

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE** questions.

1. (a) Define idempotent matrix nilpotent matrix with examples. Prove that a square matrix A is invertible if it can be written as a product of elementary matrices. Hence express

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix} \text{ as a product of elementary matrices.} \quad (20)$$

- (b) Solve the linear system using LU-factorization: (15)

$$2x_1 + 4x_2 - 2x_3 = 4$$

$$6x_1 + 3x_3 = 15$$

$$4x_1 + 2x_2 + 4x_3 = 6$$

2. (a) If $A = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 9 & 12 \\ 0 & 10 & 1 \end{bmatrix}$, find two non-singular matrices P and Q such that $PAQ = I$.

Hence find A^{-1} . (15)

- (b) Prove that the points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are collinear if and only if the rank

of the matrix $A = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ is less than 3. (8)

- (c) If $A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 2 & 7 \\ 0 & 0 & 4 \end{bmatrix}$, then find the eigenvalues of $3A^3 + 5A^2 - 6A + 2I - 8A^{-1}$. (12)

3. (a) Diagonalize the matrix $A = \begin{bmatrix} 4 & 0 & 8 \\ 0 & 12 & 0 \\ 8 & 0 & 4 \end{bmatrix}$ by means of an orthogonal transformation. (15)

- (b) Find the minimal polynomial of the matrix $A = \begin{bmatrix} 3 & 5 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$. (10)

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(c) $A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$, using Cayley-Hamilton theorem find the value of A^n . (10)

4. (a) Define quadratic form and explain its matrix representation. Deduce the quadratic form $q = x^2 + 2y^2 + 3z^2 - 2xy + 2yz$ into the sum of squares. Hence find nature, rank, index and signature of quadratic form. (20)

(b) Find AB by conformal partitioning of A and B , given (15)

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 3 & -1 \\ 2 & 0 & -4 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & -1 & 2 & 2 \\ 1 & 3 & 0 & 0 & 1 & 0 \\ -3 & -1 & 2 & 1 & 0 & -1 \end{bmatrix}$$

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE** questions.

5. (a) Let l be the line in the xy –plane that passes through the origin and makes an angle θ with the positive x – axis, where $0 \leq \theta < \pi$. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear operator that maps each vector \mathbf{X} into its orthogonal projection on l . (15)

(i) Find the standard matrix for T .

(ii) Find the orthogonal projection of the vector $\mathbf{X} = (2, 5)$ onto the line through the origin that makes an angle of $\theta = \pi/3$ with the positive x -axis.

(b) Determine whether multiplication by (10)

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 3 & -4 \end{bmatrix}$$

is a one-to-one linear transformation. Explain your reasoning.

(c) Find the eigenvalues and corresponding eigenvectors of $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ where T is the reflection about the line $y = x$. Hence check your conclusion by calculating the eigenvalues and corresponding eigenvectors from the standard matrix for T . (10)

6. (a) Determine whether the set

$$W = \{ \text{all polynomials } a_0 + a_1x + a_2x^2 + a_3x^3 \text{ for which } a_0 + a_1 + a_2 + a_3 = 0 \}$$

is a subspace of $P_3(x)$. (10)

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(b) Determine whether the following set of vectors is a basis for $P_3(x)$. (15)

$$P_1 = -1 + x, \quad P_2 = 1 + x, \quad P_3 = x^2, \quad P_4 = x^3$$

if so, find the coordinate vector of $p = 2 - x^2 + 3x^3$ relative to the basis

$$S = \{P_1, P_2, P_3, P_4\}.$$

(c) Find a basis for the row space of (10)

$$A = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$$

consisting entirely of row vectors from A .

7. (a) Let W be the subspace of \mathbb{R}^5 spanned by the vectors

$$w_1 = (2, 2, -1, 0, 1), \quad w_2 = (-1, -1, 2, -3, 1)$$

$$w_3 = (1, 1, -2, 0, -1), \quad w_4 = (0, 0, 1, 1, 1)$$

Find a basis for the orthogonal complement of W . (20)

(b) Find the QR-decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}, \text{ if possible.} \quad (15)$$

8. (a) Consider the basis $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for \mathbb{R}^3 where

$$\mathbf{v}_1 = (1, 1, 1), \quad \mathbf{v}_2 = (1, 1, 0) \text{ and } \mathbf{v}_3 = (1, 0, 0).$$

Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation such that

$$T(\mathbf{v}_1) = (1, 0), \quad T(\mathbf{v}_2) = (2, -1), \quad T(\mathbf{v}_3) = (4, 3)$$

Find a formula for $T(x_1, x_2, x_3)$; then use this formula to compute $T(5, -3, 2)$. (15)

(b) Let T be multiplication by the matrix

$$A = \begin{bmatrix} 4 & 1 & 5 & 2 \\ 1 & 2 & 3 & 0 \end{bmatrix}. \text{ Find}$$

(i) a basis for the range of T . (ii) a basis for the Kernel of T .

