TRANSIENT DYNAMIC ANALYSIS OF UNDERGROUND STRUCTURES

A Thesis
by
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DECLARATION

I hereby declare that the research reported in this thesis was performed by me and that the work has not been submitted anywhere else for any other purpose.

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Signature of the student
ABSTRACT

A nonlinear, plane-strain, dynamic finite element method of analysis computer program is developed to study the interactive behaviour of underground structures. Static problems can also be solved without loss of efficiency. An extensive description of the analytical models used in the code is given.

A planned scheme of analyses was performed to study the dynamic response of soil without buried structure and the interactive response of a buried structure subjected to impulsive loading on the surface. Base conditions of rigid and semiinfinite cases were investigated to study their influence on the response of the system. A parallel series of nonlinear static analyses was performed for the maximum magnitude of the shock loading.

An optimum position of the structure in the soil was found giving minimum moments in the structure. For rigid base case, proximity of the structure to the base severely affected the system response. Elimination of the reflected wave actions by use of viscous base yielded that no specific advantage in load sharing of the structure can be gained for high depth of burial. Inadequacy of the presently available recommendations for a unique dynamic magnification factor was proved by comparing the results of parallel dynamic and static analyses.
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CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

Almost all structures may be subjected to one form or another of dynamic loading during its lifetime. Most severe dynamic loadings which the civil engineering structures may experience, are usually of nonperiodic nature. Nonperiodic loadings may be either transient loadings (often called impulsive loadings) due to blast from an explosion or long duration general dynamic loadings.

Some structures are of necessity built to withstand high blast loads due to external explosions. Explosion of a nuclear or conventional bomb on or above the ground surface causes three types of loadings: (i) an air over pressure (the excess above atmospheric pressure) caused by a travelling shock front, (ii) a drag loading caused by the high velocity movement of the air behind the shock front, and (iii) a diffraction effect that results from the higher pressure due to reflection of the shock wave on the front face of the object and also from the time lag before the over pressure acts on the rear face. High blast resistant constructions are preferred to be built underground, since the second and third components of the blast loading effects are not active on a buried structure. Underground structures such as tunnels, shelters, silos etc. built to provide protection during warfare need to be designed to withstand blast loadings on the surface.

The behaviour of a structure buried in the ground is completely different from that of one built overground. The difference is the result of the triaxial nature of loading on the underground structure as well as the interaction
between the structure and its surrounding soil. The problems peculiar to the design and analysis of underground structures built to withstand blast loading are thus in the realms of structural dynamics, soil mechanics and wave propagation. This thesis develops an analytical model encompassing this complex manifold nature of the problem and investigates the influence of various parameters on the response of such structures.

Detailed studies have been made to select the appropriate analytical tools and finally an integrated analytical scheme has been developed to examine the behaviour of underground structures subjected to transient dynamic loading on the ground surface. A nonlinear finite element technique with a step-by-step integration procedure has been employed for this purpose. Due consideration has been given to material nonlinearity of the soil and incidence and reflection characteristics of the shock wave.

1.2 DYNAMIC EQUILIBRIUM EQUATION

Chan et. al (1) described the response of a multi-degree-of-freedom system to dynamic loading by the differential equation of motion in matrix form

\[ M\ddot{x} + C\dot{x} + Kx = f \]  

(1.1)

where \( M \) is the system mass matrix; \( C \) is the matrix of viscous damping coefficients; \( K \) is the instantaneous stiffness matrix; \( f \) is the external force vector applied at the nodal points of the system, and \( x, \dot{x} \) and \( \ddot{x} \) are vectors of nodal displacement, velocity and acceleration respectively. Vectors \( f, x, \dot{x} \) and \( \ddot{x} \) are functions of time. Among the coefficient matrices, mass matrix \( M \) is constant throughout the motion and damping coefficient matrix \( C \) may
or may not be constant depending on the approximation used in the analysis. For linear elastic systems, the stiffness matrix $K$ remains constant throughout the motion, but for systems exhibiting nonlinear behaviour, $K$ is a function of displacements and hence of time.

Mathematically eqn. (1.1) represents a system of linear differential equations of second order. Standard procedures (2) for the solution of differential equations with constant coefficients are not suitable for use in finite element analysis as the order of the matrices is large. A recursive technique for the time domain solution of the dynamic differential equations has been employed with the coefficient matrices $M, C$ and $K$, known at any instant of time.

1.3 A REVIEW OF PREVIOUS WORKS

Many advances have been made to study soil-structure interaction problems in the static regime (3,4,5). Following these rapid advancements made in the last few decades, parallel formulations were made for dynamic analysis with some expectations that the dynamic problem would lend itself to a similar approach. But the peculiar features of the dynamic problem of the present nature warrant a completely different approach free from the shortcomings of a pseudo-dynamic static solution.

One of the earliest discussions on engineering approach to blast resistant design of structures appears to be due to Newmark (6). He proposed an equivalent static loading of a nuclear detonation for such design. In fact, very crude approximations were proposed for use in the preliminary design of structures. Purposely, the procedure was not readily useful to determine dynamic response or deflection.
Szilard (7) looked into the design of underground structures for blast loading effects and presented a clear outline of the problems involved in protective construction as they were understood at that time.

Selig et. al (8) proposed a closed form solution procedure for determining the damage to underground structures induced by air over pressure. He tried to derive analytical expressions that relate the parameters of the structure, soil and loading as they affect the failure of the structure. The material parameters are assumed to remain constant throughout the time. In fact, the proposed method was a form of dynamic modification of established static analyses.

Biggs (9) discussed the blast loading effects on both overground and underground structures. The methods used for analysis are also of equivalent static nature.

Marino and Riley (10) conducted a test to determine dynamic magnification of static response for a small cylinder buried close to the surface in a uniformly graded silica sand. They made no attempt to eliminate wave reflection from the sides of the soil container and therefore, the reflection of the shock wave from the container boundary might have influenced the results severely. Allgood (11) reported a series of tests on comparatively large (24" dia.) buried cylinders and noted different dynamic magnifications for thrusts and moments.

Farhoomand and Wilson (12) developed a nonlinear finite element code with a view to simulate an experimental blast response study. This code for analyzing axisymmetric systems came out after few reports of limited scope presented by Wilson (13,14). But the latest one also had many limitations. Structural part was modeled by membrane
elements having straight edges. Material nonlinearity to soil was attributed by using variable parameters determined from a tabular information on volumetric and shear behaviour as obtained from laboratory tests. No provision was kept to take account of wave energy dissipation in the semiinfinite soil mass. Soil-structure interface characteristics were also ignored in the analysis.

Duns et al. (15) presented a dynamic analysis of buried flexible cylinders by finite element method. The structural part was included in the analysis by straight beam-column elements and the soil part was represented by constant strain triangles. Material properties of both sides were kept constant throughout the time period. Wave energy dissipation effects were taken into account by inclusion of dashpots along the finite boundary. The presented analysis was, in fact an extension of its static counterpart and a dynamic magnification of static stresses was tried to be established.

Baron et al. (16) studied the influence of constitutive models on ground motions caused by one megaton air burst. A large volume of McCormick Ranch Sand was analyzed by finite element method incorporating special boundary treatments for wave energy dissipation. The paper stressed for selecting a proper constitutive model to represent nonlinear behaviour of soil under dynamic loads.

Dumanoglu (17) proposed a mathematical model of using non-reflecting boundaries for interaction analysis of embedded structures. A complex formulation was used to express the displacement at any instant of time. The system equilibrium equations (eqn. 1.1) were transformed to a complex formulation. This method seems to be inefficient for nonlinear systems.
Zaman et. al (18) used thin layer interface elements in interactive analysis of model nuclear containment structure for strong ground motion generated by blasts. In the analysis, soil property was assumed to be elastic-plastic and no boundary treatment was applied for wave propagation. The results obtained by finite element analysis were compared with the observed results.

It appears from the above review that no one model proposed so far can simulate an actual field behaviour of underground structures subjected to transient loading. It is the purpose of this thesis to present an analytical model giving due consideration to all necessary features of the problem.

1.4 OBJECTIVES OF THE RESEARCH

The research aims at selecting and applying the essential analytical tools for analysis of underground structures subjected to transient loading on the ground surface. The principal objectives of the research are as follows:

a) Development of a plane strain finite element model for nonlinear dynamic analysis of integrated structure-soil systems paying due attention to interface behaviour, nonlinear hysteretic behaviour of soil, separation in the interface zone or in the soil, appropriate base treatment depending on the field situation etc.

b) Utilization of the above analytical tool to analyze a series of systems comprising soil and structure or soil only to gain an insight into the response of such dynamic systems.
c) Investigation of the structural performance for variations in the relevant parameters viz. cover depth, stiffness of the structure, base conditions etc.

d) Determination of dynamic magnification of static stresses, if applicable, by comparing the results from dynamic analysis with that from static analysis performed with maximum magnitude of the shock wave pressure.

1.5 OUTLINE OF THE RESEARCH

The blast response of underground structures can be studied by a nonlinear finite element analysis. This has been achieved by a step-by-step numerical integration of the basic dynamic equilibrium equation (1.1). A recursive technique proposed by Rahman (19) has been used for this purpose. The technique is based on the well-known Newmark method (20) of numerical integration.

The system selected for analysis lends itself to a plane strain treatment. Finite elements used for the soil-structure system are two noded arc elements for structural representation and eight noded isoparametric elements for soil and interface representation. A kinematically consistent formulation is used to develop mass matrices of isoparametric elements, the masses of arc elements being lumped at the connecting nodes. Material properties of the structural part are assumed constant throughout the motion. Nonlinear material properties of soil and interface are represented by using variable parameters. The parameters are normally updated at the beginning of each time step, but may also be updated during the time step if any change in loading/unloading path is perceived. A rigid or viscous base is assumed depending on the soil condition. A viscous base is used when an infinite extent is to be simulated in the
Chapter 5 presents the features of the analyses performed in the present study. The code has been verified by comparison with published experimental and predicted results. The results obtained from a series of analyses for variable parameters have been critically examined in Chapter 6.

A FORTRAN code implementing the selected analytical method has been developed. A dynamic storage scheme is adopted to provide spaces for matrices and vectors of varying size depending on the size of the problem. The program is also applicable to nonlinear static analysis without loss of efficiency. Various features of the program are outlined in Chapter 4.

Chapter 5 presents the features of the analyses performed in the present study. The code has been verified by comparison with published experimental and predicted results. The results obtained from a series of analyses for variable parameters have been critically examined in Chapter 6.

Some difficulties have been faced during the course of analyses. These difficulties may be attributed to inadequacies of the material model and the material data adopted for soil. Moreover, there are some aspects of the problem which were not included in the present research due to its limited scope. Conclusions of the research together with suggestions for inclusion of further aspects of the problem in future studies are presented in Chapter 7.
2.1 INTRODUCTION

The success of an analytical model used to simulate a real field problem depends on appropriate selection of necessary tools. The tools necessary for blast response study of underground structures include a suitable scheme for solution of dynamic equilibrium equation, a suitable constitutive model to represent the nonlinear hysteretic behaviour of soil, a correct representation of the soil-structure interface characteristics and a suitable finite boundary model to simulate semi-infinite media. Various possibilities of selecting these tools are discussed in this chapter. It should be noted here that all of these models are not suitable for a generalized modelling. These have been devised for special applications in diversified fields of structural dynamics, soil mechanics and wave propagation. This chapter is an attempt to review only the related approaches in brief.

2.2 SOLUTION OF DYNAMIC EQUILIBRIUM EQUATIONS

The procedures that are common in solution of the dynamic equilibrium equations are divided in three categories: direct integration, mode superposition and complex response. References 21-23 present detail description on all of the methods.

In direct integration approach, equilibrium equations (1.1) are integrated by using a numerical step-by-step procedure making no transformation of the equations into a different
form prior to the numerical integration. In this method the
equilibrium is satisfied at discrete time intervals $\Delta t$
apart. A variation is assumed of the displacement, velocity,
and acceleration within each time interval $\Delta t$. Depending on
the form of this assumption, many direct integration
operators have been developed.

In the mode superposition technique, the equilibrium
equations (1.1) are transformed into a more effective form
prior to the use of any suitable numerical integration
scheme. The objective of such transformation is to reduce
the bandwidth of the original stiffness, mass and damping
matrices.

In the complex response method, the equilibrium equations
are transformed into the frequency domain by expressing the
applied loads and displacements in terms of their harmonics.

The choice of any one method from among these is influenced
by its numerical effectiveness and the nature of the
problem. The complex response method is efficient for
solution of linear systems subjected to harmonic loading.
But, applied loading is generally of nonharmonic nature for
most practical problems. The applicability of the modal
analysis is limited to linearly elastic systems and to cases
in which all forces applied to the system have the same time
variation. These are severe limitations for many practical
analyses. The direct integration approach allows for
nonlinear behaviour of the system by changing the
coefficient matrices of Eqn. 1.1 at each time step. Non-
uniform distribution of damping, if considered at all, can
also be accommodated by assigning variable damping
properties in each element. Only objection that arises
against the use of a direct integration scheme is that the
probability of inaccurate prediction of the response due to
high mode shapes cannot be eliminated. However, assuming
that in finite element analysis all important frequencies are predicted accurately, little response is calculated in the higher modes of the system (which are presumably not important) and by use of the finite element system, high frequency response will not seriously affect the accuracy of the solution.

2.3 DIRECT INTEGRATION METHODS

Numerous integration schemes are available for step-by-step solution of dynamic equilibrium equation. The attention of this article is focused on the behaviour of some well-known temporal operators only.

In the explicit forms of integration, displacement solution is computed explicitly from old informations. A lumped mass approximation is usually used and the set of equations are solved with virtually no computation. Widely known Central Difference Scheme (24) is considered to be the best of the explicit methods as it allows maximum time step size. However, no explicit dynamic integration method is unconditionally stable (25).

In the implicit schemes of integration, coefficient matrices of eqn. (1.1) appear with the unknown displacement, velocity and acceleration vectors. This fact requires a costlier solution of the set of equilibrium equations. But this apparent disadvantage of the implicit methods is overcome by their allowance for larger time steps than these for explicit schemes. Houbolt (26), Wilson (21) and Newmark (20) operators are the most widely used implicit integration schemes.

Analytical investigations for stability and accuracy of the temporal operators in linear system analysis are described
in references 21 and 27. All implicit schemes are claimed to be unconditionally stable. Among the wellknown operators Newmark method seems to be the most effective because of the smallest numerical errors and the largest allowable time steps.

On the stability and accuracy of temporal operators in nonlinear problems, Weeks (28) presented an interesting debate on the findings of different investigators who studied spatial nonlinear problems. In contrast to the results reported by Tillerson (29), he found that the Newmark and Houbolt operators were always stable if the Newton Raphson method was used to coverage to a nonlinear solution at each time step. It appeared from this study that the Newmark operator was more convenient and economical than the Houbolt operator. This finding was in contrast to that reported by Stricklin (30) in which the Houbolt operator was the preferred choice. Nickel (31) argued that the Newmark method was the most effective procedure for solving nonlinear equations provided that predictor-evaluation-correction algorithms were chosen. All of these studies with spatial nonlinearity imply that some form of iteration must be performed at each time step in order to achieve a stable analysis.

However, the behaviour of temporal operators has not been extensively studied for material nonlinearity which is a major concern of the current research topic. The only references known to the author for dynamic finite element analysis with nonlinear material behaviour are due to Farhoomand et. al (12) and McNamara et. al (32). Farhoomand et. al assumed a linear variation of acceleration over two consecutive time steps. The material nonlinearity was simulated by varying elastic parameters at each time step and no additional corrector device was applied to satisfy equilibrium equations at each time step. On the other hand, McNamara et. al used Houbolt operator for step-by-step
integration with elastic-plastic stress-strain relation for the material. This investigation showed that the standard incremental equations were not sufficient in the dynamic case and a correction was suggested to be applied in the form of an equilibrium check.

From all available evidences, it seems that the selection of an integration scheme for time domain solution is critical with respect to computational efficiency and stability. The constant-average-acceleration Newmark operator was found to be most effective for linear and geometric nonlinear problems. It can be expected that a similar solution would suffice for material nonlinearity also and so this research uses the Newmark method as its basis for step-by-step integration.

