SIMPLIFIED ANALYSIS OF WALL-FRAME STRUCTURE WITH COLUMNS AND GIRDERS OF UNEQUAL LATERAL DIMENSION

BY

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A PROJECT REPORT

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DECLARATION

I hereby declare that the project work reported herein has been performed by me and that this work has not been submitted for any other degree.

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ABSTRACT

Wall-frame structures in high rise buildings very often comprise columns and girders whose lateral dimensions (i.e. the depth of column and width of girder) are unequal. A simplified method of calculating the stresses and deflections in such structures, subjected to lateral loads, is presented.

The accuracy of the method is verified by analyzing a 10-storey wall-frame and comparing the results of the simplified method with those obtained from a finite element analysis and a continuous medium analysis.

The method, originally proposed by A.H. Khan for wall-frame structures of uniform thickness, consists in dividing the original structure into a discrete number of 'modules' which are then replaced by uniform plate elements of same height and width as the parent module and same thickness as that of column of the parent module, but having analogous stiffness properties. This report presents the results of an effort to extend Khan's method to structures where the lateral dimension of columns and girders are unequal.

General computer programmes for the analysis of wall-frames, subject to lateral load, have been developed using the following three methods:

i. Simplified Method

ii. Finite Element Method

and iii. Continuous Medium Method

Wall-frames consisting of variable thickness columns and girders are analysed by the proposed simplified method and the results are compared with those obtained by continuous Medium Method and Finite Element Method. The values of stresses and deflections obtained from the simplified method agree well with these obtained from a finite element analysis and from continuous medium method.

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CONTENTS

NOTATIONS

A	Area of cross-section
B	Half-width of module at wider section
b	Half-width of module at narrower section
t	Width of girder
T	Width of column
E	Modulus of Elasticity
E'	Modified value of E
G	Modulus of Rigidity
G'	Modified value of G
H	Half-height of a module
h	Half-height of the wider section of a module, or half-depth of the girder.

CHAPTER 1

INTRODUCTION

1.1 General:

The development of highrise buildings is a consequence of the large-scale migration of agricultural population from the rural areas to the urban commercial and industrial centres. The need for more working, living and other spaces in these cities can be efficiently solved by the construction of multistory buildings, keeping much more open space at the ground level.

With the increasing height of buildings, the selection of proper structural systems to provide strength, stiffness and stability against lateral loads (viz. wind and earthquake) becomes extremely important. Shear wall, which derive their strength and stiffness from their inherent shape, have been widely used. Ideally, from the structural engineering point of view, they should not have any openings. However from functional consideration, walls, whether in the interior core or in the exterior facade, must have openings to allow circulation within the floor area and to allow natural light to enter the building.

A structure in which the vertical and horizontal framing members have similar depth (or width) to thickness ratios, may be defined as a 'wall-frame' structure[1].

During recent years, 'wall-frame' type of structures have been used increasingly in the construction of tall buildings. These structures have higher strengths and efficiency than ordinary building frames when loaded laterally and in many cases can surpass coupled shear walls in economy and aesthetics.

At one extreme a wall-frame structure becomes a slender, mutistorey multi-bay building frame (Fig. 1.1a) while at the other extreme, it becomes a thin cantilever wall with very small openings (Fig. 1.1b). Steel, concrete, timber and composite materials may be used in their construction; they can be constructed of precast units or can be cast in-situ.

The behaviour of wall-frame structures subject to horizontal loads is complex, as they possess the characteristic behaviour of both frames and shear walls. Past research efforts have produced methods for the analysis and design of both frames and plane shearwalls. The continuous connection technique[2], the equivalent frame method, the wide-column frame method[2] etc. have long been used for the analysis of almost all types of plane shear wall systems.

At present, the more general Finite Element Method (FEM) is capable of analyzing almost all plane stress/plane strain problems including the wall-frame structures. Although these modern computer oriented techniques can be used successfully for the complete analysis of these structures, the use of such techniques is virtually limited to the final analysis stage of the design process due to the heavy cost involved. This fact indicates the great need for simplified methods in the normal design offices for the analysis of complex problems.

In this thesis, the feasibility of extending Khan's method (a simple hand-method for analyzing wall-frame structures subjected to lateral loads) to wall-frame with unequal column depth and girder width is investigated. The method combines the simplicity and directness of graphical and simple analytical methods, and is suitable for design office use in performing a preliminary analysis of wall-frame and other similar structures.

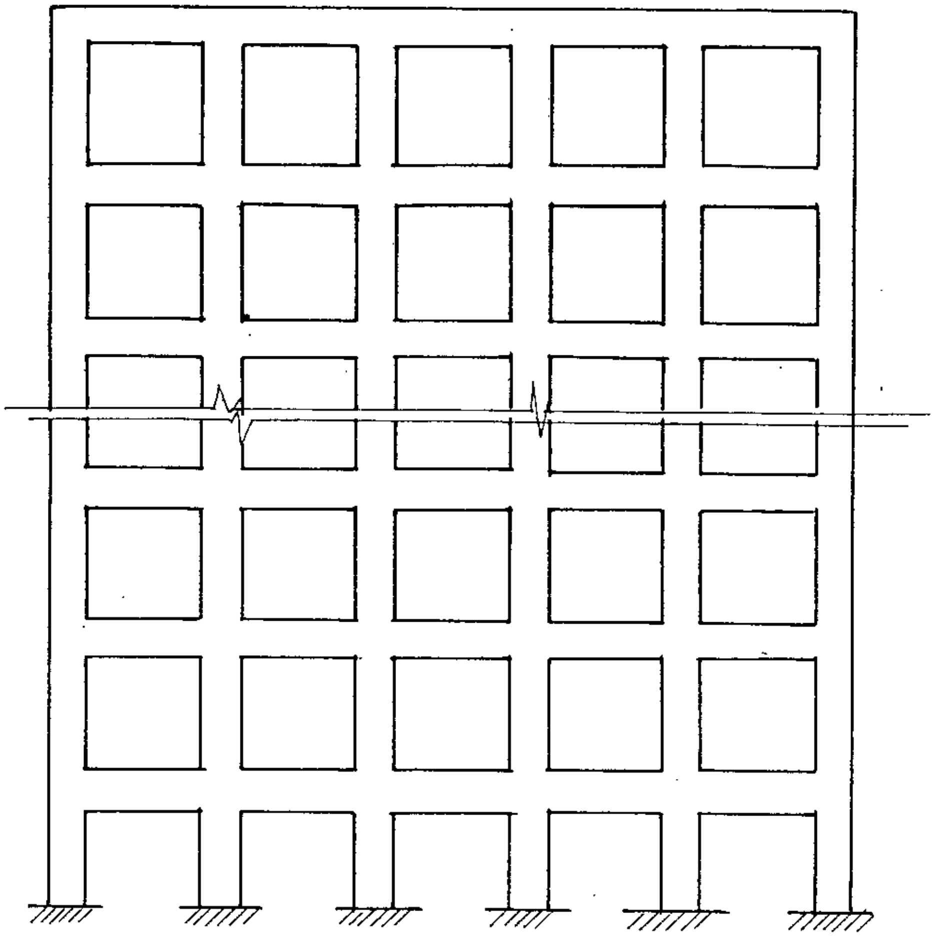
1.2 Types of wall-frame:

The types of wall-frame includes

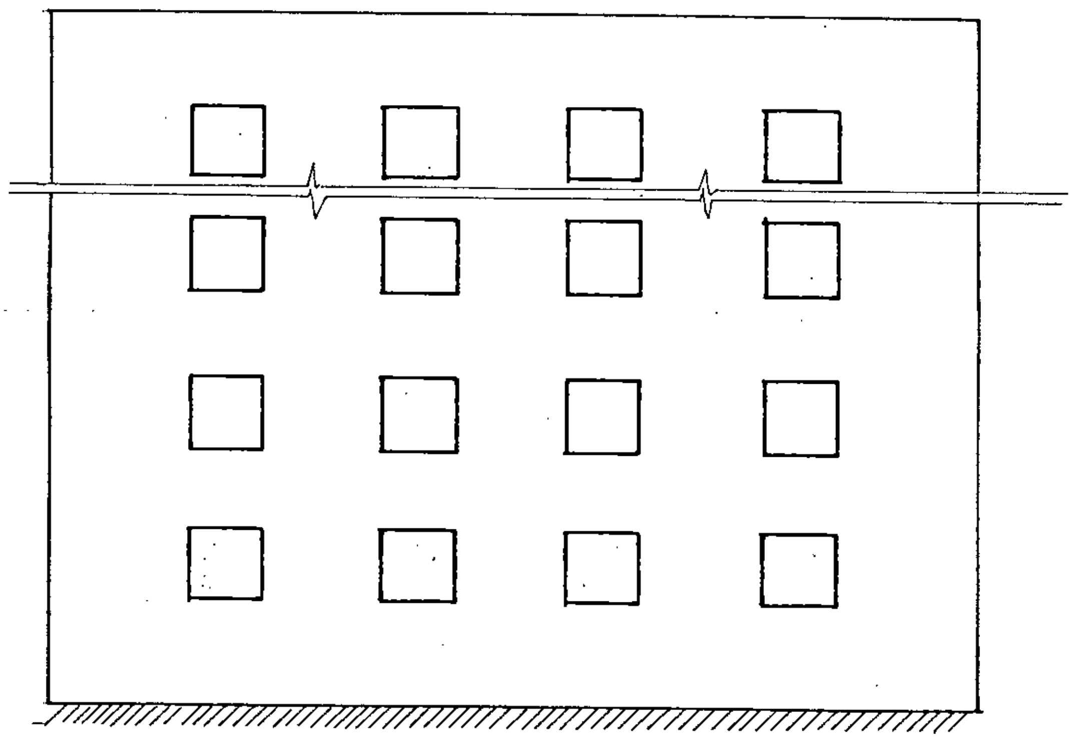
i. Multi-storey multi-bay frames,

ii. Plane shear wall with multiple bands of openings,

and iii. Facade walls with arrays of window openings.



1.1 a



1.1 b

FIG. 1.1

CHAPTER 2

LITERATURE SURVEY

2.1 Basic Mathematical Model:

When analyzing a complex structure by any simple method, an appropriate idealization of the basic structure is of vital importance. In the case of wall-frame structures, an idealization into either of two extremes, namely a frame or a plane wall, would offer scope for simple solution. In this project, the basic structure is to be carried to the latter extreme for formulating a simple solution process. Therefore, the highly redundant perforated wall is to be reduced to an unperforated, thin cantilever plate with equivalent stiffness properties. In the process, the original structure is divided into a discrete number of modules, which are then replaced by uniform plate elements of same height and width as the parent module and the thickness as that of column, but having analogous stiffness properties.

The above technique is an extension of the method originally proposed by A.H. Khan(1) for wall-frames of uniform lateral dimension. In Khan's method, the original structure was divided into a discrete number of modules which were then replaced by uniform plate elements of same height, width and thickness as the parent module.

Fig. 2.1a shows the basic wall-frame structure a facade shear wall with multiple window openings. Figs. 2.1b, 2.1c and 2.1d show a representative module, the equivalent plate element and the equivalent plane structure respectively.

A simple method for the analysis of large multistory multi-bay frame work has been presented by Kinh, Paul and Osama[5]. It is based on replacing the actual structure by an elastically equivalent orthotropic membrane which is then analysed by the finite element technique. The inflection point for the bottom storey is assumed at 2/3 of the length of column from the support. The refined expressions for the equivalent elastic properties in combination with the versatility of the finite element technique make this method well adapted to a wide range of wall-frame structures.

However, the above method is dependent on the availability of access to a computer with a plane stress analysis programme, and therefore, may not be convenient as a preliminary design tool.

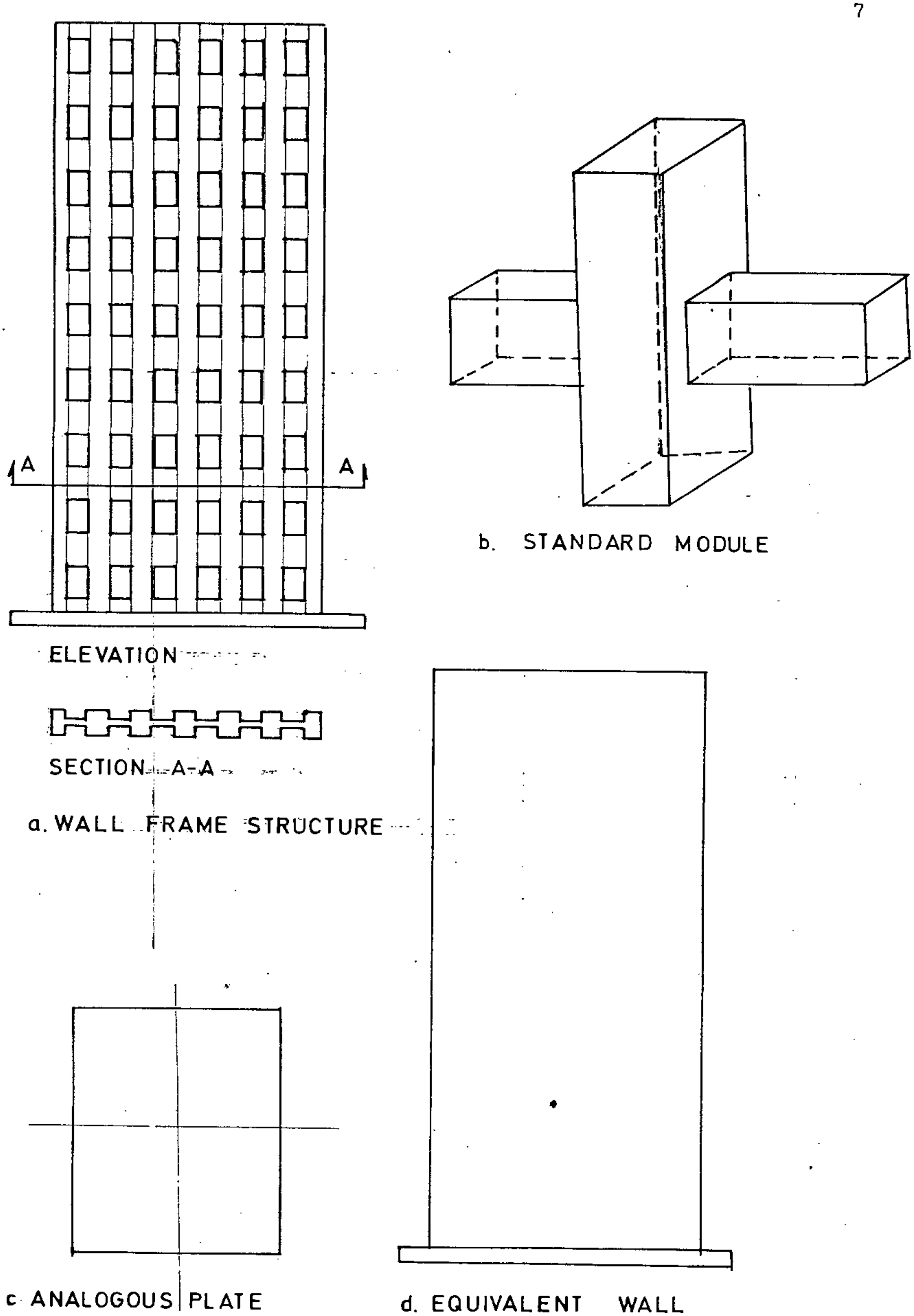


FIG. 2.1, MODELING OF WALL FRAME STRUCTURES

CHAPTER 3

METHOD OF ANALYSIS

3.1 Stiffness Properties of a Module:

A wall-frame structure behaves under load as a shear-flexure beam. When the opening sizes are very small it behaves predominantly in a flexural mode but, if the openings are large, the structure behaves in the shear mode. For intermediate case, both the shear and flexure modes are significant in the behaviour of these structures.

In devising an analogous plate for a typical module, it is necessary to consider the two primary modes of behaviour, viz. flexure and shear. The flexural stiffness of the equivalent plate is represented by the quantity 'E'I instead of EI and the shearing stiffness is represented by the quantity G'A rather than GA as parameters of the module. As the deformations and stresses in a cantilever beam depend on the two quantities, E' and G', the analogous structure is assumed to behave as a simple cantilever, rather than two parallel systems of bending and shearing components interacting together, in resisting the external loads.

The stiffness properties of a module can be found from two different methods of analysis:

a) the ordinary strength of materials theory and

b) a finite element computer analysis.

The strength of materials theory does not take into account the effect of stress concentration due to change in section, nor the nonlinear distribution of stress across the module, on the other hand the finite element analysis presents a relatively more accurate picture of the behaviour of a module by taking these effects into account. The E' and G' parameters derived from either method are capable of predicting the overall behaviour of the total structure under load - the computer method obviously giving closer and more accurate results.

3.2 Axial or Flexural Stiffness Parameter, E'.

By Strength of Materials Theory:

A representative module from the interior of a wall-frame structure with regular openings is shown in Fig. 3.1. A uniform axial stress, σ , is assumed to be applied as shown. Since the module is symmetrical, only a half or even a quarter need be considered, in deriving the basic relationships. The stress, σ , reduced to $(b/B)\sigma$ when distributed uniformly over the width 2B of the analogous plate and $\sigma/[1 + \frac{t}{T}(\frac{B}{b} - 1)]$ when distributed over the width 2B of the module.

Under these stresses,

the elongation of the narrower portion, $\delta_1 = \frac{\sigma}{E}(H-h)$ (3.1)

and the elongation of the wider portion, $\delta_2 = \frac{\sigma}{E} \frac{h}{1+\frac{t}{T}(\frac{B}{b}-1)}$ (3.2)

The total elongation of the semi-height, H, of the module is, from eqn. (3.1) and (3.2),

$$\delta_1 + \delta_2 = \frac{\sigma}{E} \left[(H-h) + \frac{h}{1 + \frac{t}{T}(\frac{B}{b} - 1)} \right] \qquad (3.3)$$

Unit elongation is, therefore,

$$\delta = \frac{\sigma}{E} \left[(H-h) + \frac{h}{1 + \frac{t}{T}(\frac{B}{b} - 1)} \right] / H$$

$$\delta = \frac{\sigma}{E} \left[(1 - \frac{h}{H}) + \frac{h/H}{1 + \frac{t}{T}(\frac{B}{b} - 1)} \right] \qquad (3.4)$$

Now it is assumed that the anologous plate for the module will have the same unit elongation under similar tensile stresses. If E' is the effective elastic modulus of the equivalent plate, its unit elongation is,

$$\delta' = \frac{b}{B} \frac{\sigma}{E'} \qquad (3.5)$$

By definition, $\delta = \delta'$

Therefore,

$$\frac{b}{B} \frac{\sigma}{E'} = \frac{\sigma}{E} \left[(1 - \frac{h}{H}) + \frac{h/H}{1 + \frac{t}{T}(\frac{B}{b} - 1)} \right] \qquad (3.6)$$

from which

$$\frac{E'}{E} = \frac{b/B}{(1 - \frac{h}{H}) + \left[\frac{h}{H} / \{ 1 + \frac{t}{T}(\frac{B}{b} - 1) \} \right]} \qquad (3.7)$$

The value E'/E gives the ratio of the overall axial stiffness of an equivalent plate to its parent module. It is seen from eqn. (3.7) that this ratio is dependent on,

i. the dimensions H and B of a module or more generally, the H/B ratio.

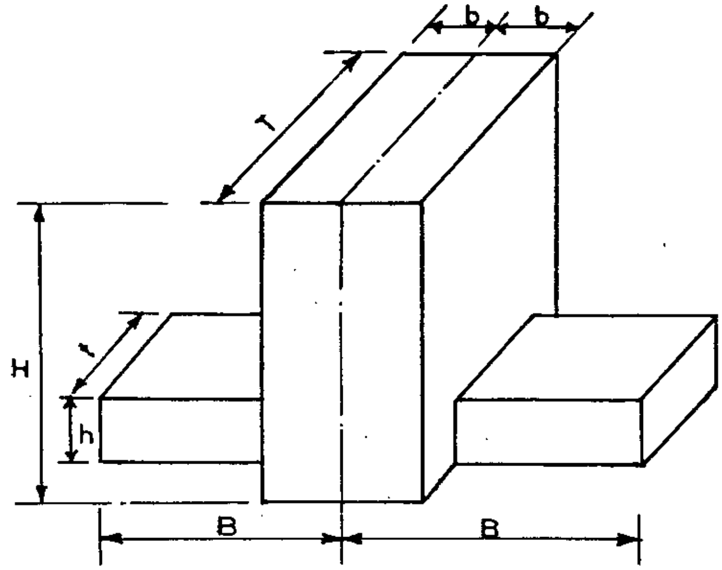
ii. the size of window openings represented by b/B and h/H ratios,

and iii. the girder and column thickness represented by t/T ratio.

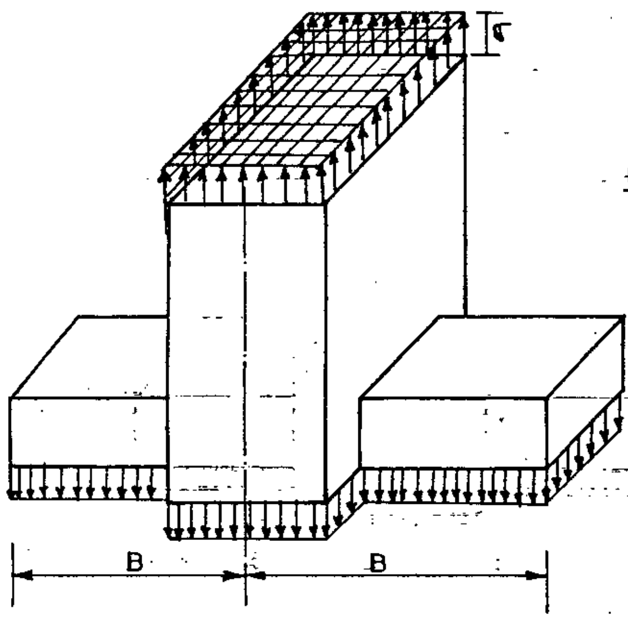
Curves for E'/E, for different values of b/B, h/H and t/T ratios can be drawn. The curves are shown in Fig. 3.6, 3.7 and 3.8 and are the same for all ranges of H/B ratios.

By Computer Analysis (Using FEM):

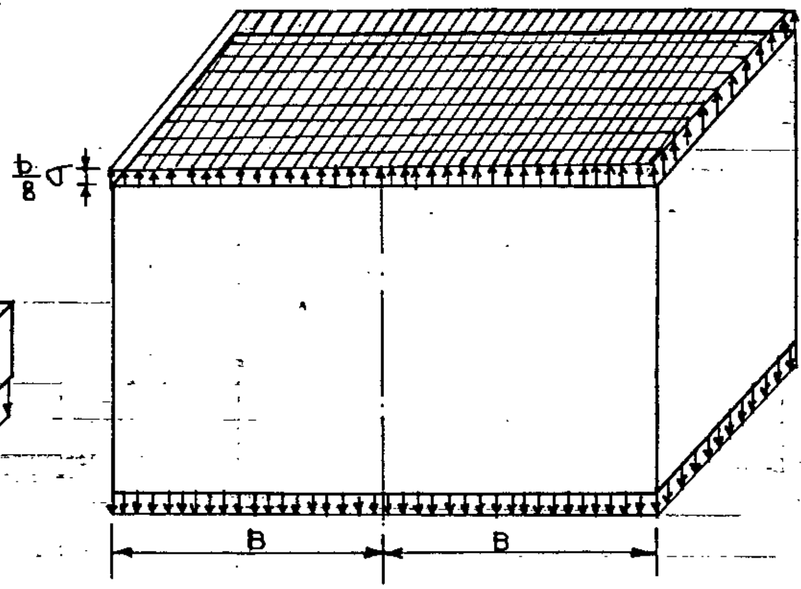
Simple strength of materials theory cannot correctly represent the true state of stress of a module because of the oversimplifications involved in its formulation. For example, it does not consider the effect of stress concentration when the module section changes from narrow to wide and assumes the distribution of stress as uniform in each section. This would overestimate the stiffness of an equivalent plate and hence the stiffness of the total structure. Due to stress concentration at the sudden change of section, the module section is not subject to uniform stress. The stress trajectories are shown in Fig. 3.2.