(iii) the rank and nullity of T . (iv) the rank and nullity of A . (10)

(c) If $T: V \rightarrow W$ is a linear transformation, then prove that (10)

(i) The Kernel of T is a subspace of V .

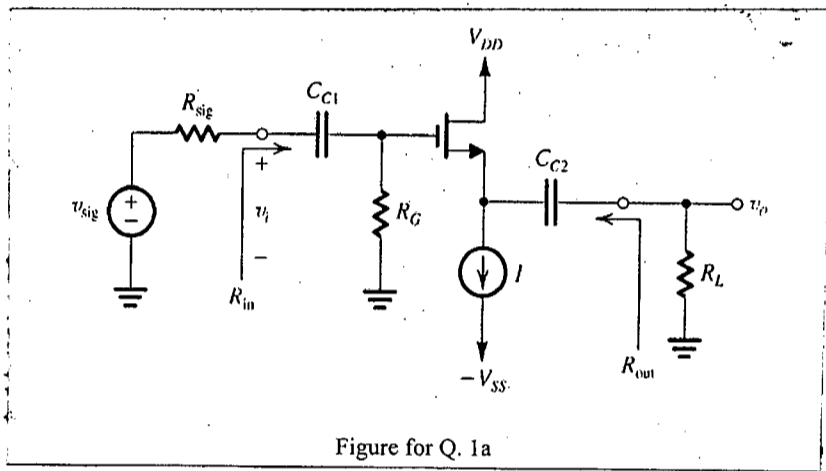
(ii) The range of T is a subspace of W .

SECTION – A

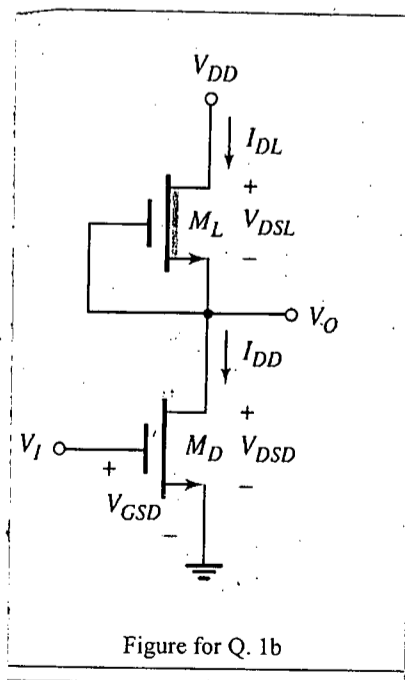
There are **FOUR** questions in this section. Answer any **THREE**.

The figures in the margin indicate full marks. Symbols have their usual meaning.

1. (a) Identify the MOS amplifier structure of Figure for Q. 1a. Draw its small signal equivalent circuit and find its voltage gain, open circuit voltage gain, overall voltage gain, input and output resistances. (2+18)



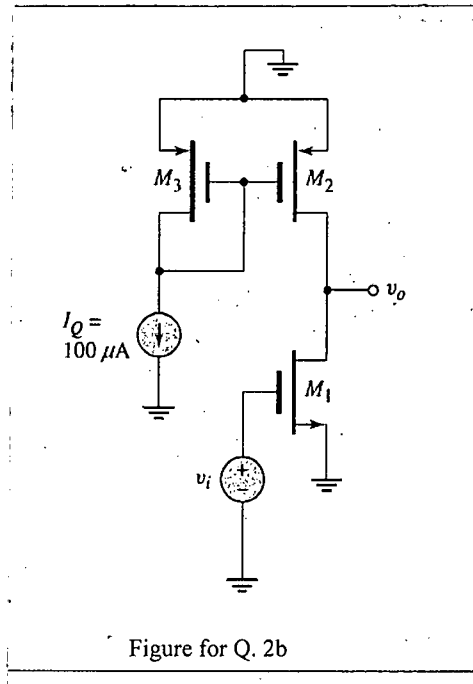
- (b) In the circuit shown in Figure for Q. 1b, let $V_{DD} = 5\text{ V}$ and assume transistor parameters of $V_{TND} = 1\text{ V}$, $V_{TNL} = -2\text{ V}$, $K_{nD}(W/L)_{nD} = 50\ \mu\text{A}/\text{V}^2$, and $K_{nL}(W/L)_{nL} = 10\ \mu\text{A}/\text{V}^2$. Determine V_O for $V_I = 5\text{ V}$. (15)



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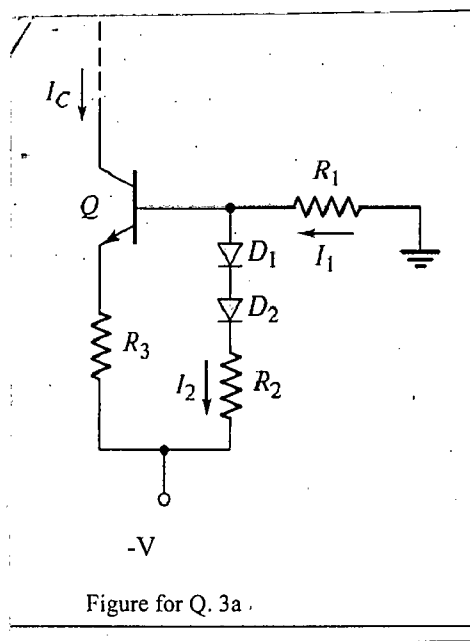
2. (a) Derive an expression of the low noise margin and high noise margin for an inverter fabricated in the CMOS technology. (15)

