SECTION – A

There are FIVE questions in this section. Answer any FOUR.

1. (a) A rectangular vertical member fixed at the base is loaded as shown in Fig. 1. Find the location for a gage in member face AB such that no longitudinal strain would occur due to the application of force \( P = 6 \text{ kN} \). Does the answer depend on the magnitude of force \( P \)? Assume elastic behavior. All dimensions are given in mm. (16 1/4)

(b) Determine the kern of a hollow circular section with outer diameter 'D' and inner diameter 'd'. (10)

2. (a) Using direct integration method, determine the equation of the elastic curve for the beam shown in Fig. 2 due to the applied loading. EI for the beam is constant. (13 1/4)

(b) Using direct integration method, determine the equation of the elastic curve for the beam shown in Fig. 3 due to the applied loading. EI for the beam is constant. (13)

3. (a) Using the moment-area method, determine the deflection and the slope of the elastic curve at point A, due to the applied load for the beam shown in Fig. 4. Specify the direction of deflection and of rotation for the calculated quantities. (13 1/4)

(b) A beam fixed at both ends carries a concentrated force \( P \), as shown in Fig. 5. Find the fixed-end moments. EI is constant. (13)

4. (a) Given the state of stress shown in Fig. 6, using Mohr’s circle transform it: (i) into the principal stresses and (ii) into the maximum shear stresses and the associated normal stresses. Show the results for both cases on properly oriented elements. (16 1/4)

(b) Using general equations for transformation of stresses, transform the stresses shown in Fig. 6 into stresses acting on the plane at an angle of 60 degrees anti clockwise with the horizontal axis. (10)

5. (a) Using strain energy method, calculate the deflection of point 'D' due to the applied loading as shown in Fig. 7. (16 1/4)

(b) A cantilever beam of span \( L \) is subjected to a downward concentrated load of \( P \) at the free end. Using strain energy method, calculate the deflection at the free end. EI for the beam is constant. (10)

Contd ……… P/2
6. (a) Determine the allowable axial loads for following W14x159 steel columns \((A = 46.7\) \text{in}^2 \text{ and } r_{min} = 4 \text{ in})\) using AISC ASD method, \(L = 40 \text{ ft}, \sigma_{yp} = 36 \text{ ksi}, \text{ pinned ends}\) \(L = 50 \text{ ft}, \sigma_{yp} = 36 \text{ ksi}, \text{ pinned ends}\) \(L = 40 \text{ ft}, \sigma_{yp} = 50 \text{ ksi}, \text{ pinned ends}\) \(L = 50 \text{ ft}, \sigma_{yp} = 36 \text{ ksi}, \text{ one end pinned and other fixed}\) Given: \(E = 29 \times 10^3 \text{ ksi}\) \[C_c = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}}\] for \((L_e/r) > C_c\) \[\sigma_{allow} = \frac{12\pi^2 E}{23(L_e/r)^2}\] \[(L_e/r) < C_c\] \[\sigma_{allow} = \frac{\left[1 - \frac{(L_e/r)^2}{2C_c^2}\right]\sigma_{yp}}{\text{F.S.}}\] (b) Discuss about the results obtained in Q. 6(a) in terms of the effect of column length, yield strength and support conditions on allowable axial loads.

7. (a) Using equation of strain energy density, derive the expression for elastic strain energy for bending stress.

(b) Calculate the strain energy of the cantilever beam for bending stress shown in Fig. 8. Also determine the end deflection of the beam using energy method. Given, \(EI = 60 \text{ N.m}^2\).

\(P = 60\text{N}\) \(L = 0.25\text{m}\)

Contd ......... P/3
8. (a) For plain stress condition ($\sigma_3 = 0$), derive and show graphically the yield criteria based on,
   (i) Maximum principal stress theory
   (ii) Maximum principal strain theory
(b) Find the maximum cable tension and cable sag at c and e for the cable shown in Fig. 9.

![Diagram of cable system](image)

9. Determine the size of the welds to resist a vertical load of 10 kips for the plate shown in Fig. 10. Given the allowable shear stress of the weld is 20 ksi.

![Diagram of plate and welds](image)

10. (a) Derive the equation of General Cable Theorem.
(b) Draw the bending moment diagram of the stiffening girder of Fig. 11.