(a) Half Module



(a) Half Module



(b) Half Analogous Plate

FIG. 3.1 MODULE UNDER AXIAL STRESS.

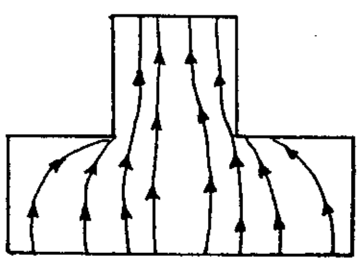


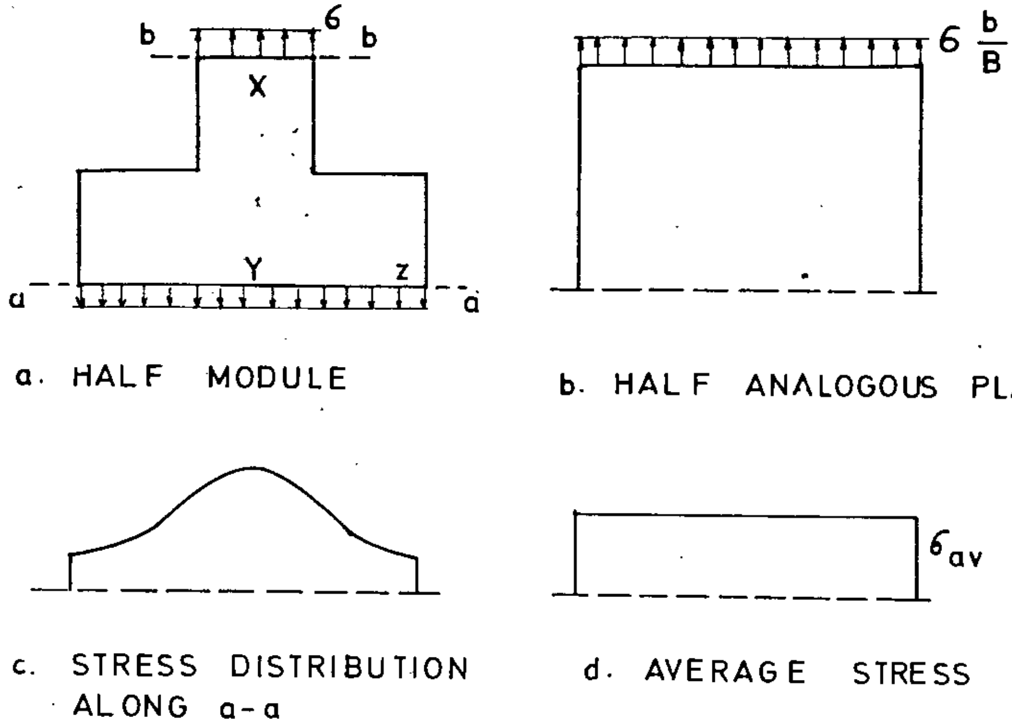
FIG. 3.2 VERTICAL STRESS TRAJECTORIES IN A MODULE.

Areas just below the re-entrant corners have low stress while the middle portions have higher stresses.

The above facts necessitate the analysis of the module by the more sophisticated and accurate finite element method using a computer. For the purpose of analysis, the whole module is divided into a large number of rectangular finite elements. A given axial stress, is applied at the nodal points along b-b as shown in Fig. 3.3a. The boundary conditions at the base and at the sides (shown in Fig. 3.4) are chosen so that the continuity with the lower half of the module as well as with the adjacent modules, is represented.

The applied axial loads give rise to axial stress throughout the module. The distribution of stress at section a-a is shown in Fig. 3.3c. If σ_{av} is assumed to be uniform stress (Fig. 3.3d) produced in the analogous plate (Fig.3.3b) due to the same applied load the value of E' representing the axial stiffness of the module can easily be calculated.

The stresses due to the applied stress are computed by using a finite element plane stress computer programme. The finite element idealization is shown in Fig. 3.4. The shape function and the resulting stiffness and stress matrices are given in Appendix-A. Repreated computer runs were made to evaluate E'/E ratios for different module



a. HALF MODULE

b. HALF ANALOGOUS PLATE

c. STRESS DISTRIBUTION ALONG a-a

d. AVERAGE STRESS

FIG. 3.3. MODULE UNDER AXIAL STRESS

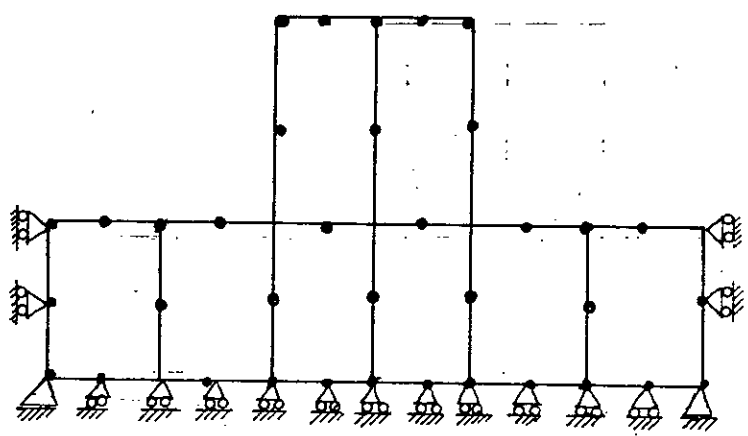


FIG. 3.4 MODULE IDEALIZATION FOR COMPUTER ANALYSIS

sizes (i.e. H/B ratios), for different b/B and h/H ratios (i.e. opening sizes) and for different t/T ratios (i.e. girder column thickness ratios).

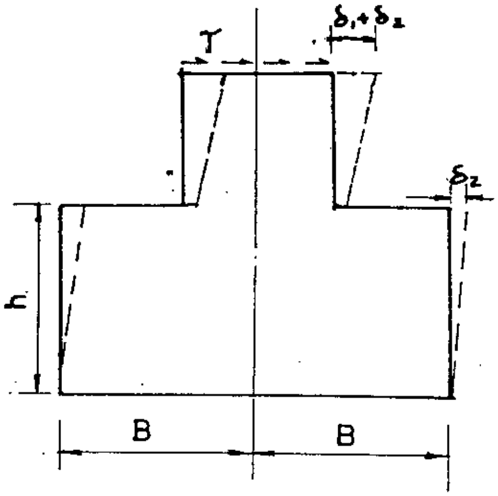
Some curves drawn with the t/T, b/B, h/H and H/B as variables are shown in Figs. 3.9, 3.10, 3.11, 3.12, 3.13, 3.14, 3.15, 3.16, 3.17 and 3.18.

Corresponding values of E'/E were found to be less than those obtained from the strength of materials theory. In other words, the analogous plate becomes less stiff and thus represents the stiffness of a wall-frame structure more accurately than by the strength of materials approximation.

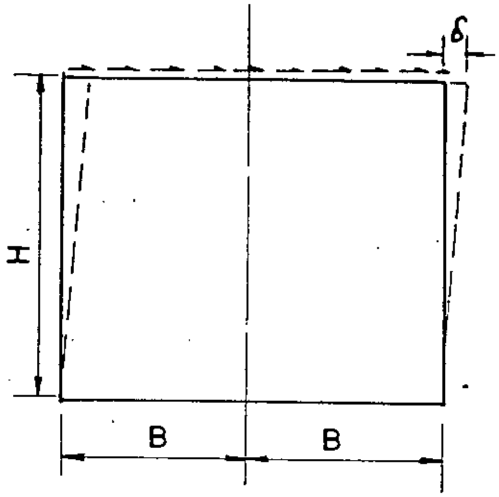
3.3 Shearing Stiffness Parameter G':

By Strength of Materials Theory:

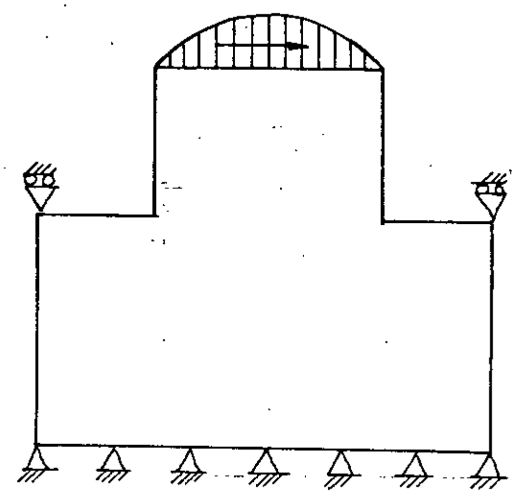
The second primary mode of deformation of a module is by shear, this is considered now so as to obtain an equivalent shearing stiffness for the analogous plate. In doing this, it is assumed that the maximum displacement



a. MODULE UNDER SHEARING STRESS



b. ANALOGOUS PLATE



c. MODULE IDEALIZATION FOR COMPUTER ANALYSIS

FIG. 3.5, MODULE SUBJECT TO SHEARING FORCE of the analogous plate is the same as the maximum displacement of the module at the narrow end.

Let a constant shearing stress, τ be applied to the module, as shown in Fig. 3.5. From Fig. 3.5a shearing deflection of the narrower portion,

$$\delta_1 = \frac{\tau}{G} (H-h) \tag{3.8}$$

shearing deflection of the wider portion,

$$\delta_2 = \frac{\tau}{G} \frac{h}{1+\frac{t}{T}\left(\frac{B}{b} - 1\right)} \tag{3.9}$$

Total deflection at narrow end,

$$\delta_1 + \delta_2 = \frac{\tau}{G} \left[(H-h) + \frac{h}{1+\frac{t}{T}\left(\frac{B}{b} - 1\right)}\right] \tag{3.10}$$

From eqn. (3.10) the average shearing strain

$$\gamma = \frac{\tau}{G} \left[(H-h) + \frac{h}{1+\frac{t}{T}\left(\frac{B}{b} - 1\right)} \right] /H \tag{3.11}$$

If the same strain produced in the analogous plate (Fig. 3.5b) with shearing modules G', and subject to the same shearing stress, then

$$\frac{b}{B} \frac{\tau}{G'} = \frac{\tau}{G} \left[(H-h) + \frac{h}{1+\frac{t}{T}\left(\frac{B}{b} - 1\right)}\right] /H \tag{3.12}$$

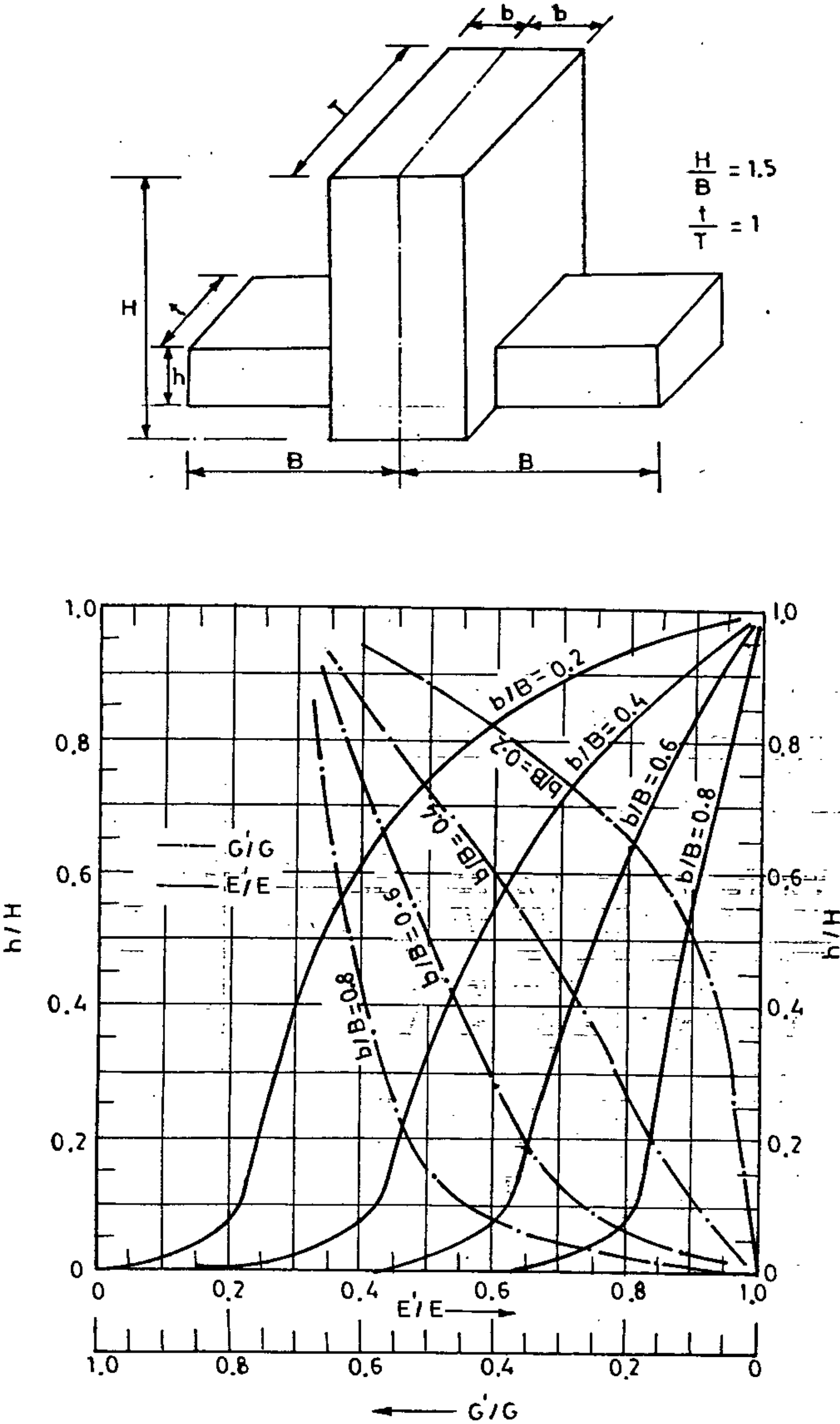


FIG. 3.6 VARIATIONS OF E'/E AND G'/G RATIOS
(by strength of material theory)

(The above curves for t/T=1 are the same as in Ref. 1)

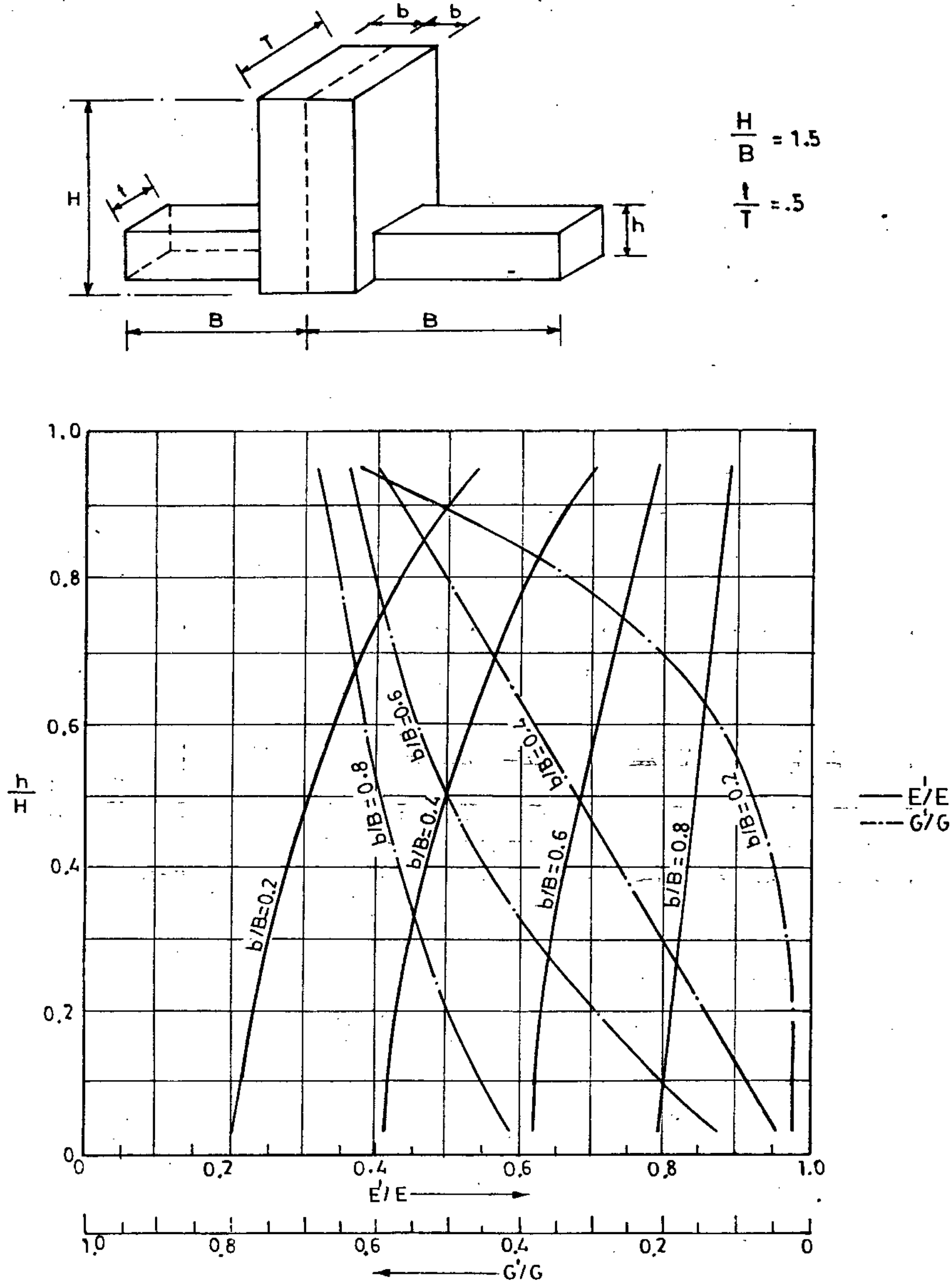


Fig.3.7 Variations of E'/E and G'/G Ratios (by strength of materials theory)

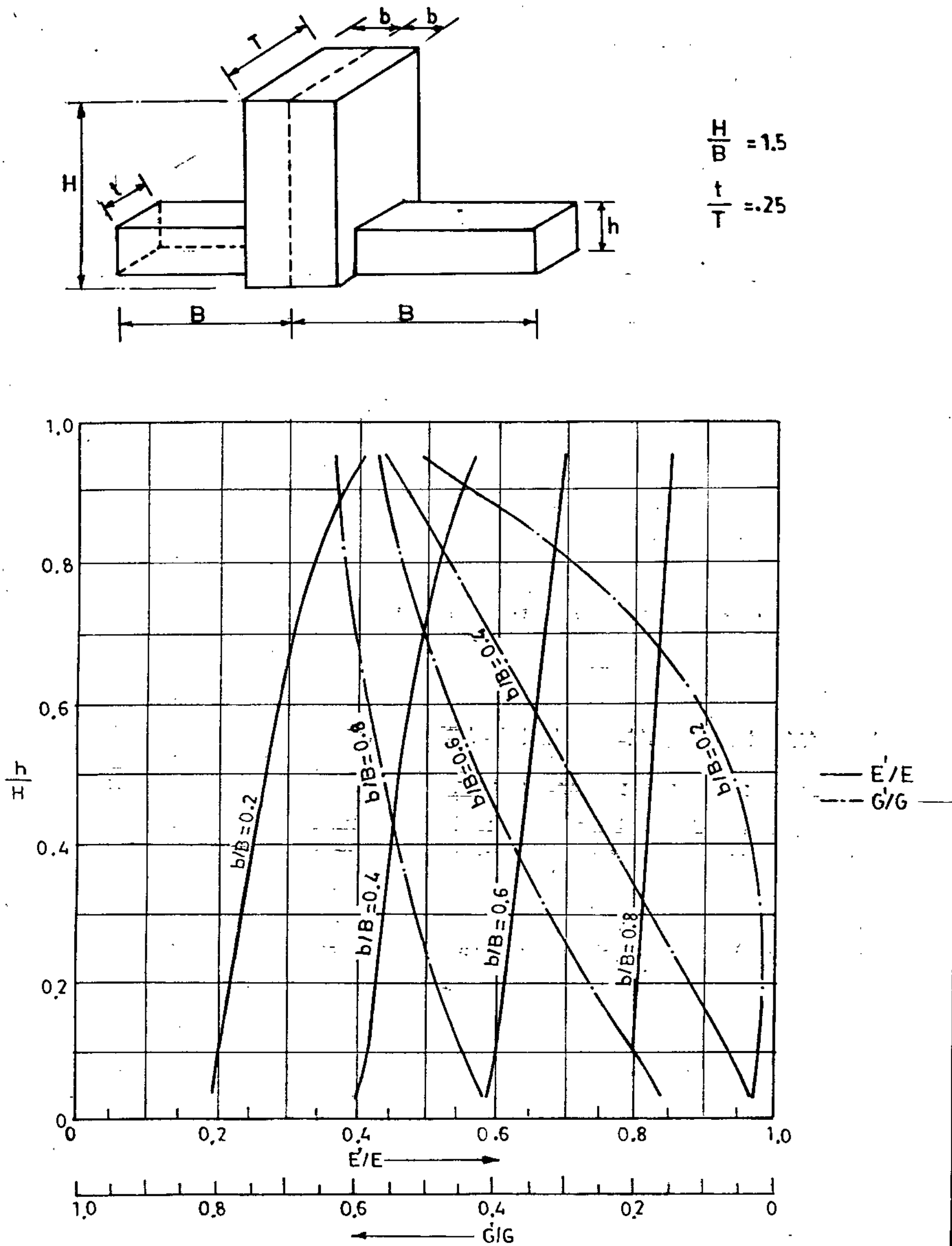


Fig. 3.8 Variations of E'/E and G'/G Ratios (by strength of materials theory)

From which,

$$G'/G = \frac{b/B}{(1-h/H)+(h/H)/(1+\frac{t}{T}(\frac{B}{b}-1))} \qquad (3.13)$$

The right hand side of eqn. (3.13) is the same as that of eqn. (3.7) indicating similar variations of both the E'/E and G'/G ratios.

In the above, the bending deflection of the module is neglected. Since the size of a module is significant, the applied shearing stress will tend to bend or rotate its segments. For consideration of the bending effects, it is assumed that the total module deformation comprises both shearing and bending actions, whilst the bending deflection of the analogous plate is negligible. This approximation is reasonable because the analogous plate, having the larger depth throughout its height (i.e. equal to 2B for the module) will possess a very high bending stiffness and consequently very little deflection will be produced due to bending.

