Analysis of Cross Phase Modulation with First- and Second Order GVD in WDM of Fiber Optic Communication System

by

Nasrin Sultana

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It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

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Nasrin Sultana

DEDICATION

To my parents

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LIST OF ABBREVIATIONS

BER	Bit Error Rate
CD	Chromatic Dispersion
FWM	Four Wave Mixing
FWHM	Full Width at Half-Maximum
GVD	Group Velocity Dispersion
ISI	Inter Symbol Interference
LEAF	Large Effective Area Fiber
NLSE	Non Linear Schrödinger Equation
SBS	Stimulated Brillouin Scattering
SMF	Single Mode Fiber
SPM	Self Phase Modulation
SRS	Stimulated Raman Scattering
SSFM	Split Step Fourier Method
SSMF	Standard Single Mode Fiber
WDM	Wavelength Division Multiplexing
XPM	Cross Phase Modulation

LIST OF SYMBOLS

$\boldsymbol{n}$	Refractive index
$\boldsymbol{n}_2$	Nonlinear refractive index of fiber
$\boldsymbol{E}$	Electric field
$\Psi(\omega)$	Spectral width of pulse
$\Delta\omega$	RMS bandwidth of pulse
$\omega_{\boldsymbol{o}}$	Carrier frequency
β	Propagation constant
β_1	Inverse group velocity
β_2	First order group velocity dispersion
β_3	Second order group velocity dispersion
α	Attenuation coefficient
λ	Wavelength of light
$\boldsymbol{L_D}$	Dispersion length
$\boldsymbol{L_{NL}}$	Nonlinear length
$\boldsymbol{L_{eff}}$	Effective length
γ	Kerr nonlinear coefficient
$\boldsymbol{z}$	longitudinal coordinate of the fiber
$\boldsymbol{A_{eff}}$	Effective cross section area of fiber
$\boldsymbol{T}_0$	Pulse width
$\boldsymbol{L_W}$	Walk of length
$\boldsymbol{L}$	Fiber length
t	Time in a framework moving at the group velocity
$\boldsymbol{A}$	Pulse amplitude
$\boldsymbol{f}(\boldsymbol{t})$	Pulse shape
$\boldsymbol{U}$	Pulse broadening factor
$\hat{\boldsymbol{L}}$	Linear operator
$\hat{\boldsymbol{N}}$	Nonlinear operator
$\theta_{\boldsymbol{m}}$	Maximum phase shift

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ABSTRACT

With increased channel capacity, launched optical powers, bit rates and number of wavelength channels, the cross phase modulation (XPM) has become the most important nonlinear effect and limits system performance. On the other hand, group velocity dispersion (GVD) is one of the linear effects in the optical fiber that also restricts bit rates. In high bit rate and first order GVD compensated system, the effect of second order GVD become significant and also play a critical role in limiting the system performance. As a result the combined effects of XPM with first- and second order GVD cause further deterioration of transmission performance in a WDM system.

In this research work, analysis has been carried out to find expressions of pulse broadening factor as well as normalized output by solving the nonlinear Schrodinger equation (NLSE) analytically, considering the effect of XPM with first- and second order GVD. The results are evaluated at various data rates, different input powers as a function of transmission distance using standard single mode fiber (SSMF) and large effective area fiber (LEAF). The pulse broadening is strongly dependent on the effects of XPM with first- and second order GVD. Though at shorter distance and low bit rates the effect of second order GVD is not noticeable but as the bit rate increases its effect becomes significant and impacts the system performance. It is observed that the data rate and fiber length have higher impact on pulse broadening than the input power. The results are computed for SSMF and LEAF using analytical derivation and numerical simulation done through split step Fourier method with same data rate and input power. It is found that the effects of XPM with first- and second orders GVD is more effective in SSMF fiber than that of LEAF fiber. Thus the findings of this work can predict the amount of performance degradation due to the effects of XPM with first- and second order GVD and may be helpful to design high speed long haul WDM fiber-optic transmission link.

CHAPTER 1
INTRODUCTION

1.1 Optical Fiber Communication

Communication means the exchange of information which may be voice, video or data. So, a communication system transmits information from one place to another place. Optical fiber communication is a communication system that employs optical fibers for information transmission. Such systems have been deployed worldwide since 1980 [1].

Twenty first century is the era of information technology (IT). IT has achieved an exponential growth through the modern telecommunication systems. Particularly, optical fiber communication plays a vital role in the development of high quality and high-speed telecommunication systems. Today, optical fibers are not only used in telecommunication links but also used in the Internet and local area networks (LAN) to achieve high signaling rates [2].

Optical transmission is a preferred medium for long distance, high bandwidth communication system running at speeds in the range of gigabit per second or higher. The important impairments in optical fiber are attenuation, dispersion and nonlinearities. Initially in 1966 optical fiber had extremely high loss which exceeded 1000 dB/km. In the third generation 1.55-μm wavelength region the attenuation in the single-mode fiber become lowest (0.2 dB/km) [3].

The impairment that limits a fiber's bandwidth is known as dispersion. Dispersion is the spreading of the optical pulses as they travel down the fiber. In late 1990s wavelength division multiplexing (WDM) systems have been widely deployed as a solution for higher bit rate transmission. Wavelength division multiplexing (WDM) solves dispersion problems by keeping the transmission rates of each channel at reasonably low levels (e.g. 10Gbps) and achieving a high total data rate by combining several channels together [4].

Light from lasers and LEDs is not highly monochromatic. It consists of some harmonics near the fundamental frequency. The different spectral components of the optical pulse travel at slightly different group velocities that lead to dispersion called group velocity dispersion (GVD). As the pulses spread or broaden due to GVD, they tend to overlap and are no longer distinguishable by the receiver as 0s and 1s. As a result errors and loss of information occur. The GVD includes first order group velocity dispersion, higher order group velocity dispersion.

The first order GVD is one of the most relevant factors that limit transmission length in high rate optical communication systems. It is possible to cancel first-order GVD using dispersion compensation fibers or fiber Gratings. As the transmission speed is increased, the influence of higher-order (second-order) dispersion is greater. The second order GVD is determinant on the pulse shape, pulse amplitude, pulse broadening and consequently in inter-symbol interference [5].

In long distance transmission links above 100 km including analog signals over optical fibers, high transmission powers are involved which give rise to nonlinear effects such as self-phase modulation (SPM) in a single channel system and cross phase modulation (XPM) between channels in a WDM system and can be highly detrimental in the presence of dispersion. These nonlinear effects limit performance in both digital and analog fiber optic communication systems [6].

The process towards ever increasing speeds in fiber encounters an obstacle in the form of the group velocity dispersion (GVD) in the optical fiber that restricts bit rates.

On the other hand, with increased channel capacity, launched optical powers, bit rates and number of wavelength channels in WDM system, the XPM has become the most important nonlinear effect and limits system performance [7].

As a result, the combined effects of XPM and GVD may cause further deterioration of transmission performance in a WDM system. Thus, it is essential to study the impact of XPM and GVD on the propagating pulse to predict the performance limitation or system outage accurately due to these effects in the WDM transmission system.

1.2 Related Research Works

Over the years, lot of research works has been carried out to study the harmful impact of fiber linearity and nonlinearity on optical transmission system. The XPM is the most important effect among the different types of fiber nonlinearities that limits system performance. On the other hand, GVD is one of the linear effects that restrict bit rates. In the following section, we are describing some of the research works those are carried out to evaluate the impact of XPM and GVD in WDM optical transmission system.

Miyagi M. et al. (1979) investigated the Pulse spreading in a single-mode optical fiber due to the second order GVD term of the waveguide when an optical source with a finite spectral width is modulated by a Gaussian-shaped pulse [8].

Hui et al. (1998) evaluated spectral characteristics of XPM in multi-span WDM systems both experimentally and theoretically in terms of crosstalk [9]. They found that the crosstalk level is dependent on optical channel spacing and fiber dispersion.

Majumder et al. (1998) evaluated the impact of GVD on the performance in a WDM optical network in terms of BER by analytically [10]. They have shown that the performance of the system highly degrades in presence of fiber dispersion and presence of dispersion imposes several restrictions on the number of nodes and the node spacing.

Hoon (2003) investigated, theoretically and experimentally the SPM and XPM induced phase noise in a DPSK system [11]. It is reported that the XPM induced phase noise becomes as large as SPM induced phase noise in a NZDSF link for channel spacing less than 100GHz. BER degradation is also observed for two channel systems as compared to a single channel system.

Sandra et al. (2003) investigated the performance for an uncharged and non-magnetic dielectric optical fiber due to higher order dispersion in terms of the pulse broadening by solving wave equation [12].

Lijun et al. (2005) investigated the effect of third-order dispersion on pulsating, erupting and creeping solitons [13]. It is shown that the effect of third-order dispersion will cause asymmetric pulse and lead to the appearance of oscillation on the trailing edge of the pulse for the positive third order dispersion. It is found that even small third-order dispersion can dramatically alter the behavior of these solitons.

Abdul-Rashid et al. (2006) investigated the performance in WDM Passive optical networks in the presence of XPM and GVD both experimentally and theoretically [14]. A general expression for electrical average noise power and electrical crosstalk level due to XPM and GVD was derived to measure the system performance for N number of WDM channels. Using the expression, they have shown that XPM and GVD causes crosstalk in the system and imposes a power penalty as the number of WDM channels increases for a given channel spacing and modulating frequency.

Sakib et al. (2006) evaluated theoretically the impact of XPM on the performance of a 2-channel WDM optical transmission system with short-period dispersion-managed fiber (SPDMF) in terms of BER [15]. The computed results show that the BER performance can be improved considerably by using an SPDMF.

Yasim et al. (2007) theoretically studied the effect of XPM induced crosstalk in WDM networks on received power and number of channels for various fiber types [16]. System performance is evaluated through determining the cumulative XPM induced crosstalk relation with both of the received power and number of channels.

Shaari et al. (2008) theoretically studied the effect of XPM crosstalk in WDM networks on received power and number of channels for various fiber types [17]. Analytical approach has been used to evaluate BER performance limitation of a WDM transmission system imposed by crosstalk due to XPM and the influence of changing channel spacing for various fiber types on the BER. Numerical results demonstrated the validity of their analysis and theoretical expressions have an insight into nonlinear effect under investigation.

Bijoy et al. (2009) carried out an analysis to find an expression for pulse broadening factor by solving nonlinear Schrodinger equation (NLSE) considering the effects of SPM and CD in [18].

Khayer et al. (2010) investigated the impact of XPM on fiber-optic communication systems in presence of first- and second order GVD. They analyzed the system performance in terms of crosstalk [19].

Bavithra et al. (2013) analyzed theoretically and experimentally that the anomalous group velocity as well as polarization mode dispersion has been effectively suppressed through optical nonlinearity in dense WDM [20].

Taopin Hu et al. (2013) investigated the modulation instability induced by XPM in dispersion decreasing fiber, whose dispersion decreases along the direction of propagation [21]. It is solved and analyzed by the perturbation method for the extended nonlinear Schrödinger equation, considering the higher order dispersion.

From the above discussion and literature review, we observed that most of the research works evaluated the impact of nonlinearity XPM without or with the presence of first order GVD in terms of BER, crosstalk, eye diagram. But at high bit rate and long haul system, the second order GVD may have destructive effect on optical transmission system. Only one work is reported about the impact of XPM considering the presence of first- and second order GVD together in a WDM system in terms of cross talk. Thus, it is essential to study and develop the impact of XPM with first- and second order GVD on the performance in WDM system in terms of pulse broadening factor analytically and numerically.

1.3 Motivation

With increasing the transmission rates, launched optical powers and link lengths of the WDM optical communication systems, the impact of nonlinear effects and dispersion on the propagating pulses increases. The nonlinear XPM effect with higher order dispersion decreases the system performance to a great extent. So, it is necessary to analyze and optimize the XPM effect with higher order dispersion, which is the main motivation of this thesis work.

Other aspects that have motivated us to analyze the effect of XPM with first- and second order GVD are as follows

a) In WDM, the frequency dependence of refractive index leads to an important nonlinear phenomenon known as XPM. When two or more optical pulses propagate simultaneously, the XPM is always accompanied by SPM and occurs because the nonlinear refractive index seen by an optical beam depends not only on the intensity of that beam but also on the intensity of other co propagating beams. The results of XPM are asymmetric spectral broadening and broadening of pulse shape. The nonlinear XPM effect limits the allowable input optical power, data rate and system capacity

b) The frequency dependence of the group velocity leads to pulse broadening simply because different spectral components of the pulse do not arrive simultaneously at the fiber output. At relative high bit rate and long distance, first order GVD limits the transmission distance and in a first order GVD compensated fiber system, second order GVD further deteriorates the performance at high bit rate.

The combination of XPM and dispersion affects the waveform and spectrum of each optical pulse during transmission and leads to both temporal and spectral changes of pulses. So it is important to analyze the influence of XPM with first- and second order dispersion on the propagating pulse for high speed transmission systems.

1.4 Objectives with specific aims and possible outcome

The goal of this research is to analysis the XPM with first- and second order GVD in WDM fiber optic transmission system. To meet the goal, the following objectives have been identified:

a) To derive the mathematical expressions for output pulse and pulse broadening factor from nonlinear Schrodinger equation (NLSE) considering the effects XPM with first- and second order GVD separately.

b) To analyze the effects of bit rates, input power and transmission distance on output pulse and pulse broadening using analytical derivation.

c) To simulate the NLSE using split-step Fourier method for finding the output pulse as well as pulse broadening factor considering XPM with first- and second order GVD separately.

d) To compare the analytical findings with numerical results in order to find the validity of the analytical expression.

Outcome

The findings of this research work will be helpful to design high speed long haul WDM fiber-optic link.

1.5 Outline of the Thesis

This thesis consists of five chapters. Brief description of its different chapter is as follows.

Chapter 1 introduces optical fiber communication. Related researches regarding an elaborate record of previous works on the effect of XPM and GVD in WDM fiber optic communication system is described. Motivation, objective with specific aims and possible outcome are presented in this chapter.

Chapter 2 describes various types of optical fiber and loss impairments in the fiber optic communication system. An insight on the fiber nonlinearity and GVD effect in WDM fiber optic transmission system is provided in this chapter. Emphasis was put on XPM and GVD.

Chapter 3 shows the analytical and numerical derivation of normalized output as well as pulse broadening factor due to the effects of XPM with first- and second order GVD.

Chapter 4 provides results of the analytical derivation and numerical simulation. The variations of results are observed considering different values of data rates and input powers for SSMF and LEAF.

Chapter 5 concludes the thesis work and also discusses further scope of future research.

CHAPTER 2
OPTICAL FIBER COMMUNICATION AND ITS VARIOUS IMPAIRMENTS

This chapter introduces the various types of optical fiber and their impairments of an optical fiber communication system. These physical phenomena are briefly summarized here in order to provide the reader with the necessary background knowledge and to set the formalisms and notations that will consistently be used throughout this thesis.

2.1 Introduction

Optical fiber is basically a solid glass rod. The diameter of rod is so small that it looks like a fiber. Optical fiber is a dielectric waveguide. The light travels like an electromagnetic wave inside the waveguide. The dielectric waveguide is different from a metallic waveguide which is used at microwave and millimeter wave frequencies. In a metallic waveguide, there is a complete shielding of electromagnetic radiation but in an optical fiber the electromagnetic radiation is not just confined inside the fiber but also extends outside the fiber. The light gets guided inside the structure, through the basic phenomenon of total internal reflection [22]. The optical fiber consists of two concentric cylinders; the inside solid cylinder is called the core and the surrounding shell is called the cladding.

For the light to propagate inside the fiber through total internal reflections at core-cladding interface, the refractive index (n_1) of the core must be greater than the refractive index $(n_2)$of the cladding. That is $n_1 > n_2$. For extra protection, the cladding is enclosed in an additional layer called the coating or buffer. The coating or buffer is a layer of material used to protect an optical fiber from physical damage. The material used for a buffer is a type of plastic.

2.2 Light Propagation through Optical fiber

When light wave enters at one end of a fiber in proper conditions, most of it is propagated down the length of the fiber and comes out from the other end of the fiber.

Simple Ray Model

A light ray is launched in a plane containing the axis of the fiber. We can then see the light ray after total internal reflection travels in the same plane i.e., the ray is confined to the plane in which it was launched and never leave the plane. In this situation the rays will always cross the axis of the fiber [22]. These are called the meridional rays and depicted in Fig. 2.1.

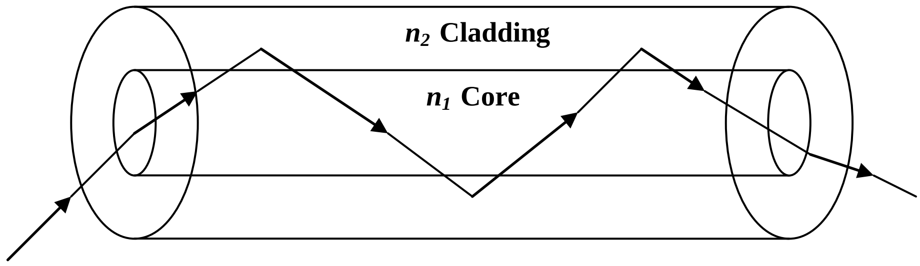

Fig. 2.1: Optical fiber with core, cladding and total internally reflected ray

2.3 Types of fibers

Fibers can be classified according to its core material composition. If the refractive index of the core is uniform and changes abruptly at the cladding boundary, then it is called as step-index fiber. If the refractive index changes at each radial distance, then it is called as graded-index fiber.

These fibers can be divided into single mode and multimode fibers. Single mode fibers operate in only one mode of propagation. Multimode fibers can support hundreds of modes. Both laser diodes and light emitting diodes (LED) can be used as light wave sources in fiber-optical communication systems. When compared to laser diodes, LEDs are less expensive, less complex and have a longer lifetime, however, their optical powers are typically small and spectral linewidths are much wider than that of laser diodes. In multimode fibers different modes travel in different speed, which is commonly referred to as intermodal dispersion, giving room to pulse spreading. In signle mode fibers, different signal frequency components travel in different speed within the fundamental mode and this result in chromatic dispersion. Since the effect of chromatic dispersion is proportional the spectral linewidth of the source, laser diodes are often used in high-speed optical systems because of their narrow spectral linewidth [23].

2.4 Impairments of Optical Fiber Communication

The physical impairments of optical fiber transmission can be categorized into two main parts irrespective of modulation/detection schemes: linear and nonlinear. Linear barriers include fiber loss and dispersion, nonlinear parts comprises nonlinear refractive index effects and inelastic scattering effects [24].

2.4.1 Losses in Fiber Optics

Attenuation is the decrease in optical power, which is measured in decibels (dB). Optical beam power traveling along the fiber decreases exponentially with distance. The mechanisms of attenuation may be classified into the following groups:

2.4.1.1 Bending Loss

The loss which exists when an optical fiber undergoes bending is called bending losses. There are two types of bending.

i) Macroscopic bending

Bending in which complete fiber undergoes bends which causes certain modes not to be reflected and therefore causes loss to the cladding. Fig. 2.3 illustrates the power loss due to macroscopic bending.

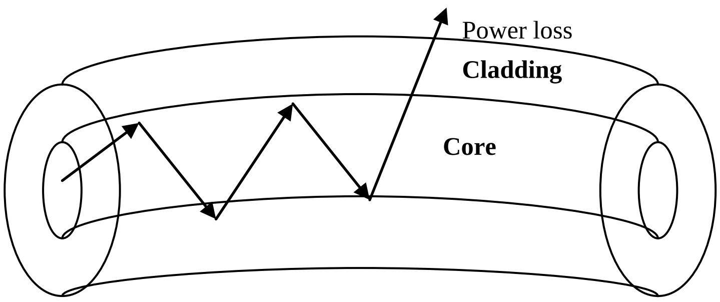

Fig. 2.2: Macroscopic bending

ii) Microscopic Bending

Either the core or cladding undergoes slight bends at its surface. It causes light to be reflected at angles when there is no further reflection. Microscopic bending loss is illustrated in Fig. 2.3.

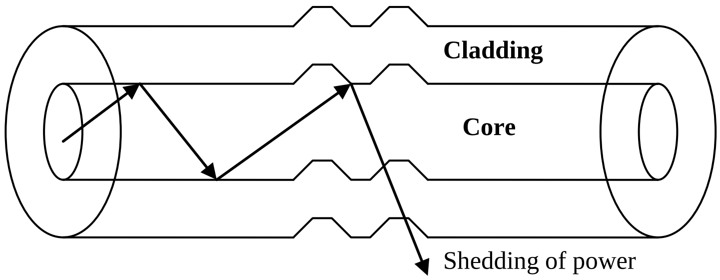

Fig. 2.3: Microscopic bending

2.4.1.2 Scattering

It occurs due to microscopic variations in the material density, compositional fluctuations, structural in homogeneities and manufacturing defects.

i. Linear Scattering

In this case the incoming signal gets scattering when it is obstructed by impurities. The scattered light frequency is same as the incoming frequency and power loss occurs due to this effect

a) **Rayleigh Scattering Losses**

These losses are due to microscopic variation in the material of the fiber. Unequal distribution of molecular densities or atomic densities leads to Rayleigh scattering losses. Glass is made up of several acids like SiO_2, P_2O_5 etc. compositions, fluctuations can occur because of these several oxides which rise to Rayleigh scattering losses. The Raleigh scattering effect is illustrated in Fig. 2.4.

