SENSITIVITY STUDY OF POWER SYSTEM RELIABILITY EVALUATION METHODS

BY

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A THESIS

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CERTIFICATE

This is to certify that this work was done by me and it has not been submitted elsewhere for award of any degree or diploma .

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ABSTRACT

In generation expansion planning , the reliability evaluation of each alternative plan is essential in order to justify that the selected plan satisfy the desired level of reliability . The commonly used reliability index is the Loss of load probability (LOLP) . Three methods , recursive , segmentation and cumulant are generally used by the utility in the evaluation of LOLP . Since the introduction of these methods , attempts are being made with an ultimate target to improve computational efficiency and flexibility . In this research these three methods are applied to two electric utilities , the IEEE Reliability Test System (medium size utility) and the Bangladesh Power System (small utility) . The methods are then compared in terms of accuracy, computational efficiency and computer memory requirements and discussed with a view to determine a suitable method for any particular type of system . In this research the sensitivity of each method to the variation of peak load and sensitivity of recursive method to step size , segmentation method to segment size and cumulant method to the number of terms are investigated and a table informing the different characteristics of each method is developed .

NOTATIONS

AC	=	Available Capacity
Ci	=	Capacity of the i-th generating Unit
E_k	=	Energy curtailed by capacity outage equal to O_k
F(L)	=	Probability distribution of load
$F^r(Le)$	=	Probability distribution of equivalent load after convolving r-th Units
$F^n(Le)$	=	Equivalent load probability distribution
$F(Lo_i)$	=	Probability distribution of outage capacity
IC	=	Installed capacity
K_n	=	n - th cumulant
L	=	Random system load
Le	=	Equivalent load
Lo_i	=	Random outage load corresponding to the i-th Unit .
m	=	Mean up time
m_n	=	n-th moment about the origin
M_n	=	n-th moment about the mean

$N(Z)$	=	Normal distribution
$N^r(Z)$	=	r-th derivative of normal distribution
O_k	=	Magnitude of capacity outage
P_k	=	Probability of capacity outage equal to O_k
p_k	=	Availability of capacity C_k of the k-th Unit
PL	=	Peak Load
q_k	=	FOR of the k-th unit
r	=	Mean down time
X_A	=	Available capacity
X_O	=	Forced outage capacity
λ	=	Generating unit failure rate
μ	=	Generating Unit repair rate .

ABBREVIATIONS

BPS	=	Bangladesh Power System
CDF	=	Cumulative Probability Distribution Function
CLC	=	Chronological Load Curve
EFOH	=	Equivalent Forced Outage Rate
ELDC	=	Equivalent Load Duration Curve
EWI	=	East - West Interconnector
FAD	=	Frequency And Duration
FOH	=	Forced Outage Hours
FOR	=	Forced Outage Rate
GT	=	Gas Turbine
LDC	=	Load Duration Curve
LOEE	=	Loss of Energy Expectation
LOEP	=	Loss of Energy Probability
LOLP	=	Loss of Load Probability
MCS	=	Moute Carlo Simulation
MTTF	=	Mean Time To Failure
MTTR	=	Mean Time To Repair
PDF	=	Probability Density function
SH	=	Service Hours

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The main concern of power system engineers is to supply customers , both large and small , with electrical energy as economically and reliably as possible. Modern society,because of its nature of working habits expects continuous electric power supply . The main hindrance to the continuous supply is the random system failures . However , the discontinuity of electric service may be reduced by increasing investment in generation and distribution sectors . Overinvestment may lead to excessive operating costs which will inflate the tariff. On the otherhand the low investment may cause unreliable supply . It is , therefore , evident that the economic and reliability constraints are competitive , and this can lead to difficult managerial decisions at the planning and operating stages .

Thus , the evaluation of reliability and economic constraints are two important aspects of generation expansion planning of a power system . Generation expansion planning begins with estimates of peak demands and associated electrical energy consumption . After identifying the need for generating capacity additions , the planners develop a number of feasible expansion alternatives on the basis of [1] .

i) Load growth
ii) Construction time
iii) Availability of sites
iv) Availability of fuel

Given these alternative plans , it is common to subject each plan to a detailed reliability analysis to ensure that all plans satisfy the desired level of reliability . Plans that do not meet the reliability criteria are eliminated or appropriately modified and plans which satisfy the required reliability level are evaluated on the basis of economics . In order to determine the economic merit of any proposed generation expansion plan , an economic indicator, usually called cost functional is introduced . For the sake of simplicity and clarity this cost functional is formulated as a sum of three major items : [2] .

i) Cost of capacity addition .
ii) Social cost arising from failure to meet the projected demand .
iii) Penalty for residual generation capacity at the end of plan period .

The cost of capacity expansion comprises generation costs, fuel costs and operation and maintenance costs . It is difficult to assess the social cost caused by loss of load or failure to meet the projected demand . However , it is clear that loss of load has a serious economic impact on production and business, in addition to inconvenience suffered by domestic consumers .In any generation expansion planning this is a major factor and hence some measure of social cost must be introduced , even if it is fictitious . The obvious reason for this is that if there is no social cost , there is no need for power generation .The penalty for residual capacity is obtained by subtracting the salvage value from the cost of residual capacity at the end of plan period . Sum of these three costs gives the cost functional. For each potential plan , financial and environmental impacts are analyzed . Finally , the alternative plans are compared in order to identify the one that impacts on the utility as a whole in the most favourable manner . In figure 1.1 the planning process is depicted in the form of block diagrams .

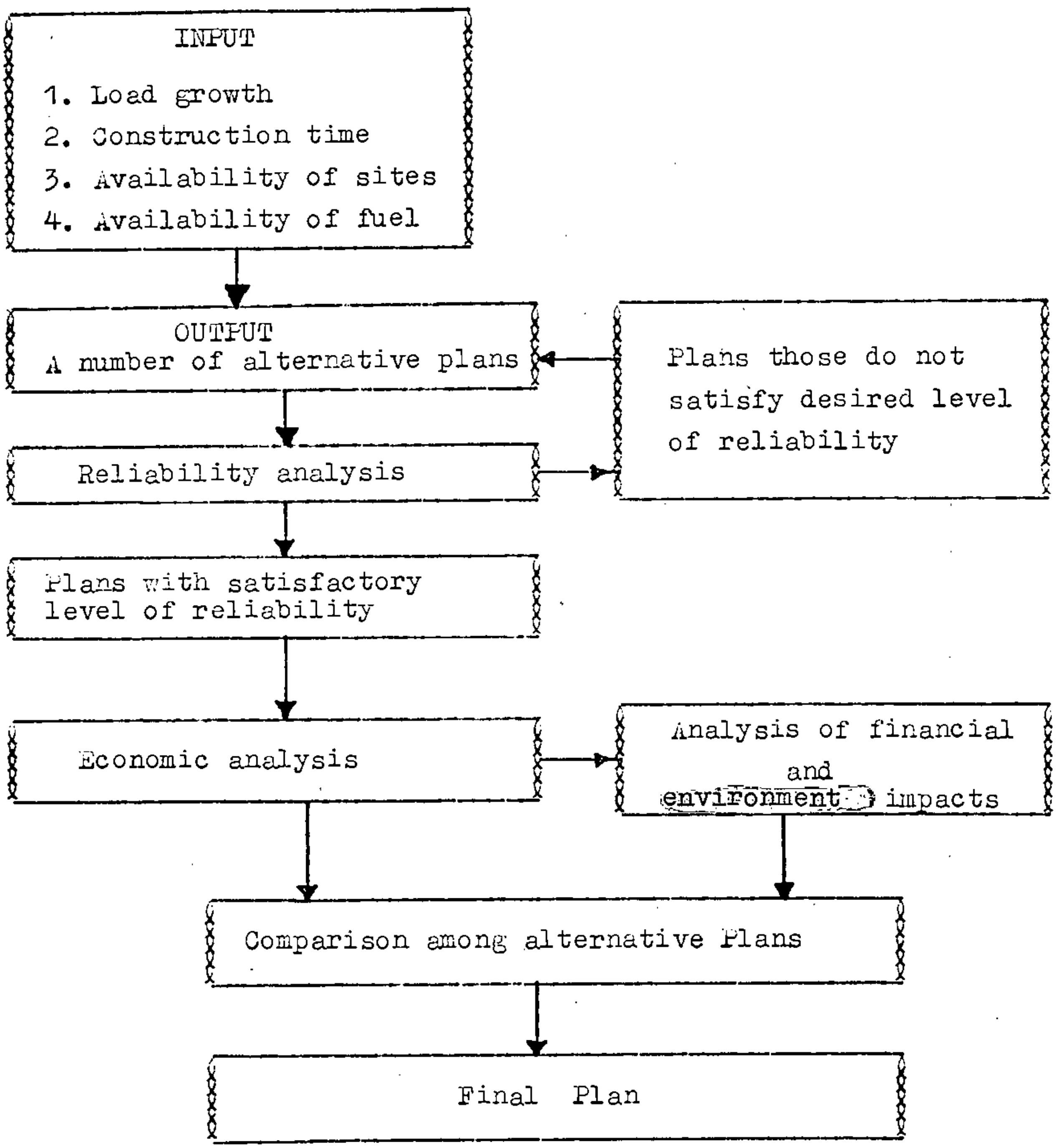

Figure 1.1 : Generation expansion planning process

1.2 BACKGROUND AND MOTIVATION

The historical development of probabilistic methods to evaluate production cost and reliability is extremely interesting . Interest in the application of probablity methods to the evaluation of capacity requirements became evident in 1933 . The first large group of papers was published in 1947. The papers by Calabrese [3] , Lyman [4] , Seelye [5] , Loane and Watchorn [6] proposed some of the basic concepts upon which some of the methods in use at the present time are based . In 1948, the first AIEE subcommittee on the application of probability methods was organized . The subcommittee submitted several reports containing comprehensive definitions of equipment outage classifications in 1949 [7] , 1954 [8] and 1957 [9] . In 1947 a group of papers proposed the methods which are now known as the 'Loss of Load Approach' , and the 'Frequency and Duration of Outage Approach' . Until 1954 most probability studies had been done either by hand or by using conventional desk calculators . The benefits of using digital computers were noted by Watchorn [10] in 1954 . In 1960 Brown , Dean and Caprez [11] published the results of a statistical study of five years of data on 387 hydroelectric generating units . Shortly after this in 1961 the AIEE subcommittee [12] produced a manual outlining reporting procedures and methods of analyzing forced outage data using digital equipment . Cook et al [13] proposed in their paper the basic method for evaluating LOLP of two interconnected systems . The initial approach to the calculation of outage frequency and duration indices in generating capacity reliability evaluation was modified by the introduction of a recursive approach . This technique is described in detail in a series of four publications by J. D. Hall , R. J. Ringlee, A. J. Wood , C. D. Galloway , L.L. Garver , V. M. Cook from September , 1968 to August , 1969 [14 , 15 , 16 , 17] .

An important advancement in probabilistic simulation for generation planning was the introduction of the concept of equivalent load by Baleriaux et al . [18] . It was refined by Booth [19] . The equivalent demand is the sum of demand and the amount of capacity on outage of the units and is obtained by a convolution formula given in terms of a recursive algorithm .

A significant breakthrough in the computational efficiency of the probabilistic method was the introduction of the method of cumulants or moments by Rau et al . [20] as well as by Stremel et al . [21] . These are approximate techniques based on the application of Gram - Charlier or Edgeworth series expansions . In this case , a given frequency function is represented by a series of orthogonal polynomials. These techniques gained increasing popularity because of their computational efficiency . However , the accuracy of these methods depends on a number of factors : unit forced outage rate , number of units , the shape of load etc .

An exact and computationally efficient technique was recently proposed by Schenk et al . [22] based on the observation that the LOLP and expected energy generation of units may be obtained from the zeroeth and first order moments of unserved demand . This technique is based on the segmentation of the demand axis into segments of equal capacity . The segmentation method is further extended by Schenk , Ahsan and Vassos [23] to incorporate the reliability evaluation of two interconnected systems . Ahsan and Schenk [24] have utilized the segmentation method to evaluate the production cost of two interconnected systems .

As mentioned earlier , the commonly used methods to evaluate alternative plans in the generation expansion planning process and also to evaluate a power system to measure the standard of service , are 'recursive' (Baleriaux and Booth) , 'cumulant' and 'segmentation' method . In this thesis , the two exact techniques , recursive and segmentation methods are compared with the approximate technique , cumulant method , by applying them to a typical system namely IEEE Reliability Test System [25] and to a small system namely Bangladesh Power System .

The methods are compared in terms of computational efficiency , computer storage requirement and also interms of accuracy of results . This thesis presents an investigation regarding the sensitivity of each method to peak load variation . The sensitiveness of recursive method to step size , segmentation method to segment size and cumulant method to number of terms in the series is also investigated in this thesis .

Finally the potential advantages and disadvantages of these techniques are discussed in an attempt to offer some guidelines to the utilities for their application to power system .

1.3 THESIS ORGANISATION

This thesis consists of six chapters . In the first chapter a brief introduction regarding the generation system planning processes and the objectives of power system engineers are presented . The step by step development of probabilistic methods to evaluate a power system is also discussed in this chapter under the heading Background and motivation .

Chapter - 2 contains the basic concepts of power system reliability . A brief description of different reliability indicies is presented in this chapter . The probabilistic model for generating unit and load are discussed and derived in Chapter -3. This chapter also contains the load probability distribution and the concept of equivalent load .

The different methodotogies used for reliability evaluation of a power system are discussed in detail with numerical examples in Chapter - 4 .

Chapter - 5 contains a brief description of the IEEE Reliability Test System as well as the electric power generation system of Bangladesh . The generating unit and load models for IEEE system used in this research as well as the results obtained by different methods are also presented in this chapter . This chapter also contains the generation data of Bangladesh Power System . Load data of this system are presented in Appendix-A. The results obtained with these data are presented in Chapter-5. Observations and discussions on the results obtained for both the system are also presented in this chapter .

Chapter - 6 is the last chapter containing the conclusions from this research . Some recommendations and suggestions for further work are also presented in this chapter .

Computer programs developed in this research for different methods to evaluate the reliability index are presented in Appendix - B .

CHAPTER 2

BASIC CONCEPTS OF POWER SYSTEM RELIABILITY

2.1 INTRODUCTION

The concept of reliability is very common . A person will be called reliable if he keeps his word . A device is reliable if it works satisfactorily . Similarly a power system will be called reliable if it serves its customers with uninterrupted electrical energy . However such qualitative notions regarding power system reliability are not sufficient . A quantitative evaluation of reliability is necessary to provide a quantitative prediction of system performance so that the reliability levels of alternate proposals can be compared along with the cost. In generation expansion planning , the reliability of a number of alternate expansion plans are first evaluated . The plans which do not satisfy the desired reliability level are either modified or rejected .

In this chapter , a basic concept of reliability and its usefulness in expansion planning is presented . This chapter also presents the different reliability indices .

2.2 POWER SYSTEM RELIABILITY

A standard definition of reliability is [26]

Reliability is the probability of a device or system performing its purpose adequately for the period of time intended under the operating conditions encountered .

This definition can be broken into four basic concepts :

i) Probability
ii) Adequate performance
iii) Time
iv) Operating conditions .

Probability :

It is the relative frequency with which an event occurs in a series of many trials or observations under constant conditions .

Probability is a key word in the definition of reliability since uncertainty is a major element in the planning of an electric power system . The most apparent source of uncertainty in the generating system , for instance, is the random failures of the generating units .

Adequate performance :

Regarding the generation system , the concept of adequate performance or the generation system adequacy relates to the amount of capacity needed to meet the demand under the random failures of the generating units . Regarding transmission system , the term adequate performance relates to the ability of the system to withstand line overloads , to maintain adequate voltage levels within the system stability limits .

Time :

For a power system an important aspect is continuity of service ; that is , how well the customers are served year after year . It is desired that the supply of electric power be continuous during the period for which the utility is responsible .

Operating Conditions :

The operating conditions are also important in determining the reliability of a power system . For the transmission system , for example , component failures may increase considerably in adverse weather periods . The maintenance work influences enormously the operating conditions .

Now the reliability of an electric power system can be defined as the probability to provide the customers with continuous service of satisfactory quality . The quality constraint refers to the requirement that the frequency and voltage of the power supply remain within prescribed tolerances .

In order to obtain quantitative assessment of reliability it is necessary to define suitable reliability criteria or indices . The determination of reliability criteria for the generation system is highly dependent upon the generation mix, unit size , load characteristics and system interconnections . The consideration of these aspects together with other less tangible elements in the planning , design and operation of a power system is usually designated 'generation capacity reliability evaluation ' .

2.3 RELIABILITY METHODS IN GENERATION PLANNING AND RELIABILITY INDICES

The following deterministic approaches are used in generation expansion planning :

i) Percentage reserve

ii) Loss of Largest unit

iii) Combination of these two

iv) Probability methods

However , these approaches can not account for the probabilistic nature of customer's demand as well as system behaviour including the random failure of system components. In probabilistic methods although time is not exposed , the stochastic nature of the system is properly taken care of and these methods are widely used now for the reliability evaluation of generation expansion plan . In order to determine the reliability of a power system a number of reliability indices have been defined . Some of the frequently used indices are :

i) Loss of Load Probability (LOLP).
ii) Loss of Energy Probability (LOEP).
iii) Frequency and Duration (FAD) .
iv) Monte Carlo Simulation (MCS).

Loss of Load Probability (LOLP)

The LOLP is the probability that the available generation capacity of a power system will be insufficient to meet the daily peak load . Thus

$$LOLP = Prob. \left\{ AC < DL \right\} \qquad (2.1)$$

where AC and DL are the available capacity and demand respectively . The basic methods to evaluate LOLP can consider forced and scheduled outage of generating units as well as load forecast uncertainty and also the assistance from interconnected systems. The reliability index LOLP does not give an indication of the magnitude or duration of generation shortfall . This only provides the probability of occurance of the loss of load . The evaluation techniques in this method will be discussed in later chapters in detail .

Loss of Energy Probability (LOEP)

The LOEP is defined as the ratio of the expected amount of unserved energy during some long period of time to the total energy required during the same period . The index LOEP therefore, reflects the frequency , magnitude and duration of the capacity outage . Clearly , as the capacity reserve margin increases the LOEP decreases .

The standard LOEP approach utilizes the daily peak load variation curve or the individual daily peak loads to calculate the expected number of days in the period that the daily peak load exceeds the available installed capacity. The index can also be calculated using the load duration curve or the individual hourly values . The area under the load duration curve represents the energy utilized during the specified period and can be used to calculate an expected energy not supplied due to insufficient installed capacity . The results of this approach can also be expressed in terms of the probable ratio between the load energy curtailed due to deficiencies in the generating capacity available and the total load energy required to serve the requirements of the system . For a given load duration curve this ratio is independent of the time period considered , which is usually a month or a year . The ratio is generally an extremely small figure less than one and can be defined as the 'Energy index of unreliability' . It is more usual , however, to subtract this quantity from unity and thus obtain the probable ratio between the load energy that will be supplied and the total load energy required by the system . This is known as the 'energy index of reliability' .

The probabilities of having varying amounts of capacity unavailable are combined with the system load as shown in figure 2.1 . Any outage of generating capacity exceeding the reserve will result in a curtailment of system load energy [27]

Let O_k = magnitude of capacity outage

P_k = probability of capacity outage equal to O_k

E_k = energy curtailed by a capacity outage equal to O_k

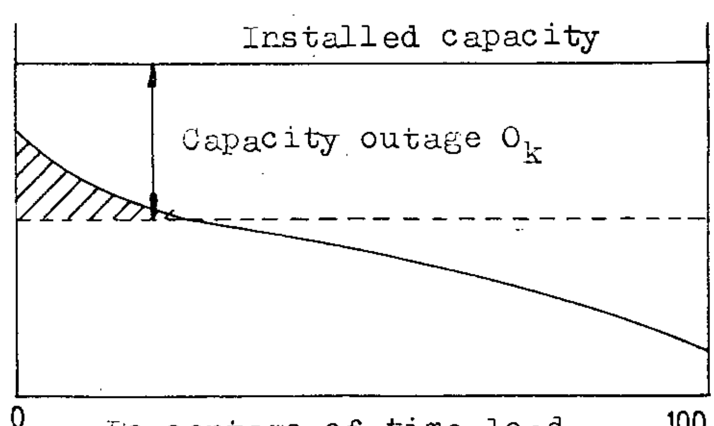

Fig. 2.1 : Energy curtailment due to a given capacity outage condition .

The probable energy curtailed is $E_k P_k$. The sum of these product is the total expected energy curtailment or loss of energy expectation (LOEE) . Thus ,

$$\text{LOEE} = \sum_{k=1}^{n} E_k P_k \tag{2.2}$$

This can be normalized by utilizing the total energy under the load duration curve designated as E and thereby giving rise to the index loss of energy probability (LOEP) . Thus ,

$$\text{LOEP} = \sum_{k=1}^{n} \frac{E_k \; P_k}{E} \tag{2.3}$$

It is important to appreciate , however , that future electric power systems may be energy limited rather than power or capacity limited and therefore future indices may be energy based rather than focused on power or capacity.

Frequency and Duration (FAD)

This reliability index gives the average number of times and the average length of time during which available generation is inadequate to meet the load. This requires consideration of the daily load cycle and data on the frequency and duration of unit outages. One problem with FAD technique is that it requires more detailed data than is usually available. In addition to failure rates of various components, repair times also be available.

Monte Carlo Simulation (MCS)

In this method , the life of a component or a system is simulated on the computer and the simulation process is observed for some times to ascertain the reliability indices. Thus the simulation is treated as a series of real experiments. During its course, events are made to occur at times determined by random processes obeying predetermined probability distributions. MCS technique is computationally expensive. However, it may produce a solution in cases where more traditional analytical techniques fail. This may happen when the failure and repair processes have non exponential distribution. MCS technique is best suited to problems in which reliability is significantly affected by system operating policies.

In the application of all these methods to evaluate any of the reliability indices, the following three steps are followed :

i) Development of a generation model

ii) Development of a load model

iii) Combination of the above two models (Convolution) to define the appropriate indices of reliability.

2.4 VALUE OF ELECTRIC POWER SYSTEM RELIABILITY

All consumers of electricity do not require same level of reliability regarding the services to them. It is found that some consumers are very sensitive to the service of the utility and require the most reliable service while others do not. Therefore, it is not possible to make a general approach to determine the valuation of reliability. However, it is usually determined in terms of the incurred losses resulting from an interruption of service .

From the customers point of view the value of reliability is dependent upon expected service and upon the customer's prediction of his losses incurred by an interruption of service. Again from utilities point of view the determination of the value of reliability due to an interruption of service may be approached by assigning a cost to the loss of revenue due to unserved demand . The parameters that have evolved as measures of the cost of interruptions are the cost per kwhr of unserved energy and the cost per kw of load . Therefore , the utility should not spend less on reliability than the value of loss and at the same time it should not spend more . This concept may be illustrated by figure 2.2 .

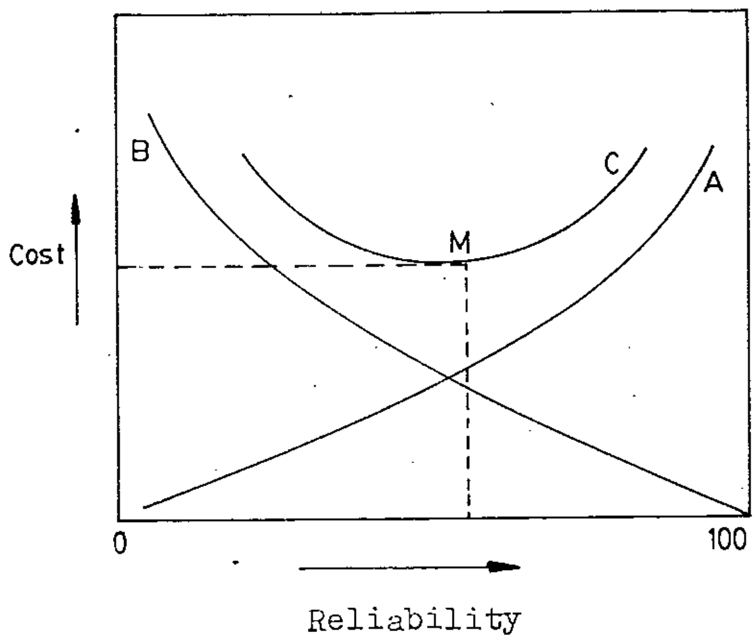

Fig. 2.2. : Reliability Vs. Cost

In Fig. 2.2 , curve A represents the utilities cost which increases exponentially as the reliability increases . Curve B represents the consumer's costs of non-supply and it is clearly zero with perfect reliability . Curve C is the sum of the two. At the point M, the costs are minimized for the specified reliability [126] .

2.5 RELIABILITY ASSESSMENT OF GENERATION EXPANSION PLANNING

A power company must take care of how much generating capacity is required to meet the demand as economically as possible with a reasonable assurance of continuity and quality of service . A key factor in the overall scheme is the provision for adequate generation reserve capacity . It is a common practice among utilities to install generating capacity in excess of forecasted peak demand . Generating reserve capacity also called reserve margin is necessary to sustain the required levels of reliability . For higher levels of rebiability the following points must be considered :

i) Proper maintenance of the equipments

ii) Sufficient margin above the peak demand

iii) Impacts of planned and forced outages of generator and the transmission lines .

The whole span of a power system is mainly devided into two sectors : the planning phase and the operating phase . Therefore , it is customary to divide reliability assessment into two categories : static reliability and spinning reliability assessment . Static reliability assessment applies to planning while spinning reliability assessment applies to the operating mode . In a static mode , the basic requirement is the planning of adequate generation to meet the forecasted demand . In an operating mode the basic requirement is the ability to operate the system as economically as possible with adequate operating reserves . These reserve may come from

i) the rapid start units
ii) the assistance from interconnected systems
iii) the interruptible loads
iv) the reduction of voltage , etc.

2.6 FACTORS AFFECTING GENERATING SYSTEM RELIABILITY

The major factors influencing reserve capacity and thus generating system reliability are :

i) Unit size and number of units

Capacity reserve requirement increases as the average unit size increases but decreases as the number of units increases.

ii) System Load Factor :

Capacity reserve requirement increases as the system load factor increases .

iii) Delayed Capacity Additions

Capacity reserve requirement increases as delays in planned capacity addition increases .

iv) Scheduled and forced outages

Capacity reserve requirement is seriously affected by forced outages of generating units . It is also affected by the scheduled outages .

v) Interconnections with neighbouring systems

Capacity reserve requirement dicreases with the addition of interconnections .

vi) Uncertainty in future load growth & peak demand

Capacity reserve requirement increases as the degree of uncertainty in future load growth and peak load demand increases .