2.4 CONSTITUTIVE MODELS FOR SOIL

Soil is perhaps the most peculiar material for its mechanical behaviour. There are numerous factors e.g. density, water content, void ratio, loading history, particle characteristics, chemical contents etc., that have made its behaviour complex. Available models to simulate soil stress-strain behaviour are also numerous. But no one model can be considered a general constitutive model which is valid under all practical circumstances. Linear stress-strain relation is the simplest among all models, but soil is not a linear elastic material. This article, therefore, reviews only such models that simulate nonlinear properties of soil.

2.4.1 Nonlinear Elastic Models

Nonlinear elastic models have been used in conjunction with the finite element method for solving some boundary value
problems. In these models, the nonlinear stress-strain curve is discretized into a number of linear curves and by advancing the solution in a step-by-step fashion, an approximation to the actual nonlinear behaviour is achieved.

Many models have been proposed depending on the representation of the states of stress and strain and the nature of soil. In some approaches (as in reference 12), piecewise linear approach involves a tabular representation, in which a set of data points on the desired curve is stored and tangent modulii are computed by interpolation of that data. In other approaches, mathematical functions such as hyperbola, spline, polynomials etc. are used to represent nonlinear stress-strain curve. Kondner (33) stated a hyperbolic relation between stresses and strains. Duncan et.al (34) used this model with some modifications in finite element analysis. Desai (35) proposed the use of spline functions to simulate a set of stress-strain data. Rahman (36) used cubic spline function in static interaction study and compared the results with those obtained by experimental studies.

But the models stated above are valid for the cases of monotonically increasing load only. For the cases involving loading, unloading and reloading, these models are not reliable at all. In order to ensure nonlinear hysteretic behaviour of soil, Baron et. al (16) used a variable modulii model where an unloading/reloading path was added to a polynomial fitting for initial loading path. Application of the model was proved to be satisfactory in predicting ground motions due to blast effects. The computer implementation of the model is simple. The required parameters can be obtained from conventional laboratory test data. However, this cannot be treated or used as a general constitutive model. The model cannot account for volume change behaviour under shear.
2.4.2 Plasticity Models

The main physical feature leading to nonlinear behaviour of soil is its irrecoverability of strain which can be conveniently modelled by applying the plasticity theory (37). This fact has led to the development of many plasticity models for describing the soil behaviour. In these models an yield surface separates stress states which give rise to both elastic and irrecoverable plastic strains. To account for complicated processes like cyclic loading, the yield surface (or cap) may move in stress space kinematically. The concept of capped models for granular soils and rocks under dynamic loading was used by Dimaggio et.al (38), Baron et. al (16) and Zaman et. al (18). Baron et.al used this model in predicting ground motions caused by blast loads. The results obtained by using this model were compared with those obtained by other material models (elastic-plastic and variable modulii models). This did not imply any significant advantage of capped model over variable modulii model. However, the concept of plasticity models has been the subject of extensive research and debate for the last few years and observations are continually changing. Latest expositions may be found in Desai et.al (39) and Murthy et.al (40). The use of such models is, however, inefficient for practical purposes in the sense that to solve a practical problem the required computer time and storage will become prohibitively large.

2.5 CHARACTERIZATION OF SOIL-STRUCTURE INTERFACE

The response of an underground structure to dynamic loading can be influenced significantly by the characteristics of interfaces between the structure and surrounding soil. The assumption of complete bonding between the structure and its surrounding soil at all stages of loading, as used in
references (12) and (15) simplifies the analysis. But it cannot account for soil-structure interaction effects accurately; because the relative motions of the two are not included in the analysis. Under dynamic loading, relative movements such as sliding, separation or debonding, rebonding (Fig. 2.1) etc. can occur at interfaces during different stages of loading. In order to take account of such deformation modes along the interface, some models have been proposed in the recent past.

Goodman and John (41) modelled jointed rock masses by special joint elements consisting of two lines each with two nodal points.

Zaman et. al (18) presented a comprehensive review of some recent works related to dynamic interaction problems. With a view to devise a generalized model which is capable of representing the true interface behaviour, they proposed a thin layer interface element. The underlying idea of the thin layer element is based on the assumption that the behaviour near the interface involves a finite thin zone, rather than a zero thickness zone as assumed in several previous investigations. The accuracy and performance of the proposed element were demonstrated by analyzing a model nuclear containment structure subjected to strong ground motion generated by blasts. The results seem to be better than those obtained by analysis with complete bond assumption.

From all the evidences stated above, it is clear that a realistic representation of the interface behaviour is necessary for dynamic interaction study. Dynamic version of the thin layer interface element proposed by Zaman et. al(18) seems to be more appropriate than other models.
Fig. 2.1 Modes of Deformation

(a) Stick or no-slip

(b) Slip

(c) Debonding

(d) Rebonding
2.6 DAMPING AND BOUNDARY PROBLEMS

Dissipation of energy occurs in soil and other solids due to inelastic behaviour of the material. This dissipation of energy depends on the velocity of motion or strain and so a velocity dependent damping force is included in the dynamic equilibrium equation (1.1).

In practice it is difficult to determine the material damping parameters for general finite element assemblages. For this reason the material damping coefficient matrix is in general not assembled from element damping properties but is expressed as a percent of the critical damping value depending on experimental findings. Christian et. al (42) represented some formulations of such damping approximations. These arbitrary approximations of damping do not give any realistic representation of the problem. Mainly for this uncertainty in deriving a proper damping matrix, material damping is ignored in the present analysis. This fact is specially justified (43) for blast response study. Because the maximum response to an impulsive load will be reached in a very short time before the damping force can absorb much energy from the system.

However, there may be another component of damping depending on the boundary condition of the system. If a rigid rock is located at a reasonable depth from the surface, the finite element mesh can be extended upto that rigid boundary. But in case of semi-infinite soil mass, since the finite element discretization cannot be continued to infinity, a finite boundary must be assumed somewhere. The shock waves that generate from blasts will be reflected at those imposed boundaries and the results will be meaningless. So an energy dissipating device must be added to those imposed boundaries. Hall et. al (44) found that material damping due to non-ideal elastic behaviour of soil was in the region of
3 percent of the critical damping value, whereas the wave energy dispersion coefficients, simulated by some form of added energy dissipation device, were normally of the order of fifty percent of critical and swamped the effects of material damping. This finding strengthened the argument for inclusion of dispersion damping and rejection of material damping in the present analytical model.

Back et al. (45) used special foundation elements in the seismic design study of a double curvature arch dam. Introduction of these elements brought the theoretical and model experiment results into better agreement than that obtained by rigid boundary assumption. However, this type of boundary treatment cannot account for shock wave transmission.

Lysmer et al. (46) proposed a finite boundary model to approximate an infinite media. Energy dissipating dashpots were used along the boundary and the model was easily adaptable to finite element analysis. They applied the model to analyze a foundation problem subjected to a harmonic excitation and the assumed viscous boundary was found to be about 95% effective in absorbing the incident normal and shear waves. The model seems to be applicable to problems involving nonlinearity and transient loads if solved by step-by-step procedure.

Dumanoglu (17) proposed a mathematical model for dynamic soil-structure interaction analysis of embedded structures. He used transmitting boundaries to absorb wave emanating from the structure. The major fault of the proposed model is the high computation cost resulting from formulation of the problem by complex variables.
3.1 INTRODUCTION

The discussion in the previous chapter has thrown light on the present state-of-art of dynamic interaction analysis. An analytical model has been chosen by combining the most suitable techniques available in the literature for the structure-soil systems to be investigated in this research. The plane strain finite element program developed to study the blast response of underground structures includes the following essential features:

a) A recursive technique based on the Newmark method of integration is used for step-by-step solution of the dynamic equilibrium equation.

b) Structural part is represented by two-noded arc elements having three degrees of freedom (two displacements and a rotation) at each node. The material properties are kept constant throughout the time and the masses of the elements are lumped at the connecting nodes.

c) Eight-noded isoparametric elements with two degrees of freedom (two displacements only) at each node, have been used to discretize the surrounding soil. Nonlinear material behaviour is simulated by changing the elastic modulii at the end of each time step and also during the time step if it is necessary. Consistent formulation is used to develop the mass matrix of soil.

d) Eight-noded isoparametric elements having two degrees of freedom (two displacements only) at each node have been used
to represent the thin layer interface zone. The element is capable of representing deformation modes such as slip, debonding and rebonding. The elastic modulii of these elements are adjusted according to the changes in the magnitudes and modes of deformation of the elements and the behaviour of the adjacent soil element.

e) Provision is kept for selecting a rigid base or a viscous base - whichever is applicable. The coefficients of dashpots used to simulate the viscous boundary effects are assumed to be constant throughout the time.

These models and tools are selected on the basis of the comparative studies presented in Chapter 2. This chapter presents detailed expositions of the selected features.

3.2 RECURSIVE TECHNIQUE FOR NONLINEAR DYNAMIC ANALYSIS

The solution of dynamic equilibrium equation (1.1) can be obtained in a step-by-step numerical procedure by replacing the derivatives by their finite difference equivalents. Rahman (19) has proposed a transformation of the equation into a recursive relationship, enabling the displacement for any time to be obtained from the known displacements during the preceding time steps.

For the derivation of this recursive relationship, the total time is divided into a number of equal intervals of length, \( h \). It is assumed that in each interval the increase in displacements is directly proportional to the increase in forces, i.e. the stiffness matrix remains constant during the interval. The relationships of displacement, velocity and
acceleration, after Newmark (20), at the end of \((n+1)\)th time intervals are

\[
\ddot{x}_{n+1} = \ddot{x}_n + (h/2). (\dddot{x}_n + \dddot{x}_{n+1})
\]

(3.1)

\[
x_{n+1} = x_n + h.\dot{x}_n + (1/2 - \beta).h^2.\ddot{x}_n + \beta.h^2.\dddot{x}_n
\]

and, similarly, at the end of \(n\)th time interval

\[
\dot{x}_n = \dot{x}_{n-1} + (h/2). (\dddot{x}_{n-1} + \dddot{x}_n)
\]

(3.2)

\[
x_n = x_{n-1} + h.\dot{x}_{n-1} + (1/2 - \beta).h^2.\ddot{x}_{n-1} + \beta.h^2.\dddot{x}_n
\]

where the dotted quantities represent the time derivatives of displacement, \(x\), and \(\beta\), the parameter of generalized accelerations. Above equations assume that the velocity at the end of a time interval is equal to the velocity at the start of the interval plus the product of the mean acceleration during the interval and the length of the interval. In accordance with the fact that the increase in velocity during any time interval is equal to the area under the acceleration-time diagram, this assumption allows a wide range of variation of acceleration with time during any particular time interval. Some examples of such admissible variations are shown in Fig. 3.1.

It is interesting to note the correspondence between \(\beta\) and the variation in acceleration during the time interval. Although a physical relationship is not possible for all values, for at least four values of \(\beta\), it is possible to define consistent variations of acceleration in the time interval; for example, a choice of \(\beta = 1/6\) corresponds to a linear variation of acceleration (Fig. 3.1a); a choice of \(\beta = 1/4\) corresponds to a uniform value (constant average) of acceleration (Fig. 3.1c); a choice of \(\beta = 1/8\) corresponds to a step function (Fig. 3.1b); and a choice of \(\beta = 0\)
Fig. 3.1 Typical variations of acceleration within a time step
corresponds to double pulses of acceleration at the beginning and end of the time interval (Fig. 3.1d).

The equation of motion (1.1) can be written at time $t = (n+1)h$, $nh$ and $(n-1)h$, respectively as

$$M \ddot{x}_{n+1} + C \dot{x}_{n+1} + K_{n+1} x_{n+1} = f_{n+1}$$

$$M \ddot{x}_n + C \dot{x}_n + K_n x_n = f_n \quad (3.3)$$

$$M \ddot{x}_{n-1} + C \dot{x}_{n-1} + K_{n-1} x_{n-1} = f_{n-1}$$

The mass and viscosity coefficient matrices, $M$ and $C$ respectively, are constant throughout the motion and the stiffness matrix $K$ is a function of displacements and hence of time. Let $x_0$ denote the displacement at time $t = 0$ and $x_1$ that at $t = ih$. Let also $K_0$ designate the stiffness matrix when the displacement is zero (and not necessarily at $t=0$). The general equation of motion during the $n$th time interval can then be written for step-by-step solution scheme, as

$$M \ddot{x}_n + C \dot{x}_n + E_n + K_n \Delta x_n = f_n \quad (3.4)$$

where

$$E_n = K_0 x_0 + K_1 (x_1 - x_0) + \ldots + K_i (x_i - x_{i-1}) + \ldots + K_n (x_n - x_{n-1}) \quad (3.5)$$

and

$$\Delta x_n = x_n - x_{n-1}$$

The quantity $E_n$ denotes the vector of internal forces carried by the elements of the system at the beginning of the $n$th time interval. It should be noted here that, $E_0 = 0$, since this actually represents the internal forces carried by the system when displacements are zero (and not necessarily at time $t = 0$).
Rewriting all three equations of (3.3) by utilizing equations (3.4) and (3.5)

\[
M \ddot{x}_{n+1} + C \dot{x}_{n+1} + E_{n+1} + K_{n+1} \Delta x_{n+1} = f_{n+1}
\]

\[
M \ddot{x}_n + C \dot{x}_n + E_n + K_n \Delta x_n = f_n \quad (3.6)
\]

\[
M \ddot{x}_{n-1} + C \dot{x}_{n-1} + E_{n-1} + K_{n-1} \Delta x_{n-1} = f_{n-1}
\]

Multiplying the first and third of equations (3.6) by \(\beta h^2\) and the second by \(2(1/2 - \beta)h^2\), and adding them,

\[
\begin{align*}
\beta h^2 M [\beta \ddot{x}_{n+1} + 2(1/2 - \beta) \dot{x}_n + \beta \ddot{x}_{n-1}] \\
+ \beta h^2 C [\beta \dot{x}_{n+1} + 2(1/2 - \beta) \dot{x}_n + \beta \dot{x}_{n-1}] \\
+ h^2 [\beta K_{n+1} \Delta x_{n+1} + 2(1/2 - \beta) K_n \Delta x_n + \beta K_{n-1} \Delta x_{n-1}] \\
+ h^2 \beta [E_{n+1} + (1/\beta - 2) E_n + E_{n-1}]
\end{align*}
\]

\[
= h^2 \beta [f_{n+1} + (1/\beta - 2)f_n + f_{n-1}] \quad (3.7)
\]

The multiplier of \(C\) in equation (3.7) contains velocity terms and that of \(M\) contains acceleration terms. All other terms in that equation are related to displacements. The velocity and acceleration terms can be eliminated and eqn. (3.7) can be expressed entirely in displacement terms by utilizing the displacement-velocity-acceleration relationships of equations (3.1) and (3.2).

Eliminating acceleration terms by using the 2nd of eqns. 3.1 and 1st and 2nd of eqns. 3.2

\[
h^2 [\beta \ddot{x}_{n+1} + 2(1/2 - \beta) \dot{x}_n + \beta \ddot{x}_{n-1}] = \Delta x_{n+1} - \Delta x_n \quad (3.8)
\]

Eliminating velocity terms by using the 1st and 2nd of eqns. 3.1 and the 1st of eqns. 3.2

\[
h^2 [\beta \dot{x}_{n+1} + 2(1/2 - \beta) \dot{x}_n + \beta \dot{x}_{n-1}] = h(\Delta x_{n+1} + \Delta x_n)/2 \quad (3.9)
\]
Substituting relations (3.8) and (3.9) into equation (3.7),

\[
\begin{align*}
[M + (h/2).C + \beta h^2.K_{n+1}] \Delta x_{n+1} &= [M - (h/2).C - (1-2\beta)h^2.K_n] \Delta x_n - h^2.K_{n-1} \Delta x_{n-1} \\
- \beta h^2 [E_{n+1} + (1/\beta - 2) E_n + E_{n-1}] \\
+ \beta h^2 [f_{n+1} + (1/\beta - 2) f_n + f_{n-1}]
\end{align*}
\]

Eqn. 3.10 is the general recurrence formula. The right hand side vector of this equation can be further modified by using the relation 3.5. The general recurrence formula is then stated simply as,

\[
[M + (h/2).C + \beta h^2.K_{n+1}] \Delta x_{n+1} = F_{n+1}
\]

where,

\[
F_{n+1} = F_n - h.C \Delta x_n - h^2.E_{n+1} \\
+ \beta h^2 [f_{n+1} + (1/\beta - 2) f_n + f_{n-1}]
\]

The recurrence formula (3.11) can be solved for the increment of displacements during any time interval by using the known values of displacement increments and forces during two preceding intervals. It is obvious that, as in all initial value problems, the initial conditions and loads within the given range of time are necessary to start off the procedure. In dynamic structure-soil interaction problem, these initial informations comprise the initial velocity \(\dot{x}_0\) (at time \(t=0\)), initial displacement \(x_0\) (at time \(t=0\)) and the time history of ground motion and applied external forces at different nodes.

