STUDY OF A BALANCED OUTPUT THREE PHASE STATIC CONVERTER FOR AN UNBALANCED SYSTEM

BY

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A THESIS

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ABSTRACT

Converter analysis presented in technical literatures have been mostly based on assumption that converter operates under balanced conditions. Evaluation of real operating conditions, however , show that this assumption is not valid for all operating conditions. It is, therefore , essential in converter analysis to accomodate unbalanced operating conditions. This thesis discusses a novel technique of producing the balanced output of the converter when the input is unbalanced in amplitude.

The proposed technique states that in order to generate balanced voltages and currents from amplitude unbalanced systems, the magnitudes of the fundamental components of three switching functions has to be inversely proportional to the amplitudes of the corresponding unbalance input phase voltages .. This thesis focuses on the analysis and design of the three phase static converter under both balanced and unbalanced operating conditions. Analytically predicted results are varified by simulation.

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APPROVAL

Accepeted as satisfactory in partial fulfillment of the requirements for the degree of Master of Science in Engineering (Electrical and Electronic).

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CERTIFICATE

This is to certify that the work presented in this thesis paper is the outcome of the investigation carried out by me under the supervision of Dr. Shahidul Islam Khan in the Department of Electrical and Electronic Engineering, Bangladesh University of Engineering and Technology, Dhaka. It is also declared that neither this thesis nor any part thereof has been submitted or is being concurrently submitted anywhere else for the award of any degree or diploma.

Signature of the Supervisor

Signature of the Author

DEDICATED TO MY PARENTS

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LIST OF SYMBOLS AND ABBREVIATIONS

w_s	=	Angular switching frequency.
w_i	=	Angular frequency of input voltage.
w_s	=	Angular frequency of output voltage.
I_a, I_b, I_c	=	Input phase currents.
I_A, I_B, I_C	=	Output phase currents.
$I_{i,1}, I_{i,2}, I_{i,3}$	=	Input line currents.
$I_{o,1}, I_{o,2}, I_{o,3}$	=	Output line currents.
V_{an}, V_{bn}, V_{cn}	=	Input phase voltages.
V_{AN}, V_{BN}, V_{CN}	=	Output phase voltages.
V_{ab}, V_{bc}, V_{ca}	=	Input line voltages.
V_{AB}, V_{BC}, V_{CA}	=	Output line voltages.
$[V_o(w_o t)]$	=	Output Voltages of the Converter.
V_i	=	Amplitude of the balanced input phase voltage.
$[F_d(w_s t)]$	=	Switching function.
I_o	=	Amplitude of the balanced output phase current.
$[I_o(w_i t)]$	=	Input currents of the Converter.
$[V_i(w_i t)]$	=	Input voltages of the Converter.
$[I_o(w_o t)]$	=	Output voltages of the Converter.
A,B,C	=	Amplitudes of the three unbalanced input phase voltages.
F_1, F_2, F_3	=	Three switching functions.

A_1, B_1, C_1	=	The amplitudes of the fundamental components of three switching functions F_1, F_2 and F_3 respectively.
A_n, B_n, C_n	=	Amplitude of the n-th harmonic component of the three switching functions.
$\delta_1, \delta_2, \delta_3$	=	Widths of the three switching functions under unbalanced input condition.
$S_1 - S_9$	=	Nine active elements of the converter
$S_1' - S_9'$	=	Nine top switches of the converter
$S_1'' - S_9''$	=	Nine bottom switches of the converter
I_{sp}	=	Peak switch current.
I_{sr}	=	rms switch current.
I_{sav}	=	Average switch current.
$g_1 - g_6$	=	Six gating signals.
L_{s1}, L_{s2}, L_{s3}	=	Three inductors in the protective circuits.
$R_{s1}, R_{s2}, R_{s3}, R_{s4}$	=	Resistors in the protective circuits.
C_{s1}, C_{s2}	=	Capacitors in the protective circuits.
UPS	=	Uniterruptible Power Supply.
FCC	=	Forced Commutated Cycloconverter.
PWM	=	Pulse Width Modulation.
P.U.	=	Per Unit.
EPROM	=	Erasable programmable Read Only Memory.
C.R.	=	Controlled Rectifier.

CHAPTER 1
INTRODUCTION

1.1 Introduction

Electrical power is processed by power electronics to make it suitable for various applications, such as dc and ac regulated power supplies, electrochemical processes, heating and lighting control, electrical machine drives, induction heating, electronic welding, active power line filtering, static var compensation, etc. The processes involve conversions (ac to ac, ac to dc, dc to dc and ac to ac) and control of power [1] using power semiconductor switches.

A power converter incorporates a matrix of power switching devices to convert the electrical power under the guidance of control electronics.The general classification of converters on functional basis are : rectifier, chopper, inverter, ac controller, cycloconverter. Often, a practical power electronic system may combine more than one conversion processes. The recent advancement of power semiconductor devices and control electronics is creating a tremendous impact on power converter technology in terms of size, cost, reliability and performance.

In a modern power electronic equipment, there are essentially two types of semiconductor elements: the power semiconductors that can be considered as the muscle of the equipment and the microelectronic control chips which provide the power of the brain. Both are digital in nature, except that one manipulates power upto gigawatts and the other handles only miliwatts. The close coordination of these end-of-the-spectrum electronics is offering large size and cost effectiveness and high level of performance in today's power supplies.

Static converter are designed to work under balanced input condition. The power frequency three phase input voltage of static converter may be unbalanced in phase or amplitude. This work discusses a novel technique of producing the output of a converter balanced when the input is amplitude unbalanced.

This thesis focuses on the analysis, design and development of a three phase to three phase static converter which produces balanced output voltage when the input is amplitude unbalance.

1.2 Review of Previous Works

Usually power electronics converters are designed to work with balanced input power. However, the input electric power may be unbalanced in phase and amplitude. The case of unbalanced voltage supply is often a perplexing and difficult application problem. Causes for such an unbalance [2] are numerous. Unsymmetrical transformer windings or transmission line impedance, unbalanced three phase loads or large single phase load, are some typical examples. Causes exist in all states of the electrical energy transformation from the generation to utilization. The cycloconverters are designed to work under balanced condition. In general, unbalanced (phase and amplitude) voltages produce various problems such as signal interference, relay malfunctioning, over voltages and excessive currents causing losses, distortion of voltages and currents (dc, in case of rectifier) and errors in monitoring meters.

Any device with nonlinear characteristics that derive their input power from a sinusoidal electrical system may be responsible for injecting harmonic currents and voltages into the electrical system. The primary sources [2] of undesired harmonics are from rectifiers and electric arc furnaces. Common applications of rectifiers are solid-state drives and uninterruptible power supplies (UPS). Unbalance voltage problem for different types of converters is studied by different researchers in different ways. Unbalance in the operation of high-voltage dc converter gives rise to harmonics in the current and voltages waveforms . The analysis and assessment [3] of uncharacteristic harmonics arising in HVDC transmission are specified in terms of asymmetry of the control pulse of the converter.

Proper understanding and detail information on unbalanced voltage operation is carried out by R.F. Woll, [4]. Here the unbalanced voltage is spilt in positive, negative and zero sequences [5]. Using this sequence network concept high voltage DC transmission when experiencing unbalanced condition was studies in reference [6].

Regardless of the cause, a small phase unbalance voltage can induce large negative sequence current, due to the relatively low negative sequence impedance the machine. Induction motors are particularly sensitive to unbalance operation since localized heating can occur in the stator and the life of the machine can be seriously affected with only a few percent voltage unbalance [7]. Induction motor phase unbalance problem solution has been proposed [7]-[9] by digital controller.

Converters, mainly rectifiers are analysed [10]-[12] for input unbalance using sequence network. In these works both phase and amplitude unbalance were considered. Complex mathematical procedure is followed to break the unbalance network into three sequence networks. Proper switching function is calculated by rigorous mathemematical calculation. An evaluation of harmonics generated by Forced Commutated Cycloconverter (FCC) under unbalance is reported [13].

This thesis is devoted only on amplitude unbalanced converter. Simple mathematical calculation is needed to find the proper switching function. The implementation of the technique is simple and effective.

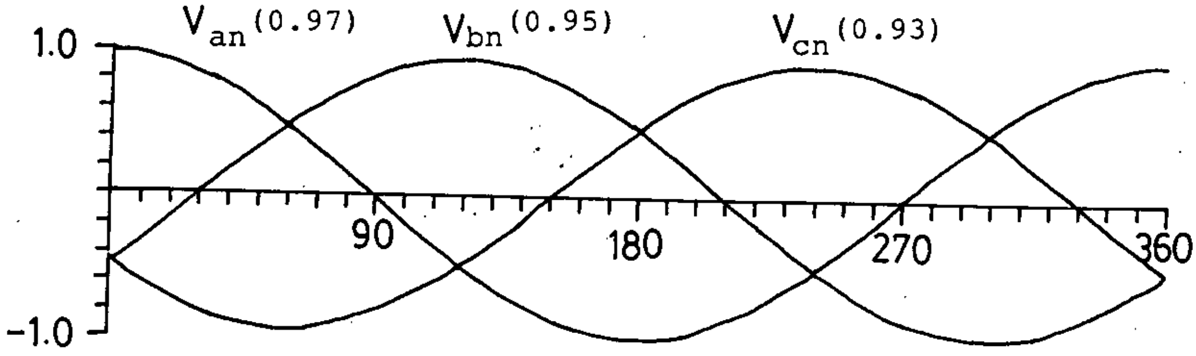
1.3 The Proposed Method

This thesis proposes and analyse a novel method for balancing the output voltage when the input voltage is amplitude unbalanced. This voltage balancing technique states that the output voltage is balanced when the fundamental component of the switching functions are equal to inverse of the amplitude of the corresponding unbalance input phase voltage. The analytical method of design and analysis of three phase to three phase static converter is presented in this thesis. A simplified block diagram of the proposed unbalance three phase to three phase static converter is shown in Fig. 1.1(b). Fig 1.1(a) and (c) shows the unbalanced input voltages and balanced output voltages respectively.

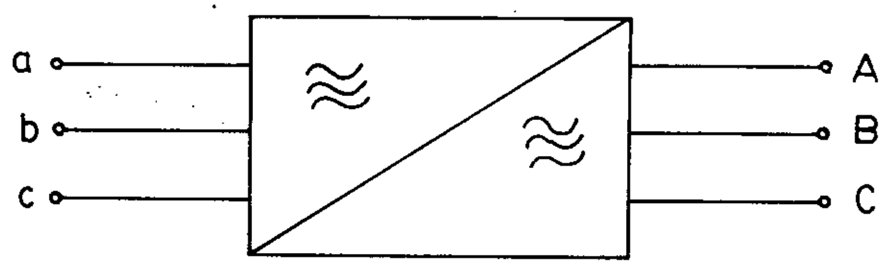
The converter consists of six bilateral switches. The switches which are controlled by switching function are synchronized with the input voltages and their ON/OFF operations are determined by the percentage of input unbalance.

1.4 Scope of the Present Work

In this thesis a novel technique is used to analyse the proposed three phase to three phase static converter conversion. The analytical method of design and analysis of three phase to three phase static converter is complicated. The voltage balancing technique proposed here states that the output voltages are balanced when the fundamental component of three switching functions are taken as inversely proportional to the amplitude of the three unbalance input phase voltages .The amplitude unbalance is corrected [14] by changing the width of the switching

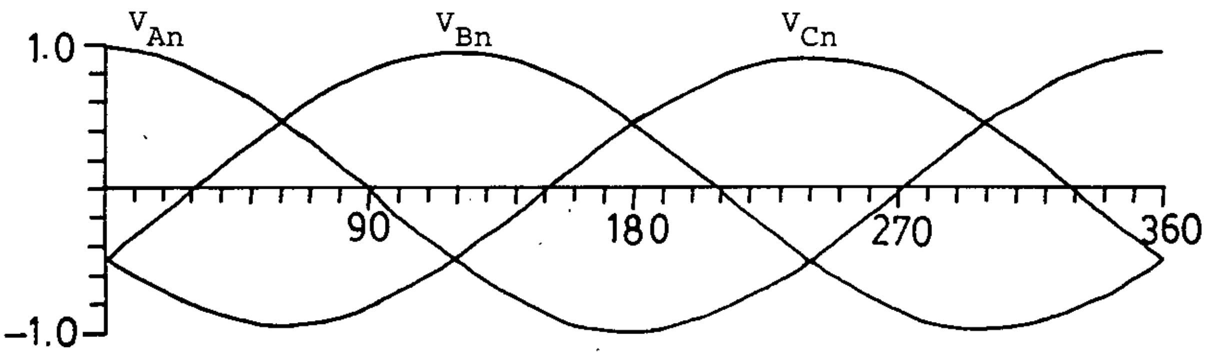


a) Unbalanced Input Voltages



b) Converter

Do you need any filter?



c) Balanced Output Voltages

Fig. 1.1: Block diagram of the proposed 3 phase to 3 phase balanced output static converter.

function according to unbalance. This type of converter may find extensive use to correct unbalance in power systems. Moreover, this converter may be used in unbalanced environment for motor drives.

The objective of this research is to analyze the simulated balanced waveforms obtained by switching function method from an unbalanced system.

In particular the contents of this thesis have been organised as follows:

A generalized converter matrix model with N-phase inputs and M-phase outputs is used to develop a three phase to three phase static converter structure. Mathametical analysis of a 3-phase to 3-phase static converter under balanced and unbalanced input condition are analysed. A graphical technique has been developed using IBM PC to findout the output voltages and input currents. The data for analysing the waveforms of input-output quantities are calculated with the help of FORTRAN program using transfer function approach.

In Chapter 3, through Fourier analysis of input and output voltage/currents are done. Both balanced and unbalanced operating conditions are considered. Computer programs were developed using the mathematical expressions detailed in chapter 2.

In Chapter 4, the operation and design of the proposed converter are discussed. The control logic algorithm development and implementation using simple simple logic blocks are also discussed.

Three phase controlled rectifier under unbalanced input condition is studied in chapter 5. It has been proved that the developed technique of voltage balancing is equally applicable to unbalanced controlled rectifier.

Chapter 6 reviews the entire work presented in this thesis and presents relevents conclusions. Suggession for using more complex and advanced PWM switching functions technique and studying the phase unbalanced cycloconverter are focused for future research work.

CHAPTER 2

MODELLING AND ANALYSIS OF THE CONVERTER

2.1 Introduction

The objective of this chapter is to develop a switching model suitable for the analysis and evaluation of a three phase to three phase converter which can handle unblanced input voltages. To achieve this objective the converter (in its ideal form) is modelled as a circuit matrix consisting of [N x M] switching elements (Fig 2.1). The matrix representation [15] of this generalized converter allows us to understand easily the process of voltage and frequency transformation. In this thesis this model is used to analyze a three phase to three phase converter. To reach the goal, first such a conveter is analysed under balanced input condition. From this analysis some basic criterion is established so that the input unbalance can be made balanced in the output circuit.

Converters are usually designed to handle balanced three and single phase input power. When the input is unbalanced conventional static conveter may not work properly. Short-circuiting, harmonic generation, unbalancing of output power are the major problems associated with unbalanced input.

The input voltage could be unbalanced by amplitudes of the three input phases or by improper phase relationships. Both amplitude and phase unbalance could be corrected. Only the amplitude unbalance is treated in this thesis.

A method for correcting the amplitude unbalance is developed in this chapter which could be applied to polyphase converters. A specific case of three phase to three phase converter is studied in details.

2.2 Mathametical Modelling of the Converter

The model of a converter structure capable of performing the voltage, current, phase, frequecny and amplitude transformation is shown in Fig 2.1. This switching model [15] comprises of an "input matrix" of [Nx1] dimension and an "output matrix" of [Mx1] dimension.

The multiplication of a set of Nx1 sinusoidal quantities (e.g. input voltage matrix) by a compatible set of MxN balanced sinusoidal quantities (e.g. converter transfer matrix) yields a third set of Mx1 sinusoidal quantities (e.g. output voltage matrix) that are balanced. Therefore, the analytical representation of the converter interms of output voltage $[v_o(w_o t)]$ and input current $[I_i\ (w_i t)]$ becomes:

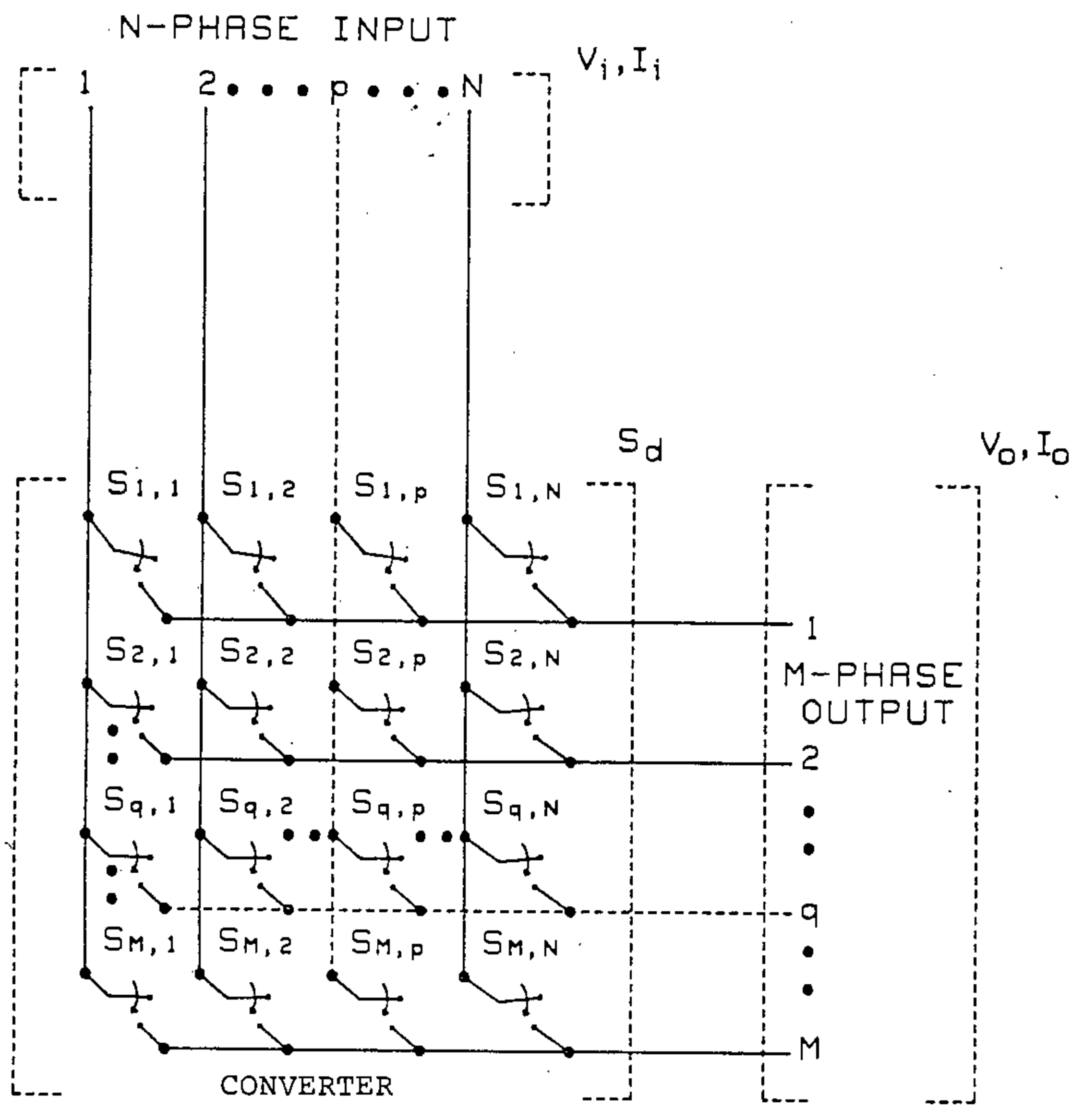


Fig. 2.1: Generalized model structure of the converter for N input and M output phases.

$$[V_o(w_o t)] = [F_d(w_s t)]\ [V_i(w_i t)]$$

$$[M\times 1] \qquad\qquad [M\times N] \qquad\qquad [N\times 1]$$

$$\begin{bmatrix} V_{o,1} \\ \vdots \\ V_{o,q} \\ \vdots \\ V_{o,M} \end{bmatrix} = A \begin{bmatrix} f_{1,1} \cdots f_{1,p} \cdots f_{1,N} \\ \vdots \\ f_{q,1} \cdots f_{q,p} \cdots f_{q,N} \\ \vdots \\ f_{M,1} \cdots f_{M,p} \cdots f_{M,N} \end{bmatrix} \cdot B \begin{bmatrix} V_{i,1} \\ \vdots \\ V_{i,p} \\ \vdots \\ V_{i,N} \end{bmatrix}$$

$$= A \begin{bmatrix} \cos(w_s t) \ldots \cos\left(w_s t - \frac{(P-1)}{N} 360^0\right) \ldots \cos\left(w_s t - \frac{(N-1)}{N} 360^0\right) \\ \vdots \\ \cos\left(w_s t + \frac{(q-1)}{M} 360^0\right) \ldots \cos\left(w_s t - \frac{(P-1)}{N} 360^0 + \frac{(q-1)}{M} 360^0\right) \ldots \cos\left(w_s t - \frac{(N-1)}{N} 360^0 + \frac{(q-1)}{M} 360^0\right) \\ \vdots \\ \cos\left(w_s t + \frac{(M-1)}{M} 360^0\right) \ldots \cos\left(w_s t - \frac{(P-1)}{N} 360^0 + \frac{(M-1)}{M} 360^0\right) \ldots \cos\left(w_s t - \frac{(N-1)}{N} 360^0 + \frac{(M-1)}{M} 360^0\right) \end{bmatrix}$$

$$B \begin{bmatrix} \cos(w_i t) \\ \vdots \\ \cos\left(w_i t - \frac{(P-1)}{N} 360^o\right) \\ \vdots \\ \cos\left(w_i t - \frac{(N-1)}{N} 360^o\right) \end{bmatrix} = \frac{NAB}{2} \begin{bmatrix} \cos(w_o t) \\ \vdots \\ \cos\left(w_o t - \frac{(q-1)}{M} 360^o\right) \\ \vdots \\ \cos\left(w_o t - \frac{(M-1)}{M} 360^o\right) \end{bmatrix}$$

(2.1a)

and

$$[I_i(w_i t)] = [F_d(w_s t)]^T \; [I_o(w_o t)]$$

$$[M\times 1] \qquad\qquad [M\times N] \qquad\qquad [N\times 1]$$

$$\begin{bmatrix} I_{i,1} \\ \vdots \\ I_{i,p} \\ \vdots \\ I_{i,N} \end{bmatrix} = A \begin{bmatrix} f_{1,1} \cdots f_{q,1} \cdots f_{M,1} \\ \vdots \\ f_{1,p} \cdots f_{q,p} \cdots f_{M,p} \\ \vdots \\ f_{1,N} \cdots f_{q,N} \cdots f_{M,N} \end{bmatrix} \cdot B \begin{bmatrix} I_{o,1} \\ \vdots \\ I_{o,q} \\ \vdots \\ I_{o,M} \end{bmatrix}$$

$$= A \begin{bmatrix} \cos(w_s t) \ldots \cos(w_s t + \frac{(q-1)}{M} 360^0) \ldots \cos(w_s t + \frac{(M-1)}{M} 360^0) \\ \vdots \\ \cos(w_s t - \frac{(p-1)}{N} 360^0) \ldots \cos(w_s t - \frac{(P-1)}{N} 360^0 + \frac{(q-1)}{M} 360^0) \ldots \cos(w_s t + \frac{(M-1)}{M} 360^0 - \frac{(p-1)}{N} 360^0) \\ \vdots \\ \cos(w_s t - \frac{(N-1)}{N} 360^0) \ldots \cos(w_s t - \frac{(N-1)}{N} 360^0 + \frac{(q-1)}{M} 360^0) \ldots \cos(w_s t - \frac{(N-1)}{N} 360^0 + \frac{(M-1)}{M} 360^0) \end{bmatrix}$$

$$B. \begin{bmatrix} \cos(w_o t) \\ \vdots \\ \cos(w_o t - \frac{(q-1)}{M} 360^o) \\ \vdots \\ \cos(w_o t - \frac{(M-1)}{M} 360^o) \end{bmatrix} = \frac{MAB}{2} \begin{bmatrix} \cos(w_i t) \\ \vdots \\ \cos(w_i t - \frac{(p-1)}{N} 360^o) \\ \vdots \\ \cos(w_i t - \frac{(N-1)}{N} 360^o) \end{bmatrix} \qquad (2.1b)$$

A generalized converter (for N input and M output phases) structure capable of performing voltage, current, phase, frequency and amplitude transformation given in eqns.(2.1a) and (2.1b) is illustrated in Fig. 2.1.

2.2.1 Practical Converter Structure

A simplified circuit topology that results from the generalized model shown in Fig 2.1 and N=M=3 is illustrated in Fig 2.2. This converter structure is cable of voltage/frequency transformation from fixed frequency/voltage 3-phase input to variable frequency 3-phase output voltage.

This topology (Fig 2.2) comprises of 9(=3x3) "active elements" since the " ideal 3-phase to 3-phase converter transfer matrix" is of dimension [3x3] (i.e. N=3, M=3). The transfer

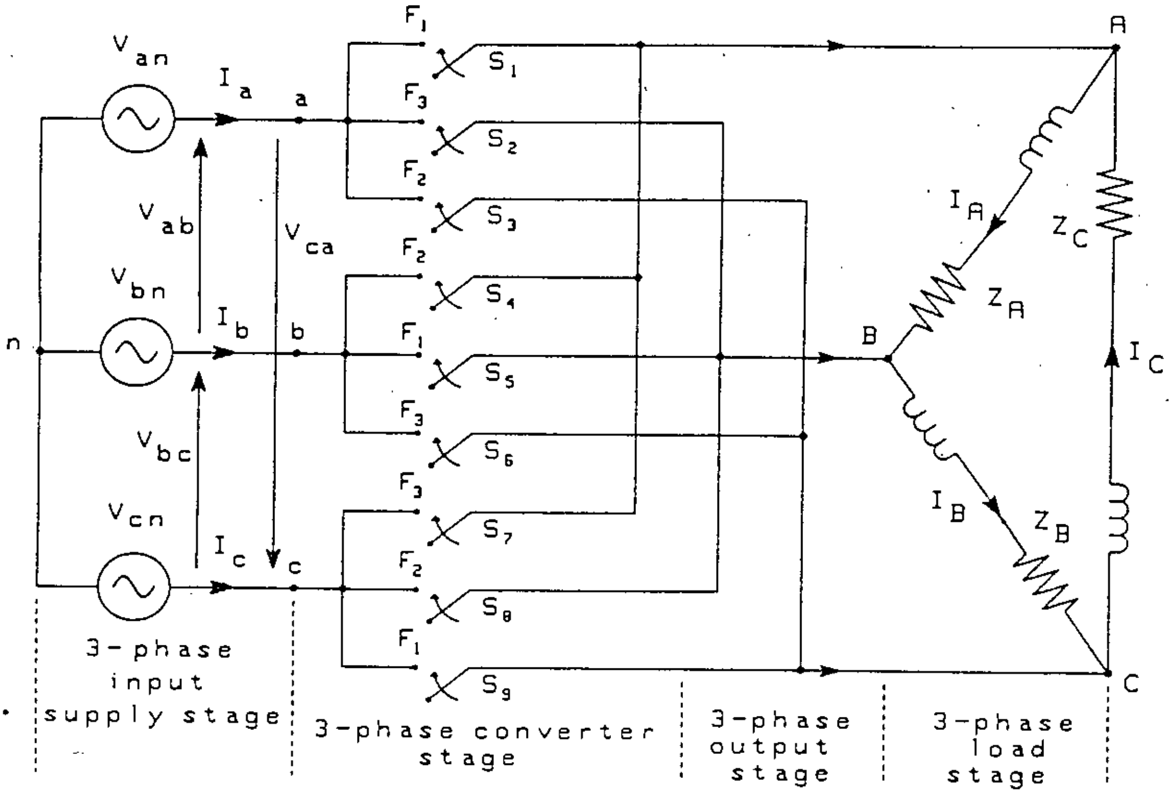


Fig. 2.2: Simplified circuit diagram of the proposed 3 phase to 3 phase static converter.

function is represented as:

$$[F_d(w_s t)] = A \begin{bmatrix} f_{1,1} & f_{1,2} & f_{1,3} \\ f_{2,1} & f_{2,2} & f_{2,3} \\ f_{3,1} & f_{3,2} & f_{3,3} \end{bmatrix} \qquad (2.3a)$$

The converter transfer matrix elements $f_{1,1}$, $f_{1,2}$,$f_{1,3}$ can be expressed as F_1,F_2 and F_3

where,

$$F_1 = f_{1,1}$$

$$F_2 = f_{1,2} = F_1 < -120^o \qquad (2.3\ b)$$

and $F_3 = f_{1,3} = F_1 < -240^o$

Equation (2.1 a) can be simplified as

$$[V_o(w_o t)] = 1 \begin{bmatrix} F_1 & F_2 & F_3 \\ F_3 & F_1 & F_2 \\ F_2 & F_3 & F_1 \end{bmatrix} . \ 1 \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

$$= \begin{bmatrix} F_1 V_{ab} + F_2 V_{bc} + F_3 V_{ca} \\ F_3 V_{ab} + F_1 V_{bc} + F_2 V_{ca} \\ F_2 V_{ab} + F_3 V_{bc} + F_1 V_{ca} \end{bmatrix}$$

or

$$V_{AB} = V_{ab}F_1 + V_{bc}F_2 + V_{ca}F_3$$
$$V_{BC} = V_{ab}F_3 + V_{bc}F_1 + V_{ca}F_2$$
$$V_{CA} = V_{ab}F_2 + V_{bc}F_3 + V_{ca}F_1$$

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} F_1 & F_2 & F_3 \\ F_3 & F_1 & F_2 \\ F_2 & F_3 & F_1 \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \quad (2.3\ c)$$

or per phase

$$V_{AN} = V_{an}\,F_1 + V_{bn}\,F_2 + V_{cn}\,F_3$$
$$V_{BN} = V_{an}\,F_3 + V_{bn}\,F_1 + V_{cn}\,F_2 \quad (2.3\ d)$$
$$V_{CN} = V_{an}\,F_2 + V_{bn}\,F_3 + V_{cn}\,F_1$$

And for a 3 phase to 3 phase converter equation (2.1 b) can also be rewritten and simplified as:

$$\begin{bmatrix} I_{i,1} \\ I_{i,2} \\ I_{i,3} \end{bmatrix} = \begin{bmatrix} F_1 & F_3 & F_2 \\ F_2 & F_1 & F_3 \\ F_3 & F_2 & F_1 \end{bmatrix} \cdot \begin{bmatrix} I_{o,1} \\ I_{o,2} \\ I_{o,3} \end{bmatrix}$$

$$= \begin{bmatrix} F_1 \cdot I_{o,1} + F_3 \cdot I_{o,2} + F_2 \cdot I_{o,3} \\ F_2 \cdot I_{o,1} + F_1 \cdot I_{o,2} + F_3 \cdot I_{o,3} \\ F_3 \cdot I_{o,1} + F_2 \cdot I_{o,2} + F_1 \cdot i_{o,3} \end{bmatrix} \quad (2.3e)$$

$$
\begin{aligned}
\text{or } I_{i,1} &= F_1 . I_{o,1} + F_3 . I_{o,2} + F_2 . I_{o,3} \\
I_{i,2} &= F_2 . I_{o,1} + F_1 . I_{o,2} + F_3 . I_{o,3} \qquad (2.3f) \\
I_{i,3} &= F_3 . I_{o,1} + F_2 . I_{o,2} + F_1 . i_{o,3}
\end{aligned}
$$

or expressing in per phase

$$
\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} F_1 & F_3 & F_2 \\ F_2 & F_1 & F_3 \\ F_3 & F_2 & F_1 \end{bmatrix} . \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \qquad (2.3\text{ g})
$$

$$
\begin{aligned}
I_a &= F_1 . I_A + F_3 . I_B + F_2 . I_C \\
I_b &= F_2 . I_A + F_1 . I_B + F_3 . I_C \\
I_c &= F_3 . I_A + F_2 . I_B + F_1 . I_C
\end{aligned}
$$

2.2.2 Operation of the Converter

Considering switch matrix element F_1, bilateral switches S_1, S_5 and S_9 are turned ON whenever F_1 has the values of one (Fig. 2.2), thus connecting the three converter input terminals a,b,c to the three respective output terminals A, B, C (Fig. 2.2). Whenever F_1 has the value of zero, two of the three switches are turned OFF (in this case of S5 and S9), while at the same time the remaining two of the three switches connected to the input terminal a, S_2 and S_3, are turned ON. With switches S_1, S_2 and S_3 simultaneously ON, all three output voltages becomes zero, thus disconnecting the output from the input terminals. This mode of ON-OFF operation lasts for 120^o interval of the period of the output voltage. For the next 120^o interval converter operation is determined by switching matrix element F_2 (instead of F_1). Consequently, the function performed by switches S_1, S_5, S_9 is now performed by S_3, S_4, S_8 and the function performed by switches S_1, S_2, S_3 is now performed by S_4, S_5, S_6. A similar switch by switch substitution occurs during the third (and final) 120^o interval of the period of the output voltage.