CHAPTER 3

PROBABILISTIC MODEL OF GENERATING UNIT AND LOAD

3.1 INTRODUCTION

In generation expansion planning process different alternative plans are subject to detailed reliability analysis to ensure that all satisfy the desired level of reliability . As it is discussed earlier , in order to quantify the reliability level ; the evaluation of reliability index is essential. All reliability evaluation techniques require the following two basic probabilistic models :

i) Generation model and

ii) Load model

In this chapter , generation and load models suitable for various probabilistic simulation techniques are presented .

3.2 PROBABILISTIC GENERATING UNIT MODEL

A generating system often consists of many different types of units , all with their own peculiarities , are randomly forced off-line due to technical problems during a normal period of operation . To take into account the random outages of a generating unit , it is essentially required to determine the probability density function that describes the probability that a unit will be forced off-line or will be available during its normal period of operation .

On the basis of historical data that the availability of the generating capacity of a given unit may be graphically represented as shown in figure 3.1 . It conveys the idea that random failure and repir of a unit can be defined as a two-state stochastic process , where a stochastic process is defined as a process that develops in time in a manner controlled by probabilistic laws .

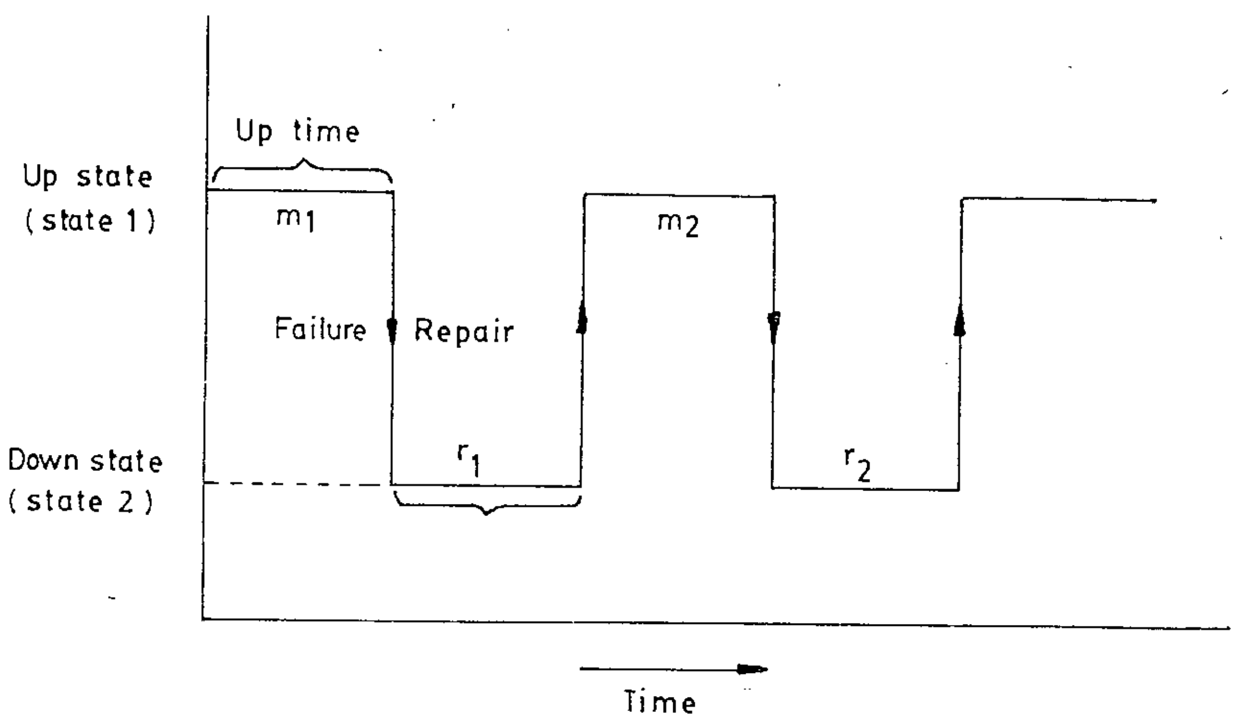

Fig. 3.1 : Run-fail-repair-run cycle for a generating unit .

In the run-fail-repair-run cycle it assumes that the system alternates between an operating state or up state (a state corresponding to maximum available capacity) followed by a failed state or down state (a state corresponding to no available capacity) , in which repair is effected . For the i-th cycle , let

$$m_i = \text{Up time}$$

$$r_i = \text{Down time}$$

The random history of a generating unit may be represented in terms of an average (mean) 'up time' and an average 'down time' as follows :

$$m = \text{mean up time} = \frac{1}{N}\sum_i m_i$$

$$r = \text{mean down time} = \frac{1}{N}\sum_i r_i$$

where N is the total number of run-fail-repair-run cycles .Thus the number of failures per unit time called the failure rate λ and the number of repairs carried out per unit time called the repair rate μ can be expressed as

$$\lambda = \text{Unit failure rate} = \frac{1}{m}$$

$$\mu = \text{Unit repair rate} = \frac{1}{r}$$

With these two parameters the random failure and repair of a generating unit may be defined by a state-space diagram (two state) as in figure 3.2.

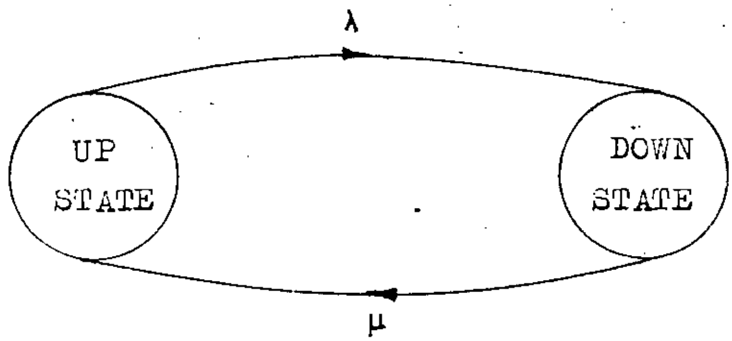

Fig. 3.2 : Generating unit state-space diagram

Now, a question may arise ,'what is the long-term average (steady-state) probability of finding the unit in the up or down state' . The answer to this question lies in the definitions of following two important quantities :

i) Unit availability : the long-term probability that the generating capacity of a unit will be available . This state is called the up state and the probability of this state is usually denoted by the variable p

ii) Unit unavailability : the long-term probability that the generating capacity of a unit will be unavailable or forced off-line and this is the down state. The probability of the down state rate is denoted by the variable q .

3.2.1 MATHEMATICAL DESCRIPTION OF GENERATING UNIT STATE-SPACE DIAGRAM

To obtain an expression for the long-term availability of the generating capacity of a unit, it is first necessary to recognize the stochastic process considered , as very special one called a zero-order , discrete state , continuous transition Markov process . Such a stochastic process has the following properties [28] :

i) Mutually exclusive and discrete state i.e., the generating unit can be either in the up or the down state only , not both simultaneously .

ii) Collectively exhaustive states i.e., since it is assumed that only possible states for a generating unit are the up and the down states , these states define all the possible states we ever expect to find a unit in .

iii) Changes of states can occur at any time .

iv) The probability of departure from a state depends only on the current state and is independent of time.

v) The probability of more than one change of state during a small time interval Δt is negligible .

Now let ,

$P_i(t)$ = Probability of finding a generating unit in state i (where i = 1 is the up state and i = 2 the down state) at time t .

$\therefore$ $P_1(t+\Delta t)$ = Probability of finding a unit in the up state at time ($t+\Delta t$) .

= [Probability of being in state 1 at time t and not leaving that state during time interval Δt] + [Probability of being in state 2 at time t and not moving to state 1 during time interval Δt]

(3. 1)

Note that in figure 3.2 ,

λ is the transition rate from state 1 to state 2 .

That is, $\frac{1}{\lambda}$ is the average time a generating unit stays in the up state .

Also note that ,

μ is the transition rate from state 2 to state 1 .

and therefore , $\frac{1}{\mu}$ is the average time a generating unit stays in the the down state .

It is assumed that the probability of a unit failure can be discribed by an exponential distribution :

$$F_1(t) = e^{-\lambda t} \triangleq \text{Probability of unit being available upto time } t \qquad (3.2)$$

Equation (3.2) can be expanded as

$$F_1(t) = 1 - \lambda \Delta t + \frac{(\lambda \Delta t)^2}{2!} - \ldots\ldots\ldots\ldots\ldots$$

$$\cong 1 - \lambda \Delta t \quad \text{(Neglecting higher order terms)}$$

$$\cong \text{Probability of unit being available during time interval } \Delta t \qquad (3.3.)$$

Where

$\lambda \Delta t \triangleq$ Probability of transfering from state 1 to state 2 in time interval Δt

Again

$$F_2(t) = e^{-\mu t} \triangleq \text{Probability of unit being unavailable upto time } t \qquad (3.4)$$

$$= 1 - \mu\Delta t + \frac{(\mu\Delta t)^2}{2!} \cdots\cdots\cdots$$

$$\cong 1 - \mu\Delta t$$

$$\triangleq \text{Probability of unit being unavailable during time interval } \Delta t \qquad (3.5)$$

Where $\mu\Delta t$ = Probability of transferring from state 2 to state 1 in time interval Δt

Now equation (3.1) becomes

$$P_1(t + \Delta t) = P_1(t)\ (1 - \lambda\Delta t) + P_2(t)\mu\Delta t \qquad (3.6)$$

Similarly,

$$P_2(t + \Delta t) = P_2(t)\ (1 - \mu\Delta t) + P_1(t)\lambda\Delta t \qquad (3.7)$$

Rearranging equations (3.6) and (3.7) we get ,

$$\frac{P_1(t+\Delta t) - P_1(t)}{\Delta t} = -\lambda P_1(t) + \mu P_2(t)$$

$$\frac{P_2(t+\Delta t) - P_2(t)}{\Delta t} = \lambda P_1(t) - \mu P_2(t)$$

If Δt approaches zero i.e. , $\Delta t \to 0$ the above two equations become ,

$$\frac{dP_1(t)}{dt} = -\lambda P_1 + \mu P_2 \qquad (3.8)$$

$$\frac{dP_2(t)}{dt} = \lambda P_1 - \mu P_2 \qquad (3.9)$$

As we are interested only in the long-term (steady-state) probabilities of being in either state 1 or state 2 , $P_1(t)$ and $P_2(t)$ must **satisfy**

$$P_1(t) + P_2(t) = 1 \qquad (3.10)$$

Equations (3.8) and (3.9) can be written in matrix form [illegible] as ,

$$\begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \end{bmatrix} = \begin{bmatrix} P_1(t) & P_2(t) \end{bmatrix} \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix} \qquad (3.11)$$

Solving equation (3.11) we get [31] ,

$$P_1(t) = \frac{\mu}{\lambda+\mu}\left[P_1(o)+P_2(o)\right] + \frac{\lambda e^{-(\lambda+\mu)t}}{\lambda+\mu}\left[P_1(o)-P_2(o)\right] \qquad (3.12)$$

$$P_2(t) = \frac{\lambda}{\lambda+\mu}\left[P_1(0)+P_2(o)\right] + \frac{\lambda e^{-(\lambda+\mu)t}}{\lambda+\mu}\left[P_2(o)-P_1(o)\right] \qquad (3.13)$$

Where

$P_1(o)$ and $P_2(o)$ represent initial states (conditions) and that $P_1(o) + P_2(o) = 1$

Now let us assume that at t = o the generating unit is in the up state i.e. , at state 1 .

$$\therefore \quad P_1(o) = 1 \text{ and } P_2(o) = o$$

Which results

$$P_1(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda e^{-(\lambda+\mu)t}}{\lambda+\mu} \qquad (3.14)$$

$$P_2(t) = \frac{\lambda}{\lambda+\mu} - \frac{\lambda e^{-(\lambda+\mu)t}}{\lambda+\mu} \qquad (3.15)$$

In generation expansion planning long-term (steady-state) probabilities are required . Thus equations (3.14) and (3.15) become ,

$$P_1(\infty) = \frac{\mu}{\lambda+\mu}$$

$$P_2(\infty) = \frac{\lambda}{\lambda+\mu}$$

Finally the long-term probabilities of unit availability and unavailability may be written as

$$\text{Prob}\left\{\text{Up state}\right\} = p = \frac{\mu}{\lambda+\mu} = \frac{m}{m+r} \qquad (3.16)$$

$$\text{Prob}\left\{\text{Down State}\right\} = q = \frac{\lambda}{\lambda+\mu} = \frac{r}{r+m} \qquad (3.17)$$

Where $\quad p + q = \frac{m}{m+r} + \frac{r}{m+r} = 1$

Unit unavailability is very often termed forced outage rate (FOR) which is an important parameter in reliability analysis and is given by ,

$$\text{FOR} = \frac{\text{Forced outage hours}}{\text{Forced outage hours} + \text{service housrs}}$$

$$\text{or , FOR} = \frac{\text{FOH}}{\text{FOH} + \text{SH}} \qquad (3.18)$$

In case of partial outages, the forced outage hours are increased by an appropriate amount of time called equivalent forced outage hours , abbreviated as 'EFOH' . This duration is obtained by multiplying the actual partial outage hours by the corresponding fractional outage capacity . The EFOH gives rise to another outage rate called 'equivalent forced outage rate', abbreviated EFOR and is expressed as ,

$$\text{EFOR} = \frac{\text{FOH} + \text{EFOH}}{\text{FOH} + \text{SH}} \qquad (3.19)$$

where the service hours (SH) include the actual partial outage duration .

3.2.2 PROBABILITY DENSITY FUNCTIONS OF AVAILABLE AND FORCED OUTAGE CAPACITY OF A GENERATING UNIT

Let us consider a generating unit whose capacity is C MW, availability p and unavailability or forced outage rate, FOR= q . The probability density functions (PDF) of available and forced outage capacity for a binary state unit is shown in figure 3.3.

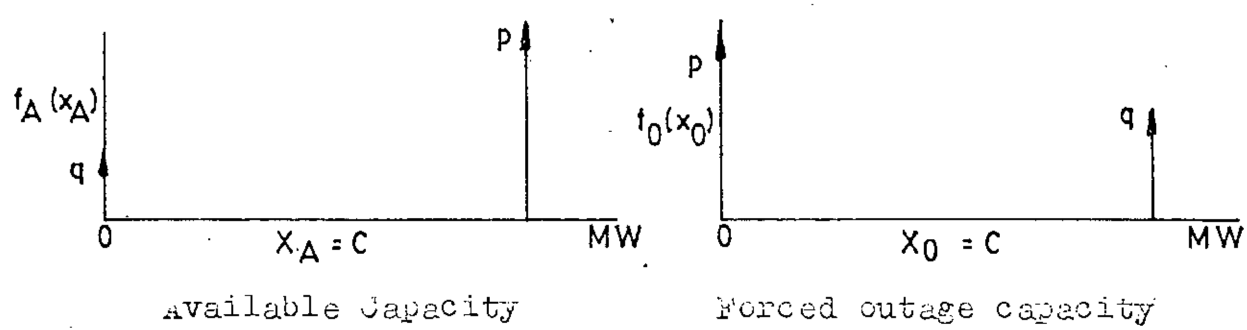

Fig. 3.3 : PDFs of available and forced outage capacity .

PDF of available capacity means that the probability of getting C MW available is p and that of 0 MW available or probability of getting no output is q .

Again PDF of forced outage capacity means that probability of not getting C MW output is q and that of O MW outage or probability of getting C MW output is p .

The PDF of forced outage capacity may be conveniently expressed by the relation .

$$f_o(X_o) = p\,\delta(x_o) + q\,\delta(X_o - C) \qquad (3.20)$$

where

f_o = PDF of forced outage capacity

$\delta(.)$ = Impulse function called Diracdelta function.

The impulse function is usually defined as

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 .$$

3.3 LOAD MODELS

To evaluate the reliability index of a generating system it is necessary to model the load properly . The load data required to develop a probabilistic load model is readily available , since continuous recordings of system demand and energy are usually made on a routine basis by electric utilities . If a recording of instantaneous demands are plotted for a certain period of time , a curve is obtained known as 'Chronological Load Curve' (CLC). From this curve another curve called the 'Load Duration Curve' (LDC) is obtained by determining what percentage of time the demand exceeds a particular level. The CLC and LDC are schematically shown in figure 3.4 and figure 3.5 respectively .

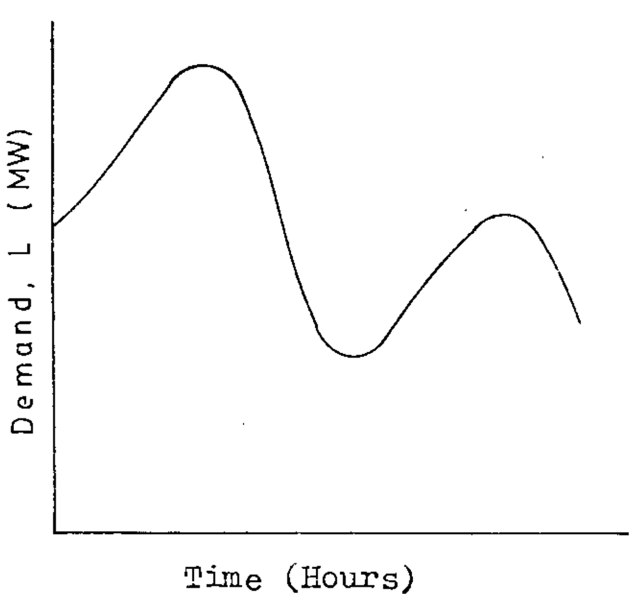

Fig. 3.4 : Chrouological load curve

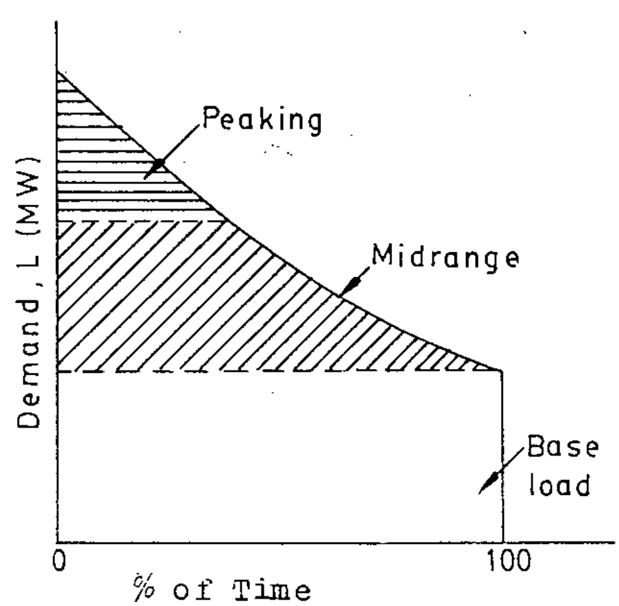

Fig. 3.5 : Load duration curve

The area under the load curve is the energy consumed . The area under the curve also defines the energy to be produced by the base-load units , peaking units and midrange units

3.3.1 LOAD PROBABILITY DISTRIBUTION

It is convenient to interchange the axis parameters of the load duration curve and normalize time , producing the load probability distribution as shown in figure 3.6. In this case the y-axis shows the probability that the load exceeds the corresponding x-axis megawatt value . This curve is also called inverted load duration curve . This load distribution is usually denoted by $F_k(L)$, where k indicates the time period for which the distribution is applicable .

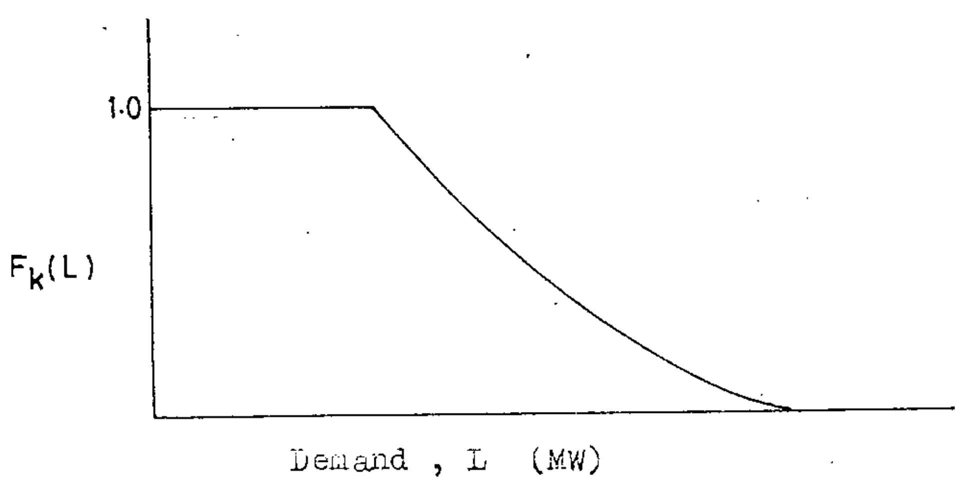

Fig. 3.6 : Load probability distribution

$F_k(l) = P_k(L \geq l)$ = Probability that the load is greater than or equal to l MW in time period k .

3.3.2 HOURLY LOAD

This load model is derived from the chronological load curve by dividing the time axis into a number of small intervals as shown in figure 3.7 .

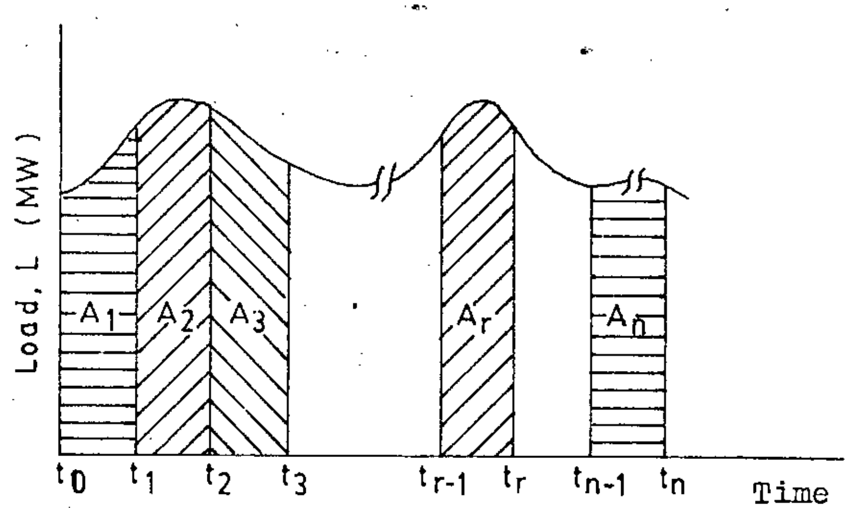

Fig. 3.7 : Chronological load curve whose time axis is divided into n small intervals .

The energy demand during the period between t_{r-1} and t_r is denoted by A_r under the chronological load curve which can be expressed as

$$A_r = \int_{t_{r-1}}^{t_r} L \, dt \qquad (3.21)$$

Average load in the interval , $t_r - t_{r-1}$ can be obtained by dividing A_r by the length of the interval. Thus,

$$L_r(avg) = \frac{A_r}{t_r - t_{r-1}} \tag{3.22}$$

In this way the average load for all other time intervals can be determined . Now if the average load for each interval is assumed to remain constant for the corresponding interval then figure 3.8 shows the approximated average load curve of the chronological load shown in figure 3.7 .

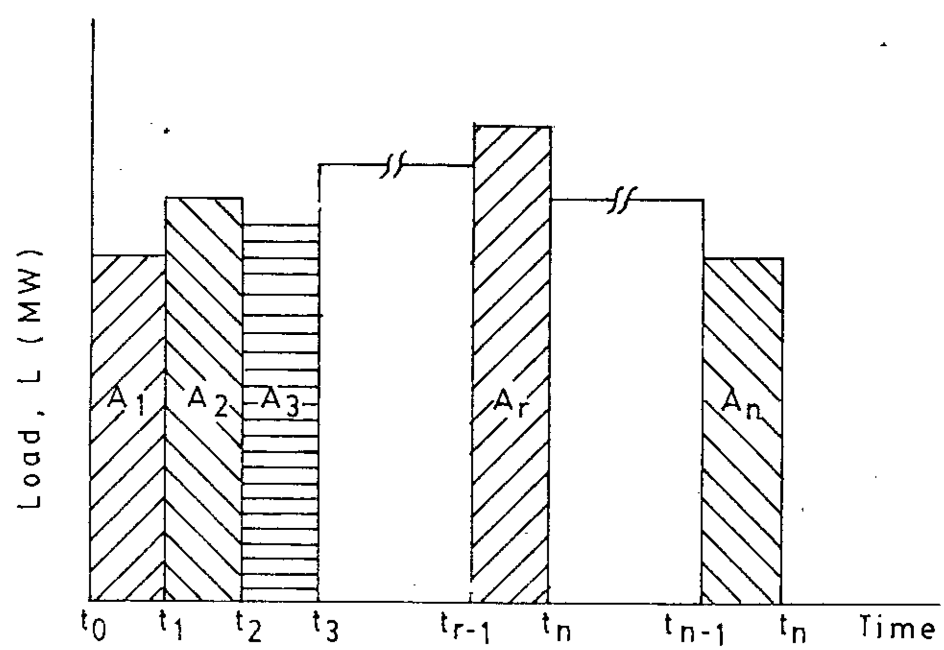

Fig. 3.8 Load curve assuming constant load for each small interval .

Note that in this process the energy demand for each interval must remain unchanged . If the time interval in figure 3.8 is one hour then the load distribution curve is called hourly load curve .

3.4 EFFECTIVE LOAD

Probabilistic models for both generating units and load have been discussed separately . Now , these two models will be combined to define the effective load of the system . The randomness in the availability of generation capacity is taken into consideration by defining a fictitious load, called equivalent load (Le) or the effective load . Figure 3.9 depicts the relationship between the system load and generating units , where actual units have been replaced by fictitious perfectly reliable units and fictitious random load ; whose probability density functions are the outage capacity density functions of the units [28].

