DYNAMIC BEHAVIOUR OF SUSPENDED FOOTBRIDGES SUBJECTED TO PEDESTRIAN INDUCED VIBRATIONS

By

MOHAMMAD MAKSUD-UL-ALAM

DEPARTMENT OF CIVIL ENGINEERING

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

DHAKA 1000, BANGLADESH

DYNAMIC BEHAVIOUR OF SUSPENDED FOOTBRIDGES SUBJECTED TO PEDESTRIAN INDUCED VIBRATIONS

By

MOHAMMAD MAKSUD-UL-ALAM

A THESIS

SUBMITTED TO THE DEPARTMENT OF CIVIL ENGINEERING

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN CIVIL AND STRUCTURAL ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

DHAKA 1000, BANGLADESH

MARCH, 2010

TO MY

PARENTS

DECLARATION

It is hereby declared that, except where specific references are made, the work embodied in this thesis is the result of investigation carried out by the author under the supervision of Dr. A.F.M. Saiful Amin, Associate Professor, Department of Civil Engineering, BUET.

Neither this thesis nor any part of it is being concurrently submitted to any other institution in candidacy for any degree.

Author

The thesis titled **"DYNAMIC BEHAVIOUR OF SUSPENDED FOOTBRIDGES SUBJECTED TO PEDESTRIAN INDUCED VIBRATIONS"** Submitted by **Mohammad Maksud-Ul-Alam, Roll: 040404310, Session: April, 2004,** has been accepted as satisfactory in partial fulfillment of the requirement for the degree of Master of Science in Civil & Structural Engineering on 30th March, 2010.

BOARD OF EXAMINERS

____

Dr. A.F.M. Saiful Amin — Chairman (Supervisor)
Associate Professor
Department of Civil Engineering
BUET, Dhaka

____

Dr. Md. Zoynul Abedin — Member (Ex-officio)
Professor and Head
Department of Civil Engineering
BUET, Dhaka

____

Dr. Syed Ishtiaq Ahmad — Member
Associate Professor
Department of Civil Engineering
BUET, Dhaka

____

Professor Dr. Sohrabuddin Ahmad — Member (External)
Apartment No. A4
House No. 61B, Road No.: 6A
Dhanmondi R.A.
Dhaka-1209

ACKNOWLEDGEMENT

The author would like to express his deepest gratitude to his supervisor, Dr. A.F.M. Saiful Amin, Associate Professor, Department of Civil Engineering, Bangladesh University of Engineering and Technology (BUET), Dhaka. The author wishes to express his sincere thanks for patiently and encouragingly guiding him towards a successful completion of this research. His assistance, advice and timely corrections proved to be very effective and will always be remembered.

Special thanks goes to the staff and author’s fellow students at the Department of Civil Engineering, BUET for their help and interesting conversations about this thesis.

The author wants to express his deepest gratitude to his parents for their support throughout his education. Above all, he is grateful to the Almighty Allah for empowering him to bring this thesis to a satisfactory completion.

ABSTRACT

Over the last few years, the trend in footbridge design has been towards greater spans and lightness. Once followed, such trend gives increased flexibility in dynamic behaviour. As a consequence, stiffness and mass sometimes decrease and lead to smaller natural frequencies. In practice, such footbridge has particularly been found to be more sensitive to dynamically imposed pedestrian loads. The reason behind the sensitivity in movement is known to be related with coincidence of fundamental natural frequency of superstructure with the dominant frequencies of the pedestrian load. In such cases footbridge has the potential to suffer excessive vibrations.

The vertical and horizontal forces that pedestrians impart to a footbridge are considered in the current work for using a modeling procedure in finite element technique to obtain design of some prototype footbridges by considering biomechanics of pedestrian movement and human-structure interaction induced synchronization effects. The work started with a literature review of dynamic loads induced by pedestrians. Design criteria and load models proposed by several widely used standards were introduced and a comparison was made. Dynamic analysis of two footbridges having different structural system has been performed using several modeling techniques to make comparisons. Available solutions to vibration problems and improvements in design procedures were exemplified.

The work further investigates the optimization of a structural system and its effect, the effect on different stiffening mechanisms, vibration modes and the fundamental natural frequencies using finite element models. Different patterns of pedestrian loading have been imposed and dynamic response of as-built structure is compared with analytical predictions. The synchronization effect due to pedestrian movement has been also investigated for the prototype cases. Human perception of vertical and horizontal vibration and their interaction with bridge movement has been studied with respect to vibration serviceability. To this end, the complex issues of human reactions to vibration and next walkup modes are discussed.

Available solutions to vibration problems and improvements of design procedures are studied. It is shown that the requirements in the codes for design of this class of structure widely varies because of the poor understanding of the complex human-structure interaction phenomena and associated bio-mechanical problems. The study and results indicate the necessity of further field measurements to analyze the human-structure dynamic interaction in footbridges to further rationalize the available design codes. In spite of this, the study indicates a better rationality in BS 5400 and ISO 10137 than other codes for taking care of lateral and vertical vibration modes. In order to resist such vibrations, structural system needs to be optimized in the way either by adjusting mass and stiffness in the dominant mode of vibration or by increasing the damping properties. Based on all such observations and keeping the variability of material, support and boundary conditions, the study have further shown the possibility of having a better performance in a hanger supported structure than in a longer span simply supported one.

CONTENTS

Title **Page**

Title	**Page**

Title **Page**

Title		**Page**

List of Notations

m	=	Mass of simple oscillator
k	=	Stiffness of a linear spring
c	=	Viscosity of a linear damper
f_0	=	Natural frequency
ξ	=	Critical damping ratio
Ω	=	Reduced or relative pulsation
$F(t)$	=	External force
F_0	=	Constant force
$\ddot{u}$	=	Acceleration of the structure
$\dot{u}$	=	Velocity of the structure
u	=	Displacement of the structure
$\tilde{m}$	=	Generalized mass of system
$\tilde{c}$	=	Generalized damping of the system
$\tilde{k}$	=	Generalized stiffness of the system
$\tilde{f}(t)$	=	Generalized force of the system
$\psi(x)$	=	Single shape function
M	=	Mass matrix of the structure
C	=	Damping matrix of the structure
K	=	Stiffness matrix of the structure
ω_n	=	Natural frequency of the vibration of the system
ϕ_n	=	Eigenvector or mode shape of the system
G_0	=	Static force or pedestrian weight for vertical component
G_i	=	i-th harmonic amplitude
ϕ_i	=	Phase angle of the i-th harmonic
f_m	=	Walking frequency
$F_v(t)$	=	Vertical component of one pedestrian load
$F_{ht}(t)$	=	Transverse component of one pedestrian load
$F_{hl}(t)$	=	Longitudinal component of one pedestrian load
α_i	=	Fourier co-efficient of the i-th harmonic

N_p	=	Number of persons present
N_L	=	Critical number of people present
$c_R(N)$	=	A correlation factor in a moderate crowd
a_g	=	Acceleration amplitude of the structure
$H(f)$	=	Frequency response function (FRF) for acceleration response
a_{crit}	=	Critical acceleration
F	=	Pulsating point load
$\alpha_{n,v}$	=	numerical coefficient corresponding to the nth harmonic, vertical direction
$\alpha_{n,h}$	=	Numerical coefficient corresponding to the nth harmonic, horizontal dir.
$\phi_{n,v}$	=	Phase angle of nth harmonic, vertical direction
$\phi_{n,h}$	=	Phase angle of nth harmonic, horizontal direction

Chapter 1

INTRODUCTION

1.1 Background

Human-structure dynamic interaction is defined not only as the influence of humans on the dynamic properties of structures they occupy, but also as forces which excite these structures. Both of these issues are becoming increasingly important for all slender civil engineering structures occupied and dynamically excited by humans, such as footbridges, long-span floors, grandstands and staircases. The problems are typically caused by excessive vibrations of such structures due to normal activities of their human occupants, such as walking, running and jumping. The human involvement in the problem is the key source of considerable randomness.

Footbridges are now becoming an integral part of the modern city infrastructures. These bridges allow safe movement of pedestrians over the urban roads, city waterways or highways by providing a grade separated transportation facility in walking mode. Furthermore, in some applications, the bridges of this class also connect urban installations at different elevations (Amin et al. 2005). In the current trend, the architects, in the design process carefully consider the aesthetic appeal of these bridges to maintain a harmony with the surrounding infrastructure of the neighborhood while the structural engineers follow the current design codes to ensure the stability, safety and durability of the structure.

Nowadays structural materials are becoming stronger and these have higher strength to weight ratio. However, live load of footbridge is quite low compared to vehicular traffic loads. For this reason, the design based on static analysis may offer slender bridge structures for pedestrian and cycle track use. As a consequence, stiffness and masses decrease and the structure becomes more flexible and easy to be excited under dynamic forces having smaller natural frequencies. The excitation of a footbridge by a pedestrian passing over it can be unpleasant for a person walking or standing on the bridge, but usually not harmful for the structure itself. Recent experiences regarding dynamic behaviour of slender footbridges have especially shown that vibration serviceability limit states are very important requirements in any such structural design (Spasojevic and Dordjevic 2002).

When the vibration frequency due to pedestrian movement synchronizes with one of the structural frequencies at 0.75 to 4 Hz range, the dynamic forces are significantly magnified and a condition of resonance occurs. The potential for these amplified forces to induce appreciable levels of motion will depend on the number of people walking on the footbridge and how well their movements are synchronized (Stoyanoff and Hunter 2005).

1.2 Different Cases

Several cases of footbridges experiencing excessive vibrations due to pedestrian induced loading have been reported in the last year, although these were not well-known and had not yet been incorporated into the relevant bridge codes. Newland (2003) reported that a German report in 1972 had described how a new steel footbridge had experienced strong lateral vibration during an opening ceremony with 300-400 people using at a time. They explained that the lateral sway of a person’s centre of gravity occurs at half the walking pace. Since the footbridge had a lowest lateral mode at about 1.1 Hz, and people typically walk at about 2 paces/second, their frequency of excitation is 1 Hz which is close to this natural frequency. Thus in this case, an almost resonating vibration occurred. Moreover, it could be supposed that in this case the pedestrian synchronized their step with the bridge vibration, thereby enhancing the vibration considerably (Bachmann 1992). The problem was said to have been solved by the installation of tuned vibration absorbers at horizontal direction.

In 1975, the north section of the Auckland Harbour Road Bridge in New Zealand experienced lateral vibrations during a public demonstration, when the bridge was being crossed by between 2.000 and 4.000 demonstrators. The span of the north section is 190 meters and the bridge deck is made of a steel box girder. Its lowest natural horizontal frequency is 0.67 Hz.

The concept of synchronization turned out to be very important, and this was presented in a later paper by Fujino et al. (1993). The paper described observations of pedestrian-induced lateral vibration of a cable-stayed steel box girder bridge. It was found that when a large number of people (about 2000 people) were crossing the Toda Park bridge at Toda city in Japan, lateral vibration of the bridge deck at 0.9 Hz could build up to an amplitude of 10 mm sway in some of the supporting cables whose natural frequencies were close to 0.9 Hz vibrating with an amplitude of up to 300mm in sway. By analyzing video recordings of pedestrians' head movement, Fujino concluded that lateral deck movement encourages pedestrians to walk in step and that synchronization increases the human force and makes it to be in resonance with the bridge deck. They summarized their findings as the growth process of the lateral vibration of the girder under the congested pedestrian movement. First a small lateral motion is induced by the random lateral human walking forces, and walking of some pedestrians is synchronized to the girder motion. Then resonant force acts on the girder, consequently the girder motion is increased. Walking of more pedestrians is then synchronized, increasing the lateral girder motion (Nakamura and Fujino 2002). In this sense, this vibration was self-excited in nature. Of course, because of adaptive nature of human being, the girder amplitude will not go to infinity and will reach a steady state with time.

The London Millennium Footbridge is a more recent example of this situation. It is a shallow suspension bridge linking St. Paul's Cathedral on the north side of the river with the Tate Modern Art Gallery on the south side. The bridge is over 325 metres long with three spans, the longest being the centre span of 144 metres. To meet the designers' artistic requirements, the suspension cables sag only 2.3 metres, a fraction of the sag of a traditional suspension bridge of the same span. As a result, the cables carry a very high tension force for a bridge of this size, totaling some 2,000 tonnes. When the bridge was opened in June 2000, it was found that the bridge swayed noticeably. With a large number of pedestrians on the bridge, the sideway movements were sufficient to cause people to stop walking and hold on to the hand-rails. Because there was danger of personal injury, it was decided to close the bridge after a few days for remedial work (Dallard et al. 2001a,b).

The dynamic stability of the structures due to human movement induced vibration came into focus. Following that event, several studies have been carried out that led to significant modifications of the code provisions for the footbridges. Nevertheless, the efforts of the architects and structural engineers in coming up with new and innovative designs have not ceased in the recent years. Recently, a cable supported footbridge has been designed and constructed over the Crescent Lake at Dhaka, Bangladesh by considering the recently improved code provisions (Amin et al. 2005).

Based on all these events and works, this thesis work focuses on the dynamic behaviour of the cable supported footbridge according to different standard codes of practices and how the use of dampers can eliminate problems occurring from unexpected dynamic feedback between people on the bridge, and the bridge structure from the viewpoint of structural dynamics.

1.3 Objective

The objective of this thesis is to study the vibration characteristics in vertical and horizontal modes that pedestrians impart to a footbridge. A numerical method e.g. finite element method will be followed. Special attention is given to the responses of a structure due to dynamic loads induced by groups or a crowd of pedestrians which can lead to the synchronization of a percentage of the persons. The work is divided into seven major tasks:

a. To conduct an in-depth literature review on structural dynamics and dynamic loads induced by pedestrians on footbridges.

b. Design criteria and load models proposed by different widely used standards like BS 5400, Euro Code and ISO 10137 will be introduced and a comparison will be made.

c. Development of a finite element model of a cable supported footbridge for dynamic analysis.

d. To compare the dynamic response of an as-built structure with the analytical predictions through eigenvalue analysis.

e. Computation of dynamic response of the cable supported footbridge when subjected to real dynamic loading events as available in different published literatures.

f. Optimization of structural system to investigate the effect of different stiffening systems on the vibration modes and the fundamental natural frequencies of the arch-deck system using the developed finite element model.

g. Study of available solutions to vibration problems and make suggestions for the improvements in design procedures.

1.4 Methodology

Every step of the pedestrian movement can be treated as one impulse, series of steps as impulses along the way and shifted in time. Therefore, load induced by walking can be assumed as sum of loads caused by continual steps, which further can be simulated with moving pulsating point load. With accurate assumptions that the load applied by every step is approximately of the same value, and that the time needed for transmission of pressure is constant for given walking pace, one can assume that this load is periodic in nature. In this way, a pedestrian creates a repeating pattern of forces as his mass rises and falls against the ground. The force has vertical, lateral and a torsional component. Figure 1.1 illustrates the methodology of human-structure dynamic interaction in footbridge structures.

In this thesis, the focus is given on the analysis of the dynamic behaviour of a cable supported footbridge and a steel girder type footbridge, using numerical tools and comparison with available experimental results. For this reason, the three dimensional finite element models of the cable supported footbridge and girder footbridge are developed using SAP 2000, general purpose finite element software (Computers and Structures Inc. 1995).

In this case, an eigenvalue analysis is performed on the bridge models (Caetano and Cunha 2004). This investigation is expected to demonstrate the complex behaviour of a cable supported footbridge and girder footbridge and also to reveal the performance of the structural system against pedestrian movement (Hauksson 2005 and Zivanovic et al. 2005). By comparing the computed eigenvalues for different cases, it is evident that the dynamic stability of the system expressed in terms of eigen-frequencies address the following aspects-

- To stiffen the structure, so that the frequency of the bridge and footsteps no longer matches.

- To increase the damping of the structure so that the energy imparting due to vibration is absorbed.

Discussions are being made on the effect of stiffening the bridge to change its frequency. The additional structure required to do this may change the appearance of the bridge significantly. In this case, increasing the damping of the structural system can be one of the promising solutions, hence considered (Breukelman 2005; Fujino et al. 1993 and Poovarodom et al. 2003).

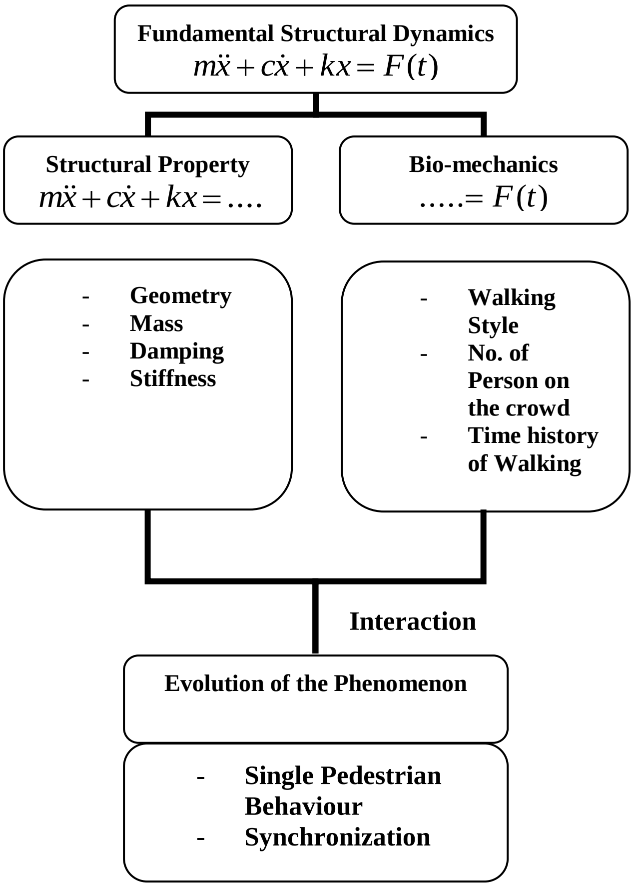


Figure 1.1: Human-structure dynamic interaction in footbridges.

In those contexts, experimental data available from the recent extensive studies (Fujino et al. 1993) on the synchronization of human walking observed during lateral vibration of a congested pedestrian bridge are being utilized to obtain frequency data. Numerical experiments are being carried out using the developed finite element model. Results obtained from numerical simulation and experimental observation is compared to evaluate the adequacy of the developed finite element procedure. Finally, efforts are being made to investigate the possibility of designing controlling devices using developed procedure.

Two types of footbridge structures are being studied in this thesis. These are:

Footbridge-I: Arch supported suspended footbridge

Footbridge-II: Girder footbridge

The study of the dynamic behavior of the arch-supported suspended-span footbridge presented in this thesis originates from a development project initiated by the Public Works Department (PWD), Government of the Peoples' Republic of Bangladesh. The footbridge was constructed over the Crescent Lake, Dhaka, Bangladesh to facilitate movement of the pedestrians from adjacent roads to the nearby Mausoleum Complex of former Bangladesh President. Since the footbridge was to be constructed within Master Plan area of well-known Bangladesh National Parliament Building Complex designed by famous Architect Louis Isadore Kahn, the architectural design of the footbridge needed to be in harmony with the masterpiece creation of Architect Kahn (Figure 1.2, 1.3 and 1.4).

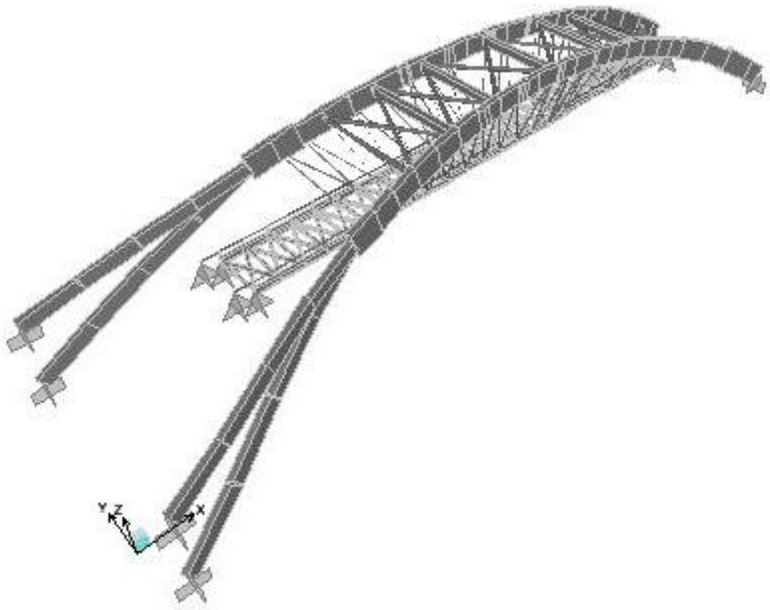


Figure 1.2: Footbridge over the Crescent Lake.

With this motivation, the architectural drawing suggested the construction of the pedestrian bridge with a special physical system where the hanging steel-framed deck (57.3 m in length and 4.572 m in width) fitted with tampered glass panels gets its support from two shallow reinforced concrete arches through hangers made of cables. The arches are connected at the top through reinforced concrete and steel ties. The arches have curvatures both in plan and elevation and are supported on 90 piles to bear the large lateral thrusts.

In this thesis, another type of footbridge is also introduced. It is defined as Footbridge-II. It is a girder bridge and constructed near Radisson Water Garden hotel, Dhaka. This structure is still under construction (Figure 1.5, 1.6 and 1.7).

Figure 1.3: Footbridge over the Crescent Lake in Dhaka.

Figure 1.4: Footbridge with two shallow reinforced concrete arches and hangers made of cables.

Figure 1.5: Perspective view of the front side of the footbridge near Radisson Water Garden Hotel.

Figure 1.6: Footbridge near Radisson Water Garden Hotel over Airport road.

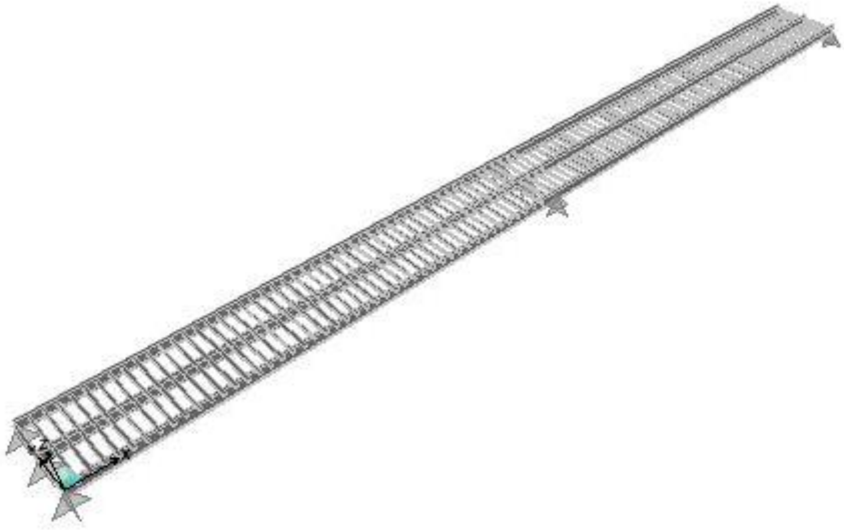


Figure 1.7: Footbridge near Radisson Water Garden Hotel (simplified model).

Table 1.1: Brief Description of Footbridges

Bridge Type	Location of the Bridge	Geographic Coordinates	Span Length (m)	Width (mm)	Structural System	Foundation System
Footbridge-I	Over Crescent Lake, Dhaka	$23^{0}45^{'}54.43^{''}$ N $90^{0}22^{'}42.01^{''}$ E	57.30	4572	Arch supported suspended footbridge	Pile Foundation
Footbridge-II	Near Radisson Water Garden Hotel, Dhaka	$23^{0}48^{'}59.65^{''}$ N $90^{0}24^{'}21.24^{''}$ E	Two span. Each span is 20.38	3000	Steel girder footbridge	Pile Foundation

It is shown that both bridges behave different from each other due to dynamic load and necessity of considering such load is clarified.

1.5 Contributions

The main contributions of this thesis can be summarized as follows:

i) Comparison of design criteria and load models proposed by different widely used standards like BS 5400, Euro Code and ISO 10137.

ii) Development of finite element models of cable supported footbridges for dynamic analysis using general purpose finite element software.

iii) An effort is made to generalize load models for one pedestrian as a load model for a group of people and for a crowded bridge.

iv) Computation of dynamic response of the cable supported footbridge when subjected to real dynamic loading events as available in different published literatures.

v) Optimization of a structural system to investigate the effect of different stiffening systems on the vibration modes and the fundamental natural frequencies of the arch-deck system using the developed finite element model.

1.6 Disposition

This thesis consists of six main parts. First there is a theoretical study of structural dynamics and dynamic loads induced by pedestrians. Chapters 2 and 3 cover these subjects and include formulation of the equation of motion and the eigenvalue problem. These chapters also include a literature study of dynamic loads induced by pedestrians.

Human movement and structural response is discussed in Chapter 4. Here two considerations are discussed. The first considers changes in dynamic properties of the footbridge, mainly in damping and natural frequency, due to human presence. The second aspect concerns a degree of synchronization of movement between the pedestrians themselves as well as between the pedestrians and the structure whose motion is perceived.

In Chapter 5, design criteria for footbridges and models for dynamic pedestrian loads set forth in various widely used standards are compared. This includes a discussion on how current standards and codes of practice deal with vibration problems of footbridges.

Chapter 6 presents dynamic analysis methodology. Here footbridge type, type of crowd density and comfort level are discussed.

In Chapter 7, finite element modeling for the dynamic analysis of the Footbridge-I and Footbridge-II is discussed. The chapter is divided into four parts. In the first section, there is a general description of the footbridge structures. The second part describes the finite element modeling of the footbridges and related software for this modeling and dynamic analysis. The third part is about the pedestrian load modeling. The fourth part describes the different types of analysis methods.

Dynamic analysis of the Footbridge-I and Footbridge-II is performed in Chapter 8. This includes a general description of the footbridge structures as well as a description of the finite element modeling of the footbridges. Here the main focus is to optimize a structural system to investigate the effect of different stiffening systems on the vibration modes and the fundamental natural frequencies of the arch-deck system using the developed finite element model.

Finally, conclusions are summarized in Chapter 9.

Chapter 2

FUNDAMENTAL STRUCTURAL DYNAMICS

2.1 General

Structural dynamics is a subset of structural analysis. It covers the behaviour of structures subjected to dynamic loading. As opposed to static analysis, dynamic analysis considers a time varying load, mass of the structure and the damping property. Dynamic loads include people, wind, waves, traffic, earthquakes, and blasts. Any structure can be a subject to dynamic loading. However, a dynamic analysis yields the time history of displacements (load) and modal behaviour. When the load on the structure is imposed slowly, hence referred to as a quasi-static process, a static analysis may yield a satisfactory result.

Dynamic analysis for simple structures can be carried out manually, but for complex structures finite element analysis is necessary to calculate the mode shapes and frequencies.

The study of a basic model referred to as simple oscillator is presented in this chapter to illustrate the dynamic analysis principles and also highlight the part played by the different structural parameters involved in the process.

Presentation of a literature review encompassing structural dynamics is presented next.

2.2 Structural Dynamics Preliminaries

2.2.1 Simple oscillator

The simple oscillator consists of mass m, connected to a support by a linear spring of stiffness k and a linear damper of viscosity c, impacted by an external force $F(t)$

(Figure 2.1). This oscillator is supposed to move only by translation in a single direction and therefore has only one degree of freedom (herein noted 'dof') defined by position *x(t)* of its mass.

The dynamic parameters specific to this oscillator are the following:

- $\omega_0 = \sqrt{k/m} = 2\pi f_0$: Natural pulsation (rad/s), f_0 being the natural frequency (Hz). Since *m* is a mass, its S.I. unit is therefore expressed in kilograms.

- $\xi = \dfrac{c}{2\sqrt{km}}$: Critical damping ratio (dimensionless) or critical damping percentage. In practice, ξ has a value that is always less than 1. It should be noted that until experimental tests have been carried out, the critical damping ratio can only be assumed. Value of damping depends on materials (steel, concrete, timber) whether the concrete is cracked (reinforced concrete, pre-stressed concrete) and the method of jointing (bolting, welding).

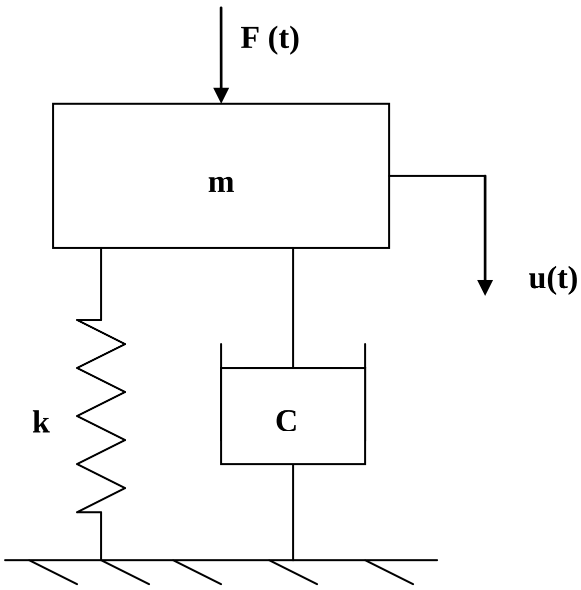


Figure 2.1: Simple Oscillator.

The resonance phenomenon is particularly clear when the simple oscillator is harmonically or sinusoidally excited under the form $F_o sin(\omega t)$.

The static response obtained with a constant force equal to F_o is:

$$u_{static} = \frac{F_o}{k} = \frac{F_o / m}{\omega_o^2} \tag{2.1}$$

The dynamic response may be amplified by a factor $A(\Omega)$ and is equal to:

$$u_{\max} = u_{static} A(\Omega) \tag{2.2}$$

where $\Omega = \omega / \omega_0$ is the reduced (or relative) pulsation and $A(\Omega) = 1/\sqrt{(1-\Omega)^2 + 4\xi^2\Omega^2}$ is the dynamic amplification.

Dynamic amplification is obtained as a function of Ω and ξ. It may be represented by a set of curves parameterized by ξ. Some of these curves are provided in Figure 2.2 for a few specific values of the critical damping ratio. These curves show a peak for the value of $\Omega_R = \sqrt{1-2\xi^2}$ characterizing the resonance and therefore corresponding to the resonance pulsation $\omega_R = \omega_0\sqrt{1-2\xi^2}$ and to the resonance frequency $f_R = \omega_R / 2\pi$. In this case the response is higher (or even much higher) than the static response (Setra 2006).

It should be noted that resonance does not occur for $\omega = \omega_0$ but for $\omega = \omega_R$. Admitting that structural damping is weak in practice, it may be considered that resonance occurs for $\omega = \omega_0$ and that amplification equals:

$$A(\Omega_R = 1) \approx \frac{1}{2\xi} \tag{2.3}$$

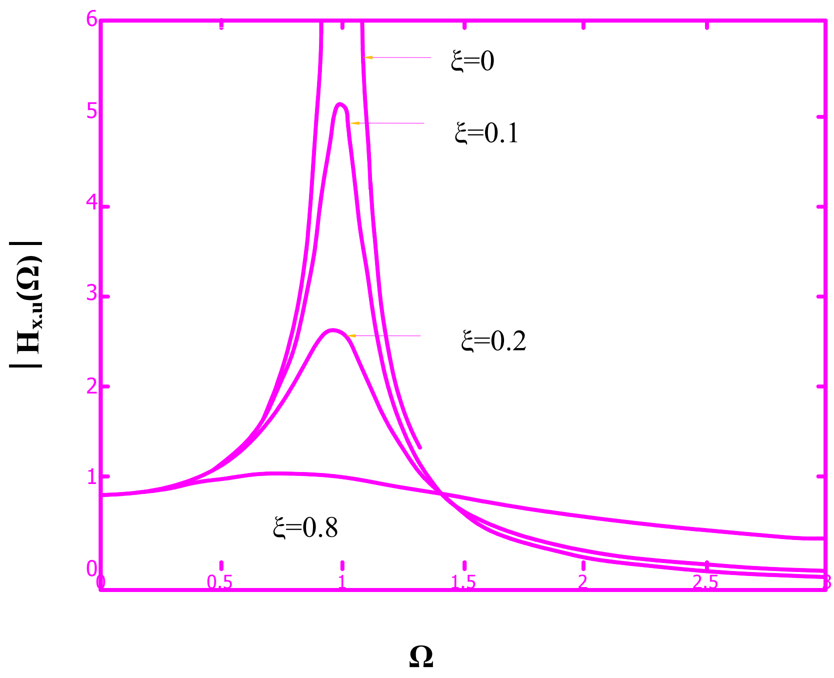


Figure 2.2: Resonance curves (after Setra 2006).

Dimensioning of the structures based on dynamic loading cannot be made using only the maximum intensity of the load impact. Thus, for example, load $F(t) = F_0 \sin(\omega_1 t)$ can generate displacements or stresses very much lower than load $F(t) = (F_0 / 10)\sin(\omega_2 t)$ which however has amplitude 10 times weaker, if this second load has a frequency much closer to the resonance frequency of the structure.

Resonance amplification being directly related to damping, it is necessary to estimate this parameter correctly in order to obtain appropriate dynamic dimensioning. It should be noted that the simple oscillator study relies on the hypothesis of linear damping (viscous, with a damping force proportional to speed), which is one damping type among others including nonlinear damping. However, this is the assumption selected by most footbridge designers and engineers.

2.2.2 Complex systems

The study of real structures, which are generally continuous and complex systems with a large number of degrees of freedom, may be considered as the study of a set of *n* simple oscillators, each one describing a characteristic vibration of the system. The new item with regard to the simple oscillator is the natural vibration mode defined by the pair constituted by the frequency and a vibratory shape (ω_i, φ_i) of the system. Computation of natural vibration modes is relatively intricate but designers nowadays have excellent software packages to obtain them, provided that they take, when modeling the system, all precautionary measures required for model analysis applications.

It should be emphasized that in some cases the problem can even be solved using a single simple oscillator. In any case, the main conclusions resulting from the simple oscillator study can be generalized to complex systems.

2.2.3 Major contributing components in structural dynamics

Structural dynamics describe the behaviour of a structure due to dynamic loads. Dynamic loads are applied to the structure as a function of time, resulting in time varying responses (e.g. displacements, velocities and accelerations) of the structure.

To obtain the responses of the structure a dynamic analysis is performed with the objective to solve the equation of equilibrium between the inertia force, damping force and stiffness force together with the externally applied force:

$$F_I + F_D + F_S = F(t) \tag{2.4}$$

where F_I is the inertial force of the mass and is related to the acceleration of the structure by $F_{\ddot{I}} = m\ddot{u}$, F_D is the damping force and is related to the velocity of the structure by $F_D = c\dot{u}$, F_S is the elastic force exerted on the mass and is related to the displacement of the structure by $F_S = ku$, where *k* is the stiffness, *c* is the damping ratio and *m* is the mass of the dynamic system. Further, $F(t)$ is the externally applied force (Maguire et al. 2002).

Substituting these expressions into Eq. 2.4 gives the equation of motion

$$m\ddot{u} + c\dot{u} + ku = F(t) \tag{2.5}$$

Pedestrian induced vibrations are mainly a subject of serviceability. In this thesis, the dynamic response can be found by solving this equation of motion.

Two different dynamic models are presented in the following sections. First the structure is modeled as a system with one degree of freedom (an SDOF-model) and a solution technique for the equations of the system is presented. Then the structure is modeled as a multi-degree-of-freedom system (an MDOF-model). Modal analysis is then presented as a technique to determine the basic dynamic characteristics of the MDOF-system.

2.2.4 Single Degrees Of Freedom (SDOF) model

In this section the analysis of generalized SDOF systems is introduced. First the equation of motion for a generalized SDOF system with distributed mass and stiffness is formulated. Then a numerical time-stepping method for solving this equation is presented. It is noted, that the analysis provides only approximate results for systems with distributed mass and stiffness.

2.2.4.1 Equation of motion

A system consisting of a simple beam with distributed mass and stiffness can deflect in an infinite variety of shapes. By restricting the deflections of the beam to a single shape function $\psi(x)$ that approximates the fundamental vibration mode, it is possible to obtain approximate results for the lowest natural frequency of the system. The deflections of the beam are then given by $u(x,t) = \psi(x)z(t)$, where the generalized coordinate $z(t)$ is the deflection of the beam at a selected location.

It can be shown that the equation of motion for a generalized SDOF-system is of the form

$$\tilde{m}\ddot{z} + \tilde{c}\dot{z} + \tilde{k}z = \tilde{f}(t) \qquad (2.6)$$

where $\tilde{m}$, $\tilde{c}$, $\tilde{k}$ and $\tilde{f}(t)$ are defined as the generalized mass, generalized damping, generalized stiffness and generalized force of the system. Further, the generalized mass and stiffness can be calculated using the following expressions

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 dx \qquad (2.7)$$

$$\tilde{k} = \int_0^L EI(x)[\psi''(x)]^2 dx \qquad (2.8)$$

where *m(x)* is mass of the structure per unit length, *EI(x)* is the stiffness of the structure per unit length and *L* is the length of the structure.

Damping is usually expressed by a damping ratio, ξ, estimated from experimental data, experience and/or taken from standards. The generalized damping can then be calculated from the expression

$$\tilde{c} = \xi(2\tilde{m}\omega) \qquad (2.9)$$

where ω is the natural frequency of the structure.

Once the generalized properties $\tilde{m}$, $\tilde{c}$, $\tilde{k}$ and $\tilde{f}(t)$ are determined, the equation of motion (Eq. 2.6) can be solved for *z(t)* using a numerical integration method. Finally, by assuming a shape function $\psi(x)$, the displacements at all times and at all locations of the system are determined from $u(x,t) = \psi(x)z(t)$ (Chopra 2001).

2.2.4.2 Response analysis

The most general approach for the solution of the dynamic response of structural systems is to use numerical time-stepping methods for integration of the equation of motion. This involves, after the solution is defined at time zero, an attempt to satisfy dynamic equilibrium at discrete points in time (Wilson 2002).

One method commonly used for numerical integration is the central difference method, which is an explicit method. Explicit methods do not involve the solution of a set of liner equations at each step. Instead, these methods use the differential equation at time t_i to predict a solution at time t_{i+1} (Wilson 2002).

The central difference method is based on a finite difference approximation of the velocity and the acceleration. Taking constant time steps, $\nabla t_i = \nabla t$ the central difference expressions for velocity and acceleration at time t_i are

$$\ddot{u}_i = \frac{u_{i+1} - u_{i-1}}{2\Delta t} \text{ and } \ddot{u}_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta t)^2} \tag{2.10}$$

Substituting these approximate expressions for velocity and acceleration into the equation of motion, Eq. 2.4, gives

$$m\frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta t)^2} + c\frac{u_{i+1} - u_{i-1}}{2\Delta t} + ku_i = f_i \tag{2.11}$$

where u_i and u_{i-1} are known from preceding time steps.

The unknown displacement at time t_{i+1} can now be calculated by

$$u_{i+1} = \frac{\hat{f}_i}{\hat{k}} \tag{2.12}$$

where

$$\hat{k} = \frac{m}{(\Delta t)^2} + \frac{c}{2\Delta t} \tag{2.13}$$

and

$$\hat{f}_i = f_i - [\frac{m}{(\Delta t)^2} - \frac{c}{2\Delta t}]u_{i-1} - [k - \frac{2m}{(\Delta t)^2}]u_i \tag{2.14}$$

This solution at time t_{i+1} is determined from the equilibrium condition at time t_i, which is typical for explicit methods (Chopra 2001).

2.2.5 Multi Degrees of Freedom (MDOF) model

All real structures have an infinite number of degrees of freedom (DOF's). It is, however, possible to approximate all structures as an assemblage of finite number of massless members and a finite number of node displacements. The mass of the structure is lumped at the nodes and for linear elastic structures the stiffness properties of the members can be approximated accurately. Such a model is called a multi degree-of-freedom (MDOF) system.

In this section the analysis of MDOF systems is introduced. First the equation of motion for a MDOF system is formulated. Then the concept of modal analysis is presented. Modal analysis includes the formulation of the eigenvalue problem and a solution method for solving the eigenvalue problem. Finally, modal analysis can be used to compute the dynamic response of an MDOF system to external forces. Equation of motion

As mentioned above, a structure can be idealized as an assemblage of elements connected at nodes. The displacements of the nodes are the degrees of freedom. By discretizing the structure in this way, a stiffness matrix **K**, a damping matrix **C** and a mass matrix **M** of the structure can be determined. Each of these matrices are of order *N x N* where *N* is the number of degrees of freedom.

The stiffness matrix for a discretized system can be determined by assembling the stiffness matrices of individual elements. Damping for MDOF systems is often specified by numerical values for the damping ratios, as for SDOF systems. The mass is idealized as lumped or concentrated at the nodes of the discretized structure, giving a diagonal mass matrix.

The equation of motion of a MDOF system can now be written on the form:

$$M\ddot{u} + C\dot{u} + Ku = F(t) \tag{2.15}$$

which is a system of *N* ordinary differential equations that can be solved for the displacements *u* due to the applied forces *F(t)*. It is now obvious that Eq. 2.15 is the MDOF equivalent of Eq. 2.6 for a SDOF system (Chopra 2001).

2.2.5.1 Modal analysis

Modal analysis can be used to determine the natural frequencies and the vibration mode shapes of a structure. The natural frequencies of a structure are the frequencies at which the structure naturally tends to vibrate if it is subjected to a disturbance. The vibration mode shapes of a structure are the deformed shapes of the structure at a specified frequency.

When performing modal analysis, the free vibrations of the structure are of interest. Free vibration is when no external forces are applied and damping of the structure is neglected. When damping is neglected the eigenvalues are real numbers. The solution for the undamped natural frequencies and mode shapes is called real eigenvalue analysis or normal modes analysis. The equation of motion of a free vibration is:

$$M\ddot{u} + Ku = 0 \tag{2.16}$$

This equation has a solution in the form of simple harmonic motion:

$$u = \phi_n \sin \omega_n t \text{ and } \ddot{u} = -\omega_n^2 \phi_n \sin \omega_n t \tag{2.17}$$

Substituting these into the equation of motion gives

$$K\phi_n = \omega_n^2 M \phi_n \tag{2.18}$$

which can be rewritten as

$$\left[K - \omega_n^2 M \right] \phi_n = 0 \tag{2.19}$$

This equation has a nontrivial solution if

$$\det \left[K - \omega_n^2 M \right] = 0 \tag{2.20}$$

Equation 2.20 is called the system characteristic equation. This equation has *N* real roots for ω_n^2, which are the natural frequencies of vibration of the system. They are as many as the degrees of freedom, *N*. Each natural frequency ω_n has a corresponding eigenvector or mode shape ϕ_n, which fulfills equation 2.19. This is the generalized eigenvalue problem to be solved in free vibration modal analysis.

