MODELING OF COLLECTOR OF POWER TRANSISTOR
INCLUDING THE EFFECTS OF MINORITY CARRIER LIFE TIME AND LOW-HIGH JUNCTION

by

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A Thesis
submitted to the Department of Electrical and Electronic Engineering
in partial fulfillment of the requirements for the degree
of
Master of Science in Engineering
(Electrical and Electronic)

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May, 1996
DECLARATION

I hereby declare that this work has been done by me and it has not been submitted elsewhere for the award of any degree or diploma.

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1 INTRODUCTION

1.1 Epitaxial Bipolar Transistor 1
1.2 Different Transistor parameters 2
1.3 Previous models 4
  1.3.1 The Ebers-Moll model 4
  1.3.2 The Gummel-Poon model 6
  1.3.3 Other models 10
1.4 Low-High junction 11
1.5 Objective of the thesis 15
1.6 Summery of the Thesis 16

2 FORMULATION OF THE COLLECTOR CURRENT AND VOLTAGE OF POWER TRANSISTOR 17

2.1 Introduction 17
2.2 Analysis of the epitaxial collector 18
2.3 Implementation of the model 31
  2.3.1 Base region model 31
  2.3.2 Emitter region model 32
2.4 Conclusion

3 RESULTS AND DISCUSSIONS

3.1 Introduction
3.2 Output characteristics of a bipolar transistor
3.3 Output characteristics of a bipolar transistor neglecting recombination current
3.4 Dependence of collector current on collector width
3.5 Conclusions

4 CONCLUSION AND SUGGESTIONS

4.1 Conclusion
4.2 Limitations of the present model
4.3 Suggestions

REFERENCES

APPENDIX A: Collector minority carrier profile for transistor in saturation.
ACKNOWLEDGMENT

The author would like to express his indebtedness and gratitude to his supervisor Dr. M. M. Shahidul Hassan, Professor of Electrical and Electronic Engineering Department, BUET, for his continuous and friendly supervision, constant inspiration, endless patience and invaluable assistance throughout the entire progress of the work.

The author also wishes to express his thanks and regards to Dr. A. B. M. Siddique Hossain, Professor and Head of Electrical and Electronic Engineering Department, BUET, for his support to complete this work successfully.

Finally the author would like to express thanks and indebtedness to his friends and colleagues for their cooperation. The author is very much grateful to his wife and the members of his family for their encouragement and constant support.
ABSTRACT

Epitaxial n⁺pn⁻n⁺ bipolar power transistors offer several advantages over the previously used npn structures. The collector of this type of transistor is more lightly doped than its base. As a result high current effects occur predominantly in the epitaxial collector region. At low voltages, the transistor operates in saturation and when the current is high, a conductivity modulated region is formed in the lightly doped collector region to support the current. Therefore, the need for understanding the dynamics of the epi-collector under different collector current densities is necessary. In order to investigate the dynamics of the epi-collector transistor, ‘regional approach’ is applied rather than the charge-control approach. The whole collector region is divided into three regions, namely i) injection region, ii) intermediate region and iii) end region. In the injection region minority and majority carrier profiles are determined by considering the finite minority carrier life time in the collector. The complex ambipolar differential equation is made analytical tractable by applying reasonable approximations. The electric field distributions and the voltage in each of these three regions are determined. The effect of the low-high junction is also incorporated in the present model. Both the drift and diffusion currents are considered in first two regions of the epi-collector but the diffusion current is neglected in the end region where the electric field is high. By using the present model, the different characteristics of bipolar transistors are studied. The results obtained by this model are in good agreement with experimental data available in the literature which demonstrates the validity and usefulness of the new model.
# LIST OF FIGURES

1.1 Cross-section of an $n^+p^+n^+$ epitaxial bipolar transistor.  
1.2 Different current components of an npn transistor.  
1.3 Circuit diagram of the Ebers-Moll model of a bipolar transistor.  
1.4 Circuit diagram of the Gummel-Poon model of a bipolar transistor.  
1.5 (a) Energy band diagram for $n^+$ and $p^+$ junctions in thermal equilibrium and (b) corresponding electron and hole densities.  
1.6 (a) Electron energy band diagram for a forward and reverse biased $n^+$ junction (b) corresponding hole densities.  
2.1 An one-dimensional $n^+p^+n^+$ structure of an epitaxial bipolar transistor showing different regions.  
3.1 Flow chart for computing the output characteristics of a bipolar transistor.  
3.2 (a) Output characteristics of an $n^+p^+n^+$ transistor indicating the quasi-saturation region for higher base currents  
3.2 (b) Output characteristics of an $n^+p^+n^+$ transistor indicating the quasi-saturation region for lower base currents  
3.3 (a) Current-Voltage characteristics for different base currents comparing compact model simulation and measurements of an npn bipolar transistor for higher base current  
3.3 (b) Current-Voltage characteristics for different base currents comparing compact model simulation and measurements of an npn bipolar transistor for lower base currents.  
3.4 Current-Voltage characteristics for different values of base-emitter voltage.
3.5 Current-Voltage characteristics for different values of base current neglecting the recombination current.

3.6 Current-Voltage characteristics for different values of collector layer thickness with constant base-emitter voltage.
LIST OF TABLES

3.1 Device make-up and parameter for Figure 3.2, Figure 3.3 and Figure 3.5 49
3.2 Device make-up and parameter for Figure 3.4 and Figure 3.6 49
1.1 EPITAXIAL BIPOLAR TRANSISTOR

The bipolar transistor, one of the most important semiconductor devices, was first invented by Bell Laboratories in 1947. Since then the transistor theory has been extended to include high-frequency, high-power and switching behaviors. Many breakthroughs have been made in transistor technology, specially in epitaxial technique. The epitaxial technique consists of growing a thin, high purity single-crystal layer of silicon on a heavily doped substrate of the same material. This augmented crystal forms the collector (Figure 1.1) on which the base and emitter may be diffused through some standard process [1]. This technique is useful in manufacturing power transistors.

The basic bipolar transistors are of npn or pnp structure in which emitters and collectors are heavily doped but the bases are very lightly doped. But this structure is not suitable for high-voltage power transistors. Because for a power transistor switch, the desired features are current handling capability in the on-state and blocking voltage in the off-state together with switching times and losses. These features can be successfully achieved in epitaxial transistors. The region adjacent to the base-collector junction is most lightly doped to support the reverse-biased collector-base voltage. Hence this region essentially determines the breakdown voltage. In regard to the collector thickness \( W_C \), the obvious choice would be to allow the depletion layer to spread freely at the specified open base breakdown voltage [1]. But a moderate reduction of \( W_C \) less than the unbounded depletion layer width can lead to an advantageous rise in maximum collector current \( I_C \). On the other hand, in order to avoid change of voltage-blocking
capability such reduction must be accompanied by a decrease in the collector impurity concentration \( N_C \), thereby increasing the collector resistance.

![Cross-section of an \( n^+pnn^+ \) epitaxial bipolar transistor](image)

Fig. 1.1. Cross-section of an \( n^+pnn^+ \) epitaxial bipolar transistor

The performance of transistors demands a combination of high doping levels and thin epitaxial layers to meet the requirements of open base breakdown voltage. Optimization of the collector layer in this respect leads to a reach-through condition at breakdown. The heavily doped (\( n^+ \)) substrate cuts the electric field abruptly at the \( n^+n^+ \) interface with zero voltage drop.

Epitaxial techniques offer great versatility for manufacturing power transistors. Multilayered epitaxial transistor may combine some of the advantages of the previous structures [1].

1.2 DIFFERENT TRANSISTOR PARAMETERS

In this section various parameters of a bipolar transistor will be discussed. For this purpose the different current components of an npn transistor are shown in Figure 1.2. The current components are related to the terminal currents by these parameters. In this introduction we shall neglect the saturation current at the collector and such effects as recombination in the transition regions because of their
small values. Under these assumptions, the collector current is made up entirely of those electrons injected at the emitter which are not lost to recombination in the base. Thus \( i_C \) is proportional to the electron component of the emitter current \( i_{En} \) and is given by [2]

\[
E \quad B \quad C
\]

\[
\begin{array}{c}
\text{n} \\
\downarrow i_{E0} \\
\text{p} \\
\uparrow i_{E} \\
\downarrow i_{E0} \\
\text{n} \\
\end{array}
\]

\[ i_C = B i_{En} \] (1.1)

B is the fraction of injected electrons which make it across the base to the collector and called the base transport factor. The total emitter current is made up of electron component \( i_{En} \) and hole component \( i_{Ep} \). The emitter injection efficiency \( \gamma \) is [2]

\[
\gamma = \frac{i_{En}}{i_{En} + i_{Ep}}
\] (1.2)

The relation between collector and emitter current is [2]

\[
\frac{i_C}{i_E} = \frac{Bi_{En}}{i_{En} + i_{Ep}} = B \gamma = \alpha
\] (1.3)

Where \( \alpha \) is called the current transfer ratio.

In accounting for the base current, we must include the rates at which holes are lost from the base by injection across the emitter junction and the rate of electron recombination with holes in the base. If the fraction of injected electrons making it across the base without recombination is B, then it follows that \( (1-B) \) is the fraction recombining in the base. Thus the base current is [2]
\[ i_B = I_{Ep} + (1-B)i_{En} \] (1.4)

neglecting the saturation current. The relation between the collector and base current is found from Eqs (1.1) and (1.4) [2]

\[
\frac{i_C}{i_B} = \frac{Bi_{En}}{i_{Ep} + (1-B)i_{En}} = \frac{B[i_{En}/(i_{En} + i_{Ep})]}{1-B[i_{En}/(i_{En} + i_{Ep})]}
\]

\[
= \frac{B\gamma}{1-B\gamma} = \frac{\alpha}{1-\alpha} \equiv \beta
\] (1.5)

The factor \( \beta \) relating the collector current to the base current is the base-to-collector current amplification factor.