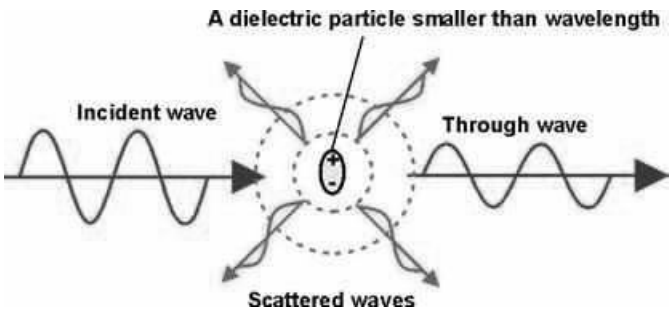

Fig. 2.4: Illustration of Rayleigh scattering effect.

b) **Mie Scattering Losses**

These losses results from the compositional fluctuations and structural inhomogenerics as well as defects created during fiber fabrications, causes the light to scatter outside the fiber.

c) **Waveguide Scattering Losses**

It is a result of variation in the core diameter, imperfections of the core cladding interface, change in refractive index of either core or cladding.

ii. Nonlinear Scattering

In this case, a new frequency is generated which is called stokes wave. The new frequency may be back propagated or forward depending on its nature. There are two types of nonlinear scattering; these are:

a) Stimulated Brillouin Scattering
b) Stimulated Raman Scattering

2.4.1.3 Absorption

Absorption of light energy due to heating of ion impurities results in dimming of light at the end of the fiber.
Two types of absorptions are usually present:

Intrinsic Absorption:

It is caused by the interaction with one or more components of the glass. It occurs when photon interacts with an electron in the valence band and excites it to a higher energy level near the ultra violet (UV) region.

Extrinsic Absorption:

It is also called impurity absorption. It results from the presence of transition metal ions like iron, chromium, cobalt, copper and from OH ions i.e., from water.

So, one important fiber parameter is a measure of power loss during transmission of optical signals inside the fiber. If Po is the power launched at the input of a fiber of length L, the transmitted power P_T is given by,

$$P_T = P_o \exp(-\alpha L) \quad (2.1)$$

where α is the attenuation constant, commonly referred to as the fiber loss. It is customary to express the fiber loss in units of dB/km by using the relation

$$\alpha_B = -\frac{10}{L}\log\frac{P_T}{P_o} \tag{2.2}$$

The fiber loss depends on the wavelength of light. The fiber exhibits a minimum loss of about 0.2dB/km near 1.55 μm. The loss is considerably higher at shorter wavelengths [25]. Fig. 2.5 illustrates the attenuation in silica optical fiber.

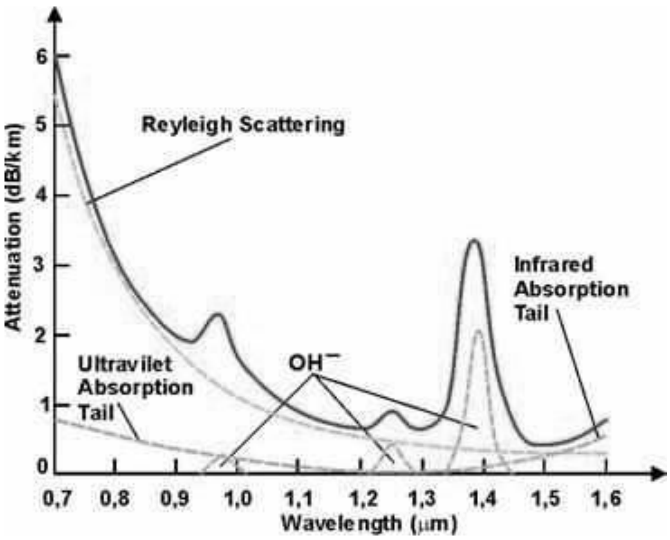

Fig. 2.5: Spectral attenuation of a silica optical fiber.

2.4.2 Dispersion in Single-Mode Fibers

Dispersion is the spreading of light pulse in time as its travels down the length of an optical fiber. Dispersion limits the bandwidth or information carrying capacity of a fiber [26]. It results short pulses broaden, which leads to significant inter-symbol interference (ISI). Therefore severely degrades the performance. Fig. 2.6 shows the pulse broadening of input pulse due to the dispersive effect of the fiber.

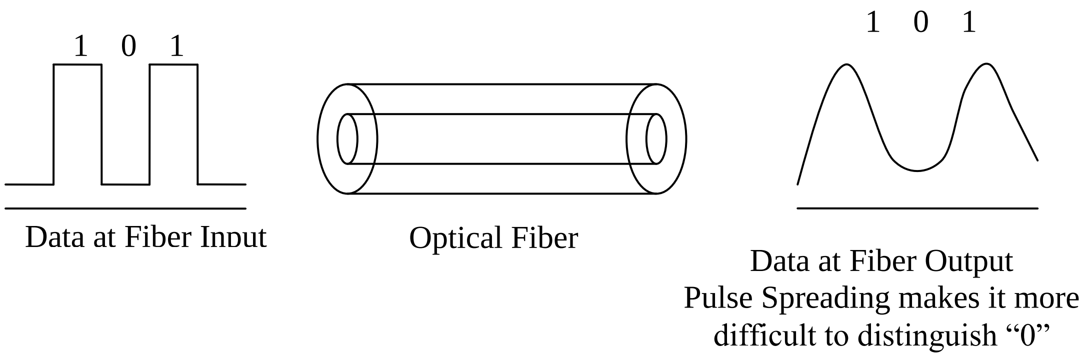

Fig. 2.6: Pulse Spreading Due to Fiber Dispersion

Fiber Dispersion is categorized in two ways:

a) Intermodal/Modal dispersion

b) Intramodal/Chromatic dispersion

- Waveguide dispersion (optical)
- Material dispersion

a) Intermodal /Modal Dispersion

In multimode fiber, different modes travel at different velocities. If a pulse is constituted from different modes then intermodal dispersion occurs. Modal dispersion is greatest in multimode step index fiber.

b) Intramodal/Chromatic Dispersion

In single mode fiber intramodal dispersion occurs because different colors of light travel through different materials and different waveguide structures at different speeds.

Waveguide dispersion

Waveguide dispersion occurs because the mode propagation constant is a function of the size of the fiber's core relative to the wavelength of operation. Waveguide dispersion also occurs because light propagates differently in the core than in the cladding.

Material dispersion

Material dispersion occurs because the spreading of a light pulse is dependent on the wavelengths' interaction with the refractive index of the fiber core. Different wavelengths travel at different speeds in the fiber material. Different wavelengths of a light pulse that enter a fiber at one time exit the fiber at different times. Material dispersion is a function of the source spectral width. The spectral width specifies the range of wavelengths that can propagate in the fiber. Material dispersion is less at longer wavelengths.

2.4.2.1 Group Velocity Dispersion (GVD)

The main advantage of single mode fibers is that intermodal dispersion is absent simply because the energy of the injected pulse is transported by a single mode. But the group velocity associated with the fundamental mode is frequency dependent because of chromatic dispersion. As a result, different spectral components of the pulse travel at slightly different group velocities, a phenomenon referred to as group-velocity dispersion (GVD). GVD plays a critical role in propagation of short optical pulses since different spectral components associated with the pulse travel at different speeds given by $\frac{c}{n(\omega)}$. Mathematically, the effect of fiber dispersion is accounted for by expanding the mode-propagation constant β in a Taylor series about the center frequency ω_o.

$$\beta(\omega) = n(\omega)\frac{\omega}{c} = \beta_o(\omega_o) + \frac{\partial\beta}{\partial\omega}(\omega-\omega_o) + \frac{1}{2}\frac{\partial^2\beta}{\partial\omega^2}(\omega-\omega_o)^2 + \frac{1}{6}\frac{\partial^3\beta}{\partial\omega^3}(\omega-\omega_o)^3 + ...$$

(2.3)

where, $\frac{\partial\omega}{\partial\beta}$ is the first order dispersion β_1, $\frac{\partial^2\omega}{\partial\beta^2}$ is the first order GVD β_2, $\frac{\partial^3\omega}{\partial\beta^3}$ is the second order GVD β_3. Thus the other terms are higher order dispersions. So the Taylor series can be expressed as,

$$\beta(\omega) = \beta_o(\omega_o) + \beta_1(\omega-\omega_o) + \frac{1}{2}\beta_2(\omega-\omega_o)^2 + \frac{1}{6}\beta_3(\omega-\omega_o)^3 + ... \quad (2.4)$$

2.4.2.1.1 First order group velocity dispersion

When optical pulses propagating in a linear dispersive medium GVD changes the phase of each spectral component of the pulse by an amount that depends, on both the frequency and the propagated distance. For first order GVD the frequency changes linearly across the pulse, i.e., a fiber imposes linear frequency chirp on the pulse.

Normal Dispersion regime (β_2 >0 or D<0)

In this regime longer wavelengths travel faster than the smaller wavelengths or red light travel faster than the blue light. On the other hand, we can say law frequency component travel faster than high frequency components. In this regime, the nonlinear dispersion is magnified by chromatic dispersion.

Anomalous Dispersion regime (β_2 <0 or D>0)

The opposite occurs in this regime. Dispersion-induced pulse broadening occurs at different frequency components of a pulse travel at slightly different speeds along the fiber because of GVD. More specifically, red components travel faster than blue components in the normal-dispersion regime (β_2 >0), while the opposite occurs in the anomalous-dispersion regime (β_2 <0). The pulse can maintain its width only if all spectral components arrive together. Any time delay in the arrival of different spectral components leads to pulse broadening.

2.4.2.1.1 Second order group velocity dispersion

Although the contribution of first order GVD (β_2) term dominates in most cases of practical interest, it is sometimes necessary to include the third-order term proportional to β_3 in this expansion. For example, if the pulse wavelength nearly coincides with the zero-dispersion wavelength λ_D, $\beta_2 \approx 0$; the β_3 term then provides the dominant contribution to the GVD effects [27]. For ultra short pulses (width T_0 < 1ps), it is necessary to include the β_3 term even when $\beta_2 \neq 0$.

2.4.3 Fiber Nonlinearity

The term linear and nonlinear, in optics, mean intensity-independent and intensity-dependent phenomena respectively. The linear and nonlinear effects are illustrated in Fig. 2.7.

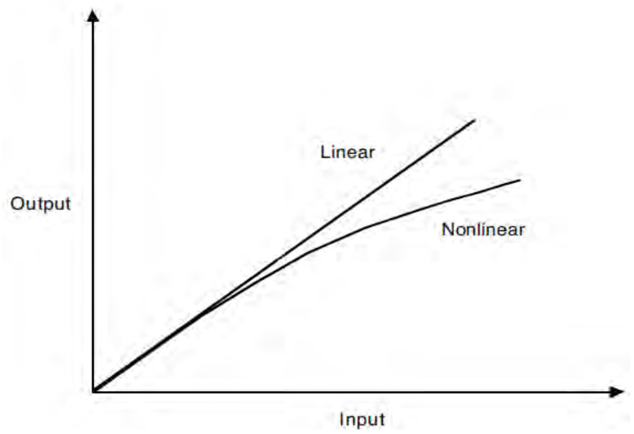

Fig. 2.7**:** Linear and nonlinear interactions.

When the optical communication systems operated at higher bit rates such as 10Gbps and above and/or at higher transmitter powers, it is important to consider the effects of nonlinearities. In the case of WDM systems, nonlinear effects can become important even at moderate powers and bit rates.

The nonlinearities in optical fibers fall into two categories. One is optical Kerr effect and the other is stimulated scattering. The classification of fiber nonlinearity is shown as tree form in Fig. 2.8.

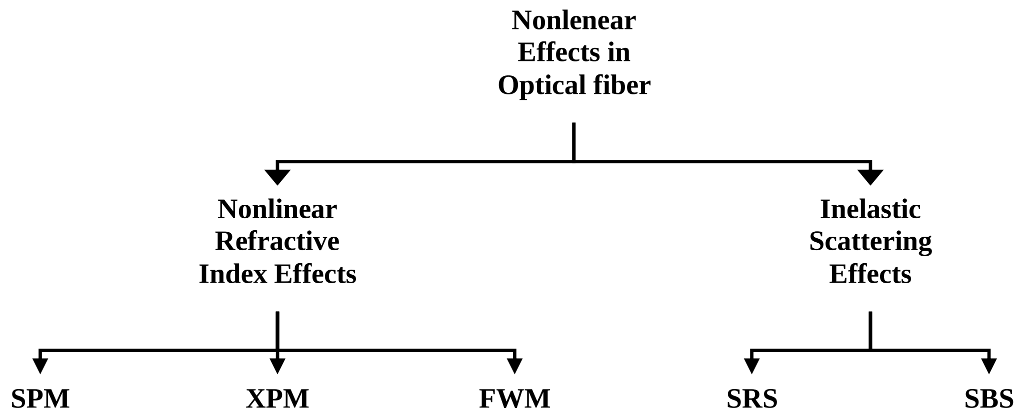

Fig. 2.8: Nonlinear effects in optical fibers.

The power dependence of the refractive index is responsible for the Kerr-effect. Depending upon the type of input signal, the Kerr-nonlinearity manifests itself in three different effects such as self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM). Stimulated scattering depends on the threshold power. If the power carried by the fiber exceed threshold, the stimulated scattering effects become effective. There are two types of stimulated scattering such as stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS).

Except for SPM and XPM, all nonlinear effects provide gains to some channel at the expense of depleting power from other channels. SPM and XPM affect only the phase of signals and can cause spectral broadening, which leads to increased dispersion.

2.4.3.1 Self Phase Modulation (SPM)

Phase modulation of an optical signal by itself is known as SPM. The nonlinear phase shift of the optical carrier signal change with respect to time because pulse intensity (power) changes over time. Generally, SPM occurs in single-wavelength systems. At high bit rates however, SPM tends to cancel dispersion. SPM increases with high signal power levels. In fiber plant design, a strong input signal helps overcome linear attenuation and dispersion losses. However, consideration must be given to receiver saturation and to nonlinear effects such as SPM, which occurs with high signal levels. SPM results in phase shift and a nonlinear pulse spread. As the pulses spread, they tend to overlap and are no longer distinguishable by the receiver. The damaging effect due to SPM depends on power transmitted, the length of the link and bit rate.

2.4.3.2 Cross Phase Modulation (XPM)

SPM is the major nonlinear limitation in a single channel system. In WDM fiber optic transmission the intensity dependence of refractive index leads to a nonlinear phenomenon known as XPM. When two or more optical pulses propagate simultaneously, the cross-phase modulation is always accompanied by SPM and occurs because the nonlinear refractive index seen by an optical beam depends not only on the intensity of that beam but also on the intensity of the other co-propagating beams [28]. In fact XPM converts power fluctuations in a particular wavelength channel to phase fluctuations in other co-propagating channels.

In a multi-channel system, the nonlinear phase shift of the signal at the center wavelength λ_i is described by,

$$\phi_{NL} = \frac{2\pi}{\lambda_i} n_2 z \left[I_i(t) + 2\sum_{i \neq j}^{M} I_j(t) \right] \tag{2.5}$$

where n_2 is the nonlinear refractive index, $n_2 z$ is known as optical path length, I is the intensity of light and M is the number of co-propagating channels in the fiber. The factor 2 in above equation has its origin in the form of nonlinear susceptibility and indicates that XPM is twice as effective as SPM for the same amount of power. The first term in above equation represents the contribution of SPM and second term that of XPM. It can be observed that XPM is effective only when the interacting signals superimpose in time. XPM hinders the system performance through the same mechanism as SPM: chirping frequency and GVD, but XPM can damage the system performance even more than SPM. XPM influences the system severely when number of channels is large. The result of XPM may be asymmetric spectral broadening and distortion of the pulse shape.

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When pulses in each channel travel at different group velocities due to dispersion, the pulses slide past each other while propagating. Fig. 2.9 illustrates how two isolated pulses in different channels collide with each other.

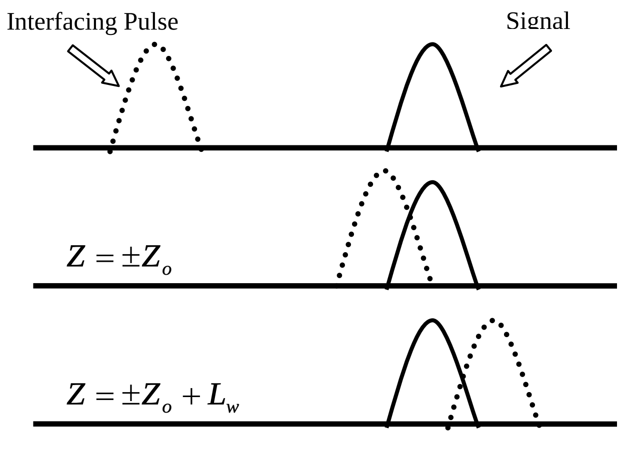

Fig. 2.9: Illustration of Cross Phase Modulation

When the faster traveling pulse has completely walked through the slower traveling pulse, the XPM effect becomes negligible. The relative transmission distance for two pulses in different channels to collide with each other is called walk-off distance, L_w

$$L_w = \frac{T_o}{\left|v_g^{-1}(\lambda_1) - v_g^{-1}(\lambda_2)\right|} \approx \frac{T_o}{|D\Delta\lambda|} \tag{2.6}$$

where T_o is the pulse width, v_g is the group velocity, and λ_1 , λ_2 are the center wavelength of the two channels. D is the dispersion coefficient and $\Delta\lambda = |\lambda_1 - \lambda_2|$.

When dispersion is significant, the walk-off distance is relatively short and the interaction between the pulses will not be significant, which leads to a reduced effect of XPM. However, the spectrum broadened due to XPM will induce more significant distortion of temporal shape of the pulse when large dispersion is present, which makes the effect of dispersion on XPM complicated. XPM can be mitigated by carefully selecting unequal bit rates for adjacent WDM channels. XPM in particular, is severe in long-haul WDM networks and the acceptable norm in system design to counter XPM effect is to take into account a power penalty that can be assumed equal to the negative effect posed by XPM. However, XPM is effective only when pulses in the other channels are synchronized with the signal of interest

2.4.3.3 Four Wave Mixing (FWM)

FWM is a nonlinear interaction that occurs in the presence of multiple wavelengths in a medium, leading to the generation of new frequencies. Thus if light waves at three different frequencies ω_2 , ω_3 and ω_4 are launched simultaneously into a medium, the same nonlinear polarization that led to a intensity dependence refractive index, leads to nonlinear polarization component at a frequency

$$\omega_1 = \omega_3 + \omega_4 - \omega_2 \tag{2.7}$$

This nonlinear polarization, under certain conditions, leads to the generation of electromagnetic waves at ω_1. This process is referred to as four wave mixing due to the interaction between four different frequencies. In a WDM system carrying multiple channels, FWM can cause severe cross talk. It is thus necessary that FWM effects are minimized in WDM systems.

2.4.3.4 Stimulated Brillouin Scattering (SBS)

The nonlinear phenomenon which occurs at the lowers power, as low as a few mW in the small core of a single mode fiber, is SBS. It occurs when an optical power reaches the level that can generate acoustic vibration in a nonlinear medium. The acoustic waves generated by the optical power affect the density material and thus change its refractive index. This refractive index fluctuation can scatter light, this effect is called brillouin scattering. Since the light wave being scattered itself also generates the acoustic waves, this process in a fiber called SBS. SBS generates stokes wave whose frequency is downshifted from an incident light by the amount set by the nonlinear medium.

SBS is strongest when the pulse width is long and the linewidth of the laser source is very narrow. It is typically harmful because it reduces signal strength by directing some portion of the light back toward the transmitter, effectively increasing attenuation.

2.4.3.5 Stimulated Raman Scattering (SRS)

When light propagates through a medium, the photons interact with silica molecules during propagation. The photons also interact with themselves and cause scattering effects, such as SRS, in the forward and reverse directions of propagation along the fiber. This results in a sporadic distribution of energy in a random direction. SRS refers to lower wavelengths pumping up the amplitude of higher wavelengths, which results in the higher wavelengths suppressing signals from the lower wavelengths. One way to mitigate the effects of SRS is to lower the input power.

2.5 Wavelength Division Multiplexing (WDM)

Wavelength division multiplexing is a technique where optical signals with different wavelengths are combined, transmitted together, and separated again. It is mostly used for optical fiber communications to transmit data in several (or even many) channels with slightly different wavelengths. In this way, the transmission capacities of fiber-optic links can be increased strongly, so that most efficient use is made not only of the fibers themselves but also of the active components such as fiber amplifiers. Apart from telecom, wavelength division multiplexing is also used for, e.g., interrogating multiple fiber-optic sensors within a single fiber.

