Analysis and Reduction of PAPR Effect using Linear Coding in OFDM system

by

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Mukta Rani Dey

DEDICATED TO MY PARENTS

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LIST OF ABBREVIATIONS

BPSK	Binary phase shift keying
QPSK	Quadrature phase shift keying
BER	Bit Error Rate
OFDM	Orthogonal Frequency Division Multiplexing
PAPR	Peak to average power ratio
FFT	Fast Fourier Transform
IFFT	Inverse Fast Fourier Transform
BER	Bit Error Rate
SNR	Signal-to-Noise Ratio
CCDF	Complementary Cumulative Distribution Function
PDF	Probability Density Function
QAM	Quadrature Amplitude Modulation
ERFC	Complementary Error Function
LDPC	Low-Density Parity-Check
FEC	Forward Error Correction
MIMO	Multiple Input Multiple Output
ICI	Inter Carrier Interference
ISI	Inter Symbol Interference
MCM	Multi Carrier Modulation

LIST OF SYMBOLS

a_i	Modulator Output
R_b	Information bit rate
T_b	Information bit period
Pb_1	Codded bit error probability for 1st system
$P_{e,block}$	Block error probability
T	OFDM symbol duration
T_S	Symbol period
N_{24}	Number of sample per day
M_{24}	Number of codded block per day
L	Modulation width
N	Total number of subcarriers
V	Number of sub block
P_m	Phase factor
Z	Threshold
$\frac{E_b}{N_0}$	System SNR
$Q(\sqrt{\cdot})$	Q-function
E_c	Coded signal
N_0	Receiver Noise Level

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ABSTRACT

A high peak-to-average power ratio (PAPR) is one of the major shortcomings in Orthogonal Frequency Division Multiplexing (OFDM) systems, as it causes nonlinearity distortion in the transmitter, degrading the performance of the system significantly. To overcome PAPR effects for OFDM signals, there are a number of proposed PAPR reduction techniques such as amplitude clipping, clipping and peak windowing, coding, Selective Mapping Technique (SLM), Partial Transmit Sequence (PTS) etc. However, all the proposed methods have some drawbacks, such as out-of-band radiation for clipping method, side information transmission for SLM and PTS methods and low coding rates for coding implementing methods. In this thesis, reduction of the PAPR of OFDM is studied. A new PAPR reduction method is proposed using linear coding technique. The new method does not distort the signal that increases out-of-band radiation, instead uses block coding of the input data and then modifies the coded data iteratively until the PAPR reduces below a certain level. In the proposed method, at first, an input data block is partitioned into sub-blocks. The bits of each sub-block are encoded using Hamming Code. The encoded data are fed to the modulator input. Outputs of Modulator are fed into IFFT blocks which form regular OFDM symbols. The PAPR values of OFDM symbols are calculated and compared with a certain threshold. If the PAPR value is below the threshold, no operation is performed on the symbol and it is directly passed to the output. However, if PAPR exceeds the threshold, the algorithm is applied to that symbol until the PAPR is reduced below threshold. The proposed technique shows better reduction performance compared to the previously proposed techniques which combat the PAPR, such as clipping and filtering, Selective mapping, LDPC code. The findings of this work may be help in the designing of efficient and effective wireless communication with lower PAPR effect.

CHAPTER 1

INTRODUCTION

1.1 Introduction

With the development of modern electrical and computer technologies, the demand for fast and reliably transmitting multimedia information over wired or wireless channels is increasing rapidly. High-speed communications must efficiently use a band limited channel to obtain a high bit-rate and must combat channel noise, distortion, fading, etc., to maintain a low Bit-Error Rate (BER) [1]. Multicarrier Modulation (MCM) is a promising technique for high-speed communications. Multi-carrier modulation (MCM) is a method of transmitting data by splitting it into several components, and sending each of these components over separate carrier signals. The individual carriers have narrow bandwidth, but the composite signal can have broad bandwidth. MCM was first used in analog military communications in the 1960s. Recently, MCM has attracted attention as a means of enhancing the bandwidth of digital communications over media with physical limitations. In a classical MCM system, the total signal frequency band is divided into N nonoverlapping frequency subchannels. Each subchannel is modulated with a separate symbol and then the N subchannels are frequency multiplexed. Spectral overlap of channels is avoided to eliminate inter channel interference. However, this leads to inefficient use of the available spectrum. To cope with the inefficiency, the ideas proposed from the 1960s were to use MCM and Frequency Division Multiplexing (FDM) with overlapping subchannels [2].

Orthogonal Frequency Division Multiplexing (OFDM) is a special case of multicarrier transmission. The word orthogonal indicates that there is a mathematical relationship between the frequencies of carriers in the system. In a normal MCM system, many carriers are spaced apart in such a way that the signals can be received using conventional filters and demodulators. In such receivers, guard bands are introduced between different carriers and in frequency domain which results in reduction of spectrum efficiency. In OFDM, the carriers are arranged such that the frequency spectrum of the individual carriers overlap and the signals are still received without adjacent carrier interference. In order to achieve this, the carriers are chosen to be mathematically orthogonal. Since 1990s, OFDM is used for wideband data communications over mobile radio FM channels, High-bit-rate Digital Subscriber Lines (HDSL, 1.6Mbps), Asymmetric Digital Subscriber Lines (ADSL, up to 6Mbps), Very-high-speed Digital Subscriber Lines (VDSL, 100Mbps), Digital Audio Broadcasting (DAB), and High Definition Television (HDTV) terrestrial broadcasting [3].

OFDM has many advantages over single carrier systems. The implementation complexity of OFDM is significantly lower than that of a single carrier system with equalizer. When the transmission bandwidth exceeds coherence bandwidth of the channel, resultant distortion may cause inter symbol interference (ISI). Single carrier systems solve this problem by using a linear or nonlinear equalization. The problem with this approach is the complexity of effective equalization algorithms. OFDM systems divide available channel bandwidth into a number of subchannels. By selecting the sub channel bandwidth smaller than the coherence bandwidth of the frequency selective channel, the channel appears to be nearly flat and no equalization is needed. Also by inserting a guard time at the beginning of OFDM symbol during which the symbol is cyclically extended, inter symbol interference (ISI) and inter carrier interference (ICI) can be completely eliminated, if the duration of guard period is properly chosen. This property of OFDM makes the single frequency networks possible. In single frequency networks, transmitters simultaneously broadcast at the same frequency, which causes inter symbol interference. Additionally, in relatively slow time varying channels, it is possible to significantly enhance the capacity by adapting the data rate per subcarrier according to the signal-to-noise ratio (SNR) of that particular subcarrier [4]. Another advantage of OFDM over single carrier systems is its robustness against narrowband interference because such interference affects only a small percentage of the subcarriers.

Beyond all these advantages, OFDM has some drawbacks compared to single carrier systems. Two of the problems with OFDM are the carrier phase noise and frequency offset. Carrier phase noise is caused by imperfections in the transmitter and receiver oscillators. Frequency offsets are created by differences between oscillators in transmitter and receiver. For single carrier systems, phase noise and frequency offsets only give degradation in the receiver SNR, rather than introducing ICI. That is why the sensitivity to frequency offsets and phase noise are mentioned as disadvantages of OFDM relative to single carrier systems. The most important disadvantage of OFDM systems is that highly linear RF amplifiers are needed. An OFDM signal consists of a number of independently modulated subcarriers, which can give a large Peak-to-Average Power Ratio (PAPR) when added up coherently. When N signals are added with the same phase, they produce a peak power that is N times the average power. In order to avoid nonlinear distortion, highly linear amplifiers are required which cause a severe reduction in power efficiency.

1.2 Motivations and Review of Previous Works

Over the years, researchers have proposed various methods to combat the PAPR. We have discussed some contribution of different authors related to PAPR and its reduction in this section.

Most widely used methods are clipping and peak windowing the OFDM signal when a high PAPR is encountered. However these methods distort the original OFDM signal resulting in an increase in the bit error probability [5,6].

There are other methods that do not distort the signal. Two of these methods can be listed as Selected Mapping [7] and Partial Transmit Sequences [8,9,10,11]. The principle behind these methods is to transmit the OFDM signal with the lowest PAPR value among a number of candidates all of which represent the same information.

Coding is another commonly used method. In this case the information bits are coded in a way that no high peaks are generated. The core of encoding method is to apply special forward error correction technique to remove the OFDM signals with high PAPR. Mainly Golay codes, Reed Muller codes and linear block codes are used [12,13,14,15].

K. Yang et al. [16] experimentally use the standard array of a linear block code in order to reduce the PAPR value of OFDM, not for error correction.

Eltholth et al. [17] suggests PEP (Peak Envelope Power) values for all possible 4 bit binary sequences normalizing the power of individual subcarriers to 1 W. Then codewords with the highest PEP values are determined.

Daoud et al. [18] suggests to split the total number of channels and applying a complementary code to each group of subchannels increasing the code rate at the cost of error correction capability and PAPR reduction.

Mukunthan et al. [19] mentioned that the correlation properties of Golay complementary sequences translate into a relatively small PAPR of 3 dB when the codes are used to modulate an OFDM signal. For OFDM with a large number of channels, it may not be feasible to generate a sufficient number of complementary codes with a length equal to the number of channels.

From the above literatures review, we observe that there is a deficiency in the performance evaluation criterion. This thesis provides a means of performance evaluation of a PAPR reducing algorithm.

In this thesis a method based on coding is proposed to reduce the PAPR value of an OFDM signal. The method uses Hamming coding and the error correction capability of the code. In the literature there are a number of similar methods proposed to reduce the PAPR value. Their performance is evaluated according to the final PAPR value achieved only. However, some of the methods result in an increase in Bit Error Rate (BER) and some of the methods require transmission of side information on a secure channel for the correct reception of the transmitted data. These effects need to be considered for accurate performance calculations.

1.3 Objective of the Thesis

The main goal of this research work is to analyze the PAPR effect in OFDM. To meet this goal, the following objectives are identified:

1. To develop analytical model to reduce PAPR using linear coding technique.
2. To derive expressions of PAPR threshold, bit error probability (BEP), block error probability and SNR for QPSK modulation format.
3. To evaluate the PAPR reduction performance through numerical simulation.
4. To compare bit error probability, block error probability and SNR of coding techniques with standard system.

1.4 Organization of the Thesis

This thesis is organized in five chapters as follows:

Chapter-1 is an introductory chapter.

Chapter-2 is a rather detailed overview of OFDM. Main equations are derived and main techniques of OFDM such as windowing, guard time & cyclic prefixing, synchronization, bit loading and PAPR are explained. Afterwards, applications of OFDM are discussed.

Chapter-3 provides a deep insight into the PAPR problem by discussing the methods used in the literature. Advantages and disadvantages of each method are discussed.

Chapter-4, the PAPR reduction method suggested in the thesis is explained. We suggest a method to compare the algorithm implementing transmission system with a plain OFDM modulating system under a very low clipping rate condition. Detailed block diagrams are given to clarify how the algorithm works. Derivations of the performance comparison equations are provided in this chapter.

Chapter-5 contains the graphs explaining system behavior. Additionally, this chapter provides the numerical results obtained by algorithm implementation and system comparison.

Finally, Chapter-6 includes some concluding remarks.

CHAPTER 2

OVERVIEW OF OFDM

In this chapter, first the general structure of an OFDM system will be mentioned. The second part will be about main techniques of OFDM. Finally, applications of OFDM will be examined.

2.1 General Structure of OFDM

The basic principle of OFDM is to split a high rate input data stream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. Because the transmission rate is slower in parallel subcarriers, a frequency selective channel appears to be flat to each subcarrier. ISI is eliminated almost completely by adding a guard interval at the beginning of each OFDM symbol. However, instead of using an empty guard time, this interval is filled with a cyclically extended version of the OFDM symbol. This method is used to avoid ICI.

OFDM is a special case of Multicarrier Modulation (MCM). In MCM, input data stream is divided into lower rate sub streams, and these sub streams are used to modulate several subcarriers. In general, the spacing between these subcarriers is large enough such that individual spectrum of subcarriers do not overlap. Therefore the receiver uses a band pass filter tuned to that subcarrier frequency in order to demodulate the signal [20]. In OFDM, subcarrier spacing is kept at minimum, while still preserving the time domain orthogonality between subcarriers, even though the individual frequency spectrum may overlap. The minimum subcarrier spacing should equal to 1/T, where T is the symbol period.

Considering, N modulated data symbols from a particular signaling constellation, to create a complex-valued symbol vector $X_k = X_0, X_1, X_{N-1}$

where, X_k is the complex value carried by the k^{th} subcarrier.
The OFDM symbol can be written as [21]

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi k f_0 t}, \quad 0 \le t \le T \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.1)}$$

where, T is the symbol interval, and $f_0 = \frac{1}{T}$ is the frequency spacing between adjacent subcarriers. Replacing $t = nT_b$, where $T_b = \frac{T}{N}$ gives the discrete time version denoted by

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/LN} \text{ , n=0, 1.....NL-1} \quad \text{.............................. (2.2)}$$

where, L is the oversampling factor.

2.1.1 Orthogonality

Signals are orthogonal if they are mutually independent of each other. Orthogonality is a property that allows multiple information signals to be transmitted perfectly over a common channel and detected without interference.

In time domain, it is given by,

$$\int_0^T x_i(t) \otimes x_j^*(t)dt = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases} \quad \text{.. (2.3)}$$

and in frequency domain

$$\int_0^T X_i(f).X_j^*(f)df = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases} \quad \text{.. (2.4)}$$

Two conditions must be satisfied for the orthogonality between the subcarriers.

1. Each subcarrier has exactly an integer number of cycles in the FFT interval.
2. The number of cycles between adjacent subcarriers differs by exactly one.

The signals are orthogonal if the integral value is zero over the interval [0 T], where T is the symbol period. Since the carriers are orthogonal to each other the nulls of one carrier coincides with the peak of another sub carrier.

Fig. 2.1 shows the construction of an OFDM signal with four subcarriers. The baseband frequency of each subcarrier is chosen to be an integer multiple of the inverse symbol time, resulting in all subcarriers having an integer number of cycles per symbol. As a consequence the subcarriers are orthogonal to each other.

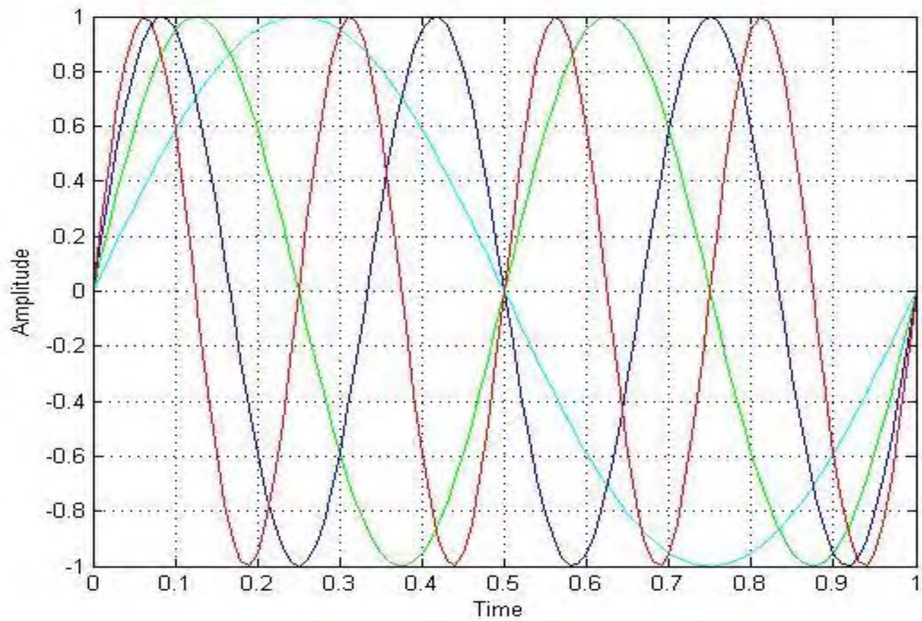


Fig. 2.1: Carrier signals in OFDM transmission

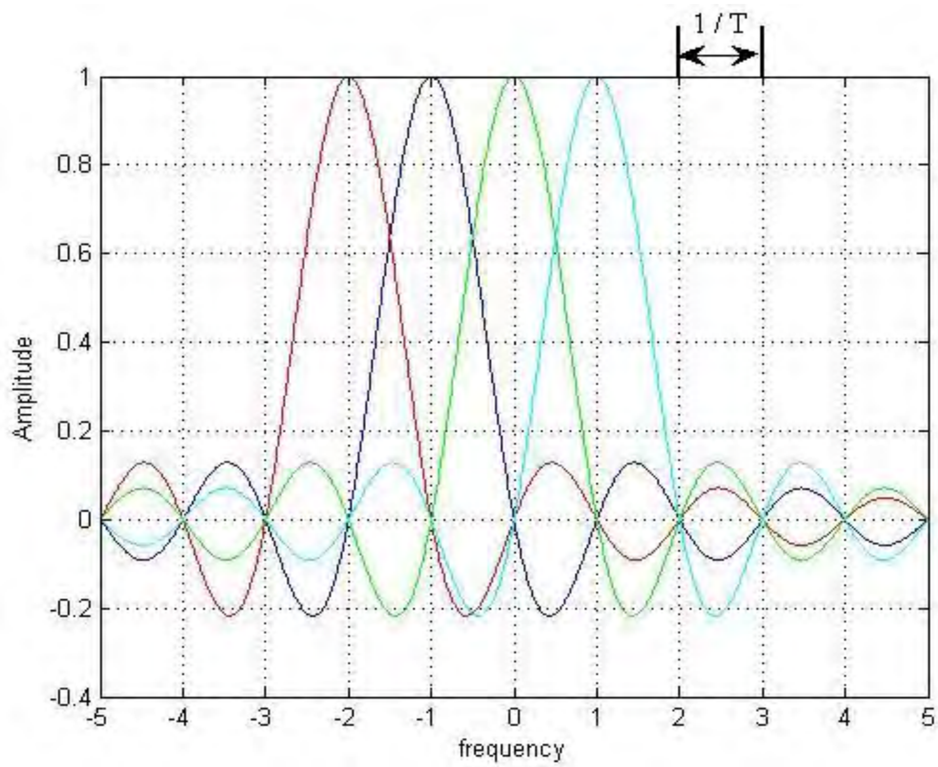


Fig. 2.2: Frequency spectrum of OFDM transmission

In the frequency domain each OFDM subcarrier has a sinc (sin(x)/x) frequency response, as shown in Fig. 2.2. This is a result of the symbol time correspondent to the inverse of the carrier spacing. This symbol time corresponds to the inverse of the subcarrier spacing of 1/T Hz. The sinc shape has a narrow main lobe, with many side lobes that decay slowly with the magnitude of the frequency difference away from the center. Each carrier has a peak at the center frequency and nulls evenly spaced with a frequency gap equal to the carrier spacing.