1.3 PREVIOUS MODELS

Device modeling aims at relating physical device parameters to device terminal characteristics. Device modeling is especially important for integrated circuits, since simple and accurate device models are needed to predict the performance of the circuits. Models representing transistors accurately are complex and difficult to study. Therefore, there is a trade-off between accuracy and complexity. Different models are analyzed until recently considering the minority carrier life time and low-high junction. The most simplified model for low level injection is the Ebers-Moll model [3].

1.3.1 The Ebers-Moll Model

Ebers-Moll model is a simplified model for bipolar transistors. This model consists of two diodes connected back to back and current sources. The current sources are driven by the diode currents which are assumed to have ideal characteristics. Forward and reverse-biased diode currents are given by

\[
I_F = I_{FD}(e^{V_{BE}/V_T} - 1)
\] (1.6)

\[
I_R = I_{RD}(e^{V_{BE}/V_T} - 1)
\] (1.7)
where, $I_{FO}$ and $I_{RO}$ are the saturation currents of normally forward and reversed-biased diodes, respectively. The terminal currents are,

$$I_E = I_F - \alpha_1 I_R$$  \hspace{1cm} (1.8) \\
$$I_C = I_R - \alpha_N I_F$$  \hspace{1cm} (1.9)

and

$$I_B = -(1 - \alpha_N)I_F - (1 - \alpha_1)I_R$$  \hspace{1cm} (1.10)

where, $\alpha_N$ and $\alpha_1$ are the forward and reversed common-base current gains, respectively. Equations (1.6)-(1.10) give the relation between terminal currents $I_E$ and $I_C$ and the terminal voltages, $V_{BE}$ and $V_{BC}$. Using equations (1.6),(1.7),(1.8) and (1.9), $I_E$ and $I_C$ can be written as,

$$I_E = \alpha_{11}(e^{V_{BC}/V_T} - 1) + \alpha_{12}(e^{V_{BC}/V_T} - 1)$$  \hspace{1cm} (1.11)

and

$$I_C = \alpha_{21}(e^{V_{BC}/V_T} - 1) + \alpha_{22}(e^{V_{BC}/V_T} - 1)$$  \hspace{1cm} (1.12)

Here,

$$\alpha_{11} = I_{FO}$$  \hspace{1cm} (1.13) \\
$$\alpha_{12} = -\alpha_1 I_{RO}$$  \hspace{1cm} (1.14) \\
$$\alpha_{21} = -\alpha_N I_{FO}$$  \hspace{1cm} (1.15) \\
$$\alpha_{22} = I_{RO}$$  \hspace{1cm} (1.16)

From the reciprocity theorem of the two-port device,

$$\alpha_{12} = \alpha_{21}$$  \hspace{1cm} (1.17)

so that,

$$\alpha_{11} I_{RO} = \alpha_N I_{FO}$$  \hspace{1cm} (1.18)

Figure 1.3 shows the circuit diagram of the Ebers-Moll model of a bipolar junction transistor.

The Ebers-Moll model was developed based on the following five assumptions (for an npn transistor).

i) Electrons diffuse from emitter to collector,
ii) drift current is negligible in the base region,

iii) the emitter current is made up entirely of electrons,

iv) the emitter injection efficiency is unity and

v) the active part of the base and the two junctions are of uniform cross-section area.

![Circuit diagram of the Ebers-Moll model of a bipolar transistor](image)

**Fig. 1.3. Circuit diagram of the Ebers-Moll model of a bipolar transistor**

Current flow in the base is essentially one dimensional from emitter to collector. In Ebers-Moll model base-narrowing (Early effect) was not incorporated. The model did not consider the collector region in the evaluation of current-voltage characteristics of the transistor.

### 1.3.2 The Gummel-Poon model

The Ebers-Moll model [3] has been the major large-signal model for bipolar transistors since its formulation in 1954. It is based directly on device physics and covers all operating regions, that is, active, saturated and cut-off operation. But various approximations limit the accuracy of the model. In 1957 Beaufoy and Sparkes [4] analyzed the bipolar transistor from a charge control point of view. The
charge control model or the equivalent charge control form of Ebers-Moll model, is
directly useful for transient analysis.

As device technology evolved over the years making possible devices of
reproducible characteristics and as better understanding was gained in device-
theories, many new effects were identified that are not represented by the Ebers-
Moll model [3]. Among these are a finite, collector-current-dependent output
conductance due to base width modulation (Early effect) [5], space-charge-layer
generation and recombination (Sah-Noyce-Shockly effect) [6], conductivity
modulation in the base (Webster effect) [7] and in the collector (Kirk effect) [8]
and emitter crowding.

The Gummel-Poon model [9] makes use of a general charge-control relation
which links junction voltages, collector current and base charge. This charge
control relation, used in conjunction with conventional charge control theory,
allows many of the effects not contained in the basic Ebers-Moll model to be
incorporated in an integral, compact form.

To obtain the integral charge control relation, consider the current
equations,

\[ J_n = q \mu_n n \frac{\delta \phi_n}{\delta x} \]  \hspace{1cm} (1.19)

\[ J_p = -q \mu_p p \frac{\delta \phi_p}{\delta x} \]  \hspace{1cm} (1.20)

where \( J_n \) and \( J_p \) are electron and hole current densities, \( \mu_n \) and \( \mu_p \) are the electron
and hole mobilities, \( \phi_n (\phi_p) \) electron (hole) Fermi level.

The electron and hole concentrations can be given by,

\[ n = n_i \exp(q(\psi - \phi_n)/kT) \]  \hspace{1cm} (1.21)

\[ p = n_i \exp(q(\phi_p - \psi)/kT) \]  \hspace{1cm} (1.22)

\( n_i \) is the intrinsic concentration and \( \psi \) is the potential.

The space derivative of the pn product can be written as
Integrating the equation (1.23) from \(x=0\) to \(x=W\), using equations (1.19) to (1.22) and considering negligible recombination, it can be shown that [9]

\[
I_w - I_p \approx J_c \int_0^W n(x) \frac{dx}{\mu_p}
\]

where, \(J_c\) is the density of current that would flow from emitter to collector if the transistor had unity gain. Substituting equations (1.21) and (1.22) into (1.24) yields

\[
\exp\left(\frac{q(\phi_p - \phi_n)}{kT}\right)_{x=0} - \exp\left(\frac{q(\phi_p - \phi_n)}{kT}\right)_{x=W} = \frac{J_c}{n_i^2 kT} \int_0^W n(x) \frac{dx}{\mu_p}
\]

In the model the electron imref \(\phi_n\) is considered constant in the base. Therefore,

\[
V_{BE} = \phi_p(0) - \phi_n(0)
\]

and

\[
V_{BC} = \phi_p(W) - \phi_n(W)
\]

These voltages differ from terminal voltages by ohmic drops. Equation (1.25) becomes

\[
J_{CC} = A_E J_{CC} = q n_i A_E D_B \left( \frac{q V_{BE}}{e kT} - \frac{q V_{BC}}{e kT} \right)
\]

where \(A_E\) is the active area. The Gummel-Poon model is based on equation (1.25) which links junction voltages, collector current and base charge. The modeling problem reduces to modeling the base charge.

\[
Q_B = q A \int_0^W n dx
\]

which consists of five components:

\[
Q_B = Q_{BO} + Q_{JE} + Q_{JC} + Q_{dE} + Q_{dC}
\]
where, $Q_{BO}$ is the zero-bias charge, $Q_{BE}$ and $Q_{IC}$ are charges associated with emitter and collector depletion capacitance, $Q_{de}$ and $Q_{dc}$ are minority-carrier charges associated with emitter and collector diffusion capacitances. As the injection level increases, the diffusion capacitance increases and gives rise to the high-injection gain degradation. Rewriting (1.28), it can be shown

$$I_{CC} = I_F - I_R$$

where,

$$I_F = I_S Q_{BO} \left( \frac{qV_{BE}}{e^{kT} - 1} \right)$$

$$I_R = I_S Q_{BO} \left( \frac{qV_{BE}}{e^{kT} - 1} \right)$$

Equations (1.32) and (1.33) resemble equations (1.6) and (1.7) in the Ebers-Moll model. The charge $Q_{de}$ can be expressed as $B\tau_F I_F$, where $\tau_F$ is the lifetime associated with minority carrier in forward current and $B$ is a factor that is usually equal to unity, but may become larger than unity from the Kirk effect. The charge $Q_{dc}$ can be expressed as $\tau_R I_R$, where $\tau_R$ is the lifetime of minority carriers in reverse current. The base current is given by

$$I_B = \frac{dQ_B}{dt} + I_{rec}$$

where the base recombination current can be separated into two parts,

$$I_{rec} = I_{BE} + I_{BC}$$

where,

$$I_{BE} = I_1 \left( \frac{qV_{BE}}{e^{kT} - 1} \right) + I_2 \left( \frac{qV_{BE}}{e^{m_e kT} - 1} \right)$$

and
In these equations, $m_e$ and $m_c$ are the emitter and collector ideality factors. For ideal currents $m_e$ and $m_c$ are both equal to 1; for depletion-recombination-generation currents, $m_e$ and $m_c$ are both equal to 2. The total emitter and collector currents can be expressed as,

$$I_E = I_{CC} + I_{BB} + r_F \left( \frac{dI_F}{dt} \right) + C_{jE} \left( \frac{dV_{BE}}{dt} \right)$$

(1.38)

$$I_C = I_{CC} - I_{BC} - r_R \left( \frac{dI_R}{dt} \right) + C_{jC} \left( \frac{dV_{CE}}{dt} \right)$$

(1.39)

Figure 1.4 shows the circuit diagram of the Gummel-Poon model, complete with series resistances. Since $Q_B$ is voltage-dependent, the effect of high injection in the base ($\tau_F I_F$ becomes larger than $Q_{BO}$) is included. The current-induced base push-out (Kirk effect) is represented by the factor $B$, which is a function of $I_C$ and $V_{BC}$. The emitter part $I_{BE}$ of the base current is modeled by two diodes in parallel, one ideal and one with an ideality factor $m_e > 1$. This makes the current gain at low current levels bias dependent. The voltage dependence of $Q_{EC} (=c_{jC} V_{BE})$ models the Early Effect. Many physical effects have been taken into account through the bias-dependent $Q_B$.