After having defined the structural properties; mass, stiffness and damping ratio and determined the natural frequencies ω_n and modes ϕ_n from solving the eigenvalue problem, the response of the system can be computed as follows. First, the response of each mode is computed by solving following equation for $q_n(t)$

$$M_n \ddot{q}_n + C_n \dot{q}_n + K_n q_n = F_n(t) \tag{2.21}$$

Then, the contributions of all the modes can be combined to determine the total dynamic response of the structure:

$$u(t) = \sum_{n=1}^{N} \phi_n q_n(t) \tag{2.22}$$

The parameters M_n, K_n, C_n and $F_n(t)$ are defined as follows

$$M_n = \phi_n^T M \phi_n,\ K_n = \phi_n^T K \phi_n,\ C_n = \phi_n^T C \phi_n \text{ and } F_n(t) = \phi_n^T F(t) \tag{2.23}$$

and they depend only on the n th-mode ϕ_n, and not on other modes. Thus, there are N uncoupled equations like Eq. 2.22, one for each natural mode (Chopra 2001).

In practice, modal analysis is almost always carried out by implementing the finite element method (FEM). If the geometry and the material properties of the structure are known, an FE model of the structure can be built. The mass, stiffness and damping properties of the structure, represented by the left hand side of the equation of motion (Eq. 2.15), can then be established using the FE method. All that now remains, in order to solve the equation of motion, is to quantify and then to model mathematically the applied forces *F(t)*. This will be the subject of the next chapter.

Chapter 3

BIO-MECHANICS OF PEDESTRIAN MOVEMENT

3.1 General

Static loads are not time variant. On the other hand, dynamic loads are time-dependent and are grouped in four categories:

Category 1: Harmonic or purely sinusoidal loads
Category 2: Periodically recurrent loads integrally repeated at regular time intervals referred to as periods
Category 3: Random loads showing arbitrary variations in time, intensity, direction etc.
Category 4: Pulsing loads corresponding to very brief loads

Pedestrian loads are time-variant and are classified in the 'periodic load' category (Category 2). One of the main features of the dynamic loading of pedestrians is its low intensity. Applied to very stiff and massive structures this load could hardly make them vibrate significantly. However, aesthetic, technical and technological developments lead to ever more slender and flexible structures, footbridge follow this general trend and they are currently designed and built with higher sensitivity to displacements. As a consequence they more frequently require a thorough dynamic analysis that considers interaction between pedestrian load and super structure movement.

Presentation of a literature review encompassing bio-mechanical loads induced by pedestrians is presented next.

3.2 Pedestrian Loading

3.2.1 Effects of pedestrian walking

Pedestrian loading, whether walking or running, has been studied rather thoroughly and is translated as a point force exerted on the support, as a function of time and pedestrian position. Noting that x is the pedestrian position in relation to the footbridge centerline, the load of a pedestrian moving at constant speed v can therefore be represented as the product of a time component $F(t)$ by a space component $\delta(x\text{-}vt)$, δ being the Dirac operator, that is:

$$P(x,t) = F(t)\delta(x - vt) \tag{3.1}$$

Several parameters may also affect and modify this load (gait, physiological characteristics and apparel, ground roughness, etc.), but the experimental measurements performed show that it is periodic, characterized by a fundamental parameter: frequency that is the number of steps per second. Table 3.1 provides the estimated frequency values (Setra 2006).

Table 3.1: Frequency values of different pedestrian walking

Designation	Specific features	Frequency range (Hz)
Walking	Continuous contact with the ground	1.6 to 2.4
Running	Discontinuous contact	2.0 to 3.5

Conventionally, for normal walking (unhampered), frequency may be described by a Gaussian distribution with 2 Hz average and about 0.20 Hz standard deviation (from 0.175 to 0.22, depending on authors). Recent studies and conclusions drawn from recent testing have revealed even lower mean frequencies, around 1.8 Hz-1.9 Hz.

The periodic function may $F(t)$, may therefore be resolved into a Fourier series, that is a constant part increased by an infinite sum of harmonic forces. The sum of all unitary contributions of the terms of this sum returns the total effect of the periodic action.

$$F(t) = G_0 + G_1 \sin 2\pi f_m t + \sum_{i-2}^{n} G_i \sin(2\pi i f_m t - \phi_i) \tag{3.2}$$

with G_0 : static force (pedestrian weight for the vertical component), G_1 : first harmonic amplitude, G_i : i-th harmonic amplitude, f_m : walking frequency, ϕ_i : phase angle of the i-th harmonic in relation to the first one, n : number of harmonics taken into account.

The mean value of 700N may be taken for G_0, weight of one pedestrian.

At mean frequency, around 2 Hz (f_m = 2 Hz) for vertical action, the coefficient values of the Fourier decomposition of *F(t)* are the following (limited to the first three terms, that is n = 3, the coefficients of the higher of the terms being less than 0.1 G_0):

$$\begin{aligned} &G_1 = 0.4G_0 ; G_2 = G_3 \approx 0.1G_0 \\ &\phi_2 = \phi_3 \approx \pi / 2 \end{aligned} \tag{3.3}$$

By resolving the force into three components, that is, a 'vertical' component and two horizontal components (one in the 'longitudinal' direction of the displacement and one perpendicular to the transverse or lateral displacement), the following values of such components may be selected for dimensioning (in practice limited to the first harmonic):

Vertical component of one-pedestrian load:

$$F_v(t) = G_0 + 0.4G_0 \sin(2\pi f_m t) \tag{3.4}$$

Transverse horizontal component of one-pedestrian load:

$$F_{ht}(t) = 0.05G_0 \sin\left(2\pi(\frac{f_m}{2})t\right) \tag{3.5}$$

Longitudinal horizontal component of one-pedestrian load:

$$F_{hl}(t) = 0.2G_0 \sin(2\pi f_m t) \tag{3.6}$$

It should be noted that, for one same walk, the transverse load frequency is equal to half the frequency of the vertical and longitudinal load. This is due to the fact that the load period is equal to the time between the two consecutive steps for vertical and longitudinal load since these steps exert a force in the same direction whereas this duration corresponds to two straight and consecutive right footsteps or to two consecutive left footsteps in the case of transverse load since the left and right footsteps exert loads in opposite directions. As a result, the transverse load period is two times higher than the vertical and longitudinal load and therefore the frequency is two times lower.

3.2.2 Effects of pedestrian running

The crossing duration of joggers on the footbridge is relatively short and does not leave much time for the resonance phenomenon to settle, in addition, this annoys the other pedestrians over a very short period. Moreover, this load case does not cover exceptional events such as a marathon race which must be studied separately. Bio-mechanically, running effects are not usually considered in these guidelines. This load case, which may be very dimensioning in nature, should not be systematically retained.

3.2.3 Random effects of several pedestrians and crowd

In practice, footbridges are submitted to the simultaneous actions of several persons and this makes the corresponding dynamic action much more complicated. In fact, each pedestrian has its own characteristics (weight, frequency, speed) and, according to the number of persons present on the bridge, pedestrians will generate loads which are more or less synchronous with each other, on the one hand, and possibly with the footbridge, on the other. Added to these, there are the initial phase shifts between pedestrians due to the different moments when each individual enters the footbridge.

Moreover, the problem induced by intelligent human behaviour is such that, among others and facing a situation differently expected, the pedestrian will modify his natural and normal gait in several ways; this behaviour can hardly be submitted to software processing.

It is very difficult to fully simulate the actual action of the crowd. One can merely set out reasonable and simplifying hypotheses, based on pedestrian behaviour studies, and then assume that the crowd effect is obtained by multiplying the elementary effects of one pedestrian, possibly weighted by a minus factor. Various ideas exist as concern crowd effects and they antedate the Solferino and Millenium Footbridge incidents. These concepts are presented in the following paragraphs together with a more comprehensive statistical study which was used as a basis for the loadings recommended in these guidelines (Setra 2006).

3.2.3.1 Random type pedestrian flow: conventional model

For a large number of independent pedestrians (that is, without any particular synchronization) which enter a bridge at a rate of arrival λ (expressed in persons/second) the average dynamic response at a given point of the footbridge submitted to this pedestrian flow is obtained by multiplying the effect of one single pedestrian by a factor $k = \sqrt{\lambda T}$, T being the time taken by a pedestrian to cross the footbridge (which can also be expressed by $T = L/V$ where L represents the footbridge length and v the pedestrian speed). In fact, this product λt presents the number of N pedestrians present on the bridge at a given time. Practically, this means that n pedestrians present on a footbridge are equivalent to $\sqrt{N}$ all of them being synchronized. This result can be demonstrated by considering a crowd with the individuals all at the same frequency with a random phase distribution.

This result takes into consideration the phase shift between pedestrians, due to their different entrance time, but comprises a deficiency since it works on the assumption that all pedestrians are moving at the same frequency.

3.2.3.2 *Experimental measurements on pedestrian flows*

Several researchers have studied the forces and moments initiated by a group of persons, using measurements made on instrumented platforms where small pedestrian groups move. Ebrahimpour et al. (1990) proposes a sparse crowd loading model on the first term of a Fourier representation the coefficient α_1 of which depends on the number N_p of persons present on the platform (for a 2 Hz walking frequency):

Table 3.2: Relation between fourier coefficient α_1 and number of pedestrians

$\alpha_1 = 0.34 - 0.09 \log(N_p)$	for $N_p < 10$
$\alpha_1 = 0.25$	for $N_p > 10$

Unfortunately, this model does not cover the cumulative random effects.

3.2.3.3 *Comprehensive simulation model of pedestrian flows*

Until recently dynamic dimensioning of footbridges was mainly based on the theoretical model loading case with one single pedestrian completed by rather crude requirements concerning footbridge stiffness and natural frequency floor values. Obviously, such requirements are very insufficient and in particular they do not cover the main problems raised by the use of footbridges in urban areas which are subject to the action of more or less dense pedestrian groups and crowds. Even the above-addressed $\sqrt{N}$ model has some deficiencies.

It rapidly appears that knowledge of crowd behaviour is limited and this makes the availability of practical dimensioning means all the more urgent. It is better to suggest simple elements to be improved on subsequently, rather than remaining in the current knowledge void.

Therefore several crowd load cases have been developed using probability calculations and statistical processing to deepen the random crowd issue. The model finally selected consists in handling pedestrians' moves at random frequencies and phases, on a footbridge presenting different modes and in assessing each time the equivalent number of pedestrians which - when evenly distributed on the footbridge, or in phase and at the natural frequency of the footbridge will produce the same effect as random pedestrians.

Several digital tests were performed to take into consideration the statistical effect (Setra 2006). For each test including N pedestrians and for each pedestrian, a random phase ϕ and a normally distributed random frequency $f = \omega / 2\pi$ centred around the natural frequency of the footbridge and with 0.175 Hz standard deviation, are selected; the maximum acceleration over a sufficiently long period (in this case the time required for a pedestrian to span the footbridge twice at a 1.5 m/s speed) is noted and the equivalent number of pedestrians which would be perfectly synchronized is calculated. The method used is explained in Figure 3.1.

These tests are repeated 500 times with a fixed number of pedestrians, fixed damping and a fixed number of mode antinodes; then the characteristic value, such as 95% of the samples give a value lower than this characteristic value (95% characteristic value, 95 percentile or 95% fractile). This concept is explained in Figure 3.2.

By varying damping, the number of pedestrians, the number of mode antinodes, it is possible to infer a law for the equivalent number of pedestrians; this law is the closest to the performed test results.

The two laws are retained:

Sparse or dense crowd: random phases and frequencies with Gaussian law distribution: $N_{eq} = 10.8\sqrt{N\xi}$ where N is the number of pedestrians present on the footbridge (density × surface area) and ξ is the critical damping ratio.

Very dense crowd: random phases and all pedestrians at the same frequency: $N_{eq} = 1.85\sqrt{N}$.

This model is considerably simplified in the calculations. We only heed to distribute the N_{eq} pedestrians on the bridge, to apply to these pedestrians a force the amplitude sign is the same as the mode shape sign and to consider this force as the natural frequency of the structure and to calculate the maximum acceleration obtained at the corresponding resonance.

3.2.4 Lock-in of a pedestrian crowd

Lock-in expresses the phenomenon by which a pedestrian crowd, with frequencies randomly distributed around an average value and with random phase shifts, will gradually coordinate at common frequency (that of the footbridge) and enters in phase with the footbridge motion. So far, known cases of crowd lock-in have been limited to transverse footbridge vibrations. The most recent two cases, now famous, are the Solférino footbridge and the Millennium footbridge which were submitted to thorough in-situ tests. Once again, these tests confirm that the phenomenon is clearly explained by the pedestrian response as he modifies his walking pace when he perceives the transverse motion of the footbridge and it begins to disturb him. To compensate his incipient unbalance, he instinctively follows the footbridge motion frequency. Thus, he directly provokes the resonance phenomenon and since all pedestrians undergo it, the problem is further amplified and theoretically the whole crowd may become synchronized. Fortunately, on the one hand, actual synchronization is much weaker and, on the other, when the footbridge movement is such that the pedestrians can no longer put their best foot forward, they have to stop walking and the phenomenon can no longer evolve.

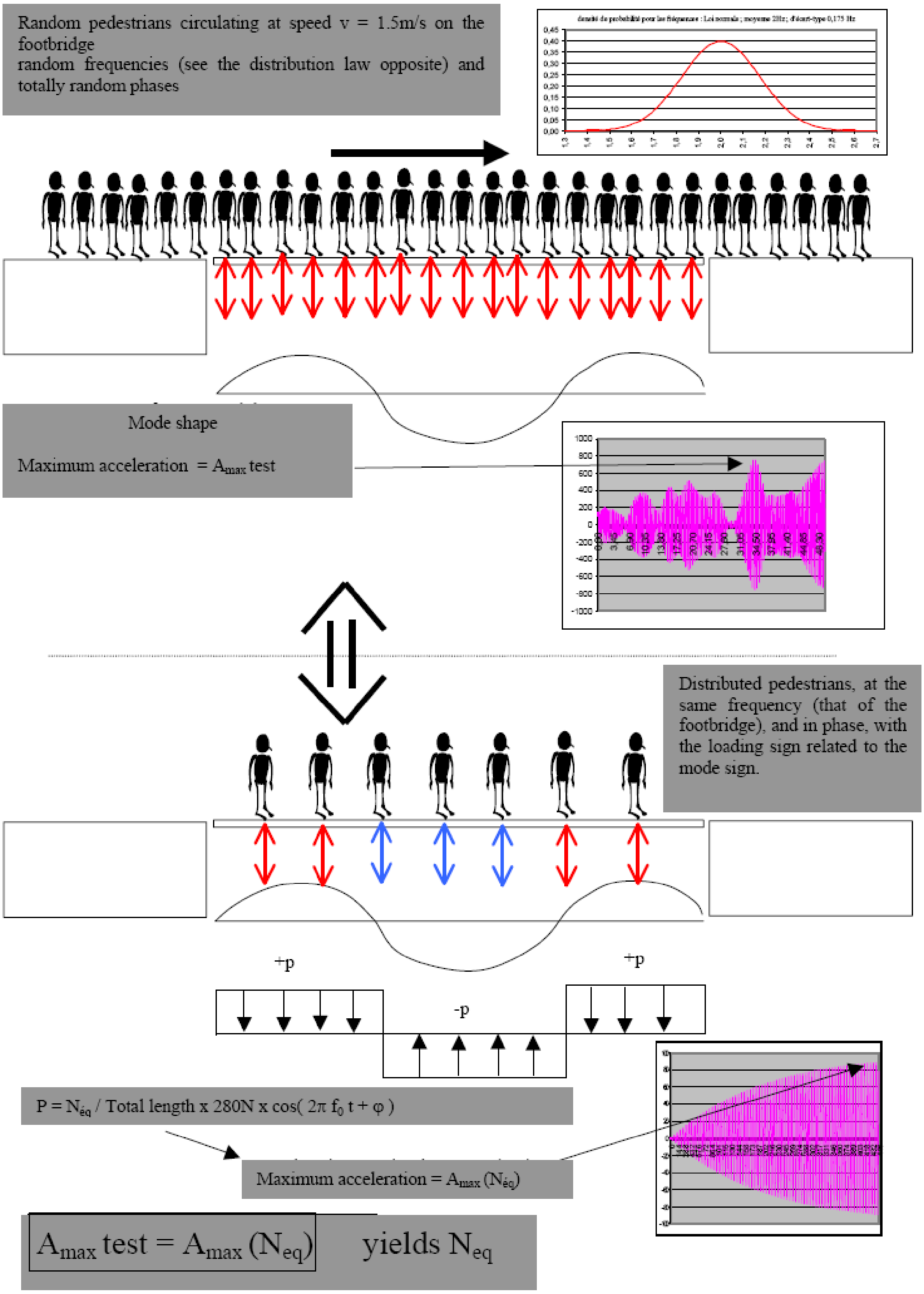


Figure 3.1: Calculation methodology for the equivalent number of pedestrians *Neq* (after Setra 2006).

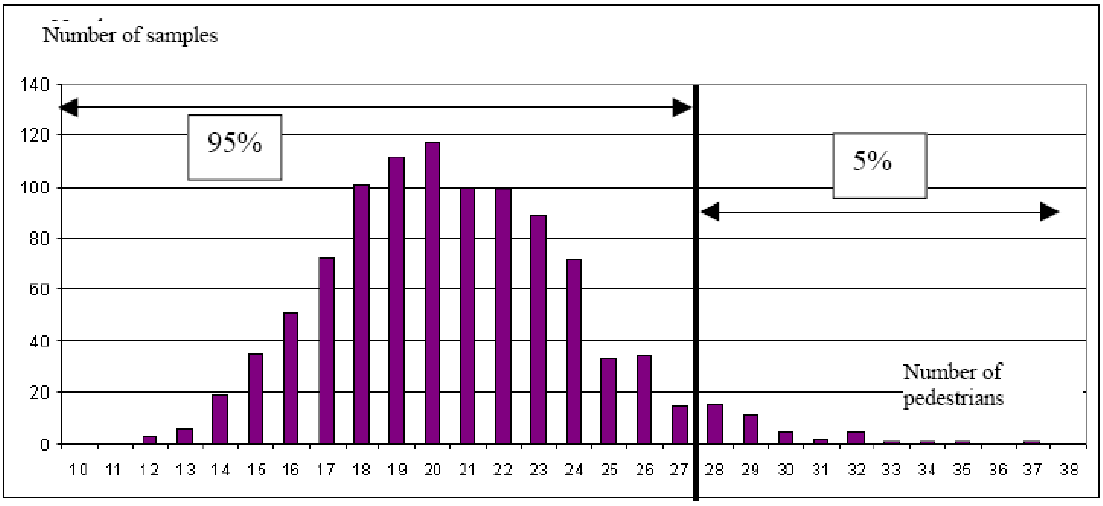


Figure 3.2: 95% fractile concept (after Setra 2006).

3.2.4.1 Pedestrian flows measured on a real footbridge structure

Using a large footbridge with a 5.25 m x 134 m main span which can be subjected to a very dense crowd (up to about 2 persons/m²), Fujino et al. (1993) observed that application of the above factor gave an under-estimation of about $\sqrt{N}$ 1 to 10 times of the actually observed lateral vibration amplitude. They formed the hypothesis of synchronization of a crowd walking in synchrony with the transverse mode frequency of their footbridge to explain the phenomenon and were thus able to prove, in this case, the measurement magnitude obtained. This is the phenomenon we call "lock-in", and a detailed presentation is provided in this study.

For this structure, by retaining only the first term of the Fourier decomposition for the pedestrian-induced load, these authors propose a $0.2N$ multiplication factor to represent any loading which would equate that of a crowd of N persons, allowing them to retrieve the magnitude of the effectively measured displacements (0.01 m).

3.2.4.2 Theory formulated for the Millennium footbridge

Arup's team issued a very detailed article on the results obtained following this study and tests performed on the Millennium footbridge (Dallard et al. 2001a & 2001b). Only the main conclusions of this study are mentioned here.

The model proposed for the Millennium footbridge study is as follows: the force exerted by a pedestrian (in N) is assumed to be related to footbridge velocity.

$F_{1pedestrian} = KV(x,t)$ where K is a proportionality factor (in Ns/m) and V the footbridge velocity at the point *x* in question and at time *t*.

Seen this way, pedestrian load may be understood as a negative damping. Assuming a viscous damping of the footbridge, the negative damping force induced by a pedestrian is directly deducted from it. The consequence of lock-in is an increase of this negative damping force, induced by the participation of a higher number of pedestrians. This is how the convenient notion of critical number appears: this is the number of pedestrians beyond which their cumulative negative damping force becomes higher than the inherent damping of the footbridge; the situation would then be similar to that of an unstable oscillator: a small disturbance may generate indefinitely-amplifying movements.

For the particular case of a sinusoidal horizontal vibration mode (the maximum amplitude of this mode being normalized to 1, f_1 representing the first transverse natural frequency and m_1 the generalized mass in this mode, considering the maximum unit displacement) and assuming an uneven distribution of the pedestrians, the critical number can then be written as:

$$N = \frac{8\pi\xi m_1 f_1}{K} \tag{3.7}$$

K is the proportionality factor, with a value of 300 N-s/m in the case of the Millennium footbridge.

It can be then noted that a low damping, a low mass, or a low frequency is translated by a small critical number and therefore a higher lock-in risk. Consequently, to increase the critical number it will be necessary to act on these three parameters.

It will be noted that the value of factor K cannot *a priori* be generalized to any structure and therefore its use increases the criterion application uncertainty.

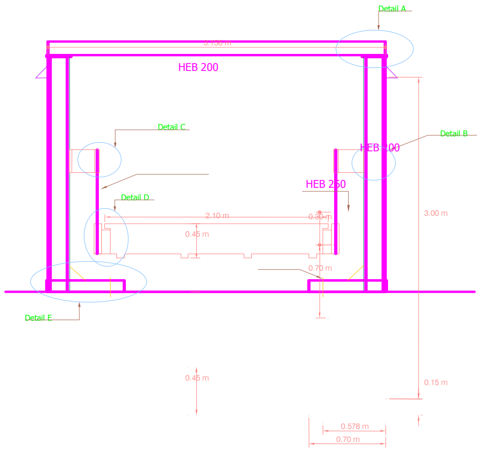
3.2.4.3 Laboratory tests on platform

To quantify the horizontal load of a pedestrian and the pedestrian lock-in effects under lateral motion, some tests were carried out on a reduced footbridge model by recreating, using a dimensional analysis, the conditions prevailing on a relatively simple design of footbridge (one single horizontal mode) (Setra 2006).

The principle consists in placing a 7-metre long and 2-metre wide slab on 4 flexible blades moving laterally and installing access and exit ramps as well as a loop to maintain walking continuity (Figure 3.3). To maintain this continuity, a large number of pedestrians is of course needed on the loop; this number being clearly higher than the number of pedestrians present on the footbridge at a given time.

By recreating the instantaneous force from the displacements measured (previously filtered to attenuate the effect of high frequencies) $[F(t) = m\ddot{x}(t) + c\dot{x}(t) + kx(t)]$. Figure 3.4 shows that, in a first step, for an individual pedestrian the amplitude of the pedestrian force remain constant, around 50N, and in any case, lower than 100N, whatever the speed amplitude. In a second step, it is observed that the force amplitude increases up to 150N, but these last oscillations should not be considered as they represent the end of the test.

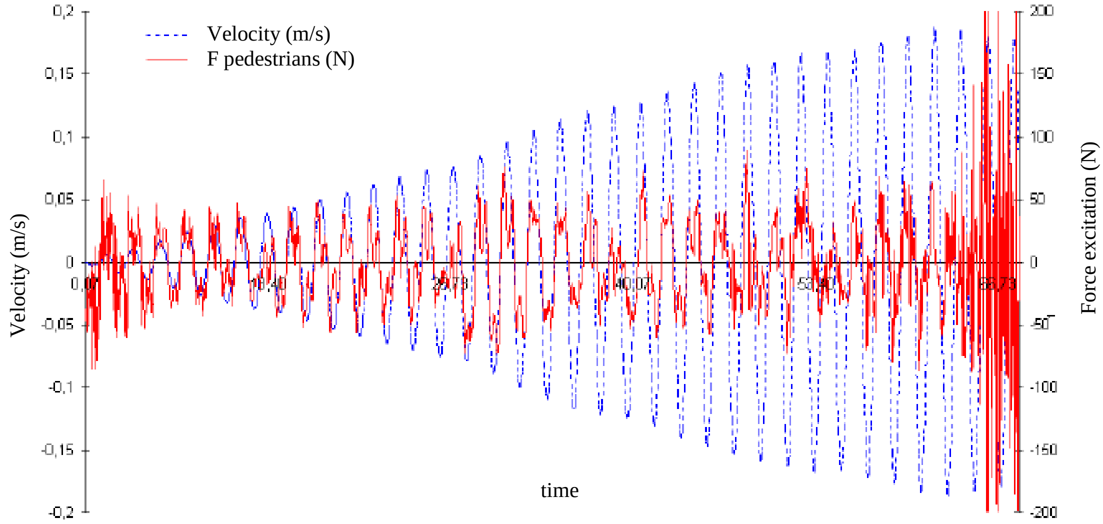
It has a peak value which does not exceed 100 N and is rather around 50N on average, with the first harmonic of this signal being around 35 N.



CROSS-SECTION VIEW (with blade 70 cm)
Scale 1/25
Detail A
Detail B
Detail C
Flexible blade (thickness 8 mm)
Detail D
Attachment axis to the slab
Detail E
HEB

Figure 3.3: Description of the model (after Setra 2006).

The graphs (Figures 3.5 and 3.6) represent, on the same figure, the accelerations in time (pink curve and scale on the RH side expressed in m/s², variation from 0.1 to 0.75 m/s²) and the 'efficient' force (instantaneous force multiplied by the speed sign, averaged on a period, which is therefore positive when energy is injected into the system and negative in the opposite case) for a group of pedestrians (blue curve, scale on the LH side expressed in N).



Comparison of F(t) and v(t)
Test of 01-10-03 – 1 pedestrian at 0.53 Hz – pedestrians following each other
Speed (m/s)
F pedestrians (N)
Time
Exciting force (N)

Figure 3.4: Force and speed at forced resonant rate (after Setra 2006).

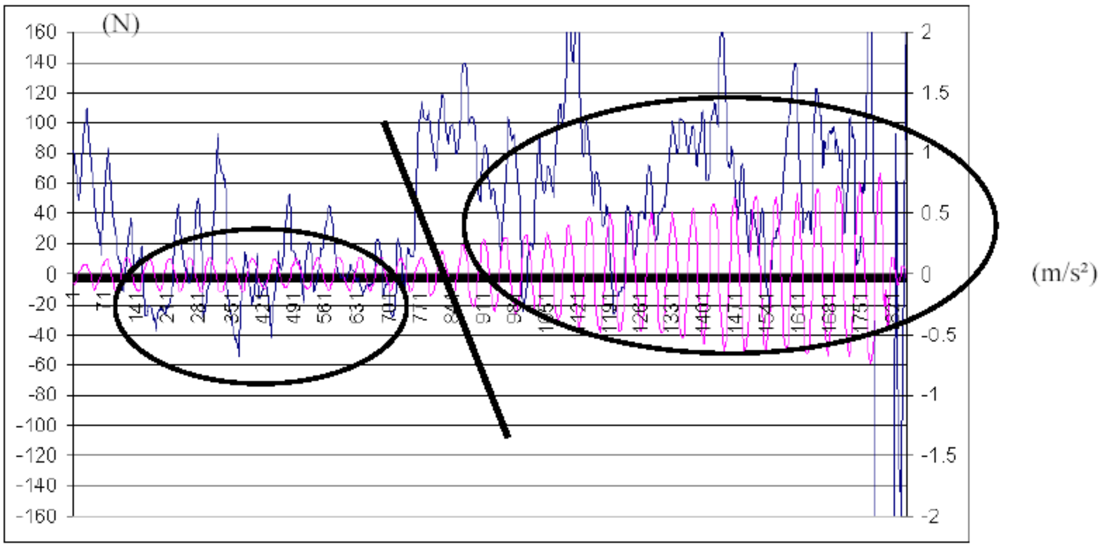


Figure 3.5: Acceleration (m/s²) and efficient force (N) with 6 random pedestrians on the footbridge (after Setra 2006).

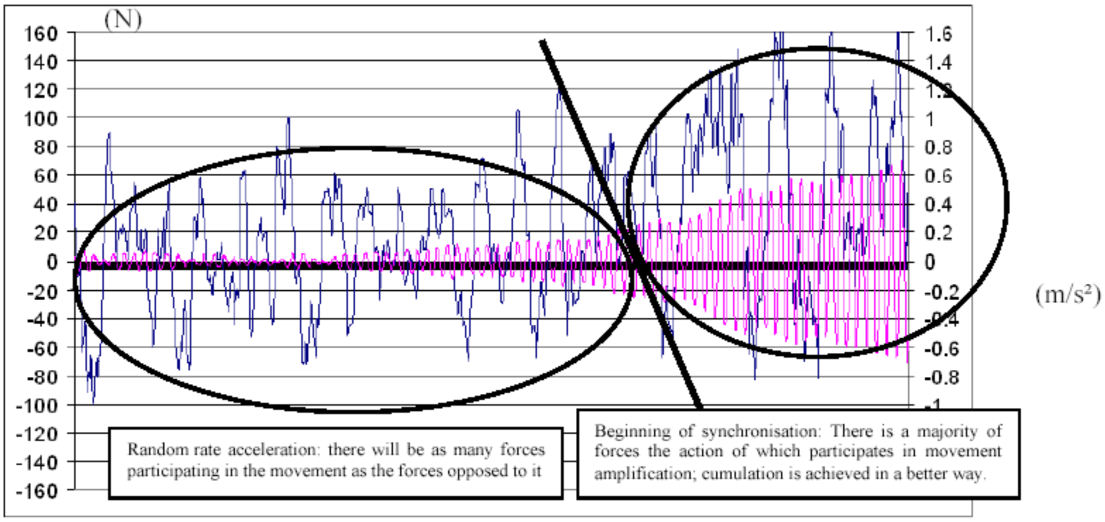


Figure 3.6: Acceleration (m/s²) and efficient force (N) with 10 random pedestrians on the footbridge (after Setra 2006).

It can be observed that, from a given value, the force exerted by the pedestrians is clearly more efficient and there is some incipient synchronization. This threshold is around 0.15 m/s²(straight line between the random rate zone and the incipient synchronization zone). However, there is only some little synchronization (maximum value of 100-150 N i.e. 0.2 to 0.3 times the effect of 10 pedestrians), but this is quite sufficient to generate very uncomfortable vibrations (>0.6m/s²).

3.2.4.4 Experience gained from the Solferino footbridge test results

Several test campaigns were carried out over several years following the closing of the Solferino footbridge to traffic, from the beginning these tests were intended to identify the issues and develop corrective measures; then they were needed to check the efficiency of the adopted measures and, finally, to draw lessons useful for the scientific and technical community.

The main conclusions to be drawn from the Solferino footbridge tests are the following:

- The lock-in phenomenon effectively occurred for the first mode of lateral swinging for which the double of the frequency is located within the range of normal walking frequency of pedestrians.

- On the other hand, it does not seem to occur for modes of torsion that simultaneously present vertical and horizontal movements, even when the test crowd was made to walk at a frequency that had given rise to resonance. The strong vertical movements disturb and upset the pedestrians' walk and do not seem to favor maintaining it at the resonance frequency selected for the tests. High horizontal acceleration levels are then noted and it seems their effects have been masked by the vertical acceleration.

- The concept of a critical number of pedestrians is entirely relative: it is certain that below a certain threshold lock-in cannot occur, however, on the other hand, beyond a threshold that has been proven various specific conditions can prevent it from occurring.

- Lock-in appears to initiate and develop more easily from an initial pedestrian walking frequency for which half the value is lower than the horizontal swinging risk natural frequency of the structure. In the inverse case, that is, when the walking crowd has a faster initial pace several tests have effectively shown that it did not occur. This would be worth studying in depth but it is already possible to explain that the pedestrian walking fairly rapidly feels the effects of horizontal acceleration not only differently, which is certain, but also less noticeably, and this remains to be confirmed.

- Clearly lock-in occurs beyond a particular threshold. This threshold may be explained in terms of sufficient number of pedestrians on the footbridge (conclusion adopted by Arup's team), but it could just as well be explained by an acceleration value felt by the pedestrian, which is more practical for defining a verification criterion.

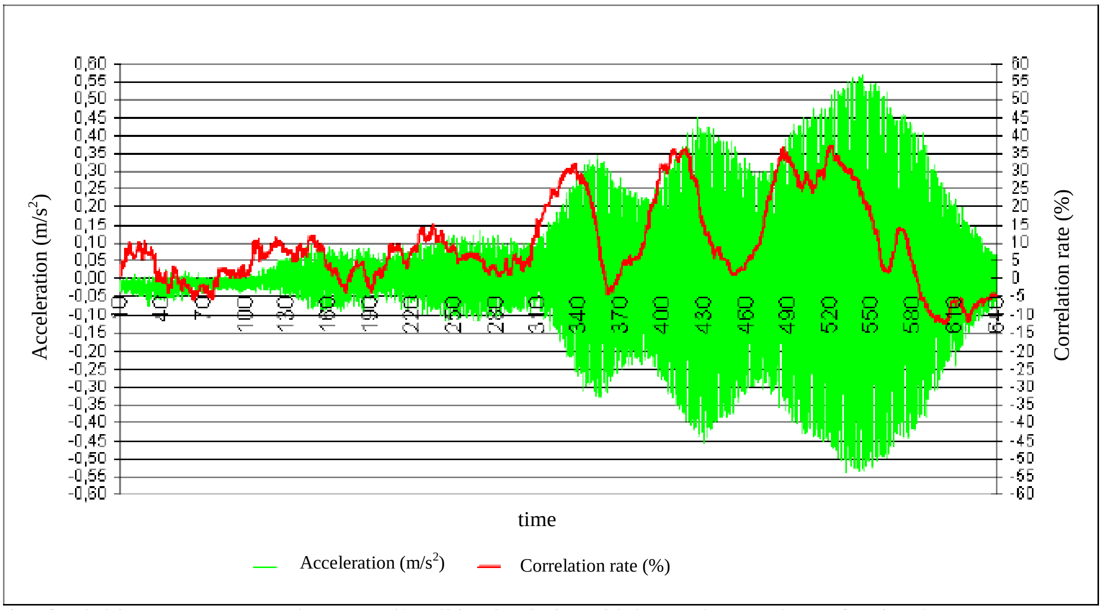
The graphs (Figures 3.7 to 3.12) present a summary of the tests carried out on the Solferino footbridge. The evolution of acceleration over time is shown (in green), and in parallel the correlation or synchronization rate, ratio between the equivalent number of pedestrians and the number of pedestrians present on the footbridge. The equivalent number of pedestrians can be deduced from the instantaneous modal force. It is the number of pedestrians who, regularly distributed on the structure, and both in phase and at the same frequency apparently inject an identical amount of energy per period into the system.

In the test shown in Figure 3.7, it can be seen that below $0.12m/s^2$, behaviour is completely random, and from $0.15m/s^2$, it becomes partly synchronized, with synchronization reaching 30-35% when the acceleration amplitudes are already high ($0.45m/s^2$). The concept of rate change critical threshold (shift from a random rate to a partly synchronized rate) becomes perceptible.

The various 'loops' correspond to the fact that the pedestrians are not regularly distributed on the footbridge, they are concentrated in groups. For this reason, it is clear when the largest group of pedestrians is near the centre of the footbridge (sag summit), or rather at the ends of the footbridge (sag trough).

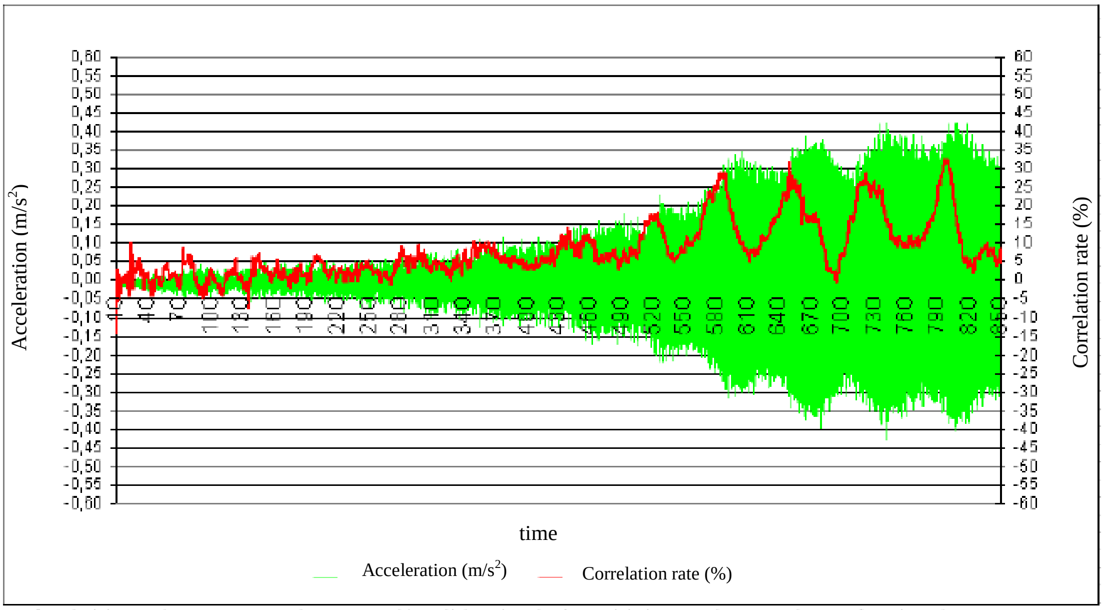
It can also be seen that the three acceleration rise sags, that occur at increasing levels of acceleration ($0.3m/s^2$ then $0.4m/s^2$ and finally $0.5m/s^2$), occur with the same equivalent number of pedestrians each time. This clearly shows the fact that there is acceleration rise, halted twice when the group of pedestrians reaches the end of the footbridge.

In the test, shown in Figure 3.8, the number of pedestrians has been increased more progressively.



Solferino footbridge: Test 1 A: Random crowd/walking in circles with increasing numbers of pedestrians
Acceleration (m/s)
Correlation rate (%)
Time (s)
Acceleration (m/s^2), Correlation rate (%)

Figure 3.7: 1A Solferino footbridge random test: a crowd is made to circulate endlessly on the footbridge with the number of pedestrians being progressively increased (69 – 138 – 207) (after Setra 2006).



Solferino footbridge: Test 1 B: Random crowd/walking in circles with increasing numbers of pedestrians
Acceleration (m/s)
Correlation rate (%)
Time (s)
Acceleration (m/s^2), Correlation rate (%)

Figure 3.8: 1B Solferino footbridge random test: a crowd is made to circulate endlessly on the footbridge with the number of pedestrians being more progressively increased (115 138 161 92 – 184 – 202) (after Setra 2006).

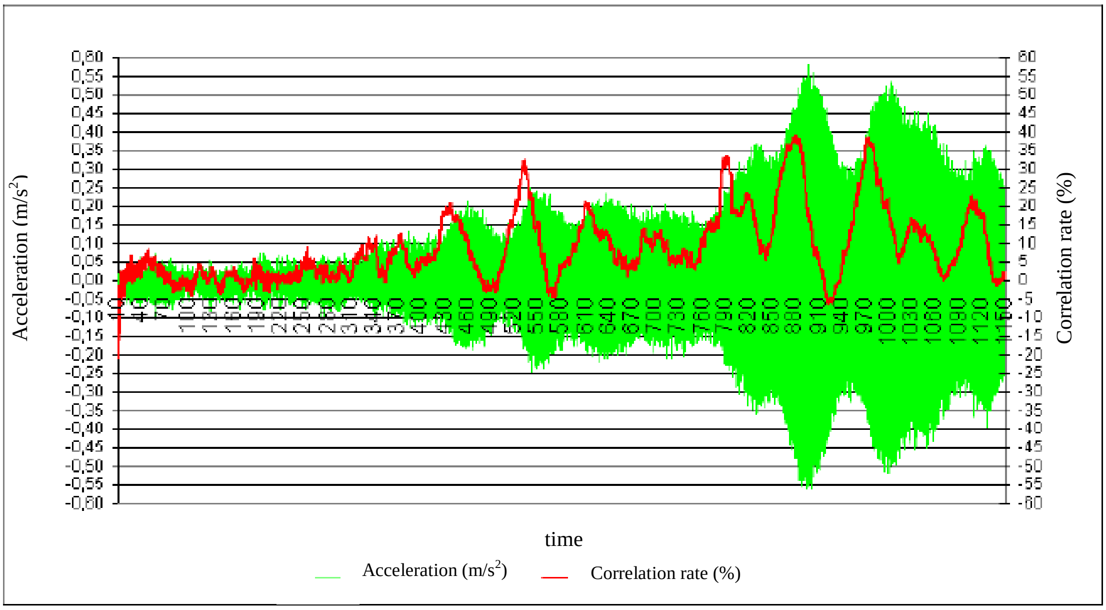
This time, the rate change threshold seems to be situated around 0.15 – 0.20 m/s^2. Maximum synchronization rate does not exceed 30%.

Test 1C, shown in Figure 3.9, leads us to the same conclusions: change in threshold between 0.10 and 0.15m/s^2, then more obvious synchronization reaching up to 35%-40%.

In the test shown in Figure 3.10, the pedestrians are more grouped together and walk from one edge of the footbridge to the other. The rise and subsequent fall in equivalent number of pedestrians better expresses the movement of pedestrians, and their crossing from an area without displacement (near the edges) and another with a lot of displacement (around mid-span). Synchronization rate rises to about 60%. This is higher than previously, however, it should be pointed out, on the one hand, that the level of vibration is higher (0.9m/s^2 instead of 0.5m/s^2) and, on the other, that the crowd is fairly compact and this favors synchronization phenomenon among the pedestrians.