2.2 OFDM System Model

A Basic OFDM system is described in Figure 2.3. Here an input data symbols are supplied into a channel encoder that data are mapped onto BPSK/QPSK/QAM constellation. The data symbols are converted from serial to parallel and using Inverse Fast Fourier Transform (IFFT) to achieve the time domain OFDM symbols. Time domain symbols can be represented as:

$x_n = \mathrm{IFFT}\,\{X_k\}$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{N}}$$

where, X_k is the transmitted symbol on the K^{th} subcarriers

N is the number of subcarriers

Time domain signal is cyclically extended to prevent Inter Symbol Interference (ISI) from the former OFDM symbol using cyclic prefix (CP).

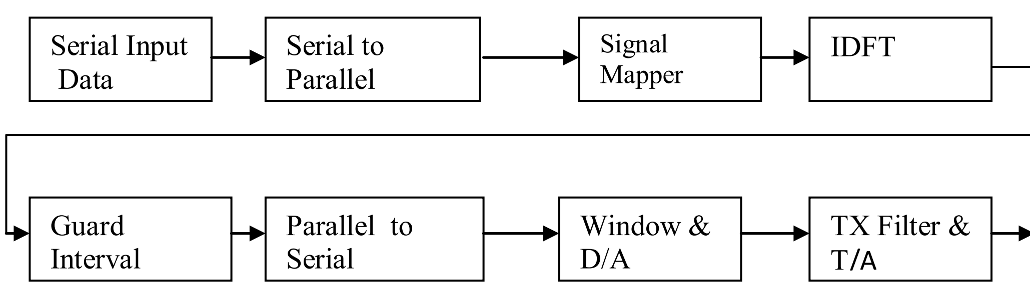


Figure 2.3 Block Diagram of an OFDM Transmitter

Figure 2.3 shows a basic OFDM transmitter structure. The serial input data stream is divided into frames of N_f bits. These N_f bits are arranged into N groups, where N is the number of subcarriers. The number of bits in each of the N groups determines the constellation size for that particular subcarrier. For example, if all the subcarriers are modulated by QPSK then each of the groups consists of 2 bits, if 16-QAM modulation is used each group contains 4 bits. This scheme is called as fixed loading. However, this is not the only way of distributing input bits among the sub channels. N_f bits could be divided among subcarriers according to the channel states. Therefore, one of the subcarriers can be modulated with 16-QAM whereas another one can be modulated with 32-QAM, etc. In this case, the former subcarrier consists of 4 bits and the latter subcarrier consists of 5 bits. This scheme is named as adaptive loading. Hence, OFDM can be considered as N independent QAM channels, each having a different QAM constellation but each operating at the same symbol rate 1/T. After signal mapping, N complex points are obtained. These complex points are passed through an IDFT block. Cyclic prefix of length v is added to the IDFT output in order to combat with ICI and ISI. After Parallel–to-Serial conversion, windowing function is applied. The output is fed into a Digital- to-Analog converter operating at a frequency of N/T. Finally transmit filter is applied in order to provide necessary spectrum shaping before power amplification.

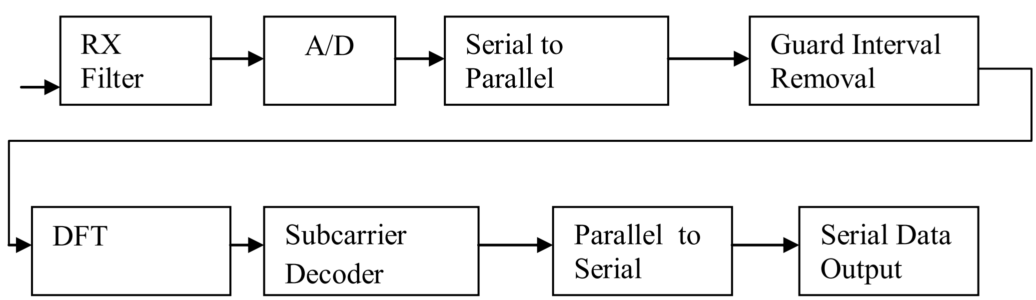


Figure 2.4 Block Diagram of an OFDM Receiver

Figure 2.4 gives the block diagram of an OFDM receiver. The receiver implements inverse operations of the transmitter. Received signal is passed through a receive filter at pass band and an Analog-to-Digital converter operating at a frequency of N/T. After these down converting and sampling operations, cyclic prefix is removed from the signal and a DFT operation is performed on the resultant complex points in order to demodulate the subcarriers. Subcarrier decoder converts obtained complex points to the corresponding bit stream.

2.2.1 Modulation

Modulation of a signal changes binary bits into an analog waveform. Modulation can be done by changing the amplitude, phase, and frequency of a sinusoidal carrier. There is several digital modulation techniques used for data transmission. A large number of modulation schemes are available allowing the number of bits transmitted per carrier. The number of bits that can be transferred using a single symbol corresponds to $\log_2(M)$ [22], where M is the number of points in the constellation, thus 256-QAM transfers 8 bits per symbol. Increasing the number of points in the constellation does not change the bandwidth of the transmission, thus using a modulation scheme with a large number of constellation points, allows for improved spectral efficiency. For example 256-QAM has a spectral efficiency of 8 b/s/Hz and spectral efficiency 1 b/s/Hz for BPSK.

2.2.2.1 Binary phase shift keying (BPSK)

BPSK is the simplest form of phase shift keying (PSK). It uses two phases which are separated by 180°. It does not particularly matter exactly where the constellation points are positioned. They are shown on the real axis, at 0° and 180°. It is shown in Fig. 2.5.

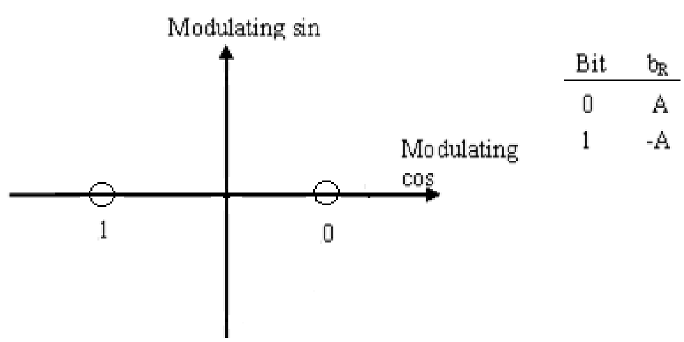
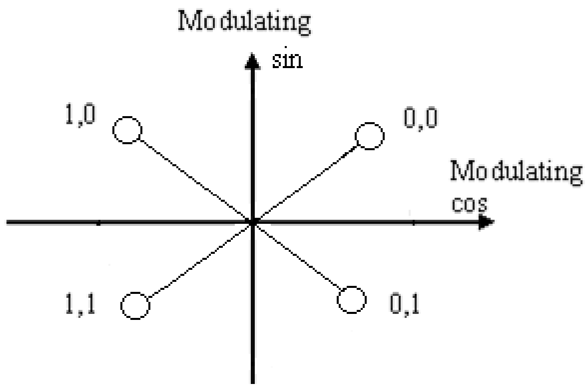


Fig. 2.5: BPSK bit-pattern

2.2.1.2 Quadrature phase shift keying (QPSK)

QPSK is a method for transmitting digital information across an analog channel. Data bits are grouped into pairs and each pair is represented by a particular waveform, called a symbol. There are four possible combinations of data bits in a pair. QPSK creates four different symbols, one for each pair, by changing the I gain and Q gain for the cosine and sine modulators. Four possible symbols of QPSK are shown in Fig. 2.13.



Bit1	Bit2	b_R	b_I
0	0	A	A
0	1	A	-A
1	0	-A	A
1	1	-A	-A

Fig. 2.6: QPSK bit-pattern

2.2.2 Serial to parallel conversion

Data to be transmitted is typically in the form of a serial data stream. In OFDM, serial to parallel conversion stage is considered to realize the concept of parallel data transmission.

Example for BPSK

input : x=[0,1,0,0,1,0,1,1,….]

The output will be a parallel: x1=[0] x2=[1] x3=[0] x4=[0] …..

Example for QPSK

input : x=[0,1,0,0,1,0,1,1,….]

The output will be a parallel : x1=[0,1] x2=[0,0] x3=[1,0] x4=[1,1] …..

In a conventional serial data system, the symbols are transmitted sequentially, with the frequency spectrum of each data symbol allowed to occupy the entire available bandwidth. When the data rate is sufficient high, symbol period is small that's why several adjacent symbols may be completely distorted over frequency selective fading or multipath delay spread channel.

The spectrum of an individual data element normally occupies only a small part of available bandwidth. An entire channel bandwidth is divided into many narrow sub channels; each sub channel is longer symbol period and the frequency response over each individual sub channel is relatively flat.

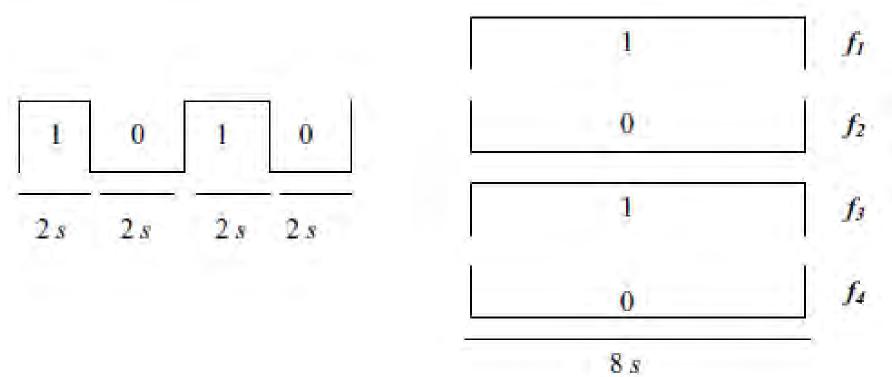


Fig. 2.7: Serial to parallel conversion

Suppose that this transmission takes eight seconds. Then, each piece of data in the left picture has duration of two second. On the other hand, OFDM would send the four pieces simultaneously as shown on the right. In this case, each piece of data has duration of eight seconds.

2.2.3 FFT and IFFT implementation

OFDM systems are implemented using a combination of fast fourier transform (FFT) and inverse fast fourier transform (IFFT) blocks that are mathematically equivalent versions of the DFT and IDFT. But we use FFT because of it's faster than a DFT and more efficient to implement. OFDM system treats the source symbols (e.g., the BPSK, QPSK or QAM symbols) at the transmitter as though they are in the frequency-domain. These symbols are used as the inputs to an IFFT block that brings the signal into the time-domain. The IFFT takes in N symbols at a time where N is the number of subcarriers in the system. Each of these N input symbols has a symbol period of T seconds. At the receiver, the OFDM message goes through the exact opposite operation in the fast fourier transform (FFT) to take from a time domain into the frequency domain. In practice, the baseband OFDM receiver performs the FFT of the receive message to recover the information that was originally sent.

The IFFT & FFT equations can be written as follows:

IFFT $$X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{j\frac{2\pi}{N}kn}$$ k=0,1,2……N-1 (2.19)

FFT $$x(n) = \frac{1}{N}\sum_{k-0}^{N-1} X(k)e^{-j\frac{2\pi}{N}kn}$$ n=0,1,2……..N-1 (2.20)

2.2.4. Linear Block Coding

In coding theory, block codes comprise the large and important family of error-correcting codes that encode data in blocks. A block code is a code in which k bits (or, more generally, symbols) are input and n bits (or, more generally symbols) are output. We designate the code as an (n,k) code. If we input k bits, then there are 2^k distinct messages [23]. Each message of n symbols associated with it is called a codeword. Linear code is an error-correcting code for which any linear combination of codewords is also a codeword. Linear codes are traditionally partitioned into block codes and convolutional codes. Linear codes allow for more efficient encoding and decoding algorithms than other codes.

2.2.4.1. Hamming Coding

Hamming code is a linear error correcting code with a (7,4) Hamming code, we have 4 information bits and we need to add 3 parity bits to form the 7 coded bits. There can be seven valid combinations of the three bit parity matrix (excluding the all zero combination) i.e.
$(001),(010),(011),(100),(101),(110),(111)$

The coding operation can be denoted in matrix algebra as follows [24]:
$c = mG$

where, m is the message sequence of dimension [1 x k], G is the coding matrix of dimension [k x n], c is the coded sequence of dimension [1 x n].

Let the coding matrix G. This matrix can be thought of as,

$$G = [I_k \mid P]$$

where, I_k is a [k x k] identity matrix and P is a [k x (n-k)] the parity check matrix. Since I_k is an identity matrix, the first k coded bits are identical to source message bits and the remaining (n-k) bits forms the parity check matrix. This type of code matrix G where the raw message bits are sent is called systematic code.

Assuming that the message sequence is $m = m_0 m_1 m_2 m_3$, then the coded output sequence is

$$c = m_0 m_1 m_2 m_3 p_0 p_1 p_2$$

where,

$$p_0 = m_0 \oplus m_1 \oplus m_2$$

$$p_1 = m_1 \oplus m_2 \oplus m_3$$

$$p_2 = m_0 \oplus m_1 \oplus m_3$$

2.2.4.2. Hamming Decoding

Hamming distance computes the number of differing positions when comparing two code words. For the coded output sequence listed in the table above, we can see that the minimum separation between a pair of code words d_{min} is 3.If an error of weight d_{min} occurs, it is possible to transform one code word to another valid code word and the error cannot be detected. So, the number of errors which can be detected is $d_{\min} - 1$.

To determine the error correction capability, let us visualize that we can have 2^k valid code words from possible 2^n values. If each code word is visualized as a sphere of radius t, then the largest value of t which does not result in overlap between the sphere is,

$$t = \left\lfloor \frac{1}{2}(d_{\min} - 1) \right\rfloor$$

where, $\lfloor x \rfloor$ is the largest integer in x. Any code word that lies within the sphere is decoded into the valid code word at the center of the sphere. So the error correction capability of the code with d_{min} distance is $t = \left\lfloor \frac{1}{2}(d_{\min} - 1) \right\rfloor$. In our example, as $d_{min} = 3$ we can correct up to 1 error.

2.2.4. 3. Parity Check Matrix

For any linear block code of (n, k) dimension, there exists a dual code of dimension [(n, k) x n]. Any code word C is orthogonal to any row of the dual code. For the chosen coding matrix G, the dual code H is,

$$H = \begin{matrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{matrix}$$

It can be seen that modulo-2 multiplication of the coding matrix G with the transpose of the dual code matrix H is all zeros i.e.

$$GH^T = \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

This dual code H is also known as parity check matrix.

2.2.4.4. Maximum Likelihood decoding

A simple method to perform maximum likelihood decoding is to compare the received coded sequence with all possible 2^k coded sequences, count the number of differences and choose the code word which has the minimum number of errors. Let the system model be,

$$y = mG + e$$

Where, y is the received code word of dimension $[1 \times n]$.

m is the raw message bits of dimension $[1 \times k]$, G is the raw message bits $[k \times n]$.

e is the error locations of dimension $[1 \times n]$

Multiplying the received code word with the parity check matrix,

$$\begin{aligned} yH^T &= (mG + e)H^T \\ &= mGH^T + eH^T \\ &= eH^T \end{aligned}$$

The term eH^T is called the syndrome of the error pattern and is of dimension $[1\times(n-k)]$. As the term $mGH^T = [000]$ the syndrome is affected only by the error sequence.

2.3 Basic Techniques in OFDM

2.3.1. Windowing

The power spectral density of the OFDM symbol (refer equation) falls off slowly, according to a sinc function. Even if the number of sub-carriers is increased, the fall off rate becomes rapid only in the beginning, but slows down after the 3dB bandwidth. In-order to make the power spectral density go down much more rapidly, time domain window is applied to the OFDM symbol. Windowing smoothens the amplitude variations due to phase changes at symbol boundaries.