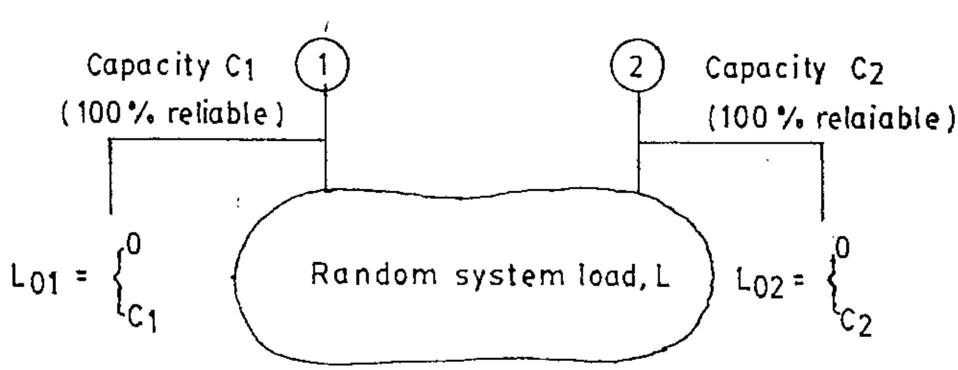

Fig. 3.9 : Fictitious generating units and system load model .

If Lo_i represents the random outage load corresponding to the i-th unit , the effective load (Le) can be defined by

$$Le = L + \sum_{i=1}^{n} Lo_i \qquad (3.23)$$

where , n is the total number of generating units . When $Lo_i = C$, the net demand injected into the system for the i-th unit is zero , just as it would be if the actual unit of capacity C_i were forced off-line . The installed capacity of the system is given by

$$IC = \sum_{i=1}^{n} C_i \qquad (3.24)$$

The outages of the generating units may be assumed independent of the system load . Therefore, the distribution of the equivalent load can be obtained after convolution of two distributions : f_{Lo} and f_L representing the PDFs of the outage capacity and the system load , respectively . For discrete case the PDFs f_L & f_{Lo} are given by

$$f_L(l) = \sum_i P_{L_i} (l - l_i) \qquad (3.25)$$

$$f_{Lo}(lo) = \sum_j P_{Lo_j} (lo - lo_j) \qquad (3.26)$$

The PDF of equivalent load f_{Le} is given by

$$f_{Le}(le) = f_L(l) * f_{Lo} (lo)$$

$$= \sum_{i,j} P_{L_i} P_{Lo_j} \delta (le - (l_i - + lo_j)) \qquad (3.27)$$

where * indicates the convolution and P_L & P_{Lo} are the probabilities of load and outages of machine respectively .

The reliability index, LOLP has already been defined in-terms of system load (L) and available capacity (AC) as

$$LOLP = Prob. \{ AC < L \}$$

LOLP can also be defined in terms of equivalent load(Le) and installed capacity as

$$LOLP = Prob. \{ Le > IC \} \qquad (3.28)$$

CHAPTER 4

METHODOLOGIES

4.1 INTRODUCTION

Probabilistic simulation method finds its wide use throughout the power industry as a useful tool in generation expansion planning . The method provides the expected energy generation , reliability index and production cost by taking into consideration the random outages of generation and demand . Since the introduction of this method , a number of different techniques have been developed with an ultimate target to improve its computational efficiency and flexibility . The probabilistic techniques that have been developed can be divided into two categories ,exact and approximate . The exact technique includes the 'Baleriaux-Booth' technique more commonly known as the 'recursive method' and the 'segmentation method' . On the other hand the approximate technique includes the 'cumulant method' also known as 'method of moments' . Thus , there are three methods to evaluate the reliability index ,

i) Recursive method

ii) Segmentation method

iii) Cumulant method

In this chapter , a brief description of all these methods will be presented to-gether with simple numerical examples for clarification .

4.2 RECURSIVE METHOD

This method depends on the construction of the load duration curve (LDC) showing the number of hours that any given load level is exceeded . The concept of equivalent load duration curve (ELDC) is introduced to take into account the random outages of units . This method starts with the probability distributions of load F(L) and the generation system $F(Lo_i)$. The probability distribution of equivalent load F(Le) is obtained by convolving F(L) and the PDF of outages of generating units . Usually generating units are convolved with load in economic merit order . That is the unit with the lowest incremental cost is convolved first , the next one with the second lowest incremental cost and so on . The figure 4.1 shows the equivalent load duration curve $F^r(Le)$, when the r th unit in the merit order is convolved with the load .

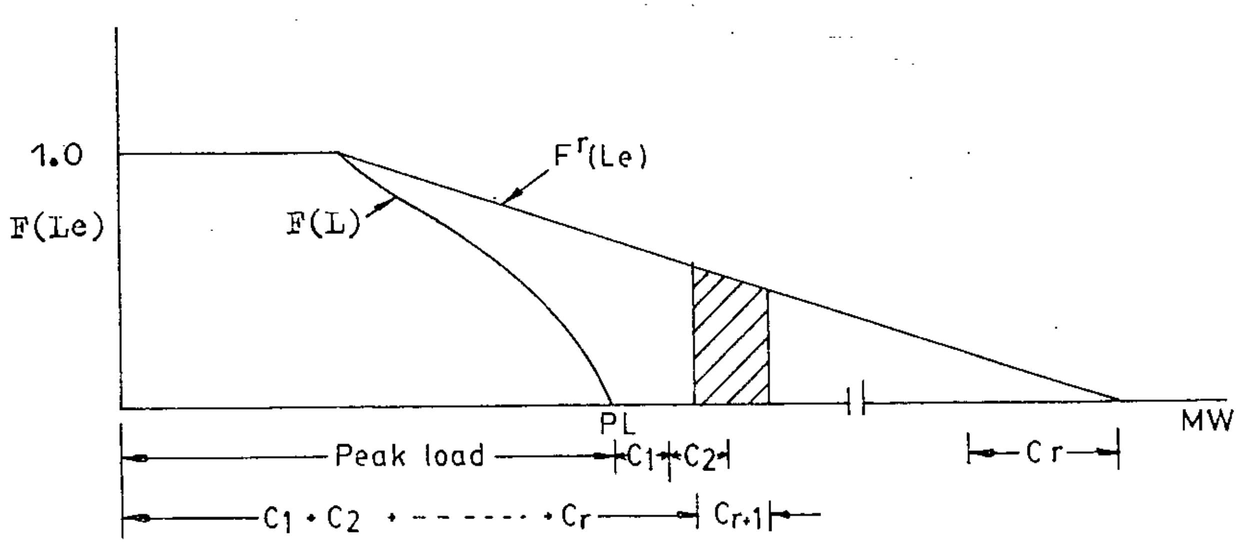

Fig. 4.1 : Probability distribution of load and equivalent load namely F(L) and F(Le).

In the figure , PL represents the peak load and C_1 , C_2.... C_r etc. represent the capacity of generating units already convolved in . The expected energy generation by the (r+1)-th unit is denoted by E_{r+1} and is represented by the area it occupies under the $F^r(Le)$ and the area is shown by be the hatched line . E_{r+1} is expressed as

$$E_{r+1} = T \; p_{r+1} \int_{k_1}^{k_2} F^r (Le) \; dLe \qquad (4.1)$$

where , T = Time period considered

p_{r+1} = availability of capacity C_{r+1} of the (r+1)-th unit = $1 - FOR_{r+1}$

$F^r(Le)$ = Cumulative probability function after convolving r-th unit

$$k_1 = \sum_{i=1}^{r} C_i \quad \text{and} \quad k_2 = \sum_{i=1}^{r+1} C_i \qquad (4.2)$$

C_i = Capacity of i-th unit

The final equivalent load probability distribution $F^n(Le)$ after convolving all the n units, is shown in figure 4.2

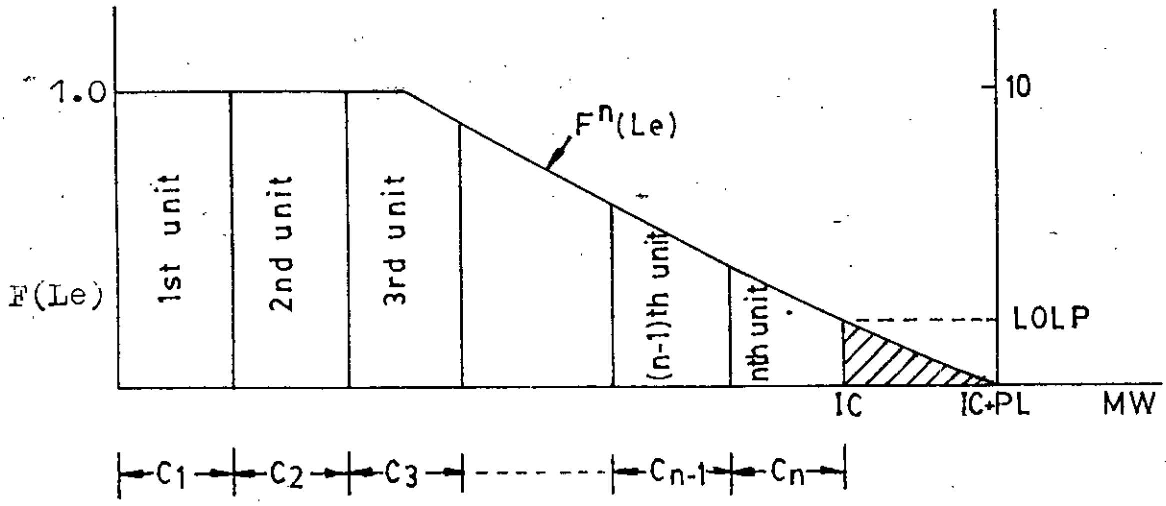

Fig. 4.2:Equivalent load probability distribution, $F^n(Le)$

In figure 4.2, IC represents installed capacity and is given by

$$IC = \sum_{i=1}^{n} C_i \qquad (4.3)$$

The reliability index LOLP is simply the cumulative probability at the point of final ELDC corresponding to installed capacity of the generating system.

4.2.1 CAPACITY OUTAGE PROBABILITY TABLES

In recursive approach , some time capacity outage tables are used especially if only peak load is used for the load model . A capacity outage probability table expresses the probability that various amounts of generating capacity will be unavailable . As the name suggests , it is a simple array of capacity levels and the associated probabilities of existence . In other words , it is a tabular representation of state probabilities usually given in terms of exact and cumulative probabilities . If all the units in the system are identical , the capacity outage probability table can be easily obtained using Binominal distribution which is explained below .[32].

4.2.1.1 BINOMIAL DISTRIBUTION METHOD FOR CAPACITY OUTAGE TABLE

Usually in reliability studies , a generating unit is represented by a two state model . The failure probability of a unit is given by unavailability or the forced outage rate, FOR and availability by p , where p =1-FOR . If a system has n identical units and g units fail then the different failed states are defined by g = 0, 1, 2, ...n , the total number of states being (n+1) . The outage probability of g units out of n units is given by [32]

$$P_g = {}^{n}C_g \, (1 - FOR)^{n-g} \, (FOR)^g \qquad (4.4)$$

Using Eq. (4.4) the state probabilities can be easily found and the outage table can be prepared . An additional column giving cumulative probabilities is often added to the table. The cumulative probability is the probability of finding a quantity of capacity on outage equal to or greater than the indicated amount . In what follows , the method is examplified for calarification .

Example - 1 : A generating system has three units of 10 MW each . FOR of each unit is 0.02. Prepare a capacity outage table for the system .

Solution : Here $n = 3$, $FOR = 0.02$

$$P_g = {}^nC_g\,(1 - FOR)^{n-g}\,(FOR)$$

when $g = 0,\ P_0 = {}^3C_0(1 - 0.02)^{3-0}\ (0.02)^0 = 0.941192$

when $g = 1,\ P_1 = {}^3C_1(1 - 0.02)^{3-1}\ (0.02)^1 = 0.057624$

when $g = 2,\ P_2 = {}^3C_2(1 - 0.02)^{3-2}\ (0.02)^2 = 0.001176$

when $g = 3,\ P_3 = {}^3C_3(1 - 0.02)^{3-3}\ (0.02)^3 = 0.000008$

The above result can be presented in tabular form as follows :

Table 4.1 : Capacity outage probability table

Number of Units out	Capacity out(MW)	Capacity available(MW)	Exact Probability	Cumulative Probability
0	0	30	0.941192	1.00000
1	10	20	0.057624	0.058808
2	20	10	0.001176	0.001184
3	30	0	0.000008	0.000008

It is extremely unlikely that all the units in a practical system will be identical and therefore the binomial distribution has limited application .

4.2.1.2 GENERALISED METHOD FOR CAPACITY OUTAGE TABLE

When a system has units of different sizes or different forced outage rates, the use of binomial approach to develope capacity outage table does not work . The procedure is to first prepare a separate table for each category of identical units . From these separate tables a combined table is prepared . In preparing the combined table every capacity outage state is , again , assumed independent. The probability of the simultaneously occurance of two or more independent events is the product of the probabilities of the respective events . The probabilities of all possible states must add to unity. Thus the units can be combined using basic probability concepts and this approach can be extended to a simple powerful recursive technique in which units are added sequentially to produce a final model . The concepts can be illustrated by a simple numerical example .

Example - 2 : A generating system consists of two 5 MW units and one 10 MW unit with forced outage rates of 0.02 . Prepare a capacity outage table .

Solution :

Two identical units of 5 MW each with FOR = 0.02 will be combined first to prepare one capacity outage table as follows :

$$0 \text{ MW out, Prob} = {}^{2}C_{0}\ (1 - 0.02)^{2-0}\ (0.02)^{0} = 0.9604$$

$$5 \text{ MW out, Prob} = {}^{2}C_{1}\ (1 - 0.02)^{2-1}\ (0.02)^{1} = 0.0392$$

$$10 \text{ MW out, Prob} = {}^{2}C_{2}\ (1 - 0.02)^{2-2}\ (0.02)^{2} = 0.0004$$

Thus for two identical unit following table can be constructed :

Table 4.2 : Capacity outage table for 2x5 MW Units

Capacity out (MW)	Exact Probability
0	0.9604
5	0.0392
10	0.0004
	1.0000

The 10 MW unit can be added to this table by considering that it can exist in two states . It can be in service with probability 1 - 0.02 = 0.98 as shown in Table 4.3 or it can be out of service with probability 0.02 as shown in Table 4.4 .

This approach can be extended to any number of unit states .

Table 4.3 : 10 MW unit in service

Capacity out (MW)	Probability			
0 + 0 = 0 MW	(0.9604)	(0.98)	=	0.941192
5 + 0 = 5 MW	(0.0392)	(0.98)	=	0.038416
10 + 0 = 10 MW	(0.0004)	(0.98)	=	0.000392
				0.980000

Table 4.4 : 10MW Unit out of Service

Capacity out (MW)	Probability			
0 + 10 = 10 MW	(0.9604)	(0.02)	=	0.019208
5 + 10 = 15 MW	(0.0392)	(0.02)	=	0.000784
10 + 10 = 20 MW	(0.0004)	(0.02)	=	0.000008
				0.020000

Tables 4.3 and 4.4 can now be combined and re-ordered resulting Table 4.5

Table 4.5 : Final capacity outage table

Capacity out of service (MW)	Individual Probability	Cumulative Probability
0	0.941192	1.000000
5	0.038416	0.058808
10	0.019600	0.020392
15	0.000784	0.000792
20	0.000008	0.000008
	1.000000	

The probability value in Table 4.5 is the probability of exactly the indicated amount of capacity being out of service. An additional column is added to give the cumulative probability. This values decrease as the capacity on outage increase. A recursive algorithm commonly used to obtain the cumulative probability is,

$$P_{new}(X) = P_{old}(X)(1 - q) + P_{old}(X-C)\,q \qquad (4.5)$$

where,

$P_{old}(0) = 1$

$P_{old}(X-C) = 1$ if $X \leq C$

$P_{old}(X)$ = Probability of a capacity outage of X MW or greater before a unit of capacity C MW is added

$P_{new}(X)$ = Probability of a capacity outage of X MW or greater after a unit of capacity C MW is added.

4.2.2 EVALUATION OF LOSS OF LOAD PROBABILITY (LOLP)

In this section , LOLP of the system described in example-2 of section 4.2.1.2 will be determined corresponding to a certain peak load . The random variable for total forced outage capacity is given by

$$X_T = X_1 + X_2 + X_3 \qquad (4.6)$$

where X_1 , X_2 , X_3 etc. denote the capacity of individual units . Since the random variables X_1 , X_2 and X_3 are independent (generating units are assumed to fail independently), the PDF of X_T is obtained by a process of convolution . The result of convolution has been given in Table 4.5 and the PDF of forced outage capacity for the system is shown in figure 4.3 and the cumulative probability distribution (CDF) is depicted in figure 4.4 .

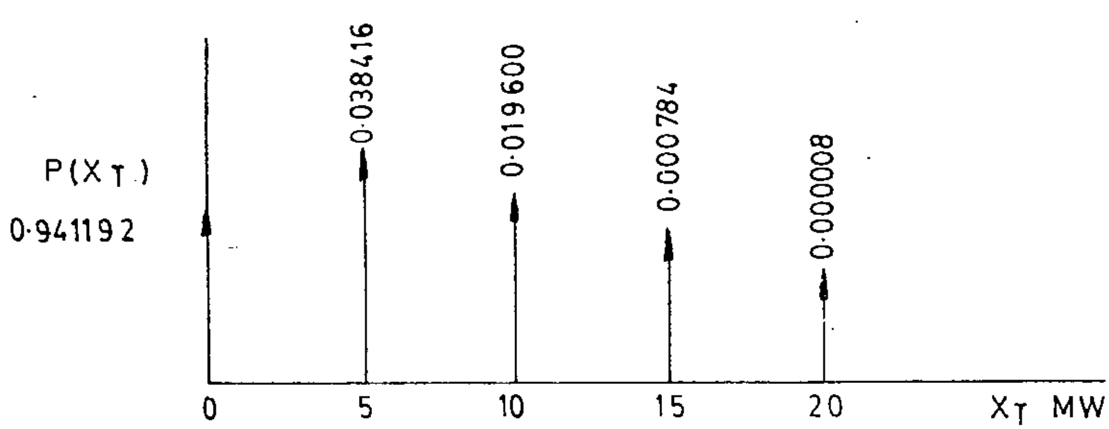

Fig. 4.3 : PDF of Forced outage capacity for the system of example 2 (impulses not to scale) .

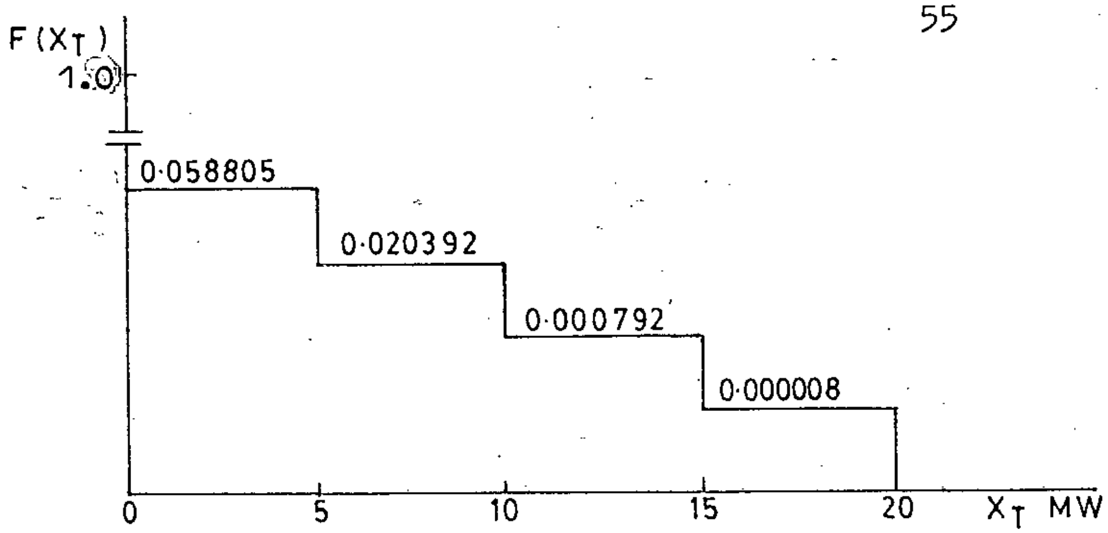

Fig. 4.4 : CDF of Forced outage capacity for the system of example 2 (not to scale) .

Now LOLP corresponding to a certain peak load is obtained by adding the PDF values of the impulses beyond that load or LOLP is the CDF value of the segment within which the peak load lies . Thus for example , if peak load is 14 MW then

LOLP = 0.000784 + 0.000008 = 0.000792

(From figure 4.3)

LOLP = 0.000792

(From figure 4.4)

For a practical system , LOLP is evaluated for each hourly load , all LOLPs are summed up and the average LOLP is obtained by dividing the sum by the total number of hours . Thus ,

$$LOLP = \frac{LOLP_1 + LOLP_2 + \ldots\ldots + LOLP_n}{\text{Total number of hours}} \quad (4.7)$$

where , $LOLP_1$ = LOLP corresponding the load of 1st hour

$LOLP_2$ = LOLP corresponding to the load of 2nd hour

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$LOLP_n$ = LOLP corresponding to the load of nth hour

4.3 SEGMENTATION METHOD

The segmentation method starts with the formation of segments of equal size by dividing the demand axis . The size of each segment depends on the largest common factor of the generating unit capacities . To each segment a probability value is attached which is equal to the sum of probabilities (zeroeth moments) of the load impulses lying in the range of the particular segment . One segment beyond the installed capacity (IC) is considered . It should be noted that the LOLP is obtained when the equivalent load is larger than the installed capacity . Therefore , the probability (zeroeth moment) attached to the last segment in the final distribution is the LOLP . Since the probability of occurrence of any load lower than the base load is zero , the formation of segments starts from the base load. Clearly it shows that the numerous number of impulses have been reduced to a few number of segments .

In order to account for the random outages of units it is necessary to get a new distribution of segments incorporating the outages of all units . Considering the k-th segment and assuming a generating unit of capacity C MW and FOR = q , to be convolved , the probability of the k-th segment , after the convolution may be expressed as [33]

$$\tilde{P}_k = P_k (1 - q) + \hat{P}_k \; q \qquad (4.8)$$

where

$\tilde{P}_k$ = Probability of the K-th segment after the convolution

$\hat{P}_k$ = Probability of the k-th segment after the shift

P_k = Probability of the k-th segment before convoluing the unit

The procedure to be followed in convolving a generating unit may be described as follows :

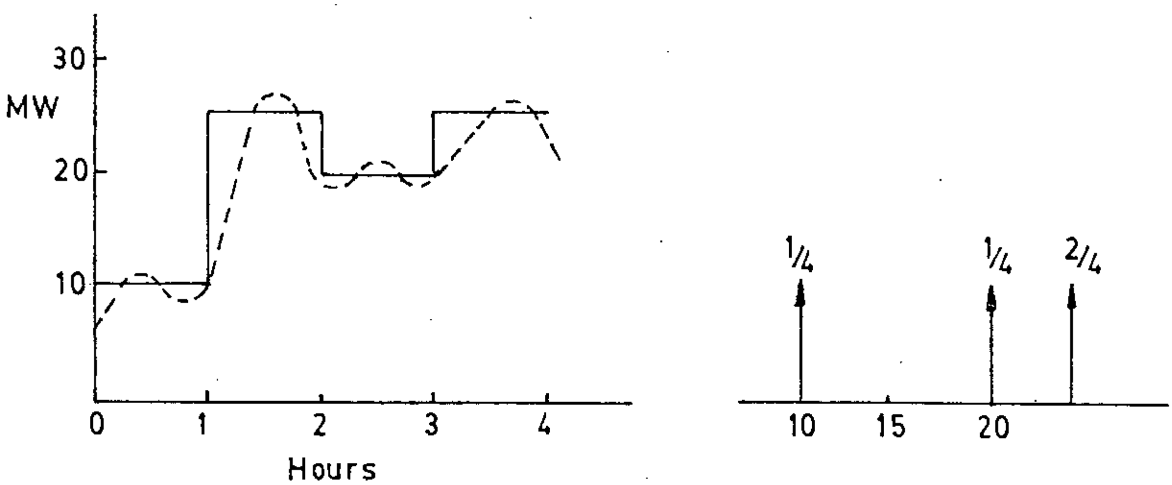
i) Multiply the original distribution of segments by the availability of the units , (1 - q) .

ii) Shift the original distribution by the unit capacity and multiply by the FOR of the unit , **q** .

iii) Add the values of the corresponding segments , obtained in (i) and (ii) above .

It should be noted that the probability value of the last segment is the sum of the probabilities of all the segments exceeding the installed capacity . Also , the segments below the already committed capacity can be deleted, since the probability values of these segments will not further contribute to the value of the last segment . Therefore , as the convolution process proceeds , the number of segments decrease. In what follows , an example is presented to clarify the method .