1.3.3 Other models

Recently, Kull et al. [10] presented a compact model which is an extension of the Gummel-Poon model [9]. This is applicable to bipolar junction transistors even exhibiting quasi-saturation or base push-out effects. When a device with a lightly doped collector region is operated at high injection level (in the collector region), dc current gain falls sharply from its maximum value. Such an operating region is generally referred as quasi-saturation. Quasi-saturation can be defined as the region where the internal base-collector metallurgical junction is forward.
biased, while the external base-collector terminal remains reverse biased. In this mode of operation, minority carriers are injected into the epitaxial region, widening the electrical base of the device and thus reducing current gain and storing excess charge in the epitaxial region.

The quasi-saturation effect has been investigated by many authors [11,12,13,14] and in general two distinct models have been developed for the ohmic and non-ohmic regions of operation. For the ohmic model, the carrier drift velocity is assumed to be linearly proportional to the electric field. For the nonohmic quasi-saturation model, the carriers in the entire collector space-charge region are assumed to be moving at their scattering -limited velocity. In the nonohmic model, Whitter and Tremers [12] have examined the importance of two dimensional effects, such as current crowding and current spreading. Kumar and Hunter [15] showed that a one-dimensional model is adequate to model the nonohmic quasi-saturation effect. Hanggen and Fossum [16] extended Kull's model to account for the possible existence of the current induced space-charge region in the epitaxial collector. Their model did not include the saturation effect and like other models requires parameters extracted from measurements.

However, minority carrier life time in the collector was not considered in any of the previous models. Very recently Hassan and Choudhury [17] adopted a regional approach taking into consideration current gain dependence on collector minority carrier life time. They have also considered the recombination velocity at the low-high (n n^+) junction.

1.4 LOW-HIGH JUNCTION

A junction between an n-region in which the doping level is low and an n-region in which the doping level is high is designated nn^+; for p material pp^+ has the same significance. Such junctions may be regarded as limiting forms of the inhomogeneous specimen in which the transition from lightly doped to heavily doped material is abrupt. They are frequently used in conjunction with a metal
contact to the heavily doped region to provide ohmic (non-rectifying) contacts to the bulk p- and n-regions of devices incorporating p-n junctions.

Fig. 1.4 Circuit diagram of the Gummel-Poon model of a bipolar transistor
The energy band diagrams for nn+ and pp+ junctions in thermal equilibrium are shown in the top part of Figure 1.5. The corresponding variation of free charge density is shown by the solid line in the lower part of the figure. The space charge density is represented by the shaded areas. Compared with the p-n junction the width of the space charge region is small, since the barrier height $V_d$ is small (this is, as for the p-n junction, simply the voltage equivalent of the difference in the electron energies at a band edge on each side of the junction). Furthermore the space charge region is not depleted of its free carriers; the space charge on the lightly doped side is in fact due to free carriers which have diffused from the heavily doped side, where they leave behind a corresponding ionic space charge of opposite sign.

Since the space charge region is not depleted of its carriers it is not a high resistance region. Applied bias is therefore developed mainly across the bulk of the lightly doped material. The situation for 'forward bias' and 'reverse bias' is shown in the top part of the Figure 1.6 (a) and (b) respectively for an nn+ junction. In both cases the current is carried predominantly by electrons and the continuity requirement for the total current density to be constant throughout both the regions is that $E_n \sigma_n = E_{n+} \sigma_{n+}$.

Fig. 1.5 (a) Energy band diagrams for nn+ and pp+ junctions in thermal equilibrium and (b) corresponding electron and hole densities
It is of interest to consider the minority carrier (hole) currents. In the n-region the hole current is greater than that in the n\(^+\)-region because (i) the hole density is greater in the n-region and (ii) so is the electric field. Since the hole current at the junctions cannot change abruptly, it is clear that additional diffusion components of hole current must arise in this region to satisfy the continuity requirements for hole current. The gradients of hole densities which produce the appropriate directions of diffusion current for this purpose are shown in the lower part of Figure 1.6. It may be noted that for forward bias the hole density in the vicinity of the junction is decreased, whereas for reverse bias it is increased. These processes are known as exclusion and accumulation respectively. To preserve electrical neutrality there are corresponding changes in the electron densities in these regions.

In general it may be said that low-high junctions are relatively impermeable to minority carriers, but offer no hindrance to majority carriers. Hence they are non-injecting contacts for minority carriers.

![Diagram of Electron Energy Band Diagram for Forward and Reverse Biased n-n\(^+\) Junction](image)

Fig. 1.6 (a) Electron energy band diagram for a forward and reverse biased n-n\(^+\) junction and (b) corresponding hole densities.
1.5 OBJECTIVE OF THE THESIS

Nowadays, epitaxial n⁺pₙn⁺ structures are extensively used as power transistors [1]. Since in this type of transistors collector is more lightly doped than base, high current effects occur predominantly in the epitaxial collector region. At low voltage, such a transistor operates in quasi-saturation and when the current is high, an injection region must form in the lightly doped collector region to support the current. Reported works on epitaxial bipolar transistor did not adequately account these effects except the work [17]. A knowledge of the injected minority carrier density profile in the lightly doped collector region is essential for predicting the transistor currents in the quasi-saturation region. To obtain an accurate profile both diffusion and drift components of the minority carriers are to be considered. Hassan and Choudhury [17] adopted a regional approach taking into account current gain dependency on collector minority carrier lifetime. They have also considered the recombination velocity at the low-high (n⁺ n⁺) junction. In the present research the model for the epitaxial collector region is derived taking into account the effect of minority carrier lifetime and low-high junction. Here, the boundary conditions that have been chosen differ from the previous model [17]. New analytical formulation for minority carrier profile has been derived considering the low-high junction effect when the collector is completely invaded by minority carriers. In [17] only the diffusion current have been considered in the intermediate region of the collector. But the present research has considered both the drift and diffusion current. The value of minority carrier density at the end of injection region is always arbitrary. In this work, the carrier density is determined from the condition of a constant electric field $E_1 (1.5 \times 10^3 \text{ V/cm})$ at the boundary between injection and intermediate regions.
1.6 SUMMARY OF THE THESIS

In chapter 1 of this thesis, literature survey of bipolar transistor has been undertaken. Recent works on epitaxial $n^+pn^+n^+$ bipolar transistors are also reviewed in this chapter.

In chapter 2, mathematical analysis for determination of carrier profiles, electric field distributions and voltages in different regions of the collector when the transistor operates in quasi-saturation is given. The minority carrier profile within the injection region adjacent to the collector base metallurgical junction is obtained considering the effect of finite collector minority carrier life time. A regional approach rather than the charge control approach is employed. When the collector is completely invaded by minority carriers, the carrier density at the $n^+n^+$ interface is obtained by applying the minority carrier blocking properties of low-high junction.

Using the mathematical formulation derived in chapter 2, the I-V characteristics of a transistor is obtained in chapter 3. Numerically obtained characteristics are found to be in agreement with experimental results.
CHAPTER 2

FORMULATION OF COLLECTOR CURRENT AND VOLTAGE OF A POWER TRANSISTOR

2.1 INTRODUCTION

In modern bipolar transistors, the collector is lightly doped than the base. High-current effects occur predominantly in the epitaxial collector region. These effects are empirically and inadequately accounted for in the old bipolar transistor models[1]-[9]. Recently, Kull et al. [10] produced a dc model which retains the ideas of Gummel-Poon model [9] and includes quasi-saturation device physics. Kull's compact extension, however, is not applicable in general because of the assumption that the entire epitaxial collector region is quasi-neutral. In the model the effect of finite life time was not considered. Hanggen and Fossum [16] extended Kull's model to account for the current-induced space-charge region in the epitaxial collector. Their model did not include the saturation effect, i.e. $V_{CE} < V_{BE}$ and like other existing models requires parameters extracted from measurements. Hassan and Choudhury [17] further extended the model of Hanggen and Fossum. They took a regional approach and considered the minority carrier lifetime in the collector. They assumed a third region adjacent to the low-high junction which is a neutral or space region.

