AN IMPROVED DESIGN APPROACH FOR MAT FOUNDATION WITH VARIABLE THICKNESS

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A THESIS BY
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Almighty Allah
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DECLARATION

Declared that, except where specified references are made to other investigators, the work embodied in this thesis is the result of investigation carried out by the author under the supervision of Professor Sohrabuddin Ahmad and Professor Abdul Muqtadir of Civil Engineering Department, BUET.

Neither the thesis nor any part thereof has been submitted or is being concurrently submitted in candidature for any degree at any other institution.

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ABSTRACT

The current practice in the design of mat foundation is to provide it with constant thickness. Analysis of mat foundation and study of the results reveal that mat foundation is extensively over-designed in the current approach. Mat foundation of constant thickness is designed by ACI method, Conventional method or computer analysis by Finite Difference method, Finite Grid method or Finite Element method. This over-design is due to uncertainty in the analysis methods and lack of understanding of mat behaviour. Use of thick shell finite elements of Ahmad lead to realistic analysis and more rational design procedure for mat foundation with variable thickness, as proposed by Morshed and Sutradhar. To improve upon this procedure, an extensive investigation has been carried out here on the behaviour of mat foundation using finite element program named ANSYS. The most significant parameters for mat foundation with variable thickness are mat thickness and modulus of subgrade reaction. A reshaping scheme proposed earlier has been improved.

The reshaping scheme of mat foundation with variable thickness is compared with flat slab having drop panels. Column loads applied on the mat foundation generates a distributed reactive soil pressure. A mat foundation can be considered as an inverted flat slab with non-uniform pressure on it. For high-rise buildings up to fifteen story, design of mat foundation is normally governed by vertical load that is dead load plus reduced live load. Usually structures have regular column arrangement. An approximate method of analysis has been proposed for mat foundation having column arrangements in a regular grid pattern. Most residential and office buildings are of this type. For these types of structures, one can design the mat foundation with variable thickness economically and speedily using hand calculation. A simplified arrangement of reinforcement distribution has also been proposed.

Finally, a complete analysis of mat foundation with variable thickness for a three bay ten storied building has been done by the proposed approximate method and Finite Element method. Results of both these methods have been compared. An economic evaluation of the mat foundation with variable thickness has been compared with mat of constant thickness. An economy of 25% to 40% can be achieved by the proposed method for the mat foundation.
## LIST OF SYMBOLS

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Plan area of mat</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Steel area</td>
</tr>
<tr>
<td>B</td>
<td>Width of the strip</td>
</tr>
<tr>
<td>$B_c$</td>
<td>C/C span width of panel</td>
</tr>
<tr>
<td>$B_{ym}$</td>
<td>y dimension of mat</td>
</tr>
<tr>
<td>$d_e$</td>
<td>Effective depth of mat</td>
</tr>
<tr>
<td>$d_s$</td>
<td>Slope width</td>
</tr>
<tr>
<td>$d_g$</td>
<td>Width of greater thickness</td>
</tr>
<tr>
<td>DL</td>
<td>Dead load</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of elasticity of materials used in structure</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of soil</td>
</tr>
<tr>
<td>$e_x$</td>
<td>$x$ eccentricity of R w.r.t. the center of the mat</td>
</tr>
<tr>
<td>$e_y$</td>
<td>$y$ eccentricity of R w.r.t. the center of the mat</td>
</tr>
<tr>
<td>$f_c'$</td>
<td>Ultimate concrete strength</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Yield strength of steel</td>
</tr>
<tr>
<td>$I_x$, $I_y$</td>
<td>Moment of inertia of mat area w.r.t. $x$ and $y$ axes</td>
</tr>
<tr>
<td>$I_F$</td>
<td>Moment of inertia of footing</td>
</tr>
<tr>
<td>$I_B$</td>
<td>Moment of inertia of structure per unit length</td>
</tr>
<tr>
<td>k</td>
<td>Modulus of subgrade reaction</td>
</tr>
<tr>
<td>LL</td>
<td>Live load</td>
</tr>
<tr>
<td>L</td>
<td>Length of the strip</td>
</tr>
<tr>
<td>$L_c$</td>
<td>C/C span length of panel</td>
</tr>
<tr>
<td>$L_{cl}$</td>
<td>Clear span length from face to face of columns</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Length of the thicker zone</td>
</tr>
<tr>
<td>$M_t$</td>
<td>Total panel moment</td>
</tr>
<tr>
<td>$M_r$, $M_t$</td>
<td>Radial and tangential moments for a unit width of mat</td>
</tr>
<tr>
<td>M</td>
<td>Acting bending moment</td>
</tr>
<tr>
<td>$M_c$, $V_c$, $w_c$</td>
<td>End correction to moment, shear and displacement respectively</td>
</tr>
<tr>
<td>$M_e$, $V_e$</td>
<td>End moment and shear respectively</td>
</tr>
<tr>
<td>$M_x$, $M_y$</td>
<td>Bending moment along $x$ and $y$ axes respectively</td>
</tr>
</tbody>
</table>
\( n \) Total number of columns

\( P's \) Column loads

\( P_i^s \) Strip column loads

\( Q \) Shear force per unit width of mat

\( Q_a \) Average load on the strip

\( q \) Soil pressure at any point under the mat

\( q_a \) Average soil pressure under the strip

\( q_i \) Soil pressure at the corners of the strip

\( q_1, q_2 \) Soil pressure intensity at the left and the right end of the strip respectively

\( R \) Resultant of all column loads

\( r's \) Distance of the strip column loads from the left end of the strip

\( r \) Radial distance of the point under investigation from \( P \)

\( t \) Thickness of mat in inch

\( v_{pu} \) Ultimate punching shear strength

\( v_{fu} \) Ultimate flexural shear strength

\( w \) Average soil pressure obtained by dividing the total axial load by the total bearing area

\( WL \) Wind load

\( X's \) x coordinates of the columns w.r.t. the corner of the mat

\( Y's \) y coordinates of the columns w.r.t. the corner of the mat

\( x, y \) Coordinates of locations where soil pressures are desired w.r.t. the center of the mat

\( v \) Poisson's ratio of concrete

\( \Psi, R's, Z's \) Functions first introduced by Schleicher (1926)

\( \phi \) Angle of the radial line passing through the point under consideration w.r.t. the positive x axis in a clockwise direction

\( \rho \) Steel ratio

\( \rho_{max} \) Maximum steel ratio

\( \rho_{min} \) Minimum steel ratio

\( \Delta t \) Change in thickness
1.1 GENERAL

Mat foundations become economical when the loads are so large that footings would occupy more than fifty percent of the projected area of the building. Mat foundations are commonly used for structures wherever the column loads or soil conditions result in conventional footings or piles occupying most of the site. For many multi-storied buildings a single mat foundation is more economical than constructing a multitude of isolated foundation elements. In a foundation on an elastic subgrade, the soil pressure and the deflection vary from point to point. Mat foundations due to their continuous nature provide resistance to independent differential column movements, thus enhancing the structure's performance. Mats can bridge across weak pockets in the non-uniform substratums, thus equalizing foundation movements. Mat foundations are predominantly used in regions where the underlying stratum consists of clayey materials with low bearing capacity. This is also used as a load distributing element placed on piles or directly on high bearing capacity soil or rock.

Mat foundation has gained widespread popularity among the engineering community of Bangladesh. Bangladesh is running its race for prosperity and urbanization has become an integral part of this endeavour. Nowadays, high rise buildings are becoming common features of major cities in Bangladesh. Many have been constructed, many are under construction and many more are on the planning table. Engineers often opt for mat as their foundation.

Construction of high rise building is not an easy task in Bangladesh. As engineers come to the soil eventually, seeking support for their superstructures, they face frequently the unfavorable reality. The soft soil of Bangladesh has relatively low bearing capacity. Heavy column loads may demand for large footing areas summing
up to more than half of the total foundation area available. In south-western region of
the country, soil has very poor bearing capacity upto 100 ft depth where cast-in-situ
pile foundation is not technically and economically sound, even for a six storied
building. The most formidable hurdle, foundation engineers often have to encounter, is
excessive settlement, specially differential settlement. Although pile foundation offers
solution to these problems, sharp jump in expense becomes a serious point to ponder
in the poverty lashed economy of Bangladesh, not to mention the disturbance to city
life and the possibility of damage to underground structures associated with pile
foundation construction.

The various advantages of mat foundations are:

(a) use of the raft as a basement floor having considerable commercial
values in urban areas,
(b) use of the flexural stiffness to reduce differential settlements due to
swelling and shrinking of active soils,
(c) use of the flexural stiffness to reduce contact pressures in regions of
higher soil compressibility,
(d) use of its own rigidity and that of the superstructure to activate
bridging effect,
(e) use of the raft in combination with piles to reduce total settlement,
(f) use of flotation effect due to displaced volume of soil.

Mat foundations are also popular for deep basements both to spread the column loads
to a more uniform pressure distribution and to provide the floor slab for the basement.
A particular advantage for basements at or below the ground water table is to provide a
water barrier. For all these advantages, mat foundations are becoming increasingly
popular.

There are several categories of mat foundation problems which by their nature require
sophisticated computer analysis, since long hand methods would not be directly
applicable. These are:

i) mats of unusual or complex shapes,
ii) mats where it is deemed necessary that a varying subgrade modulus must be used,

iii) mats with non-uniform thickness,

iv) mats where large moments or axial forces are transmitted to the mat from laterally loaded shear walls, trusses or frames, and

v) mats where rigidity of superstructure significantly affects mat behavior and stress distribution.

There are several types of mat foundation i.e.,

i) flat plate,

ii) plate thickened under column,

iii) waffle slab,

iv) plate with pedestal,

v) plates with stiffeners as basement walls,

vi) thin plate with grid floor beams.

Fig. 1.1 illustrates several mat foundation configurations as might be used for buildings.

The most common mat foundation configuration is the flat plate type. This type of foundation tends to be heavily overdesigned due to additional cost of thick slab and uncertainty in analysis.

1.2 STATEMENT OF THE PROBLEM

In Bangladesh, Mat foundation is the solution not only to high rise structures but also to low rise structures, particularly where soil has low bearing capacity like south-western part of the country. At the present state, manual design of mat foundation is quite laborious and time consuming. Finite Element Solution of it is also laborious and time consuming. It is common tendency of the designer to simplify structural design and over design the mat due to uncertainty in analysis of
Fig. 1.1. Common types of mat foundation: (a) flat plate; (b) plate thickened under columns; (c) waffle slab; (d) plate with pedestals and (e) basement wall as part of mat.
structures. It is desirable to analyse and design mat foundation speedily, rationally and economically.

1.3 BACKGROUND OF THE STUDY

A number of methods is available for the analysis of mat. None is accurate and convenient enough. There are some approximate methods which are considerably crude and are in use for a long time. Recently, there are some computer based methods available. However, these computer based methods idealize mat unrealistically. The modeling of soil also has got its own limitations.

In the most common and simplified method, known as Conventional method, column loads are distributed under the mat; then the mat is divided into strips midway between the column lines and the force system acting on each strip is adjusted to establish equilibrium. Finally, each strip is designed as a combined footing. This is repeated for the other direction. This method is recommended by ACI when adjacent spans and column loads do not vary by more than 20%. However, in this method mat is considered to be fully rigid in determining the soil pressure and also the divided strips lose their plate characteristics i.e. two way bending.

One of the most popular methods of mat analysis is the ACI Approximate Flexible method (Committee 336) which is essentially based on the analysis of Schleicher (1926). In this method, as the name implies, the mat is considered to be a flexible plate acted upon by concentrated column loads and resting on a Winkler medium. Effect of each load is calculated as if the plate were infinite. Forces for each individual column are summed up. To have the edges free from forces, the mat is divided into strips and the forces obtained by this approach on the edges are applied in opposite direction on the respective edges considering each strip to be supported on semi-infinite elastic foundation [Hetenyi (1946)]. Finally, this new set of forces are added to the previous ones. Shukla (1984) presented design aids for using this method. Later, this method was further modified by Mician (1985). However, this
complicated and lengthy approximate method fails to take into account the boundary conditions and moments from columns.

Baker (1948) proposed a method of mat foundation analysis where the mat is divided into column strips resting on Winkler medium and each strip is analyzed separately. A particular soil pressure distribution of unknown magnitude is assumed and applied on both the column strip and the supporting soil. The magnitude of soil pressure is determined by taking the maximum differential displacements of the column strip and the supporting soil as the matching criteria. However, the detailed deflected shapes of the column strip and the supporting soil do not match in this method. Also, this method can not take into account column base moments and fails to represent the plate characteristics of mat.

Bowles (1974) has proposed Finite Grid method in which the mat is reduced into a grid system consisting of beam-column elements resting on Winkler medium. Solution to the problem is obtained following the stiffness matrix approach. According to Bowles, there is a little computational improvement if the soil is modeled using its modulus of elasticity and Poisson’s ratio instead of its modulus of subgrade reaction. However, the plate characteristics of mat is lost in this method. Also, the program requires extensive data in generating the grid geometry, sectional and material properties.

There is also Finite Difference method [Bowles (1974), Deryck and Severn (1960, 1961)] where the mat is modeled as large flat plate on elastic medium. The fourth order differential equation as given by plate theory is then numerically solved using finite difference technique. Handling column moments is beyond the scope of this method. Also, interpretation of the boundary points is cumbersome in this method.

At BUET, Hossain (1993) conducted a comparative study of the available analysis methods of mat foundation and felt the need for a rigorous finite element study of the problem. Later, Molla (1995) tested the performance of Mindlin plate element in finite element analysis of mat foundation and also compared the results with those
from other available methods. The study of Molla opened a wide horizon for a rigorous finite element study of the problem with more appropriate and versatile element in search of a more economic design of mat. Sutradhar (1995) compared the analysis of mat foundation by Finite Element method using Ahmad's thick shell element with those by Finite Grid method and Finite Difference method.

Lefas, Georgiannou and Sheppard (1996) presented an iterative procedure for analysis and design of mat foundation based on a soil-structure interaction procedure developed using well-marketed programs VDISP by OASYS, STAAD-III/ISDS by Research Engineers and SuperCalc by Computer Associates. VDISP is a geotechnical program for calculating vertical displacements at specified points due to any pattern of loading on an elastic half-space representing the ground. From VDISP the spring constant $K_s$ is entered into the STAAD-III grillage and the raft is analysed.

Liou and Lai (1996) presented a simplified structural analysis model for mat foundation with grid floor beams as stiffeners. The model becomes grid floor beams on an elastic foundation with loadings applied at the intersections of floor beams. The yield line theory (Johansen 1962) for bottom slabs is employed to lump the subgrade reaction springs to the location under grid floor beams.

Morshed (1997) and Sutradhar (1999) conducted comparative analysis between finite element method using Ahmad's thick shell element with ACI method and Conventional method. Morshed suggested a mat foundation with variable thickness. His findings paved the way for developing a new design rationale for mat foundation. Sutradhar suggested a more definite guideline for selecting a mat of variable thickness and analysis with finite element methods.
1.4 OBJECTIVES OF THE PRESENT STUDY

The present research has been aimed at the following objectives:

(i) To propose a guideline for analysis and design of rectangular mat foundation by finite element methods.

(ii) To conduct significant parametric study of mat foundations with both constant and variable thickness.

(iii) To demonstrate the necessity of designing mat foundation with variable thickness.

(iv) To propose a guideline for approximate analysis and design of mat foundation with variable thickness.

1.5 METHODOLOGY

For analyzing mat foundation by finite element technique a well reputed finite element program ANSYS has been used. Soil has been modeled with spring elements and Eight noded shell element has been used to represent mat foundation. Application of these elements has been checked with closed form solution results obtained from Hetenyi (1979).

In an attempt to make foundation model a thorough survey of the related literatures has been made. Limitations and assumptions of such models has been thoroughly studied to choose the closest reasonable model. Comparative assessments has been carried out to model soil as Winkler foundation model. Efforts have been made to substantiate the findings from the sensitivity study of the parameters which are critical in mat foundation design. Study of Morshed (1997) and Sutradhar (1999) has been taken into account and verified. Efforts has been made to step ahead the proposed design procedure.

A simple analysis and design method for mat foundation with variable thickness has been proposed and critical aspects of the guidelines of this method has been verified. A number of cases has been studied.
Finally, a complete analysis and design of a mat foundation of ten storied building has been presented. At the end, a suggestion for future study has been recommended.

1.6 LAYOUT OF THE THESIS

The main body of the thesis consists of six main chapters. A thorough literature survey has been carried out in Chapter 2. As this research is expected to step ahead the study of Morshed and Sutradhar, main findings of their research is included in this chapter. Chapter 3 describes the finite element model of mat foundation used in the study. An extensive parametric study on selected parameters are carried out in Chapter 4. In Chapter 5, a number of mat foundation of different shapes on different modulus of subgrade reaction is carried out and their results have been summarized. In Chapter 6, an improved approach to the analysis and design of mat foundation is proposed. Validity of this approach is compared with finite element solution. Also, a comparative economic evaluation has been shown. Chapter 7 contains the conclusions drawn from the research.

In addition, there are four appendices in this thesis. Appendix A includes description of eight nodded Structural shell Element SHELL93 of ANSYS Library Function, Appendix B includes description of spring element COMBIN14 of ANSYS library function. Appendix C contains the MACRO used for analysis of mat with ANSYS for prospective researchers. Appendix D contains description of geometry and loading for mat and variation of deflection, bending moment and shearing stress.
CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

The role of typical raft foundation is to transmit the load coming from the super structure to the soil beneath without causing distress to any of the components of the super structure or foundation. Analysis of mat foundation requires study of two aspects, the soil-structure interaction and the various methods of analyzing the mat. The mechanical response of naturally occurring soils can be influenced by a variety of factors. These include shape, size and mechanical properties of the individual soil particles, the configuration of the soil structure, the intergranular stress history and the presence of soil moisture, the degree of saturation and the soil permeability. These factors generally contribute to stress-strain phenomena which display markedly non-linear, irreversible and time dependent characteristics. Thus an attempt to solve a soil-foundation interaction problem taking into account of all such material aspects is clearly an onerous task. In order to obtain meaningful and reliable information for practical problems it becomes necessary to idealize the behavior of soil by depicting soil models.

2.2 BACKGROUND OF THE PROBLEM

There are several methods to analyse mat foundation. These are a) Conventional method, b) ACI approximate flexible method, c) Baker’s method, d) Finite difference method, and e) Finite grid method.

Conventional method is the crudest one. Here, mat is treated as a rigid slab. Slab is assumed to be rigid, resulting in uniform or linearly varying contact pressure distribution, depending on whether the raft supports symmetric and eccentric loads.
Actual contact pressure distribution is the result of the foundation-soil interaction, which can be determined by an interaction analysis involving the elastic properties of both the foundation and soil. Soil structure interaction is an important aspect in the process of predicting overall structural response.

All the other methods are based on Winkler spring model. ACI method is based on Schleicher's solution of infinite flexible slab on continuous spring support and Baker's method is designed to simulate that solution. Finite grid and Finite difference methods are numerical analysis methods and require extensive computer support though they do not offer any substantial improvement in results over Schleicher's solution. The first two methods are generally used.

2.3 FOUNDATION MODEL

2.3.1 General

Generally the analysis of bending of beams on an elastic foundation is developed on the assumption that the reaction forces of the foundation are proportional at every point to the deflection of the beam at that point. This assumption was first introduced by Winkler (1867) and formed the basis of Zimmermann's classical work (1930) on the analysis of railroad track. It has been shown by Foppl's classical experiment (1922) and Hetenyi's analytical work (1946) that Winkler's assumption in spite of its simplicity leads to satisfactory results in stress analysis of beams on an elastic foundation.

On the other hand, by means of the hypothesis of isotropic, linearly elastic semi-infinite space, the physical properties of a natural foundation can be correctly described. To bridge the gap between these two extremes, interactions between Winkler's springs were considered by several authors. Hetenyi (1950) treated the problems of beams or plates on an elastic foundation by assuming a continuous beam or plate embedded in the material of foundation, which is itself without any continuity. Pasternak (1954) assumed that the shear
interactions exist between the springs. Vlasov and Leont'ev (1966) also considered the shear interactions in the foundation and formulated their problems by using a variational method. They solved a large number of problems involving beams, plates and shells on two parameter model elastic foundation.

2.3.2 Winkler Model

The idealised model of soil media proposed by Winkler (1867) assumes that deflection \( w \), of the soil medium at any point on the surface is directly proportional to the stress, \( q \) applied at that point and independent of stresses applied at other locations, i.e.

\[
q(x,y) = kw(x,y)
\]

where \( k \) is termed the modulus of subgrade reaction with units of stress per unit length. There are indications that this assumption is already to be found in the works of Euler, Fuss, Bubnov and Zimmermann (Korenov, 1960; Hetenyi, 1966). The equation is usually the response function or the kernel function of Winkler model. Physically Winkler's idealisation of the soil medium consists of a system of mutually independent spring elements with spring constant \( k \) (Fig. 2.1).

Fig. 2.1 Surface displacements of Winkler model due to (a) a non-uniform load, (b) a concentrated load, (c) a rigid load, (d) a uniform flexible load
The procedure for the conventional analysis consists of the following steps:

(i) The resultant of all loads acting on the mat and its location is determined from:

\[ R = \sum_{i=1}^{n} P_i \]  

(2.2)

\[ e_x = \frac{\sum_{i=1}^{n} P_i X_i}{\sum_{i=1}^{n} P_i} - L_r. \]  

(2.3)

\[ e_y = \frac{\sum_{i=1}^{n} P_i Y_i}{\sum_{i=1}^{n} P_i} - B_r. \]  

(2.4)

where

- \( R \) = resultant of all column loads,
- \( X's \) = x coordinates of the columns w.r.t. the corner of the mat,
Y's = y coordinates of the columns w.r.t. the corner of the mat,
P's = column loads,
n = total number of columns,
$L_{x_m}$ = x dimension of mat,
$L_{y_m}$ = y dimension of mat,
$e_x$ = x eccentricity of R w.r.t. the center of the mat,
$e_y$ = y eccentricity of R w.r.t. the center of the mat.

(ii) The slab is divided into strips in x and y directions, one direction at a time, halfway in-between the column lines. Each strip is assumed to act as an independent beam supported by soil pressure and acted upon by column loads. Steps (iii) to (vi) will be applicable to each of these strips.

(iii) The soil pressure distribution is calculated using the equation:

$$q = R \left( \frac{1}{A} + \frac{e_x}{I_y} + \frac{e_y}{I_x} \right)$$

where

$q$ = soil pressure at any point under the mat,
$A$ = plan area of mat,
$I_x, I_y$ = moment of inertia of $A$ w.r.t. x and y axes,
$x, y$ = co-ordinates of locations where soil pressures are desired w.r.t. the center of the mat.

