Behavior of Superelastic SMA Columns under Compression and Torsion

（圧縮と振りを受ける超弾性形状記憶合金円柱の挙動）

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Chapter 1

Introduction

Summary

This dissertation studies behaviors of the superelastic shape memory alloy (SMA) for large deformation via mechanical loading. Most of the applications and/or, researches on this unique functional material have been mainly centered to its property of superelasticity, that is, the ability to recover the original shape from the deformed state when the mechanical load, which causes the deformation, is withdrawn. Besides this unique property, however, the peculiar stress-strain curves beyond the stress-induced martensitic transformation (SIMT) region also need careful attention with the view of investigating on the large deformation of the structural elements made of the same material.

With this perspective, the buckling and postbuckling behaviors of the superelastic SMA columns have been studied under compressive loading-unloading cycles and compared with those of Al and SUS304 columns under similar test conditions. In this work several experiments were carried out to demonstrate a few unique and useful phenomena for the superelastic SMA columns which are absent in the Al and SUS304 columns. In addition to qualitative analysis of those observed phenomena, numerical simulation has also been performed for more precise quantitative analysis.

Behaviors of the superelastic SMA shafts (both solid and hollow) have also been studied and compared with those of the SUS304 shafts considering large angle of twist under loading-unloading cycles. Moreover, the interesting buckling characteristics of the slender superelastic SMA shafts under torsion have been examined and explained.
1. Introduction

1.1 Background

In the following paragraphs physical properties of the shape memory alloy (SMA) are briefly described. Some important experimental results dealing with the stress-strain relations and the stress-induced martensitic transformation (SIMT), obtained from the present study are also discussed since these are essential for interpreting the mechanical behaviors of the columns and shafts made of the superelastic SMA. Relevant works of other researchers are also cited wherever suitable.

1.1.1 Shape memory alloy (SMA) and related issues

Shape memory alloys, also termed as functional materials, show two unique capabilities, that is, the shape memory effect (SME) and superelasticity (SE) which are absent in the traditional materials. Both, SME and SE largely depend on the solid-solid, diffusion-less phase transformation process known as martensitic transformation (MT) from a crystallographically more ordered parent phase (austenite) to a crystallographically less ordered product phase (martensite) [1,2]. Typical hysteresis for the SMA, corresponding crystal lattice deformations and the stress-strain curves (during SME and SE) are depicted in Figs. 1.1-1.4. Following paragraphs sequentially describe them.

The phase transformation (from austenite to martensite or vice versa) is typically marked by four transition temperatures, named as martensite finish ($M_f$), martensite start ($M_s$), austenite finish ($A_f$), and austenite start ($A_s$). A plot of the volume fraction of martensite (or, the length of the SMA wire loaded with a constant weight $W$), as a function of temperature, provides a curve as shown in Fig. 1.1. Let us assume, $M_f < M_s < A_s < A_f$. Thus a change in the temperature within $M_s < T < A_s$ induces no phase change and both martensite and austenite may coexist within $M_f < T < A_f$. The shape of the hysteresis for a particular type of SMA (Fig. 1.1) can be controlled for a wide range by changing the
composition of the alloy and its processing. It should be noted that the transition temperatures are also influenced by the operating conditions and increase with the increasing operating stress.

Fig. 1.1 Transformation temperatures and hysteresis for the SMA.

Binary Ni-Ti alloys typically have transformation temperatures ($A_s$) between 0°C to 100°C with a width of the hysteresis loop ($H$, as shown in Fig. 1.1) of 25°C to 40°C. Copper containing Ni-Ti alloys show a narrow hysteresis ($H=7-15°C$) with transformation temperatures ($A_s$) ranging from -10°C to approximately 80°C. An extremely narrow hysteresis ($H=0-5°C$) can be found in some binary and ternary Ni-Ti alloys exhibiting a premartensitic transformation (commonly called R-phase). On the other hand, a wide hysteresis ($H=150°C$ or more) can be realized in Niobium containing Ni-Ti alloys after a particular thermomechanical treatment [3].
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**Mechanism of SME**

The crystal lattice deformation mechanisms in the SMA are shown schematically in Fig. 1.2. As seen, the phase transformations are caused by change in the temperature (SME) or, change in the stress (SE) [4]. Suppose $T > A_t$, so the SMA is in the parent austenite phase. Under stress free condition, if the SMA is cooled to any temperature $T < M_f$, multiple variants and twins (Figs. 1.2, 1.3) are formed, all crystallographically equivalent but with different orientations (different habit plane indices). Note that MT is basically a macroscopic deformation process, though actually no transformation strain is generated due to the so-called self-accommodating twinned martensite. The SMA is able to show the SME because of this self-accommodation by twinning. If a mechanical load is applied to this material and the stress reaches a certain critical value, the pairs of martensite twins begin 'detwinning' (conversion) to the stress-preferred twins (Figs. 1.2, 1.3). The 'detwinning' or conversion process is marked by the increasing value of strain with insignificant increase in stress (as shown by the idealized stress-strain curve of Fig. 1.4). The multiple martensite variants begin to convert to single variant, the preferred variant determined by alignment of the habit planes with the axis of loading. Since the single variant of martensite is thermodynamically stable at $T < A_s$, upon unloading there is no reconversion to multiple variants and only a small elastic strain is recovered, leaving the material with a large residual strain (apparently plastic). If the deformed SMA is heated above $A_t$, it transforms to parent phase (which has no variants), the residual strain is fully recovered and the original geometric configuration is recovered. It seems as if the material recalls its original shape before the deformation and fully recovers it. Thus, this phenomenon is termed as shape memory effect (SME) [2,5,6]. However, if some end constraints are used to prevent this free recovery to the original shape, the material generates large tensile recovery stress, which can be exploited as actuating force for active or passive control purpose.
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Fig. 1.2 Crystal lattice deformations during SME and SE.

Fig. 1.3 Schematic diagrams for a small block of the SMA specimen showing the mechanism of SME; (a) twinned martensite, (b) detwinned martensite due to the applied load, (c) detwinned martensite with residual deformation (d) recovery of the original shape due to heating [5].
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Fig. 1.4 Idealized stress-strain diagram showing SME.

Application of SME

The following advantages of the SMA’s have made them popular as smart actuators and sensors: very high force to weight ratio, high adaptability, and compactness. Among the SMA materials, the most popular and widely used Nickel titanium alloys (abbreviated as NITINOL or, TiNi) have strong actuating capability with almost 8% recoverable strain, and the tensile recovery stress can be as large as 700 MPa. Thus powerful actuators can be designed within these extreme limits. Besides the actuators, the major applications of SMA...
have been as pipe couplings and electrical connectors. Both, SME and SE enable the SMAs to have excellent medical applications.

**Mechanism of SE**

The superelastic SMA has the unique capability to fully regain the original shape from a deformed state when the mechanical load, which causes the deformation, is withdrawn. For some superelastic SMA materials, the recoverable strains can be on the order of 10% [5, 7]. This phenomenon, termed as the pseudoelasticity or, superelasticity (SE) is dependent on the stress-induced martensitic transformation (SIMT), which in turn depends on the states of temperature and stress of the SMA. To explain the SE, let us consider the case when the SMA that has been entirely in the parent phase \(T > A_t\), is mechanically loaded. Thermodynamic considerations indicate that there is a critical stress at which the crystal phase transformation from austenite to martensite can be induced. Consequently, the martensite is formed 'prematurely' because the applied stress substitutes for the thermodynamic driving force usually obtained by cooling (for the case of SME). When the applied load is uniaxial, only one orientation (out of many) of martensite is selectively formed, which imparts an overall deformation to the SMA specimen. During unloading, because of the instability of the martensite at this temperature in the absence of stress, again at a critical stress, the reverse phase transformation starts (from the SIM to parent phase). When it is complete the SMA returns to its parent austenite phase. The complete loading-unloading cycle shows a typical hysteresis loop (Fig. 1.5), known as pseudoelasticity or superelasticity [2,5]. Note that the detwinning of the martensitic variants during SME, and the SIMT (or the reverse SIMT) during SE, are marked by a reduction of the material stiffness (Figs. 1.4, 1.5). Usually, the austenite phase has much higher Young's modulus in comparison with the martensite phase.
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For $T < A_s$, there is no pseudoelastic recovery and the residual strain can be recovered by heating above $A_r$ (SME). For any temperature there exists a critical stress for irreversible plastic slip to occur in the material (this critical stress value decreasing with increasing temperature), and if the stress is exceeded, then the residual strain can't be recovered by heating or unloading [5].

Application of SE

Having the unique feature of superelasticity (usually at room temperature) SMAs are used in various applications. SE has been extensively used in the medical and clinical applications (orthodontic wires, self-expanding micro-structures used in the treatment of hollow-organ or duct-system occlusions). Applications of the superelastic elements have the following advantages: light weight, retention of shape even if misused and being comfortable to use. Some of the latest applications of the superelastic wires include antenna of portable phones, headband of headphones and eyeglass frames. They are also used to retain the shape of the shoes for comfort [1,2].

Since SMAs display superelastic hysteresis behavior over large strain ranges, a significant amount of energy dissipation is possible. Thus superelastic SMAs can be suitably used in the structures as dampers [8,9]. Besides these applications of the superelastic SMAs, extensive studies are also reported mostly on the theoretical constitutive modeling for the same material [10].

Two-way shape memory

Two-way shape memory (TWSM) can be realized by thermo-mechanical treatment (usually involving several transformation cycles) of the SMA. Typically, a specimen deformed in the martensitic condition is intentionally constrained during heating in order to
suppress the normal one-way shape memory. This process generates "built-in" micro-stresses in the parent phase, which in turn makes the specimen to behave as in a stress-induced martensitic transformation. That is, the micro-stresses promote only a single orientation of martensite upon subsequent cooling, which produces a spontaneous deformation. When the specimen is heated, the normal shape memory process occurs and its original shape is reproduced. The TWSM can be repeated indefinitely (as with a thermostat) as opposed to one-way shape memory, which is a one time only operation [2,6].

The effect of training on the TWSM for the Cu based SMA (polycrystalline) under the combined loading (torsion and axial load) have been studied by Tokuda, M. et al. [11, 12], and Sittner, P. et al. [13]. The authors pointed out that the internal residual stress can be easily and effectively created by thermo-mechanical cycles (training), because the used SMA materials are polycrystalline and thus non-homogenous from semi-microscopic point of view. Thus, no external bias stress is necessary to realize the TWSM effect, which in turn, makes the actuators compact.

1.1.2 Recent researches related with the present study

Extensive studies on the tensile tests of the superelastic SMA are readily available in the literature. Very recently, Tobushi et al. [7], demonstrated that during tensile tests, there appears a mobile interface between the austenite and martensite phases during the SIMT and reverse SIMT. Tobushi et al. [7] also showed that during the SIMT and reverse SIMT the Poisson’s ratio of the material increases from normal value of 0.33 to 0.46. Interested readers may refer to Appendix A of this dissertation for more details of those unique phenomena.
The studies on the behaviors of the superelastic SMA under compressive or torsional loads are rather scant. Among the notable works related to the present study, Orgeas and Favier [14] demonstrated non-symmetric tension-compression behaviors (particularly for superelasticity) for an equiatomic Nitinol SMA sample, by comprehensive experimental results. Later, Raniecki and Lexcellent [15] theoretically verified the same asymmetry. Orgeas and Favier [14] concluded that though the same martensitic fraction is formed during tension and compression, but the orientation of the martensite variants is more efficient in tension than in compression which accounts for the asymmetric behavior.

A few researchers investigated the behaviors of the SMA under combined loading as discussed below. Interestingly, all of them used tubular shaped specimen for experiments.

The effect of training on the TWSM for the Cu based SMA (polycrystalline) under the combined loading (torsion and axial load) has been studied by Tokuda, M. et al. [11, 12] and Sittner, P., et al. [13]. The maximum strain during the test was within 2% and the training behavior was demonstrated, related with SME.

Tanaka et al. [16], studied the austenite and martensitic start conditions in an Fe-based polycrystalline SMA under tension/compression-torsion loads. It was demonstrated that an oval cone in the stress-temperature space could represent the martensite start condition while a polygonal cone in the same stress-temperature space could represent the austenite start condition.

Raniecki et al. [17], studied (both experimentally and theoretically) the deformation behavior of TiNi shape memory alloy undergoing R-phase reorientation in torsion-tension (compression) tests. The maximum strain during the test was within 0.5% and the deformation behavior associated with the R-phase reorientation was studied.
Moreover, Tobushi and Tanaka [18] developed a model using a simplified torsional stress-strain diagram, for the shape memory alloy spring to represent its load-deflection behavior.

Studies on the bending of the SMA beams are also reported in the literature. The investigations of the bending problems of pseudoelastic beam were initiated by Atanakovic et al. [19], where the explicit analytical moment curvature relation was derived for rectangular beams loaded by a single pulse moment.

Raniecki et al. [20], studied the variation of stress and the phase content distribution in arbitrary symmetric cross-section of the beam for single bending cycle and derived the explicit analytical equations for the moment-curvature hysteresis loop.

By numerical simulation, Auricchio and Sacco [1] demonstrated that for pure bending of a Nitinol superelastic SMA beam, with different properties in tension and compression, the axial strain has a non-monotonous response with the bending moment during loading and unloading. The complicated movement of the neutral axis of the cross-section of the beam due to SIMT leads to such peculiar response.

Very recently, Urushiyama et al. [21], showed that short columns made of the Cu based SMA carry higher buckling loads compared with the steel columns. It was also found that when subjected to axial compression, the curved SMA columns have the tendency to become straight before buckling [21]. To the author’s knowledge, however, no study on the buckling of the superelastic SMA columns or, the behavior of superelastic SMA shafts under torsion for large angle of twist has been reported in the literature.

1.1.3 Important properties of the superelastic SMA observed from the present study

The interpretations of the mechanical behaviors of the structural elements made of superelastic SMA are closely related with the important material properties, that is, its
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stress-strain curves and the SIMT. For simplicity, the idealized stress-strain diagram (for \( T > A_f \)), that actually resembles the hysteresis of tensile test for the superelastic SMA is depicted in Fig. 1.5. To avoid any trace of plastic deformation within the material upon unloading, usually, studies on the superelastic SMA assume that the total maximum strains (about 6%-7% for the Nitinol SMA) do not exceed much beyond the SIMT region. As far as the mechanical properties are concerned, the Young’s modulus of the Nitinol superelastic SMA is more or less comparable with that of Al.

From the present study, however, tensile tests (at room temperatures of 23 °C -30 °C) for the Al, SUS304 and the superelastic SMA (Ti49.3 at% Ni50.2at% V0.5at%) show that, the SMA material exhibits pretty high stiffness (although there is plastic slip) if loading continues after the complete SIMT (Fig. 1.6). The transformation temperatures are shown in Table 1.1. The details of the test conditions are described in Chapter 2.

| Table 1.1 |
| Transformation temperatures for the SMA (Ti49.3 at% Ni50.2at% V0.5at%) |
| \( M_f \) | \( M_s \) | \( A_y \) | \( A_f \) |
| -59 °C | -34 °C | -27 °C | -3 °C |

It is noteworthy from the tensile tests that the Al rod has the lowest yield strength, while for large strains after the SIMT the SMA rod can carry the highest load. Unlike the superelastic SMA, both Al and SUS304 exhibit more or less the similar nature of stress-strain curves in compression and tension. On the other hand, it is a well-known fact that the behaviors of the Nitinol SMA in tension and compression are very much different, particularly for superelasticity [1,14,15]. Experimental results from the current study also verify that the compressive strength of the superelastic SMA is significantly higher than its
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Fig. 1.6 Experimental stress-strain curves for the SMA, SUS304, and Al rods.

Fig. 1.7 Experimental loading-unloading hysteresis for the SMA under pure tension and pure compression.
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tensile strength, particularly for large strains (Figs. 1.6, 1.7). Unlike the tensile stress-strain curve there is no distinct plateau for a particular range of strains and as such, the start and finish points of the SIMT process are rather difficult to identify. It appears, for the compression test, the SIMT process is indicated by a slight change in slope of the stress-strain curve. The complete loading-unloading cycles for pure tension and compression tests are shown in Fig. 1.7. It should be noted here that different specimens were used for tension and compression tests under similar test conditions. Following Johnson [22], to avoid any chance of buckling of the specimen during the pure compression test, \( L/k \) was kept less than 12.

1.2 Motivation for the Present Study

As discussed, besides the SE, superelastic SMA possesses another important engineering aspect, its high strength for large strains. Moreover, compared with the commonly used structural materials like Al or SUS304, its stress-strain curves also show peculiarly different characteristics in tension and compression. Thus, it is possible that the above characteristics can be exploited to yield more interesting and useful phenomena. Motivated by the above facts, the mechanical behaviors of the superelastic SMA columns and shafts are studied in this dissertation considering large deformations and a few repeated loading-unloading cycles.

1.3 Contents

In Chapter 2, buckling and postbuckling behaviors of the superelastic SMA columns are observed for a wide range of slenderness ratio \( (L/k) \) and compared with those of the SUS304 and Al columns. The experimental method comprises of compressive loading on the columns much beyond the point of instability, followed by unloading allowing them to
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recovery the shapes. It is found from the load-deformation curves that, among the three
materials, the buckling load of the SMA column increases most significantly with the
decreasing value of \( L/k \) and ultimately exceeds that of SUS304 column. Interestingly, the
SMA column with \( L/k = 38 \) exhibits two distinct peak loads, the second one being the
higher, contrary to the trend that load falls off monotonously for other columns in the
postbuckling region. Similarly, for \( L/k = 28 \), the SMA column can sustain significantly high
load after a distinct change in the mode of deformation. Furthermore, for certain values of
\( L/k \), the recovery forces increase remarkably as the buckled SMA columns are gradually
unloaded. If not too short, they can significantly recover the initial shapes when completely
unloaded. Qualitative analysis shows that the above phenomena can be attributed to the
special nature of SMA's stress-strain curve and the superelasticity itself. On the other
hand, for too high slenderness ratios, Al columns have slightly higher buckling loads
compared with the SMA columns. However, both of the Al and SUS304 columns show
large residual strains after the complete cycle.

In Chapter 3, by numerical simulation, an attempt has been made to precisely analyze
the unique buckling and postbuckling behaviors of the superelastic SMA columns, as
already discussed in Chapter 2. Stress-strain relations have been determined from pure
compressive and tensile test data for sufficiently large strains, and used in the analysis. The
commercial FEM code ANSYS has been used for the simulation purpose. It is found that
the compressive stress-strain curve is vital to analyze the load-deformation curves, in
particular, for the short SMA columns. To simulate the remarkable phenomenon that the
recovery force increases during unloading of the slender SMA columns, a special method
has been developed to trace the continuous unloading path of the buckled columns.
Theorems of Thompson and Hunt [23] have been used to explain the unique postbuckling
behaviors for the short SMA columns.
Chapter 4 deals with the behavior of the superelastic SMA shafts (both solid and hollow) under torsional loading-unloading cycles for large angle of twist. It is found from the torque-angle of twist curves that compared with the SUS304, the stiffness increases quite steeply for the SMA for large angle of twist. Buckling is usually caused by compression; but a shaft may also become unstable under the action of a torque [24]. As observed from the current study, the slender SMA shaft buckles like a column when its critical twisting moment is exceeded. The superelastic SMA shafts have been found to make a much narrower hysteresis under reverse loading in comparison with the SUS304 shafts. In general, the hollow superelastic SMA shaft makes a narrower hysteresis and fails at a lower angle of twist in comparison with the solid shaft.

Chapter 5 is for general discussions and conclusions on the overall work.

Appendix A deals with a unique phenomenon that occurs during the SIMT for a superelastic SMA rod under tension test. Deformations of the specimen were measured simultaneously by the displacements of the fixture and also by strain gage. Strain gage gives reading of the local strain while the fixture displacement gives reading of the overall strain of the specimen. It is found, as the SIMT is initiated, there is no local strain although there is end displacements! Local displacements start again after the SIMT is over. Thus, the SIMT start and finish points can be observed by plotting the data of strain gage reading versus the end displacement reading. The same phenomenon is observed during unloading when the reverse phase transformation occurs.