The present model employs a regional approach rather than charge control model. In a bipolar transistor at high currents and low base voltages, an injection region in the collector adjacent to the metallurgical collector-base junction is found to form. In the remaining part of the collector the majority carriers behave as ohmic, tepid or even as hot carriers moving with the scattering limited velocity. A detailed description of all possible carriers and field distributions within the

17
collector and their dependence on current and voltage, as well as, on doping level and width of collector are given.

The proposed model includes device physics such as collector conductivity modulation and voltage drops in low and high field collector regions under all injection level considering minority carrier lifetime.

2.2 ANALYSIS OF EPITAXIAL COLLECTOR

Equations suitable for relating collector current with collector voltages are derived in this section. One-dimensional n^+p+n^+ epitaxial bipolar transistor operating in quasi-saturation mode is shown in Figure 2.1. At relatively low voltages and high collector currents an injection mode must be used in finding expressions for collector current $I_C$ [17]. Both the base drive and the collector-emitter voltage $V_{CE}$ will determine whether the transistor operates in quasi-saturation or active mode of operation. For operation in quasi-saturation, the collector can be divided into three regions. The regions are, i) an injection region, ii) an intermediate region and iii) an end region. Regions of the collector will be treated subsequently.

i) Injection region ( $x \leq x_1$ )

In the injection region, holes are injected from the base into the lightly doped collector. Electrons also enter the collector from the left side leading to a quasi-neutral situation accompanying the hole pileup. The collector is thus conductively modulated from $x = 0$ to $x = x_1$. The recombination in the lightly doped collector cannot be neglected because the minority carrier (hole) lifetime is high but not infinite. Since the base current is comprised of this recombination current and the hole current injected in the emitter, significant deviation from the predicted characteristics can occur if the former component is ignored. Taking the hole current into account, the collector current density $J_C$ is given by
Fig. 2.1 One-dimensional epitaxial n⁺pn⁺n⁺ bipolar transistor

\[ J_C = J_{nc}(x) - J_{pc}(x) \]  \hspace{1cm} (2.1)

Where \( J_{nc} \) and \( J_{pc} \) are the electron and hole current density, respectively, at any \( x \). \( J_C \) is independent of \( x \). The basic equations written for one-dimensional geometry are

\[ J_{pc}(x) = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx} \]  \hspace{1cm} (2.2)

\[ -J_{nc}(x) = q\mu_n n(x)E(x) + qD_n \frac{dn(x)}{dx} \]  \hspace{1cm} (2.3)

\[ - \frac{1}{q} \frac{dJ_{nc}(x)}{dx} = r \]  \hspace{1cm} (2.4)

\[ - \frac{1}{q} \frac{dJ_{pc}(x)}{dx} = -r \]  \hspace{1cm} (2.5)

where \( r \) is the rate of recombination, \( q \) is the electron charge, \( \mu_n (\mu_p) \) is the electron (hole) mobility, \( D_n (D_p) \) is the diffusion co-efficient of electron (hole) and \( E(x) \) is the electric field.
Adding equations (2.2) and (2.3), the injection current density can be expressed as

\[ J_C = -q\mu_n n(x)E(x) - qD_n \frac{dn(x)}{dx} - q\mu_p p(x) + qD_p \frac{dp(x)}{dx} \]  

(2.6)

In the injection region the quasi-neutral condition will prevail and therefore the charge neutrality can be given by

\[ n(x) - p(x) - N_C \approx 0 \]  

(2.7)

Assuming that \( N_C \) is constant throughout the collector

\[ \frac{dp(x)}{dx} = \frac{dn(x)}{dx} \]  

(2.8)

Under high level injection \( n \) and \( p \) are much greater than \( N_C \), therefore

\[ n(x) = p(x) \]  

(2.9)

Applying the assumptions in eqn. (2.8) and (2.9), the eqn. (2.6) can be written as

\[ J_C = -q\mu_n E(x) (1 + m) p(x) - q\left(D_n - D_p\right) \frac{dp(x)}{dx} \]  

(2.10)

Where \( m = \frac{\mu_p}{\mu_n} \).

So, the electric field can be written as

\[ E(x) = \frac{J_C + q\left(D_n - D_p\right) \frac{dp(x)}{dx}}{q\mu_n p(x)(1 + m)} \]  

(2.11)

Substituting the result into eqn. (2.2) the hole current density becomes

\[ J_{pc}(x) = \frac{mJ_c + 2qD_p \frac{dp(x)}{dx}}{(1 + m)} \]  

(2.12)

Differentiating (2.12), we get

\[ \frac{dJ_{pc}(x)}{dx} = \frac{2qD_p \frac{dp^2(x)}{dx^2}}{(1 + m)} \]  

(2.13)

Using eqn. (2.5) and (2.13) finally we get

\[ \frac{r(1 + m)}{\mu_p} = 2V_T \frac{d^2p(x)}{dx^2} \]  

(2.14)
where \( V_T = (kT/q) \).

The recombination process is given by \( r = (p/\tau_p) \) where \( \tau_p \) is the minority carrier (hole) lifetime. So, using the above relation (2.14) becomes

\[
\frac{d^2 p(x)}{dx^2} = \frac{(1+m)p(x)}{2V_T \mu_p \tau_p} \quad (2.15)
\]

The profile of the injected hole in the injection region can be obtained from (2.15). One of the boundary conditions is \( p(x) = p_0 \) at \( x=0 \) and the other can be obtained by defining \( J_{pc0} = J_{pc}(0) \) at \( x=0 \). From eqn. (2.12) we get

\[
\frac{dp(x)}{dx} = \frac{mJ_C + (1+m)J_{pc}(x)}{2qD_p} \quad (2.16)
\]

So, the other boundary condition is

\[
\frac{dp(x)}{dx} \bigg|_{x=0} = S_{po} = \frac{-mJ_C + (1+m)J_{pc0}}{2qD_p} \quad (2.17)
\]

The hole profile in the injection region for high injection level is obtained by solving eqn. (2.15)

\[
p(x) = p_0 \cosh \left( \frac{x}{L_a} \right) + S_{po} L_a \sinh \left( \frac{x}{L_a} \right) \quad (2.18)
\]

Where \( L_a \) is the ambipolar diffusion length and is defined by

\[
L_a = \sqrt{\frac{2V_T \tau_p \mu_p}{(1+m)}} \quad (2.19)
\]

The hole concentration at \( x_1 \) i.e. \( p_1 \) can be obtained by putting \( x = x_1 \) in eqn. (2.18)

\[
p_1 = p_0 \cosh \left( \frac{x_1}{L_a} \right) + S_{po} L_a \sinh \left( \frac{x_1}{L_a} \right) \quad (2.20)
\]

The value of \( n_1 \) and \( p_1 \) is always arbitrary. The concentration \( p_1 \) can be obtained from \( n_1 \) i.e. \( n(x) \) at \( x=x_1 \). The carrier density \( n_1 \) is a function of \( J_C \) and will be
calculated in connection with the intermediate region. The analysis for determining \( p \) and \( n \) is provided in the intermediate region. In the intermediate region the hole current is neglected i.e. \( J_p(x) = 0 \). So, using this condition eqn. (2.16) becomes

\[
\frac{dp(x)}{dx} \bigg|_{x=x_1} = -\frac{mJ_c}{2qD_p}
\]  

(2.21)

Now, differentiating eqn. (2.18) and putting it in eqn. (2.21) one obtains

\[
J_C = \frac{L_a(1+m)J_{peo} \cosh \left( \frac{x_1}{L_a} \right) - 2qD_p p_0 \sinh \left( \frac{x_1}{L_a} \right)}{L_a m \left[ 1 - \cosh \left( \frac{x_1}{L_a} \right) \right]}
\]  

(2.22)

To get the voltage equation in the injection region the equation for electric field in the region is obtained first. It is obtained from eqn. (2.11) and is given by

\[
E(x) = -\frac{J_C + qV_T \mu_n (1-m) \frac{dp(x)}{dx}}{q\mu_n (1+m)p(x)}
\]  

(2.23)

Now, voltage in the injection region can be obtained by integrating equ. (2.23) w.r.t. \( x \) from \( x=0 \) to \( x=x_1 \) using the relation

\[
V_{\text{inj}} = -\int_0^{x_1} E(x) dx
\]  

(2.24)

For \([S_pL_o/p_0]^2 < 1\), the voltage in the injection region is given by

\[
V_{\text{inj}} = -\frac{J_C}{q\mu_n} \frac{2L_a}{p_0(1+m)s} \tan^{-1} \left( \frac{s \tanh \left( \frac{x_1}{2L_a} \right)}{1 + u \tanh \left( \frac{x_1}{2L_a} \right)} \right) + V_T \frac{(1-m)}{(1+m)} \ln \left( \frac{p_0}{p_1} \right)
\]  

(2.25)

where \( s = (1-u^2)^{1/2} \) and \( u = S_pL_o/p_0 \).