Figure 4.5 represents the load and Table 4.6 represents the generating system . Dotted line represents the chronological load while firm line represents the hourly load .



a) Choronological and hourly load profile

b) LDF of load

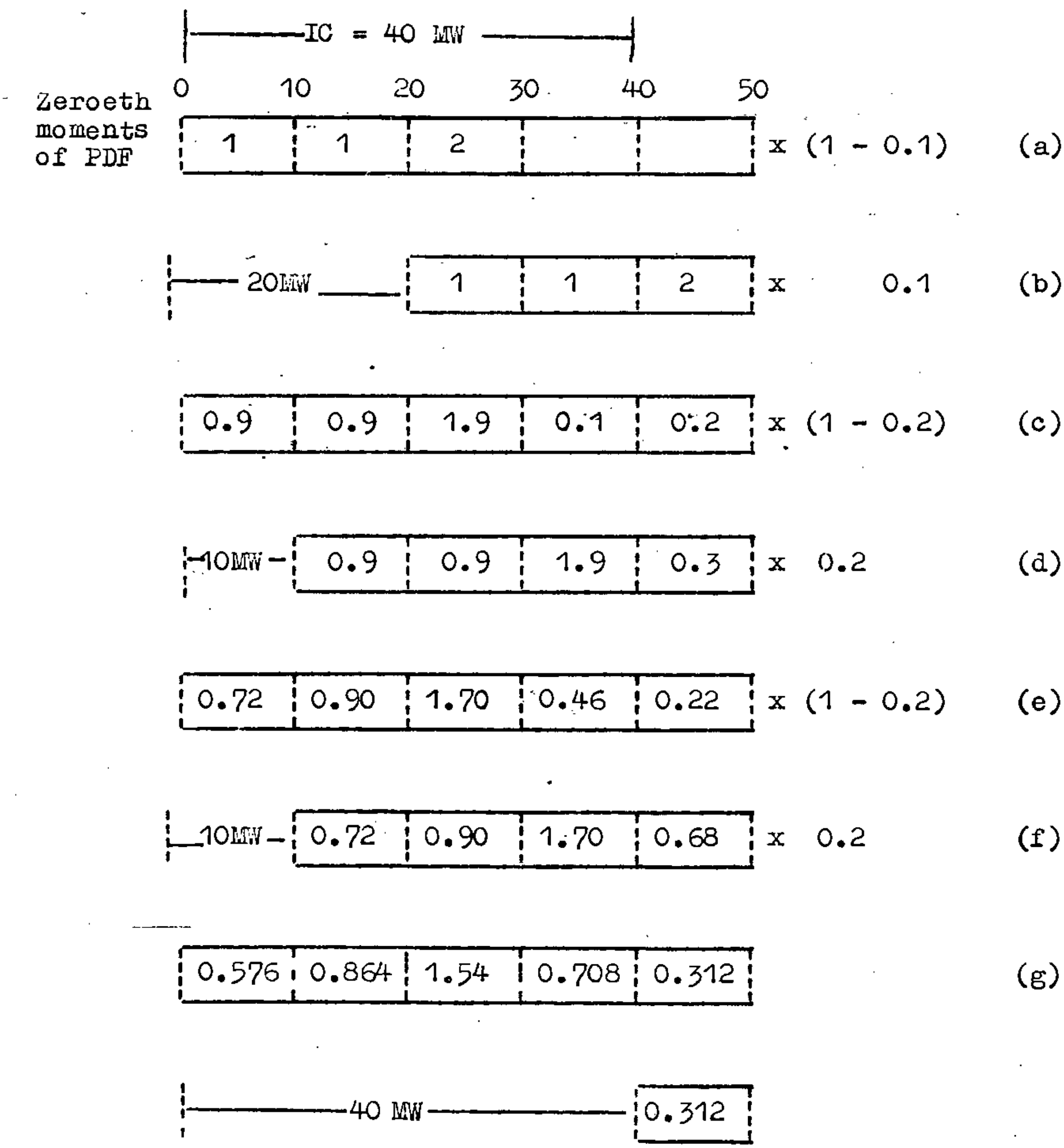
Fig. 4.5 : Hourly load representation

The hourly load of figure 4.5 (a) is sampled at an interval of one hour and by assigning to each sampled hourly load an equal probability , i.e.,1/4 in this case , the PDF of load shown in figure 4.5 (b) is obtained .

Table 4.6 : Generating system description

No. of Units	Capacity (MW)	FOR	Installed Capacity(MW)
1	20	0.10	40
2	10	0.20	

The largest common factor of the generating unit capacities of Table 4.6 is 10 MW and hence the segment size can be chosen to be 10 MW . The demand axis up to 40 MW is divided into 40 ÷ 10=4 segments each of 10 MW size . One additional segment is considered at the end which is shown in figure 4.6 (a) . The probability value of each segment corresponding to the respective impulse of figure 4.5 (b), lying in the range of the particular segment is also shown in figure 4.6(a) . Note that the numbers shown in the boxes of figure 4.6 should be divided by 4 to get the actual value of PDF .



ig. 4.6 : Schematic of convolution procedure (All numbers in the boxes to be divided by 4) .

The different steps of convolution of load and the generating units are depicted in figure 4.6 . To convolve the first 20 MW unit the segments of figure 4.6(a) are shifted towards right in figure 4.6 (b) by the unit capacity , i.e., 20 MW . The original distribution in figure 4.6 (a) is multiplied by the availability of the unit i.e., 0.9 and the shifted distribution of figure 4.6(b) is multiplied by the FOR of the unit which is 0.1 . The distribution after convolution is obtained by adding the probability values of the corresponding segments of figure 4.6(a) and figure 4.6 (b). This is shown in figure 4.6 (c) . The same procedure is followed for the rest of the units .

Note that the segments below the convolved capacity may be deleted during the convolution process since the probability values of these segments do not contribute further in the evaluation of LOLP . Also , the probability values are shifted towards the last segment and a number of them may be accumulated in this special segment . Thus , the last segment of figure 4.6(f) is the sum of last two segments of figure 4.6(e) .

Now the LOLP is simply the probability value of the last segment of figure 4.6(g) , since LOLP is obtained when the equivalent load is larger than the installed capacity . Thus ,

$$\text{LOLP} = \frac{0.312}{4} = 0.078$$

4.3.1 COMPUTATIONAL STEPS OF SEGMENTATION METHOD

The different computational steps to evaluate LOLP . Using segmentation method are as follows :

Step - 1 : Obtain hourly load for the period under study, from the chronological load (this may be predicted demand in case of planning) .

Step - 2 : Sample the hourly load at every hour or any other suitable interval by assigning equal probability to each sample and obtain the distribution of load .

Step - 3 : Obtain the distribution of segments by dividing the demand axis and assigning a probability to each segment equal to the sum of the probabilities of the load impulses lying in the range of that particular segment.

Step - 4 : Convolve the generating units one by one in any order. Because merit order loading is not required for the evaluation of LOLP

LOLP is obtained from the final distribution of segments .

4.4 CUMULANT METHOD

The cumulant method also known as the method of moment is an approximate technique which approximates the discrete distribution of load through Gramcharlier series expansion as a continuous function . In this method , convolution of unit outages with the distribution of load is performed through a very fast algorithm.

The repeated convolution of any n density functions can be expressed by the Gram Charlier's expansion series in terms of the normalized standard variable z as [20]

$$f(z) = N(z) - \frac{G_1 N^{(3)}(Z)}{3!} + \frac{G_2 N^{(4)}(z)}{4!} - \frac{G_3 N^{(5)}(z)}{5!}$$

$$\frac{(G_4 + 10G_1^2) N^{(6)}(z)}{6!} - \frac{(G_5 + 35 G_1G_2) N^{(7)}(z)}{7!}$$

$$+ \frac{(G_6 + 56 G_1G_3 + 35 G_2^2) N^{(8)}(z)}{8!} \qquad (4.9)$$

where the normal PDF N(z) and its derivatives are given by

$$N(z) = \frac{1}{\sqrt{2\pi}} \exp(-z^2/2) \qquad (4.10)$$

$$N^r(z) = \frac{d^r}{dz^r} N(z) \; ; \; r = 1,2,3,\ldots\ldots \qquad (4.11)$$

The normal PDF and its derivatives may be obtained using the following recursive relations

$$N^{(r)}(z) = -(r-1)\,N^{(r-2)}(z) - Z N^{(r-1)}(z) \qquad (4.12)$$

$$r = 3, 4, 5, \ldots\ldots\ldots\ldots$$

and

$$N^{(1)}(z) = -Z\,N(z)$$

$$N^{(2)}(Z) = (Z^2 - 1)\quad N(Z)$$

Using these recursive relations Eq (4.9) can be expressed in terms of N(z) and powers of z , the normalized variable , normalized capacity in MW in this case . The co-efficients G_1, G_2 , G_3 ,.............etc. are expansion factors expressed in terms of cumulants and the cumulants are the functions of moments . The n-th moment m_n of any PDF p(x) is given by

$$m_n = \int_{-\infty}^{\infty} x^n\, p(x)\, dx \qquad (4.13)$$

This method is general in nature and can be used for the convolution of any number of PDFs . Any PDF obeys the law that the area under it always equal to unity i.e.

$$\int_{\infty}^{\infty} p\,(x)\,dx = 1\ .$$

In case of a two-state representation of the machine say the i-th machine in a system , the PDF consists of just two impulses , one of magnitude p_i at 0 MW and other of magnitude q_i (FOR) at C_i MW and that $p_i + q_i = 1$. The moments (about the origin) of such PDFs are given by

$$m_n(i) = C_i^n q_i \; ; \quad n = 1 , 2, 3, \ldots\ldots 8 \qquad (4.14)$$

For any i-th machine the moments upto order about the origin may be calculated using the following relations .

$$\begin{aligned} m_1(i) &= C_i^1 q_i \\ m_2(i) &= C_i^2 q_i \\ m_3(i) &= C_i^3 q_i \\ m_4(i) &= C_i^4 q_i \\ m_5(i) &= C_i^5 q_i \\ m_6(i) &= C_i^6 q_i \\ m_7(i) &= C_i^7 q_i \\ m_8(i) &= C_i^8 q_i \end{aligned} \qquad (4.15)$$

For the i-th machine the central moments (moments about the mean) are calculated using the following relations

$$M_1(i) = m_1(i) \tag{4.16}$$

$$M_2(i) = v_i^2 = m_2(i) - m_1^2(i)$$

$$M_3(i) = m_3(i) - 3\, m_2(i)\, m_1(i) + 2\, m_1^3(i)$$

$$M_4(i) = m_4(i) - 4\, m_3(i)\, m_1(i) + 6 m_1^2(i) m_2(i) - 3 m_1^4(i)$$

$$M_5(i) = m_5(i) - 5\, m_4(i)\, m_1(i) + 10 m_3(i)\, m_1^2(i) - 10\, m_2(i)\, m_1^3(i) + 4 m_1^5(i)$$

$$M_6(i) = m_6(i) - 6\, m_5(i)\, m_1(i) + 15 m_4(i)\, m_1^2(i) - 20\, m_3(i)\, m_1^3(i) + 15 m_2(i)\, m_1^4(i) - 5\, m_1^6(i)$$

$$M_7(i) = m_7(i) - 7\, m_6(i)\, m_1(i) + 21 m_5(i)\, m_1^2(i) - 35\, M_4(i)\, m_1^3(i) + 35 m_3(i)\, m_1^4(i) - 21\, m_2(i)\, m_1^5(i) + 6 m_1^7(i)$$

$$M_8(i) = m_8(i) - 8\, m_7(i)\, m_1(i) + 28\, m_6(i)\, m_1^2(i) - 56 m_5(i)\, m_1^3(i) + 70\, m_4(i)\, m_1^4(i) - 56 m_3(i)\, m_1^5(i) + 28 m_2(i)\, m_1^6(i) - 7\, m_1^8(i)$$

For the i-th machine the cumulants are calculated using the following relations.

$$
\begin{aligned}
K_1(i) &= M_1(i) \\
K_2(i) &= M_2(i) = V_i^2 \\
K_3(i) &= M_3(i) \\
K_4(i) &= M_4(i) - 3 M_2^2(i) \\
K_5(i) &= M_5(i) - 10M_2(i) M_3(i) \\
K_6(i) &= M_6(i) - 15M_2(i)M_4(i) - 10M_3^2(i) + 30M_2^3(i) \\
K_7(i) &= M_7(i) - 21M_5(i)M_2(i) - 35M_4(i)M_3(i) \\
&\quad + 210 M_3(i) M_2^2(i) \\
K_8(i) &= M_8(i) - 28 M_6(i) M_2(i) - 56 M_5(i) M_3(i) \\
&\quad - 35M_4^2(i) + 420 M_4(i) M_2^2(i) + 560 M_3^2(i) M_2(i) \\
&\quad - 630 M_2^4(i)
\end{aligned}
\tag{4.17}
$$

Load moments of order n about origin may be obtained from the LDC using the following relation ;

$$
m_{nL} = \frac{1}{A} \int_0^{PL} x^n f(x)\, dx \tag{4.18}
$$

where A = Area under the LDC

PL = Peak Load .

In case of hourly load the moments of order n about origin may be calculated from the following relation

$$m_n(i) = \frac{1}{T} \sum_{i=1}^{T} (L)^n \quad n = 1, 2, \; ....8 \qquad (4.19)$$

The central moments of load may be obtained from the relation given in (4.16) . Then the cumulants may also be obtained using the same relation as those used for machine i.e. the equations (4.17) .

From the properties of statistical cumulants one knows that random variable (of equivalent load) which is the sum of independenet random variables generating machine outages and hourly load , is characterized by cumulants which are the sum of machine cumulants and load cumulants . Therefore , the process of convolution is performed by the summation of cumulants only. In a system of r units , the cumulants of equivalent load may be expressed as

$$K_k(EL_r) = K_k\,(L) + \sum_{i=1}^{r} K_k(i) \qquad (4.20)$$

$$i = 1,2,3, \; ....$$

where ,

$K_k(EL_r)$ = k-th cumulant of equivalent load when r units have been convolved .

$K_k(L)$ = k-th cumulant of the hourly load .

$K_k(i)$ = k-th cumulant of the i-th generating unit .

Note that the first cumulant of equivalent load is the mean (M) and the second cumulant is the square of standard deviation (V^2) of the distribution .

The expansion factors G_1, G_2, G_3 ...... etc. of the Gram-Charlier series are calculated by

$$G_i = K_{(i+2)} (El_r) \;/\; K_{(2)}^{(i+2)/2} (El_r) \qquad (4.21)$$

$$i = 1, 2, 3, \ldots \quad \ldots \quad \ldots$$

The values up to equation (4.17) are considered to be fundamental parameters and are stored . Convolution of additional machines will involve only recalculation of equations (4.20) and (4.21) .

Having obtained the G - coefficients as outlined so far, the Gram - Charlier series describing the convolution of LDC with the machine outages is obtained . The area under the equivalent load curve between some limits say Z_1 & Z_2 and this area is given by

$$a = \int_{Z_1}^{\infty} f(z)dz \;-\; \int_{Z_2}^{\infty} f(z)\, dz \qquad (4.22)$$

where ,

f(z) = Equivalent load distribution

Z_i = Standardized random variables (RVs)

$$= (X_i - M_i) / V_i \tag{4.23}$$

in which X_i is any capacity (MW) , and M_i and V_i are the mean and standard deviation of the equivalent load distribution .

The integral in equation (4.22) is calculated as follows :

$$I = \int_{Z_i}^{\infty} f(z)\, dz = \int_{Z_i}^{\infty} N(z)\, dz + F(Z_i) \tag{4.24}$$

Where, $N(Z_i) = \frac{1}{\sqrt{2\pi}} \exp(-Z_i^2/2)$

$$F(Z_i) = \frac{G_1 N^{(2)}(Z_i)}{3!} - \frac{G_2 N^{(3)}(Z_i)}{4!} + \frac{G_3 N^{(4)}(Z_i)}{5!} - \frac{(G_4 + 10G_1^2) N^{(5)}(Z_i)}{6!} + \frac{(G_5 + 35\, G_1 G_2) N^{(6)}(Z_i)}{7!} - \frac{(G_6 + 56\, G_1 G_3 + 35\, G_2^2)\, N^{(7)}(Z_i)}{8!} \tag{4.25}$$

Equation (4.22) consists of areas under the normal probability density function and factor $F(Z_i)$ which can be readily obtained . By building a normal table of areas in the program, a numerical integration is avoided . The area under the normal curve can be found from the following polynomial approximation for $Z \geqslant 0$. Suppose the area Q (Z) shown in figure 4.7 is required .

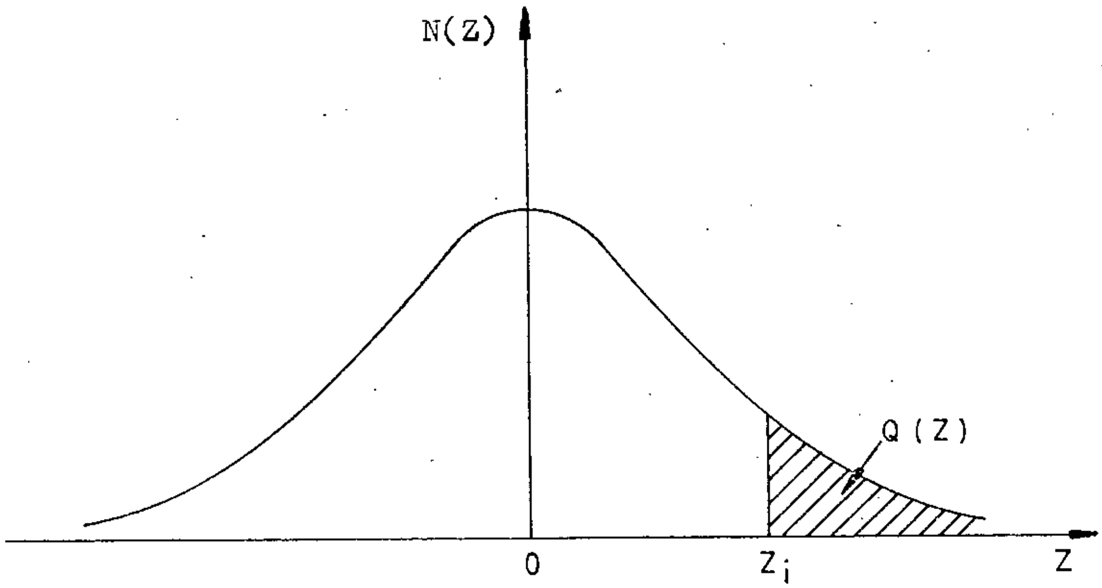


Fig. 4.7 : Area under normal density function .

The area Q(Z) is given by [29]

$$Q(Z) = y \left[b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \right] + e(Z) \qquad (4.26)$$

where ,

$$y = N(Z) = \frac{1}{\sqrt{2\pi}} \exp(-Z^2/2)$$

$t = 1/(1 + rZ)$

$r = 0.2326419$

$b_1 = 0.31938153$

$b_2 = -0.356563782$

$b_3 = 1.781477937$

$b_4 = -1.821255978$

$b_5 = 1.330274429$

and the error is $|e(z)| < 7.5 \times 10^{-8}$ and therefore , can be neglected . The expression for area under normal density function is ,

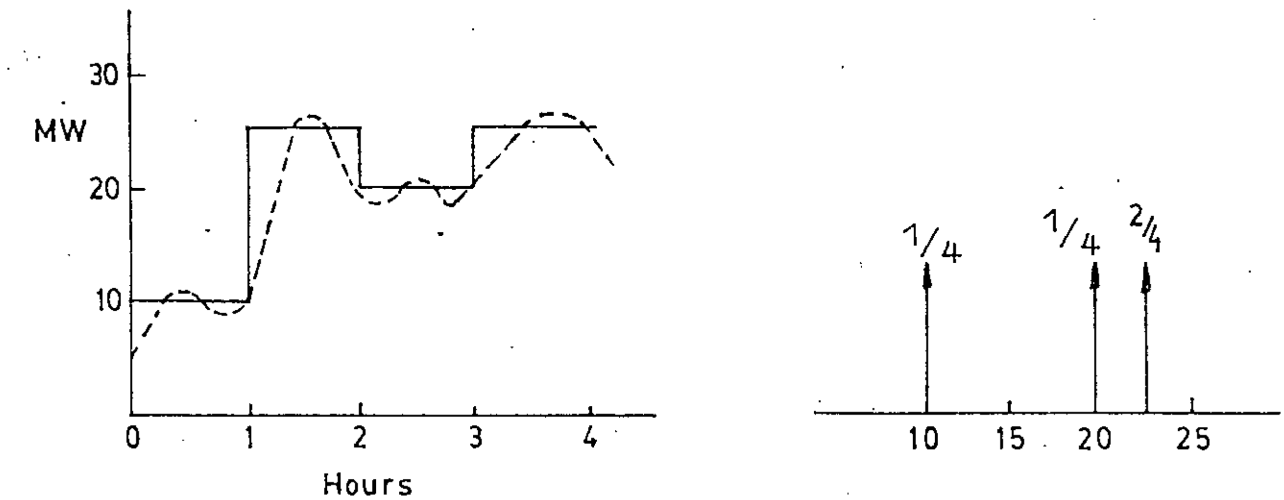
$$Q(Z_i) = N(Z_i)\left[b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5\right] \qquad (4.27)$$

Equation (4.27) is sufficiently accurate for all practical purposes . The expression is only valid for positive values of Z_i. Since normal curve is symmetrical , the values of $Q(-Z)$ can be found from

$$Q(-Z) = Q(Z) \qquad (4.28)$$

The reliability index , LOLP of the system is the value of the ordinate of the final ELDC (which is obtained after convolution of all machines in the system) at the installed capacity . An example is presented below to clarify the method .

Figure 4.7 below represents the load and PDF of load and Table 4.7 represents the generating system .



a) Chronological and hourly load profile .   b) PDF of load

Fig. 4.7 : Hourly load representation

Table 4.7: Generating System description

No. of Units	Capacity (MW)	FOR	Installed Capacity(MW)
1	20	0.10	40
2	10	0.20	

Solution :

Calculation of moments of the generating Units about the origin :

1) <u>For 20 MW Unit with FOR = 0.10</u>

$m_1(1) = (20)^1 \ (0.10) = 2$

$m_2(1) = (20)^2 \ (0.10) = 40$

$m_3(1) = (20)^3 \ (0.10) = 800$

$m_4(1) = (20)^4 \ (0.10) = 16000$

$m_5(1) = (20)^5 \ (0.10) = 3.2 \times 10^5$

$m_6(1) = (20)^6 \ (0.10) = 6.4 \times 10^6$

2) <u>For 10 MW Unit with FOR = 0.20</u>

$m_1(2) = (10)^1 \ (0.20) = 2$

$m_2(2) = (10)^2 \ (0.20) = 20$

$m_3(2) = (10)^3 \ (0.20) = 200$

$m_4(2) = (10)^4 \ (0.20) = 2000$

$m_5(2) = (10)^5 \ (0.20) = 2 \times 10^4$

$m_6(2) = (10)^6 \ (0.20) = 2 \times 10^5$

Calculation of mo-ments of the generating units about the mean :

1) <u>For 20 MW Unit</u>

$M_1(1) = 2$

$M_2(1) = 40 - (2)^2 = 36$

$M_3(1) = 800-3(40)(2) + 2(2)^3 = 576$

$M_4(1) = 16000-4(800)(2)+6(2)^2(40)-3(2)^4 = 10512$

$M_5(1) = 3.2x10^5-5(16000)(2)+10(800)(2)^2-10(40)(2)^3+4(2)^5 = 188928$

$$M_6(1) = 6.4x10^6-6(3.2x10^5)(2) + 15(16000)(2)^2 - 20(800)(2)^3 + 15(40)(2)^4 - 5(2)^6$$

$$= 3401280$$

2) <u>For 10 MW Unit</u>

$M_1(2) = 2$

$M_2(2) = 20 - (2)^2 = 16$

$M_3(2) = 200- 3(20)(2) + 2(2)^3 = 96$

$M_4(2) = 2000- 4(200)(2) + 6(2)^2(20) - 3(2)^4 = 832$

$$M_5(2) = 20000-5(2000)(2) + 10(200)(2)^2 - 10(20)(2)^3+4(2)^5$$

$$= 6528$$

$$M_6(2) = 2 x 10^5- 6(20000)(2) + 15(2000)(2)^2 - 20(200)(2)^3 + 15(20)(2)^4 - 5(2)^6$$

$$= 52480$$

Calculation of cumulants of generating Units :

1) <u>For 20 MW Unit</u>

$K_1(1) = 2$

$K_2(1) = 36$

$K_3(1) = 576$

$K_4(1) = 10512 - 3\,(36)^2 = 6624$

$K_5(1) = 188928 - 10(36)(576) = -18432$

$K_6(1) = 3401280 - 15(36)(10512) - 10(576)^2 + 30(36)^3$

$= -4193280$

2) <u>For 10 MW Unit</u>

$K_1(2) = 2$

$K_2(2) = 16$

$K_3(2) = 96$

$K_4(2) = 832 - 3\,(16)^2 = 64$

$K_5(2) = 6528 - 10(16)(96) = -8832$

$K_6(2) = 52480 - 15(16)(832) - 10(96)^2 + 30(16)^3$

$= -116480$

Calculation of total cumulants of generating units of same size :

1) <u>For 1x20 MW Unit</u>

$K(1,1) = 1(2) = 2$

$K(1,2) = 1(36) = 36$

$K(1,3) = 1(576) = 576$

$K(1,4) = 1(6624) = 6624$

$K(1,5) = 1(-18432) = -18432$

$K(1,6) = 1(-4193280) = -4193280$

2) <u>For 2x10 MW Unit</u>

$K(2,1) = 2(2) = 4$

$K(2,2) = 2(16) = 32$

$K(2,3) = 2(96) = 192$

$K(2,4) = 2(64) = 128$

$K(2,5) = 2(-8832) = -17664$

$K(2,6) = 2(-116480) = -232960$

Calculation of total cumulants of generating units :

$$K_1(g) = K(1,1) + K(2,1) = 2+4 = 6$$

$$K_2(g) = K(1,2) + K(2,2) = 36 + 32 = 68$$

$$K_3(g) = K(1,3) + K(2,3) = 576 + 192 = 768$$

$$K_4(g) = K(1,4) + K(2,4) = 6628 + 128 = 6756$$

$$K_5(g) = K(1,5) + K(2,5) = (-18432) + (-17664) = -36096$$

$$K_6(g) = K(1,6) + K(2,6) = (-4193280) + (-232960) = -4426240$$

Calculation of moments of hourly load about the origin :

$$m_1(l) = \frac{1}{4}(10 + 20 + 25 + 25) = 20$$

$$m_2(l) = \frac{1}{4}(10^2 + 20^2 + 25^2 + 25^2) = 437.50$$

$$m_3(l) = \frac{1}{4}(10^3 + 20^3 + 25^3 + 25^3) = 10062.50$$

$$m_4(l) = \frac{1}{4}(10^4 + 20^4 + 25^4 + 25^4) = 237812.50$$

$$m_5(l) = \frac{1}{4}(10^5 + 20^5 + 25^5 + 25^5) = 5707812.50$$

$$m_6(l) = \frac{1}{4}(10^6 + 20^6 + 25^6 + 25^6) = 138320312.50$$

Calculation of central moments of hourly load :

$$M_1(1) = 20$$

$$M_2(1) = 437.50 - (20)^2 = 37.50$$

$$M_3(1) = 10062.50 - 3(437.50)(20) + 2(20)^3 = -187.50$$

$$M_4(1) = 237812.50 - 4(10062.50)(20) + 6(20)^2(437.50) - 3(20)^4$$

$$= 2812.50$$

$$M_5(1) = 5707812.50 - 5(237812.50)(20) + 10(10062.50)(20)^2 - 10(437.50)(20)^3 + 4(20)^5.$$

$$= -23437.50$$

$$M_6(1) = 138320312.50 - 6(5707812.50)(20) + 15(237812.50(20)^2 - 20(10062.50)(20)^3 + 15(437.50)(20)^4 - 5(20)^6$$

$$= 257812.50$$

Calculation of cumulants of hourly load :

$$K_1(1) = 20$$

$$K_2(1) = 37.50$$

$$K_3(1) = -187.50$$

$$K_4(1) = 2812.50 - 3(37.50)^2 = -1406.25$$

$$K_5(1) = -23437.50 - 10(37.50)(-187.50)$$

$$= -10(-187.50)^2 + 30(37.50)^3$$

$$= -93750.00$$

Calculation of cumulant of equivalent load or system cumulent :