In Appendix B, experiment was carried out to investigate the stability of an eccentrically loaded steel column. The column is subjected to high environmental temperatures as well as mechanical load. Since the SMA wires can act as thermal actuators, they were externally attached with the columns. The effect of the actuating force on the buckling behaviors of the columns is observed. It is found, the buckling load and the shape of the column can be changed by using the SMA wires.
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References


1. Introduction


Chapter 2

Buckling and Postbuckling Characteristics of the Superelastic SMA Columns- Experiment and Qualitative Analysis of the Results

Summary

Some unique buckling and postbuckling behaviors of the superelastic SMA columns are observed for a wide range of slenderness ratio \((L/k)\) and compared with those of the SUS304 and Al columns. The experimental method is comprised of compressive loading on the columns much beyond the point of instability followed by unloading allowing them to recover the shapes. It is found from the load-deformation curves that among the three materials, the buckling load of the SMA column increases most significantly with the decreasing \(L/k\) and ultimately exceeds that of SUS304 column. Interestingly, during compression the SMA column with \(L/k=38\) exhibits two distinct peak loads, the second one being the higher, contrary to the trend that load falls off monotonously for other columns in the postbuckling region. Similarly, for \(L/k=28\) the SMA column can sustain significantly high load after a distinct change in the mode of deformation. Furthermore, for certain values of \(L/k\), the recovery forces increase remarkably as the buckled SMA columns are gradually unloaded. If not too short, they can significantly recover the initial shapes when completely unloaded. The above phenomena can be attributed to the special nature of SMA’s stress-strain curve and the superelasticity itself. On the other hand, for too high slenderness ratios, Al columns have slightly higher buckling loads compared with the SMA columns. Both the Al and SUS304 columns, however, show large residual strains after the complete cycle.
2.1 Introduction

Mechanical instability is an important criterion for designing structural elements. In practical applications structural elements like slender columns or thin shells often buckle at a load surprisingly lower than the theoretically predicted loads, owing to the initial imperfections (geometrical or physical). Thin and light structures are increasing for economic reasons. Consequently, today's structures are prone to failure due to mechanical instability. The following paragraphs briefly introduces the basics of mechanical instability of structures.

2.1.1 Stability of structures

The concept of stability of equilibrium is a strongly intuitive one, consequently it arises quite early in the development of classical mechanics initiated by the famous work of Euler [1]. If at any level of external cause (in the form of displacement, velocity, force, etc.), a structure can sustain a "small" disturbance from its equilibrium condition, then the structure is said to be in stable equilibrium at that level of external cause. It should be noted here that sustaining the disturbance means the structure should oscillate with a small amplitude about its equilibrium position. On the other hand, if the structure does not go back to its original equilibrium position or vibrate with ever increasing amplitude due to the disturbance, then the structure is said to be in an unstable equilibrium state at that level of external cause. If the structure remains in the disturbed state without vibration, then the equilibrium is referred to as neutral equilibrium. By "small" it is meant as small as desired.

This can best be demonstrated by the system shown in Fig. 2.1(a). This system consists of a ball of weight $W$ resting at different points on a surface with zero curvature normal to the plane of the figure. Points of zero slope on the surface denote positions of static equilibrium (points, $A$, $B$, $C$). Furthermore, the character of equilibrium at these points is substantially different. At $A$, if the system is disturbed through infinitesimal disturbances (small
displacements or small velocities), it will simply oscillate about the static equilibrium position A. Such equilibrium position is called stable in the small. At point B, if the system is disturbed, it will tend to move away from the static equilibrium position B. Such an equilibrium is called unstable in the small. Finally, at point C, if the equilibrium is disturbed, it will tend to remain in the disturbed position. Such an equilibrium position is called neutrally stable or indifferent in the small. The expression ‘in the small’ is used because the definition depends on the small size of the perturbations. If the disturbances are allowed to be of finite magnitude, then it is possible for a system to be unstable in small but stable in the large (point B, Fig. 2.1(b)) or stable in the small but unstable in the large (point A, Fig. 2.1(c)) [2].

![Diagram](image)

Figs. 2.1(a) Character of static equilibrium positions in the small. (b), (c) Character of static equilibrium positions in the large.

A close assessment of the critical load for simple mechanical stability models reveals that the system maintains its stable equilibrium states as long as the work done due to internal resisting forces is greater than that due to the external load for any disturbance from the equilibrium position. In other words, it is the balance between the potential energy due to the internal resisting forces or the strain energy, and the potential energy due to the external load or the load potential, which accounts for the stability of the system.
2.2 Objective of the Study

Slender columns buckle elastically and the Young’s modulus of the column material governs the buckling behaviors. Steel is supposed to have the highest elastic stiffness among the common engineering materials. Thus, under the same conditions, slender steel columns will buckle at loads much higher than those for columns made of other materials. On the other hand, short columns buckle plastically and the stress-strain curves beyond the elastic limit
dominate their buckling behaviors. For example, Urushiyama et al. [3], showed that short columns made of Cu based SMA carry higher buckling loads compared with steel columns.

Columns made of traditional engineering materials become practically useless once they buckle, because of the fact that usually too large deformation takes place immediately after buckling. On the other hand, at usual room temperatures, superelastic SMAs can regain the original shape when the cause of deformation, that is, the mechanical load, is withdrawn. Thus columns made of the superelastic SMA (sometimes termed only as SMA, here after) need not be replaced from the place of application even after repeated loading-unloading cycles.

Moreover, to the author's knowledge no rigorous investigation on the buckling of superelastic SMA columns has been reported in the literature. Observing the above facts, the present study extensively deals with the buckling and postbuckling behaviors of the superelastic SMA columns through a complete loading-unloading cycle, for slenderness ratio of 28-368. For valid comparison, experiments were also carried out for SUS304 and Al columns under similar test conditions.

Basic mechanical properties of the SMA and the phenomenon of stress-induced martensitic transformation (SIMT) have been discussed in Chapter 1, since those are closely related to the buckling resistance of the SMA columns. This chapter describes the details of the experimental procedure, discusses some interesting results and qualitatively explains those results.

2.3 Test Conditions and Experimental Procedure

The materials, configurations and conditions used in this experiment were as follows. Column materials: SMA (Ti49.3 at% Ni50.2at% V0.5at%), stainless steel (SUS304), and Aluminum. The unsupported column lengths were 14, 19, 24, 34, 44, 54, 64, 74, 84, 109, 134,
159, and 184 (mm). Diameter of the columns was 2mm. SMA's transformation temperatures were, -59°C, -34°C, -27°C and -3°C for $M_f$, $M_s$, $A_s$ and $A_r$, respectively. Room temperature range was 23°C - 30°C. Instron machine was used and the speed of the cross-head during loading-unloading cycle was 2 mm/min.

At first, the columns were inserted into the holes of the loading fixtures (Fig. 2.3). One of the fixtures was fixed while the other was attached to the movable cross-head of the Instron machine. As the test started, the axial deformation (more precisely, the end shortening) of the column due to the axial load $P$ was directly measured by $\delta$, the linear displacement of the moving fixture. During compressive loading on the column, the moving fixture moved towards the fixed one until its predetermined final position was reached. In the mean time, the column buckled and had enormous deformations. Immediately after the final point of
displacement was reached, the moving fixture was moved away from the fixed one. Thus, the largely deformed column was allowed to gradually recover its shape. The unloading process was stopped and the cycle ended when \( P \) became approximately zero (Fig. 2.4).

It was observed during the experiments that quite astonishingly too short superelastic SMA columns do not fail due to buckling simply after the first peak load is reached or the mode of deformation is changed. Those columns fail only after exceptionally large deformation takes place beyond the first point of instability. Thus, for columns with \( L/k=28 \) and 38, the unloading process started when the values of \( \Delta \) (that is, \( \delta L \)) were 10% and 5.5%, respectively, in order to closely observe the interesting \( P-\Delta \) curves. All other columns were unloaded from \( \Delta = 3\% \).

The value of \( \Delta \), measured after the complete cycle (at \( P=0 \)) corresponds to the residual axial strain (\( \varepsilon \)). As mentioned, \( \Delta \) was measured by the displacement of the loading fixture and thus its measured values may not be always highly accurate. Moreover, it is difficult to fully eliminate the gap that remains between the loading fixture and the column at the beginning of the loading. Though the value of \( \varepsilon \) may not be very accurate, it is a handy parameter that gives a rough evaluation of shape recovery, specially, for the SMA columns.

2.4 Results and Discussions

The stress-strain curves for sufficiently large strains, determined by pure tensile and compressive tests, are once more presented in Fig. 2.5, since they will be frequently referred to while discussing the buckling and postbuckling behaviors of the superelastic SMA, SUS304 and Al columns. The comparative buckling and postbuckling behaviors of the columns with different \( L/k \) are demonstrated by the axial compressive load (\( P \)) versus the end shortening (\( \Delta \)) curves as shown in Figs. 2.6-2.13. To precisely observe the buckling and
postbuckling behaviors of each column, the corresponding $P$-$\Delta$ curve was obtained much beyond the point of instability.

![Experimental s-s curves](image)

Fig. 2.5 Nominal stress-strain curves for the superelastic SMA, Al and SUS304 rods.

2.4.1 $P$-$\Delta$ curves for $L/k=28$ and 38

As observed from Figs. 2.6 and 2.7, the nature of $P$-$\Delta$ curves for the two short SMA columns is significantly different. For the shortest SMA column (that is, with $L/k=28$) the load increases after a distinct change in the mode of deformation (Fig. 2.6).
It should be noted that the term 'change in the mode of deformation' is used here to refer to the distinct change in the slope of the $P-\Delta$ curve for a column. The portion of the $P-\Delta$ curve connecting the two modes of deformation contains a point of instability. Ideally, the first and the second modes of deformation should correspond, respectively, to the initial straight shape and the buckled shape of the columns. Owing to the unavoidable initial imperfections, however, the column is slightly bent (may not be identified easily) during the first mode of deformation. While, the second mode of deformation corresponds to the markedly bent

Fig. 2.6 Comparative load-end shortening curves for $L/k=28$. 
configuration of the column after it passes through the point of instability. As seen, unlike the Al and SUS304 columns, quite remarkably, this SMA column can sustain significantly high load during its secondary mode of deformation (Fig. 2.6). On the other hand, for $L/k=38$, the $P-\Delta$ curve of the SMA column (Fig. 2.7) shows a valley between two distinct peak (buckling) loads (the second peak being slightly higher than the first one!). The above characteristic is contrary to the general trend that load falls off monotonously for any further compression after the first distinct peak load (which is also a point of instability) on the equilibrium configuration path ($P-\Delta$ curve) of a column.

![Graph](image.png)

Fig. 2.7 Comparative load-shortening curves for $L/k=38$. 
By numerical simulation, Auricchio and Sacco [4] demonstrated that for pure bending of a superelastic Nitinol beam, with different properties in tension and compression, the axial strain has a non-monotonous response with the bending moment during loading and unloading. The complicated movement of the neutral axis of the cross-section of the beam due to stress-induced martensite transformation (SIMT) leads to such peculiar response. Obviously, the instability of a column and pure bending of a beam are very different phenomena. However, it appears that for the present study too, the occurrence of SIMT along with the phenomenon of buckling may cause the load to change non-monotonically with deformation for the superelastic SMA columns. Thus the unique buckling and postbuckling behaviors of the short superelastic SMA columns are qualitatively explained in the following paragraphs.

The pure compression stress-strain curve (Fig. 2.5) will dominate the buckling and postbuckling behaviors particularly for too short superelastic SMA columns with $L/k=28$ (Fig. 2.6). Obviously for this column, at one stage of the prebuckling compression, the SIMT is initiated within the column material. As discussed, unlike the tensile stress-strain curve, there is no distinct plateau during the SIMT process for the compressive stress-strain curve. Rather the initiation of the SIMT may be marked by a slight and smooth decrease in the material stiffness (Fig. 2.5). It appears, because of this decrease in the material stiffness, the highly compressed SMA column (which is prone to buckling if there is any kind of disturbance) gradually approaches the first point of instability with increasing value of end shortening (Fig. 2.6). The bending effect causes the innermost fibers of the cross-section of the column to be under higher compressive strains compared with the outermost fibers. Thus the innermost fibers first complete the SIMT. After the SIMT is completed, the material stiffness again increases significantly, which in turn increases the resisting moment of the bent column. Consequently, during the secondary mode of deformation, the bending effect
due to the applied load is overcome by the resisting moment until the second point of instability is appeared. Thus, because of its peculiar material property, quite unusually, after the first point of instability, this column can assume highly stable postbuckling configuration. Numerical simulation of the buckling and postbuckling behaviors of the short superelastic SMA columns (based on compressive stress-strain curve), presented in the next chapter, has verified the above facts. The $P-\Delta$ curve of Fig. 2.6 indicates that after pretty large deformation ($\delta L=9.6\%$) this column ultimately fails at the second point of instability at a load much higher than that for SUS304 column.

Next, let us discuss the $P-\Delta$ curve with two peaks for the SMA column with $L/k=38$. The compressive stress-strain curve will also dominate the behavior of this column. Significantly high stress prior to the first peak (when $\Delta$ is about 2.7% in the $P-\Delta$ curve of Fig. 2.7) can easily initiate the SIMT within the column material. After the first peak (which is a point of instability for the column) is reached, the load falls off to a valley and the column is further deformed. It can be verified that, compared with the primary mode of deformation prior to the first peak, large deformation takes place with small change in load, between the first peak and the valley of the $P-\Delta$ curve (Fig. 2.7). Consequently, as the load slowly falls from the first peak to the valley, each strained cross-sections of the column are added with rapidly increasing strains. Due to the bending strains, for all the critically strained cross-sections of the column, the fibers that are farthest from the neutral axis, first undergo complete phase transformation. The material stiffness starts increasing as fibers gradually complete SIMT. Consequently, the bending effect due to the load $P$, is overcome by the increasing resisting moment due to significant increase in the material stiffness. Thus, the load again increases slowly from the valley until buckling occurs at the second peak (Fig. 2.7). Obviously, the increase in the material stiffness after SIMT can more significantly contribute to resist the buckling of the SMA columns having lower $L/k$. Thus, the SMA column with $L/k=28$ shows
much more buckling resistance compared to that with $L/k=38$ after the first point of instability.

It is noteworthy that the prebuckling stiffness of the SUS304 column is higher than that of the SMA column. Therefore, it can be concluded from the current discussions that the special nature of the stress-strain curves encompassing the region of the SIMT enables the short SMA columns to exhibit the above discussed exceptional behaviors.

2.4.2 $P$-$\Delta$ curves for $L/k=48$-368

For $L/k=48$, the SMA column shows a single peak but its buckling load is significantly higher than that for Al column and the residual strain is too small (Fig. 2.8).

Fig. 2.8 Comparative load-end shortening curves for $L/k=48$.

Some notable postbuckling behaviors exhibited by the SMA columns with $L/k=88$-368 are shown in the Figs. 2.9-2.13 and discussed in the following paragraphs.
Fig. 2.9 Comparative load-end shortening curves for $L/k=88$.

Fig. 2.10 Comparative load-end shortening curves for $L/k=108$. 
Fig. 2.11 Comparative load-end shortening curves for $L/k=128$.

Fig. 2.12 Comparative load-end shortening curves for $L/k=218$. 
LOAD SUSTAINING CAPABILITY FOR THE POSTBUCKLING COMPRESSION

Although the SMA column with $L/k=38$ exhibits some exceptional behaviors, it is found that load falls off monotonously for other columns if the compression is continued beyond the first peak on the $P$-$\Delta$ curves up to the final point of loading. As seen from Table 2.1, among the three materials, the load falls off most drastically for the SUS304 column during the postbuckling compression. Moreover, it can be verified that shorter is the column, more notably the load falls off. However, compared with the Al and SUS304 columns, the superelastic SMA columns, particularly the slender ones, can easily sustain the load with least change in magnitude after buckling (Table 2.1 and Figs. 2.9-2.13). For instance, for SMA columns with $L/k=368$, the load falls off from its peak value (that is, from the buckling load) only by 4% in magnitude up to the last point of loading. For the same $L/k$, the fall in load's
magnitude is 5 times more for Al columns and almost 10 times more for SUS304 columns (Table 2.1).

Table 2.1 Percent fall between the load at the point of instability and load at $\Delta = 3\%$.

<table>
<thead>
<tr>
<th>$L/k$</th>
<th>368</th>
<th>318</th>
<th>268</th>
<th>218</th>
<th>168</th>
<th>128</th>
<th>88</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMA</td>
<td>4</td>
<td>5</td>
<td>12</td>
<td>20</td>
<td>27</td>
<td>36</td>
<td>43</td>
</tr>
<tr>
<td>Al</td>
<td>20</td>
<td>26</td>
<td>31</td>
<td>44</td>
<td>51</td>
<td>57</td>
<td>58</td>
</tr>
<tr>
<td>SUS304</td>
<td>38</td>
<td>45</td>
<td>49</td>
<td>55</td>
<td>63</td>
<td>63</td>
<td>58</td>
</tr>
</tbody>
</table>

INCREASE OF THE RECOVERY FORCE DURING UNLOADING OF THE COLUMNS

Next, let us discuss another remarkable phenomenon exhibited by some of the SMA columns during the unloading process. The shape recovery process of the buckled columns actually starts with the unloading and ends when the load becomes approximately zero. It is found that for the certain range of slenderness ratio, the recovery force of the SMA columns increases, quite significantly (for $L/k=68-128$), as if the pronouncedly deformed SMA columns push the loading fixtures during the unloading period (Figs. 2.9-2.11, also Figs. 2.17-2.19). The above phenomenon is quite contrary to the general trend that load falls off monotonously during unloading of a buckled column. For too slender SMA columns ($L/k=218$ to 368), however, the recovery force does not change much during the unloading process, and starts falling linearly after closely approaching the initial portion of the loading curve (Figs. 2.12, 2.13).
It seems, for a particular $L/k$ whether the recovery force of the SMA column will increase notably or not, depends on the nature of its $P$-$\Delta$ curve (equilibrium path) during loading. As noted, slender SMA columns ($L/k=218-368$) can sustain the load with least changes in magnitude up to the last point of loading (Table 2.1). During unloading of these slender columns the recovery force do not change much. On the other hand, for the SMA columns with $L/k=68-128$, load falls off notably if the compression is continued up to $\Delta=3\%$. Although the load falls off significantly during the postbuckling deformation, due to its superelasticity, the buckled SMA column is still capable of recovering the initial shape upon unloading. As a result, at one stage during the shape recovery process the bent column starts pushing the fixture until the rest of the shape is almost recovered. Thus the $P$-$\Delta$ curve during unloading gradually increases to a maximum load. The shape is almost recovered at the maximum point and the recovery force does not increase any more with further displacement of the fixture. The remaining amount of strain is recovered linearly resembling an elastic unloading curve.

2.4.3 Buckling loads versus slenderness ratio

The buckling loads for the columns as shown in Table 2.2 and Fig. 2.14, are determined from the peak loads (after which the load falls off monotonously) of the $P$-$\Delta$ curves. Naturally, shorter is the column, higher is its buckling load and larger is the corresponding prebuckling deformation. The above fact can be easily verified from the Figs. 2.6-2.13. For a certain value of the axial load, the most critical section of the short column becomes fully plastic and initiates its failure. As verified by experiments, Al has the lowest yield strength among the three materials. Thus Al columns with $L/k=28-108$, have the lowest buckling loads. Moreover, once started, plastic deformation continues without appreciable change in the load's magnitude until fracture occurs for the Al and SUS304 rods (Fig. 2.5). That is why, the
buckling loads change by small magnitude with $L/k$ if the SUS304 and Al columns are too short (Fig. 2.14 and Table 2.2).