For \([S_pL_o/p_0]^2 > 1\)

\[
V_{\text{inj}} = -\frac{J_C}{q\mu_n} \frac{L_a}{p_0(1+m)z} \ln \left( \frac{1+(u+z) \tanh \left( \frac{x_1}{2L_a} \right)}{1+(u-z) \tanh \left( \frac{x_1}{2L_a} \right)} \right) + V_T \frac{(1-m)}{(1+m)} \ln \left( \frac{p_0}{p_1} \right)
\]  

(2.26)
where $z=(u^2-1)^{1/2}$

The electric field at $x = x_1$ for high injection can be obtained by differentiating (2.18) and putting it into (2.23) and is given by

$$E_1 = -\frac{V_T J_C}{2qD_n p_1}$$

(2.27)

In the low injection regime, the recombination term is neglected. The hole current density $J_{pc}(x)$ is neglected in comparison with the electron current density $J_{ne}(x)$ i.e. $J_{pc}(x)= 0$. This assumption is valid inside the injection region when [17]

$$\left(1+\frac{N_C}{p_0}\right) \ll \frac{\mu_n J_n}{\mu_n J_{pco}}$$

(2.28)

By putting $J_{pc}(x)=0$ in equ. (2.2)

$$E(x) = \frac{V_T}{p(x)} \frac{dp(x)}{dx}$$

(2.29)

Using equ. (2.1), (2.3), (2.7), (2.8), eliminating electric field by (2.29) and putting $J_{pc}(x)=0$ we get

$$\frac{N_C}{p(x)} \frac{dp(x)}{dx} + 2 \frac{dp(x)}{dx} = -\frac{J_C}{qD_n}$$

(2.30)

Integrating (2.30) with the boundary conditions $p(0)=p_0$ at $x=x_1$ one obtains

$$2(p_0-p(x)) - N_C \ln \left(\frac{p(x)}{p_0}\right) = \frac{J_C x_1}{qD_n}$$

(2.31)

so, the hole concentration $p_1$ at the end of the injection region under low injection condition can be obtained by putting $x = x_1$ in (2.31) as

$$2(p_0-p_1) + N_C \ln \left(\frac{p_0}{p_1}\right) = \frac{J_C x_1}{qD_n}$$

(2.32)

Under low injection condition, the voltage can be obtained in the injection region upon integration of eqn. (2.29) using eqn. (2.23) as

$$V_{\text{inj}} = V_T \ln \left(\frac{p_0}{p_1}\right)$$

(2.33)
Electric field at the end of the injection region for low injection condition can be obtained by putting \(dp(x)/dx\) from (2.30) in (2.29) and is given by

\[
E_1 = -\frac{V_{T JC}}{qD_n(2p_1 + N_C)}
\]  

(2.34)

Once \(x_1\), \(p_0\) and \(p_1\) are known, the above equations completely describe the situation prevailing in the injection region under low and high injection condition. The concentration \(p_1\) can be calculated in connection with the intermediate region. For determination of \(p_0\) the value of \(V_{BC}\) (base-collector voltage) is required.

At all level of injection prevailing in the base and the collector \(p_0\) can be obtained from the following relation [18]

\[
p_0 = -\frac{1}{2} N_C + \frac{1}{2} N_C \left(1 + 4 \frac{n_i^2}{N_C^2} e^{V_{BC}/V_T}\right)^{1/2}
\]  

(2.35)

Where, \(N_C\) is the collector doping density and \(n_i\) is the intrinsic carrier concentration.

When the transistor is saturated, the whole collector is conductively modulated and the minority carrier at the \(n^n+\) interface depends on the effective surface recombination \(S_{\text{eff}}\) at the edge of the low-high junction. The low-high junction acts as a barrier to the flow of minority carrier current from the low to the high region and the minority carrier is proportional to the carrier height at the edge of the LHJ. The minority carrier current can be assumed to be completely diffusive inside the \(n^+\) substrate and is continuous across the LHJ edges[19]. Derivation of \(p(x)\) in the quasineutral collector and the hole density at the collector edge of the base-collector junction under the saturation condition of operation are shown in Appendix A. The equation of \(p(x)\) is

\[
p(x) = \frac{p_0}{\cosh \left( \frac{W_C - x}{L_a} \right) + \left(1 + m\right)S_{\text{eff}}L_a \frac{W_C - x}{L_a} \sinh \left( \frac{W_C - x}{L_a} \right)} \frac{mJ_C L_a}{2qD_p} \sinh \left( \frac{x}{L_a} \right)
\]  

(2.36)

\[
p(x) = \frac{mJ_C L_a}{2qD_p} \sinh \left( \frac{x}{L_a} \right)
\]
where $S_{\text{eff}}$ is the effective surface recombination at the edge of the low-high junction and is given by [17]

$$S_{\text{eff}} = \left( \frac{N_C}{N_C^+} \right) \left( \frac{D_p^+}{N_C^+} \right) \left( \frac{D_p^+}{L_a^+} + \tanh \frac{W^+}{L_a^+} \right)$$

(2.37)

When $S_{\text{eff}}$ approaches infinity, $p(x)$ will be independent of $S_{\text{eff}}$. But for the intermediate situation $p(x)$ depends on $S_{\text{eff}}$.

The hole concentration at the base-collector junction $p_0$ for saturation condition in terms of $J_C$ and $J_{pco}$ is given by (see Appendix A)

$$p_0 = \frac{\cosh \frac{W_C}{L_a} + \frac{2D_p}{(1+m)} \sinh \frac{W_C}{L_a} \left( \frac{mJ_C}{(1+m)} + J_{pco} \right)}{qS_{\text{eff}} \left( \cosh \frac{W_C}{L_a} - \frac{L_a}{7pS_{\text{eff}}} \sinh \frac{W_C}{L_a} \right)}$$

(2.38)

Using the expressions (2.23), (2.24) and (2.36) the voltage across the collector under saturation condition is given by

$$V_{\text{inj}} = \frac{J_C L_a}{q \mu_n (1+m)} p_w F \ln \left[ \frac{1+D+F \tanh \frac{W_C}{2L_a}}{-V_T \left( \frac{1-m}{1-m} \right) \ln \frac{p_w}{p_0}} \right]$$

(2.39)

where,

$$p_0 = \frac{m J_C L_a}{2qD_p S_{\text{eff}}} \sinh \left( \frac{W_C}{L_a} \right)$$

(2.40)

$$p_w = \frac{1+D+F \tanh \frac{W_C}{2L_a}}{\tan \left( \frac{W_C}{L_a} \right) + \frac{(1+m)S_{\text{eff}} L_a}{2D_p} \sinh \left( \frac{W_C}{L_a} \right)}$$

(2.41)

$$D = \frac{L_a}{2qD_p p_w} \left[ (1+m)qS_{\text{eff}} p_w + m J_C \right]$$

(2.41)

$$F = \sqrt{D^2 - 1}$$

(2.42)
ii) Intermediate Region \( (x_1 < x \leq x_2) \)

The neutrality condition does not prevail all alone the collector because the hole concentration will reduce to zero before the electron concentration becomes equal to \( N_C \). Therefore, there is a transition region from the cold electron regime of the injection region to the hot electron regime of the following region which is the intermediate region. In this region, the excess holes are neglected in the Poisson's equation and the recombination current is also neglected but both the electron drift and diffusion currents are considered. A numerical work [20] was done on determining the density profile within the collector considering the drift and diffusion currents and neglecting the recombination current. The work [20] demonstrated that the distribution follows approximately an exponential form. In this work, an exponential profile for electron density in the intermediate region is assumed. Electron density at \( x = x_1 \) is assumed to be continuous. But as the hole concentration is negligible beyond the injection region it is assumed to be zero at \( x = x_1^+ \) i.e. a discontinuity in hole is assumed. So, the electron density profile in the intermediate region is assumed as follows

\[
n(x) = N_C + Ae^\frac{x_1-x}{C}
\]

(2.43)

here, \( A \) and \( C \) are two constants which are calculated by using the boundary conditions \( n(x_1) = n_1 \) and \( n(x_2) = n_2 \) and given as

\[
A = n_1 - N_C
\]

(2.44)

\[
C = \frac{x_1 - x_2}{\ln\left(\frac{n_2 - N_C}{n_1 - N_C}\right)}
\]

(2.46)

The concentration \( n_i \) i.e. \( n(x) \) at \( x = x_1 \) depends upon the collector current density \( J_c \) and the situation prevailing in the end region. In the present work, the electric field is assumed to be discontinuous at \( x = x_1 \) i.e. \( E(x_1^-) \neq E(x_1^+) \). The value of \( E_1^+ \) is assumed to be \( E_c = 1.5 \times 10^3 \). On the other hand, when collector current \( J_c < J_0 \) (where \( J_0 = qv_s N_C \)), the electron concentration in the end region is assumed to be \( N_C \).
Moreover if the collector current density $J_c < J_1$, where $J_1 = \mu n_e N_c E_c$, the electric field within the intermediate region must be less than $E_c$.

For $J_c < J_1$, the collector current density cannot support an electric field $E_c$ in the collector. Under this situation, the electron and hole concentration at the end of the injection region is chosen in this work as

\[ n_l = 2N_c \quad \text{and} \quad p_l = N_c, \quad \text{for high injection} \]  

\[ n_l = N_c, \quad p_l = 0 \quad \text{and} \quad E_l^+ = \frac{J_c}{q\mu n_c N_c}, \quad \text{for low injection} \]  

For $J_c > J_1$, $E_l^+ = E_c$ and $n_l$ is found as follows.

