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A THESIS

SUBMITTED TO THE DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF MASTER OF SCIENCE IN ENGINEERING


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## ABSTRACT

Static power converters are designed to work under balanced supply condition. In many instances static converters are subject to unbalanced condition. Conventional static converters may not work under such condition. Special unbalanced converter are required for such cases. An unbalanced converter topology is analysed and design procedure is̄ fully discussed in this thesis.

The contribution of the thesis is the development of a technique to correct the input unbalance. The proposed technique states that inorder to generate balanced output voltage and current from unbalanced (amplitude) input voltages, the magnitude of the fundamental component of the switching functions are inversely proportional to the amplitude of the corresponding unbalanced input phase voltage. The dependency of the width of switching function on the Fourier_coefficient is established by harmonic analysis. Input currents and output voltage are fully analysed both under input balaw nced and unbalanced conditions. The analytically predicted results are verified by computer aided analysis.

Finally, a complete design procedure is described for this three phase to single phase unbalanced converter.

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Dedicated to my parents
and
to my Nieces Razia and Sumi.

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| K m | $=$ The m-th harmonic Fourier Co-efficient of the switching |
| :---: | :---: |
|  | function. |
| $\delta_{1}, \delta_{2}, \delta_{3}$ | - Widths of the three switching functions under unbalanced |
|  | input condition. |
| $S_{1}, S_{2}, S_{3}, S_{4}$, | *Six bilateral switches of the converter |
| $S_{5}, S_{6}$ 。 | . |
| $g_{1}, g_{2}, g_{3}, g_{4}$, | = Six gating signals for the bilateral switches. |
| $g_{5}, g_{6}$ - |  |
| $L_{S 1}, L_{\text {S }}, L_{S 3}$ | - Three inductors in the protective circuits. |
| $\mathrm{R}_{\mathbf{S 1}}, \mathrm{R}_{\mathbf{S 2}}$ | = Registors in the protective circuits. |
| $\mathrm{C}_{\mathbf{S 1}}$ | = Capacitor in the protective circuits. |
| PWM | = Pulse width Modulation |
| DMO | $=$ Direct Mode of Operation |
| p.u. | n per unit. |

## INTRODUCTION

## 1.1 Introduction

Unbalanced a.c. electric power causes severe problem in operation of static power converters. Static power converters are usually designed to work under balanced input condition. The unbalance is usually caused due to improper loading of different phases or unbalanced phase impedance. Nature of unbalance could be i) Amplitude unbalance or ii) Phase unbalance. Unbalance in static converters causes various problems [1] - [3] like harmonic neating in motors, electromagnetic interference (EMI) in communication equipments, etc.

This thesis proposes a method for balancing the output voltage when the input voltage is amplitude unbalanced. The proposed method.is applied to a three to single phase static converter.

### 1.2 Implementation of the Proposed Method

The method proposed in this thesis to balance the input amplitude unbalance uses switching function variation technique. According to the magnitude of the input unbalance, the switching function width is varied and it is inversely proportional to the amplitude of the input phase voltage. A simplified block diagram of the proposed unbalance phase converter is shown in Fig. 1.

The converter consists of six bilateral switches. The switches are synchronized with the input voltage and their opening and closing are determined by the percentage of input unbalance. The output voltage is balanced (Fig. 1).




Fig. 1 : Block diagram of the proposed balanced output static converter.

### 1.3 Review of Previous Work

There are two types of unbalances in Three-phase supply voltages. These are amplitude unbalance and phase unbalance. Amplitude unbalance arises due to the uneven distribution of loads [4] in the three-phase circuits. phase unbalance arises due to the difference in impedances (reactive part) of loads connected in the threemphase circuits. Amplitude unbalance can be eliminated by changing the width of the switching functions. Phase unbalance can be eliminated by shifting the switching functions to the desired positions.

The problems those were previously detected due to unbalance are harmonic generation in the converter, electromagnetic interference (EMI) in the communication equipments, harmonic heating in motor drives, and in military application. Smail amount of unbalances may generate harmonics in the converter due to it's non-linear characteristics. Electrómagnetic interference (EMI) in the commanication equipments (receivers) due to unbalances does not allow faithful reproduction of message. In large motors, harmonic heating due to unbalances may cause insulation breakdown or damage the motors. Electromagnetic (EM) radiation from the submarine due to unbalances is used to detect the enemy submarine. This is very important in military application. .

Not very many work are reported in unbalanced converters, although it has good prospect with the evolution of semiconductor technology. Power Electronics group at Concordia University, Montreal, [4] is doing some work in unbalanced converter. But those works are aimed at assessing the harmonics present in converters due to unbalance rather than finding solution. The Power Electronics group at Texas A\&M University is working on unbalance converter solution by using Fortescue's theorem but their work is targeted for motor problems rather than converter problems.

### 1.4 Scope of the Present Work

A new technique is used to analyse the proposed static converter. Such' technique was not used previously in the analysis. Switching functions are used to solved the amplitude unbalance. Amplitude unbalance is corrected by changing the width of the switching functions according to the unbalances. The magnitude of the fundamental component of the three switching functions is taken as inversely proportional to the amplitude of the three unbalanced input phase voltages.

At first a graphical technique has been developed to find out the output voltage and input currents. The graphs of the above mentioned quantities are plotted with the help of FORTRAN-IV program. A general FORTRAN-IV plotting program has been developed in the analysis.

Fourier analysis is used to prove the graphical voltage and currents. Fourier comefficient for m-th harmonic is used to find out the output voltage and input currents. All analysis is carried out on the basis of three phase to single phase converter consisting of six-switches. The results of two methods are compared and found satisfactory.

## ANALYSIS OF THE CONVERTER

### 2.1. Introduction

The objective of this chapter is to analyse a converter which can handle unbalanced input voltages. To achieve this objective first a phase converter is analysed under balanced input condition. From this analysis some basic criterion is established so that the input unbalance can be made balanced in the output circuit.

Converters are usually designed to handle balanced three and single phase input power. When the input is unbalanced conventional static converter may not work properly. Short circuiting, harmonic generation, unbalancing of the output power $[5]$ are the major problems associated with unbalanced input.

The input voltage could be unbalanced due to different amplitudes of the three input phases or the improper phase relation. Both amplitude and phase unbalance could be corrected. Only the amplitude unbalance is treated in this thesis.

A method for correcting the amplitude unbalance is developed in this chapter which could be applied to different phase converters. A specific case of three phase to single phase converter is studied in details.

### 2.2 Mathematical Analysis of the Converter

The transfer function of a converter (Fig. 2.1) can be written as the ratio of output to input quantity;

$$
\begin{gathered}
H_{s}(S)=\frac{V_{0}(S)}{V_{1}(S)} \text { and }\left[H_{S}(S)\right]^{T}=\frac{I_{1}(S)}{I_{0}(S)} \\
\text { or, } H_{s}\left(w_{s} t\right)=\frac{v_{0}\left(w_{0} t\right)}{v_{i}\left(w_{i} t\right)} \text { and }\left[H_{s}\left(w_{s} t\right)\right]^{T}=\frac{I_{1}\left(w_{1} t\right)}{I_{0}\left(w_{0} t\right)}
\end{gathered}
$$

Therefore, output voltage, $\left[v_{0}\left(w_{0} t\right)\right]=\left[H_{s}\left(w_{s} t\right)\right] \bullet\left[v_{i}\left(w_{i} t\right)\right] \ldots$... (2a) and input current, $\left[I_{i}\left(w_{i} t\right)\right]=\left[H_{s}\left(w_{s} t\right)\right]^{T}\left[I_{0}\left(w_{0} t\right)\right] \quad \ldots \quad . . \quad$ (2b)


Fig. 2.1: Representation of ideal converter by transfer function.

Let $\left[H_{s}\left(w_{s} t\right)\right]=A\left[\cos \left(w_{s} t\right) \cos \left(w_{s} t-120^{\circ}\right) \cos \left(w_{s} t-240^{\circ}\right)\right]$ and

$$
\left[v_{1}\left(w_{1} t\right)\right]=v_{i}\left[\begin{array}{l}
\cos \left(w_{i} t\right) \\
\cos \left(w_{i} t-120^{\circ}\right) \\
\cos \left(w_{1} t-240^{\circ}\right)
\end{array}\right] \text { be the switching function }
$$

and input voltage of the converter.

From eqn. (aa),

$$
\begin{aligned}
& {\left[v_{0}\left(w_{0} t\right)\right]=\left[H_{s}\left(w_{s} t\right)\right] \cdot\left[v_{i}\left(w_{i} t\right)\right]} \\
& =A\left[\cos \left(w_{s} t\right) \cos \left(w_{s} t-120^{\circ}\right) \cos \left(w_{s} t-240^{\circ}\right)\right] \text {. } \\
& v_{1}\left[\begin{array}{l}
\cos \left(w_{1} t\right) \\
\cos \left(w_{1} t-120^{\circ}\right) \\
\cos \left(w_{1} t-240^{\circ}\right)
\end{array}\right] \\
& =A V_{1}\left[\cos \left(w_{s} t\right) \cos \left(w_{1} t\right)+\cos \left(w_{s} t-120^{\circ}\right) \cos \left(w_{i} t-120^{\circ}\right)\right. \\
& \left.+\operatorname{Cos}\left(w_{s} t-240^{\circ}\right) \operatorname{Cos}\left(w_{1} t-240^{\circ}\right)\right] \\
& \text { or, }\left[v_{0}\left(w_{0} t\right)\right]=\frac{A V_{i}}{2}\left[\cos \left(w_{s}+w_{i}\right) t+\cos \left(w_{s}-w_{1}\right) t+\cos \left\{\left(w_{s}+w_{i}\right) t-240^{\circ}\right\}\right. \\
& \left.+\operatorname{Cos}\left(w_{s}-w_{1}\right) t+\operatorname{Cos}\left\{\left(w_{s}+w_{1}\right) t-120^{\circ}\right\}+\cos \left(w_{s}-w_{1}\right) t\right] \\
& =\frac{3 A V_{1}}{2}\left[\operatorname{Cos}\left(w_{s}-w_{1}\right) t\right]+\frac{A V_{1}}{2}\left[\cos \left(w_{s}+w_{1}\right) t+\operatorname{Cos}\left\{\left(w_{s}+w_{1}\right) t-120^{\circ}\right\}\right. \\
& \left.+\operatorname{Cos}\left\{\left(w_{s}+w_{i}\right) t-240^{\circ}\right\}\right]
\end{aligned}
$$

Since the second term of the above equation is the three phasors of equal magnitude and displaced $120^{\circ}$ apart from each other, their sum 1.e. the resultant is zero.