As observed in eqn. 3.10, the general difference formula contains displacement quantities at three successive intervals and no velocity term. The equation is not applicable at the beginning of the time range since this
will require displacement quantities before the first time interval. It is, therefore, necessary to eliminate such displacement quantities and introduce terms of initial velocity. This is conveniently done by deriving a special initial equation of motion similar to equation (3-10).

Writing the equation of motion (1.1) at times \( t=0 \) and \( t=h \),

\[
M \ddot{x}_0 + C \dot{x}_0 + K_0 x_0 = f_0 \tag{3.13}
\]

\[
M \ddot{x}_1 + C \dot{x}_1 + K_1 (x_1-x_0) + K_0 x_0 = f_1
\]

Multiplying the first of equations (3.13) by \((1/2 - \beta )h^2\) and the second by \( \beta h^2 \) and adding, them

\[
M \left[ \beta h^2 \dot{x}_1 + (1/2-\beta )h^2 \dot{x}_0 \right] + C \left[ \beta h^2 (\dot{x}_1-\dot{x}_0) + (h^2/2) \ddot{x}_0 \right] + \beta h^2 K_1 (x_1-x_0) + (h^2/2) K_0 x_0 = \beta h^2 f_1 + (1/2-\beta )h^2 f_0 \tag{3.14}
\]

Writing the 2nd of eqns. 3.2 at \( t=h \), \( n=1 \),

\[
x_1 = x_0 + h \dot{x}_0 + (1/2-\beta )h^2 \ddot{x}_0 + \beta h^2 \dddot{x}_1 \tag{3.15}
\]

Writing the 1st of eqns. 3.2 at \( t=h \), \( n=1 \),

\[
\dot{x}_1 = \dot{x}_0 + (h/2). (\dddot{x}_0 + \dddot{x}_1) \tag{3.16}
\]

Using the eqn 3.16, multiplier of \( C \) in eqn. 3.14 can be transformed to the following form:

\[
\beta h^2 (\dddot{x}_1-\dddot{x}_0) + (h^2/2) \dddot{x}_0 = (h/2). [(x_1-x_0) + 2(\beta -1/4)h^2 \dddot{x}_0] \tag{3.17}
\]

Substituting relations 3.15 and 3.17 into eqn. 3.14,

\[
[ M + (h/2). C + ( \beta h^2 ). K_1 ] \Delta x_1 = h.M \dddot{x}_0 - (h^2/2). E_1 + \beta h^2 . f_1
\]

\[
+ (1/2-\beta )h^2 . f_0
\]

\[
- (\beta -1/4)h^3 . \dddot{x}_0 . C \tag{3.18}
\]
This is the initial difference equation of motion required to start off the recursive procedure. This equation requires the acceleration quantity, \( \ddot{x}_0 \) at time \( t = 0 \), which is not known. It will be seen later that a value of \( \beta = 1/4 \) is desirable for stability of the solution scheme. So, the term containing acceleration in equation (3.18) will be automatically eliminated. But for values of \( \beta \) other than \( 1/4 \), that term remains in the analysis. In that situation, acceleration, \( \ddot{x}_0 \) is to be obtained by solving the first of equations (3.13), where \( \ddot{x}_0 \) is the only unknown.

For the special case when all initial quantities are zero, i.e., \( \dot{x}_0 = 0 \), \( x_0 = 0 \), and \( f_0 = 0 \) at time \( t = 0 \), stiffness matrices \( K_0 \) and \( K_1 \) are identical and equation (3.18) then reduces to,

\[
\left[ M + \left( \frac{h}{2} \right) C + \beta h^2 K_1 \right] \Delta x_1 = \beta h^2 f_1
\]  

(3.19)

Using any of the start up equations (3.18) or (3.19) as applicable, the solution can be started and then using equation 3.11, solution for subsequent time intervals can be obtained by marching through the time.

3.3 STABILITY OF THE SOLUTION SCHEME AND THE TIME STEP SIZE

Stability of an integration method means that the initial condition for the equation must not be amplified artificially thus making the integration worthless. It is evident from the previous section that the recursive technique is dependent on the selection of parameter, \( \beta \) and the length of time interval, \( h \). Equation (3.18) shows that approximation of initial acceleration can be avoided by selecting a value of \( \beta = 1/4 \). Considering the various patterns of the acceleration variations and the consequent magnitude of artificially induced damping, Newmark (20) stated that reasonable results might be obtained with values
of $\beta$ in the range from $1/6$ to $1/4$ and theoretically infinite stability can be achieved by selecting a value of $\beta=1/4$. This means that with a value of $\beta=1/4$, the solution is expected to be stable with any value of the time step size. However, use of a larger time step will induce larger approximation errors. The selection of the most appropriate length of time interval is largely a matter of experience. Usually, this is determined by the shortest period of vibration of a multi-degree-of-freedom system. The shortest period can be selected by an eigen-solution of the system equilibrium equations. This is a cumbersome procedure and practically the time step is selected on trial basis from considering a balance between the cost of analysis and the accuracy of prediction.

3.4 DETERMINATION OF STIFFNESS AND MASS MATRICES AND BODY FORCE VECTOR

As stated earlier three types of elements: (i) two-noded arc element, (ii) eight-noded isoparametric element for soil, and (iii) eight-noded isoparametric element to represent interface, have been utilized in the present analytical process. These elements are discussed separately for their distinct features. Element stiffness matrices are determined by usual finite element practice. Masses of the arc elements are lumped at the connecting nodes. But the element mass matrices for soil and interface elements are determined by using the consistent formulation. In many past investigations, consistent formulation was avoided on the pretext of increased computational cost. But the use of lumped mass approximation in implicit integration of dynamic equilibrium equation, does not relieve the efforts for solving the set of equations. Although the lumped approximation relieves some computation during the development of the global mass matrix, the efforts involved
in repetitive development of stiffness matrix far exceeds that needed to develop the mass matrix only once. Body force vector, needed to find the dead load stresses in the system, is also developed by using shape functions. Following subsections give major development stages of the various matrices involved in the analysis.

3.4.1 Two Noded Arc Element

The flexibility matrix of a uniform member forming a circular arc (Fig. 3.2) in local coordinates is given by Livesley (47) as follows:

\[
[F] = \begin{bmatrix}
a & 0 & d \\
0 & b & 0 \\
d & 0 & c
\end{bmatrix}
\]

(3.20)

where

\[
a = R/EA(\phi + Sin\phi Cos\phi) + R^3/EI(\phi (1 + 2 Cos^2\phi) - 3 Sin\phi Cos\phi)
\]
\[
b = R/EA(\phi - Sin\phi Cos\phi) + R^3/EI(\phi - Sin\phi Cos\phi)
\]
\[
c = 2\phi R/EI
\]
\[
d = 2 R^2(Sin\phi - Cos\phi)/EI
\]

The stiffness matrix, \( K' \) in local coordinates, can be obtained by inverting the flexibility matrix and the stiffness matrix, \( K \) in global coordinates can then, be obtained from the following equation:

\[
[K] = [T]^T [K'][T]
\]

(3.21)

where \([T]\) is the transformation matrix.

The apparent young's modulus, \( E' \), for plane strain case, \( E' = E/(1 - \nu^2) \), is used in the analysis. Here \( E \) denotes the modulus of elasticity for simple tension and \( \nu \) denotes the Poisson's ratio. The mass matrix and the body force vector for an arc element is obtained by lumping its mass and weight at the connecting nodes.
Fig. 3.2 Two noded arc element
3.4.2 Isoparametric Element for Soil

For plane elements, the stiffness matrix in global coordinates can be obtained as

\[
K = t \int [B]^T [D] [B] \, d(\text{area}) \tag{3.22}
\]

where

\([B] = \text{strain-displacement transformation matrix}\)

\([D] = \text{elasticity matrix}\)

\(t = \text{thickness of the element}\).

For plane strain case, the [D] is given by

\[
[D] = \begin{bmatrix}
K + 4G/3 & K - 2G/3 & 0 \\
K - 2G/3 & K + 4G/3 & 0 \\
0 & 0 & G
\end{bmatrix} \tag{3.23}
\]

where \(K\) and \(G\) imply bulk modulus and shear modulus, respectively.

In finite element analysis, the [B] is obtained as

\[
[B] =
\begin{bmatrix}
\partial N_1 & \partial N_2 & \partial N_3 & \partial N_4 & \partial N_5 & \partial N_6 & \partial N_7 & \partial N_8 \\
\partial x & \partial x & \partial x & \partial x & \partial x & \partial x & \partial x & \partial x \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\partial y & \partial y & \partial y & \partial y & \partial y & \partial y & \partial y & \partial y \\
\partial N_1 & \partial N_2 & \partial N_3 & \partial N_4 & \partial N_5 & \partial N_6 & \partial N_7 & \partial N_8 \\
\partial y & \partial x & \partial y & \partial x & \partial y & \partial x & \partial y & \partial x \\
\partial N_1 & \partial N_1 & \partial N_2 & \partial N_2 & \partial N_3 & \partial N_3 & \partial N_4 & \partial N_4 \\
\partial y & \partial x & \partial y & \partial x & \partial y & \partial x & \partial y & \partial x \\
\partial N_1 & \partial N_1 & \partial N_1 & \partial N_1 & \partial N_2 & \partial N_2 & \partial N_3 & \partial N_3 \\
\partial y & \partial x & \partial y & \partial x & \partial y & \partial x & \partial y & \partial x
\end{bmatrix}
\tag{3.24}
\]

where \(N_1, N_2, \text{etc.}\) are shape functions for nodes 1, 2, etc.
Shape functions in local coordinates (Fig. 3.3), as given by Irons and Ahmad (48) are

\[
\begin{align*}
N_1 & = (1 - \xi)(1 - \eta)(-\xi - \eta -1)/4 \\
N_2 & = (1 - \xi^*)(1 - \eta)/2 \\
N_3 & = (1 + \xi)(1 - \eta)(\xi - \eta -1)/4 \\
N_4 & = (1 + \xi^*)(1 - \eta^*)/2 \\
N_5 & = (1 + \xi)(1 + \eta)(\xi + \eta -1)/4 \\
N_6 & = (1 - \xi^*)(1 + \eta)/2 \\
N_7 & = (1 - \xi)(1 + \eta)(-\xi + \eta -1)/4 \\
N_8 & = (1 - \xi^*)(1 - \eta^*)/2
\end{align*}
\]

The cartesian derivatives in eqn. 3.24 can be obtained by transformation of the local derivatives of the above shape functions to global derivatives.

The mass matrix, \( m \) of an isoparametric element is given as

\[
m = \rho t \int N^T N \ d(\text{Area}) \tag{3.25}
\]

where \( N \) represents the shape functions as given earlier and \( \rho \) is the mass density.

The body force vector can be determined as

\[
R_b = t \int N^T f_b \ d(\text{Area}) \tag{3.26}
\]

Integrals in eqns. 3.22, 3.25, and 3.26 can be evaluated by numerical integration.

3.4.3 Interface Element

Schematic diagram of a thin layer interface element adopted for use in the present study is shown in Fig. 3.4. The element can be treated like any other solid element by adopting appropriate constitutive laws and strain-displacement transformation relation.
Fig. 3.3 Eight noded isoparametric element
Fig. 3.4 (a) Idealization of interface
(b) Interface element
Since the interface element is capable of transferring only a normal stress and shear stress, eqn. 3.23 is modified as

\[
[D]_i = \begin{bmatrix}
4G_i/3 & -2G_i/3 & 0 \\
-2G_i/3 & K & 0 \\
0 & 0 & G_i
\end{bmatrix}
\] (3.27)

where \( K \) is the bulk modulus as obtained for the adjacent soil element and \( G_i \) is the shear modulus of the interface element. Since the normal and shear stresses of interface are desirable in local directions, the \([B]\) matrix is derived such as to give local strains directly from global displacements.

For the local orthogonal axes \( x' \) and \( y' \) (Fig. 3.4), the local strains are given in terms of local displacements as

\[
\begin{bmatrix}
\varepsilon_{x'} \\
\varepsilon_{y'} \\
\varepsilon_{x'y'}
\end{bmatrix} = \begin{bmatrix}
\partial u' / \partial x' \\
\partial v' / \partial y' \\
\partial u' / \partial y' + \partial v' / \partial x'
\end{bmatrix}
\] (3.28)

Using transformation rule,

\[
\begin{bmatrix}
\partial u' / \partial x' \\
\partial v' / \partial y'
\end{bmatrix} = [T]^{-1} \begin{bmatrix}
\partial u / \partial x \\
\partial v / \partial x
\end{bmatrix}
\] (3.29)

where

\[
[T]^{-1} = [T]^{-1}; \quad [T] \text{ being the matrix of direction cosines of } x' \text{ and } y' \text{ axes. Parametrically these can be obtained at any value of } \xi \text{ and } \eta, \text{ noting that directions } x' \text{ and } \xi \text{ coincide, as}
\]
\[
[T] = \begin{bmatrix}
\cos \phi & \sin \phi \\
-S\sin \phi & \cos \phi
\end{bmatrix}
= \begin{bmatrix}
\partial x'/\partial \xi & \partial y'/\partial \xi \\
\partial y'/\partial \xi & \partial x'/\partial \xi
\end{bmatrix}
\]

Thus,
\[
[\Theta] = \frac{1}{\sqrt{(\partial x'/\partial \xi)^2 + (\partial y'/\partial \xi)^2}} \times \begin{bmatrix}
\partial x'/\partial \xi & \partial y'/\partial \xi \\
\partial y'/\partial \xi & \partial x'/\partial \xi
\end{bmatrix}
\]  \quad (3.30)

Assigning
\[
\partial x'/\partial \xi = a
\]
\[
\sqrt{(\partial x'/\partial \xi)^2 + (\partial y'/\partial \xi)^2} = a
\]

and
\[
\partial y'/\partial \xi = b
\]
\[
\sqrt{(\partial x'/\partial \xi)^2 + (\partial y'/\partial \xi)^2} = b
\]

Eqn. 3.29 becomes
\[
\begin{bmatrix}
\partial u'/\partial x' & \partial v'/\partial x' \\
\partial u'/\partial y' & \partial v'/\partial y'
\end{bmatrix}
= \begin{bmatrix}
a(-a \partial u/\partial x + b \partial u/\partial y) & -b(-a \partial u/\partial x + b \partial u/\partial y) \\
+a(-b \partial v/\partial x + a \partial v/\partial y) & +b(-b \partial v/\partial x + a \partial v/\partial y)
\end{bmatrix}
\]

\[
\begin{bmatrix}
a(-b \partial u/\partial x + a \partial u/\partial y) & -b(a \partial u/\partial x + b \partial u/\partial y) \\
+a(-b \partial v/\partial x + a \partial v/\partial y) & +b(a \partial v/\partial x + b \partial v/\partial y)
\end{bmatrix}
\]  \quad (3.32)
Equation 3.28 may be rewritten as

\[
\{ \epsilon' \} = [B] \{ d \}
\]

where \( \epsilon' \) is the local strain vector and \( d \) is the global displacement vector.

Then from term by term comparison of relations 3.28 and 3.32 following local strain - global displacement transformation matrix is obtained:

\[
[B] = \begin{bmatrix}
\partial N_j & \partial N_j & \partial N_j & \partial N_j \\
(a^2 \ldots + ab \ldots) & (ab \ldots + b^2 \ldots) & \partial N_j & \partial N_j \\
\partial x & \partial y & \partial x & \partial y \\
\partial N_j & \partial N_j & \partial N_j & \partial N_j \\
(b^2 \ldots - ab \ldots) & (-ab \ldots + a^2 \ldots) & \partial x & \partial y \\
\partial x & \partial y & \partial x & \partial y \\
\partial N_j & \partial N_j & \partial N_j & \partial N_j \\
(-ab \ldots + a^2 \ldots) & (-b^2 \ldots + ab \ldots) & \partial x & \partial y \\
\partial x & \partial y & \partial x & \partial y \\
\partial N_j & \partial N_j & \partial N_j & \partial N_j \\
(-ab \ldots - b^2 \ldots) & (+a^2 \ldots + ab \ldots) & \partial x & \partial y \\
\partial x & \partial y & \partial x & \partial y \\
\end{bmatrix}
\]

Having \([B]\) matrix known from eqn. 3.33, the element stiffness matrix can be derived by numerical integration of eqn. 3.22. The derivations of mass matrix and body force vector for the interface elements are same as those for soil elements.