Putting $J_{pc}=0$ in eq. (2.1) one obtains

\[ J_c = -q\mu n_e E(x)n(x) - qD_n \frac{dn(x)}{dx} \]  

(2.49)

From eqn.(2.43) and (2.44) using $E(x_1^+) = E_l^+$ and (2.49) it can be shown that

\[ C = \frac{qD_{nc}(n_l - N_c)}{q\mu n_c n_l E_c + J_c} \]  

(2.50)

Putting $p(x)=0$ in Poisson's eqn. and using the boundary condition $E(x_1^+) = E_l^+$ we get

\[ E(x) = E_l^+ + \frac{qAC}{\epsilon} \left[ \frac{x_1-x}{C} - 1 \right] \]  

(2.51)

Now, using (2.44), (2.45) and $E(x_2) = E_2$ from (2.51) one obtains

\[ C = \frac{(E_2 - E_l^+)\epsilon}{q(n_2 - n_1)} \]  

(2.52)

Coupling eqn.(2.52) and (2.50) and simplifying

\[ n_l^2 + bn_l + c = 0 \]  

(2.53)

where,
\[ b = \frac{E_1^+ (E_2 - E_1^+) s}{qV_T} - n_2 - N_C \]  
\[ c = n_2 N_C + \frac{J_C (E_2 - E_1^+) s}{q^2 D_{nc}} \]  

Now the electron density at \( x = x_1 \) is given by

\[ n_1 = \frac{-b + \sqrt{b^2 - 4c}}{2} \]  

But to get \( n_1 \) one should know \( n_2 \) and \( E_2 \) whose values depends on the condition prevailing in the end region. For \( J_C < J_0 \), \( n_2 = N_C + n_i \) (\( n_i \) is the intrinsic concentration) independant of \( J_C \). This prevents the ln-term in eqn. (2.62) from going to infinity. This is the best boundary condition chosen in handling the situation. On the hand boundary value of \( E_2 \) is found from eqn. (2.57) [21] which relates electron velocity and electric field in the intermediate region.

\[ \frac{1}{v_n} = \frac{1}{\mu_{nc} E} + \frac{1}{v_s} \]  

For \( E < E_s \)

where, \( E_s \) is the field for which the velocity takes the scattering limited value \( v_s \). For electron, the scattering limited velocity assumed to be equal to \( 10^7 \) cm/sec and electric field \( E_s \) is considered to be \( 4 \times 10^4 \) V/cm[21]. Using \( J_C = qv_s n_2 \) and eqn. (2.57) \( E_2 \) can be found as

\[ E_2 = \frac{J_C}{q\mu_{nc} (n_2 - \frac{J_C}{qv_s})} \]  

Here, \( J_0 = qv_s N_C \) is the space-charge limited current density.

For \( J_C > J_0 \),

\[ E_2 = E_s \]  

and using the equations \( J_C = qv_s n_2 \) and \( J_0 = qv_s N_C \), \( n_2 \) can be obtained as follows

\[ n_2 = \frac{J_C}{qv_s} \]
\[ J_C = \frac{J_0}{N_C} \]  

The voltage in the intermediate region can be calculated by integrating (2.51) from \( x_1 \) to \( x_2 \) using:

\[ V_2 = -\int_{x_1}^{x_2} E(x) \, dx \quad (2.61) \]

and other boundary conditions at \( x_1 \) and \( x_2 \) as mentioned above one obtains

\[ V_2 = E_1'(x_1 - x_2) - \frac{q(x_1 - x_2)^2}{8} \left[ \ln \left( \frac{n_2 - N_C}{n_1 - N_C} \right) \ln \left( \frac{n_2 - N_C}{n_1 - N_C} + n_1 - n_2 \right) \right] \quad (2.62) \]

Knowing the value of \( n_1 \) one can calculate \( p_1 \) from the relation

\[ p_1 = n_1 - N_C \quad (2.63) \]

and then \( x_1 \) from (2.20) and (2.32) in the injection region for low and high injection respectively. Now \( x_2 \) can be obtained from (2.52) putting the value of \( C \) from (2.46)

\[ x_2 = x_1 + \frac{(E_2 - E_1')e}{q(n_1 - N_C) \ln \left( \frac{n_2 - N_C}{n_1 - N_C} \right)} \quad (2.64) \]

Since \( E_s, E_c, v_s \) and \( N_C \) are known the injection and intermediate regions are complete defined.

iii) End Region \( (x_2 < x < W_c) \)

Rest of the portion of the collector adjacent to the low-high junction is the end region. Condition prevailing in this region depends on collector current density \( J_C \).

When \( J_C < J_0 \), carrier density is constant. Therefore \( n_2 = N_C + n_i \) and hence the field in this region is also constant and \( E(x) = E_2 \). Value of \( E_2 \) is given by eqn. (2.58). So, the voltage in this region for above condition is obtained by integrating (2.58) from \( x_2 \) to \( W_c \) using
When $J_C > J_o$, the carrier and field density are no longer constant. Carrier density is given by $n_2 = J_C N_C / J_o$, and electric field $E(x)$ increases with a constant slope. Which is obtained from Poisson's equation using $n(x) = J_C N_C / J_o$ and $p(x) = 0$. So, the slope of the electric field is obtained as follows

$$\frac{dE}{dx} = \frac{q}{\varepsilon} \left( n(x) - p(x) - N_C \right)$$

$$= \frac{q}{\varepsilon} \left( \frac{J_C}{J_0} N_C - N_C \right)$$

$$= \frac{qN_C}{\varepsilon} \left( \frac{J_C}{J_0} - 1 \right)$$

(2.67)

So,

$$E(x) = E_s + \frac{dE}{dx} x$$

(2.68)

Now voltage in this region for this condition is obtained by using (2.68) and (2.65) and given as

$$V_3 = E_s \left( W_C - x_2 \right) + \frac{qN_C}{2\varepsilon} \left[ \frac{J_C}{J_0} - 1 \right] \left( W_C - x_2 \right)^2$$

(2.69)

Now the dc collector to emitter voltage at the terminals is given by

$$V_{CE} = (V_{BE} - V_{BC}) + V_{inj} + V_2 + V_3 + I_C R_{sat}$$

(2.70)

where $R_{sat}$ is the external series resistance in the emitter and collector and $V_{BC}$ and $V_{BE}$ are the base collector and base emitter junction voltages respectively.
2.3 IMPLEMENTATION OF THE MODEL

In d.c. analysis, the collector region model as proposed in the previous section need to be incorporated with the existing base and emitter models. In this section recently developed base and emitter region models are presented. The equations needed to couple the collector with the base and emitter are also given in this section. Base current density \( J_B \) can be modeled as the sum of the current density \( J_{pe} \) that is the hole current injected from the base to the emitter and the current density \( J_{peo} \) which is injected from base to the collector. i.e.

\[
J_B = J_{pe} + J_{peo}
\]  

And the collector current density \( J_C \) is related to the electron current density \( J_n \) injected from emitter and collector into the base by the following relation

\[
J_C = J_n - J_{peo}
\]  

So, to couple the collector with the base current we have to know the equations of \( J_{pe} \) and \( J_n \) and they are found from existing emitter and base region models respectively.

2.3.1 Base region model

Recently J. S. Yuan [22] has shown the electron current density injected from emitter and collector into the base as

\[
J_n = \frac{q \eta^2}{\int_0^{W_B} \frac{N_B(x) + n(x)}{D_{nb}(x)} \frac{n_e^2}{n_{ieb}(x)} dx}
\]

where \( V_{BE} \) and \( V_{BC} \) are the base-emitter and base collector junction voltages respectively, \( W_B \) is the base width, \( N_B \) is the base doping and \( n_{ieb} \) is the effective intrinsic density in the base. \( n(x) \) is the injected electron density profile in the base region.
For uniform base profile the minority carrier concentration in the base for a thin base BJT can be written by the well-known expression:

\[ n(x) = n(0) - \frac{n(0) - n(W_B)}{W_B} x \]  

(2.74)

where, \( n(0) \) and \( n(W_B) \) are the electron concentration at the edge of emitter-base space-charge region and at the edge of collector-base space-charge region respectively and they are given by

\[ n(0) = \frac{q n_i^2}{N(0)} \exp \left( \frac{V_{RE}}{V_T} \right) \]  

(2.75)

\[ n(W_B) = \frac{q n_i^2}{N(W_B)} \exp \left( \frac{V_{BC}}{V_T} \right) \]  

(2.76)

### 2.3.2 Emitter region model

The hole current density \( J_{pe} \) injected into the emitter from the base is given by the following equation \[23\]

\[ J_{pe} = J_{oc} \left\{ \exp \left( \frac{V_{BE}}{V_T} \right) - 1 \right\} \]  

(2.77)

where, \( J_{oc} \) is given by

\[ J_{oc} = \frac{q n_i^2}{G_{eff}(W_E) + \frac{N_{Deff}(W_E)}{S_e}} \left[ 1 + \frac{Q_c}{r_p(x) N_{Deff}(x)} \right] \left\{ \int_0^{W_E} \frac{G_{eff}(W_E) - G_{eff}(x)}{r_p(x) N_{Deff}(x)} \, dx + \frac{N_{Deff}(W_E)}{S_e} \int_0^{W_E} \frac{dx}{r_p(x) N_{Deff}(x)} \right\} \]  

(2.78)

and

\[ G_{eff}(x) = \int_0^x \frac{N_{Deff}(x)}{D_{pe}} \, dx \]  

(2.79)

here \( S_e \) is surface recombination velocity at the emitter contact.

The output characteristics of a bipolar transistor obtained by the present model is compared to the experimental characteristics taken from P. L. Hower \[14\]. The required device make-up and parameters for the model is also taken from the
above mentioned work which is given in Table 1 and 2. But the parameters obtained from the work [14] is not sufficient to incorporate the base and emitter model given above. So, the simplyfied equations of the base and emitter model are used to calculate the output characteristics. The simplyfied equations are given below.