Reconsidering the module subject to a uniform shearing stress, Fig. 3.5a, the deflection due to shear and bending of the different segments of a module are as follows,

Shear deformation of segment XY,

$$\delta_1 = \frac{\tau}{G}\left[(H-h) + \frac{h}{1+\frac{t}{T}(\frac{B}{b}-1)}\right] \qquad (3.14)$$

bending deformation of segment XY,

$$\delta_2 = \frac{1}{3EI_2}\ \tau\, 2b\ T\ (H-h)^3 + \frac{h}{6EI_1}\ [\tau\, 2b.\ T\ (H-h)\ \{ 2(H-h) + H \}$$

$$+ \tau\, 2b\ T\ H\ (2H + H-h)\ ]$$

$$\delta_2 = \frac{1}{3EI_2}\ \tau\, 2b.T.\ (H-h)^3 + \frac{\tau\, 2b\ T\ h}{6EI_1}\ [(H-h)\ (3H-2h) + 3H^2 - hH]$$

$$\delta_2 = \frac{\tau\, 2b\ T}{3EI_2}\ (H-h)^3 + \frac{\tau\, 2b.T.h}{6EI_1}\ (6H^2 + 2h^2 - 6hH)$$

$$\delta_2 = \frac{\tau\, 2b\ T}{3E}\ [\frac{(H-h)^3}{I_2} + \frac{h}{2I_1}\ (6H^2 + 2h^2 - 6hH)\ ] \qquad (3.15)$$

Where $I_2 = \frac{T(2b)^3}{12} = T\ B^3\ \frac{2}{3}(b/B)^3$

$$I_1 = \frac{2}{3}\ Tb^3 + [\ \frac{t(B-b)^3}{12} + t(B-b)\ (b + \frac{B-b}{2})^2\ ]\ 2$$

$$I_1 = TB^3\ \frac{2}{3}\ (b/B)^3 + \frac{1}{6}\ (\frac{t}{T})(1-\frac{b}{B}\ )\ (\ 1 - \frac{b}{B})^2 + 3(1+\frac{b}{B})^2$$

The total deflection of point X in XY,

$$\delta_1 + \delta_2 = \frac{\tau}{G}\ [(H-h) + \frac{h}{1+\frac{t}{T}(\ \frac{B}{b} - 1)}\ ]$$

$$+ \frac{\tau 2b\ T}{3E}\ [\frac{(H-h)^3}{I_2} + \frac{h}{2I_1}\ (6H^2+2h^2-6hH)\ ] \qquad (3.16)$$

The shearing deflection of the equivalent plate under similar shearing load is

$$\delta' = \frac{b}{B}\,\tau\,\frac{H}{G'} \tag{3.17}$$

where G' is the effective shearing modulus of the equivalent plate. Eqn. (3.16) and (3.17) to be equivalent,

$$\frac{b}{B}\tau\frac{H}{G'} = \frac{\tau}{G}\left[(H-h) + \frac{h}{1+\frac{t}{T}\left(\frac{B}{b}-1\right)}\right]$$

$$+ \frac{\tau\,2b\,T}{3E}\left[\frac{(H-h)^3}{I_2} + \frac{h}{2I_1}(6H^2+2h^2-6hH)\right] \tag{3.18}$$

From which

$$\frac{G}{G'} = \left[(H-h) + \frac{h}{1+\frac{t}{T}\left(\frac{B}{b}-1\right)}\right] / \left(\frac{b}{B}\,H\right)$$

$$+ \frac{2b}{3}\,\frac{T}{2(1+\nu)}\left\{\frac{(H-h)^3}{I_2}\;\frac{h}{2I_1}(6H^2+2h^2-6hH)\right\} / \frac{b}{B}\,H$$

$$\text{or } \frac{G}{G'} = \left[\left(1-\frac{h}{H}\right) + \frac{h/H}{1+\frac{t}{T}\left(\frac{B}{b}-1\right)}\right] / \frac{b}{H} + \left[\frac{1}{3(1+}\ \right)\frac{b}{B}$$

$$\left\{(H/B)^2\,\frac{\left(1-\frac{h}{H}\right)^3}{\frac{2}{3}\left(\frac{b}{B}\right)^3} + \frac{h}{H}\left(\frac{H}{B}\right)^2\,\frac{6+2\left(\frac{h}{H}\right)^2-6\frac{h}{H}}{3\left[\frac{2}{3}\left(\frac{b}{B}\right)^3+\frac{t}{T}\left(1-\frac{b}{B}\right)\left\{\left(1-\frac{b}{B}\right)^2+3\left(1-\frac{b}{B}\right)^2\right\}\right]}\right]/\frac{b}{B}$$

and $\frac{G'}{G} = \frac{1}{G/G'}$

Curves for G'/G, for different values of b/B, h/H and t/T ratios, can be drawn for each of the H/B ratios. Some nondimensional curves are shown in Fig. 3.6, 3.7 and 3.8. The value of ν was assumed as 0.15 (for concrete).

By Computer Analysis (Using EEM)

As before, the module is idealized by dividing it into a number of rectangular finite elements (Fig. 3.4). The shear load is applied in the horizontal direction at the narrow end of the module. The distribution of this load is assumed to be parabolic.

The boundary conditions, shown in Fig. 3.5c, applied along the sides and at the base of the module, ensure the necessary continuity conditions.

The displacements of the centreline of the module due to both bending and shearing actions of the load, are obtained from a plane stress finite element analysis. The equivalent G' for the analogous plate is obtained by the same assumption that the above displacement is equal to the shearing displacement of the analogous plate. The curves using non-dimensional parameters for G'/G are shown in Fig. 3.9, 3.10, 3.11, 3.12, 3.13, 3.14, 3.15, 3.16, 3.17 and 3.18.

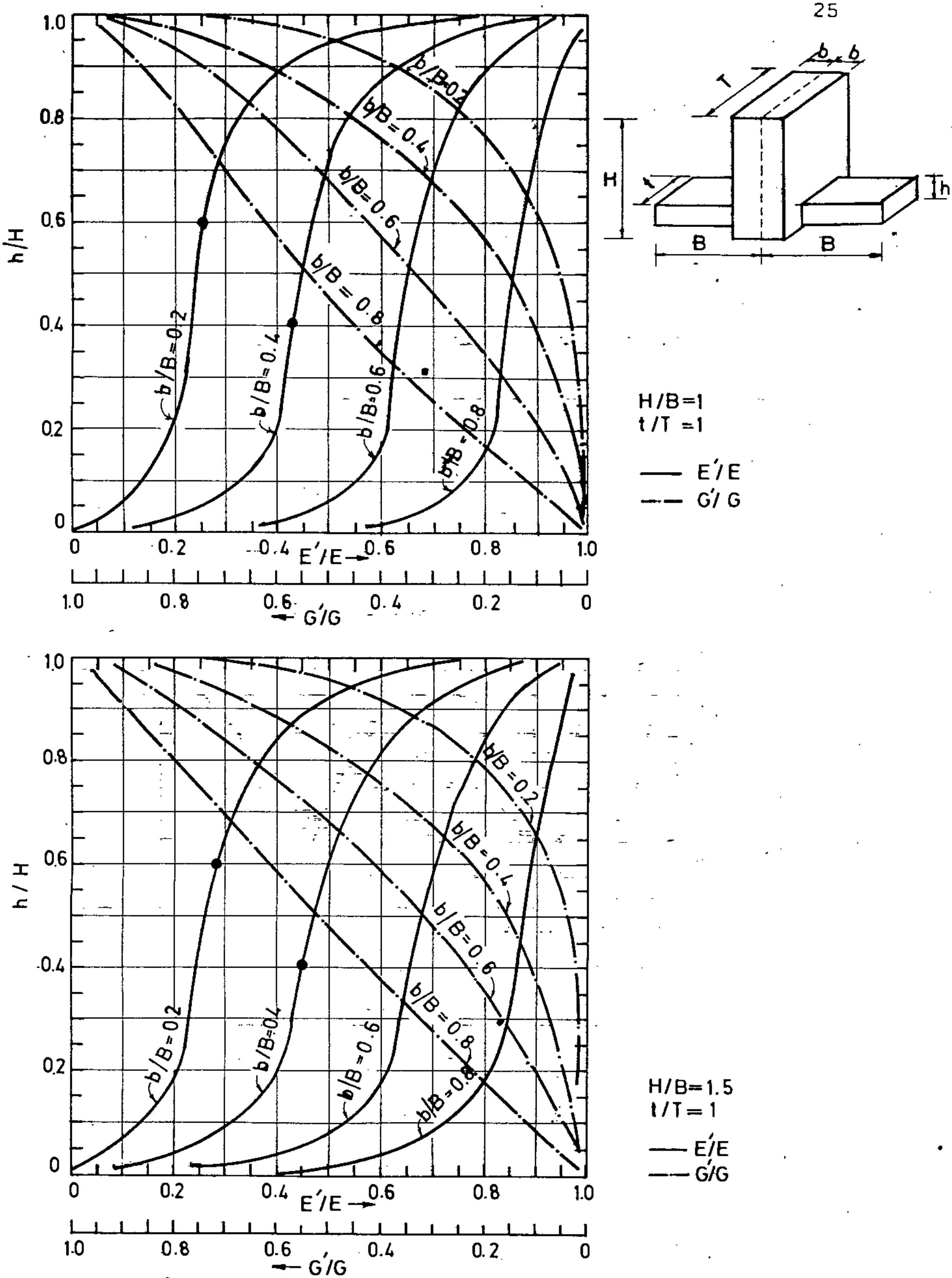


FIG. 3.9 VARIATIONS OF E′/E AND G′/G RATIOS (BY F.E.M.)

(THE ABOVE CURVES FOR t/T=1 ARE THE SAME AS IN REF.-1)

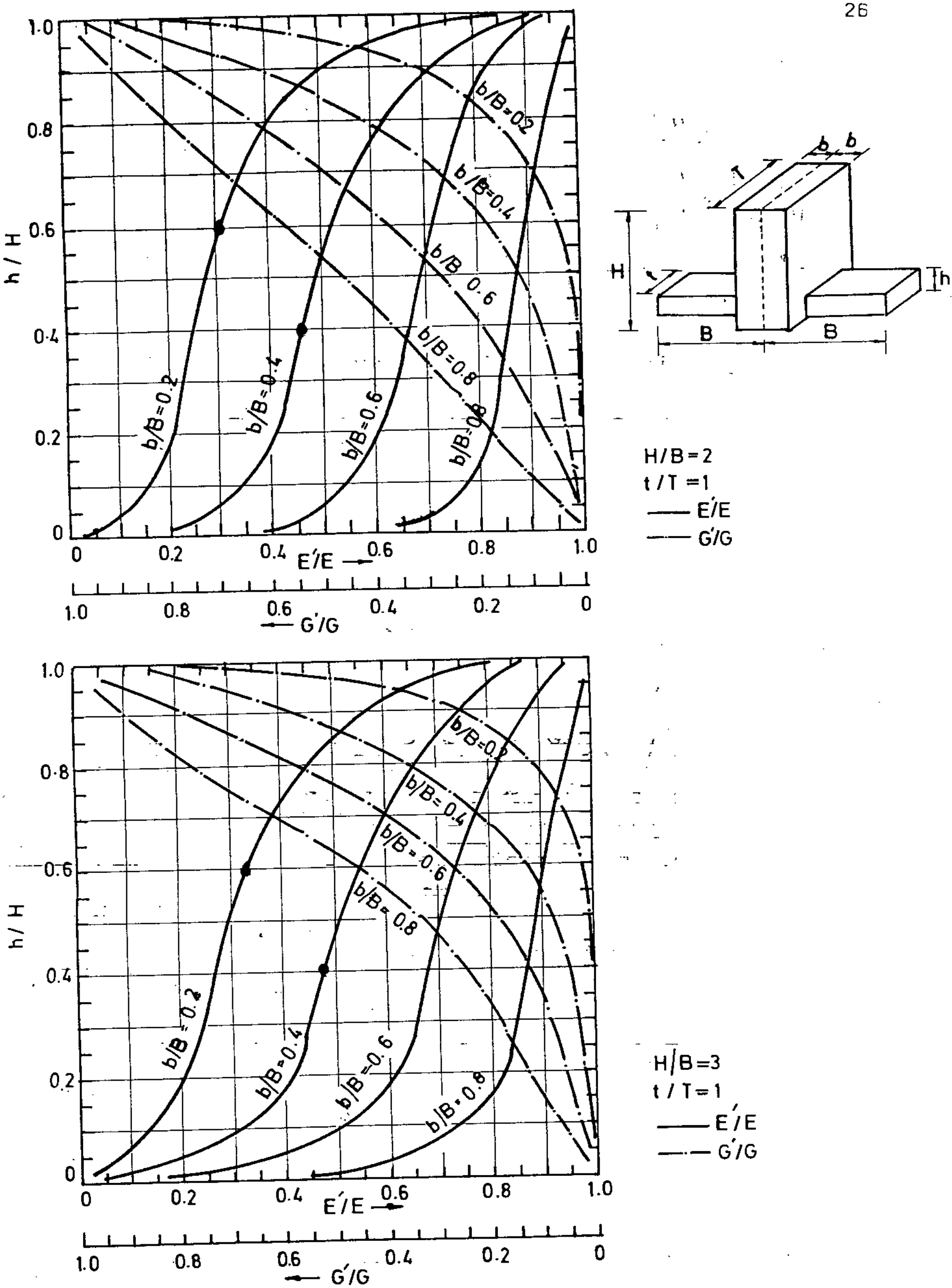


FIG. 3.10, VARIATIONS OF E'/E AND G'/G RATIOS (BY FEM)

(THE ABOVE CURVES FOR t/T=1 ARE THE SAME AS IN REF.-1)

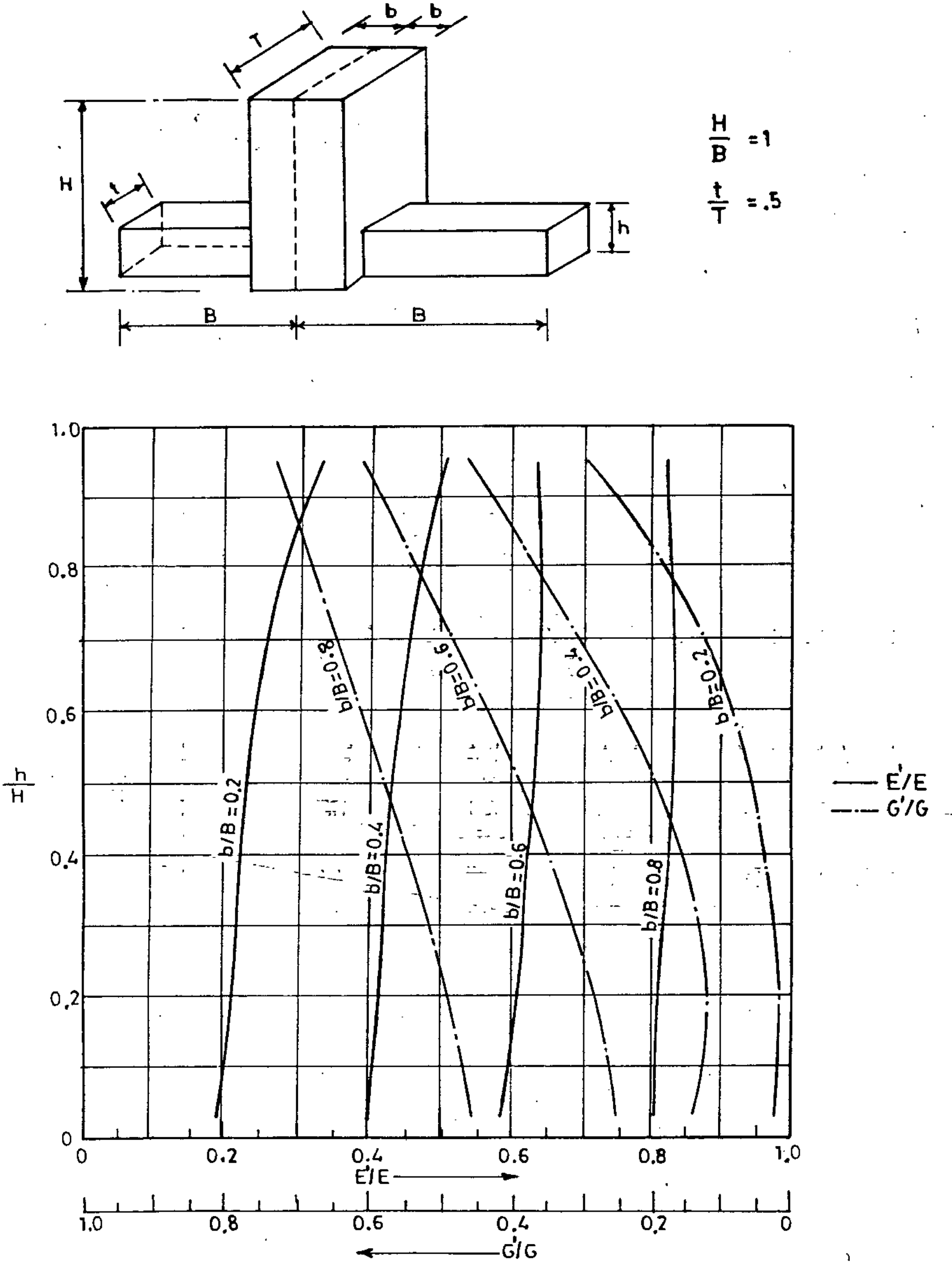


Fig.3.11 Variations of E'/E and G'/G Ratios (by FEM)

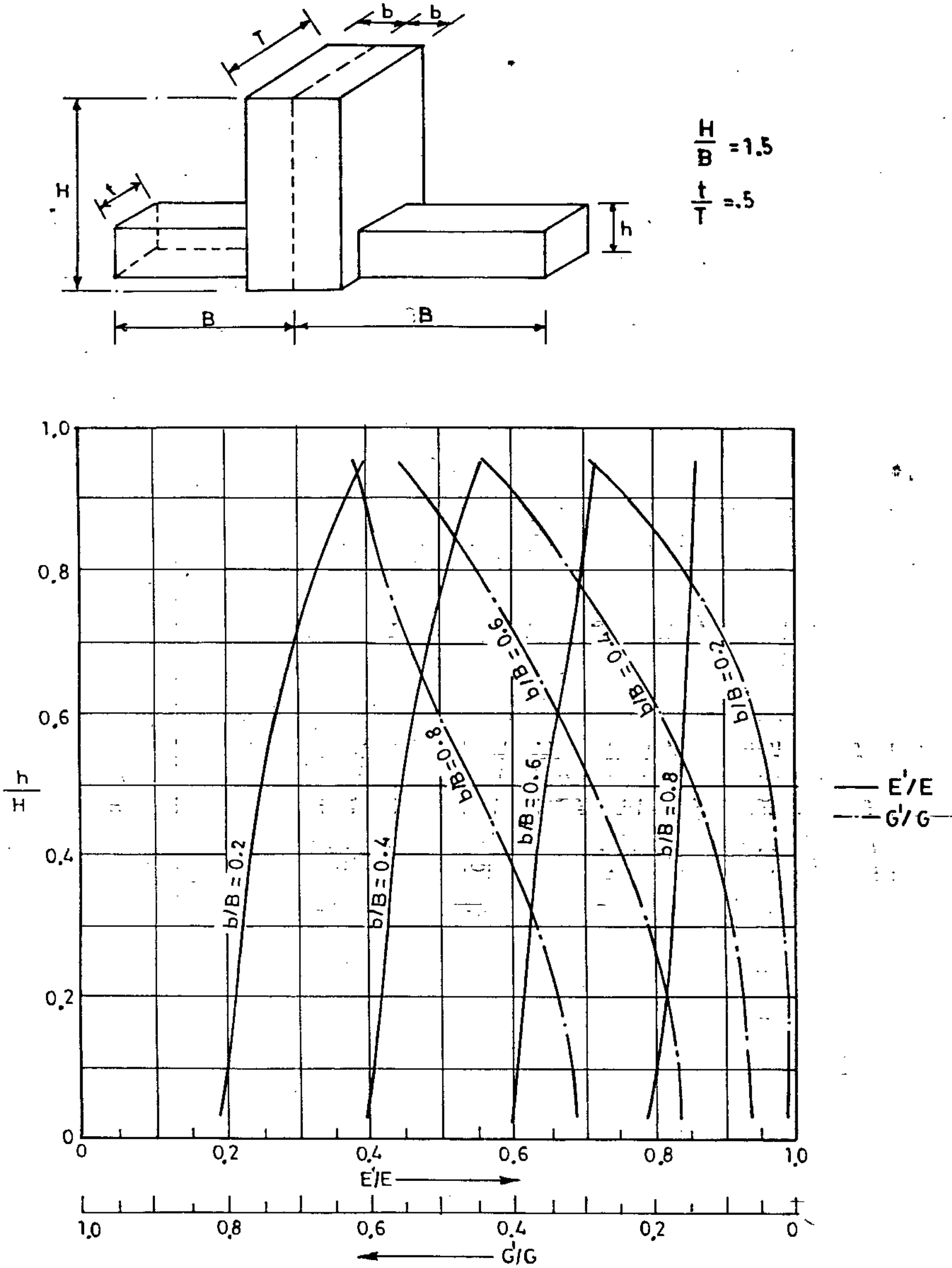
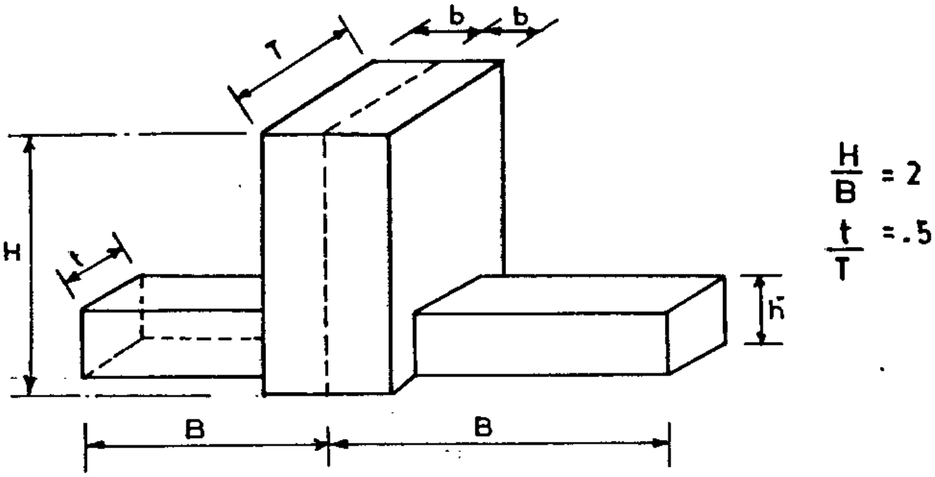


Fig. 3.12 Variations of E'/E and G'/G Ratios (by FEM)