Therefore,

$$
\begin{align*}
{\left[v_{0}\left(w_{0} t\right)\right] } & =\frac{3 A V_{i}}{2}\left[\cos \left(w_{s}-w_{i}\right) t\right] \\
& =\frac{3 A V_{i}}{2} \cos \left(w_{0} t\right) \quad\left(\because w_{s}=w_{i}+w_{0}\right) \tag{ac}
\end{align*}
$$

$\therefore v_{0}\left(w_{0} t\right)=\frac{3 A V_{i}}{2} \cos \left(w_{0} t\right) \quad \ldots$

From eqn. (ab), input current is given by

$$
\begin{aligned}
& o r,\left[\begin{array}{l}
I_{a}\left(w_{i} t\right) \\
I_{b}\left(w_{i} t\right) \\
I_{c}\left(w_{i} t\right)
\end{array}\right]=\left[\begin{array}{l}
I_{i}\left(w_{i} t\right)
\end{array}\right]=\left[\begin{array}{l}
\operatorname{Cos}\left(w_{s} t\right) \\
\operatorname{Cos}\left(w_{s} t-120^{\circ}\right) \\
\operatorname{Cos}\left(w_{s} t-240^{\circ}\right)
\end{array}\right] \cdot\left[I_{0}\left(w_{0} t\right)\right] \\
&=A I_{0}\left[\begin{array}{l}
I_{0}\left[\cos \left(w_{0} t\right)\right] \\
\cos \left(w_{s} t-120^{\circ}\right) \cos \left(w_{0} t\right) \\
\cos \left(w_{s} t-240^{\circ}\right) \cos \left(w_{0} t\right)
\end{array}\right] \\
&=\frac{A I_{0}}{2}\left[\begin{array}{l}
\cos \left(w_{s} t\right) \cos \left(w_{0} t\right) \\
\left.\cos \left\{\left(w_{s}+w_{0}\right) t+w_{0}\right) t-120^{\circ}\right\}+\cos \left(w_{s}-w_{0}\right) t \\
\cos \left\{\left(w_{s}+w_{0}\right) t-240^{\circ}\right\}+\cos \left\{\left(w_{s}-w_{0}\right) t-240^{\circ}\right\}
\end{array}\right]
\end{aligned}
$$

$$
=\frac{A I}{2}\left[\begin{array}{l}
\cos \left(w_{s}-w_{0}\right) t  \tag{ad}\\
\cos \left\{\left(w_{s}-w_{0}\right) t-120^{\circ}\right\} \\
\cos \left\{\left(w_{s}-w_{0}\right) t-240^{\circ}\right\}
\end{array}\right]+\frac{A I}{2}\left[\begin{array}{l}
\cos \left(w_{s}+w_{0}\right) t \\
\cos \left\{\left(w_{s}+w_{0}\right) t-120^{\circ}\right\} \\
\cos \left\{\left(w_{s}+w_{0}\right) t-240^{\circ}\right\}
\end{array}\right] \ldots
$$

The second part of equation (2d) is the ard harmonic term. As we are considering the ideal case (free of harmonics), the second part will be considered as zero. Therefore,

$$
\begin{align*}
{\left[\begin{array}{l}
I_{a}\left(w_{1} t\right) \\
I_{b}\left(w_{i} t\right) \\
I_{c}\left(w_{i} t\right)
\end{array}\right] } & =\frac{A I_{0}}{2}\left[\begin{array}{l}
\cos \left(w_{s}-w_{0}\right) t \\
\cos \left\{\left(w_{s}-w_{0}\right) t-120^{\circ}\right\} \\
\cos \left\{\left(w_{s}-w_{0}\right) t-240^{\circ}\right\}
\end{array}\right] \\
& =\frac{A I_{0}}{2}\left[\begin{array}{l}
\cos \left(w_{1} t\right) \\
\cos \left(w_{1} t-120^{\circ}\right) \\
\cos \left(w_{1} t-240^{\circ}\right)
\end{array}\right] \tag{2e}
\end{align*}
$$

### 2.2.1 Three to Single Phase Converter under Unbalanced Condition

The output voltage of a three to single phase converter shown in Fig. 2.2 is given by $[6]$;

$$
\left[v_{0}\left(w_{0} t\right)\right]=\left[F\left(w_{s} t\right)\right] \cdot\left[v_{1}\left(w_{1} t\right)\right]
$$

where, $\left[v_{0}\left(w_{0} t\right)\right]=$ output voltage

$$
\begin{aligned}
& {\left[v_{i}\left(w_{i} t\right)\right]=\text { Input voltage }=\left[\begin{array}{ll}
A & \cos \left(w_{i} t\right) \\
B & \cos \left(w_{i} t-120^{\circ}\right) \\
C & \cos \left(w_{1} t-240^{\circ}\right)
\end{array}\right]} \\
& {\left[\begin{array}{ll}
F\left(w_{s} t\right)
\end{array}\right]=\text { Switching function }=\left[\begin{array}{lll}
F_{1} & F_{2} & F_{3}
\end{array}\right]}
\end{aligned}
$$

Unlike three phase balanced voltages, the amplitude of the three input voltages $V_{A n}, V_{B n}, V_{C n}$ are unequal and given by $A, B$, and $C$ respectively.

Let us assume,

$$
\begin{aligned}
& F_{1}=A_{1} \cos \left(w_{s} t\right) \\
& F_{2}=B_{1} \cos \left(w_{s} t-120^{\circ}\right) \\
& F_{3}=c_{1} \cos \left(w_{s} t-240^{\circ}\right)
\end{aligned}
$$



Fig. 2.2 : Simplified circuit diagram of the proposed 3-phase to 1-phase converter.
where $A_{1}, B_{1}$ and $C_{1}$ are the amplitudes of the fundamental components of three switching functions $F_{1}, F_{2}$ and $F_{3}$ respectively.

Therefore,

$$
\begin{aligned}
& {\left[v_{0}\left(w_{0} t\right)\right]=\left[A_{1} \cos \left(w_{s} t\right) \quad B_{1} \cos \left(w_{s} t-120^{\circ}\right) c_{1} \cos \left(w_{s} t-240^{\circ}\right)\right] \text {. }} \\
& {\left[\begin{array}{ll}
A & \cos \left(w_{1} t\right) \\
B & \cos \left(w_{1} t-120^{\circ}\right) \\
C & \cos \left(w_{1} t-240^{\circ}\right)
\end{array}\right]} \\
& =\left[A_{1} A \cos \left(w_{s} t\right) \cos \left(w_{1} t\right)+B_{1} B \cos \left(w_{s} t-120^{\circ}\right) \cos \left(w_{1} t-120^{\circ}\right)\right. \\
& \left.+C_{1} C \cos \left(w_{s} t-240^{\circ}\right) \cos \left(w_{1} t-.240^{\circ}\right)\right]
\end{aligned}
$$

Assuming $A_{1} A=1, B_{1} B=1 \quad \& C_{1} C=1$

$$
\begin{aligned}
& =1 / 2\left[\operatorname{Cos}\left(w_{s}+w_{1}\right) t+\operatorname{Cos}\left(w_{s}-w_{1}\right) t+\cos \left\{\left(w_{s}+w_{1}\right) t-240^{\circ}\right\}\right. \\
& \left.+\operatorname{Cos}\left(w_{s}-w_{1}\right) t+\operatorname{Cos}\left\{\left(w_{s}+w_{1}\right) t-120^{\circ}\right\}+\cos \left(w_{s}-w_{1}\right) t\right] \\
& \left.=\frac{3}{2} \cos \left(w_{s}-w_{1}\right) t+\right\}_{1}\left[\cos \left(w_{s}+w_{1}\right) t+\operatorname{Cos}\left\{\left(w_{s}+w_{1}\right) t-120^{\circ}\right\}+\operatorname{Cos}\left\{\left(w_{s}+w_{1}\right) t-240^{\circ}\right\}\right]
\end{aligned}
$$

Since the Second term in the above equation is zero, therefore,

$$
\begin{aligned}
{\left[v_{0}\left(w_{0} t\right)\right] } & =\frac{3}{2} \cos \left(w_{5}-w_{1}\right) t \\
& =\frac{3}{2} \cos \left(w_{0} t\right) ; \text { as } w_{0}=w_{1} \text { and } w_{5}=w_{0}+w_{1}
\end{aligned}
$$

Since $\quad A_{1} A=1 \because A_{1}=1 / A$

$$
\begin{aligned}
& B_{1} B=1 \bullet B_{1}=1 / B \\
& C_{1} C=1 \bullet C_{1}=1 / C
\end{aligned}
$$

Therefore, we can conclude that in order to get a balanced output voltage from three unbalanced input voltages, the amplitude of the fundamental components of switching functions should be inversely proportional to the amplitude of three input phase voltages respectively.

## 2. 3 Operation of the Balanced 3 to 1 Phase Converter

The operation of a 3-phise to 1-phase balanced output Static Converter can be explained with the help of Fig. 2.2. The converter output voltage is given by

$$
\begin{aligned}
{\left[v_{0}\left(w_{0} t\right)\right] } & =\left[\begin{array}{lll}
S_{d} & \left(w_{s} t\right)
\end{array}\right] \cdot\left[\begin{array}{ll}
v_{1} & \left(w_{1} t\right)
\end{array}\right] \\
& =\left[\begin{array}{lll}
F_{1} & F_{2} & F_{3}
\end{array}\right] \cdot\left[\begin{array}{c}
v_{a n} \\
v_{b n} \\
v_{c n}
\end{array}\right] \\
& =\left[F_{1} \quad * v_{a n}+F_{2} * v_{b n}+F_{3} * v_{c n}\right]
\end{aligned}
$$

$\therefore V_{0}\left(w_{0} t\right)=F_{1} * V_{a n}+F_{2} * V_{b n}+F_{3} * V_{c n}$

The switching functions can have both +1 and -1 and zero values. when
$F_{1}$ has +1 value the top switch $S_{1}$ is closed.
$F_{1}$ has -1 value the bottom switch $S_{4}$ is closed.
$F_{1}$ has 0 value both top and bottom switches $S_{1} \& S_{4}$ are open.
$F_{2}$ has +1 value the top switch $S_{3}$ is closed.
$F_{2}$ has -1 value the bottom switch $S_{6}$ is closed.
$F_{2}$ has 0 value both top \& bottom switches $S_{3} \& S_{6}$ are open.
$F_{3}$ has +1 value the top switch $S_{5}$ is closed.
$F_{3}$ has -1 value the bottom switch $S_{2}$ is closed.
$F_{3}$ has 0 value both top \& bottom switches $S_{5} \& S_{2}$ are open.

The principle of operation of the converter is explained by dividing each switching function into two switch operation as follows;

Figure 2.3 shows the input voltages and switching functions graphically. For the first $30^{\circ} F_{1}$ has a value of +1 and $F_{3}$ has a value of -1 . So $S_{1}$ and $S_{2}$ is closed. The output is the combination of $F_{1} \% V_{\text {an }}$ and $F_{3}{ }^{4} V_{\mathrm{cn}}$. For $30^{\circ}$ to $60^{\circ}$, $F_{2}$ has a value of +1 and $F_{3}$ has a value of -1 . So $S_{3}$ and $S_{2}$ is closed. The output is the combination of $F_{2}{ }^{*} V_{b n}$ and $F_{3} * V_{C n}$. For $60^{\circ}$ to $90^{\circ}, F_{1}$ has a value of -1 and $F_{2}$ has a value of +1 . So $S_{3}$ and $S_{4}$ is closed. The output is the combination of $F_{1} * V_{\text {an }}$ and $F_{2}^{*} V_{b n}$ *

For $90^{\circ}$ to $120^{\circ}, F_{1}$ has a value of -1 and $F_{3}$ has a value of +1 . So $S_{4}$ and $S_{5}$ is closed. The output is the combination of $F_{1} * V_{\text {an }}$ and $F_{3}{ }^{*} V_{c n}$. For $120^{\circ}$ to $150^{\circ}, F_{2}$ has a value of -1 and $F_{3}$ has a value of +1 . So switches $S_{5}$ and $S_{6}$ is closed. The output is the combination of $F_{2} * V_{b n}$ and $F_{3} * V_{c n^{*}}$ For $150^{\circ}$ to $180^{\circ}$, $F_{1}$ has a value of +1 and $F_{2}$ has a value of -1 . So $S_{1}$ and $S_{6}$ is closed. The output is the combination of $F_{1} * V_{\text {an }}$ and $F_{2}^{*} V_{b n}$.

After $180^{\circ}$, the operation is the same as explained earlier and the cycle is repeated.

b)

c)



Fig. 2.3 : Switching functions for balanced case.
a)Balanced input voltages.
b)-d) Switching function components.


Fig. 2.4 : Switching functions for unbalanced case.
a) Unbalanced input voltages.*
b)-d) Corresponding switching functions.