![Buckling load versus slenderness ratio](image)

**Fig. 2.14** Buckling load versus slenderness ratio (28-168) for the columns.

On the other hand, among the three materials, SMA column's buckling load increases most significantly with the decreasing $L/k$ and ultimately it exceeds the buckling load of the SUS304 column (Table 2.2 and Fig. 2.14). As already discussed, for too short columns, SIMT is triggered on during loading. After the complete SIMT the stiffness of the SMA materials increases significantly. Consequently, this increase of the material stiffness helps too short SMA columns to easily sustain higher buckling loads compared with the SUS304 columns.
Table 2.2 Buckling loads ($P_{cv}$) and residual axial strains ($\varepsilon$) of the columns for the first cycle.

<table>
<thead>
<tr>
<th>$L/k$</th>
<th>Material</th>
<th>$P_{cv}$ (N)</th>
<th>$\varepsilon$ (%)</th>
<th>$L/k$</th>
<th>Material</th>
<th>$P_{cv}$ (N)</th>
<th>$\varepsilon$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>SUS304</td>
<td>2305, 2325, 2374</td>
<td>8.5</td>
<td>148</td>
<td>SUS304</td>
<td>765, 769, 769</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>2930, 2943, 2972</td>
<td>1.2</td>
<td></td>
<td>SMA</td>
<td>280, 285, 286</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Al</td>
<td>1163, 1163, 1170</td>
<td>8.5</td>
<td></td>
<td>Al</td>
<td>299, 301, 301</td>
<td>1.2</td>
</tr>
<tr>
<td>38</td>
<td>SUS304</td>
<td>2235, 2241, 2261</td>
<td>5.6</td>
<td>168</td>
<td>SUS304</td>
<td>638, 648, 648</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>1756, 1776, 1805</td>
<td>0.9</td>
<td></td>
<td>SMA</td>
<td>226, 232, 234</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Al</td>
<td>1071, 1072, 1100</td>
<td>5.6</td>
<td></td>
<td>Al</td>
<td>240, 245, 254</td>
<td>1.0</td>
</tr>
<tr>
<td>48</td>
<td>SUS304</td>
<td>2041, 2060, 2070</td>
<td>2.0</td>
<td>218</td>
<td>SUS304</td>
<td>426, 427, 427</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>1481, 1501, 1614</td>
<td>0.35</td>
<td></td>
<td>SMA</td>
<td>137, 139, 143</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>Al</td>
<td>981, 991, 991</td>
<td>2.0</td>
<td></td>
<td>Al</td>
<td>153, 154, 155</td>
<td>0.75</td>
</tr>
<tr>
<td>68</td>
<td>SUS304</td>
<td>1638, 1717, 1815</td>
<td>2.0</td>
<td>268</td>
<td>SUS304</td>
<td>294, 295, 307</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>1020, 1060, 1079</td>
<td>0.3</td>
<td></td>
<td>SMA</td>
<td>90, 91, 91</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>Al</td>
<td>814, 824, 834</td>
<td>1.9</td>
<td></td>
<td>Al</td>
<td>103, 105, 105</td>
<td>0.5</td>
</tr>
<tr>
<td>88</td>
<td>SUS304</td>
<td>1364, 1373, 1373</td>
<td>1.9</td>
<td>318</td>
<td>SUS304</td>
<td>217, 217, 218</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>697, 716, 726</td>
<td>0.2</td>
<td></td>
<td>SMA</td>
<td>65, 66, 66</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>Al</td>
<td>638, 667, 667</td>
<td>1.7</td>
<td></td>
<td>Al</td>
<td>71, 74, 76</td>
<td>0.35</td>
</tr>
<tr>
<td>108</td>
<td>SUS304</td>
<td>1118, 1158, 1177</td>
<td>1.8</td>
<td>368</td>
<td>SUS304</td>
<td>165, 167, 167</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>520, 540, 559</td>
<td>0.2</td>
<td></td>
<td>SMA</td>
<td>52, 53, 53</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>Al</td>
<td>500, 520, 520</td>
<td>1.5</td>
<td></td>
<td>Al</td>
<td>53, 56, 59</td>
<td>0.22</td>
</tr>
<tr>
<td>128</td>
<td>SUS304</td>
<td>893, 922, 932</td>
<td>1.6</td>
<td></td>
<td>SMA</td>
<td>358, 363, 367</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Al</td>
<td>378, 383, 387</td>
<td>1.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 For each type of material at least 3 tests were performed for a particular $L/k$. The highest $P_{cv}$ and the corresponding maximum $\varepsilon$ are shown. $P_{cv}$ for SMA column with $L/k=38$ corresponds to the second peak of the $P-\Delta$ curve.
Unlike the short SMA columns, the slender ones can not sustain much axial compression and buckle elastically while they are in the austenite phase. SMA has the lowest elastic stiffness among the three materials. Therefore, for $L/k = 128-368$, the buckling loads of Al columns are higher than those of SMA columns by 5%-15% (Table 2.2, Figs. 2.11-2.14). Usually, this difference increases with the increasing slenderness ratios. In the same manner, for the above range of the slenderness ratio, the buckling loads for SUS304 columns are much higher (about 2.5-3.3 times) than those for SMA columns.

ACCURACY OF THE EXPERIMENTAL RESULTS

Since the buckling loads of the columns are highly imperfection sensitive, it is essential to check the accuracy of the present experimental results by theory. The easiest way to check the accuracy of the experimental buckling loads of Table 2.2 is to make use of the famous Euler formula ($4\pi^2EI/L^2$, symbols having their usual meanings) for both ends-clamped slender columns. As shown in Table 2.2, for the columns made of stainless steel (SUS304) with $L/k = 218, 268, 318$ and 368, the highest buckling loads are 427N, 307N, 218N and 167N, respectively. The Young’s modulus ($E$) of SUS304 is around 200GPa. Accordingly, the corresponding Euler buckling loads are 522N, 345.4N, 245.3N and 183.2N, respectively. Euler formula [1], based on eigen-value solutions, is for ideal conditions. Even then the experimental buckling loads are found to be 82%-91% of their theoretical values according to the above formula. Higher is the slenderness ratio, better is the prediction by theory. Since the prebuckling stress for a particular column material exceeds the proportional limit below certain slenderness ratio, the above formula can’t be used in those cases to predict the buckling loads. However, good agreement between the theoretical and the experimental buckling loads for slender SUS304 columns indicates that the results obtained for columns of other materials and of different slenderness ratios using the same method are also accurate. In
fact, it will be verified in the next chapter that both the prebuckling and postbuckling
behaviors of the SMA columns with various $L/k$, as predicted by the numerical simulation and
as observed from experiment are in close agreement for most of the cases.

2.4.4 Consecutive loading-unloading cycles for the columns

One noteworthy postbuckling characteristic of the superelastic SMA columns (if not too
short) is that, having too small values of residual strain ($\varepsilon$) after a complete cycle, they are
able to exhibit almost the same pattern of $P-\Delta$ curves for a few repeated loading-unloading
cycles. The above excellent capability is attributed to SMA's superelasticity. Huge
postbuckling compression on the columns initiates SIMT that is again converted to pure
austenite phase upon unloading allowing the columns to recover large strains. The measured
values of $\varepsilon$ after the first complete cycle give an estimation of the shape recovery capability
of the buckled columns. Among the three materials, $\varepsilon$ increases most significantly for Al
columns with the decreasing $L/k$, although, slender Al columns show small $\varepsilon$ (Table 2.2).

Superelastic SMA columns' unique performances for consecutive cycles are
comprehensively demonstrated in the Figs. 2.15-2.19. Although, large postbuckling
deformations take place for each cycle, the cumulative residual strains are too small even for
the short SMA columns ($L/k=28-68$) after a few consecutive loading-unloading cycles (Figs.
2.15-2.17). On the other hand, both the Al and SUS304 columns suffer large permanent
deforation in all the cases as seen from Figs. 2.6-2.13, 2.16 and Table 2.2, and therefore
found to be plastically bent when completely unloaded.

For $L/k=28$, two modes of deformation is clearly observed for the first three cycles (Fig.
2.15). Remarkably, along with its very high buckling resistance, this SMA column recovers
the shape with small residual strains upon unloading, provided $\Delta_{ext}$ is not more 10% during
loading. As a result, for each of the first four cycles performed, it can easily sustain higher load than the buckling load of the SUS304 column (compare Figs. 2.6 and 2.15).

Fig. 2.15 Four consecutive loading-unloading cycles for the superelastic SMA column ($L/k=28$).

It should be mentioned here that for Fig. 2.16, $\Delta_{set}$ was 5.5% for each cycle, in order to observe the second peak load of the SMA column. As discussed, there is a valley between two distinct peak (buckling) loads for the SMA column with $L/k=38$ for the first cycle. These two
peaks (the second one being the higher) are also seen for the second cycle. It is observed, however, that the valley between the two peak loads gradually diminishes for repeated cycles. Unlike the SUS304 or Al columns, the SMA column's buckling load falls slightly for repeated cycles. As a result, though, the SUS304 column can sustain the highest buckling load for the first cycle, for the second and third cycles, SMA column's buckling load is the highest. Furthermore, it can be easily verified that SMA column's cumulative residual strains after each cycle are much smaller compared with its counterparts (Fig. 2.16). Thus it can be
concluded that, short superelastic SMA columns can be good candidates as structural elements in practical applications where high load carrying capability is required for a few repeated loading-unloading cycles.

The increase of recovery forces during unloading for a few consecutive loading-unloading cycles are observed for the SMA columns with $L/k=68$, 128 and 148 (Figs. 2.17-2.19). At the same time, it is also verified that the cumulative residual strains even after a few consecutive cycles are too small.

![Graph](image)

**Fig. 2.17** Three consecutive loading-unloading cycles for the SMA column with $L/k$ of 68 ($\Delta_{set}$ for the 1st, 2nd and 3rd cycles were 3%, 2% and 4.5% respectively).
Fig. 2.18 Consecutive loading-unloading cycles for the SMA column with $L/k$ of 128 ($\Delta_{set}$ for each cycle was 3%).

Fig. 2.19 Consecutive loading-unloading cycles for the SMA column with $L/k$ of 148 ($\Delta_{set}$ for each cycle was 2.2%).
Naturally, higher is the $L/k$, better is the shape recovery. Table 2.2 shows, for $L/k=88-168$, $\epsilon$ is within 0.2% and for too slender SMA columns ($L/k=218-368$), it is negligibly small (within 0.04%). Thus it can be concluded that those SMA columns ($L/k=88-368$) will show more or less the similar unique buckling and post buckling behaviors for a few cycles. This fact has been best reflected by Fig. 2.19. Interestingly, for this column (with $L/k=148$), $P_{cr}$ is found to be slightly higher for the $4^{th}$ cycle than that for the $2^{nd}$ cycle. For the same column, the residual strain after the $1^{st}$ cycle is the largest. But, for the next consecutive cycles, the residual strain reduces gradually. Thus, the cumulative residual strain is only 0.27% after the 6th cycle (Fig. 2.19).

2.4.5 Influence of change in the room temperature on the $P-\Delta$ curve

Finally, the effect of room temperature change on the unique buckling and postbuckling behaviors of the superelastic SMA columns should be clarified. As mentioned, for the present study the room temperature was always much higher than the austenite finish temperature of the SMA, leading to its property of superelasticity during unloading. As a result, SMA columns, particularly the slender ones can recover their shapes with small residual strains even after too large postbuckling deformations (Table 2.2, Figs.2.18, 2.19).

From the results of pure tension tests it is a well-known fact that, the critical stress necessary to induce SIMT process may increase notably and the stress-strain hysteresis change with significant increase in temperature (of course, the temperature should not be so high to soften the material) for the superelastic SMA [5-7]. Thus, if both the tensile and compressive properties of the materials change notably with changing temperatures, it is likely to affect the buckling behaviors of the columns. A comprehensive study concerning the above point will be separately carried out in near future. However, for the present case, many tests were performed for a long span of time during which the room temperature was within...
23°C-30°C. Practically, any change in the mechanical properties of the materials, due to such a small temperature variation may be treated as insignificant. As discussed slender SMA columns buckle elastically in the austenite phase while the short ones buckle plastically after the maximum strains in the column material exceed the region of SIMT.

Fig. 2.20 Load-end shortening curves for three different test temperatures for the SMA column with $L/k$ of 28.

That the buckling and postbuckling behaviors of the shortest ($L/k=28$) superelastic SMA columns (three different specimens) are very similar for small temperature variation could be clarified from Fig. 2.20. As seen, the $P$-$\Delta$ curves during the whole cycles are almost identical
for room temperatures 23°C and 26°C. Though for the room temperature of 25°C, the $P-\Delta$ curve shows slight deviations, it can be treated as negligible, as the variation in the buckling load is within 1%. Since the slight variation in room temperature has negligible effect on the buckling and postbuckling behaviors of the shortest SMA column, it can be concluded that, it (the temperature effect) would be also negligible for all other columns, in particular, for the slender ones, which buckle elastically in the parent austenite phase. Moreover, buckling behaviors of columns are highly imperfection sensitive. Thus the effect of geometrical or, physical imperfections are more important than the effect of slight temperature change.

### 2.5 Conclusions

Using a loading-unloading cycle, some interesting and useful buckling and postbuckling characteristics of the superelastic SMA columns were observed from the present study. Those phenomena depend largely on the slenderness ratio $(L/k)$. For example, the $P-\Delta$ curve for the short SMA column with $L/k=38$ shows two distinct peak loads during loading, the second one being the higher, quite contrary to the general notion that load falls off monotonously for other columns during any further compression beyond the first peak load. Similarly, for $L/k=28$, the SMA column can sustain significantly high load (even higher than that of a SUS304 column) after a distinct change in the mode of deformation. For a few consecutive cycles, these two short SMA columns ($L/k=28, 38$) have the smallest cumulative residual strains but the highest buckling loads. Thus the short superelastic SMA columns can be excellent candidates for any engineering application where high compressive load carrying capability is required for a few repeated loading-unloading cycles.

It is also found that, too slender SMA column has the lowest buckling loads among the three materials used in the investigation. Because of high material stiffness after the SIMT,
the buckling load of the superelastic SMA columns increases most significantly for decreasing $L/k$, and ultimately exceeds that of the SUS304 column.

It is also observed that the SMA columns, if not too short, can sustain the buckling load with least change in magnitude if the compression is continued much beyond the point of instability. Moreover, during unloading from the postbuckling state, the recovery force of the SMA columns (for $L/k=68-128$) increases significantly, another remarkable phenomenon, contrary to the general trend that load falls off monotonously during unloading of a buckled column. Furthermore, if the slenderness ratio is not too low, the initial shape before the deformation is recovered with negligible residual strains when the profusely deformed SMA columns are unloaded from the postbuckled state. Thus they can be used for repeated cycles to show the similar buckling and postbuckling behaviors.

The above discussed unique phenomena can be attributed to the special nature of SMA’s stress-strain curve and the superelasticity itself. On the other hand, for the whole range of slenderness ratio considered in this study, Al and SUS304 columns show large residual strains after the complete cycle.
2. Buckling and Post Buckling Characteristics of the Superelastic SMA Columns - Experiment

References


Chapter 3

Buckling and Postbuckling Characteristics of the Superelastic SMA Columns - Numerical Simulation

Summary

Numerical simulation has been carried out in order to analyze a few experimentally observed buckling and postbuckling behaviors of the superelastic SMA columns. Simulation has also been performed for the SUS304 columns. Experimentally obtained stress-strain data have been utilized to predict the unique load-deformation curves of the columns during a loading-unloading cycle, by using the FEM code ANSYS.

Simulation results verify that the shortest SMA column (L/k=28) assumes a highly stable postbuckling configuration after the first point of instability that makes it to sustain higher load in comparison with the SUS304 column.

The total strain distributions in the column materials have been predicted at different state of loading. Precise and quantitative analyses of the results verify that the SMA column’s unique behaviors could be attributed to the special nature of the stress-strain curves.

A special method has been devised to simulate the unloading path of the slender superelastic SMA columns and its usefulness has been demonstrated. It is found that because of the buckled slender SMA columns’ elastic shape recovery, the load increases when they are unloaded.
3.1 Introduction

From the previous experimental study dealing with the buckling and postbuckling behaviors of the superelastic SMA, SUS304 and Al columns, the following unique characteristics have been demonstrated for the superelastic SMA columns: (1) For decreasing value of the slenderness ratio, the buckling load increases most significantly and below certain slenderness ratio, it is higher than that of the SUS304 column. The slender superelastic SMA columns, however, buckle elastically at slightly low loads in comparison with the Al columns. (2) For higher slenderness ratio, they can sustain the load with least change in the magnitude for the postbuckling compression. (3) For a particular range of slenderness ratio, they exhibit increase of the recovery force during unloading. (4) The residual strain is too small and thus almost the original shape of a slender column is recovered by unloading.

In order to analyze the above mentioned unique phenomena of the SMA columns precisely and quantitatively, numerical simulation has been carried out and the predicted results are compared with the available experimental results. The FEM code ANSYS has been used for the simulation purpose. This study is also to devise a method to simulate the complete loading-unloading cycle of the buckled superelastic SMA column. Some basic features of the quantitative analysis for solving the problems of instability of structures are briefly discussed below.

3.1.1 Physical and geometrical nonlinearities

The governing equations for predicting the prebuckling and postbuckling behaviors of structures consist of three sets of equations, namely the equilibrium equations, the equation relating the stress and strain, and the compatibility equations. The equilibrium equations relate the external load with internally induced stress and bending moment resultants. The
material property is represented by the stress-strain equations. The compatibility equations relate the internal strains with the physical deflection of the material of the structural element.

The nonlinearity is introduced into the governing equations of the problem in three ways:

a. through the strain-displacement relations (compatibility equations),

b. through the equations of equilibrium of a volume element of the body, and

c. through the stress-strain relations.

In (a) and (b) the retention of nonlinear terms is conditioned by geometric considerations, that is, the necessity of taking into account the angles of rotation in determining the changes of dimension in the line element and in the formulation of the conditions of equilibrium of a volume element. On the other hand, the nonlinear terms appear in the third set of equations (c) if the material does not behave in a linearly elastic fashion. Hence, there are two types of nonlinearity: geometric, and physical.

The use of nonlinear strain-displacement equations is required for accurate prediction of load-deflection curves for a structure in order to study its prebuckling and postbuckling behaviors. Additionally, for dealing with the plastic deformations of the buckled structures, the use of stress-strain relations beyond the elastic limit becomes essential. Moreover, the superelastic SMA can recover large nonlinear strain through a hysteresis, when the applied load, which causes the deformation, is withdrawn. Therefore, for accurate analysis, both the nonlinear stress-strain property (that is, finite strain) and the effect of large deflection (that assumes large rotation though the strain may be small) options have been considered simultaneously for the current study.
3.1.2 Predictions of prebuckling and postbuckling behaviors of the structures

The predictions of buckling loads for structures based on eigen-value solutions usually result in ideal and therefore much higher buckling loads than their real values during applications. Alternately, the predictions of buckling loads for structures based on large deformation theory may yield more realistic solutions than the corresponding eigen-value solutions. The strategy to determine the buckling loads of the columns simply from the equilibrium study (considering the large deflection), is based on the following two theorems of Thompson and Hunt [1].

- **Theorem 1.** An initially stable equilibrium path rising monotonically with the loading parameter can not become unstable without intersecting a further distinct secondary equilibrium path.

- **Theorem 2.** An initially stable equilibrium path rising with the loading parameter can not approach an unstable equilibrium state from which the system would exhibit a finite dynamic snap without the approach of an equilibrium path (which may or may not be an extension of the original path) at values of the loading parameter less than that of the unstable state.

Thus, the buckling characteristics of a structure may be better comprehended if its load-deformation curve (mathematically, the equilibrium configuration path) can be traced for both the prebuckling and postbuckling zones. The first instability in the equilibrium equations (based on large deformations) on the stable primary equilibrium configuration path (ensuring the unique state of lowest potential energy) would correspond to the critical load of the structure, whatever may be the type of buckling (for example, limit point or branching, symmetric branching or unsymmetric branching, stable branching or unstable branching).