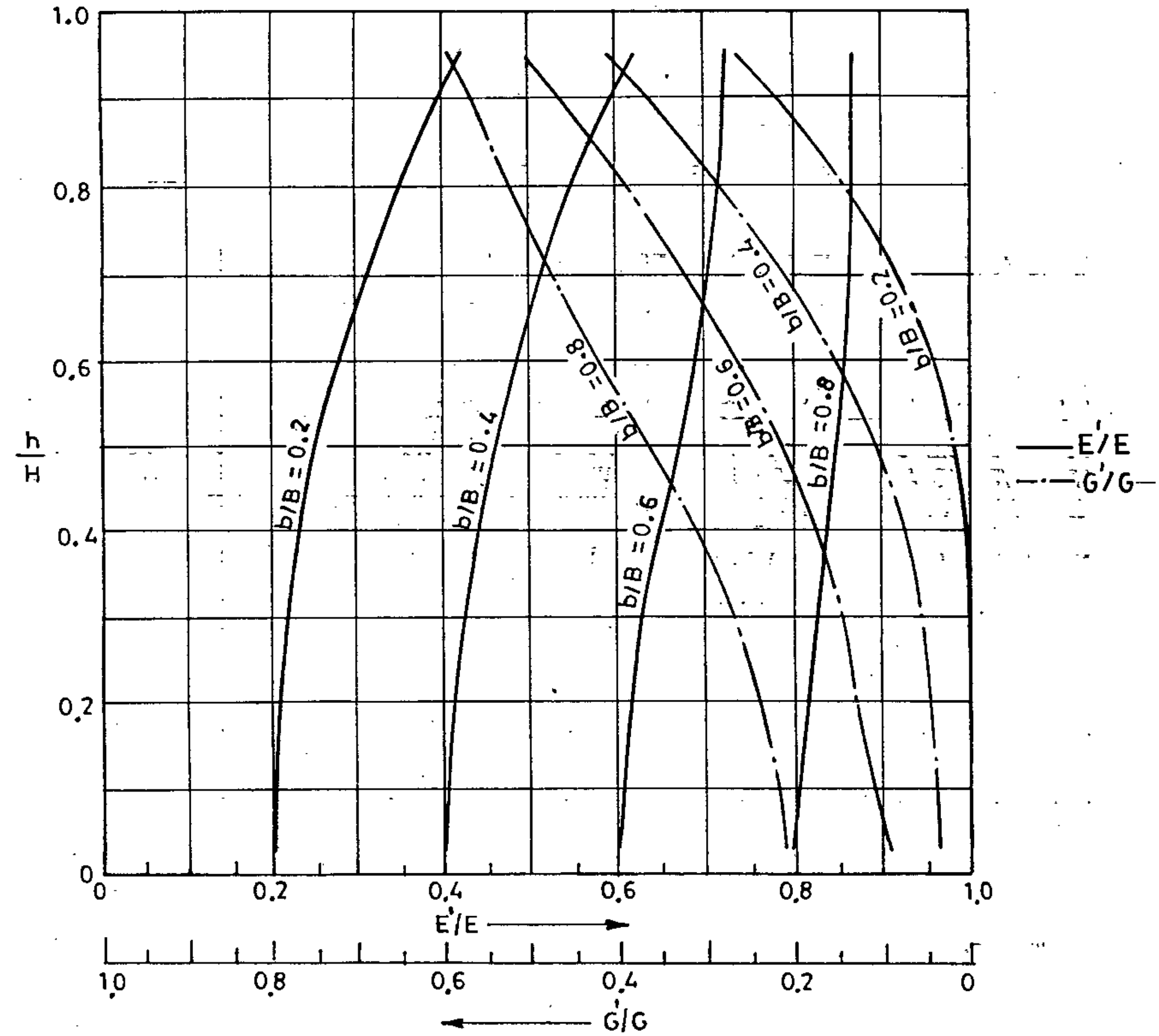
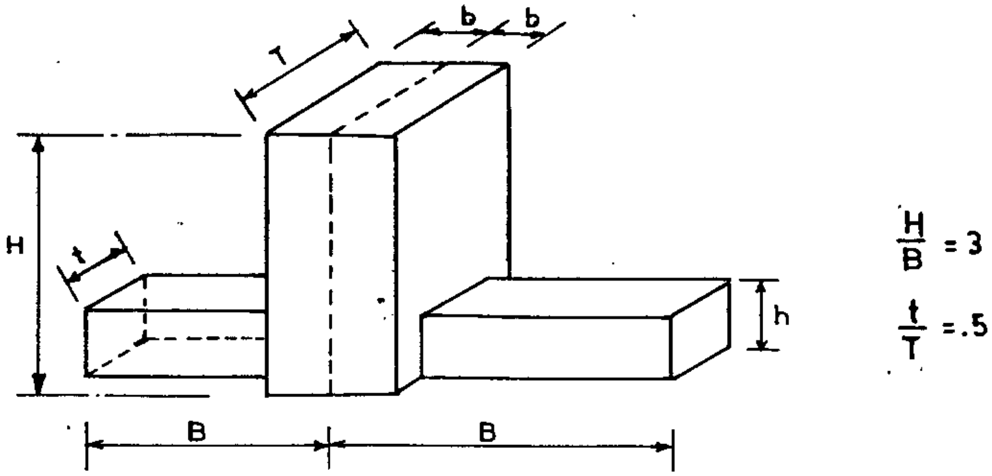


Fig. 3.13 Variations of E'/E and G'/G Ratios (by FEM)



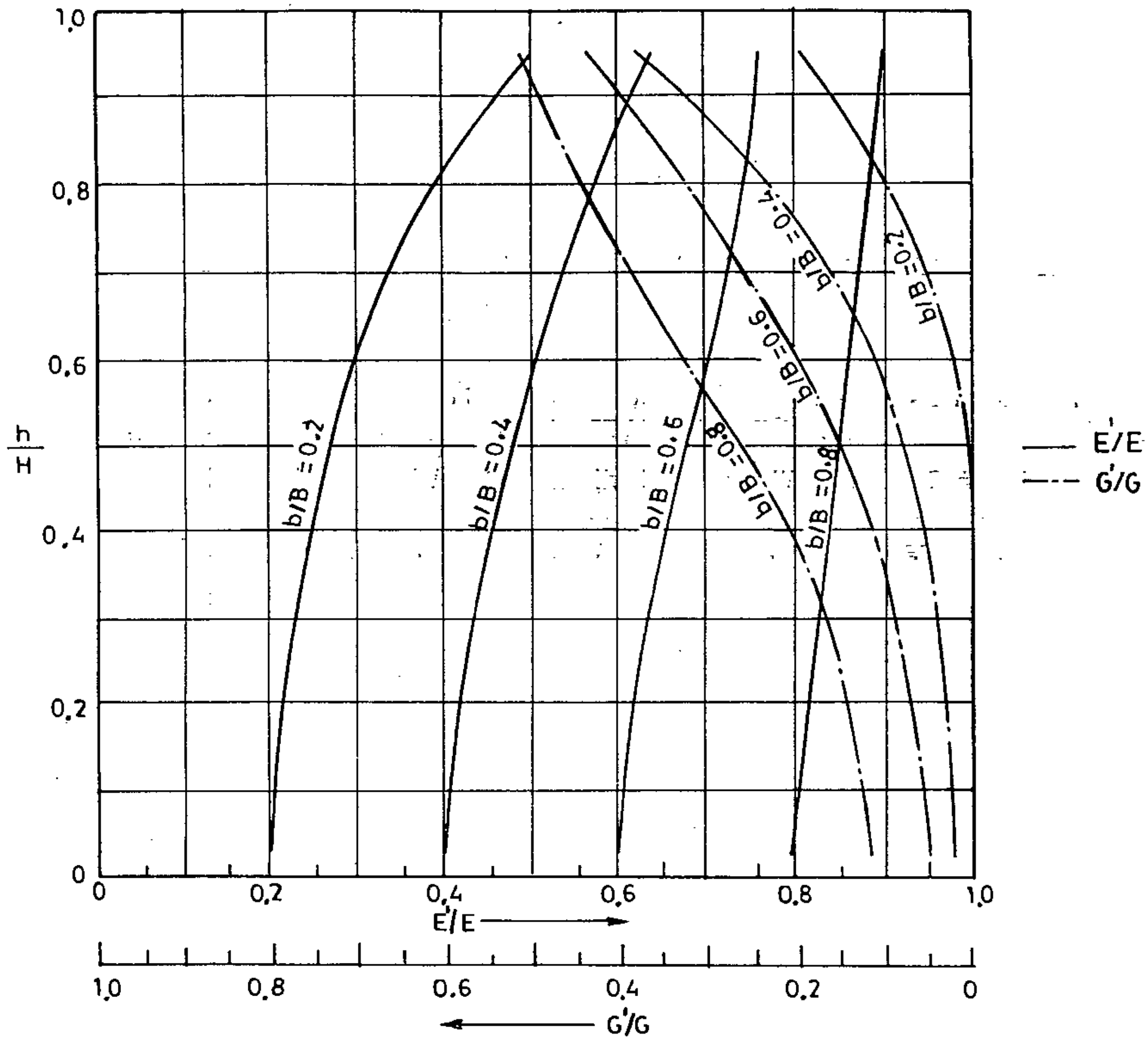


Fig.3.14 Variations of E'/E and G'/G Ratios (by FEM)

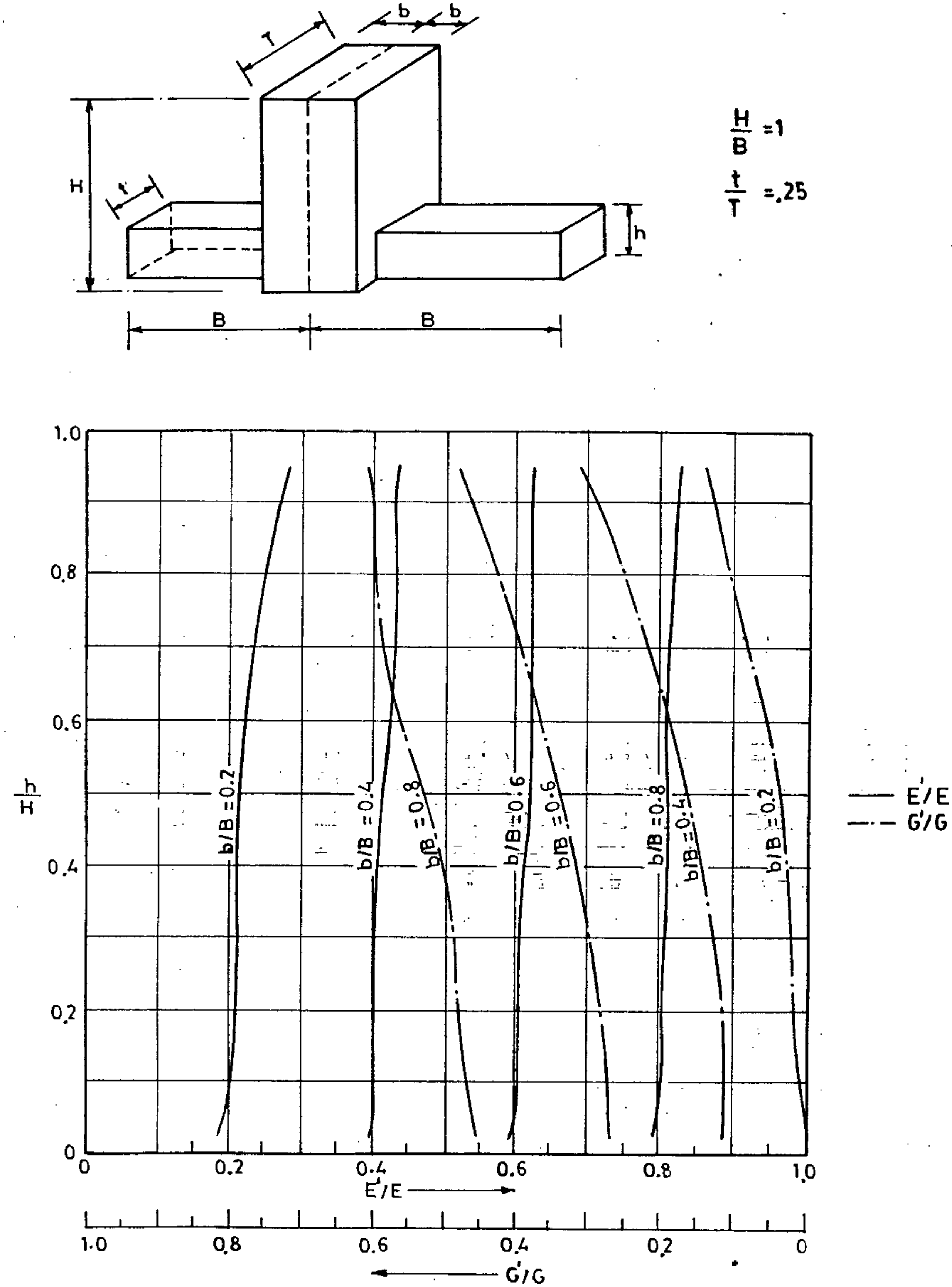


Fig. 3.15 Variations of E'/E and G'/G Ratios (by FEM)

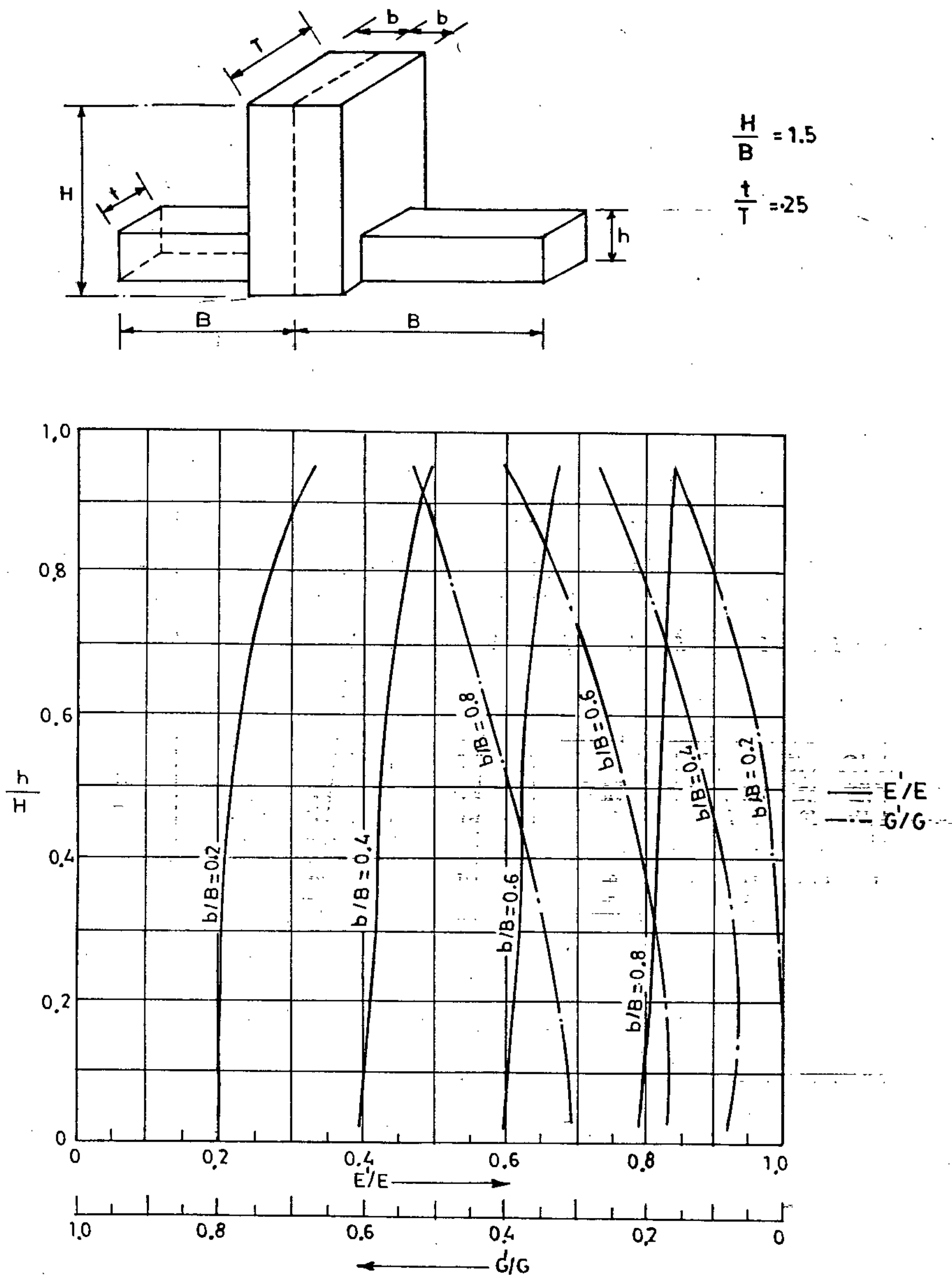


Fig.3.16 Variations of E'/E and G'/G Ratios (by FEM)

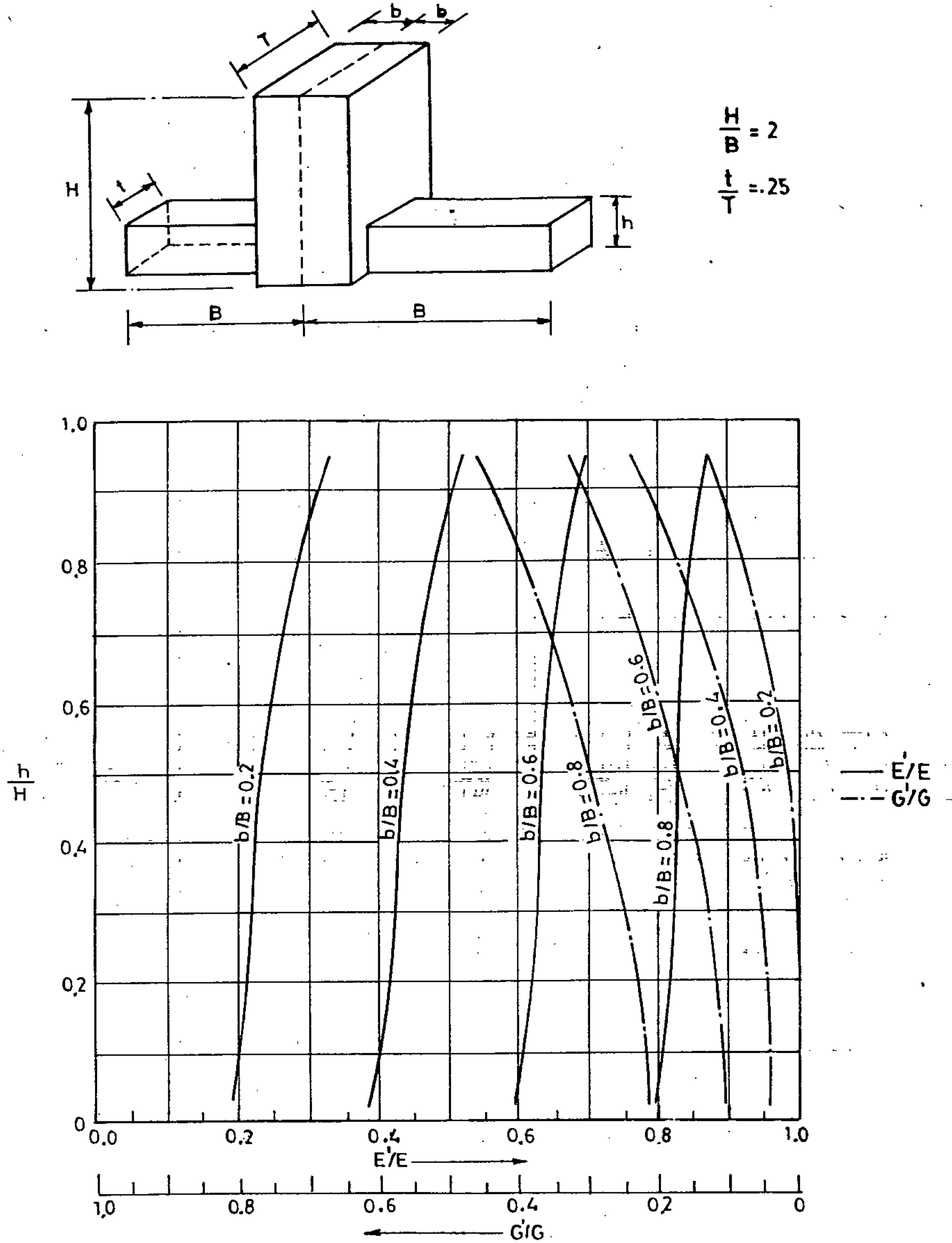
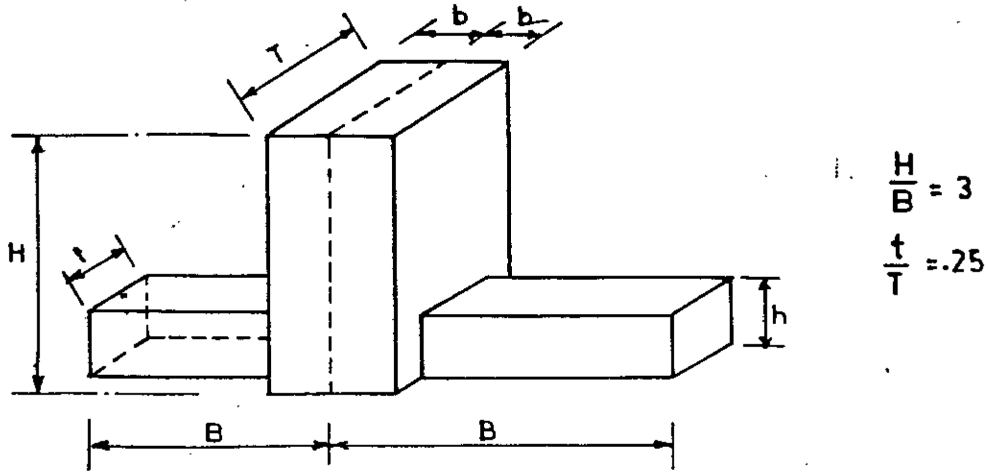


Fig. 3.17 Variations of E'/E and G'/G Ratios (by FEM)



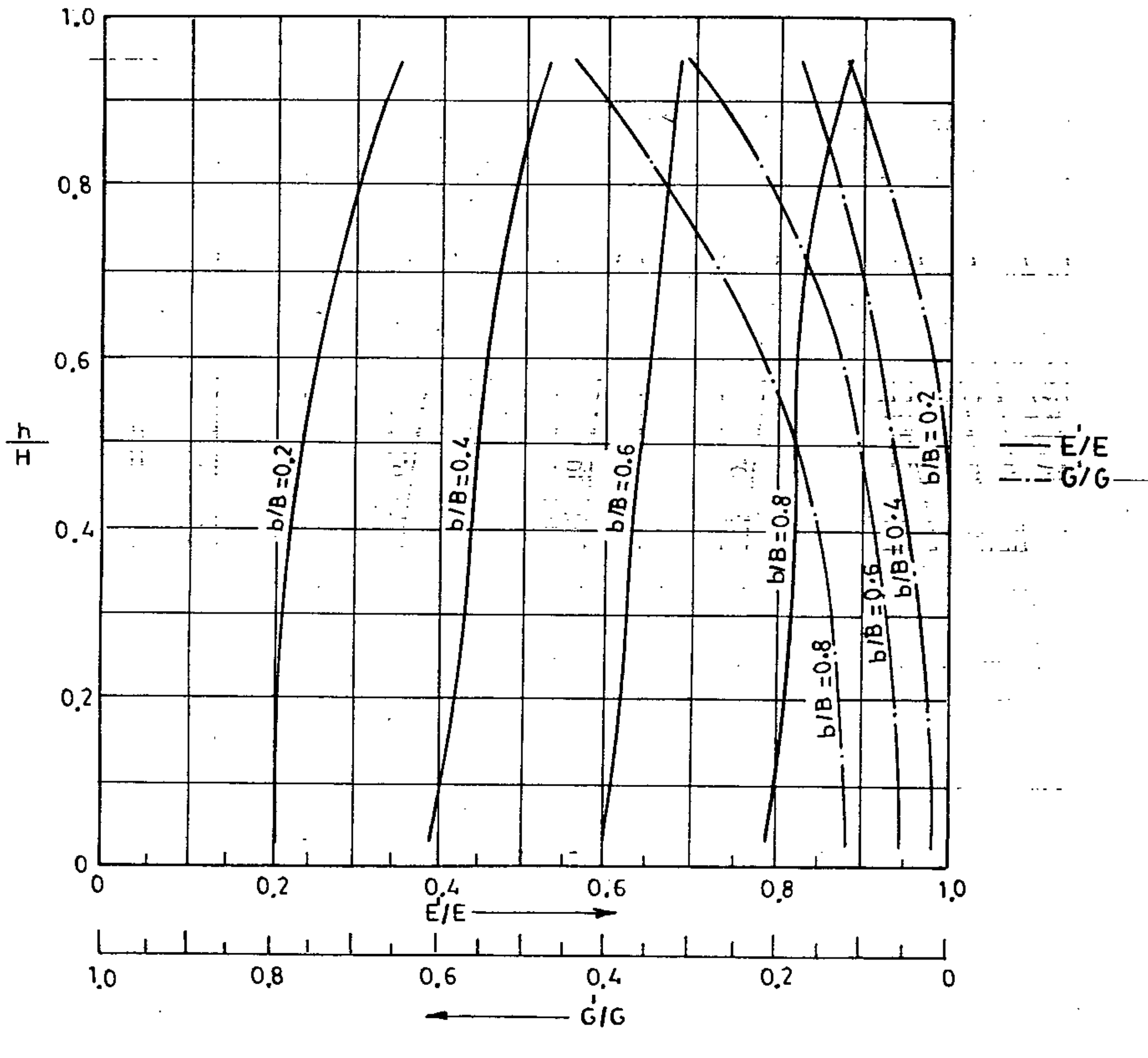


Fig. 3.18 Variations of E'/E and G'/G Ratios (by FEM)

3.4 Stress Factors:

The purpose of the so-called stress-factors, defined below, is to estimate the appropriate design stress at different points of a parent structure from the simplified analysis. From the strength of materials theory it is observed that the average stress in the analogous plate is b/B times the applied stress at the narrow section of the module. In other words, the actual stress in a module at the narrower section would be B/b times that obtained from the analogous plate. Therefore, B/b becomes the stress factor for the narrower section of the module and T/t for the end of the wider section. The stress factor at the middle of the wider portion is obviously unity.