Soil pressure is calculated at the four corners of the strip and their average is obtained from

$$q_a = \frac{1}{4} \sum_{i=1}^{4} q_i$$

where

$q_a$ = average soil pressure under the strip,
$q_i$ = soil pressure at the corners of the strip.
This time a check is made that the maximum soil pressure is less than the allowable bearing capacity.

(iv) \( q_a \) and the strip column loads are modified to enforce static force equilibrium. First, the average load on the strip is calculated by

\[
Q_a = \frac{1}{2} \left( |BLq_a| + \sum_{i=1}^{n_s} |P_i^s| \right) \tag{2.7}
\]

where

- \( Q_a \) = average load on the strip,
- \( B \) = width of the strip,
- \( L \) = length of the strip,
- \( n_s \) = no of columns in the strip,
- \( P_i^s \) = strip column loads.

The modified average soil pressure is calculated by

\[
\bar{q}_a = \frac{Q_a}{LB} \tag{2.8}
\]

The modified column loads \( F_i \) are determined from

\[
F_i = \frac{Q_a}{\sum_{i=1}^{n_s} F_i^r} P_i^r \tag{2.9}
\]

(v) To ensure moment equilibrium \( \bar{q}_a \) is then shaped as a trapezoid so that the resultants of soil pressure and column loads meet at the same point. The end magnitudes of the soil pressure trapezoid is calculated as follows:

\[
e_r = \frac{\sum_{i=1}^{n_s} F_i r_i}{\sum_{i=1}^{n_s} F_i} \tag{2.10}
\]
\[ q_1 = 2\bar{q}a \left( 2 - \frac{3e_r}{L} \right) \]  
\[ q_2 = 2\bar{q}a \left( \frac{3e_r}{L} - 1 \right) \]  

where

\( e_r \) = eccentricity of the resultant of the strip column loads w.r.t. the left end of the strip,

\( r's \) = distance of the strip column loads from the left end of the strip,

\( q_1 \) and \( q_2 \) = soil pressure intensity at the left and the right end of the strip respectively.

(vi) The shear force and bending moment are calculated at the various points of the strip for the modified column loads and soil pressure.

Arora (1992) proposed that as the analysis is approximate, the actual reinforcement provided should be twice the computed values.

**Applicability And Limitations**

According to the ACI Committee 336 (1966), the design of mats may be accomplished using the conventional method if:

a) The average of the two adjacent spans in a continuous strip is less than \( 1.75/\lambda \) (equation 2.33) and adjacent column loads and column spacing do not vary by more than 20 percent of the greater value.

b) The relative stiffness factor \( K_r \) is found to be greater than 0.5, where

\[ K_r = \frac{EI_y}{E_yB^3} \]  

(2.13)
where

\[ E = \text{modulus of elasticity of materials used in structure,} \]
\[ I_B = \text{moment of inertia of structure per unit length,} \]
\[ E_s = \text{Modulus of elasticity of soil and} \]
\[ B = \text{Width of footing.} \]

An approximate value of \( E I_B \) per unit length of building can be found by summing flexural rigidity of footing \( (E I_F) \), the flexural rigidity of each framed member \( (E I_b) \) and the flexural rigidity of any shear walls \( (E a h^3/12) \) where \( a \) and \( h \) are the thickness and height of the walls respectively; \( E I_B \) is given by:

\[
E I_B = E I_F + \sum E I_b + \sum \frac{E a h^3}{12} \tag{2.14}
\]

where

\[ I_F = \text{Moment of inertia of footing.} \]

2.4.2 ACI Approximate Flexible Method

ACI Committee 336 (1966) suggested this method for the general case of a flexible mat supporting columns at random locations with varying intensities of load. This procedure is essentially based on Schleicher's solution of circular plate on Winkler medium. Shukla (1984) provided charts for easy calculation of moments and shear following this method.

The effect of a concentrated load on a flexible infinite slab resting on continuous spring support has been found to damp out quickly away from the load. It is, therefore, possible to consider mat as a plate of infinite dimension and determine the effect of a column load in a specific region, called the zone of influence, surrounding the load. This zone of influence is generally not large and beyond this zone moments and shears in mat is insignificant. The total effect of all the column loads at any point can thus be determined by superimposing the effect of all the column loads within whose zones of influence the point lies. If moments and shears are found along the edges then these are
later applied in opposite direction at the same location and with a semi-infinite beam
analysis their effects inside the mat is calculated and superimposed on the previous
solution to have the final moments and shears. The later part of the analysis is called
end correction and is a way of getting rid of the errors acquired by treating the mat
infinite. Mat deflections, though unrealistic, can be calculated in the same manner.

This method can not take care of column base moments, does not model boundary
conditions realistically and gives unsatisfactory result near the edges.

A problem can be systematically solved through the following steps.

(i) The flexural rigidity of the mat, D is calculated.

\[
D = \frac{E t^3}{12(1 - \nu^2)}
\]  

(2.15)

where

\[ 
E = \text{modulus of elasticity,}
\]

\[ 
= 57000 \sqrt{f_c} \text{ psi for concrete (ACI 318-83, Section 8.5.1)},
\]

\[ 
f_c = \text{ultimate concrete strength in psi},
\]

\[ 
t = \text{thickness of mat in inch},
\]

\[ 
\nu = \text{Poisson's ratio of concrete (0.15 to 0.25}).
\]

(ii) The radius of effective stiffness L is then calculated as follows:

\[
L = 4\sqrt{\frac{D}{k}}
\]

(2.16)

where

\[ 
k = \text{modulus of subgrade reaction}.
\]
Radius of influence for any column load is $4L$ and calculation of deflections, bending moments, and shear forces due to any column load will be limited within this zone around that column.

(iii) Since the effect of each load is transmitted through the mat in a radial direction, polar co-ordinate system was used in the original solution. The radial and tangential moments, the shear and deflection at a point are calculated using the following formulae:

\[ M_r = -\frac{P}{4} \left[ Z_4 \left( \frac{r}{L} \right) - \frac{Z_3 \left( \frac{r}{L} \right)}{r \frac{L}{r}} \right] \]  
\[ M_t = -\frac{P}{4} \left[ \nu Z_4 \left( \frac{r}{L} \right) + \frac{Z_4 \left( \frac{r}{L} \right)}{r \frac{L}{r}} \right] \]  
\[ Q = -\frac{P}{4L} Z_4'' \left( \frac{r}{L} \right) \]  
\[ w = \frac{PL^2}{4D} Z_3 \left( \frac{r}{L} \right) \]  
\[ Z_3 \left( \frac{r}{L} \right) = \frac{1}{2} Z_1 \left( \frac{r}{L} \right) - \frac{7}{11} \left[ R_1 \left( \frac{r}{L} \right) + Z_2 \left( \frac{r}{L} \right) \log_e \left\{ 1.781 \frac{r}{L} \right\} \right] \]  
\[ Z_4 \left( \frac{r}{L} \right) = \frac{1}{2} Z_2 \left( \frac{r}{L} \right) + \frac{7}{11} \left[ R_2 \left( \frac{r}{L} \right) + Z_1 \left( \frac{r}{L} \right) \log_e \left\{ 1.781 \frac{r}{L} \right\} \right] \]  
\[ Z_1 \left( \frac{r}{L} \right) = \sum_{i=1}^{\infty} \frac{(-1)^{i+1} \left( \frac{r}{L} \right)^{4(i-1)}}{2^{4(i-1)] \left\{ 2(i-1)! \right\}^2} \]  
\[ Z_2 \left( \frac{r}{L} \right) = \sum_{i=1}^{\infty} \frac{(-1)^{i} \left( \frac{r}{L} \right)^{4(i+1)+2}}{2^{4(i-1)+2} \left\{ 2(i-1) + 1 \right\}!]^2} \]  
\[ R_1 \left( \frac{r}{L} \right) = \sum_{i=1}^{\infty} \frac{(-1)^{i+1} \psi \left\{ 2(i-1) + 1 \right\} \left( \frac{r}{L} \right)^{4(i+1)+2}}{2^{4(i-1)+2} \left\{ 2(i-1) + 1 \right\}!]^2} \]
concentrated load,
radial distance of the point under investigation from P,
radial and tangential moments for a unit width of mat,
shear force per unit width of mat,
matt displacement,

\[
R_2 \left( \frac{r}{L} \right) = \sum_{i=1}^{n} \frac{(-1)^{i+1}\Psi (2i)(\frac{r}{L})^{4i}}{2^{4i}[\{2i\}!]^2} \quad (2.26)
\]

\[
\Psi (n) = \sum_{i=1}^{n} \frac{1}{i} \quad (2.27)
\]

where

- \( P \) = concentrated load,
- \( r \) = radial distance of the point under investigation from P,
- \( M_r \& M_t \) = radial and tangential moments for a unit width of mat,
- \( Q \) = shear force per unit width of mat,
- \( w \) = mat displacement,
- \( \Psi, R \)'s and \( Z \)'s = functions first introduced by Schleicher (1926).

The \( Z \) and \( R \) functions have the characteristic features of exponential waves and it is accurate enough to calculate only 4 or 5 terms of those.

(iv) The radial and tangential moments are then converted into bending moments in cartesian co-ordinate system using the following formulae, whereas shear and displacements in the cartesian coordinate system remains the same.

\[
M_x = M_r \cos^2 \phi + M_t \sin^2 \phi \quad (2.28)
\]

\[
M_y = M_r \sin^2 \phi + M_t \cos^2 \phi \quad (2.29)
\]

where

- \( M_x, M_y \) = bending moment about y and x axes respectively,
- \( \Psi \) = angle of the radial line passing through the point under consideration w.r.t. the positive x axis in a counter clockwise direction.
(v) For the resultant effect of all the column loads, radii of influence of whose overlap at points of interest, the moments, shears and displacements due to individual columns are superimposed.

(vi) If the edge of the mat is within the influence zone of some of the columns, at the end of the afore-mentioned procedure there will be bending moments and shear forces along the edges. If the edges should be free of forces, a correction should be applied. This is done as follows:

The mat is divided into strips of unit width in both directions. Assuming the strips as semi-infinite beams; shears and moments equal and opposite to those obtained in the previous analysis is applied and their effects at various points are superimposed on the respective values obtained earlier.

For moment, shear and deflection in a semi infinite beam, the following relationships are used:

\[
M_c = M_e A_{\lambda r} - s \times \frac{V_e}{\lambda} B_{\lambda r} \quad (2.30)
\]

\[
Q_c = -s \times 2M_e \lambda B_{\lambda r} - V_e C_{\lambda r} \quad (2.31)
\]

\[
w_c = -\frac{2M_e}{k} C_{\lambda r} + s \times \frac{2V_e}{k} D_{\lambda r} \quad (2.32)
\]

\[
\lambda = \sqrt{\frac{kB}{4EI_b}} \quad (2.33)
\]

\[
A_{\lambda r} = e^{-\lambda r} (\cos \lambda r + \sin \lambda r) \quad (2.34)
\]

\[
B_{\lambda r} = e^{-\lambda r} \sin \lambda r \quad (2.35)
\]

\[
C_{\lambda r} = e^{-\lambda r} (\cos \lambda r - \sin \lambda r) \quad (2.36)
\]

\[
D_{\lambda r} = e^{-\lambda r} \cos \lambda r \quad (2.37)
\]
where

\[ M_e, V_e \& w_e = \text{end correction to moment, shear and displacement respectively}, \]
\[ M_e, V_e = \text{end moment and shear respectively}, \]
\[ B = \text{width of mat strip} = 1, \]
\[ I_b = \text{moment of inertia of mat strip}, \]
\[ s = 1 \text{ for left side end forces}, \]
\[ = -1 \text{ for right side end forces}, \]
\[ r = \text{distance of the point under consideration from the end conditioning force } M_e \text{ or } V_e. \]

Variation of \( A_{\lambda r}, B_{\lambda r}, C_{\lambda r} \text{ and } D_{\lambda r} \) with \( r \) and variation of \( C_{mr} \text{ and } C_{nr} \) with \( x \) is shown in Fig.2.2 and Fig.2.3 respectively.

### 2.4.3 Comparison of Various Methods

Results obtained by Conventional Method and ACI Approximate Flexible Method are compared with respect to finite element solution by Morshed (1997) and Sutradhar (1999). Their main findings are tabulated in Table 2.1.
Fig. 2.2 Variation of $A_{lr}$, $B_{lr}$, $C_{lr}$, $D_{lr}$ with $\lambda r$ (After Shukla, 1984).

Fig. 2.3 Variation of $C_{mr}$ and $C_{ml}$ with $x = r/L$ (After Shukla, 1984).
Table 2.1 Comparison Between Various Methods

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Location</th>
<th>Conventional Method</th>
<th>ACI Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflection</td>
<td>At Col. Edge</td>
<td>Cannot be Calculated</td>
<td>Overestimates</td>
</tr>
<tr>
<td></td>
<td>In between. Column.</td>
<td>Cannot be Calculated</td>
<td>Underestimates</td>
</tr>
<tr>
<td>Moment</td>
<td>Column Face</td>
<td>Fails to predict any moment</td>
<td>Underestimates</td>
</tr>
<tr>
<td></td>
<td>+ve moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-ve moment between cols.</td>
<td>Overestimates</td>
<td>Overestimates less at edges</td>
</tr>
<tr>
<td>Shear</td>
<td>Away from Column</td>
<td>Compatible</td>
<td>Compatible</td>
</tr>
<tr>
<td></td>
<td>Near Column.</td>
<td>Underestimates too much</td>
<td>Overestimates</td>
</tr>
<tr>
<td>Punching Shear</td>
<td>At d/2 from column face</td>
<td>Compatible</td>
<td>Underestimates, Compatible</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>in central cols.</td>
</tr>
<tr>
<td>Flexural Shear</td>
<td>At d from column face</td>
<td>Underestimates</td>
<td>Overestimates</td>
</tr>
<tr>
<td>Thickness</td>
<td>Whole Mat</td>
<td>Compatible, Governed by Punching Shear</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Overestimates, Governed by Flexural Shear</td>
<td></td>
</tr>
<tr>
<td>+ve Steel</td>
<td>+ve moment areas</td>
<td>Governed by minimum steel requirements</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Governed by minimum steel requirements</td>
<td></td>
</tr>
<tr>
<td>-ve steel</td>
<td>-ve moment areas</td>
<td>Overestimates</td>
<td>Largely Overestimates</td>
</tr>
<tr>
<td>Economic</td>
<td>In terms of concrete &amp;</td>
<td>Overestimates</td>
<td>Overestimates</td>
</tr>
<tr>
<td>Evaluation</td>
<td>steel</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.5 SENSITIVITY STUDY OF MAT FOUNDATION

An extensive parametric study on various parameters has been done by Morshed (1997) and Sutaradhar (1999). The summary of sensitivity analysis of mat foundation is given in Table 2.2.

Table 2.2  Summary of Sensitivity Analysis of Mat

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Displacement</th>
<th>Positive moment</th>
<th>Negative moment</th>
<th>Flexural Shear</th>
<th>Punching shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mat thickness Increases</td>
<td>Decreases. More decreases at corner columns than central columns.</td>
<td>Decreases. More decrease in interior columns than exterior columns.</td>
<td>Increases. More increase in interior midspan than exterior midspan.</td>
<td>Flexural shear has very little change</td>
<td>Punching shear remains almost constant</td>
</tr>
<tr>
<td>Modulus of subgrade reaction (K_s) increases</td>
<td>Displacement decreases proportionately. If K_s is 2 times. displacement is almost half</td>
<td>Positive moment increases. More increase in exterior columns than interior columns.</td>
<td>Negative moment decreases. More decrease in exterior span than interior span.</td>
<td>Flexural shear has very little change</td>
<td>Punching shear remains almost constant</td>
</tr>
<tr>
<td>Ultimate strength of concrete increases</td>
<td>Displacement decreases and the change is small.</td>
<td>Positive moment decreases slightly.</td>
<td>Negative moment increases moderately.</td>
<td>Flexural shear has very little change</td>
<td>Punching shear remains almost constant</td>
</tr>
<tr>
<td>Width of greater thickness increases</td>
<td>Displacement decreases more in interior columns than exterior columns.</td>
<td>Positive moment increases slightly.</td>
<td>Negative moment decreases moderately.</td>
<td>Flexural shear has very little change</td>
<td>Punching shear remains almost constant</td>
</tr>
</tbody>
</table>
2.6 MAT WITH NON UNIFORM THICKNESS: A CHOICE FOR ECONOMY

Morshed (1997) and Sutradhar (1999) conducted comparative analysis between finite element method using Ahmad’s thick shell element with ACI method and Conventional method. Morshed suggested a non-uniform mat foundation. His findings paved the way for developing a new design rationale for mat foundation. Sutradhar suggested a more definite guideline for selecting a mat of non-uniform thickness and its analysis with finite element methods.

The parametric studies of Morshed (1997) and Sutradhar (1999) revealed that the design of regular shaped mats with non-uniform thickness follow some well defined trends. Based on these findings, a design approach for mat with non-uniform thickness has been recommended as follows:

(i) The problem should be solved first with a non-uniform thickness geometry selected from the guideline shown in Fig.2.4. Column punching shear should be estimated as 0.95 x column axial force.

\[
t_g = \text{Greater thickness, calculate from this punching shear}
\]
\[
t_s = \text{Smaller thickness} = \frac{2}{3} \text{ of } t_g
\]
\[
d_g = \text{Width of greater thickness, (effective depth corresponding to } t_g)
\]
\[
d_s = \text{Slope width} = \frac{t_g}{\sqrt{3}}
\]

(ii) A mat analysis is performed by finite element program considering this geometry. Now, the greater thickness \(t_g\) under the columns should be designed from the column punching shear found from analysis.

(iii) The greater thickness \(t_g\) should be provided around the column peripheries over a distance equal to the effective depth corresponding to the greater thickness itself.
\[ P_{DL+LL(\text{max})} = \text{Maximum column axial load due to live load and dead load} \]

\[ t_g = \text{thickness required to encounter } 0.95 P_{DL+LL(\text{max})} \text{ as punching shear}, \]

\[ t_r = \frac{2}{3} \text{ of } t_g, \]

\[ d_e = \text{effective depth corresponding to } t_g, \]

\[ d_s = \frac{t_g}{\sqrt{3}}. \]

Fig. 2.4 Guideline for selecting cross-sectional geometry for mat with variable thickness.

(iv) Smaller thickness should be calculated from the maximum flexural shear, found from the solution, at the neck sections.

(v) Width of the transition zones (for gradual reduction of thickness) should be such that slope angle of the underside of mat in these zones is about 30°.

(vi) The problem should be analysed again with the new geometry just selected in order to calculate reinforcements. Finally, a check on the adopted geometry using the shear force diagrams and punching shears found from the new solution.

(vii) Minimum reinforcement required for the zone of greater thickness should be provided as bottom reinforcement under the columns across the entire widths of column strips. Again, the new solution may be used to check the adequacy of these reinforcements.

(viii) Reinforcements for negative moments in-between the columns should be designed using the new solution.

(ix) Top reinforcements in-between the columns (negative moment zones) and bottom reinforcements under the columns (positive moment zones) should be calculated as per minimum requirements specified by ACI code.

Case studies reveal that mat thickness away from the column faces can be reduced by about 40%. But to be on the conservative side \( \frac{1}{3} \) reduction is suggested in the guideline.
2.7 COMPARISON BETWEEN MAT WITH VARIABLE THICKNESS AND FLAT SLAB WITH DROP PANELS

Analysis and parametric study has shown that there is a basic similarity between flat slab having drop panels and mat foundation of variable thickness. In flat slab, uniformly distributed load i.e. combination of dead and live load is applied on the slab floor supported on columns. In flat slab, the slab is made somewhat thicker near the column than the rest of the slab, to reduce the shearing stress in the slab within the area of drop. The increase in effective slab thickness provided by the drop also decreases the compressive stresses in the concrete and reduces the amount of steel required over the column heads.

In mat foundation with variable thickness, a thick slab type continuous mat foundation rests on elastic subgrade. Column loads are applied on the slab to distribute the column load to the substratum and a reactive soil pressure generates at the bottom face of the slab. Mats having greater thickness near the column can actually be considered as an inverted flat slab with drop panels. The reactive soil pressure distribution at bottom face of mat is not, however, uniform as in the case of flat slab. Comparison of results in this field requires extensive study of mat behaviour.

In flat slab drop panels are square or rectangular. The ACI code specifies that the side of a drop panel shall be at least 0.33 times of the span in each direction and smaller thickness outside the drop should be at least 67% of greater thickness within the drop. This reduction in thickness has been done considering flexural shear.

In beams, simply supported or continuous, at the face of the column, flexural shear is approximately 0.5 WL where W is load/unit length and L is span length of the beam. At a distance 0.17L from the column face flexural shear is 0.33WL and bending moment is not significant. Bending moment is maximum at midspan. At a distance 0.17L, beam thickness is governed by flexural shear and flexural shear is 2/3 of shear at the column.
face. Considering flexural shear, beam thickness can be reduced to 66% of thickness at the column face. At a distance 0.2L from the column face flexural shear is 0.3WL and beam thickness can be reduced to 60% of thickness at the column face.

In flat slab, each panel is considered as a beam. Drop is extended 0.165 times of the span on each side of the column. Considering flexural shear thickness outside the drop is reduced to 66% of the greater thickness provided that each side of the drop is at least 0.33 times of the span in that direction.

A mat can be considered as inverted flat slab. To be conservative, width of thickened zone has been taken 0.4 times of the span in each direction. Mat panel can be considered as a total beam. Thickened zone is extended 0.2 times of the span on each side of the column. At the transition point of greater thickness to smaller thickness, flexural shear is 60% of the flexural shear at column face and bending moment is not significant. So, at a distance 0.2 times of the span, mathematically thickness can be reduced to 60% of greater thickness.
CHAPTER 3

FINITE ELEMENT IDEALISATION OF MAT FOUNDATION

3.1 INTRODUCTION

Finite element technique is the most powerful and versatile of all the available numerical analysis techniques. In this method, the structure to be analyzed is modeled as an assemblage of a finite number of discrete interconnected elements and the displacements of the connecting points, called the nodal points, of these elements are taken as the basic unknowns. The applied loads are transformed into equivalent nodal loads and for each element, the relationship between the nodal loads and nodal displacements is established. Then with a suitable assembling of the element load-displacement relationship, sufficient number of load-displacement relations for the whole structure can be found which, upon solution, give the nodal displacements. Once known, these displacements are used to calculate stresses at the nodes or at points within the elements. A well reputed powerful computer program ANSYS has been used for analysis of mat foundation.