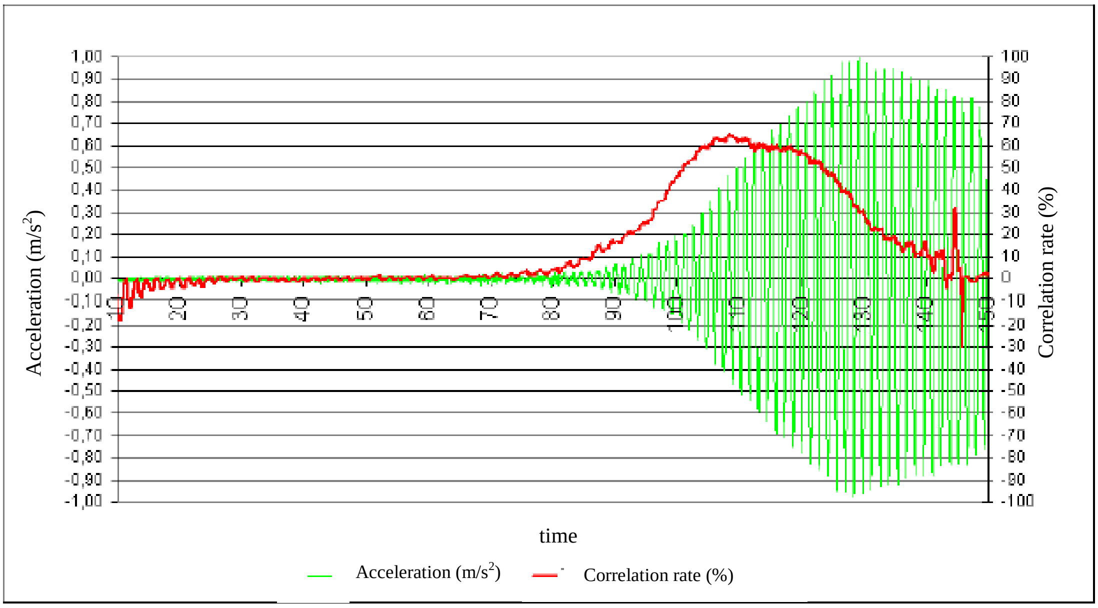
In the test shown in Figure 3.11, there are only 160 people still walking slowly from one end of the footbridge to the other. This time, synchronization rate reaches 50%. Accelerations are comparable with those obtained in tests 1A, 1B, and 1C.

This last test (Figure 3.12) is identical to the previous one except that, this time, the pedestrians were walking fast. Pedestrian synchronization phenomenon was not observed, although there was a compact crowd of 160 people. This clearly shows that where pedestrian walking frequency is too far from the natural frequency, synchronization does not occur.



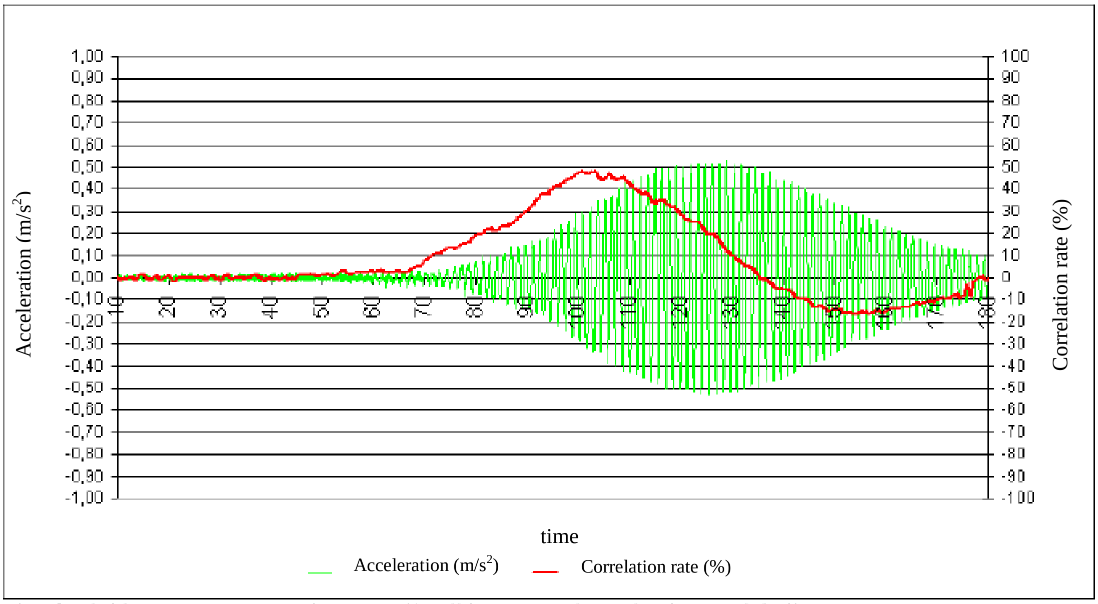
Solferino footbridge: Test 1 C: Random crowd/walking in circles with increasing numbers of pedestrians
Acceleration (m/s)
Correlation rate (%)
Time (s)
Acceleration (m/s^2), Correlation rate (%)

Figure 3.9: 1C Solferino footbridge random test (after Setra 2006).



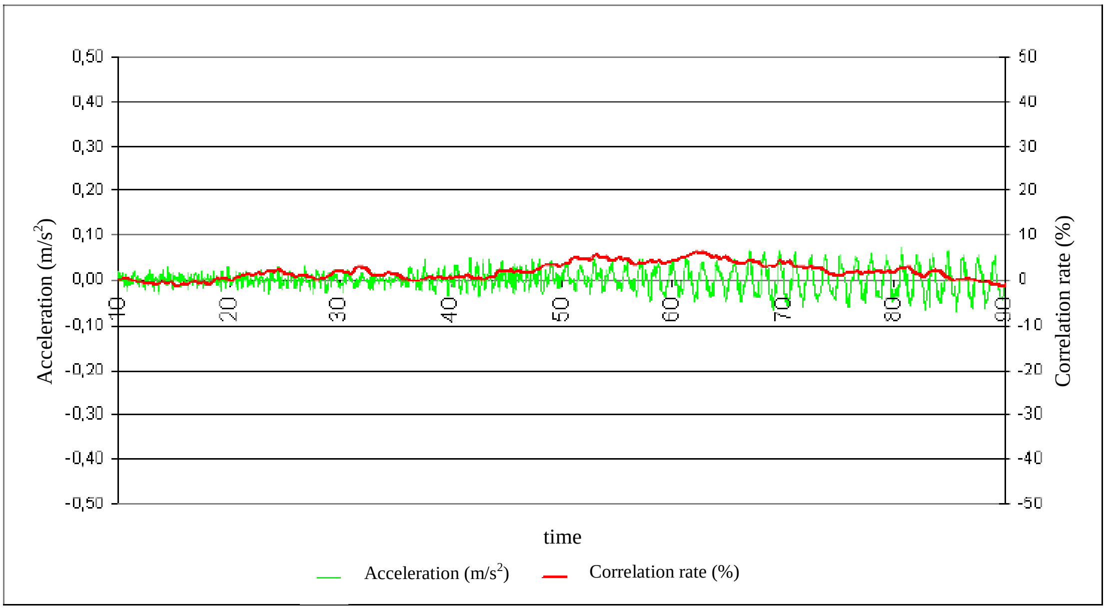
Solferino footbridge: Test 2 A1: Random crowd/walking grouped together in a straight line 229 p.
Acceleration (m/s)
Correlation rate (%)
Time (s)
Acceleration (m/s^2), Correlation rate (%)

Figure 3.10: 2A1 Solferino footbridge random test (after Setra 2006).



Solferino footbridge: Test 2 A2: Random crowd/walking grouped together in a straight line 160 p.
Acceleration (m/s)
Correlation rate (%)
Time (s)
Acceleration (m/s^2), Correlation rate (%)

Figure 3.11: 2A2 Solferino footbridge random test (after Setra 2006).



Solferino footbridge: Test 2 B: Random crowd/Rapid walking in straight line 160 p.
Acceleration (m/s)
Correlation rate (%)
Time (s)
Acceleration (m/s^2), Correlation rate (%)

Figure 3.12: 2B Solferino footbridge random test (after Setra 2006).

These tests show that there is apparently a rate change threshold in relation to random rate at around 0.10-0.15 m/s². Once this threshold has been exceeded accelerations rise considerably but remain limited. Synchronization rates in the order of 30 to 50% are reached when the test is stopped. This value can rise to 60%, or even higher, when the crowd is compact.

For dimensioning purposes, the value 0.10 m/s² shall be noted. Below this threshold, behaviour of pedestrians may be qualified as random. It will then be possible to use the equivalent random pedestrian loadings mentioned above and this will lead to synchronization rates to the order of 5 to 10%. Synchronization rate can rise to more than 60% once this threshold has been exceeded. Thus, acceleration goes from 0.10 to over 0.60 m/s² relatively suddenly. Acceleration thus systematically becomes uncomfortable. Consequently, the 0.10 m/s² rate change threshold becomes a threshold not to be exceeded.

3.2.4.5 Remarks on lock-in phenomenon

The various studies put forward conclusions that appear to differ but that actually concur on several points.

Dallard's and Nakamura's load models assume that the pedestrian force is a function of bridge velocity. However, the force proposed by Dallard increases linearly with bridge velocity (Dallard et al. 2001) whereas the force proposed by Nakamura (Nakamura and Fujino 2002) increases linearly at low velocities but its increase rate becomes smaller at higher velocities.

Actually synchronization or lock-in effect in lateral direction was more precisely described by Fujino's model (Fujino et al. 1993). In this case, a small lateral motion is induced by the random lateral human walking forces, and walking of some pedestrians is synchronized to the girder motion. Then resonant force acts on the girder, consequently the girder motion is increased. Walking of more pedestrians are synchronized, increasing the lateral girder motion. In this sense, the vibration has a self excited nature. Because of adaptive nature of human being, the girder amplitude will not go to infinity and will reach a steady-state.

As soon as the amplitude of the movements becomes perceptible, crowd behaviour is no longer random and a type of synchronization develops. Several models are available (force as a function of speed, high crowd synchronization rate) but they all lead to accelerations rather in excess of the generally accepted comfort thresholds.

Passage from a random rate on a fixed support to a synchronized rate on a mobile support occurs when a particular threshold is exceeded, characterized by critical acceleration or a critical number of pedestrians. It should be noted that the concept of a critical number of pedestrians and the concept of critical acceleration may be linked. Critical acceleration may be interpreted as the acceleration produced by the critical number of pedestrians, still random though after this they are no longer so.

Although it is true that the main principles and the variations in behaviour observed on the two footbridges concur, modelling and quantitative differences nevertheless lead to rather different damper dimensioning.

The concept of critical acceleration seems more relevant than that of a critical number of pedestrians. Acceleration actually corresponds to what the pedestrians feel whereas a critical number of pedestrians depend on the way in which the said pedestrians are organised and positioned on the footbridge. It is therefore this critical acceleration threshold that will be discussed in these guidelines, the way in which the pedestrians are organised depends on the level of traffic on the footbridge.

We devote next chapter (chapter 4) to discuss the issues of synchronization phenomena and summarizes different codes and standards in chapter 5.

Chapter 4

SYNCHRONIZATION MECHANISM

4.1 General

It is now widely accepted that during footbridge vibration some kind of human-structure interaction inevitably occurs. Often, this interaction can be neglected, but it is becoming more common that it cannot. In general, there are two aspects of this issue. The first considers changes in dynamic properties of the footbridge, mainly in damping and natural frequency, due to human presence. The second aspect concerns a degree of synchronization of movement between the pedestrians themselves as well as between the pedestrians and the structure whose motion is perceived. Both phenomena are currently not well understood and research related to them has been intensified in recent years.

In this chapter, dynamic properties of footbridges due pedestrians will be discussed Effect of vertical and lateral synchronization of people walking in groups and crowds will be also discussed. Finally, human perception due to crowd movement in the footbridge will be studied.

4.2 Dynamic Properties of Footbridges under Moving People

It is well-known that the presence of a stationary (standing or sitting) person changes the dynamic properties of a structure they occupy. The most important effect is the increase in damping in the combined human–structure dynamic system compared with the damping of the empty structure (Sachse 2002). The effect is greater if more people are present (Ellis et al. 1997). Therefore, it can be concluded that the human body behaves like a damped dynamic system attached to the main structural system.

Such a system can be described by bio-dynamics methods, structural dynamics methods or by their combination. The human body is in effect a complex nonlinear MDOF system with its parts responding in different ways to structural movement. In a simplified study of human body–structure interaction, the human body can be approximated by a linear SDOF system. One of very few reported attempts to carry out system identification of the dynamic properties of a standing person, applicable to civil engineering, was done by Zheng and Brownjohn (2001). Their SDOF human body model had a damping ratio of 39% and natural frequency of 5.24 Hz. However, the simplified SDOF human body system has been shown to be frequency-dependent and cannot be always represented by the same set of mass, stiffness and damping parameters.

The problem is even less studied in the case of moving people, which is usual for footbridges. Ellis and Ji (1994) found that a person running and jumping on the spot cannot change dynamic characteristics of the structure and, therefore, should be treated only as load (Ellis et al. 1994). However, this investigation was conducted using a simply supported beam having a high fundamental frequency of 18.68 Hz compared with typical footbridge natural frequencies. Nevertheless a similar conclusion was reached by the same researchers regarding the effects of a moving crowd on grandstands.

4.3 Dynamic Forces on Flexible Footbridges

Ohlsson (1982) reported that the spectrum of a force measured on a rigid surface differed from that measured on a flexible timber floor. The spectrum experienced a drop around the natural frequency of the structure where the motion was the highest. This could be a consequence of the interaction phenomenon and is in agreement with previously mentioned Pimentel's (1997) findings of lower Dynamic Load Factor (DLF) on real and moving footbridges in comparison with those measured on rigid surfaces. Ohlsson also claimed that a moving pedestrian increased the mass and the damping of the structure. However, it was stressed again that he investigated only light timber floors where human–structure dynamic interaction is more likely due to large ratios of the mass of the humans and the empty structure. However, Willford also mentioned a result of data analysis from pedestrian tests on the Millennium Bridge which indicated that walking crowd had increased the damping of the structure in the vertical direction.

That jumping and bouncing can change dynamic properties of a flexible structure was reported by Yao et al. (2002). They found that jumping forces are lower on a more flexible structure, but it should be noted that in their investigation the subject to structure mass ratio was very high (0.41). Further, Pavic et al. (2002) compared horizontal jumping forces directly measured on a force plate and indirectly measured on a concrete beam. They found that the force on the structure was about two times lower than that one on the force plate. This could also be a consequence of a human-structure interaction effect but no conclusive evidence for it was presented.

All these reported observations give only an indication that human–structure interaction really occurs without a more precise quantification of the phenomenon. Furthermore, with the exception of a paper by Pavic et al. (2002), all reviewed research is related to vibrations in the vertical direction. Information on possible effects of moving people on the dynamic characteristics of footbridges in the horizontal direction is very scarce.

It is clear that research into human–structure interaction involves various human activities (e. g. walking, jumping, sitting, and standing) on different types of structure. In case of footbridges, although some previous findings are quite useful, the most relevant interaction scenario appears to be a walking crowd. Considering the extremely scarce published data, this is an area that clearly requires further investigation.

4.4 People Walking in Groups and Crowds

It is hard to design against vertical and horizontal load of a structure which is likely to have to carry a dense crowd of human beings. In an attempt to consider this problem, it was noted that none of the following two extreme cases are real. Neither is an increase in load directly proportional to the number of people involved, in comparison with a single pedestrian force (i.e. the case of perfect synchronization), nor should only the static weight of the crowd be taken into account (i.e. dynamic effects be neglected). Subsequent research has shown that the solution is somewhere between these two scenarios.

The first attempts to define the load induced by several pedestrians were in terms of multiplication of the load induced by a single pedestrian. One of the first proposals was given by Matsumoto et al. (1978). Assuming that pedestrians arrived on the bridge following a Poisson distribution they stochastically superimposed individual responses and found that the total response can be obtained by multiplying a single pedestrian response by the multiplication factor $\sqrt{\lambda T_0}$, where λ is the mean arrival rate expressed as the number of pedestrians per second per width of the bridge and T_0 (s) is the time needed to cross over the bridge. Therefore, $\sqrt{\lambda T_0}$ is equal to $\sqrt{n}$, where n is the number of pedestrians on the bridge at any time instant. According to random vibration theory (Newland 1993), if the response due to n equal and randomly distributed inputs is $\sqrt{n}$ times higher than the response due to a single input, it means that inputs (in this case pedestrians) are absolutely uncorrelated (unsynchronised).

Similar to Matsumoto et al. (1978), Wheeler (1982) stochastically combined individual forces (defined deterministically using the half-sine model) assuming random arrival rate, normal distribution of step frequencies and a distribution of people's weights obtained for the Australian population. However, his simulations revealed that group loads were not a more onerous design case than a single pedestrian load, at least for footbridges with fundamental natural frequency away from approximately 2 Hz. Namely, the group load on bridges with the fundamental frequency away from the normal walking frequency range can be regarded as a non-resonant load which probably generates lower response than the one induced by a single pedestrian walking at the resonant frequency. However, the question still is if this can be applied in case of nonrandom walking of groups of pedestrians when some degree of synchronisation between people can be established.

In any case, proposal of Matsumoto et al. (1978) was regarded as appropriate at least for footbridges with natural frequencies in the range of walking frequencies (1.8–2.2 Hz), while for bridges with natural frequencies in the ranges 1.6–1.8 and 2.2–2.4 Hz a linear reduction of Matsumoto et al.'s multiplication factor $\sqrt{\lambda T_0}$ was suggested with its minimum value of 2 at the ends of these intervals in the case of more than four people present on the bridge at the same time. Mouring (1993) simulated a vertical force from walking groups in a way similar to Wheeler (1982). However, she described a single pedestrian force more precisely using the first ten coefficients of the Fourier series instead of the half-sine model. As a result, she found that the effect of group loads should be considered even in case of footbridges with fundamental frequency outside the normal walking frequency range (1.8–2.2 Hz). The response obtained agreed with the findings of Matsumoto et al. (1978). However, Pimentel (1997) measured the response under three uncorrelated people on two footbridges and confirmed the inapplicability of the proposed multiplication factor for bridges with frequencies outside the normal walking frequency range, as claimed by Bachmann and Ammann (1987). It appears that group loading becomes more important precisely in the normal walking frequency range, and in that case it should be considered. Also, Matsumoto et al.'s proposal did not consider the possibility of synchronization between people in a dense crowd, a phenomenon which has attracted a great deal of attention from researchers since the Millennium Bridge problem in London occurred in 2000.

In 1985, Eyre and Cullington (Eyre et al. 1985) noticed that the vertical acceleration recorded on a footbridge in a controlled resonance test with a single pedestrian was 1.7 times lower than the one measured in normal usage which included two or more pedestrians who were not formally synchronised in any way. They explained it as a possible consequence of the occasional and by chance synchronisation between two people. Ebrahimpour and Fitts (Ebrahimpour et al. 1996) reported that the optical sense plays an important role in the synchronisation of people's movement. Namely, two jumping persons who could see each other synchronised their movement better than when they were looking in opposite directions. In both cases the jumping frequencies were controlled by an audio signal. Eriksson (Eriksson 1994) claimed that the first walking harmonic could be almost perfectly synchronised for highly correlated people within a group, while the higher harmonics should be treated as completely uncorrelated. Not surprisingly, Ebrahimpour et al. (1990) therefore focused only on the first harmonic (Fig. 4.1) claiming that higher harmonics cannot produce significant response for a walking crowd.

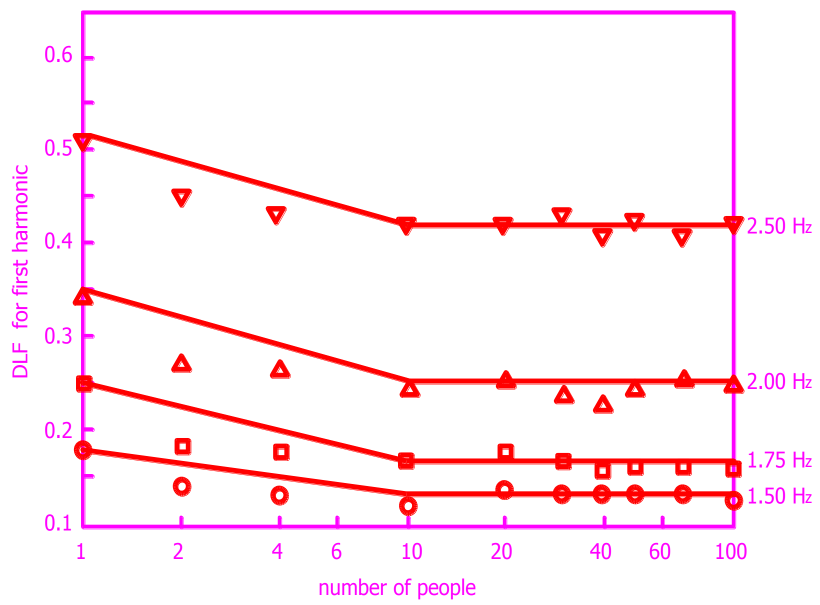


Figure 4.1: Dynamic Load Factor (DLF) for the first harmonic of the walking force as a function of number of people and walking frequency (after Ebrahimpour et al. 1990)

It is now widely accepted that people walking in a crowd, because of the limited space on the bridge deck and the possibility that thus can see each other, would subconsciously synchronise their steps. This becomes more likely if the crowd is dense. Bachmann and Ammann (1987) reported that the maximum physically possible crowd density can be 1.6–1.8 persons/m^2 of the footbridge deck. However, they concluded that a value of 1 person/m^2 is more probable. During the opening day of the Millennium Bridge in London, the maximum density was 1.3–1.5 people/m^2. The crowd density on the T-bridge in Japan (also prone to lateral movement) was between 1 and 1.5 people/m^2 (Fujino et al. 1993). In any case, crowd density influences the walking speed (Fig. 4.2), the degree of synchronisation between people and, consequently, the intensity of the human-induced force.

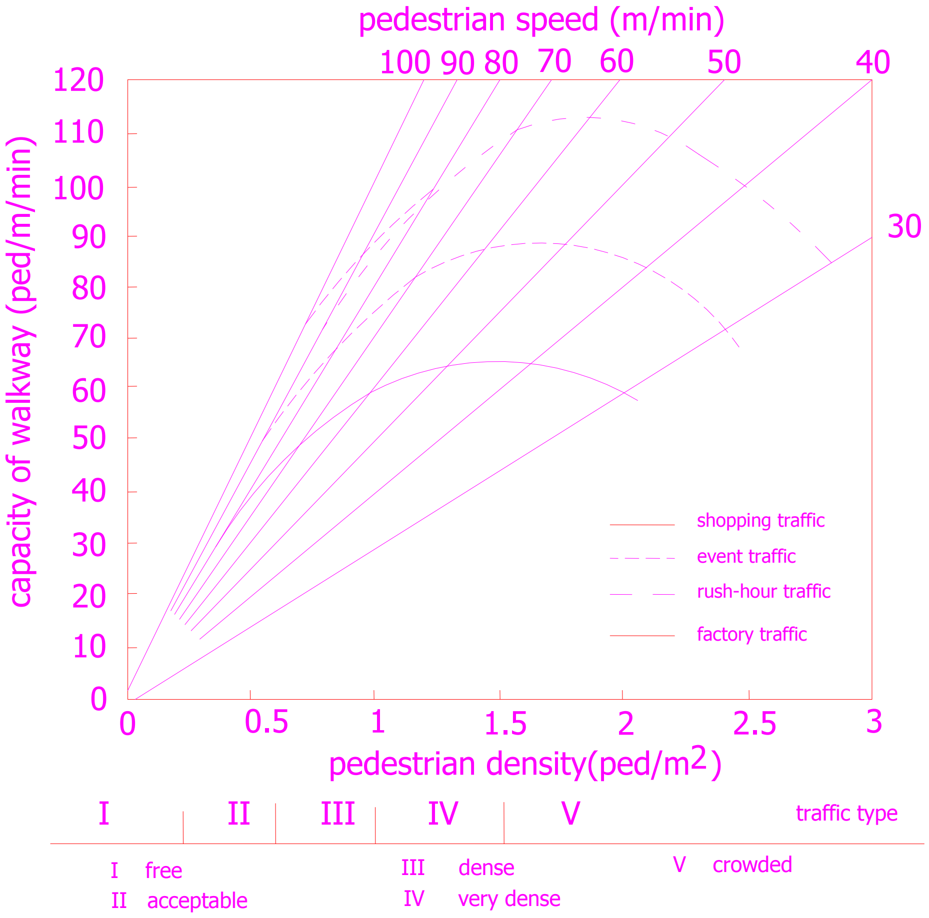


Figure 4.2: Relationship between the bridge capacity, pedestrian density and their velocity (after Schlaich 2002)

Grundmann et al. (1993) proposed three models corresponding to different pedestrian configurations on a footbridge which should be considered separately. These are:

Model 1: When people walk in small groups it is probable that they will walk with the same speed v_s, and slightly different step frequencies f_s and step length l_s according to the equation:

$$v_s = f_s l_s \tag{4.1}$$

In such cases, some synchronisation between these people is expected, but only when the bridge frequency is within the normal walking frequency range.

Model 2: On bridges with a light stream of pedestrians where people can move freely and their walking frequencies are randomly distributed. The maximum density of 0.3 pedestrians/m^2 was suggested as an upper limit for unconstrained free walking. This type of walking (i.e. free walking) was considered in the previously mentioned proposal by Matsumoto et al.'s (1978)

Model 3: If footbridges are exposed to pedestrian traffic of 0.6–1.0 pedestrians/m^2 then free unconstrained movement is practically impossible. In such circumstances, pedestrians are forced to adjust to some extent their step length and speed to the motion of other pedestrians. The previously mentioned swaying problems of the Millennium Bridge and the Japanese T-Bridge belong to this group, despite the fact that their pedestrian densities were higher than proposed by Grundmann et al.

As for the third model, it should be added that the case of crowd walking on a perceptibly moving bridge deck is related not only to synchronization between people but also to synchronization between people and the structure.

Before considering the research into the human–structure synchronization phenomenon, two terms widely used in this article will be defined. The term ''group'' of walking pedestrians is used for several people walking at the same speed as defined in Model 1 above, while the term ''crowd'' is related to densely packed walking people who have to adjust their step to suit the space available, as explained in Model 3.

4.5 Lateral Synchronization

The phenomenon when people change their step to adapt it to the vibrations of the bridge, is—for the same level of vibrations—much more probable in the horizontal than in the vertical direction. This is because of the nature of human walking and desire to maintain the body balance on a laterally moving surface. When it occurs, this is known as the synchronization phenomenon or lock-in effect. As a consequence of the adjusted step when people tend to walk with more spread legs, the motion of the upper torso becomes greater and the pedestrian-induced force becomes larger. This in turn increases the bridge response and, finally, results in structural dynamic instability. In such circumstances, only reducing the number of people on the footbridge or disrupting/stopping their movement can solve the problem. It is interesting, however, that in a laboratory experiment with a single pedestrian walking across a laterally moving platform, not every pedestrian walked in a way to boost the lateral vibrations. Some of them even managed to damp vibrations out. This fact complicates further study of pedestrian behaviour within a crowd, but also points out the need to define and investigate a factor which will describe the degree of synchronization between people.

Typically, the excessive swaying occurs on bridges with lateral natural frequencies near 1 Hz which is the predominant frequency of the first harmonic of the pedestrian lateral force (Fig. 4.3). Fujino et al. (1993) reported such a case on the previously mentioned T-bridge in Japan. During very crowded times, significant lateral movement occurred in the first lateral mode with frequency of 0.9 Hz. The procedure proposed by Matsumoto et al. (1978) underestimated the actual bridge response. By video recording and observing the movement of people’s heads in the crowd, and by measuring the lateral response, Fujino et al. concluded that 20% of the people in the crowd perfectly synchronized their walking. Fujino et al.’s assumption was also that the individual forces produced by the rest of pedestrians cancelled each other, so that their net effect was zero. Later, using image processing technique for tracking people’s movement on the same bridge, Yoshida et al. (2002) estimated the overall lateral force in the crowd of 1500 pedestrians at 5016 N, which gives an average of only 3.34N per pedestrian.

During the opening day of the Millennium Bridge in London, lateral acceleration of 0.20–0.25g was recorded. This corresponded with lateral displacement amplitudes of up to 7 cm. Dallard et al. (2001a,b) tried to define the problem analytically on the basis of observations made during tests with a gradually increasing number of people on the bridge (up to 275 people). Assuming that everybody contributed equally, they identified the amplitude of the modal lateral force per person [Fig. 4.4(a)] and the dependence of the lateral force on the footbridge velocity [Fig. 4.4(b)]. This force was

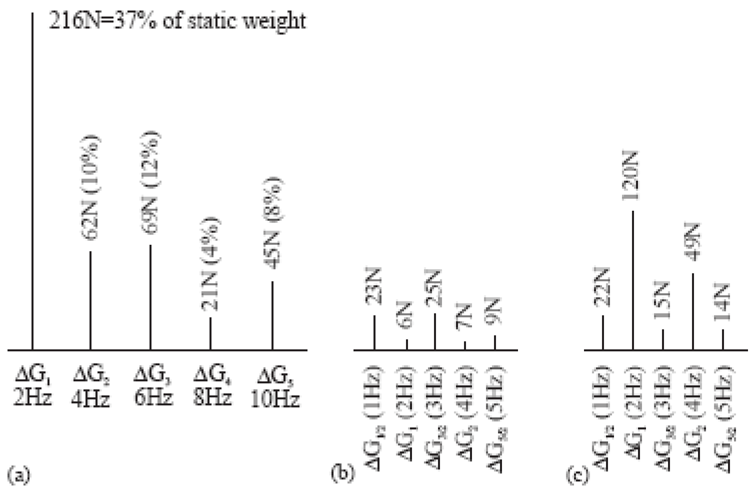


Figure 4.3: Harmonic components of the walking force in (a) vertical, (b) lateral and (c) longitudinal directions (after Bachmann and Ammann 1987).

considerably higher than the one reported by Yoshida et al. (2002). Based on results in Fig. 4.4(b), Dallard et al. concluded that people, after synchronizing their movement with the movement of the structure, produced a dynamic force $F(t)$ which was proportional to the deck lateral velocity $v(t)$:

$$F(t) = kv(t) \tag{4.2}$$

This means that moving pedestrians act as negative dampers (i.e. amplifiers) increasing the response of the structure until walking becomes so difficult, due to body balancing problems, that they have to stop. This clearly indicates the need to model differently the human-induced load before and after the synchronisation occurs. Also, it seems more relevant to investigate bridge behaviour before (and not after) the lock in occurs, in order to predict and prevent the problem in the future. Bearing in mind several other known examples of excessive lateral vibrations of crowded bridges, Dallard et al. further concluded that the same problem can happen on every bridge with a lateral frequency below 1.3 Hz and with sufficient number of people crossing the bridge. That triggering (critical) number of people N_L was defined as

$$N_L = \frac{8\pi c f M}{k} \tag{4.3}$$

where c is the modal damping ratio, f is the lateral frequency of the bridge, M is the corresponding modal mass and k (Ns/m) is the lateral walking force coefficient introduced in Eq. (4.2). For the case of the Millennium Bridge it was found by back analysis that $k = 300$Ns/m in the lateral frequency ranges 0.5–1.0 Hz. However, it would be interesting to find this factor for other bridges with the lateral swaying problem to compare with this value. Also, the shape of the force time history in Fig. 4.4(a) revealed that the lock-in started at about 900 s. However, it seems that the lock-in was unsuccessfully triggered two times between 600 and 800 s. The factors which prevented these two lock-ins are still not identified and it would be extremely beneficial to know what they are. Also, it should be emphasized that, although the predominant lateral load frequency is about 1 Hz, during the bridge opening day the first lateral mode at about 0.5 Hz was also excited. This can be caused by the reduced frequency of the lateral walking force in a crowd (down to 0.6 Hz) and by some ‘‘meandering’’ patterns in human walking on moving bridge deck surfaces, as observed by Dallard et al. (2001).

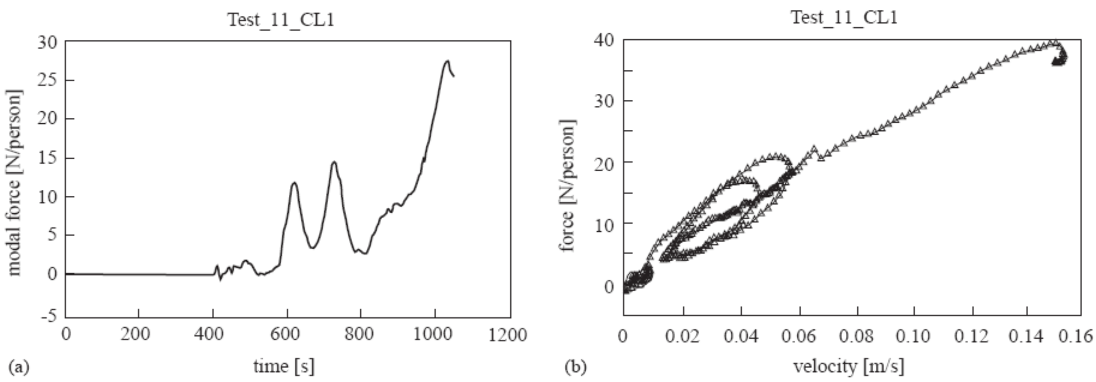


Figure 4.4: (a) Peak amplitude of the lateral modal force per person per vibration cycle, (b) Lateral force per person per vibration cycle vs deck velocity (after Dallard et al. 2001).

Research described in three papers by Dallard et al. (2001) stressed the need to investigate the dependence between the probability of synchronisation between people and the amount of bridge movement in the lateral direction. In that sense, Willford reported tests with a single walking person on a platform moving laterally. The results showed that the lateral pedestrian force was increasing when the lateral movement increased. Also, he found that in the case of structural movement at 1Hz with an amplitude of 5 mm, the probability of people adapting their step to the bridge movement is 40%. These relationships are nonlinear and dependent on frequencies of the bridge movement, even for a single person (Fig. 4.5). These observations were made for individuals and their applicability to people walking in a crowd is still unknown.

An interesting study on a lively footbridge (M-bridge) in Japan revealed that a pedestrian, walking within a crowd on a perceptibly moving deck, synchronised their movement with the bridge vibrations. A phase from 120^{o} to 160^{o} between girder and pedestrian motion was identified. This synchronisation was only spoiled at maximum measured deck amplitude of 45 mm, when it became much harder to walk. It is interesting that excessive lateral vibrations on this footbridge occurred at two different response frequencies (0.88 and 1.02 Hz) depending on the crowd density. These two frequencies corresponded to two modes as high as the sixth and seventh lateral mode of vibrations. A very low damping ratio of 0.5% and also very low bridge mass of 400 kg/m^2 certainly contributed to developing of such large vibrations. Nakamura also reported that the bridge mass was lower than the mass on other two well-known lively (in lateral direction) footbridges: Millennium Bridge (about 500 kg/m^2) and T-bridge (800 kg/m^2).

Nowadays, increasing efforts are made to quantify the vibrations due to crowds using the basis of wind engineering theory. In one such attempt, Stoyanoff et al. (2005) suggested a correlation factor $c_R(N)$ in a moderate crowd of N people when the density is below 1 pedestrian/m^2 similar to one from vortex-shedding theory:

$$c_R(N) = e^{-\gamma N} \tag{4.5}$$

where the factor γ could be obtained from a condition that $c_R = 0.2$ (20%) for the maximum congested footbridge as it was in the work by Fujino et al. (1993). Yoneda (2002) stressed that several factors influenced the synchronisation factor: the lateral natural frequency, damping, length between node points in the resonant mode, walking speed and bridge length on which synchronisation occurs. This observation was not experimentally verified on full scale structures but it deserves attention because of its generality.

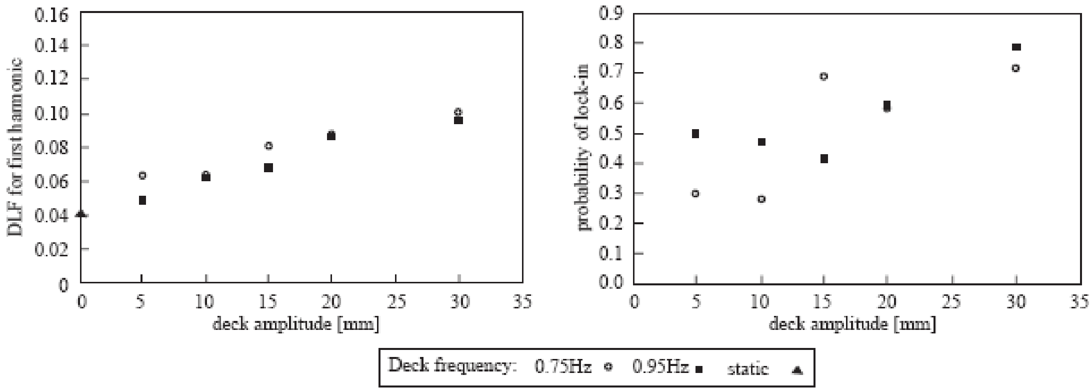


Figure 4.5: (a) DLF and (b) probability of "lock-in" for a single person as a function of the moving platform amplitude and frequency (after Dallard et al. 2001).

Interestingly enough, an entirely different theory to the one considered so far in this section which is based mainly on observations made on the Millennium Bridge, was given by Barker (2002). He reported that the response to crowd movement may increase without any synchronization between people. Further, Dinmore (2002) suggested treating the human-induced force as a wave which propagates through the structure. As a way to control bridge response and avoid synchronisation, he recommended to vary the dynamic stiffness through the structure using different materials which will provide energy loss due to wave reflection and refraction on their contact.

4.6 Vertical Synchronization

An attempt to quantify the probability of synchronization in the vertical direction was made by Grundmann et al. (1993). They defined the probability of synchronization $P_S(a_g)$ as a function of the acceleration amplitude of the structure a_g (Fig. 4.6). They proposed that the response to N people on a structure should be calculated from the following formula:

$$a_g = P_S(a_g) N_r a_{1rz} \tag{4.6}$$

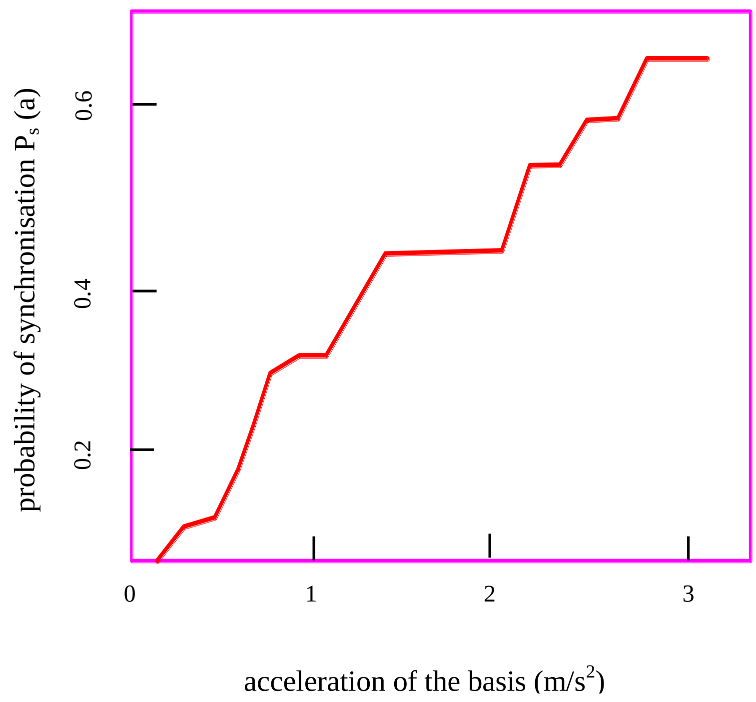


Figure 4.6: Probability of synchronisation as a function of the acceleration of the bridge (after Grundmann et al. 1993).

where a_{1rz} is the response to a single pedestrian and $N_r = NK$ is the number of people reduced by the factor $K < 1$ which takes into account that the load changes position along the structure. For a single span $K = 0.6$ was proposed. For a bridge with fundamental frequency of 2 Hz the probability of synchronization was suggested as 0.225. Therefore, for these parameters the multiplication factor $P_S(a_g)N_r$ for the single pedestrian response a_{1rz} becomes

$$P_S(a_g)N_r = 0.225 * 0.6 * N = 0.135\,N \tag{4.7}$$

This is lower then the value $\sqrt{N}$ given by Matsumoto et al. (1978) for N up to 55 people, despite the fact that Grundmann et al. took into account the synchronisation possibility, and that $\sqrt{N}$ implies N completely uncorrelated people. Grundmann et al. (1993) finally suggested that for groups of up to 10 people, the multiplication factor can be taken as presented in Fig. 4.7, with maximum value of 3 for vertical natural frequencies between 1.5 and 2.5 Hz. The same factor was proposed for the lateral direction but corresponding to two times lower natural frequencies. It should be said that synchronisation with bridge movement in the vertical direction is much less likely, although Bachmann and Ammann reported that it could happen when the vertical amplitude becomes at least 10 mm.

Dallard et al. (2001) suggested using random vibration theory to predict the bridge vertical response due to crowd. The mean square acceleration response $E(a^2)$ due to N pedestrians with normally distributed pacing rates was given as

$$E(a^2) \approx \frac{\pi N}{16c} \frac{\omega_n}{\sigma} p\left(\frac{\omega_n - \mu}{\sigma}\right)\left(\frac{F_{\omega_n}}{M}\right)^2 \tag{4.8}$$

where c, ω_n and M are the modal damping ratio, natural frequency and modal mass, $F\omega_n$ is the amplitude of the harmonic human force while p is the probability density function for normally distributed pacing frequencies with mean value μ and standard deviation σ. However, this formula was conservative even in the Millennium Bridge case. Its assumption that people were uniformly distributed across the structure and that the mode shape was a sinusoid could induce errors and should be corrected according the real conditions on the bridge considered. Also, the distribution of step frequencies within a crowd is unknown.

Finally, Mouring (1993) and Brownjohn et al. (2004) identified that a quantification of the degree of correlation between people in a crowd is a primarily task for future research. Brownjohn et al. (2004) went further and suggested a mathematical model for calculation of the bridge response under crowd of pedestrians based on theory of a turbulent wind on linear structures. They proposed that the ASD of the response in a single mode $S_z(f)$ in a degree of freedom (DOF) specified by the coordinate z should be calculated as

$$S_z = \psi_Z^2 \left|H(f)\right|^2 S_{P,1}(f) \int_0^L \int_0^L \psi_{z_1} \psi_{z_2} \cosh(f, z_1, z_2) dz_1 dz_2 \tag{4.9}$$

where ψ_z is the mode shape ordinate in the same DOF, $H(f)$ is the frequency response function (FRF) for acceleration response, $S_{p,1}(f)$ is the ASD of the pedestrian loads per unit length while ψ_{z_1} and ψ_{z_2} are mode shape ordinates related to the location of each two pedestrians on the bridge described by coordinates z_1 and z_2: Moreover, $\cosh(f, z_1, z_2)$ is the correlation factor, between 0 and 1, which should be further researched, as mentioned earlier. This method gave a good estimate of the response for the footbridge investigated, but needs wider verification.