From the plane stress finite element analysis, stresses at points x,y,z etc. (Fig. 3.3a), are calculated. The stress factors at these points are simply the ratios of these stresses to the σ_{av} of the analogous plate (Fig. 3.3b). The stress factors for longitudinal stress for x,y,and z position of the module (Fig. 3.3a) are presented in Fig. 3.19, 3.20, 3.21, 3.22, 3.23, 3.24, 3.25, 3.26 and 3.27.

3.5 Proposed Simplified Method

The mathematical modelling technique described in the aforementioned articles can be applied to the following

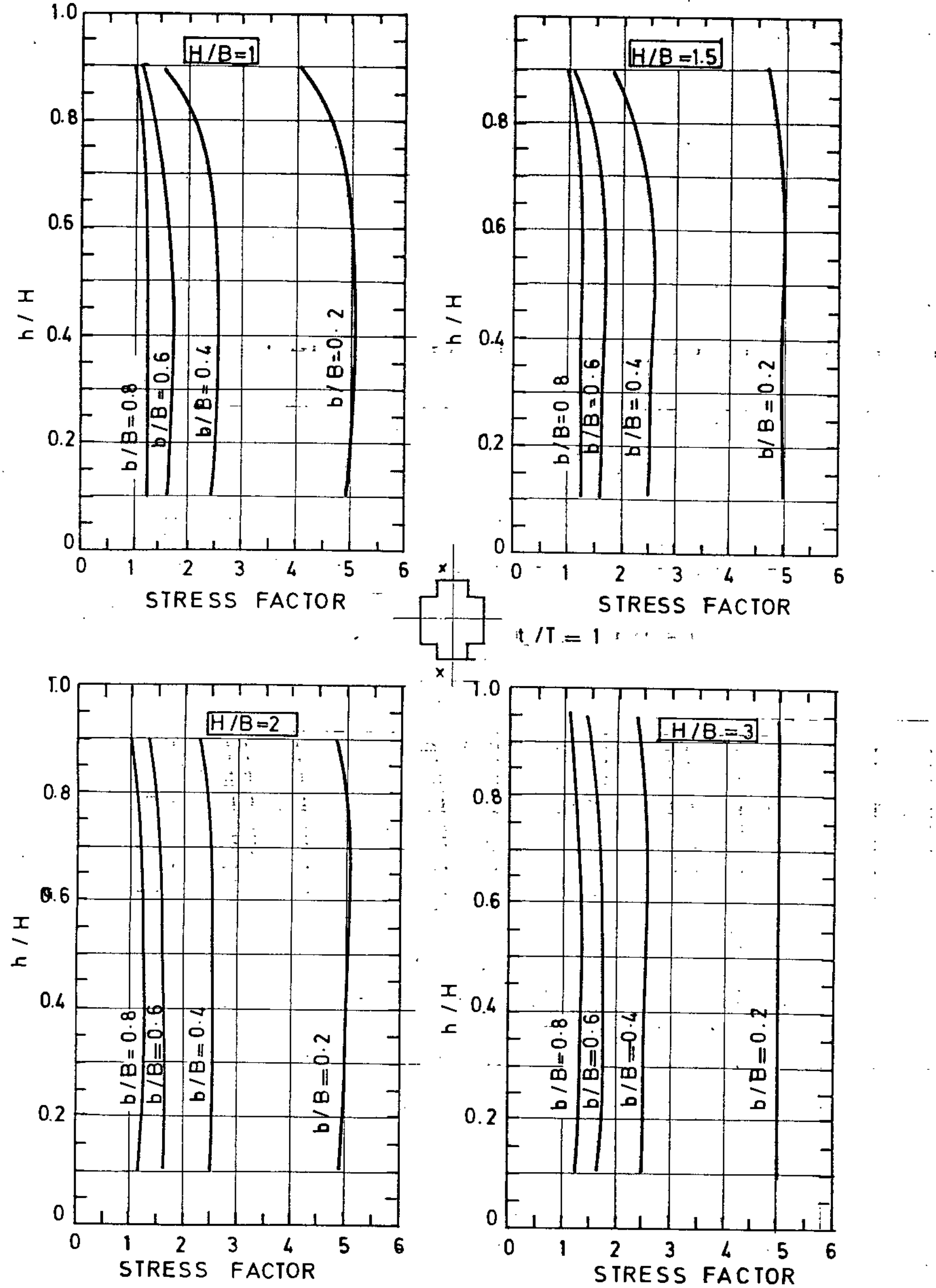


FIG. 3.19, STRESS FACTOR FOR COLUMNS (AT POSITION X)

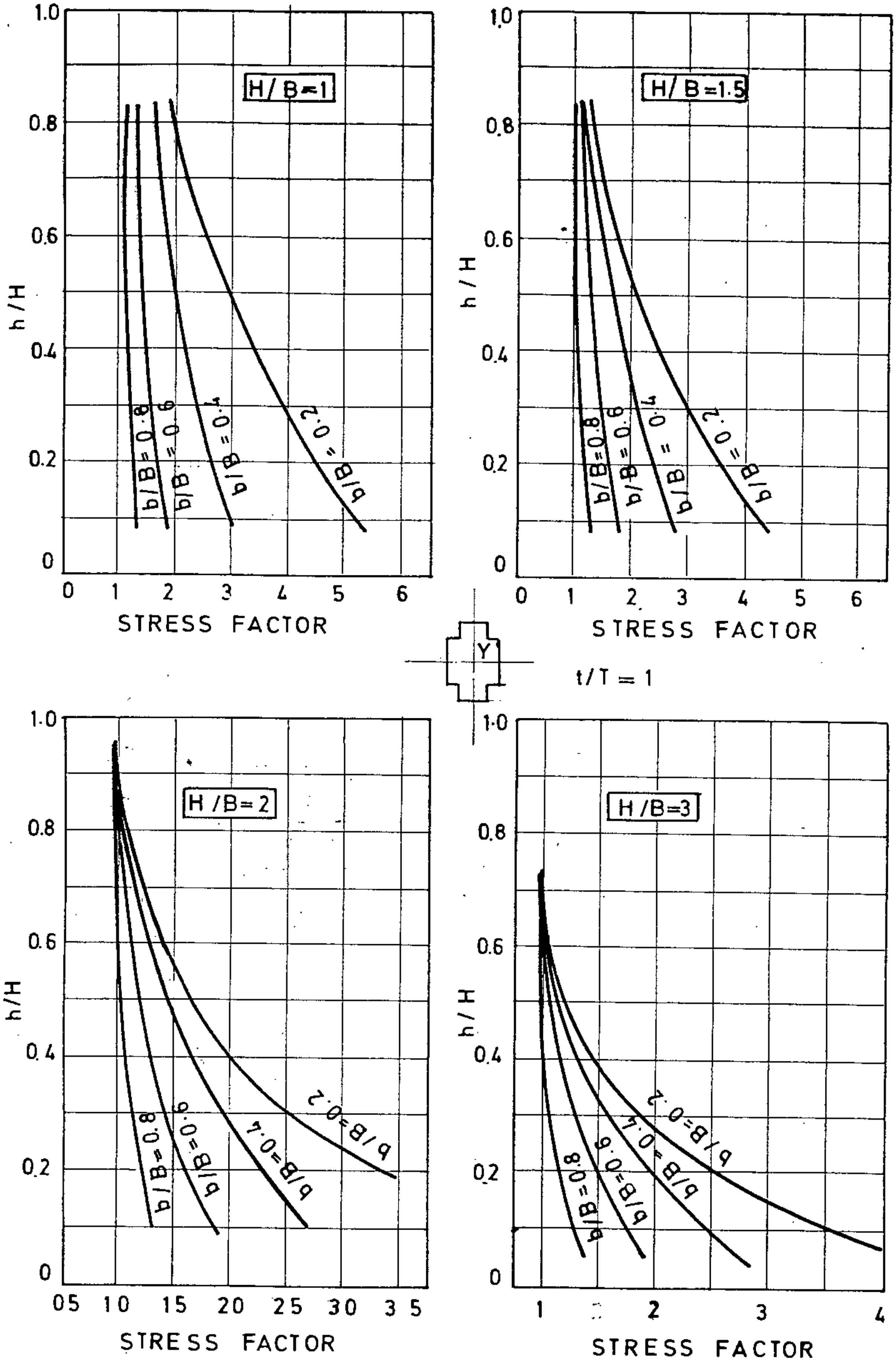


FIG. 3.20. STRESS FACTORS FOR JOINTS (AT POSITION Y)

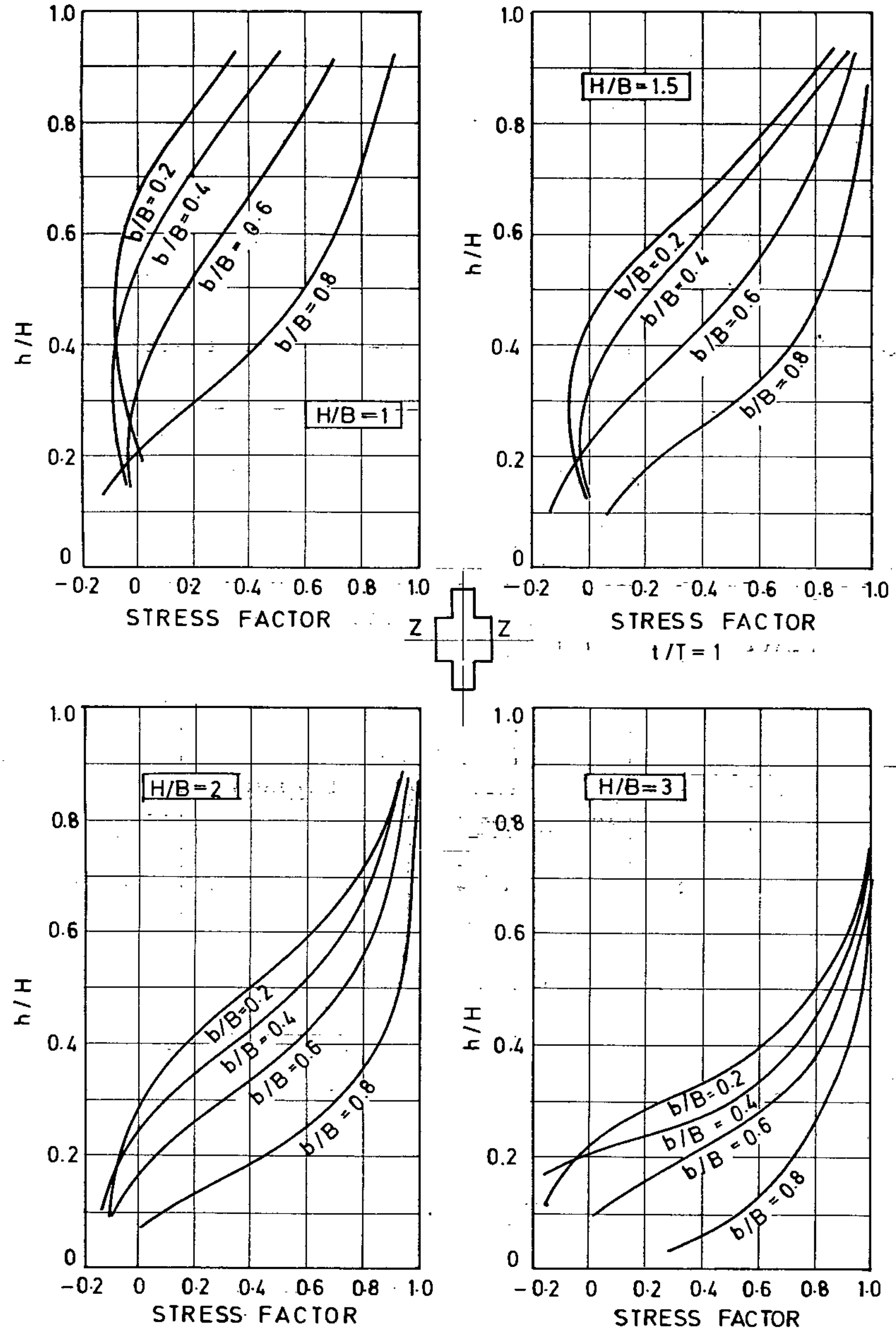


FIG. 3.21. STRESS FACTOR FOR BEAM (AT POSITION Z)

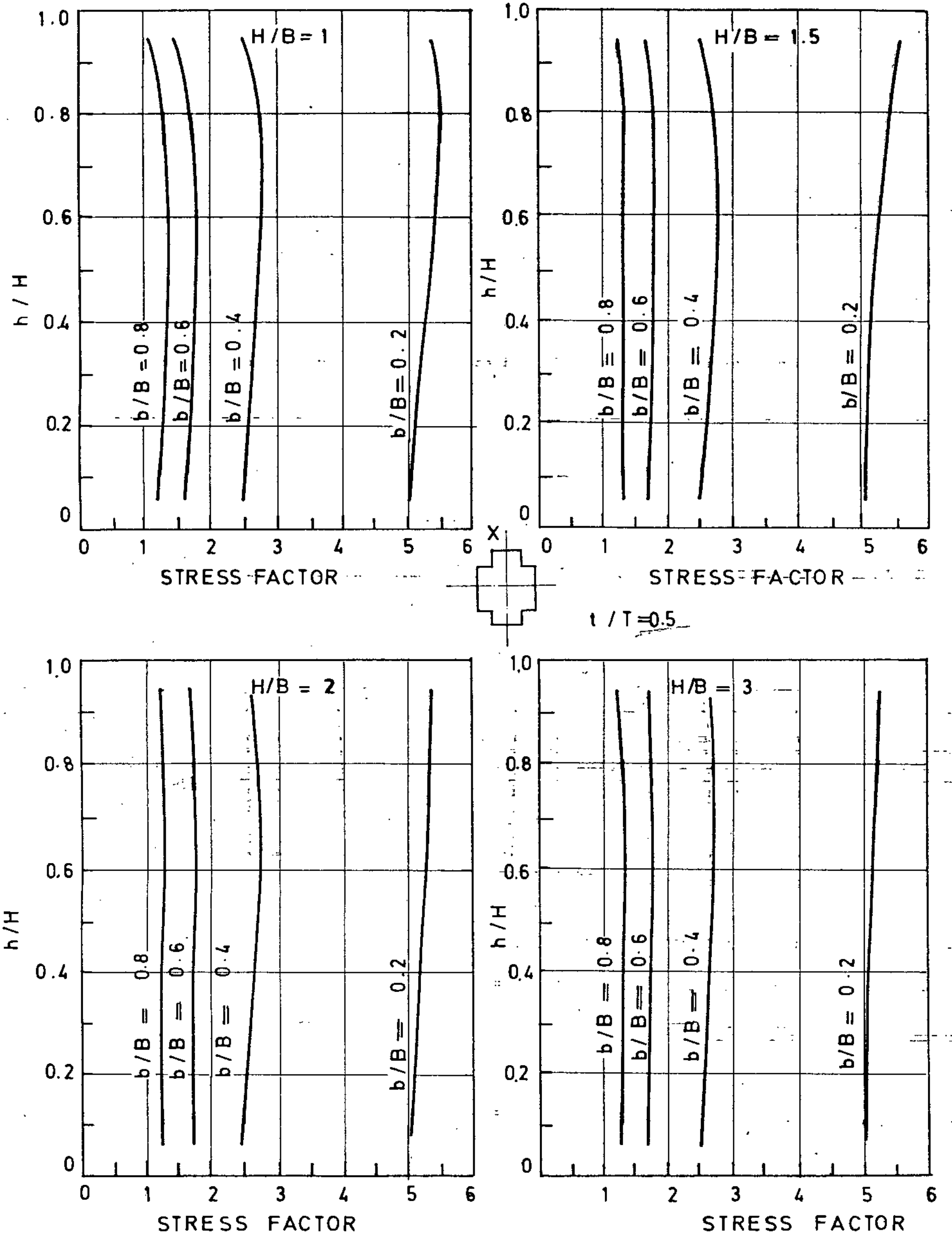


FIG. 3.22. STRESS FACTOR FOR COLUMNS (AT POSITION X)

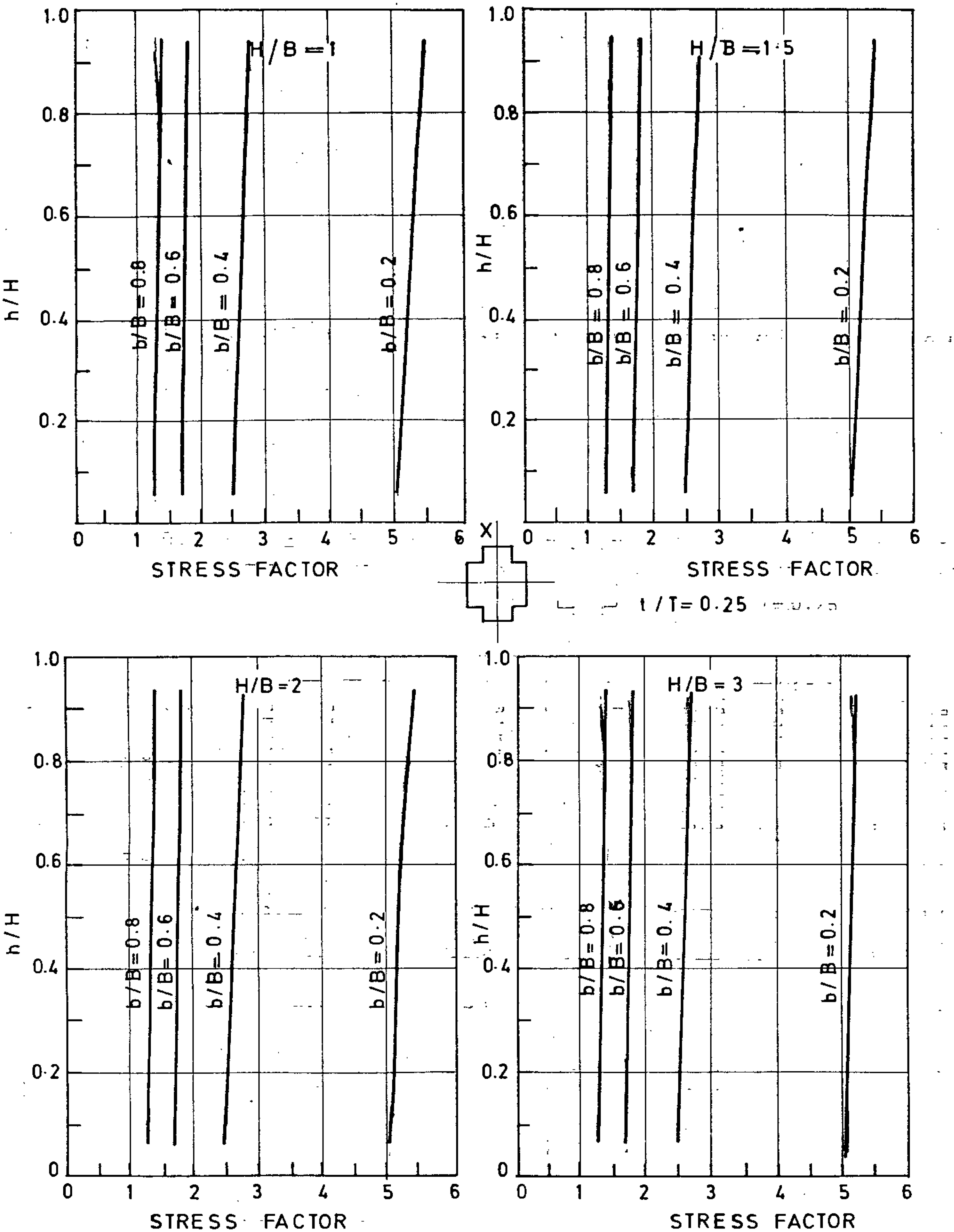


FIG. 3.23. STRESS FACTORS FOR COLUMNS (AT POSITION X)

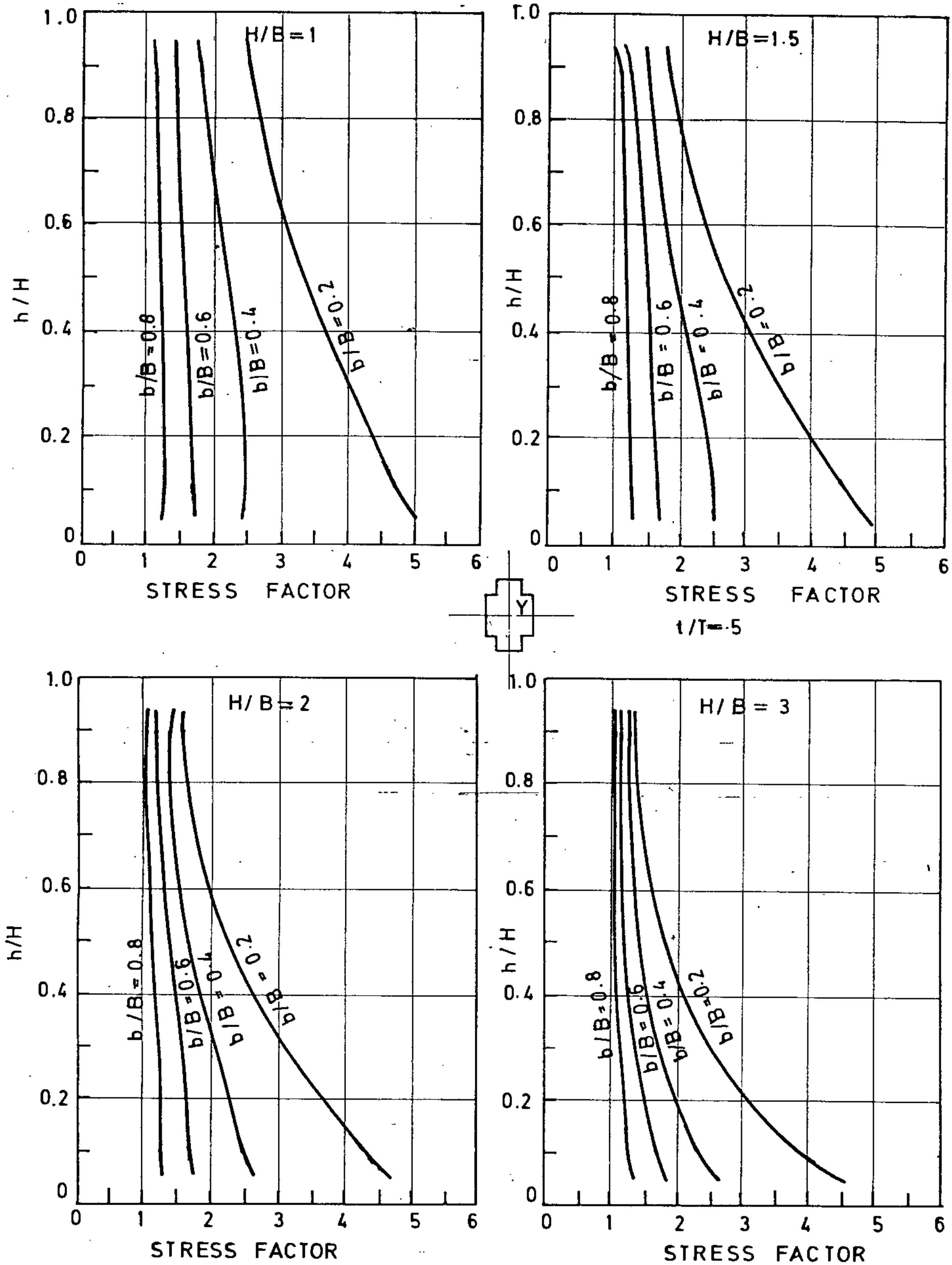


FIG. 3.24 STRESS FACTOR FOR JOINTS (AT POSITION Y)