![Diagram of stiffening girder](image)
Fig-1

Fig-2

Fig-3

Fig-4

Fig-5

Fig-6

Fig-7

$P = 45$ kips

$L = 144$ in.

$a = 36$ in.

$b = 108$ in.

$E = 29 \times 10^6$ ksi

$I = 248$ in$^4$
SECTION A

There are FOUR questions in this section. Answer any THREE.

1. (a) What is meant by production possibility frontier (PPF)? Explain how resources can be allocated in a society with the help of PPF. (13½)
   
   (b) Illustrate the following applications of PPF:
       (i) Choice between public goods and private goods.
       (ii) Choice between necessities and luxuries.
       (iii) Choice between current consumption goods and investment.

2. (a) Explain the concept of demand function. (3)
   
   (b) Discuss the factors that affect the demand for a commodity. (7)
   
   (c) (i) Calculate the equilibrium price and quantity from the following demand and supply functions and graphically show the results.
       \[ QD_x = 50 - 3P_x \]
       \[ QS_x = -30 + 5P_x \]
       
       (ii) If a per unit tax of Tk. 0.50 is imposed, how will it affect the equilibrium price and quantity?
       (iii) If government provides a subsidy of Tk. 2 per unit, what will happen to the equilibrium price and quantity?
       (iv) Briefly discuss how the state of technology affects the supply of a commodity. (3½)

3. (a) Who were the Cardinalist Economists? Derive mathematically the cardinal theory of consumer equilibrium. (8½)
   
   (b) What is meant by cross elasticity of demand? Write down the formula for cross elasticity of demand and give suitable examples with reference to the substitute and complement commodities. (8)
   
   (c) Explain any two properties of an indifference curve. (7)

4. (a) Define marginal rate of substitution. Give a suitable example. (5)
   
   (b) What is meant by budget constraint line? Explain. (5)
   
   (c) What do you understand by substitution effect and income effect of a price change? Derive a demand curve with the help of indifference curves and budget lines and show that price effect is equal to substitution effect and income effect. Present and explain all necessary diagrams. (13½)

Contd .......... P/2
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SECTION – B

There are FOUR questions in this section. Answer any THREE.
Symbols indicate their usual meaning.

5. (a) A manufacturer has a fixed cost of $40,000 and a variable cost of $1.60 per unit made and sold. Selling price is $2 per unit. (10)

(i) Find the revenue, cost and profit functions using q for the number of units.
(ii) Compute profit if 150000 units are made and sold.
(iii) Find the break-even quantity.
(iv) Construct the break-even chart. Label the cost and revenue lines, the fixed cost line, and the break-even point.

(b) Complete the following table and sketch the graph explaining the relations among the various short run cost curves: (13 1/3)

<table>
<thead>
<tr>
<th>Quantity of output</th>
<th>Total fixed cost</th>
<th>Total variable cost</th>
<th>Total cost</th>
<th>Average fixed cost</th>
<th>Average variable cost</th>
<th>Average Total cost</th>
<th>Marginal cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>70</td>
<td>120</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. (a) From the following revenue and cost functions calculate the profit maximizing level of output and maximum profit. (10)

\[ R = 111Q - 2Q^2 \]
\[ C = \frac{1}{3}Q^3 - 8Q^2 + 122Q + 50 \]

(b) Graphically explain the short run equilibrium of a firm under perfect competition. (13 1/3)

7. (a) Explain producer's equilibrium with the help of iso-cost and isoquant curves. (13 1/3)

(b) From the following functions calculate the amount of labour and capital that maximizes output. What is the maximum amount of output? (10)

\[ Q = 200L^{0.5} K^{0.05} \]
\[ 3000 = 35L + 45K \]

8. (a) What do you understand by localization of industries? What are the causes of localization of industries? (10)

(b) Explain the advantages and disadvantages of localization of industries. (13 1/3)
SECTION - A

There are FOUR questions in this section. Answer any THREE. Assume reasonable values if not given.

1. (a) Write short notes on following items:
   (i) Saturation vapor pressure
   (ii) Minor losses in pipe flow
   (iii) Surface tension
   (iv) Viscous sublayer

(b) A flat plate of 0.4 m² in area moves edgewise through oil between large fixed parallel planes 15 cm apart. If the velocity of the plate is 0.5 m/s and oil has a kinematic viscosity of 0.45 × 10⁻⁴ m²/s and specific gravity of 0.85, calculate the drag force when (i) the plate is 5 cm from one of the planes and (ii) the plate is equidistant from both the planes.

