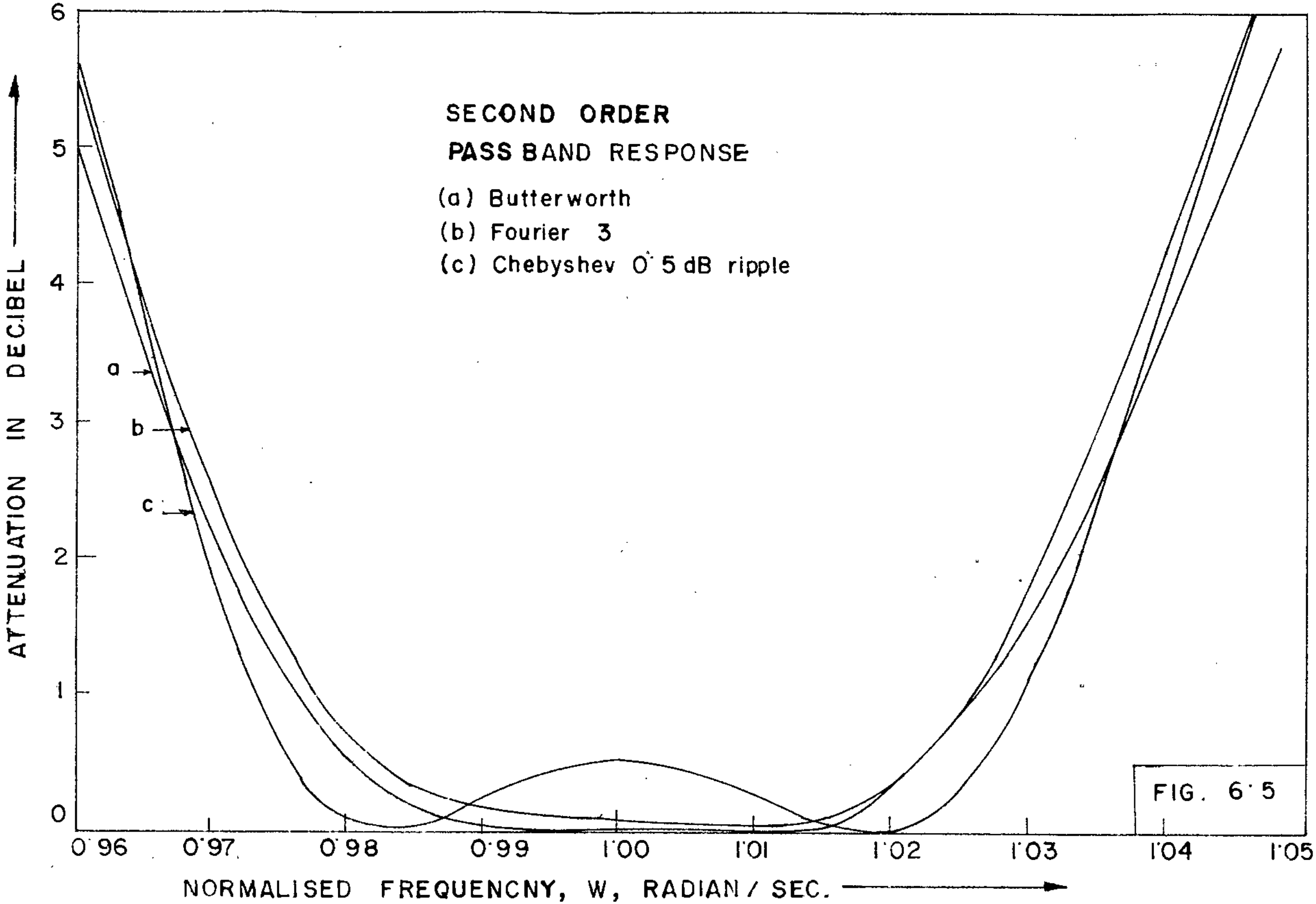

SECOND ORDER
PASS BAND RESPONSE
(a) Butterworth
(b) Fourier 3
(c) Chebyshev 0·5 dB ripple
a
b
c
ATTENUATION IN DECIBEL
0
1
2
3
4
5
6
0·96
0·97
0·98
0·99
1·00
1·01
1·02
1·03
1·04
1·05
NORMALISED FREQUENCNY, W, RADIAN / SEC.

FIG. 6·5

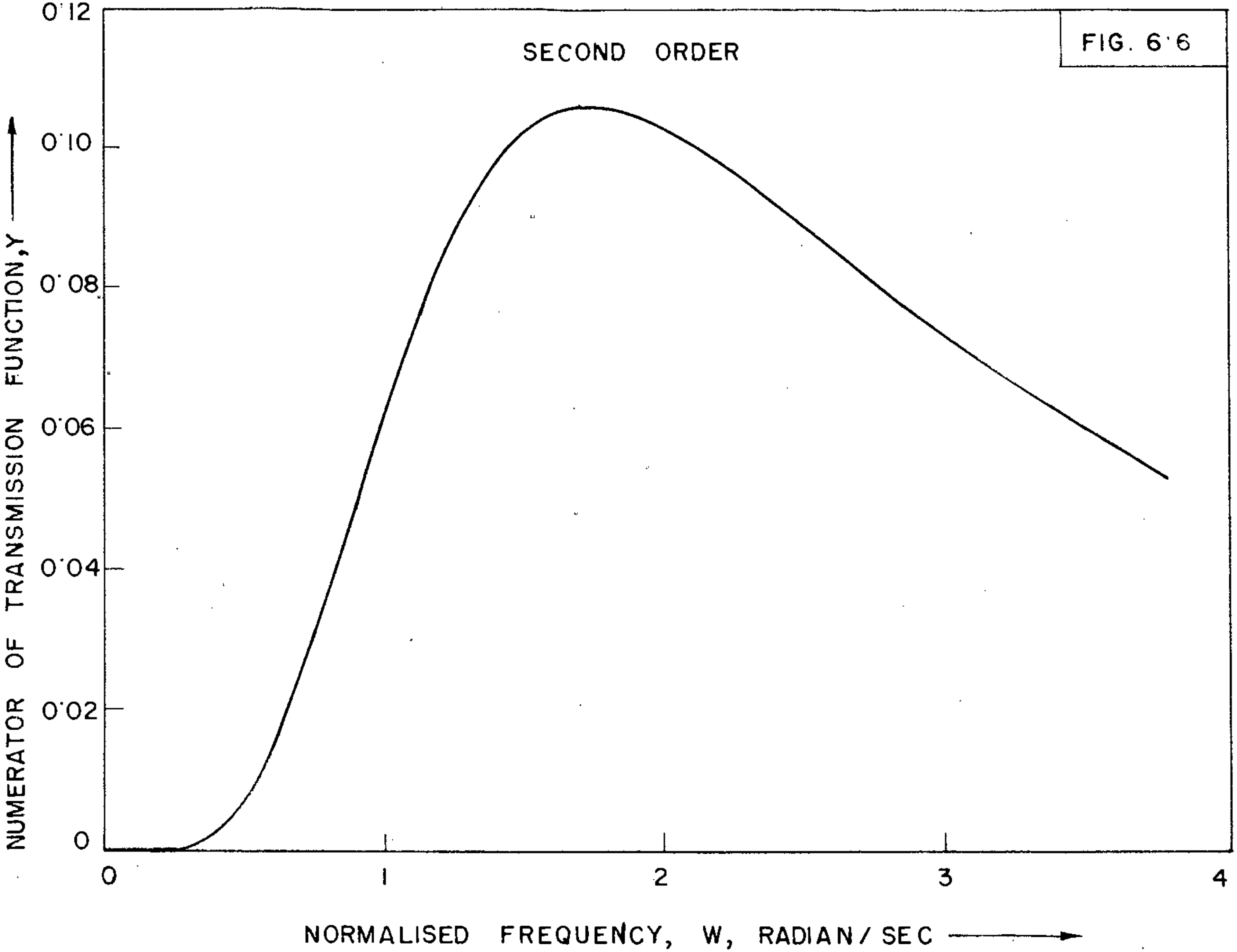

SECOND ORDER
NUMERATOR OF TRANSMISSION FUNCTION, Y
0·12
0·10
0·08
0·06
0·04
0·02
0
0
1
2
3
4
NORMALISED FREQUENCY, W, RADIAN / SEC

FIG. 6·6

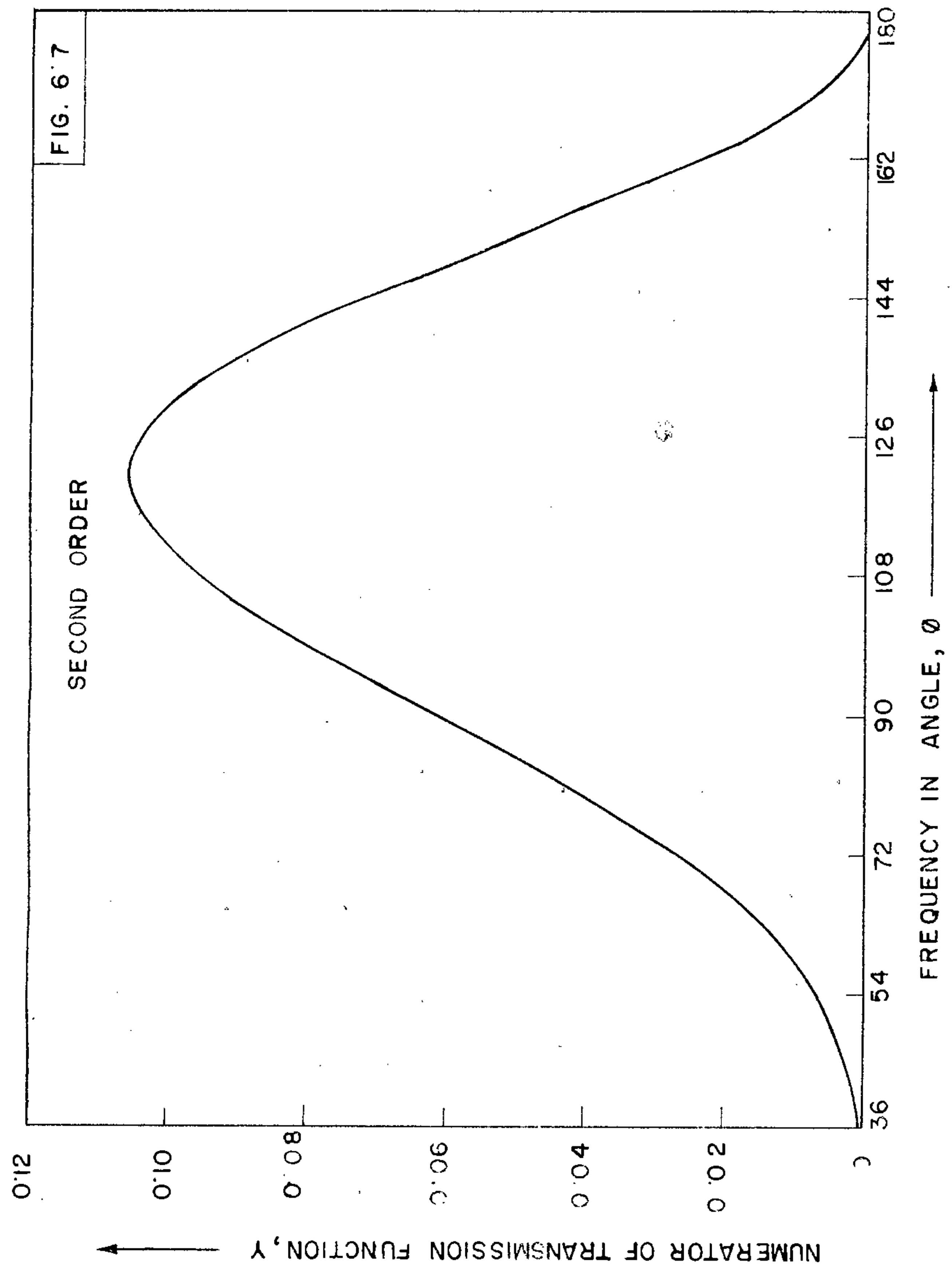

SECOND ORDER
FREQUENCY IN ANGLE, Ø
NUMERATOR OF TRANSMISSION FUNCTION, Y
36
54
72
90
108
126
144
162
180
0
0.02
0.04
0.06
0.08
0.10
0.12