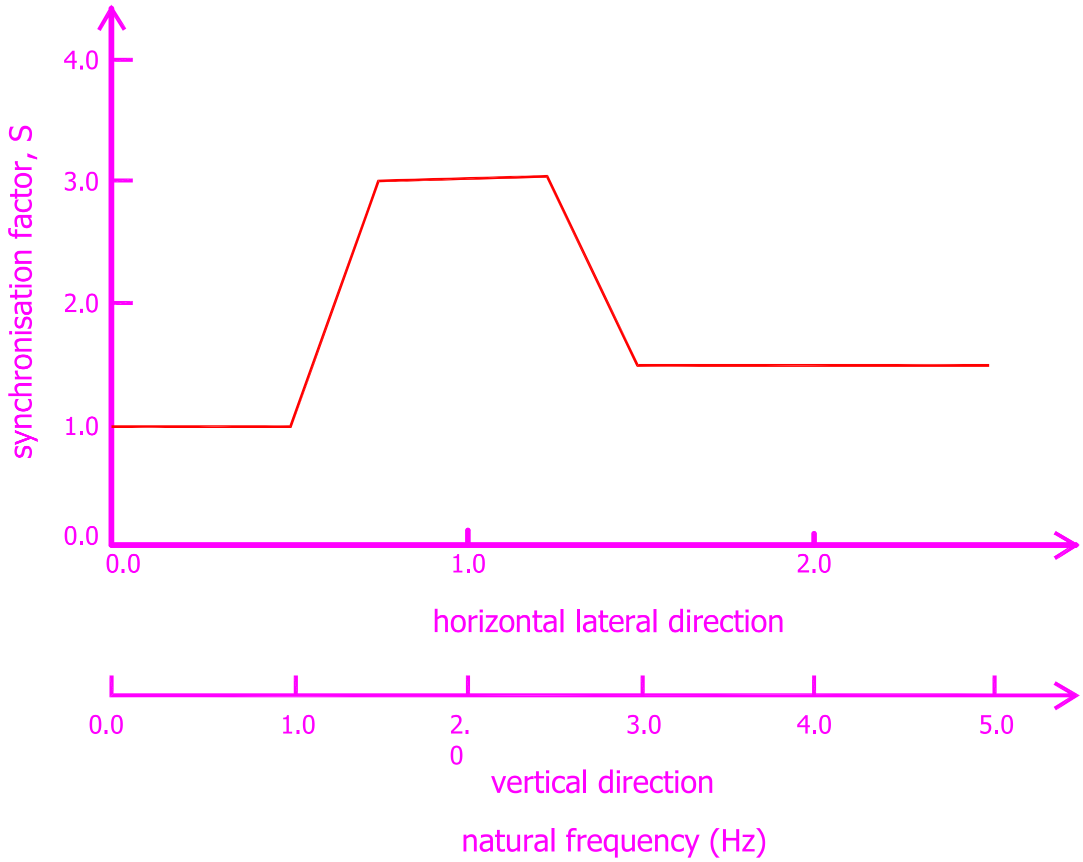


Figure 4.7: Multiplication factor for groups of up to 10 pedestrians (after Grundmann et al. 1993).

An interesting suggestion for the assessment of liveliness of footbridges in the vertical direction under large crowd load, also based on the wind engineering theory was given by McRobie et al. (2003) and McRobie and Morgenthal (2002). It was proposed that the acceptability of vertical vibrations can be assessed by comparing the pedestrian Scruton number $vPSN$ which is achieved with the one required for a particular footbridge. This number is defined as

$$vPSN = k_1 k_2 m \tag{4.10}$$

where factors $k_1 = \xi / 0.005$ and $k_2 = 0.6 / n$ take into account the damping ratio of the empty footbridge ξ (relative to the typical damping ratio of 0.5%) and the possibility that crowd density n could be different from an typical value of 0.6 persons/m^2, respectively. In Eq. (4.10) m represents the mass per unit deck area for an equivalent simply supported beam having constant cross section. To have structure which will meet vibration serviceability requirements, a larger pedestrian Scruton number (i.e. larger damping and mass and lower pedestrian densities) is preferred. Data about acceptable Scruton numbers as a function of footbridge frequency should be provided by collecting data from existing footbridges known to be lively in the vertical direction. However, this task is hampered by the fact that not many footbridges have experienced large vertical vibrations under crowd load.

In conclusion, it can be said that although the two considered types of synchronization (among people, and between the people and the structure) are different in their nature, they usually happen simultaneously and lead to the same result—an increase in the response of the structure. In order to understand better the interaction between the moving crowd and the structure it is necessary to identify:

1. the relationships between the crowd density, walking speed, walking frequency and probability of synchronisation, and
2. the probability of lock-in and effective force per person in a crowd as a function of the amplitude and frequency of the bridge motion.

4.7 Human Perception

The main receivers of vibrations on pedestrian bridges, who govern their vibration serviceability, are walking people. The reaction of human beings to vibrations is a very complex issue having in mind that humans are ‘‘the greatest variables with which anyone may deal’’ (Jacklin 1936).

Probably one of the first laboratory works and certainly the most often referenced in the future studies was conducted by Reiher and Meister (1931). They investigated the effect of harmonic vibrations on ten people having different postures (laying, sitting, standing) on a test platform driven by different amplitudes, frequencies and direction of vibrations. As a result they classified the human perception into six categories and as a function of vibration amplitude and frequency.

Motivated by the lack of research related to walking and standing people under vibrations with limited duration, Leonard (1966) conducted a laboratory experiment on a 10.7m long beam driven by sinusoidal excitation at different amplitudes (up to 0:200 i.e. 5.08 mm) and frequencies (1–14 Hz). Forty walking and standing persons helped in these tests to define the boundary between acceptable and unacceptable vibrations in individual tests lasting up to 1 min during which vibration amplitude was held on a constant level. Results clearly indicated that a standing person is more sensitive to vibrations than a walking one (Fig. 4.8). Similarly to Wright and Green (1963), it was shown that the Reiher and Meister scale is fairly inappropriate for application to bridges. Leonard further suggested using the curve applicable to stationary standing people for vibration perceptibility in the case of large numbers of pedestrians because of a prolonged duration of the vibration level. A similar recommendation was made regarding the perception of vibration in the horizontal direction because of the greater human sensitivity in this direction.

Data on human perception of horizontal vibration of bridges are very scarce. Probably most valuable information about the tolerance level to footbridge lateral vibrations due to crowd loading is given by Nakamura (2003). Based on pedestrian experience of vibrations on full-scale footbridges, he concluded that the amplitude of deck displacement of 45mm (corresponding to an acceleration of 1.35 m/s^2) is a reasonable serviceability limit. At the same time he noticed that deck displacement amplitude of 10mm (corresponding to acceleration level of 0.3 m/s^2) was tolerable by most pedestrians, while a displacement of 70mm (2.1 m/s^2) would make people to feel unsafe and prevent them from walking.

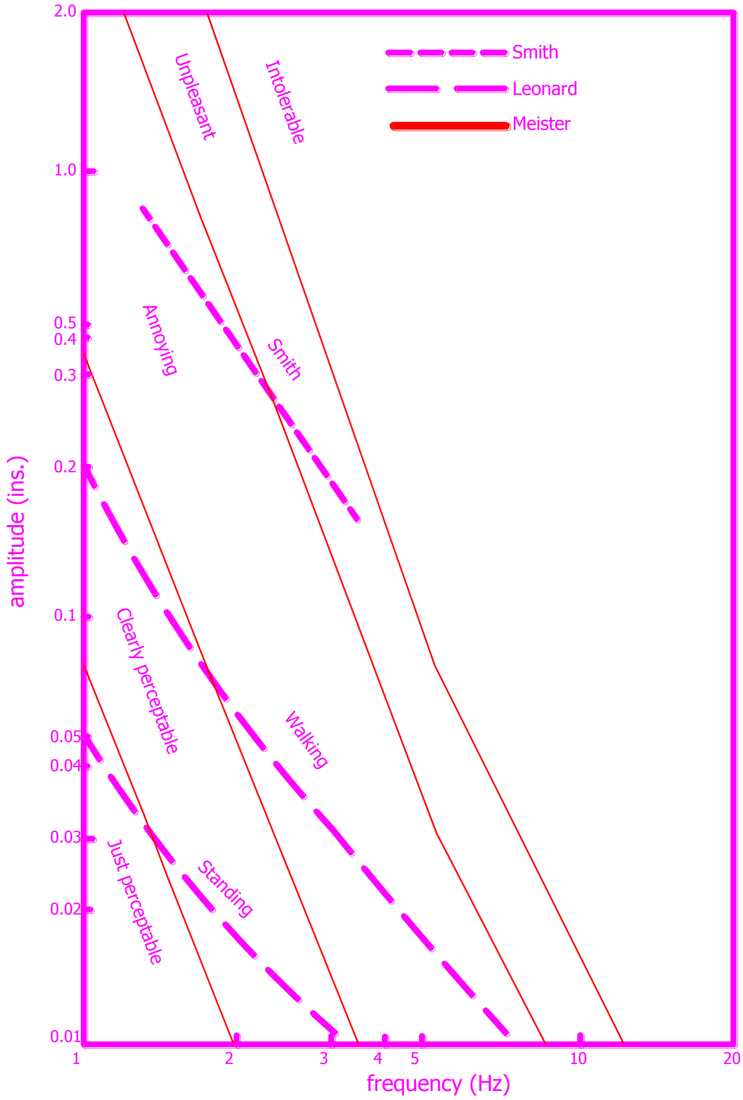


Figure 4.8: Leonard's and Smith's scales of human perception (after Smith 1969).

Chapter 5

VIBRATION CONSIDERATIONS IN DIFFERENT CODES

5.1 General

New lightweight and high-strength structural materials, longer spans and greater slenderness of footbridges have in the past years caused several problems with vibration serviceability. This chapter will discuss how these problems are dealt with in current standards and codes of practice.

The main focus in this chapter will be on the serviceability criteria and the load models proposed by four widely used standards. These codes and standards are BS 5400, Euro code, ISO 10137 and Bro 2004. Risk frequencies, comfort criteria, comfort threshold and improvement of dynamic behaviour of footbridges will be discussed. Finally, there will be a comparison of these four standards and a discussion on the similarities and the differences in vibration criteria and load models.

5.2 Parameters that affect Dimensioning: Frequency, Comfort Threshold, Comfort Criterion

The problems encountered on recent footbridges echo the well-known phenomenon of resonance that ensues from the matching of the exciting frequency of pedestrian footsteps and the natural frequency of a footbridge mode. Owing to the fact that they are amplified, noticeable movements of the footbridge ensue and their consequence is a feeling of discomfort for the pedestrians that upset their progress.

Thus, it is necessary to review the structural parameters, vital to the resonance phenomenon, represented by natural vibration modes (natural modes and natural frequencies) and the values of the structural critical damping ratio associated with each mode. In reality, even a footbridge of simple design will have infinity of natural vibration modes, frequencies and critical damping ratios associated to it. However, in most cases, it is sufficient to study a few first modes.

Along with this, it is necessary to consider pedestrian walking frequencies, since they differ from one individual to another, together with walking conditions and numerous other factors. So, it is necessary to bear in mind a range of frequencies rather than a single one.

The first simple method for preventing the risk of resonance could consist in avoiding having one or several footbridge natural frequencies within the range of pedestrian walking frequency. This leads to the concept of a range of risk frequencies to be avoided.

5.2.1 Risk frequencies noted in the literature and in current regulations

Compilation of the frequency range values given in various articles and regulations has given rise to the table 5.1, drawn up for vertical vibrations.

As concerns lateral vibrations, the ranges described in the table 5.1 are to be divided by two owing to the particular nature of walking: right and left foot are equivalent in their vertical action, but are opposed in their horizontal action and this means transverse efforts apply at a frequency that is half that of the footsteps.

However, on the Millennium footbridge it was noticed that the lock-in phenomenon appeared even for a horizontal mode with a frequency considerably beneath that of the lower limit generally accepted so far for normal walking frequency. Thus, for horizontal vibration modes, it seems advisable to further lower the lower boundary of the risk frequency range.

Table 5.1: Risk frequencies in different standards

Standards	Frequency Range
Eurocode 2	1.6 Hz and 2.4 Hz and, where specified, between 2.5 Hz and 5 Hz.
Eurocode 5	Between 0 and 5 Hz
Appendix 2 of Eurocode 0	<5 Hz
BS 5400	<5 Hz
Regulations in Japan	1.5 Hz – 2.3 Hz
ISO/DIS standard 10137	1.7 Hz – 2.3 Hz
CEB 209 Bulletin	1.65 – 2.35 Hz
Bachmann	1.6 – 2.4 Hz

Although risk frequency ranges are fairly well known and clearly defined, in construction practice, it is not easy to avoid them without resorting to impractical rigidity or mass values. Where it is impossible to avoid resonance, it is necessary to try to limit its adverse effects by acting on the remaining parameter: structural damping; obviously, it will be necessary to have available criteria making it possible to determine the acceptable limits of the resonance.

5.2.2 Comfort thresholds

Before going any further, the concept of comfort should be specified. Clearly, this concept is highly subjective. In particular:

- from one individual to the next, the same vibrations will not be perceived in the same way.

- for a particular individual, several thresholds may be defined. The first is a vibration perception threshold. This is followed by a second that can be related to various degrees of disturbance or discomfort (tolerable over a short period, disturbing, unacceptable). Finally, a third threshold may be determined in relation to the consequences the vibrations may entail: loss of balance, or even health problems.

- furthermore, depending on whether he is standing, seated, moving or stationary, a particular individual may react differently to the vibrations.

- it is also well known that there is a difference between the vibrations of the structure and the vibrations actually perceived by the pedestrian. For instance, the duration for which he is exposed to the vibrations affects what the pedestrian feels. However, knowledge in this field remains imprecise and insufficient.

Hence, though highly desirable, it is clear that in order to be used for footbridge dimensioning, determining thresholds in relation to the comfort perceived by the pedestrian is a particularly difficult task. Indeed, where a ceiling of 1 m/s² for vertical acceleration has been given by Matsumoto et al. (1978), others, such as Wheeler (1982), on the one hand, and Tilly et al. (1984), on the other, contradict each other with values that are either lower or higher. Furthermore, so far, there have only been a few suggestions relating to transverse movements.

5.2.3 Acceleration comfort criteria noted in the literature and regulations

The literature and various regulations put forward various values for the critical acceleration, noted a_{crit}. The values are provided for vertical acceleration and are shown in Figure 5.1.

For vertical vibrations with a frequency of around 2 Hz, standard walking frequency, there is apparently a consensus for a range of 0.5 to 0.8 m/s². It should be remembered that these values are mainly associated with the theoretical load of a single pedestrian.

For lateral vibrations with a frequency of around 1 Hz, Eurocode 0 Appendix 2 proposes a horizontal critical acceleration of 0.2 m/s² under normal use and 0.4 m/s² for exceptional conditions (i.e. a crowd). Unfortunately, the text does not provide crowd loadings.

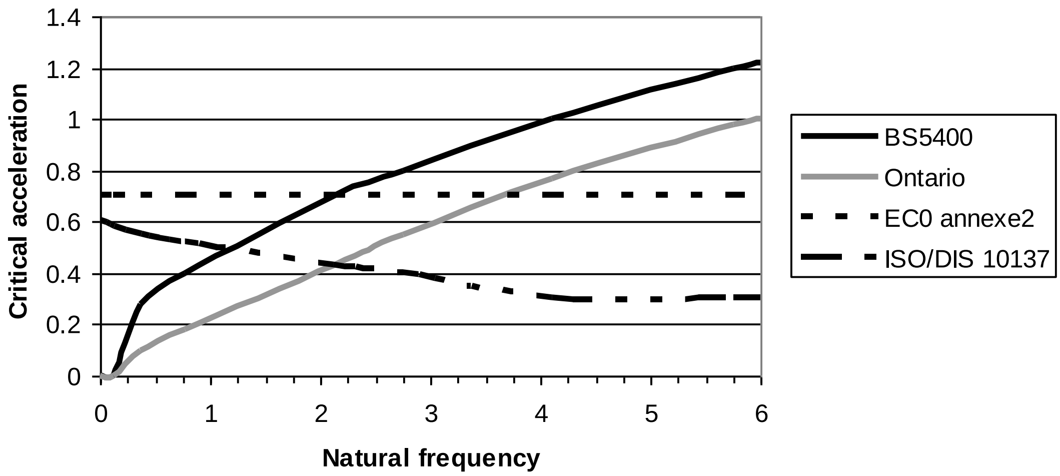


Figure 5.1: Vertical critical accelerations (in m/s²) as a function of the natural frequency for various regulations: some depend on the frequency of the structure, others do not.

It is also useful to remember that since accelerations, speeds and displacements are related, an acceleration threshold may be translated as a displacement threshold (which makes better sense for a designer) or even a speed threshold.

- *Acceleration* = (*frequency of 2 Hz*)^2 movement

- *Acceleration* = (*frequency of 2 Hz*) speed

For instance, for a frequency of 2 Hz: acceleration of 0.5 m/s² corresponds to a displacement of 3.2 mm, a speed of 0.04 m/s, acceleration of 1 m/s² corresponds to a displacement of 6.3 mm, a speed of 0.08 m/s

but for a frequency of 1 Hz: acceleration of 0.5 m/s² corresponds to a displacement of 12.7 mm, a speed of 0.08 m/s, acceleration of 1 m/s² corresponds to a displacement of 25.3 mm, a speed of 0.16 m/s

5.3 Improvement of Dynamic Behaviour

When footbridge acceleration does not respect comfort criteria, in that case it is necessary to distinguish between a footbridge at the design stage and an existing one.

In the case of a footbridge at the design stage, it is logical to try to modify its natural frequency vibrations. If it is not possible to modify them so that they are outside the resonance risk ranges in relation to excitation by the pedestrians, then attempts should be made to increase structural damping.

With an existing footbridge, it is also possible to try to modify its natural frequency vibrations. However, experience shows that it is generally more economic and hence convenient to increase damping.

5.3.1 Modification of vibration natural frequencies

A vibration natural frequency is always proportional to the square root of the stiffness and inversely proportional to the square root of the mass. The general aim is to try to increase vibration frequency. Therefore the stiffness of the structure needs to be increased. However, practice indicates that an increase in stiffness is frequently accompanied by an increase in mass, which produces an inverse result, this is a difficult problem to solve.

5.3.2 Increasing structural damping

5.3.2.1 Natural structural damping of the structures

The critical damping ratio is not an inherent fact of a material. Most experimental results suggest that dissipation forces are to all practical intents and purposes independent of frequency but rather depend on movement amplitude. The critical damping ratio also increases when vibration amplitude increases. It also depends on construction details that may dissipate energy to a greater or lesser extent (for instance, where steel is concerned, the difference between bolting and welding).

It should be noted that although the mass and rigidity of the various structure elements may be modeled with a reasonable degree of accuracy, damping properties are far more difficult to characterize. Studies generally use critical damping coefficients ranging between 0.1% and 2.0% and it is best not to overestimate structural damping in order to avoid under-dimensioning.

CEB information bulletin No. 209, an important summary document dealing with the general problem of structure vibrations provides the following values for use in projects:

Table 5.2: Critical damping ratio of the deck of the footbridge due to different materials

Type of Deck	Critical Damping Ratio	
	Minimum Value	Average Value
Reinforced Concrete	0.8%	1.3%
Pre-stressed Concrete	0.5%	1.0%
Metal	0.2%	0.4%
Mixed	0.3%	0.6%
Timber	1.5%	3.0%

As concerns timber, Eurocode 5 recommends values of 1% or 1.5% depending on the presence, or otherwise, of mechanical joints.

Where vibration amplitude is high, as with earthquakes, critical damping ratios are considerably higher and need to be SLS checked. For instance, the AFPS 92 Guide on seismic protection of bridges states in the Table 5.3.

Table 5.3: Critical damping ratio due to different materials

Material	Critical Damping Ratio
Welded Steel	2%
Bolted Steel	4%
Pre-stressed Concrete	2%
Non-reinforced Concrete	3%
Reinforced Concrete	5%
Reinforced Elastomer	7%

Finally, it should be pointed out that an estimate of actual structural damping can only be achieved through measurements made on the finished structure. This said, increased damping may be obtained from the design stage, for instance through use of a wire-mesh structure, or in the case of tension tie footbridges (stress ribbon), through insertion of elastomer plates distributed between the prefabricated concrete slabs making up the deck.

5.3.2.2 Damper Implementation

The use of dampers is another effective solution for reducing vibrations by increasing damping. Appendix 4 describes the different types of dampers that can be used and describes the operating and dimensioning principle of a selection of dampers. The table of examples (Table 5.4) shows this is a tried and tested solution to the problem:

Table 5.4: Examples of the use of tuned dynamic dampers

Country	Name	Mass (kg)	Total Effective Mass (%)	Damper Critical Damping Ratio	Critical Damping Ratio of The Structure without Dampers	Critical Damping Ratio of The Structure with Dampers	Structure frequency (Hz)
France	Passerelle du Stade de France (Football stadium footbridge) (Saint-Denis) Coordinate: 48^{0}55’32.38”N, 2^{0}21’54.68”E	2400 per span	1.6	0.075	0.2% to 0.3%	4.3% to 5.3%	1.95 (vertical)
France	Solferino footbridge (Paris) Coordinate: 48^{0}51’42.81”N, 2^{0}19’28.39”E	15000	4.7		0.4%	3.5%	0.812 (horizontal)

Country	Name	Mass (kg)	Total Effective Mass (%)	Damper Critical Damping Ratio	Critical Damping Ratio of The Structure without Dampers	Critical Damping Ratio of The Structure with Dampers	Structure frequency (Hz)
Ditto	Ditto	10000	2.6		0.5%	3%	1.94 (vertical)
Ditto	Ditto	7600	2.6		0.5%	2%	2.22 (vertical)
England	Millenium footbridge (London) Coordinate: $51^0$30’35.01”N, $0^0$05’54.01”W	2500			0.6% to 0.8%	2%	0.49 (horizontal)
Ditto	Ditto	1000 to 2500			0.6% to 0.8%		0.5 (vertical)
Japan			1		0.2%	2.2%	1.8 (vertical)
USA	Las Vegas (Bellagio-Bally) Coordinate: $36^0$06’50.65”N, $115^0$10’25.22”W				0.5%	8%	
South Korea	Seonyu footbridge				0.6%	3.6%	0.75 (horizontal)
Ditto	Ditto				0.4%	3.4%	2.03 (vertical)

Note: In the case of the Millennium footbridge, as well as ADAs, viscous dampers were also installed to dampen horizontal movement.

5.4 Codes and Standards

5.4.1 BS 5400: Design and construction of steel, concrete and composite bridges

The British Standard BS 5400 applies to the design and construction of footbridges. Each of the parts of BS 5400 is implemented by a BD standard, and some of these standards vary certain aspects of the part that they implement. There are two BD standards that relate to the design of footbridges. Design criteria for footbridges are given in BD 29/04 and loads for footbridges are given in BD 37/01.

The BS 5400 standard is one of the earliest codes of practice which dealt explicitly with issues concerning vibrations in footbridges. In BS 5400: Appendix C there is defined a procedure for checking vertical vibrations due to a single pedestrian for footbridges having natural vertical frequencies of up to 5 Hz. Based on experience with lateral vibrations of the London Millennium Bridge, an updated version of BS 5400, BD 37/01, requires check of the vibration serviceability also in the lateral direction. For all footbridges with fundamental lateral frequencies lower than 1.5 Hz a detailed dynamic analysis is now required. However, the procedure for that is not given.

The BD 29/04 standard, which deals with design criteria for footbridges, states that the designer should consider the susceptibility of any footbridge to vibrations induced by pedestrians. Particular consideration shall be given to the possibility that the passage of large numbers of people may unintentionally excite the structure into motion. It is noted that designers should be aware that footbridges having modes of oscillation with frequencies less than 5 Hz involving vertical motions of the deck, and/or less than 1.5 Hz involving horizontal motions of the deck, are particularly susceptible to unacceptably large oscillations caused by the passage of large groups of people who may synchronize their walking patterns.

The BD 29/04 further states that all footbridges shall satisfy the vibration serviceability requirements set out in BD 37/01: Appendix B5.5. There it is stated that if the fundamental natural frequency of vibration exceeds 5 Hz for the unloaded bridge in the vertical direction and 1.5 Hz for the loaded bridge in the horizontal direction, the vibration serviceability requirement is deemed to be satisfied.

If the fundamental frequency of vertical vibration, on the other hand, is less than, or equal to 5 Hz, the maximum vertical acceleration of any part of the bridge shall be limited to $0.5\sqrt{f_0}$ m/s^2. The maximum vertical acceleration can be calculated either with a simplified method or a general method.

The simplified method for deriving the maximum vertical acceleration given in BD 37/01 is only valid for single span, or two-or-three-span continuous, symmetric, simply supported superstructures of constant cross section. For more complex superstructures, the maximum vertical acceleration should be calculated assuming that the dynamic loading applied by a pedestrian can be represented by a pulsating point load F, moving across the main span of the bridge at a constant speed v_t as follows:

$$F = 180\sin(2\pi f_0 t) \quad [\text{N}] \tag{5.1}$$

$$v_t = 0.9 f_0 \quad [\text{m/s}] \tag{5.2}$$

where f_0 is the frequency of the load and t is the time.

If the fundamental frequency of horizontal vibration is less than 1.5 Hz, special consideration shall be given to the possibility of excitation by pedestrians of lateral movements of unacceptable magnitude. Bridges having low mass and damping and expected to be used by crowds of people are particularly susceptible to such vibrations. The method for deriving maximum horizontal acceleration is, however, not given.

5.4.2 EN 1990: Basis of structural design

In EN 1990: Basis of Structural Design, it is stated that pedestrian comfort criteria for serviceability should be defined in terms of maximum acceptable acceleration of any part of the deck. Also, recommended maximum values for any part of the deck are given; see Table 5.5.

Table 5.5: Maximum acceptable acceleration, EN1990

	Maximum Acceleration
Vertical vibrations	0.7 m/s^2
Horizontal vibrations, normal use	0.2 m/s^2
Horizontal vibrations, crowd conditions	0.4 m/s^2

The standard Euro code 1: Part 2 defines models of traffic loads for the design of road bridges, footbridges and railway bridges. Chapter 5.7 deals with dynamic models of pedestrian loads. It states that, depending on the dynamic characteristics of the structure, the relevant natural frequencies of the main structure of the bridge deck should be assessed from an appropriate structural model. Further, it states that forces exerted by pedestrians with a frequency identical to one of the natural frequencies of the bridge can result into resonance and need be taken into account for limit state verifications in relation with vibrations. Finally, Euro code 1 states that an appropriate dynamic model of the pedestrian load as well as the comfort criteria should be defined. The method for modelling the pedestrian loads is, however, left to the designer.

Euro code 5, Part 2 contains information relevant to design of timber bridges. It requires the calculation of the acceleration response of a bridge due to small groups and streams of pedestrians in both vertical and lateral directions. The acceptable acceleration is the same as in EN1990, 0.7 and 0.2 m/s^2 in the vertical and the horizontal directions, respectively. A verification of this comfort criteria should be performed for bridges with natural frequencies lower than 5 Hz for the vertical modes and below 2.5 Hz for the horizontal modes. A simplified method for calculating vibrations caused by pedestrians on simply supported beams is given in Euro code 5: Annex B. Load models and analysis methods for more complex structures are, on the other hand, left to the designer.

In Euro code 5, it is also noted that the data used in the calculations, and therefore the results, are subject to very high uncertainties. Therefore, if the comfort criteria are not satisfied with a significant margin, it may be necessary to make provision in the design for the possible installation of dampers in the structure after its completion.

5.4.3 ISO 10137: Basis for design of structures - serviceability of building and walkways against vibrations

The ISO 10137 guidelines (ISO 2005) are developed by the International Organization for Standardization with the objective of presenting the principles for predicting vibrations at the design stage. Also, to assess the acceptability of vibrations in structures.

ISO 10137 defines the vibration source, path and receiver as three key issues which require consideration when dealing with the vibration serviceability of structures. The vibration source produces the dynamic forces or actions (pedestrians). The medium of the structure between source and receiver constitutes the transmission path (the bridge). The receiver of the vibrations is then again the pedestrians of the bridge. According to ISO 10137, the analysis of response requires a calculation model that incorporates the characteristics of the source and of the transmission path and which is then solved for the vibration response at the receiver.

ISO 10137 states that the designer shall decide on the serviceability criterion and its variability. Further, ISO 10137 states that pedestrian bridges shall be designed so that vibration amplitudes from applicable vibration sources do not alarm potential users. In Annex C, there are given some examples of vibration criteria for pedestrian bridges. There it is suggested to use the base curves for vibrations in both vertical and horizontal directions given in ISO 2631-2 (Figures 5.2 and 5.3), multiplied by a factor of 60, except where one or more persons are standing still on the bridge, in which case a factor of 30 should be applicable. This is due to the fact that a standing person is more sensitive to vibrations than a walking one.

However, according to Zivanovic (Zivanovic et al. 2005) these recommendations are not based on published research pertinent to footbridge vibrations.

According to ISO 10137, the dynamic actions of one or more persons can be presented as force-time histories. This action varies with time and position as the persons traverse the supporting structure.

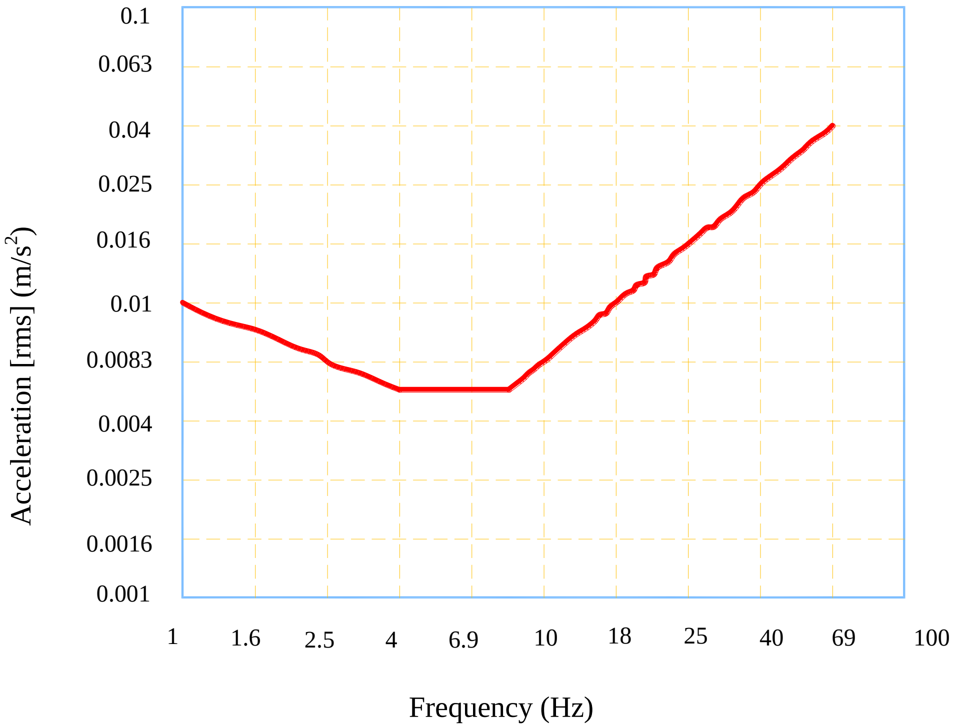


Figure 5.2: Vertical vibration base curve for acceleration.

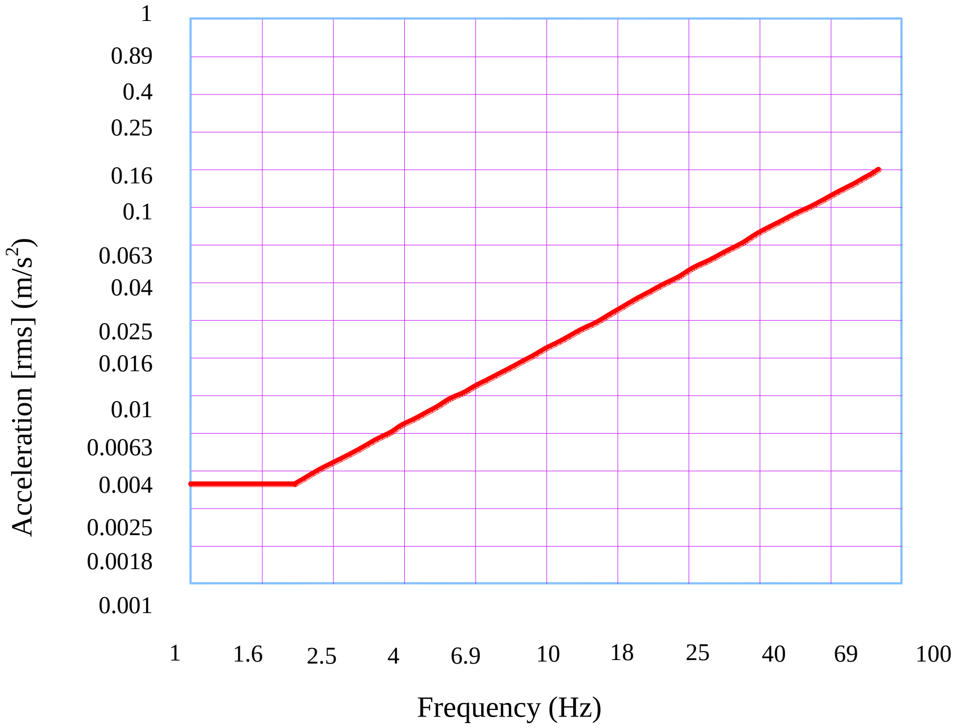


Figure 5.3: Horizontal vibration base curve for acceleration.

The design situation should be selected depending on the pedestrian traffic to be admitted on the footbridge during its lifetime. It is recommended to consider the following scenarios:

- One person walking across the bridge
- An average pedestrian flow (group size of 8 to 15 people)
- Streams of pedestrians (significantly more than 15 persons)
- Occasional festive of choreographic events (when relevant)

According to ISO 10137: Annex A, the dynamic force $F(t)$ produced by a person walking over a bridge can be expressed in the frequency domain as a Fourier series, Eq. 5.3 and 5.4.

$$F_v(t) = Q(1 + \sum_{n=1}^{k} \alpha_{n,v} \sin(2\pi nft + \phi_{n,v}))) \text{ vertical direction} \tag{5.3}$$

and

$$F_h(t) = Q(1 + \sum_{n=1}^{k} \alpha_{n,h} \sin(2\pi nft + \phi_{n,h}))) \text{ horizontal direction} \tag{5.4}$$

where, $\alpha_{n,v}$: numerical coefficient corresponding to the n^{th} harmonic, vertical direction, $\alpha_{n,h}$: numerical coefficient corresponding to the n^{th} harmonic, horizontal dir., Q : static load of participating person, f : frequency component of repetitive loading, $\phi_{n,v}$: phase angle of n^{th} harmonic, vertical direction, $\phi_{n,h}$: phase angle of n^{th} harmonic, horizontal direction, n : integer designating harmonics of the fundamental, k : number of harmonics that characterize the forcing function in the frequency range of interest

Some examples of values for the numerical coefficient *αn* are given in ISO 10137: Annex A.

Dynamic action of groups of participants depends primarily on the weight of the participants, the maximum density of persons per unit floor area and on the degree of coordination of the participants.

The coordination can be represented by applying a coordination factor $C(N)$ to the forcing function:

$$F(t)_N = F(t).C(N) \tag{5.5}$$

where N is the number of participants. For example, if the movements of a group of people are un-coordinated, the coordination factor becomes:

$$C(N) = \sqrt{N} / N \tag{5.6}$$

5.4.4 Bro 2004: Swedish standards

Bro 2004 is a general technical standard, which applies to the design and construction of bridges in Sweden. Bro 2004 is published by the Swedish Road Administration (SRA). The SRA is the national authority assigned the overall sectoral responsibility for the entire road transport system. The SRA is also responsible for the planning, construction, operation and maintenance of the state roads.

Bro 2004 states that footbridges should have fundamental frequencies of vertical modes of vibration above 3.5 Hz. Alternatively, the bridge should be checked for vibration serviceability. If any natural frequency of vertical vibration is less, or equal to 3.5 Hz, the root-mean-square vertical acceleration (a_{RMS}) of any part of the bridge shall be limited to $a_{RMS} \leq 0.5$ m/s^2. The vertical acceleration can be calculated from dynamic analysis. The dynamic analysis can be performed either with a simplified method or a general method.

The simplified method given in Bro 2004 is only applicable to simply supported beam bridges. For more complex superstructures, a detailed analysis using handbooks or computer programs is required.

The RMS-vertical acceleration should be calculated assuming that the dynamic loading applied by a pedestrian is represented by a stationary pulsating load

$$F = k_1 k_2 \sin(2\pi f_F t) \quad [\text{N}] \tag{5.7}$$

where $k_1 = \sqrt{0.1BL}$ and $k_2 = 150$ N are loading constants, f_F is the frequency of the load, t is the time, B is the breath of the bridge and L is the length of the bridge between supports.

Bro 2004 speaks only of vertical accelerations and no requirements or precautions regarding horizontal vibrations are set forth in the code.

5.5 Code Comparisons

In addition to the frequency comparison presented in Table 5.1, Table 5.6 compares the serviceability criteria in terms of acceleration set forth in the four standards discussed in this chapter. A comparison of the vertical and the horizontal vibration criteria are presented in Fig. 5.4 and Fig. 5.5 respectively. The ISO 10137 and Bro 2004 curves are obtained by converting the RMS acceleration to the maximum value by multiplying by the factor $\sqrt{2}$.

A comparison of the vertical vibration criteria show that Euro code and Bro 2004 present a frequency independent maximum acceleration limit of 0.7 m/s^2. For a footbridge with a natural vertical frequency of 2 Hz, which is the mean pacing rate of walking, the BS 5400 criteria also gives $a_{\max} \leq 0.5\sqrt{2Hz} = 0.7$ m/s^2. ISO 10137 gives, on the other hand, a slightly lower value, $a_{\max} \cong 0.6$ m/s^2.

Table 5.6: Acceleration Criteria

Standard	Vertical Acceleration	Horizontal Acceleration
BS 5400	$a_{\max} \leq 0.5\sqrt{f}$ m/s^2	No requirements
EN 1990	$a_{\max} \leq 0.7\sqrt{f}$ m/s^2	$a_{\max} \leq 0.2$ m/s^2
ISO 10137	60 times base curve, Figure 5.2	60 times base curve, Figure 5.3
Bro 2004	$a_{RMS} \leq 0.5$ m/s^2	No requirements

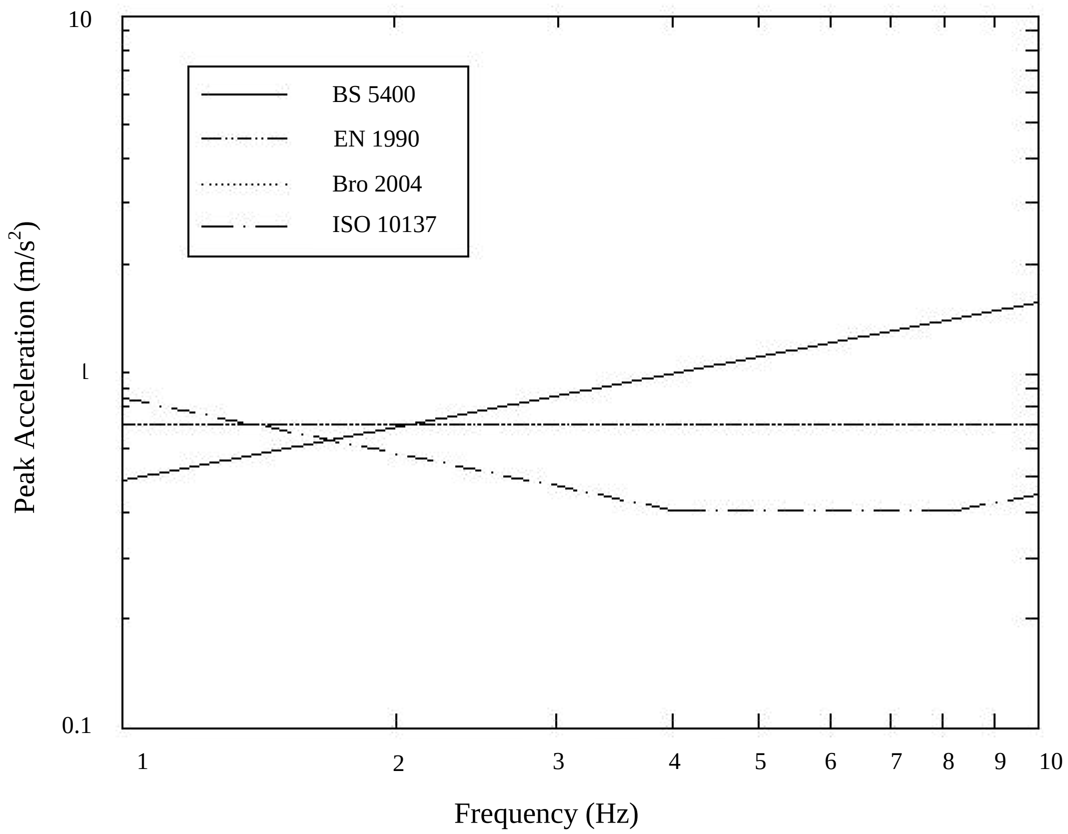


Figure 5.4: Comparison of acceptability of vertical vibration

A comparison of the horizontal vibration criteria show that Euro code presents a frequency independent maximum acceleration limit of 0.2 m/s^2. ISO 10137 gives a frequency independent maximum acceleration of $a_{max} \cong 0.31$ m/s^2 up to a frequency of 2 Hz. Neither BS 5400 nor Bro 2004 presents numerical acceleration criteria for horizontal vibration. However, BS 5400 states that if the fundamental frequency of horizontal vibration is less than 1.5 Hz, the designer should consider the risk of lateral movements of unacceptable magnitude.

The British standard BS 5400 proposes a pedestrian load model only in the vertical direction and not in the horizontal. ISO 10137 models both vertical and horizontal loads imposed by one pedestrian. It is noted that the modelling of the horizontal pedestrian load assumes that the static weight of the pedestrian, Q, acts in the horizontal direction. Euro code proposes load models for both vertical and horizontal loads only for simplified structures. For more complex structures, the modelling of pedestrian loads is left to the designer. The Swedish standard Bro 2004 proposes a load model for calculations of vertical vibrations. However, it proposes neither a load model nor design criteria for horizontal vibrations.