The hole current density injected into the emitter from the base is given by

$$J_{pe} = \frac{q n_t^2}{G_E} \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - 1 \right]$$  \hspace{1cm} (2.80)

where

$$G_E = \frac{N_E W_E}{D_{pE}}$$  \hspace{1cm} (2.81)

$W_E$, $N_E$ and $D_{pE}$ are the effective emitter width, emitter doping concentration and hole diffussion coefficient in emitter respectively.

The electron current density injected into the base from the emitter and collector is given by

$$J_n = \frac{q n_t^2}{G_B} \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - \exp \left( \frac{V_{BC}}{V_T} \right) \right]$$

$$= J_{nf} - J_{nr}$$  \hspace{1cm} (2.82)

where

$$G_B = \frac{N_B W_B}{D_{nB}}$$  \hspace{1cm} (2.83)

$J_{nf}$ is the electron current density injected from the emitter into the base region (the base region recombination is neglected) and $J_{nr}$ is the collector current density due to back injection of electron from the base. $N_B$, $W_B$ and $D_{nB}$ are the base width, base doping concentration and electron diffusion coefficient in the base respectively. The dc forward current gain is given by

$$h_{FEO} = \frac{G_E}{G_B}$$  \hspace{1cm} (2.84)

The electron current density injected into the base from the collector is related to the base and collector current through $h_{FEO}$ with the following relation
To find out the boundary between the quasi-saturation region and the active region of the output characteristics the following equations are used. At the above mentioned boundary line no hole will be injected into the collector and no electron will be injected into the base from the collector i.e. \( J_{pc0} = 0 \) and \( J_{mr} = 0 \). Using this condition, eqn.(2.82) and (2.72) we get

\[
J_{C} = \frac{q n_{l}^{2}}{G_{B}} e^{\frac{v_{BE}}{V_{T}}} \tag{2.86}
\]

The relation between the Collector current and the collector-emitter voltage at the boundary can be given by

\[
V_{CE} = V_{BE} + I_{C} R_{C} \tag{2.87}
\]

Eliminiting \( V_{BE} \) from (2.86) and (2.87) the relation can be obtained as

\[
V_{CE} = V_{T} \ln \frac{G_{B}}{q n_{l}^{2} A} + V_{T} \ln I_{C} + I_{C} R_{C} \tag{2.88}
\]

### 2.4 CONCLUSION

The current-voltage characteristics of a bipolar transistor operating in quasi-saturation have been modelled based on the relevant device physics associated with the free carrier transport within the collector layer. The carrier profile within the injection region are obtained incorporating the effect of finite collector minority carrier life time and also both drift and diffusion currents. The boundary value \( n_{l} \) is obtained considering an exponential carrier profile in the intermediate region and a constant electric field \( E_{c} \) at \( x=x_{l} \). In intermediate region both drift and diffusion currents are considered. The current-voltage characteristics of a transistor can be obtained for the present collector model and the existing emitter and base models. In the next chapter results obtained from the present model are presented.
CHAPTER 3

RESULTS AND DISCUSSIONS

3.1 INTRODUCTION

When the Collector-Base junction is internally forward biased, an injection region is formed near the metallurgical junction. In chapter 2, the equations for high as well as low injection in collector are obtained for transistor operating in quasi-saturation. Based on the derived equations, a computer program using iterative scheme is developed to study the characteristics of transistors. Different transistor characteristics obtained by the model are presented in this chapter. Required parameters have been taken from practical transistors.

3.2 OUTPUT CHARACTERISTICS OF A BIPOLAR TRANSISTOR

An iterative computer program is developed based on the flow chart shown in Figure 3.1 to study the output characteristics of a bipolar transistor with lightly doped collector. The injection level dependence of diffusion co-efficient $D_n$ is taken into account [20] using

\[
\frac{1}{\mu_n} = \frac{1}{\mu_0} + \frac{p_{av} \ln \left( 1 + 4.54 \times 10^{11} \left( p_{av} \right)^{-0.667} \right)}{1.428 \times 10^{20}}
\]

(3.1)

where $\mu_0$ is the low level electron mobility and $p_{av}$ is an average excess carrier density given by

\[
p_{av} = \frac{p(0)}{2}
\]

(3.2)

Figure 3.2, 3.3 shows the collector current as a function of collector-emitter voltage with different base currents and Figure 3.4 shows the collector current
Calculate $n$ depending on $J$

Specify $I_B$, $I_C$

Calculate $J_C$, $V_{BC}$, $I_a$

Calculate $n_1$, $p_1$ depending on $J_C$

Calculate $P_0$ using $V_{BC}$, $N_C$

Choose new $J_{peo}$

Choose new $J_{peo}$

Calculate $X_l$ and new $P_0$ for high injection using $J_C$, $J_{peo}$, $n_1$

$er = [(P_0 - \text{new } P_0) / P_0]$

$er < 0.0001$

$X_l < W_C$

$J_{peo} < 0$

$Y$

$N$

$N$

$Y$
Choose new $J_{peo}$

Calculate $V_{inj}$, $V_{2}$, $V_{3}$, $V_{CE}$

Choose $J_{peo}$

Calculate $V_{BC}$ and $P_0$ using $V_{BC}$ & $N_c$

Choose new $P_0$ using $J_{C}$, $J_{peo}$ for hard saturation

$er = (P_0 \cdot \text{new } P_0) / P_0$

$er < 0.01$?

$J_{peo} > 0$?

$J_{peo} = 0$

Calculate $V_{BC}$ and $P_0$ using $V_{BC}$ & $N_c$.

Calculate $n_1$, $p_1$, $E_1$ for low injection depending on $J_C$.

Print $V_{CE}$, $I_C$

Hard saturation

Stop

Print $V_{CE}$, $I_C$

Stop

High injection

Stop

$(1+N_c/P_0) << u_{peo}/u_{peo}$

Calculate $V_{inj}$, $V_{2}$, $V_{3}$, $V_{CE}$
Fig. 3.1 Flow chart for computing the output characteristics of a bipolar transistor
versus collector-emitter voltage as a step function of base-emitter voltage. In Figure 3.2 the I-V characteristics obtained by the proposed model are shown indicating the boundary between quasi-saturation and active regions by the dotted line. The present model is applicable only when the transistor operates under quasi-saturation condition. These plots show that quasi-saturation occurs in a wider bias range if the base current or the base-emitter voltage is increased. This is because a larger base current or a larger base-emitter voltage results in a larger collector current in the collector and thus a larger voltage drop in the quasi-neutral collector, keeping the internal base-collector junction forward biased long after the external base-collector terminal is reversed biased. Figure 3.3 shows the comparison between the characteristics obtained by the present model and the characteristics reported in a literature. The minority carrier life time $\tau_p$ depends on the minority carrier concentration. Again the minority carrier concentration in the injection region depend on emitter-base voltage and base current. So, to get the best fit to the experimental results different values of the minority carrier life time are taken[20]. The effective surface recombination velocity $S_{ef}$ at the low-high is considered to be $10^5$ cm/s to incorporate the blocking property of the low-high junction when the transistor is in saturation. It can be seen from Figure 3.3(a) and (b) that the collector characteristics predicted by the present analysis agree very well with the experimental results of Hower[14] over the entire range of $V_{CE}$ and $I_C$ values. This good agreement is possible because the effects of minority carrier life time are taken into account. The device parameters for Figure 3.2 and 3.3 have been taken from the work [14] and shown in Table 3.1. For Figure 3.4 these have been taken from the work [10] and shown in and Table 3.2.

3.3 OUTPUT CHARACTERISTICS OF BIPOLAR TRANSISTOR NEGLECTING RECOMBINATION CURRENT

Figure 3.4 shows the collector current versus collector-emitter voltage as a step of base current. Here the recombination of the minority carrier in the base is
Fig. 3.2(a) Output Characteristics of an n'pn'n+ transistor indicating the quasi-saturation region for higher base currents.
Fig. 3.2(b) Output characteristics of an $n^+pn^+n^+$ transistor indicating the quasi-saturation region for lower base currents.
Fig. 3.3(a) Current-Voltage characteristics for different base currents comparing compact model simulation and measurements of an n$^+$pnn$^+$ bipolar transistor [14] for higher base current.
Fig. 3.3(b) Current-Voltage characteristics for different base currents comparing compact model simulation and measurements of an n^+pn^+ bipolar transistor [14] for lower base currents.
Fig. 3.4 Current-Voltage characteristics for different values of base-emitter voltage.
Fig. 3.5 Current-Voltage characteristics for different values of base current neglecting the recombination current.
Fig. 3.6 Current-Voltage characteristics for different values of collector layer width with constant base-emitter voltage.
neglected i.e. minority carrier life time is assumed as infinity. The result is again compared with the experimental characteristics of Hower[14] as well. We observe a considerable amount of deviation between the analytical and experimental values. Therefore, the minority carrier life time plays an important role on the collector current.

3.4 DEPENDENCE OF COLLECTOR CURRENT ON COLLECTOR WIDTH

To examine the effect of the collector layer width on the device performance, the collector current versus collector-emitter voltage as a step function of collector layer width are plotted in Figure 3.6. Increasing collector width increases the voltage drops in the collector and makes quasi-saturation more prominent. For the plots of Fig. 3.6 the base-emitter voltage are kept constant at a value of 0.775V. The device parameters are taken from the work [10] which is shown in Table 3.2.