2.3.2. Guard Time and Cyclic Prefix

One of the main advantages of OFDM is its immunity to multi-path delay spread that causes Inter-symbol Interference (ISI) in wireless channels. Since the symbol duration is made larger (by converting a high data rate signal into 'N' low rate signals), the effect of delay spread is reduced by the same factor. Guard Time is introduced in-order to eliminate the ISI almost completely. Making the guard time duration larger than that of the estimated delay spread in the channel does this. If the guard period is left empty, the orthogonality of the sub-carriers no longer holds, i.e., Inter-Carrier Interference (ICI) comes into picture. In-order to eliminate both the ISI as well as the ICI, the OFDM symbol is cyclically extended into the guard period. This preserves the orthogonality of the sub-carriers by ensuring that the delayed versions of the OFDM symbol always have an integer number of samples within the FFT interval. Thus we can eliminate ISI and ICI by cyclically extending the OFDM symbol into the guard period and making sure that the guard time duration is larger than the delay spread.

2.3.3. Synchronization

Time and frequency synchronization are very important for the OFDM based communication system. Without correct frequency synchronization the orthogonality will not exist leading to and increase in BER. Without correct timing synchronization it is not possible to identify start of frames. The condition for orthogonality is that the OFDM subcarriers have an integer number of cycles within the FFT interval. Once that is lost due to frequency offset, Inter Channel Interference is introduced into the system. While the OFDM system can handle a timing error of a maximum of the guard interval, the system performance and robustness against delay spread decreases with timing error. So the system has to be synchronized in time also.

2.3.3.1 Synchronization using cyclic extension

The cyclic extension of the OFDM symbol can be used for synchronization purposes. When the symbol is taken and the guard time part of the symbol is correlated with the end of the symbol we get a correlation peak. The frequency offset is estimated by averaging the correlation over the guard time period and then estimating the phase.

2.3.3.2 Synchronization using training sequences

The cyclic extension method is only used for blind synchronization where the data is not known. But when it is possible to send a training sequence as in the case of a packet transmission system an easier way of synchronization would be to implement a matched filter at the receiver. The output of the matched filter will have correlation peaks, from which the timing and frequency synchronization can be achieved.

2.4 Mathematical Definition of OFDM Signal

OFDM consists of multiple carriers. Each carrier can be presented as a complex waveform like [25]:

$$s_c(t) = A_c(t)e^{j[\omega_c t + \varphi_c t]}$$

where,

$A_c(t)$ is the amplitude of the signal $s_c(t)$

$\varphi_c(t)$ is the phase of the signal $s_c(t)$

The complex signal can be described by

$$s_s(t) = \frac{1}{N}\sum_{n=0}^{N-1} A_n(t)e^{j[\omega_n t + \varphi_n t]}$$

This is a continuous signal. Each component of the signal over one symbol period can take fixed values of the variables like:

$$\varphi_n(t) \Rightarrow \varphi_n$$

$$A_n(t) \Rightarrow A_n$$

where,

n is the number of OFDM block.

T is a time interval and the signal is sampled by 1/T then it can be represented by:

$$s_s\,(kT) = \frac{1}{N}\sum_{n=0}^{N-1} A_n\, e^{j[(\omega_0+\omega\Delta n)kT+\varphi_n]}$$

Let ω_0=0 then the signal becomes:

$$s_s\,(kT) = \frac{1}{N}\sum_{n=0}^{N-1} A_n\, e^{j[(\omega\Delta n)kT+\varphi_n]}$$

The signal is compared with general Inverse Fourier Transform (IFT):

$$g(kT) = \frac{1}{N}\sum_{n=0}^{N-1} G\frac{n}{NT} e^{j[2\pi nk/N]}$$

Here,s(KT) is time frequency domain.

Both are equivalent if $\Delta f = \frac{\Delta\omega}{2\pi} = \frac{1}{NT} = \frac{1}{\tau}$

where,

τ is symbol duration period

The OFDM signal can be defined by Fourier Transform. Frequency domain OFDM symbols can be obtained by Fast Fourier Transform (FFT) and time domain symbols can be obtained by Inverse Fast Fourier Transform (IFFT). They can be written as:

Fast Fourier Transform

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j(2\pi/N)kn}$$

Inverse Fast Fourier Transform

$$x(n) = \sum_{n=0}^{N-1} X(n)\, e^{j(2\pi/N)kn}$$

where,$0 \le n \le N-1$

2.5 Mathematical Definition of PAPR

Theoretically, large peaks in OFDM system can be expressed as Peak-to-Average Power Ratio, or referred to as PAPR, in some literatures, also written as PAR. It is usually defined as [26]:

$$\mathrm{PAPR} = \frac{P_{peak}}{P_{average}} = 10\log_{10}\frac{\max[|x_n|^2]}{E[|x_n|^2}$$

Where P_{peak} represents peak output power, $P_{average}$ means average output power. $E[.]$ denotes the expected value, x_n represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols x_k. Mathematical, x_n is expressed as:

$$= \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{N}}$$

For an OFDM system with N subcarriers, the peak power of received signals is N times the average power when phase values are the same. The PAPR of baseband signal will reach its theoretical maximum at $\mathrm{PAPR(dB)} = 10\log N$. For example, for a 16 sub-carriers system, the maximum PAPR is 12 dB. Nevertheless, this is only a theoretical hypothesis. In reality the probability of reaching this maximum is very low.

Another commonly used parameter is the Crest Factor (CF), which is defined as the ratio between maximum amplitude of OFDM signal s(t) and root-mean-square (RMS) of the waveform. The CF is defined as [26]:

$$\mathrm{CF}(s(t)) = \frac{\max[|s(t)|]}{E[||s(t)|^2|]} = \sqrt{\mathrm{PAPR}}$$

In most cases, the peak value of signal x(t) is equal to maximum value of its envelope$|x(t)|$.

2.5.1 Why PAPR reductions in OFDM system?

The instantaneous output of an OFDM system often has large fluctuations compared to traditional single-carrier systems. This requires that system devices, such as power amplifiers, A/D converters and D/A converters, must have large linear dynamic ranges. If this is not satisfied, a series of undesirable interference is encountered when the peak signal goes into the non-linear region of devices at the transmitter, such as high out of band radiation and inter-modulation distortion. PAPR reduction techniques are therefore of great importance for OFDM systems.

The OFDM technique divides the total bandwidth into many narrow sub-channels and sends data in parallel. It has various advantages, such as high spectral efficiency, immunity to impulse interference and, frequency selective fading without having powerful channel equalizer. But one of the major drawbacks of the OFDM system is high PAPR. OFDM signal consists of lot of independent modulated subcarriers, which are created the problem of PAPR. It is impossible to send this high peak amplitude signals to the transmitter without reducing peaks. So we have to reduce high peak amplitude of the signals before transmitting.

2.5.2. Effect of PAPR

The power amplifiers at the transmitter need to have a large linear range of operation. When considering a system with a transmitting power amplifier, the nonlinear distortions and peak amplitude limiting introduced by the High Power Amplifier (HPA) will produce inter-modulation between the different carriers and introduce additional interference into the system. This additional interference leads to an increase in the Bit Error Rate (BER) of the system. One way to avoid such non-linear distortion and keep low BER is by forcing the amplifier to work in its linear region. Unfortunately such solution is not power efficient and thus not suitable for wireless communication.

The Analog to Digital converters and Digital to Analog converters need to have a wide dynamic range and this increases complexity.

2.6 Advantages and Disadvantages of OFDM System

OFDM has several advantages over single carrier modulation systems and these make it a viable alternative for CDMA in future wireless networks. In this section, I will discuss some of these advantages.

2.6.1 Multipath Delay Spread Tolerance

OFDM is highly immune to multipath delay spread that causes inter-symbol interference in wireless channels. Since the symbol duration is made larger (by converting a high data rate signal into 'N' low rate signals), the effect of delay spread is reduced by the same factor. Also by introducing the concepts of guard time and cyclic extension, the effects of inter-symbol interference (ISI) and inter-carrier interference (ICI) is removed completely.

2.6.2 Immunity to Frequency selective fading Channels

If the channel undergoes frequency selective fading, then complex equalization techniques are required at the receiver for single carrier modulation techniques. But in the case of OFDM the available bandwidth is split among many orthogonal narrowly spaced sub-carriers. Thus the available channel bandwidth is converted into many narrow flat- fading sub-channels. Hence it can be assumed that the sub-carriers experience flat fading only, though the channel gain/phase associated with the sub-carriers may vary. In the receiver, each sub-carrier just needs to be weighted according to the channel gain/phase encountered by it. Even if some sub-carriers are completely lost due to fading, proper coding and interleaving at the transmitter can recover the user data.

2.6.3 High Spectral Efficiency

OFDM achieves high spectral efficiency by allowing the sub-carriers to overlap in the frequency domain. At the same time, to facilitate inter-carrier interference free demodulation of the sub-carriers, the sub-carriers are made orthogonal to each other. If the number of sub-carriers is 'N', the total bandwidth required is

$$BW_{total} = \frac{(N+1)}{T_s}$$

For large values of N, the total bandwidth required can be approximated as

$$BW_{total} \approx \frac{(N)}{T_s}$$

On the other hand, the bandwidth required for serial transmission of the same data is

$$BW_{total} = \frac{(2N)}{T_s}$$

Thus we achieve a spectral gain of nearly 100% in OFDM compared to the single carrier serial transmission case.

2.6.4 Efficient Modulation and Demodulation

Modulation and Demodulation of the sub-carriers is done using IFFT and FFT methods respectively, which are computationally efficient. By performing the modulation and demodulation in the digital domain, the need for highly frequency stable oscillators is avoided.

2.6.5 Frequency Diversity

The orthogonality preservation procedures in OFDM are much simpler compared to CDMA/TDMA technique in multipath conditions. Pilot subcarriers are used in OFDM system to prevent frequency and phase shift errors. It is possible to use maximum likelihood detection with reasonable complexity.

2.7. Disadvantages of OFDM

Some of the disadvantages of an OFDM system are as follows:-

- The OFDM signal suffers high peak to average power ratios (PAPR) of transmitted signal.
- OFDM is very sensitive to carrier frequency offset.
- It is difficult to synchronize when subcarriers are shared among different transmitters.

2.8. Applications of OFDM

2.8.1 Digital Audio Broadcasting (DAB)

Digital Audio Broadcasting is a new multimedia push technology, with a good sound quality and better spectrum efficiency. This is achieved by the use of OFDM technology. The DAB system samples audio at a sample rate of 48 kHz and a resolution of 22bits. Then the data is compressed to between 32 and 384 KBPS. A rate ¼ convolution code is used with constraint length 7. The total data rate is about 2.2Mbps. The frame time is 24ms. QPSK modulation is performed at the transmitter. The advantage of using OFDM for DAB is that the OFDM suffers very little from delay spread and also that the OFDM system has high spectral efficiency.

2.8.2 Digital Video Broadcasting (DVB)

Digital Video Broadcasting (DVB) is an ETSI standard for broadcasting Digital Television over satellites, cables and thorough terrestrial (wireless) transmission. Terrestrial DVB operates in either of 2 modes called 2k and 8k modes with 1705 carriers and 6817 carriers respectively. It uses QPSK,16 QAM or 64 QAM subcarrier modulation. It also uses pilot subcarriers for recovering amplitude and phase for coherent demodulation.

2.9 Summary

OFDM has several significant advantages over traditional serial communications such as the ability to support high data rates for wide area coverage, robustness to multipath fading and a greater simplification of channel equalization. Due to these advantages, OFDM has been adopted in both wireless and wired applications in recent years including wireless networking (IEEE 802.11), digital terrestrial television broadcasting and Broadband Radio Access Network (BRAN).

CHAPTER 3

PAPR REDUCTION TECHNIQUES

3.1. Introduction

OFDM signal is essentially the sum of many independently modulated sine waves and its amplitude has an almost Rayleigh distribution. Amplitude of OFDM signal exhibits strong fluctuations and the resultant Peak-to-Average Power Ratio (PAPR) can be rather high. In the worst case, N signals with the same phase are added up resulting in a peak power that is N times the average power; i.e. PAPR may reach a value of N. For an OFDM system with 256 subcarriers, PAPR may reach to 24 dB. High value of PAPR brings disadvantages like an increased complexity of A/D and D/A converters and a reduced efficiency of RF power amplifiers. In a practical system, before transmission, OFDM signal is passed through a power amplifier that is always peak power limited. If the squared magnitude of the OFDM signal is larger than the saturation point of the power amplifier at any time instant, then the signal will be clipped. Clipping destroys the orthogonality between subcarriers resulting in an increase in the BER when compared with the non-clipped. PAPR is important for OFDM since it is a measure of the clipping probability.

There have been many new approaches developed during the last few years. Several PAPR reduction techniques have been proposed in the literature. These techniques are divided into two groups. These are signal scrambling techniques and signal distortion techniques. In this chapter, some PAPR reducing methods will be discussed.

The signal distortion techniques are:

- Peak windowing
- Envelope Scaling
- Peak Reduction Carrier
- Clipping and Filtering

The signal scrambling techniques are:

- Selective Level Mapping (SLM)
- Partial Transmit Sequences (PTS)
- Block coding

Signal scrambling techniques work with side information which minimized the effective throughput since they commence redundancy. Signal distortion techniques introduce band interference and system complexity also. Signal distortion techniques minimize high peak dramatically by distorting signal before amplification.

3.2 Signal Distortion Techniques

3.2.1 Peak Windowing

The peak windowing method proposes that it is possible to remove large peaks at the cost of a slight amount of self-interference when large peaks arise infrequently. Peak windowing reduces PAPRs at the cost of increasing the BER and out-of-band radiation. Clipping is a one kind of simple introduces PAPR reduction technique which is self-interference [27]. The technique of peak windowing offers better PAPR reduction with better spectral properties. (Peak Windowing technique provides better PAPR reduction with better spectral properties than clipping).

In peak windowing method large signal peak are multiplied with a specific window, for example; Gaussian shaped window, cosine, Kaiser and Hamming window. In view of the fact that the OFDM signal is multiplied with several of these windows, consequential spectrum is a convolution of the original OFDM spectrum with the spectrum of the applied window. Thus, the window should be as narrow band as possible, conversely the window should not be too long in the time domain because various signal samples are affected, which results an increase in bit error rate (BER). Windowing method, PAPRs can be obtained to 4dB which from the number of independent subcarriers. The loss in Signal-to-Noise Ratio (SNR) due to the signal distortion is limited to about 0.3dB. A back off relative to maximum output power of about 5.5dB is needed in spectra distortion at least 30dB below the in-band spectral density.

3.2.2 Envelope Scaling

The Envelope Scaling technique has been proposed by Foomooljareon and Fernando in [28]. They proposed a new algorithm to reduce PAPR by scaling the input envelope for some subcarriers before they are sent to IFFT. In this paper, they used 256 subcarriers with QPSK modulation technique, so that envelopes of all the subcarriers are equal. The key idea of this scheme is that the input envelope in some sub carrier is scaled to achieve the smallest amount of PAPR at the output of the IFFT. Thus, the receiver of the system doesn't need any side information for decoding the receiver sequence.

This scheme is appropriate for QPSK modulation; the envelopes of all subcarriers are equal. Results show that PAPR can be reduced significantly at around 4 dB. Finally the system of single scaling factor and number of clusters equal to number of sub carriers is recommended.

3.2.3 Peak Reduction Carrier

Peak Reduction Carrier has been proposed by Tan and Wassell to use of the data bearing Peak Reduction Carriers (PRC) to reduce the effective PAPR in the OFDM system [29]. This scheme includes the use of a higher order modulation scheme to represent a lower order modulation symbol. This permits the amplitude and phase of the PRC to be positioned within the constellation region symbolizing the data symbol to be transmitted.

For example, to use a PRC that employs a 16-PSK constellation to carry QPSK data symbol, the 16-phases of the 16-PSK constellations are divided into four regions to represent the four different values of the QPSK symbol.

This scheme is appropriate for PSK modulation; where the envelopes of all subcarriers are equal. When the QAM modulation scheme will be implemented in the OFDM system, the carrier envelope scaling will result in the serious BER degradation. To limit the bit error rate (BER) degradation, amount of the side information would also be excessive when the number of subcarriers is large.

3.2.4 Clipping and Filtering

High PAPR is one of the most common problems in OFDM. A high PAPR brings disadvantages like increased complexity of the ADC and DAC and also reduced efficiency of radio frequency (RF) power amplifier.

One of the simple and effective PAPR reduction techniques is clipping, which cancels the signal components that exceed some unchanging amplitude called clip level. However, clipping yields distortion power, which called clipping noise, and expands the transmitted signal spectrum, which causes interfering. Clipping is nonlinear process and causes in-band noise distortion, which causes degradation in the performance of bit error rate (BER) and out-of-band noise, which decreases the spectral efficiency [30].

Clipping and filtering technique is effective in removing components of the expanded spectrum. Although filtering can decrease the spectrum growth, filtering after clipping can reduce the out-of-band radiation, but may also cause some peak re-growth, which the peak signal exceeds in the clip level. The technique of iterative clipping and filtering reduces the PAPR without spectrum expansion. However, the iterative signal takes long time and it will increase the computational complexity of an OFDM transmitter [31].