FIG. 6.7

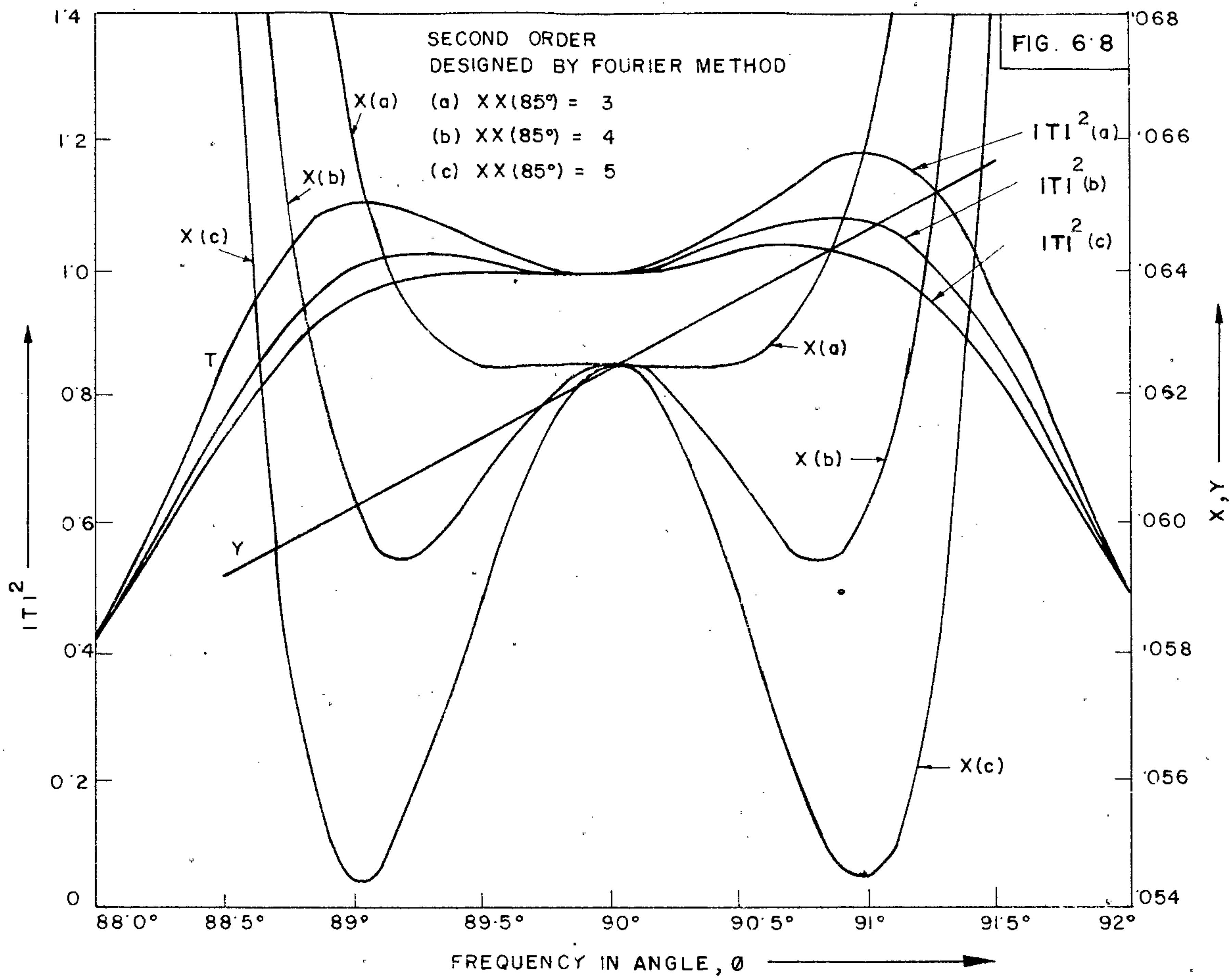

SECOND ORDER
DESIGNED BY FOURIER METHOD
(a) XX(85°) = 3
(b) XX(85°) = 4
(c) XX(85°) = 5
X(a)
X(b)
X(c)
T
Y
|T|²(a)
|T|²(b)
|T|²(c)
|T|²
X, Y
FREQUENCY IN ANGLE, Ø
1·4
1·2
1·0
0·8
0·6
0·4
0·2
0
·068
·066
·064
·062
·060
·058
·056
·054
88·0°
88·5°
89°
89·5°
90°
90·5°
91°
91·5°
92°

FIG. 6·8

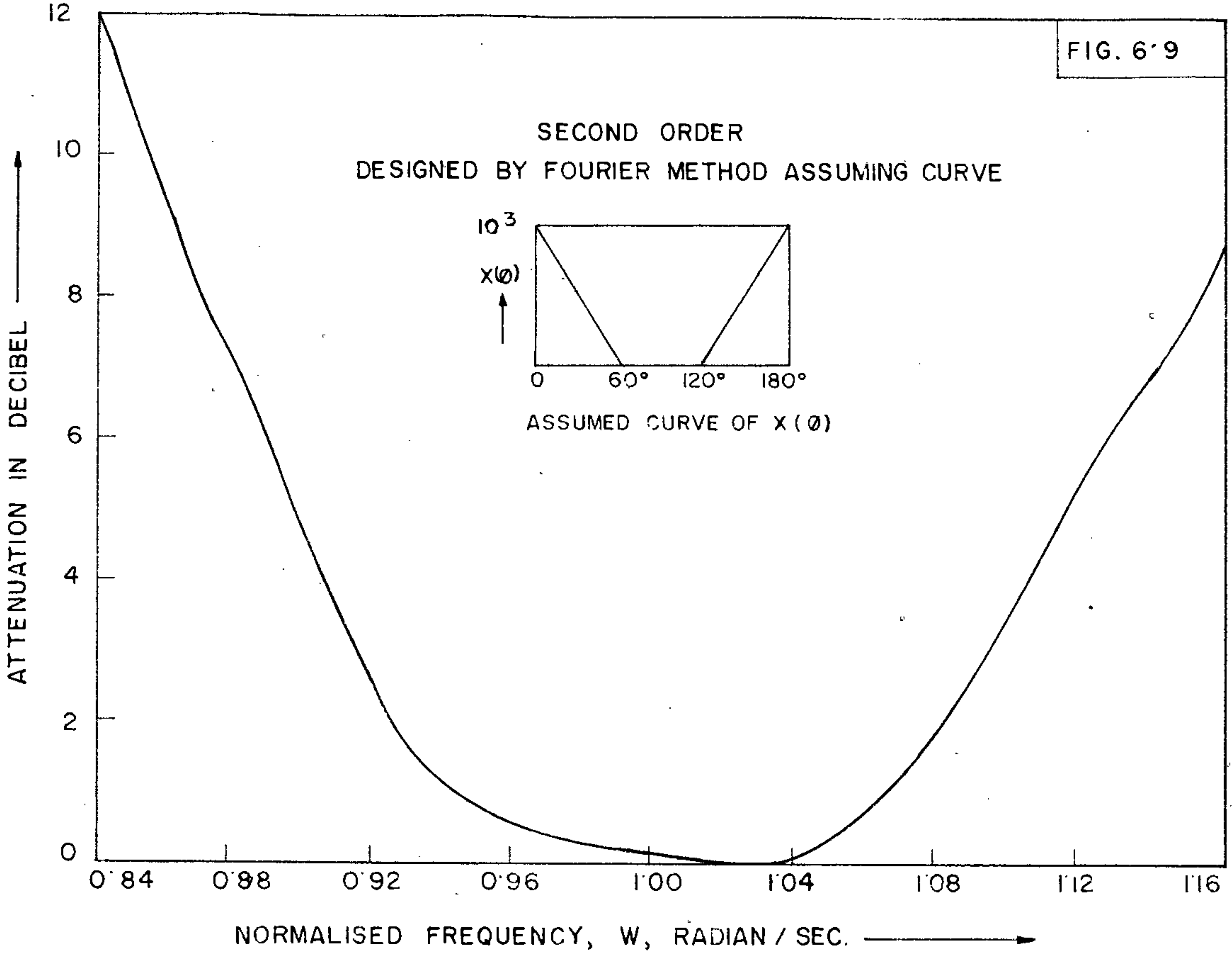
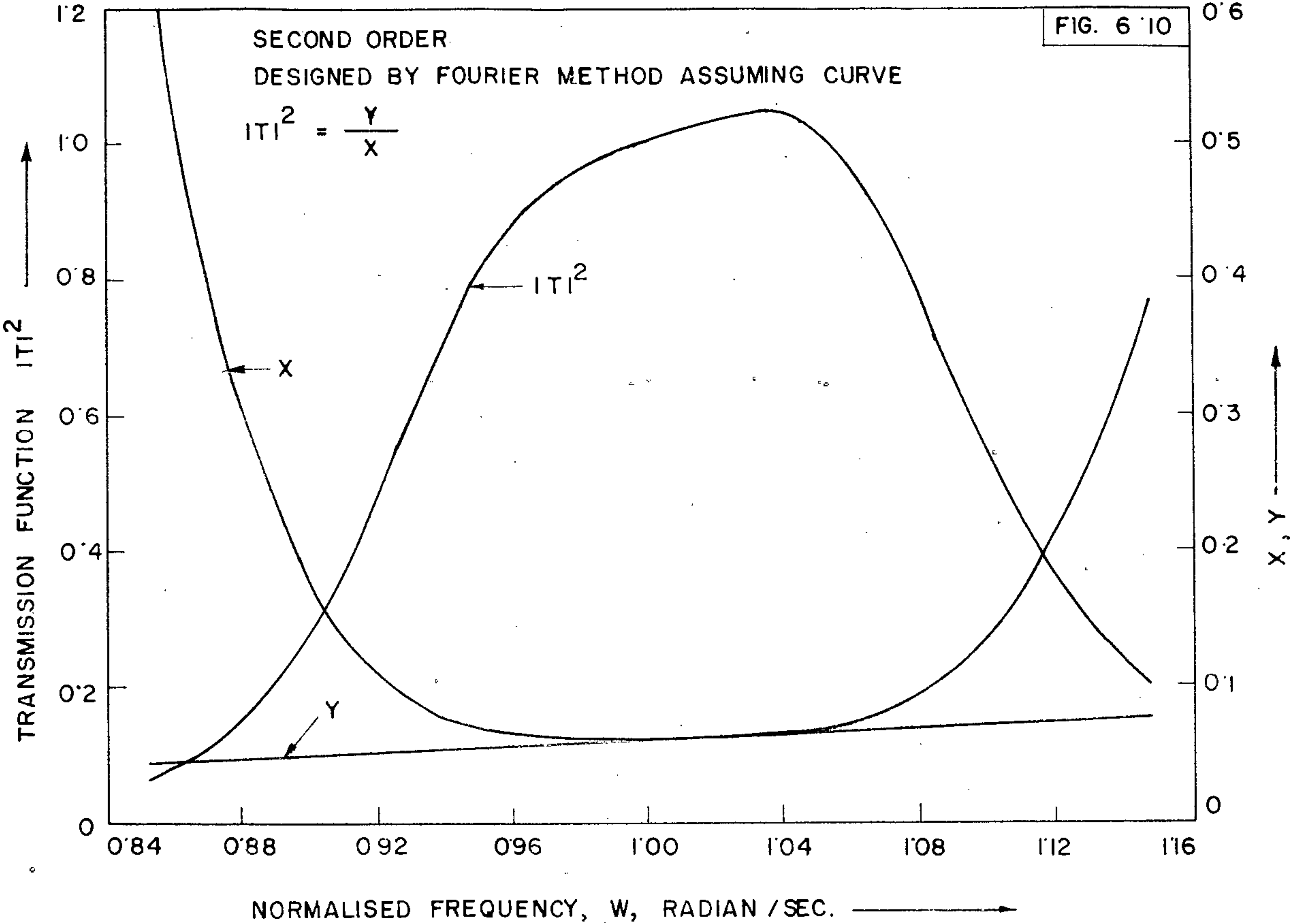


FIG. 6·9
SECOND ORDER
DESIGNED BY FOURIER METHOD ASSUMING CURVE
10^3
X(Ø)
0
60°
120°
180°
ASSUMED CURVE OF X(Ø)
ATTENUATION IN DECIBEL
12
10
8
6
4
2
0
0·84
0·88
0·92
0·96
1·00
1·04
1·08
1·12
1·16
NORMALISED FREQUENCY, W, RADIAN / SEC.