The onset of the first point of instability is indicated by a substantial increase in the displacements for very small increase of the loading parameter. At the critical point itself any
increase of the load parameter, however small, causes enormous deformations and thus the numerical technique fails to converge to any solution. It should be mentioned here that the term 'point of instability' refers to the initiation of the secondary mode of deformation, may it be a limit point or a branching point. As will be seen later, the theorems of Thomson and Hunt [1] are useful to interpret the postbuckling behaviors of the short SMA columns.

3.2 Particulars of the Experiment and Simulation

3.2.1 Experiment

The test conditions and experimental procedure (Figs. 3.1(a) and 3.1(b)) are the same as described in the previous chapter. It should be mentioned here that during loading, large end shortening was necessary to observe some important postbuckling phenomena for the columns with $L/k=38$ and 28. Thus values of $\Delta_{sel}$ were about 5.5% and 9% for those two columns, respectively. For all other slenderness ratios, however, $\Delta_{sel}$ was within 3%. The value of $\Delta$, after the complete cycle corresponds to the residual axial strain of the columns.

\[ \Delta = \frac{\delta}{L}, \quad k = \frac{D}{4}, \]

Slenderness ratio = $L/k$

Fig. 3.1 (a) Column inserted into the fixtures.
Loading starts
$P$-$\Delta$ curve is observed

Column buckles
Loading continues

$\Delta = \Delta_{\text{set}}$
Loading stops and unloading starts

$P = 0$
Unloading stops

Next cycle

Fig. 3.1(b) Experimental procedure.

$U_x = \Delta_{\text{set}}$

$U_y = 0$
for surface

$U_x = 0$
Mid-span

$h$

$L/2$

Fig. 3.1(c) Half model of the column used for simulation.
3.2.2 Simulation

Static analysis has been performed using the commercial FEM code ANSYS (Swanson analysis systems, Inc.) for the present study. The half model of the column depicting the boundary conditions and loading is shown in Fig. 3.1(c). Particulars of the geometry are explained in the appendix. A negligibly small transverse disturbance, necessary for buckling analysis, was assigned by $F_y$ (the magnitude was 1N) at any point on the mid-span of the column.

Small load steps are essential for accurate calculation of the buckling load. Therefore, during loading (in terms of end displacement $\Delta$), the total end displacement ($\Delta_{tot}$) was assigned in a sufficiently large number of steps. Similarly, the unloading path was simulated in small steps until $P$ became zero. The resulting $P-\Delta$ curve, containing the point of instability is observed and compared with that obtained from experiment.

The stress-strain (also abbreviated as s-s hereafter) curves for the superelastic SMA and SUS304 are determined by experiment for sufficiently large strain and used for the simulation. Poisson’s ratio, density and Young’s modulus were taken as 0.33, $6.5 \times 10^3$ kg/m$^3$ and $65 \times 10^9$ Pa, respectively for SMA. While for SUS304, the same set of data were taken as 0.25, $7.8 \times 10^3$ kg/m$^3$, and $210 \times 10^9$ Pa, respectively.

MATERIAL MODELS USED FOR SIMULATION

The simulation results presented in this study are based on three kinds of material models capable of handling nonlinear stress-strain curves. The following paragraphs briefly delineate how those material models have been used to predict the columns’ behaviors.

Multilinear isotropic hardening (MISO) model- MISO model uses the von Mises yield criteria coupled with an isotropic work hardening assumption. Suitable for large strain, this model can represent the highly nonlinear material behavior by a piece-wise-linear curve,
through at most 100 stress-strain points. Only positive values can be given as input. Therefore, the compressive and tensile s-s data can’t be used simultaneously. Using the input stress-strain (or, the loading curve), MISO model can accurately simulate both the loading and unloading paths for the SUS304 columns and also the loading path for the SMA columns. This model, however, should not be used to predict the unloading paths for the superelastic SMA columns.

Multilinear elastic (MELAS) model- Similar to the MISO model, the MELAS model can also represent the nonlinear stress-strain curves and accurately simulate the loading paths for both the SUS304 columns and the SMA columns. The special feature of this model is that, unloading occurs along the same loading path, so that no inelastic strains are induced. That is, no energy is lost (unlike the plasticity), or it considers conservative process. If, however, there is one stress-strain curve to define the loading path, and at the end of loading, a separate stress-strain curve to define the unloading path, a hysteresis similar to that of the superelastic SMA could be created by this model. This strategy was primarily used to simulate the complete loading-unloading cycle for the slender superelastic SMA columns.

Mooney-Rivlin (M-R) or the Hyperelastic model- Hyperelasticity refers to materials whose stresses are derived from their total strains using a strain energy density function. Mooney-Rivlin (M-R) is a material law suitable for nearly incompressible natural rubber. For this model, both the tensile and compressive stress-stain data can be used simultaneously. However, as a procedure, ANSYS software first modifies the input (test) data, while calculating some necessary strain energy constants (also called Mooney-Rivlin constants). Thus, the results of simulation will be based on the modified data. This M-R model with 5 terms and 9 terms M-R constants (see Appendix) has been tried specially to predict ‘the bifurcation points-based on large strains and large deflection’ for the short SMA columns as discussed later on.
For the MISO and the MELAS models, element type was PLANE42 (4 nodes, 2-D space, DOF: UX, UY), while for the M-R model, the element type was HYPER56 (hyperelastic mixed U-P solid, 4 nodes, 2-D space, DOF: UX, UY, UZ). For more details of the material models, element types, or the nonlinear geometric options, interested readers may refer to the ANSYS user’s manual.

3.3 Results and Discussions

3.3.1 Stress-strain (s-s) curves used for simulation

![Stress-strain curve diagram](image)

Fig. 3.2(a) Idealized stress-strain curve for the superelastic SMA.

The present quantitative analysis is based on the pure compression and tensile stress-strain data obtained from experiments. The idealized stress-strain diagram, that actually resembles the hysteresis of tensile test for the superelastic SMA is depicted in Fig. 3.2(a) to compare with the actual stress-strain curves found by experiments as shown in Figs. 3.2(b), 3.2(c).
Experimental s-s curves

Fig. 3.2(b) Stress-strain curves for the SMA (true s-s curves were used for simulation).

Those s-s curves have been discussed in the Introduction of this thesis. Note that the tension-compression asymmetry becomes prominent for the superelastic SMA (both for the nominal and true stress-strain curves) when the values of strains exceed 1% (Figs. 3.2(b), 3.2(c)). Though not shown here, it was also found from the experiment, the tensile and compressive behaviors of the SUS304 and Al rods can be considered as almost symmetric.
3. Buckling & Postbuckling Characteristics of the Superelastic SMA Columns-Numerical Simulation

Fig. 3.2(c) Nominal stress-strain curves for the superelastic SMA, SUS304 and Al rods.

Orgeas and Favier [2] also demonstrated similar non-symmetric tension-compression behaviors (particularly for superelasticity) for an equiatomic Nitinol SMA sample, by comprehensive experimental results. Later, Raniecki and Lexcellent [3] theoretically verified the same asymmetry. Orgeas and Favier [2] concluded that though the same martenstic fraction is formed during tension and compression, but the orientation of the martensite variants is more efficient in tension than in compression which accounts for the asymmetric behavior.
Practically, the tensile or the compressive strains of the specimens measured by the displacement of the moving fixture are higher than the actual strains. Thus, for the present study, the tensile strain data were measured simultaneously by the strain gage and the displacement of the cross-head of the Instron machine. For accuracy, the strain gage data was used for simulation. It was found that the characteristic plateau for SIMT in tension, starts when the strain is approximately 1% (Fig. 3.2(b)).

On the other hand, following Johnson [4], to avoid any chance of bending/buckling for the specimen during the pure compression test, $L/k$ was kept less than 12. Unfortunately, no SMA sample with larger dimension was available. Thus, strain gage could not be used because of too small gage length (4.5mm) during the compression test and the strain data for the specimen were measured only by the displacement of the moving fixture. However, accuracy of the simulation results largely depends on the correct stress-strain data. Particularly, for simulating the behaviors of the columns those are not too short, at least the initial portion of the stress-strain should be highly accurate. As mentioned the tensile s-s data were measured accurately by strain gage. Thus to compromise, while keeping continuity of the data, only the initial portion (0 to about 1% strain) of the compressive stress-strain curve (shown by blue color in Fig. 3.2(b)), was modified to make it identical with the tensile stress-strain curve, until the distinct plateau for the SIMT in tension is approached (Figs. 3.2(b), 3.2(c)). After that, SIMT starts for the tension and its stiffness becomes negligibly small so the curves deviate from each other significantly. It is found that based on the modified compressive stress-strain data (the nominal and true s-s curves for large strains are shown in Fig. 3.2(b)), the buckling and postbuckling behaviors of the SMA columns can be predicted with reasonable accuracy for any $L/k$. However, for simulation purpose it would be always desired to use the whole range of strain data, measured by strain gage or other sensitive instruments.
As can be checked from theoretical and experimental studies by Raniecki and Lexcellent [3] and Orgeas and Favier [2], there is hardly any difference between the stress-strain curves in tension and compression until the appearance of the critical point for SIMT in tension for the Nitinol SMA. Therefore, the above mentioned modification to the compressive stress-strain curve, which is in fact, essential for the present study, is justified.

Usually, for the Nitinol SMA, the SIMT for the compression can be identified with another distinct plateau but at a higher stress than that for the tension as shown by Orgeas and Favier [2] and Raniecki and Lexcellent [3]. For the current study, however, unlike the tensile stress-strain curve there is no distinct plateau for a particular range of strains and as such, the SIMT process is rather difficult to identify from the compressive stress-strain curve. It appears, however, the SIMT process is indicated by a slight change in slope of the compressive stress-strain curve (Figs. 3.2(b), 3.2(c)).

3.3.2 Simulated $P-\Delta$ curves for the short columns

As seen from Figs. 3.3(a) and 3.4(a), the nature of the $P-\Delta$ curves changes quite notably for the two short SMA columns with $L/k=28$ and 38. Experimental $P-\Delta$ curves, however, show small residual strains for these two SMA columns. Though, the unloading paths of the slender SMA columns are simulated by a special scheme using the MELAS model, the same scheme can't be applied for the short SMA columns, since the residual strains can't be practically neglected. Thus, only the loading paths, that is, the unique $P-\Delta$ curves encompassing both the prebuckling and the postbuckling zones, were simulated up to the last point of loading for the two short SMA columns with $L/k=28$ and 38.

Let us sequentially discuss the simulation results for the shortest SMA columns with $L/k=28$ as presented in Figs. 3.3(a)-3.3(e). Apparently, the experimental results ($P-\Delta$ curves) show two distinct modes of deformation for the column (Fig. 3.3(a)). Simulation result using
Fig. 3.3 (a) Load-end shortening curves (experiment and simulation) for the SMA column.

Fig. 3.3(b) Simulated load-transverse deflection curves for any point on the mid-span of the SMA column ($L/k=28$).
the pure compressive s-s curve (by MISO model) can predict the column’s behavior and the buckling loads most accurately (see also Table 3.2). It is noteworthy that the predicted load-end shortening curve based on the tensile s-s data, dips a little to a valley and again rises to a higher peak (the predicted buckling load), which is much lower than the experimental result. The reason is comprehensively explained later while analyzing the total strain distribution in the same column (Figs. 3.3(c)-3.3(e)). The initial portions of the \( P-\Delta \) curves predicted by the tensile and the compressive s-s curves merge together while that predicted by the M-R model shows less stiffness (Fig. 3.3(a)). This is because the input s-s curves are slightly modified while the M-R model is to be used (Appendix). Though, the M-R model (based on 9 terms M-R constants) predicts much lower buckling load than the experimental result, it can be proven from Fig. 3.3(b) that it is because of the interesting phenomenon, that is, the change in mode of deformation (after the first point of instability) on the equilibrium path \( P-\Delta \) curve of this column.

Fig. 3.3(b) shows only simulation results (load-transverse deflection, of a point on the mid-span) in order to verify two distinct modes of deformations for this SMA column. The results of Fig. 3.3(b) should be observed in conjunction with Fig. 3.3(a). Both the MISO model (based on the compression s-s curve) and the M-R model (based on 9 terms M-R constants) identify the first point of instability, which is apparent from the experimental result. The M-R model assumes the material to be almost incompressible, and is suitable for a material having much higher compressive strength than its tensile strength. This model predicts bifurcation type buckling but can’t predict further postbuckling deformation accurately, after the first point of instability is reached. As mentioned, though the term bifurcation is used, both large strain and large deflection options were used for all the cases in this study. As seen from Fig. 3.3(b), the M-R model (based on 9 terms Mooney-Rivlin constants) predicts that the transverse displacements of any point on the mid-span of the column become enormous and
the numerical solutions fail to converge to any solution at the first point of instability (corresponding to a load of 1.92kN). This value approximately corresponds to the junction of the two modes of deformation as can be verified from the experimental data of Fig. 3.3(a). Incidentally, at this point, the theorems of Thompson and Hunt [1] seem to apply perfectly, which state that, the onset of the first point (either a limit point or a branching point) of instability is indicated by a substantial increase in the displacements for very small increase of the loading parameter. At the critical point itself any increase of the load parameter, however small, causes enormous deformations and thus the numerical technique fails to converge to any solution.

It will be proven soon that for this too short SMA column, most of the material remains under significant compressive strains during loading. As seen from the compressive stress-strain curve of SMA, after the initial portion, for a particular range of strain, the stiffness decreases gradually (Fig. 3.2(b)). This may cause the first point of instability for the highly compressed column. However, since the stiffness again increases significantly with the increasing value of strain, quite astonishingly, this column has a highly stable postbuckling configuration that makes it to sustain increasing load in the secondary mode of deformation until the maximum load is reached. The same is verified by the solution based on the MISO model using the compressive s-s curve. The M-R model, however, can not predict this particular postbuckling behavior after the first point of stability (Figs. 3.3(a), 3.3(b)). Thus, it can be concluded from the simulation results that this short SMA column does not fail due to buckling at the primary point of instability, as it possesses an unusually stable postbuckling configuration.

As already demonstrated, the compressive s-s curve can accurately predict the buckling loads for the short SMA column, while the tensile s-s curves predict lower buckling loads (Figs. 3.3(a), 3.3(b). The main reason behind this difference in prediction is attributed to the
very significant asymmetric behaviors of the material in compression and tension particularly for large strain (Figs. 3.2(b), 3.2(c)). For more rigorous and quantitative proof of this fact, let us now analyze the distribution of total strain (in the loading direction) over the entire cross-sections for the half-model of the column, based on the compressive s-s data using the MISO model as shown in Figs. 3.3(c)-3.3(e). Note that, the half model includes the actual length of the column inside the loading fixtures for accurate prediction of results.

The total strain distributions in the column material for increasing end shortening, as shown in Figs. 3.3(c)-3.3(e), should be observed in conjunction with Figs. 3.2(b) and 3.3(a). As discussed in the last few paragraphs, the first point of instability corresponds, approximately to a load of 1920 N, after that the column bends prominently in its secondary mode of deformation. Simulation results show that only compressive strains remain in the entire column cross-sections for \( \Delta = 0 \) to 7.6\%, and the first tensile strains appear in between \( \Delta = 7.6\% \) and 7.7\%. Fig. 3.3(c) shows the appearance of the tensile strains (with the maximum magnitude of 0.0187\%) for a very small region in the outermost fibers near the mid-plane section and also slightly in the innermost fibers near the fixture. It can be shown from Fig. 3.3(c), almost 97\% of the column material remains under compressive strains, the maximum magnitude being \(-8.46\%\).

For the critical point, that is, \( \Delta = 9.4\% \) and \( P_{cr} = 3136 \) N, distribution of strains in the column material can be analyzed from Fig. 3.3(d). The maximum tensile strains (1.68\%) occur almost in the same small regions as those of Fig. 3.3(c). Most of the column material still remains under compressive strains, the maximum magnitude being \(-12.19\%\). As mentioned, the tension-compression asymmetry starts if the strain is larger than 1\% (Fig. 3.2(b)). As can be seen from Fig. 3.3(d), more than 1\% tensile strains (the maximum is 1.68\%) occupy only 3\% of the column material. Consequently, the simulation result based only on the compressive s-s
Fig. 3. 3(c) Total strain distribution in the loading direction for the half model of the SMA column ($L/k=28$) based on the MISO model and compressive s-s curve corresponding to $\Delta = 7.7\%$ and $P=2940N$. 
Fig. 3. 3(d) Total strain distribution in the loading direction for the SMA column ($L/k=28$) based on the MISO model and compressive s-s curve corresponding to the critical point ($\Delta =9.4\% , P=3136N$).
Fig. 3.3(e) Total strain distribution in the loading direction for the SMA column ($L/k=28$) based on the MISO model and compressive s-s curve corresponding to the postbuckling state ($\Delta=10\%, P=3115\,N$).
curve (MISO model) can predict the buckling load quite accurately as seen from Fig. 3.3(a).

For Fig. 3.3(e), the postbuckling configuration corresponding to the last point of loading ($\Delta=10\%, P=3115\text{N}$), the maximum tensile strain occurs in the same small region as that of Fig. 3.3(d), but the magnitude increases to 2.8%.

As noted at the critical point ($\Delta=9.4\%$ and $P_{cr}=3136\text{N}$), most of the column material is under compression and the maximum compressive strain is as high as 12.2% (Fig. 3.3(d)), which justifies the use of only the compressive s-s curve for simulation. If instead, the tensile s-s curve is used, the simulation results for this column predict much lower values of the buckling load than found by experiment (Fig. 3.3(a)) because of significant difference between the tensile and compressive s-s curves for large strains.

Fig. 3.3(f) Load-end shortening curves (experiment and simulation) for the SUS304 column.
On the other hand, because of almost symmetric tension-compression s-s curves for the SUS304, the MISO model (using the tensile s-s curve) can predict the $P-\Delta$ curve for the complete cycle of the SUS304 column for any value of $L/k$. For ready reference, Fig. 3.3(f) can be observed. As seen, the experimental buckling load and also the residual strain are predicted with reasonable accuracy, using the tensile s-s data. Since the SMA has much higher strength than that of SUS304 for large strains (Fig. 3.2(c)), thus the simulation results show that the SUS304 column buckles at a lower load compared with the SMA column for $L/k=28$ (Figs. 3.3(a) and 3.3(f)).

For $L/k=38$, the experimental $P-\Delta$ curve of the SMA column (Fig. 3.4(a)) shows a valley between two distinct peak loads (the second peak being slightly higher than the first one). The above unique characteristic is contrary to the general trend that load falls off monotonously for any further compression after the first distinct peak load (which is also a point of instability) on the equilibrium configuration path ($P-\Delta$ curve) of a column. Interestingly, it was found that the valley between two distinct peaks gradually disappears for repeated cycles, though the buckling load does not fall much and the cumulative residual strains remain very small for this SMA column [5].

The above unique phenomenon of two distinct peaks was qualitatively explained in the previous study [6]. For the current study, both the MISO model (based on the compressive s-s data) and the M-R model can predict the loading portion of the $P-\Delta$ curve, including the buckling load, though the valley between the two peaks can’t be predicted as shown in Fig. 3.4(a). The simulated load-transverse deflection curves are shown in Fig. 3.4(b). As seen the M-R model shows a distinct bifurcation point, and the load falls during the secondary mode of deformation (Fig. 3.4(b)). The MISO model based on the tensile s-s curve predicts lower buckling load (Figs. 3.4(a), 3.4(b)). The reason is similar as explained for the simulation results of SMA column with $L/k=28$. 
3. Buckling & Postbuckling Characteristics of the Superelastic SMA Columns-Numerical Simulation

Fig. 3.4 (a) Load-end shortening curves (experiment and simulation) for the SMA column.