(b) Identify the amplifier circuit shown in Figure for Q. 2b. If the transistor parameters for M_1 are $V_{TN} = 0.5 \text{ V}$, $k'_n = 85 \mu\text{A}/\text{V}^2$, $(W/L)_1 = 50$, and $\lambda = 0.05 \text{ V}^{-1}$, and for M_2 and M_3 are $V_{TP} = -0.5 \text{ V}$, $k'_p = 40 \mu\text{A}/\text{V}^2$, $(W/L)_{2,3} = 50$, and $\lambda = 0.075 \text{ V}^{-1}$, determine the small-signal voltage gain. (2+18)



3. (a) For the circuit shown in Figure for Q. 3a if the transistor voltage V_{be} and diode voltages are equal and β is very high, show that, for $R_1 = R_2$, the expression for I_C reduces to (13)

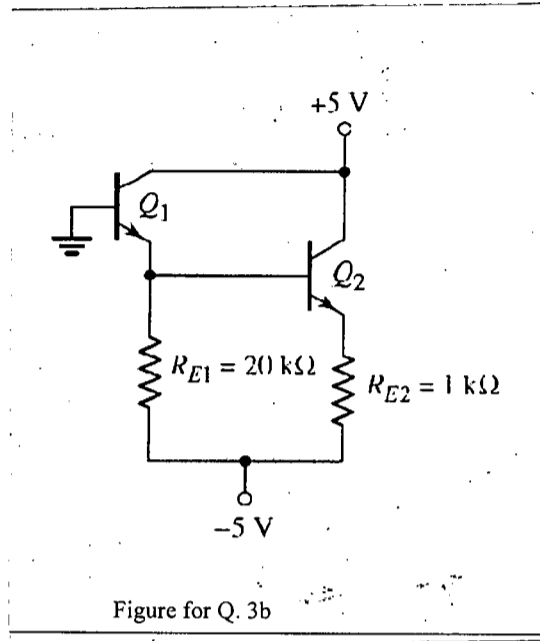
$$I_C = \frac{V}{2R_3}$$



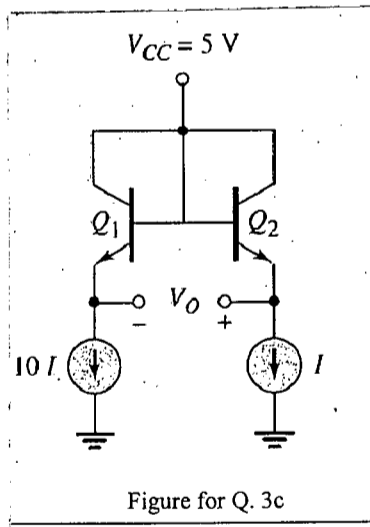
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Contd... Q. No. 3

(b) The parameters for each transistor in the circuit shown in Figure for Q. 3b are $\beta = 80$ and $V_{BE(on)} = 0.7 \text{ V}$. Determine the quiescent values of base, collector, and emitter currents in Q_1 and Q_2 . (11)



(c) Assuming that the transistors Q_1 and Q_2 in the circuit shown in Figure for Q. 3c are identical and the emitter currents have the form $I_E = I_{E0} \exp(V_{BE}/V_T)$, derive the expression for the output voltage V_O as a function of temperature T . (11)



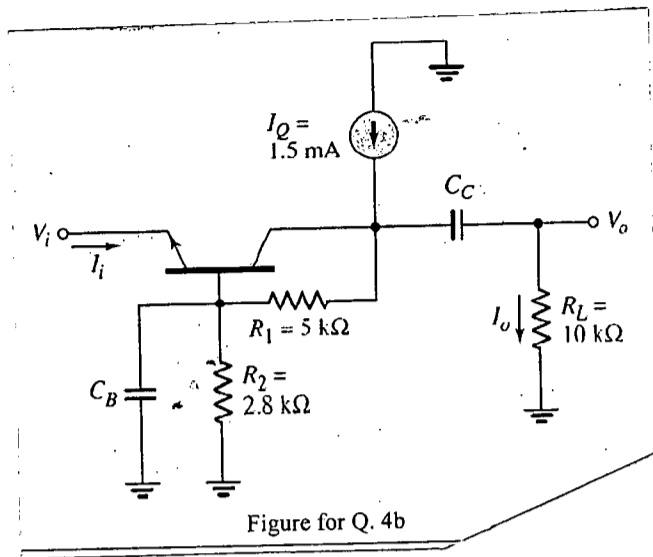
4. (a) For small signal operation of a BJT, derive an expression of voltage gain, A_V , and small-signal base resistance. (12)

(b) For the circuit shown in Figure for Q. 4b, the parameters of the transistor are $\beta = 120$, $V_{be(on)} = 0.7 \text{ V}$, and $V_A = \infty$. (10+13)

- (i) Determine the quiescent values I_{CQ} and V_{CEQ} .
- (ii) Find the small-signal voltage gain $A_V = V_o/V_i$.