### 2.3.1 Operation of the Unbalanced 3 to 1 Phase Converter

For the unbalanced case, only the switching function widths will be different, Principle remains the same. Figure. 2.4 shows the three unbalanced input voltages and three switching functions with different widths due to unbalances in the input voltages. We can explain the operation of the unbalanced converter as follows;

The unbalanced voltages are taken as

$$
\begin{aligned}
& v_{a n}=0.97 \cos \left(w_{i} t\right) \\
& v_{b n}=0.95 \cos \left(w_{i} t-120^{\circ}\right)
\end{aligned}
$$

and $V_{c n}=0.93 \operatorname{Cos}\left(w_{1} t-240^{\circ}\right)$ i.e. unbalances are $3 \%, 5 \% \& 7 \%$ respectively. Due to this unbalance, the widths of the switching functions are $\delta_{1}$, $\delta_{2}$, and $\delta_{3}$ (eqn. 2.5) which is shown in Fig. 2.4. For the balanced inputs, the width of the switching function is $\delta=\delta_{1}=\delta_{2}=\delta_{3}$

Comparing Figs $2.3 \& 2.4$ we see that the principle of operation of the converter remains the same under balanced and unbalanced input conditions, only the widths of the switching functions become different.

### 2.4 Harmonic Analysis

Practical power converters operate in ON/OFF mode rather than in continuous mode and employ static switches. Consequently, the switch elements, i.e. switching functions described earlier are actually 'trains' of rectangular pulses of uniform or sine modulated widths and as such, they posses frequency spectra comprised of infinite series of harmonic components. The harmonics associated with switching functions, output voltage and input currents are described as follows:

The Fourier-series expansion for a function $F(\theta)$ with time period 2 Tiradians is given by

$$
F(\theta)=\frac{a_{0}}{2}+\sum_{n=1,2,3}^{\infty}\left[a_{n} \cos (n \theta)+b_{n} \sin (n \theta)\right]
$$



Fig. 2.5: Switching function $F(\theta)$ with pulse width $\delta$ and time period 2IT radians.
where,

$$
\begin{aligned}
& a_{n}=\frac{1}{\pi} \int_{-\pi}^{+\pi} F(\theta) \cos (n \theta) d \theta \\
& b_{n}=\frac{1}{\pi} \int_{-\pi}^{+\pi} F(\theta) \sin (n \theta) d \theta
\end{aligned}
$$

$a_{0}=$ Average value of the function $F(\theta)$

$$
=\frac{1}{\pi} \int_{-\pi}^{+\pi} F(\theta) d \theta
$$

For the switching function shown in Fig. 2.5, the Fourier coefficients are calculated

$$
\begin{aligned}
& \text { as follows; } \\
& a_{n}=\frac{1}{\pi} \int_{-\pi}^{+\pi} F(\theta) \cos (n \theta) d \theta \\
&=\frac{2}{\pi} \int_{0}^{\pi} F(\theta) \cos (n \theta) d \theta \\
&=\frac{2}{\pi}\left[\int_{0}^{\delta / 2} \cos (n \theta) d \theta-\int_{(\pi-\delta / 2)}^{\cos (n \theta)} d \theta\right] \\
& a_{n}=\frac{2}{n \pi}\left[(\sin n \theta)^{\delta / 2}-(\sin n \theta)_{(\pi-\delta / 2)}^{\pi}\right] \\
&=\frac{2}{n \pi}\left[\sin \left(\frac{n \delta}{2}\right)-\sin (n \pi)+\sin \left(n \pi-\frac{n \delta}{2}\right)\right] \\
&=\frac{2}{n \pi}\left[\sin \left(\frac{n \delta}{2}\right)-\sin (n \pi)+\sin (n \pi) \cos \left(\frac{n \delta}{2}\right)-\cos (n \pi) \sin \left(\frac{n \delta}{2}\right)\right] \\
&=\frac{2}{n \pi}\left[\sin \left(\frac{n \delta}{2}\right)-\cos (n \pi) \sin \left(\frac{n \delta}{2}\right)\right](\ddots \sin (n \pi)=0) \\
&=\frac{2}{n \pi}(1-\cos n \pi) \sin \left(\frac{n \delta}{2}\right) \\
&=0 ; n=\operatorname{even}=2,4,6, \ldots \ldots
\end{aligned}
$$

$\therefore a_{n}=\frac{4}{n \pi} \sin \left(\frac{n \delta}{2}\right) ;$ for. $n=$ odd $=1,3,5, \ldots$

$$
\begin{aligned}
a_{0} & =\frac{2}{\pi}\left[\int_{0}^{\delta / 2} d \theta-\int_{(\pi-\delta / 2)}^{\pi} d \theta\right. \\
& =\frac{2}{\pi}\left[\frac{\delta}{2}-\pi+\pi-\frac{\delta}{2}\right]=0
\end{aligned}
$$

$$
b_{n}=\frac{1}{\pi} \int_{-\pi}^{+\pi} F(\theta) \sin n \theta d \theta
$$

$$
\therefore b_{n}=\frac{1}{\pi}\left[\int_{-\pi}^{(-\pi+\delta / 2)}-1.0 \sin n \theta d \theta+\int_{-\delta / 2}^{+\delta / 2} \sin n \theta d \theta-\int_{(n-\delta / 2)}^{\pi} \sin n \theta d \theta\right]
$$

$$
=-\frac{1}{n \pi}\left[-\cos n(\pi-\delta / 2)+\cos (n \pi)+\cos \left(\frac{n \delta}{2}\right)-\cos \left(\frac{n \delta}{2}\right)-\cos (n \pi)+\right.
$$

$$
\cos n(\pi-\delta / 2)]
$$

$$
=-\frac{1}{n \pi}\left[-\cos n \pi \cos \frac{n \delta}{2}-\sin n \pi / \sin \frac{n \delta}{2}+\cos / n \pi+\cos \frac{n \delta}{2}-\cos \frac{n \delta}{2}\right.
$$

$$
\left.-\cos (n \pi)+\cos n \pi / \cos \frac{n \delta}{2}+\sin (n \pi) \sin \frac{n \delta}{2}\right]
$$

$$
=0
$$

Therefore, the switching function $F(\theta)$ is given by

$$
\begin{aligned}
F(\theta) & =\sum_{n=1,3,5}^{\infty} a_{n} \cos n \theta \\
& =\sum_{n=1,3,5}^{\infty} a_{n} \cos (n \omega t)
\end{aligned}
$$

and the Fourier coefficient $a_{n}$ is given by

$$
\begin{aligned}
a_{n} & =\frac{4}{n \pi} \sin \left(\frac{n \delta}{2}\right) ; \text { for } n=1,3,5, \ldots \\
\text { or, } \quad K_{m} & =\frac{4}{m \pi} \sin \left(\frac{m \delta}{2}\right) ; \text { for } m=1,3,5, \ldots
\end{aligned}
$$

$$
\begin{equation*}
\because \quad K_{m}=\frac{4}{m \pi} \sin \left(\frac{m \delta}{2}\right) \tag{2.4}
\end{equation*}
$$

From eqn (2.4) we see that $K_{m}$ is dependent on $\delta$.

$$
\begin{equation*}
\text { 1.e. } \delta=\frac{2}{m}\left[\sin ^{-1}\left(\frac{m \pi K_{m}}{4}\right)\right] \tag{2.5}
\end{equation*}
$$

2.4.1 Output Voltage Spectrum

The output voltage and input current equations (2a) and (2b) are for ideal case. In an actual converter, the switches operate in ON-OFF mode, therefore the practical switching function $\left[F_{d}\left(w_{s} t\right)\right]$ contains harmonics. Therefore, the practical expression for output voltage $\left[v_{0}\left(w_{0} t\right)\right]$, (2a) becomes

$$
\begin{aligned}
& {\left[v_{0}\left(w_{0} t\right)\right]=\left[\sum_{n=1,3,5}^{\infty} \quad A_{n}^{\prime} \cos n w_{s} t \sum_{n=1,3,5}^{\infty} B_{n}^{\prime} \cos \left\{n \quad\left(w_{s} t-120^{\circ}\right)\right\}\right.} \\
& \left.\sum_{n=1,3,5}^{\infty} c_{n}^{\prime} \cos \left\{n\left(w_{s} t-240^{\circ}\right)\right\}\right] \cdot\left[\begin{array}{l}
A_{1} \cos \left(w_{1} t\right) \\
B_{1} \cos \left(w_{1} t-120^{\circ}\right) \\
c_{1} \cos \left(w_{1} t-240^{\circ}\right)
\end{array}\right] \\
& =\sum_{n=1,3,5}^{\infty} A_{1} A_{n}^{\prime} \cos \left(n w_{s} t\right) \cos \left(w_{1} t\right)+\sum_{n=1,3,5}^{\infty} B_{1} B_{n}^{\prime} \cos \left\{n\left(w_{s} t-120^{\circ}\right)\right\} \\
& \cos \left(w_{1} t-120^{\circ}\right)+\sum_{n=1,3,5}^{\infty} c_{1} c_{n}^{\prime} \cos \left\{n\left(w_{s} t-240^{\circ}\right)\right\} \cos \left(w_{1} t-240^{\circ}\right) \\
& =3_{1}\left(A_{1} A_{1}^{\prime}+B_{1} B_{1}^{\prime}+C_{1} C_{1}^{\prime}\right) \cos \left(w_{s}-w_{1}\right) t+3 \sum_{n=3,5}^{\infty} A_{1} A_{n}^{\prime}\left[\cos \left(n w_{s}+w_{1}\right) t\right. \\
& \left.+\operatorname{Cos}\left(n w_{s}-w_{1}\right) t\right]+1_{2} \sum_{n=3,5}^{\infty} B_{1} B_{n}^{\prime}\left[\operatorname{Cos}\left\{\left(n w_{s}+w_{i}\right) t-n 120^{\circ}-120^{\circ}\right\}\right. \\
& \left.+\cos \left\{\left(n w_{s}-w_{i}\right) t-n 120^{\circ}+120^{\circ}\right\}\right]+1 / \sum_{n=3,5}^{\infty} c_{1} c_{n}^{\prime}\left[\cos \left\{\left(n w_{s}+w_{i}\right) t-n 240^{\circ}-240^{\circ}\right\}\right. \\
& \left.+\operatorname{Cos}\left\{\left(n w_{s}-w_{1}\right) t-n 240^{\circ}+240^{\circ}\right\}\right]
\end{aligned}
$$

Since $w_{s}=w_{i}+w_{0}$ and $w_{i}=w_{0}$
$\therefore n w_{s}-w_{i}=(2 n-1) w_{0}$
and $A_{1} A_{1}^{\prime}+B_{1} B_{1}^{\prime}+C_{1} C_{1}^{\prime \prime}=1+1+1=3$

$$
\begin{aligned}
{\left[v_{0}\left(w_{0} t\right)\right]=\frac{3}{2} \cos \left(w_{0} t\right) } & +\left[\frac{1}{2} \sum_{n=3,5}^{\infty} A_{1} A_{n}^{\prime} \operatorname{Cos}(2 n-1) w_{0} t+\frac{1}{2} \sum_{n=3,5}^{\infty} B_{1} B_{n}^{\prime} \operatorname{Cos}\{(2 n-1)\right. \\
& \left.\left.w_{0} t-(n-1) 120\right\}^{\prime}+\frac{1}{2} \sum_{n=3,5}^{\infty} c_{1} c_{n}^{\prime} \operatorname{Cos}\left\{(2 n-1) w_{0} t-(n-1) 240^{\circ}\right\}\right] \\
& +\left[\frac{1}{2} \sum_{n=3,5}^{\infty} A_{1} A_{n}^{\prime} \operatorname{Cos}(2 n+1) w_{0} t+\frac{1}{2} \sum_{n=3,5}^{\infty} B_{1} B_{n}^{\prime} \operatorname{Cos}\right. \\
& \left\{(2 n+1) w_{0} t-(n+1) 120^{\circ}\right\}+\frac{1}{2} \sum_{n=3,5}^{\infty} c_{1} c_{n}^{\prime} \cos \left\{(2 n+1) w_{0} t-\right. \\
& \left.\left.(n+1) 240^{\circ}\right\}\right] \quad \ldots
\end{aligned}
$$

From equation (2.6) it is clear that the output voltage of the converter is the sum of the fundamental component and the harmonic components:

### 2.4.2 Input Current Spectrum

Since the input voltages are unbalanced and switching functions are unbalanced, the input currents will also be unbalanced. The input phase currents can be calculated refering to Fig. 2.2.