Fig. 3.4(b) Simulated load-transverse deflection curves for any point on the mid-span of the SMA column ($L/k=38$).
As mentioned, rigorous treatment of buckling of the superelastic SMA columns has not been reported in the literature particularly by numerical simulation. Auricchio and Sacco [7], however, demonstrated that for pure bending of a Nitinol SMA beam, with different properties in tension and compression, the axial strain has a non-monotonous response with the bending moment during loading and unloading. Auricchio and Sacco [7] showed that the neutral axis of the cross-section of the beam moves as a consequence of SIMT that leads to such peculiar response. For the present study, however, dealing with the instability of columns, apparently the peculiar nature of the stress-strain curves (encompassing the region of SIMT) enables the short SMA columns to have an unusually stable postbuckling configuration after the first point of instability. Consequently, the short SMA columns show two distinct modes of deformation or two peak loads.

### 3.3.3 Simulated $P-\Delta$ curves for the slender columns

Since for small strain, SUS304 has much higher stiffness compared with SMA, the buckling load is also much higher for the slender SUS304 columns. The MISO model is suitable for simulating behaviors of common engineering materials, assuming von Mises yield criteria. Thus, for the SUS304 columns the buckling loads and the residual strains can be predicted by the MISO model based on the loading stress-strain (tensile) curve, as can be seen from the Figs. 3.3(f), 3.6(d), 3.7(b), and 3.8(b).

As discussed, unlike the SUS304, the superelastic SMA can fully recover large strains through a typical nonlinear hysteresis. Prediction/simulation of such typical hysteresis could be possible by using the appropriate material model and defining an unloading curve compatible with the problem. Taking advantage of the MELAS model (that assumes no inelastic strains at the end of the unloading), a special method has been devised to simulate the complete loading-unloading cycle (hysteresis) for the slender SMA column as explained with the schematic $s-s$ curves of Figs. 3.5(a), 3.5(b).
Fig. 3.5(a) Stress jump for any point $H$ during simulation; defined loading s-s curve-$OAB$, defined unloading s-s curve-$CDEO$, actual unloading s-s curve (not defined) for point $H$-$HGE0$, simulated unloading path-$H1GE0$, stress jump-$H1$.

Fig. 3.5(b) Effect of the intermediate unloading curve $PQO$ during simulation (unloading in two steps); actual unloading path from simulation for point $H$-$H123E0$, smoothed unloading path-$HGE0$. 
3. Buckling & Postbuckling Characteristics of the Superelastic SMA Columns-Numerical Simulation

(1) At first, the prebuckling and the postbuckling paths until the last point of loading are simulated by the MELAS (or, the MISO) model using the loading stress-strain curve $OAB$ of Figs. 3.5(a), 3.5(b).

(2) Assuming, the slender columns fully recover the shape upon unloading, the MELAS material model is used to simulate the unloading path. In Figs. 3.5 (a) and (b), let point $C$ ($\varepsilon, \sigma$) correspond to the most critically strained elements of the buckled column at the end of loading ($\Delta = \Delta_{\text{crit}}$). At point $C$, the loading stress-strain curve $OAB$ is replaced by the unloading stress-strain curve $CDEO$, so that the MELAS model can create a closed loop hysteresis. It is important to note that prior to unloading (at $\Delta = \Delta_{\text{crit}}$), each element of the buckled column is in a different state of stress and strain (unlike the cases of pure tension or compression). This fact is also evident from the total strain distribution of the column as shown in Figs. 3.3(c)-3.3(e). Therefore, to trace the continuous unloading path of the buckled column, each element must be assigned its unique unloading stress-strain curve. For example, the s-s state for any element is defined by the point $H$, though the most critical point is $C$. Therefore, point $H$ should be assigned with its actual unloading curve $HGE0$. Defining such unique unloading stress-strain curves perfectly for all the elements is rather difficult.

On the other hand, if only a single unloading curve ($CDE0$) is defined for all the elements, there will be always a discontinuity for point $H$ and consequently, a stress jump (equal to $H1$) at the beginning of unloading. The simulated path would be $H1E0$ (Fig. 3.5(a)) instead of the actual one. It is seen, the error occurs only at the beginning of the unloading. However, it should be noted that, if $H1$ is too large, solution might not converge at all. This problem has been solved by the following scheme.

Let one intermediate unloading stress-strain curve $PQ0$ be defined between the loading ($OAB$) and the final unloading ($CDE0$) s-s curves as shown in Fig. 3.5(b). Thus unloading of point $H$ is done in two steps. In the first step, $OAB$ is changed to $PQ0$ and the amount of
unloading was \((\epsilon_f - \epsilon_s)\) and the stress jump equal to \(H_1\) or, \((\sigma_H - \sigma_I)\). Note that the amount of stress jump decreases for using the intermediate curve \(PQO\) (Figs. 3.5(a), 3.5(b)). In the second step, \(PQO\) is changed to the final unloading stress-strain curve \(CDEO\), the amount of unloading is \((\epsilon_f - \epsilon_C)\) and the stress jump equal to \((\sigma_2 - \sigma_3)\). The rest of the unloading is done with the final unloading curve so the simulated path is \(H123E0\) and if smoothed, the simulated path could resemble the actual path \(HGE0\).

Obviously, if instead of a single intermediate unloading curve \((PQO)\), a large number (say \(N\)) of unloading curves are used in between the curves \(OAB\) and \(CDEO\), the stress jump would be smaller for each step and the initial portion of the unloading curve of the columns as predicted by simulation results would be more accurate. In that case, the final unloading curve \(CDEO\) has to be assigned in \(N+1\) number of steps (each time, assigning all the elements with the closest unloading stress-strain curve). The rest of the unloading has to be done gradually with \(CDEO\) until \(P\) becomes zero. For the sake of accuracy and convergence of solution \(\Delta_{set}\) should be reduced gradually to simulate the unloading path.

The predicted unloading paths for the slender SMA columns (Figs. 3.6(a), 3.7(a)) are based on the above scheme using the unloading curves of Fig. 3.5(c). The last unloading curve was assigned in three steps (or, \(N=2\) as shown in Fig. 3.5(c)) and the small stress jump at the beginning of each step was smoothed. The usefulness of the devised method can be checked by comparing the complete loading-unloading cycles for experiment and simulation as shown in Figs. 3.6(a), 3.7(a). Obviously, for meaningful prediction, the final unloading curve must be accurate and compatible with the problem. Thus, it was chosen on the assumption that maximum total strains of the slender column material remain within 6.5% at \(\Delta=\Delta_{set}\) (Fig. 3.5(c)). This assumption is valid as can be checked from the total strain distribution of the slender column with \(L/k=108\) (Fig. 3.6(c)). Usually, it is assumed that Nitinol SMA can fully recover this amount of strain.
3. Buckling & Postbuckling Characteristics of the Superelastic SMA Columns-Numerical Simulation

Fig. 3.5(c) Stress-strain curves used to simulate the unloading path for the SMA column.

Fig. 3.6(a) Simulated load-end shortening curve for the complete cycle for the SMA column with $L/k=108$, based on the compressive stress-strain curves of Fig. 3.5(c).
Fig. 3.6(b) Total strain distribution for the SMA column ($L/k=108$) based on the compressive stress-strain curve and MISO model corresponding to the critical state ($\Delta=0.4\%, P=603N$).
Fig. 3.6(c) Total strain distribution in the loading direction for the SMA column ($L/k=108$) based on the compressive $s-s$ curve and MISO model corresponding to the postbuckling state ($\Delta=2.8\%, P=347N$).
As pointed out, the above simulation scheme is based on the valid assumption of zero residual strains for the slender columns. For a certain temperature, however, if the deformation exceeds a critical limit, upon unloading there will be permanent set in the superelastic SMA which can't be recovered by unloading or heating [8]. As shown, too short SMA columns $(L/k=28, 38)$ show small residual strains upon unloading. Thus, the unloading paths of the short superelastic SMA columns can't be predicted by the MELAS model that considers fully elastic shape recovery upon unloading.

The $P-\Delta$ curve for $L/k=108$ (Fig. 3.6(a)) during loading has been simulated by the compressive $s-s$ curve using the MELAS model. Let us analyze the distribution of total strains during loading for this column in conjunction with the stress-strain curves of Fig. 3.2(b).

Fig. 3.6(b) shows the total strain distribution in the $x$ direction (or the loading direction) for half model of the column at the critical point ($\Delta=0.4\%, P_c=603\text{N}$). At this critical state, the entire column material is in the austenite phase as the maximum compressive strain is $-0.65\%$, while the maximum tensile strain is $0.10\%$. As discussed, the tension-compression asymmetry effect starts approximately at a strain of $1\%$ (Fig. 3.2(b)). Consequently, both the compressive and tensile $s-s$ curves can be used to predict the buckling loads for the column that is not too short.

Fig. 3.6(c) shows the total strains (in the $x$ direction) in the column material at the end of loading ($\Delta=2.8\%, P=347\text{N}$). In this case, only $9\%$ of the column material is occupied by tensile strains with magnitude of $1.55\%$ to $3.14\%$. Thus, $91\%$ of the column material remains under compressive strain with the maximum magnitude of $-3.97\%$, which justifies the use of compressive $s-s$ curve during loading. However, the best result would be possible if both the tensile and compressive $s-s$ curves can be used simultaneously to predict large postbuckling deformations of this column. Since the SMA columns with $L/k=108$ buckle in the austenite phase, thus for higher slenderness ratios they will definitely buckle within the austenite phase,
and if the postbuckling deformation is not too large, they will also recover the shape upon unloading.

![Load-end shortening curves (experiment and simulation) for the SUS304 column with $L/k=108$.]

Fig. 3.6(d) Load-end shortening curves (experiment and simulation) for the SUS304 column with $L/k=108$.

As mentioned, the MISO model can be used to predict the complete loading-unloading cycle for the SUS304 column with reasonable accuracy. Only the loading curve, that is, the tensile stress-strain data, for large value of strain, was used to simulate the complete cycle for the SUS304 column with $L/k=108$ (Fig. 3.6(d)).

Next, let us discuss and compare some of the postbuckling characteristics of the slender SMA columns as observed from experiment and simulation. The shape recovery process of the buckled columns starts with the unloading and ends when the load becomes approximately zero. It is found from experiment that for $L/k=68-168$, recovery force of the buckled SMA columns increases quite significantly as if they push the loading fixtures during
3. Buckling & Postbuckling Characteristics of the Superelastic SMA Columns—Numerical Simulation

Fig. 3.7(a) Simulated load-end shortening curve for complete cycle for the SMA column ($L/k=168$) based on the stress-strain curves of Fig. 3.5(c).

Fig. 3.7(b) Simulated load-end shortening curve for complete cycle for the SUS304 column ($L/k=168$).
unloading (Figs 3.6(a), 3.7(a)). The above phenomenon, contrary to the general notion that load falls off monotonously during unloading of a buckled column, can be attributed to the superelasticity. Due to large postbuckling compression, the columns become highly unstable but still they are fully capable of recovering the shape upon unloading. Since the load falls off appreciably from its peak value at $\Delta = \Delta_{\text{set}}$, thus, at one stage during unloading the recovery force has to rise to make a closed loop hysteresis. Too slender SMA column, however, has low bending stiffness and thus can’t show this increasing recovery force during unloading.

As mentioned, the unloading path of the SMA columns (Figs. 3.6(a), 3.7(a)) was simulated using the material model MELAS based on the unloading curves of Fig. 3.5(c). Let us suppose, both loading and unloading paths are predicted by the MELAS model using only the loading curve of Fig. 3.5(c). Since $P$ falls appreciably from its peak during loading, during unloading the recovery force must increase to follow the same loading path, so the hysteresis will reduce to a line. Thus, it is obvious the recovery force increases because of elastic shape recovery. Since the defined unloading curve is different from the loading curve, the recovery force still increases, the hysteresis, however, is of finite width. As noted, the simulation results for the increase of the recovery force and the shape of the hysteresis of Figs. 3.6(a) and 3.7(a) match agreeably with those of experiment and, in turn, justify the choice of the ‘compatible’ unloading curves of Fig. 3.5(c).

Since the MELAS model is used for simulating the unloading of the slender SMA columns, the residual strain at the end of the cycle is automatically zero. On the other hand, MISO model, considering the effect of plasticity, can be used to simulate the unloading path and the residual strains of the SUS304 columns with reasonable accuracy (Figs. 3.7(b) and 3.8(b)).

Usually, load falls off drastically if the compression continues after a column buckles. In contrast, the load does not fall much for further postbuckling compression for slender superelastic SMA columns (Table 3.1 and Figs. 3.6(a), 3.7(a), 3.8(a)). For instance, for SMA
Fig. 3.8(a) Simulated load-end shortening curve for the SMA column ($L/k=318$) based on tensile stress-strain curve.

Fig. 3.8(b) Simulated load-end shortening curve for the SUS304 column ($L/k=318$).
column with $L/k=318$, the load falls from its peak value (that is, from the buckling load) only by 5% (8% by simulation) in magnitude up to the last point of loading. On the other hand, for the same $L/k$, the fall in load's magnitude is 46% for the SUS304 column. The above phenomenon can be explained from the following discussions.

Table 3.1 Percent fall between the load at the point of instability and load at the end of loading.

<table>
<thead>
<tr>
<th>$L/k$</th>
<th>Exp.</th>
<th>Simulation</th>
<th>Exp.</th>
<th>Simulation</th>
<th>Exp.</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMA</td>
<td>5</td>
<td>8.3</td>
<td>27</td>
<td>31</td>
<td>46</td>
<td>43</td>
</tr>
<tr>
<td>SUS304</td>
<td>46</td>
<td>47</td>
<td>63</td>
<td>66</td>
<td>63</td>
<td>70</td>
</tr>
</tbody>
</table>

For the present study, the columns buckle and undergo pretty large deformation during loading. Thus even if a slender column buckles elastically, its postbuckling deformations have to be treated by the stress-strain curves much beyond the elastic limit. For instance, it was shown that the maximum strains reach about 4% at the end of loading for the SMA column with $L/k=108$ (Fig. 3.6 (c)). Fig. 3.2(c) shows, after yielding the stress increases too slowly with the increasing strains for the SUS304 and Al. On the other hand, particularly, SMA's compressive strength increases very significantly for increasing values of strain. Apparently, because of this sufficiently high strength for large strains, the slender superelastic SMA columns are able to sustain the load with least change in magnitude during the postbuckling deformations.
Fig. 3.9(a) Tensile loading-unloading curves used for simulation.

Fig. 3.9(b) Simulated complete loading-unloading cycle for the SMA column ($L/k=108$) based on tensile stress-strain curves of Fig. 3.9(a).
Simulation results for the SMA columns, based on the tensile stress-strain curve are presented in the Figs. 3.8(a) and 3.9(b). As seen from Fig. 3.8(a), for the most slender SMA column ($L/k=318$), the load-deformation curve during loading can be predicted by the tensile stress-strain curve with reasonable accuracy.

Figs. 3.9(a) and 3.9(b) are presented to show that the complete loading-unloading cycles for the slender SMA columns can also be predicted based on the tensile stress-strain curves by using the same scheme as already discussed. Fig. 3.9(a) shows the three intermediate unloading curves used for simulation purpose. The loading curve was changed to the actual unloading curve in four steps. The 4th and final unloading curve is modified from experimental data.

As seen from Fig. 3.9(b), the buckling load is predicted accurately. However, for postbuckling compression ($\Delta = 0.7\% \text{ to } 2.75\%$), the simulation curve remains slightly below the experimental load-deformation curve. As shown earlier, for the same column ($L/k=108$) the predicted load-deformation curve (based on the compressive stress-strain data) remains slightly above the experimental curve (Fig. 3.6 (a)). Thus, for simulating the postbuckling behaviors of the slender columns, the best results might be obtained if both the compressive and tensile stress-strain curves are used simultaneously.

3.3.4 Buckling load versus slenderness ratio (experiment and simulation)

It is a well-known fact that, whatever may be the method of prediction, usually the theoretical buckling loads of structures are higher than those are during applications. The reasons behind this discrepancy between the theoretical and practical buckling loads are mainly attributed to the unavoidable (and often undetected as well) imperfections (physical and/or, geometrical) which can not be fully taken into account during theoretical calculations.
In this study, different material models have been used with the tensile and compressive stress-strain data to predict the columns' buckling loads using large deformation theory. It can be verified that the results of numerical simulation agree well with the experimental buckling loads (Fig. 3.10 and Table 3.2) for the whole range of the slenderness ratio ($L/k=28-318$), in particular, for the SMA columns.

From the results of simulation and experiment, it is verified that slender SMA columns buckle elastically in the austenite phase at a load much lower than that of a slender SUS304 column. However, compared with the SUS304, SMA possesses much higher strength for large strains. Thus, SMA column's buckling load increases very significantly for the decreasing value of the slenderness ratio and at one stage it exceeds the buckling load of the SUS304 columns (Fig. 3.10 and Table 3.2). It should be noted here that the term 'elastic
buckling' is justified for the slender superelastic SMA columns as they are able to recover the
shape upon unloading.

Table 3.2 Comparison of $P_{cr}$ from experiment and simulation for the SMA columns.

<table>
<thead>
<tr>
<th>$L/k$</th>
<th>Experiment</th>
<th>Simulation</th>
<th>Difference between the highest buckling loads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>28</td>
<td>2930,2943,2972</td>
<td>1890</td>
<td>3121</td>
</tr>
<tr>
<td>38</td>
<td>1756,1776,1805</td>
<td>1544</td>
<td>1794</td>
</tr>
<tr>
<td>68</td>
<td>1020,1060,1079</td>
<td>1171</td>
<td>1149</td>
</tr>
<tr>
<td>88</td>
<td>697,716,726</td>
<td>872.2</td>
<td>856</td>
</tr>
<tr>
<td>108</td>
<td>520,540,559</td>
<td>612</td>
<td>604</td>
</tr>
<tr>
<td>128</td>
<td>358,363,367</td>
<td>443.7</td>
<td>436.6</td>
</tr>
<tr>
<td>168</td>
<td>226,232,234</td>
<td>260.7</td>
<td>258</td>
</tr>
<tr>
<td>218</td>
<td>137,139,143</td>
<td>155</td>
<td></td>
</tr>
<tr>
<td>318</td>
<td>65,66,66</td>
<td>72.1</td>
<td></td>
</tr>
</tbody>
</table>

1- Simulation based on tensile s-s curves (MISO/MELAS model).
2- Simulation based on compressive s-s curves (MISO/MELAS model).
3- Simulation based on both tensile and compressive s-s curves (M-R model). Presented values of $P_{cr}$ for $L/k=28$ and 38, are calculated based on 5 terms and 9 terms M-R constants, respectively.

3.4 Conclusions

Based on the stress-strain curves of the superelastic SMA, a few unique buckling and postbuckling characteristics of the superelastic SMA columns have been comprehensively treated by numerical simulation using the commercial FEM code ANSYS. Two suitable material models, namely, Multilinear isotropic (MISO), Multilinear elastic (MELAS) have been utilized for most of the simulation problems, while the Hyperelastic Mooney-Rivlin (M-
R) model has been used only for the short SMA columns. The prediction of results is based on the large strains and large deflection options.

The compressive strength of the used superelastic SMA is significantly higher than its tensile strength, particularly for large strains. Thus, in general the compressive stress-strain curve has been used for analyzing the buckling and postbuckling behaviors using the MISO or MELAS model. Numerical simulation shows, most of the column material remains under significant compressive strains at the end of loading, which also justifies the prediction of results based on the compressive s-s curve, in particular for the short SMA columns. For too slender SMA columns or for the SUS304 columns, however, the tensile stress-strain data can be used to predict the behaviors of the columns.

Simulation results verify that the slender superelastic SMA columns buckle elastically in the austenite phase and thus at a much lower load compared with the slender SUS304 columns. It is also verified that for the shortest SMA column, there is a distinct point of instability after which the column has a unique stable postbuckling configuration, which enables it to sustain higher load than that of a SUS304 column under similar test conditions. It can be concluded that because of very high compressive strength for large strains, the buckling load of the SMA columns increases very significantly for the decreasing slenderness ratio and finally exceeds that of the SUS304 column.