$K_1(EL) = K_1(l) + K_1(g) = 20 + 6 = 26$

$K_2(EL) = K_2(l) + K_2(g) = 37.50 + 68 = 105.50$

$K_3(EL) = K_3(l) + K_3(g) = -187.50 + 768 = 580.50$

$K_4(EL) = K_4(l) + K_4(g) = -1406.25 + 6756 = 5349.75$

$K_5(EL) = K_5(l) + K_5(g) = 46875.00 - 36096 = 10779.00$

$K_6(EL) = K_6(l) + K_6(g) = -93750.00 - 4426240 = -4519990.00$

Calculation of 'G' Co-efficients :

$$G_i = K_{i+2}(EL) / K_2^{(i+2)/2}(EL) \qquad i = 1, 2, 3, \ldots\ldots$$

$$G_1 = K_3(EL)/ K_2^{3/2}(EL) = 580.50/(105.50)^{3/2} = 0.5357023$$

$$G_2 = K_4(EL)/K_2^{2}(EL) = 5349.75/(105.50)^{2} = 0.4806496$$

$$G_3 = K_5(EL)/K_2^{5/2}(EL) = 10779.00/(105.50)^{5/2} = 0.094286$$

$$G_4 = K_6(EL)/K_2^{3/2}(EL) = -4519990.00/(105.50)^{3} = -3.849285$$

Calculation of LOLP :

Installed Capacity , X_i = 40 MW

$$Z_i = (X_i - M)/V = (X_i - K_1(EL))/ \ K_2(EL)$$

$$= (40 - 26) / \ 105.50$$

$$= 1.3630187$$

$$Q(Z_i) = N(Z_i) \ (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5)$$

$$N(Z_i) = \frac{1}{2\pi} \exp\left(-\frac{(1.3630187)^2}{2}\right) = 0.1575758$$

$$t = \frac{1}{1 + rZ_i} = \frac{1}{1 + (0.2316419)(1.3630187)} = 0.760033$$

$$Q(Z_i) = 0.1575758 \Big[(0.31938153)(0.760033) + (-0.356563782) \times$$

$$(0.760033)^2 + (1.781477937)(0.760033)^3$$

$$+ (-1.821255978)(0.760033)^4$$

$$+ (1.330274429)(0.760033)^5 \Big]$$

$$= 0.086438$$

Calculation of derivatives :

$$N^{(1)}(Z_i) = Z_i\ N(Z_i) = -(1.3630187)\ (0.1575758)$$

$$= -0.2147787$$

$$N^{(2)}(Z_i) = (Z_i^2 - 1)\ N(Z_i) = \left[(1.3630187)^2 - 1\right](0.1575758)$$

$$= 0.1351717$$

$$N^{(3)}(Z_i) = 2\ N^{(1)}(Z_i) - Z_i\ N^{(2)}(Z_i)$$

$$= -2\ (0.2147787) - (1.3630187)\ (0.1351717)$$

$$= -\ 0.613799$$

$$N^{(4)}(Z_i) = -\ 3\ N^{(2)}(Z_i) - Z_i\ N^{3}(Z_i)$$

$$= -\ 3\ (0.1351717) - (1.3630187)\ (-0.613799)$$

$$= 0.431104$$

$$N^{(5)}(Z_i) = -\ 4\ N^{(3)}(Z_i) - Z_i\ N^{(4)}(Z_i)$$

$$= -\ 4\ (-0.613799) - (1.3630187)\ (0.431104)$$

$$= 1.867592$$

$$F(Z_i) = \frac{G_1 N^{(2)}(Z_i)}{3\,!} - \frac{G_2 N^{(3)}(Z_i)}{4\,!} + \frac{G_3\,N^{(4)}(Z_i)}{5\,!} -$$

$$= \frac{(G_4 + 10\ G_1^2)\ N^{(5)}(Z_i)}{6\,!}$$

$$= \frac{(0.5357023)\ (0.1351717)}{3\,!} - \frac{(0.4806496)\ (-0.613799)}{4\,!}$$

$$+ \frac{(0.094286)\ (0.431104)}{5\,!}$$

$$- \frac{(-3.849285 + 10\ (0.5357023)^2\ (1.867592)}{6\,!}$$

$$= 0.0272406$$

$$\text{LOLP} = Q(Z_i) + F(Z_i)$$

$$= 0.086438 + 0.0272406$$

$$= 0.1136787$$

4.4.1 COMPUTATIONAL STEPS FOR CUMULANT METHOD

The different computational steps to evaluate LOLP by the cumulant method are stated below :

Step - 1 : Obtain hourly load , for the period under study, from the chronological load (this may be predicted demand in case of planning) .

Step - 2 : Sample the hourly load at every hour or any other suitable interval by assigning equal probability to each sample and obtain the distribution of load .

Step - 3 : Determine the moments about the origin for each machine .

Step - 4 : Determine the central moments (moments about the mean) for each machine .

Step - 5 : Obtain the cumulants for each machine

Step - 6 : Obtain the total cumulants of generating system by summing the individual generating unit cumulants .

Step - 7 : Now , determine the moments about the origin , moments about the mean (Central moments) and cumulants of the hourly loads .

Step - 8 : Determine the system cumulants by summing the total cumulants of generating units and corresponding cumulants of load .

Step - 9 : Obtain the Gram-Charlier coefficients and the derivatives of normal PDF .

Step -10 : Calculate the standardized random variable Z_i .

Step - 11: Determine the area under the normal PDF and the factor $F(Z_i)$.

LOLP is obtained from the sum of the area under the noraml PDF and the function $F(Z_i)$.

CHAPTER 5

NUMERICAL EVALUATION

5.1 INTRODUCTION :

The different methodologies of evaluating the reliability of power system have been discussed in the previous chapter. In this chapter, the methodologies are applied to evaluate the reliability indices of IEEE reliability test system as well as Bangladesh Power System . The methods are compared in terms of

i) accuracy of the results

ii) computational efficiency and

iii) computer storage requirements .

In this chapter the observations regarding the sensitivity of the recursive method to step size , the segmentation method to segment size and the cumulant method to the number of terms in the Gram-charlier series , are also persented .

The sensitiveness of each method to the variation of peak load of the above two systems are discussed in this chapter. The sensitivity of each method in terms of accuracy of result due to types of precission in the computation are furnished in this chapter . A brief description of IEEE Reliability Test System and Bangladesh Power System are also presented in this chapter .

5.2 IEEE RELIABILITY TEST SYSTEM (IEEE-RTS) [25]

In order to provide a basis for comparision of results obtained from different methods , IEEE desired to have a reference or test system which incorporates the basic data needed in reliability evaluation . The test system has load, generation system and transmission network model . The load model provides hourly loads for one year on per unit basis expressed in chronological fashion . The generating system contains 32 units of various capacity from 12 to 400 MW. The transmission system contains 24 load/generation bus connected by 38 lines or autotransformers at two stages, 138 and 230 KV. The transmission system includes cables, lines on a common right of way and lies on a common tower . The transmission system data includes line length , impedance rating and reliability data . A brief description of the IEEE reliability test system is given below :

5.2.1 LOAD MODEL [25]

The annual peak load for the test system is 2850 MW.Table 5.1 gives data on weekly peak loads in percentage of the annual peak load . The annual peak load occurs in the 51 st week . The data in Table 5.1 shows a typical pattern , with two seasonal peaks . The second peak is in the 23 rd week (90%) .If the 1st week is taken as January , Table 5.1 describes a winter peaking system . If 1st week is taken as a summer month , a summar peaking system can be described .

Table 5.1 : Weekly peak load in percent of annual peak

Week	Peak load	Week	Peak load
1	86.2	27	75.5
2	90.0	28	81.6
3	87.8	29	80.1
4	83.4	30	88.0
5	88.0	31	72.2
6	84.1	32	77.6
7	83.2	33	80.0
8	80.6	34	72.9
9	74.0	35	72.6
10	73.7	36	70.5
11	71.5	37	78.0
12	72.7	38	69.5
13	70.4	39	72.4
14	75.0	40	72.4
15	72.1	41	74.3
16	80.0	42	74.4
17	75.4	43	80.0
18	83.7	44	88.1
19	87.0	45	88.5
20	88.0	46	90.9
21	85.6	47	94.0
22	81.1	48	89.0
23	90.0	49	94.2
24	88.7	50	97.0
25	89.6	51	100.0
26	86.1	52	95.2

Table 5.2 gives a daily peak load cycle , in percentage of the weekly peak . The same weekly peak load cycle is assumed to apply for all seasons . The data in Tables 5.1 and 5.2 , together with the annual peak load define a daily peak load model of 52x7 = 364 days , with Monday as the first day of the year .

Table 5.2 : Daily peak load in percent of weekly peak :

Day	Peak load
Monday	93
Tuesday	100
Wednesday	98
Thursday	96
Friday	94
Saturday	77
Sunday	75

Table 5.3 gives week day and weekend hourly load models for each of the three seasons . A suggested interval of weeks is given for each season . The first two columns of this table reflect a winter season (evening peak) , while the next two columns reflect a summer season(afternoor peak) . The interval of weeks shown for each season in Table 5.3 represents application to a

winter peaking system . If Table 5.1 is started with a summer month then the intervals for application of each column of the hourly load model in Table 5.3 should be modified accordingly .

Table 5.3 : Hourly Peak Load in percent of daily peak :

HOURS	Winter weeks 1 - 8 & 44 - 52		Summer Weeks 18 - 30		Spring/Fall Weeks 9 - 17 & 31 - 43	
	Wkdy	Wknd	Wkdy	Wknd	Wkdy	Wknd
12 - 1 a.m.	67	78	64	74	63	75
1 - 2	63	72	60	70	62	73
2 - 3	60	68	58	66	60	69
3 - 4	59	66	56	65	58	66
4 - 5	59	64	56	64	59	65
5 - 6	60	65	58	62	65	65
6 - 7	74	66	64	62	72	68
7 - 8	86	70	76	66	85	74
8 - 9	95	80	87	81	95	83
9 - 10	96	88	95	86	99	89
10 - 11	96	90	99	91	100	92
11 - 12	95	91	100	93	99	94
12 - 1 p.m.	95	90	99	93	93	91
1 - 2	95	88	100	92	92	90
2 - 3	93	87	100	91	90	90
3 - 4	94	87	97	91	88	86
4 - 5	99	91	96	92	90	85
5 - 6	100	100	96	94	92	88
6 - 7	100	99	93	95	96	97
7 - 8	96	97	92	95	98	100
8 - 9	91	94	92	100	96	97
9 - 10	83	92	93	93	90	95
10 - 11	93	87	87	88	80	90
11 - 12	63	81	72	80	70	85

Wkdy = Weekday , Wknd = Weekend

Combination of Tables 5.1, 5.2 and 5.3 with the annual peak load defines an hourly load model of 364 x 24 = 8736 hours. Hourly load for any hour of the weekday may be expressed as ,

$$HL = WKPK \times DPK \times PKWD \times APK \qquad (5.1)$$

Similarly hourly load for any hour of the weekend day may be expressed as ,

$$HL = WKPK \times DPK \times PKWN \times APK \qquad (5.2)$$

where ,

HL = Hourly load

WKPK = Weekly peak load in percentage of annual peak

DPK = Daily peak load in percentage of weekly peak

PKWD = Hourly peak load in percentage of daily peak for week day

PKWN = Hourly peak load in percentage of daily peak for weekend day

APK = Annual peak load

5.2.2 GENERATING SYSTEM [25]

Table 5.4 gives a list of the generating unit ratings and reliability data . In addition to forced outage rate , the parameters needed in frequency and duration calculations are also given (MTTF and MTTR) . This table gives data on full outages only .

Table 5.4 : Generating unit reliability data :

Unit size MW	Number of Units	FOR	MTTF hrs	MTTR hrs	Schedule maintenance Wks/ year
12	5	0.02	2940	60	2
20	4	0.10	450	50	2
50	6	0.01	1980	20	2
76	4	0.02	1960	40	3
100	3	0.04	1200	50	3
155	4	0.04	960	40	4
197	3	0.05	950	50	4
350	1	0.08	1150	100	5
400	2	0.12	1100	150	6

MTTF = Mean time to failure

MTTR = Mean time to repair

$$FOR = \frac{MTTR}{MTTF+MTTR} \quad (5.3)$$

5.3 BANGLADESH POWER SYSTEM (BPS) [30]

The electric power system of Bangladesh may be divided into two zones : the East zone and the West zone , separated by the rivers Padma , Jamuna and Meghna . There are a number of power stations in the East and in the West Zones . The generation cost in the East zone is cheaper than that in the West zone . In order to transmit cheaper resources from East zone to the west zone , an electrical interconnector from Tongi grid sub-station to Ishurdi grid sub-station has been constructed and thereby forming an intergrated natural grid. The East-West Interconnector (EWI) is a double circuit line operating at 132 KV . The power transmission capacity of the EWI is 180 MVA per circuit at 132 KV . The total installed capacity of BPS is 1141 MW out of which 725 MW is located in the East zone .

There are a number of power stations in the East and the West zones . The grographical locations of different power stations of BPS are shown in figure 5.1 . The simplified single line diagram of the integrated power system of Bangladesh is shown in figure 5.2. The large power stations are Karnafuli Hydro-Electric station at Kaptai , Ashuganj steam and combined cycle power plants , Ghorasal steam power station, Siddhirganj steam power station , Chittagong steam power station, Shahjibazar

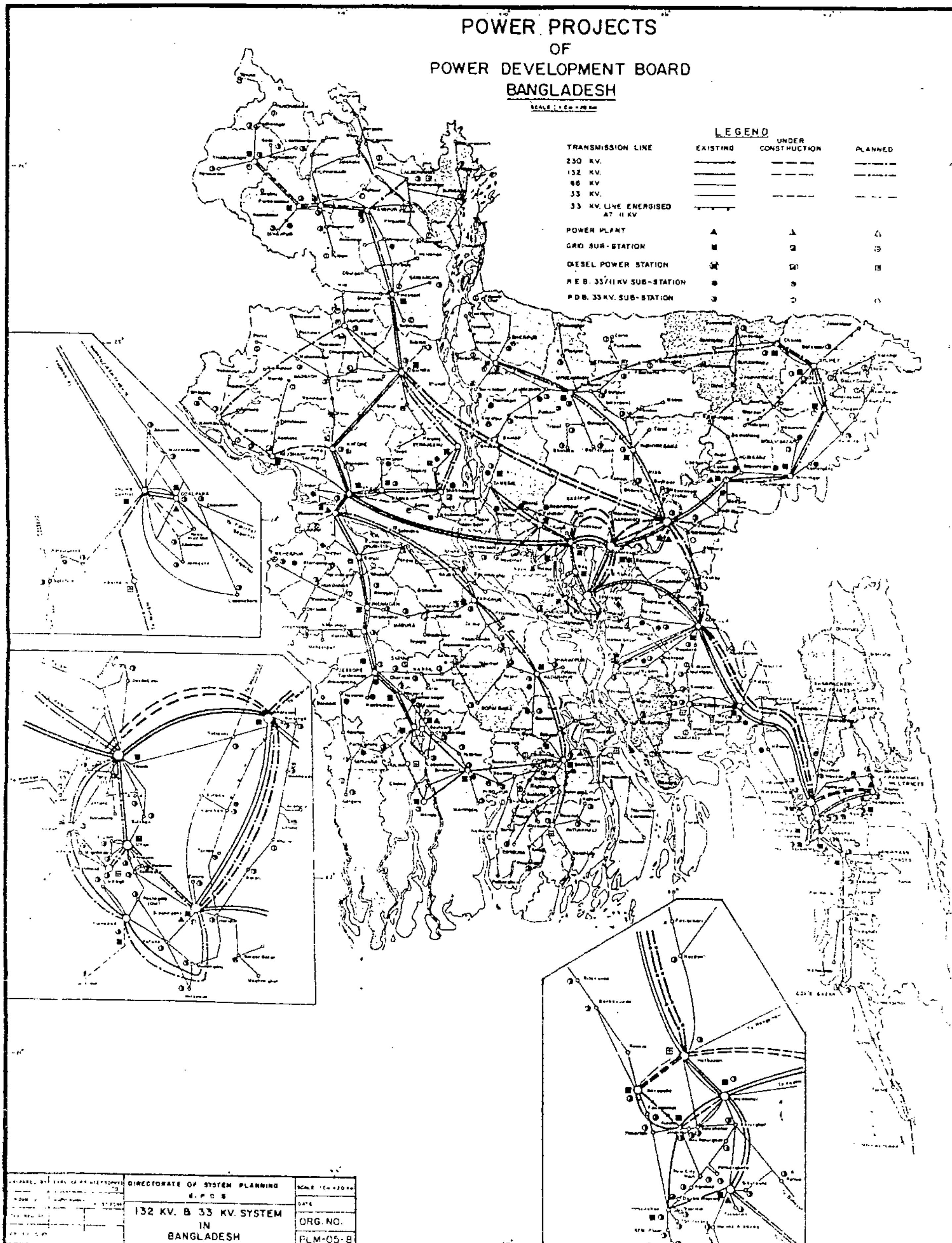


Fig. 5.1: Geographical Locations of different Power stations of BPS.

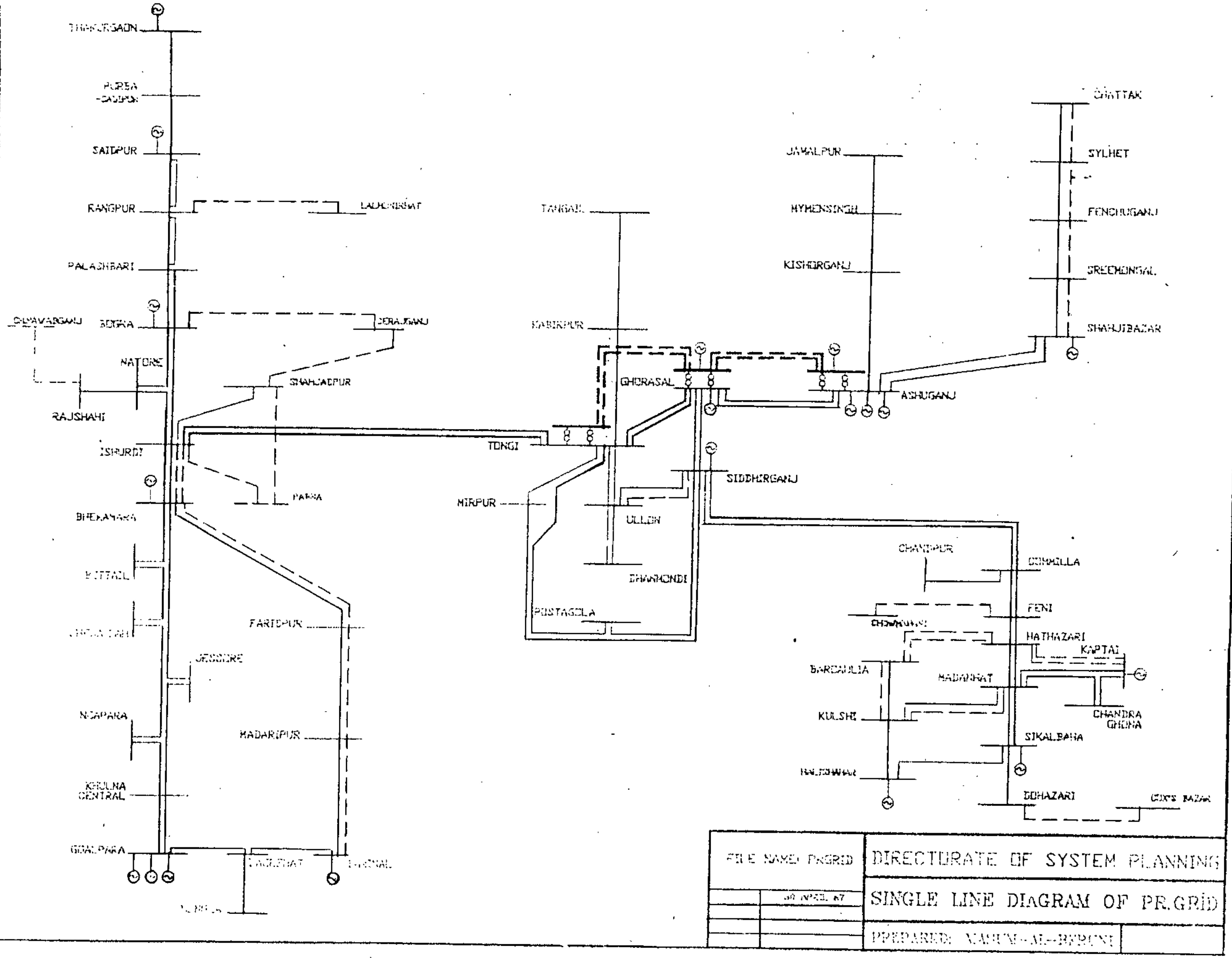


Fig. 5.2 : Simplified single line diagram of the integrated power system of Bangladesh.

gas turbine power station , Khulna steam and gas turbine power plants and Bheramara gas turbine power plants . Besides these there are number of small diesel power stations .

The power stations in the two zones have different types of generating units such as hydro , gas turbine, diesel etc. Some of the units are old and their output are now lower than the rated values . As a result , the total generating capacity of BPS is 1018 MW instead of 1141 MW . The maximum generating capability of the East zone is 672 MW and that of the West zone is 346 MW [30] . Most of the thermal power stations in the East zone use natural gas as fuel , while those in the West zone generate electricity by burning costly liquid fuel .

5.3.1 BPS GENERATION DATA :

Generation data of BPS used in this research are given in Tables 5.5 and 5.6. Capacities of some of the small units shown in this table are rounded values . This is done to decrease the computer time . In the West zone the small diesel units with capacities less than 5 MW are aggregated to form units of capacity 5 MW each .

The East zone has 23 generating units with an installed capacity of 675 MW . The generating units in this zone include 3 hydro units , 10 steam units and 10 gas turbine units . The West zone has 17 generating units with an installed capacity of

355 MW . The average incremental fuel costs of the generating units in the East zone ranges from 0.14 to 0.28 Tk/Kwh, while those of the generating units in theWest zone ranges from 1.57 to 4.17 Tk/Kwh.

Table 5.5 : BPS Generation data for East Zone :

Name of Power Station	Type of fuel	No.of Units	Capacity (MW)	FOR	Avg.inc. fuel cost Tk./Kwh.
Karnafuli Hydro	Hydro	1 2	50 40	0.01 0.01	0.0 0.0
Ashuganj Steam Turbine	Gas	2	65	0.10	0.14
Ashuganj Combined Cycle	Gas	1 (GT) 1 (ST)	55 30	0.19 0.19	0.16 0.16
Ghorasal Steam Turbine	Gas	2	55	0.10	0.16
Siddhirganj Steam Turbine	Gas	1 3	50 10	0.10 0.15	0.15 0.23
Chittagong Steam Trubine	Gas	1	60	0.10	0.16
Chittagong Gas Turbine .	Gas	2	5	0.18	0.28
Shahjibazar Gas Trubine	Gas	7	10	0.18	0.27

Table 5.6 : BPS Generation data for West Zone

Name of Power Station	Type of fuel	No. of Units	Capacity (MW)	FOR	Avg. inc. fuel cost Tk./Kwh.
Khulna Steam Turbine	F.Oil	1	110	0.10	1.57
		1	60	0.10	1.79
		2	5	0.15	3.32
Khulna Gas Turbine (Barge)	SKO	2	25	0.18	2.62
Khulna Gas Turbine	HSD	1	10	0.18	3.45
		1	10	0.18	4.17
Barisal Gas Turbine	HSD	1	20	0.15	3.01
Bheramara Gas Turbine	HSD	3	20	0.18	3.24
Small Diesel Stations *	LDO/HSD	1	5	0.12	1.93
		1	5	0.12	2.00
		1	5	0.12	2.20
		1	5	0.12	2.35
		1	5	0.12	2.49

* These are small diesel power stations located at Thakurgaon, Bogra , Goalpara , Barisal , Rajshahi and Serajganj. Several small diesel units of these stations have been aggregated to form 5 units of capacity 5 MW each .

5.3.2 LOAD DATA OF BANGLADESH POWER SYSTEM

In this research ,the hourly load data of August , 1985 and December , 1985 are used . The hourly loads of these two months are given in Appendix - A . Tha peak load usually occurs during 7:00 to 9:00 hours in the afternoon and the peak of August is 765.05 MW and that of December is 660.53MW. The base load is observed to occur around 4 a.m. and this load is 287.09 MW in August . In December the base is observed to occur at around 2 a.m. and this load is 306.32 MW .

5.4 COMPUTER PROGRAMS

For numerical evaluation computer programs are developed in FORTRAN 77 . Three different methodologies are used in this thesis to make a comparative study of the commonly used methods and for each of these method different program has been developed. During the process of developping each program , it is tested by several small examples and simultaneously these examples are worked out by hand with a view to check the results .

The developed programs are general in nature and may be used to evaluate any power system by appropriately changing the dimensions of the variables . The computer programs are given in Appendix - B .

5.5 NUMERICAL RESULTS

In this research ; two power systems are evaluated . One is the well known IEEE reliability test system (IEEE-RTS) [25] and the other is the Bangladesh Power System (BPS) . In case of IEEE-RTS, load data of the winter , summer and the whole year are used separately . In case of BPS, load data of August,1985 and December , 1985 are also used separately . The reliability indices (LOLPs)are evaluated by using previously mentioned three different methods viz ; recursive , segmentation and cumulant method considering loads of different periods .

5.5.1 COMPARATIVE STUDY OF DIFFERENT METHODS

The reliability indices are determined for IEEE - RTS with 2184 hours winter load . The peak load for this period is 2850 MW . LOLPs (in percent) obtained using the three methods are presented in Table 5.7 . The computer memory and the CPU time in IBM 4331 required for each method are compared in this table . To compare the variation in the results of each method for single and double precision the results obtained using single precision as well as double precision arithmetic are also presented in this table .