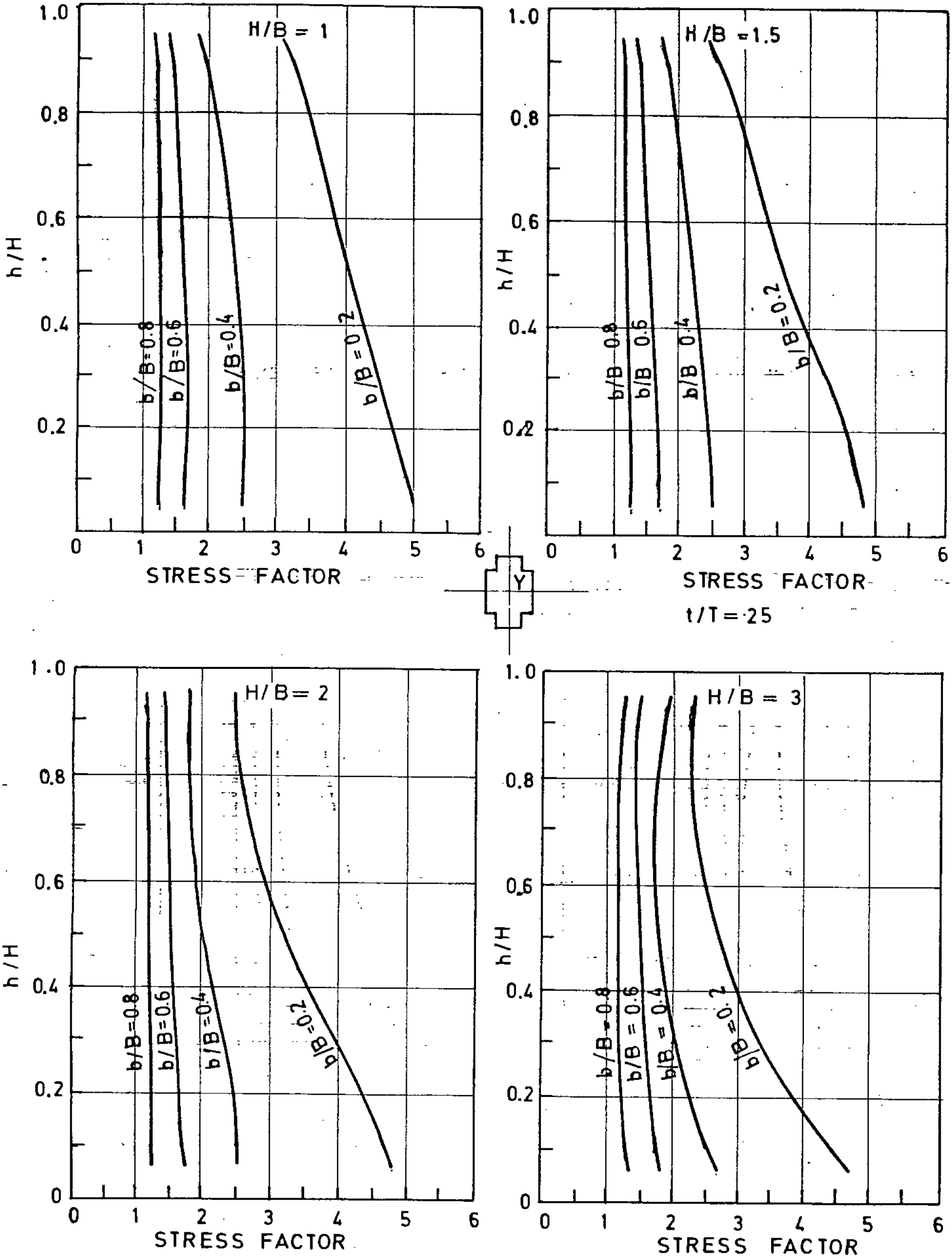


FIG 3.25. STRESS FACTORS FOR JOINTS (AT POSITION Y)

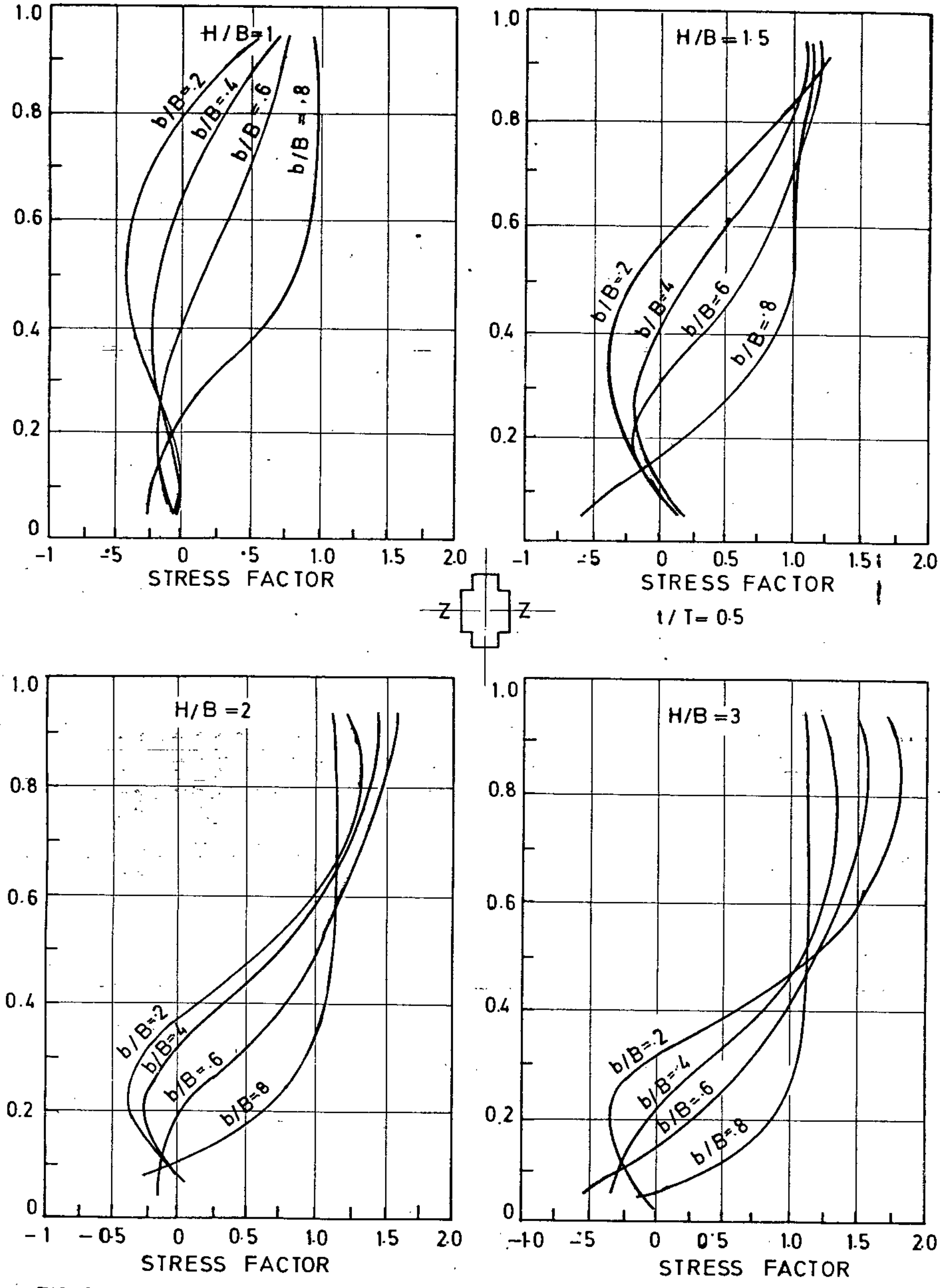


FIG. 3.26 STRESS FACTORS FOR BEAM (AT POSITION Z)

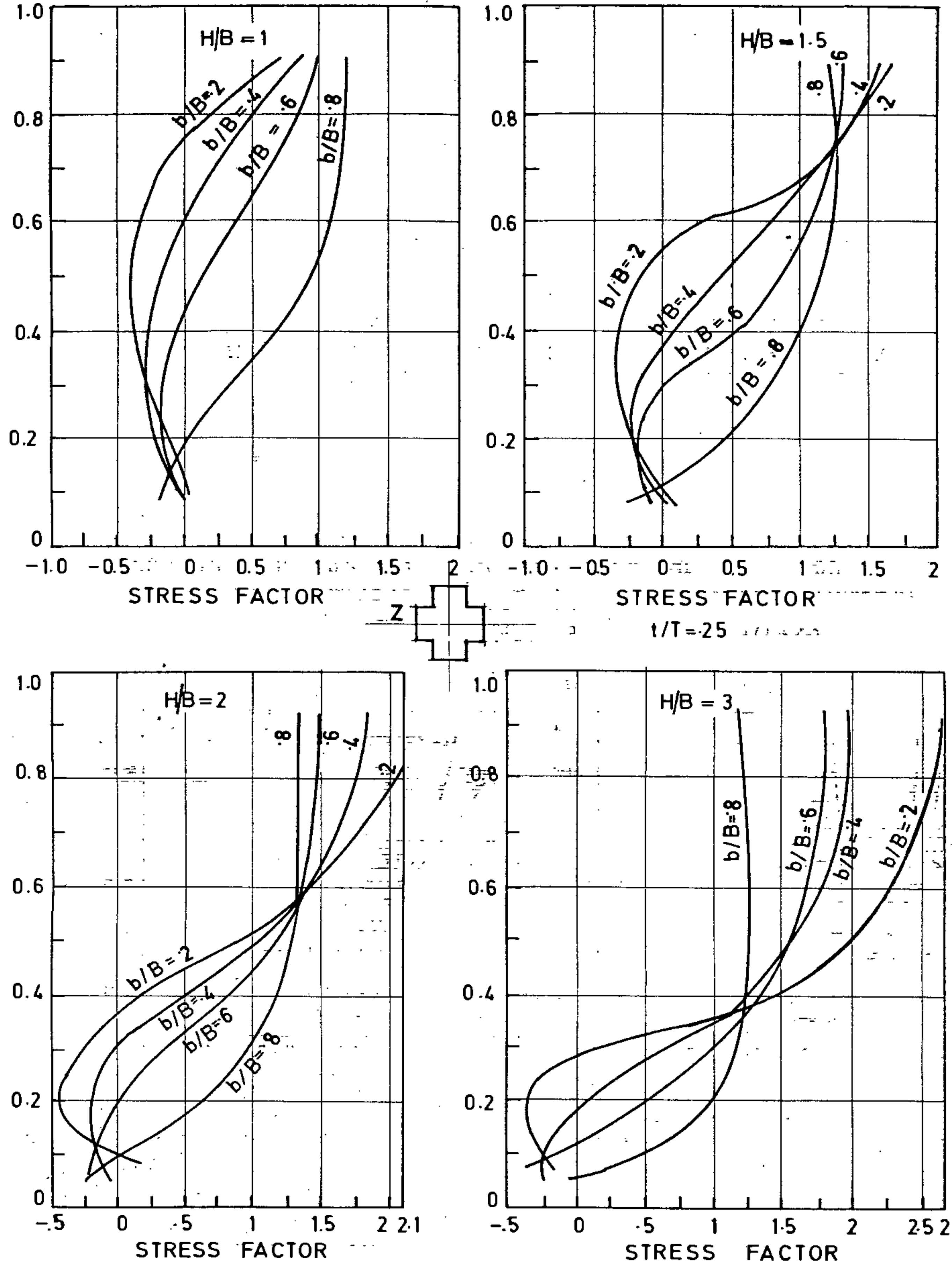


FIG. 3. 27, STRESS FACTORS FOR BEAM (AT POSITION Z)

practical structures:

i. Wall-frame structure,

ii. Shear walls with multiple bands of openings,

iii. Facade shear walls

and iv. Multi-bay multi-storey building frames (b/B and h/H ratios small).

Analogous to the well known finite element analysis, where a structure is idealized as an assumblage of large number of finite elements whose geometry and stiffness matrices are known this method considers a structure as an assemblage of equivalent plates whose geometry and predominent stiffness parameters are known. The idealized structure is analysed by hand using simple bending theory without using a computer.

3.5.1 Evaluation of Deflections and Stress

The presence of an opening in a wall reduces its overall stiffness and modifies the stress distribution across the wall section. The analogous plate modules with modified E' and G' parameters, assign the equivalent unperforated structure a uniform, reduced stiffness comparable to that of the basic wall-frame structure. Therefore, the equivalent wall will have the same defected form and amplitude as the parent structure subject to the same lateral load. The stresses obtained from the equivalent wall will, however, have to be modified to get the actual stresses in the basic structure. This is done by multiplying the stress obtained from the simplified analysis by the appropriate stress factors discussed earlier.

3.6 Other Methods of Analysis

To demonstrate the a c curacy, simplicity and effectiveness of the proposed method for analyzing wall frame structures, the following alternative methods of analysis were considered. Typical practical wall frame structures were analysed using both the proposed method and the alternative methods and values of deflections and stress were compared. The alternative methods are:

i. Finite element method (plane stress analysis)

ii. Continuous medium method

Before proceeding to a comparison of results, a brief outline of each of the numerical methods is given in the following:

3.6.1 Finite Element Plane Stress Analysis

The finite element method provides a very convenient, versatile and widely used numerical technique for various types of stress analysis problems. In case of plane stress problem it has been observed that 8-noded rectangular finite elements are preferable to 4-noded rectangular, triangular, quadrilateral or other shaped elements for the analysis of tall planar structures. As curved boundaries are seldom encountered in these types of constructions, idealization of such structures into rectangular finite elements presents nondifficulty. The degree of accuracy achieved with a few 8-noded rectangular elements is quite comparable to that obtained with a large number of, say, trangular elements.

With a view to comparing the results obtained from the simplified method described earlier with those from a finite element analysis a computer programme was developed using 8-noded rectangular elements.

A listing of the programme is given in Appendix B-1.[3]

3.6.2 Continuous Medium Method

Plane shear walls with multiple band of openings were analysed in detail by Choudhury[2]. In this method of analysis, the discrete system of connections formed by lintel beams, is replaced by a continuous medium connecting the walls for the full height and having the same bending stiffness as the beam they replace.

Assumptions:

i. The connecting beams do not deform axially and hence the lateral deflection of individual walls is the same at any level.

ii. The moment of inertia and cross-sectional areas of the walls and the connecting beams are constant throughout the height, except the connecting beam at the topmost storey which has half the moment of inertia and half the cross-sectional area of the other beams.

iii. The point of contraflexure of the connecting beams are at their midspan.

iv. Plane section of the wall before bending remains plane after bending, so that the moment curvature relations based on the simple engineers theory of bending (ETB) may be used.

Computer Programme

The sequence of operations followed in the programme is outlined in the flow diagram given in Appendix B-2. A listing of the programme is also given in Appendix B-3.

CHAPTER 4

EXAMPLE PROBLEM

4.1 Problem

A 10 storey 60' x 120' concrete shear wall with 6 bands of regular openings is chosen as the example wall-frame structure*. The structure has 6 equal bays of 10 ft each and a storey height of 12 ft. This makes a module size (i.e. H/B ratio) of 1.20. The size of opening is considered to give equal b/B and h/H ratios of 0.4. Three girder column thickness ratios (i.e. t/T) viz. 1, 0.50 and 0.25 are considered. A point load at the free end is considered.

Analysis by Simplified Method

The structure was idealized as a thin unperforated cantilever having stiffness parameters E' and G' as discussed earlier. For the structure having b/B = h/H = 0.4, stiffness parameters obtained from the Finite element method was used in the analysis.

Engineers theory of bending (ETB) was used in the deflection analysis of the structure. The expressions for the deflection of a cantilever is given in Appendix-A. Deflection due to shear is taken into account.

---

* The structure is similar in dimensions to that analysed by Khan(1), - except the overall height of 120 ft, which is half of that used in Khan's example.

Finite Element Analysis

Detailed finite element analysis of the example structures were made using the plane stress programme mentioned above. 117 8-noded rectangular elements were used to idealized the basic structure.

Analysis by Continuous Medium Method

The programme listed in Appendix-B was used to analyze the wall-frame by continuous medium method.

4.2 Results and Discussions

Deflection:

Figs. 4.1, 4.2, 4.3 and 4.4, show the deflected shape of the example structure due to a point load at the top. In case of t/T = 1, the simplified method overestimates the maximum deflection at the top by 13% and continuous medium method underestimates by 6% from that given by finite element method (FEM). In case of t/T = 0.5, simplified method overestimates the top deflection by 15% and continuous medium method overestimates by 2% from that given by FEM. And for t/T = 0.25, simplified method overestimates the top deflection by 17% and continuous medium method overestimates by 19% from that given by FEM.

The effect of the ratios of girder and column thickness (i.e. t/T) on deflection has been investigated. The value of top deflection for t/T = 0.5 is within 6% of t/T = 1 and for t/T = 0.25 is within 14%. It appears that for the example problem, the effect of t/T the deflection is not very large.

Stress:

The distributions of stresses across wall sections are shown in Fig. 4.5, 4.6, 4.7 and 4.8 of the example structure due to a pointload at the top. In case of t/T = 1, simplified method and continuous medium method of predicting stresses at X (for column) shows good agreement with the FEM, at Y (for joint) simplified method overestimates the stress by 4% and continuous medium method underestimates by 1% and at z (for girder) simplified method underestimates the stress by 3% from that of FEM. In case of t/T = 0.5, at X (for column) simplified method overestimates the stresses by 9% and continuous medium method by 10% from that of FEM, at y (for joint) simplified method overestimates the stress by 12% and continuous medium method overestimates by 9% from that given by FEM at z (for girder) simplified method underestimates the stress by 5% from that given by FEM. And for t/T = 0.25, at X (for column) simplified method overestimates the stress by 24% and continuous medium method overestimate by 30% from that given by FEM at section Y(for joint) simplified method overestimates the stresses by 20% and continuous medium method overestimates by 18% from that given by FEM and at z (for joint) simplified method understimates the stress by 10% from that given by FEM.

The effect of the ratios of girder and column thickness (i.e. t/T) on stress has been investigated. The value of stress for t/T = 0.5 is within 7% and 2% of t/T = 1 at Y and Z respectively, and stress for t/T = 0.25 is within 15% and 3% at t/T = 1 at Y and Z respectively.

From the above discussion it is seen that the example problem, the simplified method presented in this report gives results which are acceptable for a preliminary design of the wall-frame.