2.2.3 Mathematical Analysis of the 3 phase to 3 phase Converter Under Balance Input

Consider a balance case and assume

$$\begin{aligned} F_1 &= A \cos (w_s t) \\ F_2 &= A \cos (w_s t - 120^o) \\ F_3 &= A \cos (w_s t - 240^o) \end{aligned} \qquad (2.4)$$

where A is the amplitude of the fundamental components of all three function F_1,F_2 & F_3.

$$\text{and } [V_i(w_i t)] = V_i \begin{bmatrix} \cos (w_i t) \\ \cos (w_i t - 120^o) \\ \cos (w_i t - 240^o) \end{bmatrix} \qquad (2.5)$$

From equation (2.3 d) , (2.4) & (2.5)

The expression for output voltage is given by:

$$[V_o(w_o t)] = [F(w_s t)].\ [V_i(w_i t)]$$

$$= \begin{bmatrix} F_1 & F_2 & F_3 \\ F_3 & F_1 & F_2 \\ F_2 & F_3 & F_1 \end{bmatrix} \quad V_i \begin{bmatrix} \cos(w_i t) \\ \cos(w_i t-120^o) \\ \cos(w_i t-240^o) \end{bmatrix}$$

$$V_{AN} = AV_i[(\cos(w_s t).\cos(w_i t)+\cos(w_s t-120^o).\cos(w_i t-120^o) + \cos(w_s t - 240^o).\cos(w_i t-240^o)]$$

$$= \frac{3\ AV_i}{2}\cos(w_s-w_i)t + \frac{AV_i}{2}[\cos(w_s + w_i)t + \cos\{(w_s + w_i)t - 120^o\} + \cos\{(w_s + w_i)t - 240^o\}]$$

Since the second term of the above equation is the three phasors of equal magnitude and displaced 120^o apart from each other, their sum i.e. the resultant is zero.

$$V_{AN} = \frac{3}{2} AV_i \cos(w_o t) \qquad (2.6a)$$

where

$$w_s = w_i + w_o$$

Similarly

$$V_{BN} = V_iA\cos(w_st-240^o).\cos(w_it) + V_iA\cos(w_st).\cos(w_it-120^o) + V_iA\cos(w_st-120^o).\cos(w_it-240^o)$$

$$= \frac{AV_i}{2}[\{\cos((w_s-w_i)t-240^o)+\cos((w_s+w_i)t-240^o)\} +\{\cos((w_s-w_i)t+120^o)+\cos((w_s+w_i)t-120^o)\} +\{\cos((w_s-w_i)t+120^o)+\cos((w_s+w_i)t)\}]$$

$$= \frac{3AV_i}{2}[\cos((w_s-w_i)t-240^o)$$

$$+ \frac{AV_i}{2}[\cos(3w_ot-240^o)+\cos(3w_ot-120^o)+\cos(3w_ot)]$$

$$V_{BN} = \frac{3AV_i}{2}\cos(w_ot-240^o) \qquad (2.6\ b)$$

and $V_{CN} = V_iA\cos(w_st-120^o).\cos(w_it) + V_iA\cos(w_st-240^o).\cos(w_it-120^o) + V_iA\cos(w_st).\cos(w_it-240^o)$

$$= \frac{AV_i}{2}[\{\cos((w_s-w_i)t-120^o)+\cos((w_s+w_i)t-120^o)\} +\{\cos((w_s-w_i)t-120^o)+\cos((w_s+w_i)t-360^o)\} +\{\cos((w_s-w_i)t-240^o)+\cos((w_s+w_i)t-240^o)\}]$$

$$= \frac{3AV_i}{2}\cos(w_ot-120^o)$$

$$+ \frac{VA_i}{2} [\cos(3w_o t-120^o)+\cos(3w_o t-240^o)+\cos(3w_o t)]$$

$$V_{CN} = \frac{3AV_i}{2} \cos(w_o t-120^o) \qquad (2.6c)$$

From equation (2.6 a), (2.6 b) and (2.6 c) it is observed that the output voltages are balanced.

The input current expressions are derived as:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} F_1 & F_3 & F_2 \\ F_2 & F_1 & F_3 \\ F_3 & F_2 & F_1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \qquad (2.7 a)$$

$$\text{or} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = A \begin{bmatrix} \cos(w_s t) & \cos(w_s t-240^o) & \cos(w_s t-120^o) \\ \cos(w_s t-120^o) & \cos(w_s t) & \cos(w_s-240^o) \\ \cos(w_s t-240^o) & \cos(w_s t-120^o) & \cos(w_s t) \end{bmatrix}$$

$$.I_o \begin{bmatrix} \cos(w_o t) \\ \cos(w_o t-240^o) \\ \cos(w_o t-120^o) \end{bmatrix}.$$

or

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{3AI_o}{2} \begin{bmatrix} \cos(w_i t) \\ \cos(w_i t-120^o) \\ \cos(w_i t-240^o) \end{bmatrix} \qquad (2.7\ b)$$

2.2.4 Mathematical Analysis of the 3 phase to 3 phase converter under Unbalance Input

Consider an unbalance case and assume the unbalanced input voltage $V_i(wt)$ as

$[V_i(w_i t)]$=Input voltage

$$= \begin{bmatrix} A \cos(w_i t) \\ B \cos(w_i t-120^o) \\ C \cos(w_i t-240^o) \end{bmatrix}$$

and

$$F_1 = A_1 \cos(w_s t)$$
$$F_2 = B_1 \cos(w_s t-120^o) \qquad (2.8)$$
$$F_3 = C_1 \cos(w_s t-240^o)$$

where A_1, B_1 and C_1 are the amplitude of the fundamental components of three switching function F_1, F_2 and F_3 respectively. From equation (2.3 c)

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = [SF_1] \,.\, [V_i(w_i t)]$$

$$= \begin{bmatrix} F_1 & F_2 & F_3 \\ F_3 & F_1 & F_2 \\ F_2 & F_3 & F_1 \end{bmatrix} \cdot \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$= \begin{bmatrix} A_1 \cos(w_s t) & B_1 \cos(w_s - 120^o) & C_1 \cos(w_s t - 240^o) \\ C_1 \cos(w_s t - 240^o) & A_1 \cos(w_s t) & B_1 \cos(w_s t - 120^o) \\ B_1 \cos(w_s t - 120^o) & C_1 \cos(w_s - 240^o) & A_1 \cos(w_s t) \end{bmatrix}$$

$$\cdot \begin{bmatrix} A \cos(w_i t) \\ B \cos(w_i t - 120^o) \\ C \cos(w_i t - 240^o) \end{bmatrix}$$

$$= \begin{bmatrix} AA_1 \cos(w_s t)\cos(w_i t) + BB_1 \cos(w_s t - 120^o)\cos(w_i t - 120^o) \\ + CC_1 \cos(w_s t - 240^o)\cos(w_i t - 240^o) \\ AC_1 \cos(w_s t - 240^o)\cos(w_i t) + BA_1 \cos(w_s t)\cos(w_i t - 120^o) \\ + CB_1 \cos(w_s t - 120^o)\cos(w_i t - 240^o) \\ AB_1 \cos(w_s t - 120^o)\cos(w_i t) + BC_1 \cos(w_s t - 240^o)\cos(w_i t - 120^o) \\ + CA_1 \cos(w_s t)\cos(w_i t - 240^o) \end{bmatrix}$$

Therefore,

$$V_{AN} = AA_1\cos(w_s t)\cos(w_i t)+BB_1\cos(w_s t-120^o)\cos(w_i t-120^o)$$
$$+ CC_1\cos(w_s t-240^o)\cos(w_i t-240^o)$$
$$= 3/2 \cos(w_o t)$$

When $AA_1 = 1$, $BB_1 = 1$, and $CC_1 = 1$

$$A = 1/A_1$$
$$B = 1/B_1$$
$$C = 1/C_1 \qquad (2.9)$$

and where $w_s = w_o + w_i$, and $w_o = w_i$

Similarly

$$V_{BN} = AC_1\cos(w_s t-240^o)\cos(w_i t)+BA_1\cos(w_s t)\cos(w_i t-120^o)$$
$$+ CB_1\cos(w_s t-120^o)\cos(w_i t-240^o)$$

$$= \frac{1}{2}\,[AC_1\{\cos((w_s-w_i)t-240^o)+\cos((w_s+w_i)t-240^o)\}$$
$$+BA_1\{\cos((w_s-w_i)t+120^o)+\cos((w_s+w_i)t-120^o)\}$$
$$+CB_1\{\cos((w_s-w_i)t+120^o)+\cos((w_s+w_i)t-360^o)\}]$$

$$= \frac{AC_1 + BA_1 + CB_1}{2}\cos(w_o t - 240^o)$$

$$+ \frac{1}{2}\,[AC_1\cos(3w_o t-240^o)+BA_1\cos(3w_o t-120^o)$$
$$+ CB_1\cos(3w_o t-360^o)]$$

and

$$V_{CN} = AB_1\cos(w_s t-120^o)\cos(w_i t)+BC_1\cos(w_s t-240^o)\cos(w_i t-120^o)$$
$$+ CA_1\cos(w_s t)\cos(w_i t-240^o)$$
$$= \frac{1}{2} [AB_1 \{\cos((w_s-w_i)t-120^o)+\cos((w_s+w_i)t-120^o)$$
$$+ BC_1 \{\cos((w_s-w_i)t-120^o)+\cos((w_s+w_i)t-360^o)\}$$
$$+ CA_1 \{\cos((w_s-w_i)t+240^o)+\cos((w_i+w_i)t-240^o)\}]$$

$$= \frac{AB_1 + BC_1 + CA_1}{2} \cos(w_o t-120^o)$$

$$+ \frac{1}{2} [AB_1\cos(3w_o t-120^o)+BC_1\cos(3w_o t-360^o)+CA_1(3w_o t-240^o)]$$

Now, from equation (2.7a) and in which the phase rotation of output current I_B and I_C is considered.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} F_1 & F_3 & F_2 \\ F_2 & F_1 & F_3 \\ F_3 & F_2 & F_1 \end{bmatrix} . \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$\text{or} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} A_1\cos(w_s t) & C_1\cos(w_s t-240^o) & B_1\cos(w_s t-120^o) \\ B_1\cos(w_s t-120^o) & A_1\cos(w_s t) & C_1\cos(w_s t-240^o) \\ C_1\cos(w_s t-240^o) & B_1\cos(w_s t-120^o) & A_1\cos(w_s t) \end{bmatrix}$$

$$
\cdot \begin{bmatrix} A \cos(w_o t) \\ C \cos(w_o t - 240^o) \\ B \cos(w_o t - 120^o) \end{bmatrix}
$$

$$
= \begin{bmatrix} AA_1 \cos(w_s t)\cos(w_o t) + BB_1 \cos(w_{st} - 120^o)\cos(w_i t - 120^o) \\ \quad + CC_1 \cos(w_s t - 240^o)\cos(w_i t - 240^o) \\ AB_1 \cos(w_s t - 120^o)\cos(w_o t) + CA_1 \cos(w_s t)\cos(w_o t - 240^o) \\ \quad + BC_1 \cos(w_s t - 240^o)\cos(w_s t - 120^o) \\ AC_1 \cos(w_s t - 240^o)\cos(w_o t) + CB_1 \cos(w_s t - 120^o)\cos(w_o t - 240^o) \\ \quad + BA_1 \cos(w_s t)\cos(w_o t - 120^o) \end{bmatrix}
$$

Therefore

$$
I_a = AA_1 \cos(w_s t)\cos(w_o t) + BB_1 \cos(w_s t - 120^o)\cos(w_o t - 120^o)
$$
$$
+ CC_1 \cos(w_s t - 240^o)\cos(w_o t - 240^o)
$$

$$
= \frac{1}{2} [AA_1 \{\cos(w_s - w_o)t + \cos(w_s + w_o)t\}
$$
$$
+ BB_1 \{\cos(w_s - w_o)t + \cos((w_s + w_o)t - 240^o)\}
$$
$$
+ CC_1 \{\cos(w_s - w_o)t + \cos((w_s + w_o)t - 480^o)\}]
$$

$$
= 3/2 \cos w_i t \qquad [\text{ As } \quad AA_1 = 1,\ BB_1 = 1 \ \&\ CC_1 = 1]
$$

Similarly

$$I_b = AB_1\cos(w_s t-120^o)\cos(w_o t)+CA_1\cos(w_s t)\cos(w_o t-240^o)$$
$$+ BC_1\cos(w_s t-240^o)\cos(w_o t-120^o)$$

$$= \frac{1}{2}\,[AB_1\{\cos(w_s-w_o)t-120^o)+\cos((w_s+w_o)t-120^o\}$$
$$+CA_1\{\cos((w_s-w_o)t+240^o)+\cos((w_s+w_o)t-240^o)\}$$
$$+BC_1\{\cos((w_s-w_o)t-120^o)+\cos((w_s+ w_o)t-360^o)\}]$$

$$= \frac{AB_1 + CA_1 + BC_1}{2}\cos(w_i t-120^o) + \frac{1}{2}\,[AB_1\cos(3w_i t-120^o)$$
$$+ CA_1\cos(3w_i t-240^o)+BC_1\cos(3w_i t-360^o)]$$

and

$$I_c = AC_1\cos(w_s t-240^o)\cos(w_o t) + CB_1\cos(w_s t-120^o)\cos(w_o t-240^o)$$
$$+ BA_1\cos(w_s t)\cos(w_o t-120^o)$$

$$= \frac{1}{2}\,[AC_1\{\cos((w_s-w_o)t-240^o)+\cos((w_s+w_o)t-240^o)\}$$
$$+CB_1\{\cos((w_s-w_o)t+120^o)+\cos((w_s+w_o)t-360^o)\}$$
$$+BA_1\{\cos((w_s-w_o)t+120^o)+\cos((w_s+w_o)t-120^o)\}]$$

$$= \frac{AC_1 + CB_1 + BA_1}{2}\cos(w_i t-240^o)$$

$$+ \frac{1}{2}[AC_1\cos(3w_i t-240^o)+CB_1\cos(3w_i t-360^o)+BA_1\cos(3w_i t-120^o)]$$

For a 3 phase to 3 phase unbalanced (amplitude only) converter :

The Output voltages are

$$V_{AN} = \frac{3}{2}\cos(w_o t)$$

$$V_{BN} = \frac{AC_1+BA_1+CB_1}{2}\cos(w_o t-240^o) + \frac{1}{2}[AC_1\cos(3w_o t-240^o)+BA_1\cos(3w_o t-120^o) + CB_1\cos(3w_o t-360^o)] \qquad (2.10)$$

$$V_{CN} = \frac{AB_1+BC_1+CA_1}{2}\cos(w_o t-120^o) + \frac{1}{2}[AB_1\cos(3w_o t-120^o)+BC_1\cos(3w_o t-360^o) + CA_1\cos(3w_o t-240^o)]$$

$$I_a = \frac{3}{2}\cos(w_i t)$$

$$I_b = \frac{AB_1+BC_1+CA_1}{2}\cos(w_i t-120^o)$$

$$+\frac{1}{2}[AB_1\cos(3w_i t-120^o)+BC_1\cos(3w_i t-360^o) \qquad (2.11)$$

$$+CA_1\cos(3w_i t-240^o)]$$

$$I_c = \frac{AC_1+BA_1+CB_1}{2}\cos(w_i t-240^o)$$

$$+\frac{1}{2}[AC_1\cos(3w_i t-240^o)+BA_1\cos(3w_i t-120^o)$$

$$+CB_1\cos(3w_i t-360^o)]$$

2.3 Conclusions

The three phase converter structure is described in terms of the generalized matrix model. The operation of a three phase converter is described in this chapter. The three phase converter is analysed under balanced and unbalanced input conditions. It is established that the output voltages are balanced in magnitude when the fundamental component of the switching functions are equal to the inverse of the amplitude of the corresponding unbalanced input phase voltages.

CHAPTER 3

FOURIER ANALYSIS OF THE CONVERTER

3.1 Introduction

The three-phase converter model developed in chapter 2 is analysed using Fourier analysis in this chapter. Both balanced and unbalanced input case have been analysed using FORTRAN programs on personal computer. The spectra of three output voltages and input currents of the converter are presented in Tables 3.1-3.6. Using these Fourier coefficients the waveforms are reconstructed. The reconstructed waveforms matches the original waveforms, which proves the validity of the proposed technique of voltage balancing. Finally the harmonic analysis of input/output voltage/current waveforms are also done using MATLAB package program.

The analysis is done for simple single pulse modulation.

3.2 Switching Functions

The operation of the converter can be achieved only by means of a set of switches which operate according to a predetermined switching pattern. The switching function for a single switch assumes unit value whenever the switch is closed and a zero value whenever the switch is opened. In a converter, each switch is closed and opened according to a predetermined repetitive pattern; hence, its switching function will take the form of a train of pulses of unit amplitude. Neither the pulses nor the intervening zero-value periods have necessarily the same time duration; however, the requirement that a repetitive switching pattern must exist means that the function must at least consist of repetitive groups of pulses. The simplest, or unmodulated, switching functions have pulses of the same time duration and zero intervales with the same property. A more complex type with differing pulse durations and various intersperses zero times, is termed as a pulse width modulated (PWM) switching function. By modulating the pulse widths, harmonic content of the resultant waveform improves and voltage control at the output is achieved. In the case of a variable frequency converter, the selection of a suitable modulation [16] strategy is of utmost importance. This is particularly true when the output voltage shape and the frequency spectrum change with a corresponding change in output frequency.

There are various types of modulation techniques e.g. single-pulse modulation, multi-pulse modulation, sinusoidal-pulse modulation, sinusoidal pulse-width modulation, etc.

The techniques available differ in the harmonic content that they produce and in the output voltage gain. Thus the acceptable harmonic content and resulting voltage gain are the factors that determine the choice of a particular PWM technique.

The switching function waveforms of the single-pulse modulation scheme is illustrated in Fig. 3.1. For the purpose of analysis it is assumed that the pulse width can be varied by equal angular intervals on both sides from the centre of the pulse, thus resulting in variation of the pulse width δ over the range $0 < \delta < \pi$ radian.

The waveform can be described by the Fourier series,

$$SF = A_o + \sum_{n=1,2,3}^{\alpha} (A_n \cos nwt + B_n \sin nwt) \qquad (3.1)$$

where

$$A_n = \frac{2}{\pi} \int_o^{\pi} \cos nwt \; d(wt)$$

$$= \frac{4}{n\pi} \sin \frac{n\delta}{2}, \qquad n \text{ is odd}$$

and

$$A_o = 0, B_n = 0$$

and the corresponding harmonics i.e. A_n values are shown in Fig.3.1b for the value of $\delta = 120^o$. This single pulse modulation

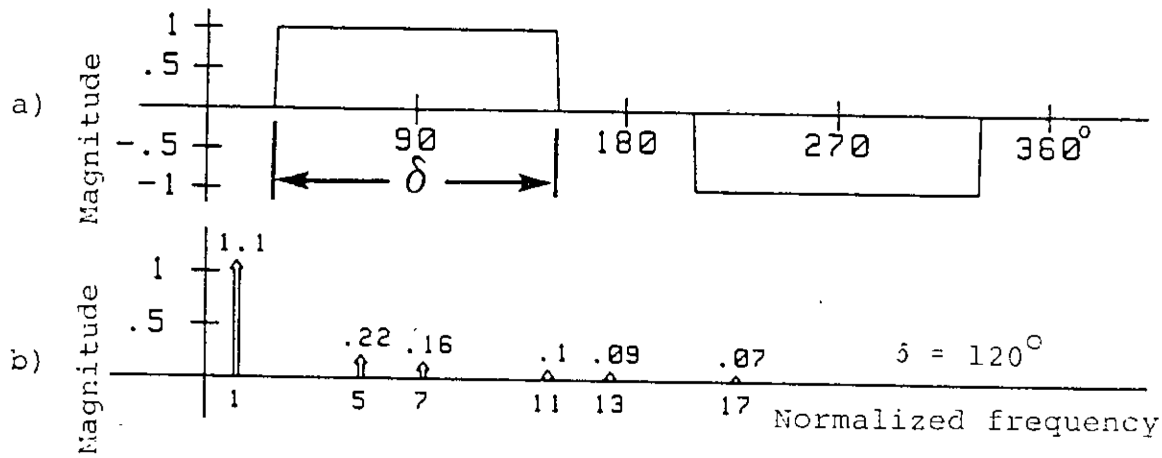


Fig. 3.1: Single pulse modulation switching function.

a) The switching function.

b) Respective frequency spectrum.

technique is used in this converter analysis.

3.3 Fourier Analysis of Output Voltage and Input Current for Balanced Input

For balanced case, the switching functions are of equal widths. The Fourier coefficients of the switching functions for this case are equal. In an actual converter (Fig.3.2),the switches operate in ON-OFF mode, therefore, the practical switching function $[F_d(w_s t)]$ cointain harmonics.

3.3.1 Output Voltage

The practical expression for output voltage under balanced case becomes

$$[V_o(w_o t)]=[F_d(w_s t)][V_i(w_i t)]$$

$$\text{where } F_d(w_s t) = \begin{bmatrix} F_1 & F_2 & F_3 \\ F_3 & F_1 & F_2 \\ F_2 & F_3 & F_1 \end{bmatrix}$$

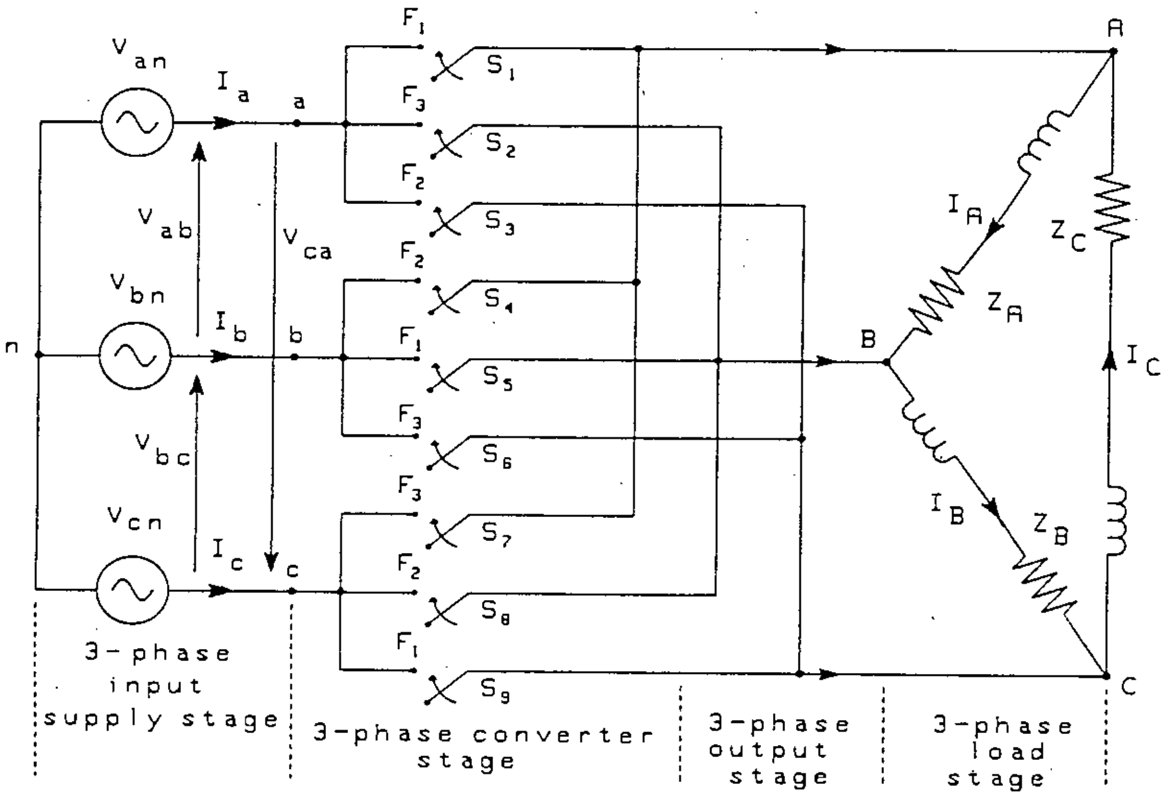


Fig. 3.2: Simplified circuit diagram of the proposed 3 phase to 3 phase static converter.

$$= \begin{bmatrix} \sum\limits_{n=1,3,5}^{\alpha} A_n \cos(n w_s t) & \sum\limits_{n=1,3,5}^{\alpha} A_n \cos n(w_s t-120^{o}) & \sum\limits_{n=1,3,5}^{\alpha} A_n \cos n(w_s t-240^{o}) \\ \sum\limits_{n=1,3,5}^{\alpha} A_n \cos n(w_s t-240^{o}) & \sum\limits_{n=1,3,5}^{\alpha} A_n \cos(n w_s t) & \sum\limits_{n=1,3,5}^{\alpha} A_n \cos n(w_s t-120^{o}) \\ \sum\limits_{n=1,3,5}^{\alpha} A_n \cos n(w_s t-120^{o}) & \sum\limits_{n=1,3,5}^{\alpha} A_n \cos n(w_s t-240^{o}) & \sum\limits_{n=1,3,5}^{\alpha} A_n \cos(n w_s t) \end{bmatrix} \quad (3.2)$$

and the analytical expression for output voltage (Figs.3.3-3.5) for 3 phase to 3 phase converter becomes

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} F_1 & F_2 & F_3 \\ F_3 & F_1 & F_2 \\ F_2 & F_3 & F_1 \end{bmatrix} \cdot \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$= \frac{3A_1V_i}{2} \begin{bmatrix} \cos(w_o t) \\ \cos(w_o t-240^{o}) \\ \cos(w_o t-120^{o})) \end{bmatrix} + [S_{dh}(w_s t)].V_i \begin{bmatrix} \cos(w_i t) \\ \cos(w_i t-120^{o}) \\ \cos(w_i t-240^{o}) \end{bmatrix} \quad (3.3)$$

where

$$[S_{dh}(w_st)]=\begin{bmatrix} \sum_{n=3,5}^{\alpha} A_n\cos(nw_st) & \sum_{n=3,5}^{\alpha} A_n\cos n(w_st-120^o) & \sum_{n=3,5}^{\alpha} A_n\cos n(w_st-240^o) \\ \sum_{n=3,5}^{\alpha} A_n\cos n(w_st-240^o) & \sum_{n=3,5}^{\alpha} A_n\cos(nw_st) & \sum_{n=3,5}^{\alpha} A_n\cos n(w_st-120^o) \\ \sum_{n=3,5}^{\alpha} A_n\cos n(w_st-120^o) & \sum_{n=3,5}^{\alpha} A_n\cos n(w_st-240^o) & \sum_{n=3,5}^{\alpha} A_n\cos(nw_st) \end{bmatrix} \quad (3.4)$$

Comparing equation(3.3) with eqn.(2.6a,b,c) of chapter 2 we can see that there is difference between practical and ideal conveter output voltages. In practical case output voltages contains an infinite series of harmonics.

3.3.2 Input Current

The input current (Figs.3.6-3.8) spectrum of the 3 phase to 3 phase converter can be calculated in the same manner as the output voltage spectrum computed in subsection 3.3.1. The equation for input current of a 3-phase to 3-phase converter for balanced case can be expressed as follows:

$$[I_i(w_it)]=[F_d(w_st)]^T[I_o(w_ot)]$$

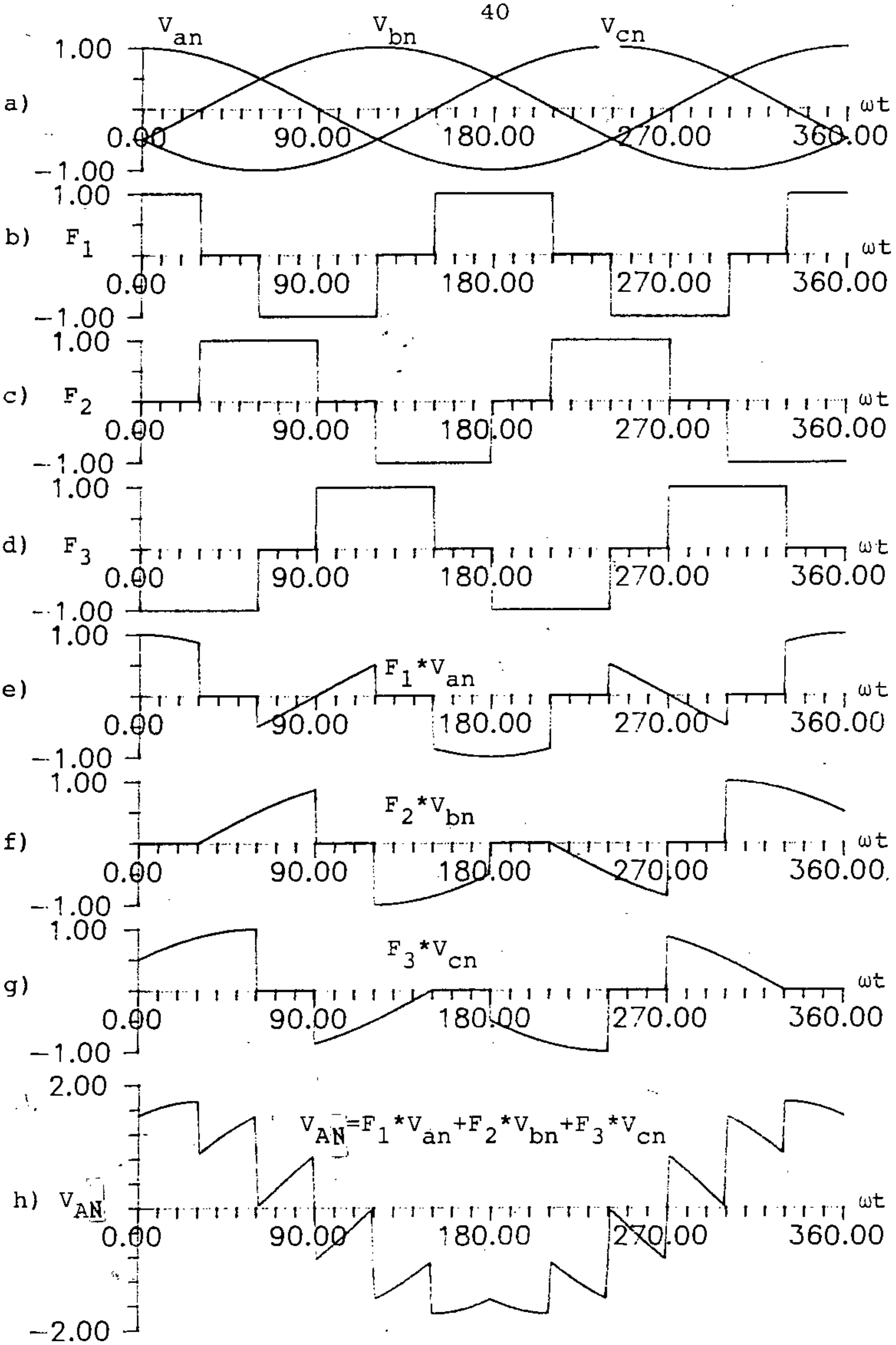


Fig. 3.3: Output voltage waveform, V_{AN} obtained with 3 phase to 3 phase converter under balanced condition.

a) Three input balanced phase voltages.

b) - d) F_1, F_2, F_3 switching function components.

h) Output voltage, V_{AN}.

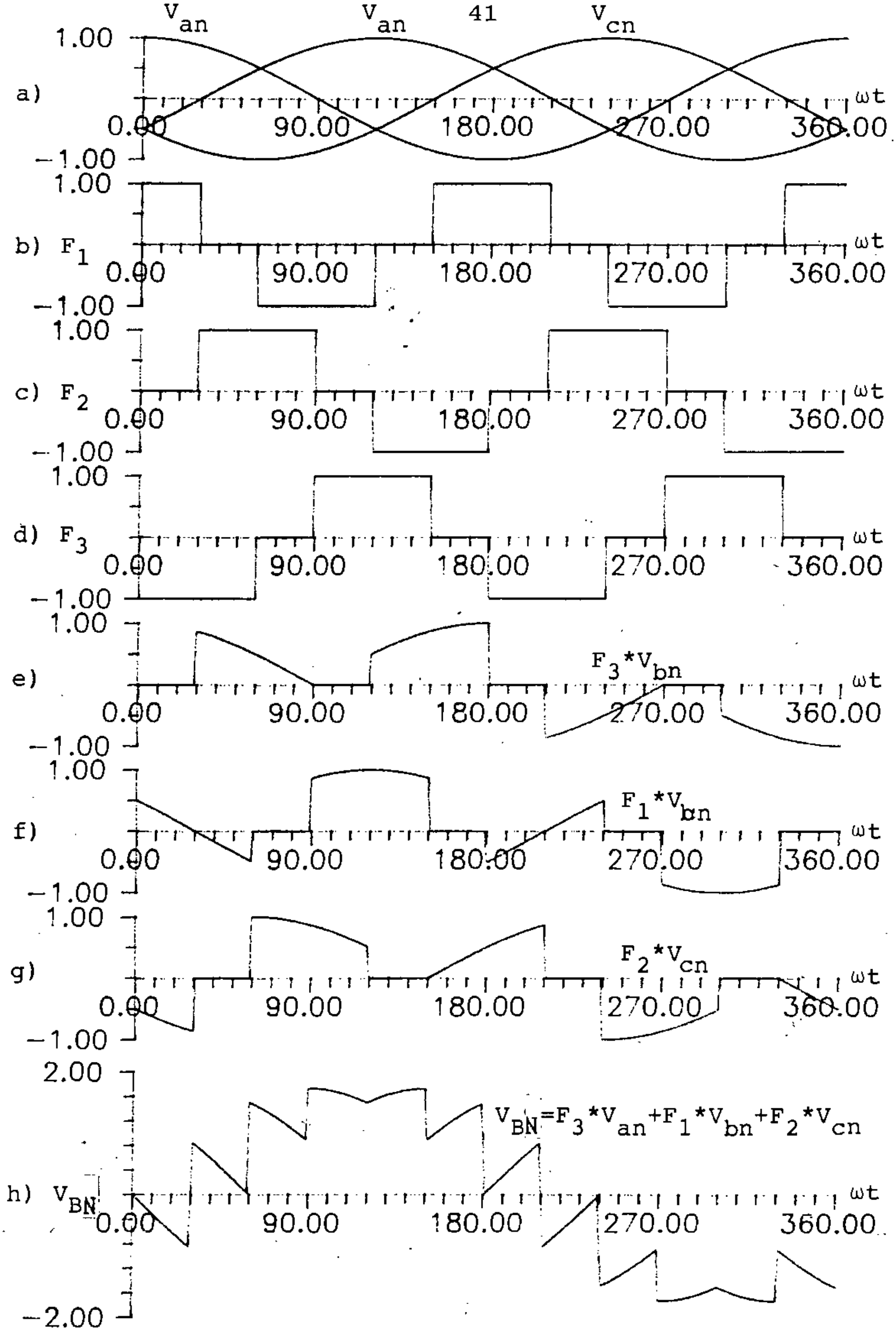
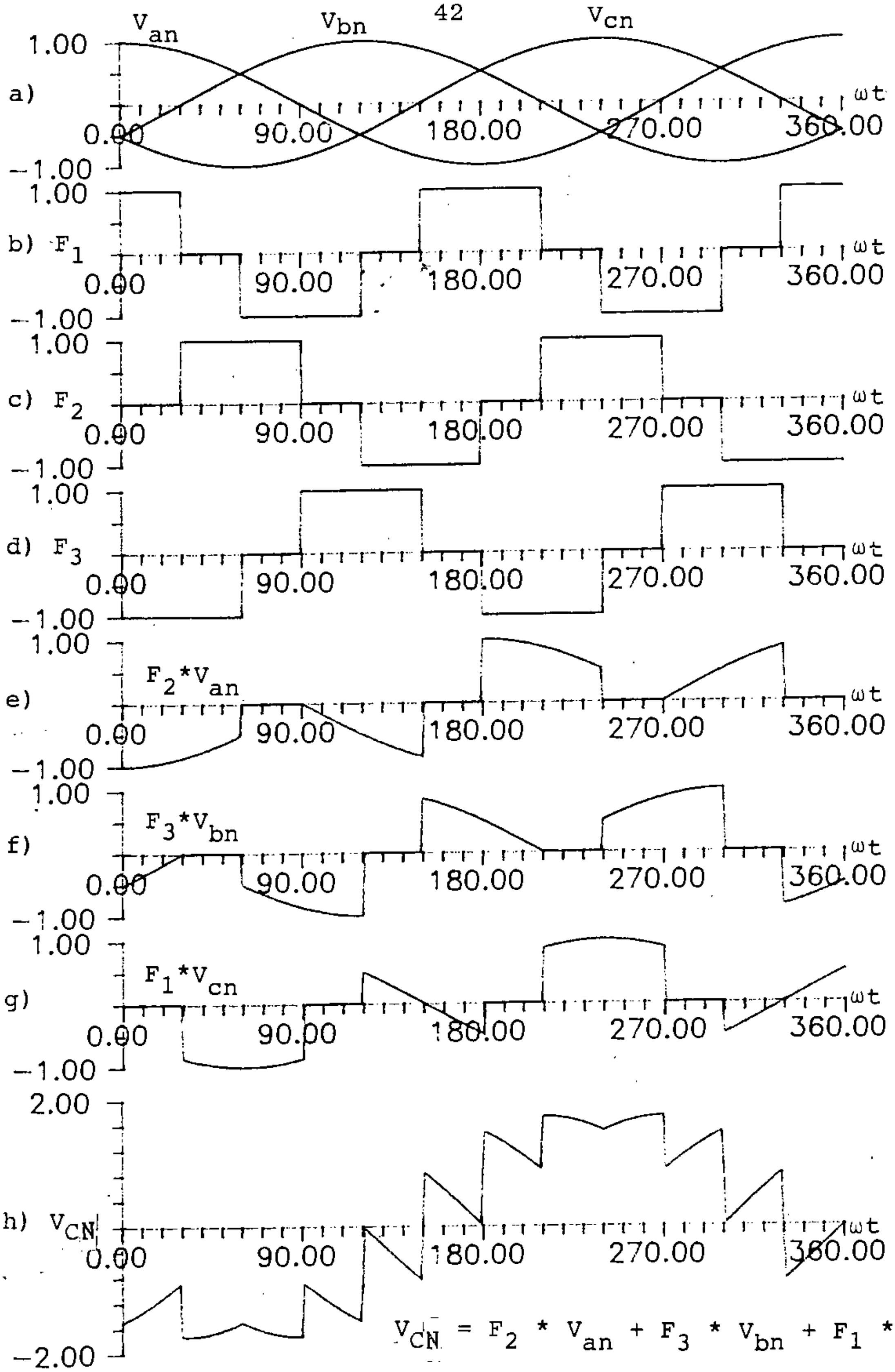


Fig. 3.4: Output voltage waveform, V_{BN} obtained with 3 phase to 3 phase converter under balanced condition.

a) Three input balanced phase voltages.

b) - d) F_1, F_2, F_3 switching function components.

h) Output voltage, V_{BN}.