3.2 IDEALIZATION OF MAT FOUNDATION ON WINKLER MODEL

3.2.1 Structural Idealization

Mat foundation is a three dimensional thick plate structure. In case of mat transverse flexural shear and bending moments are the most important internal forces produced in response to the loads it usually encounters e.g. axial column forces and column base moments. Element that can take account of shear deflection is particularly suitable for this purpose. Plate bending elements are acceptable for representing the flexural action of slab.
3.2.2 Idealization of Soil

Actual contact pressure distribution is the result the foundation soil interaction, which can be determined only by an interaction analysis involving the elastic properties of both the foundation and soil. Soil structure interaction is an important process of predicting overall structural response. Settlement profile and soil contact pressure for raft foundations are dependent on the relative stiffness of raft and supporting soil. In computer based methods, Winkler spring model has been used for representation of the soil behavior. It consists of representing the soil which supports a raft by discrete vertical springs. Winkler spring model gives satisfactory results for structural design of a raft where loading could be approximated to a single point load.

3.2.3 Selection of Element

Mat foundation, a three dimensional thick plate structure, for which transverse flexural shears and bending moments are important. Eight noded structural shell element SHELL93 in the library of ANSYS is one of the most suitable elements which matches all of these above mentioned criteria. It can represent the thickness of the mat by its very own geometry. The element has six degrees of freedom at each node: translations in X, Y, Z directions and rotations about the nodal X, Y, Z axes. Shear deflections are included in this element.

Spring element COMBIN14 has been used to represent the soil as the Winkler Spring Model. The longitudinal spring option is a uni-axial tension-compression element with three degrees of freedom at each node, translations in the nodal x, y, z directions. It's options are so used that it acts as 3-D longitudinal spring-damper which can take only deflection in the vertical, Y direction.

Soil springs may be uniform or may be zoned to incorporate the coupling effect of soil springs. Uniformly distributed soil springs are first concentrated at the nodes. This is done element wise and by lumping the total spring stiffness under an element
at its nodes. A simplified approach is followed for this purpose. Spring elements are connected at the corner nodes of the element only. Spring constants are calculated from their influence area.

3.2.4 Element Mesh Configuration

The simplest element division scheme is adopted for mat. It makes both data generation and result interpretation straightforward. The mesh used here is a rectangular grid with the option of finer elements near column loads. The mesh is characterized by the number of X and Z directional divisions of the grid and at present, each of these can be increased up to infinite numbers. Details of the mesh are shown in Fig 3.1.

3.2.5 Element and Node Numbering

The program uses frontal solution technique to evaluate nodal displacements. So in order to minimize computer memory requirement, the front must be as narrow as possible. This is accomplished by efficient numbering of elements. Front can be narrowed by numbering the elements serially from one corner of the mat to the other. The node numbering scheme is such that it makes nodal coordinate generation simple. Nodes are numbered serially from one corner of the mat to the other corner. Details of the node and element numbering scheme are given in Fig. 3.2.

3.2.6 Modeling of Soil and Boundary Conditions

Supporting soil is modeled as Winkler medium. Soil springs may be uniform or may be zoned to incorporate the coupling effect of soil springs. Spring element, from element library of ANSYS, COMBIN14 has been used to model the soil.

For vertical loading one fourth of the mat is analyzed. For vertical load in combination with wind load half of the mat is analyzed. Rotation and horizontal movement is restrained at the symmetry line to bring the effect of total structure.
Fig. 3.1 Typical finite element mesh for mat foundation.

Fig. 3.2 A typical finite element mesh for mat with element and node numbering.
3.2.7 Application of Column Loads

Both column axial loads and column base moments can be applied to the elements. Column axial load may be applied as concentrated load applying the load as concentrated force at corner nodes or it may be distributed over an area equal to the cross-section of the column by taking an element, termed as column element. Similarly, column base moments may be concentrated or distributed. In this study, column axial load and bending moments are applied to the column element at corner four nodes equally.

3.3 VERIFICATION OF FINITE ELEMENT MODEL

Results obtained by ANSYS has been verified with closed form solution of a beam on elastic foundation in Hetenyi (1974). The structure is modeled with element SHELL93 and soil is modeled with element COMBIN14 of ANSYS element library.

A beam of infinite length of unit cross section subjected to a single concentrated force, $P$ at center, has the exact solution for deflection, bending moment and rotation are given by

$$Y = \frac{P\lambda}{2k} e^{-\lambda x} (\cos \lambda x + \sin \lambda x)$$  \hspace{1cm} (3.1)

$$M = \frac{P}{4\lambda} e^{-\lambda x} (\cos \lambda x - \sin \lambda x)$$  \hspace{1cm} (3.2)

$$\theta = -\frac{P\lambda^2}{k} e^{-\lambda x} \sin \lambda x$$  \hspace{1cm} (3.3)

Example:
A beam of unit width and unit depth is modeled with SHELL93 and COMBIN14 element. The model has been analyzed with line load $P = 30 \text{ kip/ft}$ across the width at the center as shown in Fig. 3.3.
Modulus of subgrade reaction, $k$ = 30 kip/cft,
Modulus of concrete, $E_c$ = $4.32 \times 10^4$ kip/sft.
Length of the beam, $L$ = 80 ft.
Beam of X-section = $1'-0" \times 1'-0"$

For long beam, $\lambda L$ must be greater than $\pi$.

Where, $\lambda = \sqrt{\frac{k}{4E_cI}}$

For beam of X-section $1'-0" \times 1'-0"$

$\lambda = 0.120141$

$\lambda L = 0.120141 \times 80$

$= 9.61 > \pi$

P = 30 kips

Fig. 3.3 Beam resting on elastic subgrade subjected to concentrated load at the center.

Comparison of results between exact solution and ANSYS solution for deflection, bending moment and rotation is given in Fig. 3.4 to Fig. 3.6.
Fig. 3.4 Variation of bending moment against distance along beam line.

Fig. 3.5 Variation of rotation against distance along beam line.
3.4  CRITICAL ASPECTS OF FINITE ELEMENT MODEL

3.4.1  Selection of Finite Element Mesh

Finite element method is a numerical approach and the accuracy of its solution increases with the number of elements in which the mat is divided. Solutions being converging, a point comes when further refinement in mesh adds nothing significant to the accuracy. Mesh sensitivity study has been made and a final mesh is selected.

3.4.2  Element Aspect Ratio

Shape of the element affects the results. Element aspect ratio must be between 0.5 to 2.0. Study on element aspect ratio has been done. Best results are obtained for element aspect ratio equal to 1.0.
3.4.3 Effect of Column Rigidity

Columns are monolithically built with mat and they act integrally with it. Height of column highly increases the rigidity of mat at these locations. This effect is realized by increasing the value of modulus of elasticity of the portions of the mat under the columns. To quantify this effect, column deflections, column face moments and shears are plotted against the ratio of the modulus of elasticity of soil under column portions by Sutradhar (1999). It appears that effect of column rigidity dies out with increase of this modulus of elasticity ratio. Examining these, a modulus of elasticity ratio of 8 is found to be justified.
CHAPTER 4

COMPARATIVE STUDY AND SENSITIVITY ANALYSIS

4.1 INTRODUCTION

A comparative study on finite element solution of mat foundation for the variation of its different parameters has been made. For comparison a sample problem of twelve-storied building has been studied for vertical load that is for dead load and live load only. Various parameters relevant to mat are selected for parametric study. Sensitivity analysis is performed on some selected items at selected locations. Variation of modulus of subgrade reaction and variation in differential thickness have been taken into consideration.

A mat of constant thickness refers to mat foundation which has constant thickness all over the mat foundation. A mat of variable thickness refers to a mat foundation, which has greater thickness calculated from punching shear near the columns and smaller thickness away from the columns.

4.2 SENSITIVITY ANALYSIS OF MAT FOUNDATION

A twelve storied building of three bay frame in each direction with span 26'-0" has been considered for the study. The building is symmetric in both directions. Therefore, only one-fourth of the problem has been considered. The structure is assumed to have uniform live load 60 psf, slab dead load 90 psf, distributed wall load 30 psf and floor finish 25 psf. An amount of 440 lb/ft has been considered for 5" wall along all beams and the column size 24" x 24". Story height of the floor is taken as 10'-0".
With Dead load factor 1.4 and Live load factor 1.7, the considered load case is 1.4DL+1.7LL for the mat foundation shown in Fig. 4.1. Geometry of the mat foundation is shown in Fig. 4.1a and Fig. 4.1b. A commercial software STAAD-III has been used for analysis of the superstructure model. Respective column loads are given in Table 4.1. Finite element idealization of the mat model has been described in chapter three.

Table 4.1: Column loads for mat of twelve-storied building.

<table>
<thead>
<tr>
<th>Title of Column</th>
<th>Axial Force (kip)</th>
<th>Bending Moment, Mx (kip-ft)</th>
<th>Bending Moment, My (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1060</td>
<td>17.75</td>
<td>21.50</td>
</tr>
<tr>
<td>A2</td>
<td>1715</td>
<td>4.51</td>
<td>28.13</td>
</tr>
<tr>
<td>B1</td>
<td>1708</td>
<td>21.42</td>
<td>5.58</td>
</tr>
<tr>
<td>B2</td>
<td>2799</td>
<td>7.99</td>
<td>9.93</td>
</tr>
</tbody>
</table>

4.2.1 Effect of Modulus of Subgrade Reaction

The mat is analysed for constant thickness 3.5 ft and variable thickness with greater thickness 3.5 ft and smaller thickness 1.75 ft, with variation of modulus of subgrade reaction from 50 kcf to 200 kcf. Selected items for parametric study are displacement, bending moment and shearing stress along lines 1 to 8 (Fig. 4.1).
Fig. 4.1b Mat of a three bay twelve storied building (Plan and X-section).
Variation of these items on column faces and midspan for mat with constant thickness and mat with variable thickness are shown in Table 4.2. Upper and lower values in the table are for modulus of subgrade reaction 50 kcf and 200 kcf, respectively. Variation of parameters are expressed in percentage are shown in Table 4.3. The variations are plotted in Fig. 4.2 to Fig. 4.5.

As modulus of subgrade reaction increases, the following points are observed.

- Positive moment below the column increases slightly. This change is more pronounced for exterior column than for interior columns (Fig. 4.2 and Fig. 4.3).
- Negative moment at mid span decreases (Fig. 4.2 and Fig. 4.3).
- The variation of shearing stress is not very significant (Fig. 4.4).
- Displacements decrease at a fast rate. With two times the value of modulus of subgrade reaction, the displacements are reduced to almost half. This variation is less in midspan than for column faces (Fig. 4.5).

The variations for displacements and moments are more pronounced in mat with variable thickness than with uniform thickness.

### Table 4.2

Variation of displacement, moment and shearing stress when modulus of subgrade reaction is increased from 50 kcf to 200 kcf.

<table>
<thead>
<tr>
<th>Location</th>
<th>Constant thickness</th>
<th>Variable thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement (inch)</td>
<td>Bending Moment (k-ft/ft)</td>
</tr>
<tr>
<td>Column face of A2</td>
<td>0.98</td>
<td>133.77</td>
</tr>
<tr>
<td>A2 (Line 3)</td>
<td>0.28</td>
<td>141.99</td>
</tr>
<tr>
<td>Midspan of A2-B2</td>
<td>0.86</td>
<td>-205.96</td>
</tr>
<tr>
<td>(Line 3)</td>
<td>0.20</td>
<td>-175.51</td>
</tr>
<tr>
<td>Column face of B2</td>
<td>0.92</td>
<td>425.65</td>
</tr>
<tr>
<td>B2 (Line 3)</td>
<td>0.28</td>
<td>432.92</td>
</tr>
<tr>
<td>Midspan of B2-C2</td>
<td>0.86</td>
<td>-114.02</td>
</tr>
<tr>
<td>(Line 3)</td>
<td>0.22</td>
<td>-106.11</td>
</tr>
</tbody>
</table>
Fig. 4.2a  
Variation of bending moment $M_x$ along line 1 with variation of subgrade modulus $K$ from 50 kcf to 200 kcf. (CT-constant thickness, VT-variable thickness).

Fig. 4.2b  
Variation of bending moment $M_x$ along line 2 with variation of subgrade modulus $K$. 
Fig. 4.2c  Variation of bending moment $M_x$ along line 3 with variation of subgrade modulus $K$.

Fig. 4.2d  Variation of bending moment $M_x$ along line 4 with variation of subgrade modulus $K$. 
Variation of bending moment $M_x$ along line 5 with variation of subgrade modulus $K$ from 50 kcf to 200 kcf. (CT-constant thickness, VT-variable thickness).

Fig. 4.3a

Variation of bending moment $M_x$ along line 6 with variation of subgrade modulus $K$.

Fig. 4.3b
Fig. 4.3c  Variation of bending moment $M_x$ along line 7 with variation of subgrade modulus $K$.

Fig. 4.3d  Variation of bending moment $M_x$ along line 8 with variation of subgrade modulus $K$. 
Fig. 4.4a  Variation of Shearing stress $S_{xy}$ along line 1 with variation of subgrade modulus $K$ from 50 kcf to 200 kcf. (CT-constant thickness, VT-variable thickness).

Fig. 4.4b  Variation of Shearing stress $S_{xy}$ along line 2 with variation of subgrade modulus $K$. 
Fig. 4.4c  Variation of Shearing stress $S_{xy}$ along line 3 with variation of subgrade modulus $K$.

Fig. 4.4d  Variation of Shearing stress $S_{xy}$ along line 4 with variation of subgrade modulus $K$. 
Fig. 4.5a  Variation of vertical deflection along line 1 with variation of subgrade modulus $K$ from 50 kcf to 200 kcf. (CT-constant thickness, VT-variable thickness).

Fig. 4.5b  Variation of vertical deflection along line 2 with variation of subgrade modulus $K$. 
Fig. 4.5c  Variation of vertical deflection along line 3 with variation of subgrade modulus $K$.

Fig. 4.5d  Variation of vertical deflection along line 4 with variation of subgrade modulus $K$. 
Table 4.3. Variation of displacement, moment and shearing stress (expressed in percentage) when modulus of subgrade reaction is increased from 50 kcf to 200 kcf.

<table>
<thead>
<tr>
<th>Location</th>
<th>Constant thickness</th>
<th>Variable thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement</td>
<td>Bending Moment</td>
</tr>
<tr>
<td>Column face of A2 (Line 3)</td>
<td>-71.4</td>
<td>6.1</td>
</tr>
<tr>
<td>Midspan of A2-B2 (Line 3)</td>
<td>-76.7</td>
<td>-14.8</td>
</tr>
<tr>
<td>Column face of B2 (Line 3)</td>
<td>-69.6</td>
<td>1.7</td>
</tr>
<tr>
<td>Midspan of B2-C2 (Line 3)</td>
<td>-74.4</td>
<td>-6.9</td>
</tr>
</tbody>
</table>

Study of Figs 4.2 to 4.5 and Table 4.2 and Table 4.3 reveals that distribution of bending moment varies more significantly for mat with variable thickness, when modulus of subgrade reaction varies.

4.2.2 Effect of Variable Mat Thickness

Mat of variable thickness consists of thickened part in the zone of higher shear and positive moment near the column and smaller thickness in the zone of low shear and negative moment away from the columns. Greater thickness is calculated for punching shear and smaller thickness is varied for different values in the range of 40% to 70% of greater thickness. This problem is taken for comparative study. Greater thickness is calculated to be 3.5 ft from punching shear and smaller thickness is taken as 1.5 ft, 1.75 ft, 2.0 ft and 2.5 ft. For all cases modulus of subgrade reaction, \( K_s \) have been taken 100 kcf.

Selected items for parametric study are taken as displacement, bending moment and shearing stress along lines 1 to 8. Variation of these items on some column faces and
Selected items for parametric study are taken as displacement, bending moment and shearing stress along lines 1 to 8. Variation of these items on some column faces and midspan along line 3 for mat with constant thickness and mat with variable thickness with smaller thickness 1.5 ft and 2.5 ft for modulus of subgrade reaction 100 kcf are tabulated in Table 4.4. Upper values in the chart are results for mat with uniform thickness 3.5 ft and lower values are for non-uniform mat. Changes in values of parameters, expressed in percentage of results for constant thickness are given in Table 4.5. The variations of parameters for different smaller thickness along with mat with constant thickness are also plotted in Fig. 4.6 to Fig. 4.9.

As the smaller thickness increases, the following points are observed

- Positive moment decreases along column line (Fig. 4.6a and Fig. 4.6c and Table 4.3) and increases along line between columns (Fig. 4.6b and Fig. 4.6d). This variation is more for interior column than for exterior column.
- Negative moment increases (Fig. 4.7a to Fig. 4.7d). This variation is more for exterior span compared to interior span.
- Shearing stress increases at column face and decreases in region away from columns. The variation is not significant at column face (Fig. 4.8). Shearing stress is decreased in the region away from column face due to increase in effective area by increased smaller thickness.
- Displacements decrease below the column and increase at midspan. This effect in exterior span is more pronounced. Deflection diagram is more uniform for mat with constant thickness. Column to column differential settlement increases slightly for mat with variable thickness (Fig. 4.9).

Smaller thickness, t, for mat with variable thickness at midspan is governed by negative moment. As t decreases negative moment decreases, at the same time capacity of the section also decreases. Table 4.6 and Fig 4.10 show the variation of bending moment and moment capacity for unit width of the smaller section for the above example along line 1 and 3. Maximum and minimum moment capacity of a section has been calculated with maximum and minimum steel ratio for unit width of the section as singly reinforced beam. From Fig. 4.10 it appears that from moment consideration smaller thickness can be reduced to 40% of greater thickness.
Fig. 4.6a  
Variation of bending moment $M_x$ along line 1 with variation of smaller thickness (Greater thickness 3.5 ft).

Fig. 4.6b  
Variation of bending moment $M_x$ along line 2 with variation of smaller thickness.
Fig. 4.6c  Variation of bending moment $M_x$ along line 3 with variation of smaller thickness.

Fig. 4.6d  Variation of bending moment $M_x$ along line 4 with variation of smaller thickness.
Fig. 4.7a  Variation of bending moment $M_x$ along line 5 with variation of smaller thickness (Greater thickness 3.5 ft).

Fig. 4.7b  Variation of bending moment $M_x$ along line 6 with variation of smaller thickness (Greater thickness 3.5 ft).
Fig. 4.7c  Variation of bending moment $M_x$ along line 7 with variation of smaller thickness.

Fig. 4.7d  Variation of bending moment $M_x$ along line 8 with variation of smaller thickness.
Fig. 4.8a  Variation of Shearing Stress $S_{xy}$ along line 1 with variation of smaller thickness (Greater thickness 3.5 ft).

Fig. 4.8b  Variation of Shearing Stress $S_{xy}$ along line 2 with variation of smaller thickness.
Fig. 4.8c  Variation of Shearing Stress $S_{xy}$ along line 3 with variation of smaller thickness.

Fig. 4.8d  Variation of Shearing Stress $S_{xy}$ along line 4 with variation of smaller thickness.
Fig. 4.9a  Variation of Vertical Deflection along line 1 with variation of smaller thickness (greater thickness 3.5 ft).

Fig. 4.9b  Variation of Vertical Deflection along line 2 with variation of smaller thickness:
Fig. 4.9c  Variation of Vertical Deflection along line 3 with variation of smaller thickness.

Fig. 4.9d  Variation of Vertical Deflection along line 4 with variation of smaller thickness.
Table 4.4. Variation of displacement, moment and shearing stress against variation of smaller thickness ($K_s = 100$ kcf).

<table>
<thead>
<tr>
<th>Location</th>
<th>Smaller thickness 43% of greater thickness</th>
<th>Smaller thickness 71% of greater thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement (inch)</td>
<td>Bending Moment (k-ft/ft)</td>
</tr>
<tr>
<td>Column face of A2 (Line 3)</td>
<td>0.51</td>
<td>137.21</td>
</tr>
<tr>
<td></td>
<td>0.63</td>
<td>165.00</td>
</tr>
<tr>
<td>Midspan of A2-B2 (Line 3)</td>
<td>0.42</td>
<td>-192.45</td>
</tr>
<tr>
<td></td>
<td>0.32</td>
<td>-83.44</td>
</tr>
<tr>
<td>Column face of B2 (Line 3)</td>
<td>0.49</td>
<td>432.34</td>
</tr>
<tr>
<td></td>
<td>0.63</td>
<td>662.30</td>
</tr>
<tr>
<td>Midspan of B2-C2 (Line 3)</td>
<td>0.43</td>
<td>-107.25</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>-59.17</td>
</tr>
</tbody>
</table>

Flexural shear force varies from maximum at the column face and minimum at midspan. For flexural shear, most critical section is neck section i.e. at the contact point in between greater thickness and smaller thickness. In region of smaller thickness of mat with variable thickness, flexural shear force is maximum at the neck section. From Fig. 4.8a and Fig. 4.8c it is observed that at the neck section along column strip shear stress is maximum. These values are always above than flexural shear stress capacity of concrete. From Fig. 4.8b and Fig. 4.8d, it is observed that at the neck sections, along middle strip, shear stress is below than shear stress capacity of concrete. Fig. 4.11 shows the variation of flexural shearing stress across the width at an interior neck section. As mat is a plate type structure local increase in flexural shear cannot cause failure. It will fail only when ultimate flexural shearing capacity is less than the flexural shear force across the width. Fig. 4.11 shows the ultimate stress capacity in flexure of the neck section along the variation of shearing stress due to change in thickness.
Variation of bending moment and moment capacity along line along 1 and 3 at interior midspan with variation of smaller thickness (greater thickness 3.5 ft).

Variation of shearing stress and flexural shear capacity across the width at the neck section.
As the smaller thickness increases, flexural shearing stress decreases, but total flexural force remains constant. This decrease in flexural shearing stress is due to increase in effective area due to increase in thickness. As the smaller thickness increases capacity for flexural shear increases. Table 4.7 shows the capacity of the neck section along with the flexural shear force at that section due to variation of smaller thickness. Flexural shear force and flexural capacity of the neck section for different neck thickness has shown in Fig. 4.12. In Fig. 4.13, the flexural shear capacity and flexural shear at the neck section is plotted against the smaller thickness as percentage of greater thickness.

Table 4.5. Variation of displacement, moment and shearing stress expressed as percentage of results of uniform mat with change of smaller thickness from 1.5 ft to 2.5 ft.

<table>
<thead>
<tr>
<th>Location</th>
<th>Smaller thickness 43% of greater thickness</th>
<th>Smaller thickness 71% of greater thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement %</td>
<td>Bending Moment %</td>
</tr>
<tr>
<td>Column face of A2 (Line 3)</td>
<td>23.53</td>
<td>20.25</td>
</tr>
<tr>
<td>Midspan of A2-B2 (Line 3)</td>
<td>-23.81</td>
<td>-56.64</td>
</tr>
<tr>
<td>Midspan of B2-C2 (Line 3)</td>
<td>4.65</td>
<td>-44.83</td>
</tr>
</tbody>
</table>

From this study, it has been observed that for smaller thickness 54.5% of the greater thickness flexural shear capacity and flexural force are equal. Smaller thickness 55% of greater thickness is sufficient to take flexural shearing force across the width provided that column strip width at least 0.4 times of the span in that direction.
Table 4.6  Negative moment in exterior midspan along line 3 and moment capacity for unit width with variation in smaller thickness.