But without performing interpolation before clipping causes it out-of-band. To avoid out-of-band, signal should be clipped after interpolation. However, this causes significant peak re-growth. So, it can use iterative clipping and frequency domain filtering to avoid peak re-growth.

In the system used, serial to parallel converter converts serial input data having different frequency component which are base band modulated symbols and apply interpolation to these symbols by zero padding in the middle of input data. Then clipping operation is performed to cut high peak amplitudes and frequency domain filtering is used to reduce the out of band signal, but caused peak re-growth [31]. This consists of two FFT operations. Forward FFT transforms the clipped signal back to discrete frequency domain. The in-band discrete components are passed unchanged to inputs of second IFFT while out of band components are null. The clipping and filtering process is performed iteratively until the amplitude is set to the threshold value level to avoid the peak out-of band and peak re-growth.

3.3. Signal Scrambling Techniques

The basic idea of symbol scrambling is that, for each OFDM symbol, the input sequence is scrambled by a certain number of scrambling sequences. The output signal with the smallest PAPR is transmitted. For uncorrelated scrambling sequences, the resulting OFDM signals and corresponding PAPR values will be uncorrelated, so if the PAPR for one OFDM symbol has a probability p of exceeding a certain level without scrambling, the probability is decreased to p^k by using k scrambling codes. Hence, symbol scrambling does not guarantee a PAPR below some low level; rather, it decreases the probability that high PAPR values occur. Initially proposed scrambling techniques were Selected Mapping (SLM) and Partial Transmit Sequences (PTS) which will be described in detail in the following sections. SLM and PTS improve the PAPR statistics by introducing little redundancy and also solve the problem signal distortion techniques which may cause in-band distortion and out-of-band noise.

3.3.1. Selected Mapping (SLM)

Selective Mapping (SLM) approaches have been proposed by Bauml in 1965 [32]. This method is used for minimization of peak to average transmit power of multicarrier transmission system with selected mapping. A complete set of candidate signal is generated signifying the same information in selected mapping, and then concerning the most favorable signal is selected as consider to PAPR and transmitted. In the SLM, the input data structure is multiplied by random series and resultant series with the lowest PAPR is chosen for transmission. To allow the receiver to recover the original data to the multiplying sequence can be sent as 'side information'.

One of the preliminary probabilistic methods is SLM method for reducing the PAPR problem. The good side of selected mapping method is that it doesn't eliminate the peaks, and can handle any number of subcarriers. The drawback of this method is the overhead of side information that requires to be transmitted to the receiver of the system in order to recover information.

3.3.2. Partial Transmit Sequence (PTS)

Partial Transmit Sequence (PTS) technique has been proposed by Muller and Hubber in 1997 [33]. This proposed method is based on the phase shifting of sub-blocks of data and multiplication of data structure by random vectors. This method is flexible and effective for

OFDM system. The main purpose behind this method is that the input data frame is divided into nonoverlapping sub blocks and each sub block is phase shifted by a constant factor to reduce PAPR. PTS is probabilistic method for reducing the PAPR problem. It can be said that PTS method is a modified method of SLM. PTS method works better than SLM method. The main advantage of this scheme is that there is no need to send any side information to the receiver of the system, when differential modulation is applied in all sub blocks.

3.3.3. Interleaving Technique

Interleaving technique has been proposed by Jayalath and Tellambura [34], for reduction peak to average power ratio of an OFDM transmission. A data randomization technique has proposed for the minimization of the PAPR in this paper.The notion that highly correlated data structures have large PAPR can be reduced, if long correlation pattern is broken down. Also, this paper proposes an additive method to minimize the complexity.

The basic idea in adaptive interleaving is to set up an initial terminating threshold. PAPR value goes below the threshold rather than seeking each interleaved sequences. The minimal threshold will compel the Adaptive Interleaving (AL) to look for all the interleaved sequences. The main important of the scheme is that it is less complex than the PTS technique but obtains comparable result. This method does not give the assurance result for PAPR reduction. In this circumstance, higher order error correction method could be used in addition to this method.

3.3.4. Tone Reservation (TR)

Tone Reservation (TR) method is proposed for PAPR reduction. The main idea of this method is to keep a small set of tones for PAPR reduction. This can be originated as a convex problem and this problem can be solved accurately. The amount of PAPR reduction depends on some factors such as number of reserved tones, location of the reserved tones, amount of complexity and allowed power on reserved tones.

This method explains an additive scheme for minimizing PAPR in the multicarrier communication system. It shows that reserving a small fraction of tones leads to large minimization in PAPR ever using with simple algorithm at the transmitter of the system without any additional complexity at the receiver end. Here, N is the small number of tones, reserving tones for PAPR reduction may present a non–negligible fraction of the available bandwidth and resulting in a reduction in data rate. The advantage of TR method is that it is less complex, no side information and also no additional operation is required at the receiver of the system. Tone reservation method is based on adding a data block and time domain signal. A data block is dependent time domain signal to the original multicarrier signal to minimize the high peak. This time domain signal can be calculated simply at the transmitter of system and stripped off at the receiver.

3.3.5. Tone Injection (TI)

Tone Injection (TI) method has been recommended by Muller, S.H., and Huber, J.B. This technique is based on general additive method for PAR reduction. Using an additive method achieves PAPR reduction of multicarrier signal without any data rate loss. Note that Tone Injection (TI) uses a set of equivalent constellation points for an original constellation points to reduce PAPR. The main idea behind this method is to increase the constellation size. Then, each point in the original basic constellation can be mapped into several equivalent points in the extended constellation, since all information elements can be mapped into several equivalent constellation points. These additional amounts of freedom can be utilized for PAPR reduction. This method is called Tone Injection method because of replacing the points in the basic constellation for the new points in the larger constellation which corresponds to injecting a tone of the proper phase and frequency in the multi-carrier symbol. The drawbacks of this method are; need to side information for decoding signal at the receiver side, and cause extra IFFT operation which is more complex.

3.3.6. Block Coding Techniques

Coding techniques can be applied for signal scrambling, M sequences, Golay complementary sequences, Shapiro-Rudin sequences codes can be used to reduce the PAPR efficiently. This Block coding technique has been proposed by Wilkinson and Jones in 1965 for the minimization of the peak to mean envelope power ratio of multicarrier communication system [35]. The key object in this paper is that PAPR can be minimized by block coding the data. The block coding techniques have three stages for the development. The first stage works with the collection of appropriate sets of code words for any number of carriers, any M-ary phase modulation method, and any coding rate. The second stage works with the collection of the sets of code words which enable proficient implementation of the encoding/decoding. The third stage offers error deduction and correction potential.

There are different methods for the collection of the sets of code words. The mainly insignificant method, order to search the peak envelope power (PEP) for all possible code words for a certain length of given number of carriers [35]. This technique is simple and accurate for short codes because it needs extreme computation. Natural algorithms are mainly sophisticated searching techniques. It can be used for the collection of longer code words. A selection of code words select from searches for encoding and decoding can be performed with a look up table or using combinatorial logic exploiting the mathematical structure of the codes minimization when the frame sixe is bigger [36].

Large PAPR reduction can be achieved if the long information sequence is separated into different sub blocks, and all sub block encoded with System on a Programmable Chip (SOPC). There are many likely spaces, where the odd parity checking bits can be put into each frame to minimize PAPR. For further minimization of PAPR, redundant bit location optimized sub-block coding (RBLO-SBC) optimizes these locations redundant Combination optimized sub-block coding scheme (COSBC) optimizes the combination of the coded sub-blocks, where two coding schemes instead of one is used to encode the same information source.

3.3.7. Block Coding Scheme with Error Correction

This Block coding scheme with Error Correction has been proposed by Ahn and et.al in [37] to introduce a new block coding proposal for minimization of peak to average power ratio (PAPR) of an Orthogonal Frequency Division Multiplexing (OFDM) system. Block coding has error correction capability. In block coding method, the OFDM symbol can be reduced by selecting only those code words with lower PAPR. A k bit data block (e.g. 4-bit data) is encoded by a (n, k) block code with a generator matrix 'G' in the transmitter of the system. Followed by the phase rotator vector b to produce the encoded output x=a.G+b(mod 2).

To achieve the accurate generator matrix and phase rotator vector that make sure the minimum PAPR for the OFDM system, check all the 2^n codes and choose only 2^k codes that obtain the minimum PAPR. After that generator matrix 'G' and the phase rotator vector 'b' are produced; which are used mapping between these symbols combination and input data vector 'a'. The converse functions of the transmitter are executed in the receiver system. The parity check matrix 'H' is achieved from the generator matrix 'G', with an exception that the effect of the phase rotator vector b is removed before calculations of syndromes.

3.4 Overall Analysis of Different Techniques

There are several techniques has been proposed in literature. Thus, it is possible to reduce the large PAPR by using the different techniques. Note that the PAPR reduction technique should be chosen with awareness according to various system requirements.

Table 3.1 Comparison of PAPR Reduction Techniques

Name of Schemes	Name of parameters		
	Distortion less	Power increases	Data rate loss
Clipping and Filtering	No	No	No
Coding	Yes	No	Yes
Partial Transmit Sequence (PTS)	Yes	No	Yes
Selective Mapping (SLM)	Yes	No	Yes
Interleaving	Yes	No	Yes
Tone Reservation (TR)	Yes	Yes	Yes
Tone Injection(TI)	Yes	Yes	No

There are many issues to be considered before using the PAPR reduction techniques in a digital communication system. These issues include PAPR reduction capacity, power increase in transmit signal, BER increase at the receiver, loss in data rate, computational complexity increase and so on. Simultaneously most of the techniques are not proficient to obtain a large reduction in PAPR with low coding overhead, with low complexity, without performance degradation and without transmitter and receiver symbol handshake.

3.5 Summary

OFDM is a promising technique for wireless communication systems although it has some drawbacks which are given below:

- High PAPR
- Frequency offset

High PAPR is one of the major problems of OFDM system. There are several techniques to reduce the PAPR in OFDM transmission system. All PAPR reduction techniques have some advantages and disadvantages. These PAPR reduction techniques should be chosen carefully for getting the desirable minimum PAPR. All PAPR reduction techniques are based on particular situation of system.

CHAPTER 4

THEORITICAL ANALYSIS OF PAPR REDUCTION

The emergence of high peak power signal in OFDM system is due to the superposition (IFFT operation) of multiple sub-carrier signals. If multiple sequences which carry the same information are used to represent one transmission process, then the best one can be chosen among those candidates for a given PAPR threshold condition. In this way the occurrence probability of peak power signal can significantly be reduced.

In this chapter, the algorithm proposed in this thesis work to reduce PAPR value of an OFDM modulating system will be explained. The method uses block coding with phase rotation for reducing the PAPR of OFDM, however, a single or double error correcting code will be enough. Its fundamental principle is: generating multiple signal waveforms which carry the same information and then choose the waveform from those candidates with the smallest PAPR for transmission. This method has lots of merits, such as high coding rate and low redundancy, although it only optimizes the statistical characteristics of PAPR in OFDM system. Therefore, varying schemes based on this principle have a bright application prospect.

4.1 Description of the Method

In the proposed method, input data is distributed to N number of Encoder& Mapper Blocks. The operation of these blocks will be described later. Outputs of Encoder & Mapper Blocks are fed into an IFFT block. This operations form regular OFDM symbols. The Peak to Average Power Ratio (PAPR) values of OFDM symbols are calculated and compared with a certain threshold Z. If the PAPR value is below the threshold, no operation is performed on the symbol and it is directly passed to the output. However, if PAPR exceeds the threshold, the algorithm is applied to that symbol until the PAPR is reduced below Z.

Determination of the threshold is critical, since it determines the PAPR reduction achieved, but choosing a very low value will increase the number of calculations needed, which results in computational inefficiency. Hence, there is a tradeoff between PAPR reduction value and implementation complexity of the system. A criterion to choose a proper threshold value will be suggested.

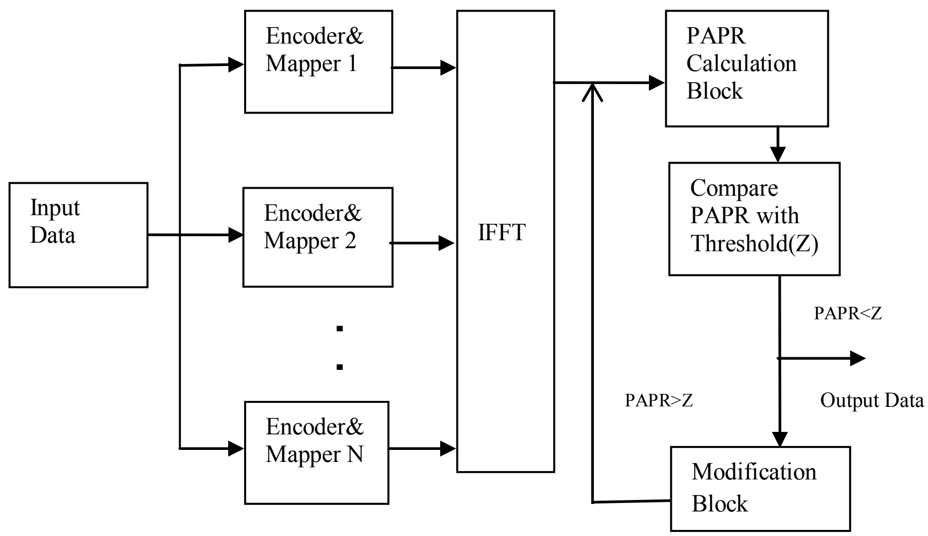


Figure 4.1 Proposed Transmitter Block Diagram

The operations performed in an Encoder & Mapper block is shown in Figure 4.2. Input data is distributed to N of these blocks and each of the block waits until k bits are collected. Then k-bits are encoded into n- bits using an (n,k) Hamming block code. The error correction capability of the selected code is important for the performance calculation of the system. Later, a parity value is calculated and added to the end of n-bits in order to obtain an even number of bits before modulation. Then, (n+1) bits are passed through a QPSK modulator and (n+1)/2 QPSK modulated symbols are obtained as Encoder & Mapper block output. In the transmitter of the system, k bit data block is encoded by (n, k) Hamming block code with a generator matrix G. However, QPSK modulation is chosen for simplicity.

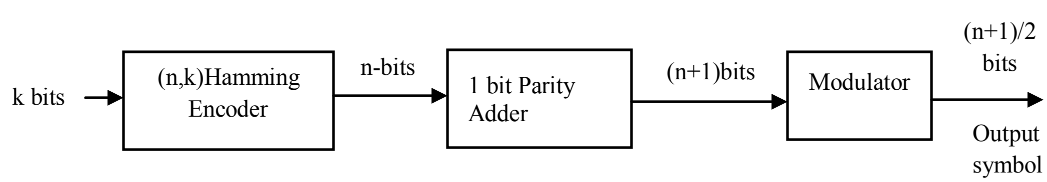


Figure 4.2 Encoder & Mapper Block Diagram

Assume that the OFDM system used has 4 channels, for illustration. The system uses (n,k) code and 1-bit parity, then the Encoder & Mapper outputs are as shown in Figure 4.3. (n+1)/2 QPSK symbols are generated on each Encoder & Mapper block for k information bits.

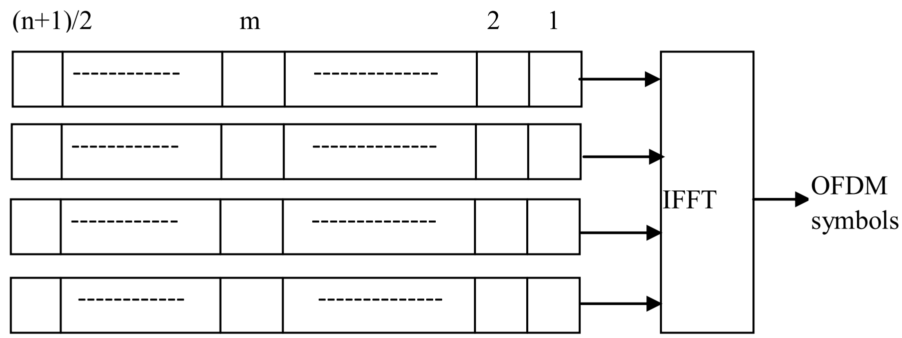


Figure 4.3 Encoder & Mapper Output for Single Coded Block

Assume that the m^{th} OFDM symbol, which corresponds to the m^{th} QPSK symbol from each Encoder & Mapper, produces a PAPR value that exceeds the threshold. m^{th} symbol consists of 4 complex numbers that are IFFT outputs. Assume that the complex number at index-1 of the m^{th} symbol, i.e. y[1], has the highest magnitude, hence causes a peak.