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Contd... Q. No. 4(b)

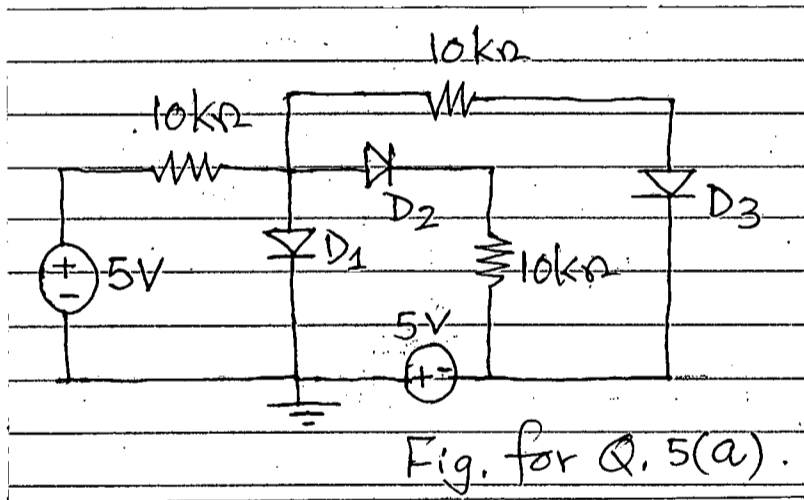


SECTION - B

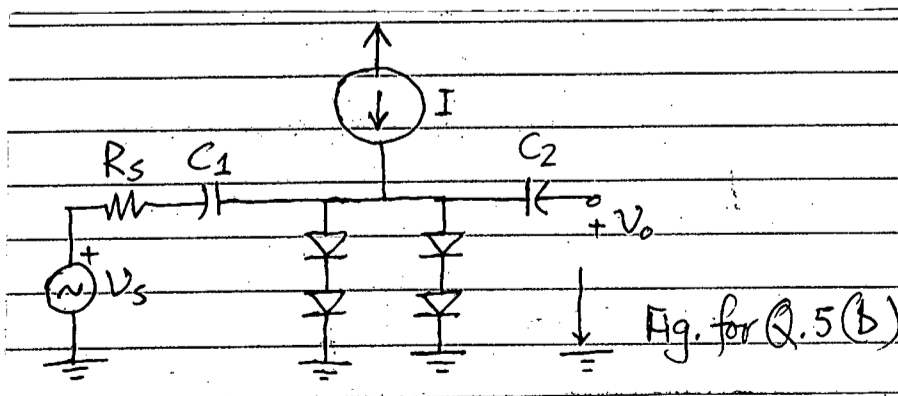
There are **FOUR** questions in this section. Answer any **THREE**.

The questions are of equal value.

5. (a) Calculate current flowing through D3 in the circuit shown in Fig. for Q. 5(a). Assume a forward drop of 0.7 V across a diode.

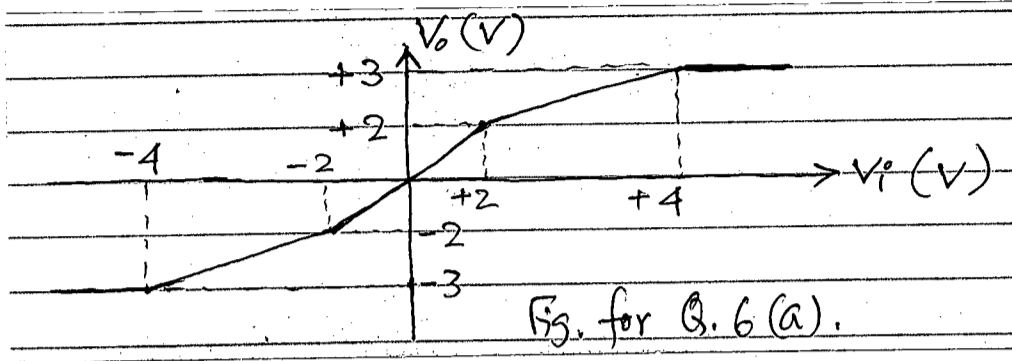


- (b) Derive an expression for V_0/V_S in terms of I , R_S , V_T (thermal voltage) and n (diode ideality factor) for the diode circuit shown in Fig. for Q. 5(b). Consider small signal model for the diodes. Here, C_1 , C_2 are coupling capacitors and the diodes are identical.