SECOND ORDER
DESIGNED BY FOURIER METHOD ASSUMING CURVE
|T|² = Y/X
|T|²
X
Y
TRANSMISSION FUNCTION |T|²
X, Y
NORMALISED FREQUENCY, W, RADIAN / SEC.
0
0·2
0·4
0·6
0·8
1·0
1·2
0·1
0·2
0·3
0·4
0·5
0·6
0·84
0·88
0·92
0·96
1·00
1·04
1·08
1·12
1·16

FIG. 6.10

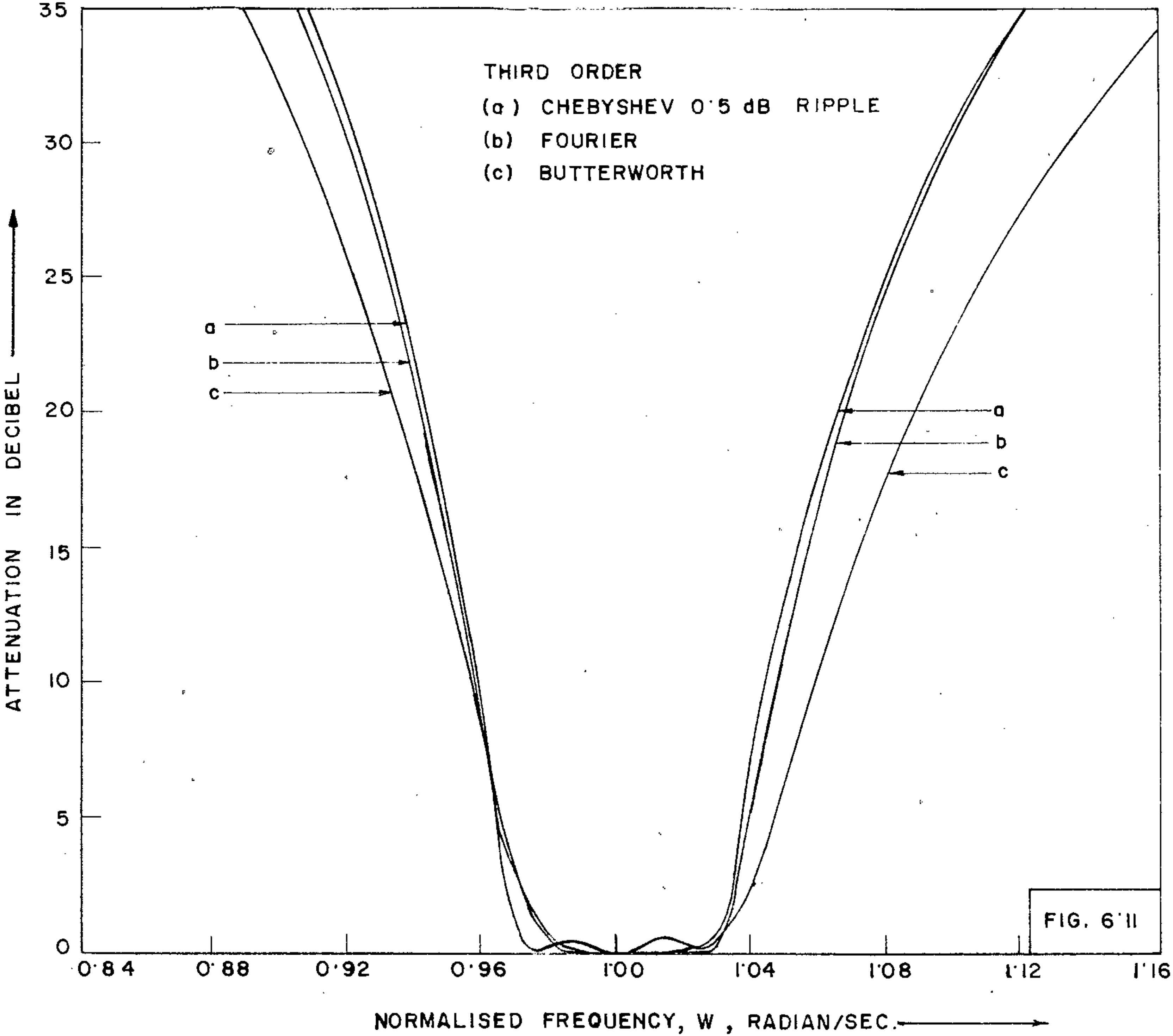

THIRD ORDER
(a) CHEBYSHEV 0·5 dB RIPPLE
(b) FOURIER
(c) BUTTERWORTH
a
b
c
35
30
25
20
15
10
5
0
0·84
0·88
0·92
0·96
1·00
1·04
1·08
1·12
1·16
ATTENUATION IN DECIBEL
NORMALISED FREQUENCY, W, RADIAN/SEC.

FIG. 6·11

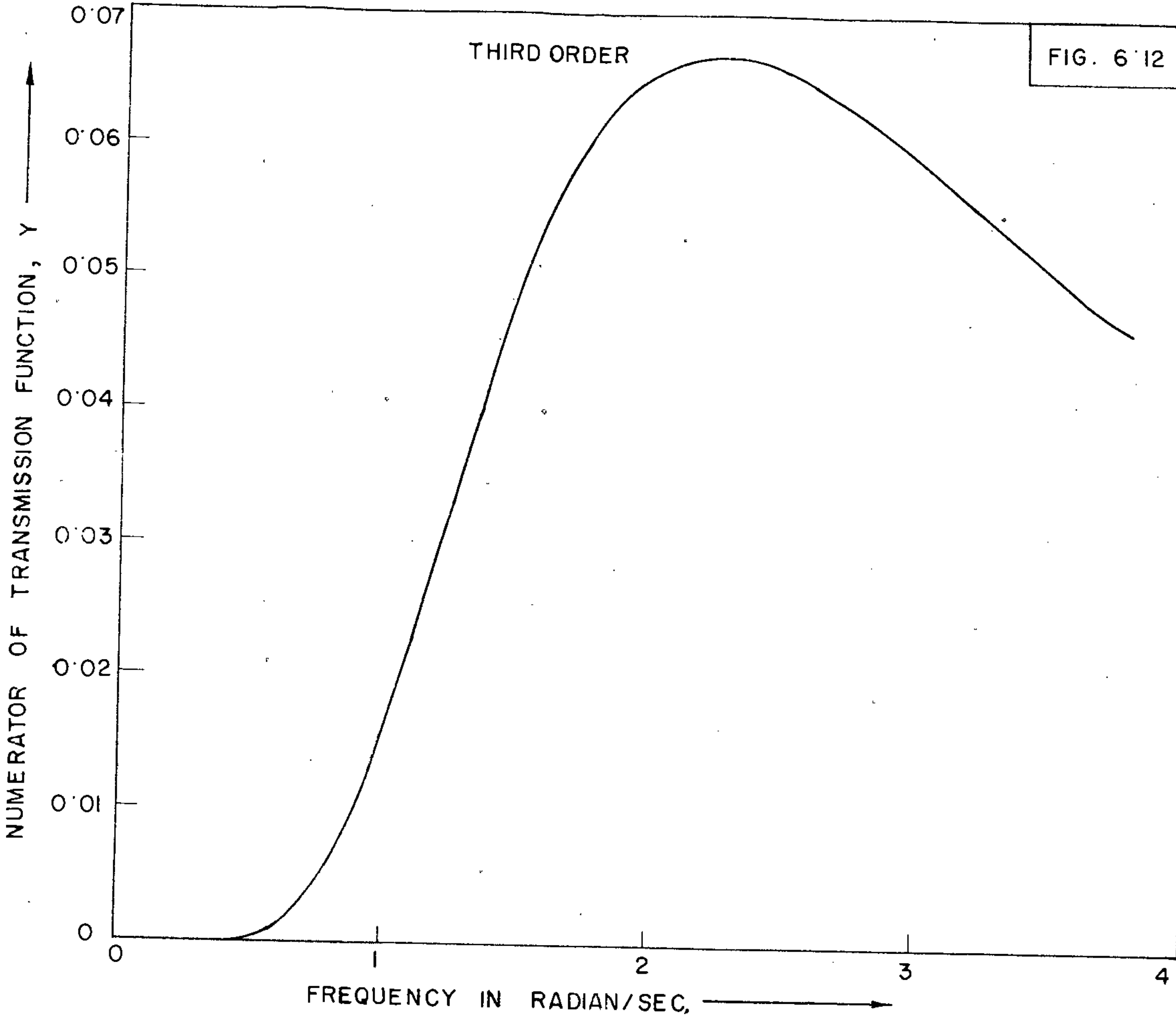

0·07
0·06
0·05
0·04
0·03
0·02
0·01
0
NUMERATOR OF TRANSMISSION FUNCTION, Y
THIRD ORDER
0
1
2
3
4
FREQUENCY IN RADIAN/SEC,