OFDM modulation can be represented as

$$y[n] = \sum_{i=0}^{N-1} ai \exp(j\ \frac{2\pi in}{N}), \qquad 0 \le n \le N-1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.1)}$$

where ai's are QPSK modulator outputs. Using (4.1), y[1] can be written explicitly as

$$y[1] = a_0 + a_1 \exp\left(j\ \frac{\pi}{2}\right) + a_2 \exp(j\ \pi) + a_3 \exp\left(j\ \frac{3\pi}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.2)}$$

The main principle of the proposed algorithm is to modify QPSK modulator outputs in a way to reduce the PAPR value of the OFDM symbol. Modification of modulator outputs results in a change of information to be transmitted. Encoders are used to eliminate this effect. Error correcting codes make the system resistant to bit errors. Therefore, if the number of bits changed at the transmitter per coded block is below the error correcting capability of the decoder, information should be received correctly at the receiver. However, it should be noted that the main objective of the error correcting code employed is to reduce the PAPR value without loss of information. Therefore, if errors occurred during transmission over the channel are to be compensated, an outer code should be used together with an interleaver or a scrambler. In simulations, only 1 bit modification per coded block is allowed since more bit modifications result in unacceptable SNR degradation.

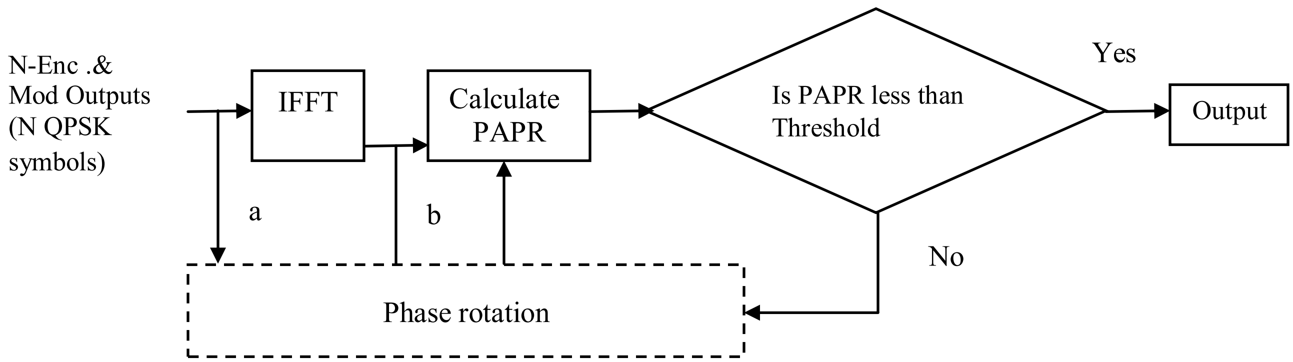


Figure 4.4 Algorithm Flow Chart

As seen in Figure 4.4, the algorithm block is enabled when the OFDM symbol PAPR value exceeds the threshold value. The block uses OFDM symbol with high PAPR value and its components before the IFFT block. The algorithm is implemented in an iterative manner until the PAPR value is reduced below the desired value. When a symbol PAPR exceeds the threshold value, the index of OFDM symbol component; i.e. IFFT output with maximum amplitude is determined which corresponds to parameter n of Equation (4.1). The phase of this component is calculated to find out its direction on the x-y plane. Using the index of the high PAPR creating component as parameter n of Equation (4.1), phase rotations on ai's caused by IFFT are determined.

4.2 Performance evaluation Criteria

The suggested algorithm reduces the PAPR of an OFDM modulating system. However, the algorithm necessitates the modification of codewords which results in an increase of the input SNR in order to satisfy the same information bit error probability as the simple OFDM system. While examining the work done in the literature, it has been observed that there is a lack of suitable performance measure criteria. The main criterion used by the suggested methods is the achieved PAPR reduction. So we have proposed a performance evaluation criterion for PAPR reduction systems.

Here, two different systems will be considered; first system is the plain OFDM system and the second system uses the algorithm described in this chapter. First, general comparison criteria will be introduced. Then, coded OFDM operation will be explained and the Probability Density

Function (pdf) of OFDM signal will be derived. Next, the bit error probability, coded block error probability and SNR relations will be derived for the two systems.

The parameters used for system evaluation are the information bit rate R_b, receiver noise level N_0 and the transmit power. Assume that there are two systems, first is the plain OFDM modulating system defined as Sys1 and the second system implements the suggested algorithm defined as Sys2. In order to compare two systems on a fair basis, a very low clipping rate will be assumed for both Sys1 and Sys2. Since the OFDM technique is used mainly for video and audio transmission, having a sufficiently low rate clipping will not cause a serious problem. The calculations in this chapter are performed for 1 clipping-per-day assumption. However, it is possible to repeat calculations for different clipping rates.

4.3. Coded OFDM Operation

Assume that the OFDM system has N channels. In the sample OFDM transmitter used during simulations, there are N Encoder & Mapper blocks. Each Encoder & Mapper uses (n,k) coding; i.e. each k bits are coded into n bits. Then a parity bit is produced and added at the end of n bits. Next, these (n+1) bits are used to modulate the subcarriers (QPSK, QAM etc.) and finally N-point IFFT is computed in order to form OFDM symbols. In order to derive the relationship between the OFDM symbol period and the information bit rate, timing diagrams of the sample system will be provided. Figure 4.5 shows the input data to the transmitter that will be distributed among Encoder & Mapper blocks.

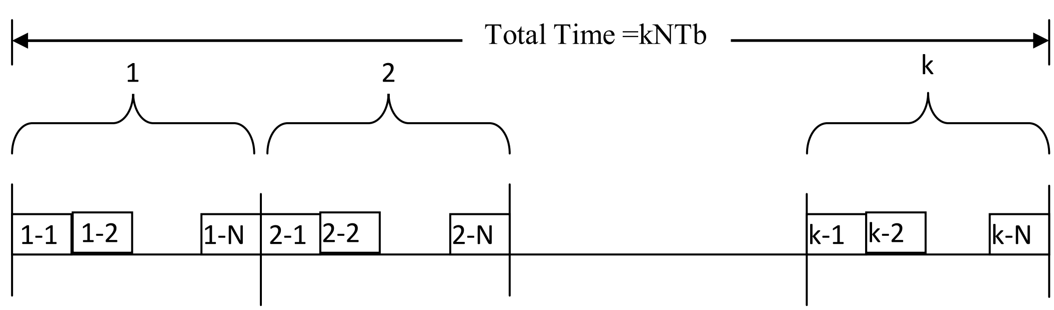


Figure 4.5: OFDM Transmitter Input Block

In Figure 4.5, N denotes the number of channels, Tb denotes the information bit period and k denotes the un-coded block width to be coded into n-bits by the encoder. After coding, adding parity and modulating (QPSK, QAM etc.) each of the k-bit blocks produce (n+1) bit blocks with one parity bit added. Finally, (n+1)/L modulated symbols are produced from each of the k-bit blocks where L is the modulation width (e.g. L=2 for QPSK). Each of these (n+1)/L modulated symbols at each of the Encoder & Mapper blocks will be called as a Coded Block during rest of this chapter. Since a modulated symbol from each of the N Encoder & Mapper block produces an OFDM symbol after IFFT stage, (n+1)/L OFDM symbols are generated from kN input bits. This procedure is shown in Figure 4.6.

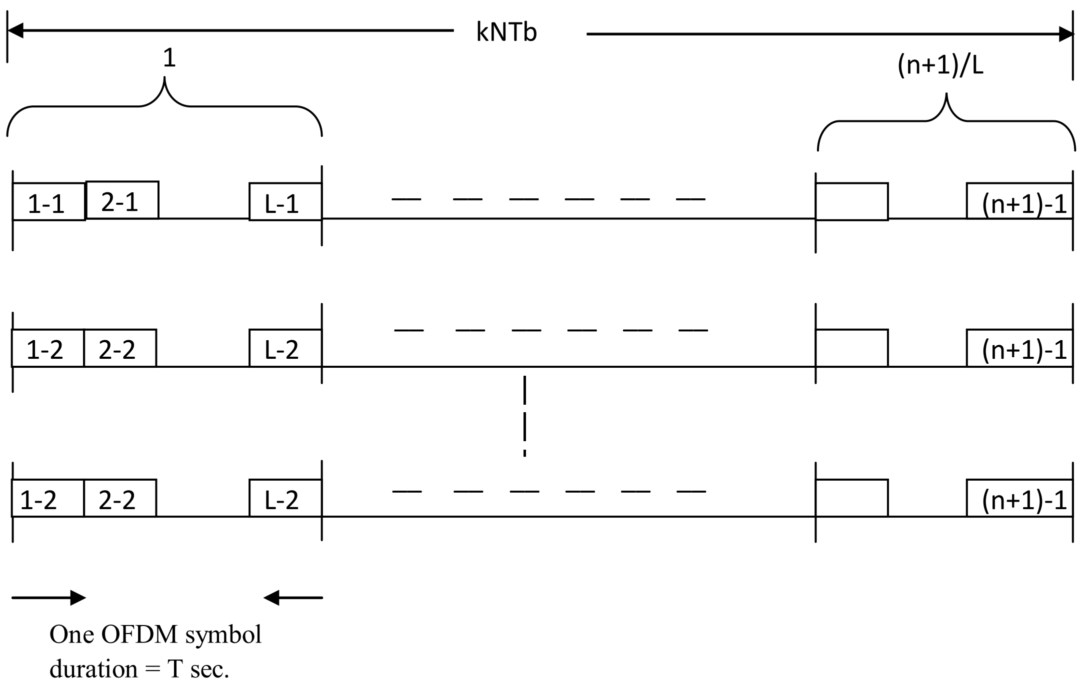


Figure 4.6: Construction of N Coded Blocks

4.4. PDF (Probability Density Function) of Instantaneous Power of OFDM Signal

According to central limit theorem, for a large number of sub-carriers in multi-carrier signal, the real and imaginary part of sample values in time-domain will obey Gaussian distribution with mean value of 0 and variance of 0.5. Therefore, the amplitude of multi-carrier signals follows Rayleigh distribution with zero mean and a variance of N times the variance of one complex sinusoid. Its power value obeys a $\chi 2$ distribution with zero mean and 2 degrees of freedom. Cumulative Distribution Function (CDF) is expressed as follows [38]

$$F\ (z)\ = 1 - \ e^{-z} \qquad \text{……………………………..… (4.3)}$$

Assuming that the sampling values of different sub-channels are mutually independent, and free of oversampling operation, the probability distribution function for PAPR less than a certain threshold (z) value, is therefore expressed as

$$P(\ PAPR < \ z)\ = F(z)^N = \ (1 - \ \ e^{-z})^N \qquad \text{…….……………………….. (4.4)}$$

In practice, it is preferred to take the probability of PAPR exceeding a threshold as measurement index to represent the distribution of PAPR. This can be described as "Complementary Cumulative Distribution Function" (CCDF), and its mathematical expression as

$$P\ (PAPR > \ z)\ = 1 - P(\ PAPR \leq \ z\ ) = 1 - F(z)^N = \ 1 - (1 - \ \ e^{-z})^N$$

where N is also the number of subcarriers

In this thesis, we will use CCDF to evaluate the performance of various PAPR reduction techniques.

Equation (4.4) is valid under the assumption that the samples are mutually uncorrelated. However, in order to calculate discrete time PAPR more accurately, at least a 4-times oversampling is necessary. And samples become correlated when oversampling is applied. It is [39] explained that the distribution for N subcarriers and oversampling can be approximated by the distribution for αN subcarriers without oversampling, with a larger than one. Hence, the effect of oversampling is approximated by adding a certain number of independent samples. The parameter α is determined to be 2.8 with computer simulations. This result lacks theoretical justification, however; it is widely used in the literature and will also be used in this thesis. Accepting this argument,(4.4) may be rewritten as:

P(PAPR not exceeding z in one OFDM symbol) = $(1 - \ \ e^{-z})^{2.8N}$ ……………….. (4.5)

Then P(PAPR exceeding threshold in one OFDM symbol) = $1 - (1 - \ \ e^{-z})^{2.8N}$

4.5. Bit Error Probability, Block Error Probability and SNR Loss Relations for Sys1

Assume that Sys1 is the plain OFDM modulating system that uses the (31, 21) 2-error correcting Hamming code. In this section, the relation between bit error probabilities and block error probabilities of plain OFDM system will be derived.

The coded bit error probability for Sys1 can be expressed as [22].

$$Pb_1 = Q\left(\sqrt{2\frac{E_c}{N_{01}}}\right) \quad \text{(4.6)}$$

where $\frac{E_c}{N_{01}}$ denote the SNR per coded bit.

Since there are (n+1) bits per codeword after parity generation and each codeword conveys k bits of information, the SNR per coded bit can be given in terms of SNR per information bit as:

$$\frac{E_c}{N_{01}} = \frac{k}{(n+1)}\frac{E_b}{N_{01}} \quad \text{(4.7)}$$

(4.6) can be rewritten as:

$$Pb_1 = Q\left(\sqrt{2\frac{k}{(n+1)}\frac{E_b}{N_{01}}}\right) = \frac{1}{2}\,erfc\left(\frac{k}{(n+1)}\frac{E_b}{N_{01}}\right) \quad \text{(4.8)}$$

For the case of (31, 21) Hamming code:

$$Pb_1 = \frac{1}{2}\,erfc\left(\frac{21}{32}\frac{E_b}{N_{01}}\right)$$

The probability of receiving a codeword in error will be defined as the block error probability. In the sample system, the generated parity bit is not used at the receiver. The only purpose of adding a parity bit is to obtain an even number of bits before QPSK modulation. Another solution to this problem may be employing BPSK modulation for the last remaining bit of the encoder output. However, we decided on using a parity bit and employing QPSK modulation for all encoder output bits. Therefore, for a t-error correcting (n,k) code (we consider a hard decision detection at the receiver):

$P_{e,block}$ = P(number of bit errors ≥ (t+1) in n bits)

=1-P(t bit error + (t-1) bit error + …….+ 1 bit error + no error in n bits).

$$P_{e,block} = 1 - \begin{bmatrix} \binom{n}{t} Pb_1{}^{t}(1 - Pb_1)^{n-t} + \binom{n}{t-1} Pb_1{}^{t-1}(1 - Pb_1)^{n-t+1} + \dots \\ \dots + nPb_1(1 - Pb_1)^{n-1} + (1 - Pb_1)^{n} \end{bmatrix} \text{………… (4.9)}$$

For the sample case of (31, 21) 2-error correcting code

$P_{e,block}$ = P(number of bit errors ≥ 3 in 31 bits)

= 1 - P(2 bit error + 1 bit error + no error in 31 bits).

Then;

$$P_{e,block} = 1 - \left[\binom{31}{2} Pb_1{}^{2}(1 - Pb_1)^{29} + 31Pb_1(1 - Pb_1)^{30} + (1 - Pb_1)^{31}\right] \text{……. (4.10)}$$

Using polynomial approximations:

$$P_{e,block} = 1 - \begin{bmatrix} \frac{31.30}{2} Pb_1{}^{2}\left(1 - 29Pb_1 + \frac{29.28}{2} Pb_1{}^{2} - \cdots\right) \\ + 31\, Pb_1\left(1 - 30Pb_1 + \frac{30.29}{2} Pb_1{}^{2} - \frac{30.29.28}{3!} Pb_1{}^{3} + \cdots\right) \\ + \left(1 - 31\, Pb_1 + \frac{31.30}{2} Pb_1{}^{2} - \frac{31.30.29}{3!} Pb_1{}^{3} + \frac{31.30.29.28}{4!} Pb_1{}^{4} - ..\right) \end{bmatrix}$$

From the above equation we find out that:

$$P_{e,block} \cong 4.495x10^{3} Pb_1{}^{3} - 9.44x10^{4} Pb_1{}^{4} \quad \text{……………………….. (4.11)}$$

for the sample case of (31,21) 2-error correcting code employed at Encoder & Mapper blocks.

4.6. Choosing Transmitter Threshold for Sys1

As described previously, in order to compare the two systems, a clipping rate of 1 per day will be used for calculations. Therefore, it is necessary to set the PAPR threshold value appropriately using this constraint. Timing diagram of Figure 4.5 will be used for this purpose. As seen from the figure, the relation between information bit duration (T_b) and OFDM symbol duration (T) is as follows:

$kNT_b = \frac{n+1}{L}T$ ……………………….…………….. (4.12)

Thus OFDM symbol duration turns out to be

$T = \frac{kNT_bL}{n+1}$ ……………………….. …………………… (4.13)

The sampling period Ts is

$T_s = \frac{T}{N}$ ………………………………………………… (4.14)

Then Ts is found as

$T_s = \frac{kT_bL}{n+1} = \frac{kL}{(n+1)R_b}$ ………………………………… (4.15)

Now, we may define a new parameter N_{24} as the number of samples per day in an OFDM system. Since a day has 86400 seconds N_{24} may be expressed as

$N_{24} = \frac{86400}{T_s} = \frac{86400}{\frac{kL}{(n+1)R_b}} = \frac{86400}{L}\frac{(n+1)}{k}R_b$ ……………………….. (4.16)

Let P_1 be the probability that a single sample exceeds the threshold. Since we expect that only one OFDM sample exceeds a certain threshold per day, P_1 may be roughly approximated as

$P_1 \cong \frac{1}{N_{24}}$ ……………………... ……………………………….… (4.17)

As found from the probability density function of OFDM that:

P(PAPR of an OFDM sample exceeds the threshold z) = e^{-z} ………… (4.18)

Using equations (4.16), (4.17) and (4.18) together, a relationship is obtained between the code rate, information bit rate and the minimum transmitter threshold value to satisfy one clip per day constraint

$$e^{-z_{min}} = \frac{kL}{86400(n+1)R_b} \quad \text{.......................... ....} \quad (4.19)$$

Therefore, the desired threshold value is obtained as

$$z_{min} = ln\left(\frac{86400(n+1)R_b}{kL}\right) \quad \text{...} \quad (4.20)$$

Note that the threshold value does not depend on the number of channels. This result may be explained in the way that for the same information bit rate as the number of channels increase, less number of OFDM symbols are produced per unit time, at the same time the probability of having a high PAPR value increases because of the increase in number of channels. These two factors compensate each other. Additionally, (4.20) gives a minimum value of the PAPR threshold that will be used for calculations. It is possible to select a larger threshold value which reduces the clipping probability; despite degrades the advantage of applying the algorithm.