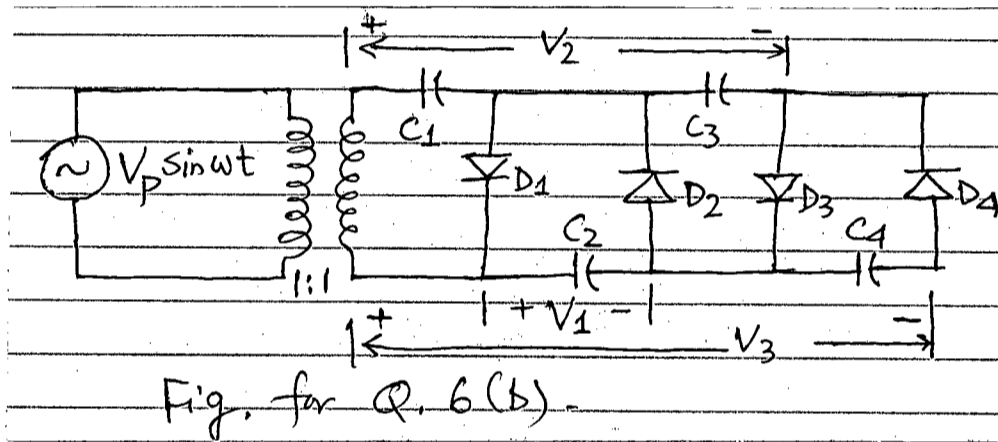


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6. (a) Using diodes and other necessary components construct a circuit to generate the voltage transfer characteristics as shown in Fig. for Q. 6(a).

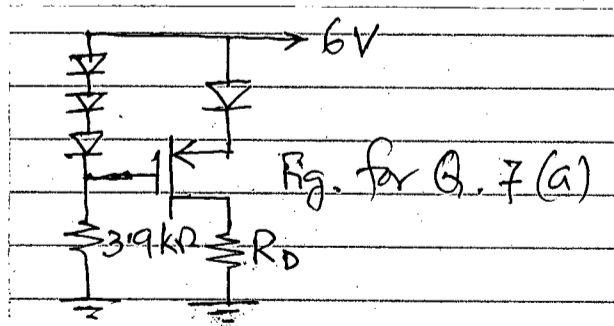


- (b) Explain the operation of the circuit shown in Fig. for Q. 6(b). Express V_1 , V_2 and V_3 in terms of V_p . Assume ideal diode.



7. (a) Design a suitable value for R_D to keep the transistor at the edge of saturation in the circuit shown in Fig. for Q. 7(a). Assume a forward drop of 0.7 V across a diode.

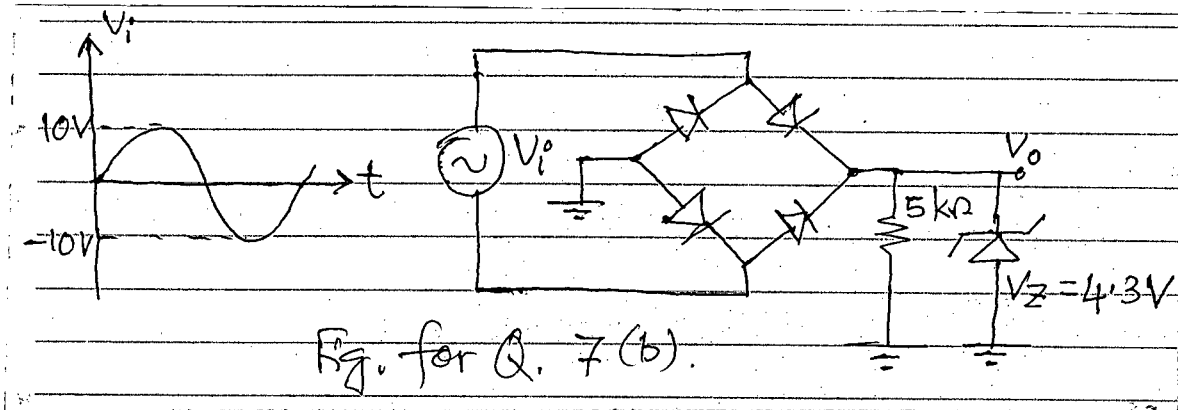
Given: $V_t = -1.0$ V and $k'_p \left(\frac{W}{L} \right) = 1 \text{ mA/V}^2$.