(c) Derive Newton's equation of viscosity. In this respect differentiate between Newtonian and non-Newtonian, ideal plastic and elastic solid with examples and figures.

(d) Explain why the viscosity of all liquids decreases and that of all gases increases with an increase in temperature.

2. (a) Derive the expression for variation of pressure in the vertical direction.

(b) Determine the diameter of steel pipe (e = 0.045 mm) to carry 30 l/s of water if the permissible head loss per meter of the pipe length is 0.05 m. Use Moody diagram. Take ν = 1 × 10⁻⁶ m²/s.

(c) A pipe 60 m long and 15 cm in diameter is connected to a water tank at one end and flows freely into the atmosphere at the other end. The height of the water level in the tank is 2.8 m above the center of the pipe. The pipe is horizontal with f = 0.04. Determine the discharge through the pipe.

(d) Explain why mercury is used in Barometer?

3. (a) Show the relationship between absolute and gage pressure with a diagram. Explain the relative advantages and disadvantages of piezometer and double column manometers.

(b) Calculate the rate of flow of water from the reservoir A to B for the system shown in Figure 1. Pipe dimensions are as follows:

\[ L_1 = 400 \text{ m}, \quad D_1 = 600 \text{ mm}, \quad e_1 = 2 \text{ mm} \]
\[ L_2 = 300 \text{ m}, \quad D_2 = 1000 \text{ mm}, \quad e_2 = 0.6 \text{ mm} \]

Consider minor losses and \( ν = 2 × 10⁻⁶ \text{ m}²/\text{s} \). Use Moody diagram.

Contd ............ P/2
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(c) Initial distribution of flows through a pipe network is shown in Figure 2. Taking n = 2
for all pipes, obtain flows in each pipe after applying correction twice. Discharge is in l/s. (15)

4. (a) Differentiate between:
(i) Adhesion and Cohesion
(ii) Solid and fluid
(iii) Compressible and incompressible fluid
(iv) Laminar and turbulent flow.

(b) The head loss in 60 m of 15 cm diameter pipe is known to be 8 m when oil (s = 0.9)
of viscosity 0.04 Ns/m² flows at 0.06 m³/s. Determine the centerline velocity, shear
stress at the wall of the pipe, friction factor and the velocity at 10 cm from the centerline. (10)

(c) For the pipes connected in parallel as shown in Figure 3, the pipe dimensions and
friction factors are as follows:

$L_1 = 900$ m $D_1 = 0.3$ m $f_1 = 0.021$
$L_2 = 600$ m $D_2 = 0.2$ m $f_2 = 0.018$
$L_3 = 1200$ m $D_3 = 0.4$ m $f_3 = 0.019$

For a total discharge of 0.34 m³/s, determine the flow through each pipe and head loss
from A to B. (15)

SECTION – B

There are FOUR questions in this section. Answer any THREE.
Use reasonable value of any missing data. Notations have their usual meanings.

5. (a) Explain local acceleration, convective acceleration, tangential acceleration and normal
acceleration in a fluid with sketch. (10)

(b) The diameter of a horizontal pipe AB reduces from 100 cm at A to 50 cm at B over a
length of 20 m. The longitudinal velocity at the center of the pipe is maximum and
reduces linearly to zero at the wall. Calculate the acceleration along the center of the pipe
section at A. $Q = 1$ m³/s. Neglect velocity components in the other directions. Water
flows from A to B. (15)

(c) For the problem above, compute the momentum correction factor at B. (10)

6. (a) Derive the general equation for steady flow of any fluid and hence derive the
Bernoulli’s equation with appropriate assumptions. (10)

(b) Water is flowing under a sluice gate of 2 m width. The flow depths are 2 m at far
upstream of the gate and 0.4 m downstream of the gate. If the head loss at the gate is 0.05
m, calculate the flow rate. (10)
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Contd ... Q. No. 6

(c) A pump is 2.5 m above the water level in the sump and has a pressure of −22 cm of mercury at the suction side. The suction pipe is of 25 cm diameter and the delivery pipe is a short 30 cm diameter pipe ending in a nozzle of 10 cm diameter. If the nozzle is directed vertically upward at an elevation of 4.0 m above the sump water level (Figure 4), determine the discharge, power input into the flow by the pump and the elevation above the sump water level to which the jet would reach (neglect all losses). Also draw the hydraulic grade line and energy line for the system.