4.7. Bit Error Probability, Block Error Probability and SNR Loss Relations for Sys2

Sys2 is the sample system applying the PAPR reduction algorithm. Sys1 and Sys2 will be compared for equal information bit error probabilities. It can be assumed that receiving an erroneous block means receiving half of the bits in error. Therefore, information bit error probability approximately equals half of the block error probability. Since the block error probability of Sys1 was written in terms of its coded bit error probability in (4.9), we need to write the block error probability ($P_{e,block}$) in terms of coded bit error probability (Pb_2) for Sys2 and we can equate block error probabilities of the two systems.

The second system implements the PAPR reduction algorithm. Therefore, the block error probability depends on the probability of observing a PAPR value higher than the threshold during that block time. Assume that Sys2 employs a t-error correcting (n,k) code. Then, the term *block* denotes the coded block of *n* bits.

$P_{e,block}$ = P(No high PAPR in the block) P(number of bit errors ≥ (t+1))
+P(1 high PAPR in the block) P(number of bit errors ≥ t)+…

Using the fact that the probability of more than 1 high PAPR in a block is approximately equal to zero, the above equation may be approximated as

$P_{e,block}$ ≈ P(No high PAPR in the block) P(number of bit errors ≥(t+1))
+P(1 high PAPR in the block) P(number of bit errors ≥ t). ……………………….. (4.21)

Again using the fact that the probability of 2 or more samples with high PAPR values in a block approximately equals zero, it may be concluded that
P(No high PAPR in the block) + P(1 high PAPR in the block) ≈ 1. ……………… (4.22)

Additionally, if a coded block is modified due to a high PAPR, only 1 bit per coded block is modified which is the assumption used in this chapter.

(4.21) can be simplified, using some short form representations of the above probabilities and (4.22). Therefore, (4.21) can be expressed as

$$P_{e,block} = P_{NoPAPR}P2 + (1 - P_{NoPAPR})P1 \quad \ldots\ldots\ldots\ldots \quad (4.23)$$

where

P2 = P(number of bit errors ≥ (t+1) in n bits)

P1 = P(number of bit errors ≥ t in n bits)

The probability P2 is in the same form as in (4.9), only the coded bit error probability of Sys1 (i.e. Pb_1) should be replaced with the coded bit error probability of Sys2 (i.e. Pb_2). Hence, P2 is in the form

$$P2 = 1 - \begin{bmatrix} \binom{n}{t} {Pb_2}^t(1 - Pb_2)^{n-t} + \binom{n}{t-1} {Pb_2}^{t-1}(1 - Pb_2)^{n-t+1} + \ldots \\ \ldots + nPb_2(1 - Pb_2)^{n-1} + (1 - Pb_2)^n \end{bmatrix} \quad \ldots\ldots\ldots \quad (4.24)$$

P1 can be written as

P1 = P(number of bit errors $\geq$ t in n bits)

= 1-P((t-1) bit error + (t-2) bit error + ….. + 1 bit error + no error in n bits).

Then;

$$P1 = 1 - \begin{bmatrix} \binom{n}{t-1} Pb_2{}^{t-1}(1 - Pb_2)^{n-t+1} + \binom{n}{t-2} Pb_2{}^{t-2}(1 - Pb_2)^{n-t+2} + \dots \\ \dots + nPb_2(1 - Pb_2)^{n-1} + (1 - Pb_2)^n \end{bmatrix} \quad \dots\dots\dots\dots (4.25)$$

In order to find the block error probability of the system using the PAPR reduction algorithm, the last step is to derive $P_{NoPAPR.}$

P_{NoPAPR} = P(a block length of M OFDM symbols have low PAPR)

Using (4.5) in this context:

P(PAPR not exceeding z in one OFDM symbol) = $(1 - e^{-z})^{2.8N}$

It may be easily seen that:

$$P_{NoPAPR} = (1 - e^{-z})^{2.8NM} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4.26)$$

where z is the PAPR threshold of the system, N is the number of channels and M=(n+1)/L for code word length of n and modulation width of L (e.g. for QPSK modulation L=2).

For the sample case of a (31, 21) 2-error correcting Hamming code, the probability P2 can be written in the form of (4.119) as

$$p2 = 4.495x10^3\, Pb_2{}^3 - 9.44x\, Pb_2{}^4 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4.27)$$

Using equation (4.25), P1 turns out to be

$$P1 = 1 - [31Pb_2(1 - Pb_2)^{30} + (1 - Pb_2)^{31}] \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4.28)$$

Again, using the polynomial approximations:

$$P1 = 1 - \left[\begin{array}{c} 31Pb_2\left(1 - 30Pb_2 + \frac{30.29}{2}{Pb_2}^2 - \frac{30.29.28}{6}{Pb_2}^3 + ..\right) + \\ (1 - 31Pb_2 + \frac{31.30}{2}{Pb_2}^2 - \frac{31.30.29}{6}{Pb_2}^3 + \frac{31.30.29.28}{24}{Pb_2}^4 - \cdots) \end{array} \right]$$

$$P1 \cong 465{Pb_2}^2 - 8.99x10^3{Pb_2}^3 + 94.365x10^3{Pb_2}^4 \quad \text{.................................} \quad (4.29)$$

Using equations (4.23), (4.26), (4.27), (4.29), the block error probability of Sys2 employing a (31, 26) 2-error correcting code for the sample case may be rewritten as:

$$P_{e,block} = (1 - e^{-z_2})^{2.8NM}(4.495x10^3\,{Pb_2}^3 - 9.44x10^4{Pb_2}^4) + (1 - (1 - e^{-z_2})^{2.8NM}) \quad (465{Pb_2}^2 - 8.99x10^3{Pb_2}^3 + 94.365x10^3{Pb_2}^4) \quad \text{...........................} \quad (4.30)$$

As seen from (4.30) for Sys2, the block error probability depends on the coded bit error probability, the PAPR threshold, the number of channels, the code word length and the modulation.

Finally, for the (31, 21) Hamming code, the coded bit error probability for Sys2 is given by:

$$Pb_2 = Q\left(\sqrt{\frac{21}{32}\frac{Eb}{N_{02}}.2}\right) = \frac{1}{2}erfc\left(\sqrt{\frac{21}{32}\frac{Eb}{N_{02}}}\right) \quad \text{.......................... ....} \quad (4.31)$$

4.8. Choosing Transmitter Threshold for Sys2

In order to be fair when comparing the two systems, the next step is to choose a proper threshold value for Sys2 employing the same constraint as the plain OFDM system. For a plain OFDM system, the system was allowed to clip the input data once a day; this was a constraint to determine its amplifier threshold value. The same criterion can be applied to Sys2. As discussed previously, it is possible to have to change almost all of the Encoder & Mapper outputs in order to reduce a high PAPR value. This is a bottleneck for the algorithm. To make this point more clear it may be argued that the algorithm may fail if there are two or more high PAPR samples during one coded block time. This restriction may be used to determine a proper threshold value for the amplifier of Sys2. For this purpose Sys2 will be allowed to clip the input signal when there are more than one high PAPR values in a single coded block. Therefore, the threshold will be chosen to make the probability of committing more than one high PAPR's in a coded block one-per-day.

Assume that the information bit rate is R_b bits/sec. If the system has N channels, each Encoder & Mapper block has R_b/N bits/sec. When using (n,k) encoders at Encoder & Mapper blocks, each k-bit of the information data is used to form a n bit codeword. After k-bits are collected at Encoder & Mapper blocks, each are coded into n-bits and one bit parity is added at the end of each codeword. This structure of (n+1) bit codewords at N-Encoder & Mapper blocks that are formed simultaneously will be named as a coded block for the rest of this section. This means that, one codeword at each Encoder & Mapper block formed simultaneously, constitutes a coded block and results in (n+1)/L OFDM symbols where L is the modulation width, e.g. L=2 for QPSK modulation. Since an OFDM symbol consists of N samples, a coded block has N(n+1)/L samples. Additionally, ignoring coding latency, since each of the Encoder & Mapper blocks should have k-bits of information to form a coded block, T_{block}, which is the coded block duration, may be given as

$$T_{block} = \frac{kN}{R_b}\ sec. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. \quad (4.32)$$

Now, we may define a new parameter M_{24} as the number of coded blocks per day, in the system implementing PAPR reduction algorithm. Since a day has 86400 seconds, M_{24} becomes

$$M_{24} = \frac{86400}{T_{block}} = \frac{86400 R_b}{kN}\ blocks. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. \quad (4.33)$$

Since we expect that for only one coded block, two or more samples exceed a certain threshold per day, we can use the same assumption as (4.17) replacing the variable N_{24} with M_{24} and we may be conclude that

$$P_1 \cong \frac{1}{M_{24}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .\ldots\ldots\ldots \quad (4.34)$$

On the other hand, P1 may be written as:

P_1= 1-[P(1 sample PAPR>z in a coded block) + P(No sample PAPR> z in a coded block)]

Since a coded block consists of (n+1)/L OFDM symbols, a new parameter M may be defined as the number of OFDM symbols per coded block.

$$M = \frac{n+1}{L} \quad \text{...} \quad (4.35)$$

Using the equation P(A sample PAPR exceeds certain threshold z)= e^{-z}

P_1 may be expressed as

$$P_1 = 1 - [MNe^{-z}(1-e^{-z})^{MN-1} + (1-e^{-z})^{MN}] \quad \text{..........................} \quad (4.36)$$

Using polynomial approximations and the assumption that e-3b << e-2b, (4.36) may be rewritten as

$$P_1 \cong \left[MNe^{-z}(1-(MN-1)e^{-z}) + (1 - MNe^{-z} + \binom{MN}{2}e^{-2z})\right]$$

$$\cong \left[MNe^{-z} - MN(MN-1)e^{-2z} + 1 - MNe^{-z} + \frac{MN(MN-1)}{2}e^{-2z})\right]$$

$$P_1 \cong \frac{MN(MN-1)}{2}e^{-2z} \quad \text{...} \quad (4.37)$$

Using equations (4.33), (4.34) and (4.37) together, a relationship between the selected code, information bit rate, clipping rate, number of channels and the transmitter threshold value may be obtained as

$$\frac{MN(MN-1)}{2}e^{-2z} \cong \frac{kN}{86400R_b} \quad \text{...} \quad (4.38)$$

Finally, the desired threshold value for the algorithm implementing system may be written approximately as

$$z \cong -\frac{1}{2}\ln\left(\frac{kN}{86400R_b}\frac{2}{MN(MN-1)}\right) \quad \text{...} \quad (4.39)$$

When the information bit rate, modulation type used, clipping rate (changes the value 86400 which denotes the number of seconds per clipping period) and the code is determined for the system, (4.39) depends only on the PAPR threshold value and the number of channels used in the OFDM system.

4.9 Summary

Assume that the information bit error probability, number of channels, employed code and the information bit rate are given. In order to compare the straightforward clipping system (Sys1) with the system implementing PAPR reduction algorithm (Sys2), the following steps should be applied:

- Block error probability equation of Sys1 is calculated for the given code in the form of (4.9). (For the (31, 21) 2-error correcting code, the equation is found out to be (4.11).)
- Pb1 is calculated using the equation found in Step 1.
- SNR of Sys1 is determined applying (4.8) for the given code.
- From the provided system parameters, PAPR threshold of Sys1 is calculated using (4.20).
- For Sys2, block error probability equation is written in the form of (4.23) using the equations (4.24), (4.25) and (4.26). (For the (31, 21) 2-error correcting code, this equation is calculated as in (4.30).)
- The PAPR threshold value of Sys2 is calculated using the previously determined parameters and equation (4.39).
- PAPR threshold value is inserted into the block error probability equation of Sys2 and the coded bit error probability, i.e. Pb2, is calculated.
- Pb2 value is used to determine the SNR of Sys2 using the equation (4.8) for the given code.

The two systems can be compared. In the next chapter, numerical results will be provided using this procedure.

CHAPTER 5

RESULT AND DISCUSSION

This chapter is devoted to the graphical and tabular results of the derivations of Chapter 4. First part will be the graphs of the previously derived equations and second part will be tables of gain parameters obtained for different conditions.

5.1. Matlab Coding Scheme

In this part, an evaluation of factors which could influence the PAPR reduction performance is performed using Matlab coding. Firstly from the perspectives of complexity and practicability, phase factor is defined as Pm, $\in[\pm1,\pm j]$. This reduces calculation complexity dramatically compared to performing miscellaneous complex multiplication.

For comparison, PAPR reduction with coding techniques is deemed. In the OFDM system under consideration, coding is applied to the sub blocks of un-coded information, which is modulated by QPSK. The performance evaluation is done in terms of complementary cumulative distribution function. PAPR reduction performance effects by number of coded block length M.

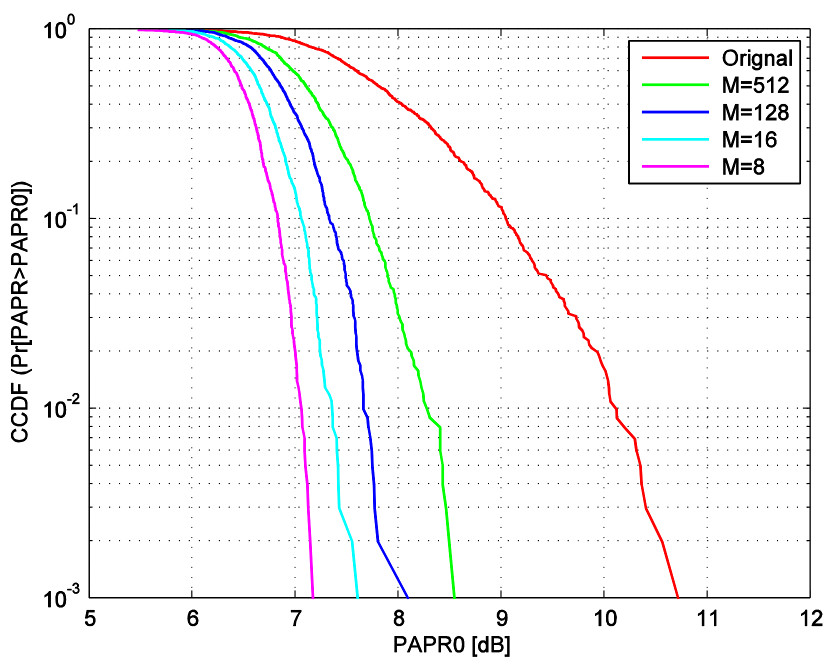


Figure 5.1 Effect of PAPR with different values of M with N=128.

From Fig. 5.1, it can be observed that the proposed method displays a better PAPR reduction performance than the original OFDM signal which is free of any PAPR reduction scheme. The probability of high PAPR is significantly decreased. Decreasing M leads to the improvement of PAPR reduction performance.

5.2. Influencing Factors of PAPR

From the previous chapter, it was shown that PAPR is closely related to number of sub-carriers, modulation schemes, and oversampling rate.

5.2.1. Number of sub-carriers (N)

Different number of sub-carrier results in different PAPR performances due to the varying information carried. In this case, we set the number of sub-carrier N equals to 256, 128, and 64, respectively. In the Fig. 5.2, the CCDF curve of original sequences's PAPR is given as the reference of comparison to the others which coding method been used. The x-axis represents the PAPR thresholds while the y-axis represents the probability of CCDF.

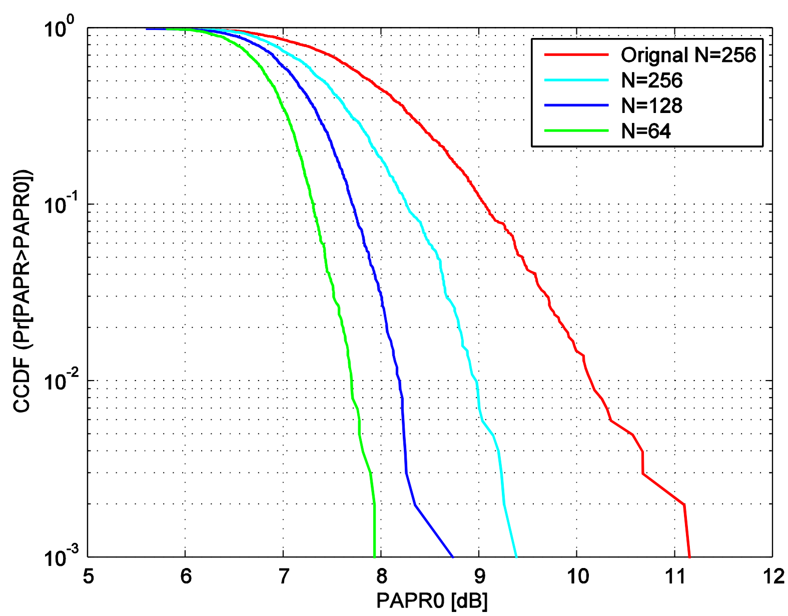


Figure 5.2 Effect of PAPR with different values of N with M=16.