The phase current $I_{a}$ for resistive (R) load is given by

$$
\begin{aligned}
I_{a} & =F_{1} * I_{A B} \\
& =F_{1} * I_{0} \cos \left(w_{0} t\right)
\end{aligned}
$$

Since $I_{o}=1.0$, therefore, we get

$$
\begin{align*}
I_{a}= & \sum_{n=1,3,5}^{\infty} A_{n}^{\prime} \cos \left(n w_{s} t\right) \times \cos \left(w_{0} t\right) \\
= & \frac{1}{2} \sum_{n=1,3,5}^{\infty} A_{n}^{\prime}\left[\cos \left(n w_{s}-w_{0}\right) t+\cos \left(n w_{s}+w_{0}\right) t\right] \\
= & \frac{A_{1}^{\prime}}{2} \cos \left(w_{1} t\right)+\frac{A_{1}^{\prime}}{2} \cos \left(3 w_{1} t\right)+\frac{1}{2} \sum_{n=3,5}^{\infty} A_{n}^{\prime}\left[\cos (2 n-1) w_{1} t+\cos (2 n+1) w_{1} t\right] \\
= & \frac{A_{1}^{\prime}}{2} \cos \left(w_{i} t\right)+\frac{A_{1}^{\prime}}{2} \cos \left(3 w_{1} t\right)+\frac{1}{2} \sum_{n=3,5}^{\infty} A_{n}^{\prime} \cos (2 n-1) w_{1} t \\
& +\frac{1}{2} \sum_{n=3,5}^{\infty} A_{n}^{\prime} \cos (2 n+1) w_{1} t \tag{2.7}
\end{align*}
$$

From equation (2.7) it is clear that for a particular value of $n$ the input current contains two harmonic components of order $(2 n-1)$ and $(2 n+1)$.

The phase current $I_{b}$ is given by

$$
\begin{aligned}
I_{b} & =F_{2^{M} \cdot I_{A B}} \\
& =F_{2^{*}} I_{0} \cos \left(w_{0} t\right)
\end{aligned}
$$

$$
\begin{aligned}
I_{b} & =F_{2^{\prime}} \operatorname{Cos}\left(w_{0} t\right) \\
& =\sum_{n=1,3,5}^{\infty} B_{n}^{\prime} \cos \left\{n\left(w_{s} t-120^{\circ}\right)\right\} * \cos \left(w_{0} t\right) \\
& =\frac{1}{2} \sum_{n=1,3,5}^{\infty} B_{n}^{\prime}\left[\cos \left\{\left(n w_{s}-w_{0}\right) t-n 120^{\circ}\right\}+\cos \left\{\left(n w_{s}+w_{0}\right) t-n 120^{\circ}\right\}\right] \\
& =\frac{1}{2} \sum_{n=1,3,5}^{\infty} B_{n}^{\prime}\left[\cos \left\{(2 n-1) w_{1} t-n 120^{\circ}\right\}+\cos \left\{(2 n+1) w_{1} t-n 120^{\circ}\right\}\right] \\
& =\frac{B_{1}^{\prime}}{2} \operatorname{Cos}\left(w_{1} t-120^{\circ}\right)+\frac{B_{1}^{\prime}}{2} \cos \left(3 w_{1} t-120^{\circ}\right) \\
& +\frac{1}{2} \sum_{n=3,5}^{\infty} B_{n}^{\prime}\left[\cos \left\{(2 n-1) w_{1} t-n 120^{\circ}\right\}+\frac{1}{2} \sum_{n=3,5}^{\infty} B_{n}^{\prime} \cos \left\{(2 n+1) w_{1} t-n 120^{\circ}\right\}\right\} \cdot(2,8)
\end{aligned}
$$

The phase current $I_{c}$ is given by

$$
\begin{align*}
& I_{c}=F_{3}{ }^{*} I_{A B} \\
& =F_{3} I_{0} \cos \left(w_{0} t\right) \\
& =F_{3}{ }^{*} \operatorname{Cos}\left(w_{0} t\right) \\
& =\sum_{n=1,3,5}^{\infty} c_{n}^{\prime} \cos \left\{n\left(w_{s} t-240^{\circ}\right)\right\} \cdots \cos \left(w_{0} t\right) \\
& =\frac{1}{2} \sum_{n=1,3,5}^{\infty} c_{n}^{\prime}\left[\cos \left\{\left(n w_{s}-w_{0}\right) t-n 240^{\circ}\right\}+\cos \left\{\left(n w_{s}+w_{0}\right) t-n 240^{\circ}\right\}\right] \\
& =\frac{1}{2} \sum_{n=1,3,5}^{\infty} c_{n}^{\prime}\left[\cos \left\{(2 n-1) w_{1} t-n 240^{\circ}\right\}+\cos \left\{(2 n+1) w_{1} t-n 240^{\circ}\right\}\right] \\
& =\frac{c_{1}^{\prime}}{2} \cos \left(w_{1} t-240^{\circ}\right)+\frac{c_{1}^{\prime}}{2} \cos \left(3 w_{1} t-240^{\circ}\right)+\frac{1}{2} \sum_{n=3,5}^{\infty} \cdot c_{n}^{\prime}\left[\operatorname { c o s } \left\{(2 n-1) w_{1} t\right.\right. \\
& \left.\left.-n 240^{\circ}\right\}\right]+\frac{1}{2} \sum_{n=3,5}^{\infty} c_{n}^{\prime}\left[\cos \left\{(2 n+1) w_{1} t-n 240^{\circ}\right\}\right] \ldots \tag{2.9}
\end{align*}
$$

From equations (2.8) and (2.9) it is observed that the input currents $I_{b}$ and $I_{c}$ contain odd harmonics only. For a particular value of $n$, there are two harmonic components of currents given by $(2 n-1)$ and $(2 n+1)$.

### 2.5 Conclusions

A three phase to single phase converter is analysed under balanced and unbalanced input conditions.

The relation between the unbalanced input voltage amplitude and the fundamental component of corresponding switching function is established. It is found that in order to make the output voltage balanced, the amplitude of the fundamental component of corresponding switching function must be inversely proportional to the amplitude of the respective input phase voltage.

The operation and analysis of the unbalanced phase converter is discussed in details. Detailed harmonic analysis of balanced output voltage and unbalanced input current is presented in this chapter.

## FOURIER ANALYSIS OF INPUT-OUTPUT VOLTAGE-CURRENT

### 3.1 Introduction

The analytical expressions developed for balancing the input amplitude unbalance in chapter 2 is thoroughly analysed in this chapter. Using this concept the proposed phase converter is studied for both balanced and unbalanced input conditions. Detailed Fourier analysis of input-output voltage and currents are performed. Comparison of spectrum of voltage and current for balanced and unbalanced condition is also done.

Dedicated computer programs in IBM-370-3278 ${ }^{2}$ using FORTRAN-IV is developed and respective results varified with analytically predicted input-output quantities. Agreement between analytical predicted and computer calculated results verify the validity of the proposed technique.

### 3.2 System Description and Mode of Operation

Fig. 3.1 shows the simplified circuit diagram of the three phase to single phase static converter. It consists of input stage, converter stage and output stage. The three phase main is assumed to be amplitude unbalanced. Switches $S_{1}$ to $S_{6}$ are ideal bilateral switches. The load could be resistive-inductive type. However, for analysis purpose only resistive load is considered.

The proposed three-phase to single-phase converter operates on direct mode of operation. The switches $S_{1}$ to $S_{6}$ are operated by the gating signals which are determined by the control scheme used.


Fig. 3.1 : Simplified circuit diagram of the proposed Three phase to Single phase static converter.

### 3.3 Switching Functions

An electrical converter contains static switches. These switches require gating signals to change the ON/OFF states. The operation of the switches depends on a particular sequence. The functions which are required for the sequential operation of the converter static switches are termed as switching functions. Switching functions may be continuous or discontinuous type.

In the design of a balanced output three phase to single phase static converter, the switching functions are of ON/OFF type. They operate on logic levels 1 and 0 . The switching pattern for a single switch assumes unit value whenever the switch is closed and a zero value whenever the switch is opened. In a converter, each switch is closed and opened according to a predetermined repetitive pattern; hence, its siwtching function will take the form of a train of pulses of unit amplitude. Neither the pulses nor the intervening zero value periods have necessarily the same duration. The requirement that a repetitive switching pattern must exist means that the function must at least consist of repetitive groups of pulses. The simplest or unmodulated switching functions have pulses of the same time duration and zero intervals with the same property. A more complex type with different pulse durations and various zero intervals, is termed as a pulse width modulated (PWM) switching function.

There are various types of modulation techniques [7] - [10] - These are single-pulse modulation, multi-pulse modulation, simusoidal pulse modulation; sinusoidal pulse-width modulation, delta modulation, etc. The modulation technique used in this thesis is described in sub-section 3.3.1.

In single-pulse modulation, there is only one pulse per half-cycle. The pulse width is varied to control the converter output voltage. The switching function obtained from single-pulse modulation is shown in Fig. 3.2(a). For the purpose of analysis, it is assumed that the start of each pulse is delayed and the end of each pulse is advanced by equal angular intervals, resulting in a variation of the pulse-width $\delta$ over the range $0 \leqslant \delta \leqslant \Pi$ radian.

The wave form of $F(\theta)$ in Fig. 3.2(a) may be described by the Fourler-series

$$
F(\theta)=\frac{a_{0}}{2}+\sum_{n=1,3,5}^{\infty}\left[a_{n} \cos (n \theta)+b_{n} \sin (n \theta)\right]
$$

where,

$$
\begin{aligned}
b_{n} & =\frac{2}{\pi} \int_{0}^{\pi} F(\theta) \sin (n \theta) d \theta \\
& =\frac{4}{n \pi} \quad \sin \left(\frac{n \delta}{2}\right), n \text { is odd }
\end{aligned}
$$

and $a_{0}=0, a_{n}=0$
and the corresponding harmonics 1.e. $b_{n}$ values are shown in Fig. 3.2(b) for $\delta=120^{\circ}$. Width of the pulse is determined by the unbalances present in the input phase voltages.


Fig. 3.2 : Single pulse modulation switching function.
a) The switching function.
b) Respective frequency spectrum.

## 3.4

 Mode of Operation (Balanced Case)For balanced case, the switching functions are of equal widths. The Fourier coefficients of the switching functions for this case are equal. The output voltage for balanced case is given by

$$
\begin{aligned}
{\left[v_{0}\left(w_{0} t\right)\right]=} & {\left[\sum_{n=1,3,5}^{\infty} A_{n} \cos \left(n w_{s} t\right) \sum_{n=1,3,5}^{\infty} B_{n} \cos \left\{n\left(w_{s} t-120^{\circ}\right)\right\}\right.} \\
& \left.\sum_{n=1,3,5}^{\infty} c_{n} \cos \left\{n\left(w_{s} t-240^{\circ}\right)\right\}\right]
\end{aligned}
$$

$$
\left[\begin{array}{l}
\cos \left(w_{i} t\right) \\
\cos \left(w_{1} t-120^{\circ}\right) \\
\cos \left(w_{i} t-240^{\circ}\right)
\end{array}\right]
$$

Since $A_{n}=B_{n}=C_{n}$ for balanced case, therefore

$$
\begin{aligned}
& {\left[v_{0}\left(w_{0} t\right)\right]=\frac{3}{2} A_{1} \cos \left(w_{0} t\right)+\frac{1}{2} \sum_{n=3,5}^{\infty} A_{n}\left[\cos (2 n-1) w_{0} t+\cos \left\{(2 n-1) w_{0} t\right.\right.} \\
& \\
& \quad-\frac{1}{2} \sum_{n=3,5}^{\infty} A_{n}\left[\cos (2 n+1) w_{0} t+\cos \left\{(2 n+1) 0_{0}^{\circ}\right\}+\cos \left\{(2 n-1) w_{0} t-(n-1) 240^{\circ}\right\}\right] \\
& \\
& \left.\left.-(n+1) 240^{\circ}\right\}\right]
\end{aligned}
$$

Equation (3.1) shows that under balanced input condition, the output voltage spectrum contains a fundamental component of amplitude $1.5 \mathrm{~A}_{1}$ and harmonic components whose frequency is determined by $\left(2 n_{ \pm} 1\right) w_{0}$ term. The amplitude of harmonic component is equal to $A_{n} / 2$.
a)
b)

c)

(1) Input phase voltages have been taken as 1 p.u. volt and $100 \%$ volt.