$V_{CN} = F_2 * V_{an} + F_3 * V_{bn} + F_1 * V_{cn}$

Fig. 3.5: Output voltage wave form, V_{CN} obtained with 3 phase to 3 phase converter under balanced condition.

a) Three input balanced phase voltages.

b) - d) $F_1, F_2 F_3$ switching function components.

h) Output voltage, V_{CN}.

Table 3.1

Frequency spectra of waveforms associated with converter output voltages V_{AN}, V_{BN} and V_{CN} shown in Figs. 3.3-3.5

Harmonic coefficients of switching function Fig.(3.3 b-d)		Harmonic coefficients of output voltages V_{AN}, V_{BN} and V_{CN} Fig.(3.3h-3.5h)		
Order (n)	Amplitude (A_n)		Amplitude V_{AN}, V_{BN} and V_{CN}	
		Order (n)	p.u.[(1)]	%[(1)]
1	1.10	1	0.87	87
3	0.00	3	-	-
5	0.22	5	-	-
7	0.16	7	-	-
9	0.00	9	-	-
11	0.10	11	0.20	20
13	0.09	13	0.14	14
15	0.00	15	-	-
17	0.07	17	-	-
19	0.06	19	-	-
21	0.00	21	-	-
23	0.05	23	0.08	8

(1) Input phase voltage have been taken as 1 p.u. volt & 100% volt.

$$= \begin{bmatrix} F_1 & F_3 & F_2 \\ F_2 & F_1 & F_3 \\ F_3 & F_2 & F_1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

or

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{3A_1 I_o}{2} \begin{bmatrix} \cos(w_i t) \\ \cos(w_i t - 120^o) \\ \cos(w_i t - 240^o) \end{bmatrix} + [S_{dh}(w_s t)]^T \cdot I_o \begin{bmatrix} \cos(w_o t) \\ \cos(w_o t - 240^o) \\ \cos(w_o t - 120^o) \end{bmatrix} \quad (3.5)$$

where

$$[S_{dh}(w_s t)] = \begin{bmatrix} \sum_{n=3,5}^{\alpha} A_n \cos(n w_s t) & \sum_{n=3,5}^{\alpha} A_n \cos n(w_s t - 120^o) & \sum_{n=3,5}^{\alpha} A_n \cos(n(w_s t - 240^o) \\ \sum_{n=3,5}^{\alpha} A_n \cos n(w_s t - 240^o) & \sum_{n=3,5}^{\alpha} A_n \cos(n w_s t) & \sum_{n=3,5}^{\alpha} A_n \cos n(w_s t - 120^o) \\ \sum_{n=3,5}^{\alpha} A_n \cos n(w_s t - 120^o) & \sum_{n=3,5}^{\alpha} A_n \cos n(w_s t - 240^o) & \sum_{n=3,5}^{\alpha} A_n \cos(n w_s t) \end{bmatrix}$$

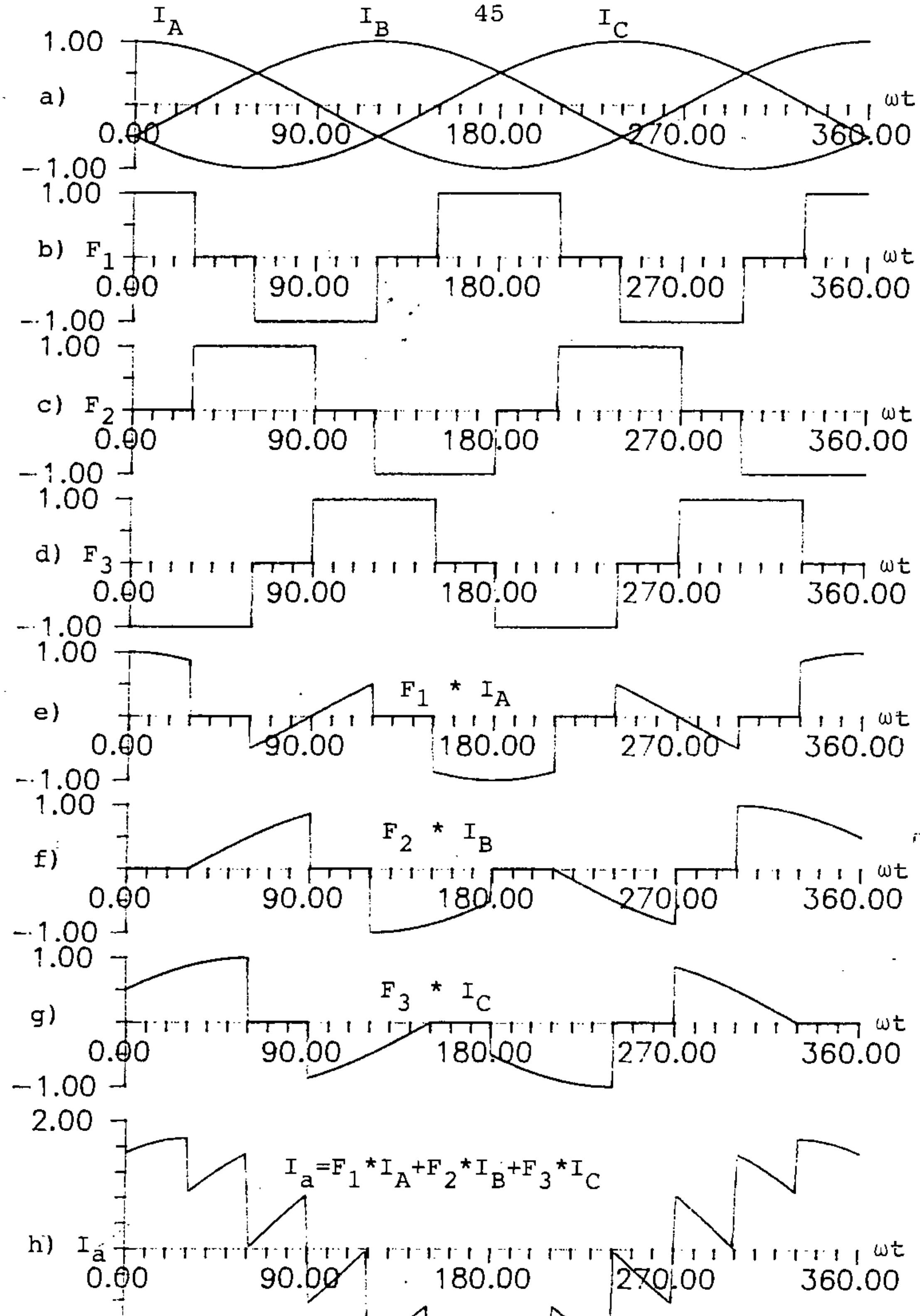
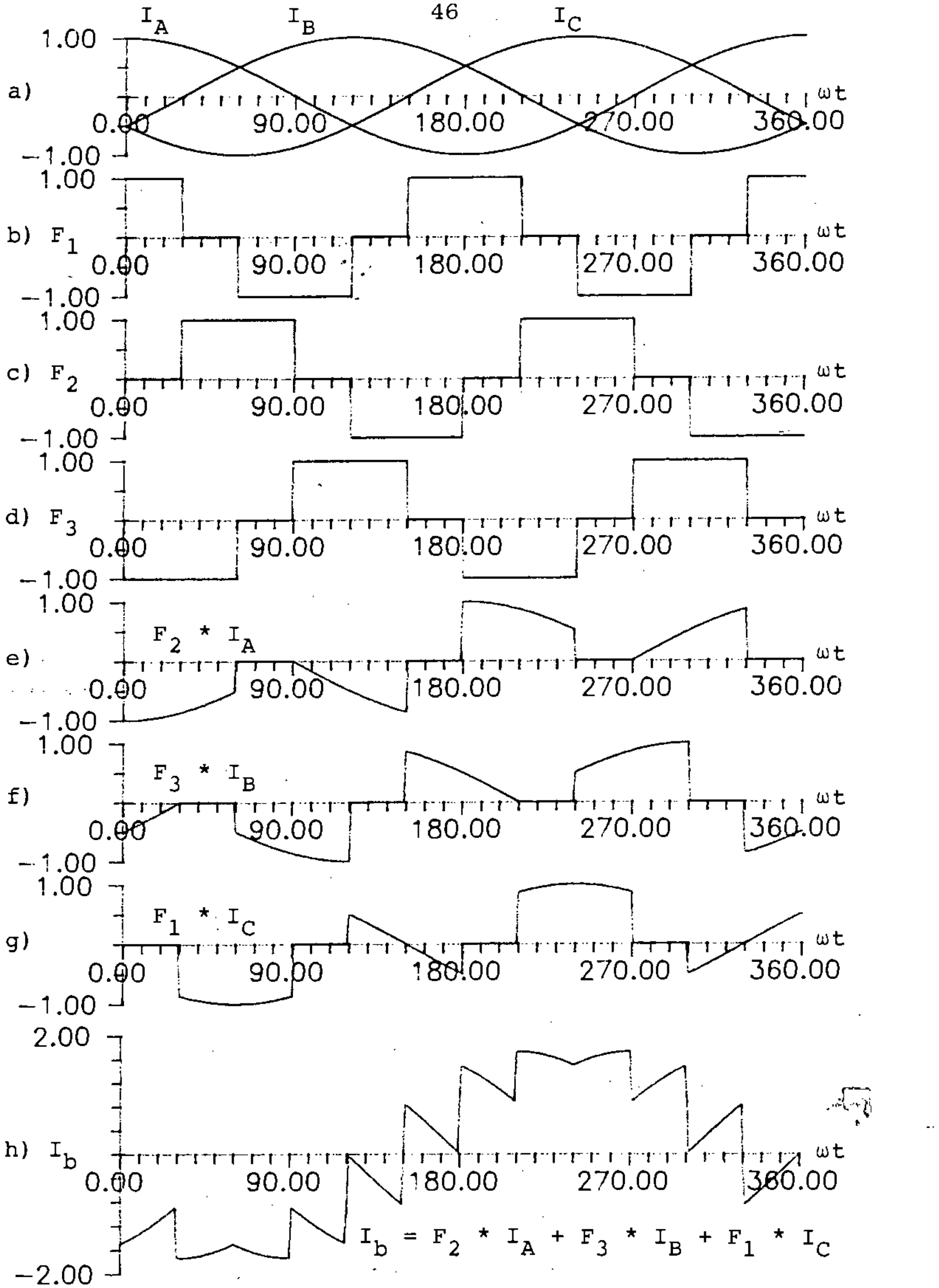


Fig. 3.6: Input current waveform, I_a obtained with 3 phase to 3 phase converter under balanced condition.

a) Three output balanced phase currents.

b) - d) F_1, F_2, F_3 switching function components.

h) Input current, I_a.



Fig, 3.7: Input current waveform, I_b obtained with 3 phase to 3 phase converter under balanced condition.

a) Three output balanced phase currents.

b) - d) F_1, F_2, F_3 switching function components.

h) Input current I_b.

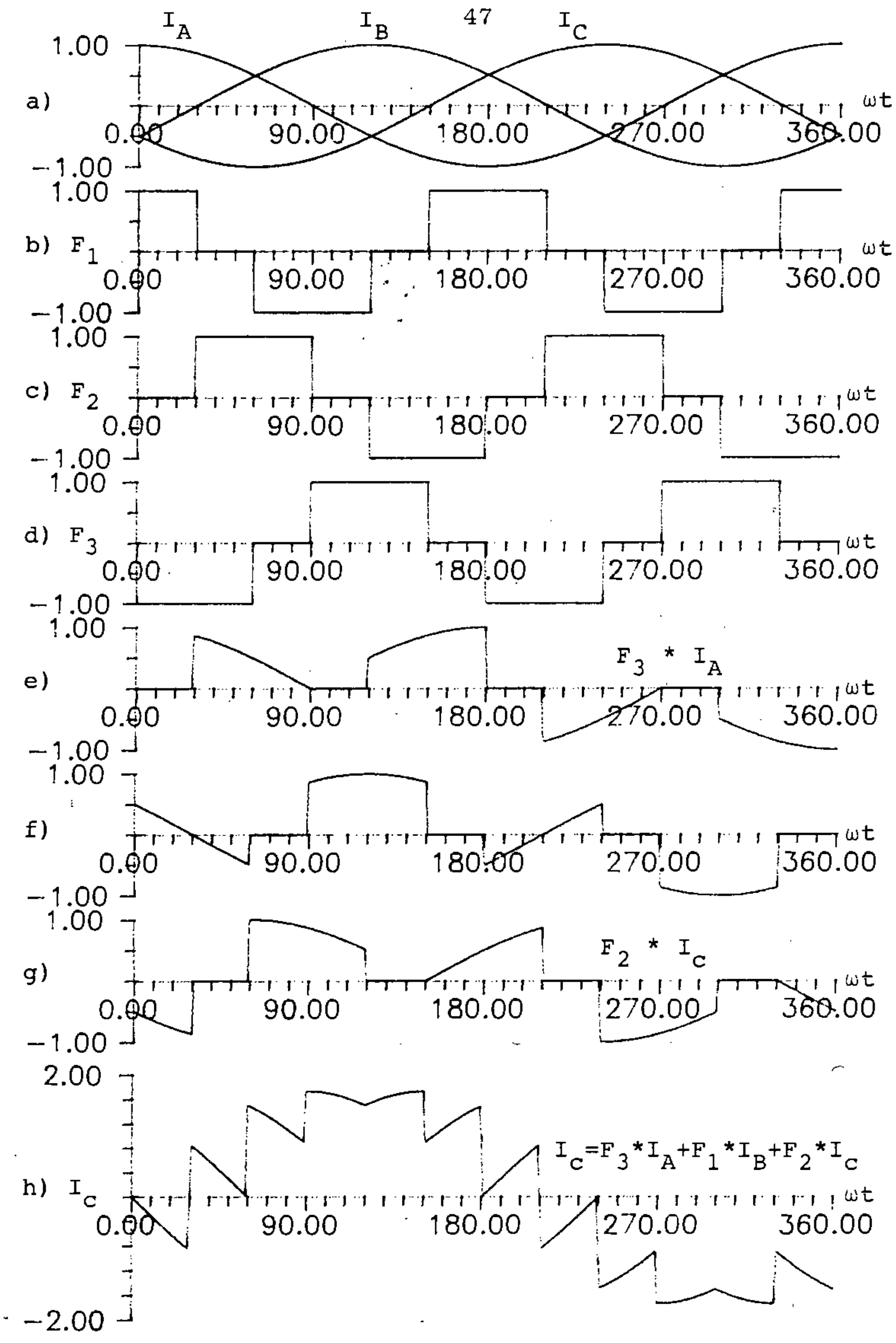


Fig. 3.8: Input current I_c, waveform obtained with 3 phase to 3 phase converter under balanced condition.

a) Three output balanced phase currents.

b) - d) F_1, F_2, F_3 switching function components.

h) Input current, I_c.

Table 3.2

Frequency spectra of waveforms associated with converter

Input currents I_a, I_b and I_c shown in Figs.(3.6-3.8)

Harmonic coefficients of switching function Fig.(3.6 b-d)		Harmonic coefficients of resulting input phase currents I_a, I_b and I_c (Fig.3.6h-3.8h)		
		Amplitude I_a, I_b and I_c		
Order (n)	Amplitude (A_n	Order (n)	p.u.[1]	%[1]
1	1.10	1	0.87	87
3	0.00	3	-	-
5	0.22	5	-	-
7	0.16	7	-	-
9	0.00	9	-	-
11	0.10	11	0.20	20
13	0.09	13	0.14	14
15	0.00	15	-	-
17	0.07	17	-	-
19	0.06	19	-	-
21	0.00	21	-	-
23	0.05	23	0.08	8

(1) Output phase current has been taken as 1 p.u. volt & 100% volt.

3.4 Fourier Analysis of Output Voltage and Input Current for Unbalanced Input

The switching functions are of unequal widths (Fig 3.9) for an unbalanced case. The widths are varied according to input unbalance.As practical power converters operate in ON/OFF mode employing switches rather than in continuous mode, they posses frequency spectra having infinite series of harmonic components. Harmonics associated with output voltages and input currents are discussed in details in the following sub-sections:

3.4.1 Output voltage

Similar to balanced case in subsection 3.3.1 the practical expression for output voltages under amplitude unbalanced input condition becomes:

$$[v_o(w_o t)]=[F_d(w_s t)].[v_i(w_i t)]$$

where

$$[F_d(w_s t)] = \begin{bmatrix} F_1 & F_2 & F_3 \\ F_3 & F_1 & F_2 \\ F_2 & F_3 & F_1 \end{bmatrix}$$

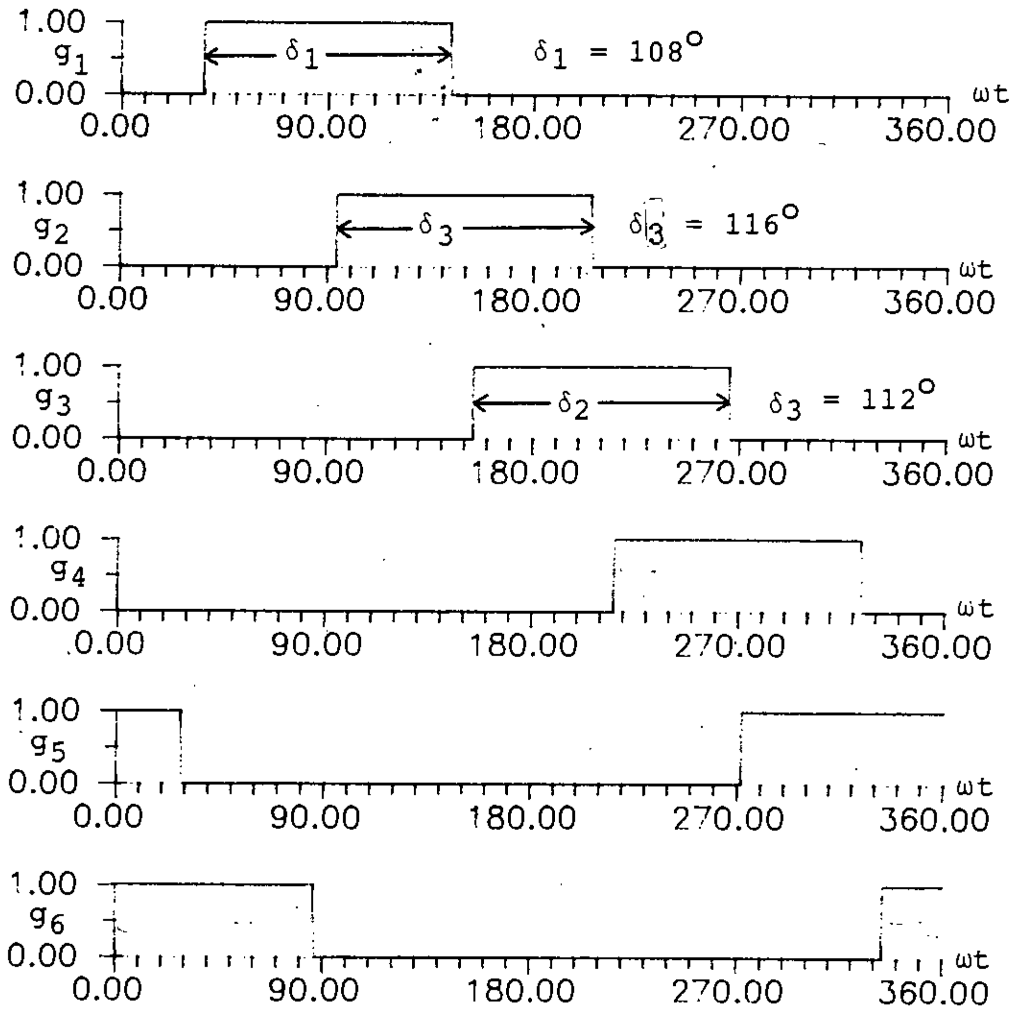


Fig. 3.9: Gating signals for the example.

$$= \begin{bmatrix} \sum_{n=1,3,5}^{\alpha} A_n \cos(nw_s t) & \sum_{n=1,3,5}^{\alpha} B_n \cos n(w_s t-120^o) & \sum_{n=1,3,5}^{\alpha} C_n \cos n(w_s t-240^o) \\ \sum_{n=1,3,5}^{\alpha} C_n \cos n(w_s t-240^o) & \sum_{n=1,3,5}^{\alpha} A_n \cos(nw_s t) & \sum_{n=1,3,5}^{\alpha} B_n \cos n(w_s t-120^o) \\ \sum_{n=1,3,5}^{\alpha} B_n \cos n(w_s t-120^o) & \sum_{n=1,3,5}^{\alpha} C_n \cos n(w_s t-240^o) & \sum_{n=1,3,5}^{\alpha} A_n \cos(nw_s t) \end{bmatrix} \quad (3.6)$$

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} \sum_{n=1,3,5}^{\alpha} A_n \cos(nw_s t) & \sum_{n=1,3,5}^{\alpha} B_n \cos n(w_s t-120^o) & \sum_{n=1,3,5}^{\alpha} C_n \cos n(w_s t-240^o) \\ \sum_{n=1,3,5}^{\alpha} C_n \cos n(w_s t-240^o)) & \sum_{n=1,3,5}^{\alpha} A_n \cos(nw_s t) & \sum_{n=1,3,5}^{\alpha} B_n \cos n(w_s t-120^o) \\ \sum_{n=1,3,5}^{\alpha} B_n \cos n(w_s t-120^o) & \sum_{n=1,3,5}^{\alpha} C_n \cos n(w_s t-240^o) & \sum_{n=1,3,5}^{\alpha} A_n \cos(nw_s t) \end{bmatrix}$$

$$. \begin{bmatrix} A \cos(w_i t) \\ B \cos(w_i t-120^o) \\ C \cos(w_i t-240^o) \end{bmatrix} \quad (3.7)$$

$$V_{AN} = \sum_{n=1,3,5}^{\alpha} AA_n \cos(nw_s t).\cos(w_i t) + \sum_{n=1,3,5}^{\alpha} BB_n \cos n(w_s t-120^o).\cos(w_i t-120^o)$$

$$+\sum_{n=1,3,5}^{\alpha} CC_n \cos n(w_s t-240^o).\cos(w_i t-240^o)$$

$$= \frac{1}{2}(AA_1+BB_1+CC_1)\cos(w_s-w_i)t+\frac{1}{2}\sum_{n=3,5}^{\alpha} AA_n \cos(nw_s+w_i)t+\cos(nw_s-w_i)t]$$

$$+\frac{1}{2}\sum_{n=3,5}^{\alpha} BB_n[\cos\{(nw_s+w_i)t-n120^o-120^o\}+\cos\{(nw_s-w_i)t-n120^o+120^o\}]$$

$$+\frac{1}{2}\sum_{n=3,5}^{\alpha} CC_n[\cos\{(nw_s+w_i)t-n240^o-240^o\}+\cos\{(nw_s-w_i)t-n240^o+240^o\}] \quad (3.8)$$

$$V_{BN}=\sum_{n=1,3,5}^{\alpha} AC_n \cos n(w_s t-240^o).\cos(w_i t)+\sum_{n=1,3,5}^{\alpha} BA_n \cos n(w_s t-120^o).\cos(w_i t-120^o)$$

$$+\sum_{n=1,3,5}^{\alpha} CB_n \cos n(w_s t-120^o).\cos(w_i t-240^o)$$

$$= \frac{1}{2}(AC_1+BA_1+CB_1)\cos\{(w_s-w_i)t-240\}+ \frac{1}{2}[AC \cos\{3(w_s+w_i)t-240^o\}$$

$$+BA_1 \cos\{3(w_s+w_i)t-120^o\}+CB_1 \cos\{3(w_s+w_i)t-360^o\}]$$

$$+\frac{1}{2}\sum_{n=3,5}^{\alpha} AC_n[\cos\{(nw_s+w_i)t-n240^o\}+\cos\{(nw_s-w_i)t-n240^o\}]$$

$$+\frac{1}{2}\sum_{n=3,5}^{\alpha} BA_n[\cos\{(nw_s+w_i)t-120^o\}+\cos\{(nw_s-w_i)t-120^o\}]$$

$$+\frac{1}{2}\sum_{n=3,5}^{\alpha} CB_n[\cos\{(nw_s+w_i)t-n120^o-240^o\}+\cos\{(nw_s-w_i)t-n120^o+240^o\}] \quad (3.9)$$

$$V_{CN}=\sum_{n=1,3,5}^{\alpha} AB_n \cos n(w_st-120^o).\cos(w_it)+ \sum_{n=1,3,5}^{\alpha} BC_n \cos n(w_st-240^o).\cos(w_it-120^o)$$

$$\sum_{n=1,3,5}^{\alpha} CA_n \cos n(w_st).\cos(w_it-240^o)$$

$$= \frac{1}{2}(AB_1+BC_1+CA_1)\cos\{(w_s-w_i)t-120^o\}+\frac{1}{2} [AC_1 \cos\{3(w_s+w_i)t-120^o\}$$

$$+BC_1 \cos\{3(w_s+w_i)t-360^o\}+CA_1 \cos\{3(w_s+w_i)t-240^o\}]$$

$$+\frac{1}{2}\sum_{n=3,5}^{\alpha} AB_n[\cos\{(nw_s+w_i)t-n120^o\}+\cos\{(nw_s-w_i)t-n120^o\}]$$

$$+\frac{1}{2}\sum_{n=3,5}^{\alpha} BC_n[\cos\{(nw_s+w_i)t-n240^o-120^o\}+\cos\{(nw_s-w_i)t-n240^o-120^o\}]$$

$$+\frac{1}{2}\sum_{n=3,5}^{\alpha} CA_n[\cos\{(nw_s+w_i)t-240^o\}+\cos\{(nw_s-w_i)t+240^o\}] \quad (3.10)$$

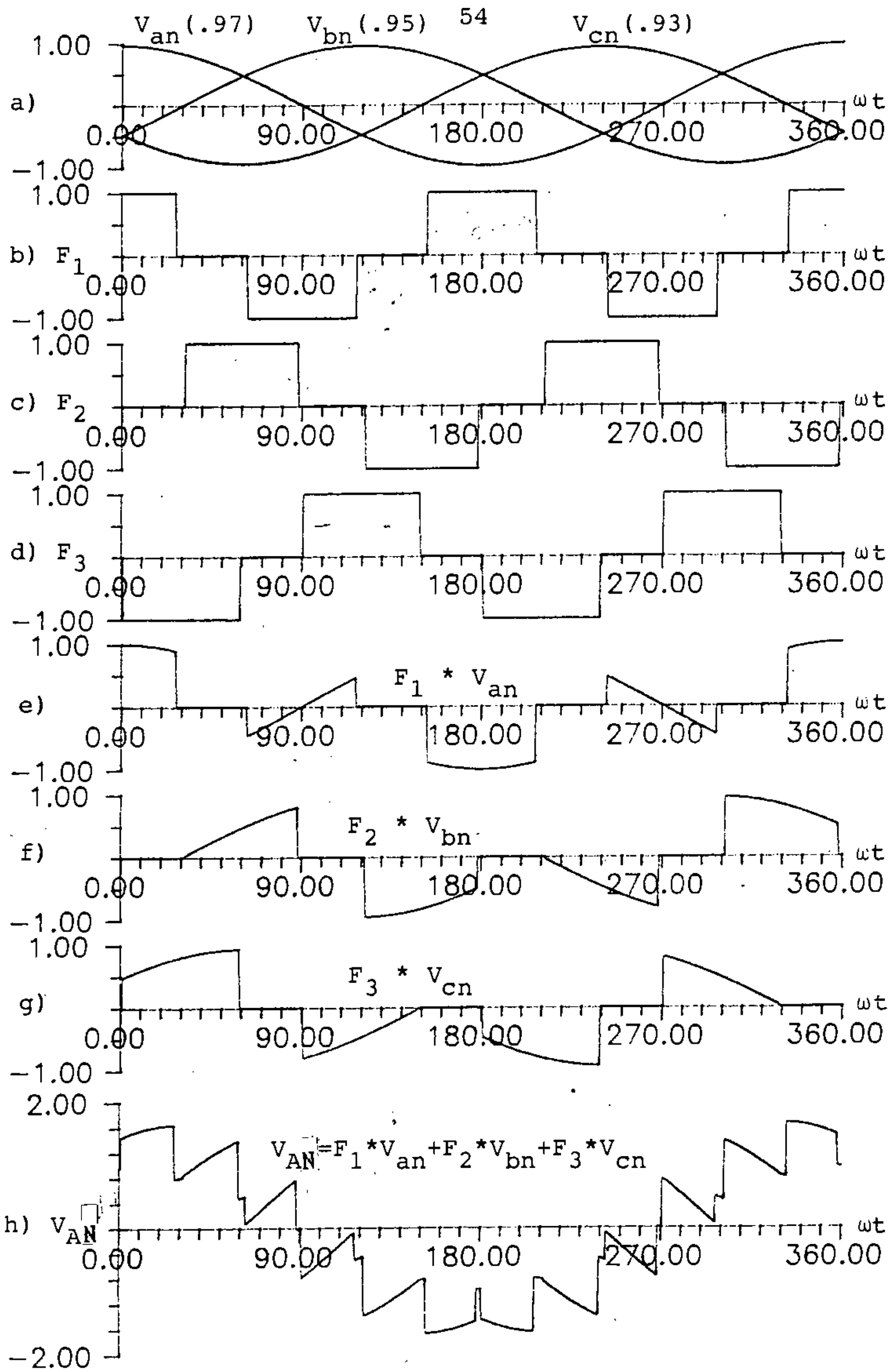


Fig. 3.10: Output voltage waveform, V_{AN} obtained with 3 phase to 3 phase converter under unbalanced condition.

a) Three input unbalanced phase voltages.

b) - d) F_1, F_2, F_3 switching function components.

h) Output voltage, V_{AN}.

Table 3.3							
Frequency spectra of waveforms associated with converter Output voltage, V_{AN} shown in Fig. 3.10							
Harmonic coefficients of switching function Fig.(3.10 b-d)				Harmonic coefficients of output phase voltage V_{AN} Fig. (3.10h)			
Order (n)	Amplitude			Amplitude, V_{AN}			
	A_n	B_n	C_n	Order	p.u.$^{(1)}$	%$^{(1)}$	
1	1.030	1.056	1.080	1	0.8682	66.82	
3	0.131	0.088	0.044	3	-	-	
5	-0.255	-0.25	-0.239	5	0.0185	1.85	
7	0.056	0.096	0.131	7	0.0185	1.85	
9	0.114	0.083	0.044	9	-0.0046	0.46	
11	-0.074	0.112	-0.115	11	-0.2044	20.44	
13	-0.030	0.014	0.055	13	0.0774	7.74	
15	0.085	0.074	0.042	15	-0.0150	1.50	
17	-0.023	-0.059	-0.075	17	0.0150	1.50	
19	-0.054	-0.018	0.025	19	0.0150	1.50	
21	0.049	0.060	0.041	21	0.0046	0.46	
23	0.017	-0.026	-0.053	23	-0.0878	8.78	
25	0.051	-0.033	0.009	25	0.0098	0.98	

(1) Input phase voltage have been taken as 1 p.u. volt & 100% volt.