### 3.5 Determination of Elastic Force Vector

The elastic forces carried by a system at any time step, \( n \), of time integration, as given by eqn. 3.5, can be written as
The same expression in terms of the nodal displacements is

\[ E_n = \sum_{i=1}^{n-1} K_i \Delta x_i \quad (3.34) \]

where \( K_i \) denotes the stiffness of the system at any step \( i \) and \( \Delta x_i \) the increment of displacement during that step. Elastic forces carried by the system after \( n \)th step may be computed as

\[ E_{n+1} = E_n + K_n \Delta x_n \quad (3.35) \]

This approach of determining the elastic force vector \( E \) requires that the stiffness matrix \( K_n \) be stored or reassembled for use at the beginning of next step.

But there may be another way of determining the elastic force vector from element levels. The elastic forces carried by an arc element can be obtained as

\[ \{ E \}_e = [K]_e \{ d \} \quad (3.36) \]

The contribution of isoparametric elements to elastic forces can be determined from virtual work consideration. Expression of virtual work is

\[ W = \int \varepsilon \sigma \ dv \quad (3.37) \]

The same expression in terms of the nodal displacements is

\[ W = \{ d \}^T \{ E \} \quad (3.38) \]

Using the relation \( \{ \varepsilon \} = [B] \{ d \} \), where \( [B] \) is the strain-displacement transformation matrix as derived by using equations (3.24) or (3.33), equation (3.37) can be modified as

\[ W = \{ d \}^T \int [B]^T \sigma \ dv \quad (3.39) \]
Now comparing the equation (3.38) and (3.39),

\[
(E) = \int [B]^T \{ \sigma \} \ dv
\]

or for plane-elements with constant thickness, \( t \),

\[
(E) = t \int_{\text{area}} [B]^T \{ \sigma \} \ d(\text{area}) \tag{3.40}
\]

At the end of each time step in the recursive solution technique presented in section 3.2, the elastic force vector \( (E) \) can be assembled from element levels by using the updated stress vector \( \{ \sigma \} \). The efficiency of the approach depends on the number of integration points used for numerical evaluation of the integral 3.40. In order to ascertain the applicability of the approach, one point integration was used to evaluate the integral. Transformation matrix \( [B] \) and the element stresses \( \{ \sigma \} \) were calculated at the centroid of the elements and the contribution of the elements to global force vector was found simply as

\[
[E]_e = [B]^T \{ [\sigma] \} \times \text{volume}
\]

\[
\zeta = 0 \quad \xi = 0
\]

\[
\eta = 0 \quad \eta = 0
\]

But the results obtained by single point integration of the eight-noded isoparametric elements became unstable. The results could have been improved by using more than one integration points. But this would require more computation. Considering all these facts, the elastic force vector is decided to be determined by using equation 3.35 , for which the stiffness matrix will have to be stored for latter use.
### 3.6 VARIABLE MODULII FOR SOIL

In the variable modulii (VM) model, the constitutive relations are separated into a deviatoric and a volumetric part by ignoring the volume change due to shear effects. The incremental stress-strain relation, given by Desai et al. (39), is

\[
d \sigma_{ij} = K \, d \varepsilon_{ij} \delta_{ij} + 2G \left( d \varepsilon_{ij} - \frac{1}{3} d \varepsilon_{kk} \delta_{ij} \right) \quad (3.41)
\]

The bulk modulus \( K \) and shear modulus \( G \) are defined as the functions of stress or strain invariants, or of both. The specific relations are established from experimental findings. Farhoomand et al. (12) reported some results obtained from standard tests conducted at Vicksburg, Mississippi. These test results are applicable to small range loading as used by them to study the blast response of a model pipe buried into a container of soil. Baron et al. (16) used a variable modulii model to predict the effects of one MT (megaton) air burst on McCormick Ranch Sand. The parameters given in the publication were obtained from a comprehensive series of uniaxial, hydrostatic and triaxial compression tests under static and cyclic loads. These parameters were essentially derived for relatively larger range stress-strain study. The model used in the present investigation is based on the published test results.

#### 3.6.1 Bulk Modulus

Bulk modulus \( K \) relates the mean stress to volumetric strain. For hydrostatic compression test, eqn. 3.41 yields

\[
K = \frac{dP}{d\varepsilon_v} \quad (3.42)
\]
where $dP$ and $d\varepsilon_v$ are increments of mean pressure and volumetric strain respectively.

Thus the bulk modulus $K$ can be obtained by fitting a suitable polynomial to $P$ vs. $\varepsilon_v$ data. It is evident from figures 3.5 and 3.6 that a third degree polynomial will suffice to fit the initial loading part. So the following polynomial is adopted

$$P = a_1 \varepsilon_v + a_2 \varepsilon_v^2 + a_3 \varepsilon_v^3$$

(3.43)

where $a_1$, $a_2$, and $a_3$ are arbitrary constants.

From a drained hydrostatic compression test, a series of data may be obtained by measuring the volumetric strain (=change in volume/total volume) for different confining pressures ($P$). The constants in eqn. 3.43 can be obtained by using least square fitting technique.

Then the bulk modulus, $K$ for initial loading part may be obtained for any value of $\varepsilon_v$ as

$$K = a_1 + 2a_2 \varepsilon_v + 3a_3 \varepsilon_v^2$$

(3.44)

For volumetric behaviour under hydrostatic pressure, unloading and reloading are simulated by using the same path. For McCormick Ranch sand, the unloading/reloading bulk modulus was proposed to be

$$K = K_0 + K_1 P , \quad P < P_m$$

$$= K_{m_1} , \quad P > P_m$$

(3.45)

where $P_m$ was a transition pressure.

Such unloading paths are shown in Fig. (3.7). Farhoomand et al (12) used bilinear unloading/reloading paths (Fig.
Fig. 3.5 McCormic Ranch sand: static hydrostat, loading only (Ref. 39)
Fig. 3.6 Stress–Strain Curve (Vicksburg's Test) Ref. 12
Fig. 3.7 Typical Unloading Curves for McCormic Ranch Sand.
3.8) leading to a residual volumetric strain of 87.4% of the total strain. Unloading curves for McCormick Ranch Sand (Fig. 3.7), fitted with the parameters given in ref.16, show that the residual strain varies between 84% to 86% of the total initial strain. In the present study, an unloading/reloading path has been defined to be a straight line leading to a residual strain of desired percentage of the total strain. This linear assumption for unloading is particularly justified for cyclic loading where bilinear or curved unloading/reloading path may lead to erroneous results. Difficulties faced during a supplementary investigation by the author with a bilinear assumption is illustrated in Fig. 3.9. However, if the loading steps are small enough to follow a correct path, relations 3.45 may be used and provision for using these relations has been kept in the model.

3.6.2 Shear Modulus

If an undrained triaxial test is performed i.e. no volume change is allowed during the test, the incremental stress-strain relation (3.41) yields

\[ d \sigma_m = 2G \, d\varepsilon \]

(3.46)

where deviatoric stress increment, \( d \sigma_m \), is related to the vertical strain increment, \( d\varepsilon \), by shear modulus, \( G \).

Desai et al (39) presented a series of deviatoric stress vs. strain curves obtained from triaxial compression tests under different confining pressures. It was evident from that tests that the state of deviatoric stresses had an influence on the shear behaviour of soil. Again confining pressure (P) also influenced the shear behaviour. The data describing the shear modulus - mean pressure relation, as used by
Fig. 3.8 Bi-linear Unloading Paths for Soil (Vicksberg Test)
Fig. 3.9 Unbound Straining in Soil When a Bi-linear Unloading/Reloading Path is used.
Farhoomand et al (12) for small range loading, are plotted in Fig. 3.10. Considering all these published test results, a general model describing the shear behaviour of soil is given as

\[ G = G_0 + b_0 J_{2D} + b_1 P + b_2 P^2 + b_3 P^3 \]  

(3.47)

where, \( J_{2D} \) is the second invariant of deviatoric stress tensor, \( G_0 \) the initial shear modulus, and \( b_0, b_1, b_2, \) and \( b_3 \) are arbitrary constants. All of these constants can be obtained from available test results by least square fitting technique.

The proposed model (eqn. 3.47) is applicable to both Vicksburg test data (where the effect of deviatoric stresses is ignored, i.e. \( b_0=0 \)) and McCormick Ranch Sand data of ref.16 (where the effect of mean pressure is limited to second order effects i.e. \( b_3=0 \)).

The loading, unloading and reloading in shear may be determined by observing the change in second deviatoric stress invariant \( (J_{2D}) \). Since, no data on unloading/reloading shear behaviour is available in the published texts and journals and no test was conducted due to limited scope of the present research, unloading and reloading in shear is ignored for the present investigation. That means the shear modulus is adjusted with the change in mean pressure only.

3.7 MODULII FOR INTERFACE ELEMENTS

The constitutive matrix (eqn. 3.27) for thin layer interface element consists of two parts - one corresponding to the normal behaviour of the interface element and the other
Fig. 3.10 Influence of mean pressure on shear modulus as found in Vicksburg test (ref. 12)
corresponding to the shear behaviour. Satisfactory results were obtained (ref. 18) by assigning the same property as the surrounding soil for the interface normal behaviour. In the present study the bulk modulus $K$ in eqn. 3.27 is assigned to be same as that for adjacent soil element. The shear modulus, $G_i$, in that equation can be obtained from a direct shear test. The functional relation (3.48), as proposed by Zaman et.al (18), has been used in the analyses.

$$
\tau = \alpha_i + \alpha_2 u_r + \alpha_3 u_r^2
$$

(3.48)

where

$$
\alpha_i = (\beta_1) + (\beta_2) + (\beta_3) + (\beta_4) N + (\beta_5) N^2
$$

$N$ being the number of cycles.

and $u_r$ = relative slip between the two bodies.

Coefficients $\alpha_i$ and $\beta_i$ can be evaluated from laboratory tests.

### 3.8 APPLICATIONS OF THE CONSTITUTIVE MODELS FOR SOIL AND INTERFACE ELEMENTS

The bulk and shear modulii for soil are updated at the end of each time step depending on the current state of stresses and strains. An element is assumed to be cracked if any of the following condition is met:

i) Total mean compression $(P + \delta P_0) < 0$; where $P$ is the current mean pressure, $P_0$ the initial mean pressure due to dead loads and $\delta$ possesses a value of 0 or 1 depending on whether the element was cracked or uncracked, respectively at the previous step,

ii) volumetric compressive strain $\varepsilon_v < 0$
The bulk modulus of a cracked element is reduced to zero and the shear modulus is taken as one-tenth of the initial value. A closure of the crack is assumed when incremental mean pressure, \( dP \) is compressive and the volumetric strain \( \varepsilon_v \) is also compressive. For a new element (which was cracked earlier) the volumetric strain at the instant of crack closure is assumed to be a residual strain and the loading bulk modulus is determined on the basis of the additional strains. The shear modulus is determined on the basis of the mean pressure state.

The modulii may be updated during the time step too. If an element, previously assumed to be on loading path signifies unloading or a previously assumed cracked element tends to reload by closure of crack, the modulii are adjusted accordingly and the solution at that time step is repeated.

The value of bulk modulus \( K \), for an interface element, is taken to be the same as that for the adjacent soil element. The shear modulus for the element in stick mode is determined by considering the history of shear behaviour. If a normal tension is developed in the interface element, the value of \( K \) is taken to be zero regardless of the stress-strain condition in adjacent soil element and the shear modulus of the element at that time is taken to be one-tenth of the initial modulus. A separation or debonding is assumed when \( (\sigma_n + \delta \sigma_n) > 0 \); where \( \sigma_n \) is the current normal stress, \( \sigma_0 \) the initial normal stress due to dead weights, and \( \delta \) possesses a value of 0 or 1 depending on whether the element was separated or not in the previous step. A rebonding is assumed when incremental normal stress \( \Delta \sigma_n \) is compressive.
3.9 DERIVATION OF VISCOUS DAMPING COEFFICIENTS MATRIX

Lysmer et al. (46) expressed the viscous boundary conditions (Fig. 3.11), as

\[ \sigma = \rho \frac{\partial v}{\partial t} \]
\[ \tau = \rho \frac{\partial u}{\partial t} \]

where \( \sigma \) and \( \tau \) are normal and shear stresses respectively; \( \frac{\partial v}{\partial t} \) and \( \frac{\partial u}{\partial t} \) are normal and tangential velocities respectively, \( \rho \) is the mass density, and \( V_s \) and \( V_p \) are the velocities of reflected shear and normal waves generated by the incident wave. These velocities are given as

\[ V_s = \sqrt{G / \rho} \]  \hspace{1cm} (3.51)

and \[ V_p = V_s / S \]  \hspace{1cm} (3.52)
In these equations $G$ is the shear modulus and $S$ is an elastic constant defined by

$$S^2 = 0.5 \times (1 - 2 \nu)/(1 - \nu)$$  \hspace{1cm} (3.53)

in which $\nu$ is the Poisson's ratio

The viscous damping forces per unit area at any point on the surface is, thus, given as

$$\begin{bmatrix} \rho V_s & 0 \\ 0 & \rho V_p \end{bmatrix} \begin{bmatrix} \partial u/\partial t \\ \partial v/\partial t \end{bmatrix} = \begin{bmatrix} \tau \\ \sigma \end{bmatrix}$$  \hspace{1cm} (3.54)

Assuming the velocities to be constant along an element boundary, total damping forces on the element boundary for unit thickness are:

$$\begin{bmatrix} \rho V_s L & 0 \\ 0 & \rho V_p L \end{bmatrix} \begin{bmatrix} \partial u/\partial t \\ \partial v/\partial t \end{bmatrix} = \begin{bmatrix} \tau L \\ \sigma L \end{bmatrix}$$  \hspace{1cm} (3.55)

These viscous damping coefficients are lumped to the boundary nodes of the element at a ratio of 1:4:1. Thus a diagonal matrix, $C$ is formed - containing viscous damping coefficients corresponding to the displacement degrees of the boundary nodes only.

The material parameters ($G$ and $\nu$), for viscous boundary are assumed to be the same as those of the adjacent soil elements in the interior zone. These parameters are variable for soil. But as the dashpots or in other words the lumped
viscous coefficients are assumed to be constant in the derivation of the recursion formula (section 3.2), reasonable constant values of the parameters should be chosen. In the present analysis, initial modulii of soil have been used to determine the dashpot coefficients.

3.10 SUMMARY OF THE STEP-BY-STEP INTEGRATION SCHEME

The solution procedure designed so far may be summarized in the following steps:

1) The initial stiffness matrix \( K_0 \), when displacement \( x_0 = 0 \), mass matrix \( M \) and a body force vector \( R_b \) due to gravitational effect, are developed.

Equations \( K_0 x = R_b \) are solved and the initial forces, stresses or moments due to dead weights are calculated and stored for each element.

2) Viscous damping coefficients matrix \( C \) is developed if a viscous boundary approximation is essential.

3) For a special start-up, an initial force vector, \( f_i \) is computed as \( f_i = h.M \dot{x}_0 - (h^2/2).K_0 x_0 + (1/2 - \beta )h^2 f_o \); where \( x_0, \dot{x}_0, \) and \( f_o \) are initial vectors of nodal displacement, velocity and force respectively at time \( t = 0 \). If all these quantities are zero, \( f_i = 0 \).

4) Step by step solution is started at this stage in the following sequence:

   a) External force vector \( f_{n+1} \) \( (n+1, \) implies the current number of time step) is determined and the effective load vector is computed as \( F_{n+1} = f_i + \beta h^2 f_{n+1} \).
b) The stiffness matrix $K_{n+1}$ is assembled from the element stiffness matrices derived on the basis of the modulii forecast for the new stage of analysis. The effective coefficient matrix $A_{n+1} = M + (h/2)C + \beta h^2 K_{n+1}$ is determined and the set of equations, $A_{n+1} \Delta x_{n+1} = F_{n+1}$, is solved to find the incremental displacement vector $\Delta x_{n+1}$.

c) Incremental strains and stresses in soil and interface elements are computed and by adding these quantities with the total quantities, a check is made on whether the element loading condition is following the predetermined path or not. If the test indicates that a wrong prediction was made and if there is provision for repetitive solution, the modulii are adjusted and step (b) is repeated with the new modulii. Repetition may be limited by the number of such iterations.

d) Total displacements, stresses in the soil and interface elements, and the forces and moments in the structural elements are updated. Modulii for soil and interface elements are updated on the basis of the current state of stresses and strains for use in the next step.

e) Elastic force vector $E_{n+1}$ is updated and an initial force vector $f_i = F_{n+1} - h \cdot C \Delta x_{n+1} - h^2 E_{n+1} + \beta h^2 [(1/\beta - 2)f_{n+1} + f_{n}]$ is computed for the next step of solution.