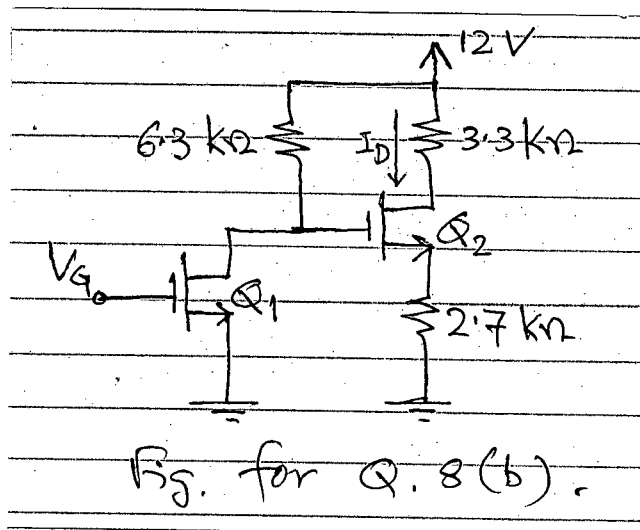
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Contd... Q. No. 7

(b) Explain the operation of the circuit shown in Fig. for Q. 7(b). Draw the output voltage (V_0) as a function of time ($V_0 \sim t$) for the given input voltage, V_i . Assume ideal diode.



8. (a) Derive an expression for the drain current of an enhancement type p-channel MOSFET using necessary diagrams. Plot I_D vs V_{DS} .
- (b) Calculate the value of V_G in the MOSFET circuit shown in Fig. for Q. 8(b). The transistors Q_1 and Q_2 are identical. Given: $V_t = 1.0$ V, $k'_n \left(\frac{W}{L}\right) = 1$ mA/V², $I_D = 1$ mA.



SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Show that if three phase currents, each of equal magnitude and differing in phase by 120° , flow in a three phase winding consisting of three separate windings spaced 120 electrical degrees apart then it will produce a rotating magnetic field of constant magnitude. (17)
- (b) Derive the equivalent circuit of a three phase induction motor. With the help of equivalent circuit explain why is the starting current high. (18)

2. (a) Draw the torque-speed characteristics of a three phase induction motor when $\frac{V}{f}$ is held constant upto a certain frequency and afterwards V is held constant. Write the expressions of maximum torque, T_{dmax} , and starting torque, T_{start} , at this condition. (20)
- (b) With a neat diagram explain the operation of a Wye-Delta starter. (15)

3. (a) Explain with appropriate diagrams what are the advantages obtained by using deep bar rotor in a three phase induction motor at starting and running conditions. (15)
- (b) The following test data are obtained from no-load, blocked-rotor and DC resistance tests of a three-phase, wye-connected, 40-hp, 60-Hz, 460-V, design B induction motor whose rated current is 57.8 A. The blocked rotor test is performed at 15 Hz. (20)

DC resistance test: $V_{DC} = 12.0$ V, $I_{DC} = 59.0$ A

No load test: $V_{line} = 460.0$ V, $I_{line} = 32.7$ A, $f = 60$ Hz, $P_{3phase} = 4664.4$ W

Locked rotor test: $V_{line} = 36.2$ V, $I_{line} = 58.0$ A, $f = 15$ Hz, $P_{3phase} = 2573.4$ W

Draw the equivalent circuit of this motor. Determine the combined core, friction and windage loss.

For class B, the reactance $X_S = 0.4 X_{BR}$ and $X_R = 0.6 X_{BR}$

4. (a) Using double revolving field theory explain how a single-phase induction motor develops its torque at starting. Explain why the forward field becomes higher than the backward field at $0 < s < 1$. (15)

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Contd... Q. No. 4

(b) A $\frac{1}{3}$ -hp, 110-V, 60-Hz, six-pole, split-phase induction motor has the following impedances: (20)

$$R_S = 1.52 \Omega \quad X_S = 2.10 \Omega \quad X_M = 58.2 \Omega$$

$$R_r = 3.13 \Omega \quad X_r = 1.56 \Omega$$

The core losses of this motor are 35 W, and the friction, windage, and stray losses are 16 W. The motor is operating at the rated voltage and frequency with its starting winding open, and the slip is 5 percent. Find the following parameters of the motor;

- (i) Speed in revolutions per minute
- (ii) Stator current in amperes
- (iii) Stator power factor
- (iv) Input power
- (v) Air gap power
- (vi) Converted power
- (vii) Induced torque
- (viii) Output power
- (ix) Load torque
- (x) Efficiency