FIG. 6·12

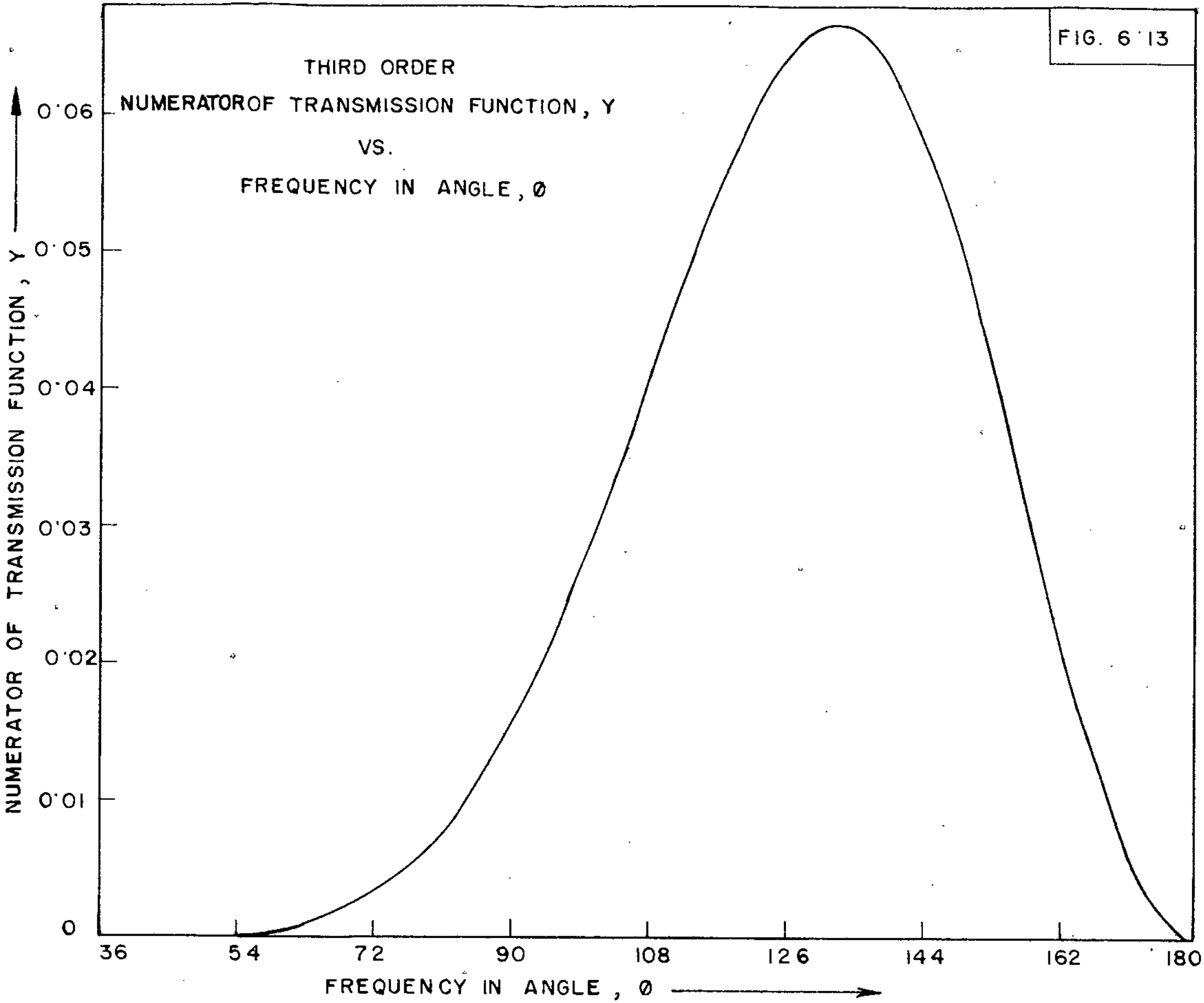


FIG. 6·13
THIRD ORDER
NUMERATOR OF TRANSMISSION FUNCTION, Y
VS.
FREQUENCY IN ANGLE, Ø
NUMERATOR OF TRANSMISSION FUNCTION, Y
0·06
0·05
0·04
0·03
0·02
0·01
0
36
54
72
90
108
126
144
162
180
FREQUENCY IN ANGLE, Ø

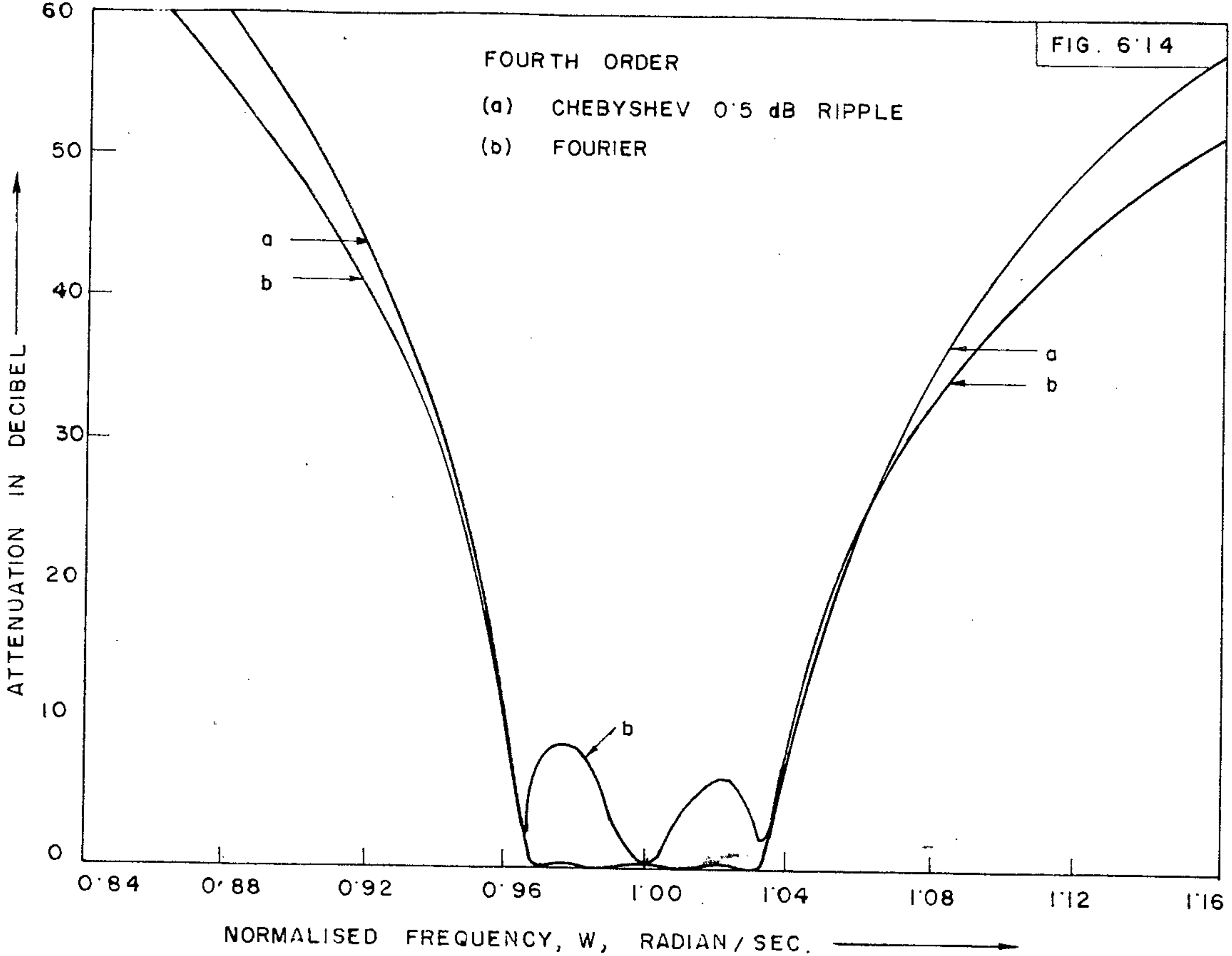

FOURTH ORDER
(a) CHEBYSHEV 0·5 dB RIPPLE
(b) FOURIER
a
b
a
b
b
60
50
40
30
20
10
0
0·84
0·88
0·92
0·96
1·00
1·04
1·08
1·12
1·16
ATTENUATION IN DECIBEL
NORMALISED FREQUENCY, W, RADIAN / SEC.

FIG. 6·14

Chapter-7

CONCLUSIONS

In this study a procedure has been developed for the synthesis of band pass filters. The realizable rational function has been obtained by approximating the bandpass response so that conventional lowpass to band pass transformation is not required for obtaining the band pass filter network. The filter network thus obtained has been transformed to a common band pass network consisting of shunt resonators coupled by capacitors.

Solution of the approximation problem by point matching technique is a laborious task though the results obtained has been observed to be satisfactory for the second and the third order filters compared to Butterworth and Chebyshev filters. For the filters of order higher than 3.the solution of the approximation problem by this technique becomes much labourious.

For the calculation of input impedance,solution of a polynomial of order 4 times the order of the filter is required. It has been observed that for some cases of higher order filters, the subroutine used to solve the polynomial is not sufficiently efficient. A more efficient subroutine is to be developed for such cases.

An alternate procedure for approximation assuming a fixed curve has also been developed which is observed to be better than the point matching technique. A second order filter has been designed by this procedure. Further improvement of this procedure may be a better procedure for the approximation.

The remaining part of the synthesis procedure developed in this study such as the realization of the network by ladder development of the input impedance and the transformation of the network into a practically attainable network is quite satisfactory.

The procedure of the synthesis of band pass filter may be further developed to design a filter having symmetrical bandpass response. The response of the conventional bandpass filters designed by lowpass to band pass transformation is not symmetrical because of the transformations required after the approximation of the low pass response, change the symmetry of the bandpass response. For this method of approximation, the band pass response may be assumed symmetrical and then the approximation problem can be solved so that the symmetry will not be changed for the final network.

APPENDIX A-1

BUTTERWORTH AND CHEBYSHEV FILTER DESIGN
BANDWIDTH = 0.07, CETRE FREQUENCY = 1 rad/sec.