Table (3.1) shows the output voltage spectrum for balanced input conditions.
The input current equation for the practical 3-phase to 1-phase converter is expressed as follows:

$$
\begin{aligned}
& I_{a}=\frac{1}{2} \sum_{n=1,3,5}^{\infty} A_{n}\left[\cos (2 n-1) w_{i} t+\cos (2 n+1) w_{i} t\right] \\
& I_{b}=\frac{1}{2} \sum_{n=1,3,5}^{\infty} A_{n}\left[\cos \left\{(2 n-1) w_{i} t-n 120^{\circ}\right\}+\cos \left\{(2 n+1) w_{i} t-n 120^{\circ}\right\}\right] \ldots \text { (3.3) } \\
& I_{c}=\frac{1}{2} \sum_{n=1,3,5}^{\infty} A_{n}\left[\cos \left\{(2 n-1) w_{1} t-n 240^{\circ}\right\}+\cos \left\{(2 n+1) w_{1} t-n 240^{\circ}\right\}\right] \ldots \text { (3.4) }
\end{aligned}
$$

Equations (3.2), (3.3) and (3.4) show that the input current spectrum for balanced input conditions contains harmonics at frequencies of $(2 n \pm 1) w_{i}$. The amplitude of $I_{a}, I_{b}$ and $I_{c}$ are equal and is given by $A_{n} / 2$. So the input current spreatra of $I_{a}, I_{b}$ and $I_{c}$ are same and is given in Table (3.2).

b)

c)

d)

e)

1
.5
.
-.5
-1
f)



g)


Fig. 3.4: Input current waveforms for balanced case.
a) Output current $I_{A B}$ for balanced inputs.
b) $-\mathrm{d}_{1} \mathrm{~F}_{1}, \mathrm{~F}_{2}, \mathrm{~F}_{3}$ Switching function components.
e) -g ) Three input phase currents.

TABLE 3.2

FREQUENCY SPECTRA OF WAVEFORMS ASSOCIATED WITH
CONVERTER INPUT CURRENTS $I_{a}$ or $I_{b}$ or $I_{c}$ SHOWN IN FIG. 3.4

(1)Output. phase current has been taken as 1.p.u. current and $100 \%$ current.

### 3.4.1 Mode of Operation (Unbalanced Case)

Output voltage is constructed by direct multiplication of input voltages by the respective converter transfer function. The practical equation for the output voltage of a 3-phase to 1-phase converter which is derived in 2.4.1, chapter 2, is once again described here to maintain continuity.

$$
\begin{align*}
& {\left[v_{0}\left(w_{0} t\right)\right]=\frac{1}{2} \sum_{n=1,3,5}^{\infty} A_{n}^{\prime} A_{1} \cdot\left[\cos \left(n w_{s}+w_{1}\right) t+\cos \left(n w_{s}-w_{1}\right) t\right]} \\
& +\frac{1}{2} \sum_{n=1,3,5}^{\infty} B_{n}^{\prime} B_{1}\left[\cos \left\{\left(n w_{s}+w_{1}\right) t-n 120^{\circ}-120^{\circ}\right\}+\cos \left\{\left(n w_{s}-w_{1}\right) t-n 128+120^{\circ}\right\}\right] \\
& +\frac{1}{2} \sum_{n=1,3,5}^{\infty} c_{n}^{\prime} c_{1}\left[\cos \left\{\left(n w_{s}+w_{1}\right) t-n 240^{\circ}-240^{\circ}\right\}+\cos \left\{\left(n w_{s}-w_{1}\right) t-n 240^{\circ}+240^{\circ}\right\}\right] \\
& =\frac{3}{2} \cos \left(w_{0} t\right)+\left[\frac{1}{2} \sum_{n=3,5}^{\infty} A_{n}^{\prime} A_{1} \operatorname{Cos}(2 n-1) w_{0} t+\frac{1}{2} \sum_{n=3,5}^{\infty} B_{n} B_{1} \operatorname{Cos}\left\{(2 n-1) w_{0} t\right.\right. \\
& \left.\left.-(n-1) 120^{\circ}\right\}+\frac{1}{2} \sum_{n=3,5}^{\infty} c_{n}^{\prime} c_{1} \cos \left\{(2 n-1) w_{o} t-(n-1) 240^{\circ}\right\}\right] \\
& +\left[\frac{1}{2} \sum_{n=3,5}^{\infty} A_{n}^{\prime} \dot{A}_{1} \cos (2 n+1) w_{0} t+\frac{1}{2} \sum_{n=3,5}^{\infty} B_{n}^{\prime} B_{1} \cos \left\{(2 n+1) w_{0} t-(n+1) 120^{\circ}\right\}\right. \\
& \left.+\frac{1}{2} \sum_{n=3,5}^{\infty} c_{n}^{\prime} c_{1} \cos \left\{(2 n+1) w_{0} t-(n+1) 240^{\circ}\right\}\right] \cdots \tag{3.5}
\end{align*}
$$

Equation (3.5) shows that output voltage spectrum contains a fundamental component of amplitude 1.5. and harmonic component whose frequency is determined by ( $2 \mathrm{n} \pm 1$ ) wo term.
a)

b)

c)
d)
e)



Fig. 3.5 : Output voltage waveform obtained with three to single phase converter.
a) Three input unbalanced phase voltages.
b) $-d) F_{1}, F_{2}, F_{3}$ switching function components.
e) Resulting output voltage.

## TABLE 3.3

FREQUENCY SPECTRA OF WAVEFOLMS ASSOCIATED WITH CONVERTER OUTPUT VOLTAGE SHOWN IN FIG. 3.5

(1) Input phase voltages have been taken as 1 p.u. volt and $100 \%$ volt.

Table 3.3 shows the output voltage spectrum from where we can compare the theoritical and analytical results for different harmonics.

The input current equation for this practical 3-phase to 1-phase converter is expressed as follows;

$$
\begin{align*}
& I_{a}=\frac{1}{2} \sum_{n=1,3,5}^{\infty} A_{n}^{\prime}\left[\cos \left(n w_{s}-w_{0}\right) t+\cos \left(n_{w_{s}}+w_{0}\right) t\right] \\
& =\frac{A_{1}^{\prime}}{2} \cos \left(w_{1} t\right)+\frac{A_{1}^{\prime}}{2} \cos \left(3 w_{1} t\right)+\frac{1}{2} \sum_{n=3,5}^{\infty} A_{n}^{\prime}\left[\cos \left\{(2 n-1) w_{i} t\right\}\right. \\
& -\cos \left\{(2 n+1) w_{1} t\right\}  \tag{3.6}\\
& I_{b}=\frac{1}{2} \sum_{n=1,3,5}^{\infty} B_{n}^{\prime}\left[\cos \left\{n\left(w_{s} t-120^{\circ}\right)\right\} \div \cos \left(w_{0} t\right)\right] \\
& =\frac{1}{2} \sum_{n=1,3,5}^{\infty} B_{n}^{\prime}\left[\cos \left\{(2 n-1) w_{i} t-n 120^{\circ}\right\}+\cos \left\{(2 n+1) w_{i} t-n 120^{\circ}\right\}\right] \ldots(3.7) \\
& I_{c}=\frac{1}{2} \sum_{n=1,3,5}^{\infty} c_{n}^{\prime}\left[\cos \left\{(2 n-1) w_{i} t-n 240^{\circ}\right\}+\cos \left\{(2 n+1) w_{i} t-n 240^{\circ}\right\}\right] \quad .(3.8)
\end{align*}
$$

From equations (3.6), (3.7) and (3.8) we see that the input current spectrum contains harmonics at frequencies of $(2 n \pm 1) w_{i}$. The input current spectra of $I_{a}, I_{b}$ and $I_{c}$ are given in table $3.4,3.5 \& 3.6$ respectively from where we can compare the theoritical \& Analytical results.


Fig. 3.6 : Input current waveform obtained with three to single phase converter.
a) Output current, $I_{A B}$.
b) $\quad F_{1}$ switching function component.
c) Resulting Input current, $I_{a}$ -

| TABLE 3.4 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FREQUENCY SPECTRA OF WAVEFORMS ASSOCIATED WITH CONVERTER INPUT CURRENT I I SHOWN IN FIG. 3.6 |  |  |  |  |
| Harmonic coefficients of switching function(Fig. 3.6b) |  | Harmonic coefficients of resulting input phase current $I_{a}$ (Fig. $3.6^{C}$ ) |  |  |
| Order <br> ( $n$ ) | $\begin{aligned} & \text { Amplitude } \\ & \left(A_{n}\right) \end{aligned}$ | Amplitude, $\mathrm{I}_{\mathrm{a}}$ |  |  |
|  |  | order <br> ( $n$ ) | $\text { poi. }{ }^{(1)}$ | $\%^{(1)}$ |
| 1 | 1.03 | 1 | 0.52 | 52 |
| 3 | 0.13 | 3 | $0.52^{\prime}$ | 52 |
| 5 | 0.25 | 5 | 0.07 | 7 |
| 7 | 0.05 | 7 | 0.07 | 7 |
| 9 | 0.11 | 9 | 0.13 | 13 |
| 11 | 0.10 | 11 | 0.13 | 13 |
| 13 | 0.03 | 13 | 0.03 | 3 |
| 15 | 0.08 | 15 | 0.03 | 3 |
| 17 | 0.02 | 17 | 0.06 | 6 |
| 19 | 0.05 | 19 | 0.06 | 6 |
| 21 | 0.05 | 21 | 0.05 | 5 |
| 23 | 0.02 | 23 | 0.05 | 5 |
|  |  | 25 | 0.02 | 2 |
|  |  | 27 | 0.02 | 2 |

(1) Output phase current has been taken as 1 p.u. current and $100 \%$ current.
a)

b)

c)


Fig. 3.7 : Input current waveform obtained with three to single phase converter.
a) Output current; $I_{A B}$.
b) $\quad F_{2}$ switching function component.
c) Resulting input current, $I_{b}$ -

| TABLE 3.5 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FREQUENCY SPECTRA OF WAVEFORMS ASSOCIATED WITH CONVERTER INPUT CURRENT $I_{b}$ SHOWN IN FIG. 3.7 |  |  |  |  |
| Harmonic coefficient of switching function (Fig. 3.7.b) |  | Harmonic coefficients of resulting input phase current $I_{b}$, (Fig. 3.7c) |  |  |
| Order <br> ( n ) | Amplitude$\left(B_{n}\right)$ | Amplitude, $\mathrm{I}_{\mathrm{b}}$ |  | $\%^{(1)}$ |
|  |  | order <br> (n) | (1) |  |
| 1 | 1.06 | 1 | 0.53 | 53 |
| 3 | 0.09 | 3 | 0.53 | 53 |
| 5 | 0.25 | 5 | 0.05 | 5 |
| 7 | 0.10 | 7 | 0.05 | 5 |
| 9 | 0.08 | 9 | 0.13 | 13 |
| 11 | 0.11 | 11 | 0.13 | 13 |
| 13 | 0.01 | 13 | 0.05 | 5 |
| 15 | 0.07 | 15 | 0.05 | 5 |
| 17 | 0.06 | 17 | 0.04 | 4 |
| 19 | 0.02 | 19 | 0.04 | 4 |
| 21 | 0.06 | 21 | 0.06 | 6 |
| 23 | 0.03 | 23 | 0.06 | 6 |
|  |  | 25 | 0.01 | 1 |
|  |  | 27 | 0.01 | 1 |

(1) output phase current has been taken as 1 p.u. current and $100 \%$ current.
a)

b)



Fig. 3.8 : Input current waveform obtained with three phase to single phase converters.
a) Output current, $I_{A B}$.
b) $\quad F_{3}$ switching function component.
c) Resulting input current, $I_{c}$ -

| TABLE 3.6 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FREQUENCY SPECTRA OF WAVEFORMS ASSOCIATED WITH CONVERTER INPUT CURRENT $I_{c}$ SHOWN IN FIG. 3.8 |  |  |  |  |
| Harmonic coefficients of switching function <br> (Fig. 3.8b) |  | Harmonic coefficients of resulting input phase current $I_{c}$ (Fig. 3.8c) |  |  |
| Order <br> ( n ) | $\begin{aligned} & \text { Amplitude } \\ & \left(c_{n}\right) \end{aligned}$ | Amplitude, $\mathrm{I}_{C}$  <br> order  |  | $\chi^{(1)}$ |
|  |  | order <br> ( $n$ ) | $\begin{aligned} & \text { (1) } \\ & \text { p.u. } \end{aligned}$ |  |
| 1 | 1.08 | 1 | 0.54 | 54 |
| 3 | 0.04 | 3 | 0.54 | 54 |
| 5 | 0.24 | 5 | 0.02 | 2 |
| 7 | 0.13 | 7 | 0.02 | 2 |
| 9 | 0.04 | 9 | 0.12 | 12 |
| 11 | 0.12 | 11 | 0.12 | 12 |
| 13 | 0.05 | 13 | 0.06 | 6 |
| 15 | 0.04 | 15 | 0.06 | 6 |
| 17 | 0.08 | 17 | 0.02 | 2 |
| 19 | 0.03 | 19 | 0.02 | 2 |
| 21 | 0.04 | 21 | 0.06 | 6 |
| 23 | 0.05 | 23 | 0.06 | 6 |
|  | . | 25 | 0.03 | 3 |
|  |  | 27 | 0.03 | 3 |

(1) Output phase current has been taken as 1 p.u. current and $100 \%$ current.