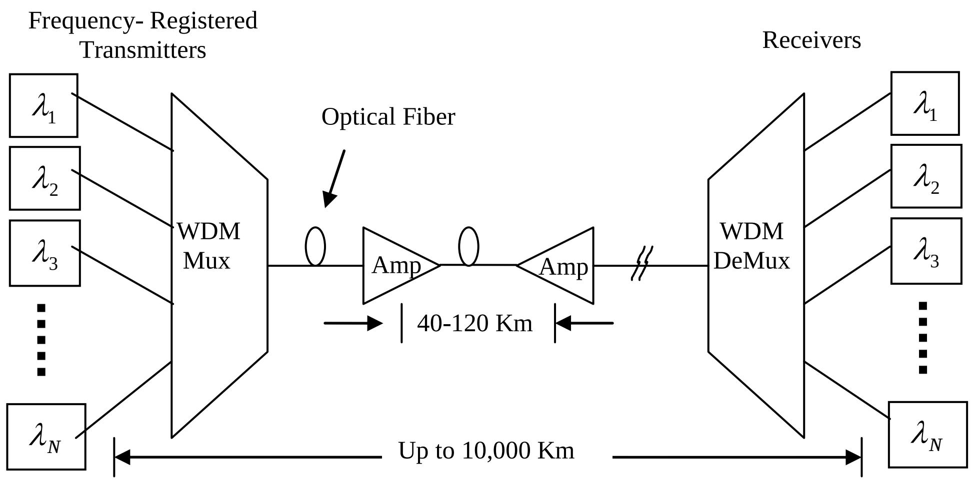

Fig. 2.10: Block diagram of WDM optical system.

Block diagram of a WDM optical system is shown in Fig. 2.10. The essential components of a WDM system include a tunable laser used at the transmitting side of the system to generate the different wavelengths. Wavelength multiplexer and demultiplexers are used to combine and separate channels in and out of the fiber respectively. A post-amplifier is used to counteract the insertion loss of the multiplexer at the transmitter. Similarly, a pre-amplifier is used to increase the sensitivity of the receiver. It is also customary to include an in-line amplifier to cater for the attenuation of the fiber. As for any other system, it is important that the system is transparent. In order to do so, international standard organizations such as (ITU-T) define standard wavelength channels for optical systems. A common standard is the laser wavelength spacing of 100 GHz between channels. This standard applies to systems that use 4, 8, 16 or 32 channels. In order to gain a better understanding of WDM systems, one needs to consider the functionality of each component in the system.

2.5 Large Effective Area Fiber (LEAF)

LEAF is an optical fiber which is developed by Corning and designed to have a large area in the core. A typical refractive index profile of LEAF is shown in the following figure.

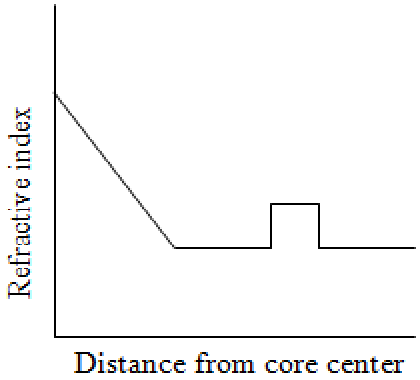

Fig. 2.11: Refractive index profile of LEAF

The core region consists of three parts. In the innermost part, the refractive index has a triangular variation. In the annular (middle) part, the refractive index is equal to that of the cladding. This is surrounded by the outermost part of the core, which is an annular region of higher refractive index. The middle part of the core, being a region of lower refractive index, does not confine the power, and thus the power gets distributed over a large area. This reduces the peak power in the core and increases the effective area of the fiber [33].

Effective area of the fiber is defined by

$$A_{eff} = \frac{\left(\int_{-\infty}^{\infty} \int |A(x,y)|^2 \, dxdy \right)^2}{\int_{-\infty}^{\infty} \int |A(x,y)|^4 \, dxdy} = \omega_o^2 \pi \tag{2.8}$$

where $2\omega_o = Mode \quad Field \quad Diameter$. The nonlinear characteristic of an optical fiber by the coefficient γ is given by

$$\gamma = \frac{k_o n_2}{A_{eff}} \tag{2.9}$$

where $k_o = 2\pi / \lambda_o$ and n_2 is the nonlinear refractive index. Thus the effect of nonlinearities can be reduced by designing a fiber with a large effective area.

2.6 Conclusion

This chapter briefly gives an overview of optical fiber communication and includes signal propagation principles, different types and impairments of optical fiber. Dispersion and nonlinearities are the main impairments of optical fiber communication. For WDM fiber optic communication system, XPM is the important nonlinear impairments that causes interference through intensity-dependent phase shifts between two optical fields. At high bit rate XPM always accompanied by first- and second order GVD.

The following chapter introduces the theory of fiber optic pulse propagation for XPM with GVD and its compressive behavior is studied both analytically and numerically.

CHAPTER 3
THEORETICAL ANALYSIS

This chapter introduces the mathematical formalism of optical pulse propagated through a nonlinear dispersive medium. The pulse broadening factor is used as performance metric. The expression of pulse broadening factor due to the effect of XPM with first-and second order GVD has been derived by solving nonlinear Schrödinger equation (NLSE).

3.1 Introduction

Optical fiber is a physical transmission medium applied for high speed and long distance data communications. With increasing channel capacity, bit rates rates, launched optical powers and wavelength channels of the WDM systems, the XPM has become the most important nonlinear effect and limits the system performance significantly [29]. The XPM produced spectral changes without the significant change in the pulse shape. On the other hand, the GVD is one of the one of the linear effect that also restricts bit rates. The first order GVD is one of the most relevant factors that limit transmission length in high bit rate optical communication systems. As the transmission speed is increased, the influence of higher-order GVD is greater. The second-order GVD is an important limiting transmission factor [16]. It is determinant on the pulse shape, pulse amplitude, pulse broadening and consequently in inter-symbol interference [15]. In high bit rate and first-order GVD compensated system, the effect of second order GVD become significant and also play a critical role in limiting the system performance. The second order GVD affects the waveform and spectrum of each optical pulse during transmission and leads to both temporal and spectral changes of pulses [30]. As a result, the combined effects of XPM with first-and second order GVD cause deterioration of transmission in a WDM system.

In this work, the influence of XPM with first-and second order GVD on the system performance is analyzed in terms of pulse broadening factor. The expression of pulse broadening factor due to the effect of XPM with first-and second order GVD has been derived by solving NLSE analytically. For checking the results of analytical technique, a numerical solution of NLSE are done using split step Fourier method (SSFM).

3.2 Nonlinear Schrödinger Equation (NLSE) for Nonlinear Dispersive Medium

The NLSE can describe propagation of electric field of optical pulse in the fiber for the case in which the two pulses propagate in the fiber. The first- and second order GVD effects can be included by adding second- and third order derivative terms on the left hand side on equation respectively.

The NLSE which takes into account both dispersion and nonlinearity can be written as [31]

$$\boldsymbol{i}\frac{\partial \boldsymbol{A}_1}{\partial \boldsymbol{z}} = -\boldsymbol{i}\frac{\alpha}{2}\boldsymbol{A}_1 + \frac{\beta_2}{2}\frac{\partial^2 \boldsymbol{A}_1}{\partial \boldsymbol{t}^2} + \boldsymbol{i}\frac{\beta_3}{6}\frac{\partial^3 \boldsymbol{A}_1}{\partial \boldsymbol{t}^3} - \boldsymbol{i}\gamma_1\left(|\boldsymbol{A}_1|^2 + 2|\boldsymbol{A}_2|^2\right)\boldsymbol{A}_1 \qquad (3.1)$$

where $\boldsymbol{A}_1(\boldsymbol{z},\boldsymbol{t})$ and $\boldsymbol{A}_2(\boldsymbol{z},\boldsymbol{t})$ are the time retarded slowly varying complex amplitude of the fields, z is the propagation direction, t is the retarded time, $\boldsymbol{i}$ is the imaginary vector, α is the attenuation coefficient, β_2 is the first order GVD, β_3 is the second order GVD, $\gamma_1\left(=\frac{\boldsymbol{n}_2\omega_0}{\boldsymbol{C}\boldsymbol{A}_{eff}}\right)$ is a nonlinear parameter, $\boldsymbol{n}_2$ is the nonlinear refractive index, ω_0 is the carrier frequency, $\boldsymbol{C}$ is the velocity of light and $\boldsymbol{A}_{eff}$ is the effective cross section area of fiber. The last two terms on the right hand side of equation (3.1) results from the fiber nonlinearity. The first term leads to SPM, while the second term is responsible for XPM. The XPM term couples the two pulses.

The equation (3.1) has been solved to derive the equations of pulse broadening factor for the effects of XPM with first- and second order GVD in WDM system in the following sections.

3.3 Pulse Broadening Factor due to the Effects of XPM with First Order GVD

In this section, the expression of pulse broadening factor due to the effects of XPM with first order GVD has been derived by solving NLSE as follows.

For an optical pulse width T_0, the walk of length L_W and dispersion length L_D and fiber length L if $L_W \le L$ and $L_D \le L$, both GVD and XPM play a significant role and XPM affect both the pulse shape and the spectrum. Now considering $L << L_D$ $L_W \le L$, so neglecting the effect of attenuation constant and dispersion terms, the equation (3.1) becomes

$$\frac{\partial A_1}{\partial z} = -i\gamma_1\left(|A_1|^2 + 2|A_2|^2\right)A_1 \tag{3.2}$$

The NLS equation (3.2) has the exact solution

$$A_1(z,t) = A_1(0,t)\exp\left[-i\gamma_1\left(|A_1|^2 + 2|A_2|^2\right)L_{eff}\right]$$

$$\Rightarrow A_1(z,t) = A_{01} f_1(t)\exp\left[-\theta_{1m} f_1^2(t) - \theta_{2m} f_2^2(t)\right] \tag{3.3}$$

where $z = 0$ is the point at which both fields are launched, $A_1(0,t) = A_{01}(0,t) f_1(t)$, $A_2(0,t) = A_{02}(0,t) f_2(t)$, A_{01} and A_{02} represent the pulse amplitude, $f_1(t)$ and $f_2(t)$ represent the pulse shape, $\theta_{1m} = i\gamma_1 A_{01}^2 L_{eff}$, $\theta_{2m} = i\gamma_1 A_{02}^2 L_{eff}$ and $L_{eff} = \dfrac{1-\exp(\alpha z)}{z}$ is the effective length of fiber and α attenuation coefficient. Thus the modulated pulse has two contributions. The first term in the equations (3.3) is due to SPM and the second term is due to XPM. The contribution of XPM changes along the fiber length because of the group velocity mismatch. Equations (3.2) and (3.3) generalize naturally to launching and receiving fields and different locations.

To add the effect of first order GVD on pulse propagation

$$i\frac{\partial}{\partial z}A_1 = \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A_1$$

$$\Rightarrow \boldsymbol{i}\frac{\partial}{\partial \boldsymbol{z}}\boldsymbol{A}_1 = -\frac{\beta_2}{2}^2 \omega^2 \boldsymbol{A}_1 \tag{3.4}$$

So the field equation in frequency domain at a distance z for the effects of XPM with first order GVD can be given by Fourier transform

$$\boldsymbol{A}_1(\boldsymbol{z},\omega) = \boldsymbol{F}\left[\boldsymbol{A}_{01}\boldsymbol{f}_1(\boldsymbol{t})\exp\left[-\theta_{1\boldsymbol{m}}\boldsymbol{f}_1^2(\boldsymbol{t}) - \theta_{2\boldsymbol{m}}\boldsymbol{f}_2^2(\boldsymbol{t})\right]\right] \times \exp\left(\frac{\boldsymbol{i}}{2}\beta_2\omega^2\boldsymbol{z}\right) \tag{3.5}$$

For both fields, Gaussian pulse has been considered as input such that

$$\boldsymbol{f}_1(\boldsymbol{t}) = \exp(-\frac{\boldsymbol{t}^2}{2\boldsymbol{T}_{\boldsymbol{o}}^2}) \quad \text{and} \quad \boldsymbol{f}_2(\boldsymbol{t}) = \exp(-\frac{\boldsymbol{t}^2}{2\boldsymbol{T}_{\boldsymbol{o}}^2})$$

Therefore, the field equation (3.5) becomes

$$\boldsymbol{A}_1(\boldsymbol{z},\omega) = \boldsymbol{F}\left[\boldsymbol{A}_{01}\exp\left(\frac{\boldsymbol{t}^2}{2\boldsymbol{T}_{\boldsymbol{o}}^2}\right)\exp\left[-\boldsymbol{i}\left(\theta_{1\boldsymbol{m}}\exp\left(-\frac{2\boldsymbol{t}^2}{2\boldsymbol{T}_{\boldsymbol{o}}^2}\right) + \theta_{2\boldsymbol{m}}\exp\left(-\frac{2\boldsymbol{t}^2}{2\boldsymbol{T}_{\boldsymbol{o}}^2}\right)\right)\right]\right]\exp\left(\frac{\boldsymbol{i}}{2}\beta_2\omega^2\boldsymbol{z}\right) \tag{3.6}$$

Let spectral width,

$$\Psi(\omega)) = \boldsymbol{F}\left\{\exp(-\frac{\boldsymbol{t}^2}{2\boldsymbol{T}_{\boldsymbol{o}}^2}) \times \exp\left[-\boldsymbol{i}\left(\theta_{1\boldsymbol{m}}\exp(-\frac{2\boldsymbol{t}^2}{2\boldsymbol{T}_{\boldsymbol{o}}^2}) + \theta_{2\boldsymbol{m}}\exp(-\frac{2\boldsymbol{t}^2}{2\boldsymbol{T}_{\boldsymbol{o}}^2})\right)\right]\right\} \tag{3.7}$$

It represents the spectral width of the pulse that encounters the effects of XPM.

And

$$\zeta(\boldsymbol{t}) = \exp(-\frac{\boldsymbol{t}^2}{2\boldsymbol{T}_0^2}) \times \exp\left[-\boldsymbol{i}\left(\theta_{1\boldsymbol{m}}\exp(-\frac{2\boldsymbol{t}^2}{2\boldsymbol{T}_0^2}) + \theta_{2\boldsymbol{m}}\exp(-\frac{2\boldsymbol{t}^2}{2\boldsymbol{T}_0^2})\right)\right] \tag{3.8}$$

So, Equation (3.6) can be written $\boldsymbol{A}_1(\boldsymbol{z},\omega) = \psi(\omega)\exp\left(\frac{\boldsymbol{i}}{2}\beta_2\omega^2\boldsymbol{z}\right)$ (3.9)

$$\Psi(\omega)) = \boldsymbol{F}[\zeta(\boldsymbol{t})] \tag{3.10}$$

the RMS band width of the pulse is given by

$$(\Delta\omega)^2 = \langle\omega^2\rangle - \langle\omega\rangle^2$$

$$\Rightarrow (\Delta\omega)^2 = \frac{\int_{-\infty}^{\infty} \omega^2 |\Psi(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} |\Psi(\omega)|^2 d\omega} - \left[\frac{\int_{-\infty}^{\infty} \omega |\Psi(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} |\Psi(\omega)|^2 d\omega} \right]^2 \tag{3.11}$$

By using Fourier transform properties, the above frequency domain equation can be written in time domain as follows

$$(\Delta\omega)^2 = \frac{\int_{-\infty}^{\infty} |\zeta'(t)|^2 dt}{\int_{-\infty}^{\infty} |\zeta(t)|^2 dt} + \left[\frac{\int_{-\infty}^{\infty} \zeta'(t)\zeta^*(t) dt}{\int_{-\infty}^{\infty} |\zeta(t)|^2 dt} \right]^2 \tag{3.12}$$

$$\text{But} \quad \int_{-\infty}^{\infty} \zeta'(t)\zeta^*(t) dt = 0$$

Equation (3.12) becomes

$$(\Delta\omega)^2 = \frac{\int_{-\infty}^{\infty} |\zeta'(t)|^2 dt}{\int_{-\infty}^{\infty} |\zeta(t)|^2 dt} \tag{3.13}$$

Now, to compute the value of $(\Delta\omega)^2$ in equation (3.13), $\int_{-\infty}^{\infty} |\zeta'(t)|^2 dt$ and $\int_{-\infty}^{\infty} |\zeta(t)|^2 dt$ have been calculated as below,

$$|\zeta(t)|^2 = \exp\left(-\frac{t^2}{T_0^2}\right) \text{ and } \int_{-\infty}^{\infty} |\zeta(t)|^2 = \sqrt{\pi} T_0 \tag{3.14}$$

$$|\zeta'(t)|^2 = \left(\frac{t^2}{T_0^4}\right) \exp\left(-\frac{t^2}{T_0^2}\right) \cdot \left[1 + 4(\theta_{1m} + 2\theta_{2m})^2 \exp\left(-\frac{2t^2}{T_0^2}\right)\right] \tag{3.15}$$

$$\text{And} \quad \int_{-\infty}^{\infty} |\zeta'(t)|^2 = \frac{\sqrt{\pi}}{2T_0} + \frac{2(\theta_{1m} + 2\theta_{2m})^2 \sqrt{\pi}}{3\sqrt{3} T_0} \tag{3.16}$$

So the equation (3.13) becomes

$$(\Delta\omega)^2 = \frac{\frac{\sqrt{\pi}}{2\boldsymbol{T}_0} + \frac{2(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\sqrt{\pi}}{3\sqrt{3}\boldsymbol{T}_0}}{\sqrt{\pi}\boldsymbol{T}_0}$$

$$\Rightarrow (\Delta\omega)^2 = \frac{1}{2\boldsymbol{T}_0^2}\left[1 + \frac{4}{3\sqrt{3}}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right] \tag{3.17}$$

Variance of the spectrum, $(\Delta\omega)^2 = \frac{1}{\boldsymbol{T}_o^2}\left[1 + \frac{4}{3\sqrt{3}}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right]$ (3.18)

Now, $\psi(\omega) = \exp\left[-\frac{\omega^2}{2(\Delta\omega)^2}\right] = \exp\left[-\frac{1}{2}.\frac{\omega^2\boldsymbol{T}_o^2}{\left[1 + \frac{4}{3\sqrt{3}}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right]}\right]$ (3.19)

So, the field equation (3.9) has been solved in frequency domain as

$$\boldsymbol{A}_1(\boldsymbol{z},\omega) = \boldsymbol{A}_{01}\exp\left[-\frac{1}{2}.\frac{\omega^2\boldsymbol{T}_o^2}{\left[1 + \frac{4}{3\sqrt{3}}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right]}\right] \times \exp\left(\frac{\boldsymbol{i}}{2}\beta_2\omega^2\boldsymbol{z}\right) \tag{3.20}$$

The field in the time domain can be obtained by inverse Fourier transform of equation (3.20)

$$\boldsymbol{A}_1(\boldsymbol{z},\boldsymbol{t}) = \frac{1}{2\pi}\boldsymbol{A}_{01}\int_{-\infty}^{\infty}\exp\left[-\frac{1}{2}.\frac{\omega^2\boldsymbol{T}_o^2}{\left[1 + \frac{4}{3\sqrt{3}}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right]}\right].\times\exp\left(\frac{\boldsymbol{i}}{2}\beta_2\omega^2\boldsymbol{z}\right).\exp(-\boldsymbol{i}\omega\boldsymbol{t})\boldsymbol{d}\omega$$

(3.21)

$$\Rightarrow \boldsymbol{A}_1(\boldsymbol{z},\boldsymbol{t}) = \frac{\boldsymbol{A}_{01}}{2\pi}\times\int_{-\infty}^{\infty}\exp\left[-\left\{\frac{\left[\boldsymbol{T}_o^2 - \boldsymbol{i}\beta_2\boldsymbol{z}\right]\omega^2}{2\left[1 + \frac{4}{3\sqrt{3}}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right]} + \boldsymbol{i}\omega\boldsymbol{t}\right\}\right]\boldsymbol{d}\omega \tag{3.22}$$

$$\boldsymbol{A}_1(\boldsymbol{z},\boldsymbol{t}) = \frac{\boldsymbol{A}_{01}}{\sqrt{2\pi}}\frac{\exp\left(-\frac{\boldsymbol{t}^2\left[9 + 4\sqrt{3}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right]}{9\boldsymbol{T}_o^2 - \boldsymbol{i}\beta_2\boldsymbol{z}\left(9 + 4\sqrt{3}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right)}\right)}{\left[9\boldsymbol{T}_o^2 - \boldsymbol{i}\beta_2\boldsymbol{z}\left(9 + 4\sqrt{3}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right)\right]^{\frac{1}{2}}}\times\left[9 + 4\sqrt{3}(\theta_{1\boldsymbol{m}} + 2\theta_{2\boldsymbol{m}})^2\right]^{\frac{1}{2}}$$

(3.23)

To simplify the equation (3.23), consider,

$$Q(z,t)=\frac{A_{01}}{\sqrt{2\pi}}\frac{\left[9+4\sqrt{3}(\theta_{1m}+2\theta_{2m})^2\right]^{\frac{1}{2}}}{\left[9T_o^2-i\beta_2 z\left(9+4\sqrt{3}(\theta_{1m}+2\theta_{2m})^2\right)\right]^{\frac{1}{2}}}. \quad (3.24)$$

And $$h(z,t)=\exp\left(-\frac{1}{2}\frac{t^2\left[9+4\sqrt{3}(\theta_{1m}+2\theta_{2m})^2\right]}{9T_o^2-i\beta_2 z\left(9+4\sqrt{3}(\theta_{1m}+2\theta_{2m})^2\right)}\right) \quad (3.25)$$

Thus, equation (3.23) can be written as, $A_1(z,t)=Q(z,t).h(z,t)$ (3.26)

where $Q(z,t)$ represents the amplitude of the pulse at distance z and $h(z,t)$ represents the shape of the pulse at distance z, that encounters the effects of XPM and first order GVD simultaneously.