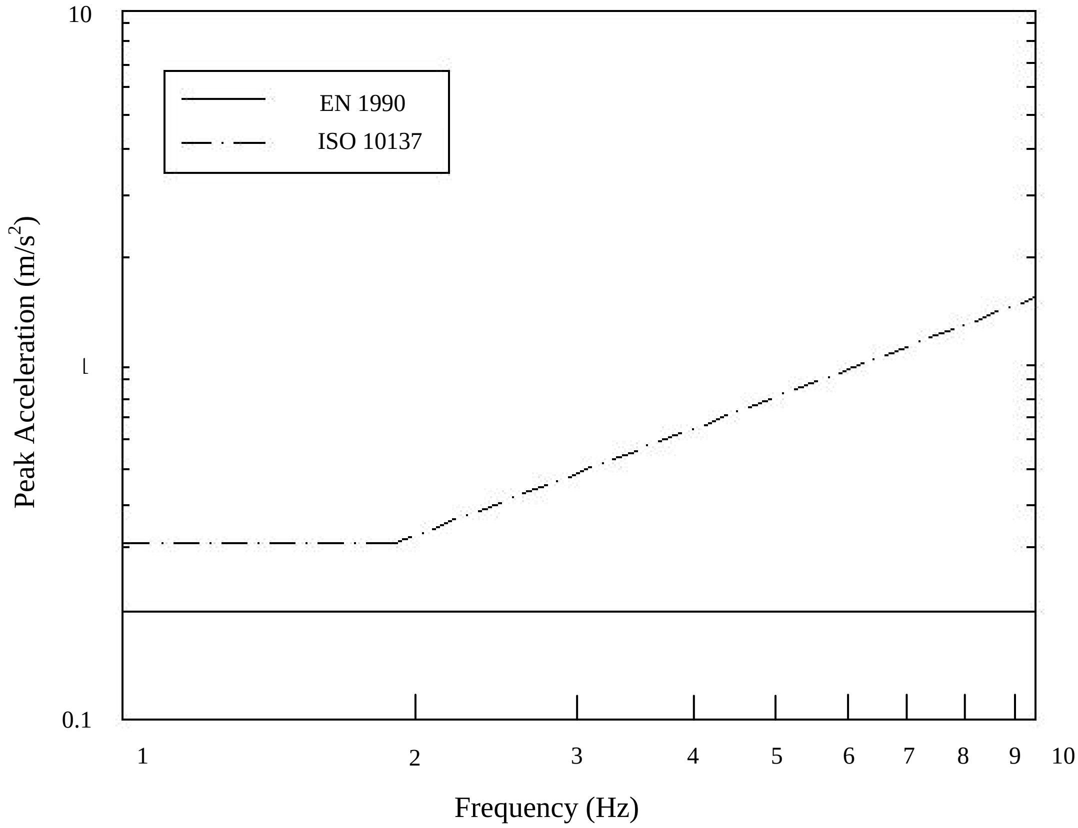


Figure 5.5: Comparison of acceptability of horizontal vibration

The load models proposed by these standards are all based on the assumptions that pedestrian loads can be approximated as periodic loads.

Chapter 6

RATIONALE FOR DEVELOPMENT OF DESIGN STANDARDS

6.1 General

Setra (2006) proposed a dynamic analysis methodology to limit risks of resonance of the structure caused by pedestrian footsteps. It clarified that, resonance apart; very light footbridges may undergo vibration phenomena.

To decide on the approach of dynamic analysis, the class of the footbridge as a function of the level of traffic needs to be defined.

Structural natural frequencies are determined. These natural frequencies lead to the selection of one or several dynamic load cases, as a function of the frequency value ranges. These load cases are defined to represent the various possible effects of pedestrian traffic. Treatment of the load cases provides the acceleration values undergone by the structure. The comfort level obtained can be qualified by the range comprising the values.

In this chapter, various methods have been proposed for analyzing footbridges by taking into account the dynamic effects caused by pedestrian traffic (Setra 2006). This chapter also includes the specific verifications to be carried out to take into account the dynamic behaviour of footbridges under pedestrian loading. These rationales have been utilized in development of different codes as summarized in chapter 7.

The dynamic analysis methodology is summarised in Figure 6.1.

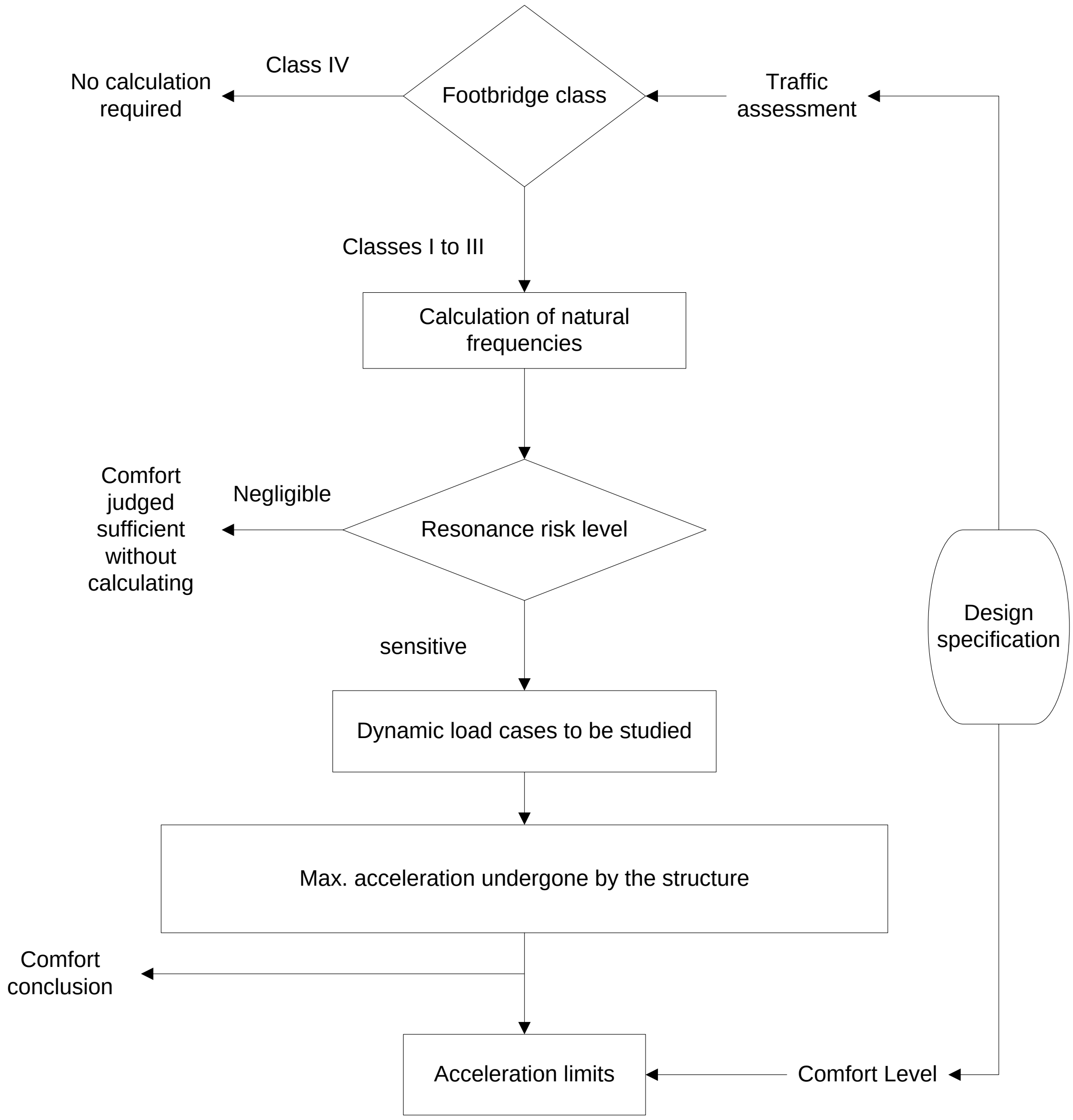


Figure 6.1: Methodology for Dynamic Analysis Methodology.

6.2 Stage 1: Determination of Footbridge Class

At this stage footbridge is classified on the basis of expected level of traffic:

Class I: urban footbridge linking up high pedestrian density areas (for instance, nearby presence of a rail or underground station) or that is frequently used by dense crowds (demonstrations, tourists, etc.), subjected to very heavy traffic.

Class II: urban footbridge linking up populated areas, subjected to heavy traffic and that may occasionally be loaded throughout its bearing area.

Class III: footbridge for standard use, that may occasionally be crossed by large groups of people but that will never be loaded throughout its bearing area.

Class IV: seldom used footbridge, built to link sparsely populated areas or to ensure continuity of the pedestrian footpath in motorway or express lane areas.

It is for the design specification to determine the footbridge class as a function of the above information and taking into account the possible changes in traffic level over time.

The design specification may also be influenced by other criteria, for instance, a higher class may be selected to increase the vibration prevention level, in view of high media expectations. On the other hand, a lower class may be accepted in order to limit construction costs or to ensure greater freedom of architectural design, bearing in mind that the risk related to selecting a lower class shall be limited to the possibility that, occasionally, when the structure is subjected to a load where traffic and intensity exceed current values, some people may feel uncomfortable.

Class IV footbridges are considered not to require any calculation to check dynamic behaviour. For very light footbridges, it seems advisable to select at least Class III to ensure a minimum amount of risk control. Indeed, a very light footbridge may present high accelerations without there being any resonance.

6.3 Stage 2: Choice of Comfort Level by the Design Specifications

6.3.1 Definition of the comfort level

The Design specification determines the comfort level to bestow on the footbridge.

Maximum comfort: Accelerations undergone by the structure are practically imperceptible to the users.

Average comfort: Accelerations undergone by the structure are merely perceptible to the users.

Minimum comfort: under loading configurations that seldom occur, accelerations undergone by the structure are perceived by the users, but do not become intolerable.

It should be noted that the above information cannot form the absolute criteria: the concept of comfort is highly subjective and a particular acceleration level will be experienced differently, depending on the individual. Furthermore, these guidelines do not deal with comfort in premises either extensively or permanently occupied that some footbridges may undergo, over and above their pedestrian function.

Choice of comfort level is normally influenced by the population using the footbridge and by its level of importance. It is possible to be more demanding on behalf of particularly sensitive users (schoolchildren, elderly or disabled people), and more tolerant in case of short footbridges (short transit times).

In cases where the risk of resonance is considered negligible after calculating structure natural frequencies, comfort level is automatically considered sufficient.

6.3.2 Acceleration ranges associated with comfort level

The level of comfort achieved is assessed through reference to the acceleration undergone by the structure, determined through calculation, using different dynamic load cases. Thus, it is not directly a question of the acceleration perceived by the users of the structure.

Given the subjective nature of the comfort concept, it has been judged preferable to reason in terms of ranges rather than thresholds. Tables 6.1 and 6.2 define 4 value ranges noted 1, 2, 3 and 4, for vertical and horizontal accelerations respectively. In ascending order, the first 3 correspond to the maximum, mean and minimum comfort levels described in the previous paragraph. The 4th range corresponds to uncomfortable acceleration levels that are not acceptable.

Table 6.1: Acceleration ranges (in m/s^2) for vertical vibrations

Acceleration Ranges	0	0.5	1	2.5
Range 1	Max			
Range 2		Mean		
Range 3			Min	
Range 4				Unacceptable

Table 6.2: Acceleration ranges (in m/s^2) for horizontal vibrations

Acceleration Ranges	0	0.1	0.15	0.3	0.8
Range 1	Max	Max			
Range 2			Mean		
Range 3				Min	
Range 4					Unacceptable

The acceleration is limited in any case to 0.10 m/s^2 to avoid "lock-in" effect.

6.4 Stage 3: Determination of Frequencies and of the Need to Perform Dynamic Load Case Calculations

For Class I to III footbridges, it is necessary to determine the natural vibration frequency of the structure. These frequencies concern vibrations in each of 3 directions: vertical, transverse horizontal and longitudinal horizontal. These are determined for 2 mass assumptions: empty footbridge and footbridge loaded throughout its bearing area, to the tune of one 700 N pedestrian per square meter (70 kg/m^2).

The ranges in which these frequencies are situated make it possible to assess the risk of resonance entailed by pedestrian traffic and, as a function of this, the dynamic load cases to study in order to verify the comfort criteria.

6.4.1 Frequency range classification

In both vertical and horizontal directions, there are four frequency ranges, corresponding to a decreasing risk of resonance.

Range 1: maximum risk of resonance.
Range 2: medium risk of resonance.
Range 3: low risk of resonance for standard loading situations.
Range 4: negligible risk of resonance.

Table 6.3 defines the frequency ranges for vertical vibrations and for longitudinal horizontal vibrations. Table 5.4 concerns transverse horizontal vibrations.

Table 6.3: Frequency ranges (Hz) of the vertical and longitudinal vibrations.

Frequency	0	1	1.7	2.1	2.6	5
Range 1						
Range 2						
Range 3						
Range 4						

Table 6.4: Frequency ranges (Hz) of the transverse horizontal vibrations.

Frequency	0	0.3	0.5	1.1	1.3	2.5
Range 1						
Range 2						
Range 3						
Range 4						

6.4.2 *Definition of the required dynamic calculations*

Depending on footbridge class and on the ranges within which its natural frequencies are situated, it is necessary to carry out dynamic structure calculations for all or part of a set of 3 load cases:

Case 1: sparse and dense crowd
Case 2: very dense crowd
Case 3: complement for an evenly distributed crowd (2nd harmonic effect)

Table 6.5 clearly defines the calculations to be performed in each case.

6.5 Stage 4 if necessary: Calculation with Dynamic Load Cases

If the previous stage concludes that dynamic calculations are needed, these calculations shall enable:

- checking the comfort level criteria in paragraph II.2 required by the Owner, under working conditions, under the dynamic load cases as defined hereafter,

- traditional SLS and ULS type checks, including the dynamic load cases.

Table 6.5: Verifications- load case under considerations.

<table>
<tr><td></td><td></td><td colspan="3">Load Cases to select for acceleration checks</td></tr>
<tr><td></td><td></td><td colspan="2">Natural frequency range</td><td></td></tr>
<tr><td>Traffic</td><td>Class</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Very Dense</td><td>I</td><td>Case 2</td><td>Case 2</td><td>Case 3</td></tr>
<tr><td>Dense</td><td>II</td><td rowspan="2">Case 1</td><td>Case 1</td><td>Case 3</td></tr>
<tr><td>Sparse</td><td>III</td><td>Nil</td><td>Nil</td></tr>
</table>

Case No. 1: Sparse and dense crowd
Case No. 2: Very dense crowd
Case No. 3: Crowd complement (2nd harmonic)

6.5.1 Dynamic load cases

The load cases defined hereafter have been set out to represent, in a simplified and practicable way, the effects of fewer or more pedestrians on the footbridge. These have been constructed for each natural vibration mode, the frequency of which has been identified within a range of risk of resonance. Indications of the way these loads are to be taken into account and to be incorporated into structural calculation software, and the way the constructions are to be modelised are given in the next chapter.

Case 1: Sparse and dense crowds

This case is only to be considered for category III (sparse crowd) and II (dense crowd) footbridges. The density d of the pedestrian crowd is to be considered according to the class of the footbridge:

Table 6.6: Footbridge class and pedestrian density

Class	Density d of the crowd
III	0.5 pedestrians/ m^2
II	0.8 pedestrians/ m^2

This crowd is considered to be uniformly distributed over the total area of the footbridge S . The number of pedestrians involved is therefore: $N = Sxd$.

The number of equivalent pedestrians, in other words the number of pedestrians who, being all at the same frequency and in phase, would produce the same effects as random pedestrians, in frequency and in phase is: $10.8x(\xi xN)^{\frac{1}{2}}$.

The load that to be taken into account is modified by a minus factor ψ which makes allowance for the fact that the risk of resonance in a footbridge becomes less likely the further away from the range 1.7 Hz – 2.1 Hz for vertical accelerations, and 0.5 Hz – 1.1 Hz for horizontal accelerations. This factor falls to 0 when the footbridge frequency is less than 1 Hz for the vertical action and 0.3 Hz for the horizontal action. In the same way, beyond 2.6 Hz for the vertical action and 1.3 Hz for the horizontal action, the factor cancels itself out. In this case, however, the second harmonic of pedestrian walking must be examined.

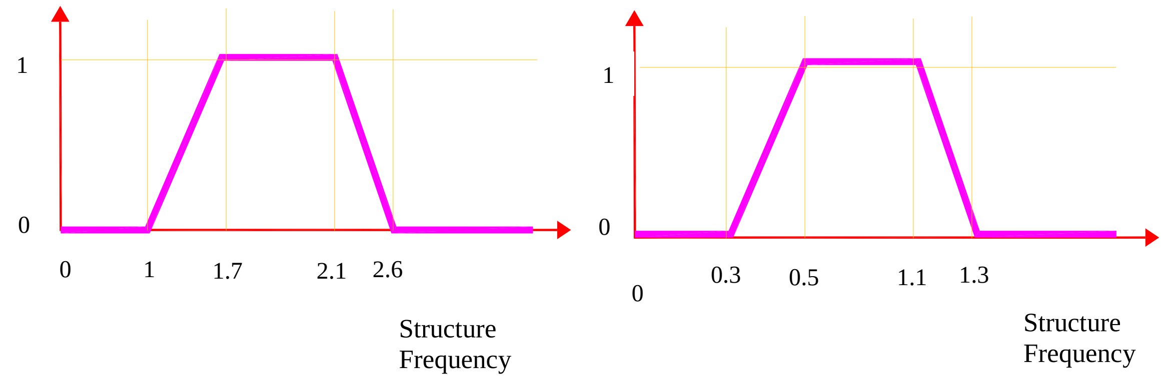


Figure 6.2: Factor ψ in the case of walking, for vertical and longitudinal vibrations on the left, and for lateral vibrations on the right.

The table below summarizes the load per unit area to be applied for each direction of vibration, for any random crowd, if one is interested in the vertical and longitudinal modes. ξ represents the critical damping ratio (no unit), and n the number of pedestrians on the footbridge (dxS).

Table 6.7: Pedestrian load in each direction

Direction	Load per m^2
Vertical (v)	d x (280N) x $\cos((2\pi f_v t)$ x 10.8 x $(\xi / n)^{\frac{1}{2}}$ x ψ
Longitudinal (l)	d x (140N) x $\cos((2\pi f_l t)$ x 10.8 x $(\xi / n)^{\frac{1}{2}}$ x ψ
Transverse (t)	d x (35N) x $\cos((2\pi f_v t)$ x 10.8 x $(\xi / n)^{\frac{1}{2}}$ x ψ

The loads are to be applied to the whole of footbridge, and the sign of the vibration amplitude must, at any point, be selected to produce the maximum effect: the direction of application of the load must therefore be the same as the direction of the mode shape, and must be inverted each time the mode shape changes direction, when passing through a node for example.

Comment (1): in order to obtain these values, the number of equivalent pedestrians is calculated using the formula $10.8 \times (\xi \times n)^{1/2}$, then divided by the loaded area S, which is replaced by n/d (reminder $n = S \times d$), which gives $d \times 10.8 \times (\xi / n)^{1/2}$, to be multiplied by the individual action of these equivalent pedestrians ($F_0 \cos(\omega t)$) and by the minus factor ψ.

Comment (2): It is very obvious that these load cases are not to be applied simultaneously. The vertical load case is applied for each vertical mode at risk, and the longitudinal load case for each longitudinal mode at risk, adjusting on each occasion the frequency of the load to the natural frequency concerned.

Comment (3): The load cases above do not show the static part of the action of pedestrians, G_0. This component has no influence on acceleration; however, it should be borne in mind that the mass of each of the pedestrians must be incorporated within the mass of the footbridge.

Comment (4): These loads are to be applied until the maximum acceleration of the resonance is obtained. The number of equivalent pedestrians is constructed so as to compare real pedestrians with fewer fictitious pedestrians having perfect resonance.

Case 2: Very dense crowd

This load case is only to be taken into account for Class I footbridges.

The pedestrian crowd density to be considered is set at 1 pedestrian per m^2. This crowd is considered to be uniformly distributed over an area S as previously defined.

It is considered that the pedestrians are all at the same frequency and have random phases. In this case, the number of pedestrians all in phase equivalent to the number of pedestrians in random phases (n) is $1.85\sqrt{n}$.

The second minus factor, ψ, because of the uncertainty of the coincidence between the frequency of stresses created by the crowd and the natural frequency of the construction, is defined by Figure 6.2 according to the natural frequency of the mode under consideration, for vertical and longitudinal vibrations on the one hand, and transversal on the other.

The following table summarizes the load to be applied per unit of area for each vibration direction. The same comments apply as those for the previous paragraph:

Table 6.8: Pedestrian load in each direction

Direction	Load per m^2
Vertical (v)	1.0 x (280N) x $\cos(2\pi f_v t)$ x 1.85 $(1/n)^{\frac{1}{2}}$ x ψ
Longitudinal (l)	1.0 x (140N) x $\cos(2\pi f_v t)$ x 1.85 $(1/n)^{\frac{1}{2}}$ x ψ
Transverse (t)	1.0 x (35N) x $\cos(2\pi f_v t)$ x 1.85 $(1/n)^{\frac{1}{2}}$ x ψ

Case 3: Effect of the second harmonic of the crowd

This case is similar to cases 1 and 2, but considers the second harmonic of the stresses caused by pedestrians walking, located, on average, at double the frequency of the first harmonic. It is only to be taken into account for footbridges of categories I and II.

The density of the pedestrian crowd to be considered is 0.8 pedestrians per m^2 for category II, and 1.0 for category I.

This crowd is considered to be uniformly distributed. The individual force exerted by a pedestrian is reduced to 70N vertically, 7N transversally and 35N longitudinally.
For category II footbridges, allowance is made for the random character of the frequencies and of the pedestrian phases, as for load case No. 1.

For category I footbridges, allowance is made for the random character of the pedestrian phases only, as for load case No. 2.

The second minus factor, ψ, because of the uncertainty of the coincidence between the frequency of stresses created by the crowd and the natural frequency of the construction, is given by Figure 6.3 according to the natural frequency of the mode under consideration, for vertical and longitudinal vibrations on the one hand, and transversal on the other.

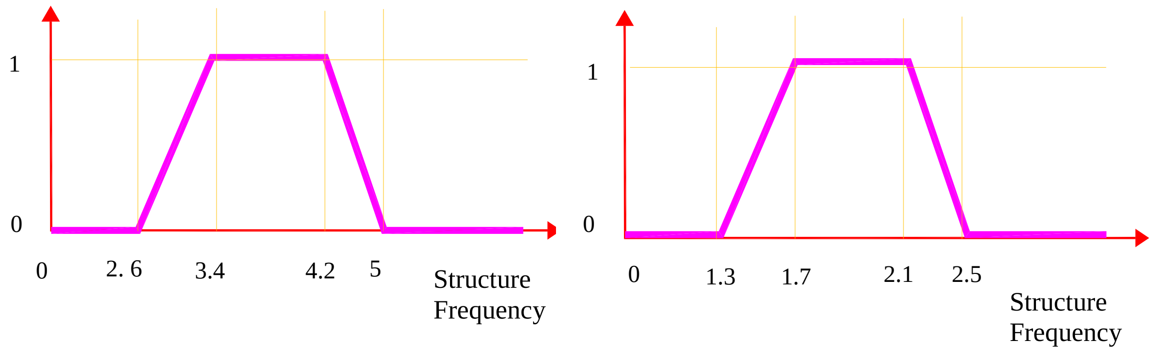


Figure 6.3: Factor ψ for the vertical vibrations on the left and the lateral vibrations on the right

6.5.2 Damping of the construction

The dynamic calculations are made, taking into account the following structural damping:

Table 6.9: Critical damping ratio to be taken into account.

Type	Critical Damping Ratio (%)
Reinforced Concrete	1.3
Pre-stressed Concrete	1
Mixed	0.6
Steel	0.4
Timber	1

In the case of different constructions combining several materials, the critical damping ratio to be taken into account may be taken as the average of the ratios of critical damping of the various materials weighted by their respective contribution in the overall rigidity in the mode under consideration:

$$\xi_{\text{mode}i} = \frac{\sum_{material m} \xi_m k_{m,i}}{\sum_{material m} k_{m,i}}$$ in which $k_{m,i}$ is the contribution of material m to the overall rigidity in mode *i*.

In practice, the determination of $k_{m,i}$ rigidity is difficult. For traditional footbridges of hardly varying section, the following formula approach can be used:

$$\xi_{\text{mode}} = \frac{\sum_{material m} \xi_m EI_m}{\sum_{material m} EI_m}$$ in which EI_m is the contribution of the material m to the overall rigidity *EI* of the section, in comparison to the mechanical centre of that section (such that $\sum_{material m} EI_m = EI$).

6.6 Stage 5: Modification of the Project or of the Footbridge

If the above calculations do not provide sufficient proof, the project is to be re-started if it concerns a new footbridge, or steps to be taken if it concerns an existing footbridge (installation or not of dampers).

6.7 Comfort Level for the Investigated Bridge

Based on the classifications presented in this chapter for the prototype bridges (Bridge I, II in chapter 2), attempts are made in the chapter 8 for optimization of structural system.

Chapter 7

FINITE ELEMENT MODELING

7.1 General

In this chapter, a finite element modeling for the dynamic analysis of the footbridge over the Crescent Lake in Dhaka city and footbridge near Radisson Water Garden Hotel will be discussed. We choose two examples of prototype footbridges (Chapter 1, Table 1.1) for detail dynamic analysis and compare the responses due to the load models proposed in different codes and recent publications. According to Chapter 6 (Rationale for Development of Design Standards), these two footbridges fall in the class I, III respectively for the present usage pattern. To this end the next sections will be devoted to model and to analyze comfort cases.

This chapter is divided into three parts. In the first section, there is a general description of the finite element modeling of the footbridge and related software for this modeling and dynamic analysis. The second part is about the pedestrian load modeling. The third part describes the different types of analysis methods.

7.2 Finite Element Models of the Pedestrian Bridges

Dynamic analysis of Footbridge-I and Footbridge-II was performed using the Finite Element Method using SAP2000, general purpose finite element software. The objective of the analysis was to investigate the response of the bridge structure due to dynamic loads applied by pedestrians.

In order to analyze the structures dynamically, 3-dimensional finite element (FE) models of the footbridge structures was established. This section follows the modeling process as well as the dynamic analysis.

7.2.1 Geometric model

The first step in an FE modeling is to consider how to represent the characteristics of the footbridge structure. The FE model of Footbridge-I consists of arch deck system. In this footbridge, there are two concrete arches which are grooved in one side and also this bridge contains steel girders, cables and concrete bracings (Figure 7.1 and 7.2).

The original bridge structure has grooved arches. However, for the sake of simplicity, the grooved portions of the two arches have been separated in the finite element model.

The coordinates of the arches, cables and the deck were taken from structural drawings of the bridge. Here the bridge model was done by considering the arches, deck etc. as the frame elements. For parametric study, different geometric arrangements of deck, tie and bracing system were considered.

After modeling the arch, hangers and the deck system, the arch was analyzed for dead loads and live loads due to pedestrians. In general, for the design loads and assigned sections, the model was found to be numerically adequate. In view of the code requirements, the models developed here are used to study the dynamic stability of the system under pedestrian movement.

The FE modeling and analysis of Footbridge-II was performed in the same way as like Footbridge-II. The FE model of the footbridge consists of steel girder deck system with three main girders and cross girders (Figure 7.3).

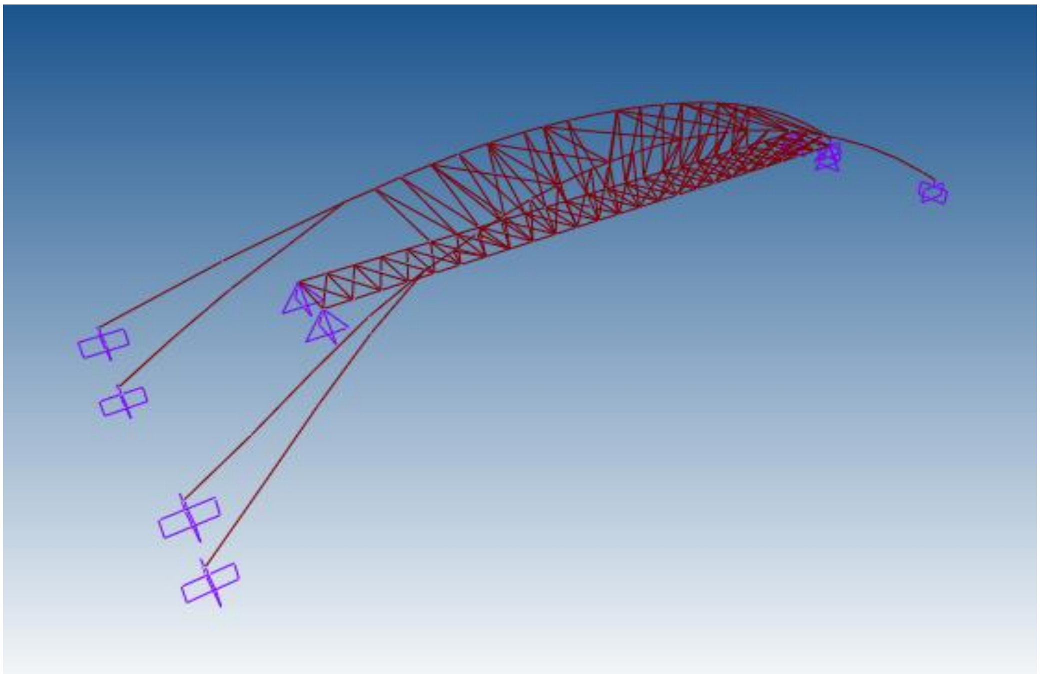


Figure 7.1: Finite Element model (Line Element) of Footbridge-I.

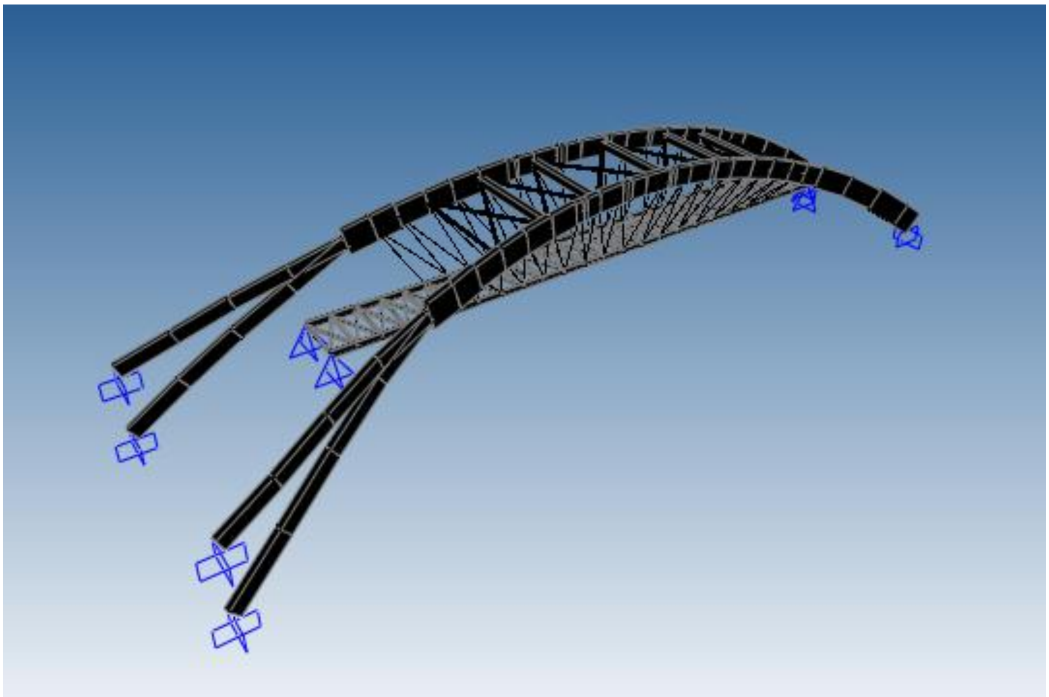


Figure 7.2: Finite Element model (Solid Element) of Footbridge-I.

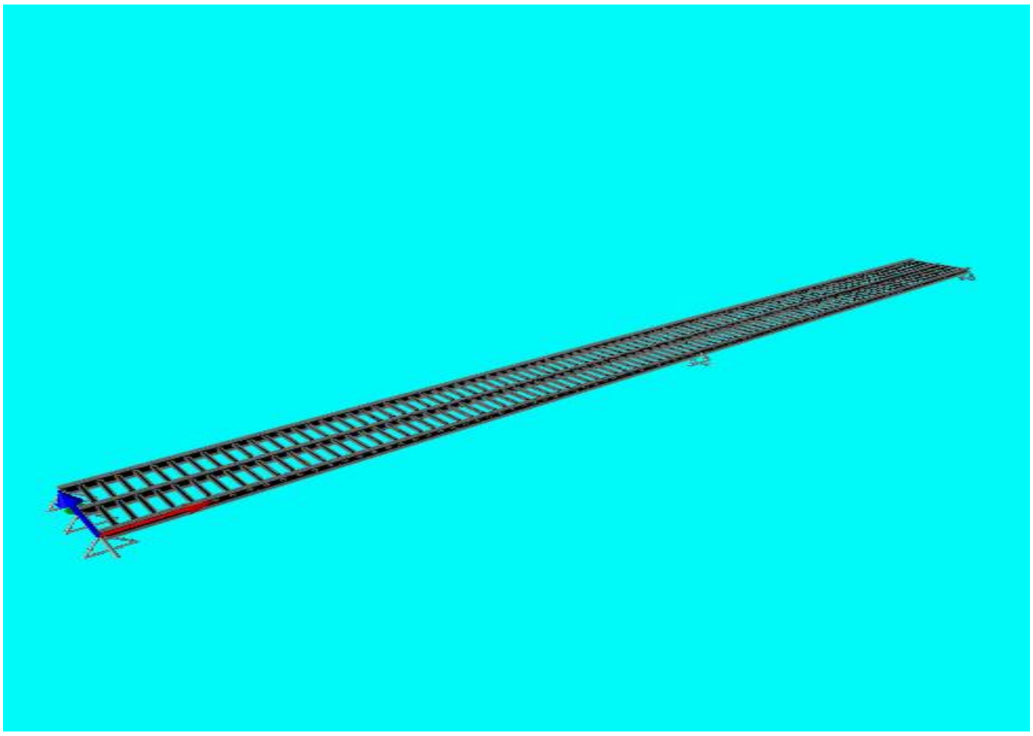


Figure 7.3: Finite Element model (Solid Element) of Footbridge-II.

7.2.2 Material and section model

An important aspect of modeling a structure is the determination of the material and section properties of its components.

In the FE model of Footbridge-I, foundation structures, arches and ties between arches at overhead locations are made of reinforced concrete. Cable of the bridge, bridge deck beams, deck bracings and cross ties between RCC ties are made of isotropic steel. Also the tampered glass panels are fitted with the hanging steel-framed deck. The material properties of this footbridge are listed in Table 7.1 and 7.2.

Table 7.1: Material properties of steel

Material Properties	
Modulus of Elasticity	200 GPa
Shear Modulus	77 GPa
Poisson's Ratio	0.30
Coefficient of Thermal Expansion	$11.7\text{x}10^{-6}$

Table 7.2: Material properties of concrete

Material Properties	
Modulus of Elasticity	25 GPa
Shear Modulus	10.3 GPa
Poisson's Ratio	0.20
Coefficient of Thermal Expansion	9.90 $x10^{-6}$
28 days Compressive strength of the concrete	31 MPa
Rebar Yield Strength	414 MPa

The deck beam sections and bracings are made of steel. The sectional properties are given in Table 7.3 and Figure 7.4.

Table 7.3: Sectional properties of steel deck and bracings of the Footbridge-I

Section Name	Shape	t3 (mm) x10	t2 (mm) x10	tf (mm)	tw (mm)	t2b (mm) x10	tfb (mm)	Area (mm^2) $x10^3$	I_{33} (mm^4) $x10^8$	I_{22} (mm^4) $x10^7$
WF-1	I/Wide Flange	60.96	30.48	25.40	25.40	30.48	25.40	29.70	17.00	10.00
WF-2	I/Wide Flange	50.80	30.48	12.70	12.70	30.48	12.70	13.90	6.00	6.00
WF-3	I/Wide Flange	30.48	15.24	12.70	12.70	15.24	12.70	7.40	1.00	0.80

where, t3 : Outside Height, t2 : Top Flange Width, tf : Top Flange Thickness, tw : Web Thickness, t2b : Bottom Flange Width, tfb : Bottom Flange Thickness.

In the SAP2000 model local axes 2-2 and 3-3 directions indicate x-x and y-y directions in the cross sections. The sectional properties of the cable (PIPE-1) and cross ties in the arches (PIPE-2) are given in the Table 7.4.

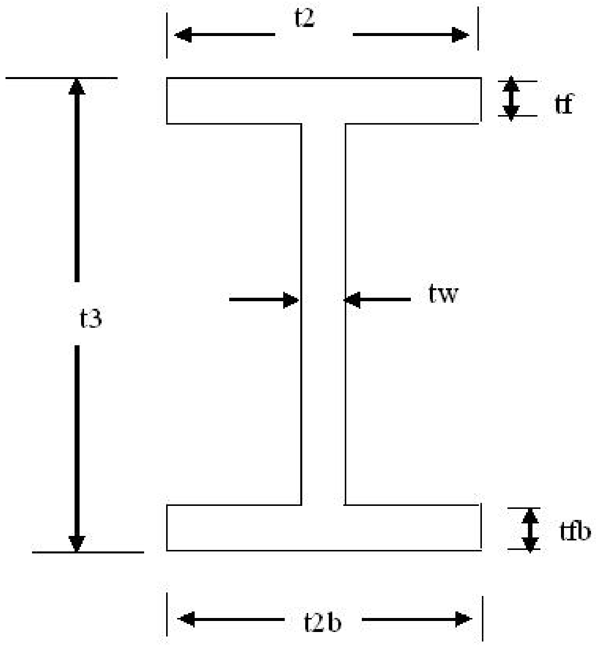


Figure 7.4: Dimensions of an I-section.

Table 7.4: Sectional properties of steel cable and cross ties in the arches of the Footbridge-I

Section Name	Shape	t3 (mm) x10	tw (mm) $x10^3$	Area (mm^2) $x10^3$	I_{33} (mm^4) $x10^7$	I_{22} (mm^4) $x10^7$
PIPE-1	Pipe	15.24	12.70	5.60	1.40	1.40
PIPE-2	Pipe	38.10	25.40	28.40	50.00	50.00

where, t3 : outside diameter of the hollow pipe section, tw : wall thickness.

The section of the arch in the finite element model has variable cross section (Table 7.5 and Figure 7.5).

Table 7.5: Sectional properties of the concrete arch sections of the Footbridge-I.

Section Name	Shape	Area (mm^2) $x10^6$	I_{33} (mm^4) $x10^{11}$	I_{22} (mm^4) $x10^{10}$
SD-10	Rectangular	1.36	6.75	3.55
SD-11	Rectangular	0.72	1.01	1.88
SD-22	Rectangular	0.68	0.84	1.77
SD-3	Rectangular	1.32	6.14	3.43
SD-33	Rectangular	0.64	0.69	1.66
SD-4	Rectangular	1.12	3.76	2.92
SD-44	Rectangular	0.59	0.56	1.55
SD-5	Rectangular	0.99	2.62	2.59
SD-6	Rectangular	0.94	2.19	2.44
SD-7	Rectangular	0.94	2.19	2.44
SD-8	Rectangular	1.01	2.73	2.62
SD-9	Rectangular	1.15	4.06	2.99
SD-T	Trapezoidal	0.39	0.19	0.88

All the material and sectional properties in the SAP2000 model are taken from the structural drawings of the pedestrian bridge.

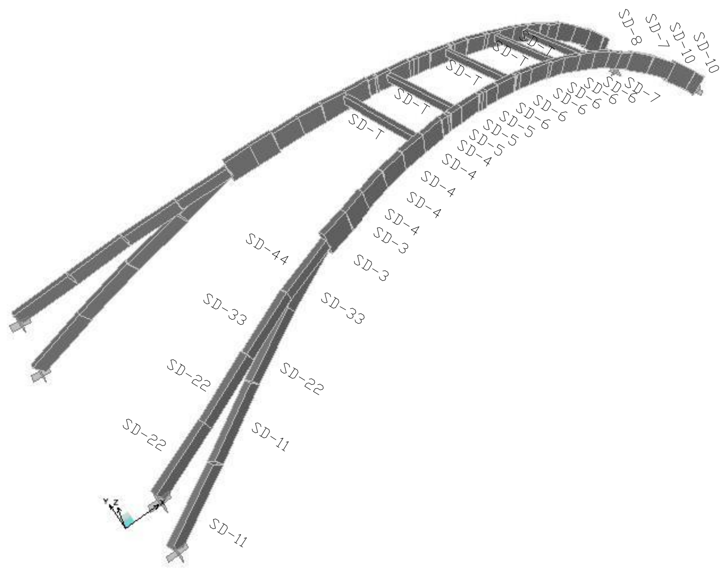


Figure 7.5: Section of the arch of the footbridge over Crescent Lake.

In the FE model of the Footbridge-II, all the structural materials are modeled as steel sections (Table 7.1). The main girder is an I-section and cross girder is made of hollow rectangular box section (Table 7.6).

Table 7.6: Sectional properties of the steel deck system of the Footbridge-II

Section Name	Shape	t3	t2	tf	tw	t2b	tfb	Area	I_{33}	I_{22}
		(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm^2)	(mm^4)	(mm^4)
		$x10^2$	$x10^2$		$x10^3$	$x10^2$	x10	$x10^3$	$x10^6$	$x10^6$
Main Beam	I/Wide Flange	4.06	1.40	10	7.00	1.40	1.00	5.50	143.00	4.58
Cross Beam	Box	0.75	1.00	3.25	3.25			1.10	1.01	1.58

where, t3 : Outside Height, t2 : Top Flange Width, tf : Top Flange Thickness, tw : Web Thickness, t2b : Bottom Flange Width, tfb : Bottom Flange Thickness of the

Main Beam and t3 : Outside Depth, t2 : Outside Width, tf : Flange Thickness, tw : Web Thickness of the Cross Beam.

7.2.3 *Boundary conditions*

A key for a successful dynamic analysis is a proper modeling of the boundary conditions of the structural system. The boundary conditions of the deck and arch system of the Footbridge-1 are illustrated in Figures 7.6 and 7.7. In the FE model of the bridge, the deck end is free to rotate, but is fixed against translation in any direction. The arches are fixed against translation and also against rotation.