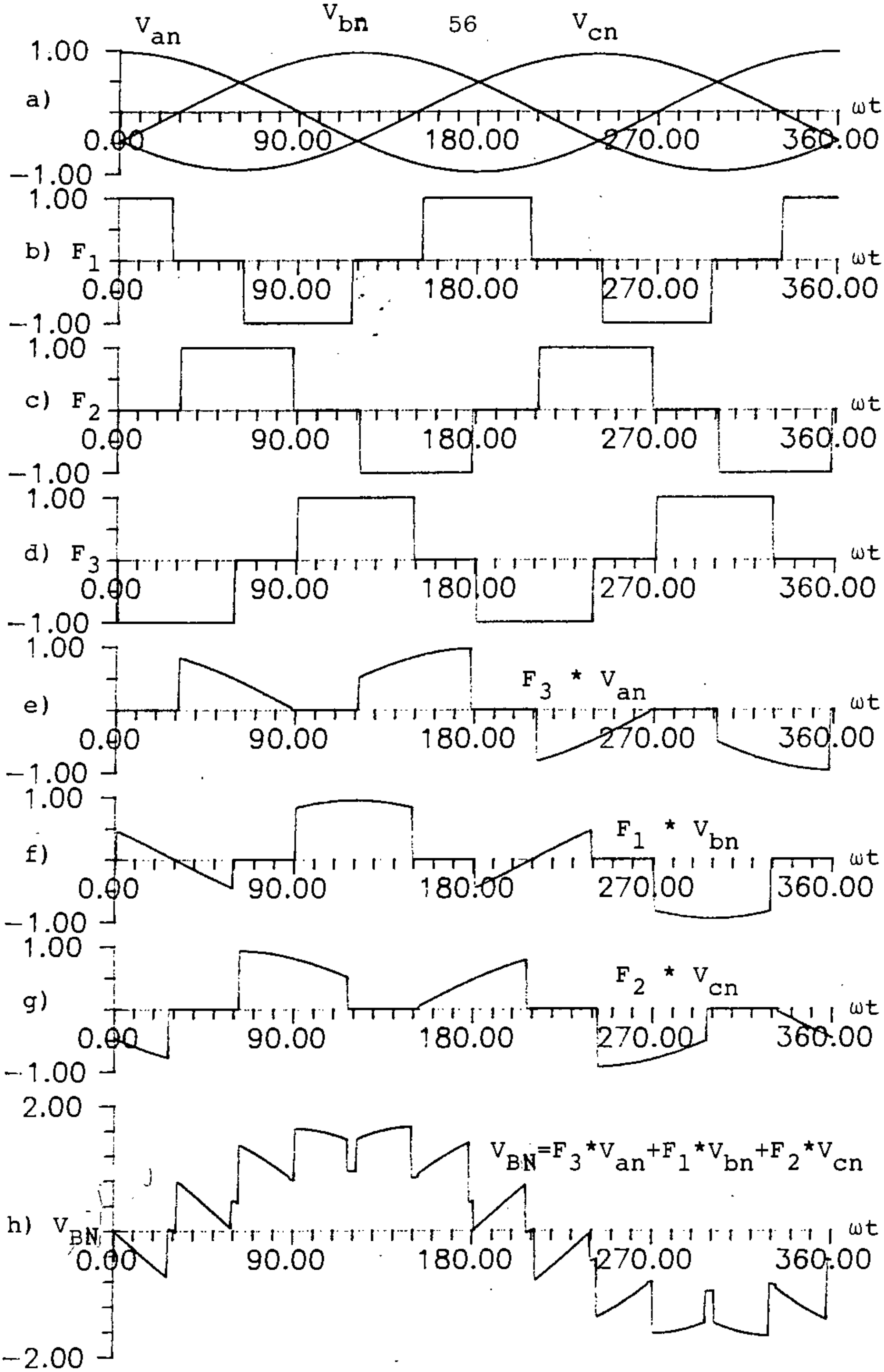


Fig. 3.11: Output voltage waveform, V_{BN} obtained with 3 phase to 3 phase converter under unbalanced condition.

a) Three input unbalanced phase voltages.

b) - d) F_1, F_2, F_3 switching function components.

h) Output voltage, V_{BN}.

Table 3.4						
Frequency spectra of waveforms associated with converter Output voltage, V_{BN} shown in Fig.3.11						
Harmonic coefficients of switching function Fig.(3.11b-d)				Harmonic coefficients of output phase voltage V_{BN} Fig. (3.11h)		
Order (n)	Amplitude			Amplitude, V_{BN}		
	A_n	B_n	C_n	Order (n)	p.u.(1)	%(1)
1	1.030	1.056	1.080	1	-0.8684	86.84
3	0.131	0.088	0.044	3	-0.0185	1.85
5	-0.255	-0.251	-0.239	5	-0.0346	3.46
7	0.056	0.096	0.131	7	-0.0346	3.46
9	0.114	0.083	0.044	9	0.0023	0.23
11	-0.094	-0.112	-0.115	11	0.2044	20.44
13	-0.030	0.014	0.055	13	-0.0774	7.74
15	-0.085	0.074	0.042	15	0.0000	0.00
17	-0.023	-0.059	-0.075	17	0.0289	2.89
19	-0.054	-0.018	0.025	19	0.0289	2.89
21	0.051	-0.060	0.041	21	0.0023	0.23
23	0.017	-0.026	-0.053	23	0.0878	8.78
25	0.051	-0.033	0.009	25	-0.0104	01.04

(1) Input phase voltage have been taken as 1 p.u. volt & 100% volt.

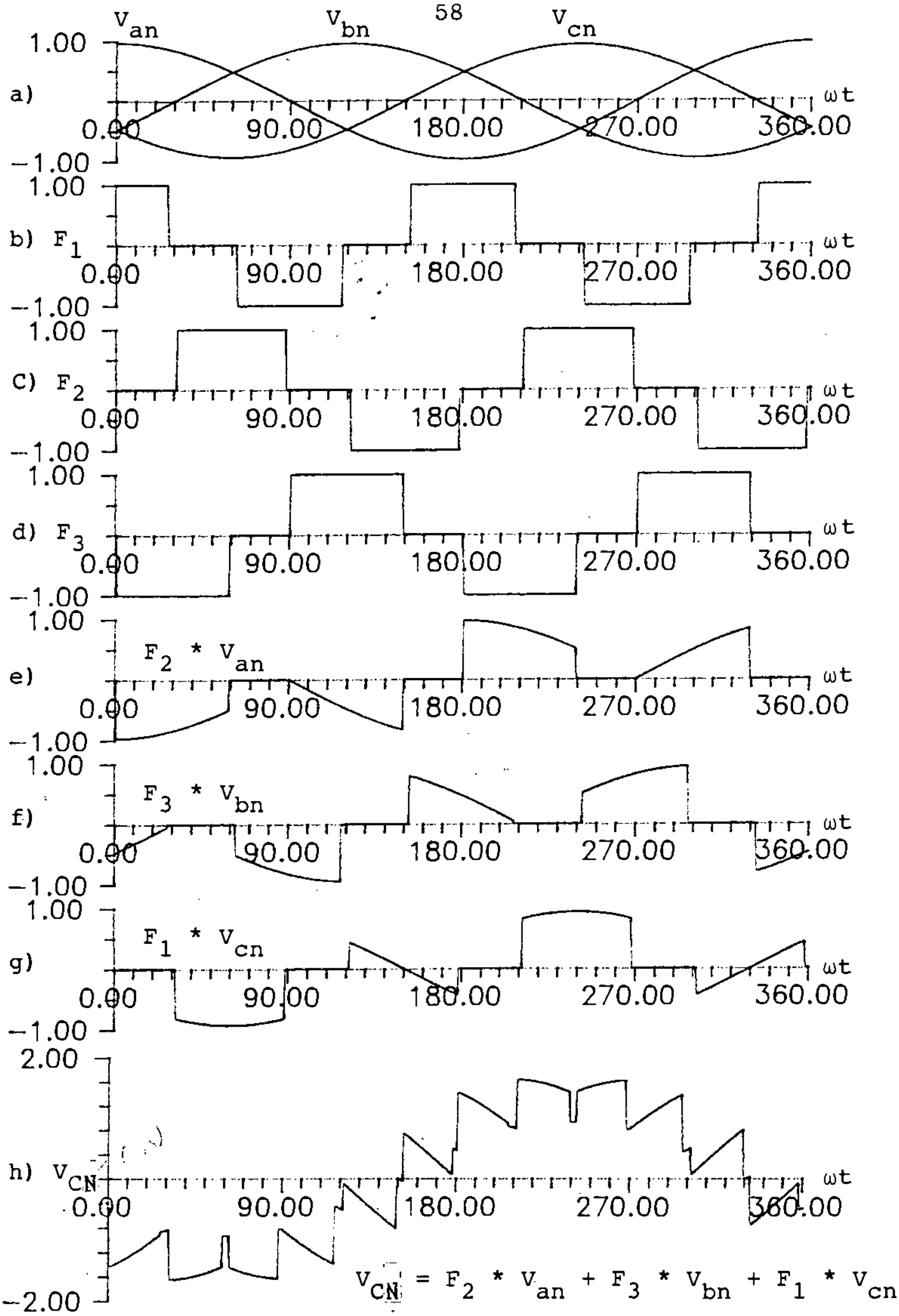


Fig. 3.12: Output voltage waveform, V_{CN} obtained with 3 phase to 3 phase converter under unbalanced condition.

a) Three input unbalanced phase voltages.

b) - d) F_1, F_2, F_3 switching function components.

h) Output voltage, V_{CN}.

Table 3.5						
Frequency spectra of waveforms associated with converter Output voltage, V'_{CN} shown in Fig. 3.12						
Harmonic coefficients of switching function Fig.(3.12b-d)				Harmonic coefficients of output phase voltage V'_{CN} Fig. (3.12 h)		
Order (n)	Amplitude			Amplitude, V'_{CN}		
	A_n	B_n	C_n	Order	p.u.[(1)]	%[(1)]
1	1.030	1.056	1.080	1	-0.8682	86.82
3	0.131	0.088	0.044	3	0.0201	2.01
5	-0.255	-0.251	-0.239	5	-0.0020	0.20
7	0.056	0.096	0.131	7	0.0020	0.20
9	0.114	0.083	0.044	9	0.0073	0.73
11	-0.094	-0.112	-0.115	11	0.2042	20.42
13	-0.030	0.014	0.055	13	-0.0779	07.79
15	0.085	0.074	0.042	15	0.0297	02.97
17	-0.023	-0.059	-0.075	17	0.0039	0.39
19	-0.054	-0.018	0.025	19	0.0039	0.39
21	0.049	0.060	0.041	21	-0.0063	0.63
23	-0.017	-0.026	-0.053	23	0.0680	6.80
25	0.051	-0.033	0.009	25	-0.0107	01.07

(1) Input phase voltage have been taken as 1 p.u volt and 100% volt.

Equations (3.8)-(3.10) show that output voltages contain fundamental components of amplitudes in the order of 1.5 and harmonic components whose frequency is determined by $(2n\pm1)w_o$ term.

3.4.2 Input Current

The input current equation for 3-phase to 3-phase converter is expressed as follows:

$$[I_i(w_i t)] = [F_d(w_s t)]^T . [I_o(w_o t)]$$

or

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} F_1 & F_3 & F_2 \\ F_2 & F_1 & F_3 \\ F_3 & F_2 & F_1 \end{bmatrix} . \begin{bmatrix} A\ \cos(w_o t) \\ B\ \cos(w_o t-240^o) \\ C\ \cos(w_o t-120^o)) \end{bmatrix} \quad (3.11)$$

where

$$[F_d(w_s t)]^T = \begin{bmatrix} F_1 & F_3 & F_2 \\ F_2 & F_1 & F_3 \\ F_3 & F_2 & F_1 \end{bmatrix}$$

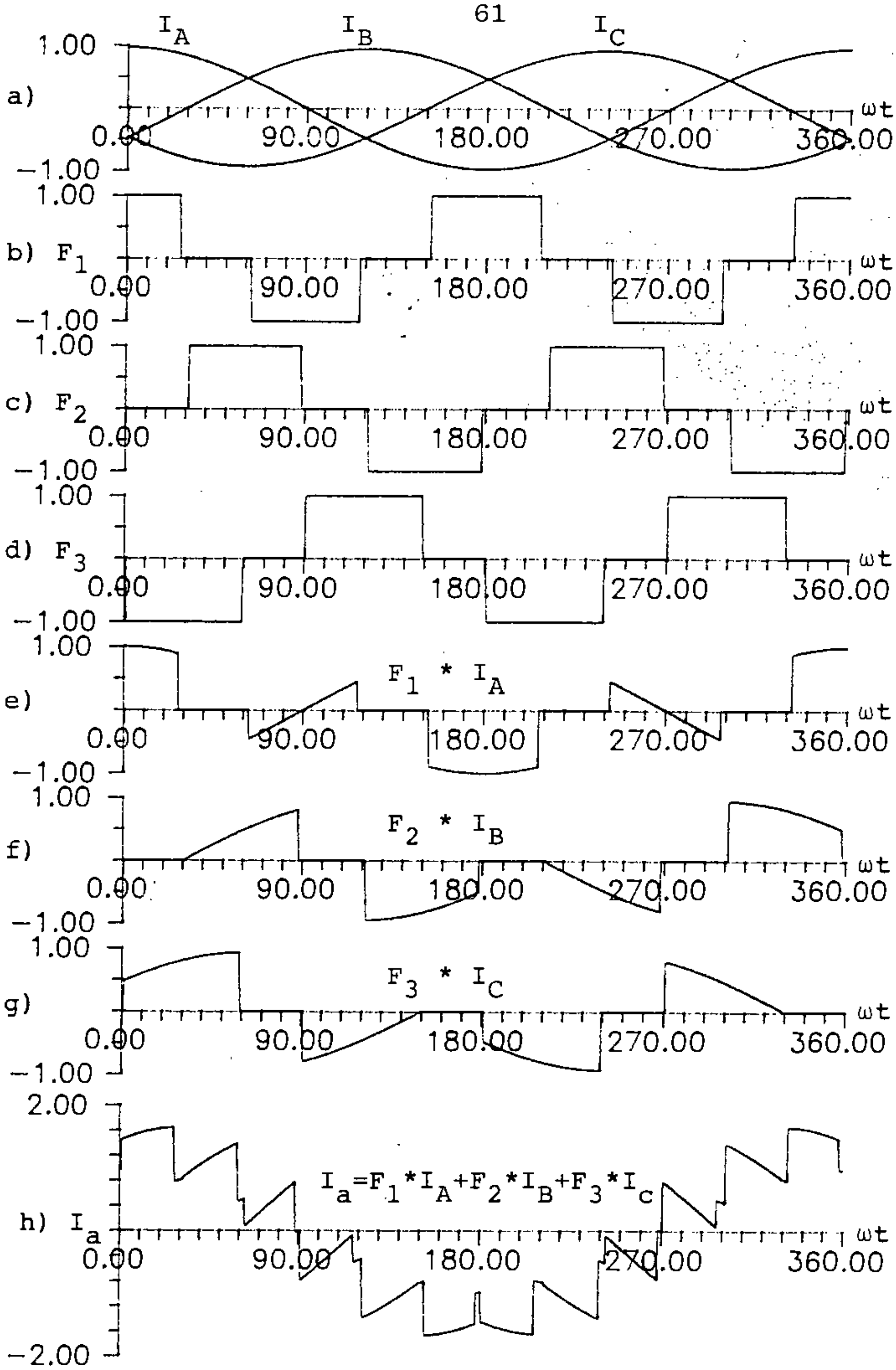


Fig. 3.13: Input current waveform, I_a obtained with 3 phase to 3 phase converters under unbalanced condition.

a) Three output unbalanced phase currents.

b) - d) F_1, F_2, F_3 switching function components.

h) Input current, I_a.

e-g) !

Table 3.6							
Frequency spectra of waveforms associated with converter Input current, I_a shown in Fig. 3.13							
Harmonic coefficients of switching function Fig.(3.13b-d)				Harmonic coefficients of input phase current, I_a Fig. (3.13h)			
Order (n)	Amplitude			Amplitude, I_a			
	A_n	B_n	C_n	Order (n)	p.u.$^{(1)}$	%$^{(1)}$	
1	1.030	1.056	1.080	1	0.8682	86.82	
3	0.131	0.088	0.044	3	-	-	
5	-0.255	-0.251	-0.239	5	-0.015	1.85	
7	0.056	0.096	0.131	7	0.0185	1.85	
9	0.114	0.083	0.044	9	-0.0046	0.46	
11	-0.094	-0.112	-0.115	11	-0.2044	20.44	
13	-0.030	0.014	0.055	13	0.0774	7.74	
15	0.085	0.074	0.042	15	-0.0150	1.50	
17	-0.023	-0.059	-0.075	17	0.0150	1.50	
19	-0.054	-0.018	0.025	19	0.0150	1.50	
21	0.049	0.060	0.041	21	0.0046	0.46	
23	0.017	-0.026	-0.053	23	-0.0878	8.78	
25	0.051	-0.033	0.009	25	-0.0098	0.98	

(1) Output phase current has been taken as 1 p.u. current & 100% current

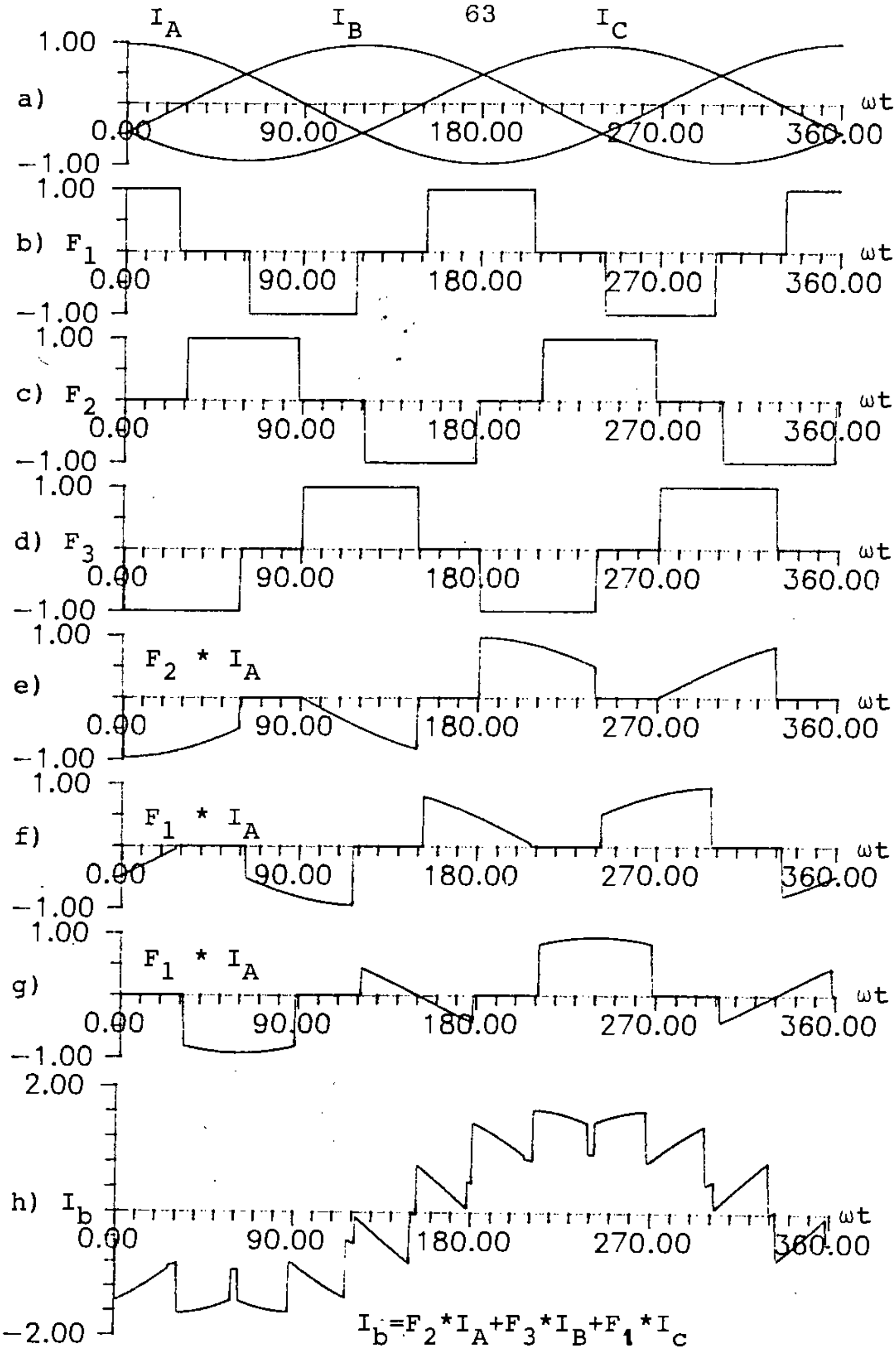


Fig. 3.14: Input current waveform, I_b obtained with 3 phase to 3 phase converter under unbalanced condition.

a) Three output unbalanced phase currents.

b) - d) F_1, F_2, F_3 switching function components.

h) Input current, I_b.

e-g) ?

85667

Table 3.7						
Frequency spectra of waveforms associated with converter Input current, I_b shown in Fig. 3.14						
Harmonic coefficients of switching function Fig.(3.14 b-d)				Harmonic coefficients of input phase current, I_b Fig. (3.14h)		
Order (n)	Amplitude			Amplitude, I_b		
	A_n	B_n	C_n	Order (n)	p.u.(1)	%(1)
1	1.030	1.056	1.080	1	-0.8682	86.82
3	0.131	0.088	0.044	3	0.0201	2.01
5	-0.255	-0.251	-0.239	5	0.0020	0.20
7	0.056	0.096	0.131	7	0.0020	0.20
9	0.114	0.083	0.044	9	0.0073	0.73
11	-0.094	-0.112	-0.115	11	0.2042	20.42
13	-0.030	0.014	0.055	13	-0.0779	7.79
15	0.085	0.074	0.042	15	0.0297	2.97
17	-0.023	-0.059	-0.075	17	0.0039	0.39
19	-0.054	-0.018	0.025	19	0.0039	0.39
21	0.049	0.060	0.041	21	-0.0063	0.63
23	0.017	-0.026	-0.053	23	0.0880	8.80
25	0.051	-0.033	0.009	25	-0.0107	1.07

(1) Output phase current has been taken as 1 p.u. current & 100% current

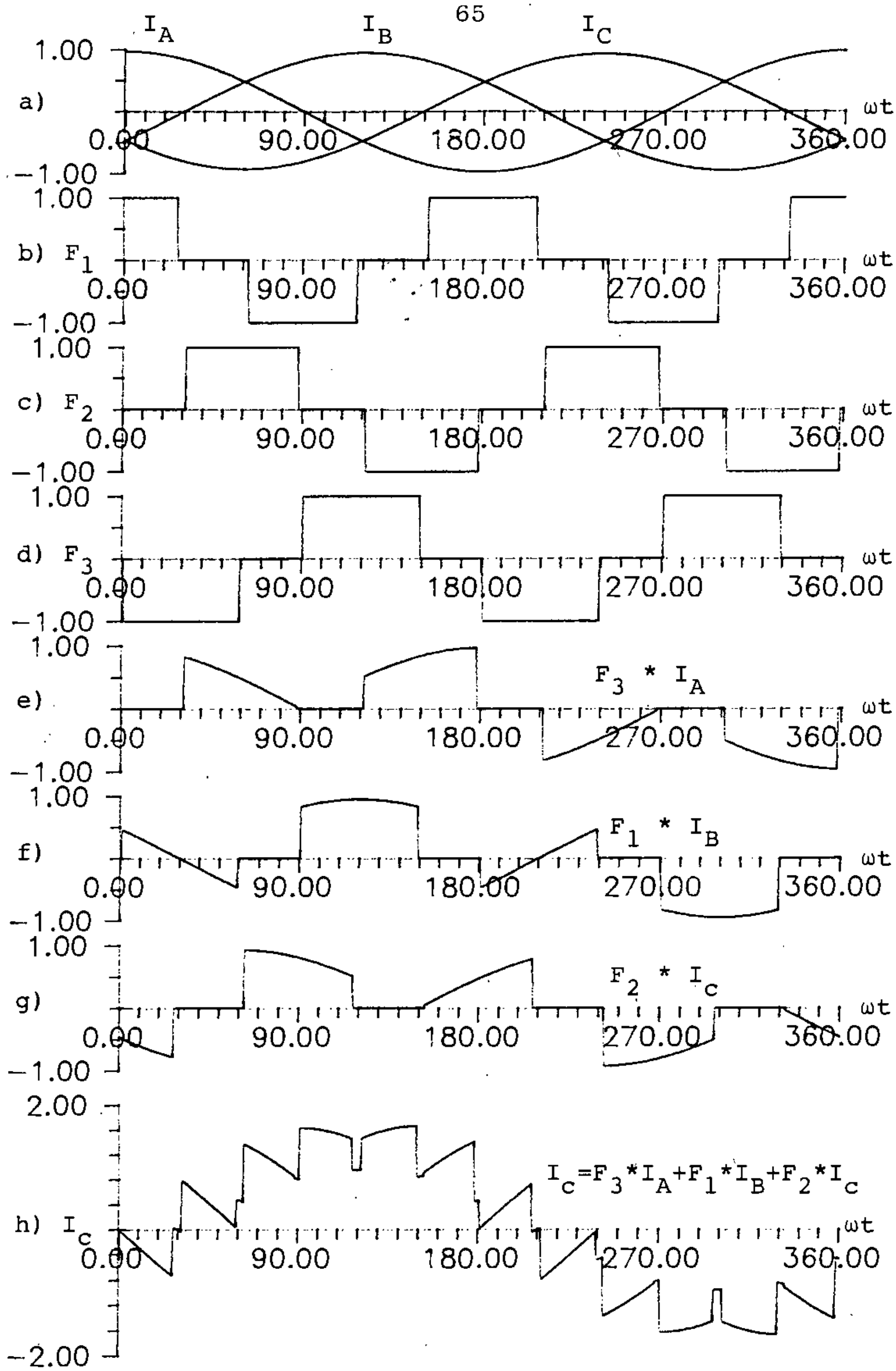


Fig. 3.15: Input current waveform, I_c obtained balanced with 3 phase to 3 phase converter under unbalanced condition

a) Three output unbalanced phase currents.

b) - d) F_1, F_2, F_3 switching function components.

h) Input current, I_c

e-g) ?

Table 3.8

Frequency spectra of waveforms associated with converter Input current, I_c shown in Fig. 3.15

Harmonic coefficients of switching function Fig.(3.15 b-d)				Harmonic coefficients of input phase current, I_c Fig. (3.15h)		
Order (n)	Amplitude			Amplitude, I_c		
	A_n	B_n	C_n	Order (n)	p.u.(1)	%(1)
1	1.030	1.056	1.080	1	-0.8684	86.84
3	0.131	0.088	0.044	3	0.0185	1.85
5	-0.255	-0.251	-0.239	5	-0.0346	3.46
7	0.056	0.096	0.131	7	-0.0346	3.46
9	0.114	0.083	0.044	9	0.0073	0.23
11	-0.094	-0.112	-0.115	11	0.2044	20.44
13	-0.030	0.014	0.055	13	-0.0774	7.74
15	-0.085	0.074	0.042	15	0.0000	0.00
17	-0.023	-0.059	-0.075	17	-0.0289	2.89
19	-0.054	-0.018	0.025	19	-0.0289	2.89
21	0.049	0.060	0.041	21	-0.0023	0.23
23	0.017	-0.026	-0.053	23	0.0878	8.70
25	0.051	-0.033	0.009	25	-0.0104	1.04

(1) Input phase current has been taken as 1 p.u. current & 100% current.

$$=\begin{bmatrix} \sum_{n=1,3,5}^{\alpha} A_n \cos(n\, w_s t) & \sum_{n=1,3,5}^{\alpha} C_n \cos n(w_s t-240^\circ) & \sum_{n=1,3,5}^{\alpha} B_n \cos n(w_s t-120) \\ \sum_{n=1,3,5}^{\alpha} B_n \cos n(w_s t-120^\circ) & \sum_{n=1,3,5}^{\alpha} A_n \cos(n\, w_s t) & \sum_{n=1,3,5}^{\alpha} C_n \cos n(w_s t-240^\circ) \\ \sum_{n=1,3,5}^{\alpha} C_n \cos n(w_s t-240^\circ) & \sum_{n=1,3,5}^{\alpha} B_n \cos n(w_s t-120^\circ) & \sum_{n=1,3,5}^{\alpha} A_n \cos(n\, w_s t) \end{bmatrix} \tag{3.12}$$

The three input currents I_a, I_b and I_c are the replica of three output voltages. Hence, the harmonic content of the input currents are similar to output voltages.

3.5 An Example

To verify the proposed technique a specific example of input voltage of magnitudes $V_{an}=0.97$ p.u. $V_{bn}=0.95$ p.u. and $V_{cn}=0.93$ p.u. i.e. 3,5 and 7 percent amplitude unbalance is considered. A simple single pulse modulation (Fig.3.1) is considered for this example. As the input phase voltages are of different amplitudes, the pulse duration will be different for the three phases. The duration, δ, of the pulses can be calculated as follows:

$$A_n = 4/n\pi \, \mathrm{Sin}(n\delta/2) \qquad \text{for } n=1,3,5$$

To get the output voltage balanced, fundamental A_1 must satisfy Eqn. 2.9 of chapter 2 which states that

$$A_1 = \frac{1}{A} = \frac{1}{0.97} = 1.03093$$

or, $A_1 = 4/\pi \sin(\delta_1/2)$

or, $1.03093 = 4/\pi \sin(\delta_1/2)$

$$\delta_1 = 108^\circ$$

$$B_1 = \frac{1}{B} = \frac{1}{0.95} \qquad \delta_2 = 112^\circ$$

and $$C_1 = \frac{1}{C} = \frac{1}{0.93} \qquad \delta_3 = 116^\circ$$

Therefore, pulse widths for a,b and c phases are $\delta_1=108^\circ$ $\delta_2=112^\circ$ and $\delta_3=116^\circ$ respectively. The converter configuration is shown in Fig.3.2 and corresponding gating signals g_1-g_6 for this specific example is depicted in Fig.3.9. The output voltages and the corresponding input currents are shown in Fig.3.10-3.15. In order to provide a detailed description of the generated converter output voltage and input current waveforms, their respective spectra have been computed and shown in Tables 3.3-3.8. This information is essential not only for the proper evaluation of the converter performance but also for the design of input output filters (when required).

3.6 Simulation of Input-Output Voltage/Currents

It is customary to verify the analytically predicted frequency spectra of unknown shaped waveform by reconstrucing the waveshapes from predicted frequency spectra. Using the harmonic coefficients shown in Tables 3.3-3.8 the three output voltages and three input currents are reconstructed and depicted in Figs 3.16 and 3.17. Their shapes matches the analytically predicted wave shapes of Figs. 3.10-3.15.

3.7 Varification of Results by MATLAB Simulation Package

To verify key analytical results, the discussed unbalanced converter are tested by simulation on IBM PC using MATLAB package program. A dedicated computer program simulating the precise opening and closing of the switches is employed to generate output voltages and input currents are shown in Figs 3.18 and 3.19. Further processing of these waveforms using MATLAB package program yields the respective frequency spectra. These waveforms are also shown in Fig 3.18 and 3.19. Comparison between analytically predicted frequency spectra (Tables 3.3-3.8) and spectra obtained by simulation shows that they are in close agreement.

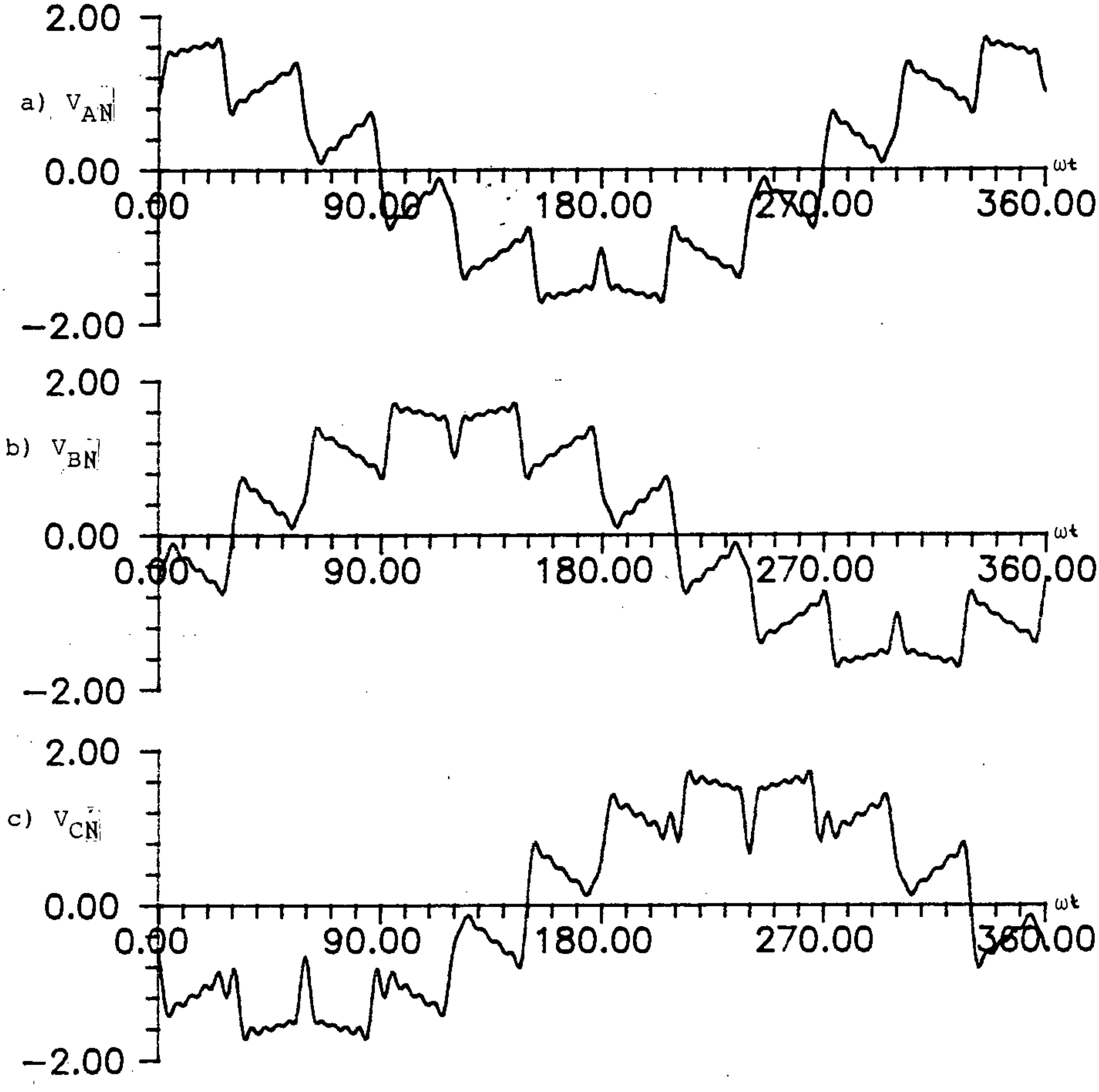


Fig. 3.16: Output voltages V_{AN}, V_{BN} & V_{CN} reconstructed from harmonic coefficients (shown in Tables 3.3 - 3.5).