RMS value of pulse width at distance z,

$$\Delta t(z)=\left[\langle t^2\rangle-\langle t\rangle^2\right]^{\frac{1}{2}} \quad (3.27)$$

Where, $\langle t^n\rangle$ is the n[th] moment defined as

$$\langle t^n\rangle=\frac{\int_{-\infty}^{\infty}t^n|A(z,t)|^2\,dt}{\int_{-\infty}^{\infty}|A(z,t)|^2\,dt} \quad (3.28)$$

But $Q(z,t)=\text{constant}$. So, $A_1(z,t)\rightarrow h(z,t)$

$$\langle t^2\rangle=\frac{\int_{-\infty}^{\infty}t^2|h(z,t)|^2\,dt}{\int_{-\infty}^{\infty}|h(z,t)|^2\,dt} \quad (3.29)$$

Now, to compute the value of RMS value of pulse width at distance z, the following calculations have been done.

$$|h(z,t)|^2 = \exp\left(-\frac{t^2\left[1+\frac{4}{3\sqrt{3}}(\theta_{1m}+2\theta_{2m})^2\right]}{T_o^2\left[1+L^2\left(1+\frac{4}{3\sqrt{3}}(\theta_{1m}+2\theta_{2m})^2\right)^2\right]}\right) \tag{3.30}$$

$$\int_{-\infty}^{\infty} t^2|h(z,t)|^2\,dt = \int_{-\infty}^{\infty} t^2 \exp\left(-\frac{\left[1+\frac{4}{3\sqrt{3}}(\theta_{1m}+2\theta_{2m})^2\right]t^2}{T_o^2\left[1+L^2\left(1+\frac{4}{3\sqrt{3}}(\theta_{1m}+2\theta_{2m})^2\right)^2\right]}\right)dt \tag{3.31}$$

$$\Rightarrow \int_{-\infty}^{\infty} t^2|h(z,t)|^2\,dt = \frac{T_0^3\sqrt{\pi}}{6\sqrt{3}}\frac{\left[27+27L^2+24\sqrt{3}L^2(\theta_{1m}+2\theta_{2m})^2+16L^2(\theta_{1m}+2\theta_{2m})^4\right]^{\frac{3}{2}}}{\left[9+4\sqrt{3}(\theta_{1m}+2\theta_{2m})^2\right]^{\frac{3}{2}}} \tag{3.32}$$

$$\int_{-\infty}^{\infty} |h(z,t)|^2\,dt = \int_{-\infty}^{\infty} \exp\left(-\frac{\left[1+\frac{4}{3\sqrt{3}}(\theta_{1m}+2\theta_{2m})^2\right]t^2}{T_0^2\left[1+L^2\left(1+\frac{4}{3\sqrt{3}}(\theta_{1m}+2\theta_{2m})^2\right)^2\right]}\right)dt \tag{3.33}$$

$$\Rightarrow \int_{-\infty}^{\infty} |h(z,t)|^2\,dt = \frac{T_0\sqrt{\pi}}{\sqrt{3}}\frac{\left[27+27L^2+24\sqrt{3}L^2(\theta_{1m}+2\theta_{2m})^2+16L^2(\theta_{1m}+2\theta_{2m})^4\right]^{\frac{1}{2}}}{\left[9+4\sqrt{3}(\theta_{1m}+2\theta_{2m})^2\right]^{\frac{1}{2}}} \tag{3.34}$$

$$\Delta t(z) = \left[\langle t^2\rangle - \langle t\rangle^2\right]^{\frac{1}{2}} \tag{3.35}$$

$$\Rightarrow \Delta t(z) = \left[\langle t^2\rangle\right]^{\frac{1}{2}} \tag{3.36}$$

$$\Rightarrow \Delta t(z) = \left[\frac{\int_{-\infty}^{\infty} t^2|h(z,t)|^2\,dt}{\int_{-\infty}^{\infty} |h(z,t)|^2\,dt}\right]^{\frac{1}{2}} \tag{3.37}$$

So, equation of RMS value of pulse width at a distance z becomes

$$\Rightarrow \Delta t(z) = \left[\frac{ \dfrac{T_0^3 \sqrt{\pi}}{6\sqrt{3}} \dfrac{\left[27 + 27L^2 + 24\sqrt{3}L^2\left(\theta_{1m} + 2\theta_{2m}\right)^2 + 16L^2\left(\theta_{1m} + 2\theta_{2m}\right)^4\right]^{\frac{3}{2}}}{\left[9 + 4\sqrt{3}\left(\theta_{1m} + 2\theta_{2m}\right)^2\right]^{\frac{3}{2}}} }{ \dfrac{T_0 \sqrt{\pi}}{\sqrt{3}} \dfrac{\left[27 + 27L^2 + 24\sqrt{3}L^2\left(\theta_{1m} + 2\theta_{2m}\right)^2 + 16L^2\left(\theta_{1m} + 2\theta_{2m}\right)^4\right]^{\frac{1}{2}}}{\left[9 + 4\sqrt{3}\left(\theta_{1m} + 2\theta_{2m}\right)^2\right]^{\frac{1}{2}}} } \right]^{\frac{1}{2}} \tag{3.38}$$

$$\Rightarrow \Delta t(z) = \frac{T_0}{\sqrt{2}} \frac{\left[27 + 27L^2 + 24\sqrt{3}L^2\left(\theta_{1m} + 2\theta_{2m}\right)^2 + 16L^2\left(\theta_{1m} + 2\theta_{2m}\right)^4\right]^{\frac{1}{2}}}{\left[3\left\{9 + 4\sqrt{3}\left(\theta_{1m} + 2\theta_{2m}\right)^2\right\}\right]^{\frac{1}{2}}} \tag{3.39}$$

RMS value of pulse width at z=0, i.e. initial pulse width

$$\Delta t(0) = \Delta t_0 = \frac{T_0}{\sqrt{2}} \tag{3.40}$$

Pulse broadening factor

$$U = \frac{\Delta t(z)}{\Delta t_0} \tag{3.41}$$

$$\Rightarrow U = \frac{\left[27 + 27L^2 + 24\sqrt{3}L^2\left(\theta_{1m} + 2\theta_{2m}\right)^2 + 16L^2\left(\theta_{1m} + 2\theta_{2m}\right)^4\right]^{\frac{1}{2}}}{\left[3\left\{9 + 4\sqrt{3}\left(\theta_{1m} + 2\theta_{2m}\right)^2\right\}\right]^{\frac{1}{2}}} \tag{3.42}$$

$$\Rightarrow U = \frac{\left[1 + L^2 + \frac{24}{9\sqrt{3}}L^2\left(\theta_{1m} + 2\theta_{2m}\right)^2 + \frac{16}{27}L^2\left(\theta_{1m} + 2\theta_{2m}\right)^4\right]^{\frac{1}{2}}}{\left[3\left\{9 + 4\sqrt{3}\left(\theta_{1m} + 2\theta_{2m}\right)^2\right\}\right]^{\frac{1}{2}}} \tag{3.43}$$

The equation (3.43) describes the pulse broadening factor for an initial Gaussian pulse experience the effects of XPM with SPM and first order GVD simultaneously during propagation through a WDM optical fiber transmission system.

3.4 Pulse Broadening Factor due to the Effects of XPM with First- and Second Order GVD

To calculate the pulse broadening factor due to the effects of XPM with first- and second order GVD, it is necessary to add the first- and second order GVD in the propagating field equation.

Now

$$\boldsymbol{i}\frac{\partial}{\partial \boldsymbol{z}}\boldsymbol{A}_1 = \boldsymbol{i}\frac{\beta_3}{6}\frac{\partial^3}{\partial \boldsymbol{t}^3}\boldsymbol{A}_1$$

$$\Rightarrow \boldsymbol{i}\frac{\partial}{\partial \boldsymbol{z}}\boldsymbol{A}_1 = \boldsymbol{i}\frac{\beta_3}{6}.\boldsymbol{i}^3\omega^3.\boldsymbol{A}_1$$

$$\Rightarrow \frac{\partial}{\partial \boldsymbol{z}}\boldsymbol{A}_1 = -\boldsymbol{i}\frac{\beta_3}{6}\;\;\omega^3\boldsymbol{A}_1 \qquad (3.44)$$

Thus the field equation in the time domain for the effect of XPM with first- and second order GVD can be expressed using equation (3.20) as follows

$$\boldsymbol{A}_1(\boldsymbol{z},\boldsymbol{t}) = \frac{1}{2\pi}\boldsymbol{A}_{01}\int_{-\infty}^{\infty}\exp\left[-\frac{1}{2}.\frac{\omega^2\boldsymbol{T}_0^2}{\left[1+\frac{4}{3\sqrt{3}}(\theta_{1\boldsymbol{m}}+2\theta_{2\boldsymbol{m}})^2\right]}\right].\exp\left[\boldsymbol{i}\left[\left(\omega\boldsymbol{t}-\frac{1}{2}\beta_2\omega^2\boldsymbol{z}-\frac{1}{6}\beta_3\omega^3\boldsymbol{z}\right)\right]\right]\boldsymbol{d}\omega$$

(3.45)

This integral is so complex to solve. So it tries to simplify as follows

Making $\boldsymbol{F}_1(\boldsymbol{z},\omega) = \exp\left(-\omega^2\left(\frac{\boldsymbol{T}_{\boldsymbol{o}}^2}{\left[1+\frac{4}{3\sqrt{3}}(\theta_{1\boldsymbol{m}}+2\theta_{2\boldsymbol{m}})^2\right]}+\boldsymbol{i}\frac{1}{2}\beta_2\boldsymbol{z}\right)\right)$ (3.46)

And $\boldsymbol{F}_2(\boldsymbol{z},\omega) = \exp\left(-\omega^3\left(\boldsymbol{i}\frac{1}{6}\beta_3\boldsymbol{z}\right)\right)$ (3.47)

Considering $\boldsymbol{F}_1(\boldsymbol{z},\omega)$ as the Fourier transform of a function denominated $\boldsymbol{f}_1(\boldsymbol{z},\tau)$ and $\boldsymbol{F}_2(\boldsymbol{z},\omega)$ As the Fourier Transform of a function denominated $\boldsymbol{f}_2(\boldsymbol{z},\boldsymbol{t}-\tau)$, using the convolution theorem, we obtain

$$f_1(z,\tau)=\left(\frac{1}{4\pi\left(\frac{T_o^2}{\left[1+\frac{4}{3\sqrt{3}}(\theta_{1m}+2\theta_{2m})^2\right]}+\frac{1}{2}\beta_2 z\right)}\right)^{\frac{1}{2}}\times\exp\left(-\frac{\tau^2}{4\left(\frac{T_o^2}{\left[1+\frac{4}{3\sqrt{3}}(\theta_{1m}+2\theta_{2m})^2\right]}+i\frac{1}{2}\beta_2 z\right)}\right)$$

(3.48)

$$f_2(z,t-\tau)=\frac{1}{3}\frac{1}{\left(-i(i\frac{1}{6}\beta_3 z)\right)^{\frac{1}{3}}}\left(\frac{t-\tau}{i\left(i\frac{1}{6}\beta_3 z\right)^{\frac{1}{3}}}\right)^{\frac{1}{2}}\frac{1}{\pi}\times besselk_{\frac{1}{3}}\left(\frac{2}{9}3^{\frac{1}{2}}\left(-\frac{t-\tau}{\left(-\frac{1}{6}\beta_3 z\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}}\right) \qquad (3.49)$$

This last expression was obtained by Matlab, solving $ifourier(\exp(-ja\omega^3))$

Where $a=i\frac{1}{6}\beta_3 z$

$$besselk_{\pm\frac{1}{3}}(l)=\pi\left(\frac{3}{\gamma}\right)Ai(\gamma) \qquad (3.50)$$

$$l=\frac{2}{9}3^{\frac{1}{2}}\left(-\frac{t-\tau}{\left(-\frac{1}{6}\beta_3 z\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}} \qquad (3.51)$$

And $\gamma=\left(\frac{3}{2}l\right)^{\frac{2}{3}}$ (3.52)

$$A(z,t)=\frac{1}{2\pi}A_o\int_{-\infty}^{\infty}\left(\frac{1}{4\pi\left(\frac{T_o^2}{2(1-iC)}+i\beta_2 z\right)}\right)\frac{1}{3}\frac{1}{\left(\frac{1}{6}\beta_3 z\right)^{\frac{1}{3}}}\left(-\frac{t-\tau}{\left(-\frac{1}{6}\beta_3 z\right)^{\frac{1}{3}}}\right)^{\frac{1}{2}}\times\left(\frac{3}{\left(\frac{3}{2}l\right)^{\frac{3}{2}}}\right)^{\frac{1}{2}}Ai\left(\left(\frac{3}{2}l\right)^{\frac{2}{3}}\right)\partial\tau$$

(3.53)

This integral has to solve numerically. So, the pulse broadening factor due to the effects of XPM with first- and second order GVD has been obtained by solving NLSE numerically using the split step Fourier method in Matlab in the following section.

3.5 Split Step Fourier Method (SSFM) for Solving NLSE Numerically due to the Effects of XPM with First- and Second Order GVD

The SSFM is a pseudo-spectral numerical method used to solve the NLSE numerically due to the effects of XPM with first- and second order GVD. The key idea behind SSFM is to apply the effects of dispersion and fiber nonlinearities separately in small partitions of propagation distance such that it approximates the actual pulse propagation under the simultaneous influence of the two effects [32].

The NLSE is described as

$$\frac{\partial \boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z})}{\partial \boldsymbol{z}} = -\frac{\alpha}{2}\boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z}) - \frac{\boldsymbol{j}}{2}\beta_2(\boldsymbol{z})\frac{\partial^2 \boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z})}{\partial \boldsymbol{t}^2} + \frac{\boldsymbol{j}}{6}\beta_3(\boldsymbol{z})\frac{\partial^3 \boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z})}{\partial \boldsymbol{t}^3} + \boldsymbol{j}\gamma\left(\left|\boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z})^2\right| + 2\left|\boldsymbol{A}_2(\boldsymbol{t},\boldsymbol{z})^2\right|\right)\boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z}) \tag{3.54}$$

As

$$\frac{\partial \boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z})}{\partial \boldsymbol{z}} = \left(\widehat{\boldsymbol{L}} + \widehat{\boldsymbol{N}}\right) \tag{3.55}$$

where, the linear operator $\widehat{\boldsymbol{L}}$ accounts for dispersion and absorption in a linear medium, and $\widehat{\boldsymbol{N}}$ is a nonlinear operator that governs the effect of fiber nonlinearities on pulse propagation

The operators are given by

$$\widehat{\boldsymbol{L}} = -\frac{\alpha}{2} - \frac{\boldsymbol{j}}{2}\beta_2\frac{\partial^2}{\partial^2 \boldsymbol{t}} + \frac{\boldsymbol{j}}{6}\beta_3\frac{\partial^3}{\partial \boldsymbol{t}^3} \tag{3.56}$$

$$\widehat{\boldsymbol{N}} = \boldsymbol{j}\gamma\left(\left|\boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z})\right|^2 + 2\left|\boldsymbol{A}_2(\boldsymbol{t},\boldsymbol{z})\right|^2\right) \tag{3.57}$$

When the electric field envelope, $\boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z})$, has propagated from z to z+Δz, the analytical solution of equation(3.54) will have a form of

$$\boldsymbol{A}\ (\boldsymbol{t},\boldsymbol{z}+\Delta\boldsymbol{z}) = \exp\left(\Delta\boldsymbol{z}(\widehat{\boldsymbol{L}}+\widehat{\boldsymbol{N}})\right)\boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z}) \tag{3.58}$$

In the SSFM, it is assumed that the two operators commute with each other. That is

$$\boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z}+\Delta\boldsymbol{z}) \approx \exp\left(\Delta\boldsymbol{z}\widehat{\boldsymbol{L}}\right)\exp\left(\Delta\widehat{\boldsymbol{N}}\right)\boldsymbol{A}_1(\boldsymbol{t},\boldsymbol{z}) \tag{3.59}$$

The exponential operator $\exp\left(\Delta\boldsymbol{z}\widehat{\boldsymbol{L}}\right)$ is evaluated in the Fourier domain using

$$\exp\left(\Delta z\hat{L}\right)B(t,z) = F_T^{-1}\exp\left[\Delta z\hat{L}(i\omega)\right]F_T B(t,z) \tag{3.60}$$

The accuracy of the SSFM can be improved by adopting a different procedure to propagate the optical pulse over one segment from z to z+Δz. In this procedure equation (3.59) is replaced by

$$A_1(t,z+\Delta z) \approx \exp\left(\frac{\Delta z}{2}\hat{L}\right)\exp\left(\int_0^{z+\Delta z}\hat{N}(z')dz'\right)\exp\left(\frac{\Delta z}{2}\hat{L}\right)A_1(t,z) \tag{3.61}$$

The accuracy of the SSFM can be further improved by approximating the integral of equation (3.61) by

$$\int_0^{z+\Delta Z} N(z')dz' \approx \frac{\Delta z}{2}\left[\hat{N}(z)+\hat{N}(z+\Delta z)\right] \tag{3.62}$$

In iterative and symmetric SSFM the fiber length is divided into large number of segments that need not to be spaced equally. The optical pulse is propagated from segment to segment using the prescription of equation (3.61). More specifically, the optical field $A_1(t, z)$ is first propagated for a distance $\Delta z/2$ with dispersion only using the FFT algorithm and equation (3.62). At the mid-plane $z + \Delta z/2$, the field is multiplied be a nonlinear term that represents the effect of nonlinearity over the whole segment length Δz. Finally, the field is propagated remaining distance $\Delta z/2$ with dispersion only to obtain $A_1(t, z+ \Delta z/2)$ [17]**.** The symmetric SSFM is illustrated in the Fig. 3.1.

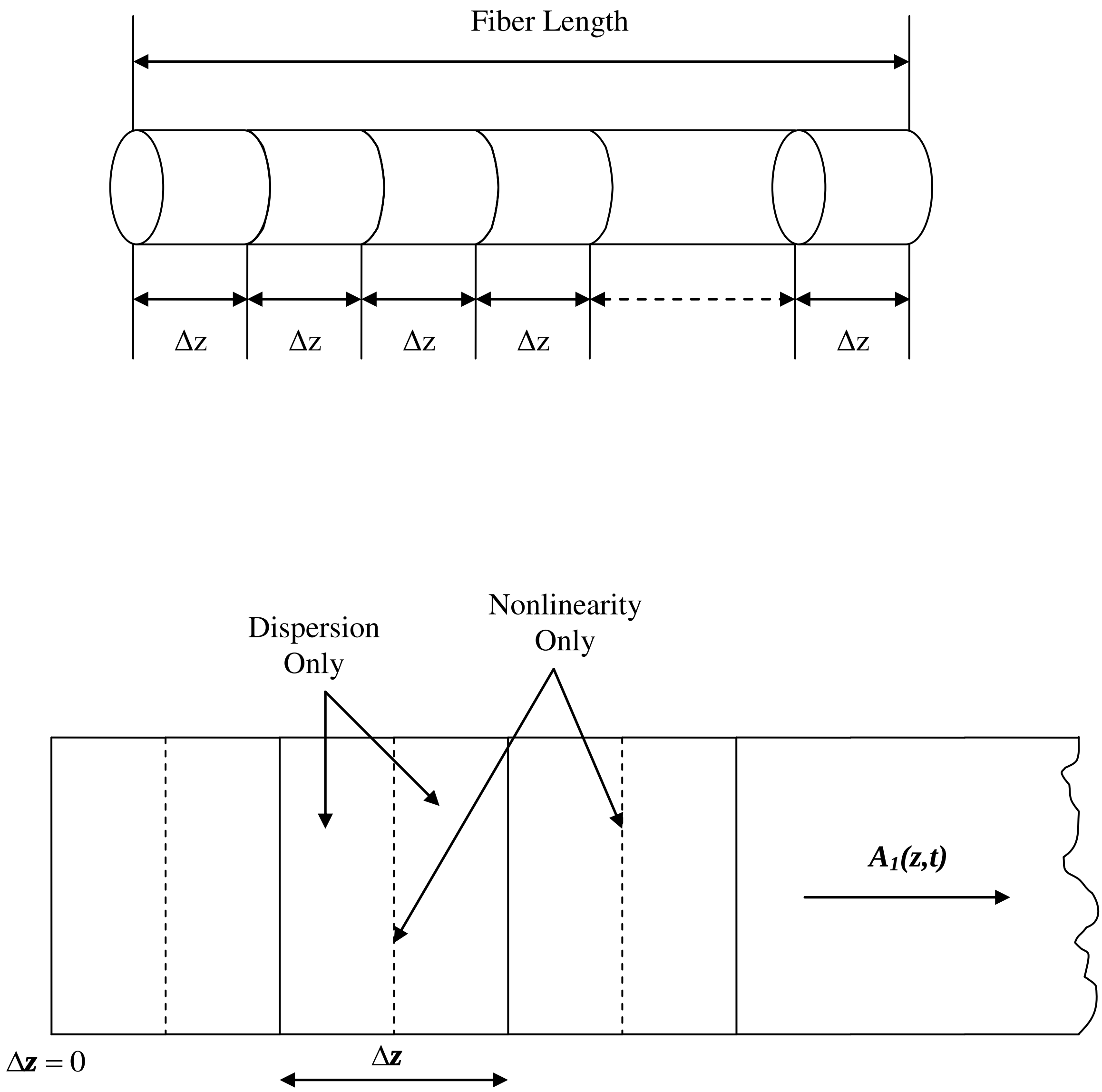

Fig. 3.1: Illustration of symmetric split step Fourier method

According to the flow chart of SSFM in Fig. 3.2, the pulse broadening factor due to the effects of XPM with first- and second order GVD has been determined by using Matlab program. The pulse broadening factor due to the effects of XPM with second order GVD could be determined when the first order GVD is absent (i.e. $\beta_2 = 0$).