<table>
<thead>
<tr>
<th>Smaller thickness</th>
<th>1.5 ft</th>
<th>1.75 ft</th>
<th>2.0 ft</th>
<th>2.5 ft</th>
<th>3.0 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative moment (along line 1) (Kip-ft/ft)</td>
<td>92.45</td>
<td>111.04</td>
<td>127.24</td>
<td>155.25</td>
<td>190.20</td>
</tr>
<tr>
<td>Negative moment (along line 3) (Kip-ft/ft)</td>
<td>92.52</td>
<td>111.26</td>
<td>128.71</td>
<td>158.73</td>
<td>200.05</td>
</tr>
<tr>
<td>Minimum moment capacity* (kip-ft/ft)</td>
<td>39.31</td>
<td>56.6</td>
<td>77.05</td>
<td>127.35</td>
<td>265.73</td>
</tr>
<tr>
<td>Maximum moment capacity* (Kip-ft/ft)</td>
<td>210.8</td>
<td>303.5</td>
<td>413.1</td>
<td>682.87</td>
<td>1424</td>
</tr>
</tbody>
</table>

* for $f'_c = 4,000$ psi, $f_y = 60,000$ psi.

Table 4.7  Capacity and flexural shear force of the neck section with change in smaller thickness.

<table>
<thead>
<tr>
<th>Smaller thickness ft</th>
<th>Smaller thickness as % of greater thickness</th>
<th>Flexural shear force kip</th>
<th>Flexural Shear Capacity* kip</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>43%</td>
<td>1287.89</td>
<td>1021.85</td>
</tr>
<tr>
<td>1.75</td>
<td>50%</td>
<td>1298.32</td>
<td>1192.15</td>
</tr>
<tr>
<td>2.0</td>
<td>57%</td>
<td>1295.32</td>
<td>1362.46</td>
</tr>
<tr>
<td>2.5</td>
<td>72%</td>
<td>1295.64</td>
<td>1703.08</td>
</tr>
<tr>
<td>3.5</td>
<td>100%</td>
<td>1283.55</td>
<td>2384.31</td>
</tr>
</tbody>
</table>

* for $f'_c = 4,000$ psi
Fig. 4.12 Variation of total flexural shear force and ultimate flexural shear capacity at the interior neck section with change in smaller thickness (neck thickness).

Fig. 4.13 Variation of total flexural shear force and ultimate flexural shear capacity at the interior neck section with change in smaller thickness expressed as percentage of greater thickness (neck thickness).
4.3 SUMMARY OF THE PARAMETRIC ANALYSIS OF MAT FOUNDATION

The summary of parametric analysis of two significant parameters for design of mat foundation is shown in Table 4.8.

Table 4.8 Summary of parametric study of mat foundation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Displacement</th>
<th>Positive moment</th>
<th>Negative moment</th>
<th>Shearing Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of subgrade reaction</td>
<td>Displacement decreases rapidly. This variation is less in midspan compared to that of column faces.</td>
<td>Positive moment increases. The change is more for exterior column face than interior column face. Variation is more for non-uniform mat than uniform mat.</td>
<td>Negative moment decreases. Variation is more for non-uniform mat compared to uniform mat.</td>
<td>Shearing stress has very little change.</td>
</tr>
<tr>
<td>increases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smaller thickness increases</td>
<td>Displacements decrease below the column and increases at midspan. This effect in exterior span is more pronounced.</td>
<td>Positive moment decreases along column line and increases along line between columns. This variation is more for interior column than exterior column.</td>
<td>Negative moment increases. This variation is more for exterior span compared to interior span.</td>
<td>Shearing stress increases at column face and decreases at midspan.</td>
</tr>
</tbody>
</table>
CHAPTER 5

ANALYSIS OF MAT FOUNDATION

5.1 INTRODUCTION

Normally mat foundations are designed with uniform thickness all over the mat. These kinds of mats are always over designed, which is also the result of uncertainty in analysis procedure. Since Finite element technique is the most powerful and versatile of all the available numerical analysis techniques, so it is possible now to look at other economic configurations of mat. In the present study mat foundation with the plate thickened under columns, termed non-uniform mat hereafter, is analysed and all parameters related to it are critically reviewed. Finally, a design guideline is proposed after doing several mat problems of regular grids.

A sample problem of twelve storied building has been studied for vertical load i.e. for dead load and live load only. Various parameters relevant to mat are selected for study.

5.2 ANALYSIS OF MAT FOUNDATION

A twelve storied building of five bay frame in each direction with span 26'-0" has been considered for the study. The building is symmetric in both directions. Therefore, only one-fourth of the problem has been considered. The structure is assumed to have uniform live load 60 psf, slab dead load 90 psf, distributed wall load 30 psf and floor finish 25 psf. An amount of 440 lb/ft is assumed for 5" wall along all beams, story height 10'-0" and the columns are of 24" x 24". Modulus of subgrade reaction, k has been considered 100 kcf.
With dead load factor 1.4 and live load factor 1.7, the considered load case is 1.4 DL + 1.7 LL for the mat foundation (Fig. 5.1). Analysis of the super structure is done by commercial software STAAD-III. Respective column loads are given in Table 5.1.

Table 5.1 Column loads for twelve storied building.

<table>
<thead>
<tr>
<th>Title of Column</th>
<th>Axial Force (kip)</th>
<th>Bending Moment, $M_x$ (kip-ft)</th>
<th>Bending Moment, $M_z$ (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1075</td>
<td>37.08</td>
<td>37.08</td>
</tr>
<tr>
<td>A2</td>
<td>1754</td>
<td>59.67</td>
<td>6.08</td>
</tr>
<tr>
<td>A3</td>
<td>1793</td>
<td>60.00</td>
<td>1.67</td>
</tr>
<tr>
<td>B1</td>
<td>1754</td>
<td>6.08</td>
<td>59.67</td>
</tr>
<tr>
<td>B2</td>
<td>2896</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>B3</td>
<td>2960</td>
<td>10.25</td>
<td>10.25</td>
</tr>
<tr>
<td>C1</td>
<td>1793</td>
<td>1.67</td>
<td>60.00</td>
</tr>
<tr>
<td>C2</td>
<td>2957</td>
<td>2.75</td>
<td>10.67</td>
</tr>
<tr>
<td>C3</td>
<td>3022</td>
<td>2.83</td>
<td>2.83</td>
</tr>
</tbody>
</table>

Selected items for study are displacement, bending moment, shearing stress, spring reaction along lines 1 to 17 (Fig. 5.1) on the column face and in midspan in between the columns.

5.2.1 Variation of Deflection

For the purpose of design of mat foundation only column to column differential deflections are important. Deflections below the column center and column to column differential deflections along lines are tabulated in Table 5.2, from which it is evident that differential settlement is higher for mat with variable thickness, but it is well below the allowable limit. Deflections of the mat foundation along the lines 1 to 9 are given in Fig 5.2. Maximum settlement occurs in the overhang portion of the mat foundation. For all practical case, due to architectural and commercial restraint, overhang portion is limited to 4 to 5 ft.
Fig. 5.1 Plan view of mat for a five bay twelve storied building.
Total settlement below the column center and column to column differential settlement along column lines.

<table>
<thead>
<tr>
<th>Line Number</th>
<th>Line A</th>
<th>Line B</th>
<th>Line C</th>
<th>Maximum Differential Settlement (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1</td>
<td>0.6711</td>
<td>0.6113</td>
<td>0.6637</td>
<td>0.0598</td>
</tr>
<tr>
<td>Line 2</td>
<td>0.6102</td>
<td>0.5709</td>
<td>0.6184</td>
<td>0.0475</td>
</tr>
<tr>
<td>Line 3</td>
<td>0.6627</td>
<td>0.6185</td>
<td>0.6697</td>
<td>0.0512</td>
</tr>
<tr>
<td>Maximum Differential Settlement (inch)</td>
<td>0.0609</td>
<td>0.0476</td>
<td>0.0513</td>
<td>Max. Col. to Col. Diff. Settlement 0.0988</td>
</tr>
</tbody>
</table>

5.2.2 Variation of Bending Moment

Mat foundation is typed to face positive bending moment below the column and negative moment at the midspan. For this example, Fig 5.3 shows the variation of bending moment, $M$, along X direction of lines 1 to 9 of Fig. 5.1 and shows positive and negative moments act at column face and midspan respectively. Fig. 5.4 shows the variation of $M_z$ along Z direction (across the width) of lines 10 to 17 of Fig. 5.1. From Fig. 5.4 it is observed that high positive and negative bending moment $M_z$ and a less positive and negative bending moment $M_x$ acts in the column line and in between the columns, respectively. Across the width, high positive and negative moment acts upto the width of thickened zone. Sharp change in moment variation across the width has been observed from the start of the smaller thickness zone. A panel can be divided into two strips across the width, a higher moment zone in the thicker zone including the slope width and a lower moment zone in between the thicker zone.

Like flat slab mat panel can be divided in two strips: column strip and middle strip. Column strip varies from starting to end of thickened zone and middle strip lies in between the column strips of the mat.
Fig. 5.2 Variation of vertical deflection of mat along different lines.

Fig. 5.3a Variation of bending moment $M_x$ of mat along lines 1,2.
Fig. 5.3b Variation of bending moment $M_x$ of mat along line 3.

Fig. 5.3c Variation of bending moment $M_x$ of mat along line 4,5.
Fig. 5.3d Variation of bending moment $M_x$ of mat along line 6.

Fig. 5.3e Variation of bending moment $M_x$ of mat along line 7,8.
Fig. 5.3f Variation of bending moment $M_x$ of mat along line 9.

Fig. 5.4a Variation of bending moment $M_x$ of mat along line 10.
Fig. 5.4b Variation of bending moment $M_x$ of mat along line 11.

Fig. 5.4c Variation of bending moment $M_x$ of mat along lines 12,13.
Fig. 5.4d Variation of bending moment $M_x$ of mat along line 14.

Fig. 5.4e Variation of bending moment $M_x$ of mat along lines 15,16.
A constant variation of bending moment can be assumed across the width of column strip and middle strip from Fig 5.4. Mat panel can be considered as total beam. Column strip and middle strip total moment at column faces and midspan is calculated by trapezoidal rule from Fig 5.4 and summarized in Table 5.3. Column face positive moment is the sum of column strip and middle strip positive moments at column faces. Midspan negative moment is the sum of negative moments for column strip and middle strip. Like flat slab, it has been observed that sum of averages of the positive moments at column faces and midspan negative moment, termed as panel moment, is equal to the midspan positive moment of a corresponding simply supported beam of the mat panel.

From Table 5.3 it is observed that total panel moment is 8052.94 kip-ft. Midspan positive moment of a corresponding simply supported beam 8278.3 kip-ft assuming column load is distributed uniformly under the mat area.

Table 5.3 Positive and negative moment for an interior panel

<table>
<thead>
<tr>
<th></th>
<th>Positive Moment (Kip-ft)</th>
<th>Negative Moment (Kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column Strip</td>
<td>4910.00</td>
<td>-897.54</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>1149.00</td>
<td>-1096.40</td>
</tr>
<tr>
<td>Total Panel</td>
<td>6059.00</td>
<td>-1993.94</td>
</tr>
<tr>
<td>Panel moment</td>
<td>8052.94</td>
<td></td>
</tr>
</tbody>
</table>

5.2.3 Variation of Shearing Stress

Variation of shear stress, $S_{xy}$ along lines parallel to X direction and along lines parallel to Z direction of Fig. 5.1 has been shown in Fig. 5.5 and Fig. 5.6 respectively. From Fig. 5.5, local increase in shearing stress has been observed in the transition zone of greater thickness to smaller thickness in the column strip. For flexural shear, neck section is the most critical section. As mat is a slab type structure, local failure due to flexural shear cannot occur. For a failure due to
Fig. 5.4f Variation of bending moment $M_x$ of mat along lines 17.

Fig. 5.5 Variation of Shearing Stress $S_{xy}$ of mat along Different lines.
Flexural shear, it must occur along a line across the width. Fig 5.7 shows the variation of shearing stress along with capacity due to flexure across the width at the start of smaller thickness (neck section). This situation is same for a single isolated footing under a column, where local flexural shearing stress near the column is higher than the average shear stress and it is designed assuming an average distribution across the width. From Fig. 5.7, capacity of the interior neck section for flexural shear is 2167.6 kip and flexural shear force is 1900.8 kip. For design consideration it is safe to provide a smaller thickness 0.60 \( t_s \) where \( t_s \) is greater thickness near the column face. Fig. 5.6 clearly shows the variation of shearing stress across the width, in column strip and in middle strip along lines 10 to 17.

Punching shear should be checked at \( d/2 \) distance from the column face and at the sections which can be critical. Column C3 has the highest loading. Critical sections are at \( d/2 \) distance from column face and at the neck section. Punching shear force has been calculated by deducting the spring reactions in the inner perimeter of the critical section from the column load. For column C3, punching shear force observed at \( d/2 \) distance away from column face and at neck section is 2576.8 kip and 2269.1 kip, respectively. Capacity for the punching shear at \( d/2 \) distance away from the column face and at the neck section are 2600.1 kip and 2477.20 kip, respectively. For punching shear consideration, smaller thickness 0.60 \( t_s \) is sufficient.

5.2.4 Variation of Spring Reaction

Variation of spring reaction is shown in Fig. 5.8 in X-direction along lines 1 to 8 of Fig. 5.1. Variation of spring reaction is pronounced in exterior panel. Spring reaction shows more variation along the column lines (lines 2, 5 & 8) than that of lines in between the columns (lines 3, 6). Spring reaction is higher near the columns. This reveals that reactive soil pressure variation beneath the mat foundation is not uniform, more pressure below the column and nearby areas and less pressures away from the columns, than the average pressure, obtained by dividing summation of the column loads by the plan area of the mat. Therefore, panel moment calculated by assuming an average pressure distribution for the corresponding simply supported beam will always be on the conservative side compared to the actual panel moment.
Fig. 5.6 Variation of Shearing Stress $S_{xy}$ of mat along different lines.

Fig 5.7 Variation of flexural shearing stress and flexural shear capacity of the neck section at interior span.
Fig. 5.8  Variation of Spring reaction along (a) lines 1-4 (b) lines 6-8
5.3 PARAMETRIC STUDY ON DISTRIBUTION OF BENDING MOMENT OF MAT FOUNDATION

Sensitivity study in chapter 4 reveals that positive and negative bending moment varies significantly for mat with variable thickness with variation in modulus of subgrade reaction. Distribution of these moments to column strip and middle strip depends on the length to width ratio of the mat panel. Study of mat foundation analysis on different L/B ratio has been carried out for different modulus of subgrade reaction.

In flat slab construction, in a column supported slab, requirement of statics says that 100 percent of the applied load must be carried in each direction. Mat can be considered as an inverted flat slab and 100% of the load should be carried in each direction. Panel moment should be calculated considering the mat spanning in either direction. This panel moment is distributed in column strip and middle strip critical sections. To identify the distribution of moments in between column strip and middle strip total nine mat of different L/B ratio has been studied with variation of modulus of subgrade reaction. L/B ratio has been varied in the most practicable range for 0.615, 0.77, 1.0, 1.30 and 1.625. For each case modulus of subgrade reaction is varied for 50 kcf, 100 kcf and 200 kcf. Mat geometry, loading of mat from columns and variation of deflection, bending moment and shear stress for different L/B ratio and for different modulus of subgrade reaction are shown in Appendix D.

A uniformly distributed load corresponding to vertical load is calculated dividing summation of column loads by the mat area. Total panel moment, $M_i$, is calculated by the following formula

$$M_i = w \times L \times B \times L_{ci} / 8$$

Where

- $w = \frac{\sum \text{Column Loads}}{\text{Total mat area}}$
- $L = \text{C/C span length of the span}$
- $B = \text{C/C width of the panel}$
- $L_{ci} = \text{Clear Span from column face to face.}$
For different L/B ratio, average pressure, span length and width, panel moment is given in Table 5.4.

Table 5.4  Panel moment for different L/B ratio.

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Span Length, L, ft</th>
<th>Panel Width, B, ft</th>
<th>Average Soil Pressure, ksf</th>
<th>Panel Moment, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.615</td>
<td>16</td>
<td>26</td>
<td>4.625</td>
<td>3552.2</td>
</tr>
<tr>
<td>0.77</td>
<td>20</td>
<td>26</td>
<td>4.492</td>
<td>5390.4</td>
</tr>
<tr>
<td>1.0</td>
<td>26</td>
<td>26</td>
<td>4.082</td>
<td>8278.3</td>
</tr>
<tr>
<td>1.30</td>
<td>26</td>
<td>20</td>
<td>4.492</td>
<td>7007.5</td>
</tr>
<tr>
<td>1.625</td>
<td>26</td>
<td>16</td>
<td>4.625</td>
<td>5771.7</td>
</tr>
</tbody>
</table>

Total moment for column strip and middle strip across the width at column faces and midspan has been calculated by trapezoidal rule. This calculation has been done for interior panel and exterior panel for interior span and exterior span (Fig 5.9) separately. Summary of total moment in column strips and middle strips for different L/B ratio has been given in Table 5.5 to Table 5.7 for different modulus of subgrade reaction. Column face and midspan moment varies for interior span and exterior span of interior panel and exterior panel.

Ratio of column strip and middle strip moments to total panel moment is termed as distribution coefficients. To understand the distribution of panel moments to different sections, total moments for column strip and middle strip sections has been expressed as the fraction of total panel moment. Distribution coefficients for positive and negative bending moments for column strip and middle strip is given in Table 5.8 to Table 5.10.
Definition of column strip and middle strip of interior panel and exterior panel.
Table 5.5  Positive and negative bending moment in column strip and middle strip for $k_5 = 50$ kcf.

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
</tr>
<tr>
<td>1.625</td>
<td>2572.92</td>
<td>-433.68</td>
</tr>
<tr>
<td>1.3</td>
<td>3586.11</td>
<td>-574.38</td>
</tr>
<tr>
<td>1.0</td>
<td>5043.00</td>
<td>-869.82</td>
</tr>
<tr>
<td>0.77</td>
<td>3932.58</td>
<td>-689.19</td>
</tr>
<tr>
<td>0.615</td>
<td>3097.00</td>
<td>-560.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
</tr>
<tr>
<td>1.625</td>
<td>436.02</td>
<td>-513.96</td>
</tr>
<tr>
<td>1.3</td>
<td>748.60</td>
<td>-556.62</td>
</tr>
<tr>
<td>1.0</td>
<td>1231.53</td>
<td>-1070.62</td>
</tr>
<tr>
<td>0.77</td>
<td>1166.92</td>
<td>-954.75</td>
</tr>
<tr>
<td>0.615</td>
<td>1030.73</td>
<td>-831.74</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
</tr>
<tr>
<td>1.625</td>
<td>1484.80</td>
<td>-464.17</td>
</tr>
<tr>
<td>1.3</td>
<td>2443.38</td>
<td>-562.92</td>
</tr>
<tr>
<td>1.0</td>
<td>3537.79</td>
<td>-784.15</td>
</tr>
<tr>
<td>0.77</td>
<td>3008.41</td>
<td>-663.12</td>
</tr>
<tr>
<td>0.615</td>
<td>2656.62</td>
<td>-578.24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
</tr>
<tr>
<td>1.625</td>
<td>-82.58</td>
<td>-530.04</td>
</tr>
<tr>
<td>1.3</td>
<td>176.36</td>
<td>-619.51</td>
</tr>
<tr>
<td>1.0</td>
<td>515.74</td>
<td>-921.71</td>
</tr>
<tr>
<td>0.77</td>
<td>639.18</td>
<td>-870.89</td>
</tr>
<tr>
<td>0.615</td>
<td>671.66</td>
<td>-806.35</td>
</tr>
</tbody>
</table>
Table 5.6  Positive and negative bending moment in column strip and middle strip for $k = 100$ kcf.

| L/B ratio | Interior Panel - Column Strip |  |  |
|-----------|-------------------------------|-------------------------------|
|           | Column face Moment (kip-ft)   | Midspan (kip-ft)              | Ext. Face of Int. Column (kip-ft) | Ext. Face of Ext. Column (kip-ft) |
| 1.625     | 2540.85                       | -269.76                       | 2120.31                        | -937.61                        |
| 1.3       | 3503.98                       | -567.82                       | 3199.18                        | -1106.06                       |
| 1.0       | 4910.00                       | -897.54                       | 4935.00                        | -975.20                        |
| 0.77      | 3827.77                       | -707.45                       | 3892.64                        | -957.22                        |
| 0.615     | 3005.93                       | -571.23                       | 3101.44                        | -703.67                        |

| L/B ratio | Interior Panel - Middle Strip |  |  |
|-----------|-------------------------------|-------------------------------|
|           | Column face Moment (kip-ft)   | Midspan (kip-ft)              | Ext. Face of Int. Column (kip-ft) | Ext. Face of Ext. Column (kip-ft) |
| 1.625     | 405.50                        | -435.6                        | 2042.34                        | -937.35                        |
| 1.3       | 723.50                        | -621.56                       | 523.66                         | -1453.72                       |
| 1.0       | 1149.00                       | -1096.4                       | 1188.98                        | -1650.52                       |
| 0.77      | 3827.77                       | -707.45                       | 1146.44                        | -1414.12                       |
| 0.615     | 979.24                        | -851.91                       | 1032.80                        | -1078.20                       |

| L/B ratio | Exterior Panel - Column Strip |  |  |
|-----------|-------------------------------|-------------------------------|
|           | Column face Moment (kip-ft)   | Midspan (kip-ft)              | Ext. Face of Int. Column (kip-ft) | Ext. Face of Ext. Column (kip-ft) |
| 1.625     | 1489.75                       | -392.66                       | 1316.4                         | -848.08                        |
| 1.3       | 2375.4                        | -478.60                       | 2093.77                        | -1012.88                       |
| 1.0       | 3434.5                        | -785.4                        | 3483.20                        | -1107.80                       |
| 0.77      | 2919.9                        | -671.75                       | 2962.95                        | -938.40                        |
| 0.615     | 2489.78                       | -583.88                       | 2553.04                        | -761.24                        |

| L/B ratio | Exterior Panel - Middle Strip |  |  |
|-----------|-------------------------------|-------------------------------|
|           | Column face Moment (kip-ft)   | Midspan (kip-ft)              | Ext. Face of Int. Column (kip-ft) | Ext. Face of Ext. Column (kip-ft) |
| 1.625     | 36.76                         | -3674.66                      | -229.40                        | -1033.32                       |
| 1.3       | 187.08                        | -547.14                       | 42.40                          | -1268.64                       |
| 1.0       | 514.58                        | 907.30                        | 556.58                         | -1423.48                       |
| 0.77      | 618.99                        | -874.98                       | 669.74                         | -1305.66                       |
| 0.615     | 647.29                        | -810.38                       | 695.03                         | -1096.38                       |

52
Table 5.7 Positive and negative bending moment in column strip and middle strip for \( k = 200 \text{ kcf} \).

### Interior Panel - Column Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
</tr>
<tr>
<td>1.625</td>
<td>2490.26</td>
<td>-383.52</td>
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<td>1.3</td>
<td>3421.10</td>
<td>-583.42</td>
</tr>
<tr>
<td>1.0</td>
<td>4763.99</td>
<td>-895.11</td>
</tr>
<tr>
<td>0.77</td>
<td>3698.40</td>
<td>-699.52</td>
</tr>
<tr>
<td>0.615</td>
<td>2885.3</td>
<td>-552.22</td>
</tr>
</tbody>
</table>

### Interior Panel - Middle Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
</tr>
<tr>
<td>1.625</td>
<td>1470.26</td>
<td>-343.38</td>
</tr>
<tr>
<td>1.3</td>
<td>2305.05</td>
<td>-506.39</td>
</tr>
<tr>
<td>1.0</td>
<td>3309.31</td>
<td>-759.44</td>
</tr>
<tr>
<td>0.77</td>
<td>2814.02</td>
<td>-656.44</td>
</tr>
<tr>
<td>0.615</td>
<td>2388.64</td>
<td>-561.93</td>
</tr>
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### Exterior Panel - Column Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
</tr>
<tr>
<td>1.625</td>
<td>57.66</td>
<td>-321.59</td>
</tr>
<tr>
<td>1.3</td>
<td>205.60</td>
<td>-501.48</td>
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<tr>
<td>1.0</td>
<td>570.56</td>
<td>-852.69</td>
</tr>
<tr>
<td>0.77</td>
<td>592.79</td>
<td>-841.80</td>
</tr>
<tr>
<td>0.615</td>
<td>611.86</td>
<td>-770.66</td>
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</tbody>
</table>

### Exterior Panel - Middle Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
</tr>
<tr>
<td>1.625</td>
<td>57.66</td>
<td>-321.59</td>
</tr>
<tr>
<td>1.3</td>
<td>205.60</td>
<td>-501.48</td>
</tr>
<tr>
<td>1.0</td>
<td>570.56</td>
<td>-852.69</td>
</tr>
<tr>
<td>0.77</td>
<td>592.79</td>
<td>-841.80</td>
</tr>
<tr>
<td>0.615</td>
<td>611.86</td>
<td>-770.66</td>
</tr>
</tbody>
</table>
Table 5.8 Distribution coefficients for positive and negative bending moments in column strip and middle strip for $k = 50$ kcf.