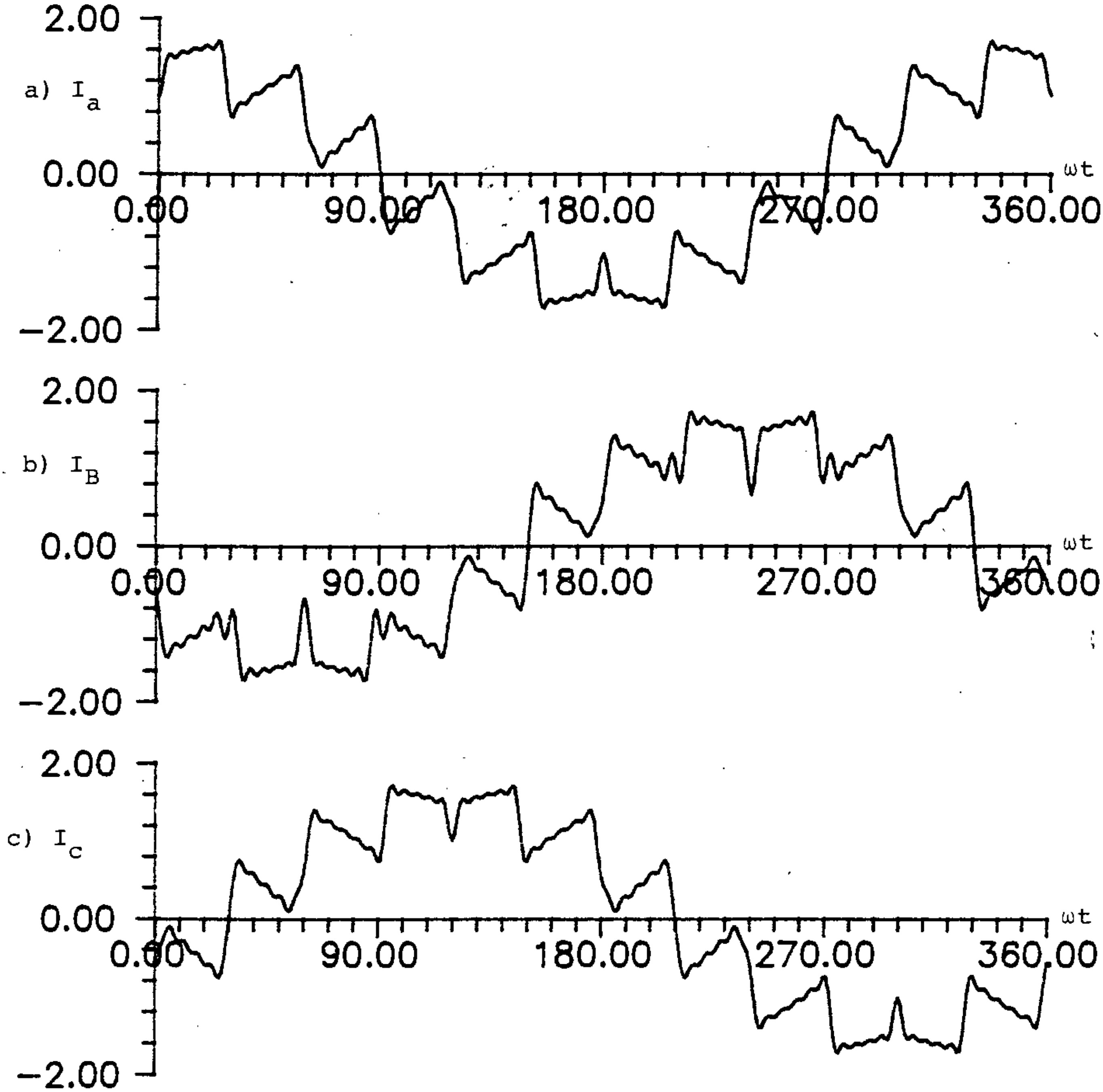


Fig. 3.17: Input currents I_a, I_b & I_c reconstructed from harmonic coefficients (shown in Tables 3.6 - 3.8).

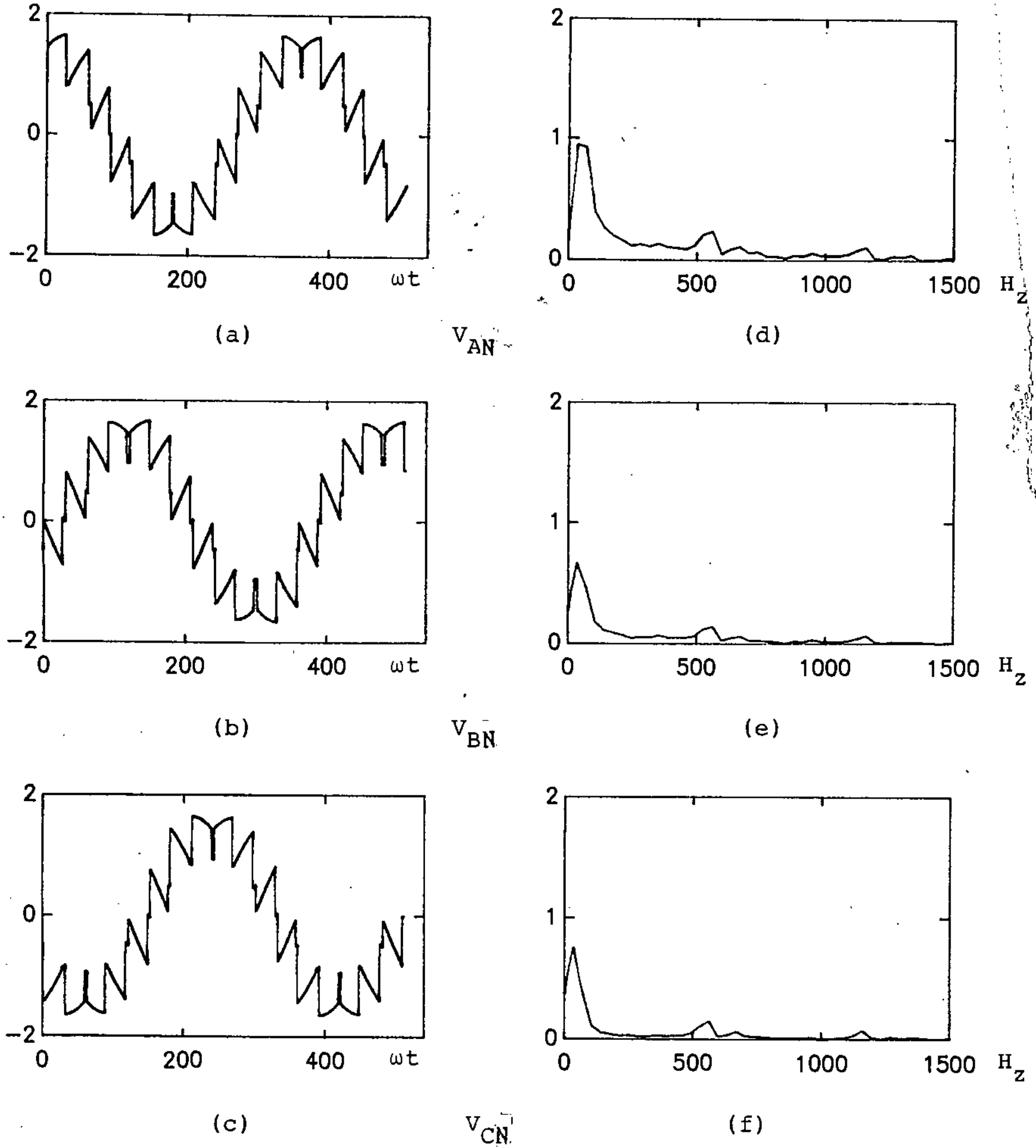


Fig. 3.18: Output voltages V_{AN}, V_{BN} and V_{CN} (a, b & c) and their spectra (e, f & g) by MATLAB simulation package.

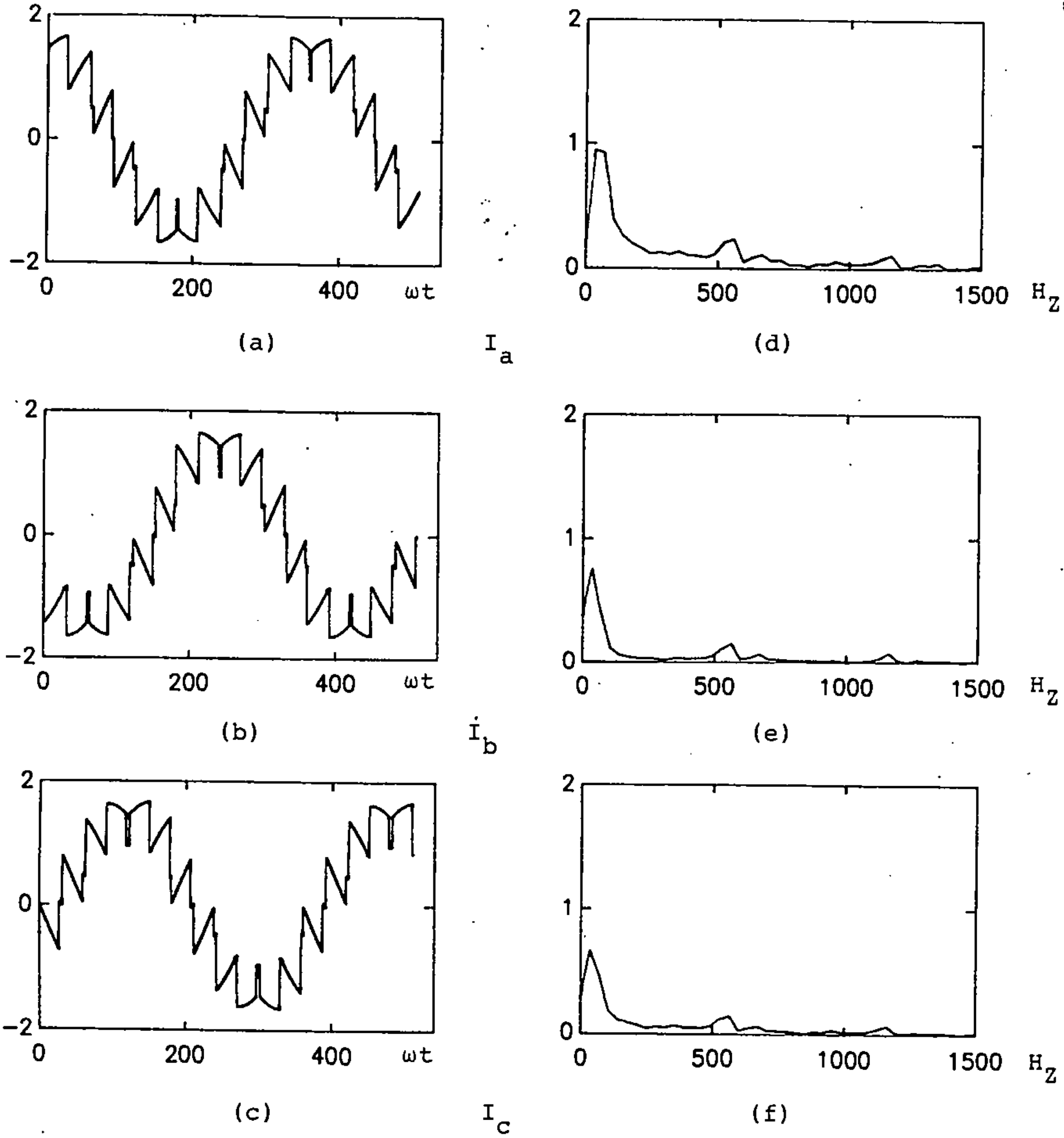


Fig. 3.19: Input currents I_a, $_b$ & I_c (a, b & c) and their spectra (e, f & g) by MATLAB simulation package.

3.8 Conclusions

A detailed Fourier analysis of output voltages and currents for the unbalanced converter is presented in this chapter. The gating signal required to get a balanced output is derived. The voltages and currents are reconstructed using the Fourier coefficients. Finally their spectrum are verified by using MATLAB package.

The frequency spectrum of output voltages and currents proves that the output of the proposed unbalanced converter is balanced. This verifies the validity of proposed technique.

CHAPTER 4

OPERATION AND DESIGN OF THE CONVERTER

4.1 Introduction

This chapter discusses the operation and design of the converter. The aspects of the active elements which are used for switching purpose and also the design aspects of logic-control circuit of the unbalanced phase converter are discussed. These aspects include, the defination and derivation of the appropriate active element i.e switching element, switching function, the processing of the gating signals from their respective functions and the developement of the necessary circuitry to implement these functions and signal processing.

As the input voltages are unbalanced, the appropriate switching fucntion or gating signal generation is much more complex [11]-[12] than the generation of respective function or signals for a balanced 3 phase to 3 phase converter. The unbalance present suggests that the implementation and performance of such converter depends to a large extent on their respective logic control boards. Slight mismatch of the gating signals will result in short-circuiting and blow-up of switches.

Some discussion is made regarding the active element i.e. elements of converter transfer matrix, before the discussion of operation of the converter.

Finally, a complete design data for the converter is provided.

4.2 Operation of the Converter

The operation of a 3-phase to 3 phase balanced output static converter can be explained with the help of Fig. 4.1. Before going in the operation of the converter it is essential to discuss about active element and switching function .

4.2.1 Active Elements & Switching Functions

The realization of the `Converter Transfer Matrix' can be achived only by means of a set of switches which operates according to a predetermined switching pattern. This topology (Fig 4.1) comprises of 9(=3x3) `active element'. In this specific case of converter as switching function have both positive(+) and negative (-) values and all nine active elements (S_1 to S_9) has to operate both in positive (+) and negetive (-) values, so it is clear that each of nine active elements (S_1 to S_9) consists of two static converter switch, one in top and another at the bottom called top switch and bottom switch respectively. For a 3-phase to 3-phase converter a total no. of 9x2(=18) static switch are required. So one switch (top switch) operate on positive value of switching function and other switch (bottom switch) operate on negative value of the switching function.

The phase A of the converter (Fig. 4.1) can be redrawn as

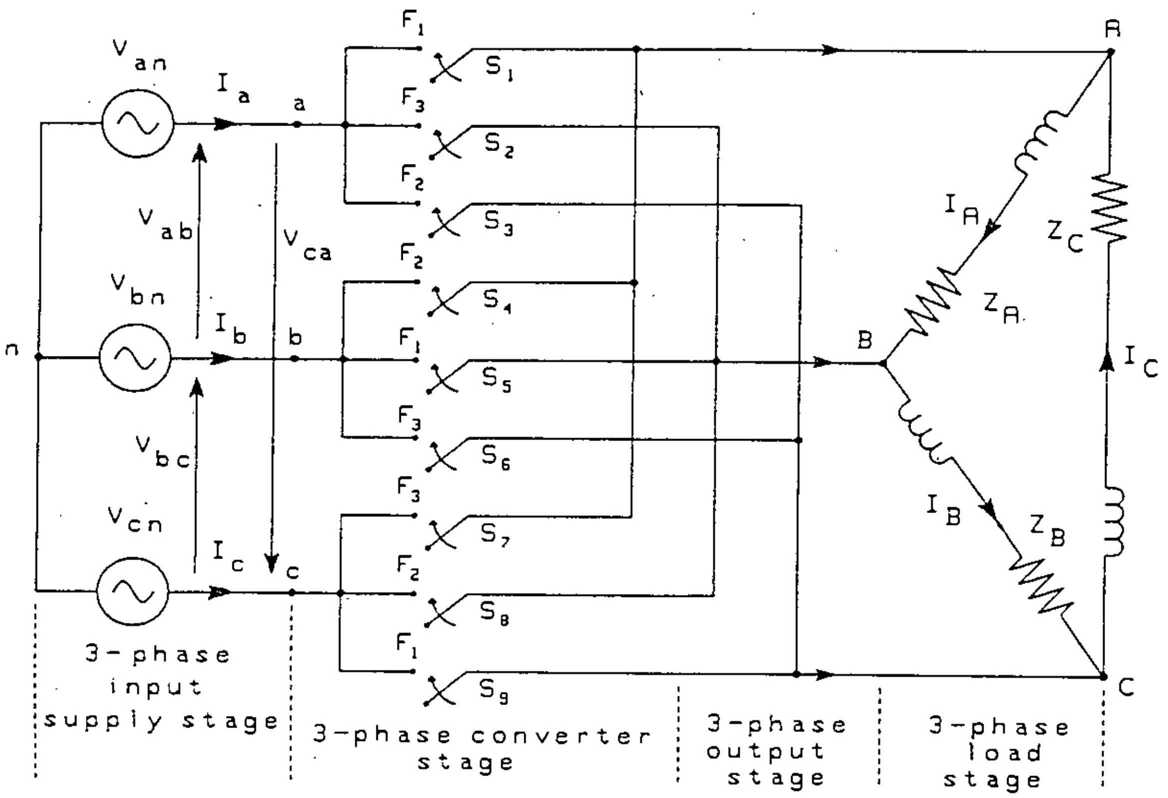
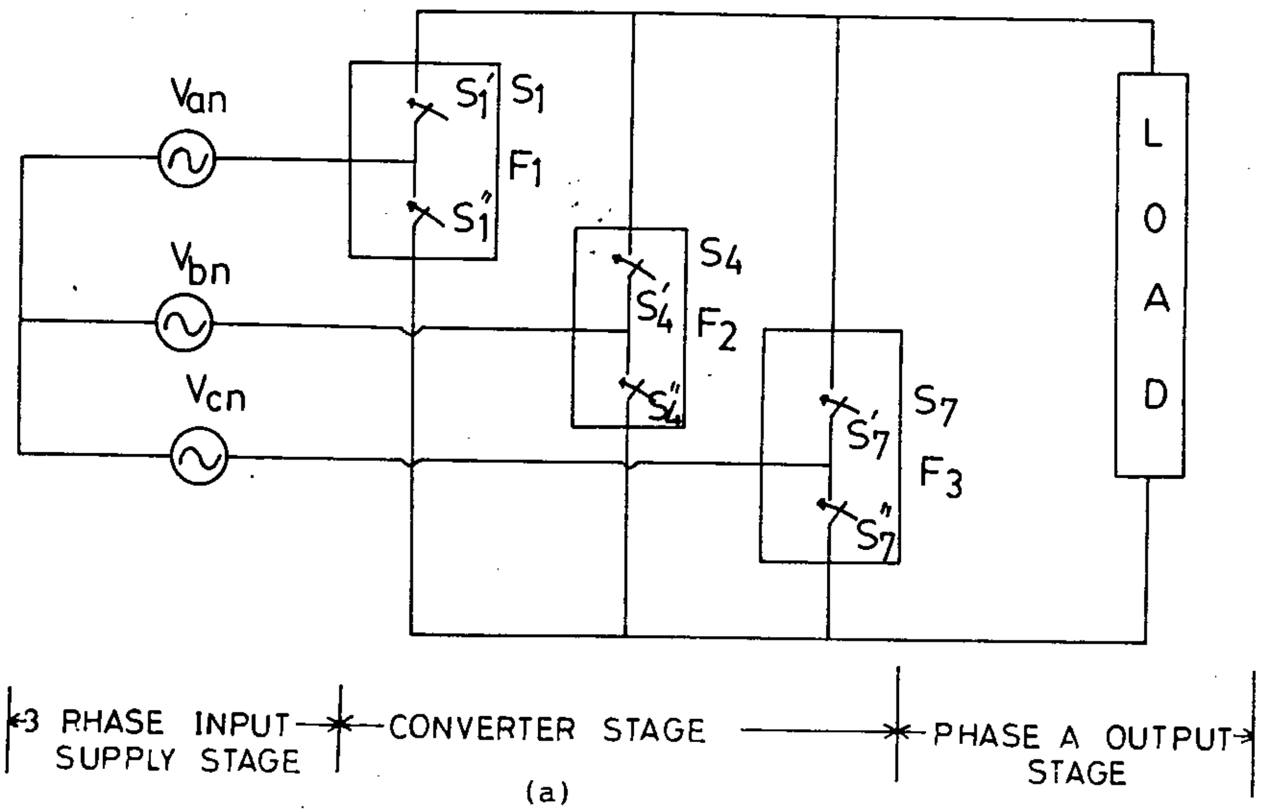


Fig. 4.1: Simplified circuit diagram of the proposed 3 phase to 3 phase static converter.

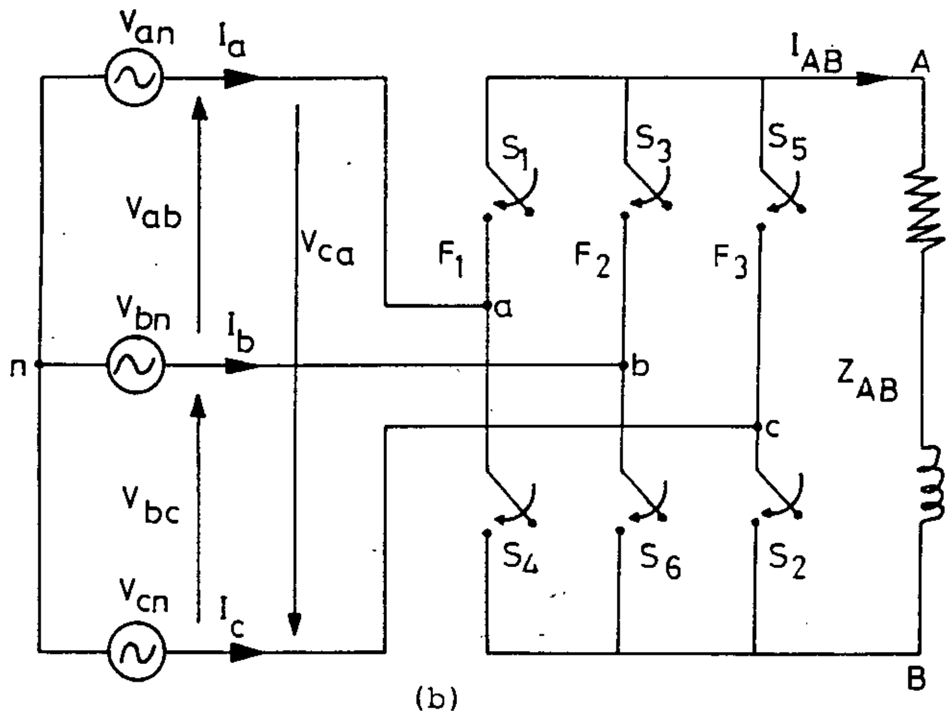
shown in Fig. 4.2 considering the top and bottom switch concept (negative and positive value).

The switching function for a single switch assumes unit value whenever the switch is closed and a zero value whenever the switch is open. But here switching function for an active element have a positive unit, zero and negative unit value; whenever the top and bottom switch are open the switching function have a zero value and whenever the bottom switch of the active element is closed the switching function have a negative unit value. The function which are required for the operation of the converter active elements/switches are termed as switching function. In the design and analysis of a balanced output 3 phase to 3 phase static converter, the three switching function F_1,F_2 and F_3 have +1, 0 and -1 value. Consider an active element S_1. Top switch S_1' operate when the switching function have a positive unit value and top switch S_1',remain inoperative when the switching function have zero or negative unit value. Similarly the bottom switch S_1'' operate when the switching function have a negative unit value and the bottom switch S_1'' remain inoperative when the switching function have a positive or zero value. All nine active elements operate like active element S_1 as described above.



(a)

a) Switches are numbered according to Fig. 4.1.



(b)

b) After renumbering the switches.

Fig. 4.2: Simpliefied circuit diagram of the proposed 3 phase to to 3 phase static converter. (Here phase 'A' has taken into account.)

4.2.2 Operation of the Converter Under Balanced Input Condition

The operation of phase `A′ according to Fig. 4.2 can be explained as follows:

For the first 30^o, F_1 has a value of +1 and F_3 has a value of -1. So S_1' and S_7'' are closed. The output is the combination of F_1*V_{an} and F_3*V_{cn}. For 30^o to 60^o F_2 has a value of +1 and F_3 has a value of -1. So S_4' and S_7'' are closed. The output is the combination of F_2*V_{bn} and F_3*V_{cn}. For 60^o to 90^o, F_1 has a value of -1 and F_2 has a value of +1. So S_1'' and S_4'' are closed. The output is the combination of F_1*V_{an} and F_2*V_{bn}. For 90^o to 120^o, F_1 has a value of -1 and F_3 has a value of +1. So S_1'' and S_7' are closed. The output is the combination of F_1*V_{an} and F_3*V_{cn}. For 120^o to 150^o, F_2 has a value of -1 and F_3 has a value of +1. So switches S_4'' and S_7' are closed. The output is the combination of F_2*V_{bn} and F_3*_{cn}. For 150^o to 180^o F_1 has a value of +1 and F_2 has a value of -1. So S_1' and S_4'' are closed. The output is the combination of F_1*V_{an} and F_2*V_{bn}.

After 180^o the operation is the same as explained earlier and the cycle is repeated.

For phases B and C the operations are similar to phase A.

4.2.3 Operation of the Converter Under Unbalanced Input Condition

For the unbalanced input condition, only the width of the switching function will be different, principle of operation remains the same as explained for balanced case disscussed in section 2.3. Fig 4.4. shows three switching function with different widths due to unbalances in the input voltages, we can explain the operation of the converter under unbalanced case as follows;

The unbalanced voltages are taken as

$V_{an} = A \cos w_i t$

$V_{bn} = B \cos(w_i t-120^o)$

$V_{cn} = C \cos(w_i t-240^o)$

Consider a specific example of input voltages V_{an}=0.97 p.u., V_{bn}=0.93 p.u. i.e 3,5 and 7 percent amplitude unbalanced is considered. The switching function angle for this specific case are calculated as $\delta_1=54^o$, $\delta_2=56^o$ and $\delta_3=58^o$ using the formula $A_n = \frac{4}{n\pi} \sin(\frac{n\delta}{2})$ and dividing the value of δ by 2(two) because $w_s=2\ w_i$.

For the balanced case the width of the switching function is $\delta=\delta_1=\delta_2=\delta_3$.

Comparing figures 4.3 and 4.5 we see that the principle of operation of the converter remains the same under balanced and unbalanced input conditions, only the widths of the switching functions become different.

4.3 <u>Design Criteria</u>

Some converter design aspects regarding component rating

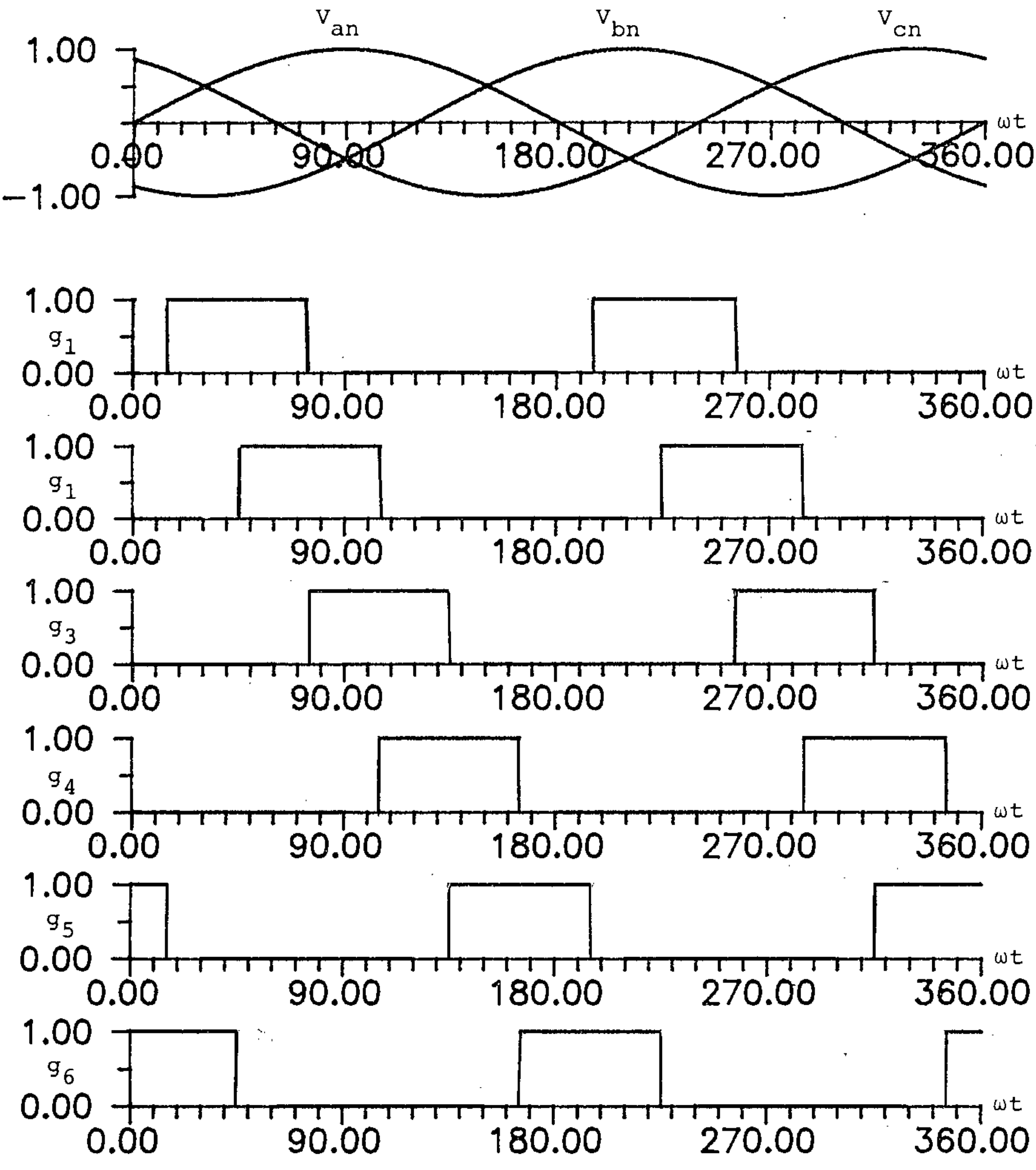


Fig. 4.3: Six gating signals relationship with balanced input voltages for the converter.

control logic and component profection are discussed in this section.

4.3.1 Component Ratings

In determining the ratings of the converter switches, the worst case condition is considered. It is noted that the worst case condition interval is 120^o conduction. Conduction of more than 120^o interval for any switch is equivalant to short circuiting the source which is not desirable. For this case the peak, rms and average switch currents I_{sp}, I_{sr} and I_{sav} are given by [13];

$I_{sp} = (\sqrt{3})*(\sqrt{2})$ p.u Amps

$I_{sr} = (\sqrt{3})*/(\sqrt{2})$ p.u. Amps

$I_{sav} = (\sqrt{6})/\pi$ p.u. Amps

With these ratings a simple design is provided here for the phase converter.

For the case of simplisity consider phase "A" only. (Fig. 4.2)

Now considering the design of 30 KVA three phase to three phase converter (Fig. 4.1). it is assumed that

Nominal ac voltage V_{an}=220 volts (rms)

1 p.u. volt =220 volts (rms)

and the fundamental component of output (Table 3.1) voltage

V_{AB}, 1 = 220*0.87=191.4 volts (rms)

and the load current I_{AB}(Fig 4.1) is given by

$I_{AB},1$ = 30,000/(√3*191.4)=90.49 Amps(rms)

By using computed per unit voltage, the actual converter switch voltage and current ratings (without safety margin) can be computed as follows:

Peak switch voltage = 220*√2=311 volts.

Peak switch current = 90.49*(√6) p.u

= 221.66 Amps.

Average switch current =90.49*(√6/π)

=70.55 Amps

RMS switch current =90.49x(√3/√2)

=110.83 Amps.

4.3.2 Design of the Control Circuit

Converter have special logic control requirements because of the complexities of associated power circuits. The design of the control circuit includes:

- the derivation of the appropriate switching functions
- the processing of the gating signals from their respective functions
- the development of the circuitry required to implement the above functions and signal processing.

To implement the schemes proper relationship between input voltages and gating signals is required. Such a gating signal which have relationship with balanced input for the phase converter (Fig 4.1 & Fig 4.2) is shown in Fig 4.3. This gating signals can be realized by using digital components.

A delta-wye step down transformer (Fig. 4.4) is used for input line voltage sensing. The output of this transformer provides the three zero crossing points for the three input line voltages. The zero-cross sensing is implemented by employing three properly biased voltage comparators.

The six gating signals g_1-g_6 (Fig 4.3) are then applied to the gates of the swithes of the converter in proper synchronization with zero crossing signals. The gating signals for balanced case and unbalanced case is depicted in Fig. 4.3 and Fig 4.5 respectively.

The derivation of unbalanced gating signals are described as follows:

Let

$$V_{an} = 0.97 \text{ Sin}wt$$
$$V_{bn} = 0.95 \text{ Sin}(wt-120^o)$$
$$V_{cn} = 0.93 \text{ Sin}(wt-240^o)$$

i.e the unbalances are 3%, 5% and 7%.