The Fourier coefficients for balanced and unbalanced cases are different due to different widths of the switching functions. Therefore, the spectrum for the two cases will also be different. The comparison of spectrum for balanced and unbalanced phase converter is explained in the following sections.

### 3.5.1 Output Voltage Spectrum

Tables 3.1 and 3.3 show the output voltage spectrum of balanced and unbalanced case respectively. For the balanced case first harmonics appears as 11th harmonics and its magnitude is $20 \%$ of the fundamental. As expected in unbalanced case, the output voltage contains 5 th, 7 th and 9 th harmonics but their magnitudes are insignificant i.e. only 2,2 and 1 percent respectively. These harmonics come from the 2nd and 3 rd terms of eqn (3.5). For example, 5 th harmonics is the sum of $\frac{1}{2} A_{3}^{\prime} A_{1}$. $\frac{1}{2} B_{3}^{\prime} B_{1}$ (with $2 \times 120^{\circ}$ phase delay), and $\frac{1}{2} C_{3}^{\prime} C_{1}$ (with $2 \times 240^{\circ}$ phase delay) terms. The 7 th harmonics is the sum of $\frac{1}{2} A_{3}^{\prime} A_{1}, \frac{1}{2} B_{3}^{\prime} B_{1}$ (with $4 \times 120^{\circ}$ phase delay), $\frac{1}{2} \kappa_{3}^{\prime} C_{1}$ (with $4 \times 240^{\circ}$ phase delay) terms. Similarly, the 9 th harmonics can be explained as above. However these harmonics can easily be filtered out by appropriate filter.

### 3.5.2 Input Currents Spectra

The input current spectra for balanced and unbalanced cases are depicted in Tables $3.2,3.4,3.5$ and 3.6 respectively. In Table 3.2 the fundamental and 3rd harmonics have the same amplitude. This phenomena is expected, as shown in eqn(3.6). It also contains 9 th, 11 th, 13 th and 15 th of amplitude $11,11,8$ and 8 percent respectively. As it is a balanced case, all the three input currents have the same spectra. For unbalanced case, the input currents are expected to be unbalanced as the input voltages are unbalanced. This is reflected in Tables 3.4, 3.5 and 3.6.

From table 3.4, we see that the 9 th, 11 th, 13 th and 15 th harmonics of input current $I_{a}$ are $13,13,3$ and 3 percent respectively. Table 3.5 shows that they are 13, 13, 5 and 5 percent respectively for input current $I_{b}$. For phase current $I_{c}$, they are $12,12,6$ and 6 percent respectively. Therefore, the input currents spectra are different due to the unbalanced input voltages. The magnitude of harmonics are dependent on the amount of amplitude unbalance present in the corresponding input phase voltages.

### 3.6 Recuction of Higher Harmonics

Output voltage of the converter contains higher harmonics. These harmonics must be eliminated in order to retrieve the fundamental frequency component. The converter will supply harmonic (Table 3.4) components to the supply (source) due to its non-linear characteristics. These harmonics will disturbe other consumers connected to the same source. Particular modulation technique may be employed to eliminate the higher harmonics. Only the modulation technique is not sufficient to eliminate the harmonics completely. So low-pass filter circuits are to be added prior to and after the sampler.

The distortion factor ( $D F$ ) of a fllter indicates the amount of harmonic distortion that remains in a waveform after the harmonics of that waveform have been subjected to a secondmrder attenuation. Thus DF is a measure of effectiveness in reducing unwanted harmonics and is defined as

$$
\begin{equation*}
\left.\left.D F=\frac{1}{v_{1}}\left[\sum_{n=2,3, \ldots}^{n^{2}}\right)^{\frac{v}{n}}\right]^{2}\right]^{1 / 2} \tag{3.9}
\end{equation*}
$$

where $V_{1}$ is the rms value of the fundamental component and $V_{n}$ is the rms value of the $n$-th harmonic component.

The distortion factor of an individual (or $n$-th) harmonic component is defined as

$$
\begin{equation*}
D F_{n}=\frac{v_{n}}{V_{1} n^{2}} \tag{3.10}
\end{equation*}
$$

From Table 3.3 , we see that fundamental component of output voltage is 0.87 p.u. whereas, $3 \mathrm{rd}, 5 \mathrm{th}, 7 \mathrm{th}, 9 \mathrm{th}, 11 \mathrm{th}$, and 13 th harmonics are $0.00,0.02,0.02,0.01$, 0.20 , and 0.08 p.u. respectively.

The total distortion factor is given by eqn. (3.9);

$$
\begin{aligned}
& \mathrm{DF}=\frac{1}{\mathrm{v}_{1}}\left[\left(\frac{\mathrm{v}_{3}}{3^{2}}\right)^{2}+\left(\frac{\mathrm{v}_{5}}{5^{2}}\right)^{2}+\left(\frac{\mathrm{v}_{7}}{7^{2}}\right)^{2}+\left(\frac{\mathrm{v}_{9}}{9^{2}}\right)^{2}\right. \\
& \left.+\left(\frac{v_{11}}{11^{2}}\right)^{2}+\left(\frac{v_{13}}{13^{2}}\right)^{2}\right]^{1 / 2} \\
& =\frac{1}{0.87}\left[0+\left(\frac{0.02}{25}\right)^{2}+\left(\frac{0.02}{49}\right)^{2}+\left(\frac{0.01}{81}\right)^{2}+\left(\frac{0.20}{121}\right)^{2}+\left(\frac{0.08}{169}\right)^{2}\right]^{1 / 2} \\
& =\frac{1}{0.87}\left[3.78 \times 10^{-6}\right]^{1 / 2} \\
& =2.234 \times 10^{-3} \\
& =0.2234 \%
\end{aligned}
$$

The distortion factor of 11 th harmonic is

$$
\mathrm{DF} 11=\frac{\mathrm{v}_{11}}{(11)^{2} v_{1}}=\frac{\mathrm{v}_{11}}{121 v_{1}}=\frac{0.20}{121 \times 0.87}=0.19 \%
$$

From Table 3.3, we see that the 11th harmonic is $20 \%$ of input phase voltage and is dominant one. Other harmonics are negligible. So a low-pass TT-section filter may be used before the load.


Fig. 3.9 : Low-pass $\Pi$-Section Filter.

The cut-off frequency, $f_{c}=\frac{1}{T \sqrt{L C}}=11 \times 50 \mathrm{c} / \mathrm{S} . \quad, \ldots$

Assuming load resistance, $R_{L}=1000$ ohms
$\therefore$ load resistance $R_{L}=\sqrt{\frac{L}{C}}=1000$ ohms.
or, $\frac{L}{C}=10^{6}$
or,
$L=C \times 10^{6}$

Putting the value of $L$ in eqn (3.11) we get,

$$
\begin{aligned}
& \frac{1}{\pi \sqrt{C \times 10^{6} \times C}}=11 \times 50=550 \\
& \text { or, } \frac{1}{\Pi C 10^{3}}=550 \\
& \text { or, } c=\frac{1}{\pi \times 10^{3} \times 550}=5.78 \times 10^{-7} \mathrm{~F} \\
&=0.58 \mu \mathrm{~F} \\
& L=c \times 10^{6}=0.58 \times 10^{-6} \times 10^{6} \mathrm{H} \\
& \therefore \quad L=0.58 \mathrm{H} \\
& \therefore \quad C=0.58 \mu \mathrm{~F}
\end{aligned}
$$

New from Table 3.4, we see that input current $I_{a}$ contains 3 rd harmonics 0.52 p.u. of output current and this is dominant. We will require a Low-pass $\Pi$ - section filter before the converter to cut-off this 3rd harmonic term.

Here, cut-off frequency, $f_{c}=\frac{1}{T T \sqrt{L C}}=3 \times 50=150 \mathrm{c} / \mathrm{s} \quad \ldots$
Now load resistance $=\sqrt{\frac{L}{C}}=1000$ ohms. $\quad$..

From eqn. (3.13) we get,

$$
\begin{aligned}
& \frac{1}{\pi \sqrt{C \times 10^{6} \times c}}=150 \\
& \text { or; } C=2.12 \mu F \\
& L=C \times 10^{6}=2.12 \times 10^{-6} \times 10^{6} \mathrm{H} \\
& =2.12 \mathrm{H} \\
& \therefore L=2.12 \mathrm{H} \\
& \text { and } C=2.12 \mu F
\end{aligned}
$$

3.7 Conclusions

The analytical expressions developed in the earlier chapter is utilized to study the phase converter under balanced and unbalanced conditions. The proposed tecinique produces balanced output voltage. The input current unbalance is dependent on input voltage unbalance. By choosing appropriate switching function the harmonic content of the output voltage can be recuced. Detailed input current, output voltage harmonic analysis has shown that the proposed technique produces balanced output voltage with effetive suppression of lower order harmonics.

## DESIGN OF THE CONVERTER

This chapter focuses on the design aspects of logic-control circuit of the unbalanced phase converter. These aspects include, the derivation of the appropriate switching function, the processing of the gating signals from their respective functions and the development of the circuitry required to implement these. functions and signal processing.

As the input voltages are unbalanced, the principle and process of deriving the proper gating signals is much more ecomplex[11]-[12] than deriving respective signals for a balanced phase converter. The undalance present suggests that the implementation and performance of such phase converter depends to a large extent on their respective logic control boards. Slight mismatch of the gating signals will result in short-circuiting and blow-up of switches.

Finally, a complete design data for the converter is provided.

### 4.2 Control Logic Design

The design of control circuit includes the derivation of the appropriate switching functions, the processing of the gating signals from their respective functions and finally the development of the circciitry required to implement the above functions and signal processing.

To implement the schemes proper relationship between input voltages and gating signals is required. Such a gating signal relationship with balanced input for the phase converter (Fig. 4.1) is shown in Fig. 4.2. This gating signals can be realized by using digital components.


Fig. 4.1: Simplified circuit diagram of the proposed 3-phase to 1-phase converter in full-bridge configuration.



Fig. 4.2 : Six gating signals relationship with balanced input voltages for the converter.


Fig. 4.3 : Logic circuit block diagram for the converter.

A delta-wye step down transformer (Fig. 4.3) is used for input line voltage sensing. The output of this transformer provides the three zero crossing points for the three input line voltages. The zero-cross sensing is implemented by employing three properly biased voltage comparators.