7. (a) What is cavitation? Write its causes and disadvantages.
(b) What are the relations between absolute and relative velocities?
(c) A fluid (sp. gravity = 1.26) is flowing through horizontal pipe AB with a constant diameter of 30 cm. If a pitot tube and a piezometer connected by a manometer are inserted at A and B respectively, find the manometer reading. What happens to the manometer reading if the point B is lower or higher than point A? Q = 1 m³/s, sp. gravity = 0.86 for the manometer fluid. Assume velocity distribution is uniform over a cross-section.
(d) The diameter of a horizontal pipe at point A is 90 cm and reduces linearly to 30 cm at point B. The pressure at point A is 70 kPa. Compute the force of a flowing fluid (sp. gravity = 0.86) on the pipe section AB if cavitation just occurs at point B. Neglect head loss. Vapor pressure of the fluid is 26 kPa, abs.

8. (a) Write short notes on
   (i) Force upon a single and series of moving blades.
   (ii) Reaction of a jet
   (b) A 5 cm-diameter jet with a velocity of 30 m/s impinges on a single vane moving in the same direction at a velocity of 18 m/s. If β₂ = 150° and friction losses over the vane are such that v₂ = 0.9v₁, compute the force exerted by the water on the vane. Also compute the power transferred to the blade and frictional loss.
   (c) When a turbine runner is held so that it cannot rotate, the discharge under a head of 15 m is found to be 0.85 m³/s. α₁ = 35°, β₂ = 155°, r₁ = 0.2 m, r₂ = 0.12 m. The area of the water jet at the entrance and exit are 0.078 m² and 0.082 m² respectively. Find the torque at zero speed. Neglect shock loss.
Values of \( \frac{D^2 V}{
u} \) for water at 60°F

Values of \( \frac{D^2 V}{
u} \) for atmospheric air at 60°F

Figure: Friction factor for pipes (Moody Diagram)
Figure 1 for Question 3(b)

Figure 2 for Question 3(a)

Figure 3 for Question 4(a)

Figure 4 for Question 4(c)
1. (a) Find the root of the following equation using Newton-Raphson method.
\[ x \sin x + \cos x = 0 \]
Use an initial guess of \( x = 1 \) and perform 4 iterations. Calculate the approximate error after each iteration.
(8)
(b) Use linear regression to fit a straight line to the data shown in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>25</td>
<td>70</td>
<td>380</td>
<td>550</td>
<td>610</td>
<td>1220</td>
<td>830</td>
<td>1450</td>
</tr>
</tbody>
</table>
(12)
Also compute the correlation coefficient (R² value) of the fit.
(c) Give two instances (with diagrams) where Newton-Raphson method fails to converge.
(3 \( \frac{1}{3} \))

2. (a) The equation of the deflection curve of a beam subjected to linearly increasing load is as follows:
\[ y = \frac{w_0}{120 \, EL} \left( -x^5 + 2L^2x^3 - L^4x \right) \]
where, \( L = 600 \) cm, \( E = 50000 \) KN/cm², \( I = 30000 \) cm⁴ and \( w_0 = 2.5 \) kN/cm. Apply bisection method to determine the point of maximum deflection (that is, the value of \( x \) where \( y = y_{\text{max}} \)) using any logical initial bracketing values. Perform 8 iterations. Also determine the value of the maximum deflection. The deflection curve of the beam is shown in Figure 1.

(b) Fit the curve \( y = ae^{bx} \) to the following data:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>15.3</td>
<td>20.5</td>
<td>27.4</td>
<td>36.6</td>
<td>49.1</td>
<td>65.6</td>
<td>87.8</td>
</tr>
</tbody>
</table>
(9 \( \frac{1}{3} \))
(Find the value of coefficients \( a \) and \( b \) only, no need to determine the goodness of fit).

Contd ........... P/2
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3. (a) Solve the following equation for \( y(0.4) \)

\[
10 \frac{d^2 y}{dx^2} + \left( \frac{dy}{dx} \right)^2 + 6x = 0
\]

using a step size of 0.2 with \( y(0) = 1 \) and \( y'(0) = 0 \). Use Heun's method.