### Interior Panel - Column Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment</td>
<td>Midspan</td>
<td>Ext. Face of Int. Column</td>
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<tr>
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<td>0.5365</td>
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### Interior Panel - Middle Strip

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<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>Column face Moment</td>
<td>Midspan</td>
<td>Ext. Face of Int. Column</td>
</tr>
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<td>1.625</td>
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### Exterior Panel - Column Strip

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</tr>
</thead>
<tbody>
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<td>Midspan</td>
<td>Ext. Face of Int. Column</td>
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<td>0.44742</td>
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### Exterior Panel - Middle Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment</td>
<td>Midspan</td>
<td>Ext. Face of Int. Column</td>
</tr>
<tr>
<td>1.625</td>
<td>-0.02325</td>
<td>-0.1492</td>
<td>-0.10613</td>
</tr>
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<td>1.3</td>
<td>0.03272</td>
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<td>-0.02427</td>
</tr>
<tr>
<td>1.0</td>
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<td>0.06065</td>
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<tr>
<td>0.77</td>
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<td>0.092625</td>
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<td>0.11637</td>
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</table>
Table 5.9  Distribution coefficients for Positive and negative bending moments in column strip and middle strip for k = 100 kcf.

### Interior Panel - Column Strip

<table>
<thead>
<tr>
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<th>Interior Span</th>
<th>Exterior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
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<td>Column face Moment</td>
<td>Midspan</td>
<td>Ext. Face of Int. Column</td>
</tr>
<tr>
<td>1.625</td>
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<td>0.596</td>
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<td>0.5935</td>
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<td>0.596</td>
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### Interior Panel - Middle Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment</td>
<td>Midspan</td>
<td>Ext. Face of Int. Column</td>
</tr>
<tr>
<td>1.625</td>
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<td>0.77</td>
<td>0.1566</td>
<td>-0.1401</td>
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<td>0.615</td>
<td>0.1697</td>
<td>-0.1476</td>
<td>0.179</td>
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### Exterior Panel - Column Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment</td>
<td>Midspan</td>
<td>Ext. Face of Int. Column</td>
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<tr>
<td>1.625</td>
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<td>0.4149</td>
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<td>0.615</td>
<td>0.4314</td>
<td>-0.10116</td>
<td>0.4423</td>
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</table>

### Exterior Panel - Middle Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment</td>
<td>Midspan</td>
<td>Ext. Face of Int. Column</td>
</tr>
<tr>
<td>1.625</td>
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<td>0.0672</td>
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<td>0.09558</td>
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<td>0.615</td>
<td>0.11215</td>
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<td>0.1204</td>
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</table>
Table 5.10 Distribution coefficients for Positive and negative bending moments in column strip and middle strip for $k = 200$ kcf.

### Interior Panel - Column Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment</td>
<td>Midspan</td>
</tr>
<tr>
<td>1.625</td>
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<td>0.52777</td>
<td>-0.09982</td>
</tr>
<tr>
<td>0.615</td>
<td>0.4999</td>
<td>-0.09567</td>
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</tbody>
</table>

### Interior Panel - Middle Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment</td>
<td>Midspan</td>
</tr>
<tr>
<td>1.625</td>
<td>0.10312</td>
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<td>0.115965</td>
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<tr>
<td>1.0</td>
<td>0.12763</td>
<td>-0.12915</td>
</tr>
<tr>
<td>0.77</td>
<td>0.14555</td>
<td>-0.13742</td>
</tr>
<tr>
<td>0.615</td>
<td>0.15787</td>
<td>-0.14237</td>
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</tbody>
</table>

### Exterior Panel - Column Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment</td>
<td>Midspan</td>
</tr>
<tr>
<td>1.625</td>
<td>0.4139</td>
<td>-0.09666</td>
</tr>
<tr>
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<tr>
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<tr>
<td>0.77</td>
<td>0.40157</td>
<td>-0.09368</td>
</tr>
<tr>
<td>0.615</td>
<td>0.4139</td>
<td>-0.09736</td>
</tr>
</tbody>
</table>

### Exterior Panel - Middle Strip

<table>
<thead>
<tr>
<th>L/B ratio</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment</td>
<td>Midspan</td>
</tr>
<tr>
<td>1.625</td>
<td>0.01623</td>
<td>-0.0905</td>
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<td>0.03814</td>
<td>-0.09303</td>
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<td>0.06895</td>
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<td>0.77</td>
<td>0.08177</td>
<td>-0.12013</td>
</tr>
<tr>
<td>0.615</td>
<td>0.106009</td>
<td>-0.13352</td>
</tr>
</tbody>
</table>
5.3.1 Summary of Variation of L/B Ratio of Mat

Length to width ratio has been varied from 0.615 to 1.625, which covers the most of the practical ranges. Distribution coefficients vary for interior panel and exterior panel.

(i) Effect on Distribution Coefficients for Interior Panel

As the length to width span ratio increases, interior panel distribution coefficients for
• column strip positive moment increases rapidly for interior span (Fig. 5.10a).
• column strip positive moment initially increases for L/B ratio upto 1.0, after that remains constant for interior column of exterior span (Fig. 5.10a).
• column strip positive moment increases for exterior column for exterior span (Fig. 5.10a).
• column strip negative moment decreases slightly for interior span (Fig. 5.10a).
• column strip negative moment increases rapidly for exterior span (Fig. 5.10a).
• middle strip positive moment decreases rapidly for interior column of interior span (Fig. 5.10b).
• middle strip positive moment decreases rapidly for interior column of exterior span (Fig. 5.10b).
• exterior end of exterior span for middle strip has negative moment and it increases rapidly (Fig. 5.10b).
• middle strip negative moment decreases for interior span (Fig. 5.10b).
• middle strip negative moment for exterior span remains unchanged for L/B ratio equal to 1.0 and after that value increases rapidly (Fig. 5.10b).

(ii) Effect on Distribution Coefficients for Exterior Panel

As the length to width span ratio increases, exterior panel distribution coefficients for
Fig 5.10 Distribution Coefficients at critical sections of interior panel (a) column strip (b) middle strip against L/B ratio for k=50 kcf.
• column strip positive moment decreases slightly for interior span (Fig. 5.10c).
• column strip positive moment for interior column of exterior span decreases (Fig. 5.10c).
• column strip positive moment for exterior column of exterior span remains more or less unchanged (Fig. 5.10c).
• column strip negative moment increases slightly for interior span (Fig. 5.10c).
• column strip negative moment for exterior span remains unchanged up to L/B ratio equal to 1.0, after that value it increases rapidly with increasing L/B ratio (Fig. 5.10c).
• middle strip positive moment decreases rapidly for interior span (Fig. 5.10d)
• middle strip positive moment for exterior face of interior column for exterior span decreases at a very fast rate (Fig. 5.10d).
• interior face of exterior column for middle strip of exterior span has negative moment and it increases rapidly (Fig. 5.10d).
• middle strip negative moment decreases for interior span (Fig. 5.10d).
• middle strip negative moment of exterior span decreases at a faster rate for exterior span (Fig. 5.10d).

5.3.2 Summary on Variation of Modulus of Subgrade Reaction

Modulus of subgrade reaction varies from 50 kcf to 200 kcf for different L/B ratio, which covers the most practical ranges. Modulus of subgrade reaction effects the distribution coefficients for non-uniform mat foundation. The values of distribution coefficients for different modulus of subgrade reaction for L/B ratio equal to 1.0 are tabulated in Table 5.11 and Table 5.12.
Fig 5.10  Distribution Coefficients at critical sections of exterior panel (c) column 
Strip (d) middle strip against L/B ratio for k=50 kcf.
Fig 5.11  Distribution Coefficients at critical sections of interior panel (a) column strip (b) middle strip against L/B ratio for k=100 kcf.
Fig 5.11  Distribution Coefficients at critical sections of exterior panel (c) column Strip (d) middle strip against L/B ratio for k=100 kcf.
Fig 5.12 Distribution Coefficients at critical sections of interior panel (a) column Strip (b) middle strip against L/B ratio for k=200 kcf.
Fig 5.12  Distribution Coefficients at critical sections of exterior panel (c) column Strip (d) middle strip against L/B ratio for k=200 kcf.
Table 5.11 Distribution coefficients for positive and negative bending moments of interior panel with variation of different modulus of subgrade reaction.

<table>
<thead>
<tr>
<th>Modulus of Subgrade Reaction kcf</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face</td>
<td>Midspan</td>
</tr>
<tr>
<td>50</td>
<td>0.60918</td>
<td>-0.10507</td>
</tr>
<tr>
<td>100</td>
<td>0.59400</td>
<td>-0.10840</td>
</tr>
<tr>
<td>200</td>
<td>0.57548</td>
<td>-0.10813</td>
</tr>
</tbody>
</table>

Table 5.12 Distribution coefficients for positive and negative bending moments of exterior panel with variation of different modulus of subgrade reaction.

<table>
<thead>
<tr>
<th>Modulus of Subgrade Reaction kcf</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face</td>
<td>Midspan</td>
</tr>
<tr>
<td>50</td>
<td>0.14877</td>
<td>-0.12933</td>
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<tr>
<td>200</td>
<td>0.12763</td>
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</table>

Exterior Panel- Column Strip

<table>
<thead>
<tr>
<th>Modulus of Subgrade Reaction kcf</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Column face</td>
<td>Midspan</td>
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<td>100</td>
<td>0.41490</td>
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<tr>
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<td>0.39976</td>
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Exterior Panel- Middle Strip

<table>
<thead>
<tr>
<th>Modulus of Subgrade Reaction kcf</th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face</td>
<td>Midspan</td>
</tr>
<tr>
<td>50</td>
<td>0.06230</td>
<td>-0.11134</td>
</tr>
<tr>
<td>100</td>
<td>0.06220</td>
<td>-0.10960</td>
</tr>
<tr>
<td>200</td>
<td>0.06895</td>
<td>-0.10300</td>
</tr>
</tbody>
</table>
(i) **Effect on Distribution Coefficients For Interior Panel**

As the modulus of subgrade reaction increases interior panel distribution coefficients for

- column strip positive moment decreases slightly for interior span (Table 5.11).
- column strip positive moment remains more or less unchanged for interior column of exterior span (Table 5.11).
- column strip positive moment increases slowly for exterior column of exterior span (Table 5.11).
- column strip negative moment remains unchanged for interior span (Table 5.11).
- column strip negative moment decreases rapidly for midspan of exterior span (Table 5.11).
- middle strip positive moment decreases for column face of interior span (Table 5.11).
- middle strip positive moment increases for interior column of exterior span (Table 5.11).
- exterior end of exterior span for middle strip has negative moment and it decreases moderately (Table 5.11).
- middle strip negative moment decreases for interior span (Table 5.11).
- middle strip negative moment decreases rapidly for exterior span (Table 5.11).

(ii) **Effect on Distribution Coefficients for Exterior Panel**

As the modulus of subgrade reaction increases, exterior panel distribution coefficients for

- column strip positive moment decreases moderately for interior span (Table 5.12).
- column strip positive moment remains unchanged for interior column of exterior span (Table 5.12).
• column strip positive moment for exterior column of exterior span increases (Table 5.12).
• column strip negative moment decreases slightly for interior span (Table 5.12).
• column strip negative moment decreases moderately for exterior span (Table 5.12).
• middle strip positive moment increases slightly for interior span (Table 5.12).
• middle strip positive moment for exterior face of interior column for exterior span changes slightly (Table 5.12).
• interior face of exterior column for middle strip of exterior span has negative moment and it decreases rapidly (Table 5.12).
• middle strip negative moment for interior span decreases (Table 5.12).
• middle strip negative moment decreases rapidly for exterior span (Table 5.12).

5.3 DISCUSSION ON ANALYSIS OF MAT FOUNDATION

Mat foundation has loading from column as concentrated load and reactive soil pressure has variation, which has high value near the column face and lower value near the midspan. The mat panel is assumed as a total beam and total moment in a panel is calculated for the panel as simply supported beam. The total panel moment is on the conservative side. Panel moment is calculated for each panel for each direction. This panel moment is distributed in column strip and middle strip critical sections.

Distribution of total panel moment for mat with variable thickness depends largely on the length to width span ratio of the panel and in some extent on the modulus of subgrade reaction. The distribution of this total panel moment as positive and negative moments to critical sections of column strip and middle strip can be calculated from Fig. 5.10 to 5.12 depending on L/B span ratio and modulus of subgrade reaction.

This method is valid until vertical load governs the design. In most buildings up to fifteen story, dead load along with reduced live load is the governing case. Up to that range a mat of regular grid design is very simple. When a complicated case arises, one should go for finite element solution.
CHAPTER 6

DESIGN OF MAT FOUNDATION

6.1 INTRODUCTION

Relative economy in design can be achieved by finite element analysis. Confidence will be gained in more economic shaping and dimensioning of mat foundation. Ultimate strength design method of American Concrete Institute is followed.

6.2 DIFFICULTIES IN FORMULATING THE MAT BEHAVIOR

Some structures are complicated to analyze but simple to categorize. For example, cassion foundation analysis is difficult but its load pattern and geometry do not vary much. So, an approximate analysis based on parametric study is simple to develop and simpler and efficient in use.

Mat is simple. Once everything is set by engineers and architects for a particular project, it has a relatively small number of parameters to be dealt with, such as elastic properties of mat material, modulus of subgrade reaction of the supporting soil and thickness. Difficulty with the formulation of the characteristics of mat arises from its loading pattern.

Structurally mat acts integrally with the building frame it supports and resists loads which do not act on it directly but comes through the frame. To analyze mat, separately moments and axial forces produced in response to external loads acting on the frame is calculated first and applied on mat later. Locations of these columns i.e. locations of these loads depend on architectural planning of the building and
column spacing. Two other important factors involved in load calculation is number of stories and the number of bays of the building frame supported by the mat.

Usually, columns having more or less uniform spacing follows a regular grid pattern specially for up to fifteen storied building structures. Magnitude of load varies depending on column spacing and number of stories. Number of bays also can vary. Mat foundation is provided beneath the super structure to distribute the load to substratum with uniform pressure.

It is possible to obtain an approximate solution for mat foundation for having column spacing and location in a regular grid pattern which is very common in most cases. For unusual, irregular cases, one must go for finite element solution.

Non-uniform mat foundation is compared with flat slabs with drop panels. Flat slab is subjected to uniform pressure from dead load and live load where slab is supported on columns at the corner of the slab. Similar is the situation in the case of non-uniform mat foundation. It can be considered as an inverted flat slab. Column load is applied on the thick slab and reactive soil pressure generates on the bottom face of the slab.

This reactive soil pressure is not uniform but varies a lot along the span. High reactive soil pressure acts under the column and its surrounding zone. This pressure is less away from the column face, near the midspan.

In finite element analysis, soil is modeled with spring elements. Modulus of subgrade reaction is concentrated at the corner nodes of element. Reaction of spring represents the soil pressure. Variation of spring reaction in mat foundation and in an interior panel is shown by contour diagram of Fig. 6.1 and Fig. 6.2 respectively. In Fig. 6.3 spring reaction variation is shown along column line and line in between the columns.
Fig. 6.1 Variation of spring reaction in the plan area of mat.

Fig. 6.2 Variation of spring reaction in an interior panel of mat.
Fig 6.3 Variation of spring reaction along column lines and in centerline in between the columns.

6.3 GENERAL DESIGN REQUIREMENTS FOR MAT FOUNDATION

Mat is a two way flexural member supporting super structures and directly supported by soil. Naturally, design of mat is governed by deflection, bending moment and shear force considerations. In order to be structurally safe, mat must be strong enough to resist the moments and shears produced by the column loads. Deflection of mat must be within allowable limits to avoid any damage to superstructure.

6.3.1 Deflection Requirements For Mat Foundation

Flotation effect and plate behavior enable most settlements to be limited to tolerable magnitudes. Also, it is easier to conceal any reasonable amount of overall settlement by providing higher plinth level initially. A problem of considerable concern is differential settlement, which may induce detrimental stresses in the superstructure. This is why strict control of differential settlement is necessary.
Mc Donals and Skempton (1955) made a study of 98 buildings which was later substantiated by Grant et. Al. (1974) from a study of 95 buildings of more recent construction. Also, Fled (1965) investigated a large number of specific structures. All these sources, give rise to a conclusion that for mat foundation on clay, recommended design differential settlement is 2.5 to 4 inches and for satisfactory differential settlement should never be greater than 3 to 5 inches. For mat on sand the respective ranges are 1.5 to 2.5 inches and 2 to 3 inches. These are however safe ranges determined from only statistical surveys. According to USSR building code, acceptable slope between two points of mat supporting concrete frame structures is 0.002, with an average maximum settlement of 4 inches.

According to BNBC, allowable or limiting settlement of a building structure will depend on nature of the structure, the foundation and the soil. As a general rule, a total settlement of 25 mm and a differential settlement of 20 mm between the columns in most buildings shall be considered safe for buildings on isolated footings. Buildings on raft can usually tolerate greater total settlements.

6.3.2 Design of Mat Under Bending Moment

Design of mat for bending moment is straightforward. Since mat suffers from two directional bending, strips of mat is selected in two perpendicular directions and the strips are designed as beams for critical positive (upside compression) and negative (bottom side compression) moments. ACI code restricts that a minimum steel ratio be maintained for any section, both near the top and near the bottom of mat. Although design positive moments should be those at the faces of the columns according to ACI code. Formula used for calculation are given below.

Effective Depth, \( d \) = \( t \) - clear cover.

Steel Ratio, \( \rho \) = \( \frac{A_s}{bd} \)

Maximum Steel ratio, \( \rho_{\text{max}} \) = \( 0.75 \times 0.85 \times \beta \times \frac{f_c}{f_y} \times \frac{87000}{87000 + f_y} \)
6.3.3 Design of Mat Under Shear Force

Perhaps the most important design criteria for mat foundation is the shear force. Since mat is a plate structure, shear reinforcements is avoided for practical considerations. Instead mat is made sufficiently thick to encounter shear. Presence of heavy column loads concentrated on very small areas makes considerations of punching shear, in addition to flexural shear, essential. As per ACI code, punching shear is calculated as the total shear force acting on the surface of a peripheral section around the column under consideration, taken at distance \( \frac{d}{2} \) from each face of that column, where \( d \) is the effective depth of mat. Physically, punching shear is the column load less the total soil pressure under the punching area of that column. Both the flexural and punching shear stresses must be lower than the respective ACI shear capacities of the mat section. According to ACI code, flexural shears are to be considered are those at a distance \( d \) away from the column faces. ACI design formula for shear capacities are

\[
\begin{align*}
\beta_i & = 0.85 - 0.05 \times \frac{f_c - 4000}{1000} \quad \text{and} \quad 0.65 \leq \beta_i \leq 0.85 \\
\text{Minimum Steel ratio, } \rho_{\text{min}} & = \frac{200}{f_y} \quad (f_y \text{ in psi}) \\
\text{Ultimate Moment capacity, } M_u & = \phi \sigma_{f_y} bd^2 \left(1 - 0.59 \frac{\rho_{f_y}}{f_c}\right)
\end{align*}
\]

Where \( \phi \) is the strength reduction factor and equal to 0.85.

6.4 DESIGN REQUIREMENTS FOR MAT FOUNDATION WITH VARIABLE THICKNESS

For structural design of mat, differential settlement is more important than total settlement. For structural consideration of a building structure, column to column differential settlement is the main concern. Use of non-uniform mat foundation
slightly increases this differential settlement but it remains well below the allowable range.

Plan area of mat can be divided into two distinct zones. The first of these includes neighborhoods of the columns where high shears and positive moments occur and the second region consists of the areas away from the columns which are subject to high negative moments but low shear forces.

Morshed (1996) and Sutradhar (1999) proposed that thicker part of the mat foundation is designed considering the punching shear force. This thicker part is proposed to extend equal to effective depth away from the column face and sloping part width equal to 0.577 times the greater thickness. This proposal for length of thicker part is constant for all span length. It should vary with the span length. In flat slab, drop panel or thickened part shall be at least 0.33 times of the span. But it should be sufficient to keep the punching shear stress below the allowable limit. Smaller slab thickness is taken as 2/3 of greater thickness for flat slab with drop panel. In mat foundation of variable thickness Sutradhar proposed to keep smaller thickness 2/3 of greater thickness as in flat slab. From parametric study on smaller thickness in Chapter 4 it has been observed that smaller thickness can be taken 60% of greater thickness.