Flow Diagram of Split-Step Fourier Method to Determine Pulse Broadening Factor

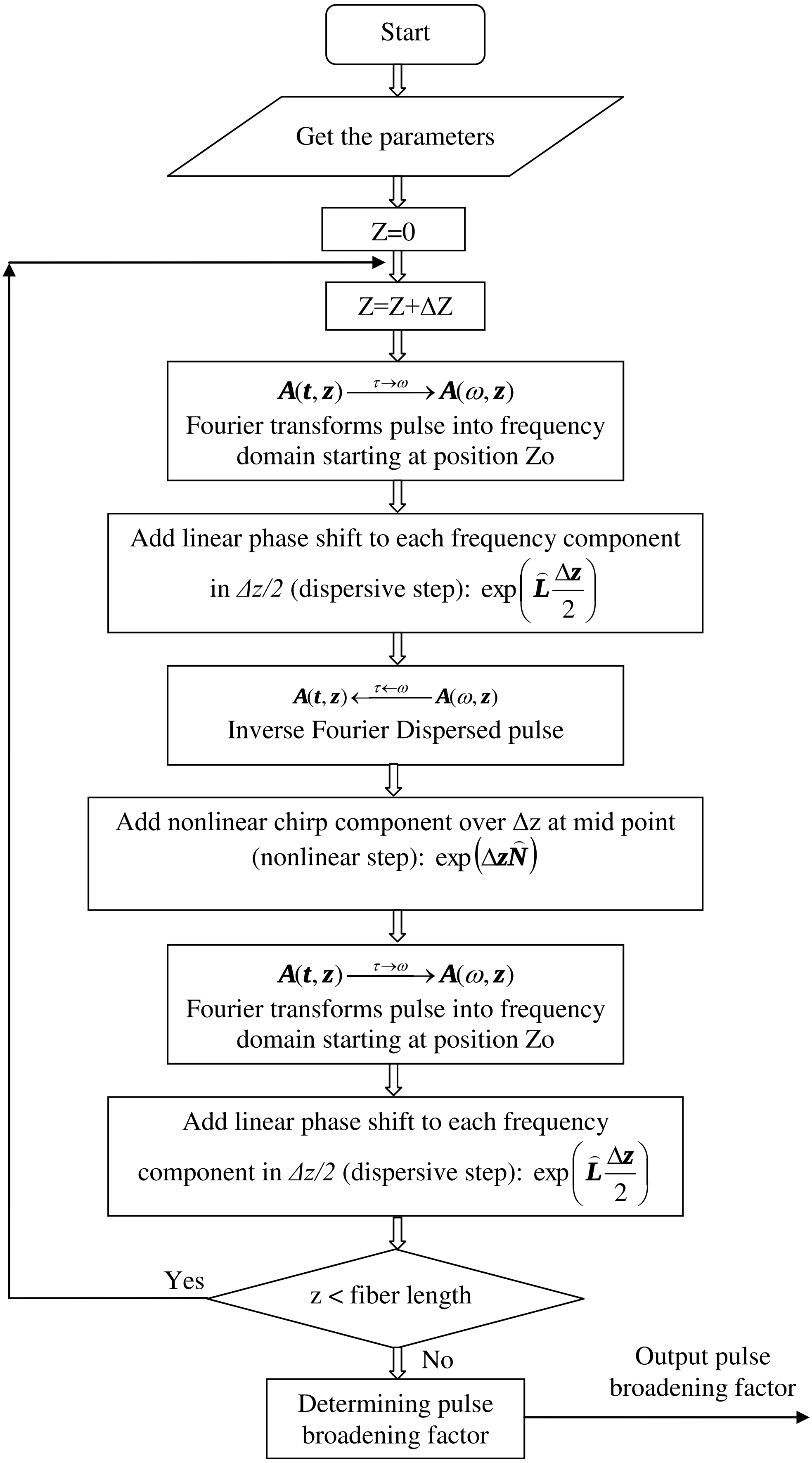

Fig. 3.2: Flow chart to determine Pulse Broadening factor using split-step Fourier method

3.6 Conclusion

In this chapter the expression of pulse broadening factor due to the effects of XPM with SPM and first order GVD has been derived analytically from NLSE in WDM fiber optic communication system. For obtaining pulse broadening factor due to the effects of XPM with first and second order GVD, NLSE has been solved numerically using SSFM. The pulse broadening factor due to the effects of XPM with second order GVD has been determined at high bit rate when the first order GVD is absent. The SSFM method is also used to achieve the full numerical simulation to validate the accuracy of proposed analytical model.

CHAPTER 4
RESULTS AND DISCUSSIONS

4.1 Introduction

Following the theoretical analysis presented in chapter 3, performance results of an optical transmission system are presented in this chapter. The performance result is carried out in terms of the pulse broadening factor for a Gaussian pulse input that encounters the effect of XPM with first- and second order GVD. The results are evaluated at different bit rates and input powers for standard single mode fiber (SSMF) and LEAF fiber. Different system parameters of SSMF and LEAF fibers are shown in Table 4.1.

Table 4.1: Parameter values used for theoretical calculations

Parameters(unit)	Values	
	SSMF	LEAF
Non-linear refractive index, $n_2\left(m^2/w\right)$	2.35×10^{-20}	2.35×10^{-20}
Bit rate, $B(Gb/s)$	$10-40$	$10-40$
Wavelength, $\lambda(nm)$	1550	1550
Attenuation coefficient, $\alpha(dB/km)$	0.25	0.25
Power, $P_1=A_{01}^2(mW)$	$10-90$	$10-90$
Power, $P_2=A_{02}^2(mW)$	$10-90$	$10-90$
Effective area, Power, $A(m^2)$	5.5×10^{-11}	7.2×10^{-11}
First order GVD, $\beta_2(ps^2/nm)$	0.206	0.3349
Second Order GVD, $\beta_3(ps^3/nm)$	0.192295	0.1957
Fiber length, (km)	varied	varied

4.2 Effect of XPM with SPM and First Order GVD on Pulse Broadening in SSMF Fiber

Using the derived analytical expressions, pulse broadening factor caused by the effect of XPM with SPM and first order GVD for SSMF is visualized using Matlab. We have observed the variation of pulse broadening factor for various data rates and input powers.

Fig. 4.1 shows the plots of analytical pulse broadening factor caused by the effect of XPM with SPM and first order GVD versus fiber length at the data rates 10Gbps and 40Gbps. While the input powers are kept constant.

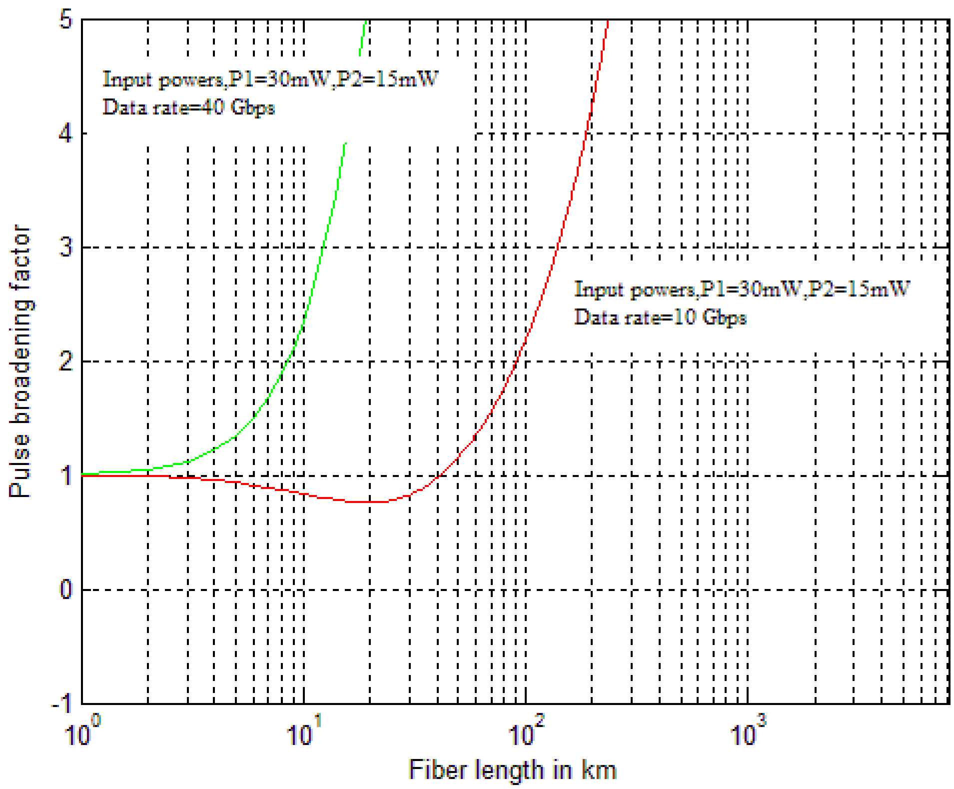

Fig. 4.1: Plots of analytical pulse broadening factor versus fiber length at the data rates 10 Gbps and 40 Gbps in SSMF.

From figure it is observed that at bit rate 40 Gbps the pulse broadening factor is almost constant up to distance 0.5 km and then increases rapidly. At 10 Gbps, this factor increases rapidly after 13km. Thus it can be said that as the data rates increases, pulse broadening factor caused by the effect of XPM with SPM and first order GVD increases and limits the transmission distance.

Fig. 4.2 shows the plots of pulse broadening factor versus fiber length at input powers p1=90mW, p2=60mW and p1==60mW, p2=30mW.

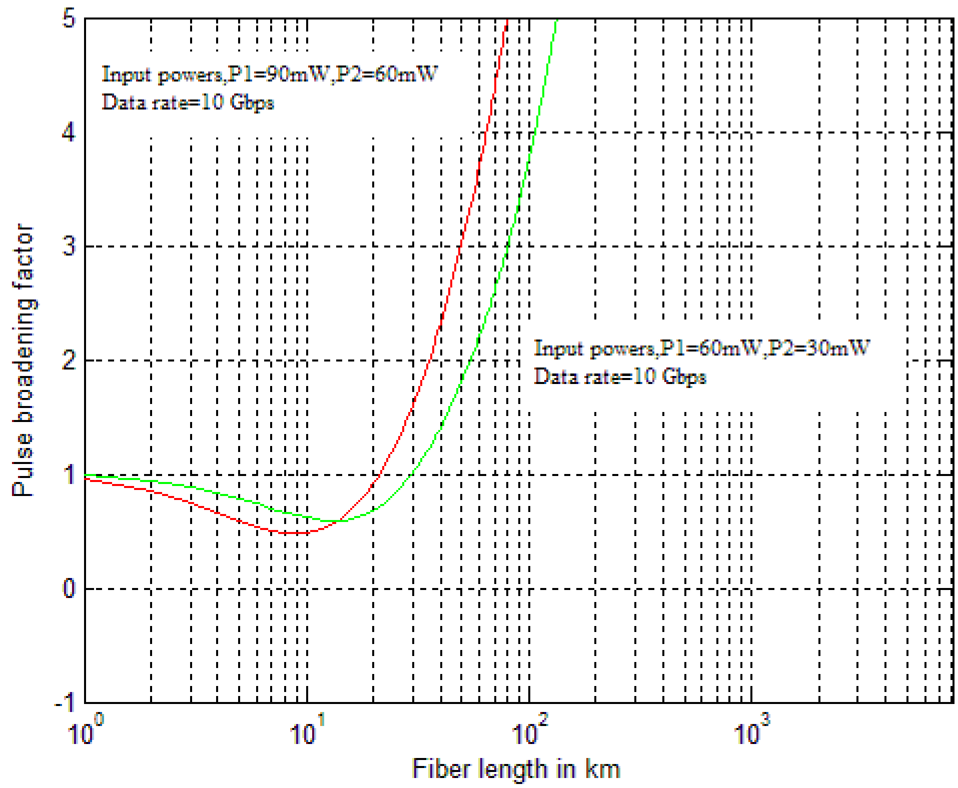

Fig. 4.2: Pulse broadening factor with varying input powers P1 and P2 for SSMF fiber operating at data rate 10 Gbps

Here data rate is kept constant at 10 Gbps. It has been observed that for input powers P1=60mW, P2=30mW, the pulse broadening factor initially decreases from 1 with fiber length. The minimum value of the factor is approximately 0.66 and occurs at 10.5km. Then the factor increases to 1 at 12 km and then keeps on increasing. The decreasing indicates compression of pulse. For P1=90mW, P2=60mW, the minimum value of the factor is 0.5 at 10km and the value is 1 at 11.1km. Thus the pulse compression and broaden increase with increase input powers.

From the above discussion the following conclusion can be drawn about the effect of XPM with first order GVD on pulse broadening

i. As, the bit rate increases the amount of pulse broadening increases. The broadening of the frequency spectrum of the pulse caused by the effect of XPM with first order GVD increases the temporal spreading of the pulse in SSMF.

ii. Pulse broadening factor increases slightly with an increase of input powers. The effect of input powers is less effective on the pulse broadening factor.

iii. The increase of fiber length increases pulse broadening.

4.3 Effect of XPM with SPM and First Order GVD on Pulse Broadening in LEAF Fiber

Fig. 4.3 shows the plots of pulse broadening factor due to the effect of XPM with SPM and first order GVD versus fiber length at the data rates 10Gbps and 40Gbps in LEAF. While the input powers are kept constant.

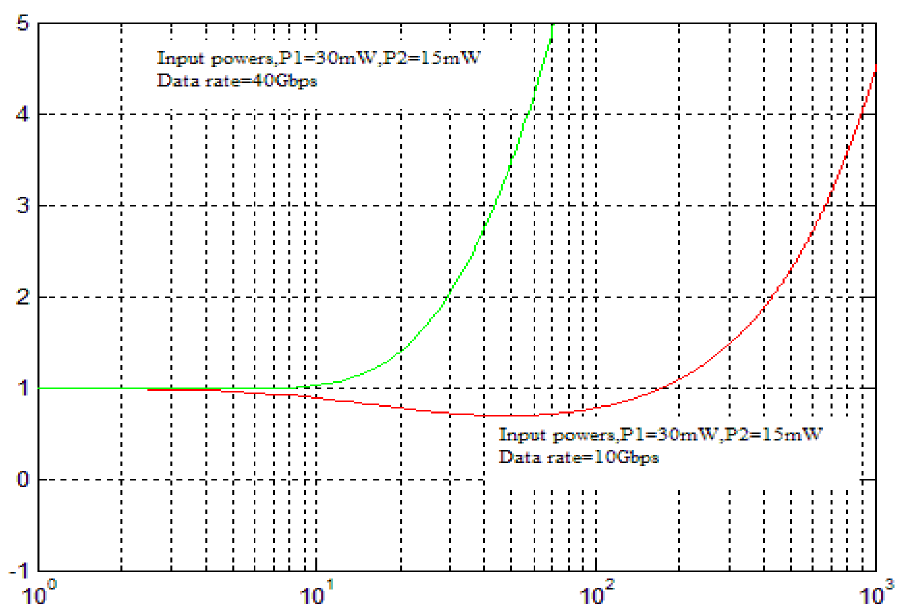

Fig. 4.3: Plots of pulse broadening factor versus fiber length at the data rates 10 Gbps and 40 Gbps in LEAF.

It is found from Fig. 4.3 that as the data rates increases, pulse broadening factor caused by the effect of XPM with SPM and first order GVD increases and limits the transmission distance.

Fig. 4.4 shows the plots of pulse broadening factor versus fiber length at input powers p1=90mW, p2=60mW and p1==60mW, p2=30mW.

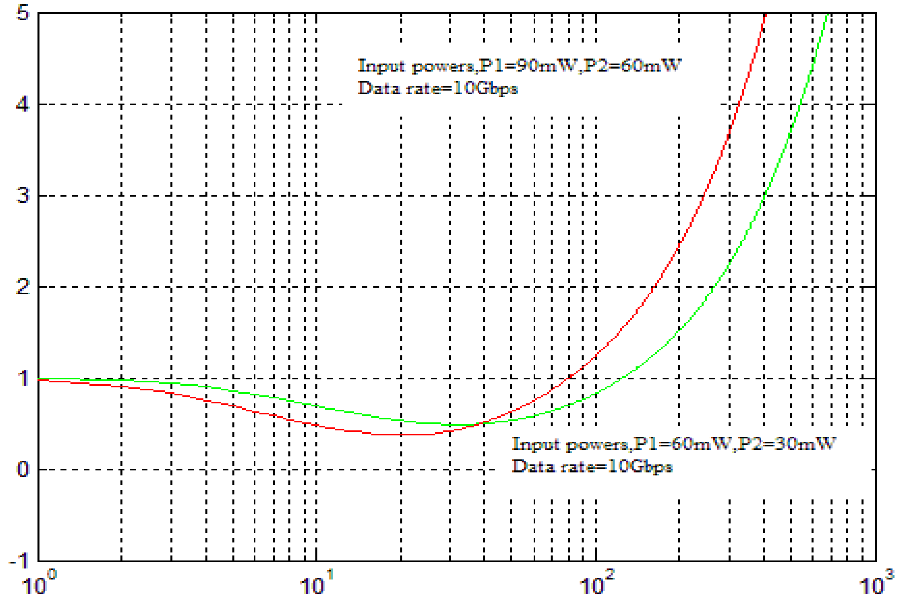

Fig. 4.4: Pulse broadening factor with varying input powers P1 and P2 for LEAF fiber operating at data rate 10 Gbps

It is found from Fig. 4.4 that as the change of input powers is less effective on the pulse broadening factor in presence of the effect of XPM with SPM and first order GVD in LEAF.

4.4 Comparison between the Effect of XPM with First Order GVD and the Effect of SPM with CD on Pulse Broadening in SSMF

In this section the present work is compared with previous work Bijoy et al. (2009) analyzed the effects of SPM and CD on the pulse broadening factor in [18]. Pulse broadening factor due to the effect of XPM with first order GVD and that due to the effect of SPM with CD are visualized and compared here. The variation of pulse broadening factors due to the both effects have been observed for various data rates and input powers in Fig. 4.5, Fig. 4.6, Fig. 4.7 and Fig. 4.8.

Fig. 4.5 shows the comparison between pulse broadening factor due to the effect of XPM with first order GVD and that due to SPM with CD at the data rate 10 Gbps.