Table 5.7 : LOLPs obtained using three different methods with IEEE-RTS winter loads

METHODS	SINGLE PRECISION			DOUBLE PRECISION		
	LOLP(%)	STORAGE (Bytes)	CPU (Sec)	LOLP(%)	STORAGE (Bytes)	CPU (Sec)
Recursive	0.276158	75278	87	0.276183	130776	111
Segmentation	0.275135	48384	19	0.275144	76400	27
Cumulant	-0.529782	35408	18	0.281973	46536	21

In table 5.7 , it is observed that in either precision of calculation , the recursive and the segmentation methods provide almost the same LOLP . The LOLP differs in or beyond the 3rd place after the decimal point . However, in case of cumulant method the LOLP obtained using double precision arithmetic is close to that obtained in recursive or segmentation method while the single precision arithmetic provides LOLPs which varies widely from the results of recursive or segmentation method . Note that in the cumulant method negative LOLP is obtained when single precision arithmetic is used , which is absurd .

Regarding the storage requirement , it is observed that recursive method requires the largest memory locations while the cumulant method requires the least. Obviously , double precision arithmetic requires larger memory storage than the single precision arithmetic .This is confirmed in Table 5.7 .

Regarding computational efficiency , it is clearly observed from Table 5.7 that computationally recursive method is the least efficient while cumulant method is the most . This table shows that cumulant method is slightly faster than the segmentation method . Here the segmentation and cumulant methods are about five times faster than the commonly used recursive method.

The LOLPs are also evaluated using the recursive , segmentation and cumulant methods for the summer load of IEEE-RTS . In this case , again the 2184 hours load data are used . The peak load for this period is 2565 MW . The LOLP , computer storage and CPU time both for single and double precision arithmetic are presented in Table 5.8 .

Table 5.8 : LOLPs obtained with IEEE-RTS summer loads

METHODS	SINGLE PRECISION			DOUBLE PRECISION		
	LOLP(%)	STORAGE (BYTES)	CPU (Sec)	LOLP(%)	STORAGE (BYTES)	CPU (Sec)
Recursive	0.095945	75278	87	0.095965	130776	112
Segmentation	0.095962	48384	19	0.095965	76400	27
Cumulant	-0.814398	35408	18	0.152714	46536	21

In Table 5.8 , it is observed that storage requirements and CPU times for corresponding methods are same as in Table 5.7 .This is expected because of same number of load impulses and the same generating system . However , the LOLP is less , compared to Table 5.7 . This is due to the smaller peak load & on the average, lower load level .

It is also observed comparing Tables 5.7 and 5.8 that the LOLPs obtained using the recursive and the segmentation methods are more close to the LOLPs obtained when the summer loads are used . Note that in case of summer loads , the number of load impulses , encountered by the available generation failing to meet the load is less . Therefore , the LOLP , which is produced as a result of the summation of the probabilities of the load impulses causing loss of load , suffers less round of error in computation .

The LOLPs are also evaluated using the recursive , segmentation and cumulant methods with the one year load of IEEE-RTS . In this case the hourly load impulses are 8736 and the peak load is 2850 MW . The LOLP , computer storage and CPU time both for single and double precision arithmetic , obtained using the so called three different methods are presented in Table 5.9 .

Table 5.9 : LOLPs obtained using three different methods with IEEE-RTS one year loads

METHODS	SINGLE PRECISION			DOUBLE PRECISION		
	LOLP(%)	STORAGE (Bytes)	CPU (Sec)	LOLP(%)	STORAGE (Bytes)	CPU (Sec)
Recursive	0.111759	76008	321	0.111788	131216	392
Segmentation	0.111525	48648	20	0.111528	76816	30
Cumulant	-1.047283	61896	24	0.142510	99392	36

Comparing Tables 5.9 with Table 5.7 and Table 5.8 it is observed that the storage requirements in case of recursive and segmentation method are almost the same while that in case of cumulant method increases by 75% for single precision arithmetic and 113% for double precision. It is also observed that the CPU time requirement in case of segmentation method remains almost the same and in case of cumulant method it increases slightly, that is 33% for single precision and 71% for double precision arithmetic. However in case of recursive method the CPU time requirement increases more than 3.5 times when one year load data is used.

As it is observed in Table 5.7 and Table 5.8, the LOLPs obtained using the recursive and the segmentation method are almost the same while the LOLPs obtained using cumulant method are still negative for single precission arithmetic . Comparing Table 5.8 and Table 5.9 it is observed that in case of double precision arithmetic the closeness in the LOLPs of cumulant method with those of segmentation or recursive method, are more when one year load data are used . Similar observation is made when Tables 5.7 and 5.8 are compared . It indicates that the cumulant method will provide good result in case of higher LOLPs , that is , if the results obtained by summing the wider area of the probability plane .

The proposed techniques are then applied to evaluate the LOLP of a small power system like Bangladesh Power System .LOLP is determined for the month of August , 1985 with 765.05 MW peak load . The LOLPs along with computer storage and CPU time requirement for three methods are presented in Table 5.10 .

Then the load data of December , 1985 with 660.53MW peak are used in the evaluation . The results obtained using the three different methods are presented in Table 5.11 .

Table 5.10 : LOLPs obtained using three different methods with BPS load of August , 1985 .

METHODS	SINGLE PRECISION			DOUBLE PRECISION		
	LOLP(%)	STORAGE (Bytes)	CPU (Sec)	LOLP(%)	STORAGE (Bytes)	CPU (Sec)
Recursive	0.071315	39632	22	0.071318	59496	25
Segmentation	0.071317	31448	12	0.071319	43344	15
Cumulant	0.093713	33424	15	0.076579	43576	17

Table 5.11 : LOLPs obtained using three different methods with BPS load of December , 1985 .

METHODS	SINGLE PRECISION			DOUBLE PRECISION		
	LOLP(%)	STORAGE (Bytes)	CPU (Sec)	LOLP(%)	STORAGE (Bytes)	CPU (Sec)
Recurisive	0.004679	39632	21	0.004679	59496	16
Segmentation	0.004679	31448	12	0.004679	43344	16
Cumulant	0.089593	33424	15	0.004185	43576	18

Comparing the storage requirements for about one third of the load data of the IEEE-RTS used in Tables 5.7 and 5.8 it is observed in case of BPS that the computer memory requirement has decreased to half for the recursive method . However , the memory requirements in other two methods do not change appreciably . Note that the installed capacity of BPS is one third of that of IEEE-RTS .

From Tables 5.10 and 5.11 it is observed that the LOLPs obtained using cumulant method are close to those obtained using recursive or segmentation method in case of double precision arithmetic . In case of single precision the LOLPs of cumulant method vary from those of segmentation or recursive method . However , in this case the negative LOLPs are not obtained which was the case in IEEE-RTS. Note that on the average the units of BPS are of higher FOR .

It is also observed that for the same number of load data, Table 5.10 provides higher LOLPs for each method . This is because of higher peak load in the month of August .

5.5.2 SENSITIVITY STUDY OF DIFFERENT METHODS

In this research , sensitivity of the different methods used for the evaluation of the reliability are investigated . Sensitivity of each method to the variation of peak load is studied for the IEEE - RTS with one year load . In this study, peak load is varied from 1000 MW to 2850 MW at a step of 200 MW LOLP for different peak loads and corresponding CPU times are presented in Table 5.12 . Storage requirements does not change with the variation of peak load and hence not shown in this table . The variation of LOLP and the variation of CPU time with the variation of peak load are also depicted in figure - 5.3 and 5.4 respectively .

Table 5.12 : Sensitivity of LOLP to peak load

PEAK LOAD (MW)	IEEE ONE YEAR LOAD					
	RECURSIVE		SEGMENTATION		CUMULANT	
	LOLP(%)	CPU (Sec)	LOLP(%)	CPU (Sec)	LOLP(%)	CPU (Sec)
1000	0.4193E-11	643	0.4185E-11	30	0.1496E-14	36
1200	0.2617E-09	614	0.2616E-09	30	0.1104E-11	36
1400	0.1002E-07	576	0.1002E-07	30	-0.2600E-09	36
1600	0.2598E-06	549	0.2598E-06	31	-0.1354E-06	36
1800	0.4759E-05	523	0.4744E-05	29	-0.9253E-05	36
2000	0.6243E-04	502	0.6224E-04	29	-0.1696E-03	36
2200	0.5834E-03	475	0.5832E-03	29	-0.8182E-03	38
2400	0.3932E-02	455	0.3930E-02	29	0.2136E-02	36
2600	0.1963E-01	415	0.1963E-01	29	0.2909E-01	36
2800	0.8177E-01	403	0.8175E-01	29	0.1103E-00	36
2850	0.1118E-00	388	0.1115E-00	30	0.1425E-00	36

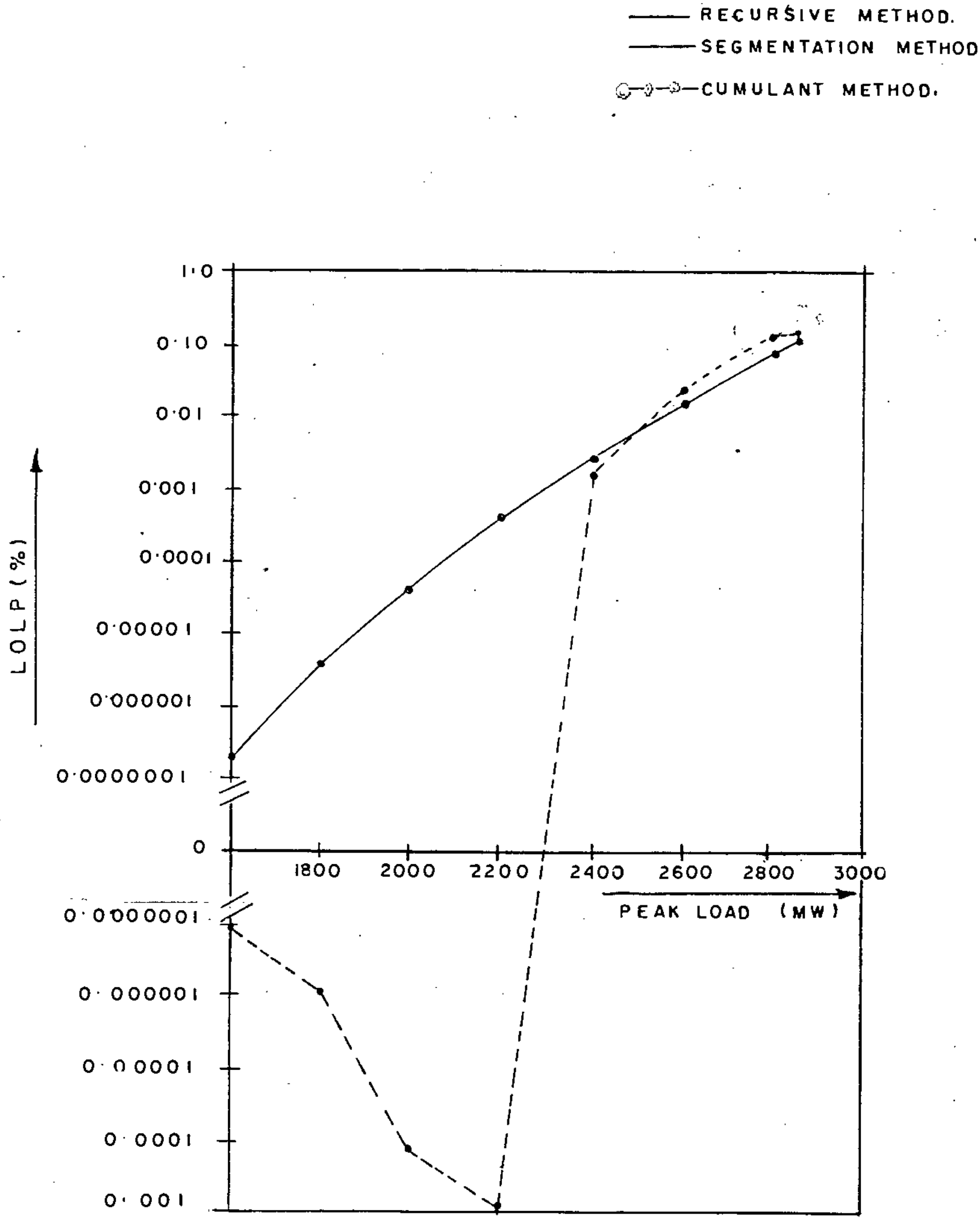


FIG. 5.3: LOLP VS. PEAK LOAD FOR IEEE SYSTEM.

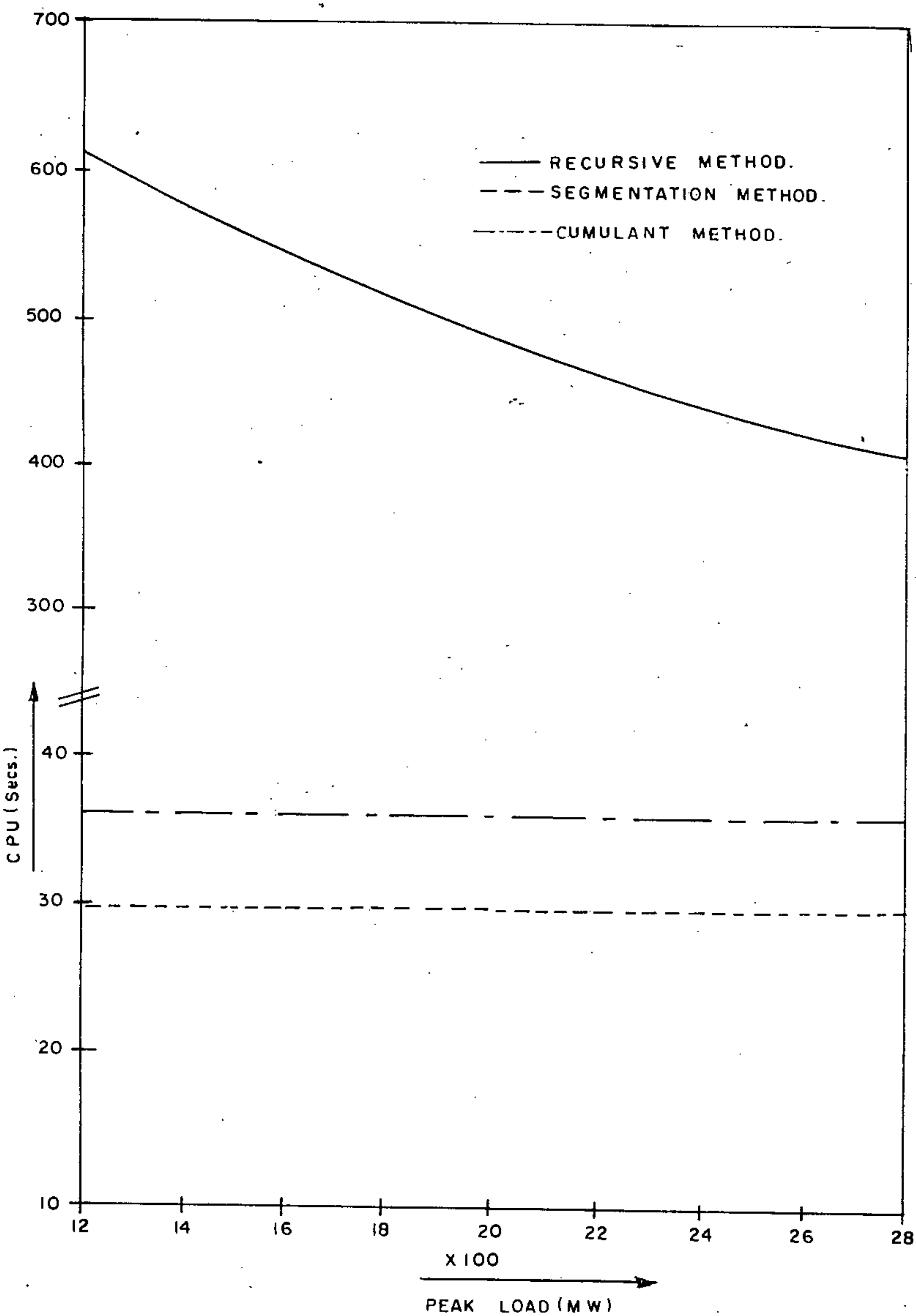


FIG. 5·4: CPU TIME VS. PEAK LOAD OF IEEE SISTEM

It is observed in columns 2 and 4 of table 5.12 as well as in figure 5.3 that LOLP increases rapidly with the increase of peak load for both in case of recursive and segmentation method. Note that these two methods provide almost the same LOLPs.However, in case of the cumulant method the LOLPs show some peculiarities. For the lower peak load the LOLP decreases , more specifically it provides negative values . But for the higher peak load the LOLPs are close to those obtained using recursive or segmentation method.

Regarding the CPU time , it is observed in Table 5.12 as well as in figure 5.4 that in case of recursive method CPU time is very high for lower peak loads and it decreases with the increase of peak loads.On the other hand, in case of segmentation or cumulant method CPU time remains almost the same for all peak loads.

In this thesis , sensitivity of the recursive and that of segmentation method , respectively , to the step and segment size are investigated . In this case , winter loads of IEEE RTS is used . The LOLPs , storage requirements and CPU times are presented for each method for different step or segniment size in Table 5.13 . The variations of LOLPs , CPU time and memory requirements with step size in case of recursive method and segment size in case of segmentation method are depicted in figures 5.5, 5.6 and 5.7 respectively .

Table 5.13 : Sensitivity of LOLP to step/segment size

STEP OR SEGMENT SIZE (MW)	IEEE WINTER LOAD (13 WEEKS)					
	RECURSIVE			SEGMENTATION		
	LOLP (%)	STORAGE (bytes)	CPU(Sec)	LOLP(%)	STORAGE (bytes)	CPU(Sec)
01	0.2762	130776	106	0.2751	76400	23
02	0.2733	76312	59	0.2757	49184	18
03	0.2667	58136	43	0.2667	40112	15
04	0.2715	49080	38	0.2738	35568	14
05	0.2736	43608	30	0.2725	32832	13
10	0.2650	32712	20	0.2772	27344	11
20	0.2408	27272	15	0.2496	24552	10
30	0.2024	25440	13	0.2262	23640	10
40	0.1802	24512	12	0.1913	23192	10
50	0.2168	24000	12	0.2315	22920	10
60	0.1123	23680	12	0.1679	22728	10
70	0.1211	23456	11	0.1672	22600	10
80	0.1567	23296	11	0.2243	22504	10
90	0.0901	23136	11	0.1752	22424	10
100	0.1417	23040	11	0.1492	22368	10

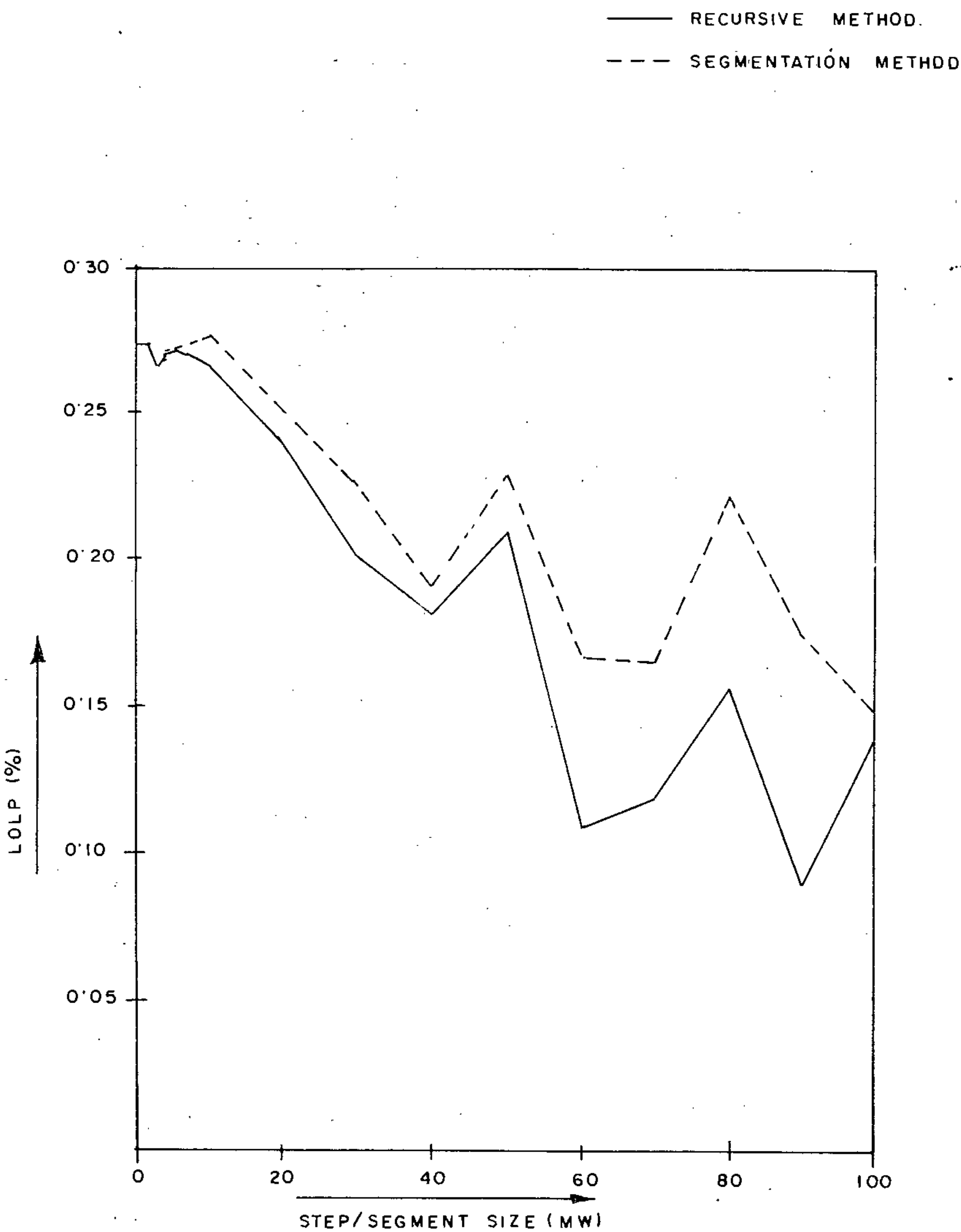


Fig. 5·5: LOLP VS. STEP/SEGMENT SIZE.

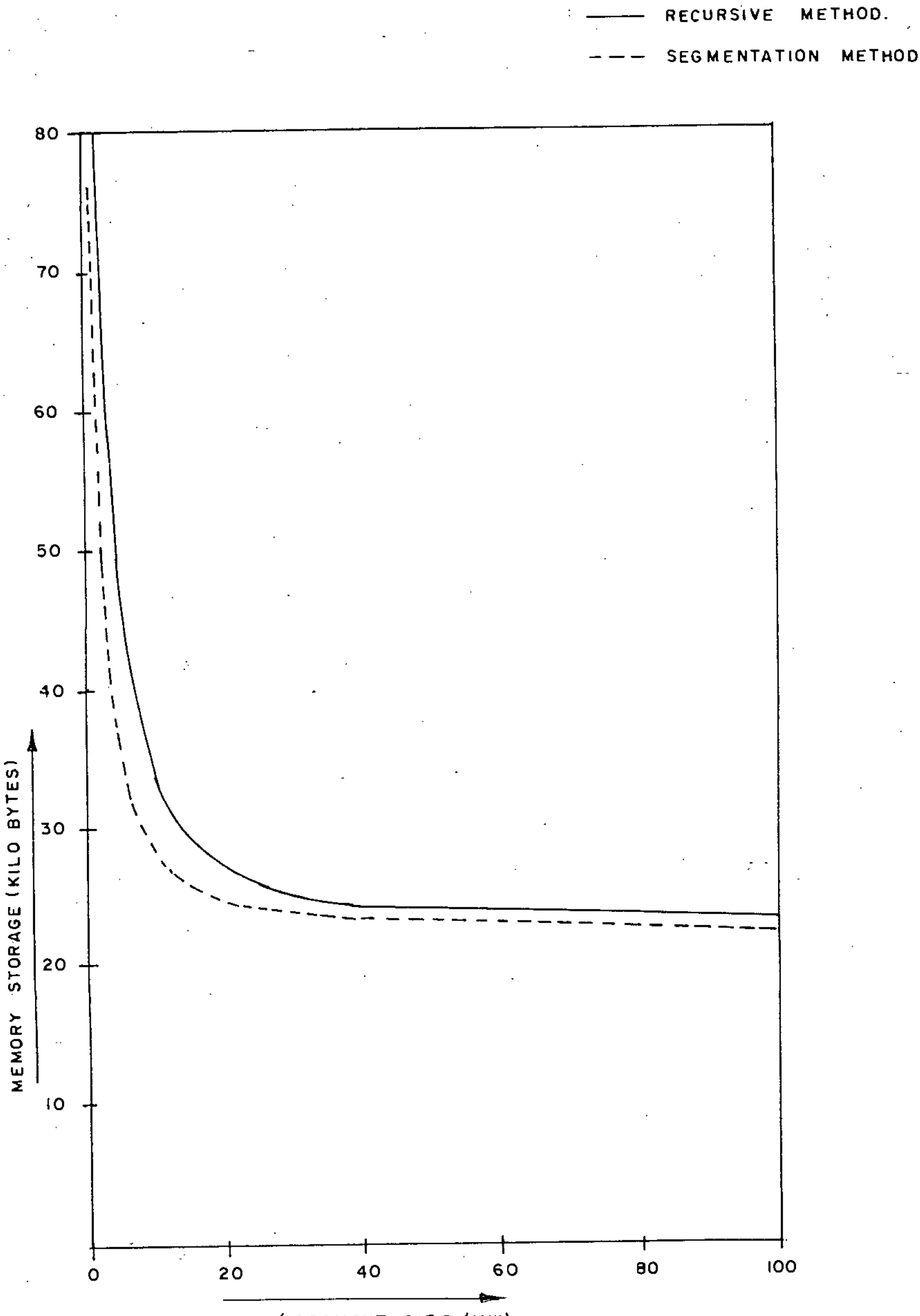


Fig. 5·6: MEMORY STORAGE VS. STEP/SEGMENT SIZE

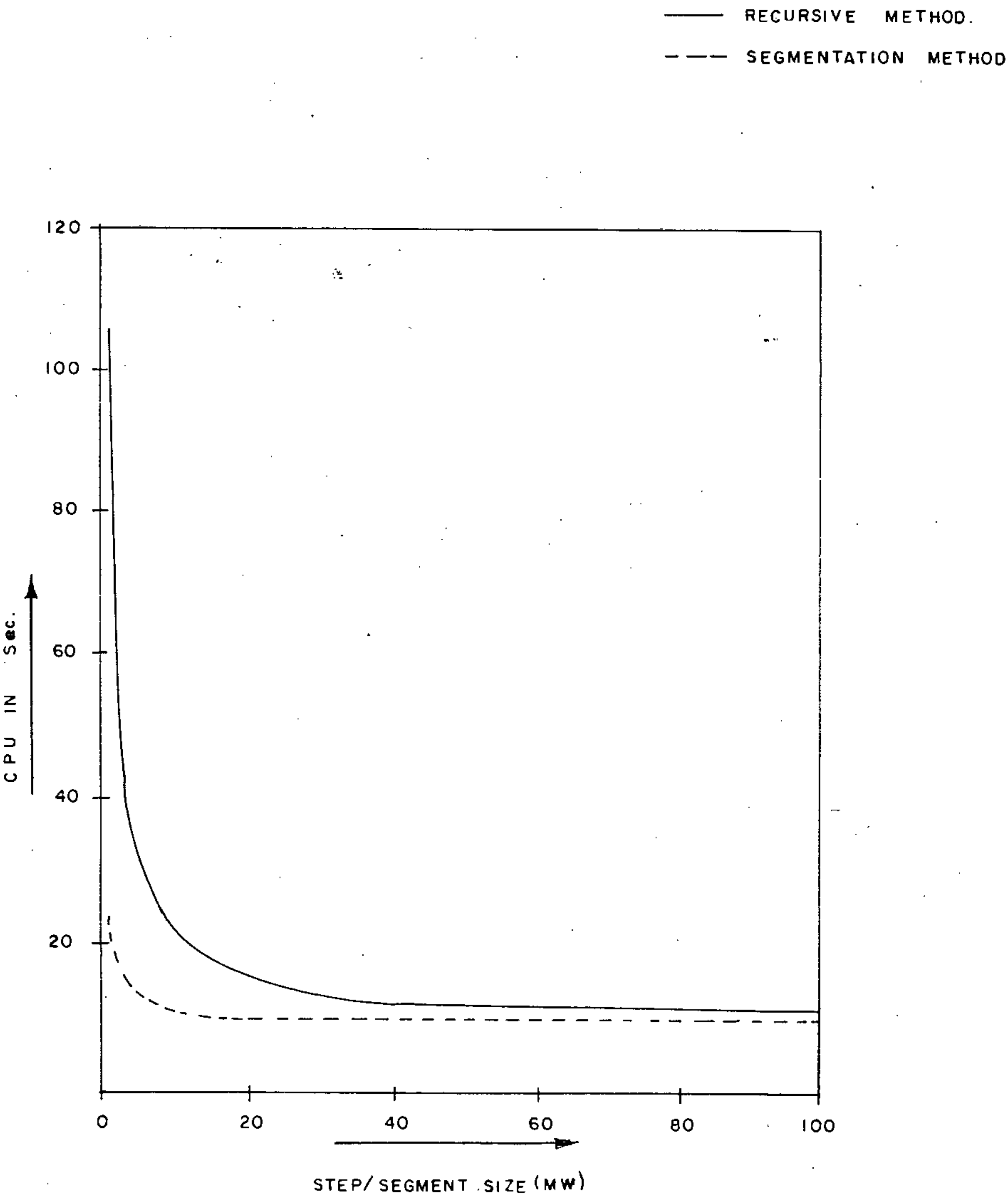


Fig. 5·7: CPU TIME VS. STEP/SEGMENT SIZE

In Table 5.13 , it is observed that with the increase of step or segment size the value of LOLP deviates irragularly from its true value . Note that in this case the true value of LOLP is obtained at 1 MW step or segment size .