N= 2

Butterworth

Low pass values $q_1 = 1.4142$

$q_2 = 1.4142$

$K_{12} = 0.7071$

$L = \frac{.07}{1.4142} = 0.0495$

$C = 20.2$

$C_{12} = K_{12} \Delta w . C = 0.7071 \times .07 \times 20.2 = 1.0$

$= C_{c1}$

$L_1 = L_2 = .0495$

$C_1 = C_2 = 20.2 - 1.0 = 19.2$

N=2

Chebyshev 0.5 db ripple

$q_1 = 1.9497$

$q_2 = 1.9497$

$K_{12} = 0.7225$

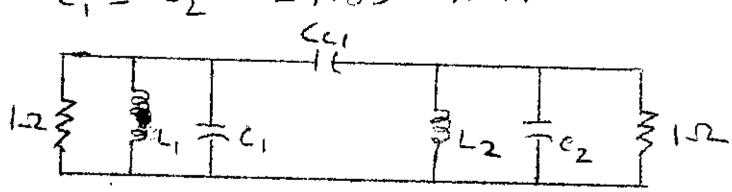
$L = \frac{.07}{1.9497} = 0.0359$

$C = 27.85$

$C_{12} = 0.7225 \times .07 \times 27.85 = 1.41 = C_{c1}$

$L_1 = L_2 = .0359$

$C_1 = C_2 = 27.85 - 1.41 = 26.44$



N=3

Butterworth

$q_1 = 1.00$

$q_3 = 1.00$

$K_{12} = .7071$

$K_{23} = .7071$

$L = \frac{.07}{1} = \frac{.07}{1} = .07$

$C = 14.29$

$C_{12} = .07 \times .7071 \times 14.29 = .7071 = C_{23}$

$L_1 = L_2 = L_3 = 0.07$

$C_1 = 14.29 - 0.7071$
$= 13.48 = C_3$

$C_2 = 14.29 - (0.7071 + 0.7071)$
$= 12.77$

N=3

Chebyshev 0.5 dB ripple

$q_1 = 1.8636$

$q_3 = 1.8636$

$K_{12} = 0.6474$

$K_{23} = 0.6474$

$L = \frac{.07}{1.8637} = 0.0376$

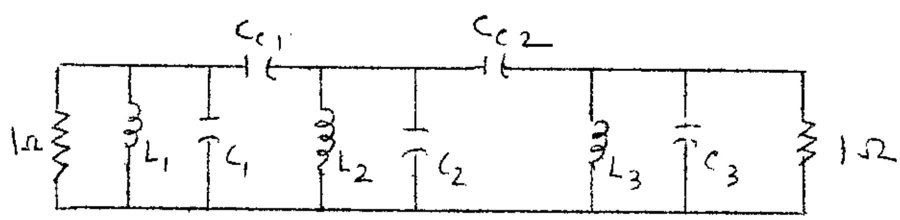
$C = 26.6$

$C_{12} = 0.6474 \times .07 \times 26.6 = 1.204 = C_{13}$

$L_1 = L_2 = L_3 = 0.0376$

$C_1 = 26.6 - 1.204$
$= 25.396 = C_3$

$C_2 = 26.6 - (1.204 + 1.204)$
$= 24.192$



N=4

Butterworth

$q_1 = 0.7654$

$q_4 = 0.7654$

$K_{12} = 0.8409$

$K_{23} = 0.5512$

$K_{34} = 0.8409$

$L = \frac{.07}{0.7654} = .0913$

$C = 10.95$

$C_{12} = 0.8409 \times .07 \times 10.95 = 0.645$

$C_{23} = 0.5512 \times .07 \times 10.95 = 0.421$

$L_1 = L_2 = L_3 = L_4 = .0913$

$C_1 = 10.95 - 0.645 = 10.305 = C_4$

$C_2 = 10.95 - (0.645 + 0.421) = 9.884 = C_3$

$C_{c_1} = 0.645$, $C_{c_2} = 0.421$

N-4

Chebyshev 0.5 dB ripple

$q_1 = 1.8258$

$q_4 = 1.8258$

$K_{12} = 0.6482$

$K_{23} = 0.5446$

$K_{34} = 0.6482$

$L = \frac{.07}{1.8258} = 0.0384$

$C = 26.07$

$C_{12} = 0.6482 \times .07 \times 26.07 = 1.185$

$C_{23} = 0.5446 \times .07 \times 26.07 = 0.992$

$L_1 = L_2 = L_3 = L_4 = 0.0384$

$C_1 = 26.07 - 1.185$
$= 24.885 = C_4$

$C_2 = 26.07 - (1.185 + 0.$[illegible]
$= 23.895 = C_3$

$C_{c_1} = 1.185$

$C_{c_2} = 0.992$

APPENDIX A2

COMPUTER PROGRAMMES

```
      DIMENSION IAW(11,11),IBW(11,11),ICW(11,11),ICP(11,11),IPW(11,11),
     1ICT(11,11),ITW(11,11)
      DOUBLE PRECISION A(11,11),CR(11,11), A(11),AB(11),XX(11),YY(11),
     1TT(11),AR(11),EM(11),G(11),H(11),GG(11),ZIR(11),ZII(11),TTWS(11),
     2ZL(11),XR(12),R(12),XCOF(22),COF(22),ROOTR(22),ROOTI(22),ZAA(5),
     3ZA3(5),CCC(2),PP(5),QQ(5),DG(11,2),Z C(4),ZSC(4),P(6),ARR(6,6),
     4AI(6,6),B(6,6),C(6,6),AFI(6,6),AII(6,6),AFFI(6,6),AFXI(6,6),
     5AXI(6,6),Q,S,BR,ABR,XXA,XXW,AFMX,YII,YIR,CNXW,TMAX
     6,W(150),AW(150),TW(150),TTW(150),XW(150),YW(150),ADB(150),PHAI(150
     7),WW(11)
     8,ZLL(2),ZC(3)
  100 READ(1,10)MA
   10 FORMAT(I10)
      IF(MA-10) 104,102,102
  102 GO TO 900
  104 CONTINUE
      N=(MA*2)+1
      READ(1,12)(AB(I),I=1,N)
   12 FORMAT(7F10.5)
      READ (1,13)(XX(I),I=1,N)
   13 FORMAT(7E11.4)
      MB=(MA*2-1)*2
      MC=MA*2
      WRITE(3,20)
   20 FORMAT(9X,'I',8X,'AB',13X,'WW',13X,'YY',13X,'XX',13X,'TT')
      DO 110 I=1,N
      AA(I)=AB(I)*1.57079633/90.
      WW(I)=DSIN(AA(I)/2.)/DCOS(AA(I)/2.)
      YY(I)=(WW(I)**MB)/((1.+WW(I)**2)**MC)
      TT(I)=YY(I)/XX(I)
      WRITE(3,22)I,AB(I),WW(I),YY(I),XX(I),TT(I)
      XXA=0.00
      DO 110 J=1,N
      A(I,J)=DCOS(XXA)
      XXA=XXA+AA(I)
  110 CONTINUE
   22 FORMAT(I10,5F15.8)
      BW=WW(MA+2)-WW(MA)
      WRITE(3,14)
   14 FORMAT(5X,'ORDER OF THE FILTER',5X,'BANDWIDTH OF THE FILTER')
      WRITE(3,16)MA,BW
   16 FORMAT(I15,F40.5)
      WRITE(3,28)
   28 FORMAT(15X,'MATRIX A FOR FOURIER SERIES EXPANSION')
      WRITE(3,26)((A(I,J),J=1,N),I=1,N)
   26 FORMAT(7E17.7)
      MD=MA+1
      DO 106 I=1,MD
      DO 106 KJ=1,MD
      J=2*KJ-1
  106 ARR(I,KJ)=A(I,J)
      DO 318 I=1,N
      DO 318 J=1,N
  318 IAW(I,J)=0
```