5) Step 4 is now repeated.
CHAPTER 4
THE FINITE ELEMENT COMPUTER PROGRAMS

4.1 INTRODUCTION

The essential features of dynamic interaction study have been described in Chapters 1 and 2. Chapter 3 has described the finite element idealization of the problem, the recursive technique adopted for solution of dynamic equilibrium equation, and the methods to incorporate nonlinear soil behaviour, soil-structure interface characteristics, and a viscous boundary to simulate the infinite extent of soil. Special features of the computer programs developed to investigate the blast effects on soil and underground structures by implementing the above mentioned tools and methods are presented in this chapter.

The program is general in nature and can be used to solve plane-strain structure-soil interaction problems for both static and dynamic loading. The program discussed here is written in FORTRAN77 and has been tested and run on a microcomputer having 80286 microprocessor at the microcomputer laboratory of the Department of Civil Engineering in B.U.E.T. It was also tested on the IBM 4331-K02 mainframe at the computer centre of B.U.E.T. The program uses a few ancillary subroutines originally developed by Farhoomand and Wilson [12] for handling axisymmetric finite elements and modified by the author.

4.2 DETERMINATION OF MATERIAL PARAMETERS

Parameters governing the stress-strain behaviour of materials related to the problem are input through a subroutine MATDTA at the beginning of the program.
Properties are read separately for all different materials. The basic material models that can be handled by the present program are linear elastic model, variable modulii model for soil and a special variable modulii model for interface elements. Different types of soil can be included in the same problem by providing separate series of parameters for each soil type. The variable modulii model simulating the hysteretic stress-strain behaviour of soil has been outlined in Chapter 3. The parameters required for implementation of the model may be input directly or derived from the supplied data by using least square fitting technique. The parameters in equation 3.43 describing the volumetric behaviour of soil can be derived from a set of mean pressure-volumetric strain data. And the parameters of equation 3.47 describing the shear behaviour of soil can be obtained from a set of shear modulus-mean pressure data. The unloading path in volumetric behaviour may be defined by assigning a suitable ratio of the strain offset at zero mean pressure level to the total initial strain. However, a more generalized model as applied by Baron et.al (16) to McCormick Ranch Sand may also be used by providing the parameters that describe loading and unloading/reloading in both volumetric behaviour and shear behaviour.

The parameters defining the variable shear modulus (eqn. 3.48) for interface elements are input directly. The elastic model can be described by providing modulus of elasticity and Poisson's ratio for two-noded arc elements or bulk modulus and shear modulus for eight-noded isoparametric elements, if applicable.

4.3 SEMI-AUTOMATIC DATA GENERATION SCHEME

To reduce the manual effort in data preparation, a semi-automatic data generation scheme is employed by calling the
subroutine DATAIN. This routine was developed by Farhoomand and Wilson (12) for four-noded elements. It has been adjusted for the present study with eight-noded isoparametric elements, two noded arc elements having three degrees-of-freedom at each node, and viscous boundary conditions. Geometric properties of the finite element mesh, load-time function and the boundary conditions of the problem are read through this routine.

The coordinates and the boundary conditions of a node are read from a single data record, the nodes being in numerical sequence. If records are omitted, the omitted nodal points are generated in equal intervals along a straight line between the defined nodal points. The boundary conditions of the missing nodes are assumed to be free. The element connectivity is given in a counter-clockwise direction around the element. Element records must also be in number sequence. If intermediate records are missing, the program automatically generates the connectivity of the missing elements by incrementing the connectivity number of the previous element successively. The pressure boundary conditions, load-time function and the viscous boundary conditions are read by the same subroutine.

4.4 DYNAMIC STORAGE ALLOCATION FOR ARRAYS AND MATRICES

Within the program, a method of dynamic storage allocation is used. For a given problem, all required data is compressed into the smallest possible storage area. All matrices and vectors involved in the generation are stored in two arrays – one for storing real values and the other for integer values. The scheme allows the capacity of the program to be increased or decreased by only changing two numbers defining the dimensions of real and integer arrays in the program MAIN. Total spaces, \(d_1\) and \(d_2\), required in
the arrays, A and IA, for a given problem are computed as

\[ d_1 = 3n^2b + 8n + 18e + 21m + 3p + 2l + 7c \]
\[ d_2 = 11e + n_3 + n_4 + 3c \]

where,
- \( n = \) no. of equations
- \( b = \) half band width of the stiffness and mass matrices
- \( e = \) no. of elements
- \( m = \) no. of materials
- \( p = \) no. of nodal points
- \( l = \) no. of points given on the load-time function
- \( c = \) no. of pressure boundary conditions
- \( n_3 = \) no. of nodes having 3 degrees-of-freedom
- \( n_4 = \) no. of dashpots along the base.

The maximum size of the total dimension \( d = d_1 + d_2 \), that can be permitted, depends on the particular computer being utilized. For a microcomputer having 640K RAM, the permissible value for \( d \) was found to be approximately 62000.

4.5 CONSTRUCTION OF MASS AND STIFFNESS MATRICES

The global mass matrix of the system is developed along with initial stiffness matrix and body force vector at the beginning of the solution scheme and kept unaltered throughout the solution. The upper half band of the matrix is stored in a portion of the main dynamic array.

The element stiffness matrices of structural parts that remain unchanged throughout the solution are developed during the assembling of initial stiffness matrix. These are then stored in a direct access file for latter use. The stiffness matrices of the isoparametric elements are updated at the beginning of each time step. The global stiffness
matrix is assembled by taking elements one-by-one. The stiffness matrix may be reconstructed during a time step if a revised solution is needed due to change in loading/unloading path of elements. The upper half band of the global stiffness matrix is stored in the main dynamic array.

4.6 THE SOLUTION ROUTINES

The routines, TRIA and BACKS, for solution of simultaneous linear equations by Gaussian elimination technique modified to suite the one dimensional storage of the upper half band of the coefficient matrix developed by Farhoomand and Wilson (12) is used in this study without any modification.

4.7 APPLICABILITY OF THE PROGRAMS

The computer programs developed for nonlinear dynamic analysis are also applicable to static analysis without loss of efficiency. The load applied to the surface may be varied at different points and the arrival time of dynamic pressure at different points may also be varied. The soil-structure interaction study may be carried out with or without interface elements. Any combination of structure and surrounding soil with different base and fill materials may be employed. The shape of the structure can be varied widely. An equivalent nonlinear static analysis is possible for maximum magnitude of the load-time function by using the same data file submitted for a dynamic analysis.

4.8 STRUCTURE OF THE COMPUTER PROGRAM

The general finite element program for both static and dynamic analyses of soil and underground structures has been
developed by a logical combination of a number of subroutines. The operation of the program is controlled by program MAIN. The material properties are input by calling the subroutine MATDTA. The control informations are then read and the spaces for geometric data, load-time function and boundary conditions are allocated from the dynamic storage. The subroutine DATAIN reads and generates these data. Spaces required by all vectors and matrices generated during execution, are then computed and the requirement is compared with the available space in the dynamic array. If the space requirement is satisfied, the program switches to a static or dynamic analysis in accordance with the option given in the data file. A simplified flow chart of the main program is given in Fig. 4.1. The flow charts of the routines SOLVD and SOLVS that actually do a dynamic analysis and a static analysis, respectively are shown in Figs. 4.2 and 4.3. The flowcharts are self-explanatory.

4.9 LIMITATIONS OF THE PROGRAMS

The capacity of the program is limited by the available core storage in a computer. The global stiffness and mass matrices are stored in the core. A copy of the stiffness matrix has to be maintained separately for later use after the original has been destroyed during a step of solution. This additional storage requirement is a limitation of the present recursive technique. Moreover, there is some unnecessary space occupancy by zero elements while storing the matrices in half band form. However, numerical accuracy is not affected by these limitations of the programs.
Start

Read total no. of materials and the number assigned to interface elements

Allocate space for storing material properties and call MATDATA to read and/or generate the parameters

Allocate spaces for storing geometric informations, boundary conditions and load-time function and call DATAIN to read and/or generate the data

Allocate spaces for matrices and vectors that will be generated during execution

Is the declared storage size sufficient for the present problem?

No

Write 'DIMENSION EXCEEDED'

Stop

Yes

2.1

FIGURE 4.1 MAIN PROGRAM SHEET 1 OF 2.
2.1

Initialize the spaces required in dynamic arrays

Read ITYPE

Is ITYPE = 0?

No

Call SOLVS for a static solution

Yes

Call SOLVD for a dynamic solution

STOP

FIGURE 4.1
Enter subroutine SOLVD for dynamic solution

Develop body force vector \( R \), mass matrix \( M \) and stiffness matrix \( K_0 \) at time \( TT = 0 \)

Solve equation \( Kx = R \) for body forces only

Find the axial thrusts and moments in the structure caused by dead weights

Find the mean pressure at the centroid of each soil element and the normal pressure at the centroid of each interface element.

FIGURE 4.2 SUBROUTINE SOLVD FOR DYNAMIC SOLUTION  SHEET 1 OF 4
If the number of dashpots at the base is not zero, call FDASH to find the diagonal matrix C containing the viscous damping coefficients.

Are the displacements, velocities and forces zero at time \( T = 0 \)?

Read initial quantities

Compute the contributions of initial quantities to the solution of next step

Begin step by step solution

\( T = T + h \)

Find the load vector at time \( T \) by calling LOAD

Determine the effective load vector, \( B \), from the current load vector and the contributions of the initial quantities

\( \text{ITER} = 0 \)
ITER = ITER + 1

Assemble the current global stiffness matrix by calling elements one by one

Find effective coefficient matrix,
\[ A = M + B h^2 K \]

If the number of dashpots is not zero call AJDC to find \((A + h/2 C)\)

Solve \(Ax = B\) by calling subroutines THIA and BACKS

\[ \text{ITEST} = 0 \]

For the incremental displacements \(B\), find strains at the centroid of each isoparametric element

Find the incremental stresses at the centroid of each isoparametric element. If the element proceeds through a wrong stress path, set \(\text{ITEST} = \text{ITEST} + 1\)

\[ \text{Is ITEST} = 0 \text{ or ITEST} \leq \text{allowable No. of iterations?} \]

Update the modulii of iso-\(P\) element

\[ 4.1 \]

\[ 3.1 \]
Add the incremental displacement vector to total displacements

Call subroutine PIPEC to calculate forces and moments at the ends of arc elements

Calculate total stresses at the centroid of each soil element. Call BULKB and SHEARG to update the modulii.

Calculate stresses at the centroid of each interface element. Call SIGEPI to update the shear modulus of the element.

Call FORCE to calculate the contributions of initial quantities to the solution of next step

2.2
Enter subroutine SOLVS for static solution

Assemble stiffness matrix $K_o$ and the body force vector $R$

Solve $K_o x = R$ for the body force vector

Find forces and moments in the structure caused by dead weights

Calculate stresses at the centroid of each isoparametric element. Store mean pressure at the centroid of each soil element and the normal pressure at the centroid of each interface element

Find the maximum magnitude of the load function and set the increment for each step

FIGURE 4.3 SUBROUTINE SOLVS FOR STATIC SOLUTION
Begin incremental solution. Set ITER = 0

Develop load vector B for incremental pressure dP on the surface

ITER = ITER + 1

Assemble the global stiffness matrix

Solve K x = B

ITEST = 0

For the incremental displacements B, find strains at the centroid of each isoparametric element

Find the incremental stresses at the centroid of each isoparametric element. If the element proceeds through a wrong stress path, set ITEST = ITEST + 1

FIGURE 4.3
3.1 Continue

Is ITEST = 0 or ITER > allowable no. of iterations? 

No

Update the moduli of isoparametric elements

Yes

Add incremental displacement vector to total displacements

2.2

Call Subroutine PIPEC to calculate forces and moments at the ends of arc elements

Calculate total stresses at the centroid of each soil element. Call BULKB and SHEARG to update the moduli

Calculate total stresses at the centroid of each interface element. Call SIGEPI to update shear modulus of the element

2.1

FIGURE 4.3
CHAPTER 5
ANALYSIS OF SOIL AND STRUCTURE-SOIL SYSTEMS

5.1 INTRODUCTION

Blast loading on ground surface causes propagation of shockwave through the soil. Any obstruction in the ground in the form of a buried man-made structure or of a natural rock mass reflects and deflects this wave and in turn is itself subjected to a shock loading. The structure, however responds to this shock not by itself alone, but by bringing in the surrounding soil mass to interact with it in resisting the shock. The soil thus acts both as a medium for propagation of the wave and as an element contributing to the strength and stiffness of the structure.

In order to investigate the nature of wave propagation through the soil and its effect on the stresses in the soil and the structure, two classes of analyses have been performed. Firstly, the soil has been analysed as a homogeneous medium without any structure buried into it. Secondly, a series of structure-soil systems, viz. cylindrical structures of varying stiffnesses buried at different depths within the soil, has been analysed. The cylindrical structures can be considered as buried pipe lines, aircraft shelters etc. which lend themselves to plane strain analysis. Results of these analyses offer an insight into the dynamic interaction that takes place in such systems.

All the analyses have been performed on systems having dimensions of a model rather than on life-size systems. This limitation was imposed because of the necessity of verifying the results with published experimental and analytical reports. This chapter describes the various pertinent
features and properties of the systems analyzed. The validity of the present finite element program is demonstrated by comparing the results of a model analysis with published experimental and analytical results (12). A summary of the various analyses performed for the present investigation is also included in this chapter.

5.2 LOADING USED IN THE ANALYSES

As stated earlier, only the airoverpressure component of the blast effects is to be considered in predicting the ground motions or the response of buried structures. The nature of blast loading is very much unpredictable and even minor variations in the details of the blast loading may cause significant changes in the response of the system. So the analysis performed by using an idealized presentation of a particular size of bomb blast as suggested by Selig et. al (7) and Biggs(9), does not provide a general understanding of the problem. For the present study, an experimentally obtained overpressure-time curve (Fig. 5.1), as reported in reference (12), has been selected. In plane-strain analysis, a slice of unit thickness is considered and the pressure at any instant of time is applied uniformly over the surface. Although a shockwave attenuates with distance from the point of occurrence, the variation of pressure on the surface is negligible for the case under study. This is because the influence of an underground pipe in lateral direction is not large and the variation of shock wave pressure over that small region of pipe influence may be considered negligible. In addition to the time-dependent loading of Fig. 5.1, the dead weights of the soil and the structure were also included in the analyses. The unit weights of soil and steel were assumed to be 110 lbs/ft³ and 480 lbs/ft³ respectively.
Fig. 5.1 Airoverpressure varying with time (Vicksburg Test) (Ref. 12)
5.3 MATERIAL PROPERTIES

For all interaction analyses, the stress-strain property of the structure is assumed to be linear elastic having a modulus of elasticity $E$ and Poisson's ratio $\nu$ equal to $30 \times 10^6$ psi and 0.33, respectively. The soil properties, as used in the present study, are described by two relations: a mean pressure vs. volumetric strain curve (Fig. 3.6) and a shear modulus vs. mean pressure curve (Fig. 3.10). The constants in equation 3.43 describing the curve of Fig. 3.6 are found to be $a_1 = 44357.0$ psi, $a_2 = -32455150.0$ psi, and $a_3 = 0.125836 \times 10^{11}$ psi. The constants in equation 3.47 describing the curve of Fig. 3.10 are obtained as $G_0 = 17958.31$ psi, $b_1 = -982.398$, $b_2 = 26.633/\text{psi}$ and $b_3 = -0.12628/\text{psi}$. The unloading path for mean pressure-volumetric strain relation is a straight line proceeding towards a residual strain of 87.4% of the total strain at the onset of unloading. The reloading path is assumed to be the same as that for unloading. The parameters describing the above mentioned curves (Figs. 3.6 and 3.10) were stored in the computer and the bulk and shear modulii were updated depending on the state of mean pressure. The extent of soil included in the analyses was assumed to be homogeneous at the onset of analysis. For the shear modulus of interface elements, the parameters given by Zaman et al (18) were selected. Table 5.1 presents these parameters.