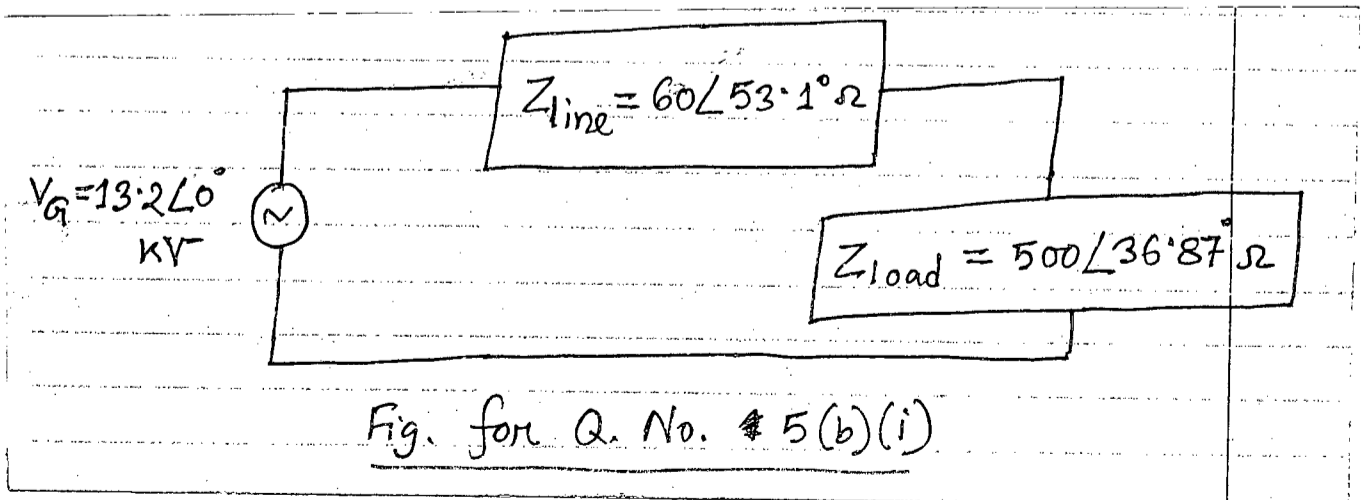
SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Derive the exact equivalent circuit of a non-ideal transformer. (18)

(b) A 13.2 kV single phase generator supplies power to a load through a transmission line. The load impedance is $Z_{load} = 500 \angle 36.87^\circ \Omega$ and the transmission line impedance is $Z_{line} = 500 \angle 53.1^\circ \Omega$. (17)

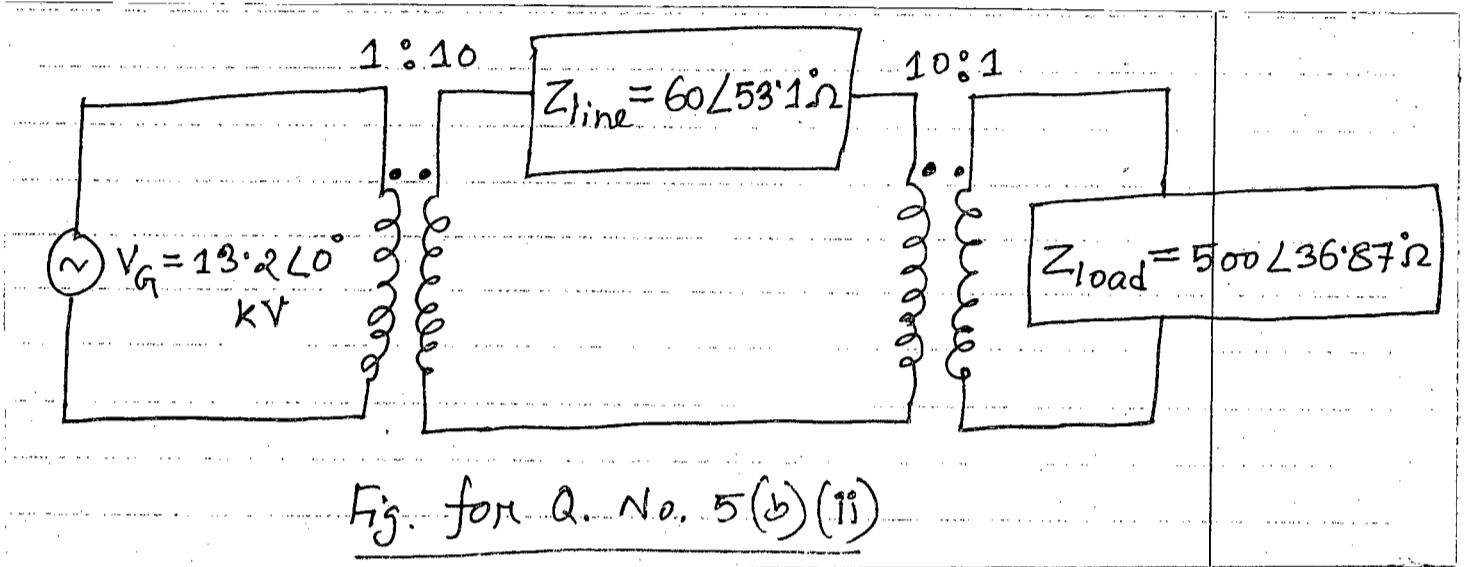
(i) If the generator is directly connected to the load (Fig. for Q. No. 5(b)(i)), what is the ratio of the load voltage to the generated voltage? What are the transmission losses of the system?



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Contd... Q. No. 5(b)

- (ii) If a 1:10 step-up transformer is connected at the output of the generator and a 10:1 transformer is connected at the load end of the transmission line, what is the new ratio of the load voltage to the generated voltage? What are the transmission losses of the system now (Fig. for Q. No. 5(b)(ii))? The transformers may be assumed to be ideal.