The widths of the gating signal will be changed according to the amplitude of the unbalanced line voltages. The width of the gating signals are calculated as follows:

- For signal g_1 and g_4, the width = 54.1^o

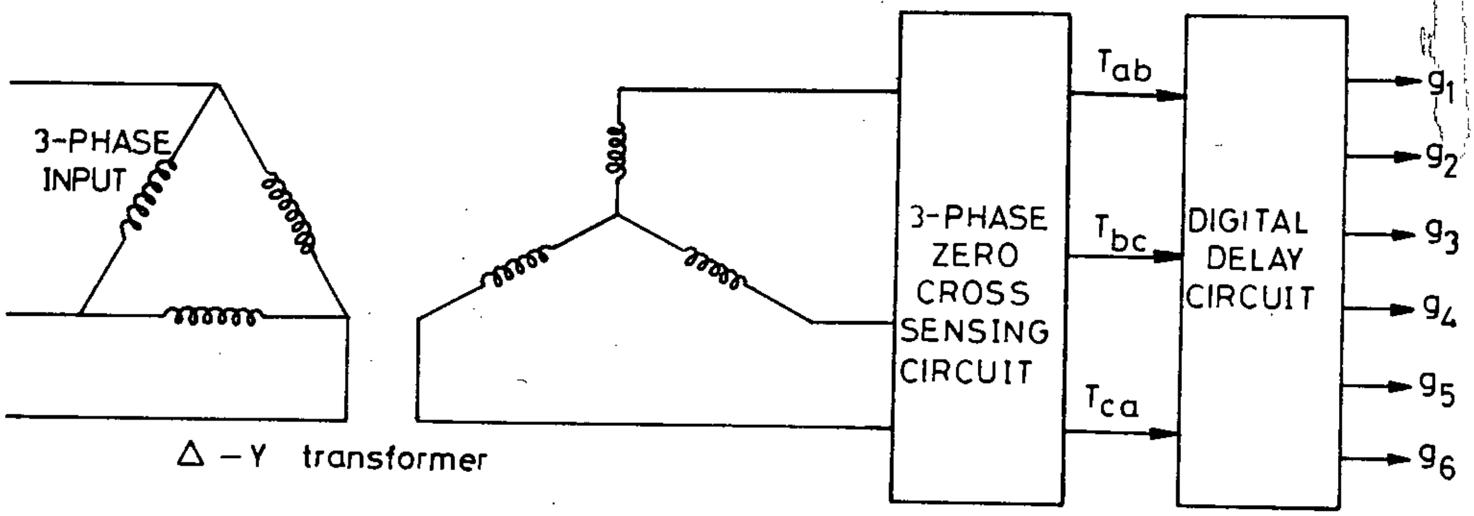


Fig. 4.4: Logic circuit block diagram for the converter.

- For signal g_3 and g_6, the width = 55.8^o
- For signal g_5 and g_2, the width = 57.6^o

The relationship between the gating signals and the unbalanced line voltages are shown in Fig 4.5.

Therefore, the width of the gating signals changes according to the unbalances present in the input line voltages.

If the amplitude of the unbalanced input voltages are previously known then the corresponding gating signals can be stored in EPROM and can be synchronized with the zero crossing. A dedicated computer program for loading (burning) EMROM for known amplitude unbalance is developed in FORTRAN language.

When the amplitude of the input unbalance voltages change with time, i.e. varying continuously, then a dedicated microporcessor can be used. Various combinations of gating signals for corresponding amplitude unbalance can be stored (Fig 4.6) in a look up table. According to the unbalance the output (Fig 4.6).

4.3.3 Component Protection

Providing effective switch protection for the converter shown in Fig 4.1 is very difficult task, as the load current commutation must be done without free-wheeling diode. Reffering to [14] a snubber circuit which can provide adequate protection for the switches shown in Fig 4.7. The function of

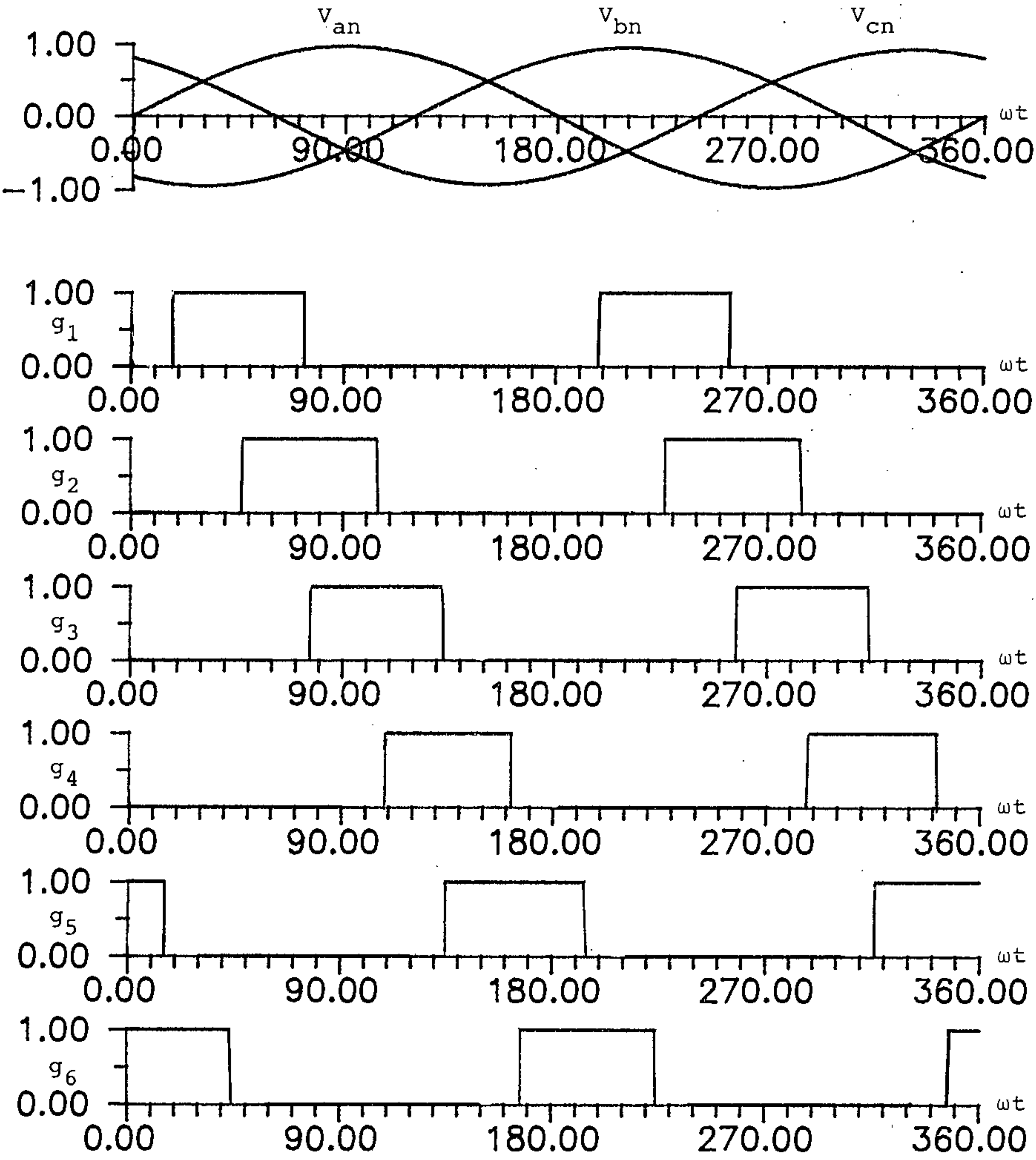


Fig. 4.5: Six gating signals relationship with unbalanced input voltages for the converter.

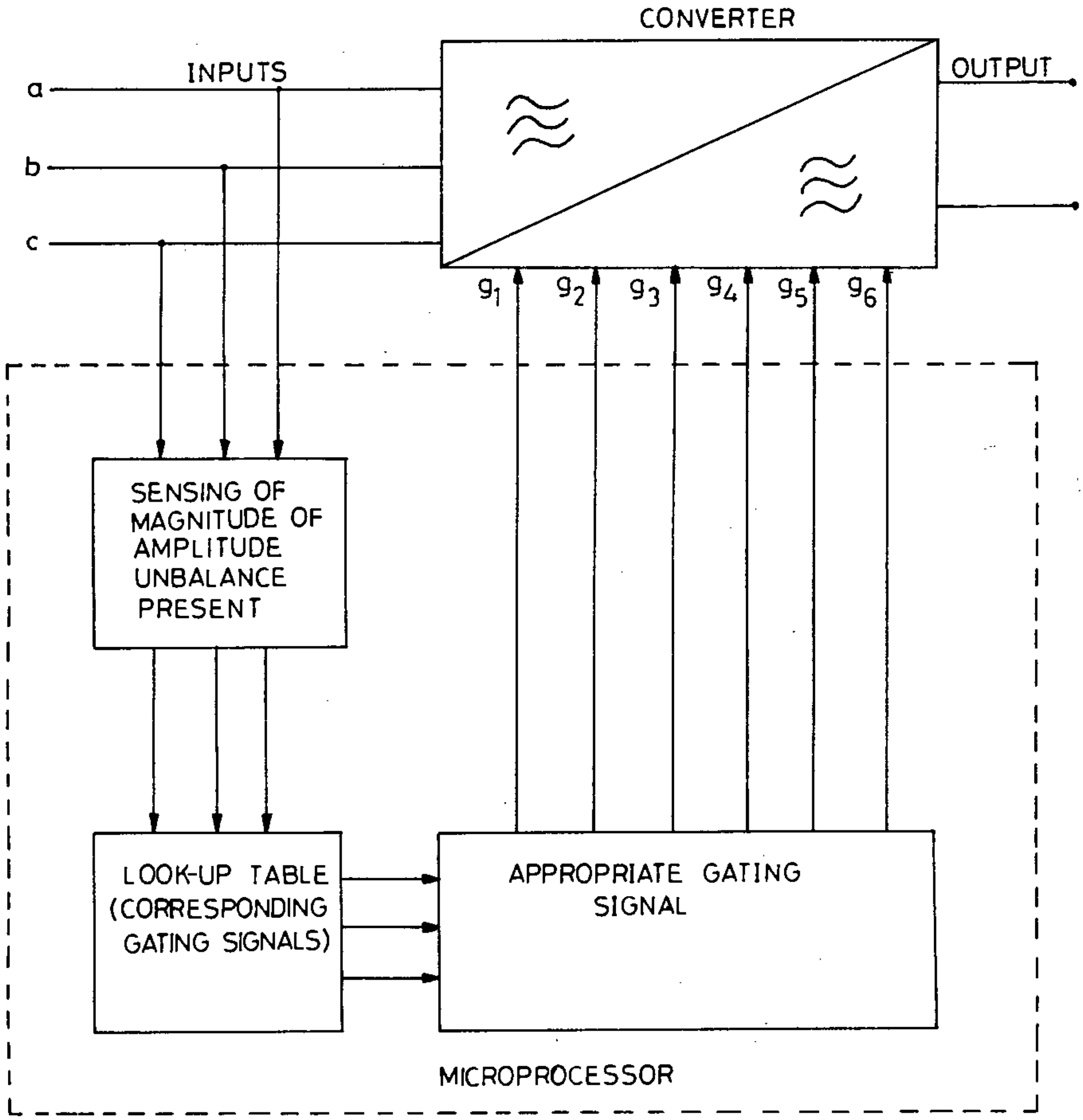


Fig. 4.6: Microprocessor based control circuitry.

LOOK-UP TABLE

Amplitude Unbalance present in input Line voltage V_{ab} in percent	Correspon-ing gating signal width g_1, g_4	Amplitude Unbalance present in input Line voltage V_{bc} in percent	Correspon-ing gating signal width g_3, g_6	Amplitude Unbalance present in input Line voltage V_{ca} in percent	Correspon-ing gating width g_5, g_2
1	52.5°	1	52.5°	1	52.5°
2	53.3°	2	53.3°	2	53.3°
3	54.1°	3	54.1°	3	54.1°
4	54.9°	4	54.9°	4	54.9°
5	55.8°	5	55.8°	5	55.8°
6	56.7°	6	56.7°	6	56.7°
7	57.6°	7	57.6°	7	57.6°
8	58.6°	8	58.6°	8	58.6°
9	59.7°	8	59.7°	8	59.7°

each (protective circuit) component is as follows:

- Reactors $L_{s\ 1-2-3}$ facilitate current transition (i.e. commutation) from a turning off to a turning on switching device.
- Front end snubber rectifier diverts input currents to *storage elements C_{s1} during converter normal or accidental switching transients.
- Load end snubber rectifier diverts load currents to storage element C_{s2} during converter normal or accidential switching transients.
- Snubber capacitors C_{s1} and C_{s2} limit resulting over voltages during above mentioned transients.
- Resistors R_{s2} and R_{s4} act as energy 'bleeding' element for C_{s1} and C_{s2}.
- Resistors R_{s1} and R_{s3} provide critical damping to L-R-C path comprised by the snubber circuit compoents.

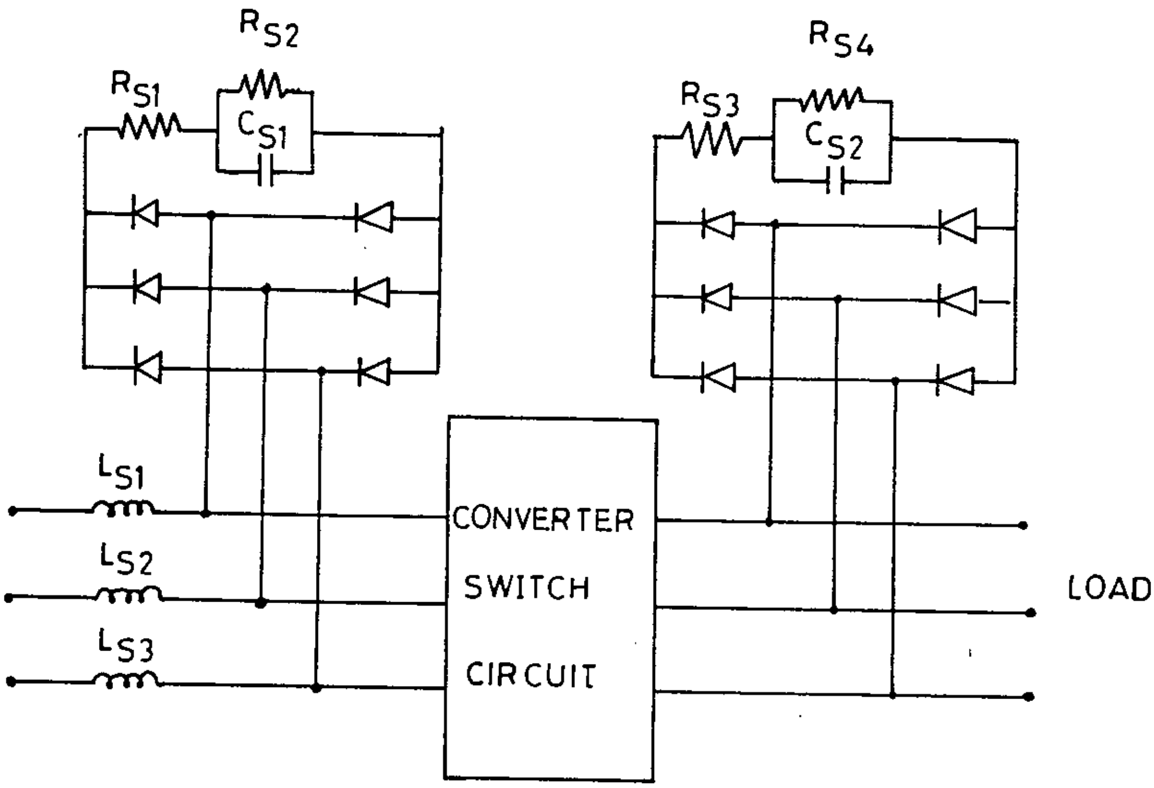


Fig. 4.7: The converter circuit showing protective elements.

4.4 Conlusions

This chapter discussed the operation and design of the converter. Derivation of the gating signals and their implementation by using microprocessor is also discussed. Component ratings and a snubber circuit for providing effective switch protection have also been discussed. Due to inadequate laboratory facilities the circuit could not be constructed.

CHAPTER 5

ANALYSIS OF AN UNBALANCED INPUT THREE PHASE CONTROLLED RECTIFIER

5.1 Introduction

Developments in microelectronics, power semiconductors and fast switches have made possible the widespread use of ac to dc converters in the control of d.c. motors, in voltage and current source inverters to provide the front end dc link, in reactive power control and harmonic current compensation schemes. Due to the increase in application of these rectifier the effect of unbalance input operation condition study has become necessary. The novel technique discussed for the study of a balanced output three phase static converter for an unbalanced system can also be applied to produce near perfect dc (output) from a three input phase voltages when they are amplitude unbalanced.

Controlled Rectifiers like all other power electronics converters are designed to work in balanced input condition. Evaluation of real operating conditions, however, shows that this assumption is not true in most cases [12]. It is, therefore, essential for the validity of typical results, converter analysis methods should accomodate unbalanced operation conditions.

This chapter focuses on the analysis and design of a three phase controlled rectifier (CR) which produces near perfect dc output voltages with the amplitude unbalanced inputs. According to novel technique to have a desired output it has been proved that the fundamental components of the switching functions are inversely proportional to the corresponding unbalanced input phase voltage magnitudes. However the output contains insignificant harmonics.

5.2 Mathematical Analysis of the Controlled Rectifier

The simplified circuit diagram of the CR capable to produce a near perfect dc is shown in Fig. 5.1. This circuit structure consists of six bilateral switches. The basic principle of operation for this CR can be derived from the genral equation (2.1a & 2.1b) in Chapter 2 by setting the number of input phases N=3 and the number of output phase M=1. In particular, the direct multiplication of the input phase voltage V_{an}, V_{bn} and V_{cn} by the corresponding switching function components F_1,F_2 and F_3 yields the near perfect dc voltage, V_{dc} (Fig. 5.3). Similarly unbalanced input phase currents Ia,Ib and Ic (Figs. 5.4, 5.5 and 5.6) are obtained by multiplying the output current I_{dc} by respective converter switching function components F_1,F_2 and F_3.

5.2.1 Analysis of the balanced three phase controlled rectifier

Consider a balanced case and assume, switching functions are:

$$F_1 = A \cos (w_s t)$$
$$F_2 = A \cos (w_s t - 120^o) \qquad (5.1)$$
$$F_3 = A \cos (w_s t - 240^o)$$

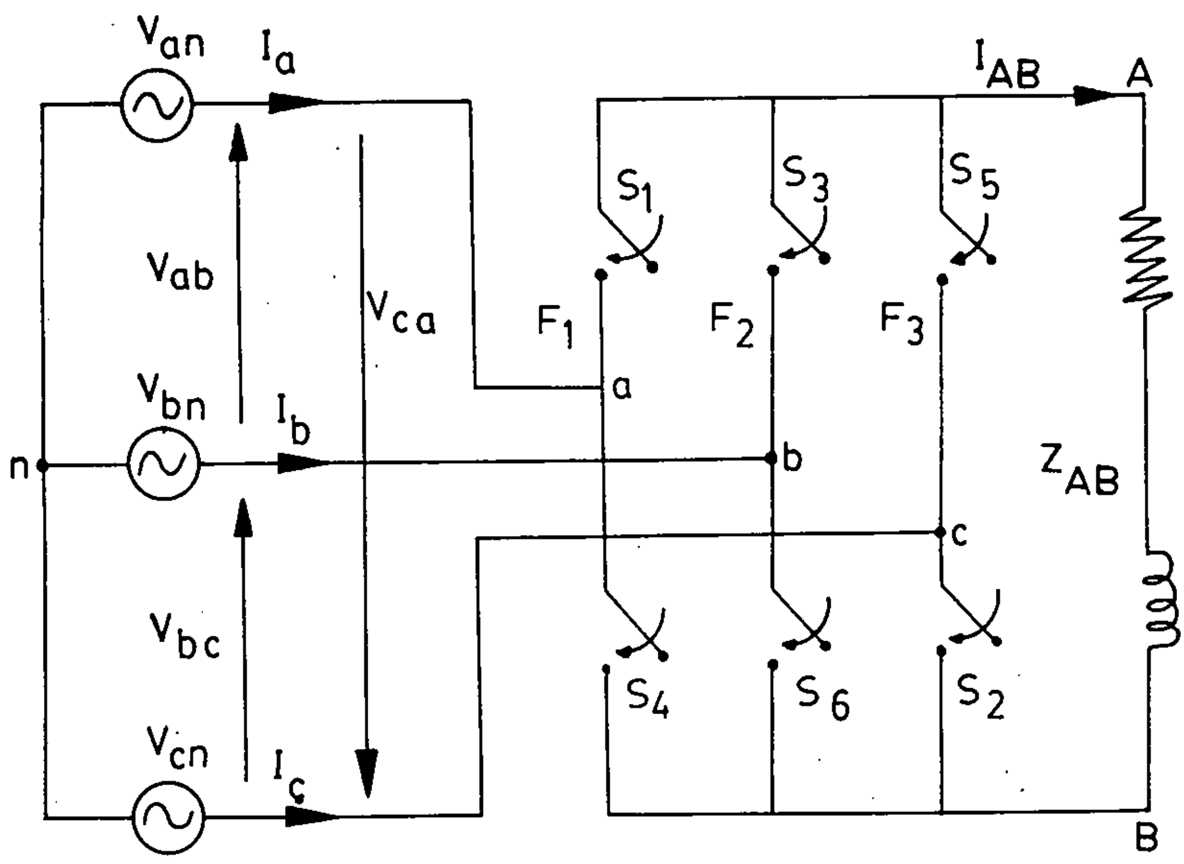


Fig. 5.1: Proposed controlled rectifier.

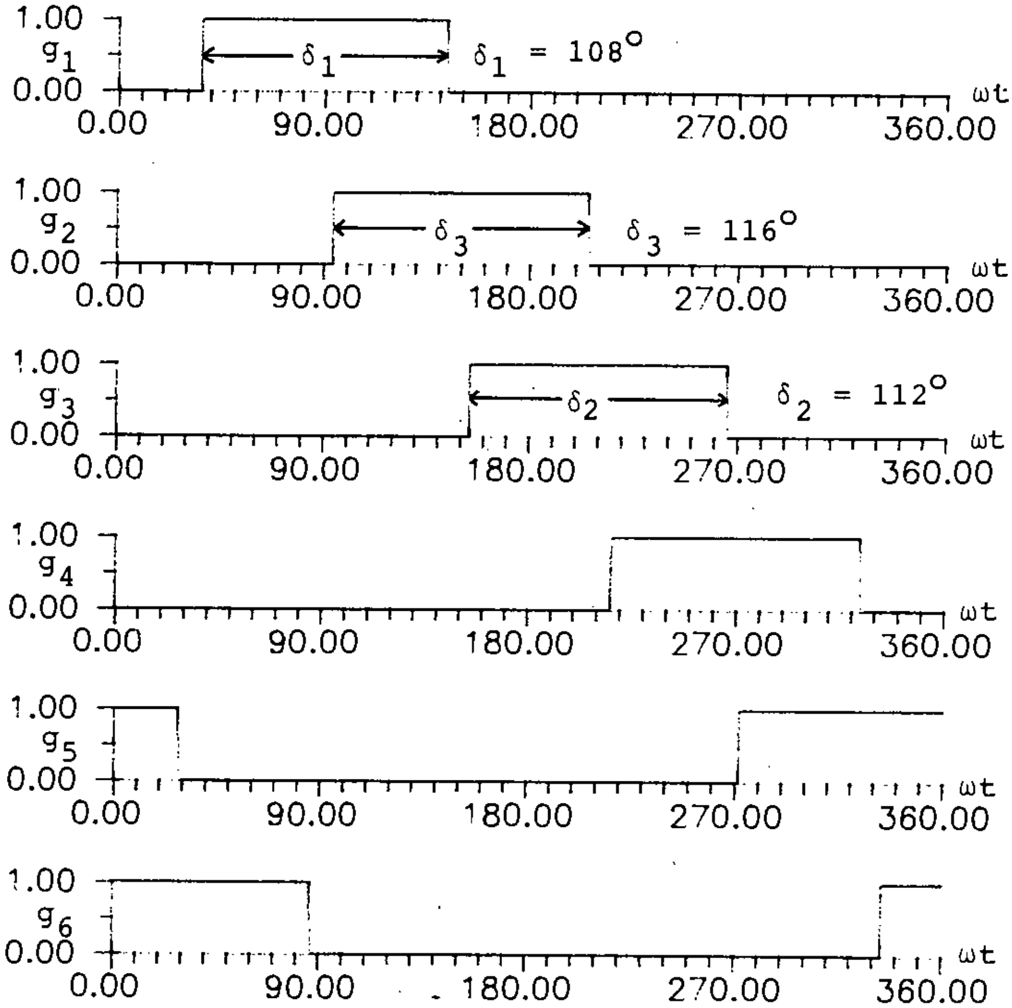


Fig. 5.2: Gating signals for the proposed controlled rectifier.

where A is the amplitude of the fundamental components of all the function F_1, F_2 & F_3.

and Input voltages

$$[V_i(w_i t)] = V_i \begin{bmatrix} \cos(w_i t) \\ \cos(w_i t - 120^o) \\ \cos(w_i t - 240^o) \end{bmatrix} \qquad (5.2)$$

Now the theoritical input, output quantities for a three phase CR with balanced input can be derived using spectrum multiplication as follows:

$$[V_o(w_o t)] = [SF].[V_i(w_i t)] \qquad (5.3)$$

$$= AV_i[\cos(w_s t)\cos(w_i t) + \cos(w_s t - 120^o)\cos(w_i t - 120^o) + \cos(w_s t - 240^o)\cos(w_i t - 240^o)]$$

If $w_s = w_i$ then

$$[V_o(w_o t)] = V_{dc} = \frac{3}{2} \qquad (5.4)$$

$$[I_i(w_i t)] = [F_d(w_s t)]^T [I_i] \qquad (5.5)$$

$$\text{or} \begin{bmatrix} I_a(w_i t) \\ I_b(w_i t) \\ I_c(w_i t \end{bmatrix} = \begin{bmatrix} A\cos(w_s t) \\ A\cos(w_s t - 120^o) \\ A\cos(w_s t - 240^o) \end{bmatrix} . [I_o]$$

$$= A \; . I_o \begin{bmatrix} \cos(w_s t) \\ \cos(w_s t - 120^o) \\ \cos(w_s t - 240^o) \end{bmatrix} \qquad (5.6)$$

5.2.2 **Analysis of the unbalanced three phase controlled rectfifer**

Consider an unbalanced case and assume switching functions are:

$F_1 = A_1 \cos (w_s t)$

$F_2 = B_1 \cos (w_s t - 120^o)$

$F_3 = C_1 \cos (w_s t - 240^o)$

and the input voltages

$$[V_i(w_i t)] = \begin{bmatrix} A \cos (w_i t) \\ B \text{ Cos } (w_i t - 120^o) \\ C \text{ Cos } (w_i t - 240^o) \end{bmatrix}$$

Now using the same equation (5.3) for a three phase CR with unbalanced input we have-

$$[V_o(w_o t)] = [SF].[V_i(w_i t)]$$

$$= A_1 A[(\cos(w_s t).\cos(w_i t)+B_1 B\cos(w_s t-120^o).\cos(w_i t-120^o)$$
$$+C_1 C\cos(w_s t-240^o).\cos(w_i t-240^o)]$$

$$= \frac{3}{2} \qquad (5.7)$$

When

$$A_1 A=1,\ B_1 B=1\ \&\ C_1 C=1$$

$$A_1 = \frac{1}{A}, B_1 = \frac{1}{B},\ C_1 = \frac{1}{C} \qquad (5.8)$$

Similarly the corresponding input currents can be derived as follows using the same equation (5.5),

$$[I_o(w_i t)]=[F_d(w_s t)]^T[I_o]$$

$$\text{or} \begin{bmatrix} I_a(w_i t) \\ I_b(w_i t) \\ I_c(w_i t \end{bmatrix} = \begin{bmatrix} A_1\ \cos(w_s t) \\ B_1\ \cos(w_s t-120^o) \\ C_1\ \cos(w_s t-240^o) \end{bmatrix} .[I_o]$$

$$=I_o \begin{bmatrix} A_1\ \cos(w_s t) \\ B_1\ \cos(w_s t-120^o) \\ C_1\ \cos(w_s t-240^o) \end{bmatrix} \qquad (5.9)$$

In the above analysis w_i is the frequency of input quintities (i.e. 50 Hz) w_s is the operating frequency of the CR (e.g. 50 Hz) and $w_o=w_s-w_i=0$.

Equation (5.4) and (5.7) shows that the output voltage is theoritically harmonic free. It can be concluded from equations (5.7) and (5.8) that the output voltage is pure dc when the fundamental component of the switching function are equal to the inverse of the amplitude of the corresponding input phase voltage. The amount of unbalance that can be corrected depends on the choice of proper switching function (SF). In this analysis a single pulse modulation is considered (Fig. 5.4b). Though theoritically the output is harmonic free, but the output contains some insignificant harmonics, because SF is a square wave instead of a cosine wave as considered during analysis. The various waveforms are depicted in Figs. 5.3 - 5.6 and the associated frequency spectra are shown in Tables 5.1 and 5.2.

The input currents are balanced for balanced case and the input currents are unbalanced for unbalanced case.

The complete expression for the output voltage and input currents can be written as follows:

$$V_o=[F_d(w_s t)].[V_i(w_i)]$$

$$= \left[\sum_{n=1,3,5}^{\infty} A_n \cos(nw_s t) \quad \sum_{n=1,3,5}^{\infty} B_n \cos n(w_s t-120^o) \quad \sum_{n=1,3,5}^{\infty} C_n \cos n(w_s t-240^o) \right]$$

$$. \begin{bmatrix} A \cos(w_i t) \\ B \cos(w_i t-120^o) \\ C \cos(w_i t-240^o) \end{bmatrix}$$

$$= \sum_{n=1,3,5}^{\infty} AA_n \cos(nw_s t).\cos(w_i t) + \sum_{n=1,3,5}^{\infty} BB_n \cos n(w_s t-120^o).\cos(w_i t-120^o)$$

$$+ \sum_{n=1,3,5}^{\infty} CC_n \cos n(w_s t-240^o).\cos(w_i t-240^o)$$

$$= \frac{3}{2} + \frac{1}{2}\left[\sum_{n=3,5}^{\infty} AA_n \left(\cos(nw_s+w_i)t + \cos(nw_s-w_i)t\right) \right.$$

$$+ \sum_{n=3,5}^{\infty} BB_n \left(\cos((nw_s+w_i)t-(n+1)120^o) + \cos((nw_s-w_i)t-(n-1)120^o)\right)$$

$$\left. + \sum_{n=3,5}^{\infty} CC_n \left(\cos((nw_s+w_i)t-(n+1)240^o) + \cos((nw_s-w_i)t-(n-1)240^o)\right) \right]$$

...(5.10)

and the input current expression becomes ;

$$[I_i(w_i t)=[F_d(w_s t)]^T .[I_o]$$

or

$$\begin{bmatrix} I_a(w_i t) \\ I_b(w_i t) \\ I_c(w_i t) \end{bmatrix} = \begin{bmatrix} \sum_{n=1,3,5}^{\infty} A_n \cos(nw_s t) \\ \sum_{n=1,3,5}^{\infty} B_n \cos n(w_s t-120^o) \\ \sum_{n=1,3,5}^{\infty} C_n \cos n(w_s t-240^o) \end{bmatrix} .[I_o]$$

$$\begin{bmatrix} I_a(w_i t) \\ I_b(w_i t) \\ I_c(w_i t) \end{bmatrix} = I_o \begin{bmatrix} \sum_{n=1,3,5}^{\infty} A_n \cos(n w_s t) \\ \sum_{n=1,3,5}^{\infty} B_n \cos n(w_s t - 120^o) \\ \sum_{n=1,3,5}^{\infty} C_n \cos n(w_s t - 240^o) \end{bmatrix} \qquad (5.11)$$

5.3 An Example

To veryfy the proposed technique a specific example of input voltage of magnitudes V_{an}=0.97 P.U. V_{bn}=0.95 P.U. and V_{cn}=0.93 p.u. i.e. 3.5 and 7 percent amplitude unbalance is considered. As the input phase voltages are of different amplitudes, the pulse duration will be different for the three phases. The duration δ of the pulses can be calculated as follows

$$A_n=(4/n\pi)\sin(n\,\delta/2) \quad \text{for } n=1,3,5...$$

To get the output balanced $A_1 = \frac{1}{A} = \frac{1}{A} = 1.03093$

$$\delta_1=108^o$$

Corresponding pulses widths for b and c phases are $\delta_2=112^o$ and $\delta_3=116^o$ respectively. The CR converter configuration shown in Fig 5.1 and corresponding gatting signals g_1-g_6 for this specific example is depicted in Fig 5.2. The output voltage V_{dc} and the corresponding input currents are shown in Fig 5.3 - 5.6 . Respective Fourier spectrum is shown in Table 5.1. For comparison purpose the output voltage and input currents waveforms for balanced C.R. are shown in Figs 5.7 - 5.10. Corresponding Fourier spectrum is shown in Table 5.2.