Table 5.1 Parameters to find the shear modulus of interface element (Ref. 18)

<table>
<thead>
<tr>
<th>$\beta_i$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.6350</td>
<td>143.0300</td>
<td>-287.940</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.0207</td>
<td>4.70000</td>
<td>-0.09645</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-0.0005</td>
<td>-0.12054</td>
<td>0.00253</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>0.0380</td>
<td>8.93700</td>
<td>-18.660</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>-0.000088</td>
<td>0.02235</td>
<td>-0.004587</td>
</tr>
</tbody>
</table>

* Units are in lb and inch.
5.4 VERIFICATION OF THE MODEL

The computer program model was verified by comparing the results with those of an experimental study and a prediction of the experiment by Farhoomand and Wilson (12). The experimental study was conducted in the blast load simulator at Vicksburg, Mississippi. The material properties of the soil and the structure used in their experiments were the same as those described in sections 5.2 and 5.3. Figure 5.2 shows the finite element discretization of the Vicksburg model test. The size of the soil extent is 132"x120" and the thickness of the steel pipe is 0.5 inch. The observed displacement at a point and a prediction of that test result were reported by Farhoomand and Wilson (12). They used an axisymmetric finite element model with a linear acceleration integration scheme. Although the location of the displacements reported in reference (12) was not mentioned, the published results can be utilized for verification of the present code by comparing the differences in the arrival time. Vertical displacements reported by Farhoomand and Wilson (12) and those at a number of points in the present analysis are shown in Fig. 5.3. All the curves show permanent set in displacements which is a qualitative verification of nonlinearity. The experimental result in Fig. 5.3 is seen to be attaining a peak value at about 18 msecs. The time lag between the peak response and the maximum loading is thus found to be about 15 msecs. The lag predicted by Farhoomand and Wilson was about 12.5 msecs and that by present analysis is 12.0 msecs. The close agreement between the two predictions implies that the mathematical functions, as used in the present study, defining the material characteristics presented in ref.12 give excellent results. However, the difference of experimental and analytical time lag could not be reduced. This discrepancy may be attributed to the inexactness of the material
Fig. 5.2 Finite Element Discretization of the Vicksburg Model Test.
Fig. 5.3 Time lag of peak vertical displacements in soil

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0</td>
<td>120.0</td>
</tr>
<tr>
<td>B</td>
<td>20.0</td>
<td>88.0</td>
</tr>
<tr>
<td>C</td>
<td>104.0</td>
<td>78.0</td>
</tr>
<tr>
<td>D</td>
<td>104.0</td>
<td>68.0</td>
</tr>
</tbody>
</table>
parameters and the inadequate information about the test pit as given in ref.12. Despite this, the results of the present analysis may be considered as reasonably accurate.

5.5 SUMMARY OF THE ANALYSES PERFORMED FOR THE PRESENT INVESTIGATION

The first series of analyses comprised the soil mass only without any buried structure. For these analyses, the soil was discretized into elements of equal size. The depth of the finite boundary at the bottom and the boundary conditions along the base were varied. Either a rigid base simulating bed rock or a viscous base simulating an infinite depth of soil was selected to represent the two extremes of ground condition. Table 5.2 shows the various cases that have been studied.

Table 5.2: Features of the Soil Analyses

<table>
<thead>
<tr>
<th>Analysis no.</th>
<th>Depth of base D, inches</th>
<th>Boundary conditions at the base</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120.0</td>
<td>Rigid</td>
</tr>
<tr>
<td>2</td>
<td>200.0</td>
<td>Rigid</td>
</tr>
<tr>
<td>3</td>
<td>300.0</td>
<td>Rigid</td>
</tr>
<tr>
<td>4</td>
<td>120.0</td>
<td>Viscous</td>
</tr>
<tr>
<td>5</td>
<td>200.0</td>
<td>Viscous</td>
</tr>
<tr>
<td>6</td>
<td>300.0</td>
<td>Viscous</td>
</tr>
</tbody>
</table>

The second series of analyses was aimed at understanding the structural response due to structure-soil interaction and considered the complete system of soil and the buried structure. Due to the symmetry of the structure and the loading, only half of the structure-soil systems was
included in the analyses. The depth of the bottom boundary was kept constant in all the analyses. A plot of the maximum displacements (Fig. 5.4) at ground surface, as obtained from the code verification run shows that a rigid lateral boundary may be placed at a distance of eight to nine times the radius of the pipe from the pipe centreline. This distance is greater than the distance (five to six radii) usually used in static analysis. It was, therefore, decided to place the lateral boundary at a distance of 8.33 \( R \) (\( R \) being the radius of the pipe) from the pipe centreline for all the subsequent interaction analyses. The boundary condition along the base was considered to be either rigid or viscous depending on the case under consideration. The diameter of the pipe (\( d \)) was kept constant at 24 inches. Keeping the \( \frac{d}{t} \) ratio ('\( t \)' being the thickness of the pipe) constant, the cover depth on the crown was varied between 0.25 \( d \) to 4.50 \( d \). Then by placing the pipe at an optimum position, the stiffness of the pipe (expressed by the non-dimensional parameter \( \frac{d}{t} \)) was varied from 50 to 100.

In order to understand the dynamic magnification of stresses a series of nonlinear static analyses corresponding to the positions and parameters used in the dynamic analyses was also performed. The static analyses were carried out for the maximum magnitude of the time dependent shock loading by using incremental stiffness method. The features of the interaction analyses performed are shown in Table 5.3.

For all the analyses performed in the present study, the nodes on the lateral boundary and line of symmetry were allowed to displace vertically, but were restrained against lateral displacement. On the line of symmetry, the nodes falling on the structure were additionally restrained against rotation. The interface elements were assumed to be of constant thickness having a value of 0.5 inch.
Fig. 5.4  Vertical displacements at the ground surface
For dynamic analyses, a time interval of 1.5 msec. was used for integration. The parameter, $\beta$ which governs the pattern of variation of acceleration within the time step and the artificial damping in the solution scheme was taken as 0.25. The method of analysis thus reduced to the constant-average-acceleration Newmark method having infinite stability.

### Table 5.3 Features of the Interaction Analyses

<table>
<thead>
<tr>
<th>Analyses No.</th>
<th>Type of Analysis</th>
<th>Cover Depth</th>
<th>Base Condition</th>
<th>(d/t^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Dynamic</td>
<td>0.25 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>Dynamic</td>
<td>1.25 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>Dynamic</td>
<td>2.00 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>Dynamic</td>
<td>3.00 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>Dynamic</td>
<td>4.50 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>Dynamic</td>
<td>0.25 d</td>
<td>Viscous</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>Dynamic</td>
<td>1.25 d</td>
<td>Viscous</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>Dynamic</td>
<td>2.00 d</td>
<td>Viscous</td>
<td>50</td>
</tr>
<tr>
<td>9</td>
<td>Dynamic</td>
<td>3.00 d</td>
<td>Viscous</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>Dynamic</td>
<td>4.50 d</td>
<td>Viscous</td>
<td>50</td>
</tr>
<tr>
<td>11</td>
<td>Dynamic</td>
<td>1.25 d</td>
<td>Rigid</td>
<td>80</td>
</tr>
<tr>
<td>12</td>
<td>Dynamic</td>
<td>1.25 d</td>
<td>Rigid</td>
<td>100</td>
</tr>
<tr>
<td>13</td>
<td>Dynamic</td>
<td>1.25 d</td>
<td>Viscous</td>
<td>80</td>
</tr>
<tr>
<td>14</td>
<td>Dynamic</td>
<td>1.25 d</td>
<td>Viscous</td>
<td>100</td>
</tr>
<tr>
<td>15</td>
<td>Static</td>
<td>0.25 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
<tr>
<td>16</td>
<td>Static</td>
<td>1.25 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
<tr>
<td>17</td>
<td>Static</td>
<td>2.00 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
<tr>
<td>18</td>
<td>Static</td>
<td>3.00 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
<tr>
<td>19</td>
<td>Static</td>
<td>4.50 d</td>
<td>Rigid</td>
<td>50</td>
</tr>
</tbody>
</table>

\* \(d\) is the diameter of the pipe and \(t\) the thickness.

Note: Total depth of the soil mass is kept constant at 8.33d.
CHAPTER 6
DISCUSSION OF RESULTS

6.1 INTRODUCTION

A summary of the analysis schemes undertaken in this research has been provided in chapter 5. Results of these analyses are presented and critically examined in this chapter. The influence of various parameters on the displacements and stresses in soil and the response of the underground structure are discussed sequentially. In all presentations, upward displacements and compressive stresses are considered to be positive. Moments causing tension inside the cylindrical structure are taken to be positive, unless otherwise stated. In all the relevant diagrams, rigid and viscous bases are represented by hatched and dotted lines, respectively.

6.2 ANALYSES OF SOIL WITHOUT BURIED STRUCTURE

To investigate the influence of wave propagation due to a surface blast on displacements and stresses in soil, a series of analyses has been performed for a soil mass without buried structure with both rigid and viscous bases. This section describes the results of these analyses.

6.2.1 Displacements in Soil

Figure 6.1 shows the downward displacements varying with time at various depths in the soil for analysis with a rigid rock base. As can be expected, the downward displacements are maximum at the ground surface and diminish towards the base. The interesting point to note in Fig. 6.1 is that the displacements at all depths reach peak values at the same time which is about 24 msecs for the present case. This
Fig. 6.1 Variation of vertical displacements with time (rigid base)
means that it suffices to observe the displacements at any arbitrary point in a soil mass overlying a rigid rock base in order to obtain the response time lag.

Time domain solution for displacements at the ground surface showing the influence of the proximity of rigid rock base is presented in Fig. 6.2. It is observed that the arrival time of peak response at surface as well as its magnitude increases with increasing depth of the rock base from the ground surface. It is more important, however, to study the displacements at a point below surface where the underground structure would lie. Figure 6.3 shows the influence of the depth of soil cover on the vertical displacements of a point located at a fixed height above the rigid rock. It is seen that while the proximity of the rigid base from the point under consideration is kept constant, the peak response increases in magnitude and is delayed more with an increase in cover depth. Therefore, the peak vertical displacement at a location is determined not by its proximity to the base rock, but by the total depth of deposit over the base. The phenomenon can be explained by studying the propagation of wave through the soil. As the shock wave, induced by the blast effects, proceeds downward, the downward displacement at any point increases with time. Reflection of the wave at the rigid base causes an upward thrust which retards the downward movement of the overlying soil. The time required to observe this retarding effect increases with increasing depth of soil deposit; because the wave will take longer time to reach the bottom in a deeper deposit. This fact is demonstrated clearly by Fig. 6.2. The increase in the so-called peak displacement due to addition of greater depth of cover soil, as seen in Fig. 6.3, has occurred because the system is now exposed to shock wave propagation for longer time span. In smaller depth of deposits (e.g. 120" depth shown in Fig. 6.4) the shock wave reflection from the rigid base occurs earlier and the resultant thrust is greater than
Fig. 6.2 Influence of proximity of rigid base on vertical displacements at ground surface
Fig. 6.3 Influence of cover soil on vertical displacements (rigid base case).
Note: The values at the centroid of each element signifies the instant of time at which initiation of crack is detected.

Fig. 6.4 Propogation of cracking tendency from bottom towards top.
the confining pressure due to dead weight and incident dynamic pressure at that time. So the elements in the bottom region of the mesh get cracked. Due to this cracking of elements, the displacements are again reversed in the downward direction by showing a permanent set as seen in curve (1) of Fig. 6.2.

The question that now arises logically from the above observations is that whether the downward displacement at any point will increase indefinitely if the propagation of shock wave is not interrupted by any rigid layer. To get an answer to this, the homogeneous semi-infinite nature of soil deposit has been simulated by using a viscous boundary at a certain depth below the ground surface. Fig. 6.5 shows the influence of that viscous boundary assumption on downward displacement at a point on the ground surface. The displacement obtained by using viscous boundary is found to be increasing with time. The influence of location of that boundary is illustrated in Fig. 6.6. Since the shock wave has not reached the boundary in the mesh of depth 300" within the time span of 30 msecs, the displacement is found to be increasing with time. The other solutions with smaller depths show slight deviations from that path. These deviations have been caused by imperfect absorption of wave at the viscous bases. However, since the deviations are insignificant, the model can be accepted as reasonably accurate.

The vertical displacements at locations with varying depth below the ground surface are shown in Fig. 6.7 for a viscous base condition. The displacements are as before increasing with time passes. Since no material damping has been imposed on the solution, the wave front proceeds downward without any decay. This fact has led to an indefinite increase in displacements with time. Another important observation from this figure is that, after a certain interval of time the
Fig. 6.5 Influence of base condition on vertical displacements at a point on the ground surface
Fig. 6.6 Influence of viscous base location on vertical displacements at surface
Fig. 6.7 Variation of vertical displacements with time (viscous base)
curves become parallel indicating that the differences between the vertical displacements of various layers in the soil mass become constant. Figure 6.8 presents the relative displacements between top and bottom boundaries for different depths of soil mass. The relative displacements tend to attain constant values after the times at which the displacements in rigid base solution (Fig. 6.2) have been reversed in the upward direction. Thus, for the present loading case having single shock in the time span, the entire mass travels in unison as a rigid body with a constant velocity after the shock passes away from the finite boundary. Then onwards the stress levels within the body are expected to remain unchanged. The step-by-step solution, therefore, can be ceased when the shock wave passes away from the viscous base.

6.2.2 Stresses in Soil

To ascertain the influence of base condition on soil stresses, the vertical stress levels at different depths below the ground surface were considered. Fig. 6.9 shows these stresses in a soil deposit of depth 200" over a rigid rock. To eliminate the natural fluctuations exhibited in Fig. 6.9, suitable polynomial fittings of the points are shown in Fig. 6.10. The stress-time curves show two pulses within the time span of 30 msecs. The first one, caused by the incident wave, is found to be progressively delayed with depth below the ground surface, whereas the second, caused by wave reflection effects at the base, arrives almost simultaneously at all depths. The overstressing caused by reflection effects is minimum at points closest to the ground surface (curve 1 in Fig. 6.10) because these points get relieved of reflection effects by moving upward. The first pulse in stress-time curve is almost nonexistent for points close to the base because the time lag between incident wave and the reflection effects is insignificant.
Fig. 6.8 relative downward displacement between top boundary and bottom boundary (viscous base)
Fig. 6.9 Vertical Stress in Soil varying with Depth for a Rigid Rock at the Base
Fig. 6.10 Vertical Stress in Soil varying with Depth for a Rigid Rock at the Base
The interference of the two effects causes high stress concentrations at points close to the base.

Horizontal stresses at the same points as those of Fig. 6.10 are plotted in Fig. 6.11. The observations regarding vertical stresses are confirmed for the horizontal stresses in this figure.

The influence of the depth of cover soil over a point at a fixed distance from the rigid base on the vertical and the horizontal stresses is demonstrated by Figs. 6.12 and 6.13 respectively. Contrasting the curves for the maximum and the minimum cover soil, it is seen that the accumulation of covering soil restrains the pulsating nature of stress and also the resultant stress accumulation due to reflection effects becomes significant.

The peak magnitudes of vertical and horizontal stresses at a point close to the rigid base (Fig. 6.14) are found to be approximately constant, but the arrival time increases with increase in the depth of cover soil. This observation is consistent with the fact that no material damping has been allowed in the system. Regaining of high stresses after cracking of an element, as observed in Fig. 6.14 for D/C=12.0, is not a realistic result. Stresses at a point close to the surface (Fig. 6.15) on the other hand, are seen to be oscillating with a decaying amplitude. The rigid boundary disturbance is maximum for the smallest depth of deposit over the base. Fig. 6.16 compares the vertical stresses at the same point for a D/c ratio of 12.0 obtained with a viscous base and a rigid base. The nature of stress oscillation with decaying amplitude is observed for both the base conditions with the oscillation centering about the decaying applied surface pressure up to a time span of about 20 msecs. This decay of amplitude is considered to be caused by the energy loss that results from hysteretic stress-
Fig. 6.11 Horizontal Stress in Soil varying with Depth for a Rigid Rock at the Base
Fig. 6.12 Influence of cover Soil on Vertical Stresses for Rigid Base Case
Fig. 6.13 Influence of Cover soil on Horizontal Stresses for Rigid Base Case
Fig. 6.14 Vertical and Horizontal Stresses in soil at a point close to the Base for varying cover depth
Fig. 6.15 Stresses in Soil at point close to the Surface
Fig. 6.16 Influence of Boundary Condition on Vertical Stresses at a point close to the Surface
strain behaviour. But in the latter part of the solution (time span 20 to 30 msecs), the stress is again amplified; Fig. 6.17 shows the vertical and the horizontal stresses near the ground surface for a soil mass of varying depth overlying a viscous base. It is observed that for all the soil depths the stress-time curves almost coincide and exhibit the same decaying amplitude type of oscillation with subsequent amplification at higher time levels. This later amplification can be considered as a divergence in the solution which is not influenced by the distance of the viscous base from the surface. It can, therefore, be argued that amplification of high mode shapes has not contributed to this divergence; since otherwise different distances of the base would have caused different magnitudes of amplification. The divergence may be attributed to the inadequacy of the material model.