A special method has been devised using the MELAS model to simulate the complete loading-unloading paths of the slender superelastic SMA columns, by defining a loading curve and a suitable unloading curve. To minimize the stress-discontinuity, the same method has been accomplished in a number of steps defining a few intermediate unloading stress-strain curves. The effectiveness of the devised method has been comprehensively demonstrated by comparing the simulation results (based on compressive and tensile stress-strain data) with the experimental results. It is evident from the simulation results that the
recovery force increases during unloading because of elastic shape recovery of the buckled SMA columns.

Numerical simulation reveals the fact that, the total strains become large enough at the end of loading, even for the slender columns. Therefore, because of SMA’s too high compressive strength for large strains, for any further postbuckling compression, the slender SMA columns can easily sustain the load without appreciable change in magnitude.

SUS304 has an almost symmetric stress-strain behavior in tension and compression, and also follows the von Mises yield criteria. Thus, based on the tensile s-s curve and using the MISO model, both the buckling load and the large residual strain have been predicted with reasonable accuracy.

Lastly, it should be pointed out that perhaps it would be possible to predict some of the results more accurately, including the two peaks of the short SMA column with $L/k=38$, if the whole range of the compressive s-s data of the superelastic SMA could have been obtained more accurately and both the tensile and compressive s-s curves could be used simultaneously.
References


Appendix

Simplified column model for simulation

From the elementary theory, columns having the same slenderness ratio and same cross-sectional area (but may be of different shapes) will exhibit the same buckling characteristics.

For a circular cross-sectional area, with a diameter \( D \), the least radius of gyration is, \( k_c = D/4 \).

For a rectangular cross-sectional area, with its sides \( b \) and \( h \) \((b > h)\), the least radius of gyration is, \( k_r = h/3.4641 \). For the present study, \( D = 2 \text{mm} \). Thus, having the same length, the slenderness ratio and area will be the same for a circular cross-sectional area and a rectangular cross-sectional area, if, \( h = 1.732 \text{mm} \) and \( b = 1.814 \text{mm} \). The geometric half model of the column (with \( h = 1.732 \text{mm} \), and \( b = 1.814 \text{mm} \) in the z-direction), as represented in Fig. 3.1(c), is used for the simulation purpose. It should be mentioned here that the length of the column inserted between the fixture, that is, value of \( a \) was \( 8 \text{mm} \) (Fig. 3.1(c)) and the element size for finite element meshing was \( 0.20 \text{mm} \). Accuracy of the results using this half model of column can be checked from the present study.

True and nominal stress-strain curves

Experimentally obtained s-s curve is called the nominal or engineering s-s curve. For accuracy of the simulation involving large strains, the nominal s-s curve must be changed to the true s-s curve as shown in Fig. 3.2(b). As mentioned, the engineering/nominal s-s curves for the SMA in tension and compression are the same up to 1% strain. However, when the nominal curve is changed to the true curve, the tensile and compressive curves slightly differ from each other even before 1% strain is reached. This is why, when the tensile s-s curve is used to predict the buckling loads of slender SMA column, the results are about 2% higher than those predicted by using the compressive s-s curve. However, this difference diminishes for increasing values of slenderness ratio as can be verified from Table 3.2.
Modified stress-strain curves for the Hyperelastic model

As a procedure for the M-R hyperelastic model, the nominal stress-strain data are required as input. ANSYS software at first calculates the 2 terms, 5 terms or the 9 terms Mooney-Rivlin constants.
Rivlin constants (strain energy constant) from the given stress-strain data. Next, based on these constants, the stress-strain data are modified. There is no fixed rule for selecting the number of strain energy constant terms. However, modified data, which suit the actual data best in tension and compression should be chosen. Figs. 3.11(a), (b) show the modified data based on 5 terms and 9 terms M-R constants. The load-deformation curve is traced based on the modified data. It should be mentioned here that in the input data, the tensile stress-strain portion was linearized for simplicity only for the M-R model.

This model is suitable for incompressible natural rubber and recommends Poisson's ratio of 0.499. It was found, SMA column's buckling load increases if the Poisson's ratio is chosen as 0.499 instead the actual value of 0.33. To minimize the number of figures, however, those simulation results are not shown in this study.
Chapter 4

Behaviors of the Superelastic Shape Memory Alloy Shafts under Torsion

Summary

Observing the unique stress-strain curves of the superelastic SMA in tension and compression for large strains, the primary intention of this study is to investigate on the behaviors of the shafts made of the same material, under torsion for large angle of twist. Experiments have been performed for the superelastic SMA shafts with different unsupported lengths and angles of twist and the results are compared with those of SUS304 shafts under similar test conditions. Results show that for large angle of twists, the torsional strength of the superelastic SMA increases nonlinearly and exceeds that of SUS304. It is found that for the superelastic SMA, the residual strains are much smaller after each cycle and consequently, the hysteresis under loading-reverse loading is much narrower than that for the SUS304. Hollow superelastic SMA shafts are found to show interestingly different performance compared with the solid shafts under similar test conditions. The slender superelastic SMA shafts are found to buckle like a column when the critical twisting moment is exceeded.
4. Behaviors of the Superelastic Shape Memory Alloy Shafts under Torsion

4.1 Introduction

Studying the behaviors of the superelastic SMA under torsion will be practically useful and necessary as the same material is being proposed/used in many practical applications. So far numerous studies have been performed involving the tensile behaviors of the SMA. Fortunately, a few studies, some of which are listed in the reference [1-6], dealt with the behaviors of the SMA under torsion and/or combined loading. Interestingly, tubular shaped specimen was used by most of the researchers for experiments [1-5].

The effect of training on the two-way shape memory (TWSM) for the Cu based polycrystalline SMA under combined loading (torsion and axial load), has been studied by Tokuda, M. et al. [1,2] and Sittner, P., et al. [3]. The maximum strain during the test was within 2% and the training behavior was demonstrated, related with SME.

Tanaka et al. [4], studied the austenite and martensitic start conditions in an Fe-based polycrystalline SMA under tension/compression-torsion loads. It was demonstrated that an oval cone in the stress-temperature space could represent the martensite start condition while a polygonal cone in the same stress-temperature space could represent the austenite start condition.

Raniecki et al. [5], studied the deformation behavior of TiNi shape memory alloy undergoing R-phase reorientation in torsion-tension (compression) tests. The maximum strain was within 0.5% and the deformation behavior associated with the R-phase reorientation was studied.

Besides these studies, Tobushi and Tanaka [6] developed a model using a simplified torsional stress-strain diagram, for the shape memory alloy spring to represent the load-deflection relation for the spring.

It can be seen, in all of the above-mentioned investigations [1-5] the maximum strain was too small (within 2%), meaning the angle of twist for the SMA specimen was too small. This
is due to the fact that, perfect applications of the TWSM training, or other proposed theories related to SME, are possible if the strain and stress in the SMA remain within such a small range. From mechanical engineering point of view, however, in practical applications the stress and strain can be sufficiently large, in particular, if the shaft material is superelastic SMA. In addition, there is possibility of loading and reverse loading of the shaft under torsion for a finite number of cycles.

As discussed in the previous chapters, the highly nonlinear stress-strain curves of the superelastic SMA (sometimes termed only as SMA hereafter) in tension and compression demonstrate significantly high strength for large strains. As soon as the SIMT is initiated in tension, the behaviors in tension and compression also exhibit notable asymmetry. Thus, it is perhaps important to study SMA’s behavior under torsion for large strains. Moreover, in the rare case, if the critical twisting moment is exceeded, any slender shaft may buckle like a column. Considering the above facts, extensive experiments under torsion have been performed in this study for the superelastic SMA, and the results are compared with those of SUS304 under similar test conditions.

**Buckling of a shaft due to torsion**

Buckling is usually caused by compression; but a shaft may also become unstable under the action of a torque. As an example, let us consider the shaft of Fig. 4.1, acted upon by a torque \( M \) and supported (fixed ends) in the manner of an Euler column.

Since buckling is characterized by a deflection of the initially straight axis, for the present we may neglect the compliance of the shaft with respect to compression and torsion; moreover, the influence of shear may be neglected. So long as the axis is straight, the bending moment is zero as in the case of Euler buckling. However, as soon as deflections occur, there appear bending moments in the various sections of the shaft. When they become sufficiently
strong, they may result in buckling [7]. Greenhill [8] was first to solve this problem. For both ends fixed shafts, mathematical calculation gives the critical twisting moment $= \frac{2.861 \pi E I}{L}$, symbols having their usual meanings.

![Diagram of shaft subjected to torsion](image)

Critical twisting moment $= 2.861 \pi EI/L$

Fig. 4.1 Shaft subjected to torsion.
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4.2 Experiment

4.2.1 Test conditions

The materials, configurations and conditions used in this experiment are as follows. The chemical composition for the 2mm solid Superelastic SMA rods is Ti49.3 at% Ni50.2 at% V0.5 at% (same as mentioned in Chapter 2). While, for 1mm solid superelastic SMA rods and the tubes (OD=1mm, ID=0.8mm) the chemical composition is Ni54-58%, Ti balance.

For comparison of the torsional behaviors, 2mm diameter solid stainless steel (SUS304) rods were also used. Room temperature range was 23°C - 26°C. The Instron machine was used and the speed during loading-unloading cycle was 0.5 to 1 rpm for the 2mm rods and 0.25rpm to 0.5rpm for the 1mm solid rods and tubes.

4.2.2 Procedure

For the 2mm rods, reverse loading was performed for consecutive cycles for L of 15mm and 40mm. While, loading-unloading cycles were performed and the buckling behavior was observed for L=80-165mm. Theoretical buckling loads for the shafts were calculated before the tests. Snaps were taken by a digital camera at different states of loading (below and above the theoretically calculated critical twisting moments) to demonstrate that the shape of the deformed shafts changes distinctly in the vicinity of the theoretical buckling loads. For the 1mm rods and tubes, loading-unloading cycles were performed for consecutive cycles for increasing values of angle of twist.

It should be mentioned here that the term residual strain is used to refer to the angle of twist at the end of the loading-unloading cycle when the torque becomes zero. Though it does not actually represent the true residual strain of any element of the shaft material, it can, to some extent, evaluate the overall shape recovery capability of the shaft.
4.2.3 **Difficulties and solutions**

Tanaka et al. [9], pointed out the fact that often preparation for a suitable sized SMA specimen, which fits the tests under the loading conditions and exhibits a stable response under repeated loading, is rather a difficult problem to solve before starting the experiment. For the present study, avoiding slip of the specimen at the gripping fixture during the tests was the first problem to solve. Obviously, because of slip the test data show high residual strains at the end of loading-unloading cycle. As mentioned, the specimens were prepared from 2mm solid rods and 1mm solid and hollow rods (all along the same diameter). To clamp the specimens rigidly in the gripping fixtures, suitable shaped or larger diameter superelastic SMA specimens are expected, which are not available commercially. Thus, it becomes quite difficult to avoid slip at the gripping fixture when the torque becomes sufficiently large (particularly for 1mm diameter solid shaft specimen). In order to avoid slip during the tests, two ends of the solid rods (diameters of 1mm, 2mm) were given the suitable shapes by grinding. While for the tubes (OD=1mm, ID=0.8mm), two small straight wires of diameter 0.7mm were inserted at the two ends, before setting the specimens in the gripping fixtures.

It should be noted here, however, using small diameter specimen (meaning, short gage length) has one plus point since we are also interested to observe the buckling of slender shafts. That is, for larger diameter specimens critical twisting moment (proportional to the area moment of inertia) will also increase tremendously, and eventually it might be difficult to avoid slip at high torque before the buckling occurs. The length of the specimen must be increased with the increasing diameter to observe buckling of the shaft. Too long specimens are difficult to set within two gripping fixtures of the machine.

4.3 **Results and Discussions**

Test results are separately discussed for the 2mm solid shafts and 1mm diameter (solid and hollow) shafts. Behaviors of the 2mm diameter solid shafts (with relatively short test lengths)
under loading-reverse loading cycles are discussed in the subsection 4.3.1. For clarity of explanation, buckling and postbuckling behaviors of the slender Superelastic SMA shafts are dealt exclusively for 2mm diameter solid shafts in the subsection 4.3.2. The comparative behaviors between the 1mm solid and hollow shafts are dealt in the subsection 4.3.3.

**CALCULATION OF MAXIMUM SHEAR STRAIN FROM, \( \gamma = \tan^{-1}(\rho \theta L) \)**

For 2mm diameter shafts, the maximum strains on the surface of the rods (\( \rho = \) radius or \( C \)) for one complete revolution (\( \theta = 360 \) degree) are as follows: \( \gamma = 39.6\% \) for \( L = 15\)mm, \( \gamma = 15.5\% \) for \( L = 40\)mm, \( \gamma = 7.8\% \) for \( L = 80\)mm, \( \gamma = 6.3\% \) for \( L = 100\)mm, \( \gamma = 5.2\% \) for \( L = 120\)mm, \( 4.48\% \) for \( L = 140\)mm, \( 3.8\% \) for \( L = 165\)mm.

**4.3.1 Hysteresis for loading-reverse loading cycles**

The superelastic SMA shafts' behaviors (far beyond the elastic limit) under torsion are studied and compared with those of SUS304 shafts. Figs. 4.2, 4.3 show behaviors of the shafts under reverse loading. The corresponding numerical values are given in Table 4.1. As seen (Fig. 4.2), compared with the SUS304 shaft, too narrow hysteresis occurs for the SMA shafts for \( L = 15\)mm and the residual strain is also too small. Thus, for reverse loading, compared to the SMA, the SUS304 consumes enormous amount of energy and also deforms plastically.

Compared with Fig. 4.2, Fig. 4.3 shows a wider hysteresis for the SMA with \( L = 40\)mm for larger angle of twist. The hysteresis is still much wider for SUS304. However, behaviors of the SMA and SUS304 are almost symmetric under reverse loading (Fig. 4.3). It can be also noted that the strength of the SMA increases steeply and is higher than that of the SUS304 for large angle of twist (Table 4.1). Upon unloading the SMA's residual strains are quite small. For too large angle of twist, however, there will be large residual strains and the hysteresis will no longer be symmetric.
4. Behaviors of the Superelastic Shape Memory Alloy Shafts under Torsion

Fig. 4.2 Loading-reverse loading cycles for \( L=15\text{mm} \).

Fig. 4.3 Loading-reverse loading cycles for \( L=40\text{mm} \).
Table 4.1 Test results for 2mm diameter shafts (loading-reverse loading cycles)

<table>
<thead>
<tr>
<th>$L$ (mm)</th>
<th>Material</th>
<th>Torque (J)</th>
<th>Residual strains (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$90^\circ$</td>
<td>$-90^\circ$</td>
</tr>
<tr>
<td>15</td>
<td>SMA</td>
<td>2.76</td>
<td>-2.40</td>
</tr>
<tr>
<td></td>
<td>SUS304</td>
<td>2.84</td>
<td>-3.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$360^\circ$</td>
<td>$-360^\circ$</td>
</tr>
<tr>
<td>40</td>
<td>SMA</td>
<td>4.38</td>
<td>-4.52</td>
</tr>
<tr>
<td></td>
<td>SUS304</td>
<td>3.31</td>
<td>-3.1</td>
</tr>
</tbody>
</table>

4.3.2 Torsional buckling of the slender shafts

Since, the stiffness of the SMA increases quite steeply with the increasing angle of twist, the critical twisting moment is approached easily and the slender SMA shafts buckle like a column. Upon unloading, however, the shape is recovered due to superelasticity and the shafts again become straight.

Let us discuss buckling and postbuckling behaviors for the SMA shaft with $L=165$mm. The theoretical critical torque for this shaft is $2.78J$. To identify the buckled shape and the corresponding load clearly, the shaft was loaded up to $4J$ and then unloaded. Figs. 4.4-4.9 correspond to snaps taken at different states of loading. Fig. 4.4 shows the straight configuration of the shaft at $0J$. The shaft is found to be slightly deformed at $2.6J$ (Fig. 4.5) as the mid-portion becomes convex upward symmetrically. Fig. 4.6 shows the buckled shape of the shaft like a sine curve at $2.9J$. To make sure of the fact that the shaft has buckled elastically the torque was further increased and the distinct buckled shapes can be observed from Fig. 4.7 (at a torque of $3.1J$) and Fig. 4.8 (at a torque of $3.17J$).
Fig. 4.4 Shape at 0 J (L=165mm).

Fig. 4.5 Shape at 2.6J (L=165mm).
Fig. 4.6 Buckled shape at 2.9J (L=165mm).

Fig. 4.7 Buckled shape at 3.1J (L=165mm).
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Fig. 4.8 Buckled shape at 3.17\( J \) (\( L=165\text{mm} \)).

Fig. 4.9 Shape at the end of cycle (\( L=165\text{mm} \)).
4. Behaviors of the Superelastic Shape Memory Alloy Shafts under Torsion

Fig. 4.10 Torque-angle of twist curves for $L=165$mm.

Fig. 4.11 Torque-angle of twist relations for too large angle of twist for $L=165$mm.
Upon unloading, the shaft becomes straight again as shown by Fig. 4.9. The corresponding torque-angle of twist curve can be seen from Fig. 4.10. It is seen that, after unloading, the residual strain is also very small for this shaft, meaning it can be cycled for a number of times.

On the other hand, the torque-angle of twist curve rises too slowly for large angle of twist for the SUS304 shafts (Figs. 4.10, 4.11). The elastic buckling load for this SUS304 shaft ($L=165\text{mm}$) is 8.98J. Usually, either the shaft material fails or slip occurs at the grips (Fig. 4.11) before this high torque can be reached.

Let us next discuss the buckling of the SMA shaft with $L=140\text{mm}$. The theoretical critical load for this shaft is 3.28J. To identify the buckled shapes and the corresponding loads clearly, the shaft was loaded up to 4J and then unloaded. Figs. 4.12-4.16 correspond to snaps taken at different state of loading. Fig. 4.12 shows the straight configuration of the shaft at 0J. The shaft is slightly deformed at 3.3J (Fig. 4.13) as the mid-portion becomes convex upward symmetrically. Fig. 4.14 shows the buckled shape of the shaft at 3.4J. The buckled shape looks like a sine curve. To make sure of the fact that the shaft has really buckled elastically and within 3.4J, the torque was further increased and the distinct buckled shapes can be observed from Fig. 4.15 (3.65J) and Fig. 4.16 (4J). The corresponding torque-angle of twist curve can be observed from Fig. 4.17. It is seen that, after unloading, the residual strain is quite small for this shaft, implying the shafts becomes almost straight upon unloading and can be cycled for a number of times to exhibit similar behaviors.

On the other hand, for SUS304 shafts of $L=140\text{mm}$, the buckling load is 10.1J and thus too large angle of twist is required to reach such a high torque. Figs. 4.18 and 4.19 show the shapes of the SUS304 corresponding to 0J and 4J, respectively. As seen no deformation can be observed from Fig. 4.19. In comparison, having much lower Young's modulus, the corresponding SMA column is buckled much below 4J (Figs. 4.14-4.16).
Fig. 4.12 Shape at 0J ($L=140\text{mm}$).

Fig. 4.13 Deformed shape at 3.3J ($L=140\text{mm}$).
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Fig. 4.14 Buckled shape at 3.4J ($L=140\text{mm}$).

Fig. 4.15 Buckled shape at 3.65J ($L=140\text{mm}$).
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Fig. 4.16 Buckled shape at 4J (L=140mm).

Fig. 4.17 Torque-angle of twist curves for L=140mm.
Fig. 4.18 Shape of the SUS304 shaft (L=140mm) at 0J.

Fig. 4.19 Shape of the SUS304 shaft (L=140mm) shape at 4J.
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![Graph showing torque vs. angle of twist for SMA and SUS304 shafts.](image)

**Fig. 4.20** Torque-angle of twist relation for the SUS304 shaft for large angle of twist.

Fig. 4.20 shows that the load increases quite slowly for increasing angle of twist for the SUS304 shafts. Thus, either the material fails or slip occurs at the grips before such a high torque (corresponding to the torsional buckling) is reached.