(b) A steady-state heat balance for a rod can be represented as \( \frac{d^2 T}{dx^2} - 0.15T = 0 \). Write down the finite difference equations in matrix form to obtain a solution for a 10-m long road with \( T(0) = 240 \) and \( T(10) = 150 \). Use \( \Delta x = 1 \) m. No need to solve the problem.

4. (a) Solve the following initial value problem over the interval \( t = 0 \) to 2 using midpoint method and a step size of 0.5 where \( y(0) = 1 \):

\[
\frac{dy}{dt} = yt^3 - 1.5y
\]

(b) The steady state two-dimensional heat-flow in a 6 cm \( \times \) 6 cm metal plate can be described by the equation \( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \). Given the boundary conditions as shown in Figure 2 below, find the temperatures \( T_1, T_2, T_3, T_4 \). The grids are square having a size of 2 cm \( \times \) 2 cm and the temperature values in the boundary are in deg. Celsius.

\[\text{Figure 2}\]

\[\begin{array}{cccc}
0 & 100 & 100 & 0 \\
0 & T_1 & T_2 & 0 \\
0 & T_3 & T_4 & 0 \\
0 & 50 & 100 & 0 \\
\end{array}\]

SECTION - B

There are FOUR questions in this section. Answer any THREE.

5. (a) Discuss the followings:

(i) Difference Table

(ii) Pivotal condensation and Bach substitution

Contd .......... P/3
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Contd ... Q. No. 5

(b) Experimentally observed deflections for a beam are shown below. Given \( E = 30 \times 10^6 \) psi; \( T = 1500 \) in\(^4\). Find Bending Moment and Shear Force at points \(-3, -1, +1\) and \(+3\). (13 1/3)

6. (a) Derive the weighing coefficients and associated points for \( n = 3 \) in Gauss Quadrature method. (10)

(b) Solve the following using Romberg's Quadrature method: \( \int_1^3 \frac{1}{x + 0.5} \, dx \). (13 1/3)

7. (a) Derive the ground expression of \( \int_a^b f(x) \, dx \) using Simpson's Rule. (10)

(b) Solve the following system of linear equations by Crout's method (13 1/3)

\[
\begin{align*}
x + 5y + 6z &= 26 \\
3x + 4y + z &= 16 \\
2x + y + 5z &= 12 
\end{align*}
\]

8. (a) Solve the following using Gauss Quadrature method. (10)

\[
\int_0^3 x^2 \cos x \, dx
\]

(b) Derive the final expression for Gregory-Newton Interpolation method. (6)
(c) The following data have been obtained from a tensile test on a steel specimen. Calculate the elongation corresponding to 2700 lbs. (7 1/3)

<table>
<thead>
<tr>
<th>Loads in lbs</th>
<th>Elongation in inch</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000</td>
</tr>
<tr>
<td>600</td>
<td>0.0245</td>
</tr>
<tr>
<td>1200</td>
<td>0.0586</td>
</tr>
<tr>
<td>1800</td>
<td>0.0962</td>
</tr>
<tr>
<td>2400</td>
<td>0.1511</td>
</tr>
<tr>
<td>3000</td>
<td>0.2415</td>
</tr>
</tbody>
</table>

-------------------------------
1. A rotatory vibrating machine (Fig. 1) is supplied to exert perfect sinusoidal force. However, due to some technical glitch, the machine fails to exert full force in one direction. Thus a distorted sinusoidal force is produced as shown in Fig. 2. Determine the motion of the mass, governed by the equation of motion of a damped simple oscillator given below.

\[ m\ddot{u} + c\dot{u} + ku = p(t) \]

where, \( m = 10 \text{ kg}, c = 2 \text{ kg/s} \) and \( k = 100 \text{ kg/s}^2 \).

2. (a) Determine if the following functions are periodic or non-periodic. If periodic, then find the fundamental periods. Find the even and odd parts of the functions.

(i) \( \frac{x}{x + 2} \) \hspace{1cm} (ii) \( \sin^2 x \) \hspace{1cm} (iii) \( 6 \cos x - 4 \sin 3x \) \hspace{1cm} (iv) \( e^{\sin 3x} \)

(b) Determine the Fourier integral representation of \( f(x) = e^{-|x|} \).
3. (a) Evaluate the following:

(i) \( F^{-1}\left\{ (\omega + 3)e^{-4(\omega + 3)^2} \right\} \)

(ii) \( F^{-1}\left\{ e^{-|\omega|} \cos \omega \right\} \)

(b) Distinguish between ordinary differential equation and partial differential equation. What is the major difference between the homogeneous solution of partial differential equation and that of ordinary differential equation? What type of PDE does the following second order equation represent?