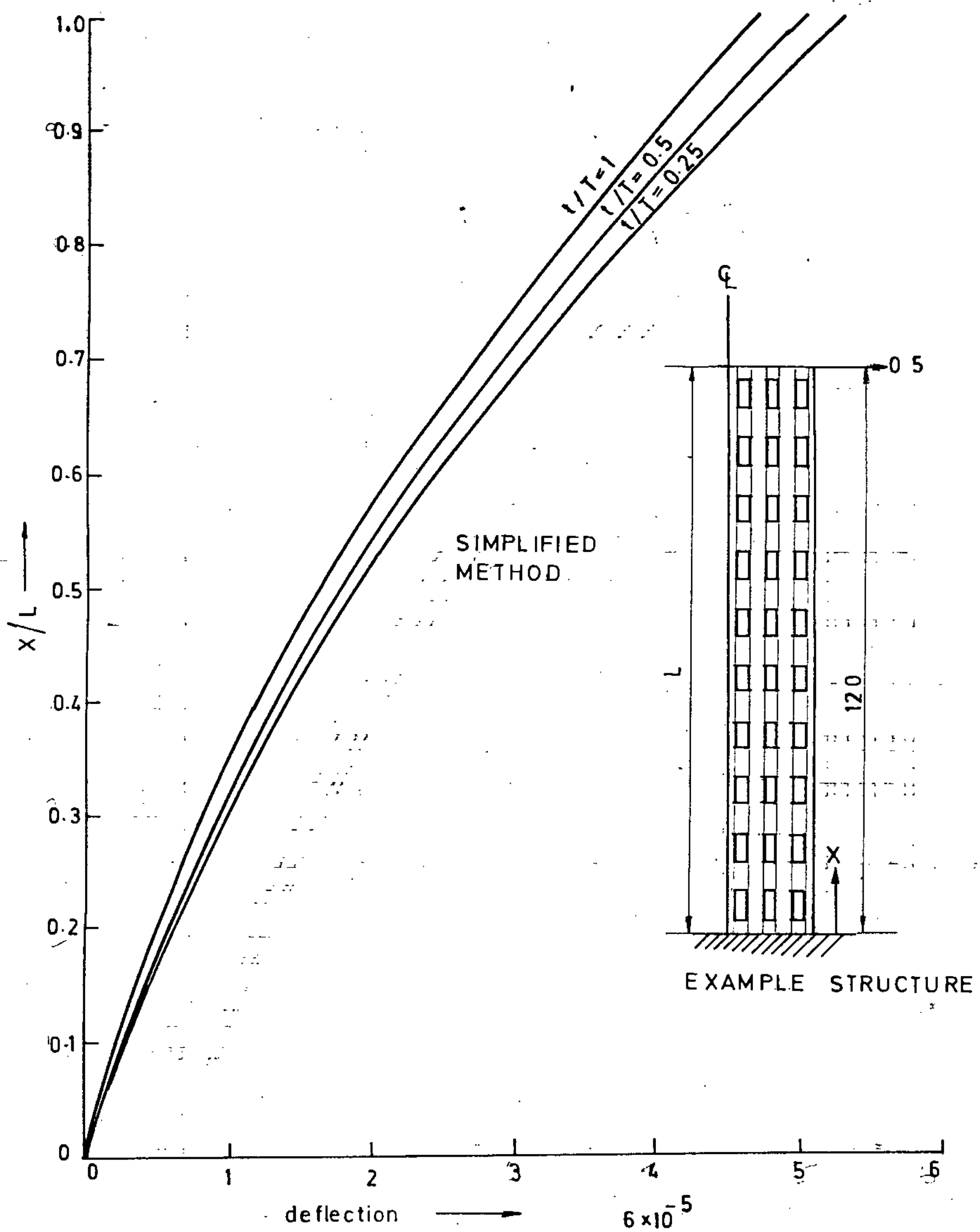


FIG 4.1 — DEFLECTED SHAPES OF THE WALL FRAME STRUCTURE

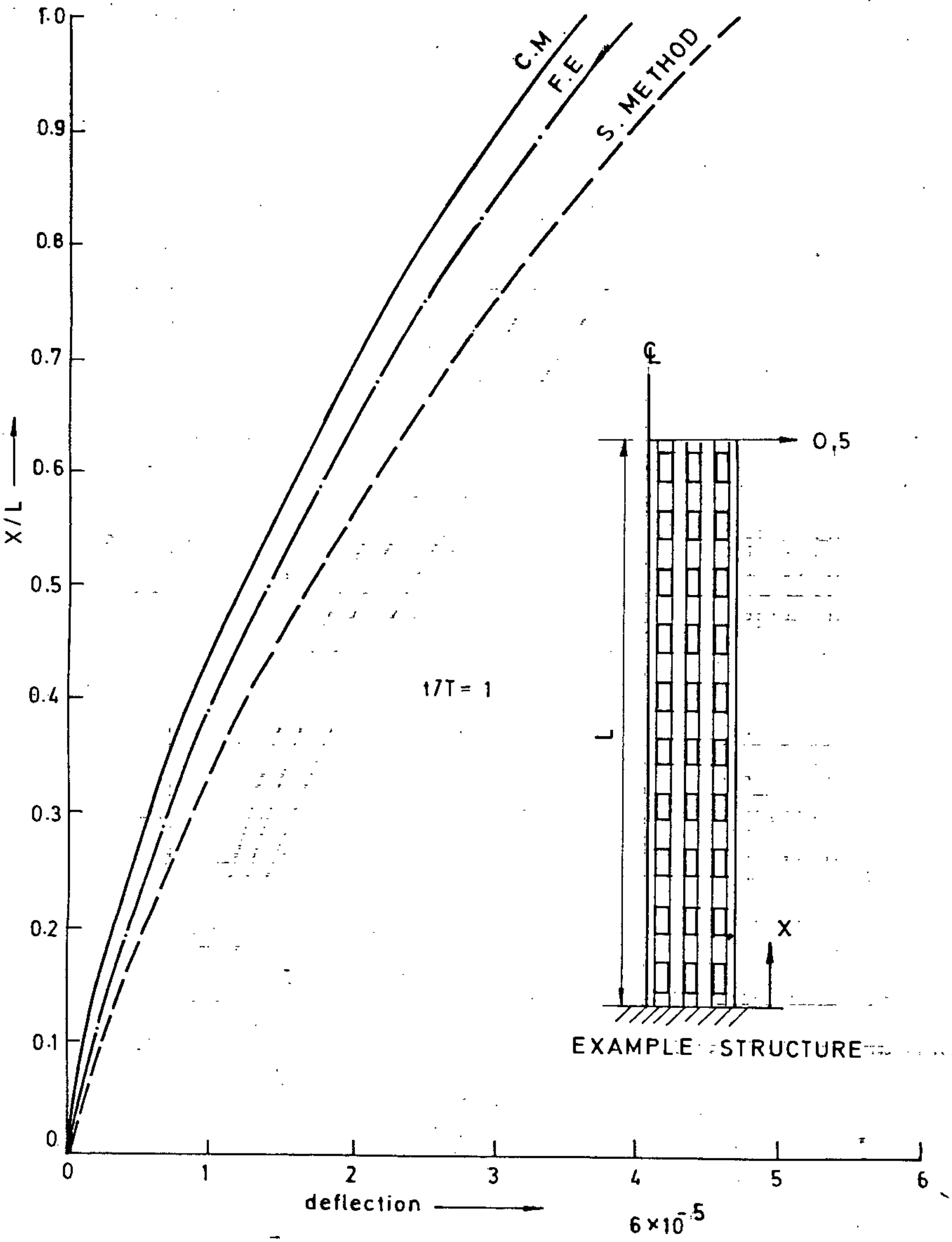


FIG. 4-2 DEFLECTED SHAPES OF THE WALL FRAME STRUCTURE

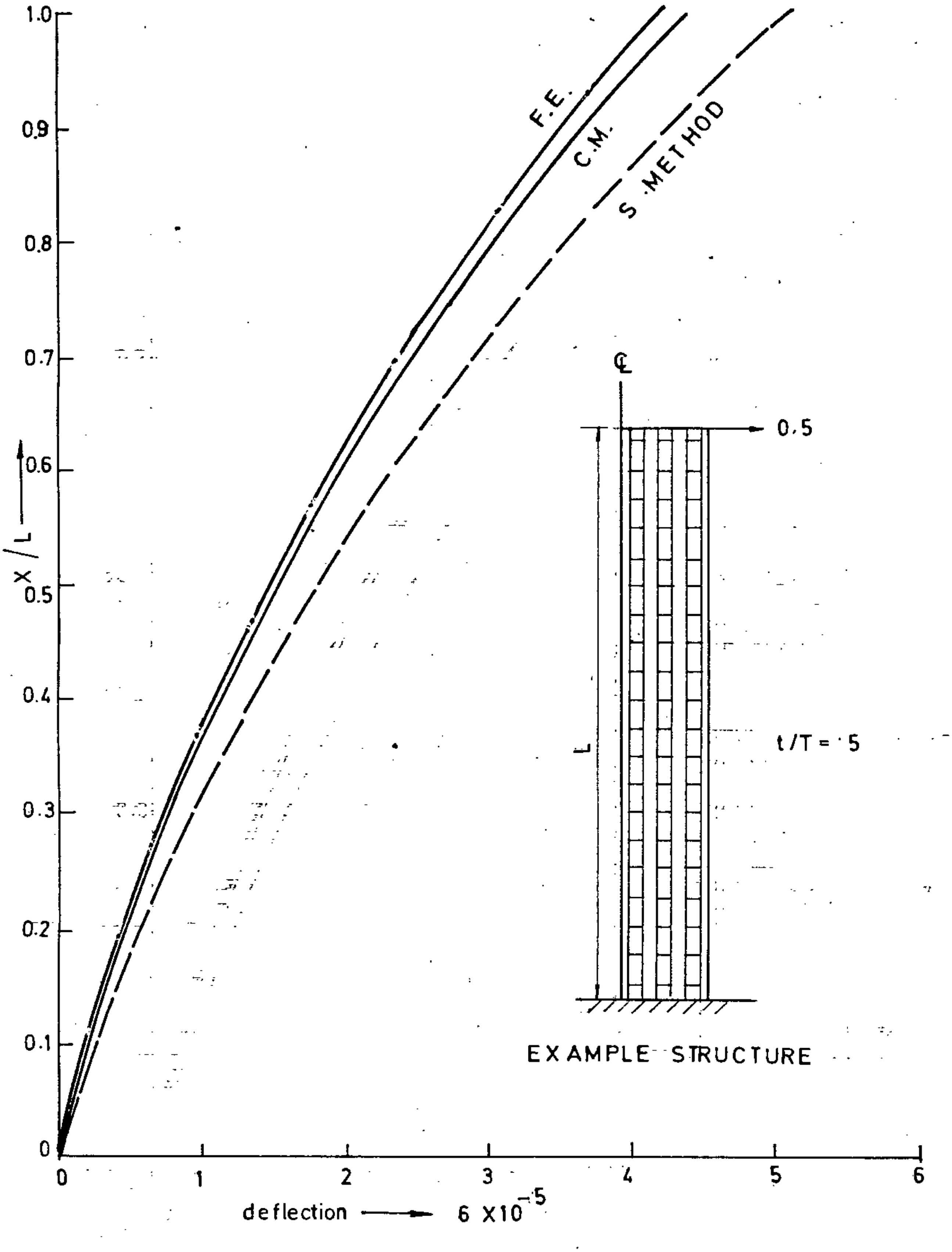


FIG. 4.3. DEFLECTED SHAPES OF THE WALL FRAME STRUCTURE

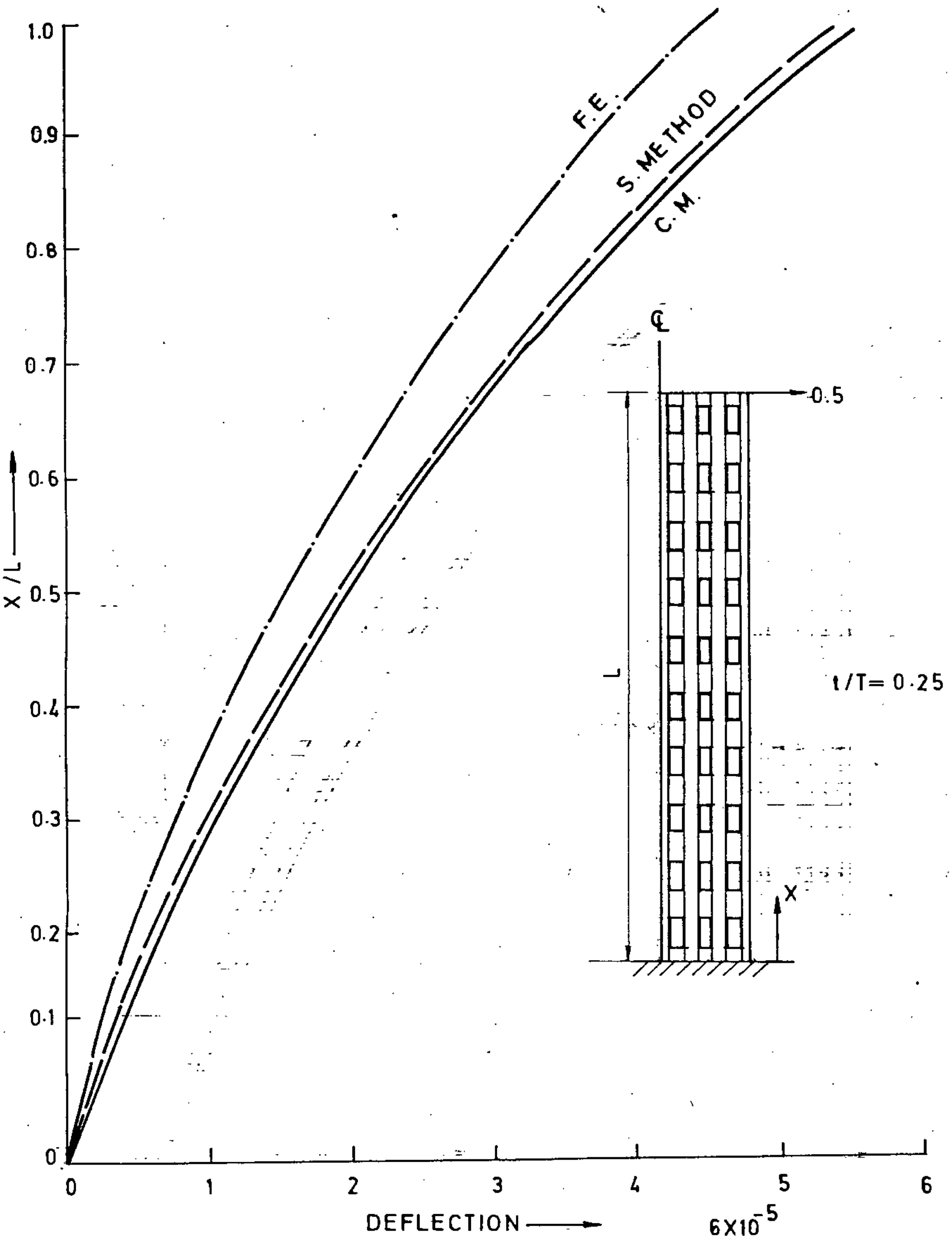


FIG.4.4, DEFLECTED SHAPES OF THE WALL FRAME STRUCTURE

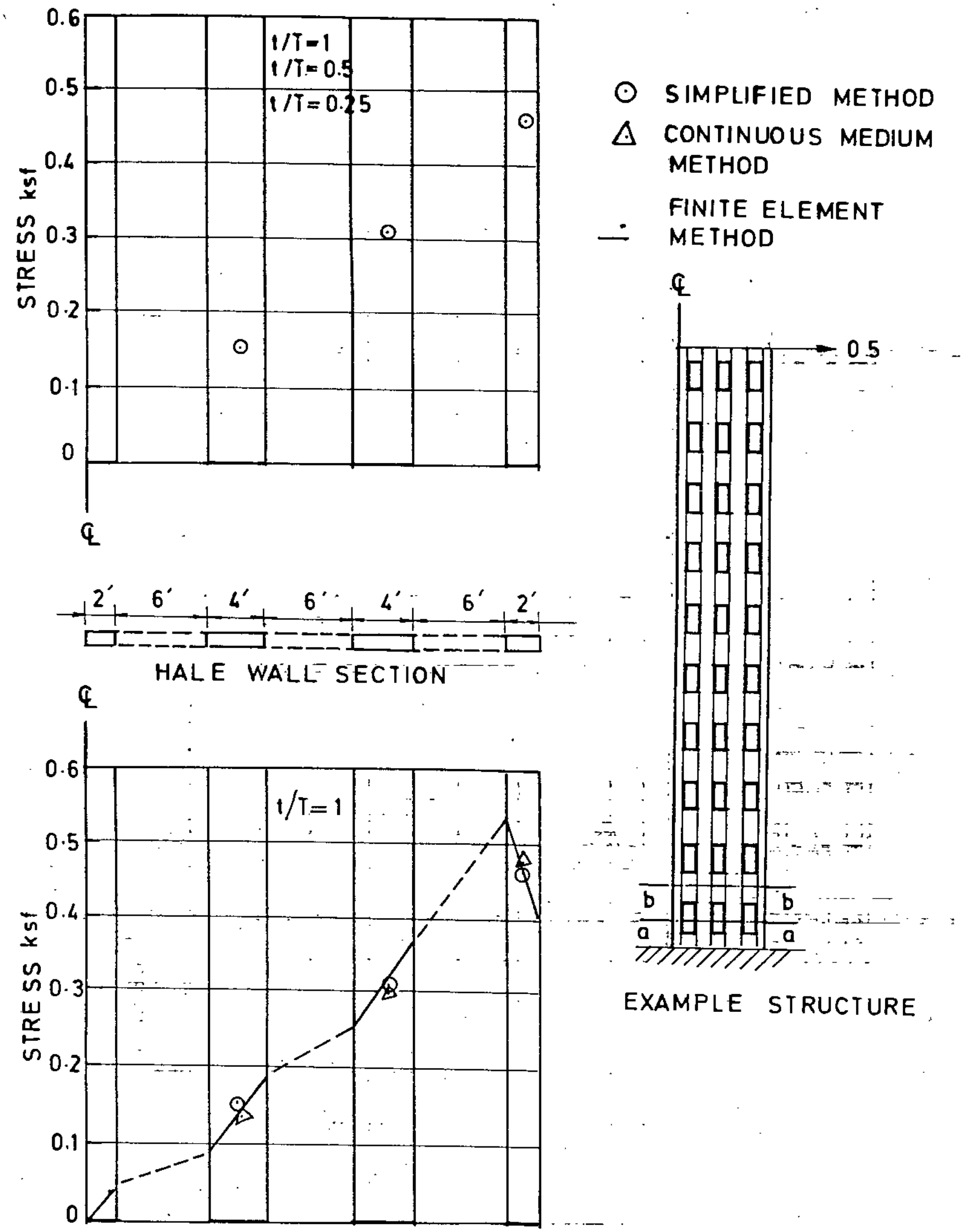


FIG. 4.5 STRESS DISTRIBUTION IN EXAMPLE STRUCTURE ACROSS THE SECTION a-a

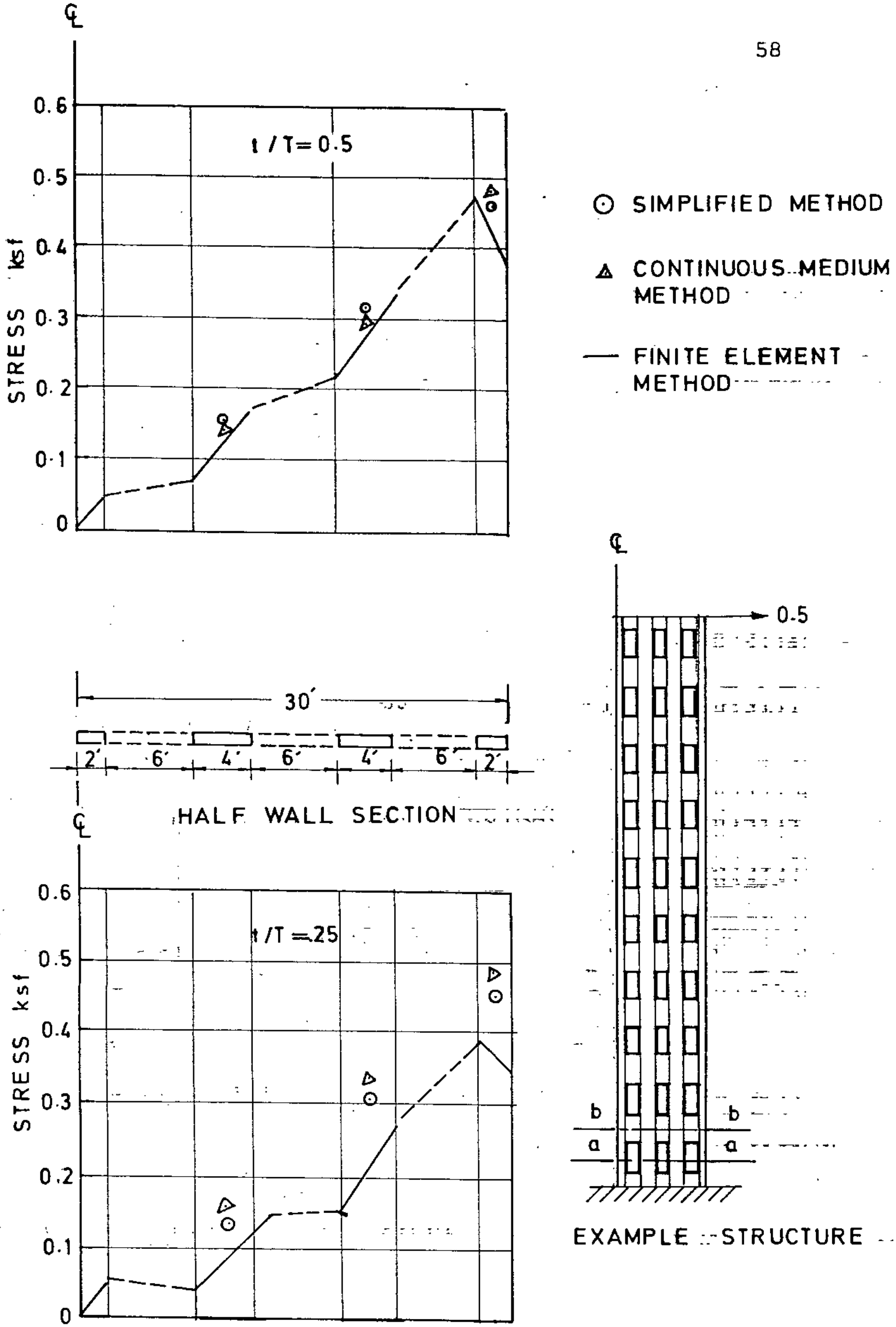


FIG. 4.6. STRESS DISTRIBUTION IN EXAMPLE STRUCTURE ACROSS THE SECTION a-a

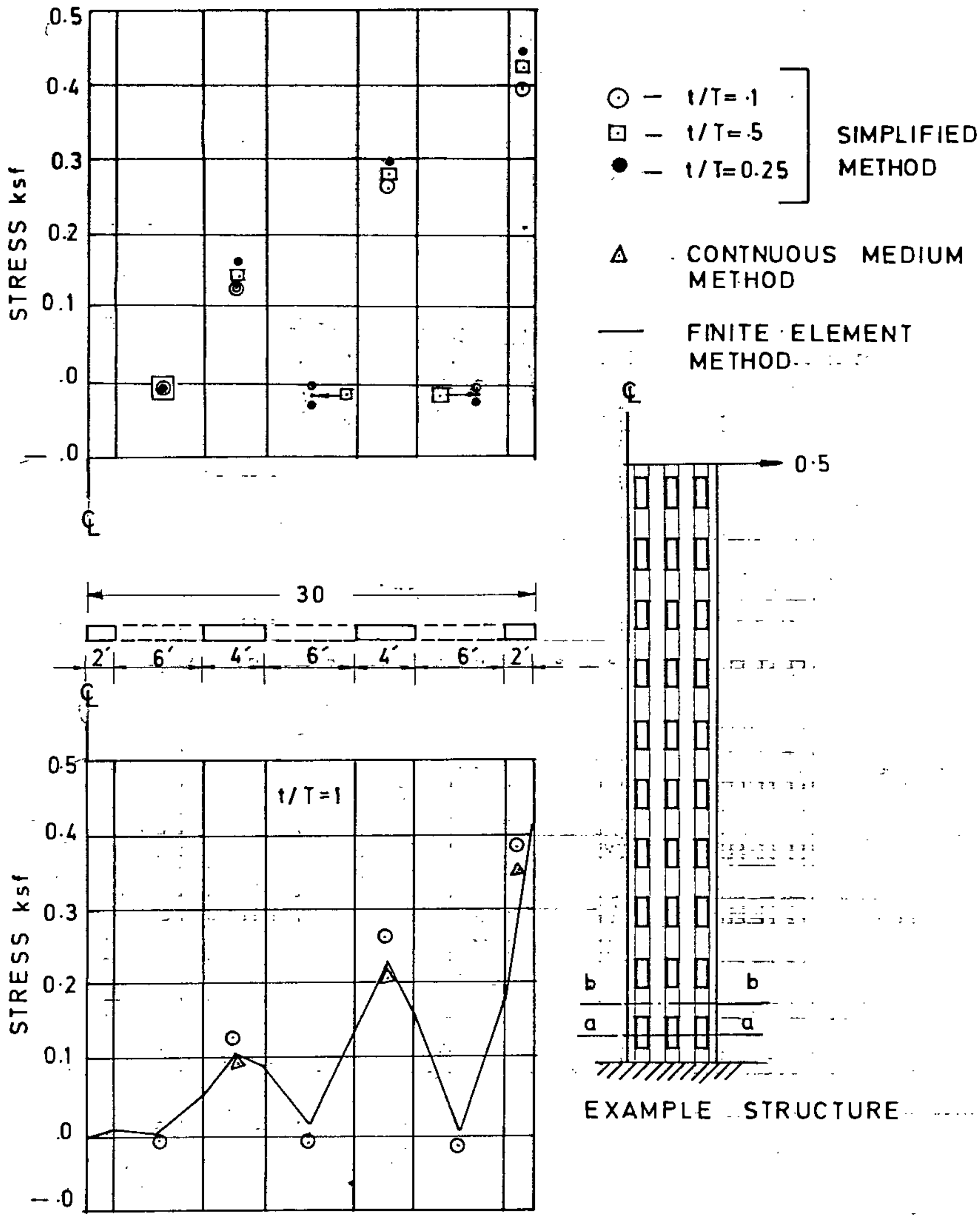


FIG. 4.7 STRESS DISTRIBUTION IN EXAMPLE STRUCTURE ACROSS THE SECTION b–b

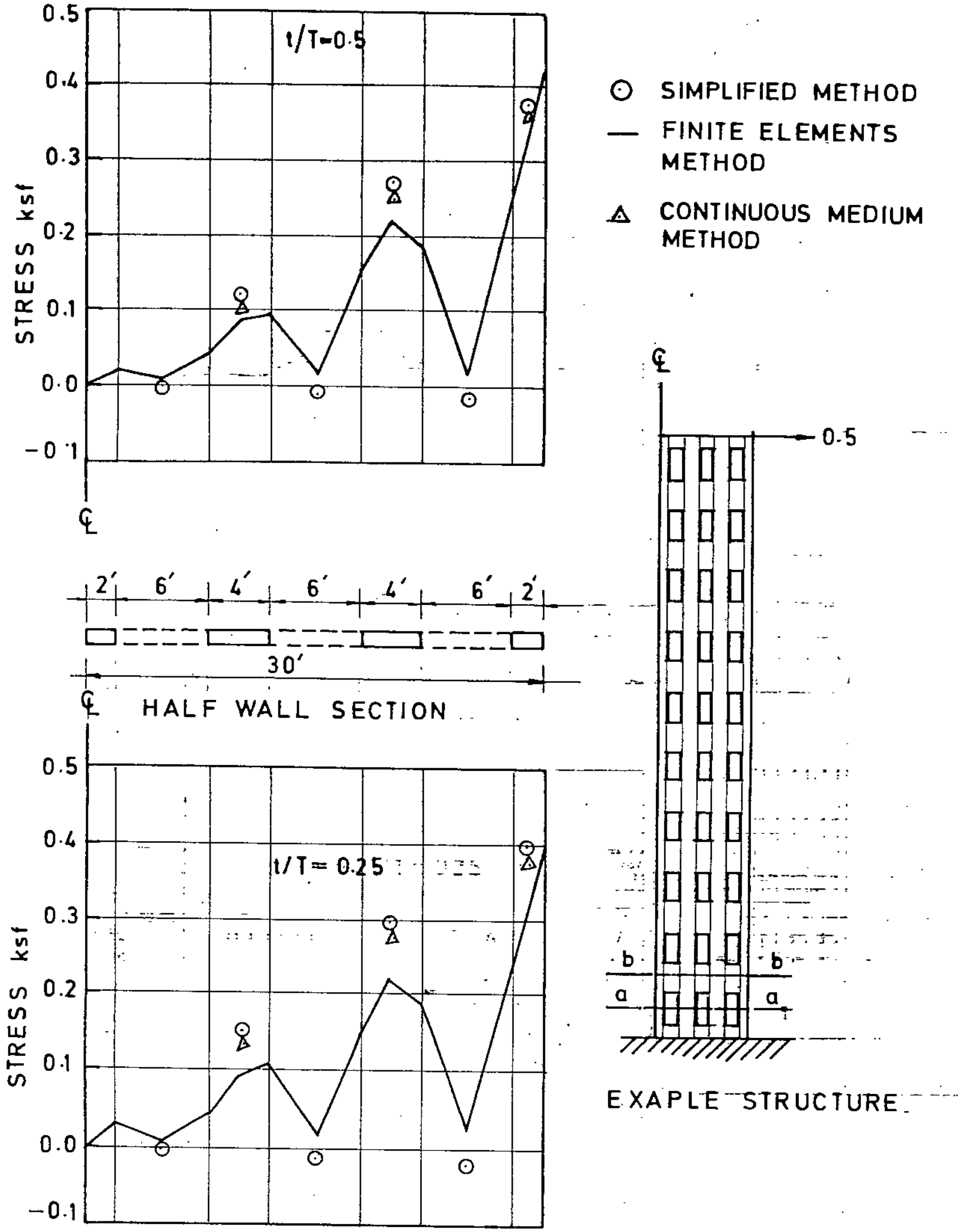


FIG. 4:8 STRESS DISTRIBUTION IN EXAMPLE STRUCTURE ACROSS THE SECTION b-b.