Let us next observe the changing shapes for increasing loading for the SMA shaft with $L=100$mm. The theoretical critical load for this shaft is 4.59J. To identify the shapes in the vicinity of the theoretical buckling load, the shaft was loaded up to 5.3J and then unloaded. Figs. 4.21-4.24 correspond to snaps taken at different state of loading. Fig. 4.21 shows the straight configuration of the shaft at 0J. The shaft is found to be slightly deformed at 3.5J (Fig. 4.22) as the mid-portion becomes concave upward symmetrically. Fig. 4.23 shows the buckled shape of the shaft at 4.9J. The buckled shape looks like a sine curve. To make sure of the fact that the shaft has really buckled within 4.9J, the torque was further increased and the distinct buckled shapes can be observed from Fig. 4.24 (5.3J). Upon unloading, this shaft also becomes straight because of small residual strains as shown in Fig. 4.25. It can be also seen
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Fig. 4.21 Shape at the start of loading ($L=100\text{mm}$).

Fig. 4.22 Deformed shape at 3.5J ($L=100\text{mm}$).
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Fig. 4.23 Buckled shape at 4.9J (L=100mm).

Fig. 4.24 Buckled shape at 5.3J (L=100mm).
that the maximum torque required by the SUS304 shaft to reach the same angle of twist is about 60% of that for the SMA shaft.

The shortest shaft chosen for studying the torsional buckling has an unsupported length of 80mm. Theoretical critical load for this shaft is 5.74J. To identify the buckled shapes, the shaft was loaded up to 6.1J and then unloaded. Figs. 4.26-4.29 correspond to snaps taken at different state of loading. Fig. 4.26 shows the straight configuration of the shaft at 0J. The mid-portion of the shaft becomes convex upward symmetrically at 4.5J (Fig. 4.27). Fig. 4.28 shows the buckled shape of the shaft at 5.9J. The buckled shape looks like a sine curve. To make sure of the fact that the shaft has really buckled within 5.9J, the torque was further increased and the distinct buckled shapes can be observed from Fig. 4.29 (6.1J). From Fig. 4.30, it is seen that the residual strain is small for this shaft, implying it would become almost straight upon unloading. Consequently, it can be cycled for a number of times within the load.
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Fig. 4.26 Shape at the start of loading ($L=80\text{mm}$).

Fig. 4.27 Deformed shape at 4.5$J$ ($L=80\text{mm}$).
Fig. 4.28 Buckled shape at 5.9J (L=80mm).

Fig. 4.29 Buckled shape at 6.1J (L=80mm).
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Fig. 4.30 Torque-angle of twist curves for $L=80\text{mm}$.

It can be also seen that the maximum torque required by the SUS304 shaft to reach the same angle of twist is about half of that for the SMA shaft (Fig. 4.30).

Table 4.2 Critical twisting moments observed from experiment

<table>
<thead>
<tr>
<th>$L$ (mm)</th>
<th>Theory</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>165</td>
<td>2.78</td>
<td>Distinct buckled shape observed at 2.9J</td>
</tr>
<tr>
<td>140</td>
<td>3.28</td>
<td>Distinct buckled shape observed at 3.4J</td>
</tr>
<tr>
<td>120</td>
<td>3.82</td>
<td>Distinct buckled shape observed at 3.9J</td>
</tr>
<tr>
<td>100</td>
<td>4.59</td>
<td>Distinct buckled shape observed at 4.9J</td>
</tr>
<tr>
<td>80</td>
<td>5.74</td>
<td>Distinct buckled shape observed at 5.9J</td>
</tr>
</tbody>
</table>
Table 4.2 gives a comparison of theoretically calculated twisting moment with the experimentally observed critical twisting moments for the shafts. Of course, the shafts are likely to buckle at a load lower than the calculated values. However, it should be noted here that the critical twisting moment was observed from the pictures of the deformed SMA shafts taken at different state of loading. Since the determination of the buckled shape depends on the eye observation, it is rather difficult to determine whether the shaft has prominently changed its shape exactly at the theoretically calculated twisting moment. Thus, to clearly demonstrate the buckled configuration, the load was slightly increased (Table 4.2) above the calculated critical value and the picture was taken at that state. It can be checked, the experimentally observed buckling moment reduces almost linearly with the increasing values of the unsupported length of the shafts.

It should be mentioned here that the critical twisting moment for the SMA shaft with \( L=120\text{mm} \) was determined in similar fashion but the pictures are not presented in order to minimize the number of figures. It can be concluded that because of unusually steep torque-angle of twist relation, the critical twisting moment is reached for increasing angle of twist and the slender superelastic SMA shafts buckle like a column. If ordinary SMA (with SME) shafts have similar torque-angle of twist relation, they are also likely to buckle under torsion.

### 4.3.3 Comparison of behaviors for the solid and hollow shafts

Under torsion, the maximum strain occurs on the surface of the shaft, while the strain is zero on its axis. Consequently, for saving materials shafts are often made hollow. Thus, study on the hollow superelastic SMA shafts under torsion is also important. This section compares behaviors of the solid and hollow superelastic SMA shafts. The solid shaft has a diameter of 1mm while for the hollow shaft OD and ID were 1mm and 0.8mm, respectively. According to the supplier (NILACO, Japan) data, both the solid and hollow specimens have almost the same chemical composition as described in the sub section 4.2.1.
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Fig. 4.31 Torque-angle of twist curves for the solid and tubular shafts $(L=30\text{mm})$ for $0-150^\circ$.

Fig. 4.32 Torque-angle of twist curves for the solid and tubular shafts $(L=30\text{mm})$ for $0-180^\circ$. 

In the following paragraphs, test results are discussed for the unsupported lengths \((L)\) of 30mm, 60mm and 90mm for both the solid and tubular shafts. Sometimes, two consecutive cycles were performed for the same angle of twist. It was found that in most of the cases for any value of \(L\), the nature/shape of the torque-angle of twist curve remains similar particularly for the first few cycles. Therefore, only the interesting results are presented and discussed.

Let us first discuss the behavior of the shafts with \(L\) of 30mm. Since the length is too short, the strain will be high for the specimens under torsion. Thus, for the tubular shafts, the angle of twist was increased in small increments and two consecutive loading and unloading cycles were performed for the maximum angles of twist of 50°, 55°, 60°, 90°, 120°, 150°, 180°. Fig. 4.31 shows the comparative torque-angle of twist relations for the solid and hollow shafts for the angles of twist of 90°, 120° and 150°. Because of lower torsional stiffness, the tubular shafts show hysteresis with lower magnitude and lower width. However, the hysteresis is similar in shape for the solid and tubular shafts. For the same specimens, hysteresis for higher angles of twist is shown in Fig. 4.32. The tubular shaft can sustain the torque when twisted up to 180° for the first time. It fails, however, when loaded for the second time and shows small residual strain upon unloading. Under similar conditions, the solid shafts can sustain higher angle of twist before failure.

For the 60mm tube two consecutive cycles were performed for each of the following maximum angles of twist (in degrees): 50, 65, 90, 120, 180, 240, 300, 360, 540. While for the solid shafts, the maximum angles of twist (in degrees) were 70, 150, 210, 240, 270, 300, 360, 540, 630.

Fig. 4.33 shows the hysteresis for the angle of twist ranging from 0-360° degrees for \(L = 60mm\). The solid shafts show slightly wider hysteresis with higher magnitude compared with the tubular shafts. Fig. 4.34 shows the test results for the same specimens, when the maximum angles of twist during the test were from 300°-630°. It was observed, solid shaft buckles like a
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Fig. 4.33 Torque-angle of twist curves for the solid and tubular shafts (L=60mm) for 0-360°.

Fig. 4.34 Torque-angle of twist curves for the solid and tubular shafts (L=60mm) for 0-630°.
column (approximately, after the angle of twist exceeds 400°). It was found, the solid shaft fails at the grip when the angle of twist is 630° and after unloading, the residual strain is quite large. On the other hand, buckling (like that of a column) was not observed for the tubular shaft, and it fails at lower angle of twist compared with the solid shaft. Fig. 4.35 specifically shows the same tubular shaft’s behavior for large angle of twist for the last three consecutive cycles (that is, cycle numbers 16th, 17th and 18th). The 17th cycle can be performed for the maximum angle of twist of 540°. However, it fails for the 18th cycle at an angle of twist of 520°. Note that, in spite of the distinct failure, upon unloading the residual strain is quite small for the tubular shaft (Figs. 4.34 and 4.35). Moreover, the torque is seen to rise slightly during unloading. This can be explained in the following way. The load drops sharply when the tube material fails. The tube materials are not totally departed even after failure and has the property of superelasticity. At one stage during unloading, because of the shape recovery tendency of parts of the tubular shaft, which are still in contact, the torque slightly increases. Similar failure pattern of the tubular specimen is discussed later in more detail.

For the 90mm tubes single loading-unloading cycle was performed for each of the following maximum angles of twist (in degrees): 180, 360, 540, 720, 900, 900. While for the solid shafts, the maximum angles of twist (in degrees) were 180, 360, 540, 720, 900, 1080, 1260.

Fig. 4.36 shows the tubular shaft’s behavior for the angle of twist of 0 to 720°. It is interesting to note the distinct difference in the hysteresis for the first two cycles, that is, when maximum the angle of twist is increased from 180° to 360°. However, for the next consecutive cycles (with the increasing angle of twist), the maximum width of the hysteresis does not change so distinctly. This shaft buckles like a column when the twisting moment is approximately 0.13 J (corresponding to an angle of twist of 540°).
Fig. 4.35 Torque-angle of twist curves for the tubular shaft (L=60mm) for the last three consecutive cycles.

Fig. 4.36 Torque-angle of twist curves for the tubular shaft (L=90mm) for 0-720°.
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Fig. 4.37 Torque-angle of twist curves for the tubular shaft ($L=90\text{mm}$) for $0-90^\circ$.

Fig. 4.38 Torque-angle of twist curves for the solid and tubular shafts ($L=90\text{mm}$) for $0-720^\circ$. 
Fig. 4.39 Torque-angle of twist curves for the solid and tubular shafts ($L=90\text{mm}$) for $0-1260^\circ$.

For the same $90\text{mm}$ long tubular specimen, Fig. 4.37 shows the hysteresis for the maximum angle of twist of $720^\circ-900^\circ$. The hysteresis becomes distinctly wider than the previous one when it is loaded up to $900^\circ$ and then unloaded. The shaft can sustain an angle of twist of $900^\circ$ for the first time but fails for the next cycle. Though the shaft buckles and is severely deformed for each of the three cycles, the residual strain is almost zero upon unloading (Fig. 4.37). The behavior of the same tubular shaft can be compared with that of the solid shaft from Figs. 4.38 and 4.39. As seen from Fig. 4.38, the twisting moment increases too slowly for the tubular shaft after the angle of twist of $200^\circ$, while it increases distinctly up to $400^\circ$ for the solid shaft. The solid shaft shows larger residual strain upon unloading and buckles like a column at a twisting moment of $0.43J$ (corresponding to an angle of twist of $400^\circ$). Fig. 4.39 depicts the behavior of the shafts for the maximum angle of twist of $720^\circ$ to $1260^\circ$. The hysteresis becomes distinctly wider than the previous one and the residual strain
increases for the solid shaft when it is loaded up to $1080^\circ$ and then unloaded. The solid shaft fails at the grip for the next cycle. It is seen that the tubular shaft fails at a lower angle of twist.

For specifically examining the behavior of the tubular shaft, another test was performed with a different specimen having $L=90\text{mm}$. To identify the buckled shape and the failure pattern, the shaft was loaded until failure occurred without any unloading. Figs. 4.40-4.43 correspond to snaps taken at different state of loading. Fig. 4.40 shows the straight configuration of the shaft at $0J$. For increasing loading, the shaft buckles like a column. Fig. 4.41 shows the distinct buckled shape of the shaft at $0.17J$ (corresponding to an angle of twist of $790^\circ$). The buckled shape looks like a sine curve. More distinct buckled shapes can be observed from Fig. 4.42 that corresponds to a load of $0.19J$ and an angle of twist of $900^\circ$. Twisted wrinkles appear on the surface near the fixed end of the fixture as if the shaft is made of fibrous elements. Fig. 4.43 shows the deformed shape of the tube after failure. The shaft fails at $0.22J$ corresponding to an angle of twist of $1040^\circ$. Fig. 4.44 shows the corresponding torque-angle of twist curve for the concerned tubular shaft. It can be noted from Fig. 4.43, that the materials are not totally departed after failure. The material still possesses the property of superelasticity even after failure of the tube. Thus, during unloading (not done in this case), the torque remains low but the tubular shape can be retained because of the superelastic action of the parts of the tube materials, which are still in contact after failure. For this reason, the torque was found to slightly increase for some of the cases (for example, for 60mm long tubular shaft, Figs. 4.34-4.35) upon unloading.
4. Behaviors of the Superelastic Shape Memory Alloy Shafts under Torsion

Fig. 4.40 Shape of the tubular shaft at the start of loading ($L=90\text{mm}$).

Fig. 4.41 Buckled shape at $0.17J$ ($L=90\text{mm}$).
4. Behaviors of the Superelastic Shape Memory Alloy Shafts under Torsion

Fig. 4.42 Buckled shape at 0.19J (L=90mm).

Fig. 4.43 Shape after failure at an angle of twist=1044°.
4. Behaviors of the Superelastic Shape Memory Alloy Shafts under Torsion

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Fig. 4.44 Torque-angle of twist until failure of the tubular shaft ($L=90$mm).

4.4 Conclusions

A few important characteristics of the solid and hollow superelastic SMA shafts have been demonstrated through experimental results. It is found that, if the angle of twist is not too large, because of small residual strain, the superelastic SMA shafts make a much narrower hysteresis than that of the SUS304 shafts under loading-reverse loading cycles. Interestingly, the torsional stiffness of the superelastic SMA increases nonlinearly and exceeds that of the SUS304.

Since, the stiffness of the SMA increases quite steeply with the increasing values of the angle of twist, the slender SMA shafts are found to buckle like a column when the critical twisting moment is exceeded. Examinations of the pictures of the deformed shaft at different state of loading shows that the slender shafts buckle in the vicinity of the theoretically calculated critical twisting moment. Upon unloading, however, the shape is recovered due to superelasticity and the shafts become straight again. On the other hand, having much higher Young’s modulus, the critical twisting moment for the slender SUS304 shaft is about thrice.
than that for a slender superelastic SMA shaft. Torque-angle of twist curve shows that, after the initial portion, the load increases too slowly for increasing angle of twist for the SUS304 shafts. Therefore, in the usual cases of applications either the material fails or slips at the grips before such a high torque (corresponding to the torsional buckling) is reached. Any trace of bent shape could not be observed examining the pictures of the highly twisted slender SUS304 shaft. Thus, it can be concluded that because of unusually steep torque-angle of twist relation, the slender superelastic SMA shafts buckle like a column for increasing value of the angle of twist.

Hollow/tubular Superelastic SMA shafts are found to show interestingly different performance compared with the solid shafts for different unsupported lengths. For the complete loading-unloading cycles, the hysteresis is similar in shape for the solid and tubular shafts. Because of lower torsional stiffness, however, the tubular shafts show hysteresis with lower magnitude and lower width. The torque-angle of twist curve remains steep for large angle of twist for the long solid shaft compared with the tubular shaft. Alike the solid shafts, slender tubular shafts also buckle like a column. Distinct twisting wrinkles appear on the surface and the tubular shafts fail because of it at a lower angle of twist compared with the solid shafts. After complete unloading tubular specimens show much better shape recovery.
References


Chapter 5

Discussions and Conclusions

Mechanical behaviors of the superelastic SMA for large deformation have been investigated particularly for columns and shafts. A few important and unique behaviors of the same material have been demonstrated by experiments and explained in detail. Practical applications of the columns and shafts often involve cyclic loading. Observing the above fact, a few loading-unloading cycles were performed during the experiment. Incidentally, it also helps to demonstrate the superelastic SMA's capability of shape recovery upon unloading. The following points can be noted from the results of the present study.

Contrary to general notions, experimental observations verify that upon too large deformation (far beyond the first point of instability), the shortest SMA column ($L/k=28$) shows a distinct and highly stable secondary modes of deformation. More astonishingly, the short column ($L/k=38$) shows two distinct peak loads, the second peak being higher than the first one. For a few repeated loading-unloading cycles, these short SMA columns have much higher buckling loads but the smallest cumulative residual strains, in comparison with the SUS304 and Al columns, under similar test conditions. Though the slender SMA column buckles at low load, the buckling load increases very significantly with the decrease in slenderness ratio and exceeds that of the short SUS304 column. On the other hand, the slender superelastic SMA columns also show a few unique behaviors. For example, the magnitude of load is least changed during the postbuckling compression and in some cases the load (recovery force) increases during unloading of the buckled column. The shape is almost fully recovered upon unloading.

To qualitatively explain the above unique phenomena, the fundamental properties of the used superelastic SMA, that is, the stress-strain curves in compression and tension were obtained by experiment for pretty large strains (beyond the region of SIMT) and compared
with those of SUS304 and Al. It can be concluded that the slender SMA column buckles in the austenite phase and because of the low material stiffness, its buckling load is lower than that of a slender Al column. On the other hand, too short SMA column buckles plastically after SIMT. Because of too high strength after the SIMT, short SMA column's buckling load exceeds that of the SUS304 column.

For more precise and quantitative analysis of the unique performance of the superelastic SMA columns, numerical solution was carried out. Static analysis was performed considering large deflection option and nonlinear stress-strain curves of the SMA obtained by experiment. The suitable material models of the FEM code ANSYS were used for simulation. Simulation results verify that particularly for the short SMA columns, most of the column material remains under compression even after buckling. The maximum compressive strains exceed the range of SIMT. Because of used SMA’s significant asymmetry of the tensile and compressive stress-strain curves, particularly for large strains, compressive stress-strain data are necessary for simulation of the short columns. It is verified that there is a point of instability after which the shortest SMA column has a stable postbuckling configuration.

Simulation results show that at the critical point of the load-deformation curve, the maximum compressive strains remain within a small value and thus the slender column buckles in the austenite phase. However, it is important to note that during the postbuckling compression, the strain becomes large even for the slender columns. Because of SMA’s high compressive strength for large strains, the slender column can sustain the load with least change in magnitude.

A special method was devised to simulate the unloading path of the slender superelastic SMA column. It is found that defining a few intermediate unloading curves between the actual loading and the unloading curve can help to minimize stress discontinuity at the
beginning of unloading. It can be concluded that the recovery force increases because of elastic shape recovery of the slender SMA column.

Better results from the simulation might be possible if both the compressive and tensile stress-strain curves could be used simultaneously, for predicting the postbuckling phenomena of the slender columns, in particular.

Under torsional loading, superelastic SMA shafts could be subjected to strains much beyond the elastic limit. Therefore, several experiments were carried out to study the behaviors of the solid and hollow shafts under torsion. Experimental evidence reveals the fact that if the angle of twist is not too large, the SMA shafts make a much narrower hysteresis compared with the SUS304 shafts, under repeated loading-reverse loading cycles.