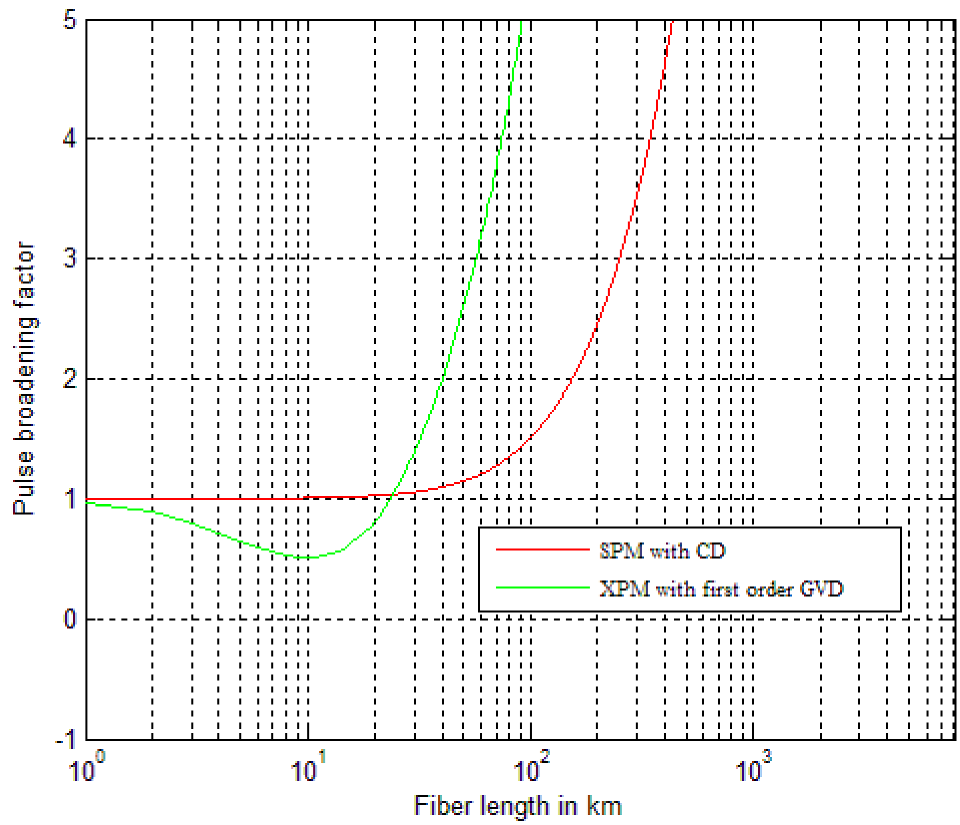

Fig. 4.5: Comparison between pulse broadening factor due to the effect of XPM with first order GVD and that due to SPM with CD at the data rate 10 Gbps.

It is found from Fig. 4.5 that the pulse broadening factor due to the effect of SPM with with CD remains constant at 1 up to the fiber length 12km and then increases with fiber length. But due to the effect of XPM with first order GVD, the pulse broadening factor initially decreases from 1 with fiber length then increases to 1 at 11.5km and continue rapidly increasing. Thus the amount of pulse compression and broaden due to effect of XPM with first order GVD is more than SPM with CD. In can also say that the XPM with GVD is more effective on the transmission distance than SPM with CD.

Fig. 4.6 shows the comparison between pulse broadening factor due to the effect of XPM with first order GVD and that due to the SPM with CD at the data rate 40 Gbps.

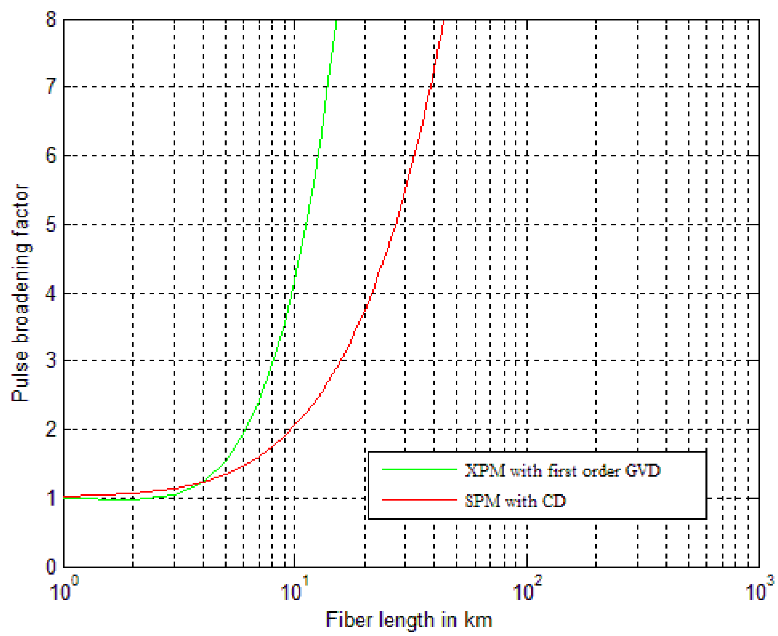

Fig. 4.6: Comparison between pulse broadening factor due to the effect of XPM with first order GVD and that due to the SPM with CD at the data rate 40 Gbps

By comparing Fig. 4.5 and Fig. 4.6, it is found that as the data rate increases, broadening factor for both effects increases. It is also noticed that as the data rate is increased from 10Gbps to 40Gbps, the pulse compression due to the effect of XPM with first order GVD has been decreased significantly.

Fig. 4.7 and Fig. 4.8 show the pulse broadening factor due to the effect of XPM with first order GVD and that due to the effect of SPM with CD at input powers p1=30mW, p2=30mW and p1==60mW, p2=60mW respectively.

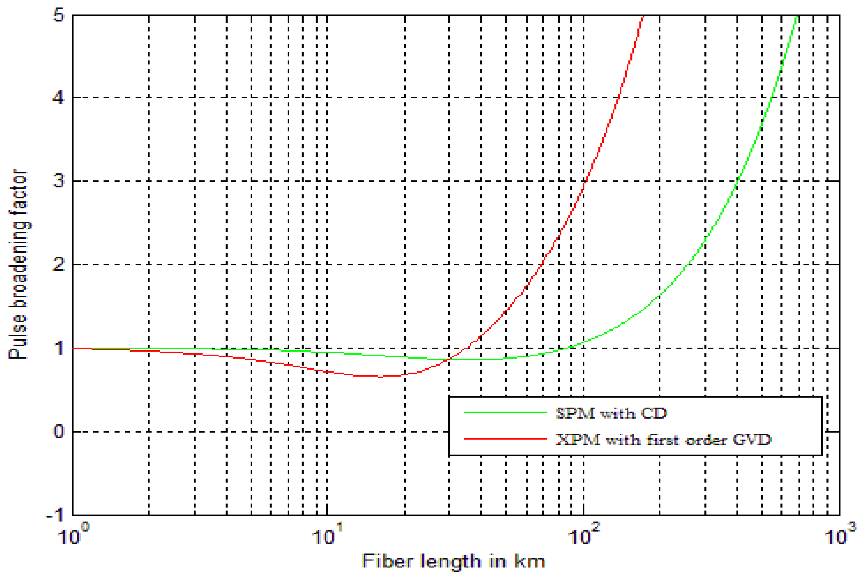

Fig. 4.7: Comparison between pulse broadening factor due to the effect of XPM with first order GVD and that due to the SPM with CD at input powers, P1=30mW, P2=30mW.

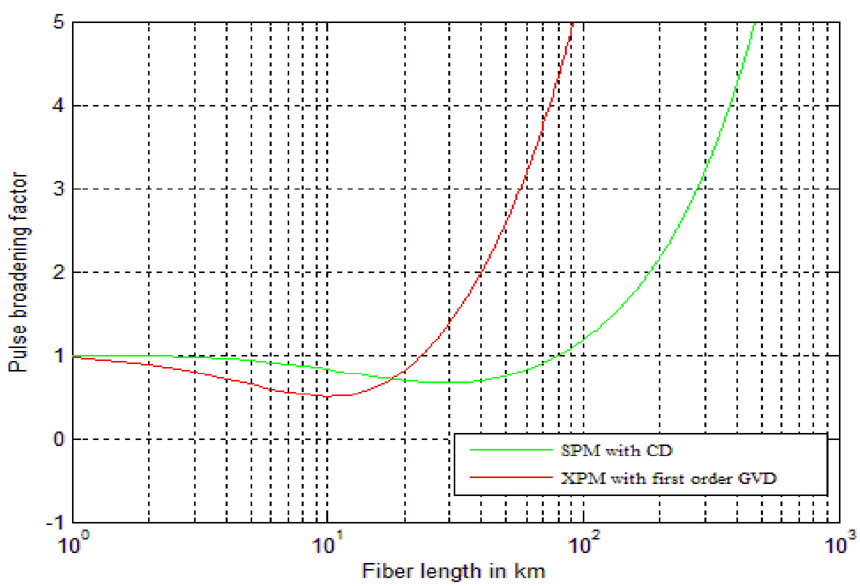

Fig. 4.8: Comparison between pulse broadening factor due to the effect of XPM with first order GVD and that due to the SPM with CD at the input powers, P1=60mW, P2=60mW.

It has been noticed from Fig.4.7 and Fig.4.8 that the pulse broadening factor due to the effect of XPM and GVD remain approximately same for changing input powers from P1=30 mW, P2=30 mW to P1=60 mW, P2=60mW. Similarly, the pulse broadening factor due to the effect of SPM with CD remain approximately same for changing input powers from P1=30 mW, P2=30 mW to P1=60 mW, P2=60mW. Thus the input powers have very small effect on the effects of XPM with GVD, SPM with CD and hence on pulse broadening factor at fixed data rate.

So, from the comparison between the effect of XPM with first order GVD and SPM with CD on pulse broadening factor, the important results can be summarized as follows:

i. For the same fiber as the data rate increases the broadening factor for both effect increases.

ii. The input power has very small effect on pulse broadening factor.

iii. The effect of XPM with first order GVD is always greater than the effect of SPM with CD on pulse broadening. Because the phase of optical pulse modified not only by itself but also of other co-propagating pulses. Hence we can say that, XPM is twice as effective as SPM for same amount of power.

4.5 Comparison of Analytical derivation with Numerical Results for XPM with first order GVD

The analytical expression of pulse broadening is derived by mathematical analysis of NLSE considering the effects of XPM and first order GVD. The analytical results are obtained by visualization of the analytical expression using Matlab. The numerical results are obtained by solving nonlinear Schrödinger equation with split step Fourier method and Matlab. The analytical and numerical results for SSMF fiber are compared in order to investigate the accuracy of the derived analytical formulas.

Fig. 4.9 shows the comparison between analytical and numerical results for SSMF fiber.

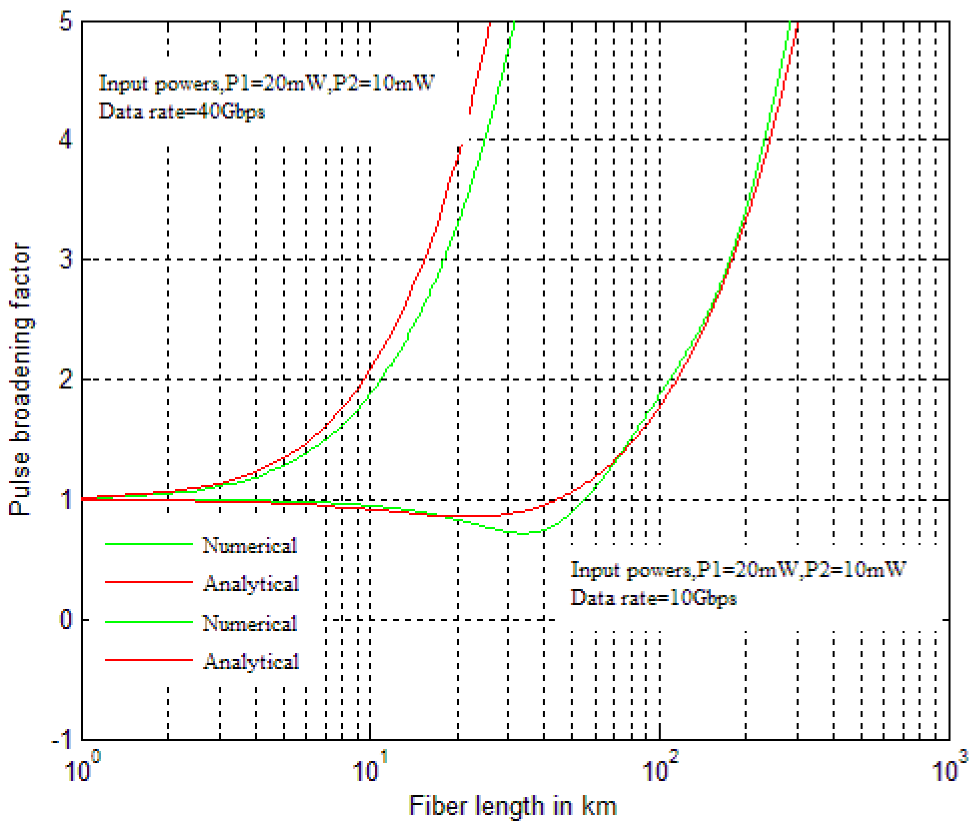

Fig 4.9: Pulse broadening factor for the input powers, P1=20 mW, P2=10 mW and data rate=10Gbps and 40Gbps in SSMF.

The effect of XPM with first order GVD on pulse broadening with the variation of bit rate is shown in Fig. 4.9. It is observed that the analytical prediction is supported by the numerical results.

4.6 Effect of XPM with Second Order GVD on Pulse Broadening in SSMF Fiber

The pulse broadening factor due to the effect of XPM with second order GVD for SSMF is obtained by solving nonlinear Schrödinger equation numerically with split step Fourier method and Matlab.

Fig. 4.10 shows the plots of pulse broadening factor due to the effect of XPM with second order GVD versus fiber length for the input powers, P1=30 mW, P2=10 mW and data rate=10 Gbps.

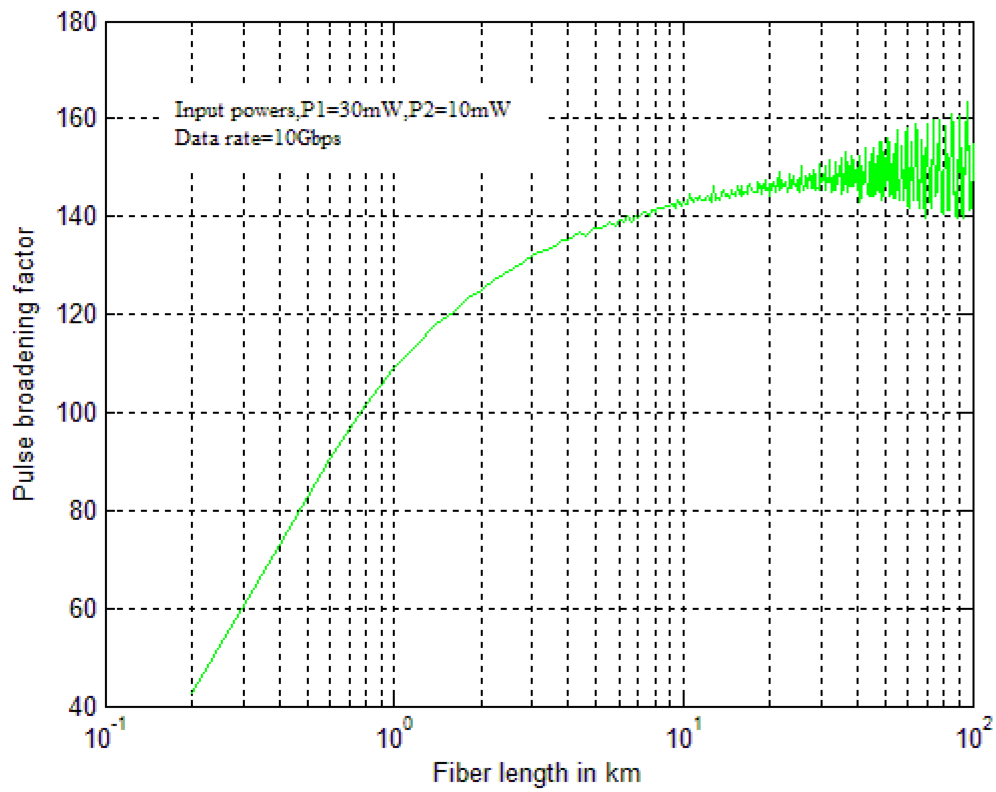

Fig 4.10: Pulse broadening factor for the input powers, P1=30 mW, P2=10 mW and data rate=10 Gbps in SSMF.

From Fig. 4.10, it is found that the spectral broadening of pulse occurred due to the combined effects of XPM with second-order GVD. It is observed in Fig.4.10 that at fiber length 0.2 km, pulse broadening factor is 40, then pulse broadening factor increases rapidly with fiber length up to 12 km. After 12 km, the curve oscillates and creates multiple ripples. Thus from these observations and analysis, we can say that the combined effects of XPM and second-order dispersion on the transmitting pulse increases with fiber length and thus limits transmission distance. After transmitting certain distance, pulse broadening factor oscillates and create multiple ripples. The oscillation ripples indicate asymmetric spectral broadening of transmitting pulse.

However, when second order GVD is present, different parts of the prove pulse propagate at different speed because of the XPM-induced chirp imposed on the probe pulse. This results in an asymmetric shape with considerable structure. The probe pulse develops rapid oscillation near the trailing edge while the leading edge is largely unaffected.

The physical origin of temporal oscillation is related to optical wave breaking mechanism. Both GVD and XPM impose frequency chirp on the pulse as it travel down the fiber. The GVP-induced chirp is linear with time but the XPM-induced chirp is far from being linear across the entire pulse.

Because of the nonlinear nature of the composite chirp, different parts of the pulse propagate at different speeds. In the case of normal GVD the red-shifted light near the leading edge travels faster and overtakes the un-shifted light in the forward tail of the pulse. The opposite occurs for the blue shifted light near the trailing edge. In both cases, the leading and trailing region of the pulse contain light at two different frequencies that interfere. So, oscillation occurs near the pulse edge, result of such interference.

Fig. 4.11 shows the pulse broadening factor due to the effect of XPM with second order GVD for input powers P1=30 mW, P2=10 mW and bit rate 40 Gbps.

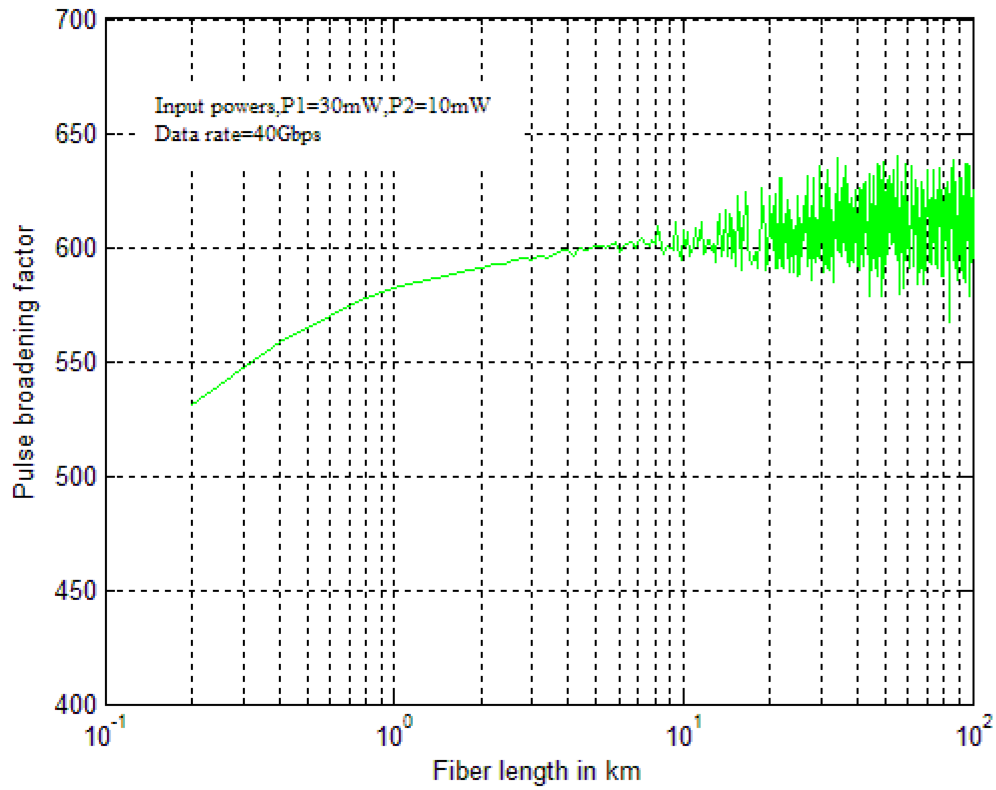

Fig 4.11: Pulse broadening factor for the input powers, P1=30 mW, P2=10 mW and data rate=40 Gbps in SSMF.

The effect of change in the data rates on the pulse broadening factor have been investigated and analyzed characterized by comparing Fig.4.10 and Fig. 4.11. It is observed from this comparison that for data rate 40Gbps, pulse broadening factor is 530 at fiber length 0.2 km , while for bit rate 10 Gbps, pulse broadening factor is 40 at fiber length 0.2 km. Thus the pulse broadening factor due to the combined effect increases with increasing data rates. The change of data rate is very effective on the pulse broadening factor due to the effect of XPM with second order GVD.

Fig. 4.12 shows the pulse broadening factor for a SSMF fiber for input powers P1=60 mW, P2=30 mW and data rate 40 Gbps.