It shows that when the number of sub-carriers increases, the PAPR also increase. As shown in Fig. 5.2, when modulation scheme set as QPSK mode, the PAPR exceeds 10 dB accounts for only 0.1% of transmitted OFDM signals when the sub-carrier number is 64, approximately. But when the sub-carrier number rises up to 256, the PAPR exceeds 10 dB accounts for almost 1% of transmitted OFDM signals. Therefore, the number of sub-carrier is a very important influence factor on the PAPR.

5.2.2. Modulation schemes

Different modulation schemes produce different PAPR performance. Figure 5.2 displays a set of CCDF curves which are processed by several commonly used modulation schemes like BPSK, QPSK, 16QAM and 64QAM with the number of sub-carriers N=128 and the block length M =16. Results show that there is only small difference between different modulation schemes. Thus, different modulation schemes have minimum influence on PAPR performance.

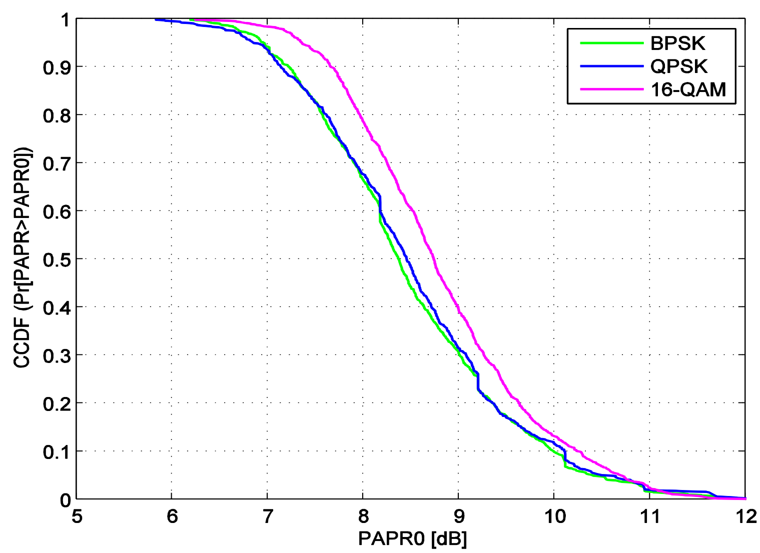


Figure 5.3 Effect of PAPR with different Modulation schemes.

5.2.3. Oversampling rate

In real implementation, continuous-time OFDM signal cannot be described precisely due to the insufficient N points sampling. Some of the signal peaks may be missed and PAPR reduction performance is unduly accurate [39]. To avoid this problem, oversampling is usually employed, which can be realized by taking s·N point IFFT/FFT of original data with (s-1)·N zero-padding operation. Oversampling plays an important role for reflecting the variation features of OFDM symbols in time domain. As shown in Fig. 5.4 for a fixed probability, higher oversampling rate leads to higher PAPR value and good PAPR reduction performance. Generally, oversampling factor s = 4 is sufficient to catch the peaks.

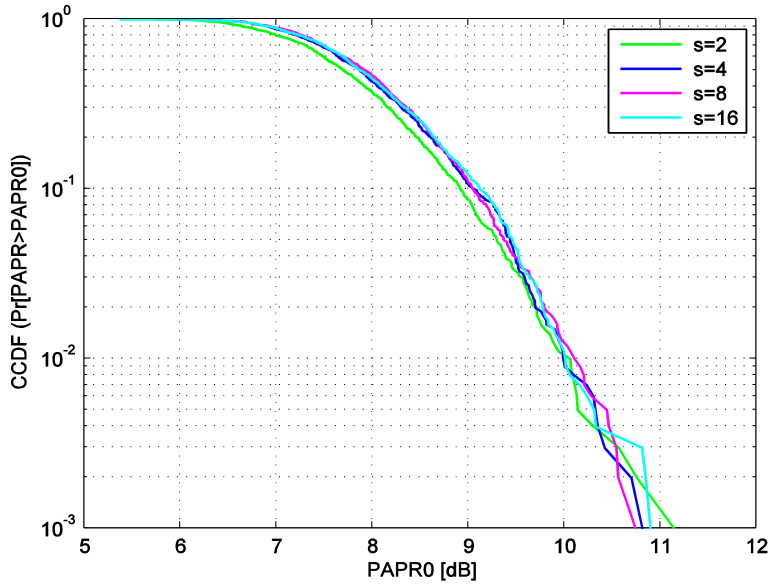


Figure 5.4 Effect of PAPR with different values of s.

5.3 Comparison with other coding technique

Figure 5.5 demonstrates the performance comparisons of an OFDM system with PAPR reduction techniques. Here Hamming and LDPC coding are considered for performance comparisons.

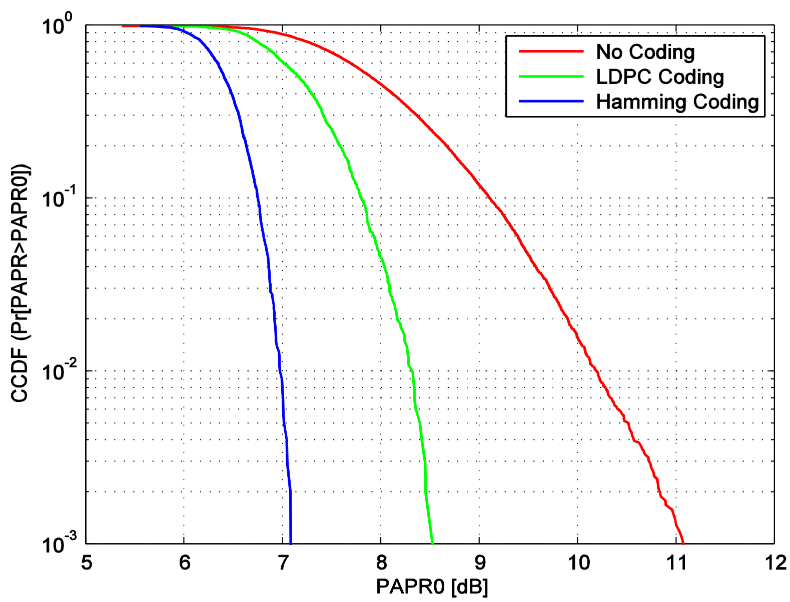


Figure 5.5 Effect of PAPR performances with Hamming and LDPC coding

From the Figure 5.5, we learned that with the same CCDF probability 1%, the PAPR value equals to 7.2dB when Hamming is employed, while the PAPR raise up to 8.5dB when LDPC is employed under the same circumstance. It shows clearly that Hamming coding method provides a better PAPR reduction performance compared to LDPC coding method.

5.4. Basic System Behavior

Figure 5.6 shows SNR versus codded bit error probability for the two sample systems. In order to reflect the effect of information bit error probability to SNR, equations (4.8) and (4.31) are used. As seen from the figure, as the desired information bit error probability level is decreased, Sys2 should operate at a somewhat higher SNR than the plain OFDM system.

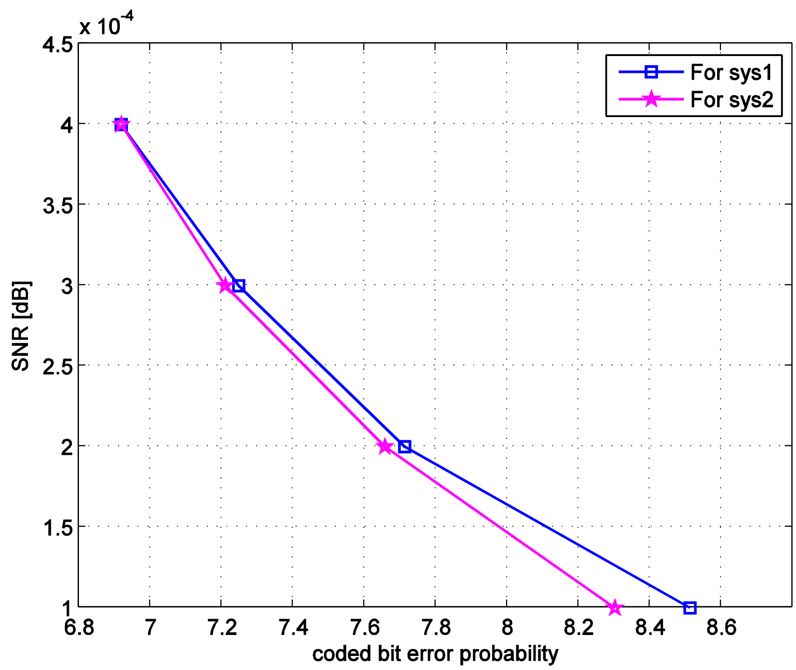


Figure 5.6 Coded Bit Error Probability versus SNR

Figure 5.7 shows the block error probability and coded bit error probability relations for the two sample systems of (31,21) Hamming code employing systems with 32 channels having an information bit rate of 8 Mbits/sec and using a clipping rate of 1-per-day. Equations (4.11) and (4.30) are employed and the PAPR threshold of Sys2 is calculated from the sample system parameters using (4.39).

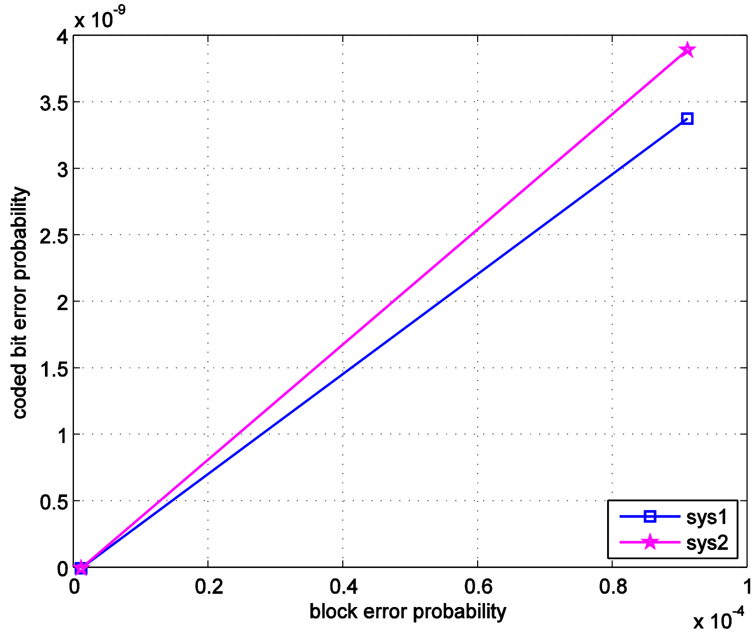


Figure 5.7 Coded Bit Error Probability versus Block Error Probability

As discussed before, information bit error probability may be assumed to be equal to the half of the block error probability. Therefore, block error probability is a direct indication of the information bit error probability. As seen from Figure 5.7, if low information bit error probabilities are desired, the system using PAPR reduction algorithm is to have a much lower coded bit error probability than the simple OFDM modulating system in order to have the same information bit error probability.

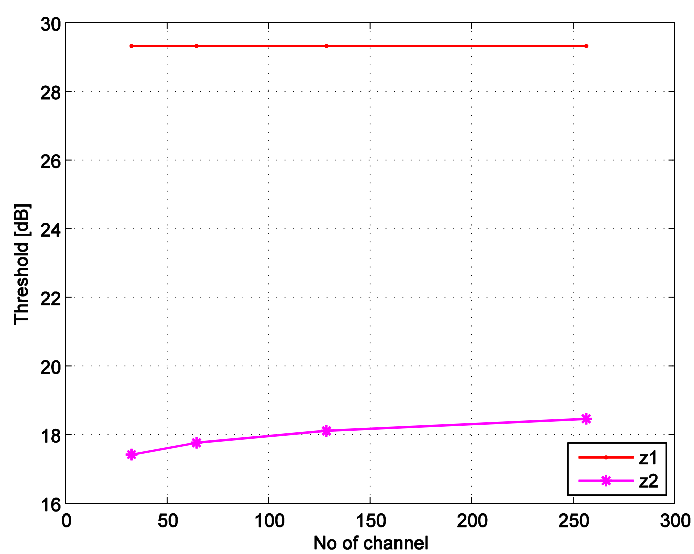


Figure 5.8 Number of Channel versus Threshold for two system

Figure 5.8 is plotted to show how the PAPR threshold value of Sys2 changes as the number of channels changes. As seen from Figure 5.8, the suitable transmitter threshold value increases much less slowly than the number of channels for sys2, this is due to very small probabilities of committing a high PAPR value whereas threshold is constant for sys1. Additionally, the PAPR threshold value for a 32-channel system is found out to be 16.18, which results in the clipping probability of one-per-day.

5.5. Tables of Gain Parameters

In order to better understand the effect of system parameters on the algorithm performance, six tables are provided (Table 5.1 – Table 5.6). The tables are used to compare a plain OFDM system; i.e.Sys1, with a system implementing PAPR reduction algorithm; i.e. Sys2. The Z1 and Z2 columns correspond to the PAPR threshold values of Sys1 and Sys2, respectively. These values are calculated using equations (4.20) and (4.39). Additionally, Eb/No1 and Eb/No2 are the system SNR of Sys1 and Sys2. The information bit error probability is set to a value of 10^{-6} and kept constant for all of the tables. Therefore, the tables can be compared based on the equal information bit error probability condition. The tables are obtained for the cases of 32 and 128 system subchannels, 1-per-day and 1-per-month clipping rates and 8 Mbits/sec and 100 Mbits/sec information bit rates.

In order to obtain the values in the tables, initially block error probability equations are written. Equations (4.11) and (4.30) are derived for the special case of (31,21) 2-error correcting Hamming code. (4.9) and (4.22) should be modified for the selected code. Coded bit error probabilities, Pb_1 and Pb_2 are calculated using the previously derived block error probability equations and used for the calculation of SNR values of Sys1 and Sys2.

Each of the six tables includes four small tables. First three tables can be used to compare codes of different lengths with the same error correction capability. The fourth table indicates the case of approximately equal rate codes with various lengths and error correction capabilities. The effect of system parameters will be analyzed in the following sections. The tables will be compared to observe the effect of the number of channels, clipping rate, information bit rate and different coding schemes on the system performance. These effects will be considered separately.

5.5.1. Number of Channels versus Threshold

The effect of number of channels can be observed by comparison of Table 5.1 to Table 5.3 and alternatively, Table 5.4 to Table 5.6. The comparison is made with all other system parameters, i.e. clipping rate and information bit rate, are kept constant. Also, equivalent rows of the tables are compared which corresponds to keeping the employed code constant also.

As can be seen from the tables, threshold Z1 does not change with the number of channel variations. This result is due to (4.20) which shows that Z1 is independent of the number of channels. SNR of Sys1 is equal for corresponding rows of all tables, i.e. when the same code is applied to the system. This can be explained in the way that, the coefficients of the block error probability equation of Sys1 in the form of (4.9) depends only on the applied code. Since the information bit error probability is constant for all cases, the block error probability is also constant for equal code case. Therefore, the coded bit error probabilities are also equal resulting in equal SNR values. As a result, the gain parameter of Sys1 is constant when the code, the clipping rate and information bit rate are constant and the number of channels is varying. In other words, the transmit power is constant for Sys1 when system parameters other than number of channels are kept constant.

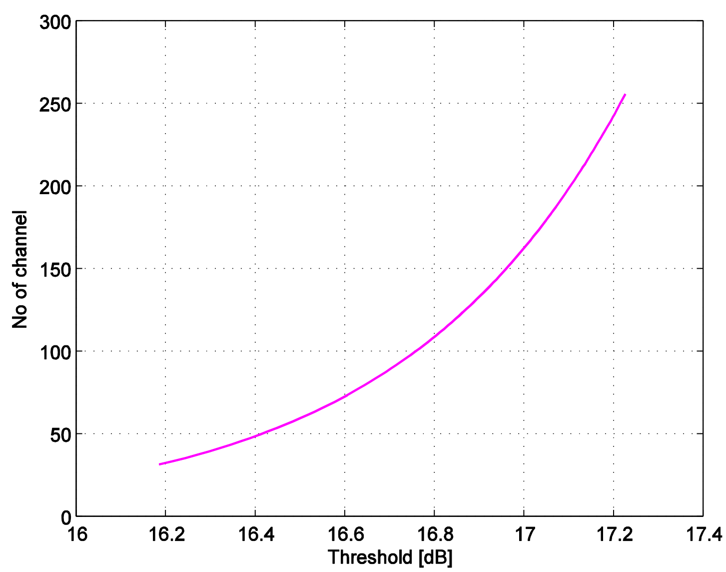


Figure 5.9 Number of Channel versus Algorithm Threshold

For Sys2, it is observed from Figure 5.9 that, Z2 is increased as the number of channels increases. Also the SNR of Sys2 shows a slight increase for increasing number of channels. Therefore, the gain parameter of Sys2 is also increased as the number of channels is increased which results in an increase of transmit power.