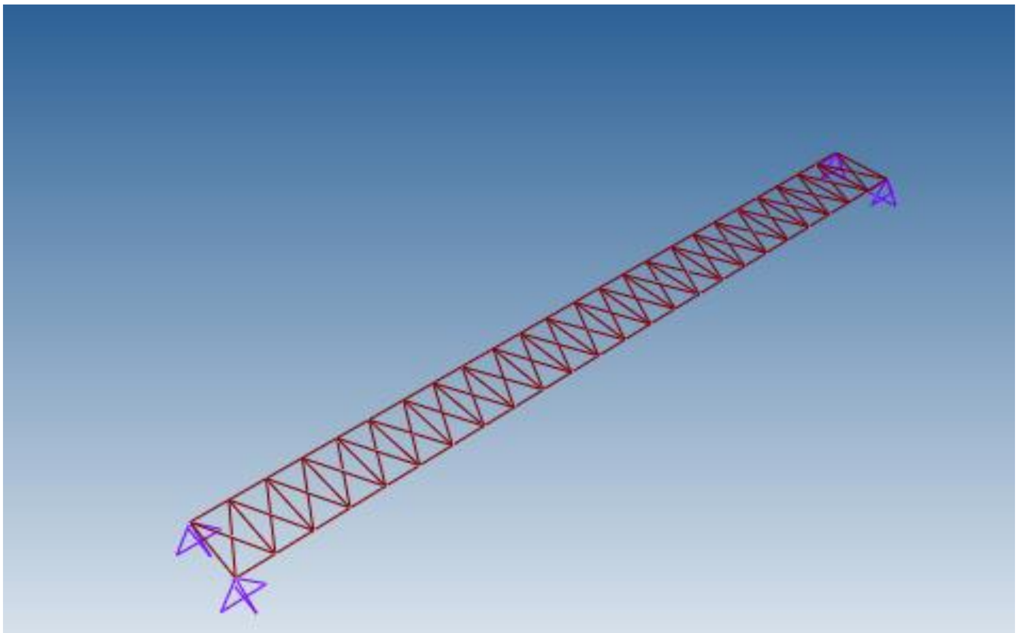


Figure 7.6: Boundary conditions of the FE model of the deck system of the Footbridge-I.

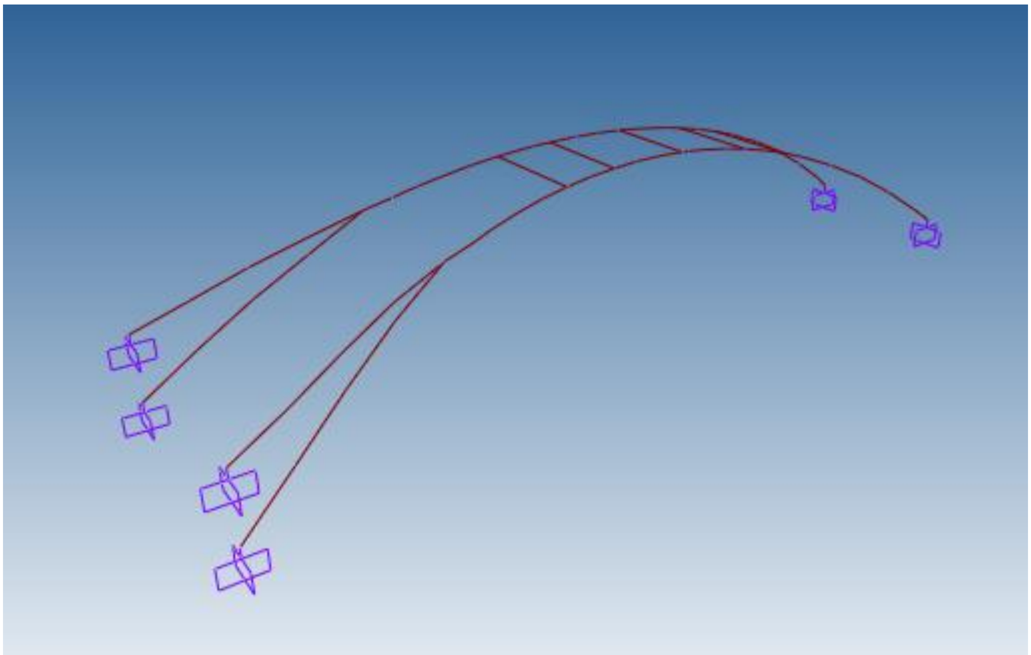


Figure 7.7: Boundary conditions of the FE model of the arch system of the Footbridge-I.

The boundary conditions of the deck system of the footbridge near Radisson Water Garden Hotel are shown in Figure 7.8.

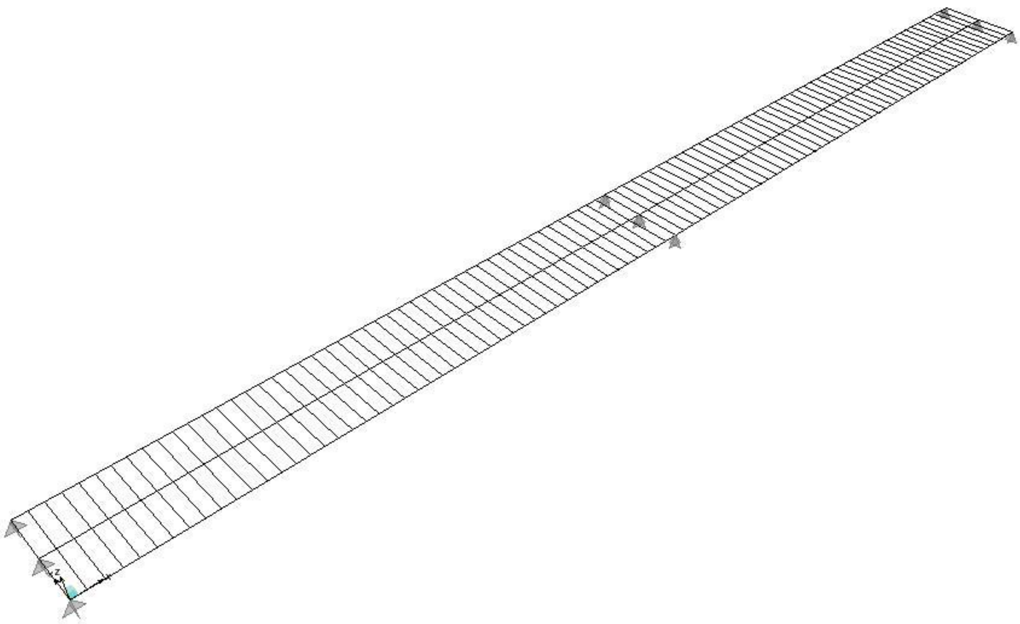


Figure 7.8: Boundary conditions of the FE model of the Footbridge-II.

7.3 Modeling of Vertical Load

A lot of research into walking forces has been done in the field of biomechanics, usually with the aim to investigate differences in the step patterns between patients who are healthy and those with abnormalities. In this way, a very comprehensive research into human forces relevant to footbridge dynamic excitation was conducted by Wheeler (1982) who systematized the work of other researchers related to different modes of human moving from slow walking to running (Figure 7.9).

7.4 Fujino's Load Modeling for Lateral Load

In this load model, observation of human-induced large-amplitude lateral vibration of an actual pedestrian bridge in an extremely congested condition is reported. Walking motions of pedestrians recorded by a video camera are analyzed. It is found that walking among 20 percent or more of the pedestrians on the bridge was synchronized to the girder lateral vibration. With this synchronization, the total lateral force from the pedestrians to the girder is evidently increased and it acts as a resonant force on the girder lateral vibration.

Here, human passage on the bridge during congested periods was recorded by an 8 mm video camera from a fixed point. Pedestrians were randomly selected and the motions of their heads were digitized from the video monitor with the help of a microcomputer. Digitization was made for around 15-20 s. Distance correction was made on the digitized motions of human walking. Some examples of the human head motion corresponding to the different pedestrian conditions are given in the Figure 7.10.

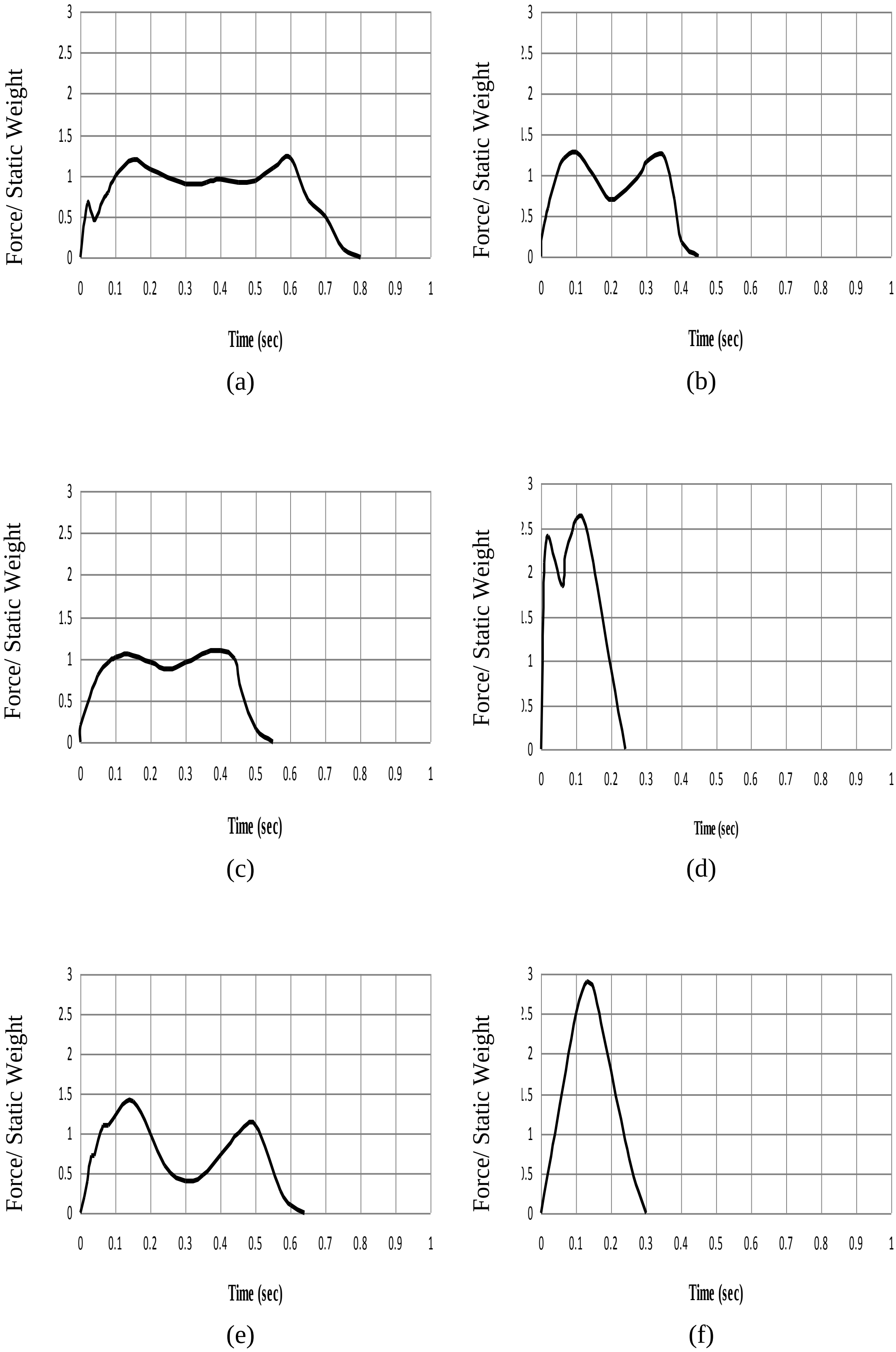
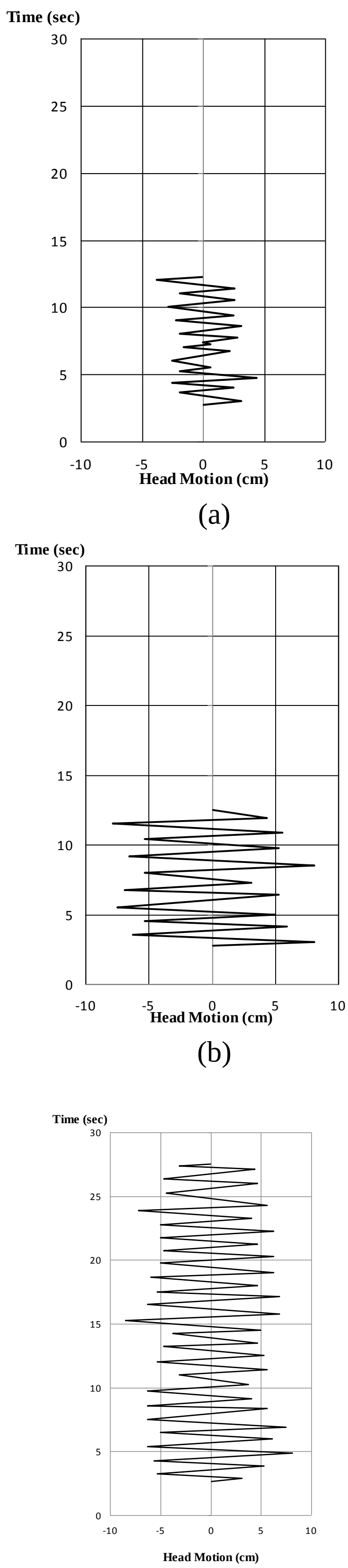


Figure 7.9: Changes of the force over time for (a) slow walk, (b) fast walk, (c) normal walk, (d) slow jog, (e) brisk walk, (f) running.



(a)

(b)

(c)

Figure 7.10: Human head motion: (a) not congested and small lateral girder vibration, (b) extreme and large lateral girder vibration and (c) extreme and large lateral girder vibration-measured from cable vibration.

Chapter 8

OPTIMIZATION OF STRUCTURAL SYSTEM FOR DYNAMIC STABILITY

8.1 General

Recently, engineers and academics have become much more interested in the dynamic behaviour of footbridges. This interest has increased after detecting the vibration problems in the new Millennium footbridge in London just after opening in 2000. With current design practice for footbridges, vibrations are becoming an important issue. This is due to several reasons such as high resistant materials, smaller cross sections or larger spans. All this causes a reduction of the stiffness leading to smaller natural frequencies and therefore the structure exhibits a higher risk of resonance with pedestrian excitation. As a consequence, the vibration issue becomes now a main reason for extending the design process beyond the statics.

Again, suspended footbridge is one of the most important structural forms of modern footbridges, and it gets its popularity around the world because of the low material consumption, ease of construction and the ability to cross longer spans. Since the main function of footbridges is to carry pedestrians and cyclists, the design load is much smaller than that of highway bridge structures. Again the structural stiffness of suspension bridges is mainly provided by the tension forces in the cable system. For this reason, they are always slender and weak with low natural frequencies. Such slender footbridges are prone to vibration induced by human activities. Research also has shown that slender suspension footbridges with shallow cable profiles often exhibit coupled vibration modes (Huang et al. 2005) and have different dynamic performance in the lateral and vertical directions (Huang et al. 2005). However, such vibration of slender footbridges with coupled modes has not yet been fully investigated.

In this chapter, dynamic analysis of different types of footbridges will be performed. Attempts are also made to explore the possibility of improving the vertical and lateral dynamic stability of the system by optimization of the different geometric structural system.

8.2 Dynamic Behaviour of Footbridge-I

8.2.1 Eigenvalue analysis

To achieve an adequate system, the fundamental vibration modes and natural frequencies of the structure with different stiffening systems need to be studied in details. Once studied, this would reveal the performance of the structural system against pedestrian movement. Eigenvalue analysis determines the undamped free-vibration mode shapes and frequencies of the system. These natural modes provide an excellent in sight into the behaviour of the structure. In this research work, a parametric study is carried out to investigate the effect of different stiffening systems on the vibration modes and the fundamental natural frequencies of the arch-deck system using the finite element model. Eigenvalues are calculated for choosing the most suitable arch-deck system.

The eigenvalues determined from the finite element model for first horizontal mode and first vertical mode is presented in Figure 8.1 in relation to the code recommended values. By comparing the computed eigenvalues for different cases, it is evident that the dynamic stability of the system expressed in terms of eigen frequencies improves for the following changes in the model geometry:

1. Decrease of deck width (Option D)
2. Increase of the lateral stiffness of the deck system by adding additional cross-bracings (Option E)
3. Adoption of inclined hangers instead of straight vertical hangers (Option F, G)
4. Increase the number of ties and cross bracings between arches at the top (Option F, G)

5. Incorporating additional ties between the deck and lake bed
6. Change of sectional properties of the members (Option H, I)
7. Adoption of deck railing (Option J, K)

Table 8.1: Different options for Footbridge-I model.

Bridge Model	Option	Hanger System		Deck Width			Deck Bracings		Deck Railing		Ties Between Arches at Overhead Locations		
		Straight	Inclined	4.27 m	4.57 m	7.9 m	Yes	No	Yes	No	3 RCC ties	5 RCC ties	5 RCC ties, 7 steel ties and bracings
Curved arch Bridge	A		√		√		√		√				√
	B		√		√		√			√			√
Curved arch Bridge (Different options)	C	√				√		√		√	√		
	D	√		√				√		√	√		
	E	√		√			√			√	√		
	F		√	√			√			√		√	
	G		√	√			√			√			√
Footbridge with Vertical Suspension Cables	H	√			√		√			√			
	I	√			√		√			√			
Straight arch Footbridge	J		√		√		√		√			√	
	K		√		√		√			√		√	

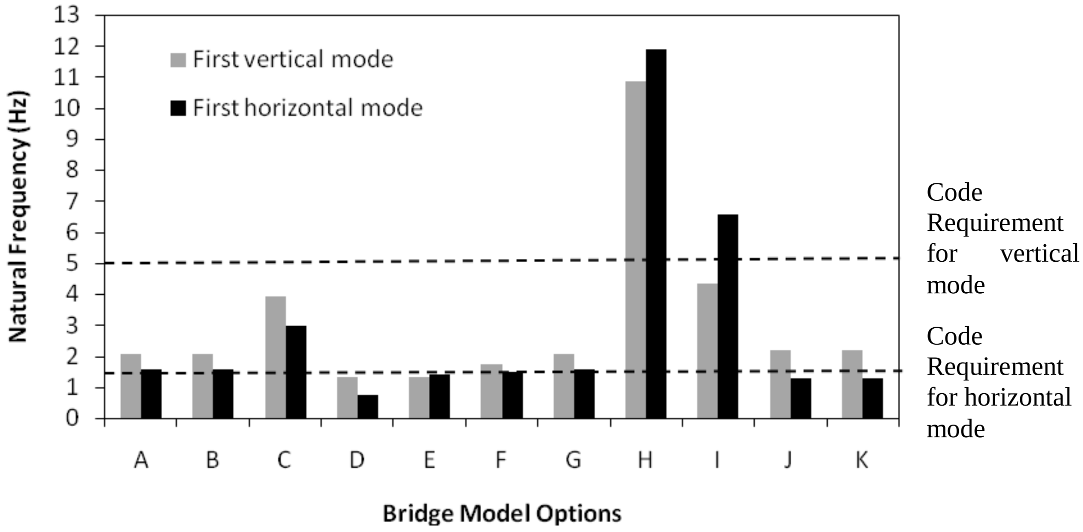
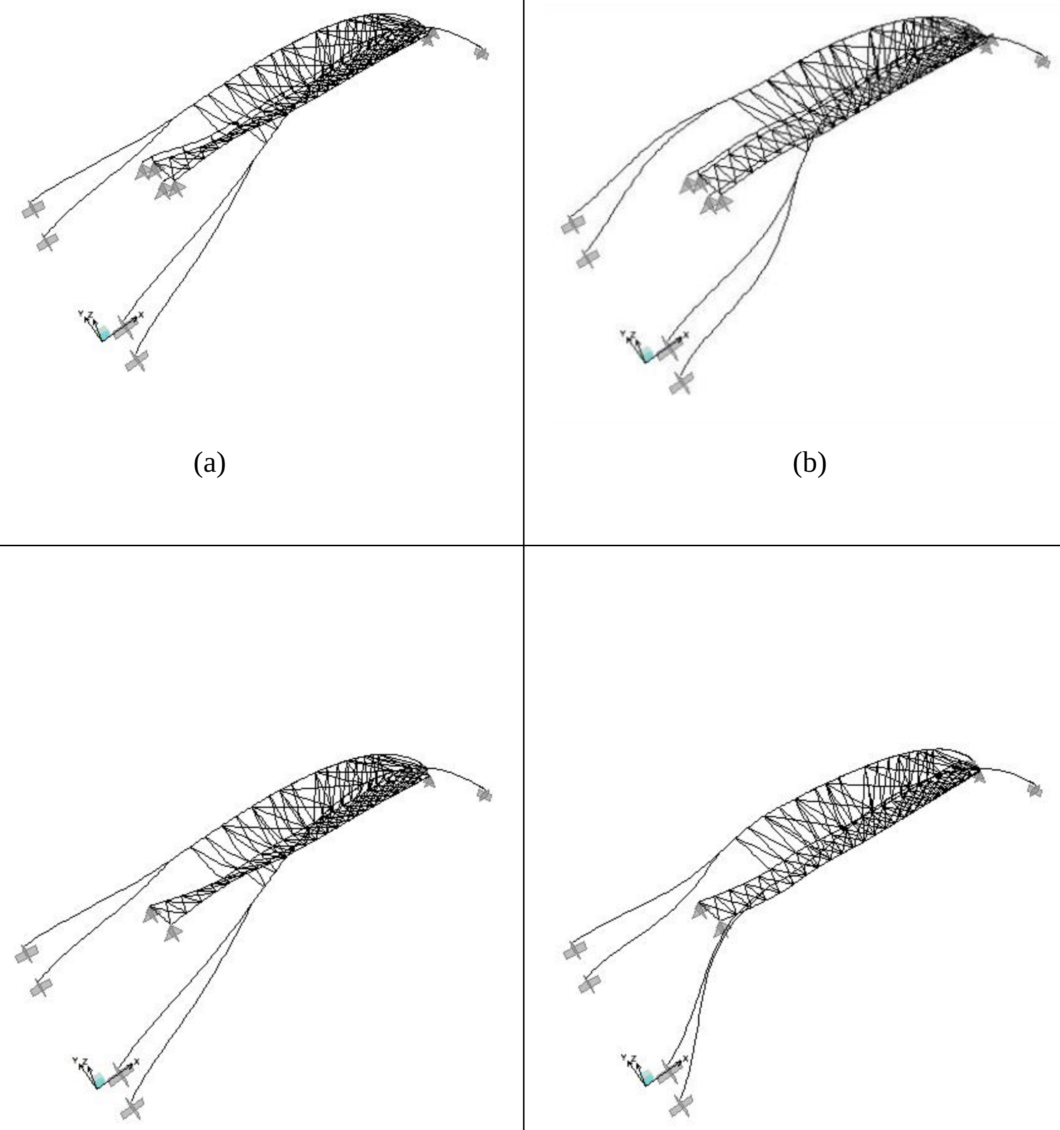


Figure 8.1: Eigen frequencies determined for different options and presented against the code requirements.



(a) (b) (c) (d)

Figure 8.2: Different mode of vibrations in the arch-deck system - (a) 1st torsional mode of vibration for option A, (b) 1st vertical mode of vibration for option A, (c) 1st torsional mode of vibration for option B, (d) 1st vertical mode of vibration for option B.

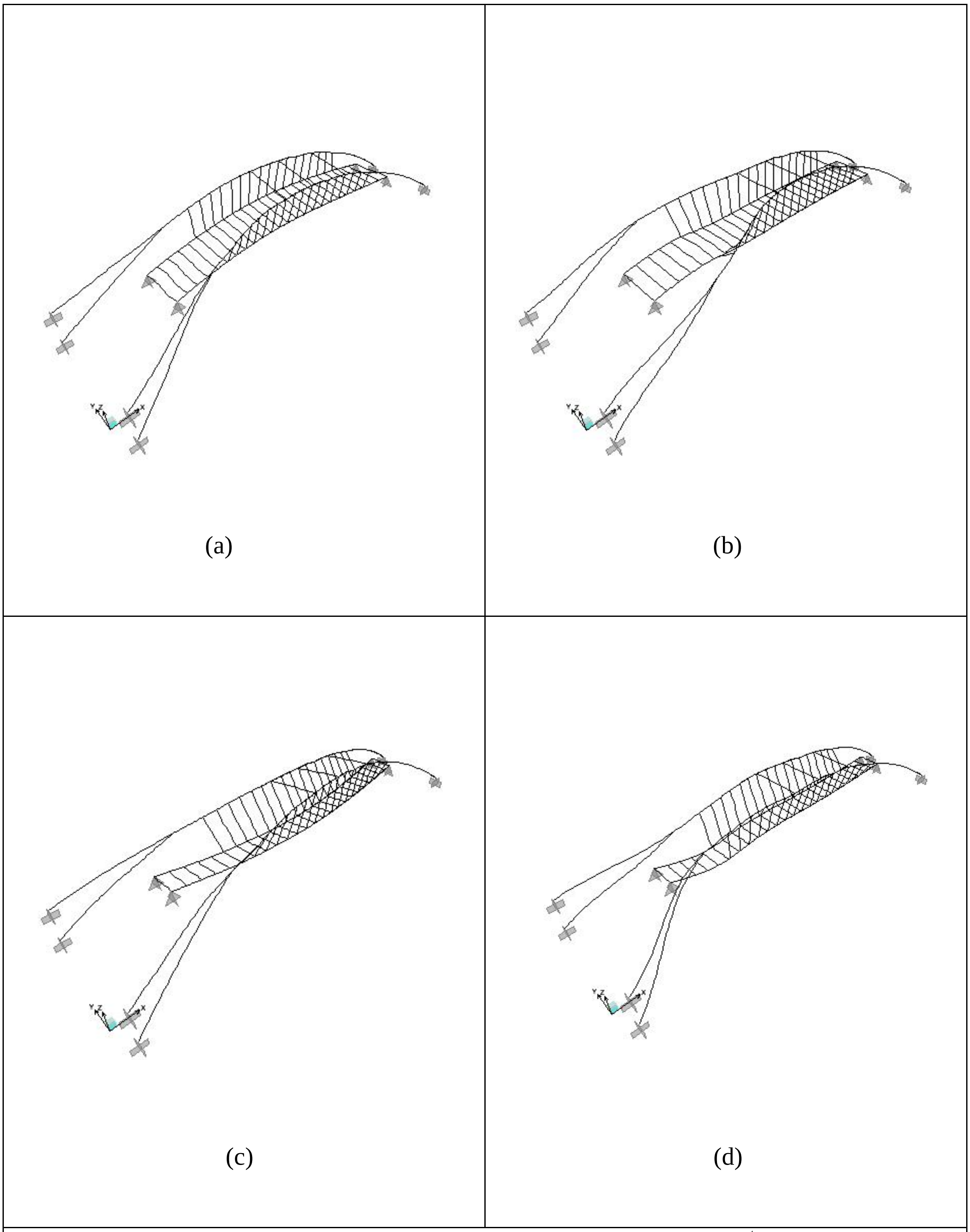


Figure 8.3: Different mode of vibrations in the arch-deck system - (a) 1st horizontal mode of vibration for option C, (b) 1st vertical mode of vibration for option C, (c) 1st horizontal mode of vibration for option D, (d) 1st vertical mode of vibration for option D.

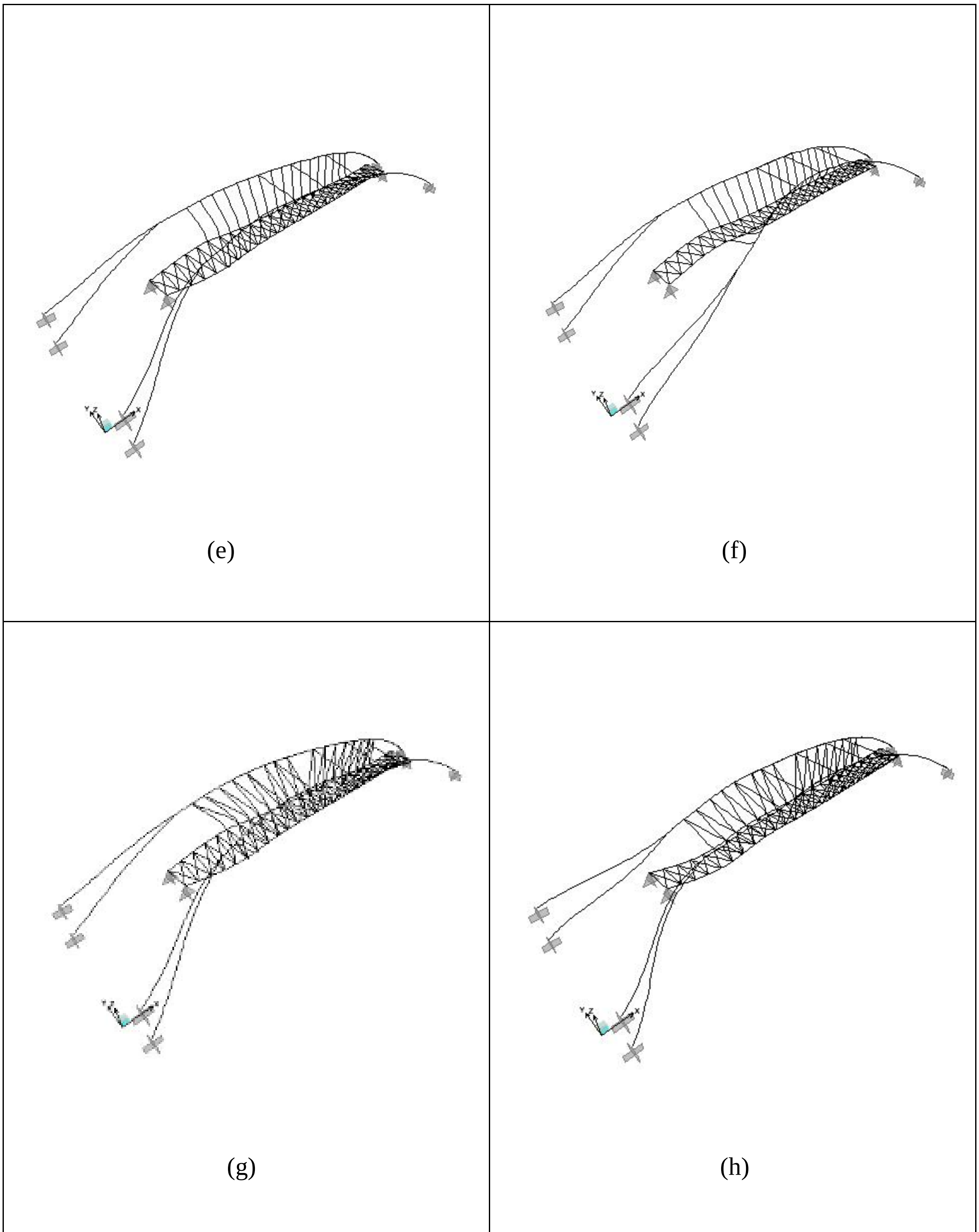
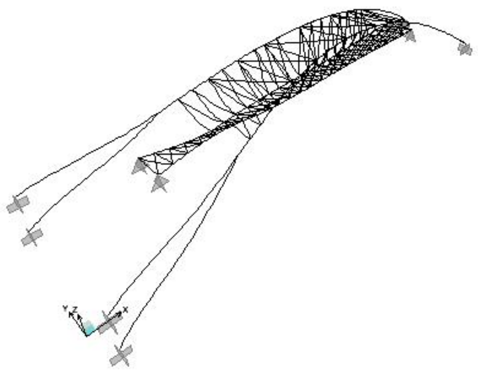
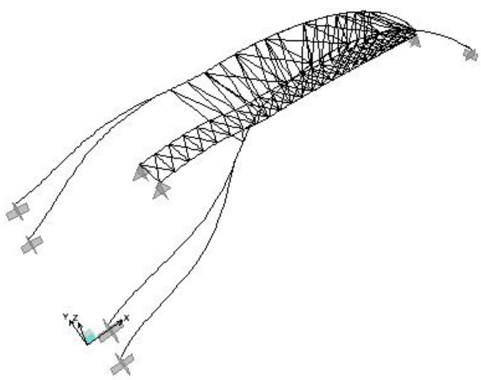


Figure 8.3: Different mode of vibrations in the arch-deck system - (e) 1st torsional mode of vibration for option E, (f) 1st vertical mode of vibration for option E, (g) 1st torsional mode of vibration for option F, (h) 1st vertical mode of vibration for option F.

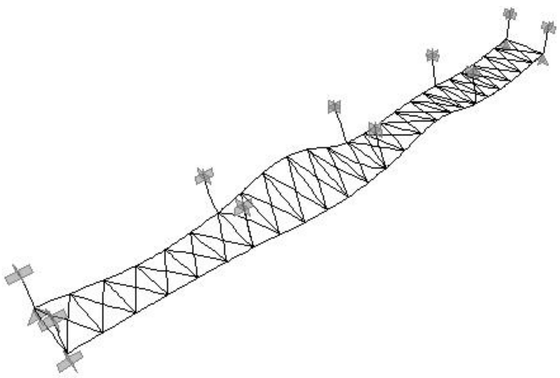


(i)

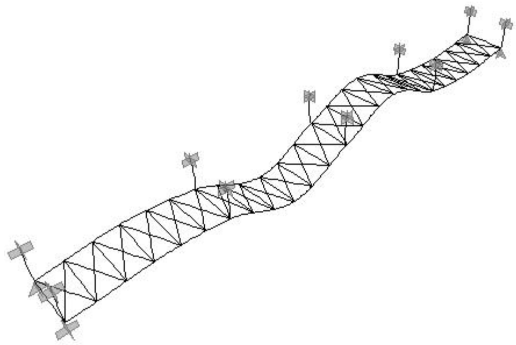


(j)

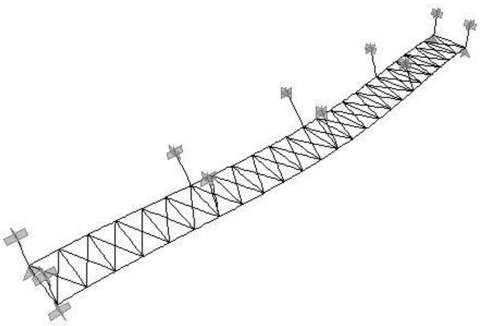
Figure 8.3: Different mode of vibrations in the arch-deck system - (i) 1st torsional mode of vibration for option G, (j) 1st vertical mode of vibration for option G.



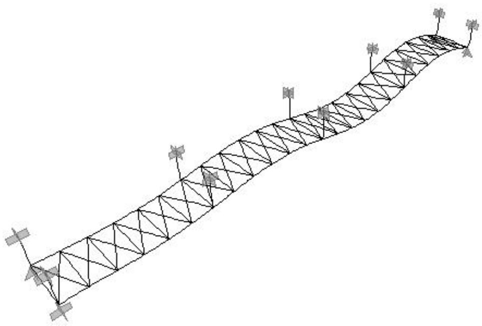
(a)



(b)



(c)



(d)

Figure 8.4: Different mode of vibrations in the arch-deck system - (a) 1st coupled lateral-torsional mode of vibration for option H, (b) 1st vertical mode of vibration for option H, (c) 1st horizontal mode of vibration for option I, (d) 1st vertical mode of vibration for option I.

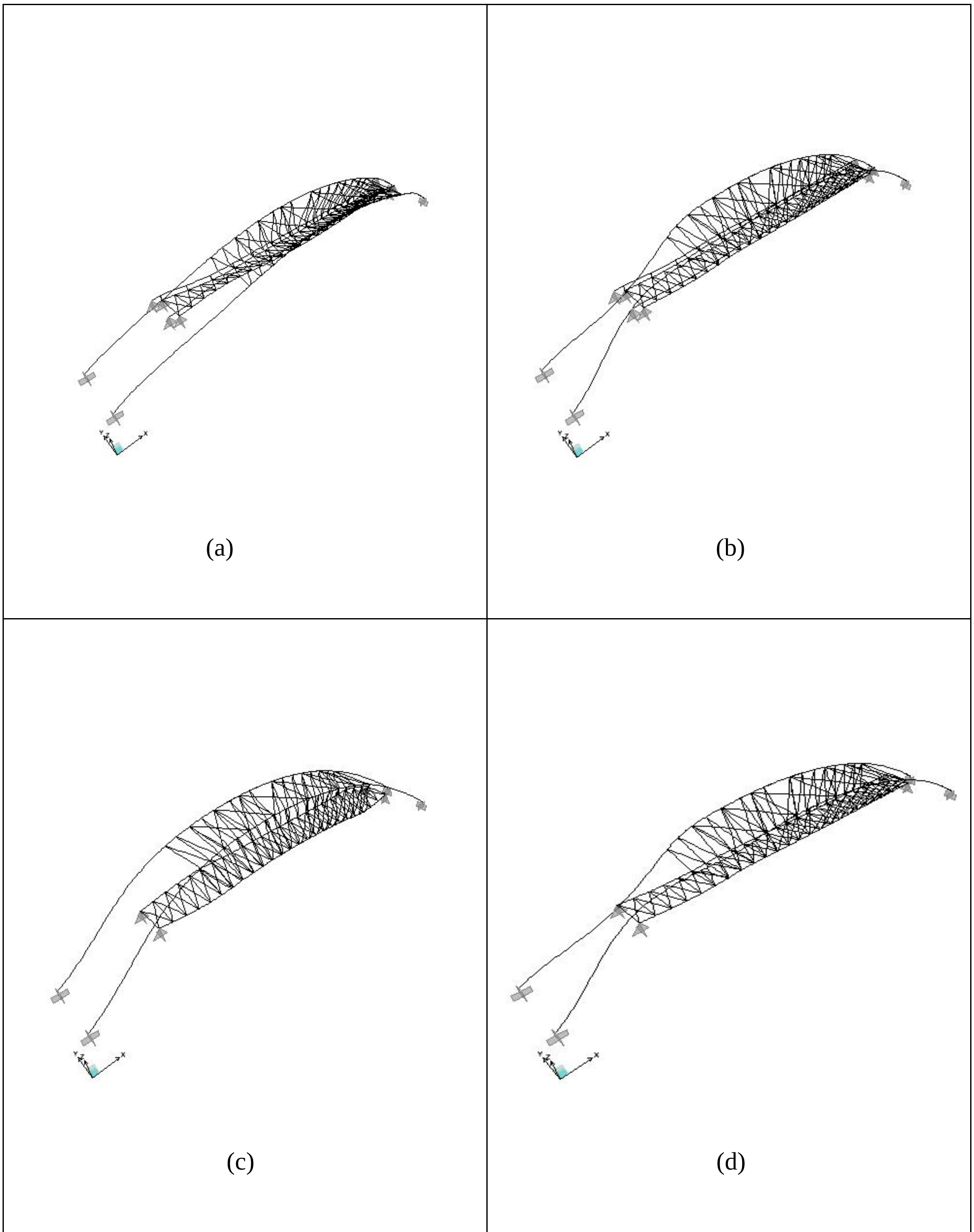


Figure 8.5: Different mode of vibrations in the arch-deck system - (a) 1st torsional mode of vibration for option J, (b) 1st vertical mode of vibration for option J, (c) 1st torsional mode of vibration for option K, (d) 1st vertical mode of vibration for option K.

Option A is the actual footbridge model for Footbridge-I and difference between option A and B is only in the deck railing. Figure 8.2(a) shows the torsional mode for option A. Deck of the footbridge model plays the dominating role in this mode. Figure 8.2(c) also shows torsional mode. Deflection is slightly higher in Figure 8.2(c) than in Figure 8.2(a). Figure 8.2(b) and 8.2(d) show the first vertical mode.

There is no bracing in the deck and in between arches of option C and D. In those two models, first horizontal modes have been found (Figure 8.3(a) and 8.3(c)). But torsional modes can be seen in the footbridge model options E, F and G (Figure 8.3(e), 8.3(g), 8.3(i)). Deck deflection in the vertical mode of option G is relatively less than that of the other models (option C, D, E and F).

Option H and I have same geometric configurations, but those have different steel sections. For this reason, option H has first coupled lateral-torsional mode shape.

Option J and K have almost same mode shapes and the deflected shapes in the vertical modes are low (Figure 8.5(b), 8.5(d)).

From all those models, it is very much clear that increasing the lateral and vertical stiffness of the models can minimize the deflected shape of the lateral and vertical mode.

8.2.2 Dynamic behaviour due to human induced vertical vibration

This section focuses on dynamic loads especially vertical forces (Wheeler 1982) induced by a single pedestrian. It is well known that a pedestrian applies dynamic forces to the surface on which he walks. The vertical component is applied at the footfall frequency (typically 2 Hz) and is about 40% of their body weight.

8.2.2.1 Dynamic stability check

According to different standards (Figure 5.4), Footbridge-I has been checked by using different loading conditions.

Table 8.2: Dynamic acceptability of Footbridge-I model.

Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Peak Acceleration (m/s^2)	Dynamic Acceptability (According to BS 5400)
Curved arch Bridge (Option A)	Vertical	normal walk	2.1	0.41	Acceptable
		running	2.1	2.18	Unacceptable
		slow jog	2.1	1.00	Acceptable
		slow walk	2.1	0.40	Acceptable
		brisk walk	2.1	0.69	Acceptable
		fast walk	2.1	1.67	Unacceptable
Curved arch Bridge (Option B)	Vertical	normal walk	2.1	-0.58	Acceptable
		running	2.1	2.19	Unacceptable
		slow jog	2.1	-1.31	Acceptable
		slow walk	2.1	-0.43	Unacceptable
		brisk walk	2.1	0.71	Acceptable
		fast walk	2.1	1.66	Unacceptable
Curved arch Bridge (Different options)- Option C	Vertical	normal walk	3.9	0.67	Acceptable
		running	3.9	1.11	Acceptable
		slow jog	3.9	1.05	Acceptable
		slow walk	3.9	-0.32	Acceptable
		brisk walk	3.9	0.47	Acceptable
		fast walk	3.9	0.61	Acceptable
Curved arch Bridge (Different options)- Option D	Vertical	normal walk	1.3	1.03	Unacceptable
		running	1.3	1.93	Unacceptable
		slow jog	1.3	1.89	Unacceptable
		slow walk	1.3	-0.52	Acceptable
		brisk walk	1.3	0.74	Acceptable
		fast walk	1.3	0.92	Acceptable
Curved arch Bridge (Different options)- Option E	Vertical	normal walk	1.3	1.07	Unacceptable
		running	1.3	2.08	Unacceptable
		slow jog	1.3	1.98	Unacceptable
		slow walk	1.3	-0.54	Acceptable
		brisk walk	1.3	-0.79	Acceptable
		fast walk	1.3	0.96	Acceptable

Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Peak Acceleration (m/s^2)	Dynamic Acceptability (According to BS 5400)
Curved arch Bridge (Different options)- Option F	Vertical	normal walk	1.7	-0.62	Acceptable
		running	1.7	1.82	Unacceptable
		slow jog	1.7	1.96	Unacceptable
		slow walk	1.7	-0.49	Acceptable
		brisk walk	1.7	0.67	Acceptable
		fast walk	1.7	1.38	Unacceptable
Curved arch Bridge (Different options)- Option G	Vertical	normal walk	2.1	-0.59	Acceptable
		running	2.1	2.19	Unacceptable
		slow jog	2.1	-1.32	Unacceptable
		slow walk	2.1	-0.46	Acceptable
		brisk walk	2.1	0.73	Acceptable
		fast walk	2.1	1.66	Unacceptable
Footbridge with Vertical Suspension Cables- Option H	Vertical	normal walk	10.8	0.51	Acceptable
		running	10.8	-0.56	Acceptable
		slow jog	10.8	5.68	Unacceptable
		slow walk	10.8	1.25	Acceptable
		brisk walk	10.8	-0.57	Acceptable
		fast walk	10.8	0.69	Acceptable
Footbridge with Vertical Suspension Cables- Option I	Vertical	normal walk	4.3	-1.75	Acceptable
		running	4.3	2.42	Unacceptable
		slow jog	4.3	-5.64	Unacceptable
		slow walk	4.3	-1.38	Acceptable
		brisk walk	4.3	2.55	Unacceptable
		fast walk	4.3	3.12	Unacceptable
Straight arch Footbridge- Option J	Vertical	normal walk	2.2	0.79	Acceptable
		running	2.2	0.98	Acceptable
		slow jog	2.2	2.40	Unacceptable
		slow walk	2.2	-0.44	Acceptable
		brisk walk	2.2	0.89	Acceptable
		fast walk	2.2	1.17	Unacceptable

Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Peak Acceleration (m/s^2)	Dynamic Acceptability (According to BS 5400)
Straight arch Footbridge-Option K	Vertical	normal walk	2.2	0.86	Acceptable
		running	2.2	0.97	Acceptable
		slow jog	2.2	2.55	Unacceptable
		slow walk	2.2	-0.49	Acceptable
		brisk walk	2.2	0.96	Acceptable
		fast walk	2.2	1.26	Unacceptable

8.2.2.2 Dynamic response due to single pedestrian

A detailed time history analysis has been performed in the Footbridge-I (Option A). Here typical force patterns for different types of human activities were used. The main focus of this analysis is to evaluate the serviceability requirement of footbridges.