3.5 CONCLUSION

The different characteristics of an epitaxial n⁺pn⁻n⁺ bipolar transistor have been studied by using the model proposed in this thesis. In the model, the dependence of mobility and minority carrier life time upon the minority carrier concentration within the injection region have been considered. A new expression for minority carrier profile for transistors operating in hard saturation condition incorporating the minority carrier blocking property of the low-high junction has been used. Also a new approach has been adopted to determine the boundary values at the end of the injection region and both drift and diffusion currents have been considered in the intermediate region. The results obtained using the present model considering the recombination current have been compared to those of without recombination current and a noticeable deviation has been observed. The results obtained using the present model have also been compared to the
experimental results available in literature [14] and a good agreement has been found. The results obtained by the model of reference [17] are also in good agreement with the experimental results. This is because voltage across the injection region is very small and the accurate width of the region does not affect the output characteristics. But the present model has some better aspects compared to [17] which are discussed in the next chapter.
Table 3.1 Device make-up and parameter for the transistor of Figure 3.2, Figure 3.3 and Figure 3.5

<table>
<thead>
<tr>
<th>$N_C$ (cm$^3$)</th>
<th>$D_C$ (cm$^2$/s)</th>
<th>$Q_E/D_E$ (cm$^4$-s)</th>
<th>$Q_B/D_B$ (cm$^4$-s)</th>
<th>$W_C$ (µm)</th>
<th>$A_E$ (cm$^2$)</th>
<th>$R_{set}$ (ohm)</th>
<th>$\mu_n$ (cm$^2$/V-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.16x10$^{14}$</td>
<td>22.3</td>
<td>4.72x10$^{13}$</td>
<td>2.56x10$^{12}$</td>
<td>50</td>
<td>0.077</td>
<td>0.085</td>
<td>1350</td>
</tr>
</tbody>
</table>

Table 3.2 Device make-up and parameter for the transistor of Figure 3.3 and Figure 3.6

<table>
<thead>
<tr>
<th>$N_B$ (cm$^3$)</th>
<th>$N_E$ (cm$^3$)</th>
<th>$N_C$ (cm$^3$)</th>
<th>$W_B$ (µm)</th>
<th>$W_E$ (µm)</th>
<th>$W_C$ (µm)</th>
<th>$A_E$ (cm$^2$)</th>
<th>$D_{nx}$ (cm$^2$/s)</th>
<th>$D_{nx}$ (cm$^2$/s)</th>
<th>$D_{nx}$ (cm$^2$/s)</th>
<th>$\mu_n$ (cm$^2$/V-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5x10$^{16}$</td>
<td>10$^{20}$</td>
<td>10$^{13}$</td>
<td>2.15</td>
<td>3.7</td>
<td>6</td>
<td>180</td>
<td>27</td>
<td>10</td>
<td>20</td>
<td>1075</td>
</tr>
</tbody>
</table>
CHAPTER 4

CONCLUSION AND SUGGESTION

4.1 CONCLUSION

Epitaxial n⁺pn⁻n⁺ bipolar transistors are extensively used as power transistors, specially in high speed switch. Modeling of this type of transistors is important with its increased use. Device modeling aims at relating physical device parameter to device terminal characteristics. Models representing transistors accurately are complex and difficult to study. Therefore, there is a trade-off between accuracy and complexity. Different models have been analyzed till date considering low and high injection but these models are not generally applicable specially at high collector current densities. Therefore, an improved model based on physical principles is now essential for accurate simulation of the high current effects.

The collector of modern bipolar transistor is more lightly doped than its base. As a result, high current effects occur predominantly in lightly doped epitaxial collector. At low voltage and high current densities the transistor operates in quasi-saturation. The minority carrier recombination current within the collector cannot be neglected when the transistor is operating in quasi-saturation region and high injection condition. Considering drift, diffusion and recombination currents within the injection region the ambipolar second order differential equation for minority carrier concentration becomes complex and analytically non tractable. In the present model this complex differential equation is made analytically tractable by some reasonable approximations and proper boundary values.

The accuracy of the solution of minority carrier profile in the injection region p(x) depends upon the selection of boundary values. The value of p(x₁)=p₁
at \( x = x_1 \) is always arbitrary. In the work of Hassan and Choudhury [17] the boundary value \( p_1 \) for high injection level was determined from the condition

\[
\frac{\varepsilon}{q} \left. \frac{dE(x)}{dx} \right|_{x_1} = p_1 \left( \frac{J_Q}{J_C} \right)
\]

(4.1)

The authors did not give any reasonable explanation for this assumption. Using the above condition the authors of the work [17] obtained \( p_1 \) given by

\[
p_1 = \frac{1}{q} \left( \frac{\varepsilon V_T}{4D_n^2 J_O} \right)^{1/2} J_C
\]

(4.2)

For \( p_1 = N_C \) the minimum allowable collector current density for high injection condition can be given by

\[
J_{C_{\text{min}}} = qN_C \left( \frac{4D_n^2 J_O}{\varepsilon V_T} \right)^{1/2}
\]

(4.3)

So, their model cannot be applied for high injection condition to determine the I-V characteristics for \( J_C < J_{C_{\text{min}}} \). A similar conclusion can also be drawn for low injection condition. In the present model the above drawback is eliminated.

I-V characteristics obtained by the present model and by the model of [17] are in similar agreement compared to the experimental results of Hower [14]. This is because voltage across the injection region is very small and the accurate width of the injection region does not play an important role in determining \( V_{CE} \). But an accurate profile is always required particularly for determining the collector transit time and cut-off frequency. The present model will enable to determine the injection region width and give more accurate value to calculate the collector transit time and cut-off frequency which are the functions of the width \( x_1 \). These are beyond the scope of the present work.

### 4.2 LIMITATIONS OF THE PRESENT MODEL

The present model has some limitations. By this model the output characteristics of a transistor can be obtained only when the transistor operates in quasi-saturation condition. In the present work current crowding effects has not
been considered. In modern transistor, this effect is usually minimized by
digitization in the emitter diffusion and not considered in modeling of transistors.

4.3 SUGGESTIONS

In the proposed model the recombination current is considered only when
the hole density $P_0$ at the base-collector junction is much higher than the collector
doping density $N_C$. On the other hand, the recombination current is neglected when
the condition

$$\left(1 + \frac{N_C}{P_0}\right) \ll \frac{\mu_p}{\mu_n} \frac{J_n}{J_{pc0}}$$

is satisfied. But in practical cases a small fraction of $J_B$ will be flowing through the
collector which is not incorporated in the present model. A study can be carried out
in considering this hole current.

Using the present model width of the injection region can be obtained for
any collector current density $J_C$ and this can be used to determine the collector
transit time and the cut-off frequency.
REFERENCES


APPENDIX A

COLLECTOR MINORITY CARRIER PROFILE FOR
TRANSISTOR IN SATURATION

When the epitaxial bipolar transistor operates in the saturation condition, the whole collector is invaded by minority carriers. The structural requirement of the low-high junction (LHJ) at the rear contact results in a boundary condition which causes the minority carrier giving blocking property of the LHJ. The expression for \( p(x) \) within the collector is derived as follows.

The equation considering drift and diffusion currents within the collector has the general solution

\[
p(x) = A \cosh\left(\frac{W_C - x}{L_a}\right) + B \sinh\left(\frac{W_C - x}{L_a}\right) \quad \text{for} \quad 0 < x < W_C \quad (A1)
\]

using the boundary conditions \( p(0) = p_0 \) and \( p(W_C) = p_w \) we get

\[
p_0 = A \cosh\frac{W_C}{L_a} + B \sinh\frac{W_C}{L_a} \quad (A2)
\]

\[
p_w = A \quad (A3)
\]

Now the slope of the minority carrier profile within the collector is given by eqn.(2.16). For convenience it is given below

\[
\frac{dp(x)}{dx} = -\frac{mJ_C + (1+m)J_{pc}(x)}{2qD_p} \quad (A4)
\]

The hole current density at the LHJ can be written as
\[ J_{pc}(W_c) = q\left(\frac{W_c}{S_{eff}}\right) \] (A5)

where \( S_{eff} \) is the surface recombination velocity and is given in [17].

Using eqn.(A1) to (A5) we get the hole profile \( p(x) \) in the lightly doped collector

\[
p(x) = \frac{p_0 \left[ \cosh \left( \frac{W_c - x}{L_a} \right) + \frac{(1+m)S_{eff}L_a}{2D_p} \sinh \left( \frac{W_c - x}{L_a} \right) \right] \cdot \frac{mJ_cL_a}{2qD_p} \sinh \left( \frac{W_c}{L_a} \right)}{\left[ \cosh \left( \frac{W_c}{L_a} \right) + \frac{(1+m)S_{eff}L_a}{2D_p} \sinh \left( \frac{W_c}{L_a} \right) \right]} \] (A6)

The total hole current entering the collector from the base can be given by

\[
J_{pco} = J_{pc}(W_c) + \frac{q}{\tau_p} \int_0^{W_c} p(x) \, dx \] (A7)

Using eqn.(A5) to (A7) the hole concentration \( p_0 \) at the base-collector edge can be given as

\[
p_0 = \frac{\left( \cosh \frac{W_c}{L_a} + \frac{(1+m)S_{eff}L_a}{2D_p} \sinh \frac{W_c}{L_a} \right) \left[ \frac{mJ_c}{(1+m)} + J_{pco} \right] - \frac{mJ_c}{(1+m)}}{qS_{eff} \left[ \cosh \frac{W_c}{L_a} + \frac{L_a}{\tau_p S_{eff}} \sinh \frac{W_c}{L_a} \right]} \] (A8)