5.4 Simulated Results

To verify key analytical results, the discussed CR was tested by simulation on an IBM PC. A dedicated computer program simulating

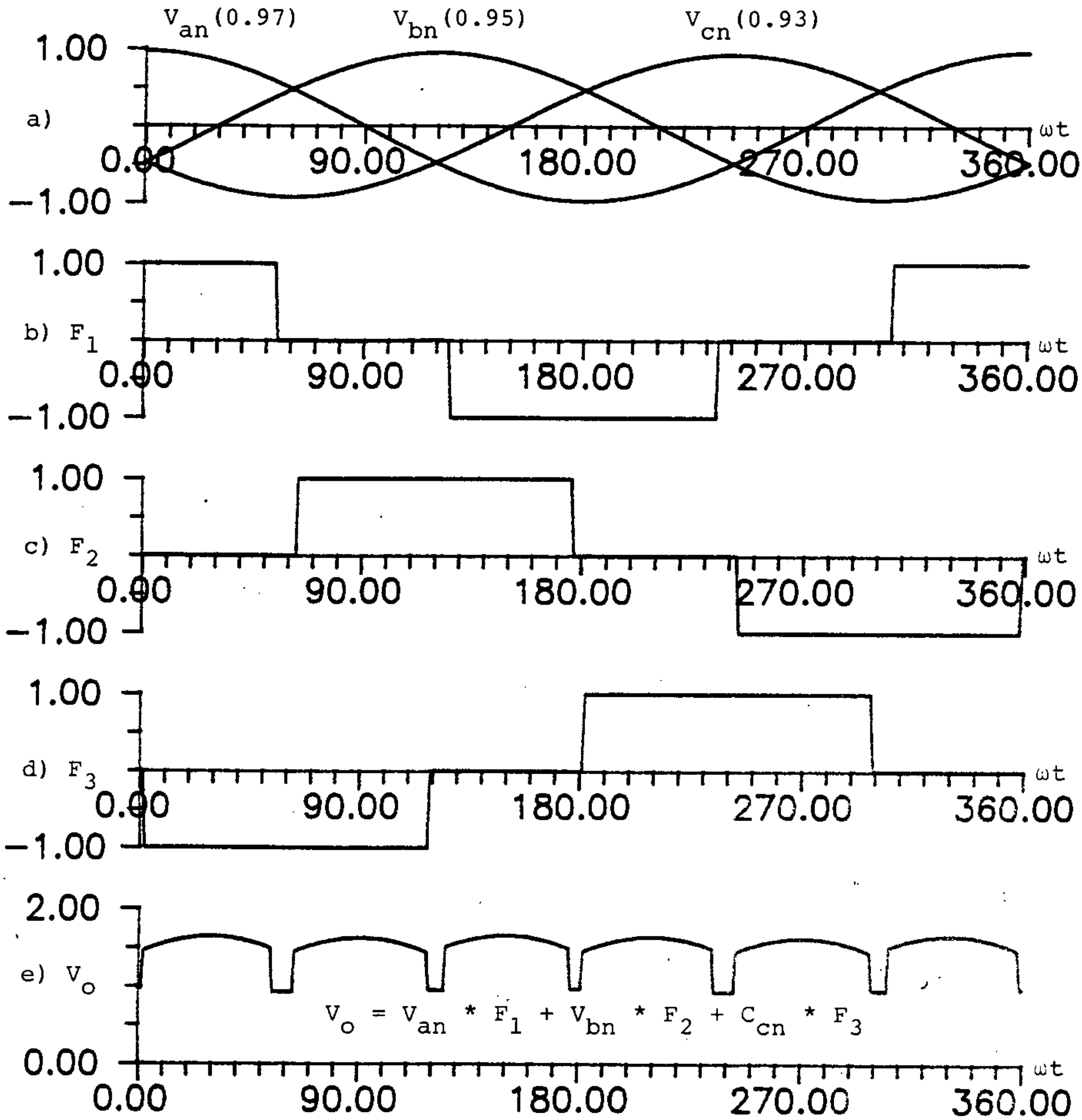


Fig. 5.3: Output voltage waveform, V_o obtained with CR under unbalanced input condition.

a) Three input unbalanced phase voltages.

b) - d) F_1, F_2, F_3 switching function components.

e) Output D.C. voltage, V_o.

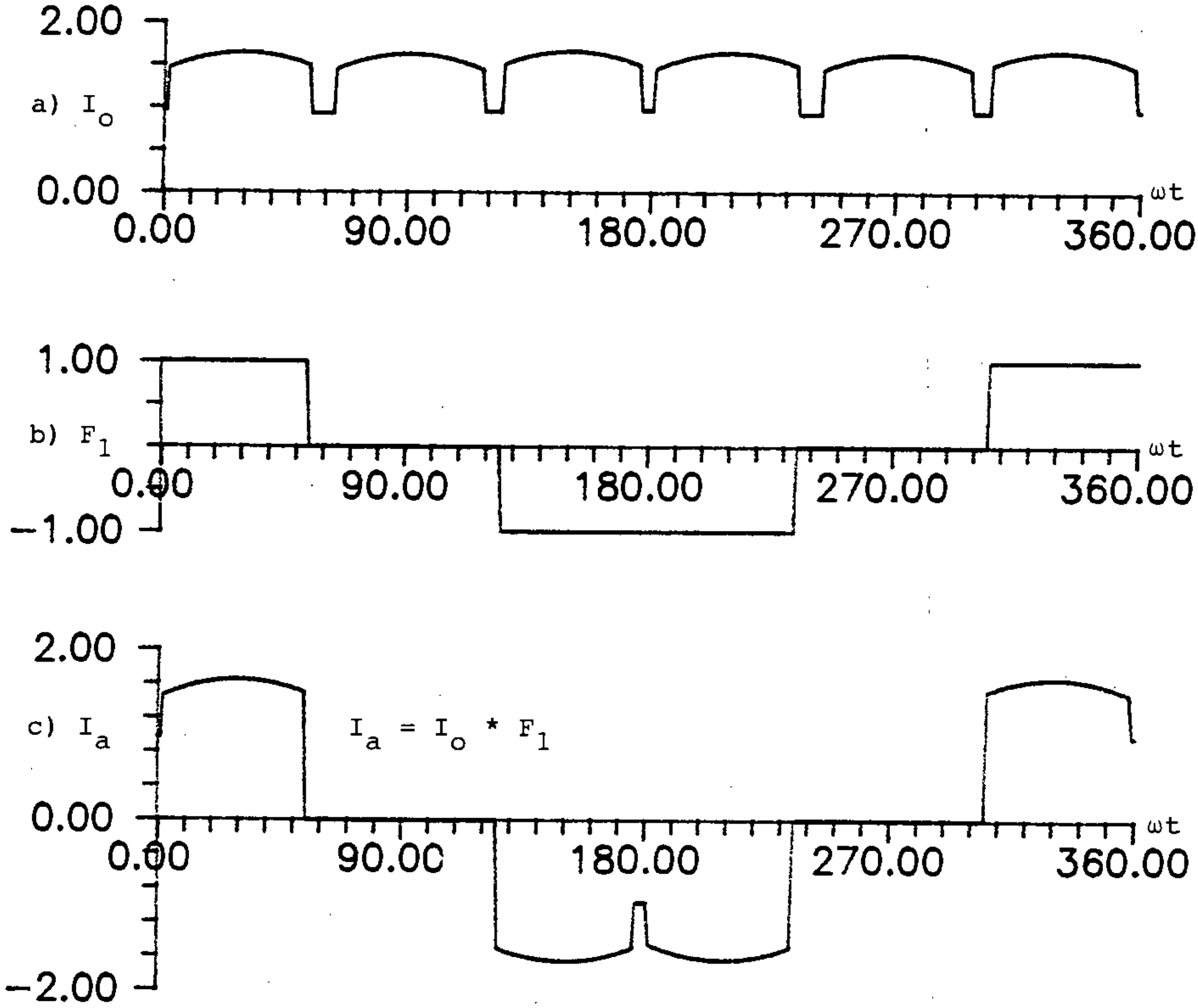


Fig. 5.4: Input current waveform, I_a obtained with CR under unbalanced condition.

a) Output current, I_o of CR.

b) Switching function, F_1

c) Input current, I_a of CR.

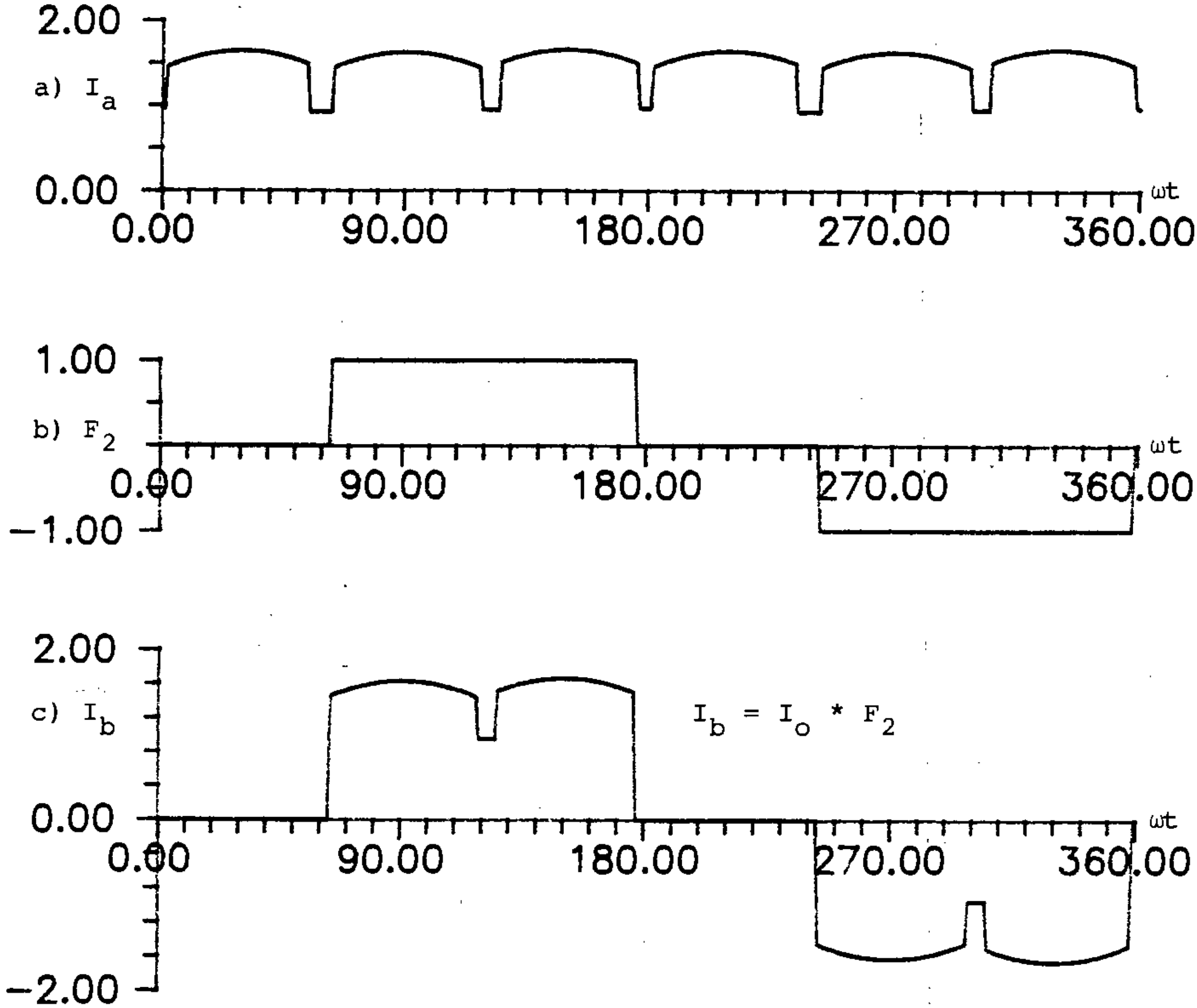


Fig. 5.5: Input current waveform, I_b obtained with CR under unbalanced condition.

a) Output current, I_o of CR.

b) Switching function, F_2.

c) Input current, I_b of CR.

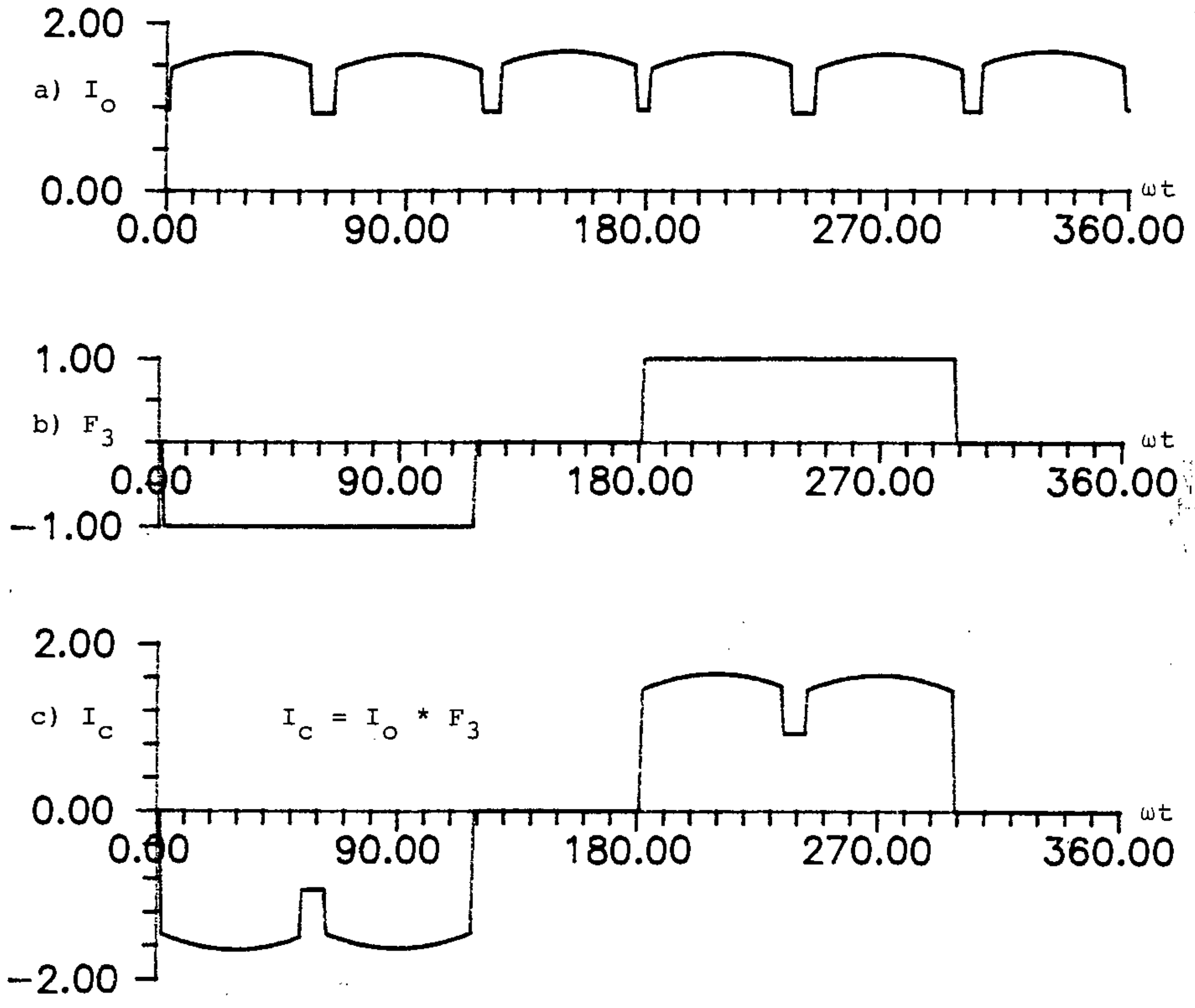


Fig. 5.6: Input current waveform, I_c obtained with CR under unbalanced condition.

a) Output current, I_o of CR.

b) Switching function, F_3.

c) Input current, I_c of CR.

Table 5.1									
Frequency spectra of waveforms Associated with unbalanced C.R. output voltage and Input currents shown in Fig 5.3 - 5.6 .									
Harmonic coefficients of switching function (Fig.5.3b-d)				Harmonic coefficients of output voltage V_{dc}(Fig.5.3e)		Harmonic coefficient of input currents I_a or I_b or I_c (Figs.5.4c,5.5c,5.6c)			
Order	Amplitude			Order	Amplitude	Order	Amplitude		
(n)	A_n	B_n	C_n	(n)	V_{dc}	(n)	I_a	I_b	I_c
1	1.03	1.06	1.08	0	1.65	1	0.52	0.53	0.54
3	0.13	0.09	0.04	2	0.03	3	0.07	0.05	0.02
5	0.25	0.25	0.24	4	0.01	5	0.13	0.13	0.12
7	0.06	0.09	0.13	6	0.29	7	0.03	0.05	0.07
9	0.11	0.08	0.04	8	0.05	9	0.06	0.04	0.02
11	0.09	0.11	0.11	10	0.02	11	0.05	0.06	0.06
13	0.03	0.01	0.05	12	0.09	13	0.02	0.01	0.03
15	0.08	0.07	0.04	14	0.03	15	0.04	0.04	0.02
17	0.02	0.06	0.07	16	0.04	17	0.01	0.03	0.04
19	0.05	0.02	0.03	18	0.11	19	0.03	0.01	0.02

(1) Input phase voltage and output current have taken as 1 P.U.

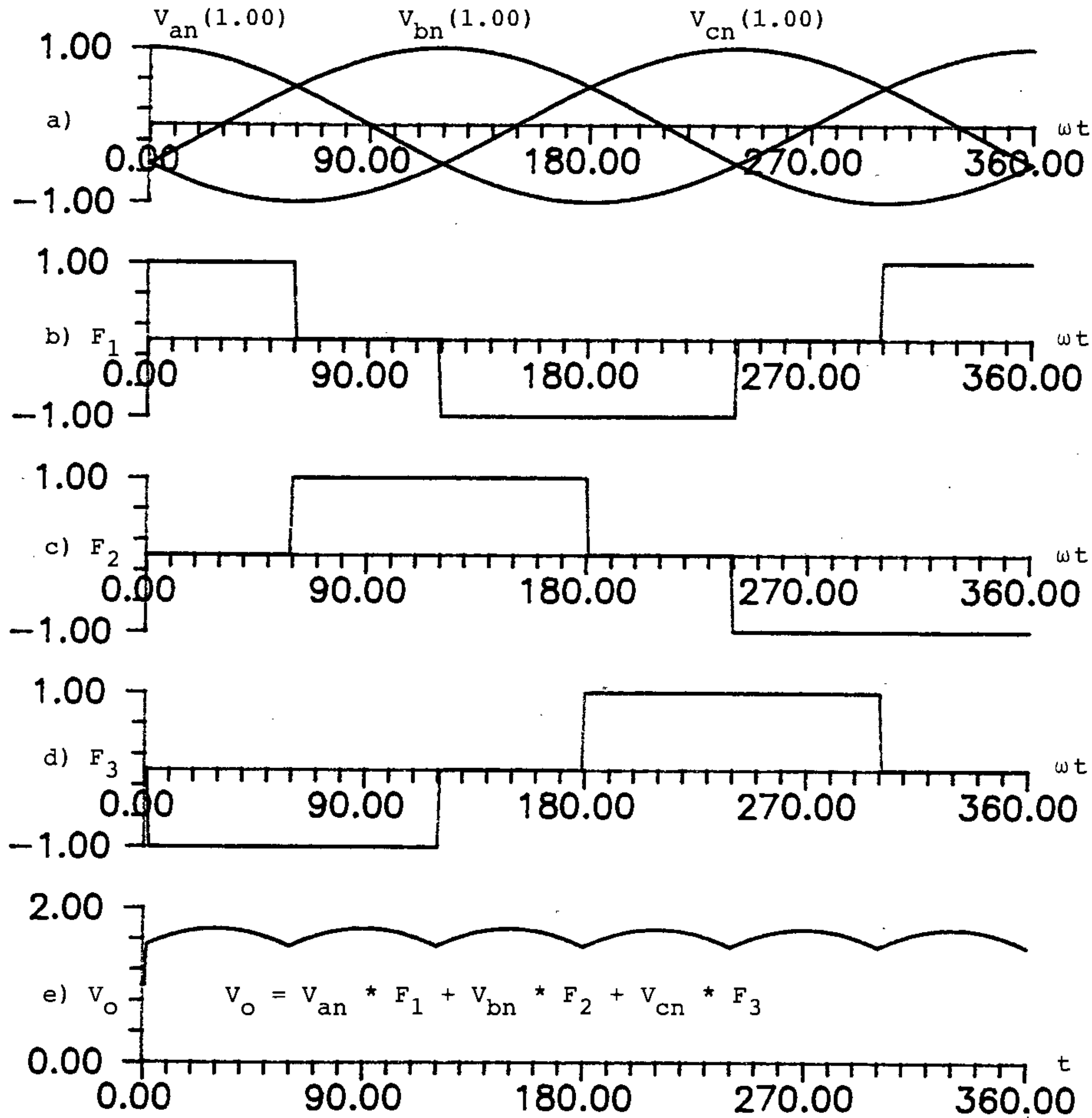


Fig. 5.7: Output voltage waveform, V_o obtained with CR under balanced input condition.

a) Three input balanced phase voltages.

b) - d) F_1, F_2, F_3 switching function components.

e) Output near perfect D.C. voltage, V_o.

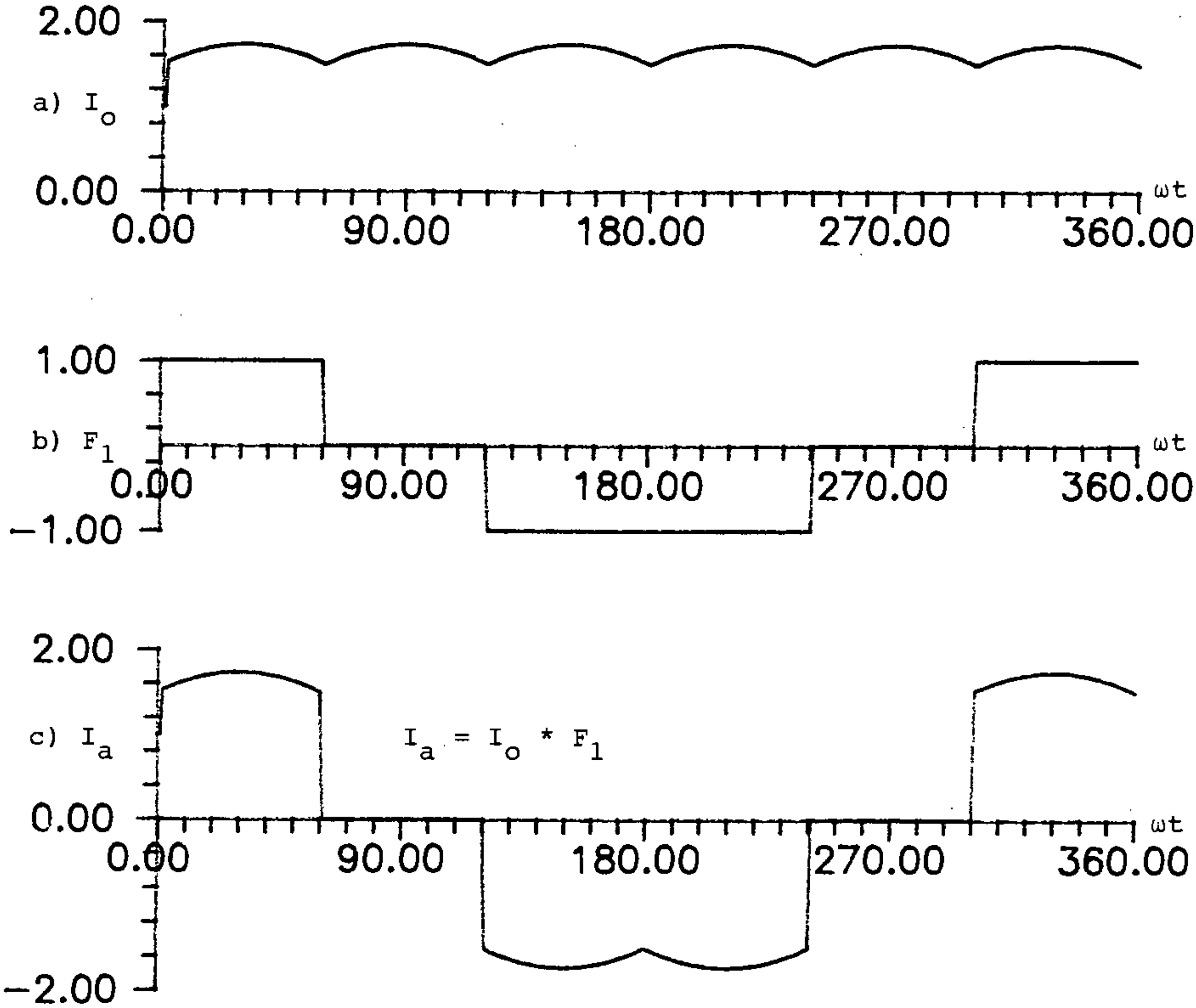


Fig. 5.8: Input current waveform, I_a obtained with CR under balanced condition.

a) Output current, I_o of CR.

b) Switching function, F_1.

c) Input current, I_a of CR.

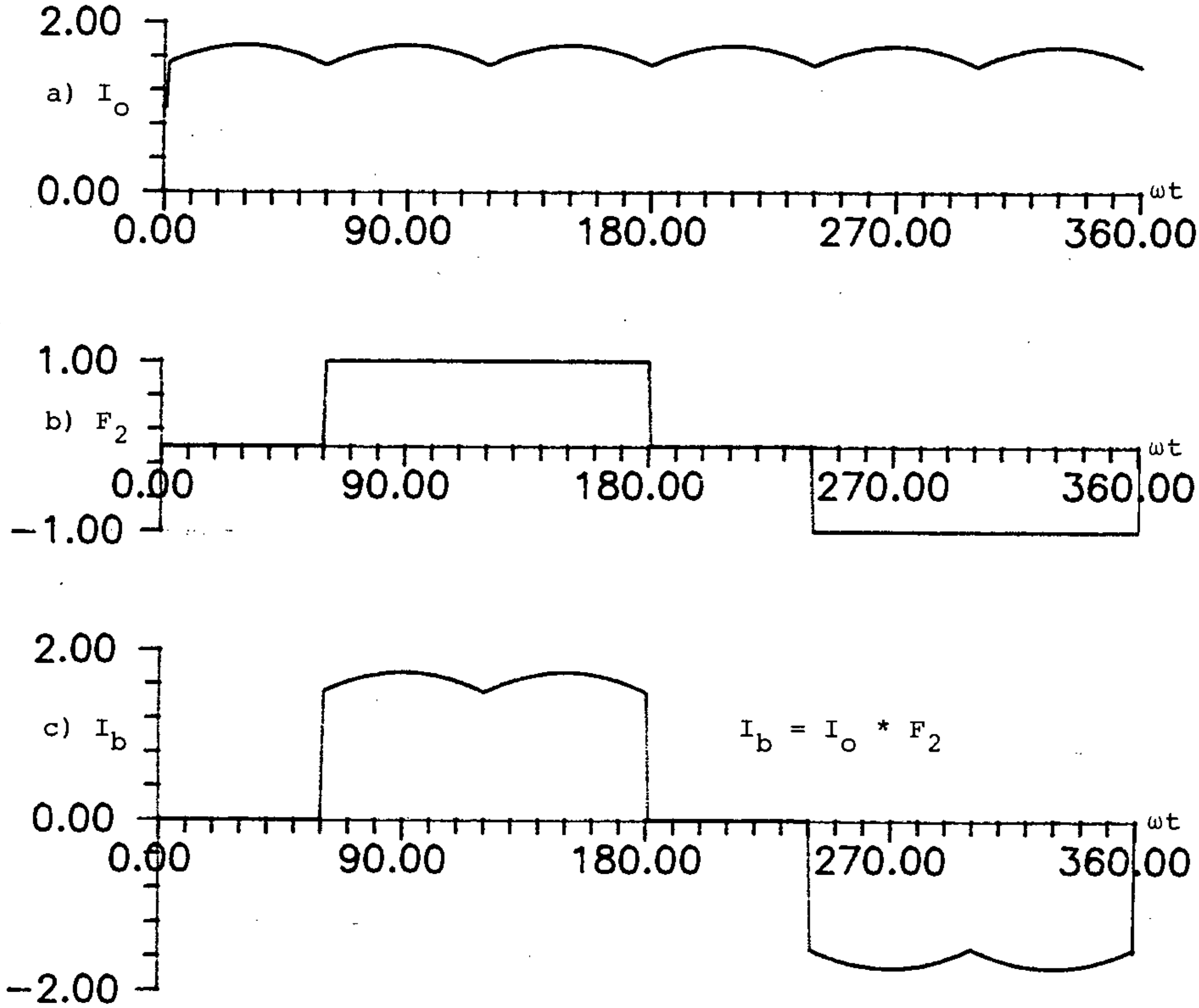


Fig. 5.9: Input current waveform, I_b obtained with CR under balanced condition.

a) Output current, I_o of CR.

b) Switching function, F_2.

c) Input current, I_o of CR.

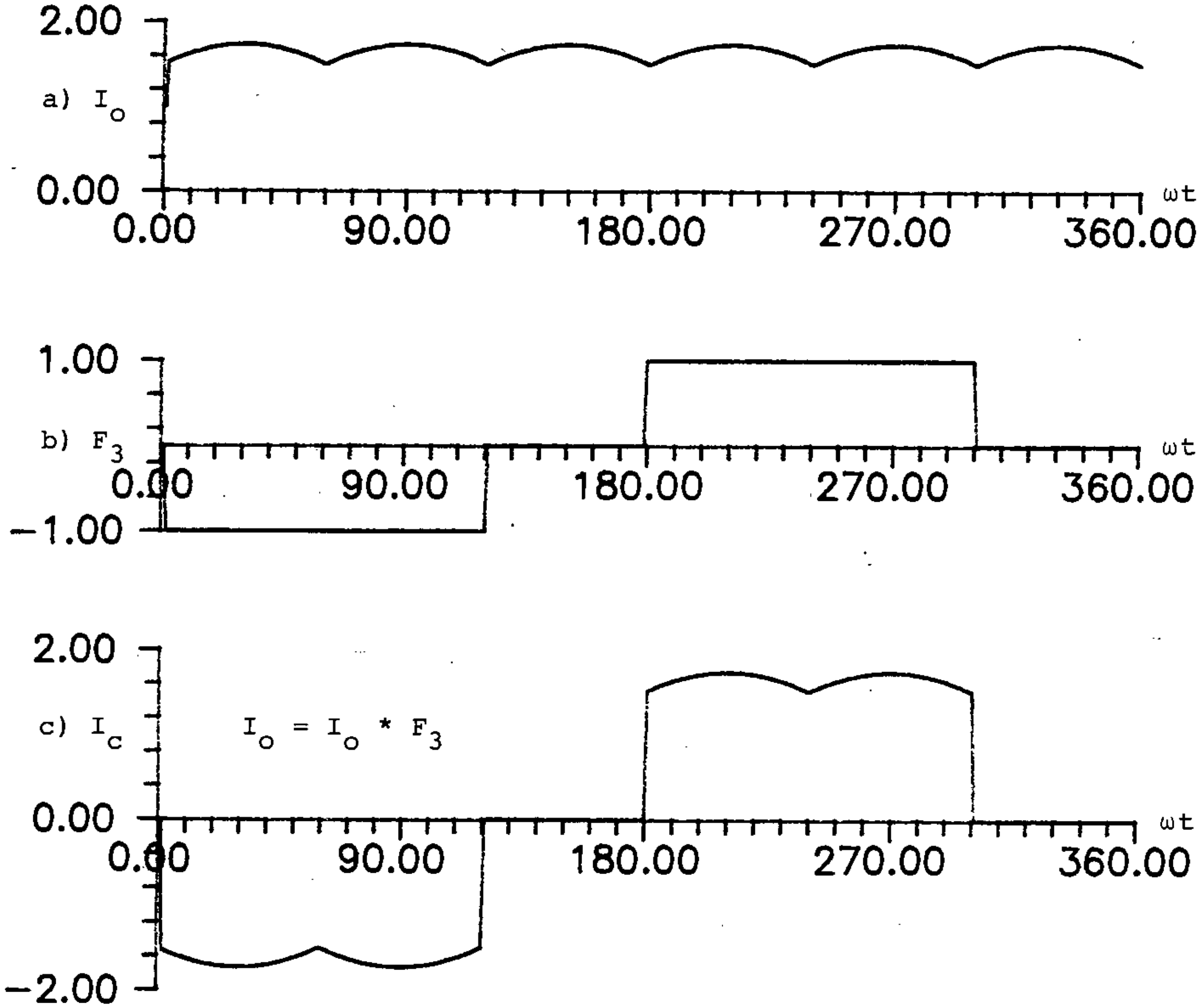


Fig. 5.10: Input current waveform, I_c obtained with CR under balanced condition.

a) Output current, I_o of CR.

b) Switching function, F_3.

c) Input current, I_c of CR.

Table 5.2					
Frequency spectra of waveforms associated with balanced controlled rectifier.					
Harmonic coefficients of switching function (Figs. 5.7b-d)		Harmonic coefficients of output voltage V_{dc}(Fig.5.7e)		Harmonic coefficient of input currents I_a or I_b or I_c (Figs.5.8c or 5.9c or 5.10c)	
Order (n)	Amplitude (A_n)	Order (n)	Amplitude (V_{dc})	Order (n)	Amplitude in p.u. (I_a or I_b or I_c)
1	1.10	0	1.65	1	0.55
3	0.00	2	0.00	3	0.00
5	0.22	4	0.00	5	0.11
7	0.16	6	0.36	7	0.08
9	0.00	8	0.00	9	0.00
11	0.10	10	0.00	11	0.05
13	0.09	12	0.18	13	0.05
15	0.00	14	0.00	15	0.00
17	0.07	16	0.00	17	0.03
19	0.06	18	0.11	19	0.03

(1) Input phase voltage and output current have taken as 1 P.U.

the opening and closing of the six CR switches in example to generate the output voltage and input current. Waveforms are shown in Figs. 5.11A, 5.12A, 5.13A, 5.14A. Further processing of these waveforms by using MATLAB package yields the respective frequency spectra as shown in Figs. 5.11B, 5.12B, 5.13B and 5.14B. Comperison between analytically predicted (Table 5.1) frequency spectra and spectra obtained by simulation shows that they are in close agreement.

5.5 Design Criteria

This CR requires only 6 gating signals for 6 bilateral switches. The swithces are composed of 4 diodes and a gate turn-off divice. In designing the digital control (firing) circuit the following need to be considered:

i) Deriving the 6 gating signals.

ii) Applying the 6 gating signals.

The 6 gating signal for the specific example are shown in Fig. 5.2. These six gating signals are applied directly to the gates of the six swithces in proper time relative to zero crossing of input voltages.

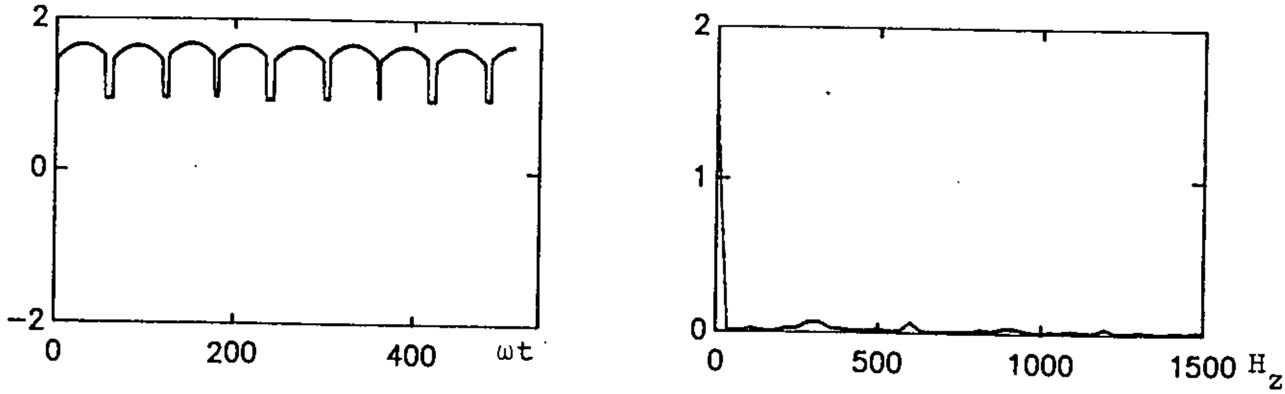


Fig. 5.11: Output voltage, V_o of CR (A) & its spectrum (B) under unbalanced condition.