A2.1 GENERALISED PROGRAMME FOR BANDPASS FILTER DESIGN BY FOURIER METHOD USING POINT MATCHING TECHNIQUE(Contd.)

```
      DO 319 J=1,N
  319 IAW(J,1)=1
      IAW(2,2)=1
      DO 320 I=3,N
      DO 320 J=2,N
      K=J-1
  320 IAW(I,J)=IAW(I,K)*(I-K)/K
      DO 321 I=1,N
      DO 321 J=1,N
  321 IBW(I,J)=IAW(N-I+1,J)*(-1)**(J+1)
      DO 322 I=1,N
      ICW(I,1)=IAW(I,1)*IBW(I,1)
      DO 322 J=2,N
      ICW(I,J)=0
      DO 322 K=1,J
  322 ICW(I,J)=ICW(I,J)+IAW(I,K)*IBW(I,J-K+1)
      WRITE(3,313)
  313 FORMAT(15X,'MATRIX ICW')
      WRITE (3,354)((ICW(I,J),J=1,N),I=1,N)
  354 FORMAT(7I15)
      DO 325 I=1,N
      DO 325 J=1,N
  325 ICP(I,J)=0
      DO 326 I=1,MA
      K=2*I -1
      KK=2*I
      ICP(K,1)=1*(-1)**(I+1)
  326 ICP(KK,1)=0
      ICP(2,2)=1
      ICP(N,1)=(-1)**MA
      DO 328 K=3,N
      LL=K-1
      LLL=K-2
      DO 328 I=2,N
      J=I-1
  328 ICP(K,I)=ICP(LL,J)*2-ICP(LLL,I)
      WRITE (3,354)((ICP(I,J),J=1,N),I=1,N)
      DO 327 I=1,N
      DO 327 J=1,N
  327 ITW(I,J)=ICW(N-I+1,J)
      DO 329 I=1,N
      DO 329 J=1,N
      ICW(I,J)=ITW(J,I)
  329 ICT(I,J)=ICP(J,I)
      DO 330 I=1,N
      DO 330 J=1,N
      IPW(I,J)=0
      DO 330 K=1,N
  330 IPW(I,J)=IPW(I,J)+ICW(I,K)*ICT(K,J)
      WRITE(3,310)
  310 FORMAT(15X,'MATRIX IAW')
      WRITE (3,354)((IAW(I,J),J=1,N),I=1,N)
      WRITE(3,311)
  311 FORMAT(15X,'MATRIX IBW')
```

```
      WRITE (3,354)((IBW(I,J),J=1,N),I=1,N)
      WRITE(3,312)
  312 FORMAT(15X,'MATRIX ICW')
      WRITE (3,354)((ICW(I,J),J=1,N),I=1,N)
      WRITE(3,314)
  314 FORMAT(15X,'MATRIX ICP')
      WRITE (3,354)((ICP(I,J),J=1,N),I=1,N)
      WRITE(3,315)
  315 FORMAT(15X,'MATRIX IPW')
      WRITE (3,354)((IPW(I,J),J=1,N),I=1,N)
      DO 332 J=1,N
      DO 332 I=1,N
  332 CR(I,J)=IPW(I,J)
      WRITE(3,24)
   24 FORMAT(15X,'MATRIX CR FOR CONVERSION OF A INTO W')
      WRITE(3,26)((CR(I,J),J=1,N),I=1,N)
      N=MD
      DO 113 I=1,N
      DO 113 J=1,N
  113 A(I,J)=ARR(I,J)
      P(1)=0.0
      DO 114 I=1,N
  114 P(1)=P(1)+A(I,I)
      DO 116 I=1,N
      DO 116 J=1,N
  116 C(I,J)=A(I,J)
      DO 128 K=2,N
      DO 118 I=1,N
      DO 118 J=1,N
  118 B(I,J)=C(I,J)
      DO 120 I=1,N
  120 B(I,I)=B(I,I)-P(K-1)
      DO 122 I=1,N
      DO 122 J=1,N
      C(I,J)=0.00
      DO 122 L=1,N
  122 C(I,J)=C(I,J)+A(I,L)*B(L,J)
      Q=0.00
      DO 124 I=1,N
  124 Q=Q+C(I,I)
      S=K
  128 P(K)=Q/S
      DO 136 I=1,N
      DO 136 J=1,N
  136 AI(I,J)=B(I,J)/P(N)
      WRITE(3,32)
   32 FORMAT(10X,' THE MATRIX AI ')
      WRITE(3,26)((AI(I,J),J=1,N),I=1,N)
      DO 340 I=1,N
      DO 340 J=1,N
      AFI(I,J)=0.
      DO 340 K=1,N
  340 AFI(I,J)=AFI(I,J)+A(I,K)*AI(K,J)
      DO 345 I=1,N
```

```
      DO 345 J=1,N
      AII(I,J)=0.
      IF(I-J)345,342,345
  342 AII(I,J)=1.
  345 CONTINUE
      WRITE(3,26)((AII(I,J),J=1,N),I=1,N)
      DO 346 I=1,N
      DO 346 J=1,N
      AFFI(I,J)=0.
  346 AFFI(I,J)=AII(I,J)-AFI(I,J)
      WRITE(3,26)((AFFI(I,J),J=1,N),I=1,N)
      AFMX=1.
      DO 348 I=1,N
      DO 348 J=1,N
      IF(AFMX-AFFI(I,J))350,348,348
  350 AFMX=AFFI(I,J)
  348 CONTINUE
      WRITE(3,40)AFMX
      IF(AFMX-1.)364,364,362
  362 WRITE(3,33)
   33 FORMAT(5X,'CORRECTION DOES NOT CONVERGE')
      GO TO 990
  364 CONTINUE
      DO 360 IK=1,N
      DO 352 I=1,N
      DO 352 J=1,N
  352 AFXI(I,J)=AII(I,J)+AFFI(I,J)
      DO 351 I=1,N
      DO 351 J=1,N
      AXI(I,J)=0.
      DO 351 K=1,N
  351 AXI(I,J)=AXI(I,J)+AI(I,K)*AFXI(K,J)
      WRITE(3,26)((AFI(I,J),J=1,N),I=1,N)
      WRITE(3,353)
  353 FORMAT(5X,'CORRECTED AI=AXI')
      WRITE(3,26)  ((AXI(I,J),J=1,N),I=1,N)
      DO 356 I=1,N
      DO 356 J=1,N
      AFI(I,J)=0.
      DO 356 K=1,N
  356 AFI(I,J)=AFI(I,J)+A(I,K)*AXI(K,J)
      DO 358 I=1,N
      DO 358 J=1,N
      AI(I,J)=AXI(I,J)
      AFFI(I,J)=0.
  358 AFFI(I,J)=AII(I,J)-AFI(I,J)
  360 CONTINUE
      DO 138 K=1,N
      R(K)=0.0
      DO 138 J=1,N
      BR=AI(K,J)*XX(J)
  138 R(K)=R(K)+BR
      DO 139 I=1,N
      IK=2*I-1
```

```
      IJ=2*I
      XR(IJ)=0.0
  139 XR(IK)=R(I)
      N=2*MA+1
      DO 141 I=1,N
  141 R(I)=XR(I)
      WRITE(3,34)
   34 FORMAT(5X,'R,COEFFICIENT OF FOURIER SERIES ')
      WRITE(3,26)(R(I),I=1,N)
      WRITE(3,38)
   38 FORMAT(6X,'W',17X,'AW',16X,'PHAI ',15X,'YW',15X,'XAW',15X,'TW')
      W(1)=0.98
      DO 142 J=1,150
      AW(J)=2.*DATAN(W(J))
      YW(J)=(W(J)**MB)/((1.+W(J)**2)**MC)
      XXW=0.000
      XW(J)=0.00
      K=1
      DO 140 I=1,N
      XW(J)=XW(J)+R(K)*DCOS(XXW)
      XXW=XXW+AW(J)
  140 K=I+1
      TW(J)=YW(J)/XW(J)
      PHAI(J)=(180./3.1416)*AW(J)
      WRITE(3,39)W(J),AW(J),PHAI(J),YW(J), XW(J),TW(J)
   39 FORMAT(6E18.5)
      L=J+1
  142 W(L)=W(J)+0.0005
      TMAX=TW(1)
      DO 148 K=2,150
      IF(TW(K)-TMAX)148,146,146
  146 TMAX=TW(K)
  148 CONTINUE
      TMAX=TMAX+0.001
      WRITE(3,40) TMAX
   40 FORMAT(F30.16)
      DO 154 J=1,N
      AR(J)=0.0
      DO 154 K=1,N
      ABR=CR(J,K)*R(K)
  154 AR(J)=AR(J)+ABR
      WRITE(3,48)
   48 FORMAT(5X,'AR,COEFFICIENT OF POLYNOMIAL OF W')
      WRITE(3,26)(AR(J),J=1,N)
      CNXW=1./(AR(N)*TMAX)
      WRITE (3,40) CNXW
      WRITE(3,50)
   50 FORMAT(6X,'W',16X,'XW',15X,'AOB',14X,'TW',15X,'TTW',14X,'PHAI',13
     1X,'YW')
      W(1)=0.10
      DO 162 J=1,150
      AW(J)=2.*DATAN(W(J))
      W(J)=DSIN(AW(J)/2.)/DCOS(AW(J)/2.)
      XW(J)=AR(1)
```

```
      DO 158 I=2,N
  158 XW(J)=(XW(J)+AR(I)*W(J)**(2*(I-1)))
      XW(J)=XW(J)/(1.+W(J)**2)**MC
      YW(J)=(W(J)**MB)/((1.+W(J)**2)**MC)
      TW(J)=YW(J)/XW(J)
      TTW(J)=TW(J)/TMAX
      ADB(J)=-10.0*DLOG10(TTW(J))
      PHAI(J)=(180./3.1416)*AW(J)
      L=J+1
      W(L)=W(J)+0.0250
  162 WRITE(3,60)W(J),XW(J),ADB(J),TW(J),TTW(J) ,PHAI(J) ,YW(J)
   60 FORMAT(7E17.5)
      KM=2*N-1
      DO 168 J=1,N
      JJ=2*J-1
      JK=2*J
      JL=J+1
      XCOF(JJ)= (AR(J)/AR(N))*(-1.)**JL
  168 XCOF(JK)=0.00
      M=2*(N-1)
      DO 264 J=1,2
      CALL DPOLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
      WRITE(3,56)(XCOF(K),K=1,KM)
   56 FORMAT(9E13.5)
      DO 160 I=1,M
  160 WRITE(3,54)ROOTR(I),ROOTI(I)
   54 FORMAT(2(15X,E25.6))
      K=0
      L=0
      CCC(1)=0.000
      DO 240 I=1,M
      IF(ROOTR(I))246,242,240
  242 IF(ROOTI(I))244,240,240
  244 K=K+1
      CCC(K)=ROOTI(I)
  246 IF(ROOTI(I))248,248,240
  248 L=L+1
      ZAA(L)=ROOTR(I)
      ZAB(L)=ROOTI(I)
  240 CONTINUE
      IF(CCC(1))250,252,250
  252 DO 254 I=1,MA
      PP(I)=-2.*ZAA(I)
  254 QQ(I)=ZAA(I)**2+ZAB(I)**2
      GO TO 255
  250 MAA=MA-1
      DO 259 I=1,MAA
      PP(I)=-2.*ZAA(I)
  259 QQ(I)=ZAA(I)**2+ZAB(I)**2
      PP(MA)=0.
      QQ(MA)=CCC(1)
  255 DG(1,J)=1.0
      DO 256 I=2,N
  256 DG(I,J)=0.0
```

```
      DO 262 I=1,MA
      EM(1)=DG(1,J)*QQ(I)
      EM(2)=DG(2,J)*QQ(I)+DG(1,J)*PP(I)
      NNN=N-1
      DO 260 K=2,NNN
  260 EM(K+1)=DG((K+1),J)*QQ(I)+DG(K,J)*PP(I)+DG((K-1),J)
      DO 262 K=1,N
  262 DG(K,J)=EM(K)
      XCOF(2.*N-3)=XCOF(2*N-3)+1./(AR(N)*TMAX)
  264 CONTINUE
      WRITE (3,26)((DG(I,J),I=1,N),J=1,2)
      NK=N+2
      DO 265 J=1,NK
      G(J)=0.0
  265 H(J)=0.0
      DO 266 I=1,N
      G  (I)=(DG(I,1)+DG(I,2))/2.
  266 H  (I)=(DG(I,1)-DG(I,2))/2.
      WRITE(3,26)(G  (I),I=1,N)
      WRITE(3,26)(H  (I),I=1,N)
      NB=N-3
      NG=N-2
      DO 280 I=1,NB
      ZL(I)=H(I+1)/G(I)
      IF(I-N+5)268,268,272
  268 DO 270 K=1,NG
  270 G(K+I-1)=G(K+I-1)-H(K+I)/ZL(I)
      GO TO 276
  272 DO 274 K=1,3
  274 G(K+I-1)=G(K+I-1)-H(K+I)/ZL(I)
  276 DO 278 K=1,N
      GG(K)=G(K)
      G(K)=H(K)
  278 H(K)=GG(K)
      WRITE(3,26)(G(K),K=1,N)
      WRITE(3,26)(H(K),K=1,N)
  280 CONTINUE
      ZL(N-2)=H(N-1)/G(N-2)
      ZL(N-1)=G(N)/H(N-1)
      ZL(N)=H(N-1)/G(N-1)
      WRITE(3,26)(ZL (I),I=1,N)
      MAA=MA-1
      ZSL=ZL(N-2)/ZL(N)
      ZLL(2)=ZL(N)
      ZC(3)=ZL(N-1)
      DO 610 I=1,MAA
      IF(MAA-I)614,612,614
  612 ZLL(1)=1.0
      GO TO 616
  614 ZLL(1)=ZL(N-2*I-2)/ZSL
  616 CONTINUE
      ZC(1)=ZL(N-2*I-1)
      ZC(2)=-ZC(1)
      ZC(3)=ZC(3)+ZC(1)
```

```
      ZC(1)=ZC(1)*ZLL(1)
      ZC(2)=DSQRT(ZLL(1))*DSQRT(ZLL(2))*ZC(2)
      ZC(3)=ZLL(2)*ZC(3)
      ZCC(I)=-ZC(2)
      ZSC(I)=ZC(3)+ZC(2)
      ZLL(2)=ZLL(1)
  610 ZC(3)=(ZC(1)+ZC(2))/ZLL(1)
      ZSC(MA)=ZC(3)
      WRITE(3,288)
  288 FORMAT(5X,'COUPLING CAPACITANCE,ZCC')
      ZIR(I)=YIR/(YIR**2+YII**2)
      WRITE(3,13)(ZSC(I),I=1,MA)
      WRITE(3,13)(ZCC(I),I=1,MAA)
      WRITE(3,13)ZSL
      WRITE(3,26)(ZCC(I),I=1,MAA)
      WRITE(3,26)(ZSC(I),I=1,MA)
      WRITE(3,26)ZL(1),ZSL
      DO 292 I=1,N
      ZIR(I)=1.0
      ZII(I)=0.0
      DO 290 K=1,MAA
      YIR=ZIR(I)/(ZIR(I)**2+ZII(I)**2)
      YII=-ZII(I)/(ZIR(I)**2+ZII(I)**2)-(1.-(WW(I)**2)*ZSL*ZSC(K))/
     1(WW(I)*ZSL)
      ZIR(I)=YIR/(YIR**2+YII**2)
      WRITE(3,26)YIR,YII,ZIR(I),ZII(I)
  290 ZII(I)=-YII/(YIR**2+YII**2)  -1./(ZCC(K)*WW(I))
      YIR=ZIR(I)/(ZIR(I)**2+ZII(I)**2)
      YII=-ZII(I)/(ZIR(I)**2+ZII(I)**2)-(1.-(WW(I)**2)*ZL(1)*ZSC(MA))/
     1(WW(I)*ZL(1))
      ZIR(I)=YIR/(YIR**2+YII**2)
      ZII(I)=-YII/(YIR**2+YII**2)
      WRITE(3,26)YIR,YII,ZIR(I),ZII(I)
  292 TTWS(I)=1.-((1.-ZIR(I))**2+ZII(I)**2)/((1.+ZIR(I))**2+ZII(I)**2)
      WRITE(3,26)(TTWS(I),I=1,N)
  990 GO TO 100
  900 CALL EXIT
      END
```

```
      SUBROUTINE DPOLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
      DIMENSION XCOF(1),COF(1),ROOTR(1),ROOTI(1)
      DOUBLE PRECISION X0,Y0,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,
     1 DX,DY,TEMP,ALPHA
      DOUBLE PRECISION XCOF,COF,ROOTR,ROOTI
      IFIT=0                                                        POL 6
      N=M
      IER=C
      IF(XCOF(N+1)) 10,25,10                                        POL 10
10    IF(N) 15,15,32
15    IER=1
20    RETURN
25    IER=4
      GO TO 20
30    IER=2
      GO TO 20
32    IF (N-36)35,35,30
35    NX=N
      NXX=N+1
      N2=1
      KJ1=N+1
      DO 40 L=1,KJ1                                                 POL 2
      MT=KJ1-L+1
40    COF(MT)=XCOF(L)
45    X0=.C05001C1
      YC=0.01000101
      IN=C
50    X=X0
      XC=-10.0*Y0
      Y0=-10.0*X
      X=X0
      Y=Y0
      IN=IN+1
      GO TO 59
55    IFIT=1
      XPR=X
      YPR=Y
59    ICT=C
60    UX=0.0
      UY=0.0
      V=0.C
      YT=0.0
      XT=1.0
      U=COF(N+1)
      IF(U) 65,13C,65
65    DO 7C I=1,N
      L=N-I+1
      TEMP=COF(L)
      XT2=X*XT-Y*YT
      YT2=X*YT+Y*XT
      U=U+TEMP*XT2
      V=V+TEMP*YT2
      F1=I
      UX=UX+F1*XT*TEMP
```

```
      UY=UY-F1*YT*TEMP
      XT=XT2
   70 YT=YT2
      SUMSQ=UX*UX+UY*UY
      IF(SUMSQ) 75,110,75
   75 DX=(V*UY-U*UX)/SUMSQ
      X=X+DX
      DY=-(U*UY+V*UX)/SUMSQ
      Y=Y+DY
   78 IF(DABS(DY)+DABS(DX)-1.0D-10) 100,80,80
   80 ICT=ICT+1
      IF(ICT-500) 60,85,85
   85 IF(IFIT) 100,90,100
   90 IF(IN-5) 50,95,95
   95 IER=3
      GO TO 20
  100 DO 105 L=1,NXX
      MT=KJ1-L+1
      TEMP=XCOF(MT)
      XCOF(MT)=COF(L)
  105 COF(L)=TEMP
      ITEMP=N
      N=NX
      NX=ITEMP
      IF(IFIT) 120,55,120
  110 IF(IFIT) 115,50,115
  115 X=XPR
      Y=YPR
  120 IFIT=0
  122 IF(DABS(Y/X)-1.0D-08) 135,125,125
  125 ALPHA=X+X
      SUMSQ=X*X+Y*Y
      N=N-2
      GO TO 140
  130 X=0.0
      NX=NX-1
      NXX=NXX-1
  135 Y=0.0
      SUMSQ=0.0
      ALPHA=X
      N=N-1
  140 COF(2)=COF(2)+ALPHA*COF(1)
  145 DO 150 L=2,N
  150 COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
  155 ROOTI(N2)=Y
      ROOTR(N2)=X
      N2=N2+1
      IF(SUMSQ) 160,165,160
  160 Y=-Y
      SUMSQ=0.0
      GO TO 155
  165 IF(N) 20,20,45
      END
```

```
C     BANDPASS FILTER DESIGN.APPROXIMATED BY ASSUMED RESPONSE CURVES.
      DIMENSION IAW(11,11),IBW(11,11),ICW(11,11),ICP(11,11),IPW(11,11),
     1ICT(11,11),ITW(11,11)
      DOUBLE PRECISION A(11,11),CR(11,11),AA(11),AB(11),XX(11),YY(11),
     1TT(11),AR(11),EM(11),G(11),H(11),GG(11),ZIR(11),ZII(11),TTWS(11),
     2ZL(11),XR(12),R(12),XCOF(22),COF(22),ROOTR(22),ROOTI(22),ZAA(5),
     3ZAB(5),CCC(2),PP(5),QQ(5),DG(11,2),ZCC(4),ZSC(4),     ARR(6,6),
     4 ZBB(2),                RN(12),AI,P,X(40),AK,RABS,
     5AXI(6,6),Q,S,BR,ABR,XXA,XXW,AFMX,YII,YIR,CNXW,TMAX
     6,W(150),AW(150),TW(150),TTW(150),XW(150),YW(150),ADB(150),PHAI(150
     7),WW(11),ZLL(2),ZC(3),SYM,Y(20)
     8,ZSR,ZSL,BW,X2
      NN=1
      IORD=2
   26 FORMAT(7E17.7)
      P=4.0*DATAN(0.10D 01)
      DO 1 I=1,10
    1 RN(I)=0.0
      X2=80.0*(P/180.0)
   20 AB(1)=00.00
      DO 900 IJK=1,NN
      BW=2.0*(DSIN(P/4.0)/DCOS(P/4.0)-DSIN(X2/2.0)/DCOS(X2/2.0))
      WRITE(3,970)IORD,BW
  970 FORMAT(1H1,///40X,'ORDER OF THE FILTER=',I2,' ASSUMED BANDWIDTH=',
     1E11.4/)
      MA=IORD
      MB=(MA*2-1)*2
      MC=MA*2
      MD=MA+1
      N=2*IORD+1
      SYM=0.0
      AFMX=10.0*1.0D 02
   21 I=IJK
      X(I)=(P/180.00)*AB(I)
      Y(I)=(P/180.00)*(180.00-AB(I)-SYM)
      DO 2 K=1,MC
      AI=K
      RN(K)=(2.0/P)*((1.0/(AI*AI*(X(I)-X2)))*(DCOS(AI*X2)-DCOS(AI*X(I)))
     1+(1.0/(AI*AI*(Y(I)-P+X2)))*(DCOS(AI*Y(I))-DCOS(AI*(P-X2))))
    2 CONTINUE
    3 FORMAT(10E12.5)
      RABS=0.0
      DO 5 I=1,N
      IM=I+1
    5 R(IM)=RN(I)*AFMX
      DO 6 I=1,MA
      K=2*I+1
      L=I+1
    6 RABS=RABS+R(K)*(-1.0)**L
      R(1)=RABS+1.0/(2.0**(2*IORD))
      DO 318 I=1,N
      DO 318 J=1,N
  318 IAW(I,J)=0
      DO 319 J=1,N
```