The most important characteristic of the SMA shaft is that its torsional strength increases nonlinearly and exceeds that of the SUS304 shafts for increasing value of the angle of twist. Consequently, when the torque exceeds a critical value (that can be found by theoretical calculation), the slender SMA shafts are found to buckle like a column. During experiments, the theoretical buckling torque could be verified by taking pictures of the shafts at different values of loading. Slender tubular shafts also buckle like a column but shows better shape recovery upon unloading. It appears that the superelastic SMA shaft behaves like a fibrous material under torsion as distinct twisting wrinkles appear on the surface of the tubular shaft which cause its failure.
Appendix A

Direct Method to Estimate Stress Induced-Martensitic Transformation Points by Tensile Test

Summary

During the stress-induced martensite transformation (SIMT), tensile test results of the superelastic SMA show that the local strains significantly differ from that of the overall strains of the specimen. The above phenomenon starts as soon as the SIMT is initiated and terminates when the SIMT is over. There appears an interface between the austenite phase and the martensite phase during the loading process. The movement of the interface causes such unique behavior. The same astonishing incidence occurs during the unloading when the reverse phase transformation occurs. This study briefly discusses the above phenomenon. The local strains were measured by strain gages while the overall strains of the specimens were measured simultaneously by displacements of the fixture. Thus the start and finish points of the SIMT and the reverse SIMT might be identified by simply plotting a curve local strain versus the overall strain of the specimen.
A.1 Introduction

Experimental results reveal the fact that, a unique phenomenon occurs during the SIMT for the SMA. During loading there appears an interface between the austenite (parent phase) and the martensite (product phase) due to the SIMT [1-3]. The interface that appears due to stress, moves during the deformation process. By tensile test of a wire, Tobushi et al. [3] also showed that during the SIMT and the reverse SIMT, the SMA behaves like an incompressible material. In the experimental procedure of reference [3], photographs of the wire specimen were taken by a digital camera. Necessary measurements were done on the enlarged photograph.

The present study, dealing with the similar phenomenon [1-3], was primarily intended to accurately measure the stress-strain (s-s) curve of the superelastic SMA by using strain gages.

A.2 Experimental Details

The materials, configurations and conditions used in this experiment are as follows.

Material: superelastic SMA (Ti49.3 at% Ni50.2 at% V0.5 at%). Diameter of the tensile test specimens and the Ar were 2 mm and -3°C, respectively. Room temperature range was 23°C- 30°C. The Instron machine was used and the speed of the cross-head during loading-unloading cycle was 2 mm/min. Gage lengths for specimens #1 and #2 were 130mm and 153mm respectively. The strain gage used for the experiment can measure up to 15% strain. The positions of the strain gages from the loading end of the specimens are shown in Figs. A.1 and A.2.

The strains were measured simultaneously by the strain gages and displacement of the fixtures. For specimen #1 the strain was large (about 22% as measured by the displacement of the fixture). For specimen #2, two loading-unloading cycles were
performed for a maximum strain of 6.5% (as measured by the displacement of the fixture).

Fig. A.1 Position of the strain gage for specimen #1 ($L=130\text{mm}$).

Fig. A.2 Positions of two strain gages for specimen #2 ($L=153\text{mm}$).

**A.3 Results and Discussions**

Using the data of strain gage and the fixture displacements simultaneously the curves as shown by Figs. A.3-A.7, are obtained. Results are discussed sequentially.

Fig. A.3 shows change of the local deformation (strain gage reading at the mid-position of the specimen) with respect to the total end displacement for specimen#1. As seen, initially the relationship between the two readings is linear. However, after 1% strain (approximately) is exceeded, quite astonishingly, there is almost no local strain although there is end displacement! It indicates that the SIM transformation is
A. Direct Method to Estimate SIMT Points by Tensile Test

Fig. A.3 Local deformation vs total deformation for specimen #1.

Fig. A.4 Tensile stress-strain (nominal) curves for specimen #1.
initiated. When the SIMT is about to finish, the part, which did not expand earlier suddenly increases rapidly. Thus, the two points may verify the starting and finishing end points of the transformation process. Tobushi et al. [3] showed that during the SIMT, the Poisson's ratio approaches 0.5, meaning, the condition of no-volume change (or, incompressibility) has been reached locally. The horizontal portion of Fig. A.3 also verifies the above fact.

Fig. A.4 shows the engineering stress-strain curves measured simultaneously by end displacement and strain gage for specimen #1. As mentioned, this study was intended to measure the strain accurately by the strain gage. As seen, initial slopes of the two curves deviate a little because the displacement reading is less accurate than the strain gage reading. However, more importantly, critical stress to initiate the local SIMT is higher than that required to initiate the overall SIMT (or, for the whole specimen) as shown by the displacement reading (Fig. A.4).

Fig. A.5 shows the stress-strain hysteresis for two consecutive cycles for the specimen #2. This Figure bears the evidence that the SIMT start and finish conditions are different for different region of the same tensile test specimen. As found, during the first cycle, for the total strain of the specimen (measured by the displacement of the fixture), the SIMT starts gradually at 1.6% strain and a critical stress of 510 MPa. The SIMT is over approximately at 5.5% strain and a stress of 535 MPa.

For a region near the loading end (measured by the strain gage), the SIMT starts at 1.2% strain and a critical stress of 519 MPa. The SIMT is over approximately at 3.8% strain and a stress of 529 MPa.

There is an increase of stress from 517 MPa (corresponding to a strain of 1.35%) to 528 MPa (corresponding to a strain of 1.4%), before the SIMT is initiated for a region near the mid-portion of the specimen (measured by the strain gage).
Fig. A.5 Tensile stress-strain (nominal) curves for specimen #2.

Thus, the SIMT starts at 1.4% strain and a critical stress of 528 MPa. The SIMT is over approximately at 4.2% strain and a stress of 535 MPa.

Evidently, the critical stress must be exceeded to initiate the SIMT. Therefore, compared to the mid-portion, the SIMT starts earlier, that is, at lower values of the critical stress for the region near the loading end. As a result, when SIMT is initiated
for the mid-portion (for the critical stress of approximately 528Mpa), by that time, the SIMT is almost over for the region near the loading end (approximately, at 3.8% strain and a stress of 529 MPa).

As can be seen from Fig. A.5, the plateau stress is decreased for the second cycle, a characteristic of the superelastic SMA. However, the SIMT start and finish conditions for different parts of the specimen are similar to those of the first cycle. It is also noteworthy that the residual strains measured by the strain gages and by the displacement of the fixture are almost the same, though the maximum strains are different.

Fig. A.6 shows the variation of the local deformations (strain gage readings near the loading end and near the mid-position of the specimen) and total end displacement for specimen#2 for the first cycle. The behaviors of the two local regions are similar as explained for Fig. A.3. A few more points can be noted. During loading, after the initial slope, the strain for the mid-portion of the specimen remains almost horizontal up to 5.5% end deformation, meaning, the SIMT for the mid-portion finishes when the total end deformation measured by the displacement of the fixture is 5.5%. In comparison, for the region near the loading end, SIMT finishes at a lower values (about 4.5%) of the total end deformation. During unloading, the reverse SIMT starts at lower values of strains.

Similar to Fig. A.6, Fig. A.7 shows the variation of the local deformations and total end displacement for specimen#2 for the second cycle. Compared with the first cycle, the SIMT starts and finishes at lower values of strains. Due to superelasticity, however, the shape of the hysteresis remains similar to that of the first cycle.
A. Direct Method to Estimate SIMT Points by Tensile Test

Fig. A.6 Local deformation vs total deformation for specimen #2 (1st cycle).

Fig. A.7 Local deformation vs total deformation for specimen #2 (2nd cycle).
A.4 Conclusions

Simultaneous measurements of strains by local strains and the total strain show that the SIMT start and finish conditions are different for different region of the tensile test specimen. The critical stress required to initiate the SIMT for the mid-portion is higher than that required for the region near the loading end. The SIMT starts at the loading end and when it is about to finish, the SIMT starts for the mid-portion. Similar phenomena can be observed for the second cycle but for a lower plateau stresses. Thus the SIM start and finish points for the tensile test specimen can be identified easily by the used procedure.
References


Appendix B

Stability Analysis of Eccentrically Loaded Slender Columns Actuated by Shape Memory Alloy Wires

Summary

In practical applications inherent physical and geometrical imperfections make structural elements prone to instability at loads surprisingly lower than the theoretically predicted loads. Moreover, structural elements are often subjected to the combined action of thermal and mechanical loads. This paper, therefore, investigates on how to control and increase buckling loads of eccentrically loaded steel columns subjected to high environmental temperatures, using shape memory alloy (SMA) actuators. Since SMAs are thermal actuators, they are well suited for these types of applications. Experimental results show that buckling loads for those columns can be increased using pre-strained SMA actuators.
B. Stability Analysis of Eccentrically Loaded Slender Columns Actuated by SMA Wires

B.1 Introduction

Usually, structures become unstable because of external compressive loads. The unstable equilibrium state is often accompanied with an abrupt change in shape of the structure and leads to its failure often at stresses much lower than the yield strength of the material. Thin and light structures are increasing for economic reasons. Consequently, today’s structures are prone to failure due to mechanical instability. Moreover, in practical applications inherent physical and geometrical imperfections cause the structures to buckle at loads surprisingly lower than the theoretically predicted load. Thus, the practicing engineers have to consider the ‘imperfection sensitivity’ in designing structural elements like slender columns or thin shells. Furthermore, these imperfection-sensitive structures are sometimes subjected to high environmental temperatures (due to aerodynamic heating, for example) in addition to the mechanical loads. Hence, this paper studies how to control and increase the buckling loads of the steel columns, taking into account the inherent geometrical imperfections associated with them. By use of the axial load versus the axial deformation curves, the buckling loads are determined for these columns subjected to high environmental temperatures. The concept of large deformation theory has been utilized to anticipate the buckling load. The tensile recovery force of the shape memory alloy wires, which are excellent thermal actuators, are exploited in an attempt to increase the buckling loads.

A number of studies concerning the buckling control of structures have been reported in the literature [1-5]. Among them, a finite element method has been presented to demonstrate that enhancement of the critical buckling load is possible for composite plates bonded with piezoelectric actuators in reference [1]. Birman [2] analyzed the passive control of instability for functionally graded shape memory alloy sandwich panels. Baz et al. [3], demonstrated that buckling load for a composite beam, reinforced with SMA wires through sleeves, can be
increased by active control. Lagoudas et al. [4], analyzed the deformation of a flexible beam embedded with SMA wires. It was shown that the beam can buckle by means of too high actuating force of the SMA itself embedded in the structure. Recently, Choi et al. [5], analyzed the shape and buckling control of composite beams with embedded SMA wires. It has been verified that in some cases buckling load can be enhanced and the shape can be controlled for the composite beam by active control.

Although a composite structure embedded with SMA wire is quite compact and light during its application, manufacturing difficulties always arise while making such a structure. Within a composite structure embedded with SMA, in spite of all the precautions, growth of voids is inevitable, which alarmingly hampers the performance of the structures. Often nucleation of the voids initiates near the vicinity of the embedded wire [6]. Moreover, during manufacturing of these composites, prevention of possible phase transition from the martensite to austenite is another problem [6]. In case of one way shape memory effect associated with the embedded SMA, it would be almost impossible to give pre-strain and then actuate the embedded SMA for repeated applications. Observing the above facts, the present analysis is aimed at the buckling control of imperfect steel columns with externally actuated SMA wires. Moreover, for compactness, the SMA wires are directly fixed to the structure itself, unlike those of references [1-5]. Use of the pre-strained SMA wire as external actuator has some distinct advantages like:

1. Fast convection cooling is possible, which is important for high frequency response.
2. Actuators can be placed at any suitable offset distance and consequently large moment is available for actuation.
3. Heating of the SMA wires does not affect the host structure's material property.

Following advantages of the SMA's have made them popular as smart actuators and sensors: very high force to weight ratio, high adaptability, and compactness. Among all the
SMA materials, the most popular and widely used Nickel titanium alloys (NITINOL) have good actuating capability with almost 8% recoverable strain and the tensile recovery stress can be as large as 700 MPa. Thus powerful actuators can be designed within these extreme limits.

The shape memory effect (SME) largely depends on the solid-solid, diffusion-less phase transformation process known as martensitic transformation (MT). MT takes place if the SMA material is cooled under stress free condition from the high temperature of austenite phase to below the low temperature of martensite phase (Fig. B.1). Although MT is basically a macroscopic deformation process, actually no transformation strain is generated due to the so-called self-accommodating twinned martensite. Detwinned martensite is induced if the SMA material in the twinned martensite state is mechanically deformed (extended, bent or twisted) as shown in Fig. B.1.

If the mechanical load is withdrawn, only a small elastic strain is recovered leaving the
material with a large residual strain. At this state, whenever the material is heated above the austenite finish temperature, the material recalls from 'memory' of its original shape before the deformation and fully recovers it (Fig. B.1) [7-9]. This recovering of the original shape is known as the shape memory effect (SME). However, if some end constraints are used to prevent this free recovery to the original shape, the material generates large tensile recovery stress, which can be exploited as actuating force for active or passive control purpose [7,8]. The above mentioned tensile recovery force of the pre-strained SMA wires has been used to increase the buckling load of the steel columns in the present case.

**B.2 Experimental Details**

The configuration and material property of the test piece, and experimental conditions are given as follows: Material of the column: Steel, Width=30mm, Thickness=1mm, Test length=190mm, Slenderness ratio (l/k)=658, Eccentricity in loading=2mm, Composition of SMA = Ni 54-58%, Ti balance, Dia of SMA wires= 0.70mm (#22), Off-center distance of the SMA wire’s axis from the mid-plane of the test section=2.35mm, residual strain imparted to the SMA wires=6.4%, No of wires affixed with the specimen=2, Room temp=28 degree C, Austenite finish temperature for SMA=60°C, Loading rate=0.01 mm/min, Temperatures during mechanical loading = 70°C-76°C.

Since it is practically impossible to apply perfectly axial load on the column, a known load eccentricity is considered to model the inherent geometrical imperfection (Figs. B.2-B.4). The buckling control is planned in the following way: The SMA wire with residual strain is attached directly to the column itself at a reasonable off-center distance opposite to the load eccentricity measured from the mid-plane of the test section (Fig. B.4). The column with SMA wires is now subjected to heating due to environmental temperature rise. The intuition is that with increasing temperature beyond its phase transition temperature the pre-strained
SMA wires will try to contract in length causing an opposite moment created by the mechanical load. Thus followings are the experimental steps:

1. SMA wire is imparted a residual strain by tensile loading-unloading cycle using the Instron Machine.

2. SMA wires with the residual strain are now attached to the specimen parallel to the load direction with slight pretension as shown in Fig. B.3.
3. The Instron Machine, used in the experiment, in addition to its loading and unloading capability, has also the facility of simultaneously heating or cooling the specimen inside a closed chamber. The specimen is inserted into the holes of the two loading fixtures (Fig. B.3), inside the closed chamber of the Instron Machine. Next, heating is started in the closed chamber. Since a sufficient gap is provided between the upper end of the column and loading surface of the fixture, thermal expansion does not cause any compressive force on the column. Heating is stopped when chamber temperature becomes higher than the SMA’s phase transformation temperature ($A_t$). Since the SMA wires are rigidly fixed to the columns as shown in Figs. B.3- B.4, the shape recovery can not take place freely. As a result, the pre-strained SMA wires generate tensile recovery forces causing a higher pretension on the specimen before the mechanical loading starts.

4. Mechanical loading is raised slowly. Fig. B.2 schematically shows the forces and the moments acting on the column at this instant. The corresponding compressive load-axial deformation curve is observed.

5. The loading is stopped when it reaches the buckling load, that is the maximum point as observed from the load-deformation curves. The specimen is unloaded for the next test.

It should be mentioned here that two steel column specimens with the same parameters have been used in the experiment. Prior to the above steps 1-5, buckling loads are determined for both of them without attaching any SMA wire (and therefore no pretension) using steps 3-5. It has been observed that in all the cases elastic buckling occurs and almost the same characteristic is repeated. For the same test piece under the same testing conditions, the variation in the buckling loads is found to be within 2%.

**B.3 Results and Discussions**

The controlled column (Fig. B.2) has its own physical stiffness. If resisting (or, actuating)
moments are created, (as done in the experimental steps,1-3), this may act to increase the stiffness of the column and enhance its buckling load, or to change the shape of the deformed column.

Practically, the following buckling modes are possible for the present case.

1. The actuating moment \( M_{SMA} \) is not large. After the initial equilibrium, the applied moment \( M \) dominates over the actuating moment and the column will buckle in a symmetric mode overcoming the resistance of the actuating moment. This will result in enhancement of the buckling load.

2. The actuating moment is too large to cause a prominent initial curvature. As a result, when the axial force \( P \) is applied, buckling occurs in the opposite direction of the applied moment. This may not increase the buckling load.

For the present case, it is found that the controlled column can overcome the resistance of the actuating moment and thus sustains higher buckling loads compared to the uncontrolled column, as verified from Figs. B.5(a), B.5(b) and Table B.1.

It is obvious from Table B.1 and also from Figs. B.5(a) and B.5(b), that the buckling load is increased when the columns are stiffened with the SMA wires. The tensile recovery force generated due to the SME plays the key role in this case. In the present case the steel columns are quite stiff and the buckling load is as high as 380 N. Even though about 4% increase in the buckling load is possible with arbitrarily chosen SMA wires and a residual strain of 6.4%.

It is anticipated from the present experience that for columns with higher slenderness ratios better performance is possible. Various types of SMA wires with different material properties are commercially available. Thus, any SMA wire with higher Young's modulus in its austenite phase than that used in this experiment is likely to show better results.

Since the steel column has an eccentric loading it starts deflecting with the axial load. The buckling loads for the specimens with and without SMA wires are defined as the maximum
B. Stability Analysis of Eccentrically Loaded Slender Columns Actuated by SMA Wires

Table B.1 Experimental buckling loads for the columns with and without SMA wires.

<table>
<thead>
<tr>
<th>Specimen ID No</th>
<th>Without wires</th>
<th>With wires</th>
<th>% Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>36.6</td>
<td>38</td>
<td>3.8</td>
</tr>
<tr>
<td>#2</td>
<td>38.7</td>
<td>40.2</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table B.2 Experimental and numerical buckling loads for the columns without SMA wires.

<table>
<thead>
<tr>
<th>Specimen ID No</th>
<th>Simulation</th>
<th>Experiment</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>36.6</td>
<td>38</td>
<td>31</td>
</tr>
<tr>
<td>#2</td>
<td>48.1</td>
<td>38.7</td>
<td>24</td>
</tr>
</tbody>
</table>

loads in the load versus deformation curves shown in Figs. B.5(a) and B.5(b). The experimental results for the same column without SMA wires is then compared with the numerical simulation results obtained by using the commercial software ANSYS as shown in Fig. B.6 and Table B.2. For accuracy, the large deformation theory has been used to predict the buckling load. The critical load, which causes excessive deformation without appreciable change in the load, is anticipated from the load deformation curve (Fig. B.6). Often numerical solution fails to converge at this unstable state. Although it is a well-known fact that the theoretical buckling load is much higher than the actual one, the difference between two loads is within 24% in this case as shown in Table B.2.

Although the load eccentricity has been taken into account during the numerical simulation, the discrepancy in the results can be attributed mainly due to the shape imperfection of the columns used in the experiments.
B. Stability Analysis of Eccentrically Loaded Slender Columns Actuated by SMA Wires

Fig. B.5(a) Load-deformation curves from experimental data for the first specimen

Fig. B.5(b) Load-deformation curves from experimental data for the second specimen.
It can be seen from Table B.2 and also from Figs. B.5(a) and B.5(b) that the first specimen becomes unstable at a lower load than the second one, although both of them have the same parameters. This is due to the fact that slender columns can buckle at unpredictable loads due to physical and geometrical imperfections associated with them.

It has been observed from the experimental results that in the absence of SMA wires, the uncontrolled column starts deflecting until buckling occurs in the direction of the applied moment. For the controlled column the same characteristic but at a higher buckling load, has been observed when the pretension in the wires is not too much. As mentioned, the tensile recovery force of SMA is exploited as the pretension in the experimental procedures for buckling control. If the pretension is too large to cause a prominent initial curvature, buckling will occur in the opposite direction of the applied moment. This may cause an adverse effect with a lower buckling load.
B.4 Conclusions

The buckling loads of slender steel columns with eccentric loading subject to high environmental temperatures have been determined by experiments and numerical simulation. It is found that the experimental buckling loads are lower than the numerical results, obviously, due to various kinds of imperfections. Moreover, experimental results verify that the buckling loads for the above mentioned columns can be increased by using SMA wires as thermal actuators.
References


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