5.5.2. Clipping Rate versus Threshold

The effect of clipping rate can be observed by the comparison of Table 5.1 to Table 5.4 and Table 5.2 to Table 5.5 and alternatively, Table 5.3 to Table 5.6. When the clipping rate is reduced from 1 clip-per-day to 1 clip-per-month, the Z1 parameter of Sys1 is increased which can be seen from (4.20). Also the SNR of Sys1 is constant when the rows of tables with the same code are compared.

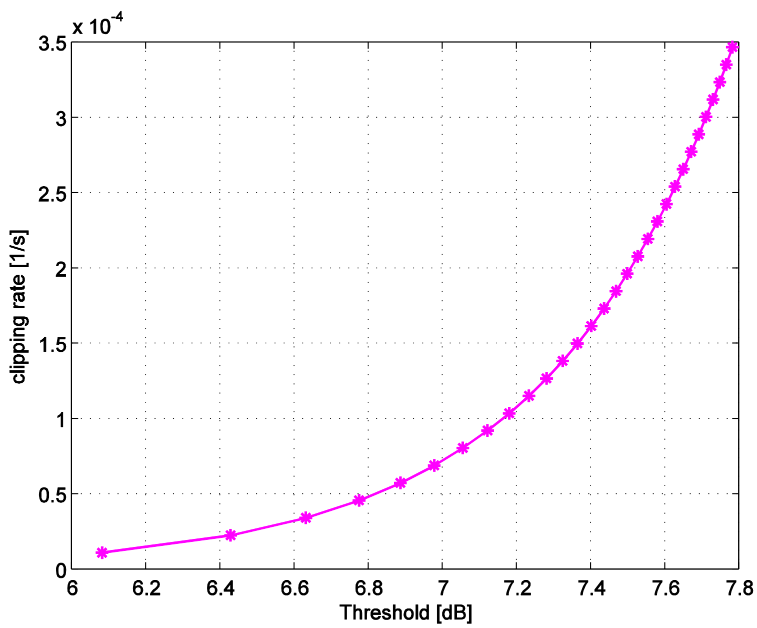


Figure 5.10 Clipping Rate versus Threshold

For Sys2, Z2 is increased for reduced clipping rate as can be observed form (4.39). The gain parameter is also increased which results in an increase in the transmit power of Sys2 for the case of reduced clipping rate. The gain parameters of the two systems are both increased when the clipping rate is reduced. When the clipping rate is reduced, the transmit power of both systems are increased.

5.4.3. Information Bit Rate versus Threshold

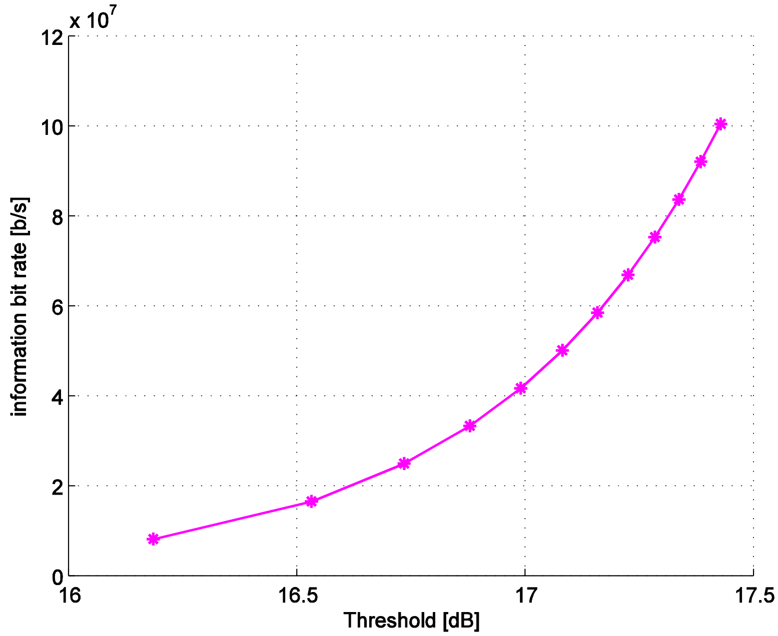


Figure 5.11 Information Bit Rate versus Threshold

The effect of information bit rate can be observed by the comparison of Table 5.1 to Table 5.2 and Table 5.4 to Table 5.5. As the information bit rate is increased, the same discussion applies as the clipping rate reduction case. In summary, when the information bit rate is increased, Z1 of Sys1 is increased. Sys2 also behaves in the same way. In other words, both systems require more transmit power, as the information bit rate is increased.

5.5.4. Code Rate versus Threshold

In order to observe the effect of the code rate, the codes having the same error correction capability in the same table will be compared with each other. For example, single error correcting codes of Table 5.1 can be compared with each other. From this comparison, it may be stated that Z1 is slightly decreased for increasing code rate of the same error correction capacity. This result can be observed from (4.20). Z1 is related approximately to the reciprocal of the code rate. Therefore, an increase in the code rate, results in a decrease in Z1.

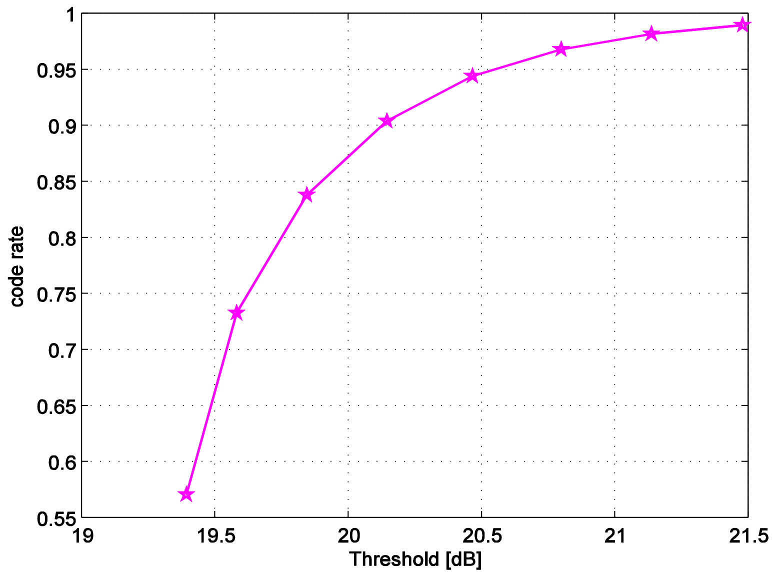


Figure 5.12 Code Rate versus Threshold

Table 5.1 is examined, if the plain OFDM system is to be used, the (31,26) code seems to be the most advantageous among four single error correcting codes.

For the case of Sys2, as the code rate is increased, Z2 is also increased. It can be seen from (4.39) that inside the logarithm expression, there is a factor of approximately k/n2. An increasing code rate results in a decrease of this factor, therefore, Z2 is increased.

5.5.5. Code Length versus Threshold

In this section, the effect of code length will be examined. The codes with equal rates will be considered. The fourth part of the tables shows the case of two equal rate codes. First is a (255,247) single error correcting code and the second one is the (1023,993) 3-error correcting code with approximately equal rate of 0.97. Since the code rates are equal, Z1 does not change for different codes. As the Z1 value is constant, if the longer code is applied a lower transmit power will be enough. For Sys2, as described before, Z2 is approximately related to the ratio of k/n2. This factor can be written as rate/n. Since the rate is constant for both codes, rate/n factor is smaller for the longer code. Hence, Z2 is increased for the longer code of equal rate. As for Sys1, the SNR value of Sys2 is much lower for the longer code. The reduction of SNR is higher than the increase in Z2. As for Sys1, Sys2 also requires a lower transmit power for the longer code of same rate.

Table 5.1 Gain Parameter for **32-Channels**, Clipping Rate of **One-per-Day** and Information Bit Rate of **100 Mbits/sec**

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,11) 1 err	29.5164	17.1823	10.0590	9.9288	0.7333
(31,26) 1 err	29.3493	17.4463	8.5114	8.4013	0.8387
(255,247) 1 err	29.1775	18.4009	7.1675	7.0748	0.9686
(1023,1013) 1 err	29.1525	19.0817	6.9906	6.9002	0.9902

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,7) 2 err	29.9684	17.4083	15.8070	15.6024	0.4667
(31,21) 2 err	29.5629	17.5531	10.5380	10.4016	0.6774
(255,239) 2 err	29.2104	18.4174	7.4074	7.3116	0.9373
(1023,1003) 2 err	29.1624	19.0866	7.0603	6.9689	0.9804

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,231) 3 err	29.2445	18.4344	7.4074	7.3116	0.9059
(1023,993) 3 err	29.1725	19.0917	7.0603	6.9689	0.9707

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,247) 1 err	29.1775	18.4009	7.1675	7.0748	0.9686
(1023,993) 3 err	29.1725	19.0917	7.0603	6.9689	0.9707

Table 5.2 Gain Parameter for **32-Channels**, Clipping Rate of **One-per-Day** and Information Bit Rate of **8Mbits/sec**

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,11) 1 err	26.9907	15.9194	10.0590	9.9288	0.7333
(31,26) 1 err	26.8236	16.1834	8.5114	8.4013	0.8387
(255,247) 1 err	26.6518	17.1381	7.1675	7.0748	0.9686
(1023,1013) 1 err	26.6268	17.8188	6.9906	6.9002	0.9902

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,7) 2 err	27.4427	16.1454	15.8070	15.6024	0.4667
(31,21) 2 err	27.0372	16.2902	10.5380	10.4016	0.6774
(255,239) 2 err	26.6847	17.1545	7.4074	7.3116	0.9373
(1023,1003) 2 err	26.6367	17.8238	7.0603	6.9689	0.9804

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,231) 3 err	26.7187	17.1716	7.4074	7.3116	0.9059
(1023,993) 3 err	26.6467	17.8288	7.0603	6.9689	0.9707

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,247) 1 err	26.6518	17.1381	7.1675	7.0748	0.9686
(1023,993) 3 err	26.6467	17.8288	7.0603	6.9689	0.9707

Table 5.3 Gain Parameter for **128-Channels**, Clipping Rate of **One-per-Day** and Information Bit Rate of **100 Mbits/sec**

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,11) 1 err	29.5164	17.8769	10.0590	9.9288	0.7333
(31,26) 1 err	29.3493	18.1402	8.5114	8.4013	0.8387
(255,247) 1 err	29.1775	19.0942	7.1675	7.0748	0.9686
(1023,1013) 1 err	29.1525	19.7749	6.9906	6.9002	0.9902

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,7) 2 err	29.9684	18.1029	15.8070	15.6024	0.4667
(31,21) 2 err	29.5629	18.2470	10.5380	10.4016	0.6774
(255,239) 2 err	29.2104	19.1106	7.4074	7.3116	0.9373
(1023,1003) 2 err	29.1624	19.7798	7.0603	6.9689	0.9804

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,231) 3 err	29.2445	19.1277	7.4074	7.3116	0.9059
(1023,993) 3 err	29.1725	19.7848	7.0603	6.9689	0.9707

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,247) 1 err	29.1775	19.0942	7.1675	7.0748	0.9686
(1023,993) 3 err	29.1725	19.7848	7.0603	6.9689	0.9707

Table 5.4 Gain Parameter for **32-Channels**, Clipping Rate of **One-per-Month** and Information Bit Rate of **100 Mbits/sec**

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,11) 1 err	32.9176	18.8829	10.0590	9.9288	0.7333
(31,26) 1 err	32.7505	19.1469	8.5114	8.4013	0.8387
(255,247) 1 err	32.5787	20.1015	7.1675	7.0748	0.9686
(1023,1013) 1 err	32.5537	20.7823	6.9906	6.9002	0.9902

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,7) 2 err	33.3696	19.1089	15.8070	15.6024	0.4667
(31,21) 2 err	32.9641	19.2537	10.5380	10.4016	0.6774
(255,239) 2 err	32.6116	20.1180	7.4074	7.3116	0.9373
(1023,1003) 2 err	32.5636	20.7872	7.0603	6.9689	0.9804

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,231) 3 err	32.6457	20.1350	7.4074	7.3116	0.9059
(1023,993) 3 err	32.5736	20.7923	7.0603	6.9689	0.9707

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,247) 1 err	32.5787	20.1015	7.1675	7.0748	0.9686
(1023,993) 3 err	32.5736	20.7923	7.0603	6.9689	0.9707

Table 5.5 Gain Parameter for **32-Channels**, Clipping Rate of **One-per-Month** and Information Bit Rate of **8 Mbits/sec**

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,11) 1 err	30.3919	17.6200	10.0590	9.9288	0.7333
(31,26) 1 err	30.2248	17.8840	8.5114	8.4013	0.8387
(255,247) 1 err	30.0530	18.8387	7.1675	7.0748	0.9686
(1023,1013) 1 err	30.0280	19.5194	6.9906	6.9002	0.9902

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,7) 2 err	30.8439	17.8460	15.8070	15.6024	0.4667
(31,21) 2 err	30.4384	17.9908	10.5380	10.4016	0.6774
(255,239) 2 err	30.0859	18.8551	7.4074	7.3116	0.9373
(1023,1003) 2 err	30.0379	19.5244	7.0603	6.9689	0.9804

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,231) 3 err	30.1199	18.8722	7.4074	7.3116	0.9059
(1023,993) 3 err	30.0479	19.5294	7.0603	6.9689	0.9707

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,247) 1 err	30.0530	18.8387	7.1675	7.0748	0.9686
(1023,993) 3 err	30.0479	19.5294	7.0603	6.9689	0.9707

Table 5.6 Gain Parameter for **128-Channels**, Clipping Rate of **One-per-Month** and Information Bit Rate of **100 Mbits/sec**

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,11) 1 err	32.9176	19.5775	10.0590	9.9288	0.7333
(31,26) 1 err	32.7505	19.8408	8.5114	8.4013	0.8387
(255,247) 1 err	32.5787	20.7948	7.1675	7.0748	0.9686
(1023,1013) 1 err	32.5537	21.4754	6.9906	6.9002	0.9902

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(15,7) 2 err	33.3696	19.8035	15.8070	15.6024	0.4667
(31,21) 2 err	32.9641	19.9476	10.5380	10.4016	0.6774
(255,239) 2 err	32.6116	20.8112	7.4074	7.3116	0.9373
(1023,1003) 2 err	32.5636	21.4804	7.0603	6.9689	0.9804

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,231) 3 err	32.6457	20.8283	7.4074	7.3116	0.9059
(1023,993) 3 err	32.5736	21.4854	7.0603	6.9689	0.9707

Code(n,k)	Z1	Z2	Eb/No1	Eb/No2	Code rate
(255,247) 1 err	32.5787	20.7948	7.1675	7.0748	0.9686
(1023,993) 3 err	32.5736	21.4854	7.0603	6.9689	0.9707

5.6 Summary

It is observed from the tables that applying the PAPR reduction algorithm reduces the transmit power of the system when compared to the plain OFDM modulating system. The tables demonstrate that when all the other system parameters are kept constant, the transmit power of the system applying the PAPR reduction algorithm is reduced for the cases of

- Less number of channels,
- Higher clipping rate,
- Lower information bit rate,
- Longer codes (more error correcting codes) among equal rate alternatives.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

There are many different methods proposed in the literature for the purpose of reducing the PAPR of an OFDM signal. The methods can be grouped under three main categories as clipping, scrambling and coding. In this thesis, an algorithm to reduce PAPR of an OFDM signal is proposed. The method is based on block coding the input data and introducing deliberate errors prior to transmission iteratively, until the PAPR goes below a previously determined threshold value.

In this thesis, the performance is measured by the comparison of an OFDM system applying the PAPR reduction algorithm, with a simple clipping rate. Two systems are compared under the assumption that equal information bit error probabilities should be satisfied for both. Additionally, a very low probability of clipping the OFDM signal is allowed for both of the systems, since a very low clipping rate can be acceptable for OFDM services. Using the clipping rate as a constraint, two different PAPR threshold values are determined for the systems. The algorithm tries to reduce the PAPR value below the selected threshold.

The simulation results indicate that, application of the algorithm results in significant reduction in the PAPR values. In order to examine the performance potential of the algorithm, extreme conditions were generated by injecting high PAPR producing symbols in two simulation cases. Through this method it was observed that a PAPR reduction of approximately 1.6 dB was achieved for a 64-channel system whereas approximately 1.4 dB was achieved for 128-channel system. The provided values indicate merely the achieved PAPR reduction.

6.2 Future Work

As a future work, the QPSK modulation used in simulations may be replaced with other modulations such as QAM and the effect of modulation type can be examined. Moreover, it is possible to implement different coding schemes such as convolutional codes at various rates and error correction capabilities. Consequently, a better implementation may be to skip those modifications which do not contribute much to PAPR reduction.

Furthermore, another improvement may be realized when more than two bit modifications per coded block are allowed using a code with more error correction capability. This flexibility lets the algorithm to reduce the PAPR values of the two OFDM symbols per coded block which indicates that the transmitter power of the system implementing the PAPR reduction algorithm could be further reduced.

Through the actual simulation process, we realize that the simulation routines used in our thesis is time consuming and results are fallible due to the limitation of Matlab simulation system. In subsequent researching, simulation software - Visual C (VC) can be used to improve the simulation accuracy and efficiency.

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