A2.2. GENERALISED PROGRAMME FOR BANDPASS FILTER DESIGN BY FOURIER METHOD APPROXIMATED ASSUMING A RESPONSE CURVE. (Contd.)

```
  319 IAW(1,1)=1
      IAW(2,2)=1
      DO 320 I=3,N
      DO 320 J=2,N
      K=J-1
  320 IAW(I,J)=IAW(I,K)*(I-K)/K
      DO 321 I=1,N
      DO 321 J=1,N
  321 IBW(I,J)=IAW(N-I+1,J)*(-1)**(J+1)
      DO 322 I=1,N
      ICW(I,1)=IAW(I,1)*IBW(I,1)
      DO 322 J=2,N
      ICW(I,J)=0
      DO 322 K=1,J
  322 ICW(I,J)=ICW(I,J)+IAW(I,K)*IBW(I,J-K+1)
  354 FORMAT(7I15)
      DO 325 I=1,N
      DO 325 J=1,N
  325 ICP(I,J)=0
      DO 326 I=1,MA
      K=2*I -1
      KK=2*I
      ICP(K,1)=1*(-1)**(I+1)
  326 ICP(KK,1)=0
      ICP(2,2)=1
      ICP(N,1)=(-1)**MA
      DO 328 K=3,N
      LL=K-1
      LLL=K-2
      DO 328 I=2,N
      J=I-1
  328 ICP(K,I)=ICP(LL,J)*2-ICP(LLL,I)
      DO 327 I=1,N
      DO 327 J=1,N
  327 ITW(I,J)=ICW(N-I+1,J)
      DO 329 I=1,N
      DO 329 J=1,N
      ICW(I,J)=ITW(J,I)
  329 ICT(I,J)=ICP(J,I)
      DO 330 I=1,N
      DO 330 J=1,N
      IPW(I,J)=0
      DO 330 K=1,N
  330 IPW(I,J)=IPW(I,J)+ICW(I,K)*ICT(K,J)
      DO 332 J=1,N
      DO 332 I=1,N
  332 CR(I,J)=IPW(I,J)
      WRITE(3,24)
   24 FORMAT(15X,'MATRIX CR FOR CONVERSION OF A INTO W')
      WRITE(3,26)((CR(I,J),J=1,N),I=1,N)
      WRITE(3,34)
   34 FORMAT(5X,'R,COEFFICIENT OF FOURIER SERIES')
      WRITE(3,26)(R(I),I=1,N)
      WRITE(3,38)
```

```
   38 FORMAT(6X,'W',17X,'AW',16X,'PHAI',15X,'YW',15X,'XAW',15X,'TW')
      WRITE(3,40)TMAX
   40 FORMAT(F30.16)
      PHAI(1)=90.0
      DO 142 J=1,101
      AW(J)=PHAI(J)*(P/180.0)
      W(J)=DSIN(AW(J)/2.)/DCOS(AW(J)/2.)
      YW(J)=(W(J)**MB)/((1.+W(J)**2)**MC)
      XXW=C.COC
      XW(J)=0.00
      K=1
      DO 140 I=1,N
      XW(J)=XW(J)+R(K)*DCOS(XXW)
      XXW=XXW+AW(J)
  140 K=I+1
      TW(J)=YW(J)/XW(J)
      K=J+1
      WRITE(3,39)W(J),AW(J),PHAI(J),YW(J), XW(J),TW(J)
   39 FORMAT(6E18.5)
  142 PHAI(K)=PHAI(J)+0.05
      TMAX=TW(1)
      DO 148 K=2,100
      IF(TW(K)-TMAX)148,146,146
  146 TMAX=TW(K)
  148 CONTINUE
      TMAX=TMAX+0.001
      DO 154 J=1,N
      AR(J)=0.0
      DO 154 K=1,N
      ABR=CR(J,K)*R(K)
  154 AR(J)=AR(J)+ABR
      WRITE(3,50)
   50 FORMAT(6X,'W',16X,'XW',15X,'ADB',14X,'TW',15X,'TTW',14X,'PHAI',13
     1X,'YW')
      W(1)=0.85
      DO 162 J=1,150
      AW(J)=2.*DATAN(W(J))
      W(J)=DSIN(AW(J)/2.)/DCOS(AW(J)/2.)
      XW(J)=AR(1)
      DO 158 I=2,N
  158 XW(J)=(XW(J)+AR(I)*W(J)**(2*(I-1)))
      XW(J)=XW(J)/(1.+W(J)**2)**MC
      YW(J)=(W(J)**MB)/((1.+W(J)**2)**MC)
      TW(J)=YW(J)/XW(J)
      TTW(J)=TW(J)/TMAX
      ADB(J)=-10.0*DLOG10(TTW(J))
      PHAI(J)=(180./P)*AW(J)
      L=J+1
      W(L)=W(J)+0.002
  162 WRITE(3,60)W(J),XW(J),ADB(J),TW(J),TTW(J) ,PHAI(J) ,YW(J)
   6C FORMAT(7E17.5)
      KM=2*N-1
      DO 168 J=1,N
      JJ=2*J-1
```

```
      JK=2*J
      JL=J+1
      XCOF(JJ)= (AR(J)/AR(N))*(-1.)**JL
  168 XCOF(JK)=0.C0
      WRITE(3,48)
   48 FORMAT(5X,'AR,COEFFICIENT OF POLYNOMIAL OF W')
      WRITE(3,26)(AR(J),J=1,N)
      CNXW=1./(AR(N)*TMAX)
      WRITE (3,40) CNXW
      M=2*(N-1)
      DO 264 J=1,2
      WRITE(3,56)(XCOF(K),K=1,KM)
   56 FORMAT(9E13.5)
      CALL DPOLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
      IF(IER)170,172,170
  170 GO TO 100
  172 CONTINUE
      WRITE(3,52)IER
   52 FORMAT(5X,'IER=',I2)
      DO 160 I=1,M
  160 WRITE(3,54)ROOTR(I),ROOTI(I)
   54 FORMAT(2(15X,E25.6))
      K=0
      L=0
      JJ=0
      ZBB(1)=0.0
      CCC(1)=0.000
  241 DO 240 I=1,M
      IF(ROOTR(I))246,242,240
  242 IF(ROOTI(I))244,240,240
  244 K=K+1
      CCC(K)=ROOTI(I)
      GO TO 240
  246 IF(ROOTI(I))248,247,240
  248 L=L+1
      ZAA(L)=ROOTR(I)
      ZAB(L)=ROOTI(I)
      GO TO 240
  247 JJ=JJ+1
      ZBB(JJ)=ROOTR(I)
  240 CONTINUE
      IF(ZBB(1))249,251,249
  249 MAA=MA-1
      DO 253 I=1,MAA
      PP(I)=-2.0*ZAA(I)
  253 QQ(I)=ZAA(I)**2+ZAB(I)**2
      QQ(MA)=ZBB(1)*ZBB(2)
      PP(MA)=-ZBB(1)-ZBB(2)
      GO TO 255
  251 IF(CCC(1))250,252,250
  252 DO 254 I=1,MA
      PP(I)=-2.*ZAA(I)
  254 QQ(I)=ZAA(I)**2+ZAB(I)**2
      GO TO 255
```

```
  250 MAA=MA-1
      DO 259 I=1,MAA
      PP(I)=-2.*ZAA(I)
  259 QQ(I)=ZAA(I)**2+ZAB(I)**2
      PP(MA)=0.
      QQ(MA)=CCC(1)**2
  255 DG(1,J)=1.0
      DO 256 I=2,N
  256 DG(I,J)=0.0
      DO 262 I=1,MA
      EM(1)=DG(1,J)*QQ(I)
      EM(2)=DG(2,J)*QQ(I)+DG(1,J)*PP(I)
      NNN=N-1
      DO 260 K=2,NNN
  260 EM(K+1)=DG((K+1),J)*QQ(I)+DG(K,J)*PP(I)+DG((K-1),J)
      DO 262 K=1,N
  262 DG(K,J)=EM(K)
      XCOF(2.*N-3)=XCOF(2*N-3)+1./(AR(N)*TMAX)
  264 CONTINUE
      NK=N+2
      DO 265 J=1,NK
      G(J)=0.0
  265 H(J)=0.0
      DO 266 I=1,N
      G  (I)=(DG(I,1)+DG(I,2))/2.
  266 H  (I)=(DG(I,1)-DG(I,2))/2.
      NB=N-3
      NG=N-2
      DO 280 I=1,NB
      ZL(I)=H(I+1)/G(I)
      IF(I-N+5)268,268,272
  268 DO 270 K=1,NG
  270 G(K+I-1)=G(K+I-1)-H(K+I)/ZL(I)
      GO TO 276
  272 DO 274 K=1,3
  274 G(K+I-1)=G(K+I-1)-H(K+I)/ZL(I)
  276 DO 278 K=1,N
      GG(K)=G(K)
      G(K)=H(K)
  278 H(K)=GG(K)
  280 CONTINUE
      ZL(N-2)=H(N-1)/G(N-2)
      ZL(N-1)=G(N)/H(N-1)
      ZL(N)=H(N-1)/G(N-1)
      ZSR=0.10D 01
      IRX=0
      IRY=0
      WRITE(3,971)AFMX,ZL(N)
  971 FORMAT(9X,'A=',E14.6,'R=',E14.6)
      WRITE(3,972)(ZL(I),I=1,MC)
  972 FORMAT(1X,'L(1)=',E14.7,'C(1)=',E14.7,'L(2)=',E14.7,'C(2)=',E14.7,
     1'L(3)=',E14.7,'C(3)=',E14.7,'L(4)=',E14.7,'C(4)=',E14.7)
      WRITE(3,973)
  973 FORMAT(6X,'RL',9X,'L1',9X,'C1',9X,'CC1',8X,'L2',9X,'C2',9X,'CC2',
```

```
      18X,'L 3',9X,'C 3',9X,'CC3',8X,'L4',9X,'C4')
      MAA=MA-1
      ZSL=ZL(N-2)/ZL(N)
      ZLL(2)=ZL(N)
  281 ZC(3)=ZL(N-1)
      DO 610 I=1,MAA
      IF(MAA-I)614,612,614
  612 ZLL(1)=1.0
      GO TO 616
  614 ZLL(1)=ZL(N-2*I-2)/ZSL
  616 CONTINUE
      ZC(1)=ZL(N-2*I-1)
      ZC(2)=-ZC(1)
      ZC(3)=ZC(3)+ZC(1)
      ZC(1)=ZC(1)*ZLL(1)
      ZC(2)=DSQRT(ZLL(1))*DSQRT(ZLL(2))*ZC(2)
      ZC(3)=ZLL(2)*ZC(3)
      ZCC(I)=-ZC(2)
      ZSC(I)=ZC(3)+ZC(2)
      ZLL(2)=ZLL(1)
  610 ZC(3)=(ZC(1)+ZC(2))/ZLL(1)
      ZSC(MA)=ZC(3)
      XR(1)=ZL(1)
      XR(2)=ZSC(MA)
      XR(3)=ZCC(MAA)
      DO 618 I=1,MAA
      KIL=3*I+1
      KIC=3*I+2
      KCC=3*I+3
      KI= MAA-I+1
      KC=KI-1
      XR(KIL)=ZSL
      XR(KIC)=ZSC(KI)
  618 XR(KCC)=ZCC(KC)
      WRITE(3,288)
  288 FORMAT(5X,'COUPLING CAPACITANCE,ZCC')
      WRITE(3,26)(ZCC(I),I=1,MAA)
      WRITE(3,26)(ZSC(I),I=1,MA)
      WRITE(3,26)ZL(1),ZSL, 8XL, SYM, X 2
      WRITE(3,974)ZSR,(XR(I),I=1,KIC)
  974 FORMAT(1X,12E11.4)
      IRX=IRX+1
      IF(IRX-2)282,282,283
  282 ZLL(2)=ZL(N)*1.0OD 01**IRX
      ZSR=ZSR/1.0OD 01
      ZSL=ZSL/1.0OD 01
      GO TO 281
  283 IRY=IRY+1
      IF(IRY-1)285,284,285
  284 ZLL(2)=ZL(N-2)/ZL(1)
      ZSL=ZL(1)
      ZSR=ZL(N)/ZLL(2)
      GO TO 281
  285 CONTINUE
```

```
      AFMX=AFMX*1.00D 01
      IF(AFMX-0.10D 04)21,21,790
  790 CONTINUE
      SYM=SYM+1.0
      IF(SYM-0.00)21,21,800
  800 IK=IJK+1
  900 AB(IK)=AB(IJK)+10.0
      X2=X2-5.0*(P/180.0)
      IF(X2-60.*(P/180.0))990,20,20
  990 CONTINUE
      IORD=IORD+1
      IF(IORD-4)20,20,100
  100 CALL EXIT
      END
```

```
      SUBROUTINE DPOLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
      DIMENSION XCOF(1),COF(1),ROOTR(1),ROOTI(1)
      DOUBLE PRECISION XO,YO,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,
     1 DX,DY,TEMP,ALPHA
      DOUBLE PRECISION XCOF,COF,ROOTR,ROOTI
      IFIT=0                                                            POL 6
      N=M
      IER=0
      IF(XCOF(N+1)) 10,25,10                                            POL 10
      IF(N) 15,15,32
      IER=1
      RETURN
      IER=4
      GO TO 20
      IER=2
      GO TO 20
      IF (N-36)35,35,30
      NX=N
      NXX=N+1
      N2=1
      KJ1=N+1
      DO 40 L=1,KJ1                                                     POL 2
      MT=KJ1-L+1
      COF(MT)=XCOF(L)
      XO=.00500101
      YO=0.01000101
      IN=0
      X=XO
      XO=-10.0*YO
      YO=-10.0*X
      X=XO
      Y=YO
      IN=IN+1
      GO TO 59
      IFIT=1
      XPR=X
      YPR=Y
      ICT=0
      UX=0.0
      UY=0.0
      V=0.0
      YT=0.0
      XT=1.0
      U=COF(N+1)
      IF(U) 65,130,65
      DO 70 I=1,N
      L=N-I+1
      TEMP=COF(L)
      XT2=X*XT-Y*YT
      YT2=X*YT+Y*XT
      U=U+TEMP*XT2
      V=V+TEMP*YT2
      F1=I
      UX=UX+F1*XT*TEMP
```