Regarding storage requirement it is observed from the table and from figure 5.6 that the difference of memory requirement is tremendously high when the step size or sigment size is varied in the lower range of steps or segments . However , the difference is almost negligible in higher range of steps or segments.

The variation in CPU time with the variation of segment size in case of segmentation method is not prominent . However , the salient variation is observed in case of the recursive method with variation of step size in the lower range of step size . In the higher range , the CPU time is almost constant with the variation of steps .

The sensitivity of the cumulant method to the number of Gram - Charlier coefficients (G) is also investigated in this thesis . In Table 5.14 the LOLPs , CPU time and storage requirements with different number of G coefficients used in the Gram-Charlier series are presented . The variation of LOLP with G coefficients is depicted in figure 5.8. In this figure the LOLPs obtained using recursive method is also presented for comparision . It is observed from Table 5.14 as well as from figure 5.8 that LOLP varies irregularly to the increase of number of G coefficients. However , the LOLP approaches to the true value when maximum number of terms are considered in the Gram-Charlier expansion series .

Table : 5.14 : Sensitivity of LOLP to number of terms :

NO.OF GRAM-CHARLIER CO-EFFICIENTS	IEEE WINTER LOAD(13 WEEKKS)		
	LOLP(%)	STORAGE(bytes)	CPU (Sec)
0(With out 'G' co-efficient)	0.31973	46872	22
1 (G_1)	0.52725	46872	21
2 (G_1, G_2)	0.24954	46880	22
3 (G_1.....G_3)	0.23456	46880	21
4 (G_1.....G_4)	0.19162	46880	22
5 (G_1,....G_5)	0.14579	46880	21
6 (G_1,....G_6)	0.28197	46880	21

This table also shows that storage requirements and CPU time do not practically change with the increase of number of 'G' coefficients .

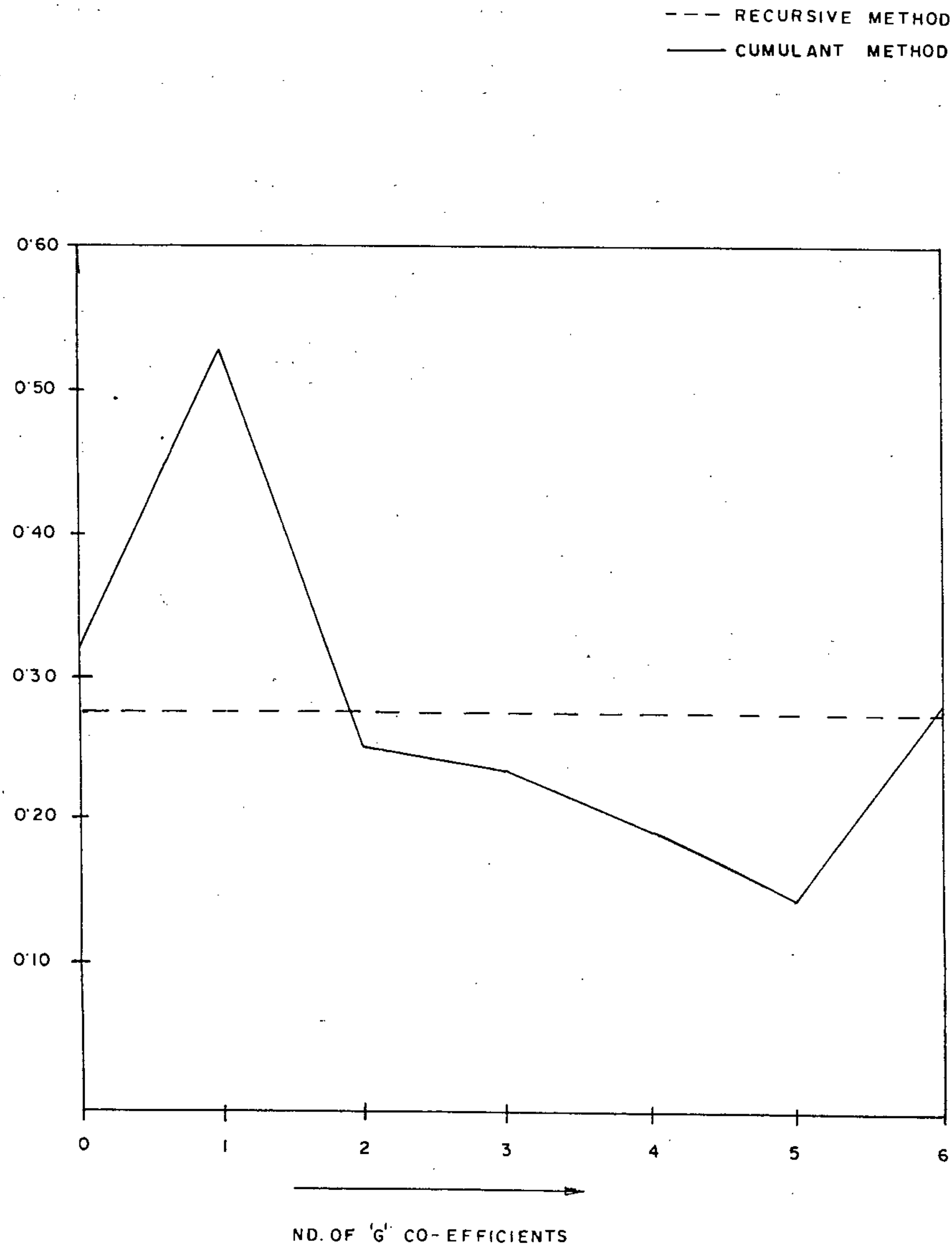


Fig. 5·8 : LOLP VS. NO. OF 'G' CO-EFFICIENTS.

CHAPTER 6

CONCLUSIONS

6.1 CONCLUSIONS

The evaluation of reliability is one of the important aspects of generation expansion planning . On the basis of the requirements, the planner develops a number of feasible expansion alternative plans . It is a common practice for power system engineers to evaluate each plan on the basis of reliability to ensure that the adopted plans satisfy desired level of reliability . Several measures have been devised to evaluate the reliability index of a given expansion plan . The simplest and most common of all is the loss of load probability (LOLP) , The three probabilistic methods are commonly used by the utilities to evaluate LOLPs . These are recursive , segmentation and cumulant methods .

In this thesis , the three methods are applied to two different power systems , one is small BPS and the other is medium size IEEE-RTS . The methods are compared in terms of accuracy and computational efficiency i.e, CPU time requirement as well as memory requirement. The senisitivity of each method to the following features are investigated .

i) Precision arithmetic in computation

ii) Peak load variation .

The sensitivity of the recursive and the segmentation methods to the variation of step size in case of recursive method and segment size in case of segmentation method are also investigated . The sensitivity of the cumulant methods to the number of terms in the Gram-Charlier series is also studied in this research .

On the basis of the results and observations described in details in the previous chapter , following conclusions may be made :

1) For probabilistic simulation , the segmentation method seems to be more flexible and computationally more efficient than the conventional recursive method in the evaluation of LOLP .

2) Cumulant method is also computationally more efficient than the recursive method but the accuracy of the result is highly system dependent . If fast calculation is desired , cumulant method may be used for large and small system . However , the results obtained using this method may be questionable . The results obtained by cumulant method also depends on the type of precision used in calculation . It is observed that cumulant method provides good result only if double precision arithmetic is used .

3) The segmentation method provides accurate results as compared to the cumulant method and simultaneously, is computationally very efficient . This method may be applied to evaluate LOLP of any size of system . For recursive and segmentation method, it is not essential to use double precision arithmetic .

4) Considering all, the segmentation method is optimum in terms of accuracy as well as computational requirements .

Characteristics of the different probabilistic methods are tabulated below for ready reference :

Table 6.1 : Characteristic of the reliability evaluation techniques

ITEMS OF CHARACTERISTICS	RECURSIVE METHOD	SEGMENTATION METHOD	CUMULANT METHOD
1. Computational efficiency.	Very Poor	Fast-very fast(System dependent)	Very fast (System dep-endent)
2. Computational accuracy on LOLP	Accurate	Accurate	System dependent
3. Computer memory requirement	Very large	Small (System dependent)	Small (System dependent)

6.2 RECOMMENDATION FOR FURTHER RESEARCH

The following recommendations may be made in this thesis :

i) In this research , the sensitivity of cumulant method to the number of terms of Gram-Charlier series has been investigated varying only upto six so called 'G' co-efficients . The sensitivity may be investigated varying more number of terms in the series . The recursive approach to calculate cumulants recently developed by Duran [34] may be used in this case .

ii) Sensitivity of each method to the variation of peak load of IEEE-RTS has been investigated in this thesis. Comprehensive investigation may be made using different types of load with different seasonal peak load and with different load factor .

iii) Sensitivity of each method may also be investigated to different types of generation system like hydro units dominated system , thermal unit dominated system etc.

iv) Each method may be compared using more practical system as well as using a number of alternative plans of each system .

APPENDIX - A

TABLE A.1 : HOURLY DEMAND (MW) OF BPS OF AUGUST, 1985.

TIME:	1:00	2:00	3:00	4:00	5:00	6:00	7:00	8:00	9:00	10:00	11:00	12:00
a.m.	433.11	411.53	[illegible]	392.92	400.57	439.61	501.01	521.65	547.31	428.80	571.50	[illegible]
p.m.	513.57	[illegible]	449.95	460.45	490.19	531.55	704.73	722.15	667.85	677.53	555.52	452.58
a.m.	444.10	403.88	393.07	381.43	[illegible]	378.81	404.02	512.54	407.67	398.12	426.21	415.18
p.m.	365.38	338.64	324.98	352.81	390.48	452.24	645.84	673.07	643.64	600.59	521.35	451.53
a.m.	405.51	379.81	370.15	344.53	354.11	392.89	467.02	477.18	505.61	503.55	553.18	554.04
p.m.	516.35	478.81	476.17	491.14	517.06	576.79	726.41	734.28	723.15	723.15	571.44	494.18
a.m.	551.70	535.10	411.83	408.67	410.45	439.35	530.31	510.03	546.98	549.94	577.13	562.01
p.m.	525.40	485.63	479.25	486.71	519.83	555.49	473.21	729.50	715.91	652.37	570.22	483.69
a.m.	453.11	432.14	420.45	405.64	410.71	439.97	514.68	529.48	546.87	554.08	565.72	558.00
p.m.	515.97	483.82	475.90	498.21	519.05	568.89	720.29	765.05	738.41	654.54	573.30	514.06
a.m.	450.08	422.79	410.92	319.56	400.29	432.56	498.74	518.39	528.39	537.99	567.67	557.82
p.m.	525.60	502.26	459.31	475.41	514.59	564.73	736.06	756.51	706.00	595.02	529.49	515.92
a.m.	439.39	414.21	391.28	385.57	400.75	419.23	488.74	498.86	523.55	540.38	558.31	554.36
p.m.	515.66	481.97	547.14	480.75	512.83	600.37	678.81	712.50	655.65	616.73	546.81	431.31
a.m.	431.17	410.50	383.49	394.23	395.19	435.96	504.05	510.10	552.63	554.27	561.82	561.21
p.m.	516.29	452.35	486.69	492.17	531.14	567.50	726.81	733.98	699.93	647.77	538.34	480.83
a.m.	426.52	406.71	403.24	394.22	374.18	389.82	440.79	443.90	421.20	423.73	430.89	428.15
p.m.	382.13	364.88	363.42	392.24	416.84	485.74	640.42	647.84	641.20	586.06	528.50	484.18
a.m.	431.26	408.42	399.13	383.52	379.20	428.80	510.35	533.52	543.34	562.61	584.71	571.72
p.m.	531.81	479.98	503.73	496.77	540.21	563.67	735.12	726.05	687.75	650.34	571.68	495.24
a.m.	451.07	426.05	418.78	415.73	422.16	438.47	509.56	520.78	537.39	568.80	566.90	574.68
p.m.	520.60	496.88	477.94	488.68	525.44	479.50	727.75	720.50	717.38	653.05	587.28	511.95
a.m.	461.66	442.62	430.73	417.34	425.65	463.92	528.94	550.49	578.53	574.03	658.97	578.54
p.m.	543.13	506.44	498.07	485.03	480.61	589.45	716.47	753.84	706.47	639.68	560.31	516.91
a.m.	434.41	414.29	402.03	403.14	406.30	439.39	466.53	513.38	545.53	571.28	578.86	578.53
p.m.	544.88	515.33	507.74	511.44	552.76	600.41	713.61	719.15	679.53	674.16	587.82	518.88
a.m.	461.46	439.05	437.75	414.23	413.71	441.28	510.34	533.82	528.45	564.25	573.52	559.38
p.m.	525.99	489.07	492.01	493.31	535.47	547.06	679.50	706.91	689.39	665.48	575.20	511.39
a.m.	456.85	442.84	429.66	415.88	426.76	452.77	529.68	533.04	569.81	517.52	593.66	590.33
p.m.	542.45	491.56	491.98	490.47	527.88	583.33	722.66	703.00	710.04	680.75	572.91	521.67

TABLE A.1 (CONTD.)

TIME:	1:00	2:00	3:00	4:00	5:00	6:00	7:00	8:00	9:00	10:00	11:00	12:00
a.m.	479.90	456.33	434.84	419.24	406.87	399.68	441.20	448.22	406.58	410.72	409.62	415.34
p.m.	377.89	354.66	359.07	379.83	422.98	469.24	622.55	523.09	614.87	601.30	517.70	473.48
a.m.	440.61	422.69	417.56	401.73	400.37	432.39	547.91	528.13	550.37	564.47	582.20	568.88
p.m.	535.38	490.15	503.25	512.74	553.36	601.65	617.82	627.72	622.09	626.39	564.39	516.60
a.m.	392.84	372.93	361.50	353.77	365.72	390.70	532.54	544.96	554.98	568.99	588.04	561.24
p.m.	531.19	467.95	487.17	486.88	525.93	568.65	655.46	660.64	679.30	646.42	569.23	498.51
a.m.	478.52	452.19	428.43	420.78	437.77	450.46	526.48	557.00	560.62	574.14	581.94	576.66
p.m.	548.33	586.29	429.51	492.97	522.35	596.08	675.98	659.40	652.88	641.62	568.36	485.53
a.m.	446.45	430.58	503.87	409.66	417.38	442.05	515.20	536.52	545.79	553.07	586.44	571.66
p.m.	540.47	479.83	489.94	506.18	539.20	589.48	645.05	562.93	649.29	546.27	498.69	498.35
a.m.	419.06	426.17	415.76	414.33	417.74	435.54	527.46	541.63	556.58	555.50	557.24	558.02
p.m.	549.39	493.08	480.01	503.27	528.37	586.65	575.97	609.03	678.52	633.04	553.69	478.48
a.m.	456.71	432.13	422.00	411.02	414.37	441.56	432.69	441.37	550.83	558.20	599.83	593.50
p.m.	560.49	514.53	508.52	505.99	546.91	600.61	731.90	728.99	703.27	661.34	558.72	496.66
a.m.	447.73	429.77	413.73	408.76	420.89	386.38	459.45	469.48	469.65	472.70	474.18	450.80
p.m.	405.00	390.65	394.65	425.97	485.31	570.22	696.47	703.65	637.47	602.76	548.82	489.67
a.m.	463.48	439.95	422.54	410.02	408.78	428.15	490.00	524.32	534.39	547.87	578.10	561.96
p.m.	532.24	489.90	467.20	479.60	525.55	593.68	606.06	700.94	697.40	628.87	528.69	460.51
a.m.	451.14	431.56	408.05	396.26	397.05	407.75	461.33	494.39	516.49	520.56	544.46	529.92
p.m.	484.92	451.42	440.00	446.12	476.10	573.03	688.02	675.11	635.64	593.63	518.61	453.20
a.m.	412.34	393.54	378.67	361.36	364.62	366.42	404.02	414.77	412.69	403.97	433.15	437.47
p.m.	421.97	383.16	373.93	375.66	412.16	468.21	584.87	587.50	584.60	577.70	499.57	444.69
a.m.	400.31	380.32	359.93	350.77	358.45	365.80	371.41	333.65	303.59	290.21	287.09	287.30
p.m.	294.66	293.73	317.25	334.89	352.84	415.49	579.02	586.89	601.00	531.25	554.50	395.93
a.m.	366.88	345.08	336.51	332.24	326.81	321.95	348.84	365.35	344.95	338.07	342.08	357.84
p.m.	348.84	324.09	315.09	317.50	344.22	415.56	608.80	596.41	553.44	520.19	445.88	416.80
a.m.	381.78	347.38	346.24	333.96	390.56	322.65	352.72	375.30	367.80	373.03	379.40	396.67
p.m.	391.94	341.12	323.61	323.66	367.78	423.20	642.26	604.42	572.11	526.90	447.96	393.96
a.m.	373.75	357.70	347.34	340.09	344.39	347.27	362.24	372.65	362.29	370.89	387.33	366.53
p.m.	340.81	307.47	293.98	311.55	363.77	473.10	656.05	657.38	603.14	545.77	468.83	406.68
a.m.	373.37	365.09	339.62	335.66	342.05	356.18	407.50	435.12	438.87	458.04	478.07	483.21
p.m.	453.96	425.94	405.97	411.24	446.55	514.82	707.62	706.94	631.83	575.62	485.84	403.72

TABLE A.2 : HOURLY DEMAND (MW) OF BPS OF DECEMBER, 1985.

TIME:	1:00	2:00	3:00	4:00	5:00	6:00	7:00	8:00	9:00	10:00	11:00	12:00
a.m.	411.79	391.10	380.10	374.57	400.61	442.11	484.37	500.48	485.38	489.60	510.80	503.02
p.m.	469.70	446.49	439.13	460.63	530.72	553.05	606.81	641.50	632.80	605.02	531.30	435.62
a.m.	379.08	364.96	354.73	348.64	356.74	392.60	458.95	453.99	431.23	428.04	443.29	454.99
p.m.	420.39	365.84	390.53	419.40	493.11	654.30	660.53	628.16	643.11	518.71	433.96	387.45
a.m.	338.36	325.56	316.20	316.58	331.93	375.62	464.71	465.43	469.56	469.09	483.16	476.33
p.m.	437.49	399.34	395.82	440.39	525.29	634.06	641.46	642.44	600.56	540.71	445.10	390.09
a.m.	364.51	352.20	340.44	340.90	356.85	422.95	557.47	503.14	475.51	503.84	506.16	526.70
p.m.	490.14	432.44	422.82	474.92	534.69	623.82	634.80	627.77	618.66	552.92	461.90	416.80
a.m.	387.38	377.51	373.73	375.27	387.72	441.13	523.30	514.89	498.16	492.84	515.54	505.55
p.m.	454.95	418.40	428.50	464.38	531.65	625.72	645.54	637.08	609.02	557.89	470.75	416.64
a.m.	381.64	366.97	357.50	355.90	360.32	415.00	425.29	432.29	408.67	374.40	391.51	379.18
p.m.	347.85	317.55	329.13	350.38	415.01	631.14	625.75	601.10	565.55	496.57	422.63	368.73
a.m.	353.52	337.30	330.10	337.00	349.18	412.56	494.15	501.03	478.46	496.49	519.63	511.50
p.m.	485.37	419.45	436.00	474.08	543.19	612.46	602.59	595.39	579.50	540.01	484.40	418.64
a.m.	385.98	366.14	359.73	355.64	374.67	415.53	477.62	481.62	478.69	487.47	500.56	489.39
p.m.	466.36	422.22	425.43	490.51	590.14	587.24	594.26	592.01	594.34	540.07	478.40	422.67
a.m.	391.65	380.46	365.45	358.96	371.50	436.19	492.88	496.05	492.12	482.08	463.87	470.47
p.m.	482.64	426.18	438.33	478.61	540.79	567.77	516.16	600.35	572.94	522.63	470.89	359.18
a.m.	369.16	355.93	360.50	359.67	380.29	436.59	489.35	504.90	502.91	501.63	493.95	501.38
p.m.	472.32	425.25	431.76	455.22	516.37	593.37	596.28	602.97	596.59	555.97	461.92	419.60
a.m.	354.21	367.32	360.57	363.27	374.38	437.69	468.26	476.90	472.05	469.25	485.37	485.97
p.m.	461.68	433.91	431.85	494.27	571.03	579.01	599.63	607.24	570.88	528.73	471.98	416.13
a.m.	383.93	360.86	371.06	366.21	374.90	432.96	473.66	485.13	474.63	476.78	468.01	471.15
p.m.	446.01	428.02	423.91	464.32	538.98	583.39	589.45	524.85	524.41	533.86	467.93	424.96
a.m.	394.51	384.55	363.98	362.98	372.16	385.17	415.34	411.80	409.13	390.25	401.30	400.01
p.m.	344.40	324.92	326.17	372.08	486.10	508.36	507.34	489.14	500.34	473.12	443.01	399.00
a.m.	379.29	372.36	360.46	356.18	378.27	416.19	430.43	483.07	485.29	486.65	506.95	513.86
p.m.	473.87	437.72	421.48	474.26	555.38	617.62	610.73	633.75	596.07	543.77	470.20	410.19
a.m.	394.60	377.75	373.89	368.81	384.29	420.29	503.75	513.59	490.21	495.03	490.20	507.66
p.m.	482.85	432.97	428.44	466.12	456.70	595.84	600.54	621.26	587.64	585.97	471.07	408.10

TABLE A.2 (CONTD.)