Thicker part of the mat can be provided in two ways, extra thickness below the slab or extra thickness over the slab.

From structural point of view second one is always preferable. For this type extra thickness always remains on the compression side. In addition to shear and vertical compression stresses, horizontal compressive stresses caused by the bending moments act above the neutral plane on a perimeter section. When the extra thickness is below the slab, it is on the tension side. Horizontal tensile stresses caused by the bending moments act below the neutral plane.
From architectural and commercial point of view, extra thickness of the mat over the slab is not preferable. Since this place is used for storage and car parking. The limitation of this type can be overcome very easily by filling the hollow over the slab with low cost materials like sand, bricks and providing a thin layer of concrete over it to get a level surface.

From construction point of view, second one is preferable. It is always easy to construct additional thickness over the slab.

Mat of variable thickness will not only reduce the concrete volume but also reduce the reinforcement where minimum reinforcement governs.

6.5 A NEW DESIGN APPROACH FOR MAT WITH VARIABLE THICKNESS

The parametric study reveals that the design of regular shaped mat with variable thickness follow some well defined trends. The behavior of such a mat is compared with flat slab having drop panel and it is considered as inverted flat slab. Based on these findings, a design approach for mat with variable thickness is presented in the following steps:

Step 1: Mat foundation is modeled in the following way

(i) The problem should be idealised first with a geometry of variable thickness where extra thickness is on the top face of the slab.

(ii) Greater thickness, $t_g$ for mat is calculated from punching shear estimated as $0.95 \times$ maximum column axial force.

(iii) Smaller thickness, $t_s$ can be taken as $0.60 \times t_g$. 

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(iii) Width of greater thickness shall not be less than \( d_e \) from the column face where \( d_e \) is effective depth.

(iv) Width of the sloping part from the greater thickness to smaller thickness should be such that the length of thicker zone is at least equal to 0.40 times the span in that direction (Fig. 6.4).

Step 2: An approximate analysis can be done instead of finite element analysis as follows

(i) An equivalent distributed load, \( w \) for the vertical load i.e. dead load along with live load can be obtained by

\[
w = \frac{\sum \text{Column Loads}}{\text{Total Mat Area}}
\]

(ii) Mat is assumed to span in each direction. Steps (ii) to (vii) shall be repeated in each direction.

(iii) Mat panel is divided into column strip and middle strip. Column strip width ranges from start of the sloping part to the end of sloping part including the width of constant greater thickness. Middle strip width ranges in between column strips.
Total panel moment, $M_t$, can be obtained by

$$M_t = \frac{w \times L_c \times B_c \times L_{cl}}{8}$$

where  
- $L_c = \text{Center to center span distance of columns}$,
- $B_c = \text{Center to center distance of columns along width of panel}$,
- $L_{cl} = \text{Clear span length, face to face distance of columns}$.

Calculate modulus of subgrade reaction for the supporting soil.

Calculate span ratio, $m = L_c/B_c$.

Considering the modulus of subgrade reaction and span ratio, $m$ total panel moment is distributed to column strip and middle strip positive and negative moments multiplying the panel moment by distribution coefficients according to Fig. 5.10 to Fig. 5.12 or from Table 5.8 to 5.10. Values other than those indicated in the charts, interpolation can be made.

Reinforcement for column strip shall be placed in the entire column strip width with an uniform distribution. Reinforcement for middle strip shall be placed uniformly for the entire middle strip.

In any section, reinforcement shall not be smaller than minimum reinforcement according to ACI code.

This simplified method is subjected to the following restrictions:

There shall be a minimum of three continuous spans in each direction.

The panel shall be rectangular, with the ratio of the longer to shorter spans within a panel not greater than 2.

The successive span lengths in each direction shall not differ by more than $1/3$ the span.

This method is applicable for high rise buildings where mat foundation design governs due to vertical load i.e. dead load and live load.

Mat has column arrangement in a regular grid pattern.

Foundation soil condition is uniform below the mat area.
To evaluate the performance of the proposed method, a complete design example of a mat with variable thickness has been carried out for a ten storied building with three bay in each direction. This mat is assumed to have 16 columns, each 20 inches square in cross section, spaced 20 ft apart and story height 10'-0". The structure is located in Khulna city.

Other design data for the analysis are

Modulus of subgrade reaction is 100 kcf.
Basic wind speed = 230 km/hour
Slab Dead load = 90 psf
Distributed wall load = 30 psf
Floor Finish = 30 psf
Live load = 60 psf

An amount of 440 lb/ft for 5" wall is assumed along all the beams.

Plan of the mat is shown in Fig. 6.5.
The analysis of the superstructure has done with STAAD-PRO a well known software. Two loading cases are considered.

LOADING 1 = 1.4 x Dead load + 1.7 x Live load
LOADING 2 = 0.75 x (1.4 x Dead load + 1.7 x Live load +1.7 x Wind load)

Column loads for both load cases are given in Table 6.1

Maximum estimated Punching Shear = 0.95* 1562.9 kip
= 1484.75 kip

For $f'_{c} = 4000$ psi,
Greater thickness required for punching shear = 2'-8"
Smaller thickness required = 0.60x 2.66 = 1'-7"
Width of Greater Thickness = 1'-7"
Width of column Strip = 7.7 ft
Fig. 6.5 Plan area of mat of a ten storied building with three bay.
Only half of the mat is analysed due to symmetry of the structure. Table 6.1 shows the column loads obtained from frame analysis of the building frame.

Table 6.1  Column loads from the super structure.

<table>
<thead>
<tr>
<th>Column No</th>
<th>Loading 1</th>
<th>Loading 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axial Force (kip)</td>
<td>Moment about X-axis (kip-ft)</td>
</tr>
<tr>
<td>C1</td>
<td>604</td>
<td>8.82</td>
</tr>
<tr>
<td>C2</td>
<td>969</td>
<td>9.92</td>
</tr>
<tr>
<td>C3</td>
<td>969</td>
<td>9.92</td>
</tr>
<tr>
<td>C4</td>
<td>604</td>
<td>8.82</td>
</tr>
<tr>
<td>C5</td>
<td>969</td>
<td>2.08</td>
</tr>
<tr>
<td>C6</td>
<td>1563</td>
<td>3.58</td>
</tr>
<tr>
<td>C7</td>
<td>1563</td>
<td>3.58</td>
</tr>
<tr>
<td>C8</td>
<td>969</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Total Column Load for Loading I = 8202 kip
Plan Area of the Mat = 33.88’ x 67.76’
= 2295.7 sft.
Equivalent soil pressure = 8202/2295.7 ksf
= 3.576 ksf

Total panel Moment,
\[ M = \frac{w \times L_c \times B_c \times L_{cl}}{8} \]
\[ = 3.576 \times 20 \times 20 \times (20-1.5) \]
\[ = 3300.65 \text{ kip-ft} \]

L/B ratio of the span, m = 20/20 = 1.0
Modulus of subgrade reaction = 100 kcf

For these two parameters, m and modulus of subgrade reactions, coefficients for Table 5.10 is applicable.

Column Strip Width = 7.7 ft
Middle Strip Width = 12.3 ft
For interior panel, Interior Span

Column Strip Positive Moment = 3300.65 x 0.594 kip-ft
= 1960.58 kip -ft

Column Strip Negative Moment = 3300.65 x 0.1084 kip-ft
= 357.79 kip -ft

Middle Strip Positive Moment = 3300.65 x 0.1388 kip-ft
= 458.13 kip-ft

Middle Strip Negative Moment = 3300.65 x 0.1324 kip-ft
= 437. kip -ft

Moments for all the critical sections are obtained by multiplying the corresponding coefficients of Table 5.9 and and total moments for column strip and middle strips are shown in Table 6.2. These are the total positive or negative moments for the column strip and middle strip for the critical sections those govern the design. Critical section moments are divided by corresponding width of the strips to obtain the design moment per unit width (Table 6.3). Reinforcement calculation has been done for the strips in each direction for the moment shown in Table 6.3 considering the ACI minimum requirement. Required reinforcement for unit width has been presented in Table 6.4.

Table 6.2 Bending Moment for Column Strip and Middle Strip

<table>
<thead>
<tr>
<th></th>
<th>Interior Span</th>
<th>Exterior Span</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
<td>Ext. Face of Int. Column (kip-ft)</td>
</tr>
<tr>
<td>Interior Panel-Column Strip</td>
<td>1960.58</td>
<td>-357.79</td>
<td>1967.18</td>
</tr>
<tr>
<td>Interior Panel-Middle Strip</td>
<td>458.13</td>
<td>-437.00</td>
<td>475.29</td>
</tr>
<tr>
<td>Exterior Panel-Column Strip</td>
<td>1369.44</td>
<td>-313.23</td>
<td>1370.76</td>
</tr>
<tr>
<td>Exterior Panel-Middle Strip</td>
<td>205.30</td>
<td>-361.75</td>
<td>221.80</td>
</tr>
</tbody>
</table>
Table 6.3  Bending Moment/ unit width for Column Strip and Middle Strip.

<table>
<thead>
<tr>
<th></th>
<th>Interior Span</th>
<th>Exterior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (kip-ft/ft)</td>
<td>Midspan (kip-ft/ft)</td>
</tr>
<tr>
<td><strong>Int. Pan.- Col. Strip</strong></td>
<td>255.00</td>
<td>-46.47</td>
</tr>
<tr>
<td><strong>Int. Pan.- Middle Strip</strong></td>
<td>59.50</td>
<td>-35.47</td>
</tr>
<tr>
<td><strong>Ext. Pan.- Column Strip</strong></td>
<td>177.85</td>
<td>-40.68</td>
</tr>
<tr>
<td><strong>Ext. Pan.- Middle Strip</strong></td>
<td>16.66</td>
<td>-29.36</td>
</tr>
</tbody>
</table>

Table 6.4  Required reinforcement/ unit width for Column Strip and Middle Strip.

<table>
<thead>
<tr>
<th></th>
<th>Required Reinforcement / unit width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column face Moment (sq.in/ft)</td>
</tr>
<tr>
<td><strong>Int. Pan.- Col. Strip</strong></td>
<td>2.059</td>
</tr>
<tr>
<td><strong>Int. Pan.- Middle Strip</strong></td>
<td>0.891</td>
</tr>
<tr>
<td><strong>Ext. Pan.- Column Strip</strong></td>
<td>1.413</td>
</tr>
<tr>
<td><strong>Ext. Pan.- Middle Strip</strong></td>
<td>0.620</td>
</tr>
</tbody>
</table>

* Governed by minimum reinforcement.

Design of mat foundation shows that design of a considerable amount of area is governed by minimum reinforcement. Use of mat with variable thickness, in those areas of smaller thickness zone, governed by minimum reinforcement, significant economy is achieved not only from amount of concrete but also from amount of steel.
6.7 COMPARISON OF RESULTS WITH FINITE ELEMENT SOLUTION

The moments across the width of critical sections are not constant. For design purposes, mat panel is divided into column strips and middle strips. Moments are not constant across the width of column strip and middle strip. Since, mat is plate type structure, a small part of the structure cannot fail alone. Moments may be considered constant within the bounds of column strip and middle strip. The total moment along the strip remains the same. Such an approximation is always done in flat slab.

This approximation and finite element solution for both load cases is shown in Fig. 6.6 to Fig. 6.10. The variation of bending moment, \( M_x \) for unit width along column lines and center lines in between columns in X direction is shown Fig. 6.6. Fig. 6.7 shows the variation of \( M_x \) for unit width across the width. From Fig. 6.6 and Fig 6.7 it is observed that there is local increase in bending moment at the column faces. At column faces it is always higher than the approximation. Away from the column face with in the column strip it is always below the approximation. Table 6.5 shows the total bending moment of column strip and middle strip critical sections for loading 1 and loading 2 along with the approximation. From Table 6.5 and Fig. 6.6 and Fig. 6.7 it can be concluded that assumed approximation fits the finite element solution well.

This example is analysed with conventional method for loading 1. Results of Finite Element analysis, Proposed approximate method and Conventional method for strips equal to panel width centered on column line are shown in Table 6.6, from which, it appears that proposed approximate method gives better approximation of moments of mat foundation with variable thickness than conventional method. In conventional method, uniform distribution of moment is assumed for column strip and middle strip which is far away from the actual solution. In the proposed method bending moment is different for column strip and middle strip and close to finite element solution.
Fig. 6.6a Variation of bending moment, $M_x$ along line 1 for loading 1, loading 2 and approximation.

Fig. 6.6b Variation of bending moment, $M_x$ along line 2 for loading 1, loading 2 and approximation.
Fig. 6.6c  Variation of bending moment, $M_x$ along line 3 for loading 1, Loading 2 and approximation.

Fig. 6.6d  Variation of bending moment, $M_x$ along line 4 for loading 1, Loading 2 and approximation.
Fig. 6.6e Variation of bending moment, $M_x$ along line 5 for loading 1, loading 2 and approximation.

Fig. 6.6f Variation of bending moment, $M_x$ along line 6 for loading 1, loading 2 and approximation.
Fig 6.7a Variation of bending moment, $M_x$ along lines 7 for loading 1, loading 2 and approximation.

Fig 6.7b Variation of bending moment, $M_x$ along lines 8 for loading 1, loading 2 and approximation.
Fig 6.7c  Variation of bending moment, $M_x$ along lines 9 for loading 1, loading 2 and approximation.

Fig 6.7d  Variation of bending moment, $M_x$ along lines 10 for loading 1, loading 2 and approximation.
Fig 6.7e Variation of bending moment, \( M_x \) along lines 11 for loading 1, loading 2 and approximation.

Fig 6.7f Variation of bending moment, \( M_x \) along lines 12 for loading 1, loading 2 and approximation.
Fig 6.7g Variation of bending moment, $M_x$ along lines 13 for loading 1, loading 2 and approximation.

Fig 6.7h Variation of bending moment, $M_x$ along lines 14 for loading 1, loading 2 and approximation.
Fig 6.7i Variation of bending moment, $M_x$ along lines 15 for loading 1, loading 2 and approximation.

Fig 6.7j Variation of bending moment, $M_x$ along lines 16 for loading 1, loading 2 and approximation.
Table 6.5  Bending Moment for Column Strip and Middle Strip for loading 1, loading 2 and Approximation

<table>
<thead>
<tr>
<th>Strip Location</th>
<th>Load Case</th>
<th>Interior Span</th>
<th>Exterior Span</th>
<th>Int. face of Ext. Column (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
<td>Ext. Face of Int. Column (kip-ft)</td>
</tr>
<tr>
<td>Interior Panel-Column Strip</td>
<td>1.4D+1.7L</td>
<td>1902.00</td>
<td>-305.20</td>
<td>1891.94</td>
</tr>
<tr>
<td></td>
<td>0.75(1.4D+1.7L+1.7W)</td>
<td>1459.78</td>
<td>-228.75</td>
<td>1311.49</td>
</tr>
<tr>
<td></td>
<td>Approximation</td>
<td>1960.58</td>
<td>-357.79</td>
<td>1996.38</td>
</tr>
<tr>
<td>Interior Panel-Middle Strip</td>
<td>1.4D+1.7L</td>
<td>495.22</td>
<td>-364.39</td>
<td>483.67</td>
</tr>
<tr>
<td></td>
<td>0.75(1.4D+1.7L+1.7W)</td>
<td>363.99</td>
<td>-273.05</td>
<td>330.13</td>
</tr>
<tr>
<td></td>
<td>Approximation</td>
<td>458.13</td>
<td>-437.00</td>
<td>475.29</td>
</tr>
<tr>
<td>Exterior Panel-Column Strip</td>
<td>1.4D+1.7L</td>
<td>1345.18</td>
<td>-286.62</td>
<td>1329.19</td>
</tr>
<tr>
<td></td>
<td>0.75(1.4D+1.7L+1.7W)</td>
<td>1026.79</td>
<td>-215.10</td>
<td>880.90</td>
</tr>
<tr>
<td></td>
<td>Approximation</td>
<td>1369.44</td>
<td>-313.23</td>
<td>1370.76</td>
</tr>
<tr>
<td>Exterior Panel-Middle Strip</td>
<td>1.4D+1.7L</td>
<td>230.27</td>
<td>-286.62</td>
<td>215.89</td>
</tr>
<tr>
<td></td>
<td>0.75(1.4D+1.7L+1.7W)</td>
<td>159.33</td>
<td>241.56</td>
<td>124.07</td>
</tr>
<tr>
<td></td>
<td>Approximation</td>
<td>205.30</td>
<td>-361.75</td>
<td>221.80</td>
</tr>
</tbody>
</table>

Table 6.6  Bending Moments for interior strip and exterior strip for loading 1.

<table>
<thead>
<tr>
<th>Strip Location</th>
<th>Analysis Method</th>
<th>Interior Span</th>
<th>Exterior Span</th>
<th>Int. face of Ext. Column (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Column face Moment (kip-ft)</td>
<td>Midspan (kip-ft)</td>
<td>Ext. Face of Int. Column (kip-ft)</td>
</tr>
<tr>
<td>Interior Strip Width 20'</td>
<td>Finite Element</td>
<td>2397.2</td>
<td>-669.6</td>
<td>2375.6</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>1578.4</td>
<td>1436.4</td>
<td>1533.4</td>
</tr>
<tr>
<td></td>
<td>Approximate</td>
<td>2418.7</td>
<td>-794.8</td>
<td>2442.5</td>
</tr>
<tr>
<td>Exterior Strip Width 13'</td>
<td>Finite Element</td>
<td>1460.3</td>
<td>-429.9</td>
<td>1437.2</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>1012.0</td>
<td>-1498.6</td>
<td>975.4</td>
</tr>
<tr>
<td></td>
<td>Approximate</td>
<td>1472.1</td>
<td>-494.1</td>
<td>1481.7</td>
</tr>
</tbody>
</table>
According to BNBC, maximum allowable differential settlement for foundation is 20 mm. Maximum differential settlement for this example is 0.31 inch, which is well below the allowable limit. The variation of deflection along lines 1-6 of Fig.6.5 is shown in Fig. 6.9.

Flexural shearing stress is critical at the neck section. Fig. 6.10c shows the variation of flexural shearing stress at interior neck section across the width for load cases 1 and 2 and capacity in flexure. Total capacity of the neck section across the width in flexure is 786 kip whereas total flexural shear force for load 1 and load 2 are 709.49 and 507.10 kip respectively. From flexural point of view this model is able to take flexural shear.

Punching shear in columns are tabulated in Table 6.7, which shows the punching shear force at d/2 distance away from the column face and at the neck section around the column. From Table 6.7, it has been observed that maximum punching shear is 1477 kip and greater thickness has been calculated from punching shear 1485 kip. So, greater thickness is adequate. At the neck section around the column maximum punching shear 1231 kip whereas the capacity is 1431 kip. So, smaller thickness at neck section is adequate for punching shear.

This approximate method can be used for non-uniform mat foundation with column spacing in a regular pattern.
Fig 6.7k Variation of bending moment $M_x$ along lines 17 for loading 1, loading 2 and approximation.

Fig 6.8a Variation of bending Moment $M_x$ along lines 8,16 for loading 1, loading 2 and approximation.
Fig 6.8b Variation of bending Moment $M_x$ along lines 9, 15 for loading 1, loading 2 and approximation.

Fig 6.8c Variation of bending moment $M_x$ along lines 11, 13 for loading 1, loading 2 and approximation.
Fig 6.8d Variation of bending moment $M_z$ along line 12 for Loading 1, Loading 2 and approximation.

Fig 6.9a Variation of vertical deflection along line 1 for loading 1 and Loading 2.
Fig. 6.9b  Variation of vertical deflection along line 2 for Loading 1 and Loading 2.

Fig. 6.9c  Variation of vertical deflection along line 3 for Loading 1 and Loading 2.

Fig. 6.9d  Variation of vertical deflection along line 4 for Loading 1 and Loading 2.
Fig. 6.9e  Variation of vertical deflection along line 5 for loading 1 and Loading 2.

Fig. 6.9f  Variation of vertical deflection along line 6 for loading 1 and Loading 2.
Fig. 6.10a Variation of Shear Stress $S_{xy}$ along lines 1-6 for loading 1.

Fig. 6.10b Variation of Shear Stress $S_{xy}$ along lines 1-6 for loading 2.
Fig. 6.10c Variation of flexural shearing stress and capacity at the interior neck section.
6.8 ECONOMY OF PROPOSED MAT WITH VARIABLE THICKNESS

Cost estimate for the above example is done with mat of constant thickness and mat with variable thickness. Economic evaluation is shown in Table 6.8.

Thickened part of the mat at the upper portion restricts the use of top surface of mat for commercial purpose like parking, store house etc. This problem can be handled in two ways.

(i) For this example, only 15% area of the total mat is thickened under the column. Structural cost of non-uniform mat is Tk. 18,93,500. Cost comparison of mat with variable thickness and uniform mat is done in Table 6.8. It is seen that, efficiency of the flat area is reduced by 15% with saving in structural cost about 39%. 

Table 6.7 Column axial forces, Punching shear at d/2 distance from the column face and at the neck section around the column.

<table>
<thead>
<tr>
<th>Column No.</th>
<th>Load Case 1</th>
<th>Load Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial load</td>
<td>Punching shear at d/2 distance</td>
<td>Punching shear at neck section</td>
</tr>
<tr>
<td>kin</td>
<td>kip</td>
<td>kin</td>
</tr>
<tr>
<td>C1</td>
<td>604</td>
<td>494.57</td>
</tr>
<tr>
<td>C2</td>
<td>969</td>
<td>877.68</td>
</tr>
<tr>
<td>C3</td>
<td>969</td>
<td>877.65</td>
</tr>
<tr>
<td>C4</td>
<td>604</td>
<td>493.96</td>
</tr>
<tr>
<td>C5</td>
<td>969</td>
<td>872.02</td>
</tr>
<tr>
<td>C6</td>
<td>1562.9</td>
<td>1476.26</td>
</tr>
<tr>
<td>C7</td>
<td>1562.9</td>
<td>1476.27</td>
</tr>
<tr>
<td>C8</td>
<td>969</td>
<td>871.92</td>
</tr>
</tbody>
</table>
Table 6.8 Economic evaluation of mat with variable thickness against mat with constant thickness.