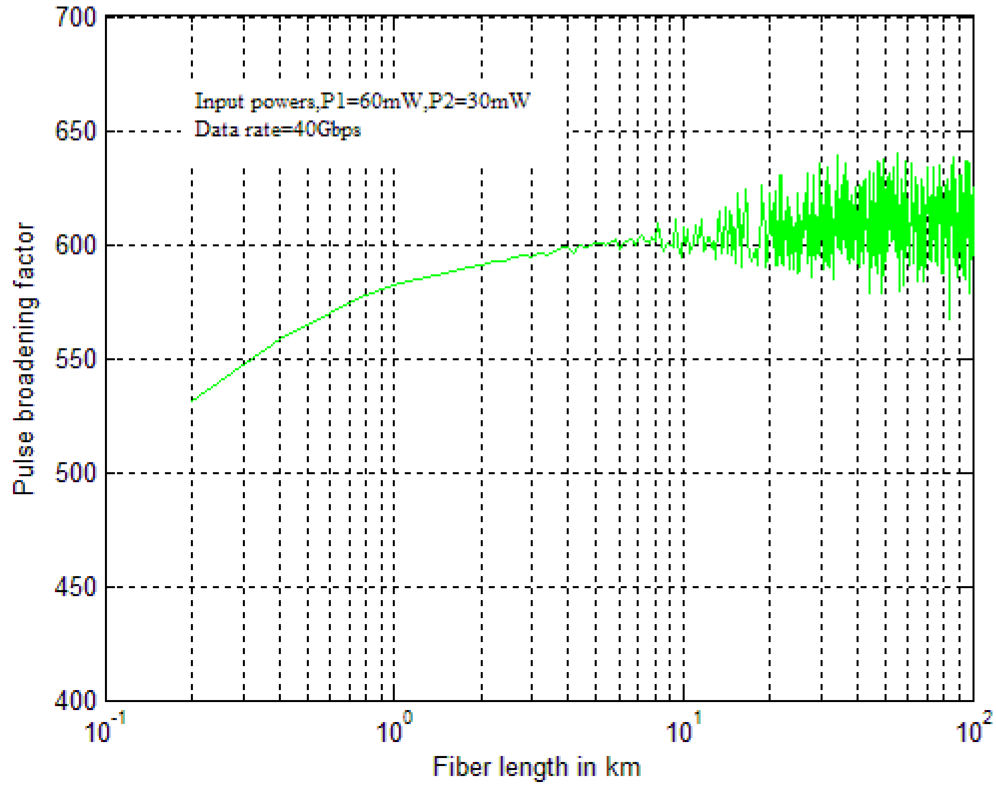

Fig 4.12: Pulse broadening factor for the input powers, P1=60 mW, P2=30 mW and data rate=40 Gbps

The effect of change in the input powers on the pulse broadening factor have been investigated and characterized by comparing Fig.4.11 and Fig. 4.12. From these comparisons, it is observed that as the input power P1 is changed from 30mW to 60mW and P2 is changed from 10mW to 30mW, the pulse broadening factor remain approximately same for fixed data rate. Thus the change in the input powers is less effective on the transmitting pulse and the pulse broadening factor with the present of XPM and second order GVD in the WDM transmission system.

From the above discussion the following conclusion can be drawn about the effect of XPM with second order GVD on pulse broadening

i. The increase of data rate increases pulse broadening.
ii. The change of input powers has low affect on pulse broadening factor in presence of XPM with second order GVD.

4.7 Effect of XPM with Second Order GVD on Pulse Broadening in LEAF Fiber

Fig. 4.13 and Fig. 4.14 show the pulse broadening factor due to the effect of XPM with second order GVD for data rate 10 Gbps and 40 Gbps respectively.

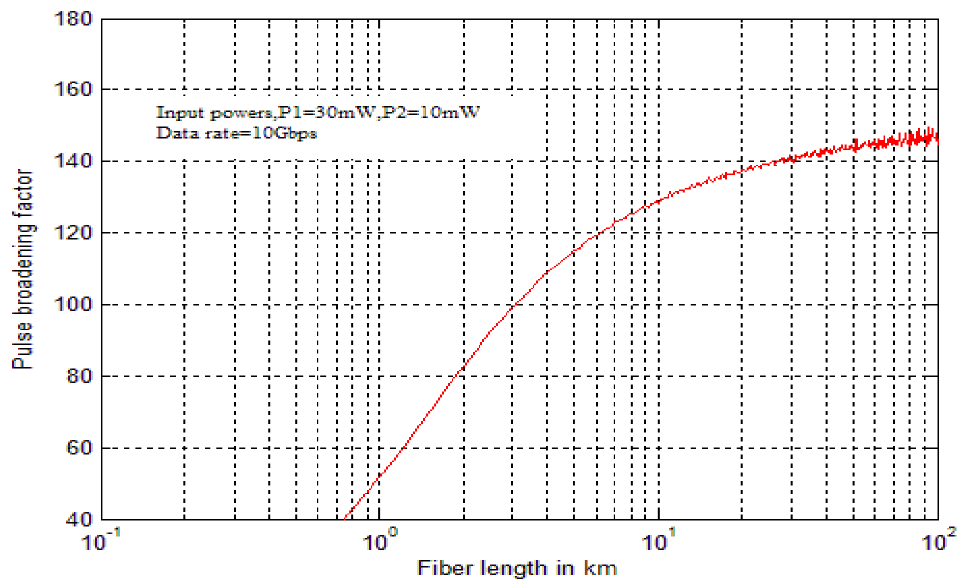

Fig. 4.13: Pulse broadening factor due to the effect of XPM with second order GVD for input powers P1=30 mW, P2=10 mW and data rate 10 Gbps in LEAF.

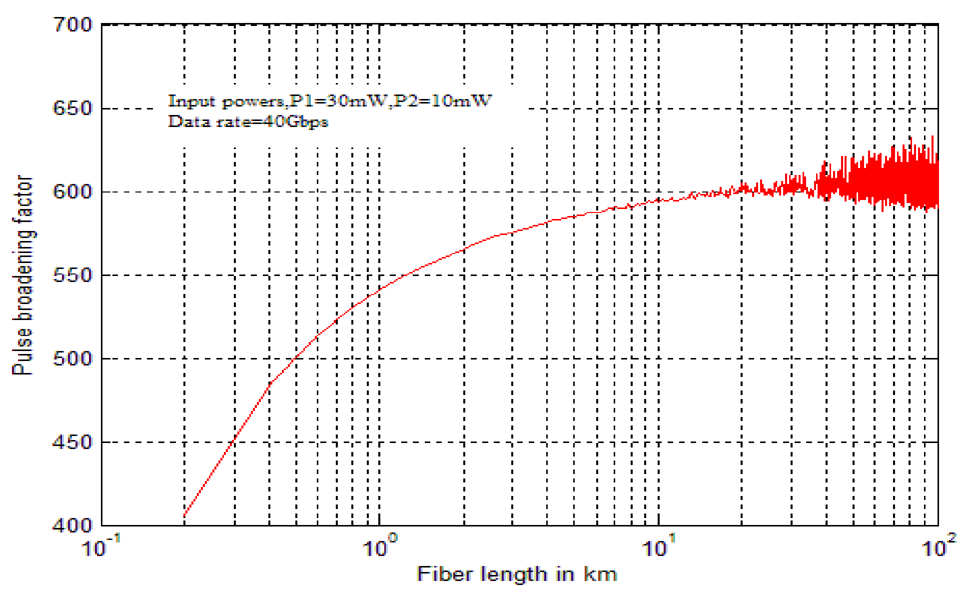

Fig. 4.14: Pulse broadening factor due to the effect of XPM with second order GVD for input powers P1=30 mW, P2=10 mW and data rate 40 Gbps in LEAF.

By comparing Fig. 4.13 and Fig. 4.14 it is found that as the data rate increases the pulse broadening factor due to XPM with second order GVD increases in LEAF fiber.

Fig. 4.15 shows the pulse broadening factor due to the effect of XPM with second order GVD for input powers, P1=60mW and P2=30mW.

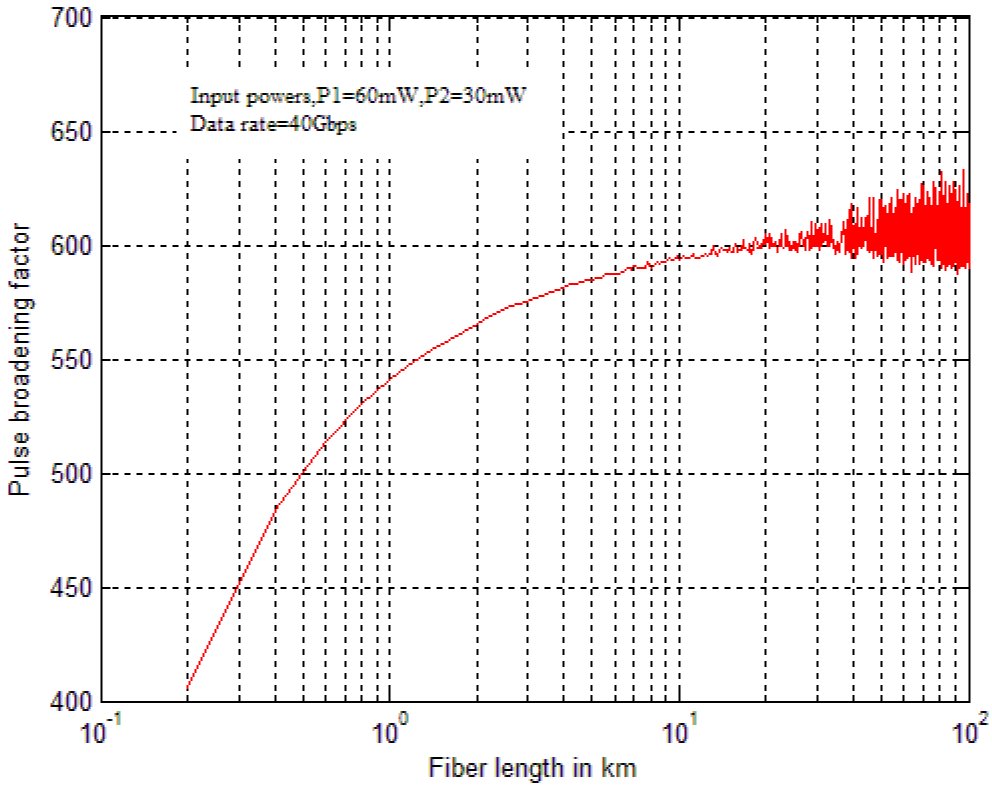

Fig. 4.15: Pulse broadening factor due to the effect of XPM with second order GVD for input powers P1=60 mW, P2=30 mW and data rate 40 Gbps in LEAF.

It is observed by comparing Fig. 4.14 and Fig. 4.15 that the change of input power is less effective on the pulse broadening factor in LEAF fiber in presence of XPM with second order GVD.

4.8 Comparison of XPM with First Order GVD and XPM with Second Order GVD on Pulse Broadening Factor in SSMF

Pulse broadening factor due to the effect of XPM with first order GVD and that due to the effect of XPM with Second Order GVD for the same data rate and input powers are visualized and compared in the following Fig. 4.16.

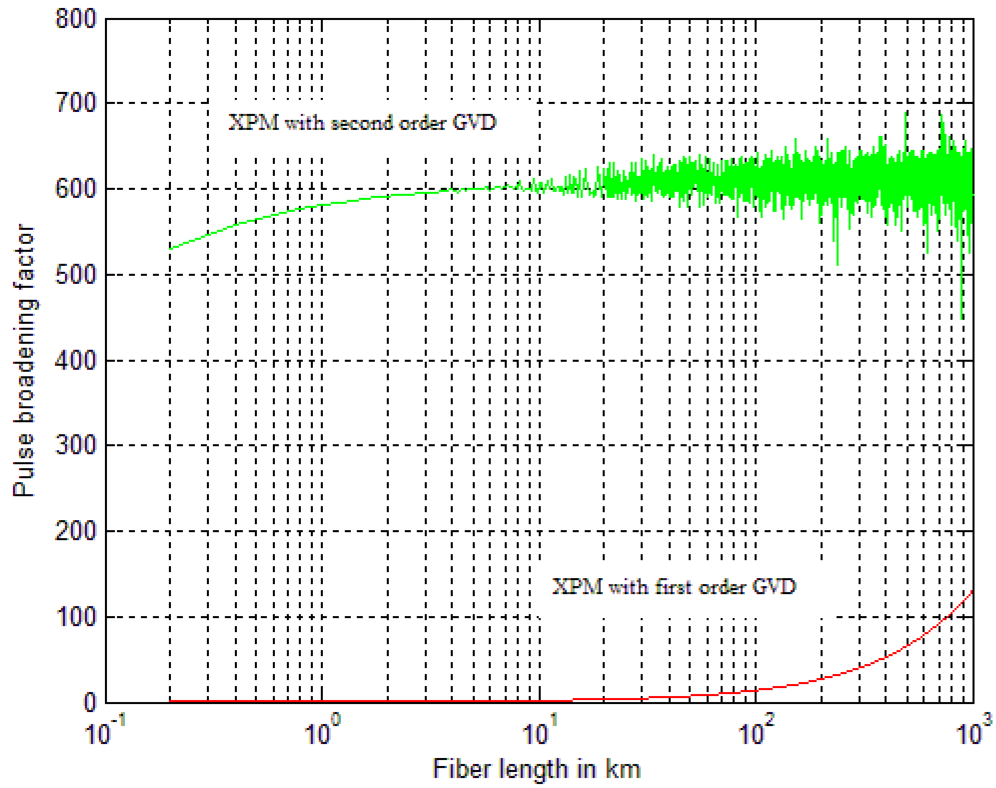

Fig 4.16: Pulse broadening factor due to XPM with first order GVD and that due to XPM with second Order GVD in SSMF

By comparing the two curves, it is observed that up to fiber length 10 km, pulse broadening factor due to XPM with second order GVD increases very much with fiber length while that due to XPM with first order GVD remains almost constant with fiber length. At fiber length 10 km, the pulse broadening factor due to the effect of XPM with second Order GVD is 600 while that due to XPM with first order GVD is about 1. After 10 km, the broadening factor due to the effect of XPM with Second Order GVD oscillates and creates multiple ripples while that due to XPM with first order GVD increases nonlinearly with fiber length. Thus, XPM with second order GVD is very much effective than XPM with first order GVD on the pulse broadening factor. The oscillation ripples that indicate asymmetric pulse broadening are introduced by the effect of XPM with Second Order GVD.

4.9 Effect of XPM with First- and Second order GVD on Pulse Broadening in SSMF

The pulse broadening factor due to the effect of XPM with first- and second order GVD are visualized for various data rates and input powers.

Fig. 4.17 shows the plot of pulse broadening factor due to the effect of XPM with first- and second order GVD versus fiber length for the input powers, P1=30 mW, P2=10 mW and data rate=10 Gbps.

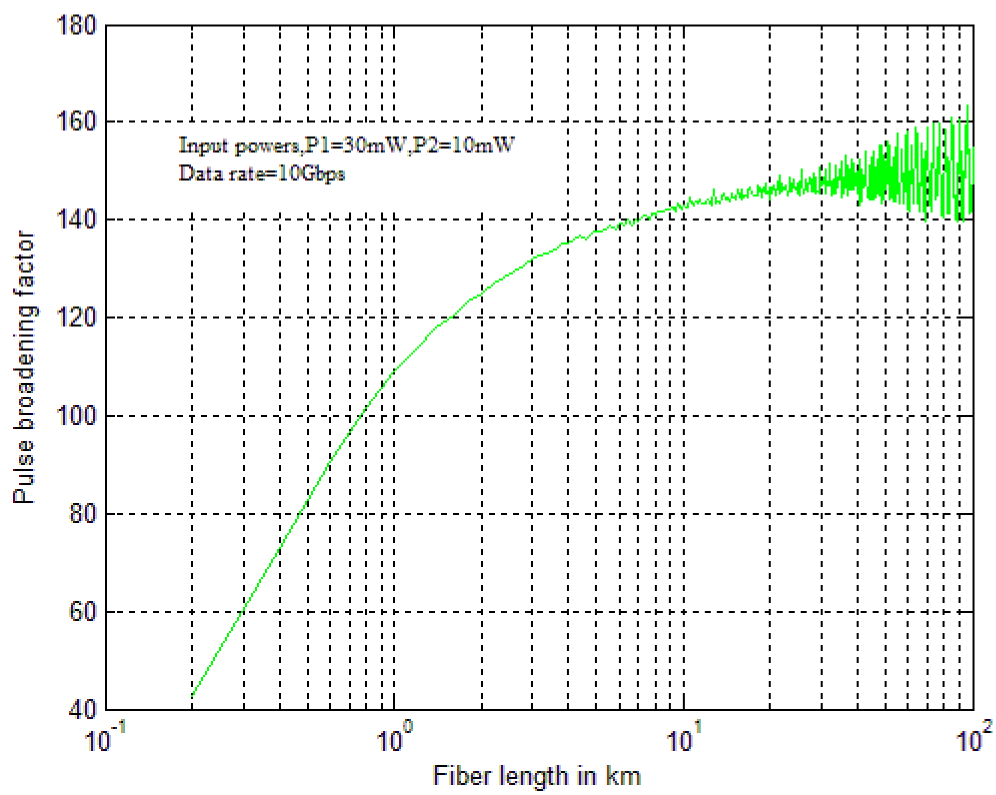

Fig 4.17: Pulse broadening factor for the input powers, P1=30 mW, P2=10 mW and data rate=10 Gbps .

It is observed from Fig. 4.10 and Fig. 4.17 that pulse broadening factor due to the effect of XPM with first- and second order GVD is almost same to that due to the effect of XPM with first- and second order GVD for the same data rate and input powers. Thus at high data rate the impact of second order GVD is dominant on the first order GVD in presence of XPM.

Fig. 4.18 shows the plot of pulse broadening factor due to the effect of XPM with first- and second order GVD versus fiber length for the input powers, P1=30 mW, P2=10 mW and data rate=40 Gbps.

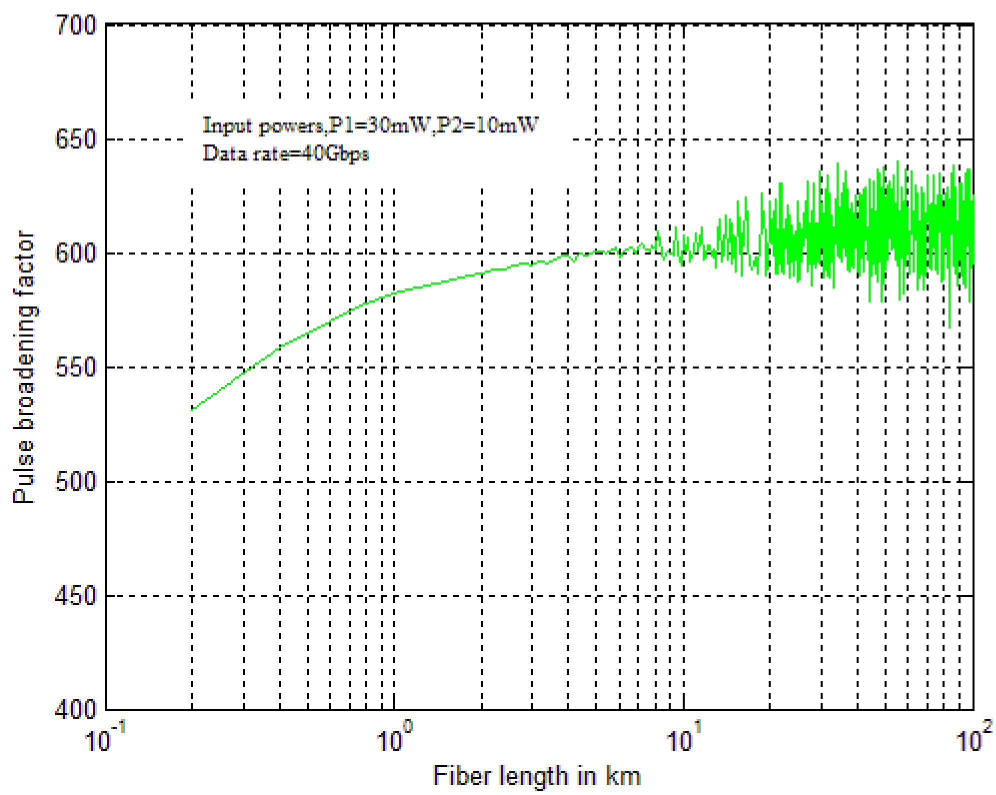

Fig 4.18: Pulse broadening factor due to XPM with first- and second order GVD for the input powers, P1=30 mW, P2=10 mW and data rate=40 Gbps

The change of data rates on the pulse broadening factor due to the effect of XPM with first- and second order GVD have been investigated by comparing Fig.4.17 and Fig. 4.18. It has been observed that at fiber length 5.0 km, pulse broadening factor is 600 for data rate 40Gbps while is 140 for bit rate 10Gbps. Thus pulse broadening factor due to the combined effect of XPM with first- and second order GVD increases with increasing data rates. The change of data rate is very effective on the pulse broadening factor due to XPM with first- and second orders GVD.

4.10 Effect of XPM with first order GVD on Pulse Compression Factor

Fig. 4.19 shows the plots of pulse compression factor due to the effect of XPM with first order GVD versus fiber length for various input powers.