Fig. 6.18 shows a mean pressure-volumetric strain relation obtained from the present analysis for an element close to the surface. Time values beside the curve indicate that during the time step 3.0-4.5 msecs, the stress jumps suddenly to a higher level. This has apparently happened due to lack of smooth transition from loading to unloading path. As shown earlier in Fig. 5.1, the externally applied load reduces suddenly in the time range 3.0 to 4.5 msecs. So the elements near the ground surface tend to follow the unloading path. Supplementary numerical experiments performed by the author have shown that if a loading bulk modulus is assigned to an element near the surface at that time period, it shows unloading in volumetric strain behaviour; whereas if an unloading modulus is assigned, the element exhibits signs of loading. In order to rescue the solution scheme from this numerical deadlock, caused by the non-smooth loading/unloading transition of the material model, such elements have been forced to follow the unloading path and thus a continuity problem has arisen.
Fig. 6.17 Amplitude Decay of Stresses in Soil at a point close to the Surface for a Viscous Base
Fig. 6.18 Divergence in Stress–Strain Behaviour.

NOTE: Values beside points on the curve signify the time of occurrence in msecs.
between the loading and unloading paths. It is thought that this instability of solution has been caused by Poisson's ratio effect. When an element shows unloading in volumetric behaviour, the bulk modulus is adjusted accordingly. Since no check or adjustment was possible for shear behaviour, the use of the same shear modulus with loading and unloading bulk modulii at the transition point causes an instability in the solution. Due to the aforementioned artificial modulus assignment, the stresses in the element tend to attain an initial stability regaining the original stress at about 16.5 msecs (Fig. 6.18). However, the errors imposed by inadequate representation of material property accumulate and eventually cause divergence after this time. Nevertheless, such disturbances are localized occurring at different levels at different instants of time as the shock wave propagates downward.

Suitable polynomial fittings of vertical and horizontal stress-time results obtained for viscous base case are shown in Figs. 6.19 and 6.20. Due to imperfect absorption of wave at the viscous base, slight amplification of stresses is observed in the figures. Table 6.1 presents the peak stresses varying with depth in a soil mass of depth 200" and their arrival time. The table shows that the rigid base effects may cause peak stress to be up to 223% of that obtained by viscous base, depending on the depth below surface.

6.3 INFLUENCE OF COVER DEPTH AND BASE CONDITIONS ON STRUCTURAL RESPONSE

Two types of boundary conditions viz. - a rigid base and a viscous base have been used to study their influence on the structural response. For a semiinfinite soil mass simulated by using a viscous base, the cover depth on a cylindrical
Fig. 6.19 Vertical Stresses in Soil at various depths for a Viscous Base
Fig. 6.20 Horizontal Stress in Soil at various depths for a Viscous Base.
Table 6.1: Peak stresses varying with depth in a soil mass of depth D=200"

<table>
<thead>
<tr>
<th>D/C</th>
<th>Horizontal stresses</th>
<th>Vertical stresses</th>
<th>Arrival time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in psi</td>
<td>in psi</td>
<td>in msec</td>
</tr>
<tr>
<td></td>
<td>rigid base</td>
<td>viscous base</td>
<td>rigid base</td>
</tr>
<tr>
<td>20.00</td>
<td>29.82</td>
<td>29.82</td>
<td>52.46</td>
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<tr>
<td>4.00</td>
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<td>28.17</td>
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<td>45.61</td>
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<tr>
<td>1.05</td>
<td>51.69</td>
<td>21.63</td>
<td>99.44</td>
</tr>
</tbody>
</table>

* 'C' being the depth below the ground surface
** The values are for horizontal and vertical stresses respectively.
structure has been varied between 0.25d and 4.5d (d being
the diameter of the cylinder). Since the position of the
viscous base does not influence significantly the response
of a system, it was placed at a fixed depth (8.33d) below
the surface for all analyses. For the rigid base case, as
found earlier, the total depth of the soil deposit above the
base plays an important role on the response of the system.
Therefore, instead of fixing the position of the structure
with respect to the base, the rigid base has been fixed at a
depth of 8.33d below ground surface. The position of the
structure was then varied within the soil deposit resulting
in a cover depth of 0.25d to 4.5d. For all the analyses,
reported in the following subsections, the value of d and
the d/t ratio ('t' being the wall thickness of the
structure) have been kept constant at 24 inches and 50
respectively. Axial forces and moments at three critical
points of the circular section viz. crown, spring line and
invert (Fig. 6.21) are considered in the following
discussions.

6.3.1 Axial forces in the Structure

Axial forces in the structure placed close to the ground
surface are shown in Figs. 6.22 and 6.23 for rigid base and
viscous base conditions respectively. Axial forces at all
three locations of the circular section are found to be
almost the same. The similarity of shapes of the curves in
Figs. 6.22 and 6.23 for a particular point on the cylinder
indicates that the influence of rigid base effects is not
much significant on axial forces of a structure buried close
to the surface. The pulsating nature of axial forces at
crown and invert in Fig. 6.23 matches the trend of vertical
stresses at a point close to the surface (Fig. 6.16). The
amplification of axial forces at later time steps appears as
a result of the vibration of the structure and the
oscillations in the surrounding soil.
Fig. 6.21 Key diagram of the structure-soil system
Fig. 6.22 Axial forces for $C/B=0.035$ (rigid base)
Fig. 6.23  Axial forces for $C=0.25d$
(viscous base)
When the structure is buried deep into the ground, the rigid base effects get pronounced as illustrated in Fig. 6.24 where the axial forces attain large values at the instant when the shock wave strikes the rigid base. The axial forces at all three points under consideration are again found to be almost the same. To study the influence of rigid base effects on axial forces in the structure, axial force-time curves for various positions of the structure are shown in Fig. 6.25. Like the stresses in soil (Fig. 6.10), the rigid base influence is found to be increasing with decreasing clearance between the invert of the circular section and the rigid base.

If the rigid base is replaced by a viscous base, the wave reflection effect causing subsequent amplification of structural stresses is expected to be eliminated. Axial force-time curves obtained by using viscous base for different cover depths are shown in the Fig. 6.26. A second pulse in the force-time curves is still apparent, though of a less significant amplitude. This is considered to be due to imperfect absorption of wave at the viscous base. With an ideal wave absorbing model, it is expected that a force-time curve would have reached a steady state after the first pulse caused by the incident shock wave. Therefore, if the second peak is ignored, the peak response is found to be delayed with increasing cover depth but the magnitude remains almost the same. The non-existence of decaying peak axial forces with increasing cover depth can be attributed to the absence of material damping in the analytical model. On the other hand, the peak axial forces do not increase either with increase in cover soil mass, due to its insignificance compared to the high shock front pressure. Axial forces at other points of the structure deserve no separate presentation as it has been seen earlier that the forces at crown, springline and invert are almost the same.
Fig. 6.24 Axial forces for C/B=0.692 (rigid base)
Fig. 6.25  Axial forces at crown (rigid base)
Axial forces at crown
(viscous base)

Fig. 6.26
Peak axial thrusts with their arrival time for both boundary conditions are shown in Table 6.2. For rigid base cases, the peak response attained in the time span of 30 msecs (within which the reflection effects get pronounced) has been taken into consideration. And for viscous base cases, the first peak shown in an axial force-time history is taken into consideration. The envelopes of maximum axial forces for various positions of the structure in a soil deposit over a rigid base are shown in Fig. 6.27. The peak response increases with an increasing ratio of cover depth to bottom clearance (C/B) until approaching a steady state beyond a C/B ratio of about 0.75. The benefit of structure-soil interaction for deeply buried structures in relieving the structural stresses is offset by the effect of wave reflection suffered by the structure. The wave reflection effect is less near the surface, and although the interaction is also less, the structural stresses are smaller, indicating the dominance of wave reflection effect over interaction effect. The argument is substantiated by the envelopes of maximum axial force in the structure for the viscous base situation presented in Fig. 6.28. As the wave reflection effect is eliminated here, interaction between the structure and the surrounding soil plays its expected role. With the structure near the surface the interaction is little and the stiffer element of the structure-soil systems, i.e., the structure carries a larger share of force. With the increase in cover depth, the interaction becomes pronounced and since there is no wave reflection from the base the share of stresses carried by the structure is reduced. However, such reduction in stresses does not continue indefinitely and the peak axial forces become constant beyond a C/d ratio of about 2.0. This is because the entire ground surface is loaded uniformly limiting the scope for lateral load dispersion. In addition, the increased weight of cover soil with increasing depth of the structure below surface has its effects. If material
Table 6.2 Peak axial thrusts with their arrival time for d/t=50*

<table>
<thead>
<tr>
<th>Point on circular pipe</th>
<th>Cover depth</th>
<th>axial thrusts in lbs / inch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>rigid base at a depth of 8.33d</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Peak magnitude Time in msec</td>
</tr>
<tr>
<td>Crown</td>
<td>0.25 d</td>
<td>560.99 4.50</td>
</tr>
<tr>
<td></td>
<td>1.25 d</td>
<td>764.02 22.50</td>
</tr>
<tr>
<td></td>
<td>2.00 d</td>
<td>937.02 24.00</td>
</tr>
<tr>
<td></td>
<td>3.00 d</td>
<td>1124.41 22.50</td>
</tr>
<tr>
<td></td>
<td>4.50 d</td>
<td>1215.67 24.00</td>
</tr>
<tr>
<td>Spring line</td>
<td>0.25 d</td>
<td>624.90 4.50</td>
</tr>
<tr>
<td></td>
<td>1.25 d</td>
<td>828.36 24.00</td>
</tr>
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<td></td>
<td>2.00 d</td>
<td>910.94 24.00</td>
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<td></td>
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<td>1117.01 22.50</td>
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<tr>
<td></td>
<td>4.50 d</td>
<td>1281.74 24.00</td>
</tr>
<tr>
<td>Invert</td>
<td>0.25 d</td>
<td>588.51 4.50</td>
</tr>
<tr>
<td></td>
<td>1.25 d</td>
<td>879.49 24.00</td>
</tr>
<tr>
<td></td>
<td>2.00 d</td>
<td>892.88 22.50</td>
</tr>
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<td></td>
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<td>1057.58 24.00</td>
</tr>
<tr>
<td></td>
<td>4.50 d</td>
<td>1214.44 24.00</td>
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</tbody>
</table>

* 'd' and 't' are diameter and wall thickness of cylinder cross section
Fig. 6.27 Envelope of maximum axial forces (rigid base)
Fig. 6.28 Envelope of maximum axial forces (viscous base)
damping were introduced, some reduction in peak response at greater depths might have been observed.

It can, therefore, be concluded from the above discussion that where a rigid rock occurs at shallow depths, it is safer to place the structure near the surface away from the rock. On the other hand, if there is no rigid rock even at great depths below surface, the structure is better placed at greater depths. However, no benefit can be gained, as far as the axial forces are concerned, by placing the structure with a cover of more than twice the cylinder diameter.

6.3.2 Bending Moments in the Structure

Bending moments in a buried circular cylinder occur due to unequal thrusts in the horizontal and vertical directions. Variations of moment with time in structure buried at shallow depth in soil with a rigid rock at a depth of 8.33d are shown in Fig. 6.29. Due to higher compression in the vertical direction, a pipe tends to expand laterally and this causes tension outside at the spring line. The moment-time plots show two pulses within the time span of 30 msecs. For the pulse of moment arriving about 5 msecs latter than the peak of load diagram (Fig. 5.1), the moment at crown is seen to be higher than that at the invert. This is considered to have happened due to unequal normal forces on crown and invert. As described by the schematic diagram in Fig. 6.30, vertical forces caused by a downward travelling shock are partially balanced by interface friction which results in a lesser reaction on the invert than on the crown.

The second pulse in the moment-time plot may appear to have been caused by wave reflection from the rigid base. But this is not so, since the same pulsating nature of moment is observed for a cylinder with shallow burial in a soil
Note: Springline moments causing tension outside are shown positive.
Other moments causing tension inside are shown positive.

Fig. 6.29 Moments for structure near the surface (C/B=0.035) (rigid base)
Fig. 6.30 Surface forces on a cylinder caused by a shock travelling downward.
overlying a viscous base, Fig. 6.31. The phenomenon can be explained by considering that due to insufficient interaction with the surrounding soil because of its shallow burial, the structure continues oscillating in the vertical direction even after the shock front passes away from the structure. Since the ground surface is still subjected to a high pressure, the structure suffers a rebound effect at the surface and thereby a pulsating tendency in moment-time plot is observed. When a structure is deeply buried, the period of vibration becomes large due to better interaction with the surrounding soil. The above mentioned pulsating nature of moment is, therefore, not observed in Fig. 6.32 which demonstrates the time domain solution of moments in a cylinder buried into semi-infinite soil deposit at a depth of three times the diameter of the structure. On the other hand, if the structure is buried under the same thickness of cover in a soil deposit overlying a rigid rock at a certain depth (in the present study 8.33d), the reflection of wave from the base influences the response of the structure. This is demonstrated in Fig. 6.33. Since the reflection effects come from the base upward, the phenomenon demonstrated by Fig. 6.30 is reversed, as shown schematically in Fig. 6.34. For this reason, the moment due to reflection effects is found to be greater at invert than at crown, which is the reverse of the shallow cover situation shown in Fig. 6.29. The springline moment also comes down due to larger confining effect of surrounding soil.

Bending moments at crown, invert and springline of a cylinder for various positions in the soil deposit overlying a rigid rock are shown in Figs. 6.35, 6.36 and 6.37 respectively. Initial pulse caused by incident shock in deeply buried structure is less than that in a shallow buried structure due to interaction effects. As for the second pulse in the moment-time plots, it has been seen earlier (Figs. 6.29 and 6.31) that the reflected wave from a
Fig. 6.31 Moments for cover depth = 0.25 d (viscous base)

Note: Springline moments causing tension outside are shown positive. Other moments causing tension inside are shown positive.
Fig. 6.32 Moments for cover depth = 3d (viscous base)

Note: Springline moments causing tension outside are shown positive. Other moments causing tension inside are shown positive.
Fig. 6.33 Moments for $C/B=0.692$, $C=3d$
(rigid base)

Note: Springline moments causing tension outside are shown positive. Other moments causing tension inside are shown positive.
Fig. 6.34 Surface forces on a cylinder caused by reflected wave from rigid base
Fig. 6.35 Moment—time plots for crown varying with C/B ratio
(rigid base)
Fig. 6.36 Moment–time plots for invert varying with C/B ratio (rigid base)
Fig. 6.37 Moment–time plots for springline varying with C/B ratio (rigid base)
rigid base has no influence on the pulsating nature of moment-time plots (curves(1) in Figs. 6.35, 6.36 and 6.37) for a structure buried close to the surface. For deeply buried structures, the wave reflection effects increase with increasing depth of burial. This is why, the second pulse in the curves of Fig. 6.35 is found to be increasing with increasing C/B ratio. The same general tendency of an increasing second pulse of invert bending moment with increasing C/B ratio is also observed in Fig. 6.36. However, curve (5) in this figure shows a deviation from this observation which may have been caused by its excessive proximity to the rigid base. As for the springline moments, at the instant of reflected wave action the increasing confining effects due to increasing depth of cover has reduced the outward expansion of the cylinder at the springline. The spring line moments, then, generally decreases with increasing C/B ratio (Fig. 6.37). The deviation of curve(5) from the general tendency can be explained in the same light as for the case of the invert moment.