<table>
<thead>
<tr>
<th>Mat Foundation</th>
<th>Concrete cft</th>
<th>Reinf. ton</th>
<th>Filler Mats. cft</th>
<th>Extra finishing with concr. cft</th>
<th>Total Cost Tk</th>
<th>Savings (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mat with constant thickness</td>
<td>12214</td>
<td>20</td>
<td>------</td>
<td>------</td>
<td>30,87,500</td>
<td>0%</td>
</tr>
<tr>
<td>Mat with variable thickness</td>
<td>7607</td>
<td>11</td>
<td>------</td>
<td>------</td>
<td>18,93,500</td>
<td>39%</td>
</tr>
<tr>
<td>Without filler</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mat with variable thickness (a)</td>
<td>7607</td>
<td>11</td>
<td>4607</td>
<td>1530</td>
<td>23,29,600</td>
<td>25%</td>
</tr>
<tr>
<td>With brick fill and 4&quot; slab on the full top surface (Fig 6.11b).</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mat with variable thickness (b)</td>
<td>7607</td>
<td>11</td>
<td>3234</td>
<td>1372</td>
<td>22,61,500</td>
<td>27%</td>
</tr>
<tr>
<td>With brick fill and 4&quot; slab on the top surface of reduced thickness (Fig 6.11c).</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mat with variable thickness (a)</td>
<td>7607</td>
<td>11</td>
<td>4607</td>
<td>2295</td>
<td>22,65,400</td>
<td>25%</td>
</tr>
<tr>
<td>With sand fill and 6&quot; slab on the full top surface (Fig 6.11b).</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mat with variable thickness (b)</td>
<td>7607</td>
<td>11</td>
<td>2548</td>
<td>2058</td>
<td>22,17,500</td>
<td>27%</td>
</tr>
<tr>
<td>With sand fill and 6&quot; slab on the top surface of reduced thickness (Fig 6.11c).</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Usually basement floor is used for storage and car parking. Non-uniform top surface is not a problem for a store house. It restricts the use of basement floor as car parking. Non-uniform top surface can be filled with low cost filler material like sand, bricks and whole or partially top surface can be finished with 4" to 6" thick concrete to provide a useful flat surface (Fig. 6.11). Cost comparison reveals that about 25%
Fig. 6.11a

Fig. 6.11b

Fig. 6.11c

Fig. 6.11d

Fig. 6.11 Use of filler material to provide a flat surface for mat foundation of variable thickness.
economy can be achieved with variable mat foundation over mat having constant thickness.

For economic evaluation assumed unit cost of materials are given in Table 6.9.

Table 6.9 Unit cost of construction material

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>Cost/Tk/unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced cement concrete (1:1.5:3)</td>
<td>cft</td>
<td>220</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>Ton</td>
<td>20,000</td>
</tr>
<tr>
<td>Concrete for extra finishing (1:3:6)</td>
<td>cft</td>
<td>150</td>
</tr>
<tr>
<td>Brick fill with mortar (1:4)</td>
<td>cft</td>
<td>50</td>
</tr>
<tr>
<td>Sand filler</td>
<td>cft</td>
<td>6</td>
</tr>
</tbody>
</table>

6.8 REMARKS

The proposed improved method of modeling and analysis with finite element method and approximate method has been verified with nine number of mats analysed for different length to width span ratio for different modulus of subgrade reaction.

In this study, reshaping scheme of mat foundation with variable thickness proposed by Morshed and Sutradhar has been improved in different ways,

(i) Previous method proposed extra thickness is to be provided below the slab. In present method, extra thickness is to be provided above the slab.

(ii) Previous method proposed that smaller thickness is to be taken 2/3rd of greater thickness. In the new proposed method, smaller thickness is to be taken 60% of greater thickness.
(iii) The greater thickness is to be provided for a distance equal to the effective depth corresponding to greater thickness itself. In the new approach, width of greater thickness, $L_s$ shall be at least 0.4 times the span length in that direction.

(iv) The new approach proposed an approximate analysis method for mat with variable thickness and having column arrangements in a regular grid.

Case studies reveal that

- Differential settlement is increased but it is well below the allowable limit. This will not pose any harm to foundation design.
- Shear stress consideration has shown that greater thickness should be designed for punching shear and punching shear stress diminishes rapidly away from the column face. Maintaining greater thickness up to $d_e$ from the column face is sufficient to encounter punching shear.
- Maintaining length of the thicken zone up to 0.40 times the span is sufficient to take care of flexural shear.
- Maximum negative moment occurs at the midspan. Smaller thickness $0.60 t_g$ is sufficient to resist the negative moment at midspan.
CHAPTER 7

CONCLUSION

7.1 GENERAL

Mat foundation is a relatively heavy and costly structure. Economy in mat has been engineer's dream for many years. This was not possible for lack of appropriate design method and guideline. Design of mat foundation is still a cumbersome job to most engineers. Possibility of economy has been hinted by several authors in the form of changing the configuration of mat geometry. In Fig 1.1 such configurations are shown. Since we have the most powerful Finite Element tools in our hand, it is our privilege to discover economy in such configurations. The specific objective of this study was to develop aids for analysis and design of mat foundation with variable thickness. Findings regarding the mat behavior under vertical loading conditions and variable thickness are based on extensive analysis by finite element method.

An extensive literature review has also been done to identify the significant parameters that influence the behavior of mat with variable thickness. Parametric study on two significant parameters, modulus of subgrade reaction and smaller thickness, has been done in this investigation. Each parameter has been widely varied within the practical range. On the basis of this extensive study design rationale proposed by Morshed and Sutradhar has been improved.

An improved guideline for modeling and analysing mat with variable thickness using finite element method has been presented. Such type of mat foundation offers a way of attaining substantial economy.
An approximate method of analysis of mat foundation with variable thickness has finally been presented. This approximate method would enable the designer to design mat with variable thickness with regular grids in most practicable cases, with relative ease. When a complicated case arises one should go for finite element solution.

7.2 FINDINGS FROM THE INVESTIGATION

From this study the actual behaviour of mat foundation with variable thickness has become obvious. A mat of three bay frames has been analyzed by varying its smaller thickness and modulus of subgrade reaction.

Parametric study on variation of smaller thickness shows that smaller thickness can be reduced up to 54% of greater thickness. To be more conservative it is maintained at 60% of greater thickness. Modulus of subgrade reaction affects the distribution of moments to column strip and middle strip for mat with variable thickness. Distribution of moments to column strip and middle strip is significantly influenced by L/B ratio of the mat panel.

Complete analysis of nine mat foundations with five bay has been performed by finite element package program ANSYS. Results obtained from these analysis have also been summarized.

7.3 THE IMPROVED DESIGN APPROACH FOR MAT WITH VARIABLE THICKNESS

Based on the investigation on nine mat foundations by varying L/B ratio and modulus of subgrade reaction by finite element method a simple modeling guideline for economic shaping of mat foundation has been proposed. This model
can be used for finite element analysis. A simple and straightforward way of analysis and design has been presented in chapter 6.

Three sets of distribution coefficients have been proposed for different subgrade modulus of reaction with variation of five different L/B ratio of mat panel. Using the proposed distribution coefficients, a designer can find out the moments at the critical sections of column strip and middle strip for interior panel and exterior panel. This method is limited up to the range where vertical loads govern the design and spacing of columns in a regular pattern. Specifically this method is applicable upto fifteen storied building.

7.3.1 RECOMMENDATION FOR BANGLADESH NATIONAL BUILDING CODE

Bangladesh National Building Code (1993) is a complete and comprehensive building code covering all aspects of planning, design and construction of buildings. These regulations, ordinances, rules and practices need updating, rationalization and unification.

South-western part of Bangladesh has highly compressible soil up to great depth. Cast-in-situ foundation is not reliable for this soil condition even for a six storied residential building. Therefore, there is a need for low-cost foundation design for buildings up to six storied residential buildings in that area with reliability and simplicity.

Usually this type of structure is more likely to have column arrangements in a regular grid pattern and uniform loading. Governing load case is the vertical load i.e. dead load plus reduced live load. Mat foundation with variable thickness for this situation is ideal one. Need for finite element solution may hinder the applicability of this economic approach. Approximate method of analysis for mat
design, as recommended here, will encourage the designers to go for this type of solution. This will cause substantial reduction in cost for foundation design.

This approach for the design of mat with variable thickness may be adopted in BNBC. Calculating the total settlement for a long period from consolidation test, plinth level of the structure can be raised with a factor of safety. Additional advantage is that one can use the top face of the mat foundation as basement floor, storage. Underground parking is not a requirement in those areas.

7.4 RECOMMENDATION FOR FUTURE STUDY

The fields of future study are

- Approximate analysis of mat foundation with non-uniform spacing of columns.
- Approximate method of analysis for load cases where lateral load is a governing criteria.
- Approximate method of analysis for mat with constant thickness.
- Effect of shear wall on design of mat foundation.
- Effect of shear wall on edges of mat foundation.
- Modeling Guideline for mat with variable thickness with shear walls.
- Behavior of irregularly shaped mat, mat with punches, mat with irregular column arrangements may be investigated.
REFERNECES


APPENDIX A

SHELL93: 8 NODE STRUCTURAL SHELL ELEMENT

A.1 BRIEF DESCRIPTION OF SHELL93 ELEMENT

Shell93 is an eight node structural element of ANSYS library function. Shell93 is well suited to model curved elements. Typical shell elements are shown in Fig A.1. The element has six degrees of freedom at each node: translations in the nodal x, y, z directions and rotations about the x, y, z axes. The deformed shapes are quadratic in both in-plane directions. The element has bending effect, transverse shear deformation, plasticity, stress stiffening, large deflection and large strain capabilities.

Fig. A.1 SHELL93: 8 node structural shell

A.1.1 Geometric Definition of the Element:
The geometry, node locations and the co-ordinate system for this element are shown in Fig. A.1. The element is defined by eight nodes, four thickness and the orthotropic
material properties. Mid-side nodes may not be removed from this element. A triangular shaped element may be formed by defining the same node number for nodes K, L and O. Orthotropic material directions correspond to the element coordinate directions.

The element x and y axis are in the plane of the element. The element may have variable thickness. The thickness is assumed to vary smoothly over the area of the element, with the thickness input at the corner nodes.

A.1.2 Loading of the Element

Pressures may be input as surface loads on the element faces. Positive pressures act into the element. Edge pressures are input as force/unit length. Temperatures may be input as element body loads at the corner.

Fig. A.2 SHELL 93 stress output
A.2 OUTPUT DATA

The solution output associated with the elements is in two forms: (a) nodal displacements included in the overall nodal solution, and (b) additional element output is shown in table A.1. Several items are illustrated in Fig. A.2. The element stress directions and force resultants are parallel to the element coordinate system.

Table A.1 Shell93 Element input and output definitions

<table>
<thead>
<tr>
<th>Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>EL</td>
<td>Element number and name</td>
</tr>
<tr>
<td>NODES</td>
<td>Nodes -I, J, K, L, M, N, O, P</td>
</tr>
<tr>
<td>MAT</td>
<td>Material number</td>
</tr>
<tr>
<td>THICK</td>
<td>Average thickness</td>
</tr>
<tr>
<td>S: INT</td>
<td>Stress Intensity</td>
</tr>
<tr>
<td>S: EQV</td>
<td>Equivalent stress</td>
</tr>
<tr>
<td>EPEL: X, Y, Z, XY, YZ, XZ</td>
<td>Elastic Strains</td>
</tr>
<tr>
<td>S: 1, 2, 3</td>
<td>Principal Stresses</td>
</tr>
<tr>
<td>T (X, Y, XY)</td>
<td>In plane element X, Y, and XY forces</td>
</tr>
<tr>
<td>M (X, Y, XY)</td>
<td>Element X, Y, and XY moments.</td>
</tr>
<tr>
<td>N (X, Y)</td>
<td>Out-of-plane element X and Y shear forces.</td>
</tr>
</tbody>
</table>

A.3 ASSUMPTIONS AND RESTRICTIONS

Zero area elements are not allowed. This occurs most often whenever the elements are not numbered properly. Zero thickness elements or elements tapering down to zero thickness at any corner are not allowed. The applied transverse thermal gradient is assumed to vary linearly through the thickness. Shear deflections are included in this element. The element may produce inaccurate stress under thermal loads for double curved warped domains.
APPENDIX B

COMBIN14: SPRING DAMPER ELEMENT

B.1 BRIEF DESCRIPTION OF THE ELEMENT

Combin14 has longitudinal or torsional capability in one, two, or three dimensional applications. The longitudinal spring-damper option is a uniaxial tension-compression element with up to three degrees of freedom at each node: translations in the nodal x, y and z directions, no bending or torsion is considered. The torsional spring damper option is a purely rotational element with three degrees of freedom at each node: rotations about the x, y, and z axis. No bending or axial load is considered.

![Diagram of COMBIN14 spring damper element]

*Note - Two-dimensional elements must be in the X-Y plane*

Fig. B.1 COMBIN14 spring damper

The spring damper element has no mass. Masses can be added by using the appropriate mass element. The spring or the damping capability may be removed from the element.
B.1.1 Geometric Definition of the Element

The geometry, node locations and the coordinate system for this element is shown in Fig. B.1. The element is defined by two nodes, a spring constant and damping coefficient $c_{v1}$ and $c_{v2}$. The damping capability is not used for static and undamped modal analyses. The longitudinal constant has units of force/unit length.

The element has option for defining the element as a one dimensional element. With this option the element operates in the nodal coordinate system.

B.2 OUTPUT DATA

The solution output associated with the element is in two forms: (a) nodal displacements included in the overall nodal solution, and (b) additional element output is shown in table B.1.

Table B.1 COMBIN14 element input and output definitions.

<table>
<thead>
<tr>
<th>Name</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>EL</td>
<td>Element Number</td>
</tr>
<tr>
<td>NODES</td>
<td>Nodes - I, J</td>
</tr>
<tr>
<td>CENT: X, Y, Z</td>
<td>Center Location XC, YC, ZC</td>
</tr>
<tr>
<td>FORC or TORQ</td>
<td>Spring Force or Moment</td>
</tr>
<tr>
<td>RATE</td>
<td>Spring Constant</td>
</tr>
</tbody>
</table>

B.3 ASSUMPTIONS AND RESTRICTIONS

The length of the element must not be zero, since the node locations determine the spring orientation. The longitudinal element stiffness of the spring acts along the length of the spring. The element allows only a uniform stress in the spring. The element can be modified to act along only one degree of freedom and no moment effects are included.
In this study the element has only one degree of freedom. This degree of freedom is specified in the nodal coordinate system and is the same for both nodes.
APPENDIX C

MACRO USED FOR ANALYSIS

C.1 GENERAL

Macro is a text file, which can be called from ANSYS command line. Commands are written in this file. This is used for modeling, solution and viewing output of the structure. This is one type of sub-programming. A macro file can invoke another macro file. A set of macro files has been used for modeling, solution and post processing purpose. Text files as macro files are given for prospective researchers.

Following six files are used to model a mat foundation of twelve storied building of five bays with variable thickness, solution and for post processing purpose.

C.2 DESIGN DATA FOR THE MAT MODEL

A twelve storied building of five bay frame in each direction with span 26'-0" has been studied. The building is symmetric in both directions. Therefore, only one-fourth of the problem has been considered. The structure is assumed to have uniform live load 60 psf, slab dead load 90 psf, distributed wall load 30 psf, floor finish 25 psf. An amount of 440 lb/ft is assumed for 5" wall along all beams. Modulus of subgrade reaction is taken 100 kcf.

With dead load factor 1.4 and live load factor 1.7, the considered load case is 1.4DL+1.7LL for the mat foundation (Fig. 5.1). Respective column loads are given in Table 5.1. Model shape for mat foundation in an isometric view is given in Fig. C.1.

C.3 MODEL OF MAT FOUNDATION WITH VARIABLE THICKNESS.

File 1: Param.txt

!***************************
!SETTING THE PARAMETERS

C-1
Generating the nodes

\[ a(i) \text{ is the spacing of nodes in X direction.} \]

*dim,a,,80
*set,a(1),1,1,1,1,1,1,1,1,1,1,1
*set,a(11),1,1,1,1,1,1,1,1,1,1,1
*set,a(21),1,1,1,1,1,1,1,1,1,1,1
*set,a(31),1,1,1,1,1,1,1,1,1,1,1
*set,a(41),1,1,1,1,1,1,1,1,1,1,1
*set,a(51),1,1,1,1,1,1,1,1,1,1,1
*set,a(61),1,1,1,1,1,1,1,1,1,1,1
*set,a(71),1,1,1,1,1,1,1,1,1,1,1

\[ b(i) \text{ is the spacing of rows in Z direction.} \]

*dim,b,,80
*set,b(1),1,1,1,1,1,1,1,1,1,1,1
*set,b(11),1,1,1,1,1,1,1,1,1,1,1
*set,b(21),1,1,1,1,1,1,1,1,1,1,1
*set,b(31),1,1,1,1,1,1,1,1,1,1,1
*set,b(41),1,1,1,1,1,1,1,1,1,1,1
*set,b(51),1,1,1,1,1,1,1,1,1,1,1
*set,b(61),1,1,1,1,1,1,1,1,1,1,1
*set,b(71),1,1,1,1,1,1,1,1,1,1,1

! row 1 starting node in x-dir
*set,ndr1st,1
*set,x1,0
*set,y1,0
*set,z1,0

! row 1 ending node in x-dir
*set,ndr1end,71
*set,x2,70
*set,y2,0
*set,z2,0

! Total number of rows in z-dir
*set,totrow,71
! spacing of rows in Z-dir
*set,spzrow,1.00
!generating nodes (copying) in y-dir.
!number of copy in Y-dir
*set,nycopy,2
!spacing of rows in y-dir
*set,spyrow,-4

!Starting node of free end nodes of springs of row1
*set,spndst,ndr1end*totrow+1

!Ending node of free end nodes of springs of row1
*set,sp1nden,(ndr1end*totrow+ndr1end)

!Defining the Material Properties
!material properties ex, Poisson’s ratio for Shell Element
*set,ex1,5.19e5
*set,nuxy1,0.15

!Defining Element Type to be used
!shell93,combin14 with keyopt2 (2) is used
!For one dimensional longitudinal spring damper keyopt(2)= 2

! setting element numbers
!Starting shell element of first row
*set,elrlst,1

! End shell element of first row
*set,elrlend,(ndr1end-1)/2

!Last shell element, last shell element of last row
*set,elshlst,((ndr1end-1)/2)*((totrow-1)/2)

!Number of repetitions in Z-dir of Spring row1 in X-dir
*set,spr1rep,(totrow+1)/2

! setting first spring number for first row
*set,spr1,elshlst+1
!setting last spring number of first row
*set,spr1end,(elshlst+(ndr1end+1)/2)

! setting spring number for first spring for last row
*set,sprlast,spr1+(spr1rep-1)*(spr1end-spr1+1)
! setting spring number for last spring for last row
*set,sprlaend,spr1end+(spr1rep-1)*(spr1end-spr1+1)

C-3
file 2: realcon.txt

! Declaration of element real constants
! number of real constant sets for shell
*set,shset,10
! Defining Various thickness
*set,a1,3.5
*set,b1,2.00
*set,spring,400
! Shell thickness Set1
*set,th11,a1
*set,th12,a1
*set,th13,a1
*set,th14,a1
! Shell thickness Set2
*set,th21,b1
*set,th22,b1
*set,th23,b1
*set,th24,b1
! Shell thickness Set3
*set,th31,b1
*set,th32,b1
*set,th33,a1
*set,th34,b1
! Shell thickness Set4
*set,th41,b1
*set,th42,a1
*set,th43,a1
*set,th44,b1
! Shell thickness Set5
*set,th51,b1
*set,th52,a1
*set,th53,b1
*set,th54,b1
! Shell thickness Set6
*set,th61,b1
*set,th62,b1
*set,th63,a1
*set,th64,a1
! Shell thickness Set7
*set,th71,b1
*set,th72,b1
*set,th73,b1
*set,th74,a1
!Shell thickness Set8
*set,th81,a1
*set,th82,b1
*set,th83,b1
*set,th84,a1
!Shell thickness Set9
*set,th91,a1
*set,th92,b1
*set,th93,b1
*set,th94,b1
!Shell thickness Set10
*set,th101,a1
*set,th102,a1
*set,th103,b1
*set,th104,b1
! spring constant sets for mat central, side, corner
!spring Constant set for central
*set,sprcon1,spring
!spring Constant set for side
*set,sprcon2,spring/2
!spring Constant set for corner
*set,sprcon3,spring/4

!Real Constant Sets For Shells upto number of shell set.

r,1,th11,th12,th13,th14
r,2,th21,th22,th23,th24
r,3,th31,th32,th33,th34
r,4,th41,th42,th43,th44
r,5,th51,th52,th53,th54
r,6,th61,th62,th63,th64
r,7,th71,th72,th73,th74
r,8,th81,th82,th83,th84
r,9,th91,th92,th93,th94
r,10,th101,th102,th103,th104

! Spring Constant Set for springs after shell set

r,shset+1,sprcon1
r,shset+2,sprcon2
r,shset+3,sprcon3
File 3: Load.txt

! This Page is Used for applying Nodal Loads
! f, node, fy, value, value2, nend, ninc

! Application of vertical load-fy
FLST,2,4,1,ORDE,4
FITEM,2,289
FITEM,2,291
FITEM,2,431
FITEM,2,433
F,P51X,FY,-269,
FLST,2,4,1,ORDE,4
FITEM,2,2135
FITEM,2,2137
FITEM,2,2277
FITEM,2,2279
F,P51X,FY,-438.5,
FLST,2,4,1,ORDE,4
FITEM,2,315
FITEM,2,317
FITEM,2,457
FITEM,2,459
F,P51X,FY,-438.5,
FLST,2,4,1,ORDE,4
FITEM,2,3981
FITEM,2,3983
FITEM,2,4123
FITEM,2,4125
F,P51X,FY,-448.25,
FLST,2,4,1,ORDE,4
FITEM,2,341
FITEM,2,343
FITEM,2,483
FITEM,2,485
F,P51X,FY,-448.25,
FLST,2,4,1,ORDE,4
FITEM,2,291
FLST,2,4,1,ORDE,4
FITEM,2,2305
F,P51X,FY,-724,
FLST,2,4,1,ORDE,4
FITEM,2,431
F,P51X,FY,-755,
F,P51X,FY,-739,
FLST,2,4,1,ORDE,4
FITEM,2,2277
F,P51X,FY,-438.5,
FLST,2,4,1,ORDE,4
FITEM,2,2279
F,P51X,FY,-438.5,
FLST,2,4,1,ORDE,4
FITEM,2,2279
FITEM,2,2329
F,P51X,FY,-739,
FLST,2,4,1,ORDE,4
FITEM,2,2331
F,P51X,FY,-739,
FLST,2,4,1,ORDE,4
FITEM,2,4033
F,P51X,FY,-755,
Application of vertical load-mz