The results have been presented as different graphs showing values of acceleration, displacement and velocity due to periodic pedestrian function (Figure 8.9).

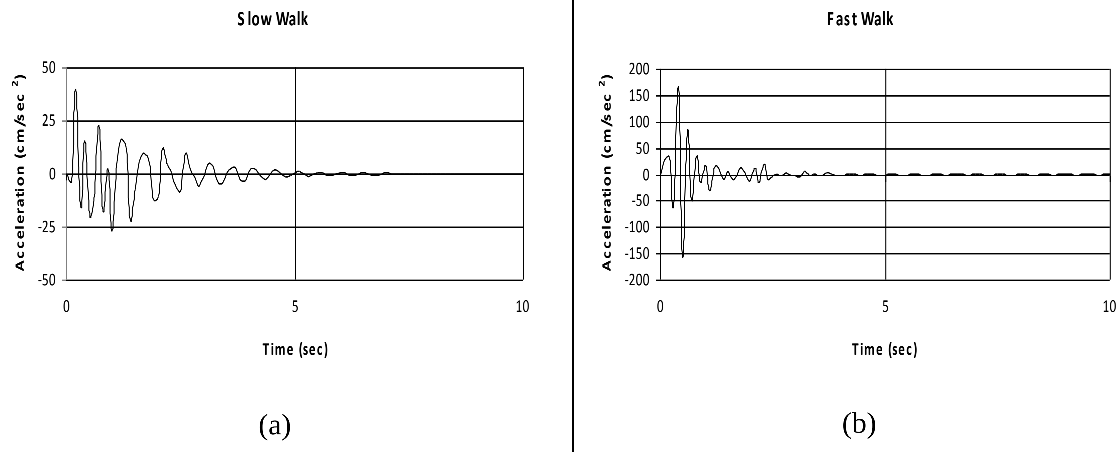
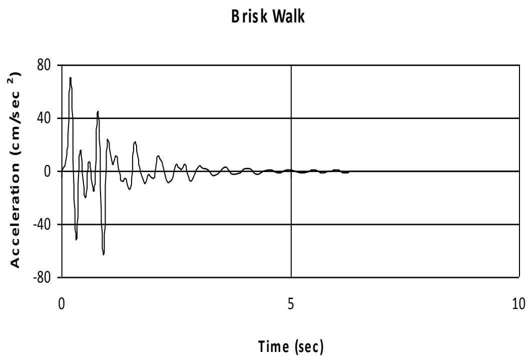
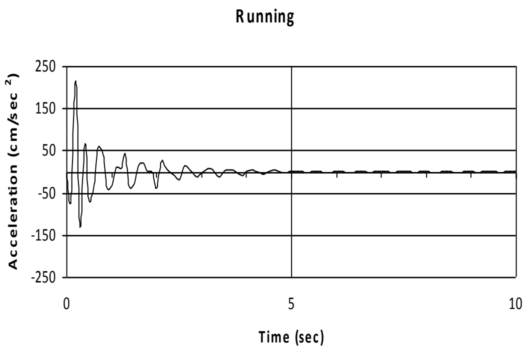


Figure 8.6: Acceleration time history at the centre of the deck due to (a) slow walk, (b) fast walk, (c) normal walk, (d) slow jog.



(e)



(f)

Figure 8.6: Acceleration time history at the centre of the deck due to (e) brisk walk, (f) running.

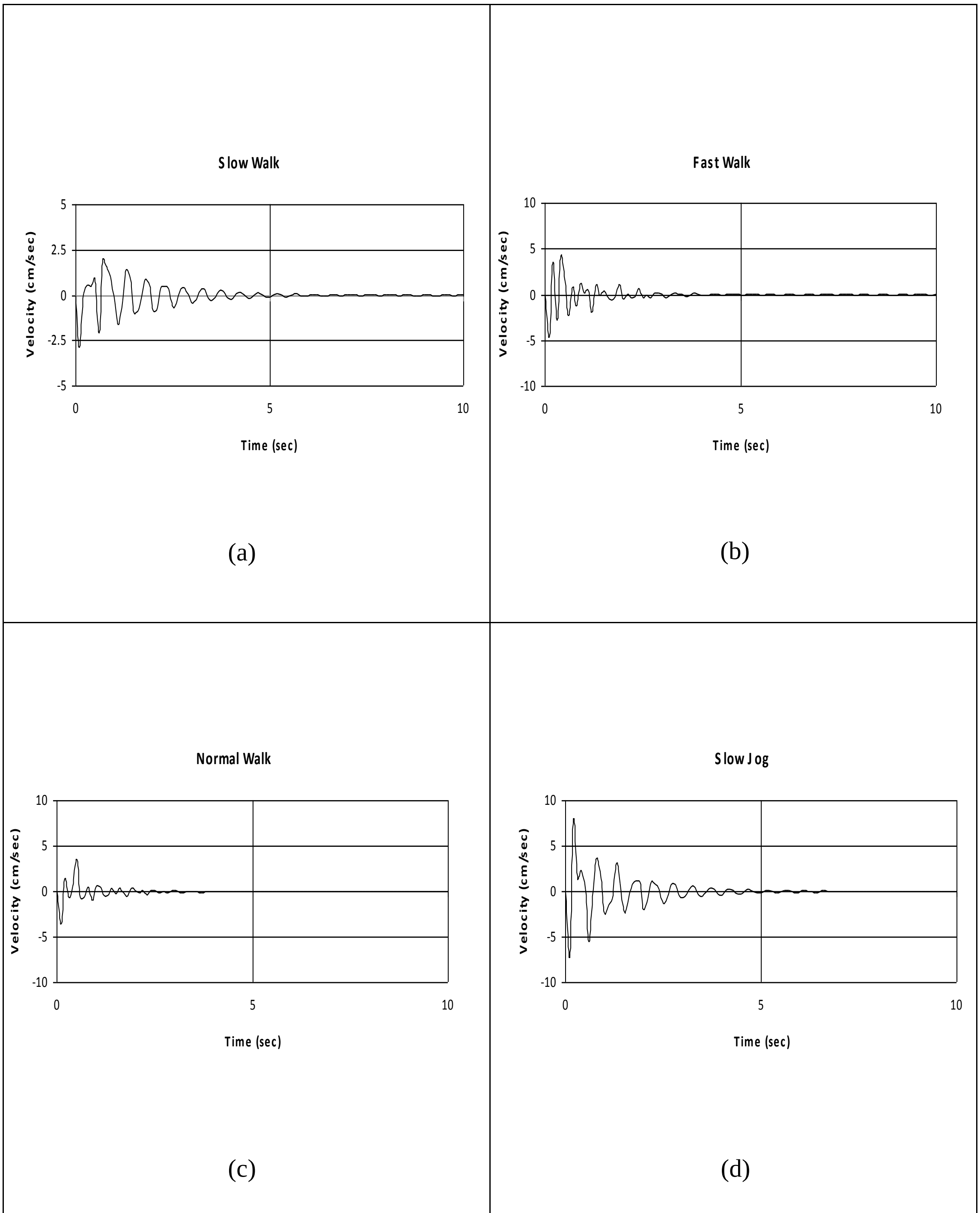
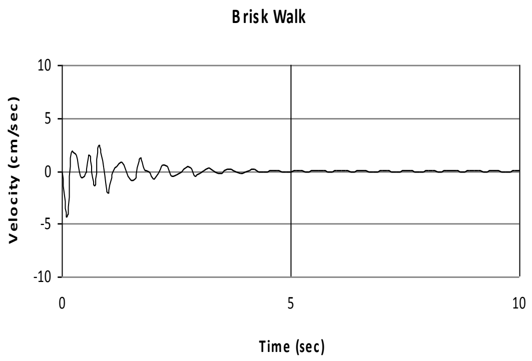
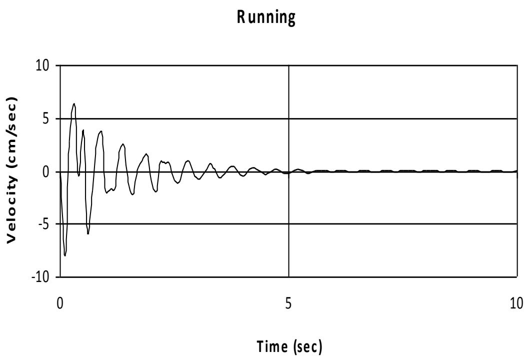


Figure 8.7: Velocity time history at the centre of the deck due to (a) slow walk, (b) fast walk, (c) normal walk, (d) slow jog.

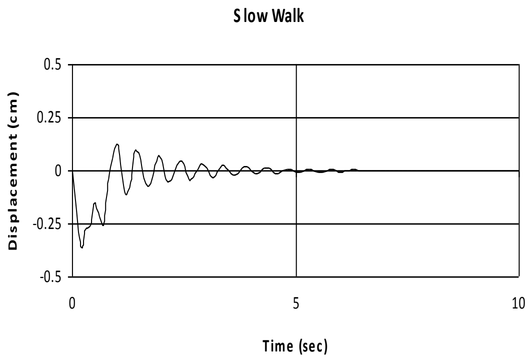


(e)

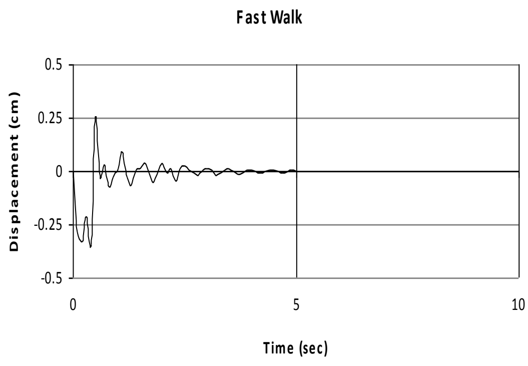


(f)

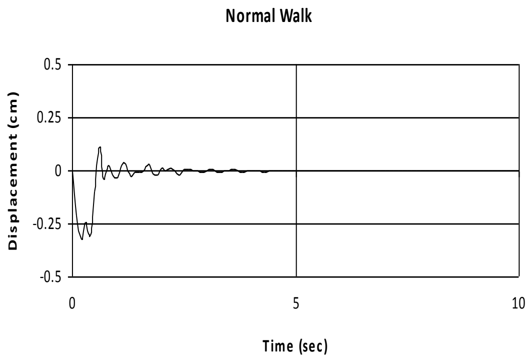
Figure 8.7: Velocity time history at the centre of the deck due to (e) brisk walk, (f) running.



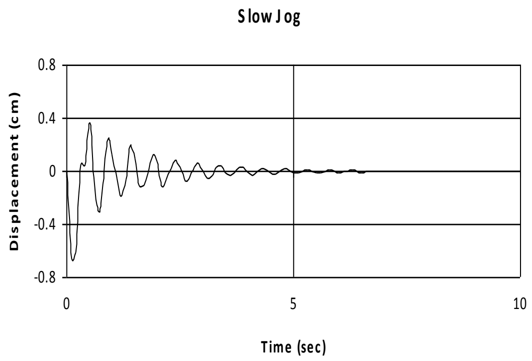
(a)



(b)

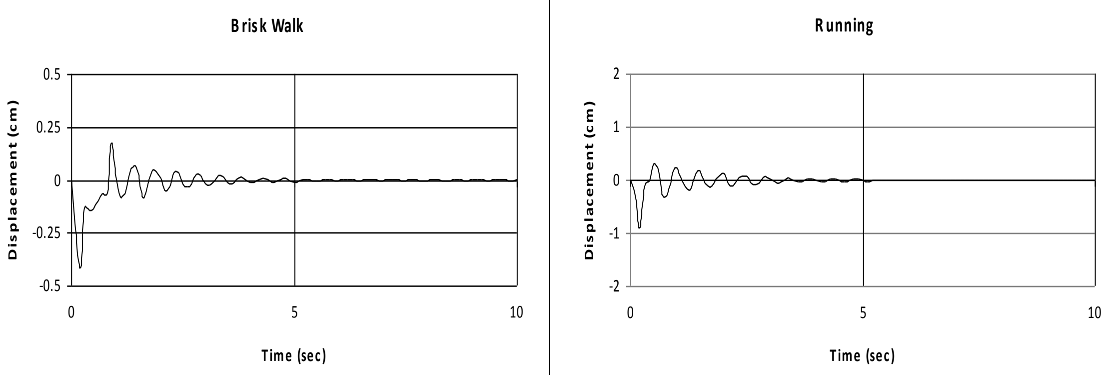


(c)



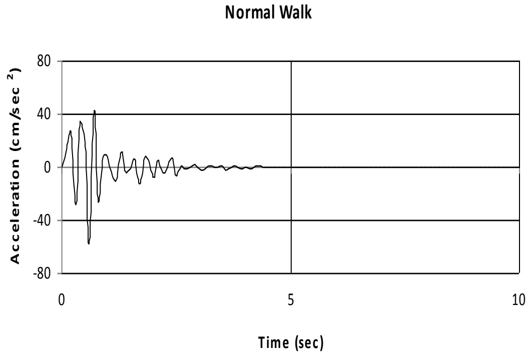
(d)

Figure 8.8: Displacement time history at the centre of the deck due to (a) slow walk, (b) fast walk, (c) normal walk, (d) slow jog.

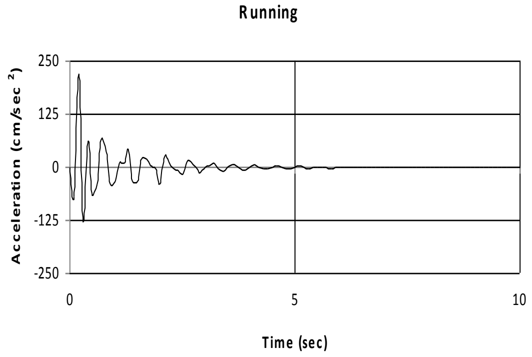


(e) (f)

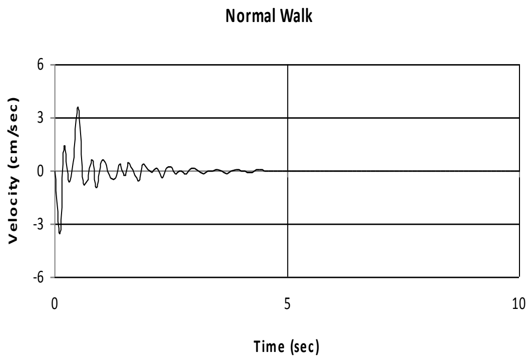
Figure 8.8: Displacement time history at the centre of the deck due to (e) brisk walk, (f) running.



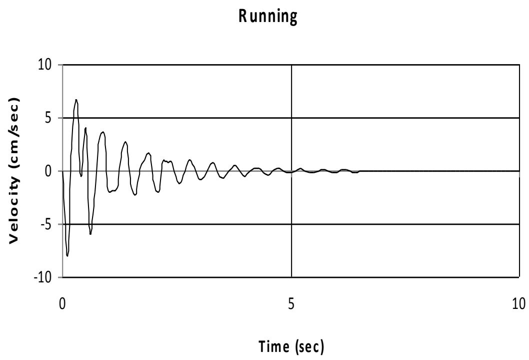
(a)



(b)

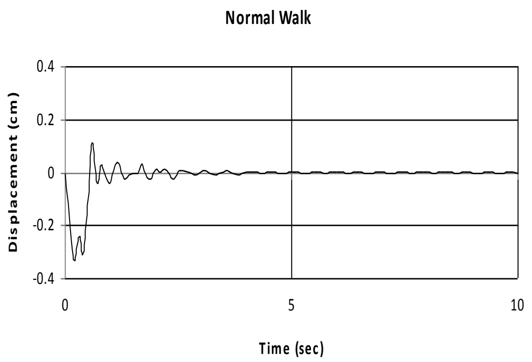


(c)

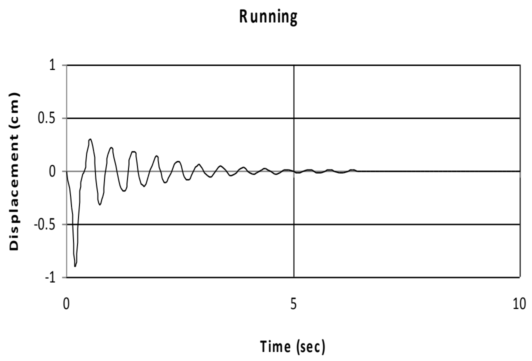


(d)

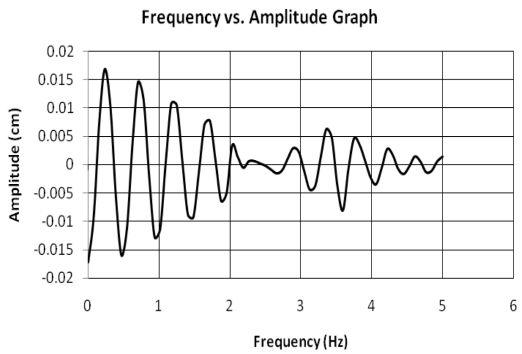
Figure 8.9: (a) Acceleration time history at the centre of the arch due to normal walk, (b) Acceleration time history at the centre of the arch due to running, (c) Velocity time history at the centre of the arch due to normal walk, (d) Velocity time history at the centre of the arch due to running.



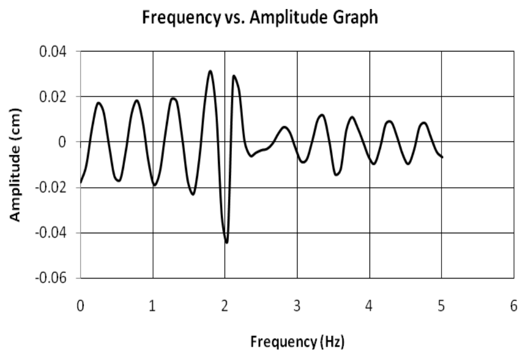
(e)



(f)



(g)



(h)

Figure 8.9: (e) Displacement time history at the centre of the arch due to normal walk, (f) Displacement time history at the centre of the arch due to running, (g) Frequency vs. amplitude graph at the centre of the deck due to normal walk, (h) Frequency vs. amplitude graph at the centre of the deck due to running.

It has been shown in Figure 8.6(a) to 8.6(f) that the acceleration responses of Footbridge-I model in the vertical direction. Only running and fast walk give higher responses. Velocity time history of different walking modes specially running mode gives higher response and it has been shown in Figure 8.7(a) to Figure 8.7(f). Higher displacement value has been shown in Figure 8.8(f). It is very much clear that walking modes affect the responses of footbridge models. Walking mode which has higher frequency gives higher response values.

8.2.2.3 Human perception

According to Leonard's and Smith's scales of human perception (Figure 4.8), crescent lake footbridge has been checked.

Table 8.3: Human perception of Footbridge-I model.

Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Amplitude (inch)	Human Perception (According to Leonard's and Smith's scale)
Curved arch Bridge (Option A)	Vertical	normal walk	2.1	0.13	Clearly Perceptible
		running	2.1	0.35	Annoying
		slow jog	2.1	0.26	Annoying
		slow walk	2.1	0.14	Clearly Perceptible
		brisk walk	2.1	0.16	Clearly Perceptible
		fast walk	2.1	0.14	Clearly Perceptible

Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Amplitude (inch)	Human Perception (According to Leonard's and Smith's scale)
Curved arch Bridge (Option B)	Vertical	normal walk	2.1	0.14	Clearly Perceptible
		running	2.1	0.34	Annoying
		slow jog	2.1	0.26	Annoying
		slow walk	2.1	0.26	Annoying
		brisk walk	2.1	0.15	Clearly Perceptible
		fast walk	2.1	0.16	Clearly Perceptible
Curved arch Bridge (Different options)- Option C	Vertical	normal walk	3.9	0.38	Unpleasant
		running	3.9	0.63	Unpleasant
		slow jog	3.9	0.56	Unpleasant
		slow walk	3.9	0.39	Annoying
		brisk walk	3.9	0.34	Annoying
		fast walk	3.9	0.39	Annoying
Curved arch Bridge (Different options)- Option D	Vertical	normal walk	1.3	0.51	Annoying
		running	1.3	0.91	Annoying
		slow jog	1.3	0.82	Annoying
		slow walk	1.3	0.52	Annoying
		brisk walk	1.3	0.49	Annoying
		fast walk	1.3	0.53	Annoying
Curved arch Bridge (Different options)- Option E	Vertical	normal walk	1.3	0.54	Annoying
		running	1.3	0.98	Annoying
		slow jog	1.3	0.86	Annoying
		slow walk	1.3	0.56	Annoying
		brisk walk	1.3	0.36	Annoying
		fast walk	1.3	0.56	Annoying

Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Amplitude (inch)	Human Perception (According to Leonard's and Smith's scale)
Curved arch Bridge (Different options)-Option F	Vertical	normal walk	1.7	0.30	Annoying
		running	1.7	0.57	Annoying
		slow jog	1.7	0.59	Annoying
		slow walk	1.7	0.32	Annoying
		brisk walk	1.7	0.29	Annoying
		fast walk	1.7	0.30	Annoying
Curved arch Bridge (Different options)-Option G	Vertical	normal walk	2.1	0.14	Clearly Perceptible
		running	2.1	0.34	Annoying
		slow jog	2.1	0.27	Annoying
		slow walk	2.1	0.15	Clearly Perceptible
		brisk walk	2.1	0.16	Annoying
		fast walk	2.1	0.15	Clearly Perceptible
Footbridge with Vertical Suspension Cables-Option H	Vertical	normal walk	10.8	0.02	Unpleasant
		running	10.8	0.04	Intolerable
		slow jog	10.8	0.05	Intolerable
		slow walk	10.8	0.02	Intolerable
		brisk walk	10.8	0.02	Unpleasant
		fast walk	10.8	0.02	Unpleasant
Footbridge with Vertical Suspension Cables - Option I	Vertical	normal walk	4.3	0.15	Unpleasant
		running	4.3	0.33	Unpleasant
		slow jog	4.3	0.40	Unpleasant
		slow walk	4.3	0.14	Annoying
		brisk walk	4.3	0.17	Unpleasant
		fast walk	4.3	0.20	Unpleasant

Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Amplitude (inch)	Human Perception (According to Leonard's and Smith's scale)
Straight arch Footbridge-Option J	Vertical	normal walk	2.2	0.20	Annoying
		running	2.2	0.15	Annoying
		slow jog	2.2	0.24	Annoying
		slow walk	2.2	0.07	Clearly Perceptible
		brisk walk	2.2	0.02	Annoying
		fast walk	2.2	0.11	Annoying
Straight arch Footbridge-Option K	Vertical	normal walk	2.2	0.09	Clearly Perceptible
		running	2.2	0.16	Annoying
		slow jog	2.2	0.25	Annoying
		slow walk	2.2	0.07	Clearly Perceptible
		brisk walk	2.2	0.11	Annoying
		fast walk	2.2	0.12	Annoying

8.2.3 Dynamic behaviour due to human induced lateral vibration

This section focuses on dynamic loads especially lateral loads (Fujino et al. 1993) induced by a single pedestrian. Footbridge vibration is mainly due to vertical forces, but recently there is some research study (Dallard et al. 2001b, Fujino et al 1993) which has shown that a significant amount of vibration is also caused by lateral loads due to pedestrians.

8.2.3.1 Dynamic stability check

According to Eurocode and ISO 10137, actual footbridge model has been analyzed. Though these acceleration values exceed the required value, vibration is still minimum.

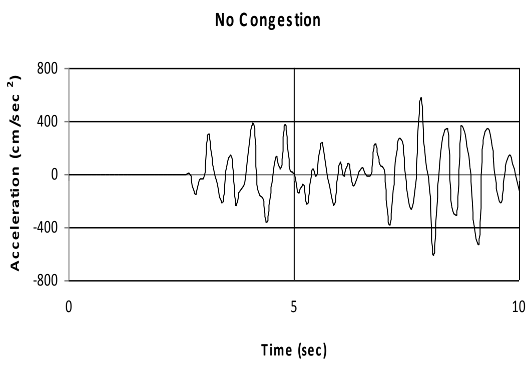
Table 8.4: Dynamic acceptability of Footbridge-I model.

Bridge Model	Loading Direction	Loading condition	Frequency (Hz)	RMS Acceleration (m/s^2)	Dynamic Acceptability (According to Eurocode and ISO 10137)
Curved arched Bridge (Option A)	Lateral	Not congested	1.6	19.24	Unaccepatble
		Congested	1.6	25.30	Unaccepatble
		Densely congested	1.6	46.47	Unaccepatble

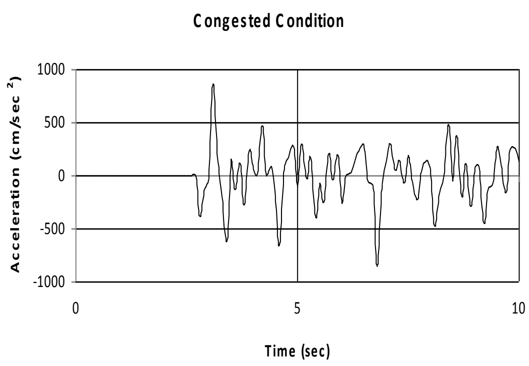
8.2.3.2 Dynamic response due to single pedestrian

A time history analysis of the footbridge using three different types of lateral loading has been used. These loads are taken from non congested, congested and densely congested condition (Figure 7.10).

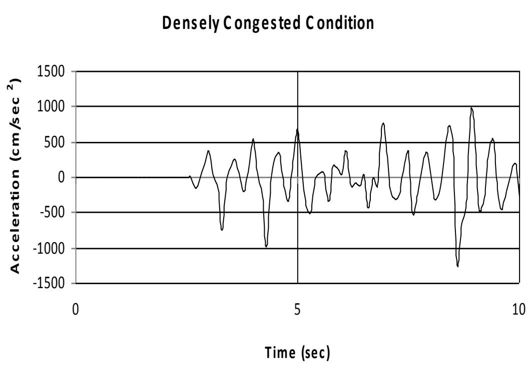
In this section, different types of time history acceleration, velocity and displacement graphs have been shown.



(a)

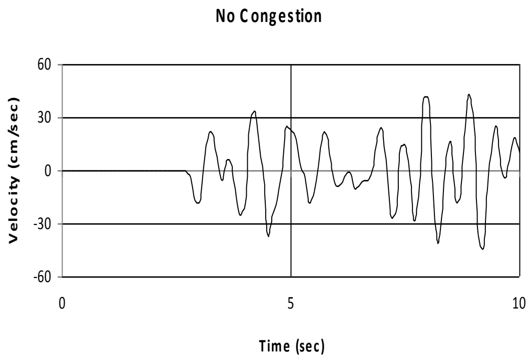


(b)

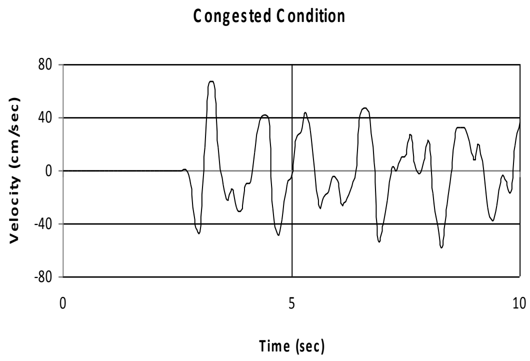


(c)

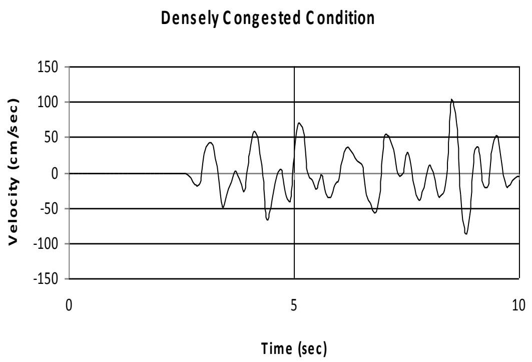
Figure 8.10: Acceleration time history at the centre of the deck due to lateral load in the (a) non congested condition, (b) congested condition, (c) densely congested condition.



(a)

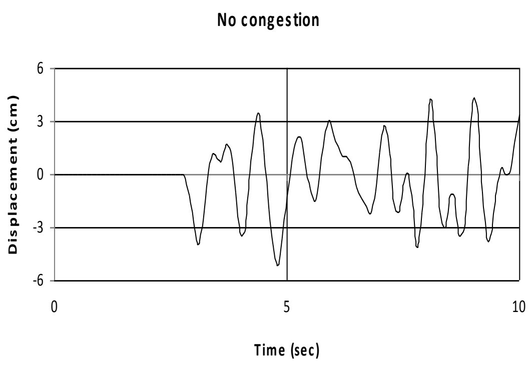


(b)

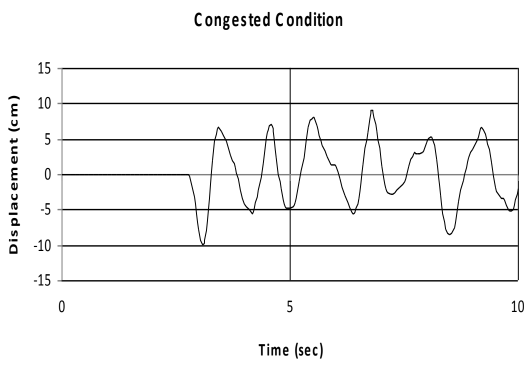


(c)

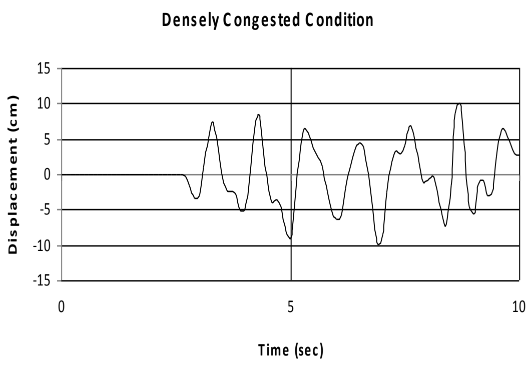
Figure 8.11: Velocity time history at the centre of the deck due to lateral load in the (a) non congested condition, (b) congested condition, (c) densely congested condition.



(a)

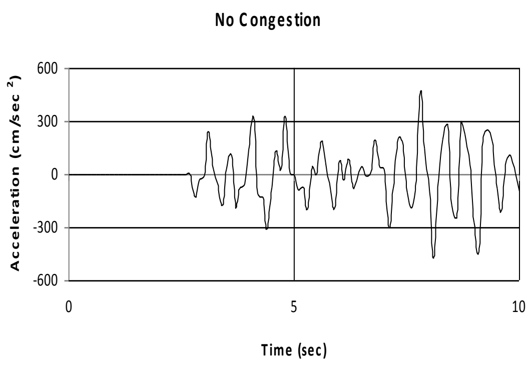


(b)

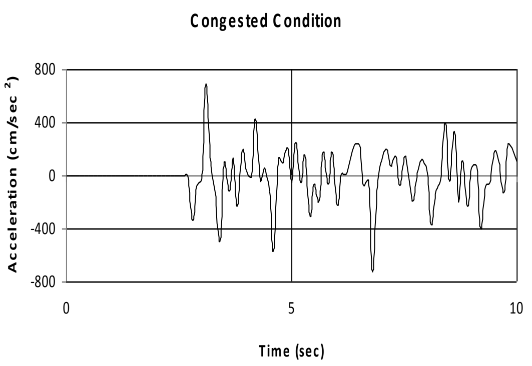


(c)

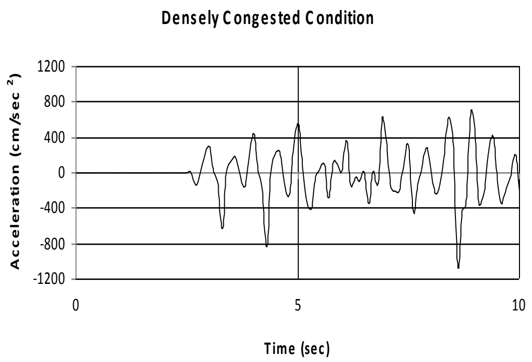
Figure 8.12: Displacement time history at the centre of the deck due to lateral load in the (a) non congested condition, (b) congested condition, (c) densely congested condition.



(a)

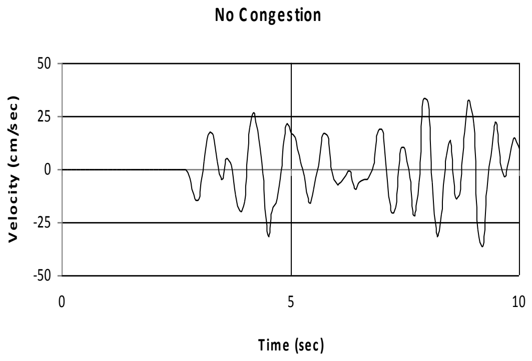


(b)

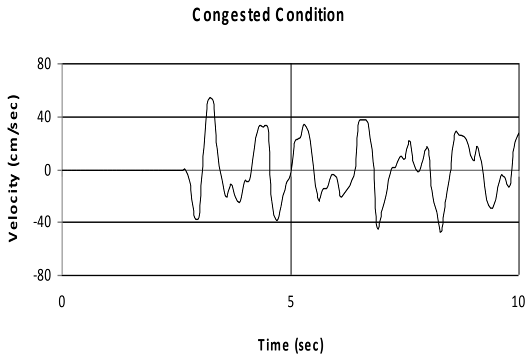


(c)

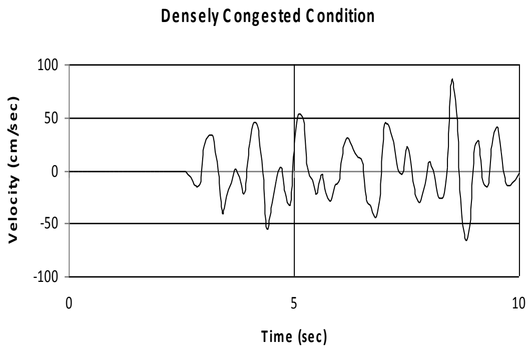
Figure 8.13: Acceleration time history at the centre of the arch due to lateral load in the (a) non congested condition, (b)congested condition, (c) densely congested condition.



(a)

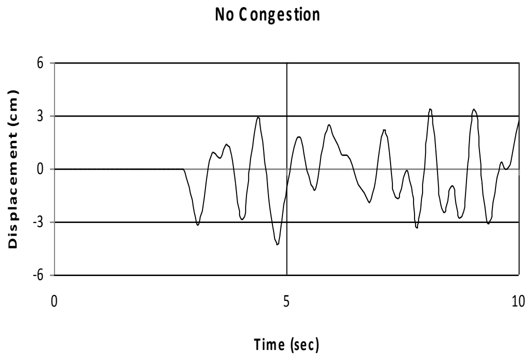


(b)

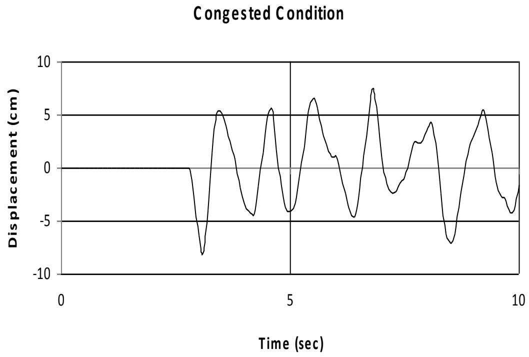


(c)

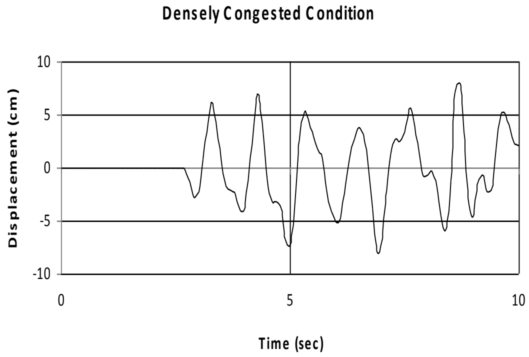
Figure 8.14: Velocity time history at the centre of the arch due to lateral load in the (a) non congested condition, (b) congested condition, (c) densely congested condition.



(a)

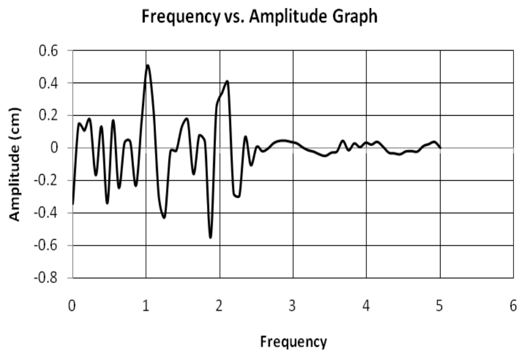


(b)

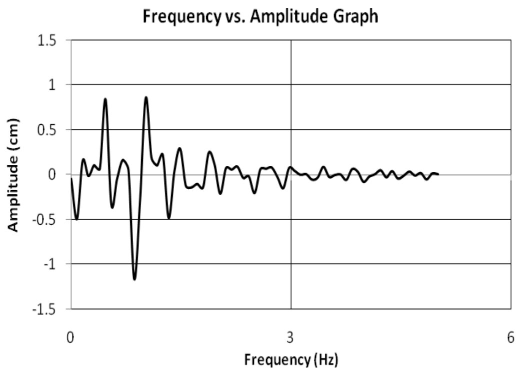


(c)

Figure 8.15: Displacement time history at the centre of the arch due to lateral load in the (a) non congested condition, (b) congested condition, (c) densely congested condition.



(a)



(b)

Figure 8.16: Frequency vs. amplitude graph at the centre of the deck due to lateral load in the (a)non congested condition, (b) densely congested condition.

It has been shown in Figure 8.10(a) to Figure 8.15(c) that pedestrian numbers affect the responses due to different loading conditions. Densely loaded conditions give higher values. But initial value is zero and then it increases after around 2.5 to 3 sec. Excitation period of the arch is much more than that of the deck.

8.2.3.3 Human perception

According to Nakamura's scale, human perception of the actual model of the footbridge has been analyzed.

Table 8.5: Human perception of Footbridge-I model.

Bridge Model	Loading Direction	Loading condition	Bridge Frequency (Hz)	Amplitude (mm)	Human Perception (According to Nakamura's scale)
Curved arched Bridge (Option A)	Lateral	Not congested	1.6	37.43	Reasonably Accepted
		Congested	1.6	81.28	Annoying
		Densely congested	1.6	75.53	Reasonably Accepted

8.3 Synchronization of Human Walking Observed in a Congested Condition

It is now widely accepted that people walking in a crowd, because of the limited space on the bridge deck and the possibility that they can see each other, would subconsciously synchronize their steps. In this section synchronization of 5 and 10 people have been investigated (Table 8.6).

Table 8.6: Dynamic stability of Footbridge-I in synchronized condition.

No. of Persons Synchronized	Loading Direction	Time History Load Name	Freq. (Hz)	Peak Acceleration (m/s^2)	RMS Acceleration (m/s^2)	Dynamic Acceptability (According to Eurocode and ISO 10137)
5	Vertical	normal walk	2.074	-2.920	-4.13	Unacceptable
		running	2.074	11.061	15.64	Unacceptable
		slow jog	2.074	-6.661	-9.42	Unacceptable
		slow walk	2.074	2.071	2.93	Unacceptable
		brisk walk	2.074	3.565	5.04	Unacceptable
		fast walk	2.074	8.435	11.93	Unacceptable
	Lateral	Not congested	1.64	68.038	96.22	Unacceptable
		Congested	1.64	89.470	126.53	Unacceptable
		Densely congested	1.64	164.311	232.36	Unacceptable
10	Vertical	normal walk	2.074	-5.841	-8.26	Unacceptable
		running	2.074	22.122	31.28	Unacceptable
		slow jog	2.074	-13.322	-18.84	Unacceptable
		slow walk	2.074	4.142	5.86	Unacceptable
		brisk walk	2.074	7.129	10.08	Unacceptable
		fast walk	2.074	16.870	23.86	Unacceptable
	Lateral	Not congested	1.64	136.076	192.44	Unacceptable
		Congested	1.64	178.941	253.06	Unacceptable
		Densely congested	1.64	328.622	464.74	Unacceptable

8.4 Dynamic Behaviour of Footbridge-II

8.4.1 Eigenvalue analysis

Eigenvalue analysis of Footbridge-II has been analyzed and found satisfactory. The first vertical and horizontal frequencies partially fulfilled the minimum requirement of the codes (Figure 8.17).