The moment-time plots for a structure buried into semiinfinite soil mass at different cover depths are shown in Figs. 6.38, 6.39 and 6.40. A better interaction for deeply buried structures is again evident from these figures. Peak response is caused by only incident wave, but the second peak for shallow cover, curves (1) in Figs. 6.38 to 6.40 is caused by insufficient interaction as explained earlier. The peak dynamic moments and their arrival times for different cover depths are summarized in Table 6.3. It is evident that the rigid base effects may cause a magnification up to 178% of the moment obtained by using a viscous base.

The envelopes of peak moments - not all occurring at the same time - are shown in Figs. 6.41 and 6.42. Since for
Fig. 6.38 Crown moment varying with cover depth (viscous base)
Fig. 6.39 Invert moment varying with cover depth (viscous base)
Fig. 6.40 Spring line moment varying with cover depth (viscous base)

Note: Moments causing tension outside are shown positive.
Table 8.3  Peak dynamic moments with their arrival time for d/t=50*

<table>
<thead>
<tr>
<th>Point on circular pipe</th>
<th>Cover depth</th>
<th>dynamic moments in lb-inch/inch</th>
<th></th>
<th>viscous base</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>rigid base at a depth of 8.33d</td>
<td>Peak magnitude</td>
<td>Arrival Time in msec.</td>
<td>Peak magnitude</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crown</td>
<td>0.25 d</td>
<td>276.04</td>
<td>9.00</td>
<td>275.88</td>
<td>9.00</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<td>159.85</td>
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</tr>
<tr>
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<td>24.00</td>
<td>153.89</td>
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<tr>
<td>Spring line</td>
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<td>155.59</td>
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<td>Invert</td>
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<tr>
<td></td>
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<td></td>
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<td>24.00</td>
<td>145.37</td>
<td>15.00</td>
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</table>

* 'd' and 't' imply diameter and wall thickness for circular cross section
Fig. 6.41 Envelopes of maximum moments (rigid base)

Note: Springline moments causing tension outside are shown positive. Other moments causing tension inside are shown positive.
Fig. 6.42 Envelopes of maximum moments (viscous base)

Note: Springline moments causing tension outside are shown positive. Other moments causing tension inside are shown positive.
cases having a rigid base, the peak response is influenced both by the cover depth and by the distance of the invert above the base, this dual effect has been taken into consideration by expressing the position of the structure by a ratio of cover depth to bottom clearance (C/B). Figure 6.41 shows that the optimum position giving minimum moments at crown and invert is at a C/B value of 0.205. This ratio for giving minimum spring line moment is about 0.53. However, since the magnitude of springline moment is much less than that at crown and invert, the C/B ratio giving minimum moment at spring line is of no practical importance. These optimum positions, however, can be confirmed conclusively only after a thorough and elaborate investigation with structures of a variety of shape and size and rigid base located at different depths below the ground surface. Figure 6.42 giving envelopes of peak moments for the structure buried into homogeneous semiinfinite media, shows that the cover depth should not be less than 1.25d to achieve benefits of interaction between the structure and the surrounding soil. It is interesting to note that this cover depth corresponds to a C/B ratio of 0.205 which coincides with that for minimum moments in a structure buried into the ground overlying a rigid base. However, to conclusively recommend this cover depth, a thorough investigation with different sizes and shapes of the structure is needed for this case too.

6.4 INFLUENCE OF STRUCTURAL STIFFNESS ON AXIAL FORCES AND BENDING MOMENTS

To investigate the influence of structural stiffness on axial forces and bending moments, the d/t ratio (‘d’ and ‘t’ being diameter and wall thickness, respectively of the cylindrical structure) was varied between 50 to 100. The position of the structure was kept fixed at a cover depth of
1.25d which gave minimum rigid base effects and better interaction. The base condition was again taken as either rigid or viscous. In section 6.3, it was observed that for a d/t ratio of 50, the axial force in a circular pipe is almost the same at all points. This property was also observed even when the d/t ratio was doubled. Fig. 6.43 shows the axial stresses in such a flexible pipe with d/t = 100, buried under a cover depth of 1.25d. The increased stresses caused by wave reflection effects are well within the range of critical buckling stress obtained by using closed form solution (Ref. 49).

Axial forces are governed by the membrane action developed in the cylindrical structure. The membrane action is not observed to be influenced appreciably by change of the d/t ratio in the range considered in the present investigation, as shown in Figs. 6.44 and 6.45.

In contrast to the above, the flexural behaviour which governs the bending moments in the structure, is expected to be greatly influenced by the change in d/t ratio. This is because the flexural stiffness of the cylinder varies inversely with the cube of the d/t ratio, whereas the axial stiffness has a linear inverse variation with d/t. The influence of flexural stiffness on moments at crown and invert for rigid and viscous base conditions are shown in Figs. 6.46 to 6.49. The general tendency is seen to be one of decreasing moment with decreasing stiffness, i.e. increasing d/t ratio. This is consistent with common interaction phenomena because in a composite system of structure and soil, as the relative stiffness of the structure increases its share of stresses would increase. For a smaller d/t ratio of 50 the moments caused by incident shock wave are larger than those caused by reflected waves (Figs. 6.46 and 6.47). But with higher values of the d/t ratio, the reflection effects of the rigid base result in
Fig. 6.43 Axial stresses in cylinder
cover depth = 1.25d
D/t = 100
Fig. 6.44 Influence of stiffness on axial forces at crown (rigid base)
Fig. 6.45 Influence of stiffness on axial forces at crown (viscous base)
Fig. 6.46 Influence of stiffness on crown moments (rigid base)
Fig. 6.47 Influence of stiffness on invert moments (rigid base)
Fig. 6.48 Influence of stiffness on crown moments (viscous base)
Fig. 6.49 Influence of stiffness on invert moments (viscous base)
larger moments than those caused by incident waves. This fact implies that the optimum position as found in section 6.3.2, giving minimum moments at crown and invert may be influenced by the structural stiffness. The reflection effects being insignificant for the viscous base condition, no well defined second peak of moments can be observed in Figs. 6.48 and 6.49.

The influence of structural stiffness on springline moments is shown in Figs. 6.50 and 6.51 for viscous and rigid bases respectively. In both the cases, as the d/t ratio increases, or in other words the structural stiffness decreases, the horizontal thrusts, exerted by the confining soil, retard the lateral expansion of the structure and at a certain stage, the spring line is pushed inward. Thus a moment causing tension outside at a smaller value of d/t ratio changes to a moment causing tension inside at higher values of the ratio. To demonstrate this more clearly, bending moment diagrams for three values of d/t ratio are plotted for a symmetric half of the circular section in Fig. 6.52. The envelopes of peak moments, not all occurring simultaneously, are shown in Fig. 6.53. Beyond a certain value of d/t ratio, the peak response becomes almost constant and moments anywhere converge within a narrow band. In the early parts of the curves the peak values have been caused by incident wave and thus the moments are almost equal at corresponding points for both the base conditions. In the latter parts of the curves, for rigid base condition, reflection effects increase the moments over those for the viscous base condition. As the curves become horizontal at high d/t ratios, decreasing the stiffness further is likely to have no effect on bending moment. But the resulting bending stresses coupled with the increased axial stresses would render the structure unsafe and unserviceable.
Fig. 6.50 Influence of stiffness on spring line moments (viscous base)

Note: Moments causing tension inside are shown positive.
Note: Moments causing tension inside are shown positive

Fig. 6.51 Influence of stiffness on spring line moments (rigid base)
Fig. 6.53 Influence of stiffness on maximum moments
FIG. 6.52 BENDING MOMENT AT 12 MSEC FOR VARIOUS STRUCTURAL STIFFNESS (VISCOUS BASE CONDITION).
6.5 STATIC ANALYSIS VS. DYNAMIC ANALYSIS.

The use of finite element technique in nonlinear dynamic analysis of underground structures is computationally extensive. It is worthwhile to explore whether a dynamic magnification factor can be devised to reasonably predict dynamic response from static analysis. A series of nonlinear static analyses has, therefore, been performed using the peak amplitude of the load-time curve as the static load. An incremental solution technique with ten equal load steps has been used. The d/t ratio ('d' and 't' being the diameter and wall thickness of the cylindrical structure) has been kept constant at a value of 50 and the cover depth has been varied between 0.25d to 4.5d. The static bending moment diagram for a cover depth of 1.25d is shown in Fig. 6.54. The shape of the diagram is the same as that of diagram (1) in Fig. 6.52 which gives dynamic bending moments for the same position of the structure. The static axial thrusts and bending moments are presented in tables 6.4 and 6.5 respectively, with the dynamic peak response obtained for the same position of the structure. The magnification factor, as given in the tables is defined as

\[
\text{M.F.} = \frac{\text{Peak dynamic response}}{\text{Corresponding static response.}}
\]

The results are graphically presented in figures 6.55 to 6.60. In almost all the cases, the magnification factor for results obtained by viscous boundary assumption are less than 1.0. In dynamic loading, the structure can dissipate some energy through vibration which cannot be accounted in static analysis. So to find an equivalent solution by static analysis, the dynamic loading may be factored down to a
Fig. 6.5.4 Static Bending Moment Diagram

Cover depth = 1.25d

$d/t = 50$
Table 6.4 Dynamic magnification of axial thrusts for d/t=50

<table>
<thead>
<tr>
<th>Point on circular pipe</th>
<th>Cover depth</th>
<th>Static thrust in lbs/inch</th>
<th>Peak</th>
<th>Dynamic response</th>
<th>viscous base</th>
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<td>rigid base at a depth of 8.33d</td>
<td>magnitude factor</td>
<td>magnitude factor</td>
</tr>
<tr>
<td>Crown</td>
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<td>526.5</td>
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* 'd' and 't' imply diameter and wall thickness for circular cross section
Table 6.5 Dynamic magnification of bending moments for d/t=5u*

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<thead>
<tr>
<th>Point on circular pipe</th>
<th>Cover depth</th>
<th>Static moment in lb-inch/ inch</th>
<th>peak dynamic response</th>
</tr>
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<td>rigid base at a depth of 8.33d</td>
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<td>magnitude magnification</td>
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* 'd' and 't' imply diameter and wall thickness for circular cross section
Fig. 6.55 Envelopes of maximum axial forces at crown
Fig. 6.56 Envelopes of maximum axial forces at spring line
Fig. 6.57 Envelopes of maximum axial forces at invert
Fig. 6.58 Envelopes of maximum moments at crown
Note: Moments are shown positive for tension outside.

Fig. 6.59. Envelope of maximum moments at spring line.
Fig. 6.60 Envelopes of maximum moments at invert

(1) dynamic response: rigid base (D=8.33d)
(2) dynamic response: viscous base
(3) static response
lower value. The higher dynamic response obtained for rigid boundary assumption is not a dynamic magnification in true sense, in fact this magnification occurs due to reflected wave actions.

For shallow buried structure (at a cover depth of 0.25d), the magnification factor for moments varies between 1.044 to 1.251. This is similar to the findings of Duns et al. (15), who proposed a dynamic load factor of 1.20 for design purposes. However, their recommendation cannot be justified for generalized use in the light of the findings of the present investigation.

The observed magnification factor of greater than unity for shallow buried structures appears to be the result of inadequate interaction effects. Due to lesser contribution to interaction by the surrounding soil coupled with the effect of continued residual shock, a structure buried close to the surface suffers higher stresses. In the static analysis, the residual loading being absent and the load steps being more refined, the structural stresses become smaller.

For structures with deeper burial, rigid base reflection causes higher dynamic responses. On the other hand, the absence of wave reflection in the viscous base case coupled with lesser dynamic effect of residual surface shock results in smaller dynamic responses than static for deeper buried structures.

The use of a unique magnification factor, suggested by some investigators (10,11,15) cannot possibly be derived for all situations. This is because the magnifications are found to be different for axial thrusts and bending moments which in turn vary with the depth of cover. Furthermore, the base condition has also significant influences on magnification
of stresses and moments. The influence of shape, size and stiffness of the structure should also receive due consideration in the recommendation for a proper magnification factor. It was not possible in the limited scope of the present investigation to address all these variables and parameters. Nevertheless, the inadequacy of the presently available recommendations for general use has been proven from the analyses performed here.
CHAPTER 7
CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

7.1 CONCLUSIONS

7.1.1 Analytical Scheme

Main features of the dynamic interaction study have been listed in Chapters 1 and 2. One of them is that the hysteretic stress-strain behaviour of soil must be modelled for dynamic analysis. Proper boundary conditions should also be included in the analysis. Numerical integration scheme used for solution of the dynamic equilibrium equation must be stable for nonlinear analysis and at the same time, it should allow reasonably large time steps for economy of the solution.

The recursive technique (19), as discussed in Chapter 3 for marching solution through the time, does not require computing the velocity and acceleration vectors at each time step and thus provide considerable saving of computational efforts. The constant-average-acceleration Newmark method coupled with the recursive technique, is, thus proved to be an efficient and stable method when finite element technique is used for nonlinear dynamic analysis. A viscous boundary model was used with success for effective absorption of shock wave at the finite boundary — imposed by practical limitations of the finite element technique.

7.1.2 Impact Loading Behaviour of Soil and Structure

The response of a soil mass without a buried structure and the response of a structure buried in the ground to shock loading applied on the surface were investigated separately in Chapter 6. The results obtained for viscous base cases were compared with those obtained for rigid base cases.
The following conclusions are made from the study:

1) The reflected wave effects from a rigid base on stresses in soil are maximum near the base and minimum near the surface. For deep burial, this effect is far more severe than that caused by incident shock wave. Use of a viscous base eliminates these base effects and the vertical stresses developed in soil approach the applied surface pressure. Emphasis must, therefore be given on the proper judgment of the existing base condition in a field problem.

2) The behaviour of a structure buried into a soil mass overlying a rigid base, is subjected to a dual effect of interaction with surrounding soil and wave reflection from the rigid base. As far as the axial thrust is concerned, the latter effect supersedes the benefit of better interaction for increasing depth of cover. But from bending moment consideration, the cover depth to bottom clearance ratio of 0.205 is found to be optimum giving minimum moments in the structure.

3) If a rigid base is non-existent in a soil deposit, only the interaction effects influence the response of the structure. Then a structure buried under a cover depth not less than two times the diameter of the cylinder is safer than shallow buried structures from both axial thrust and bending moment considerations.

4) For rigid base cases, the critical response of a structure is governed not by the incident shock loading but by the reflection effects.

5) The membrane and flexural actions in the cylindrical structure behave differently for change in structural stiffness within the range of stiffnesses considered.
6) A unique dynamic magnification factor to perceive the dynamic response from a static analysis cannot be given for all practical situations. This finding is in contrast to the presently available recommendations for general dynamic factors.

7.2 RECOMMENDATIONS FOR FUTURE STUDY

To increase the applicability of the computer programs developed, certain modifications can be done. The half band form of storing the stiffness and mass matrices can be replaced by a variable band storage scheme which will provide considerable saving of core storage requirement during execution. If facilities can be incorporated to discretize the soil mass, sufficiently away from the curved boundaries of the structure, into four noded isoparametric elements, a major reduction in computation might be achieved.

In the recursive technique used for step-by-step solution of the dynamic equilibrium equation, the time intervals were assumed to be of equal size. But as the nonlinear behaviour of a system progresses, the fundamental time periods of the system also increase. So, the time interval of integration may be increased from step to step without loss of accuracy. The variable time interval will require special adjustments of the recursive technique.

The constitutive model for soil used in the investigation suffered from lack of smooth transition from loading path to unloading path in volumetric behaviour. During adjustment for unloading volumetric behaviour, a simultaneous check and adjustment may also be performed for shear behaviour of the soil. The unloading shear behaviour can be ascertained from appropriate laboratory tests.
Further investigation for dynamic response study of soil and structure-soil systems can be made in the following regimes:

1) Soil structures (e.g. dams) can be analyzed to study their dynamic response.

2) The optimum shape and position of the underground structure to be resistant to blast loading on the surface may be found out by an extensive series of analyses of structures of various sizes and shapes buried under different depths of cover soil and with rigid base, if exists, at different depths below the surface.

3) Different properties for fill and base materials may be assigned to find their influence on the structural response.

4) Life size systems may be analyzed to verify the findings of the model analysis.

5) Large deformations in the system may be allowed for appropriate case by including suitable treatments for geometric nonlinearity in the analysis.

6) Model experiments may be performed and constructed structures may be monitored to verify the results of the analytical procedure presented in this thesis.
REFERENCES