FITEM,2,341
FITEM,2,343
FITEM,2,483
FITEM,2,485
F,P51X,mx,15,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mx,2.5,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,291
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-0.1,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-2.5,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,291
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-9.3,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,291
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-2.6,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,291
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-14.9,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,291
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-2.66,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,291
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-15,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,291
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-0.7,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,291
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-0.71,
FLST,2,4,1,ORDE,4
FITEM,2,2161
FITEM,2,291
FITEM,2,2163
FITEM,2,2303
FITEM,2,2305
F,P51X,mz,-0.71,
File 4: Model.txt

!Entering into Pre-Processor mode

/prep7
/TITLE, MAT FOUNDATION OF VARIABLE THICKNESS
! Calling input parameters
*use, param.txt
!pnum, lab, key (0 : off, 1 : on)
/pnum, node, 0

!Generating nodes in X-direction
i=1
*do, ndx, ndr1st, ndr1end, 1
n, ndx, x1, y1, z1
x1 = x1 + a(i)
i = i + 1
*enddo

!fill, ndr1st, ndr1end
!ngen, itime, inc, node1, node2, ninc, dx, dy, dz, space

!Generating nodes Z direction copying row 1
i2 = 0
i = 1
*do, ndz, 2, totrow, 1
spzrow = b(i)
gen, 2, ndr1end, ndr1st + i2*ndr1end, ndr1end + i2*ndr1end, 1, 0, 0, spzrow
i2 = i2 + 1
i = i + 1
*enddo
!copying nodes in X-Z plane to another X-Y plane
ngen, nycopy, ndr1end*totrow, ndr1st, ndr1end*totrow, 1, 0, spyrow, 0
/units, si

!Setting material properties for Shell elements
mp, ex, 1, ex1
mp, nuxy, 1, nuxy1

! Defining element type
et, 1, shell93
et, 2, combin14, 0, 2

!Calling real constants for elements
*use, realcon.txt
/pnum,elem,0 !(0: off, 1: on)
!Generation of structural shell elements
type,1
mat,1
real,1
en,elr1st,2*ndrlend+1,ndrlst,ndrlst+2,2*ndrlend+3,ndrlst+1,ndrlend+3,2*ndrlend+2
egen,itime,ninc,iel1,iel2,ielnc,rinc,cinc
egen,(ndrlend-1)/2,2,elr1st,elr1st
gen,(totrow-1)/2,2*ndrlend,elr1st,elr1end

!Generation of spring elements
type,2
real,shset+1
en,sprl,ndrlst,sprndst
egen,(ndrlend+1)/2,2,sprl,sprl
egen,sprlrep,2*ndrlend,sprl,sprlend

/eshape,1
! Modification of uniform mat to non uniform mat foundation.
*use,emod.txt

! Setting spring constants for all springs
*use,emodsp.txt

! Defining support condition and symmetry condition.
*use,support.txt

! Calling loads on the model.
*use,load.txt

File 5 : Support.txt
! Defining support condition
*do,i,0,totrow-1,2
d,sprndst+i*ndrlend,all,,sprlnden+i*ndrlend,2
*enddo

!Defining symmetry condition
d,ndrlst+(totrow-1)*ndrlend,rotx,,ndrlend*rotrow,2
d,ndrlst+(totrow-1)*ndrlend,uz,,ndrlend*rotrow,2
d,ndrlend,rotz,,ndrlend*rotrow,2*ndrlend
d,ndrlend,ux,,ndrlend*rotrow,2*ndrlend
This page will be used to convert the shell real constants

This page will be used to convert the mat foundation with constant thickness to mat foundation with variable thickness.

```
FLST,2,27,2,ORDE,18
FITEM,2,142
FITEM,2,-144
FITEM,2,155
FITEM,2,-157
FITEM,2,168
FITEM,2,-170
FITEM,2,597
FITEM,2,-599
FITEM,2,610
FITEM,2,-612
FITEM,2,623
FITEM,2,-625
FITEM,2,1052
FITEM,2,-1054
FITEM,2,1065
FITEM,2,-1067
FITEM,2,1078
FITEM,2,-1080
EMODIF,P5IX,REAL,4
```

This page will be used to convert the spring real constants

```
*do,spnum,sp1+1,sp1end-1,1
emodif,spnum,real,12
*enddo
*do,spnum,sprlast+1,sprlaend-1,1
emodif,spnum,real,12
*enddo
*do,spnum,sp1+spxnum,sprlaend-spxnum,spnum
emodif,spnum,real,12
*enddo
*do,spnum,sp1end+spxnum,sprlaend-spxnum,spnum
emodif,spnum,real,12
*enddo
emodif,sp1,real,13
emodif,sp1end,real,13
```
C.4 SOLUTION OF THE MODEL

File 1: Solu.txt
/solu
/stat,solu
Solve

C.5 POST PROCESSING OF THE RESULTS

Post processing of the results is done by graphical editor.
Fig. C.1 Structural model of mat foundation with variable thickness with SHELL93 and COMBIN14 elements for analysis.
APPENDIX D

DESIGN DATA: INPUT AND OUTPUT

D.1 INTRODUCTION

In this study, three mat foundations for different length to width ratio and for three different modulus of subgrade reaction has been studied. A total number of nine mat foundations have been analysed and their results have been summarized in chapter 5 in Table 5.5 to Table 5.7. L/B ratio of mat foundation has been varied in the range of 0.615 to 1.625, which covers the most practical ranges. Modulus of subgrade reaction has been varied from 50 kcf to 200 kcf, which covers soft to hard soils.

D.2 SUPERSTRUCTURE AND IT'S LOADING

The super structure is a twelve storied building with five bays in each direction. Only gravity loading i.e. live load and dead load have been considered. Each structure is assumed to have uniform loading of live load 60 psf, slab dead load 90 psf, distributed wall load 30 psf and floor finish 25 psf. An amount of 440 lb/ft is assumed for 5" wall along all beams.

With dead load factor 1.4 and live load factor 1.7 the considered load case is 1.4DL+1.7LL for the mat foundation.

The mat foundation is symmetric in both directions considering its geometry, loading and support conditions. As the structure is symmetric in both directions, only one fourth of the mat foundation has been considered. Boundary conditions at the symmetry line have been so selected at the symmetry line to include the effect of total structure.
Actually mat of three panel length to width ratio has been analysed but results of five L/B ratio has been summarized. For example, a mat having interior panel of 26' x 16' provide results for L/B ratio of 1.625 and 0.615. When the analysis results are summarized in the longer direction considering the span 26' and width 16', L/B ratio is 1.625. Again, when the results are summarized in the shorter direction, the span is 16' and width is 26', then L/B ratio is 0.615. Column loads for three different panel length to width ratio has been presented in Table D.1.

Table D.1  Column axial Forces ($F_x$) and bending moments ($M_x$ & $M_y$) for different length to width ratio.

<table>
<thead>
<tr>
<th>Col. Title</th>
<th>Length to Width Ratio of Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>$F_x$</td>
</tr>
<tr>
<td></td>
<td>(kip)</td>
</tr>
<tr>
<td>C1</td>
<td>1075</td>
</tr>
<tr>
<td>C2</td>
<td>1754</td>
</tr>
<tr>
<td>C3</td>
<td>1793</td>
</tr>
<tr>
<td>C4</td>
<td>1754</td>
</tr>
<tr>
<td>C5</td>
<td>2896</td>
</tr>
<tr>
<td>C6</td>
<td>2957</td>
</tr>
<tr>
<td>C7</td>
<td>1793</td>
</tr>
<tr>
<td>C8</td>
<td>2960</td>
</tr>
<tr>
<td>C9</td>
<td>3022</td>
</tr>
<tr>
<td>Total Load</td>
<td>20004</td>
</tr>
</tbody>
</table>


Table D.2  Mat area, panel dimensions, total loading and uniform loading of the panel for a/b ratio.

<table>
<thead>
<tr>
<th>Mat Panel Length to Width Ratio</th>
<th>Total Mat Area (ft)</th>
<th>Panel Dimensions (ft)</th>
<th>Total Column Loads (kip)</th>
<th>Equivalent Avg. Soil Pressure (ksf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>70' x 70'</td>
<td>26' x 26'</td>
<td>20,004</td>
<td>4.082</td>
</tr>
<tr>
<td>1.3</td>
<td>70' x 53'</td>
<td>26' x 20'</td>
<td>16,689</td>
<td>4.491</td>
</tr>
<tr>
<td>1.625</td>
<td>70' x 43'</td>
<td>26' x 16'</td>
<td>13,937</td>
<td>4.624</td>
</tr>
</tbody>
</table>

D.3  GEOMETRY OF MAT FOUNDATIONS

Total plan area of mat, panel dimensions, total column loads and equivalent average soil pressure are shown in Table D.2 for different length to width ratio of mat panel. Greater thickness and smaller thickness for mat foundations are given in Table D.3

Table D.3  Selected geometry of mat foundation for different mats.

<table>
<thead>
<tr>
<th>Length to Width Ratio</th>
<th>Greater Thickness (ft)</th>
<th>Smaller Thickness (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>3.50</td>
<td>2.00</td>
</tr>
<tr>
<td>1.3</td>
<td>3.50</td>
<td>2.00</td>
</tr>
<tr>
<td>1.625</td>
<td>3.25</td>
<td>1.875</td>
</tr>
</tbody>
</table>

D.4  OUTPUT OF MAT FOUNDATIONS

Only results of mat foundation for L/B ratio equal to 1.0 and for modulus of subgrade reaction 100 kcf have been presented in Chapter 5. Variation of deflections, bending moments and shear stress along lines 1 to 17 (shown in Fig. 5.1) are presented in Fig. D.1 to Fig. D.48. Descriptions of Fig. D.1 to Fig. D.48 are shown in Table D.4.
Table D.4 Description of figures presented in Appendix D.

<table>
<thead>
<tr>
<th>Figure Nos.</th>
<th>Modulus of Subgrade Reaction (kcf)</th>
<th>Panel Length to Width Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. D.1 to Fig. D.6</td>
<td>50</td>
<td>1.0</td>
</tr>
<tr>
<td>Fig. D.7 to Fig. D.12</td>
<td>50</td>
<td>1.30</td>
</tr>
<tr>
<td>Fig. D.13 to Fig. D.18</td>
<td>50</td>
<td>1.625</td>
</tr>
<tr>
<td>Fig. D.19 to Fig. D.24</td>
<td>100</td>
<td>1.30</td>
</tr>
<tr>
<td>Fig. D.25 to Fig. D.30</td>
<td>100</td>
<td>1.625</td>
</tr>
<tr>
<td>Fig. D.31 to Fig. D.36</td>
<td>200</td>
<td>1.00</td>
</tr>
<tr>
<td>Fig. D.37 to Fig. D.42</td>
<td>200</td>
<td>1.30</td>
</tr>
<tr>
<td>Fig. D.43 to Fig. D.48</td>
<td>200</td>
<td>1.625</td>
</tr>
</tbody>
</table>
Fig. D.1 Variation of deflection along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.0 and K=50 kcf
Fig. D.2  Variation of bending moment, $M_y$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.0 and K=50 kcf.
Fig. D.2  Variation of bending moment, $M_z$ along (d) line 3, (e) line 6 (f) Line 9 for panel L/B ratio 1.0 and K=50 kcf.
Fig. D.3  Variation of bending moment, $M_b$, along (a) line 10, (b) lines 12,13
(c) Lines 15, 16 for panel L/B ratio 1.0 and K=50 kcf.
Fig. D.3  Variation of bending moment, $M_x$, along (d) line 11, (e) line 14
(f) Line 17 for panel $L/B$ ratio 1.0 and $K=50$ kcf.
Fig. D.4 Variation of bending moment, $M_z$, along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.0 and K=50 kcf.
Fig. D.4  Variation of bending moment, $M_z$ along (d) line 3, (e) line 6
(f) Line 9 for panel $L/B$ ratio 1.0 and $K=50$ kcf.
Fig. D.5 Variation of bending moment, $M_z$ along (a) line 10, (b) lines 12, 13 (c) Lines 15, 16 for panel L/B ratio 1.0 and $K=50$ kcf.
Fig. D.5  Variation of bending moment, $M_z$ along (d) line 11, (e) line 14
(f) Line 17 for panel L/B ratio 1.0 and K=50 kcf.
Fig. D.6 Variation of Shearing stress, $S_{xy}$, along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.0 and $K=50$ kcf.
Fig. D.7  Variation of deflection along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.30 and K=50 kcf.
Fig. D.8  Variation of bending moment, $M_x$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.30 and K=50 kcf.
Fig. D.8 Variation of bending moment, $M_x$, along (d) line 3, (e) line 6
(f) Line 9 for panel L/B ratio 1.30 and $K=50$ kcf.
Fig. D.9 Variation of bending moment, $M_x$ along (a) line 10, (b) lines 12, 13 (c) Lines 15, 16 for panel L/B ratio 1.30 and $K=50$ kcf.
Fig. D.9  Variation of bending moment, $M_z$ along (d) line 11, (e) line 14
(f) Line 17 for panel L/B ratio 1.30 and $K=50$ kcf.
Fig. D.10  Variation of bending moment, $M_x$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.30 and K=50 kcf.
Fig. D.10  Variation of bending moment, $M_z$ along (d) line 3, (e) line 6 (f) Line 9 for panel L/B ratio 1.30 and K=50 kcf.
Fig. D.11  Variation of bending moment, $M_b$, along (a) line 10, (b) lines 12,13 (c) Lines 15, 16 for panel L/B ratio 1.30 and $K=50$ kcf.
Fig. D.11 Variation of bending moment, $M_z$ along (d) line 11, (e) line 14 (f) Line 17 for panel L/B ratio 1.30 and K=50 kcf.
Fig. D.12 Variation of Shearing stress, $S_{xy}$ along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.30 and $K=50$ kcf.
Fig. D.13  Variation of deflection along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.625 and K=50 kcf.
Fig. D.14 Variation of bending moment, $M_y$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.625 and $K=50$ kcf.
Fig. D.14 Variation of bending moment, $M_x$ along (d) line 3, (e) line 6, (f) Line 9 for panel L/B ratio 1.625 and $K=50$ kcf.
Fig. D.15  Variation of bending moment, $M$, along (a) line 10, (b) lines 12,13  
(c) Lines 15, 16 for panel L/B ratio 1.625 and $K=50$ kcf.
Fig. D.15 Variation of bending moment, $M_e$ along (d) line 11, (e) line 14, and (f) line 17 for panel L/B ratio 1.625 and K=50 kcf.
Fig. D.16  Variation of bending moment, $M_z$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.625 and $K=50$ kcf.
Fig. D.16 Variation of bending moment, $M_z$ along (d) line 3, (e) line 6 (f) Line 9 for panel L/B ratio 1.625 and K=50 kcf.
Fig. D.17  Variation of bending moment, $M_z$ along (a) line 10, (b) lines 12, 13 (c) Lines 15, 16 for panel L/B ratio 1.625 and $K=50$ kcf.
Fig. D.17  Variation of bending moment, $M_z$ along (d) line 11, (e) line 14 (f) Line 17 for panel L/B ratio 1.625 and K=50 kcf.
Fig. D.18  Variation of Shearing stress, $S_{xy}$ along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.625 and K=50 kcf.
Fig. D.19 Variation of deflection along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.30 and K=100 kcf.
Fig. D.20 Variation of bending moment, $M_x$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.30 and $K=100$ kcf.
Fig. D.20  Variation of bending moment, M, along (d) line 3, (e) line 6, (f) line 9 for panel L/B ratio 1.30 and K=100 kcf.
Fig. D.21 Variation of bending moment, $M_x$ along (a) line 10, (b) lines 12, 13 (c) Lines 15, 16 for panel L/B ratio 1.30 and K=100 kcf.
Fig. D.21 Variation of bending moment, $M_x$, along (d) line 11, (e) line 14 (f) Line 17 for panel L/B ratio 1.30 and K=100 kcf.
Fig. D.22  Variation of bending moment, $M_z$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.30 and K=100 kcf.
Fig. D.22 Variation of bending moment, $M_2$ along (d) line 3, (e) line 6, (f) line 9 for panel L/B ratio 1.30 and $K=100$ kcf.
Fig. D.23  Variation of bending moment, $M_z$ along (a) line 10, (b) lines 12, 13  
(c) Lines 15, 16 for panel L/B ratio 1.30 and K=100 kcf.
Fig. D.23 Variation of bending moment, $M_z$, along (d) line 11, (e) line 14 (f) line 17 for panel L/B ratio $1.30$ and $K=100$ kcf.
Fig. D.24 Variation of Shearing stress, $S_y$, along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.30 and $K=100$ kcf.
Fig. D.25: Variation of deflection along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.625 and K=100 kcf.
Fig. D.26  Variation of bending moment, $M_x$, along (a) lines 1, 2, (b) lines 4, 5, (c) Lines 7, 8 for panel L/B ratio 1.625 and $K=100$ kcf.
Fig. D.26 Variation of bending moment, $M_x$ along (d) line 3, (e) line 6, (f) Line 9 for panel L/B ratio 1.625 and $K=100$ kcf.
Fig. D.27  Variation of bending moment, $M_x$ along (a) line 10, (b) lines 12,13 (c) Lines 15, 16 for panel L/B ratio 1.625 and K=100 kcf.
Fig. D.27  Variation of bending moment, $M_x$ along (d) line 11, (e) line 14 (f) Line 17 for panel L/B ratio 1.625 and $K=100$ kcf.
Fig. D.28  Variation of bending moment, \( M_{z} \) along (a) lines 1, 2, (b) lines 4, 5, (c) Lines 7, 8 for panel L/B ratio 1.625 and K=100 kcf.
Fig. D.28 Variation of bending moment, $M_z$ along (d) line 3, (e) line 6
(f) Line 9 for panel L/B ratio 1.625 and $K=100$ kcf.
Fig. D.29  Variation of bending moment, $M_y$ along (a) line 10, (b) lines 12,13 (c) Lines 15, 16 for panel L/B ratio 1.625 and $K=100$ kcf.
Fig. D.29  Variation of bending moment, $M_z$ along (d) line 11, (e) line 14
(f) Line 17 for panel L/B ratio 1.625 and K=100 kcf.
Fig. D.30  Variation of Shearing stress, $S_{xy}$ along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.625 and $K$=100 kcf.
Fig. D.31  Variation of deflection along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.0 and $K = 200$ kcf.
Fig. D.32  Variation of bending moment, $M_x$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.0 and $K = 200$ kcf.
Variation of bending moment, $M_x$, along (d) line 3, (e) line 6, (f) line 9 for panel L/B ratio 1.0 and $K = 200$ kcf.
Fig. D.33  Variation of bending moment, $M_x$, along (a) line 10, (b) lines 12, 13
(c) Lines 15, 16 for panel L/B ratio 1.0 and $K = 200$ kcf.
Fig. D.33  Variation of bending moment, $M_x$ along (d) line 11, (e) line 14
(f) Line 17 for panel L/B ratio 1.0 and $K = 200$ kcf.
Fig. D.34  Variation of bending moment, $M_z$, along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.0 and $K = 200$ kcf.
Fig. D.34 Variation of bending moment, $M_z$ along (d) line 3, (e) line 6
(f) Line 9 for panel L/B ratio 1.0 and $K = 200$ kcf.
Fig. D.35  Variation of bending moment, $M_z$ along (a) line 10, (b) lines 12,13 (c) Lines 15, 16 for panel L/B ratio 1.0 and $K = 200$ kcf.
Fig. D.35  Variation of bending moment, $M_x$ along (d) line 11, (e) line 14 (f) Line 17 for panel L/B ratio 1.0 and $K = 200$ kcf.
Fig. D.36  Variation of Shearing stress, $S_y$, along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.0 and $K = 200$ kcf.
Fig. D.37 Variation of deflection along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.30 and K = 200 kcf.
Fig. D.38  Variation of bending moment, $M_x$, along (a) lines 1, 2, (b) lines 4, 5, (c) lines 7, 8 for panel L/B ratio 1.30 and $K = 200$ kcf.
Fig. D.38  Variation of bending moment, $M$, along (d) line 3, (e) line 6, (f) line 9 for panel L/B ratio 1.30 and $K = 200$ kcf.
Fig. D.39  Variation of bending moment, \( M_x \) along (a) line 10, (b) lines 12,13 (c) Lines 15, 16 for panel \( L/B \) ratio 1.30 and \( K = 200 \) kcf.
Fig. D.39 Variation of bending moment, $M_y$ along (d) line 11, (e) line 14
(f) Line 17 for panel L/B ratio 1.30 and $K = 200$ kcf.
Fig. D.40 Variation of bending moment, $M_z$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.30 and $K = 200$ kcf.
Fig. D.40 Variation of bending moment, $M_z$ along (d) line 3, (e) line 6
(f) Line 9 for panel L/B ratio 1.30 and $K = 200$ kcf.
Fig. D.41 Variation of bending moment, $M_z$ along (a) line 10, (b) lines 12, 13 (c) Lines 15, 16 for panel L/B ratio 1.30 and $K = 200$ kcf.
Fig. D.41 Variation of bending moment, $M_z$ along (d) line 11, (e) line 14 (f) Line 17 for panel L/B ratio 1.30 and $K = 200$ kcf.
Fig. D.42 Variation of Shearing stress, $S_{xy}$ along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.30 and $K = 200$ ksf.
Fig. D.43  Variation of deflection along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.625 and $K = 200$ kcf.
Fig. D.44 Variation of bending moment, $M_x$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.625 and $K = 200$ kcf.
Fig. D.44 Variation of bending moment, $M$, along (d) line 3, (e) line 6
(f) line 9 for panel L/B ratio 1.625 and $K = 200$ kcf.
Fig. D.45  Variation of bending moment, $M_x$, along (a) line 10, (b) lines 12,13 (c) Lines 15, 16 for panel L/B ratio 1.625 and $K = 200$ kcf.
Fig. D.45  Variation of bending moment, $M_y$ along (d) line 11, (e) line 14
(f) Line 17 for panel L/B ratio 1.625 and $K = 200$ kcf.
Fig. D.46  Variation of bending moment, $M_x$ along (a) lines 1, 2, (b) lines 4, 5 (c) Lines 7, 8 for panel L/B ratio 1.625 and $K = 200$ kcf.
Fig. D.46 Variation of bending moment, $M_z$, along (d) line 3, (e) line 6 (f) Line 9 for panel L/B ratio 1.625 and $K = 200$ kcf.
Fig. D.47  Variation of bending moment, $M_y$ along (a) line 10; (b) lines 12, 13; (c) Lines 15, 16 for panel L/B ratio 1.625 and $K = 200$ kcf.
Fig. D.47  Variation of bending moment, $M_z$ along (d) line 11, (e) line 14
(f) Line 17 for panel L/B ratio 1.625 and $K = 200$ kcf.
Fig. D.48 Variation of Shearing stress, $S_{y}$ along (a) lines 1-5, (b) lines 6-9 for panel L/B ratio 1.625 and $K = 200$ kcf.