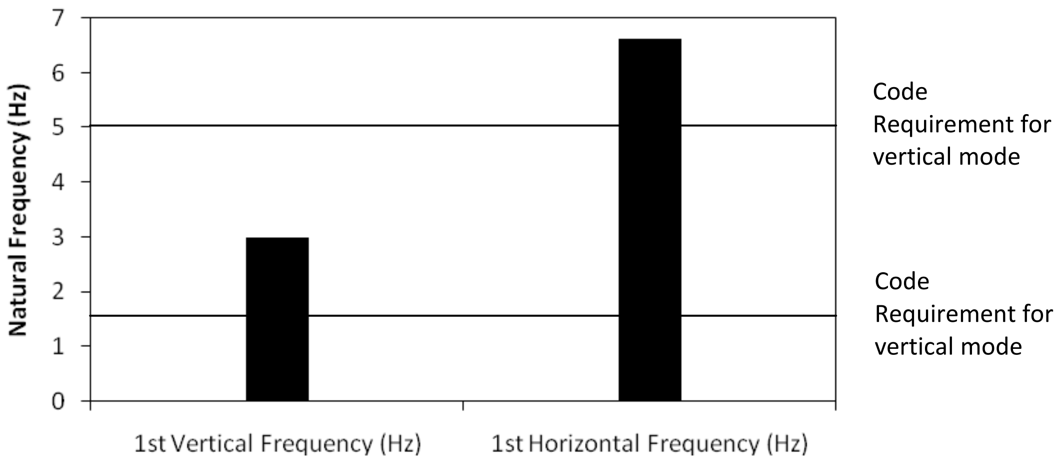
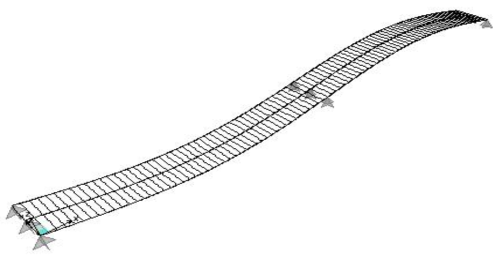
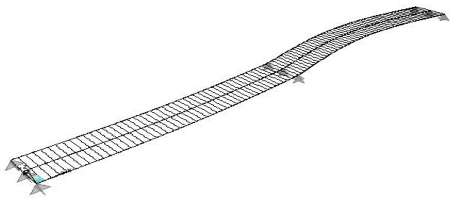


Figure 8.17: Eigen frequencies of Footbridge-II determined for simplified actual model and presented against the code requirements.



(a)



(b)

Figure 8.18: (a) First Vertical mode of the Footbridge-II (simplified model), (b) First Lateral mode of Footbridge-II (simplified model).

Footbridge-II has two spans. Deflection patterns are different for both spans in the vertical and lateral mode shapes.

8.4.2 Dynamic behaviour due to human induced vertical vibration

8.4.2.1 Dynamic stability check

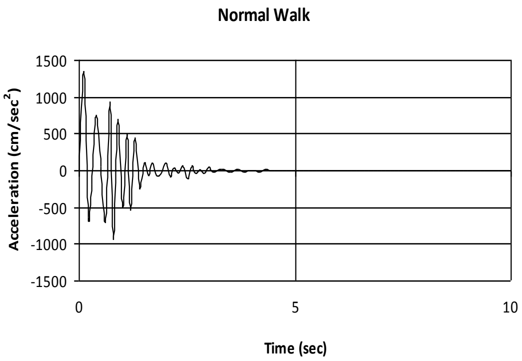
According to different standards, Footbridge-II has been analysed and checked for different loading conditions (Wheeler 1982) of a single pedestrian (Table 8.7)

Table 8.7: Dynamic acceptability of Footbridge-II model.

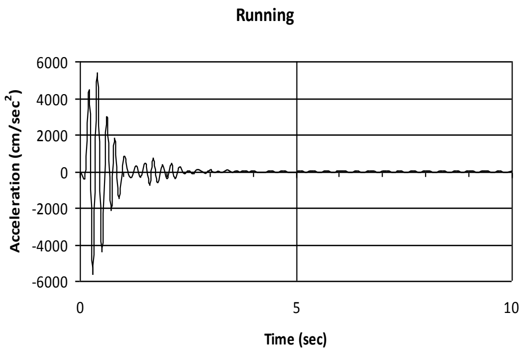
Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Peak Acceleration (m/s^2)	Dynamic Acceptability
Footbridge-II	Vertical	normal walk	3.0	13.82	Unacceptable
		running	3.0	-58.38	Unacceptable
		slow jog	3.0	51.69	Unacceptable
		slow walk	3.0	-8.99	Unacceptable
		brisk walk	3.0	-16.30	Unacceptable
		fast walk	3.0	47.56	Unacceptable

8.4.2.2 Dynamic response due to single pedestrian

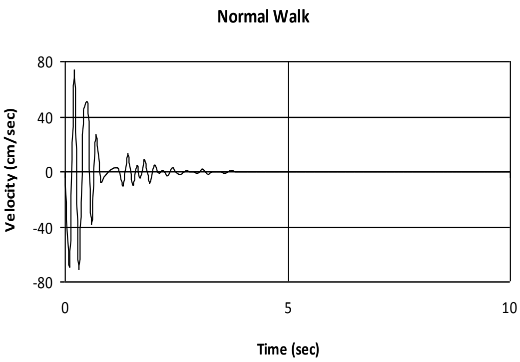
A detailed time history analysis of the Footbridge-II has been performed. The results of the time history analysis of the footbridge have been shown in the Figure 8.19 to Figure 8.20.



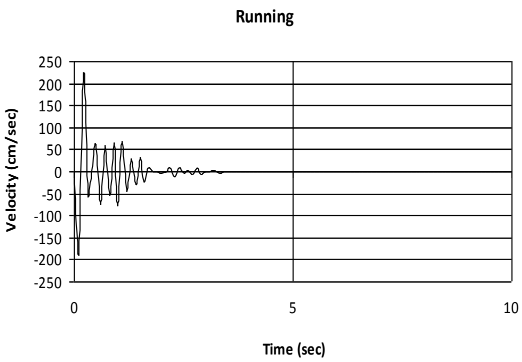
(a)



(b)

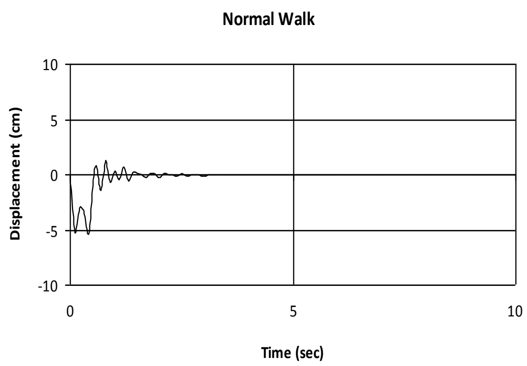


(c)

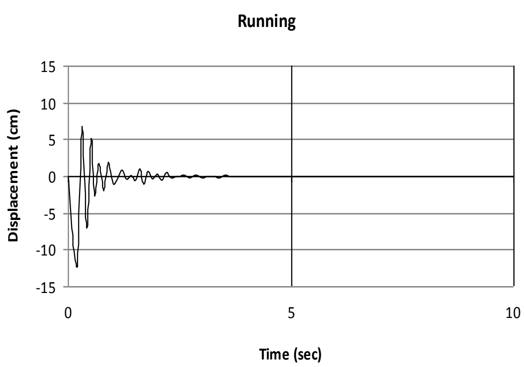


(d)

Figure 8.19: (a) Acceleration time history at the centre of the deck due to normal walk, (b) Acceleration time history at the centre of the deck due to running, (c) Velocity time history at the centre of the deck due to normal walk, (d) Velocity time history at the centre of the deck due to running.

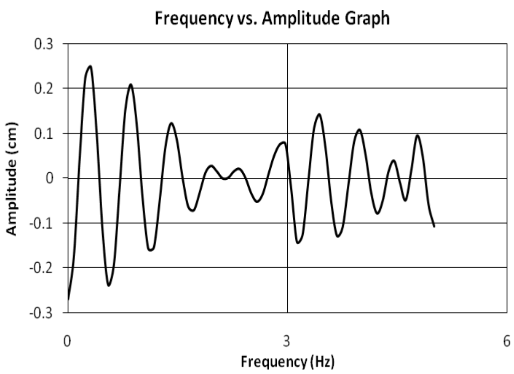


(e)

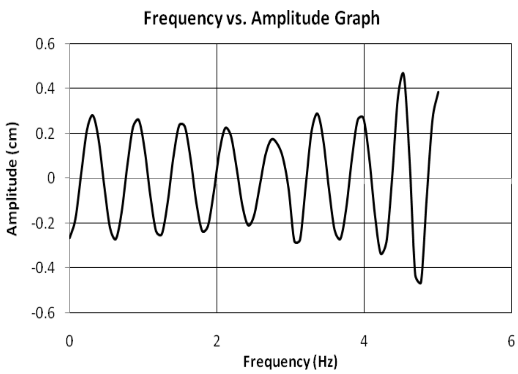


(f)

Figure 8.19: Displacement time history at the centre of the deck due to (e) normal walk, (f) running.



(a)



(f)

Figure 8.20: Frequency vs. amplitude graph at the centre of the deck due to (a) normal walk, (b) running.

It has been shown in Figure 8.19(a) to Figure 8.19(f) that different walking mode changes the footbridge time history responses. Walking pattern with higher frequency (running) induces higher responses.

8.4.2.3 Human perception

According to Leonard's and Smith's scales of human perception (Figure 4.8), Footbridge-II has been checked (Table 8.8).

Table 8.8: Human perception of Footbridge-II.

Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Amplitude (inch)	Human Perception (According to Leonard's and Smith's scale)
Footbridge-II	Vertical	normal walk	3.0	-2.25	Unpleasant
		running	3.0	-6.00	Intolerable
		slow jog	3.0	-6.71	Intolerable
		slow walk	3.0	-2.45	Intolerable
		brisk walk	3.0	-2.53	Intolerable
		fast walk	3.0	-2.98	Intolerable

8.4.3 Dynamic behaviour due to human induced lateral vibration

8.4.3.1 Dynamic stability check

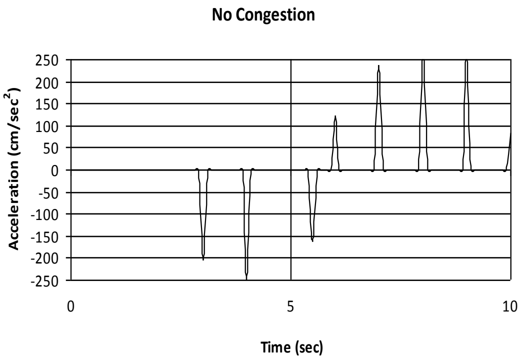
According to different standards, footbridge has been analyzed for a single pedestrian movement.

Table 8.9: Dynamic stability of Footbridge-II.

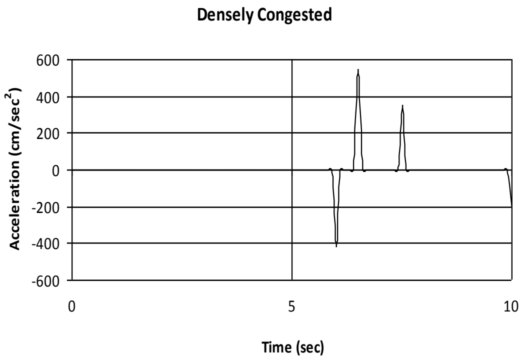
Bridge Model	Loading Direction	Time History Load Name	Frequency (Hz)	Peak Acceleration (m/s^2)	RMS Acceleration (m/s^2)	Dynamic Acceptability (According to Eurocode and ISO 10137)
Footbridge-II	Lateral	Not congested	High (>10)	2.67	3.78	Acceptable
		Congested	High (>10)	-6.12	-8.66	Acceptable
		Densely congested	High (>10)	5.51	7.79	Acceptable

8.4.3.2 Dynamic response due to single pedestrian

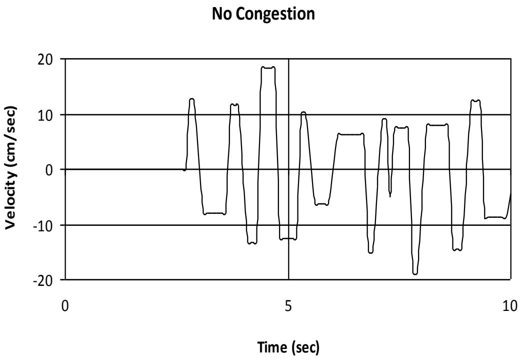
The graphical representation of the time history analysis of the Footbridge-II has been shown in Figure 8.21 to 8.22.



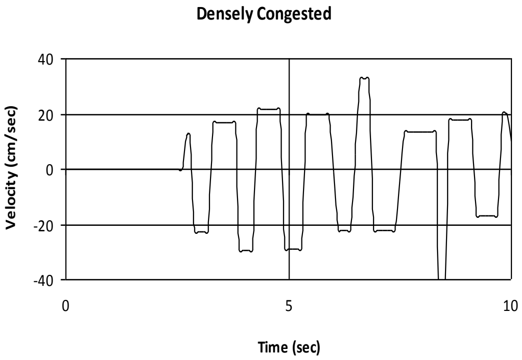
(a)



(b)

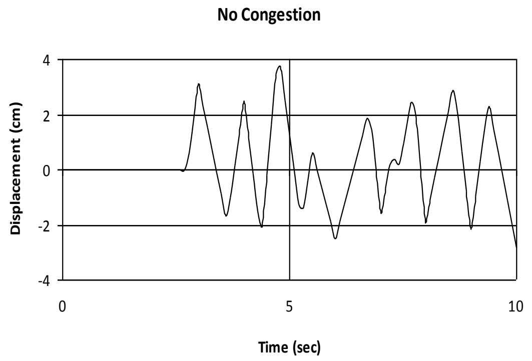


(c)

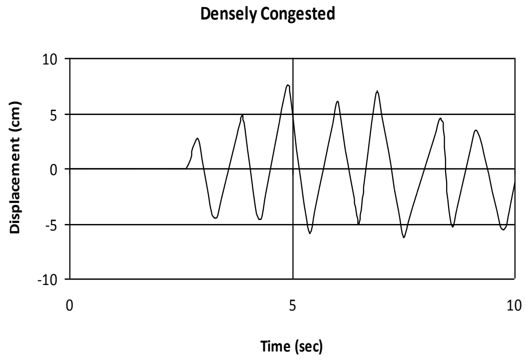


(d)

Figure 8.21: (a) Acceleration time history at the centre of the deck due to lateral load in the non congested condition, (b) Acceleration time history at the centre of the deck due to lateral load in the densely congested condition, (c) Velocity time history at the centre of the deck due to lateral load in the non congested condition, (d) Velocity time history at the centre of the deck due to lateral load in the densely congested condition.

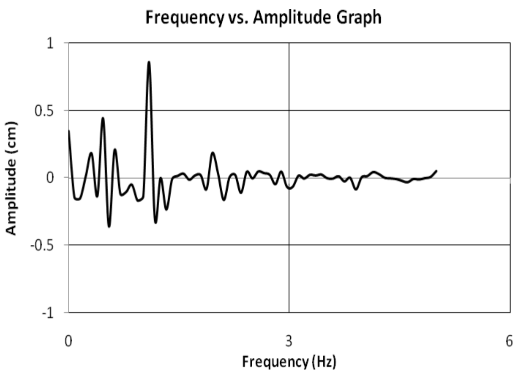


(e)

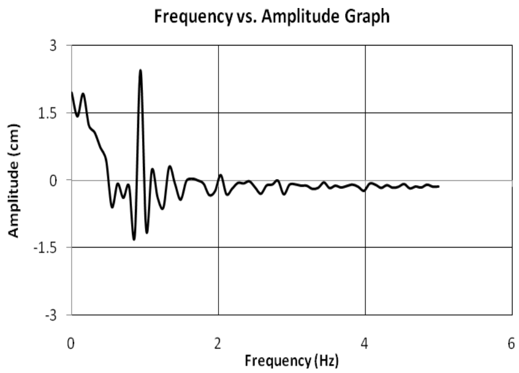


(f)

Figure 8.21: Displacement time history at the centre of the deck due to lateral load in the (e) non congested condition, (f) densely congested condition.



(a)



(f)

Figure 8.22: Frequency vs. amplitude graph at the centre of the deck due to the (a) no congested condition, (b) densely congested condition.

Responses start after few seconds and pedestrian density condition affects response values (Figure 8.21(a) to Figure 8.21(f)). Congested situation induces higher responses.

8.4.3.3 Human perception

Human perception level has been analyzed and tabulated in Table 8.10.

Table 8.10: Human perception of Footbridge-II.

Bridge Model	Loading Direction	Loading Type	Bridge Frequency (Hz)	Amplitude (mm)	Human Perception (According to Nakamura's scale)
Footbridge-II	Lateral	Not congested	High (>10)	37.44	Tolerable
		Congested	High (>10)	81.28	Tolerable
		Densely congested	High (>10)	75.53	Reasonably Tolerable

8.5 Synchronization Effect of Footbridge-II

Synchronization Behaviour of Footbridge-II has been investigated.

Table 8.11: Dynamic stability of Footbridge-II.

Bridge Model	No. of Persons Synchronized	Loading Direction	Time History Load Name	Freq. (Hz)	Peak Acceleration (m/s^2)	RMS Acceleration (m/s^2)	Dynamic Acceptability (According to Eurocode and ISO 10137)
Footbridge-II	5	Vertical	normal walk	3.0	69.08	97.69	Unacceptable
			running	3.0	-176.74	-249.95	Unacceptable
			slow jog	3.0	258.48	365.55	Unacceptable
			slow walk	3.0	-44.99	-63.63	Unacceptable
			brisk walk	3.0	-81.52	-115.28	Unacceptable
			fast walk	3.0	237.86	336.38	Unacceptable
		Lateral	Not congested	High (>10)	13.36	18.91	Unacceptable
			Congested	High (>10)	-30.63	-43.32	Unacceptable
			Densely congested	High (>10)	27.56	38.98	Unacceptable
Footbridge-II	10	Vertical	normal walk	3.0	138.16	195.39	Unacceptable
			running	3.0	-583.82	-825.64	Unacceptable
			slow jog	3.0	516.97	731.10	Unacceptable
			slow walk	3.0	-89.98	-127.25	Unacceptable
			brisk walk	3.0	-163.04	-230.57	Unacceptable
			fast walk	3.0	-472.06	-667.59	Unacceptable
		Lateral	Not congested	High (>10)	26.74	37.81	Unacceptable
			Congested	High (>10)	-61.26	-86.64	Unacceptable
			Densely congested	High (>10)	55.14	77.97	Unacceptable

8.6 Comparison of Codes

This section focuses on dynamic loads proposed by different codes especially BS 5400, ISO 10137 and Bro 2004 and the effects of these loads on structures.

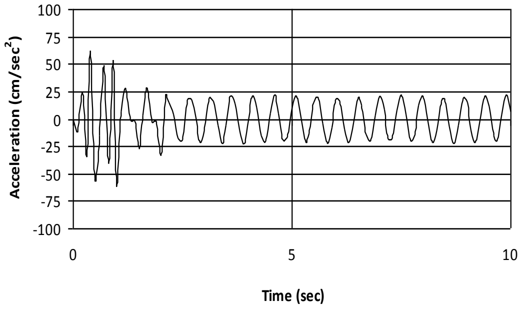
8.6.1 Dynamic behaviour of Footbridge-I model

According to different standards (Figure 5.4), Footbridge-I has been checked by using different loading conditions.

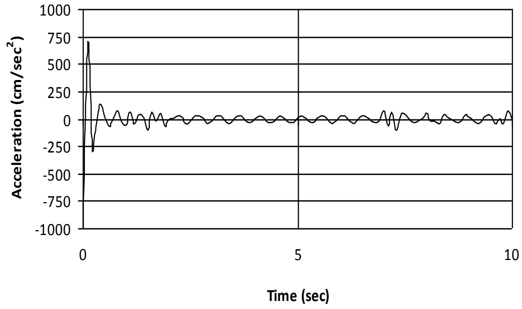
Table 8.12: Dynamic acceptability of Footbridge-I model.

Bridge Model	Loading Direction	Loads proposed by Codes	Bridge Frequency (Hz)	Peak Acceleration (m/s^2)	Dynamic Acceptability
Footbridge-I	Vertical	BS 5400	2.1	0.62	Acceptable
		ISO 10137	2.1	6.98	Unacceptable
		Bro 2004	2.1	9.42	Unacceptable
	Lateral	ISO 10137	1.6	0.14	Acceptable

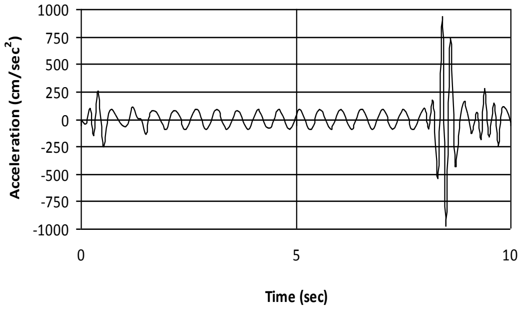
A detailed time history analysis has been performed of Footbridge-I. Here typical force patterns from different codes were used. The main focus of this analysis is to evaluate the serviceability requirement of footbridges.



(a)

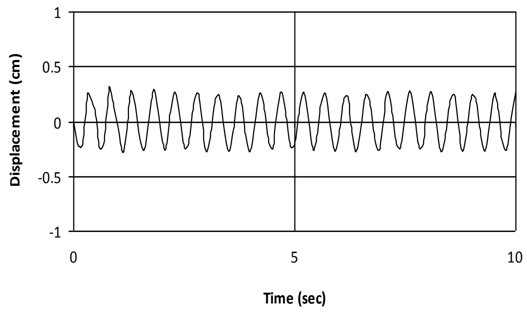


(b)

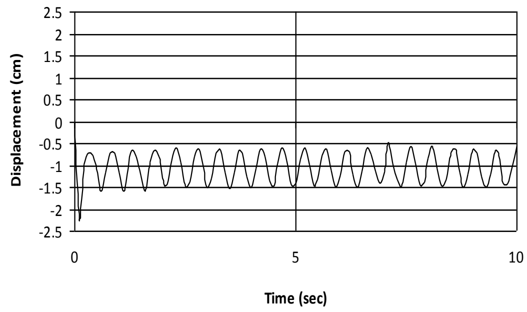


(c)

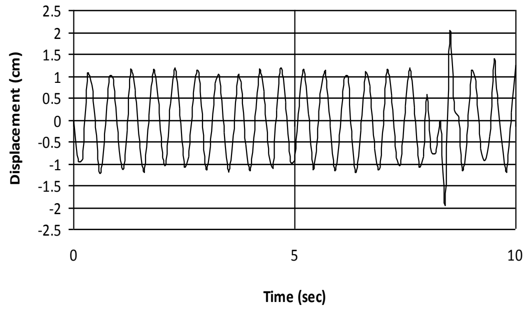
Figure 8.23: Acceleration time history at the centre of the deck due to vertical load proposed by (a) BS 5400, (b) ISO 10137, (c) Bro 2004.



(a)

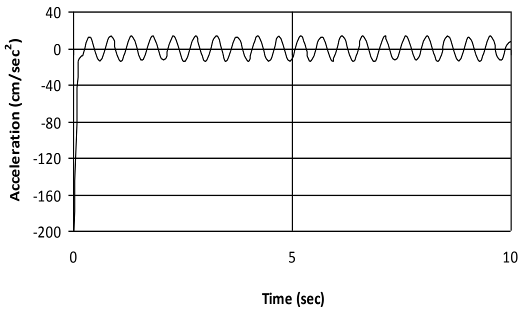


(b)

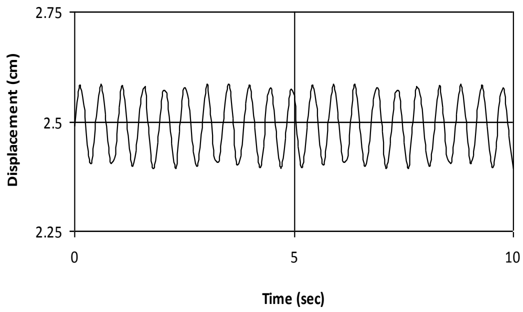


(c)

Figure 8.24: Displacement time history at the centre of the deck due to vertical load proposed by (a) BS 5400, (b) ISO 10137, (c) Bro 2004.



(a)



(b)

Figure 8.25: (a) Acceleration time history at the centre of the deck due to lateral load proposed by ISO 10137, (b) Displacement time history at the centre of the deck due to lateral load proposed by ISO 10137.

BS5400 load produces less acceleration responses than that of ISO 10137 and Bro 2004 in the vertical direction (Figure 8.23(a) to Figure 8.23(c)). Bro 2004 response graph shows that acceleration peak value is higher after some time. Bro 2004 also gives higher responses. Acceleration response starts with a large value and then it decreases much and produces quite regular value for some time (Figure 8.25(a)). Displacement response is not higher in Figure 8.25(b)

8.6.2 Dynamic behaviour of Footbridge-II model

Footbridge-II has been checked by using different loading conditions. In Option B model, extra cross tie and cross beam have been installed.

Table 8.13: Different options for Footbridge-II model

Bridge Model	Option	Total Deck Width	Deck Bracings		Deck Railing		Extra Cross girder	
		3.00 m	Yes	No	Yes	No	Yes	No
Footbridge-II	A	√		√		√		√
Footbridge-II	B	√	√			√	√	

Table 8.14: Dynamic acceptability of Footbridge-II model.

Bridge Model	Loading Direction	Loads proposed by Codes	Bridge Frequency (Hz)	Peak Acceleration (m/s^2)	Dynamic Acceptability
Footbridge-II (Option A) Actual Model	Vertical	BS 5400 (British Standard 2001)	3.0	1.38	Acceptable
		ISO 10137 (ISO 2005)	3.0	14.47	Unacceptable
		Bro 2004 (Swedish Standards 2004)	3.0	16.40	Unacceptable
	Lateral	ISO 10137	6.6	0.24	Acceptable
Footbridge-II (Option B)	Vertical	BS 5400	2.1	0.78	Acceptable
		ISO 10137	2.1	3.46	Unacceptable
		Bro 2004	2.1	7.45	Unacceptable
	Lateral	ISO 10137	25.9	0.23	Acceptable

8.7 Results and Possible Solutions

From various time history analysis and eigen value analysis, it is very much clear that lighter bridge structures give higher response to dynamic actions.

When compared between the cases, it is evident that the option A of Footbridge-I gives the possible performance in the light of the code. But here arch of the footbridge is more susceptible to vibrations. Inserting new ties and cross ties in between the arches is the best solutions to fulfill the vibration serviceability requirement. But it is more vulnerable to the aesthetic point of view. Increasing of the lateral stiffness generally increases the performance of the footbridges.

Comparing both footbridges, it is very much clear that Footbridge-I is more serviceable against the vertical and horizontal vibration. Again, Footbridge-II is relatively less serviceable against the horizontal vibration loads.

From time history vibration analysis of the footbridges, the following steps should be taken to minimize the vibration problem of the footbridge:

- To further increase the lateral stiffness of the Footbridge-I, ties, cross ties can be installed in the deck and arch.

- Increasing the section of the ties and other members which is already installed.

- To further increase the vertical stiffness of the Footbridge-II, ties and cross beams should be installed.

- Since both bridges have failed to withstand the synchronization effect, a more refined design method should be used.

- Changing the sections of the members of the Footbridge-II will be a better option.

Chapter 9

CONCLUSION

9.1 General

The main focus of this thesis has been on the vertical and horizontal forces that pedestrians impart to a footbridge and how these loads can be modeled for dynamic design of footbridges. The work has been divided into seven subtasks:

- A literature study of dynamic loads induced by pedestrians has been performed.

- Design criteria and load models proposed by different widely used standards like BS 5400, Euro Code and ISO 10137 have been introduced and a comparison has been made.

- Development of a finite element model of a cable supported footbridge for dynamic analysis.

- Dynamic response of an as-built structure with the analytical predictions through eigenvalue analysis has been compared.

- Dynamic response of the cable supported footbridge when subjected to real dynamic loading events as available in different published literatures has been computed.

- The optimization of structural system to investigate the effect of different stiffening systems on the vibration modes and the fundamental natural frequencies of the arch-deck system using the developed finite element model has been conducted.

- Available solutions to vibration problems and improvements of design procedures have been studied.

The next section will discuss about this.

9.2 Observations from Literature Review

Vibration measurement and serviceability assessment of two different footbridges have been conducted in this thesis. Effect of different walking modes and density of pedestrian movement have been studied. Actually, pedestrians induce both vertical and horizontal dynamic loads on the structure they traverse. The frequency range of the vertical force is 1.4 – 2.4 Hz and 0.7 – 1.2 Hz for the horizontal force. If pedestrian frequency matches with the natural frequency of the footbridge, it will give the resonance condition.

Apart from a single person walking, a group of pedestrians walking at the same speed to maintain the group consistency are a very frequent load type on footbridges. In this thesis, synchronization effects of footbridges in both vertical and lateral condition are also studied. Different responses of different vibration modes have been investigated.

Human perception of footbridges has been studied. How the source of vibration of the footbridge structure affects the same receiver of the footbridges has also been investigated.

9.3 Observations from Study

In this study, different vertical and lateral walking modes have been imposed on structures. Vertical load models proposed by Wheeler (1982) and lateral loads proposed by Fujino et al. (1993) are studied and also different load models from different standards like BS 5400, ISO 10137 and Bro 2004 are thoroughly investigated.

Existing vibration limits presented in standards are most probably sufficient to prevent vertical synchronization between structure and pedestrians. However, observations indicate that horizontal synchronization can start when the amplitude of the footbridge vibration is only a few millimeters.

The British standard BS 5400 requires a check of vibration serviceability in both vertical and horizontal directions. However, it only proposes a load model and a design criterion for vertical vibrations. The load modeling and the evaluation of a design criterion for horizontal vibrations are left to the designer.

The standard ISO 10137 proposes load models for calculation of vertical and horizontal vibrations due to one pedestrian. It also proposes design criteria for vertical and horizontal vibrations. It does not, however, take into account the phenomenon of pedestrian synchronization.

Eurocode proposes load models for both vertical and horizontal loads only for simplified structures. For more complex structures, the modeling of pedestrian loads is left to the designer. Eurocode proposes frequency independent maximum acceleration limits both for vertical and horizontal vibrations.

The load models proposed by the above mentioned standards are all based on the assumptions that pedestrian loads can be approximated as periodic loads. However, it is not perfectly periodic and it is not shown in the standards. They also seem to be incapable of predicting structures sensitivity to excessive horizontal vibrations due to a crowd of pedestrians.

Dynamic analysis of the Footbridge-I according to BS 5400 showed good serviceability in the vertical direction. An attempt to model the horizontal load resulted in accelerations that exceeded the criteria proposed in Eurocode (as no criteria for horizontal vibrations are presented in BS 5400).

Dynamic analysis of the Footbridge-II according to ISO 10137 showed good serviceability in the horizontal direction. An attempt to model the vertical load resulted in accelerations that exceeded the criteria proposed by the standard. However, in accordance with the vibration assessment, Footbridge-I is more serviceable than Footbridge-II.

Natural frequencies which are in range coinciding with frequencies typical for human-induced dynamic loading can be avoided by increasing structural stiffness. Increasing stiffness can be an expensive measure and will almost always have negative effects on the aesthetics of the structure. However, in this case the current model is an optimum model.

The trend in footbridge design over the last few years, which has been described in the beginning of this thesis, has led to several cases of excessive vibrations of footbridges due to pedestrian-induced loading. It is hoped that identification of these problems will lead to improved design of footbridges in the future.

9.4 Scope for Future Studies

Based upon the work described in this thesis, some suggestions for future work within this field are noted.

- The level of pedestrian synchronization as a function of the amplitude and frequency of bridge motion has been proposed for future studies.

- Mathematical quantification of maximum number of pedestrian allowed in a particular footbridge needs to be studied.

- Quantification of the horizontal load factor of pedestrian load as a function of the amplitude and frequency of bridge motion are also needed.

- Synchronization effect of footbridge in different damping ratios and in different conditions will provide further insight to the problem.

- Development of a simpler mathematical model for horizontal pedestrian loads can be useful for dynamic design of a footbridge.

References

Amin, A.F.M.S., Hossain, T.R., Habib, A. (2005). Vibration serviceability requirement in the design of arch-supported suspended footbridge, Japan-Bangladesh joint seminar on Advances in bridge engineering, Dhaka, 10th August 2005, pp. 13-22.

Bachmann, H., Ammann, W. (1987). Vibrations in Structures - Induced by Man and Machines, Structural Engineering Documents, Vol. 3e, International Association of Bridge and Structural Engineering (IABSE), Zurich.

Bachmann, H. (1992). Case-studies of structures with man-induced vibrations, J. Engineering Mechanics, 118,631-647.

Barker, C. (2002). Some observations on the nature of the mechanism that drives the self-excited lateral response of footbridges, Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges, Paris, France.

British Standard (BS 5400): Part 2 (2002). Design Manual for Roads and Bridges, Highway Agency, London: The Stationary Office, February 2002. (Reference of amendment: BD 37/01 August 2001)

Breukelman, B. (2005). Damping Systems. RWDI Technotes, issue no. 10. http:// www. rwdi.com/. (Accessed on 09 April 2006).

Brownjohn, J.M.W., Pavic, A., Omenzetter, P. (2004). A spectral density approach for modeling continuous vertical forces on pedestrian structures due to walking, Canadian Journal of Civil Engineering 31 (1), 65–77.

Caetano, E. and Cunha, A. (2004). Experimental and numerical assessment of the dynamic behaviour of a stress-ribbon footbridge.

CEB, B. I. 209, August 1991. Practical guidelines.

Chopra, A.K. (2001). Dynamics of Structures, Theory and Applications to Earthquake Engineering, Prentice Hall, Second Edition, New Jersey, USA.

Computers and Structures, Inc. (1995), SAP2000, Analysis Reference Manual, Berkeley, California, USA.

Dallard, P., Fitzpatrick, A.J., Flint, A., Bourva, S.L., Low, A., Smith, R.M.R., Willford

(2001a). The London millennium Footbridge, The Structural Engineer, 79, 17-33.

Dallard, P., Fitzpatrick, T., Flint, A., Low, A., Smith, R.R., Willford, M., Roche, M. (2001b). The London millennium bridge: Pedestrian-induced lateral vibration, J. Bridge Engineering, 6, 412-417.

Dinmore, G. (2002). Dynamic wave behaviour through dense media of varied dynamic stiffness, Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges, Paris, France.

Ebrahimpour, A, Sack, R.L. (1990). Design live loads for coherent crowd harmonic movements, ASCE Journal of Structural Engineering, Vol. 118-4, pp.1121-1136.

Ebrahimpour, A., Fitts, L.L., (1996). Measuring coherency of human-induced rhythmic loads using force plates, Journal of Structural Engineering 122 (7), 829–831.

Ellis, B.R., Ji, T., (1994). Floor vibration induced by dance-type loads: verification, The Structural Engineer 72 (3), 45–50.

Ellis, B.R., Ji, T., (1997). Human–structure interaction in vertical vibrations, Structures and Buildings 122 (1) 1–9.

Eriksson, P.E., (1994). Vibration of Low-Frequency Floors - Dynamic Forces and Response Prediction, PhD Thesis, Unit for Dynamics in Design, Chalmers University of Technology, Goteborg, Sweden.

Eurocode, Basis of Structural Design- prAnnex A2. EN1990: 2002. European Committee for Standardization, Brussels, Belgium 2002.

Eurocode 1, General Actions Traffic loads on bridges. EN1991- 2: 2003. European Committee for Standardization, Brussels, Belgium 2003.

Eyre, R., D.W. Cullington, D.W., (1985). Experience with vibration absorbers on footbridges, TRRL Research Report No. 18, Transport and Road Research Laboratory, Crowthrone.

Fujino, Y., Sun, L., Pacheco, B.M. and Chaiseri, P. (1992). Tuned liquid damper (TLD) for suppressing horizontal motion of structures, J. Engineering Mechanics, ASCE, 118, 2017-2030

Fujino, Y., Pacheco, B.M., Nakamura, S. and Warnitchai, P. (1993). Synchronization of human walking observed during lateral vibration of a congested pedestrian bridge, Earthquake Engineering and Structural Dynamics, 22, 741-758

Grundmann, H., Kreuzinger, H., Schneider, M. (1993). Dynamic calculations of footbridges, Bauingenieur 68, 215–225.

Hauksson, F. (2005), Dynamic Behaviour of footbridges subjected to pedestrian-induced vibrations, M.Sc Thesis, Division of Structural Mechanics, Lund University, November 2005

Hauksson, F., November 2005, Dynamic Behaviour of footbridges subjected to pedestrian induced vibration, M.Sc thesis, Lund University, Sweden.

Huang, M.H., Thambiratnam D.P., and Perera N.J. (2005). Resonant vibration of shallow suspension footbridges, Bridge Engineering, 158 BE4, pp. 201-209.

ISO, 2005. Bases for design of structures Serviceability of buildings and pedestrian walkways against vibration, ISO/CD 10137, International Standardization Organization, Geneva, Switzerland.

Jacklin, H.M. (1936). Human reactions to vibration, S.A.E. Journal 39 (4) , 401–407.

Leonard, D.R. (1966). Human tolerance levels for bridge vibrations, TRRL Report No. 34, Road Research Laboratory.

Maguire, J.R., Wyatt, T.A. (2002), Dynamics, An introduction for civil and structural engineers, Thomas Telford, Second Edition, London, UK.

Matsumoto, Y., Nishioka, T., Shiojiri, H. and Matsuzaki, K. (1978). Dynamic design of footbridges, IABSE Proceedings, 1978, P-17/78, pp. 1-15.

McRobie, A., Morgenthal, G. (2002). Risk management for pedestrian-induced dynamics of footbridges, Proceeding of the International Conference on the Design and Dynamic Behaviour of Footbridges, Paris, France.

McRobie, A., Morgenthal, G., Lasenby, J., Ringer, M. (2003). Section model tests on human-structure lock-in, Bridge Engineering 156 (BE2), 71–79.

Mouring, S.E. (1993). Dynamic Response of Floor Systems to Building Occupant Activities, PhD Thesis, The Johns Hopkins University, Baltimore, MD, USA.

Nakamura, S. and Fujino, Y. (2002). Lateral vibration on a pedestrian cable stayed bridge, Structural Engineering International, 12, 295-300.

Nakamura, S. (2003). Field measurements of lateral vibration on a pedestrian suspension bridge, The Structural Engineer 81 (22), 22–26.

Newland, D.E. (1993). An Introduction to Random Vibrations and Spectral Analysis, Longman Group, Harlow.

Newland, D. E. (2003). Pedestrian Excitation of Bridges-Recent Results, Tenth International Congress on Sound and Vibration, Stockholm, Sweden, 7-10 July 2003.

Ohlsson, S.V., 1982. Floor Vibration and Human Discomfort, PhD Thesis, Chalmers University of Technology, Goteborg, Sweden, (in English).

Pavic, A., Yu, C.H., Brownjohn, J., Reynolds, P. (2002). Verification of the existence of human-induced horizontal forces due to vertical jumping, Proceedings of IMAC XX, Vol. 1, Los Angeles, CA, February 4–7, 2002, pp. 120–126.

Pimentel, R.L., 1997. Vibrational Performance of Pedestrian Bridges Due to Human-Induced Loads, PhD Thesis, University of Sheffield, Sheffield, UK.

Poovarodom, N., Kanchanosot, S. and Warnitchai, P. (2003). Application of non-linear multiple tuned mass dampers to suppress man-induced vibrations of a pedestrian bridge, Earthquake Engineering and Structural Dynamics. 32: 1117-1131.

Reiher, H. and Meister, F.J. (1931). “The Effect of Vibration on People.” Translated from Forsch. Geb. Ing. Wes: 2(11) 381-6, by U.S. Air Material Command, Translation F-TS-616-RE, Wright Field, Ohio, AMC, 1946.

Sachse, R. (2002). The Influence of Human Occupants on the Dynamic Properties of Slender Structures, PhD Thesis, University of Sheffield, Sheffield, UK.

Schlaich, M. (2002). Planning conditions for footbridges, Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges, Paris, France, November 20–22, 2002, pp. 40–52.

Setra, Technical Guide. (2006). Footbridges - Assessment of Vibrational Behaviour of Footbridges under Pedestrian Loading, Service d’Etudes Techniques des Routes et Autoroutes, Association Francaise De Genie Civil, Paris, France.

Spasojevic, A., Dordjevic, D., Spasojevic, M. and Spasojevic, N., (2002), Some relevant aspects of footbridge vibrations, Architecture and Civil Engineering, Vol. 2, 2002, 281-289.

Smith, J.W. (1969). The Vibration of Highway Bridges and the Effects on Human Comfort, PhD Thesis, University of Bristol, Bristol, UK.

Stoyanoff, S. and Hunter, M. (2005). Footbridges: Pedestrian induced vibrations. RWDI Technotes, issue no. 15. http://www.rwdi.com/. (Accessed on 09 April 2006).

Tilly,G.P., Cullington, D.W. and Eyre, R., February 1984. Dynamic behaviour of footbridges, IABSE periodical, S-26/84, page 13 et seq.

Wheeler, J., September, 1982. Prediction and control of pedestrian-induced vibration in footbridges, ASCE Journal of structural engineering, Vol. 108, No. ST9.

Willford, M. (2002). Dynamic actions and reactions of pedestrians, in: Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges, Paris, France, November 20–22, 2002, pp. 66–73.

Wilson, Ed. (2002), Three-Dimensional Static and Dynamic analysis of Structures, Computers & Structures, Inc., Third Edition, Berkeley, California, USA.

Wright, D.T., Green, R. (1963). Highway bridge vibrations - part II: Ontario test programme, Report No. 5, Queen’s University, Kingston, Ontario, Canada.

Yao, S., Wright, J., Pavic, A., Reynolds, P. (2002). Forces generated when bouncing or jumping on a flexible Structure, Proceedings of the International Conference on Noise and Vibration, Vol. 2, Leuven, Belgium, September 16–18, 2002, pp. 563–572.

Yoneda, M. (2002). A simplified method to evaluate pedestrian-induced maximum response of cable-supported pedestrian bridges, Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges, Paris, France.

Yoshida, J., Abe, M., Fujino, Y., Higashiuwatoko, K. (2002). Image analysis of human induced lateral vibration of a pedestrian bridge, Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges, Paris, France.

Zivanovic, S., Pavic, A., and Reynolds, P. (2005). Vibration serviceability of footbridges under human-induced excitation: a literature review. Journal of Sound and Vibration 279.

Zheng, X., Brownjohn, J.M.W. (2001). Modelling and simulation of human-floor system under vertical vibration, Proceedings of SPIE, Smart Structures and Materials, Vol. 4327, March 5–8, Newport Beach, CA, USA, 2001, pp. 513–520.