6. (a) Describe open circuit and short circuit tests of a single phase transformer with appropriate circuit diagram. (18)

- (b) 1000 VA, 230/115 V transformer has been tested to determine its equivalent circuit. The test results are shown below. All data are measured in H.T side. (17)

Open circuit test	Short circuit test
$V_{OC} = 230 \text{ V}$	$V_{SC} = 19.1 \text{ V}$
$I_{OC} = 0.45 \text{ A}$	$I_{SC} = 8.7 \text{ A}$
$P_{OC} = 30 \text{ W}$	$P_{SC} = 42.3 \text{ W}$

Determine (i) the equivalent circuit of this transformer referred to the low-voltage side of the transformer, (ii) the voltage regulation and efficiency when operating at rated load and 0.8 power factor lagging.

7. (a) Explain the working principle of an autotransformer. What are the advantages and disadvantages of an autotransformer if compared with a conventional transformer? Why can an autotransformer handle more apparent power than conventional transformer of the same size? Mention the areas of application of autotransformers. (20)

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Contd... Q. No. 7

(b) A 100 VA, 120/12 V transformer is to be connected so as to form a step-up autotransformer. A primary voltage of 120 V is applied to the transformer (across the common winding). (15)

- (i) What is the secondary voltage?
- (ii) What is the maximum VA rating in this mode of operation?
- (iii) Calculate the rating advantages of this autotransformer over the transformer's rating in convention (two winding) 120/12 V operation.

8. (a) Explain the problems and their solutions in a three phase Y-Y connected transformer, without the neutral grounding. (17)

(b) What is the inrush current of a transformer? Explain why Δ - Δ connected three phase transformer is derated to 57.7% of its power rating when one of the phases is damaged and removed. (18)

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Distinguish between 'change in supply' and 'change in quantity supplied'. (10)
 (b) Define market equilibrium. Illustrate the mechanism through which market equilibrium is achieved in the free market economy. (15)
 (c) The demand and supply functions of a commodity X are respectively. (10)

$$Q_{dx} = 2380 - 32P_x$$

$$Q_{sx} = 1455 + 25P_x$$
 Calculate the equilibrium price and output of the commodity. If Government imposes 13% VAT on unit price, what will be the new equilibrium price and output.
2. (a) Illustrate the concept of marginal rate of substitution (MRS) with the help of an indifference curve? (10)
 (b) Explain graphically the equilibrium of the consumer under the ordinal theory of utility analysis. (15)
 (c) Derive the conditions for equilibrium using the 'Lagrangian multipliers' method. (10)
3. (a) What do you understand by localization of industries? What are the causes of localization of industries? (10)
 (b) Explain the advantages and disadvantages of localization of industries. (15)
 (c) What do you know about division of labour? Explain different types of division of labour. (10)
4. Write short notes on any **THREE** of the following: (35)
 - (i) 'substitution effect' and 'income effect' of a price change
 - (ii) Point elasticity and arch elasticity
 - (iii) Basic determinants of elasticity of demand
 - (iv) Fundamental economic problems that every economy has to face.

SECTION – BThere are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What are the assumptions of a perfectly competitive market? Explain them. (10)
 (b) Explain the long run equilibrium of a firm under perfect competition. (15)
 (c) From the following revenue and cost functions, calculate the profit maximizing level of output and maximum profit. (10)

$$R = 100Q - Q^2$$

$$C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 90$$

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6. (a) When does a firm emerge as a monopolist? (10)
 (b) Explain the short run equilibrium of a firm under monopoly. (10)
 (c) What is the relation among marginal revenue (MR), price (P) and price elasticity of demand (e). (10)
 (d) What are the conditions of profit maximization? (5)
7. (a) Explain any two methods for measuring national income. (10)
 (b) What are the problems of measuring national income in a developing country like Bangladesh? (15)
 (c) Given that, $C = 100 + 0.75 Y_d$ (10)
 $I = 100, G = 100, X = 70, M = 250, TR = 200, T = 0.15 Y$
 (i) Calculate the equilibrium level of income and multiplier in this model.
 (ii) If tax rate is increased to 20%, then what will be the new equilibrium level of income and multiplier?
 (iii) What will happen to the equilibrium level of income if investment is increased to 300?
8. (a) How would you derive the long run average cost curve of a firm from its short run average cost curves? Explain graphically. (10)
 (b) Define fixed cost and variable cost. (5)
 (c) A manufacturer has a fixed cost of \$40,000 and a variable cost of \$1.60 per unit made and sold. Selling price is \$2 per unit. (10)
 (i) Compute profit if 150000 units are made and sold.
 (ii) Find the break-even quantity.
 (iii) Construct the break-even chart. Label the cost and revenue lines, the fixed cost line, and the break-even point.
 (d) Complete the following table and sketch the graph explaining the relations among the various short run cost curves. (10)

Quantity of output	Total fixed cost	Total variable cost	Total cost	Average fixed cost	Average variable cost	Average total cost	Marginal cost
1	80	30					
2	80	40					
3	80	45					
4	80	55					
5	80	75					
6	80	120					