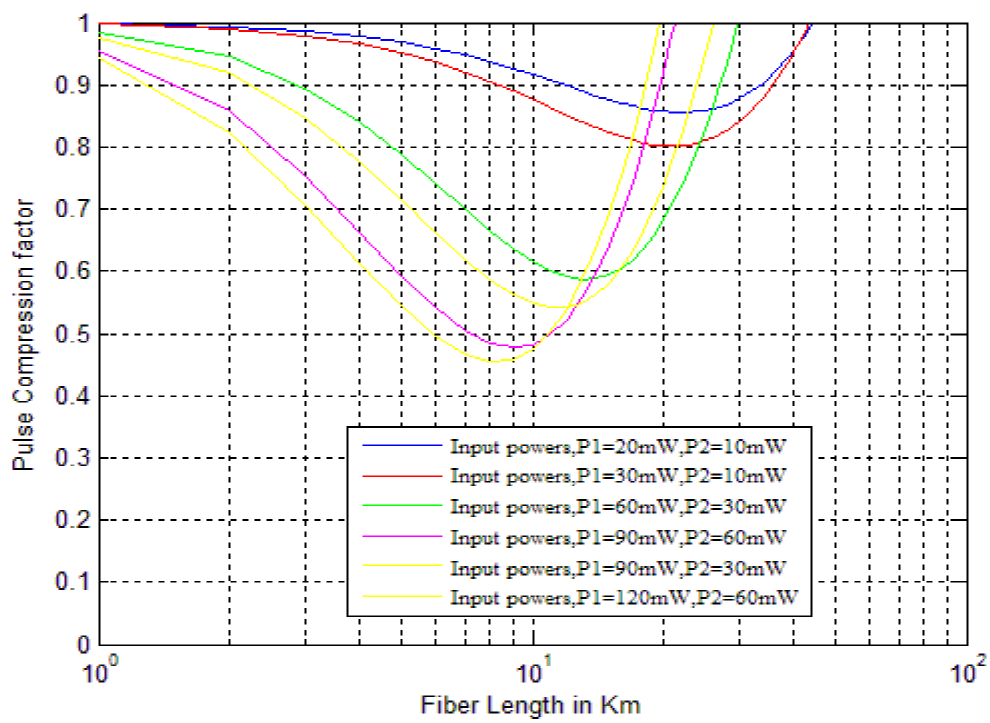

Fig. 4.19: Plots of pulse compression factor due to the effect of XPM with first order GVD versus fiber length for various input powers

Since the pulse fluctuation introduced by XPM is intensity dependent, different parts of the pulse undergo different phase shift. This leads frequency chirping, in which the leading edge experience down shift (red shift) and trailing edge up shift (blue shift). On contrary, chirping introduce by GVD in the anomalous-dispersion regime is up shift in the leading edge and down shift in the trailing edge. The two chirping are opposite in sign and cancel each other at some distance. The pulse shape adjusts itself during propagation to make such cancellation as complete as possible. At relatively shorter distance XPM induced chirping is higher than GVD induced chirping. As a result, the pulse get compressed due to the interplay of XPM and GVD up to a particular distance and gets its minimum width which is shown in Fig. 4.19, after this distance the pulse width further broadens. From the plots, it is revealed that as the input power increases, the amount of pulse compression also continues up to certain length.

4.11 Effect of XPM with First- and Second Order GVD on Output Pulse

The input Gaussian pulse shown in Fig. 4.20 is launched into fiber to transmit. In Fig. 4.21 it is observed that the transmitting pulse is broadened due to the effect of XPM with first order GVD. Fig.4.22 shows the transmitting pulse due to the combined effect of XPM with second order GVD.

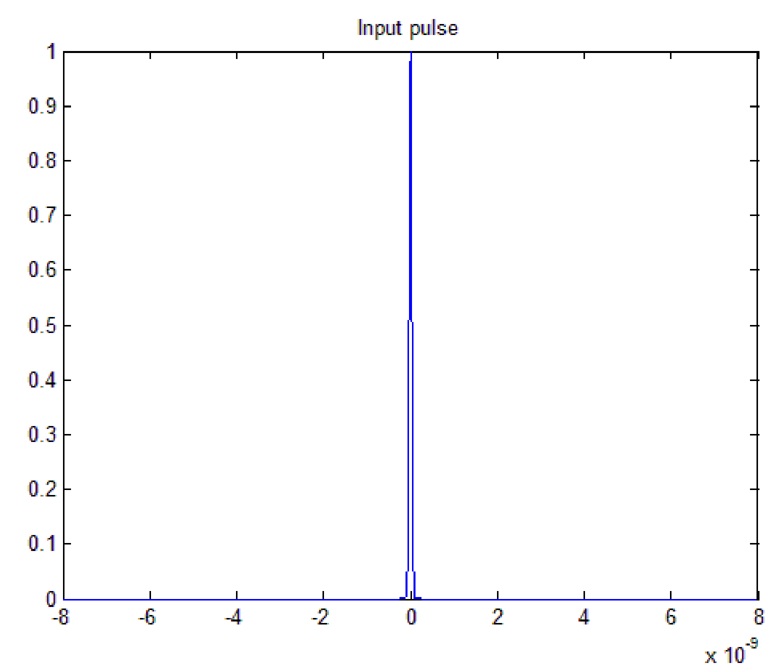

Fig. 4.20: Visualization of an input Gaussian pulse.

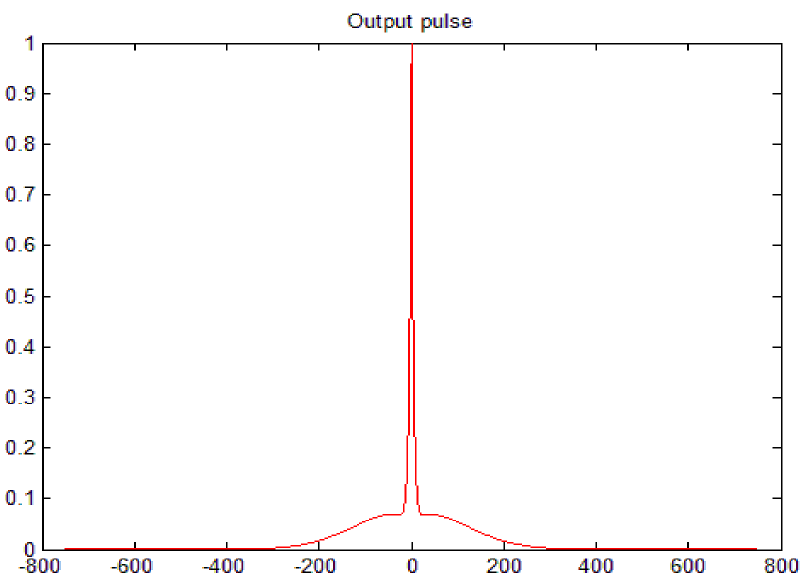

Fig. 4.21: Visualization of output pulse in presence of XPM with first order GVD

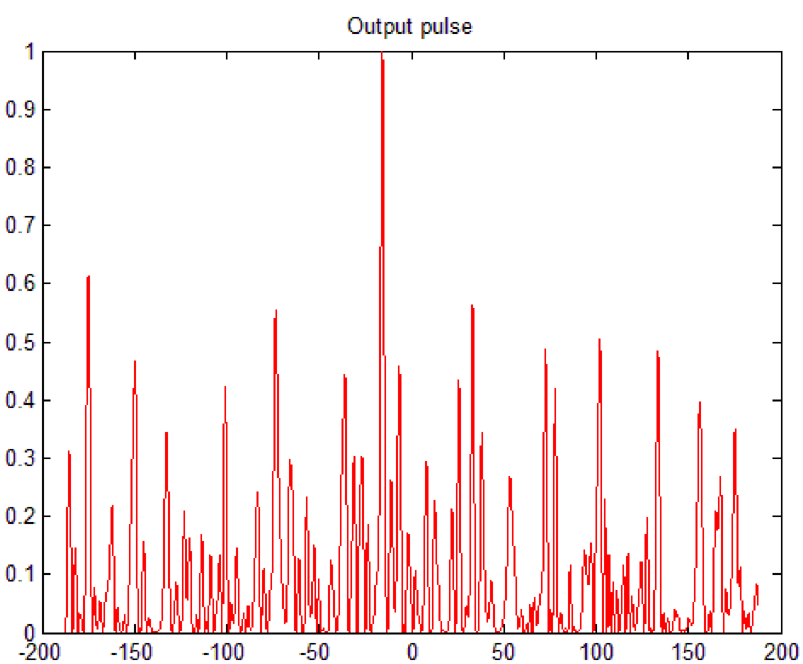

Fig. 4.22: Visualization of output pulse in presence of XPM with second order GVD

It has been observed that at high data rate the second order GVD can dramatically change the behavior of the transmitting pulse as well as transform Gaussian pulse into multiple-shaped pulses in presence of XPM. Asymmetric spikes have been created on the output pulse.

4.12 Comparison between SSMF and LEAF Fiber in presence of XPM with First- and Second Order GVD

The plots of pulse broadening factor versus fiber length in presence of the effect of XPM with first order GVD are shown in Fig. 4.23 for SSMF and LEAF Fiber.

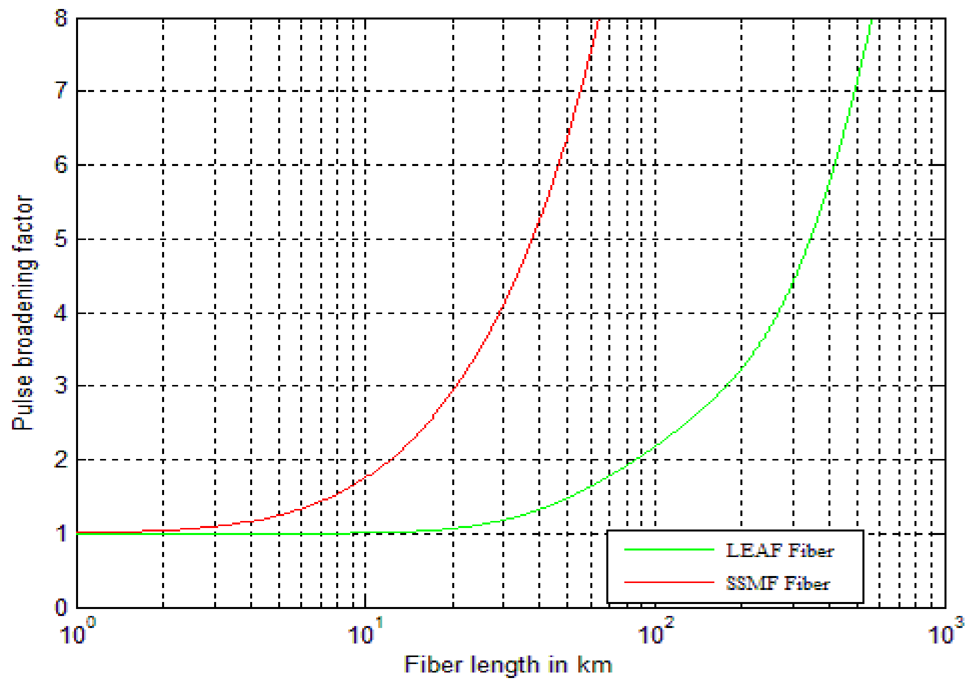

Fig. 4.23: Comparison between SSMF and LEAF fiber due to XPM with first order GVD

It is found from Fig. 4.23 that the pulse broadening factor for a SSMF fiber remains constant up to fiber length 0.2 Km and then increases with fiber length. For LEAF fiber this factor remains constant up to 11 Km and then increases with fiber length.

The plots of pulse broadening factor versus fiber length in presence of the effect of XPM with second order GVD are shown in Fig. 4.24 for SSMF and LEAF fiber.

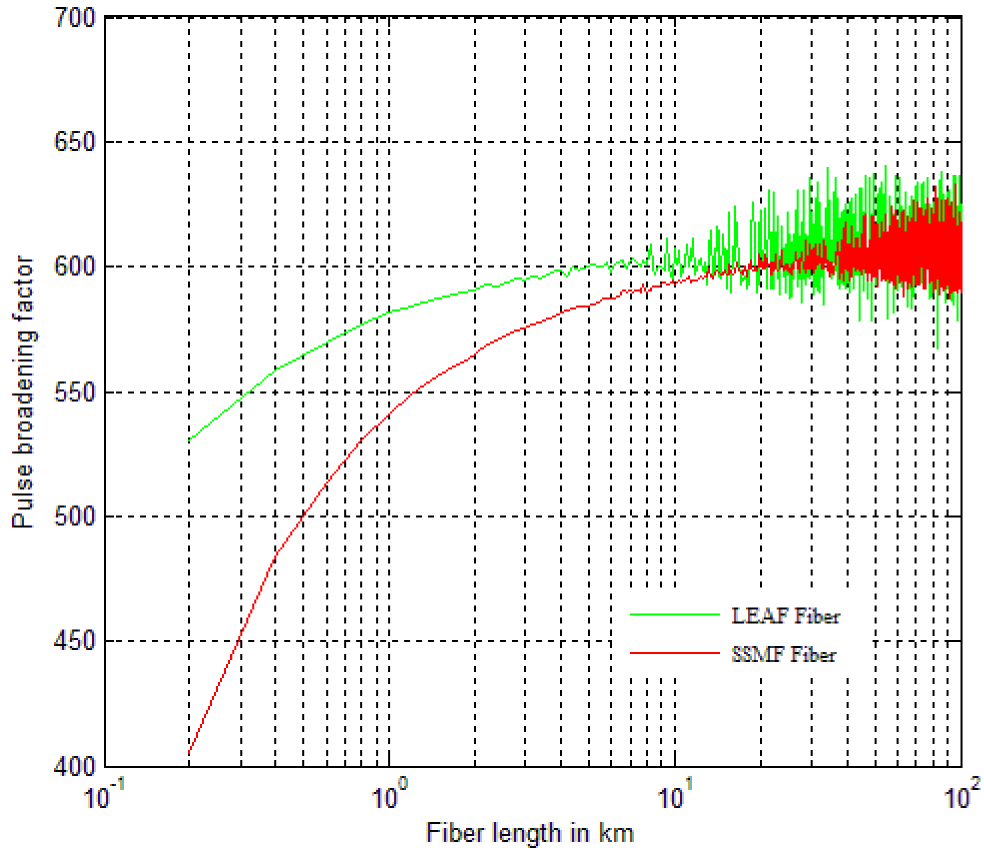

Fig. 4.24: Comparison between SSMF and LEAF fiber due to XPM with second order GVD

It is observed in Fig.4.24 that after fiber length 0.2km, the pulse broadening factor is 400 for a SSMF fiber while that for LEAF fiber is 530 In SSMF the curve oscillates and creates multiple ripples after 10 km and in LEAF fiber this oscillation and multiple ripple creation occur after 12 km.

For the same data rates and input powers, the amount of pulse broadening for two different fibers is different since they have different dispersion profile. The pulse broadening factor in SSMF fiber is more than LEAF fiber because the effect of XPM with first- and second orders GVD in SSMF fiber is more than the LEAF fiber.

4.13 Conclusion

The visualization of pulse broadening factor versus fiber length for SSMF and LEAF is done and analyzed. The variation of this factor due to effect of XPM with first- and second order GVD has been observed for various data rates and input powers. It is found that the impact of data rate and fiber length on the pulse broadening factor is more than input power. The amount of pulse broadening factor due to the effect of XPM with SPM and first order GVD is higher than that of SPM with CD.

It is observed that at high bit rate the effect of second order GVD becomes significant and limits the system performance. It is also found that XPM with first- and second order GVD is more effective in SSMF than that of LEAF fiber.

CHAPTER 5
CONCLUSION

In this chapter, the conclusion is summarized that can be drawn from the researcher formed for this dissertation, then provide suggestions for future research.

5.1 Conclusion

At high data rate, channel capacity and launched optical powers, the effect of XPM effect with GVD limits the system performance significantly in WDM system. So, the main motivation of this research work is to study the effect of XPM in presence of first– and second order GVD to optimize system performance. A detailed analysis has been carried out analytically and numerically to obtain pulse broadening factor for a pulse that is affected by the effects of XPM with first– and second order GVD in WDM optical fiber communication system.

The findings of this research work are given below:

i. The spectral broadening of pulse is strongly dependent on the effect of XPM with first order GVD in WDM system. It is observed that the broadening factor increases with increase of data rates, fiber length and input powers. But the data rate and fiber length have higher impact on the pulse broadening factor than the input powers.

ii. At high data rate, the impact of second order GVD is significant on the system performance. Asymmetric pulse broadening as well as multiple ripples is occurred due to the effect of XPM with second order GVD. The effect of XPM with second order GVD on pulse broadening factor increases with increase data rate and fiber length.

iii. The XPM with second order GVD is very much effective and shows different behavior on pulse broadening factor than XPM with first order GVD.

iv. The effect of XPM with first- and second order GVD on pulse broadening factor is almost same to the effect of XPM with second order GVD because at high data rate the impact of second order GVD is dominant on first order GVD in presence of XPM.

v. The output pulse broadens as well as creates multiple ripples due to the effect of XPM with second order GVD while the output pulse only broadens due to the effect of XPM with first order GVD. The ripples in the pulse represent asymmetrical behavior of second order GVD in presence of XPM.

vi. XPM with first- and second order GVD is very effective in SSMF fiber than LEAF fiber.

vii. XPM with first order GVD on the pulse broadening is more effective than SPM with CD.

5.2 Recommendations for Future Work

Research work is a continuous process. So it is important to think about the scope of further extension of this work. We have analyzed a WDM system with inter-channel XPM considering two channels only. Further research may be carried a large number (>16) of channels with various channel spacing even as small as 0.1 nm and considering the intra-channel XPM effect. It is also important to look into the methods and techniques for compensate the XPM impairment. Other nonlinear effects may be also considered to reflect the true performance limitations due to nonlinear phenomenon.

Appendix

DERIVATION

$$\int_{-\infty}^{\infty} e^{-ax^2 - bx - c}\, dx = \int_{-\infty}^{\infty} \exp\left\{-(ax^2 + bx + c)\right\} dx$$

$$= \int_{-\infty}^{\infty} \exp\left\{-a(x^2 + 2.x.\frac{b}{2a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + ac)\right\} dx$$

$$= \int_{-\infty}^{\infty} \exp\left[\left\{-a(x^2 + 2.x.\frac{b}{2a} + \frac{b^2}{4a^2})\right\} \times \left(\frac{b^2}{4a} - c\right)\right] dx$$

$$= \exp\left(\frac{b^2}{4a} - c\right) \int_{-\infty}^{\infty} \exp\left\{-a\left(x + \frac{b}{2a}\right)^2\right\} dx$$

$$= \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2 - 4ac}{4a}\right)$$

The solution $\int_{-\infty}^{\infty} \exp\left\{-a\left(x + \frac{b}{2a}\right)^2\right\} dx$ is given below

Let,

$$x + \frac{b}{2a} = u$$
$$\Rightarrow dx = du$$

$$\int_{-\infty}^{\infty} \exp\left\{-a\left(x + \frac{b}{2a}\right)^2\right\} dx = \int_{-\infty}^{\infty} \exp\left(-au^2\right) du$$

$$= \sqrt{\left(\int_{-\infty}^{\infty} e^{-au^2}\, du\right)^2}$$

$$= \sqrt{\left(\int_{-\infty}^{\infty} e^{-au^2}\, du\right)\left(\int_{-\infty}^{\infty} e^{-au^2}\, du\right)}$$

$$= \sqrt{\left(\int_{-\infty}^{\infty} e^{-au^2} du\right)\left(\int_{-\infty}^{\infty} e^{-av^2} dv\right)}$$

$$= \sqrt{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-a(u^2+v^2)} dudv}$$

Using the following transform,

$$u = r\cos\theta$$
$$v = r\sin\theta$$

$$J\left(\frac{u,v}{r,\theta}\right) = \begin{vmatrix} \frac{du}{dr} & \frac{du}{d\theta} \\ \frac{dv}{dr} & \frac{dv}{d\theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r$$

$$= \sqrt{\int_{0}^{2\pi}\int_{0}^{\infty} e^{-ar^2} Jdrdv}$$

$$= \sqrt{\int_{0}^{2\pi}\int_{0}^{\infty} e^{-ar^2} rdrdv}$$

$$= \sqrt{\int_{0}^{2\pi}\left[-\frac{1}{2a}\left(e^{-\infty} - e^{0}\right)\right]d\theta}$$

$$= \sqrt{\int_{0}^{2\pi}\frac{1}{2a}d\theta}$$

$$= \sqrt{\frac{1}{2a}\left[\theta\right]_{0}^{2\pi}}$$

$$= \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} \exp\left(-au^2\right)du = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} \exp\left\{-a\left(x + \frac{b}{2a}\right)^2\right\}dx = \sqrt{\frac{\pi}{a}}$$

$$\therefore \int_{-\infty}^{\infty} \exp\left\{-(ax^2 + bx + c)\right\}dx = \sqrt{\frac{\pi}{a}}\exp\left(\frac{b^2 - 4ac}{4a}\right)$$

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