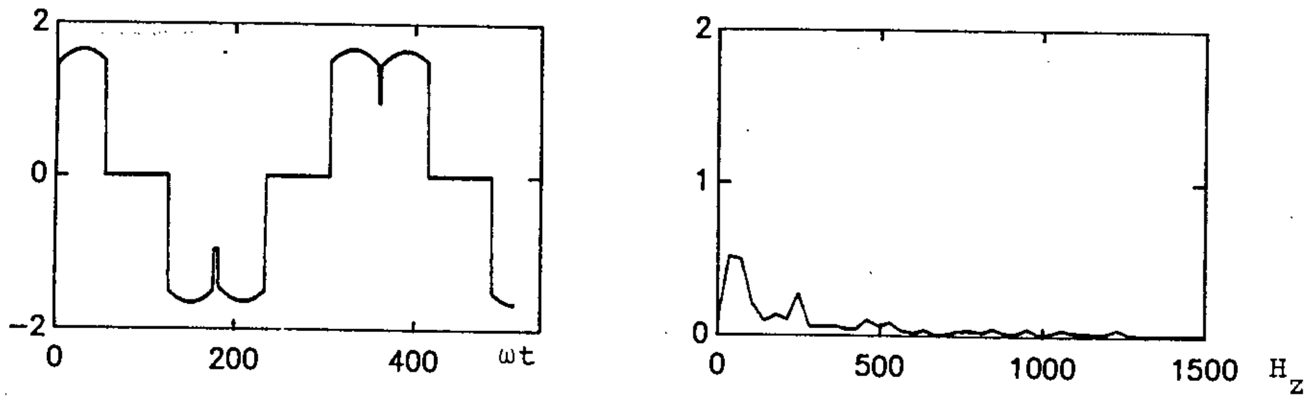


Fig. 5.12: Input current, I_a (A) & its spectrum (B) under unbalanced condition.

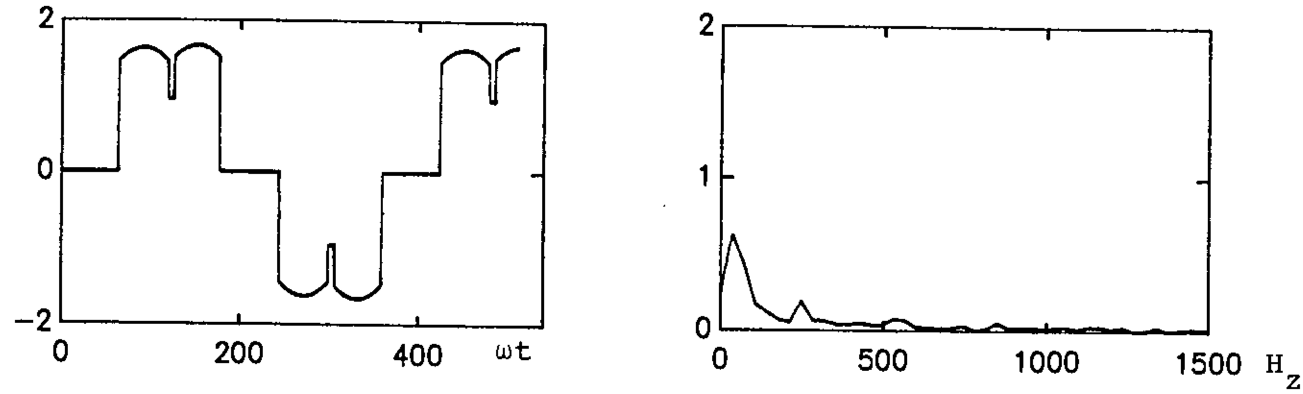


Fig. 5.13: Input current, I_b (A) & its spectrum (B) under unbalanced condition.

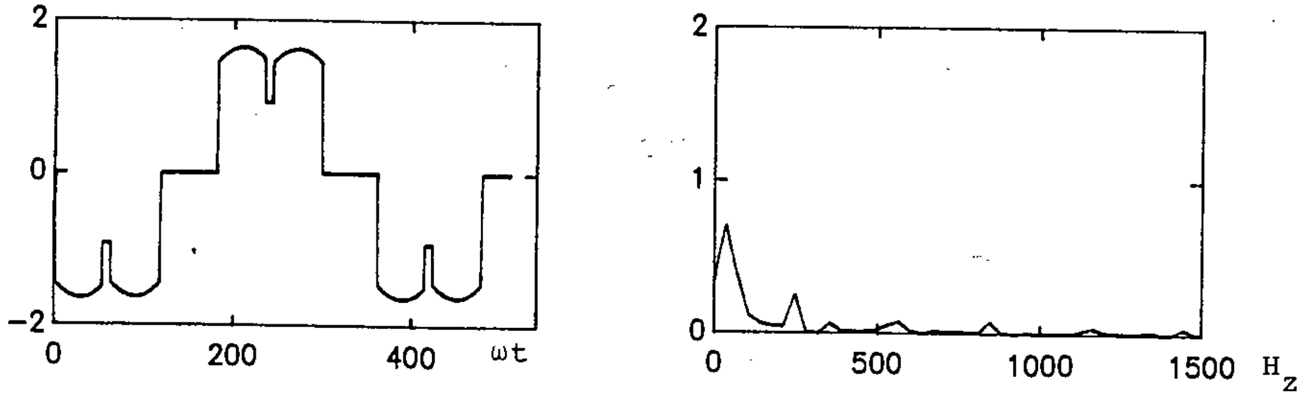


Fig. 5.14 Input current, I_c (A) & its spectrum (B) under unbalanced condition.

5.6 Conclusions

This chapter provides a comprehensive analysis of a three phase rectifier under amplitude unbalanced input voltages. A simple example is used to illustrate the validity of the principle and is supported by simulation.

CHAPTER 6

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary and Conclusions

A three phase converter under unbalanced input condition has been studied in this thesis . The converter is modelled and analysed using Fourier analysis . The technique used for correcting the input unbalance is proved to be effective.

Contributions of this thesis by chapter are as follows:

In Chapter 2 , the three phase converter model is derived using generalized matrix model. The mathematical relationship of input/output quantities are derived for both balanced and unbalanced input conditions.

Fourier analysis of the proposed converter for balanced and unbalanced conditions is provided in Chapter 3. To prove the effectiveness of the proposed technique the output and input voltages/currents are reconstructed using the predicted Fourier coefficients. Finally the waveforms are analysed in a personal computer using MATLAB package. The results proves the vality of the proposed technique.

In Chapter 4 operation of the converter and necessary design and protection data are provided. Input voltage sensing is used to synchronize the precise opening and closing of the converter switches. Microprocessor can also be used to correcy various amount of input unbalance.

Next, the search for using this proposed technique for frequency changer application has led to a novel unbalanced controlled rectifier is analysed using Fourier coefficients. Design data are also provided for its implementation.

6.2 Suggestion for Future Work

The analysis and design of an unbalanced three phase converter using a simple single switch is presented in this thesis. During this analysis only unbalance in amplitude is considered . Advanced PWM switching function may be used to study the improvement of quality (spectrum) of output voltages/currents considering both amplitude and phase unbalance operating condition. Further study may be carried out to investigate the validity of this voltage balancing technique for this converter as a frequency changer. Further study may also be focussed on the microprocessor based control circuitry and the filter circuit necessary for such a converter.

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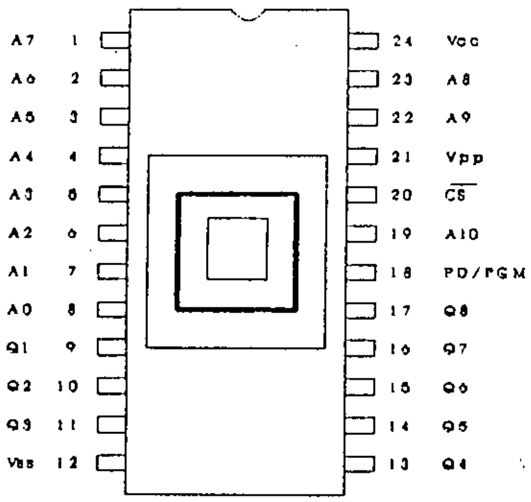
APPENDIX A
CIRCUIT IMPLEMENTATION

A.1 INTRODUCTION

In implementing a converter configuration the Erasable Programmable Read Only Memory (EPROM) can be used as a means of storing the gate signals applied to the different SCR's and power transistors. The gate signals can be stored in the memory locations of the EPROM by means of a device called the EPROM programmer or burner. A logic circuit can be used to retrieve the gate signals from the memory locations of the EPROM. Both the storing of gate signals into EPROM and retrieving of the same is discussed in this chapter.

A.2 EPROM

EPROM is a field-programmable and field erasable device. The package has a quartz window over the chip. Fig. A.1 shows a pictorial view of TMS 2516 or 2716 EPROM device. It is possible to erase this device by exposing it to ultraviolet light at or near a wavelength of 0.2537 μm for about a minimum of 21 minutes. Before programming, the device is erased. After erasure all bits are in the "1" state (assuming a high level output corresponding to logic "1").



(a) Top View

PIN NOMENCLATURE	
A(N)	Address Inputs
CS	Chip Select
PD/PGM	Power Down/Program
Q(N)	Input/Output
Vcc	+5V Power Supply
Vpp	+25V Power Supply
Vss	0 V Ground

(b)Pin Nomenclature

FIGURE A-1 TMS 2516 EPROM Device

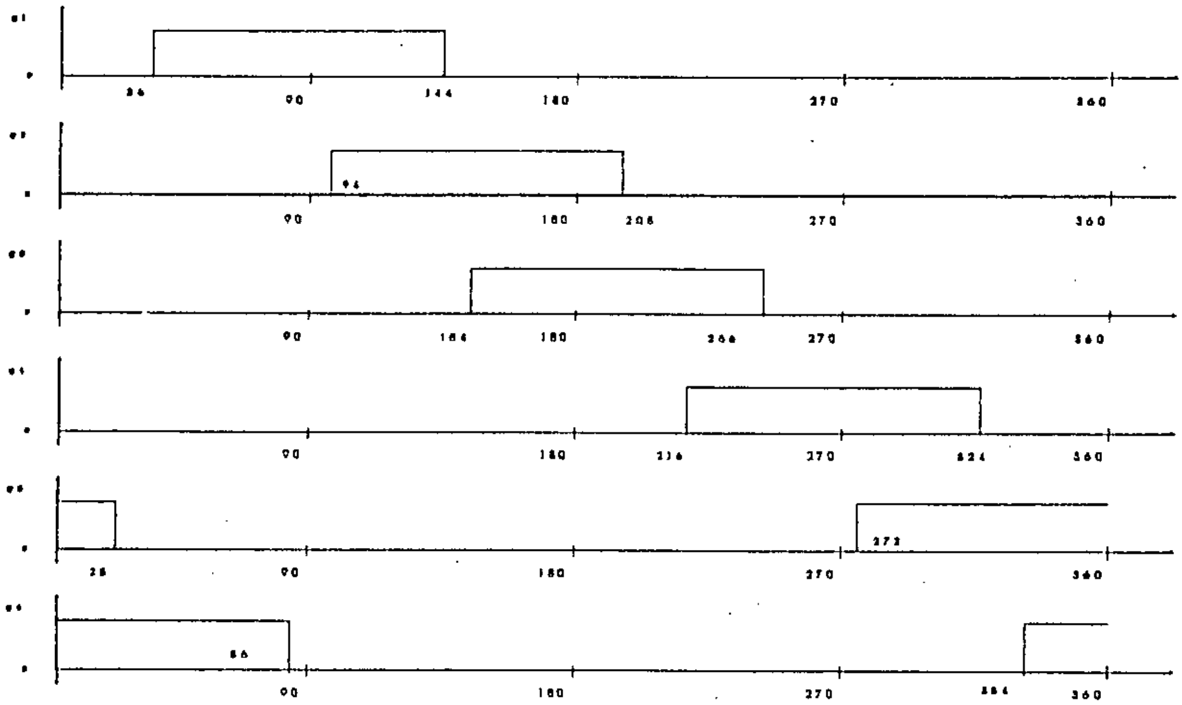


FIGURE A-2 Converter Gate Pulses Stored in the EPROM

After erasure logic "0's" are programmed into the desired locations. EPROM 2516 and 2716 are organized into 16K memory locations. A "0" can be erased only by Ultra Violet light. The programmable mode is achieved when V_{pp} is 25 volts and NOT CS is at V_{1H}. Data is presented in parallel (8 bits) on pins Q_1 through Q_8 . Once address and data are stable a 50ms TTL high level pulse should be applied to the PGM pin at each address location to be programmed. Programming is usually performed by placing the EPROM in a programmer or burner.

A.3 PROGRAMMING THE EPROM

The EPROM programmer used to store data on the different memory locations of an EPROM consists of an PC add on card, test socket box and software based on a IBM PC/XT/AT or compatible PC under PC-DOS or MS-DOS environment. The main software is EPROMX.EXE and using this software the EPROM can be programmed, read, verified, blank checked, and the memory buffer contents can be edited, displayed, printed etc. The EPROM 2716 and 2516 contains a total of 16K bits of memory space and each has 0 to 7FF (hex) or 0 to 2047 memory locations. Each location can store up to 8 bits of binary information. The EPROM programmer software consists of a built in editor through which the desired 8 bit data can be stored in the desired location. If the binary bits 00011100 (1C Hex) is to be stored in the memory location 6FF(Hex) of the EPROM then after running the EPROMx.EXE programe under the DOS prompt the editor is invoked and the data 1C (Hex) is written against the correspondig address 6FF. This memory area is on the PC system memory and is called the memory buffer. The buffer is allocated by main software through the arrangement of DOS. Any portion of this buffer can be saved to diskette or programmed to the EPROM.

The six gate signals as shown in fig. A.2 which are to be applied to a converter are stored in the 2048 memory locations of the EPROM 2516 or 2716 by dividing each signal into 2048 sub-divisions. Each of these sub-divisions contains either a "1" state or a "0" state according to the corresponding signal. Then for each of the 2048 8-bit location from 0 to 2047 the six gate signals g_1 through g_6 and two other signals g_7 and g_8, assumed to be zero at all time, are combined to form the 8-bit binary information to be stored in that location. As for example, suppose after dividing the gate signals into 2048 sub-divisions the gate signals corresponding to location 005 are g_1=0,g_2= 0,g_3=0,g_4=0,g_5=1,g_6=1, and g_7=g_8=0. So, at the memory location 005 of the EPROM the 8-bit byte 00001100 or 0C Hex corresponding the gate signals is to be stored. In this way every memory locaton of the EPROM can be programmed to store certain data. A computer program is developed to determine the 8-bit binary

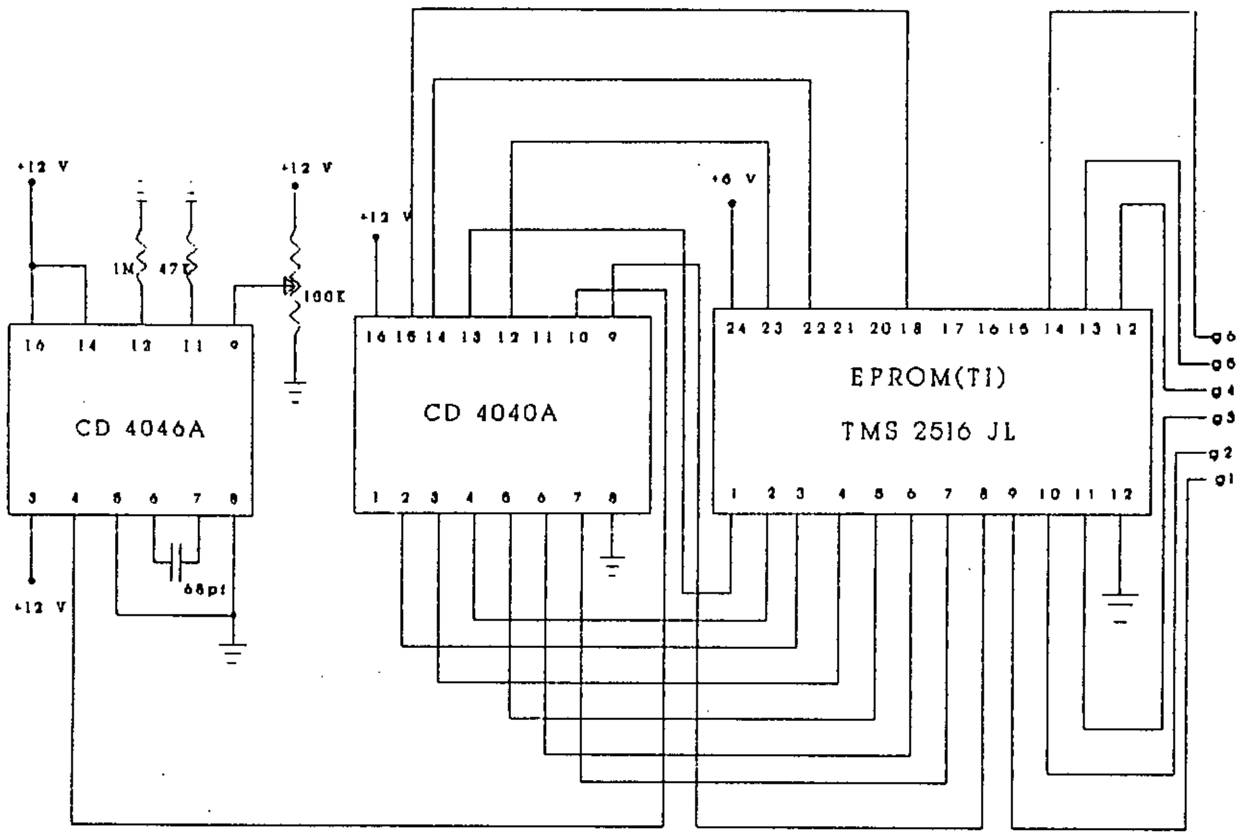


FIGURE A-3 Approximate Logic Circuit for Reading the EPROM digits corresponding to the six gate signals for each memory location from 0 TO 7FE (Hex) of the TMS 2516 or 2716 EPROM. The output of the program is a binary file containing the required 8-bit byte to be stored at each memory address. The binary file was loaded into the memory buffer of the PC using the EPROM main software and later loaded into the corresponding EPROM memory locations through the EPROM programmer or burner.

The programming of the EPROM is briefly as follows:-

1. Execution of the main software of the EPROM burner under DOS command.
2. Selection of the desired EPROM type numbe and corresponding manufacturer of the EPROM.
3. Placing of an erased EPROM on the master socket of the burner and blank-checking (all bits high) of the EPROM.
4. Loading of the desired contents of each memory locations into the memory buffer from diskette containing the binary file.
5. Programming the EPROM (i.e. loading the contents of memory buffer into the EPROM) by using the programming option of the main software of EPROM programmer.

A.4 RETRIEVING DATA FROM THE EPROM

Once the EPROM TMS 2516 or TMS 2716 has been programmed, the six gate signals, as discussed previously, can be retrieved by using an approximate logic circuit as shown in the figure A.3. The logic circuit consists of a counter and a clock. If the desired output frequency is 50 Hz then the clock has to be set to a frequency of 50*2048=102.4 KHz for reading all the 2048 memory locations in the required 50 Hz interval. The TMS 2516 or 2716 contains 2048(=2^{11}) memory locations; i.e. 11 address bits are required to obtain the content of each memory location. The CD 4040 is a 11 bit binary counter and changes the address of the 11 EPROM address pins at the clock frequency of 102.4 KHz. As the addresses of the EPROM memory location is continuously varied from 0 to 2047 the output pins of the EPROM 9,10,11,13,14,15 contains the gate signals previously stored, at the desired output frequency of 50 Hz.

APPENEDIX B

COMPUTER PROGRAM - 1

```
C     THIS PROGRAM IS DONE BY SADIQUR RAHMAN KHAN
C
C     DETERMINATION OF NECESSARY DATA TO PLOT OUTPUT VOLTAGES
C     AND  INPUT  CURRENTS  UNDER  BALANCED  INPUT  CONDITION

      DIMENSION TA(9),TB(9),TC(9),FA(3000),FB(4500),FC(5000),VA(5000)
      DIMENSION XA(5000),XB(5000),XC(5000),XDC1(5000),VB(5000),VC(5000)
      DIMENSION XDC2(5000),XDC3(5000)
      OPEN(UNIT=3,FILE='OUT9.DAT',STATUS='OLD')
      DEL1=120.0/2.0
      TA(1)=(DEL1/2.0)
      TA(2)=90.0-(DEL1/2.0)
      TA(3)=TA(2)+DEL1
      TA(4)=180.0-(DEL1/2.0)
      TA(5)=180.25
      I=1
      J=1
1     XF=.25*J
      FA(J)=1
      J=J+1
      IF(XF.LT.TA(I))GO TO 1
      FA(J)=0
      IF(I.EQ.5) GO TO 2
      I=I+1
      J=J
3     J=J+1
      XF=.25*J
      FA(J)=0
      IF(XF.LE.TA(I))GO TO 3
      I=I+1
      J=J
      IF(J. GE.720) GO TO 2
      GO TO 1
2     J=0
4     J=J+1
      IF (J.LE. 180) GO TO 5
      IF (J.GT. 540) GO TO 5
      FA(J)=-FA(J)
      GO TO 7
5     FA(J)=FA(J)
7     IF(J. EQ. 720) GO TO 6
      GO TO 4
6     J=J+1
```

```
      FA(J)=FA(J-720)
      IF(J. EQ. 1440) GO TO 8
      GO TO 6
8     DEL2=120.0/2.0
      TB(1)=60.0+(DEL2/2.0)
      TB(2)=90.0-(DEL2/2.0)+60.0
      TB(3)=TB(2)+DEL2
      TB(4)=180.0-(DEL2/2.0)+60.0
      TB(5)=180.25+60.0
      I=1
      J=241
11    XF=.25*J
      FA(J)=1
      J=J+1
      IF(XF.LT.TB(I))GO TO 11
      FB(J)=0
      IF(I.EQ.5) GO TO 12
      I=I+1
      J=J
13    J=J+1
      XF=.25*J
      FB(J)=0
      IF(XF.LE.TB(I))GO TO 13
      I=I+1
      J=J
      IF(J.GE.960) GO TO 12
      GO TO 11
12    J=240
14    J=J+1
      IF(J.LE.420)GO TO 15
      IF(J.GT.780) GO TO 15
      FB(J)=-FB(J)
      GO TO 17
15    FB(J)=FB(J)
17    IF(J.EQ.960) GO TO 16
      GO TO 14
16    J=720
18    J=J+1
      FB(J-720)=FB(J)
      IF(J.EQ.960) GO TO 19
      GO TO 18
19    J=240
20    J=J+1
      FB(J)=FB(J)
      IF(J.EQ.720) GO TO 50
      GO TO 20
50    J=J+1
      FB(J)=FB(J-720)
```

```
      IF(J.EQ.1920) GO TO 60
      GO TO 50
 60   DEL3=120.0/2.0
      TC(1)=120.0+(DEL3/2.0)
      TC(2)=90.0-(DEL3/2.0)+120.0
      TC(3)=TC(2)+DEL3
      TC(4)=180.0-(DEL3/2.0)+120.0
      TC(5)=180.25+120.0
      I=1
      J=481
 21   XF=.25*J
      FC(J)=1
      J=J+1
      IF (XF .LT. TC(I)) GO TO 21
      FC(J)=0
      IF(I. EQ. 5) GO TO 22
      I=I+1
      J=J
 23   J=J+1
      XF=.25*J
      FC(J)=0
      IF (XF .LE. TC(I)) GO TO 23
      I=I+1
      J=J
      IF (J. GE. 1200) GO TO 22
      GO TO 21
 22   J=480
 24   J=J+1
      IF (J .LE. 660) GO TO 25
      IF (J .GT. 1020) GO TO 25
      FC(J)=-FC(J)
      GO TO 27
 25   FC(J)=FC(J)
 27   IF(J .EQ. 1200) GO TO 26
      GO TO 24
 26   J=720
 28   J=J+1
      FC(J-720)=FC(J)
      IF(J .EQ. 1200) GO TO 29
      GO TO 28
 29   J=480
 30   J=J+1
      FC(J)=FC(J)
      IFC( J .EQ. 720) GO TO 61
      GO TO 30
 61   J=J+1
      FC(J)=FC(J-720)
      IF(J .EQ. 2400) GO TO 100
```

```
      GO TO 61
100   J=1
      PI=3.141592654
31    YA=.25*J
      VA(J)=1.00*COS((YA*PI)/180.0)
      XA(J)=FA(J)*VA(J)
      YB=YA-120.0
      VB(J)=1.00*COS((YB*PI)/180.0)
      XB(J)=FB(J)*VB(J)
      YC=YA-240.0
      VC(J)=1.00*COS((YC*PI)/180.0)
      XC(J)=FC(J)*VC(J)
      XDC1(J)=XA(J)+XB(J)+XC(J)
      XDC2(J)=FC(J)*VA(J)+FA(J)*VB(J)+FB(J)*VC(J)
      XDC3(J)=FB(J)*VA(J)+FC(J)*VB(J)+FA(J)*VC(J)
      WRITE(3,230)YA,FA(J),FB(J),FC(J),XDC1(J), XDC2(J), XDC3(J)
230   FORMAT(1X,F7.2,10(1X,F6.3)
      IF(J .EQ. 1440) GO TO 200
      J=J+1
      GO TO 31
200   STOP
      END
```

COMPUTER PROGRAM - 2

```
C     THIS PROGRAM IS DONE BY SADIQUR RAHMAN KHAN
C
C     DETERMINATION OF NECESSARY DATA TO PLOT OUTPUT VOLTAGES
C     AND INPUT CURRENTS  UNDER  UNBALANCED  INPUT  CONDITION

      DIMENSION TA(9),TB(9),TC(9),FA(3000),FB(4500),FC(5000),VA(5000)
      DIMENSION XA(5000),XB(5000),XC(5000),XDC1(5000),VB(5000),VC(5000)
      DIMENSION XDC2(5000),XDC3(5000)
      OPEN(UNIT=3,FILE='OUT9.DAT',STATUS='OLD')
      DEL1=116.4/2.0
      TA(1)=(DEL1/2.0)
      TA(2)=90.0-(DEL1/2.0)
      TA(3)=TA(2)+DEL1
      TA(4)=180.0-(DEL1/2.0)
      TA(5)=180.25
      I=1
      J=1
1     XF=.25*J
      FA(J)=1
      J=J+1
      IF(XF.LT.TA(I))GO TO 1
      FA(J)=0
      IF(I.EQ.5) GO TO 2
      I=I+1
      J=J
3     J=J+1
      XF=.25*J
      FA(J)=0
      IF(XF.LE.TA(I))GO TO 3
      I=I+1
      J=J
      IF(J. GE.720) GO TO 2
      GO TO 1
2     J=0
4     J=J+1
      IF (J.LE. 180) GO TO 5
      IF (J.GT. 540) GO TO 5
      FA(J)=-FA(J)
      GO TO 7
5     FA(J)=FA(J)
7     IF(J. EQ. 720) GO TO 6
      GO TO 4
6     J=J+1
```

```
      FA(J)=FA(J-720)
      IF(J. EQ. 1440) GO TO 8
      GO TO 6
8     DEL2=114.0/2.0
      TB(1)=60.0+(DEL2/2.0)
      TB(2)=90.0-(DEL2/2.0)+60.0
      TB(3)=TB(2)+DEL2
      TB(4)=180.0-(DEL2/2.0)+60.0
      TB(5)=180.25+60.0
      I=1
      J=241
11    XF=.25*J
      FA(J)=1
      J=J+1
      IF(XF.LT.TB(I))GO TO 11
      FB(J)=0
      IF(I.EQ.5) GO TO 12
      I=I+1
      J=J
13    J=J+1
      XF=.25*J
      FB(J)=0
      IF(XF.LE.TB(I))GO TO 13
      I=I+1
      J=J
      IF(J.GE.960) GO TO 12
      GO TO 11
12    J=240
14    J=J+1
      IF(J.LE.420)GO TO 15
      IF(J.GT.780) GO TO 15
      FB(J)=-FB(J)
      GO TO 17
15    FB(J)=FB(J)
17    IF(J.EQ.960) GO TO 16
      GO TO 14
16    J=720
18    J=J+1
      FB(J-720)=FB(J)
      IF(J.EQ.960) GO TO 19
      GO TO 18
19    J=240
20    J=J+1
      FB(J)=FB(J)
      IF(J.EQ.720) GO TO 50
      GO TO 20
50    J=J+1
      FB(J)=FB(J-720)
```

```
      IF(J.EQ.1920) GO TO 60
      GO TO 50
60    DEL3=111.6/2.0
      TC(1)=120.0+(DEL3/2.0)
      TC(2)=90.0-(DEL3/2.0)+120.0
      TC(3)=TC(2)+DEL3
      TC(4)=180.0-(DEL3/2.0)+120.0
      TC(5)=180.25+120.0
      I=1
      J=481
21    XF=.25*J
      FC(J)=1
      J=J+1
      IF (XF .LT. TC(I)) GO TO 21
      FC(J)=0
      IF(I. EQ. 5) GO TO 22
      I=I+1
      J=J
23    J=J+1
      XF=.25*J
      FC(J)=0
      IF (XF .LE. TC(I)) GO TO 23
      I=I+1
      J=J
      IF (J. GE. 1200) GO TO 22
      GO TO 21
22    J=480
24    J=J+1
      IF (J .LE. 660) GO TO 25
      IF (J .GT. 1020) GO TO 25
      FC(J)=-FC(J)
      GO TO 27
25    FC(J)=FC(J)
27    IF(J .EQ. 1200) GO TO 26
      GO TO 24
26    J=720
28    J=J+1
      FC(J-720)=FC(J)
      IF(J .EQ. 1200) GO TO 29
      GO TO 28
29    J=480
30    J=J+1
      FC(J)=FC(J)
      IFC( J .EQ. 720) GO TO 61
      GO TO 30
61    J=J+1
      FC(J)=FC(J-720)
      IF(J .EQ. 2400) GO TO 100
```

```
      GO TO 61
100   J=1
      PI=3.141592654
31    YA=.25*J
      VA(J)=0.97*COS((YA*PI)/180.0)
      XA(J)=FA(J)*VA(J)
      YB=YA-120.0
      VB(J)=0.95*COS((YB*PI)/180.0)
      XB(J)=FB(J)*VB(J)
      YC=YA-240.0
      VC(J)=0.93*COS((YC*PI)/180.0)
      XC(J)=FC(J)*VC(J)
      XDC1(J)=XA(J)+XB(J)+XC(J)
      XDC2(J)=FC(J)*VA(J)+FA(J)*VB(J)+FB(J)*VC(J)
      XDC3(J)=FB(J)*VA(J)+FC(J)*VB(J)+FA(J)*VC(J)
      WRITE(3,230)YA,FA(J),FB(J),FC(J),XDC1(J), XDC2(J), XDC3(J)
230   FORMAT(1X,F7.2,10(1X,F6.3)
      IF(J .EQ. 1440) GO TO 200
      J=J+1
      GO TO 31
200   STOP
      END
```

```
C                   COMPUTER  PROGRAM -3
C
C     THIS PROGRAM IS DONE BY SADIQUR RAHMAN KHAN
C
C     DETERMINATION OF NECESSARY DATA TO PLOT GATING SIGNALS
C
      DIMENSION X(725),Z(725),F1(725),F2(725),F3(725),F4(725)
     1,F5(725),F6(725)
      DO 11 M=0,720
      X(M)=FLOAT(M)/2.0
   11 CONTINUE
      DO 22 I=0,720
      A=FLOAT(I)/2.0
      IF(A.LE.36.)THEN
      F1(I)=0.0
      ELSEIF((A.GT.36.).AND.(A.LT.144.))THEN
      F1(I)=1.0
      ELSEIF((A.GE.144.).AND.(A.LE.360.))THEN
      F1(I)=0.0
      ENDIF
   22 CONTINUE
      DO 33 J=0,720
      B=FLOAT(J)/2.0
      IF(B.LE.94.)THEN
      F2(J)=0.0
      ELSEIF((B.GT.94.).AND.(B.LT.206.))THEN
      F2(J)=1.0
      ELSEIF((B.GE.206.).AND.(B.LE.360.))THEN
      F2(J)=0.0
      ENDIF
   33 CONTINUE
      DO 44 K=0,720
      C=FLOAT(K)/2.0
      IF(C.LE.154.)THEN
      F3(K)=0.0
      ELSEIF((C.GT.154.).AND.(C.LT.266.))THEN
      F3(K)=1.0
      ELSEIF((C.GE.266.).AND.(C.LE.360.))THEN
      F3(K)=0.0
      ENDIF
   44 CONTINUE
      DO 23 I=0,720
      D=FLOAT(I)/2.0
      IF(D.LE.216.)THEN
      F4(I)=0.0
      ELSEIF((D.GT.216.).AND.(D.LT.324.))THEN
      F4(I)=1.0
      ELSEIF((D.GE.324.).AND.(D.LE.360.))THEN
```

```
      F4(I)=0.0
      ENDIF
   23 CONTINUE
      DO 34 J=0,720
      E=FLOAT(J)/2.0
      IF(E.LE.28.)THEN
      F5(J)=1.0
      ELSEIF((E.GT.28.).AND.(E.LT.272.))THEN
      F5(J)=0.0
      ELSEIF((E.GE.272.).AND.(E.LE.360.))THEN
      F5(J)=1.0
      ENDIF
   34 CONTINUE
      DO 45 K=0,720
      F=FLOAT(K)/2.0
      IF(F.LE.86.)THEN
      F6(K)=1.0
      ELSEIF((F.GT.86.).AND.(F.LT.334.))THEN
      F6(K)=0.0
      ELSEIF((F.GE.334.).AND.(F.LE.360.))THEN
      F6(K)=1.0
      ENDIF
   45 CONTINUE
      DO 55 L=0,720
      Z(L)=X(L)
      WRITE(3,20)Z(L),F1(L),F2(L),F3(L),F4(L),F5(L),F6(L)
   20 FORMAT(1X,F6.2,2X,3(F4.1,2X),3(F4.1,2X))
   55 CONTINUE
      STOP
      END
```