A2.3. SUBROUTINE FOR SOLUTION OF POLYNOMIALS.

```
      UY=UY-F1*YT*TEMP
      XT=XT2
 70   YT=YT2
      SUMSQ=UX*UX+UY*UY
      IF(SUMSQ) 75,110,75
 75   DX=(V*UY-U*UX)/SUMSQ
      X=X+DX
      DY=-(U*UY+V*UX)/SUMSQ
      Y=Y+DY
 78   IF(DABS(DY)+DABS(DX)-1.0D-10) 100,80,80
 80   ICT=ICT+1
      IF(ICT-500) 60,85,85
 85   IF(IFIT) 100,90,100
 90   IF(IN-5) 50,95,95
 95   IER=3
      GO TO 20
 100  DO 105 L=1,NXX
      MT=KJ1-L+1
      TEMP=XCOF(MT)
      XCOF(MT)=COF(L)
 105  COF(L)=TEMP
      ITEMP=N
      N=NX
      NX=ITEMP
      IF(IFIT) 120,55,120
 110  IF(IFIT) 115,50,115
 115  X=XPR
      Y=YPR
 120  IFIT=0
 122  IF(DABS(Y/X)-1.0D-08) 135,125,125
 125  ALPHA=X+X
      SUMSQ=X*X+Y*Y
      N=N-2
      GO TO 140
 130  X=0.0
      NX=NX-1
      NXX=NXX-1
 135  Y=0.0
      SUMSQ=0.0
      ALPHA=X
      N=N-1
 140  COF(2)=COF(2)+ALPHA*COF(1)
 145  DO 150 L=2,N
 150  COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
 155  ROOTI(N2)=Y
      ROOTR(N2)=X
      N2=N2+1
      IF(SUMSQ) 160,165,160
 160  Y=-Y
      SUMSQ=0.0
      GO TO 155
 165  IF(N) 20,20,45
      END
```

```
      SUBROUTINE MTINV(A,XX,N,AI,R)
      DOUBLE PRECISION A(11,11),B(6,6),C(6,6),AI(6,6),XX(11),R(12),P(6
     1Q,S,BR
      P(1)=0.0
      DO 106 I=1,N
  106 P(1)=P(1)+A(I,I)
      DO 107 I=1,N
      DO 107 J=1,N
  107 C(I,J)=A(I,J)
      DO 108 K=2,N
      DO 109 I=1,N
      DO 109 J=1,N
  109 B(I,J)=C(I,J)
      DO 110 I=1,N
  110 B(I,I)=B(I,I)-P(K-1)
      DO 111 I=1,N
      DO 111 J=1,N
      C(I,J)=0.0
      DO 111L=1,N
  111 C(I,J)=C(I,J)+A(I,L)*B(L,J)
      Q=0.0
      DO 112 I=1,N
  112 Q=Q+C(I,I)
      S=K
  108 P(K)=Q/S
      IF(P(N))114,115,114
  115 WRITE (3,98)
   98 FORMAT('THE MATRIX IS SINGULAR')
      GO TO 104
  114 CONTINUE
      DO 116 I=1,N
      DO 116 J=1,N
  116 AI(I,J)=B(I,J)/P(N)
      DO 117 K=1,N
      R(K)=0.0
      DO 117 J=1,N
      BR=AI(K,J)*XX(J)
  117 R(K)=BR+R(K)
  104 RETURN
      END
```

A2.4. SUBROUTINE FOR MATRIX INVERSION AND SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS.

```
      SUBROUTINE CFCBPL(MA,N,ICP)
C     SUBROUTINE CFCBPL FOR CHEBYSHEV POLYNOMIAL
      DIMENSION ICP(11,11)
      DO 325 I=1,N
      DO 325 J=1,N
  325 ICP(I,J)=0
      DO 326 I=1,MA
      K=2*I -1
      KK=2*I
      ICP(K,1)=1*(-1)**(I+1)
  326 ICP(KK,1)=0
      ICP(2,2)=1
      ICP(N,1)=(-1)**MA
      DO 328 K=3,N
      LL=K-1
      LLL=K-2
      DO 328 I=2,N
      J=I-1
  328 ICP(K,I)=ICP(LL,J)*2-ICP(LLL,I)
      RETURN
      END
```

A2.5. SUBROUTINE FOR CHEBYSHEV POLYNOMIALS.

```
      SUBROUTINE CFBNPL(K,IAW,IBW)
C     SUBROUTINE CFBNPL FOR BINOMIAL EXPANSION
      DIMENSION IAW(11,11),IBW(11,11)
      DO 2 J=1,K
    2 IAW(J,1)=1
      IF(K-2)3,4,4
    3 GO TO 10
    4 IAW(2,2)=1
      IF(K-3)5,6,6
    5 GO TO 10
    6 CONTINUE
      DO 7 I=3,K
      DO 7 J=2,K
      L=J-1
    7 IAW(I,J)=IAW(I,L)*(I-L)/L
      DO 8 I=1,K
      DO 8 J=1,K
    8 IBW(I,J)=IAW(I,J)*(-1)**(J+1)
   10 RETURN
      END
```

A2.6. Subroutine For Binomial Expansion.

REFERENCES

1. Zobel, O. — Theory and Design of Electric Wave Filters, Bell System Tech.J., January, 1923.

2. Norton, E.L., — Constant Resistance Networks with Applications to filter groups. Bell System Tech. J. Vol.16, April, 1937.

3. Foster, R.M., — A Reactance Theorem, Bell System Tech., J. Vol.3, 1924.

4. Cauer, W, — Die Verwirklinchung Von Weshel Wechsel Stromwiderstander Vorgechrichumer Frequenzabhongighuit," Arch.Electrotech., Vol. 17, 1927.

5. Bode, H.W. — Network Analysis and Feedback Amplifier Design, D.Van Nostrand Company, Princeton, N.J. 1945.

6. Brune, O. — Synthesis of a Finite Two terminal network whose Driving point Impedance is a prescribed Function of Frequency. J.Math. Phy. Vol.10, August 1931.

7. Guillemin, E.A. — Synthesis of Passive Network, John Wiley and Sons, 1957.

8. Darlington, S., — Synthesis of Reactance Reactance Four Poles which produce prescribed Insertion low. Charactertics, J.Math.Phys. 30 September 1939.

9. Darlington, S. — Network Synthesis Using Tchebyshev Polynomial series. Bell System Tech. J. 31, July 1952.

10. S.Butterworth, — On theory of Filter Amplifiers, Wireless Engg., Vo.7, 1930.

11. Cohn.S.B.: — Direct Coupled Resonator Filter, Proc. IRE, Feb. 1957.

12. Dishal M, — Design of Dissipative Bandpass filters producing Desired exact Amplitude Frequency Characteristics. Proc. IRE Vol.37, September 1949.

13. Ruston, H & Bordoga, J — "Electric Networks: Functions, Filters, Analysis McGrawhill Book Company, 1966.

14. Mathaei, Young, Jones, Microwave Filters, Impedance Matching Networks, and Coupling Structures, McGraw-Hill Book Company, 1964.

15. Anatol I.Zverev — Handbook of Filter Synthesis, John Wily & Son, 1967.

16. Guellemin, E.A. — The Mathematics of Circuit Analysis, M.I.T. Press, 1949.

17. Pennington, H.R. — Computer Methods and Numerical Analysis, The Macmillan Company, 1970.