The six gating signals $g_{1}-g_{6}$ (Fig. 4.2 ) are then applied to the gates of the six switches $S_{1}-S_{6}$ in synchronization with zero crossing signals. The gating signals for balanced and unbalanced case is depicted in Fig. 4.2 and 4.4 respectively.

The derivation of unbalanced gating signals are described as follows:

Let,

$$
\begin{aligned}
& v_{a b}=0.97 \sin w t \\
& v_{b c}=0.95 \sin \left(w t-120^{\circ}\right) \\
& v_{c a}=0.93 \sin \left(w t-240^{\circ}\right)
\end{aligned}
$$

i.e. the unbalances are $3 \%, 5 \%$ and $7 \%$.

The widths of the gating signal will be changed according to the amplitude of the unbalanced line voltages. The width of the gating signals are calculated as follows;

For signal $g_{1} \& g_{4}$, the width $=90^{\circ} \times 0.97=87.3^{\circ}$
For signal $g_{3} \& g_{6}$, the width $=90^{\circ} \times 0.95=85.5^{\circ}$
For signal $g_{5} \& \dot{g}_{2}$, the width $=90^{\circ} \times 0.93=83.7^{\circ}$

The relationship between the gating signals and the unbalanced line voltages are shown in Fig. 4.4.

Therefore, the width of the gating. signals changes according to the unbalances present in the input line voltages.


Fig. 4.4 : Six gating signals relationship with unbalanced input voltages for the converter.


Fig. 4.5 : Microprocessor based control circuitry.


If the amplitude of the unbalanced input voltages are previously known, then the corresponding gating signals can be stored in EPROM and can be synchronized with the zero crossings. A computer program for loading (burning) EPROM for known amplitude unbalance is written in BASIC language.

When the amplitude of the input unbalance voltages change with time i.e. varying continuously, then a dedicated microprocessor can be used. Various combinations of gating signals for corresponding amplitude unbalance can be stored (Fig. 4.5) in a lookup table. According to the unbalance present the corresponding gating signals will be applied to the switches to balance the output (Fig. 4.5).

### 4.3 Component Ratings

To select the ratings of the switches, the worst case condition (Fig. 4.2) is considered. The worst case condition is $120^{\circ}$ conduction and for this case the peak, rms and average switch currents $I_{s p}, I_{s r}$ and $I_{\text {save }}$ are given by $[13]$;

$$
\begin{array}{ll}
I_{s p}=(\sqrt{3}) *(\sqrt{2}) & \text { p.u॰ Amps. } \\
I_{s r}=\sqrt{3} / \sqrt{2} & \text { p.u. Amps. } \\
I_{s a v}=\sqrt{6} / \pi & \text { p.u. Amps. }
\end{array}
$$

with these ratings a simple design is provided here for the phase converter.

Considering the design of 30 KVA three phase to single phase converter (Fig. 4.1), it is assumed that ;

$$
\begin{aligned}
\text { Nominal input ac voltage } V_{a n} & =220 \text { volts (rms) } \\
\text { 1.p.u. volt } & =220 \text { volts (rms) }
\end{aligned}
$$

and the fundamental component of output (Table 3.1) voltage

$$
V_{A B}, 1=220 * 0.87=191.4 \text { volts (rms) }
$$

and the load current, $I_{A B}$ (Fig. 4.1) is given by

$$
I_{A B}{ }^{\prime} 1=30,000 /(3 * 191.4)=52.25 \mathrm{Amps}(\mathrm{rms})
$$

By using computed per unit voltage and current values, the actual converter switch voltage and current ratings (without safety margin) can be computed as follows:

$$
\begin{aligned}
\text { Peak switch voltage } & =220 * \sqrt{2}=311 \text { volts. } \\
\text { Peak switch current } & =52.25 *(\sqrt{6}) \text { p.u. } \\
& =128 \text { Amps. } \\
\text { Average switch currente } & 52.25 *(\sqrt{6} / \pi) \\
& =40.74 \text { Amps. } \\
\text { RMS switch current } & =52.25 *(\sqrt{3} / \sqrt{2}) \\
& =64 \text { Amps. }
\end{aligned}
$$



Fig. 4.6 : The converter circuit showing protective elements.

### 4.4 Component protection

It is very difficult to provide effective protection for the converter shown in Fig. 4.1 as the load current commutation must be done without freewheeling diode. Referring to [14] a snubber circuit (Fig. 4.6) which can provide adequate protection for the switches is discussed below :

1) When the switch is turned $O N$ from the OFF condition, a current transient takes place. Reactors $L_{s}-1-2-3$ facilitate current transient (i.e. commutation) from a turning off to a turning on switch. .
2) The snubber circuit consists of a full-wave diode rectifier circuit. During normal or accidental switching transient, the snubber rectifier diverts input currents to storage element $\mathrm{C}_{\mathrm{s} 1}$.
3) Over-voltages occur during transient. Snubber capacitor $C_{s 1}$ limits resulting over-voltages during transients.
4) Once the storage element $C_{s 1}$ is fully charged, it mast be discharged for further application. The charge stored in $C_{s 1}$ is discharged through the energy "bleeding" resistor $\mathrm{R}_{\mathrm{s} 2}{ }^{\circ}$
5) During transient, a series L-R-C circuit is formed by the smuber circuit components. Resistor $\mathrm{R}_{\mathrm{s} 1}$ provides critical damping for $\mathrm{L}-\mathrm{R}-\mathrm{C}$ series path.

### 4.5 Conclusions

A complete design for the proposed unbalanced phase converter is provided in this chapter. Design for both fixed and variable amplitude unbalance is discussed. Using these design data and control logic circuitry a laboratory prototype can easily be built.

## SUMMARY, CONCLUSIONS AND RECOMMÉNDATIONS

A three to single phase converter is investigated under balanced and unbalanced supply conditions. A complete analytical description and relevant design data are provided. The analysis is carried out for amplitude unbalance and the load is considered as resistive. In particular the contributions of this thesis by chapter are as follows:

In chapter 2 , a converter which can handle unbalanced input voltages is analysed. To make the output balanced, the converter is first analysed under balanced input condition and from this analysis some basic principle is established for unbalanced input condition.

The principle states that inorder to generate balanced output voltage (and current) from unbalanced (amplitude) input voltages, the magnitudes of fundamental component of the switching functions are inversely proportional to the amplitude of the corresponding unbalanced input phase voltage. Operational feature of the converters is described under both balanced and unbalanced input conditions. Principle of operation remains the same for both conditions; only width of the switching functions varies.

The dependency of the width of switching function on the Fourier-coefficient is established by harmonic analysis. The output voltage and input current spectra are described in this chapter. These spectra show that output voltage and input currents contain fundamental of 0.87 and 0.55 respectively and the harmonics content is not very high.

In chapter 3, complete Fourier analysis for balanced and unbalanced phase converter is done. The system description and mode of operation are described. The switching function and modulation technique of the switching function used in this thesis are presented. Computer simulated results of output voltage and input currents spectra under balanced and unbalanced conditions are compared. It is seen that the harmonic content of output voltage increases with the increase of unbalances.

The fundamental component of input currents decreases with the increase of unbalances. For balanced case, the fundamental component of $I_{a}$ or $I_{b}$ or $I_{c}$ is $55 \%$, whereas they are $52 \%, 53 \%$ and $54 \%$ for $I_{a} ; I_{b}$ and $I_{c}$ respectively for unbalanced case. But the harmonic content of input currents increases with the increase in unbalances.

In chapter 4, complete design data is presented. Input voltage sensing is used to synchronize the opening and closing of the converter switches. Control logic circuit using simple logic blocks is designed to handle both fixed and varying amplitude unbalance. Dedicated microprocessor is suggested to correct the continuous variation of input unbalance. Component ratings and their protection scheme are also discussed.

The analysis and design of the three to single phase converter under unbalanced operating condition is presented in this thesis. As a first step simple switching function is considered for this study. More complex and advanced PWM switching functions may be used to study further the behaviour of this converter. Moreover, three phase to single phase converter study may be extended to cover the three phase to three phase unbalanced converter.

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APPENDICES

## APPENDIX-A

## BASIC PRINCIPLE OF CONVERTERS

The output of a converter depends on the switching pattern of the converter switches and the input voltage (or current). Similar to linear system, the output quantities of a converter can be expressed in terms of the input quantities by spectrum multiplication. The arrangement of a single-phase converter is shown in Fig. A-1a. If $V_{i}\left(W_{i} t\right)$ and $I_{i}\left(W_{i} t\right)$ are the input voltage and current, respectively, the corresponding output voltage and current are $v_{0}\left(W_{0} t\right)$ and $I_{0}\left(W_{0} t\right)$, respectively. The converter could be either a voltage source or a current source converter[15].

Voltage Source: For a voltage source, the output voltage $v_{0}\left(W_{0} t\right)$ can be related to input voltage $v_{i}\left(w_{i} t\right)$ by

$$
v_{0}\left(W_{0} t\right)=F\left(W_{s} t\right) v_{i}\left(W_{i} t\right) \quad \therefore \quad \text {. } \quad \text { (A-1) }
$$

where $F\left(W_{s} t\right)$ is the switching function of the converter as shown in Fig. A-1b. $F\left(W_{s} t\right)$ depends on the type of converter and the gating pattern of the switches. If $g_{1}, g_{2}, g_{3}$ and $g_{4}$ are the gating signals for switches $Q_{1}, Q_{2}, Q_{3}$ and $Q_{4}$, respectively, the switching function is

$$
F\left(w_{s} t\right)=g_{1}-g_{4}=g_{3}-g_{2}
$$

Neglecting the losses in the converter switches gives us

$$
\begin{aligned}
& V_{1}\left(W_{i} t\right) I_{i}\left(W_{i} t\right)=V_{0}\left(W_{0} t\right) I_{0}\left(W_{0} t\right) \\
& F\left(W_{s} t\right)=\frac{V_{0}\left(W_{0} t\right)}{V_{i}\left(W_{i} t\right)}=\frac{I_{i}\left(W_{1} t\right)}{I_{0}\left(W_{0} t\right)} \quad \cdots \\
& I_{i}\left(W_{i} t\right)=F\left(W_{8} t\right) I_{0}\left(W_{0} t\right) \quad \cdots \quad \cdots \quad(A-2)
\end{aligned}
$$

once $F\left(W_{s} t\right)$ is known, $v_{0}\left(W_{0} t\right)$ can be determined. $V_{0}\left(W_{0} t\right)$ divided by the load impedance, gives $I_{0}\left(W_{0} t\right)$; and $I_{i}\left(W_{i} t\right)$ can be found from eqn. (A-3).

(a) Converter Structure.


(b) Switching function

Fig. A-1 : Single-phase converter structure.