\( \alpha^2 u_{xx} = u_t + V u_x + H u \)

4. (a) A stone falls from a leaning tower under gravitational force from rest and gradually accelerates to a velocity \( v(t) \) in air. Explain the phenomenon and identify the factors affecting the fall. List the variables and identify the dependent and independent variables.

(b) Differentiate between Implicit Solution and Explicit Solution.

(c) What is indicial equation? Derive it from Bessel's equation and describe the roots with physical significance.

SECTION - B

There are FOUR questions in this section. Answer any THREE.

5. (a) If \( X_1, X_2, \ldots, X_n \) are independent Normal random variables with each having the same parameters \( \mu \) and \( \sigma \), determine the maximum likelihood estimators of \( \mu \) and \( \sigma \) (Derive expressions).

(b) Buses arrive at a specified stop at 15 minute intervals starting at 7:00 am. That is, they arrive at 7:00, 7:15, 7:30, 7:45 am etc. If a passenger arrives at a stop a time uniformly distributed between 7:00 and 7:30 am, find the probability that he waits: (i) less than 5 minute; (ii) at least 12 minute.

(c) Sample of 10 grains of metallic fibre taken from a large heap has following lengths (mm): 2.2, 3.4, 1.6, 0.8, 2.7, 3.3, 1.6, 2.8, 2.5, and 1.9. It is known that the size of an individual particle will have an approximate lognormal distribution. Estimate the percentage of fibre with length between 2 and 3 mm.

6. (a) A plane is missing and it is presumed that it was equally likely to have gone down in any of 3 possible regions. Let \( 1-\beta_i \) denote the probability that the plane will be found upon a search of the \( i \)th region when the plane is, in fact, in that region, \( i = 1, 2, 3 \). (The constants \( \beta_i \) are called overlook probabilities because they represent the probability of overlooking the plane, they are generally attributable to the geographical and environmental conditions of the regions). What is the conditional probability that the plane is in the \( i \)th region, given that a search of region 1 is unsuccessful, \( i = 1, 2, 3 \)?
(b) Civil Engineers believe that \( W \), the amount of weight (in units of 1000 pounds) that a certain span of a bridge can withstand without structural damage, is normally distributed with mean 400 and standard deviation 40. Suppose that the weight (in units of 1000 pounds) of a car is a random variable with mean 3 and standard deviation 0.3. How many cars would have to be on the bridge span for the probability of structural damage to exceed 0.1?

(c) Daily probability that a major earthquake occurs \( P[E] = 10^{-5} \). Probability that premonitory event A or B occurs given that major earthquake occurs is \( P[A|E] = P[B|E] = 0.1 \). Probability that premonitory event A or B occurs given that major earthquake does not occur is \( P[A|\overline{E}] = P[B|\overline{E}] = 0.001 \).

(i) Determine the probability of a major earthquake, given that premonitory event A is observed.

(ii) Also, determine the probability of a major earthquake, given that both premonitory events A and B are observed at the same time.

7. (a) What do you mean by standard residuals? Discuss with diagram how plot of residuals can be helpful in assessing validity of the linear regression model.

(b) A car is advertised as having a gas mileage rating of at least 30 miles/gallon in highway driving. If the miles per gallon obtained in 10 independent experiments are 26, 24, 25, 27, 25, 30, 26, 33, should you believe the advertisement? What assumptions are you making?

(c) The following data indicate the gain in reading speed versus the number of weeks in the program of 10 students in a speed-reading program.

<table>
<thead>
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<th>Number of weeks</th>
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(i) Plot a scatter diagram to see if a linear relationship is indicated.

(ii) Find the least squares estimates of the regression coefficients.

(iii) Estimate the expected gain of a student who plans to take the program for 7 weeks.

8. (a) List and explain the valid operations on power series.

(b) What is the fundamental condition for existence of a power series solution of an equation?

(c) Apply power series method and solve the following differential equation:

\[ y' - y = 0 \]
### STANDARD NORMAL DISTRIBUTION

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Other significant probabilities:

\[ \chi^2_0 = 4.6, \quad \chi^2_{4.61} = 4.03, \quad \chi^2_{1.78} = 4.97. \]