TIME:	1:00	2:00	3:00	4:00	5:00	6:00	7:00	8:00	9:00	10:00	11:00	12:00
a.m.	382.95	363.27	358.46	355.76	366.83	381.19	421.32	416.76	393.19	378.20	386.46	392.39
p.m.	366.90	316.51	306.22	334.04	404.21	572.49	604.30	593.51	555.99	511.51	435.53	375.72
a.m.	342.98	330.08	325.14	324.23	343.96	407.48	479.84	514.85	509.13	500.90	510.81	510.96
p.m.	461.66	427.46	439.64	493.06	564.98	590.46	594.10	582.50	561.64	537.68	492.93	418.28
a.m.	390.95	375.13	368.33	365.47	374.57	420.85	491.84	495.00	503.45	493.92	514.71	518.85
p.m.	482.35	424.95	426.81	484.01	581.11	596.64	603.16	597.97	597.00	548.55	454.56	399.77
a.m.	384.10	374.40	361.43	364.71	384.70	414.26	491.16	491.76	474.81	486.33	491.53	495.90
p.m.	476.00	414.30	423.64	471.78	546.15	581.29	586.94	597.36	591.21	534.32	459.50	422.54
a.m.	385.31	368.41	367.08	355.40	360.59	393.01	432.93	436.27	332.15	387.00	402.90	375.61
p.m.	334.96	308.14	330.21	375.88	460.34	553.60	570.90	564.00	548.25	491.52	432.55	432.55
a.m.	426.88	347.56	345.86	348.86	367.43	408.93	491.62	498.01	495.45	479.75	491.05	495.20
p.m.	476.84	429.80	416.95	467.05	556.50	612.25	617.12	614.03	601.93	549.04	489.77	444.08
a.m.	409.36	400.87	397.37	364.50	383.90	434.50	500.82	504.88	495.58	491.60	514.08	513.20
p.m.	481.14	405.53	421.03	490.64	547.30	589.29	597.44	630.28	602.27	542.16	465.62	406.12
a.m.	381.79	370.69	357.63	356.73	372.69	412.80	502.17	517.37	506.73	510.35	510.27	519.10
p.m.	487.02	424.67	420.60	475.15	546.78	604.59	618.16	616.10	604.72	555.63	477.65	414.56
a.m.	390.10	375.27	365.50	362.10	386.10	428.75	507.70	520.80	512.85	507.25	529.25	521.96
p.m.	487.04	424.64	429.01	472.25	559.05	531.21	534.30	561.85	595.65	566.80	486.71	421.22
a.m.	386.11	369.96	357.49	368.32	375.28	410.51	498.35	511.80	498.11	495.13	512.16	510.50
p.m.	461.37	404.77	423.57	483.57	483.33	608.03	624.65	621.58	579.45	547.63	463.21	392.15
a.m.	387.75	368.20	358.57	363.65	375.32	389.02	389.38	509.62	489.42	494.95	417.44	519.00
p.m.	484.05	431.04	428.45	475.53	549.90	575.81	617.48	614.18	591.68	556.02	485.79	408.79
a.m.	384.97	375.78	371.88	357.90	363.26	374.19	423.12	440.41	412.75	404.17	404.62	401.02
p.m.	354.13	316.67	321.01	374.30	475.22	612.42	627.07	603.67	550.37	500.54	449.59	398.50
a.m.	355.70	336.07	324.20	320.70	331.33	360.08	434.48	459.92	469.73	473.96	487.17	486.06
p.m.	435.20	413.63	408.18	437.58	529.90	619.80	615.40	603.25	574.99	518.96	433.51	386.01
a.m.	362.20	346.50	342.90	340.54	349.80	380.88	468.25	493.80	488.30	487.73	494.11	493.22
p.m.	457.47	408.61	403.36	442.37	518.26	637.50	625.67	616.66	587.55	541.50	447.60	400.69
a.m.	366.09	354.69	346.04	343.64	360.54	400.14	474.34	505.32	491.34	484.24	498.85	502.95
p.m.	482.24	417.94	415.50	465.51	529.67	610.70	597.66	593.52	579.05	530.94	445.77	393.84
a.m.	369.44	354.16	347.72	343.77	356.37	397.71	469.28	497.49	502.31	493.08	506.36	596.50
p.m.	482.91	424.82	414.86	451.97	536.32	621.53	631.52	620.62	600.50	545.90	462.99	409.07

APPENDIX - B

COMPUTER PROGRAMS

1. RECURSIVE METHOD

```
C
C.                          FILE  NAME  IS   RCVE-5
C
C           P D B 3   >>>>>    RCVE-3    >>>>>    RCVE-5
C                  IEEE   SUMMAR  LOAD ( 13 WEEKS)
C        CALCULATION OF CAPACITY OUTAGE TABLE BY RECURSIVE METHOD
C        USING UNITS OF DIFFERENT CAPACITIES
         IMPLICIT REAL*8 (A-H,O-Z)
         COMMON /ONE/HDIST(2200),PKLOAD,DEMAND
         COMMON /INT/NH
         DIMENSION PO(3500),CUM(3500),PROB(3500),C(99),Q(99),X(3500)
C
         CALL LOAD
C
         WRITE(3,3012) NH,PKLOAD,DEMAND
         NO=1
         L=1
         TOTCAP=0.
         READ(1,55) STEP
         WRITE(3,59) STEP
      1  READ(1,2,END=300) C(L),Q(L)
C     2  FORMAT(2F5.0)
C
         CALL RCSIVE(C,Q,TOTCAP,NO,L,STEP,PO,K,X,CUM)
         GO TO 1
  300    L=L-1
         DO 600 I=1,K
         M=I+1
         IF(M.GT.K) CUM(M)=0
         PROB(I)=CUM(I)-CUM(M)
  600    CONTINUE
C
         WRITE(3,7)
         DO 500 I=1,L
  500    WRITE(3,511) I,C(I),Q(I)
         WRITE(3,4)
         DO 400 I=1,K
  400    WRITE(3,410) X(I),PROB(I),PO(I)
         TLOLP=0.00
         DO 551 I=1,NH
         PKLOAD=HDIST(I)
C
         CALL    LOLP2(PKLOAD,X,CUM,TOTCAP,ALOLP)
  551    TLOLP=TLOLP+ALOLP
         XNH=NH
         ALOLP=TLOLP/XNH
         WRITE(3,552) ALOLP
         WRITE(3,6)
    6    FORMAT('1')
         WRITE (3,77) TOTCAP
C
   55    FORMAT(2F5.0)
    7    FORMAT(/////,20X,'SL.NO',7X,'CAP.(MW)',7X,'F.O.R.',//)
  511    FORMAT(1H0,20X,I3,10X,F5.0,9X,F5.2)
    4    FORMAT(/////,11X,'CAP.OUT',9X,'EXACT PROB.',12X,'CUM.PROB.',//)
  410    FORMAT(1H0,10X,F6.1,5X,E16.7,7X,E16.7)
  552     FORMAT(//,10X,'   A L O L P  ='E16.7//)
   59    FORMAT(//30X,'* STEP =',F4.0//)
 3012    FORMAT(//,30X,'* NUMBER OF HOURS ='I5///30X,'* PEAK LOAD ='F6.
        +////30X,'* EXPECTED TOTAL ENERGY DEMAND ='F10.4,1X,'(GWHR)')
   77    FORMAT(//,35X,'INSTALLED  CAPACITY =',F8.2//)
         STOP
         END
```

RECURSIVE METHOD(CONTD.)

```
      C
            SUBROUTINE RCSIVE(C,Q,TOTCAP,NO,L,STEP,PO,K,X,CUM)
            IMPLICIT REAL*8 (A-H,O-Z)
            DIMENSION PO(3500),CUM(3500),C(99),Q(99),X(3500)
            PO(1)=1.
            TOTCAP=TOTCAP+C(L)
            NO=NO+1
            K=1
            X(K)=0.
       20   IF(X(K)-C(L)) 160,160,170
      160   J=1
            GO TO 120
      170   J=J+1
      120   IF(K.GE.NO) PO(K)=0.
            CUM(K)=PO(K)*(1.-Q(L))+PO(J)*Q(L)
            IF(K.GE.NO.AND.X(K).GE.TOTCAP) GO TO 100
            K=K+1
            X(K)=X(K-1)+STEP
            GO TO 20
      100   DO 110 I=1,K
      110   PO(I)=CUM(I)
            L=L+1
            NO=K
            RETURN
            END
```

```
      C
            SUBROUTINE LOLP2(PKLOAD,X,CUM,TOTCAP,ALOLP)
            IMPLICIT REAL*8 (A-H,O-Z)
            DIMENSION X(3500),CUM(3500)
            I=1
            RESCAP=TOTCAP-PKLOAD
      200   IF(RESCAP-X(I)) 202,202,201
      201   I=I+1
            GO TO 200
      202   ALOLP=CUM(I)
            RETURN
            END
```

RECURSIVE METHOD (CONTD.)

```
C
      SUBROUTINE LOAD
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /ONE/HDIST(2200),PKLOAD,DEMAND,
      COMMON /INT/NH
      DIMENSION WEEK(52),DAY(7),HOUR(24),SHOUR(24)
C
      READ(1,300) NOWEEK
      READ(1,405) (WEEK(I),I=1,NOWEEK)
      READ(1,405) (DAY(I),I=1,7)
      READ(1,405) (HOUR(I),I=1,24)
      READ(1,405) (SHOUR(I),I=1,24)
      DEMAND=0.0
C
C     CALCULATION OF THE HOURLY LOADS
C
      KI=1
      MM=1
      DO 307 J=1,NOWEEK
      DO 301 KK=KI,7
      DO 301 L=1,24
      IF(KK.GT.5) GO TO 3000
      HDIST(MM)=WEEK(J)*DAY(KK)*HOUR(L)*2850
      MM=MM+1
      GO TO 303
 3000 HDIST(MM)=WEEK(J)*DAY(KK)*SHOUR(L)*2850
      MM=MM+1
  303 IF(MM.GT.2184) GO TO 304
  301 CONTINUE
      KI=1
  307 CONTINUE
  304 NH=MM-1
C     WRITE(3,502) NH
C 502 FORMAT(//,40X,'NUMBER OF HOURS  =',I5,//)
      DO 500 I=1,NH
  500 DEMAND=DEMAND+HDIST(I)
      DEMAND=DEMAND/1000.
C     WRITE(3,501) DEMAND
      PKLOAD=HDIST(1)
      DO 11 I=2,NH
   11 IF(HDIST(I).GT.PKLOAD) PKLOAD=HDIST(I)
C     WRITE(3,508)PKLOAD
C 501 FORMAT(//,40X,'EXP. TOTAL ENERGY DEMAND  =',F9.4,1X,'(GWHR)',//
C 508 FORMAT(//,40X,'PEAK LOAD      =',F7.2,1X,'(MW)',//)
  405 FORMAT(12F6.0)
  300 FORMAT(I2)
      RETURN
      END
```

2. SEGMENTATION METHOD

```
C      SEGMENTATION METHOD FOR EVALUATION OF POWER SYSTEM RELIABILITY
C
C                          FILE  NAME IS  SG-IEYR
C     *****************************************************************
C     * TO USE 1 MW BLOCK SIZE CHANGE THE DIMENSION OF AMOM,BMOM,CMOM *
C     *****************************************************************
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /ONE/ HDIST(8736),NH
      COMMON /TWO/ AMOM,CMOM,SUMOM,BLOCK,DCAPP,NBLK,K,IDEL
      DIMENSION AMOM(3405),BMOM(3405),CMOM(3405),CAP(27),FOR(27),
     + DCAP(27),TCAP(27),NBC(27)
C
      BLOCK=1.
      DCAPP=0.
      TOTEN=0.
      TOTCAP=0.
      XCAP=0.
      N=0
      M=0
      K=1
      LIM=0
      NHB=0
C
      CALL LOAD
C
      PRINT 7003
      BSLOAD=HDIST(1)
      PKLOAD=HDIST(1)
C
C     READ DATA OF THE GENERATING UNITS
C
      READ 3000,NBB,NOUNIT
      DO 14 I=1,NOUNIT
      TCAP(I)=0.
      NBC(I)=0
 14   DCAP(I)=0.
      DO 105 NU=1,NBB
      READ 1310,FOR(NU),CAP(NU)
      PRINT 7000,NU, FOR(NU),CAP(NU)
      TOTCAP=TOTCAP+CAP(NU)
 105  CONTINUE
      NBLK=TOTCAP/BLOCK+1
C
C     CALCULATION OF MOMENTS
C
      DO 115 K2=1,NBLK
 115  AMOM(K2)=0.
      DO 120 I=1,NH
      NBL=HDIST(I)/BLOCK+0.999999
      AMOM(NBL)=AMOM(NBL)+1.
      IF(HDIST(I).LT.BSLOAD) BSLOAD=HDIST(I)
      IF(HDIST(I).GT.PKLOAD) PKLOAD=HDIST(I)
      TOTEN=TOTEN+HDIST(I)
 120  CONTINUE
      TOTEN= TOTEN/1000.
C
C     CONVOLUTION OF GENERATING UNITS
C
      DO 250  J=1,NBB
 200  XCAP=XCAP+CAP(J)
 15   N=CAP(J)/BLOCK
      M=M+N
C
      CALL CONV(CAP,FOR,M,N,J)
C
 250  CONTINUE
      ALOLP=AMOM(NBLK)*100./FLOAT(NH)
      PRINT 340, PKLOAD,TOTEN,BSLOAD,TOTCAP,NH,ALOLP
 7001 FORMAT(/,23X,F9.0,5X,F6.4)
 3000 FORMAT(2I3)
 301  FORMAT(10X,F10.0,5X,F10.0)
 340  FORMAT(6(/),20X,'PEAK LOAD   =',F9.2,3X,' (MW)',//,20X,'E (DEMAN
     + =',F11.4,' (GWHR)',//,20X,'BASE LOAD  =',F9.2,2X,' (MW)',//,
```

SEGMENTATION METHOD(CONTD.)

```
     +X,'INST.CAP.   =',F9.2,4X,'(MW)'//,20X,'TIME     =',I6,7X,'(HO
     +S)'//,20X,'L O L P     =',E15.6,3X,'( % )')
C
 1310 FORMAT(F5.0,F10.0)
 7000 FORMAT(/,15X,I3,5X,F9.2,5X,F6.1)
 7003 FORMAT('1',23X,'INPUT DATA',//,15X,'SL.NO.',5X,'F O R',5X,'CAPAC
     +Y')
      STOP
      END


C
      SUBROUTINE CONV(CAP,FOR,M,N,J)
C     THIS SUBROUTINE CONVOLVE THE GENERATING UNIT
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /TWO/ AMOM,CMOM,SUMOM,BLOCK,DCAPP,NBLK,K,IDEL
      DIMENSION AMOM(3405),CMOM(3405),FOR(27),CAP(27),BMOM(3405)
C
      DO 205 I2=K,NBLK
  205 BMOM(I2)=0.
      DO 220 L=K,NBLK
      IF((L+N).GT.NBLK) GO TO 215
      BMOM(L+N)=AMOM(L)
      IF(AMOM(L).LE.0.0) GO TO 213
  213 GO TO 220
  215 BMOM(NBLK)=BMOM(NBLK)+AMOM(L)
  220 CONTINUE
      IDEL=M+1
      DO 230 I2=K,NBLK
      AMOM(I2)=AMOM(I2)*(1.-FOR(J))+BMOM(I2)*FOR(J)
  230 CONTINUE
C
      SUMOM=0.
      DO 23 I2=IDEL,NBLK
      SUMOM=SUMOM+AMOM(I2)
   23 CONTINUE
      CAP(J)=CAP(J)-DCAPP
      DCAPP=0.
      RETURN
      END
```

SEGMENTATION METHOD(CONTD.)

```
      SUBROUTINE LOAD
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /ONE/HDIST(8736),NH
      DIMENSION WEEK(52),DAY(7),HOUR(24),SHOUR(24)
C
      READ(1,405) (DAY(I),I=1,7)
      READ(1,300) NOWEEK
      READ(1,405) (WEEK(I),I=1,NOWEEK)
      K1=1
      MM=1
      KL=1
      DO 1121 II=1,5
      READ(1,1122) NOWEEK
      READ(1,405) (HOUR(I),I=1,24)
      READ(1,405) (SHOUR(I),I=1,24)
C
C     CALCULATION OF THE HOURLY LOADS
      NEWEEK=NOWEEK+KL-1
      DO 307 J=KL,NEWEEK
      DO 301 KK=K1,7
      DO 301 L=1,24
      IF(KK.GT.5) GO TO 3000
      HDIST(MM)=WEEK(J)*DAY(KK)*HOUR(L)*2850.
      MM=MM+1
      GO TO 303
 3000 HDIST(MM)=WEEK(J)*DAY(KK)*SHOUR(L)*2850.
      MM=MM+1
  303 IF(MM.GT.8736) GO TO 304
  301 CONTINUE
      K1=1
  307 CONTINUE
      KL=KL+NOWEEK
 1121 CONTINUE
  304 NH=MM-1
C
  405 FORMAT(12F6.0)
  300 FORMAT(I2)
 1122 FORMAT(I2)
      RETURN
      END
```

3. CUMULANT METHOD

```
C                              FILE  NAME  IS  GC-AUG
C
C                    PDB  LOAD FOR THE MONTH OF AUGUST , 1985
C
C   .   CALCULATION OF LOLP USING CUMULANTS  FROM HOURLY LOAD
C       IMPLICIT REAL*8 (A-H,O-Z)
        COMMON /NAME1/C,Q,UN,G(40,8),AM(40,8),K
        COMMON /NAME2/SUM(8),AG(8),RC,TRC,ALOLP
        DIMENSION ELK(40,8),BM(8),TM(8),TG(8),P(744),X(744)
C
        DATA CI,NU/0.,0/
        AG(7)=0.
        AG(8)=0.
        K=0
        NH=744
        READ(1,200) (X(I),I=1,NH)
        WRITE(3,111) (X(I),I=1,NH)
        DEMAND=0.
        DO 500 I=1,NH
  500   DEMAND=DEMAND+X(I)
        DEMAND=DEMAND/1000.
C
        BSLOAD=X(1)
        PKLOAD=X(1)
        DO 1550 I=2,NH
        IF(X(I).LT.BSLOAD) BSLOAD=X(I)
 1550   IF(X(I).GT.PKLOAD) PKLOAD=X(I)
        WRITE(3,444) NH,BSLOAD,PKLOAD,DEMAND
        XNUMB=NH
        DO 206 I=1,NH
        P(I)=1./XNUMB
  206   CONTINUE
        DO 106 I=1,8
        TM(I)=0.
        DO 106 J=1,NH
  106   TM(I)=TM(I)+P(J)*X(J)**I
        DO 400 I=1,8
  400   WRITE(3,399) I,TM(I)
C
        CALL MISFA(TM,TG)
C
        DO 11 I=1,8
   11   SUM(I)=0.
    1   READ(1,2,END=20) MU,Q,C
        UN=MU
        K=K+1
        CI=CI+UN*C
        NU=NU+UN
        DO 10 I=1,8
   10   AM(K,I)=Q*C**I
C
        CALL MOMENT
C
C   CALCULATION OF TOTAL CUMULANTS
C
        DO 3 I=1,8
        ELK(K,I)=UN*G(K,I)
    3   CONTINUE
        WRITE(3,1017)K,MU,Q,C
        GO TO 1
   20   CONTINUE
C
C   CALCULATION OF TOTAL SYSTEM CUMULANTS
C
        DO 133 I=1,8
        DO 13 M=1,K
        SUM(I)=SUM(I)+ELK(M,I)
   13   CONTINUE
  133   SUM(I)=SUM(I)+TG(I)
C
C   CALCULATION OF G.C. COEFFICIENTS
C
        DO 12 I=1,[illegible]
        [illegible]=I+[illegible]
```

CUMULANT METHOD(CONTD.)

```
      A=M
 12   AG(I)=SUM(M)/(SUM(2)**(A/2))
 14   CONTINUE
      RC=CI
C
C     LOLP OF SYSTEM A  WITHOUT INTERCONNECTION
C
      TRC=RC
      CALL LOLP
      FLOLP=ALOLP
      WRITE(3,509) FLOLP
      DO 35 I=1,8
 35   WRITE(3,37) I,SUM(I),I,AG(I)
      WRITE(3,36) CI,NU
C
 509  FORMAT(////,30X,' L O L P  =',E17.7)
 1017 FORMAT(//,10X,I2,5X,I2,5X,F5.2,5X,F5.0)
  444 FORMAT(/////,40X,'*  NUMBER OF HOURS =',I8///40X,'* BASE LOAD ='
     +F10.4,1X,'(MW)'///40X,'* PEAK LOAD =',F10.4,1X,'(MW)'///40X,'*
     +XP.TOTAL ENERGY DEMAND ='F10.4,'(GWHR)',//)
C
 37   FORMAT(//,20X,'K(',I1,') = ',E15.7,10X,'G(',I1,') = ',E15.7)
 399  FORMAT(//,20X,'K(',I1,') = ',E15.7)
 36   FORMAT(//////,20X,'INSTALLED CAPACITY (MW) = ',F8.2////
     +20X,'NUMBER OF UNITS',9X,'=',I4)
 111  FORMAT(//,10X,12F9.2)
 230  FORMAT(12F6.2)
 2    FORMAT(I2,F5.0,F5.0)
 199  STOP
      END

C
      SUBROUTINE MOMENT
C     IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /NAME1/C,Q,UN,G(40,8),AM(40,8),K
      DIMENSION CM(40,8)
C
C  CALCULATION OF CENTRAL MOMENTS
C
      CM(K,1)=AM(K,1)
      CM(K,2)=AM(K,2)-AM(K,1)**2
      CM(K,3)=AM(K,3)-3*AM(K,2)*AM(K,1)+2*AM(K,1)**3
      CM(K,4)=AM(K,4)+6*AM(K,2)*AM(K,1)**2-4*AM(K,3)*AM(K,1)
     $3*AM(K,1)**4
       CM(K,5)=AM(K,5)-5.*AM(K,4)*AM(K,1)+10.*AM(K,3)*AM(K,1
     $-10.*AM(K,2)*AM(K,1)**3+4*AM(K,1)**5
       CM(K,6)=AM(K,6)-6.*AM(K,5)*AM(K,1)+15.*AM(K,4)*AM(K,1
     $-20.*AM(K,3)*AM(K,1)**3+15*AM(K,2)*AM(K,1)**4-5*AM(K,1
      CM(K,7)=AM(K,7)-7.*AM(K,1)*AM(K,6)+21.*AM(K,1)**2*AM(K      35.*
     $,1)**3*AM(K,4)+35.*AM(K,1)**4*AM(K,3)-21.*AM(K,1)**5*       ,2)+6.
     $(K,1)**7
      CM(K,8)=AM(K,8)-8.*AM(K,1)*AM(K,7)+28.*AM(K,1)**2*AM(   6)-56.*AM(
     $,1)**3*AM(K,5)+70.*AM(K,1)**4*AM(K,4)-56.*AM(K,1)**5*  (K,3)+28.*
     $M(K,1)**6*AM(K,2)-7.*AM(K,1)**8
C
C  CALCULATION OF CUMULANTS
C
      G(K,1)=CM(K,1)
      G(K,2)=CM(K,2)
      G(K,3)=CM(K,3)
      G(K,4)=CM(K,4)-3*CM(K,2)**2
      G(K,5)=CM(K,5)-10.*CM(K,2)*CM(K,3)
      G(K,6)=CM(K,6)-15.*CM(K,2)*CM(K,4)-10.*CM(K,3)**2+30.*CM(K,2)**3
      G(K,7)=CM(K,7)-21.*CM(K,5)*CM(K,2)-35.*CM(K,4)*CM(K,3)+210.*CM(K.
     $)*CM(K,2)**2
      G(K,8)=CM(K,8)-28.*CM(K,6)*CM(K,2)-56.*CM(K,5)*CM(K,3)-35.*CM(K,4
     $**2+420.*CM(K,4)*CM(K,2)**2+560.*CM(K,3)**2*CM(K,2)-630.*CM(K,2)*
     $4
      RETURN
      END
```

CUMULANT METHOD(CONTD.)

```
C
      SUBROUTINE LOLP
C     IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /NAME2/SUM(8),AG(8),RC,TRC,ALOLP
      Z=(TRC- SUM(1))/SQRT(SUM(2))
      T=1./(1.+0.2316419*Z)
      AN=EXP(-(Z**2)/2)/(2*3.14159265)**(0.5)
      A1=AN*T*(((((1.330274429*T-1.821255978)*T+1.781477937)*T-
     $0.35656382)*T+0.31938153)
C
      AN1=-Z*AN
      AN2=(Z**2-1.)*AN
      AN3=-2.*AN1-Z*AN2
      AN4=-3.*AN2-Z*AN3
      AN5=-4.*AN3-Z*AN4
      AN6=-5*AN4-Z*AN5
      AN7=-6*AN5-Z*AN6
C
  303 AK=AG(1)*AN2/6.-AG(2)*AN3/24.+AG(3)*AN4/120.-(AG(4)+10.*AG(1)**2
     $AN5/720.+(AG(5)+35.*AG(1)*AG(2))*AN6/5040.-(AG(6)+56.*AG(1)*AG(3
     $+35.*AG(2)**2)*AN7/40320.
  305 ALOLP=A1+AK
      RETURN
      END
```

```
C
C     SUBROUTINE MISRA(AM,G)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION AM(8),BM(8),G(8)
C
C     CALCULATION OF CENTRAL MOMENTS FROM MOMENT ABOUT ANY POINT
C
      BM(1)=AM(1)
      BM(2)=AM(2)-AM(1)**2
      BM(3)=AM(3)-3*AM(2)*AM(1)+2*AM(1)**3
      BM(4)=AM(4)+6*AM(2)*AM(1)**2-4*AM(3)*AM(1)-
     $3*AM(1)**4
      BM(5)=AM(5)-5.*AM(4)*AM(1)+10.*AM(3)*AM(1)**2
     $-10.*AM(2)*AM(1)**3+4*AM(1)**5
      BM(6)=AM(6)-6.*AM(5)*AM(1)+15.*AM(4)*AM(1)**2
     $-20.*AM(3)*AM(1)**3+15*AM(2)*AM(1)**4-5*AM(1)**6
      BM(7)=AM(7)-7.*AM(1)*AM(6)+21.*AM(1)**2*AM(5)-35.*AM(1)
     $**3*AM(4)+35.*AM(1)**4*AM(3)-21.*AM(1)**5*AM(2)+6.*AM
     $(1)**7
      BM(8)=AM(8)-8.*AM(1)*AM(7)+28.*AM(1)**2*AM(6)-56.*AM(1)
     $**3*AM(5)+70.*AM(1)**4*AM(4)-56.*AM(1)**5*AM(3)+28.*AM
     $(1)**6*AM(2)-7.*AM(1)**8
C
C  CALCULATION OF CUMULANTS FROM CENTRAL MOMENTS
C
      G(1)=BM(1)
      G(2)=BM(2)
      G(3)=BM(3)
      G(4)=BM(4)-3*BM(2)**2
      G(5)=BM(5)-10.*BM(2)*BM(3)
      G(6)=BM(6)-15.*BM(2)*BM(4)-10.*BM(3)**2+30.*BM(2)**3
      G(7)=BM(7)-21.*BM(5)*BM(2)-35.*BM(4)*BM(3)+210.*BM(3
     $)*BM(2)**2
      G(8)=BM(8)-28.*BM(6)*BM(2)-56.*BM(5)*BM(3)-35.*BM(4)
     $**2+420.*BM(4)*BM(2)**2+560.*BM(3)**2*BM(2)-630.*BM(2)**
     $4
      RETURN
      END
```

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