Current Source: In the case of ourrent source, the input current remains constant, $I_{i}\left(W_{i} t\right)=I_{i}$ and the output current $I_{0}\left(W_{0} t\right)$ can be related to input current $I_{i}$,

$$
\begin{aligned}
& I_{0}\left(W_{0} t\right)=F\left(W_{s} t\right) I_{i} \cdots \\
& V_{0}\left(W_{0} t\right) I_{0}\left(W_{0} t\right)=V_{i}\left(W_{i} t\right) I_{i}\left(W_{i} t\right)
\end{aligned}
$$

which gives

$$
\begin{array}{ll}
V_{i}\left(W_{i} t\right)=F\left(W_{s} t\right) V_{0}\left(W_{0} t\right) & \cdots \\
F\left(W_{s} t\right)=\frac{V_{i}\left(W_{i} t\right)}{V_{0}\left(W_{0} t\right)}=\frac{I_{0}\left(W_{0} t\right)}{I_{i}\left(W_{i} t\right)} \cdots & \cdots(A-5) \tag{A-6}
\end{array}
$$

A-1 SINGLE-PHASE BRIDGE RECTIFIERS
If the input voltage to the single-phase bridge rectifier is

$$
\begin{aligned}
& v_{i}\left(W_{i} t\right)=v_{m} \sin \left(w_{i} t\right), \text { then output voltage is given by } \\
& v_{0}\left(W_{0} t\right)=F\left(w_{s} t\right) v_{i}\left(w_{i} t\right)=\frac{4 v_{m}}{\pi} \sum_{n=1,3,5, \ldots}^{\infty} \frac{\sin \left(w_{i} t\right) \sin \left(n w_{i} t\right)}{n} \ldots(A-7) \\
&=\frac{4 v_{m}}{\pi} \sum_{n=1,3,5, \ldots}^{\infty} \ldots
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{2 V_{m}}{\pi}\left[1-\cos \left(2 W_{i} t\right)+\frac{1}{3} \cos \left(2 W_{i} t\right)-\frac{1}{3} \cos \left(4 V_{i} t\right)+\frac{1}{5} \cos \left(4 W_{i} t\right)\right. \\
& \left.-\frac{1}{5} \cos \left(6 W_{i} t\right)+\frac{1}{7} \cos \left(6 W_{i} t\right)-\frac{1}{7} \cos \left(8 W_{i} t\right)+\cdots\right] \\
& =\frac{2 v_{m}}{\pi}\left[1-\frac{2}{3} \cos \left(2 w_{i} t\right)-\frac{2}{15} \cos \left(4 w_{i} t\right)-\frac{2}{35} \cos \left(6 w_{i} t\right)-\cdots\right] \\
& =\frac{2 v_{m}}{\pi}-\frac{4 V_{m}}{\pi} \sum_{m=1,2,3, \ldots}^{\infty} \frac{\cos \left(2 \pi w_{1} t\right)}{4 m^{2}-1} \quad \cdots \quad . \quad .(A-9)
\end{aligned}
$$

The first term of eqn. (A-9) is the average output voltage and the second part is the ripple constant on the output voltage.

- For a three-phase rectifier, the switching functions are $F_{1}\left(W_{s} t\right), F_{2}\left(W_{s} t\right)$ and $F_{3}\left(W_{S} t\right)$ respectively. If the three input phase voltages are $V_{a n}\left(W_{i} t\right), V_{b n}\left(W_{i} t\right)$, and $V_{c n}\left(W_{i} t\right)$, the output voltage becomes

$$
v_{0}\left(W_{0} t\right)=F_{1}\left(W_{s} t\right) v_{a n}\left(W_{i} t\right)+F_{2}\left(W_{s} t\right) v_{b n}\left(W_{1} t\right)+F_{3}\left(W_{s} t\right) v_{c n}\left(W_{i} t\right) \ldots(A-10)
$$

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AJSTCIATE PROFESSOR，DEPARTMEHT OF E．F．E．RUET，DHAKA
DETERMINATIDY．OF OUTPUT VULTAGE WAVEFORPA
under unbalances ivput congitidin
WITH THE HEL？DF FOURIER COEFFICIENT
DIMENSION B（100），BC（200），XX（360），YY（360），A（100），C（100）
OPEN（UNIT＝3，FILE＝＇OUTPUT＇，STATUS＝＇NEW＇）
PI＝3．1415．727
OELI＝108．ヶPI／180．
DELZ $=112 . * \mathrm{PI} /$／ 80 ．
JEL $3=115 . \% \mathrm{PI} / 190$ ．
DJ． $44 k=1,99,2$

$B(K)=4 . \dot{5}$（N（K）OEL $2 / 2.) /(K \approx P()$
$C(K)=4 . \hat{S I N}(K \approx D E L 3 / 2) /.(K \div P I)$
44 CONTINUE
WRITE（3，31）（K，K，A（K），K，3（K），K，C（K），K＝1，97，2）
 1．C（＇，I2，＇）＝＇FL2．8）
A1＝0．${ }^{27}$
$\mathrm{Cl}=.2 .75$
CI $=7.9 .3$
DO 61 14＝3，77，2
$\triangle N=A 1 \div A(M)$
$3 \mathrm{~N}=3 \mathrm{I}=3(\mathrm{M})$
$C N=$ こ $1 * C(M)$
$\mathrm{N}=2: \mathrm{M}-1$

$K=2 \div M+1$
$B C(K)=0.5 *(A N * 1.0+3 N=C O J(-(M+1) \div 2 . \therefore P I / 3.1+C N=\operatorname{COS}(-(M+1) \div 4 . * P I / 3.1)$
51 EONTINJE
WRITE（3，72）
72 FORMATI＇I＇，5X，．START FQOM NEN PASE＇I
HRITE 3,32$)(M, H, B C(M), M=3,97,2)$
32．FURMATI $2 X, \cdot \cdot M=\cdot, 12,2 X, \cdot 3 C(\cdot, 12, \cdot 1=\cdot, F 7.5)$
DO $71 \quad 1=1,360$
$V O V=0.0$
RAD＝I音PI／1RO．
DO $72 \mathrm{~J}=3,93,2$
$\operatorname{VOV}=B C(J) \approx \operatorname{COS}(J * R A O)+V O V$
92 CUNTINUE
XX（I）＝FLOAT（I）
YY（I）$=3 . \hat{*} \operatorname{COS}(R A D) / 2 .+V O V$
91．CONTINUE
CALL ZPLDT（XX，YY，360）
STOP
END
C SUBROUTINE FOP ABIVE FUNCTION
SUBRDUTIAE ZPLOT（ZL，DO，N）
DIMENSIDN ZZ（W），QQ（M），PLOT（ICI）
CHARACTER G．RLANK，H，PLOT
DATA G．BLANK，H／＇i＇，＇•＇••／

WQITㄷ(3,35)

DO $80 \mathrm{~K}=0,101$
8J PLOT(K)=3LANK
QQMAX=QJ(1)
QQMIN=QQ(L)
DO $31 I=2, N$
IF (QD\{I).CT, QQMAXIQQ:MAX=Q2(I)
(F(QQ(I).LT•JQMIN) J.JMIN=JJ(I)
gl CONTINJE
IAXIS = 1.5+(-QOMIN)/(QQMAX-QQMIN):25.0
DO 9 ? $1=1$, H
plot(IAXIS) $=\mathrm{H}$
$L=1.5+($ QD(I)-OQNIN)/(QQHAAX-QQMIII) $\div 25.0$
PLOT(L) =G

83 FORNAT $3 \times, F S, 2,3 X, F 6,3,2 X, 101.111$
PLOT(L)=BLAAIK
82 CDNTITIUE RETURN
END

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ASSUCIATE PRJFESSOP，DEPARTMEVT OF E．E．F．BJFT，DHAKA •
JETEPMIIATITV TF INPIT PHAST EJRPENTSIIA，IGEICINAVFFJRMS
UNDER UNBALANCEO IVIUT COHNITIUN
WITH THE HFLP DF FOURIER CDEFFICIENT
DIMENSION B（100），XB（360）；YB（36J），ᄃ（107），XC（360），Y（1760），A（100），XA（
$1360)$ YA（360）
OPEN（UNITシ3，FILE＝＇OUTPUT＊，STATUS＝＇NEW＊）
PI＝ 3.1415927
DELI＝109．※PI／180．
DEL2＝112．$\%$ PI／180．

DO $01 \mathrm{~N}=1.37,2$
$n(N)=4 . \approx S I N(N O n E L 1 / 2 \cdot 1 /(N)=P I)$
$3(N)=4 . * 5[H(N+D E L 2 / 2) /.(N \div \Gamma I)$

OL CONTINUE
WRITE（3，51）（N，H，A（N），N，Z（N），N，（N（1），H＝1，7n，2）
 $1^{\prime},\left[2,{ }^{\circ} 1=, F 8.51\right.$
Dr） $03 \quad[=1,363$
RAD＝I F FI／190．
こOA $=0.3$
C03 $=0.0$
こいこ $=0.0$
DO $04 \mathrm{~N}=1,7 \mathrm{n}, 2$
$\operatorname{COA}=(A(H) / 2)=.(C O S((2 \div N+1) \because R A))+C O S((2 O N-1) \div R A D))+C O A$

1PI／3．1）＋COB

10I／3．）I＋COC
O4 CONTIMU：
XA（I）＝FLOAT（I）
$Y A(I)=C O A$
XB（I）＝FLOAT（I）
$\mathrm{YB}(I)=C O B$
XC（I）＝FLOAT（I）
$Y C(I)=C O C$
03 CONTIPNUE
CALL APLOT（XA，YA，353）
CALL EPLOT（X3，YJ，360）
CALL CPLOT（XL，YC，350）
STOP
END
SURRDUTINE FOR CURPENT．
SUSROUTINE A？LOT（LA，TA，NA）
DIMEVSION ZA（NA），DA（NA），PLOTA（1OI）
CHARACTER GA，ABLANK，HA，PLOTA
DATA GA，ABLANK，HA／＇，＂，＂，＂．＂／
WRITE（ 1,55$)$
 DO $05 K=0,101$

```
05 PLOTA(K)=ABLANK
    QAMAX=2A(1)
    QaM[N=OA(1)
    J0 06 I=2,NA
    IF(QAII).GT.QA\AXIQA:AX=TA(I)
    IF(QAII).LT.JA*GIN)QANII=OA(I)
OS CONTINJE
    IAX=1.5+(-QAM[N)/(QAMAX-QANIN) %50.0
    DO 07 I= I.NA
    PLOTA(IAX)=HA
    LA=1.5+IQA(I)-714[:1)/(2A'4AX-2A'4(N)=50.0
    PLOTA(LA)=GA
    WRITF(3,56)ZA(I), 2A(I),PLOTA
55 CORMATIIX,FS.2,2X,FR.5,4X,101A1)
    PLOTA(LA)=ABLANK
OT LONTINJE
    2ETURN
    END
    SUBPDUTINF RPLUT(ZO,2R,NB)
    DIME!NSION 2.(VN),Q3(M!3),PLIJT3(101)
    CHADACTER G3,BMLA:NK,HE,PLOTTB
    DATA GR,3:3LANK,+3/';'9'*''.'/
    WRIT:(3,57)
```



```
    DO O& K=0.1.11
08 PLOTM(K)=YRLANK
    2,1^x=3:(1)
    *9M.19=27(1)
    D] 29 [=2,N3
    I=(QR(I).\sigmaT.QSMAX)D:MAX=Q(3(I)
```



```
0) covir:uc
    IOX=1.5+(-GBM(N)/(D34AX-2BM[N) =50.0
    Ju 10 I= L,N3
    PLOTH(I:3x)=H3
    L3=1.5+(J3(I)-QBMIN)/(JEMAX-2SHIN)*50.0
    P(QT3(LB)=GB
    WRITE(3,58)2!3(I),NB(I),PLOTS
53 5JXMAT(IX,F5.2,2X,F8.5,4X,101A1)
    PLOTB(LB)=B?LANK
10 こJNTINUE
    PETURN
    EVD
    SUTROUTINE CPLOTIZC,OC,NEI
    DIMENSIJN ZCINC),QC(NCI.DLOTC(1O1)
    CHAPACTER GC,CBLANIK,HC,PLOTC
    DATA GC,CBLANK,HC/'!',' ','.'/
    WRITE(?,59)
57 FOPMATI'1',2X,'IMPUT DHASE EURREVT IC'//2X,'XCIII',4X,'YCIII'//.
    DO 11 K=0,171
11 PLOTE(K)=CBLANK
    QCMAX=QC(1)
    QCMIN=QCI1)
    DO 12 I=2,NC
    IF(QCII).CT.QCMAX)QCMAX=QC(I)
```

IF（OC（I）．LT．OCMIN）2CMIN＝2C（I）
12 こク＇小TIUE

3C $13 \mathrm{I}=1$ ，NC
PLOTC（ICX）＝HE
$\left.L \Sigma=1.5+\left(2 C(1)-254 \Psi^{\prime}\right\}\right) /(2 C M A X-Q C M I V)=50.0$
PLORC（LC）＝GC
WRITE（3，60）2CII）．RC（I），PLOTC
SU FORHAT（1X，F5．2，2X，F3．5，4X，101A1）
PLOTC（LC）＝CBLANく
13 CONTINUE
RETUPII
END