CONCLUSIONS AND RECOMMENDATION FOR FURTHER STUDY

Conclusion

On the basis of the results obtained for a 10 storey wall-frame structure with unequal lateral dimensions of columns and girders, it may be concluded that the simplified method of analysis proposed in the study may be conveniently used to rapidly evaluate the deflections and stress for the type of wall-frame investigated. It has been shown that even with a t/T ratio of 0.25 acceptable levels of accuracy may be obtained by analyzing the analogous plate with E'/E, G'/G and stress factor values obtained from graphs presented in this report.

Recommendation for further Study

In this study the accuracy of the simplified method has been studied using only one example with H/B ratio of 1.2, h/H ratio of 0.4 and b/B ratio of 0.4. Before recommending this method for general analysis of wall-frame structures further comparative studies must be performed with a wide spectrum of H/B, h/H and b/B ratios.

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APPENDIX-A

A-1 Rectangular element under plane stress

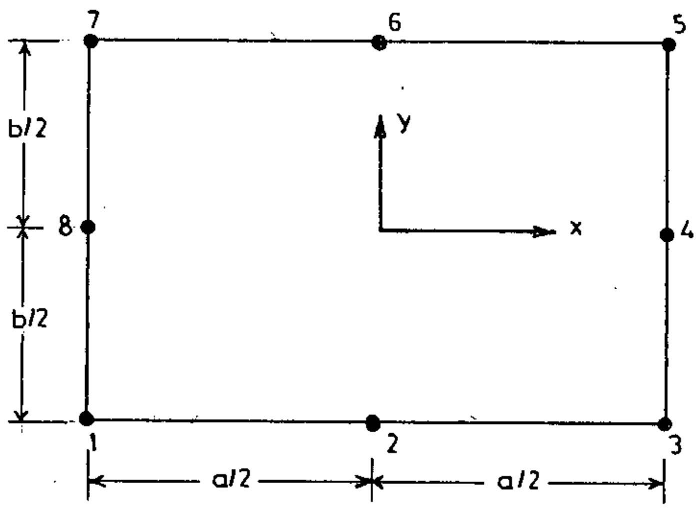
Displacement function

$$u = N_1u_1 + N_2u_2 + N_3u_3 + N_4u_4 + N_5u_5 + N_6u_6 + N_7u_7 + N_8u_8$$

$$v = N_1v_1 + N_2v_2 + N_3v_3 + N_4v_4 + N_5v_5 + N_6v_6 + N_7v_7 + N_8v_8$$

Displacement vector

$$u = \{ u_1\ v_1\ u_2\ v_2\ u_3\ v_3\ u_4\ v_4\ u_5\ v_5\ u_6\ v_6\ u_7\ v_7\ u_8\ v_8 \}$$



where

$$N_1 = \tfrac{1}{4}(1-\tfrac{2x}{a})(1-\tfrac{2y}{b}) - \tfrac{1}{4}(1-(\tfrac{2x}{a})^2)(1-\tfrac{2y}{b}) - \tfrac{1}{4}(1-(\tfrac{2y}{b})^2)(1-\tfrac{2x}{a})$$

$$N_2 = \tfrac{1}{2}(1-(\tfrac{2x}{a})^2)(1-\tfrac{2y}{b})$$

$$N_3 = \tfrac{1}{4}(1+\tfrac{2x}{a})(1-\tfrac{2y}{b}) - \tfrac{1}{4}(1-(\tfrac{2x}{a})^2)(1-\tfrac{2y}{b}) - \tfrac{1}{4}(1-(\tfrac{2y}{b})^2)(1+\tfrac{2x}{a})$$

$$N_4 = \tfrac{1}{2}(1-(\tfrac{2y}{b})^2)(1+\tfrac{2x}{a})$$

$$N_5 = \tfrac{1}{4}(1+\tfrac{2x}{a})(1+\tfrac{2y}{b}) - \tfrac{1}{4}(1-(\tfrac{2y}{b})^2)(1+\tfrac{2x}{a}) - \tfrac{1}{4}(1-(\tfrac{2x}{a})^2)(1+\tfrac{2y}{b})$$

$$N_6 = \tfrac{1}{2}(1-(\tfrac{2x}{a})^2)(1+\tfrac{2y}{b})$$

$$N_7 = \tfrac{1}{4}(1-\tfrac{2x}{a})(1+\tfrac{2y}{b}) - \tfrac{1}{4}(1-(\tfrac{2x}{a})^2)(1+\tfrac{2y}{b}) - \tfrac{1}{4}(1-(\tfrac{2y}{b})^2)(1-\tfrac{2x}{a})$$

$$N_8 = \tfrac{1}{2}(1-(\tfrac{2y}{b})^2)(1-\tfrac{2x}{a})$$

Stiffness matrix

$$
E't \, * \begin{bmatrix}
A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} & A_8 & A_{12} & -A_6 & A_{13} & A_{14} \\
 & A_{15} & A_{16} & -A_3\xi & -A_6 & A_{17} & A_8 & A_7/\xi & A_{10} & A_{18} & A_8 & \xi A_{11}\xi & A_6 & A_{19} & A_4 & A_{13}/\xi \\
 & & A_{20} & 0.0 & -A_3 & -A_{14} & 0.0 & 4A_8 & A_{11} & A_8 & A_{21} & 0.0 & A_{11} & -A_8 & 0.0 & -4A_8 \\
 & & & A_{22} & -A_4 & -A_3\xi & 4A_8 & 0.0 & A_8 & \xi A_{11} & 0.0 & A_{23} & -A_8 & \xi A_{11}\xi & -4A_8 & 0.0 \\
 & & & & A_1 & -A_2 & A_{13} & -A_{14} & A_{24} & A_6 & A_{11} & -A_8 & A_9 & -A_{10} & A_7 & -A_8 \\
 & & & & & A_{15} & -A_4 & A_{13}/\xi & -A_6 & A_{19} & -A_8 & A_7 & -A_{10} & A_{18} & -A_8 & A_7/\xi \\
 & & & & & & A_{25} & 0.0 & A_{13} & A_4 & 0.0 & -4A_8 & A_7 & -A_8 & A_{26} & 0.0 \\
 & & & & & & & A_{27} & A_{16} & A_{13}/\xi & -4A_8 & 0.0 & -A_8 & A_7/\xi & 0.0 & A_{28} \\
 & & & & & & & & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 \\
 & & \text{Symmetry} & & & & & & & A_{15} & A_{16} & -A_3\xi & -A_6 & A_{17} & A_8 & A_7/\xi \\
 & & & & & & & & & & A_{20} & 0.0 & -A_3 & -A_{14} & 0.0 & 4A_8 \\
 & & & & & & & & & & & A_{22} & -A_4 & -A_3\xi & 4A_8 & 0.0 \\
 & & & & & & & & & & & & A_1 & -A_2 & A_{13} & -A_{14} \\
 & & & & & & & & & & & & & A_{15} & -A_4 & A_{13}/\xi \\
 & & & & & & & & & & & & & & A_{25} & 0.0 \\
 & & & & & & & & & & & & & & & A_{27}
\end{bmatrix}
$$

where

$E' = E/(1-\nu^2)$ $\xi = \frac{1-\nu}{2}$

$A_1 = \frac{11}{18ab}(b^2 + \xi a^2)$ $A_2 = \frac{17}{36}(\nu + \xi)$

$A_3 \ \frac{8}{9}\ \frac{b}{a}$ $A_4 = \frac{1}{9}(\nu - 5\xi)$

$A_5 = -\frac{1}{18ab}(5b^2 + \xi 4a^2)$ $A_6 = \frac{1}{12}(\xi - \nu)$

$A_7 = -\frac{4}{9}\ \frac{a}{b}\xi$ $A_8 - \frac{1}{9}(\nu + \xi)$

$A_9 = \frac{2}{9ab}(b^2 + \xi a^2)$ $A_{10} = \frac{7}{36}(\nu + \xi)$

$A_{11} = -\frac{4}{9}\ \frac{b}{a}$ $A_{12} = \frac{1}{18ab}(4b^2 + 5a^2\xi)$

$A_{13} = -\frac{8}{9}\ \frac{a}{b}\xi$ $A_{14} = \frac{1}{9}(\xi - 5\nu)$

$A_{15} = \frac{11}{18ab}(a^2 + \xi b^2)$ $A_{16} = \frac{1}{9}(\xi - 5\nu)$

$A_{17} = \frac{1}{18ab}(4a^2 + \xi 5b^2)$ $A_{18} = \frac{2}{9ab}(\xi b^2 + a^2)$

$A_{19} = \frac{1}{18ab}(\xi 4b^2 + 5a^2)$ $A_{20} = \frac{1}{9ab}(16b^2 + \xi 6a^2)$

$A_{21} = \frac{1}{9ab}(8b^2 - 6a^2\xi)$ $A_{22} = \frac{1}{9ab}(6a^2 + 16b^2\xi)$

$A_{23} = \frac{1}{9ab}(\xi 8b^2 - 6a^2)$ $A_{24} = \frac{1}{18ab}(4b^2 + 5a^2\xi)$

$A_{25} = \frac{1}{9ab}(6b^2 + \xi 16a^2)$ $A_{26} = \frac{1}{9ab}(6b^2 + \xi 8a^2)$

$A_{27} = \frac{1}{9ab}(16a^2 + \xi 6b^2)$ $A_{28} = \frac{1}{9ab}(\xi 6b^2 - 8a^2)$

Stress matrix :

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [S]\{U\}$$

$$[S] = F' \begin{bmatrix} S_1 & \nu S_9 & S_2 & \nu S_{10} & S_3 & \nu S_{11} & S_4 & \nu S_{12} & S_5 & \nu S_{13} & S_6 & \nu S_{14} & S_7 & \nu S_{15} & S_8 & \nu S_{16} \\ \nu S_1 & S_9 & \nu S_2 & S_{10} & \nu S_3 & S_{11} & \nu S_4 & S_{12} & \nu S_5 & S_{13} & \nu S_6 & S_{14} & \nu S_7 & S_{15} & \nu S_8 & S_{16} \\ \xi S_9 & \xi S_1 & \xi S_{10} & \xi S_2 & \xi S_{11} & \xi S_3 & \xi S_{12} & \xi S_4 & \xi S_{13} & \xi S_5 & \xi S_{14} & \xi S_6 & \xi S_{15} & \xi S_7 & \xi S_{16} & \xi S_8 \end{bmatrix}$$

where

$$S_1 = -\tfrac{1}{2}a\left(1 - \frac{2y}{b}\right) + \frac{2x}{a^2}\left(1 - \frac{2y}{b}\right) + \tfrac{1}{2}a\left(1 - \frac{4y}{b^2}\right)$$

$$S_2 = -\frac{4x}{a^2}\left(1 - \frac{2y}{b}\right)$$

$$S_3 = \tfrac{1}{2}a\left(1 - \frac{2y}{b}\right) + \frac{2x}{a^2}\left(1 - \frac{2y}{b^2}\right) - \tfrac{1}{2}a\left(1 - \frac{4y^2}{b^2}\right)$$

$$S_4 = \frac{1}{a}\left(1 - \frac{4y^2}{b^2}\right)$$

$$S_5 = \tfrac{1}{2}a\left(1 + \frac{2y}{b}\right) - \tfrac{1}{2}a\left(1 - \frac{4y}{b^2}\right) + \frac{2x}{a^2}\left(1 - \frac{2y}{b}\right)$$

$$S_6 = -\frac{4x}{a^2}\left(1 + \frac{2y}{b}\right)$$

$$S_7 = -\tfrac{1}{2}a\left(1 + \frac{2y}{b}\right) + \frac{2x}{a^2}\left(1 + \frac{2y}{b}\right) + \tfrac{1}{2}a\left(1 - \frac{4y^2}{b^2}\right)$$

$$S_8 = -\frac{1}{a}\left(1 - \frac{4y^2}{b^2}\right)$$

$$S_9 = -\tfrac{1}{2}b\left(1 - \frac{2x}{a}\right) + \tfrac{1}{2}b\left(1 - \frac{4x^2}{a^2}\right) + \frac{2y}{b^2}\left(1 - \frac{2x}{a}\right)$$

$$S_{10} = -\frac{1}{b}\left(1 - \frac{4x^2}{a^2}\right)$$

$$S_{11} = -\tfrac{1}{2}b\left(1 + \frac{2x}{a}\right) + \tfrac{1}{2}b\left(1 - \frac{4x^2}{2}\right) + \frac{2y}{b^2}\left(1 - \frac{2x}{a}\right)$$

$$S_{12} = -\frac{4y}{b^2}\left(1 + \frac{2x}{a}\right)$$

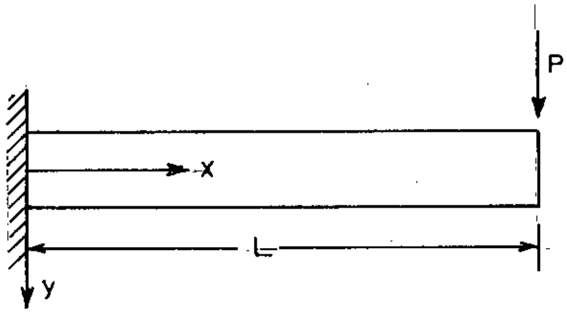
$$S_{13} = \tfrac{1}{2}b\left(1 + \frac{2x}{a}\right) + \frac{2y}{b^2}\left(1 + \frac{2x}{a}\right) - \tfrac{1}{2}b\left(1 - \frac{4x^2}{a^2}\right)$$

$$S_{14} = \frac{1}{b}\left(1 - \frac{4x^2}{a^2}\right)$$

$$S_{15} = \tfrac{1}{2}b\left(1 - \frac{2x}{a}\right)\left(1 - \frac{4x^2}{a^2}\right) + \frac{2y}{a^2}\left(1 - \frac{2x}{a}\right)$$

$$S_{16} = -\frac{4y}{b^2}\left(1 - \frac{2x}{a}\right)$$

A-2 Expressions for deflection and stress by Engineer's Theory of Bending



For a point load at the free end, P

Deflection

$$Y = \frac{Pl^3}{6EI} \quad 3\,R^2 - R^3 + \frac{1.2\,Pl}{GA}\,R$$

Stress

$$= \pm \frac{P(l-x)c}{I}$$

where $R = \frac{x}{l}$

c = distance from the centraidal axis.

```
C
C
      COMMON/CDAT/NCN,NED,IBC,NE3,NN3
      COMMON/CDATA/NRMX,NCMX,NDFEL,NN,NE,NLN,NBN,NDF,NNE,N,MS,E,G
      DIMENSION X(10 ),Y(10 ), CON(100),PROP(10 ),IB(60),TK(82 ,22),
     +AL(100),FORC(100),REAC(300),ELST(16,16),     NODES(100),V(40)
     +,R(3,16),NC(8),KC(16),REA(300)
C
C
      NED=8
      NNE=8
      NCN=NED*NNE
      IBC=60
      NE3=NED*3
      NN3=123
      NDF=2
      NRMX=82
      NCMX=22
      NDFEL=NDF*NNE
C
   10 READ(1,4) NPROB
    4 FORMAT(I2)
      IF(NPROB.EQ.0) GO TO 2
      WRITE(3,5) NPROB
    5 FORMAT(//5X,'PROBLEM NUMBER 'I3)
C
      CALL  INPUT(X,Y,CON,PROP,AL,IB,REAC,NPROB,REA)
C
      CALL  ASSEM(X,Y, CON,PROP,TK,ELST,AL)
C
      CALL  BOUND(TK,AL,REAC,IB,REA)
C
      CALL  SLESI(TK,AL,V,N,MS,NRMX,NCMX)
C
      CALL  FORCE( CON,PROP,FORC,REAC,X,Y,AL,NODES)
C
      CALL  OUTPT(AL,FORC,REAC)
      GO TO  10
    2 STOP
      END
```

```
      SUBROUTINE INPUT(X,Y, CON,PROP,AL,IB,REAC,NPROB,REA)
C
      COMMON/CDAT/NCN,NED,IBC,NE3,NN3
      COMMON/CDATA/NRMX,NCMX,NDFEL,NN,NE,NLN,NBN,NDF,NNE,N,MS,E,G
      DIMENSION X(NED),Y(NED),CON(NCN),PROP(NED),AL(NRMX),IB(IBC),
     +IC(8),NCDES(NCN),NV(8),REAC(NN3),W(8),REA(NN3)
C
      WRITE(3,20)
   20 FORMAT(' ',130('*'))
      IF(NPROB.GT.1) GO TO 31
      READ(1,1) NN,NE,NLN,NBN,E,G
   31 WRITE(3,21)NN,NE,NLN,NBN,E,G
   21 FORMAT(//'   INTERNAL DATA'//
     +' NUMBER OF NODES           =',I5/
     +' NUMBER OF ELEMENTS        =',I5/
     +' NUMBER OF LOADED NODES    =',I5/
     +' NUMBER OF SUPPORT NODES=',I5/
     +' MODULUS OF ELASTISITY     =',F15.0/
     +' POISSON COEFFICIENT       =',F15.4///
     +'  ELEMENT DIMENSION'/5X,'ELEMENT',6X,'X',9X,'Y',9X,'T')
    1 FORMAT(4I10,2F10.2)
C
      READ(1,2) (I,X(I),Y(I),PROP(I),I=1,NE)
      WRITE(3,2) (I,X(I),Y(I),PROP(I),I=1,NE)
    2 FORMAT(I10,3F10.3)
C
      IF(NPROB.GT.1) GO TO 10
C
      WRITE(3,22)
   22 FORMAT(/' ELEMENT CONNECTIVITY '/2X,'ELEMENT',1X,'(
     +    NODES                          )')
C
      DO 3 J=1,NE
      READ(1,4)I,IC(1),IC(2),IC(3),IC(4),IC(5),IC(6),IC(7),IC(8)
      WRITE(3,34)I,IC(1),IC(2),IC(3),IC(4),IC(5),IC(6),IC(7),IC(8)
C
      N1=NNE*(I-1)
      CON(N1+1) =IC(1)
      CON(N1+2)=IC(2)
      CON(N1+3)=IC(3)
      CON(N1+4)=IC(4)
      CON(N1+5)=IC(5)
      CON(N1+6)=IC(6)
      CON(N1+7)=IC(7)
    3 CON(N1+8)=IC(8)
    4 FORMAT( 9I4)
   34 FORMAT( 9I6)
   10 N=NN*NDF
      DO 5 I=1,N
    5 AL(I)=0.0
      WRITE(3,23)
   23 FORMAT(/' NODAL LODES'/7X,'NODES',5X,'PX',8X,'PY')
      DO 6 I=1,NLN
      READ(1,2) J,(W(K),K=1,NDF)
      WRITE(3,2) J,(W(K),K=1,NDF)
      DO 6 K=1,NDF
      L=NDF*(J-1)+K
    6 AL(L)=W(K)
C
      IF(NPROB.GT.1) GO TO 41
C
      WRITE(3,24)
   24 FORMAT(/' BOUNDARY CONDITION DATA'/23X,'STATAS',14X,'PRESCRIBED V
     +LUES'/15X,'(0 PRECRIBED,   1 FREE)'/7X,'NODE',8X,'U',9X,'V',16X,'U
     +,9X,'V')
      DO 7 I=1,NBN
      READ(1,8) J,(IC(K),K=1,NDF),(W(K),K=1,NDF)
      WRITE(3,9) J,(IC(K),K=1,NDF),(W(K),K=1,NDF)
      L1=(NDF+1)*(I-1)+1
      L2=NDF*(J-1)
      IB(L1)=J
      DO 7 K=1,NDF
      N1=L1+K
      N2=L2+K
      IB(N1)=IC(K)
      REA(N2)=W(K)
    7 REAC(N2)=REA(N2)
    8 FORMAT(3I10,2F10.4)
```

```
    9     FORMAT(3I10,10X,2F10.4)
C
   41     RETURN
          END
```

```
      SUBROUTINE ASSEM(X,Y, CON,PROP,TK,ELST,AL)
C
      COMMON/CDAT/NCN,NED,IBC,NE3,NN3
      COMMON/CDATA/NRMX,NCMX,NDFEL,NN,NE,NLN,NBN,NDF,NNE,N,MS,E,G
      DIMENSION X(NED),Y(NED),CON(NCN),TK(NRMX,NCMX),ELST(16,16),
     +AL(NRMX),PROP(NED)
C
      N1=NNE-1
      MS=0
      DO 7 I=1,NE
      L1=NNE*(I-1)
      DO 7 J=1,N1
      L2=L1+J
      J1=J+1
      DO 7 K=J1,NNE
      L3=L1+K
      L=ABS(CON(L2)-CON(L3))
      IF(MS-L) 6,7,7
    6 MS=L
    7 CONTINUE
      MS=NDF*(MS+1)
      WRITE(3,29) MS
   29 FORMAT(///7X,'HALF BAND WIDTH='I5)
C
      DO 10 I=1,N
      DO 10 J=1,MS
   10 TK(I,J)=0.0
C
      DO 20 NEL=1,NE
C
      CALL STIFF(NEL,X,Y,PROP,CON,ELST,AL)
C
   20 CALL ELASS(NEL, CON,TK,ELST)
C
      RETURN
      END
```

```
      SUBROUTINE ELASS(NEL, CON,TM,ELMAT)
C
      COMMON/CDAT/NCN,NED,IBC,NE3,NN3
      COMMON/CDATA/NRMX,NCMX,NDFEL,NN,NE,NLN,NBN,NDF,NNE,N,MS,E,G
      DIMENSION   CON(NCN),TM(NRMX,NCMX),ELMAT(16,16)
C
      L1=NNE*(NEL-1)
      DO 50 I=1,NNE
      L2=L1+I
      N1=CON(L2)
      I1=NDF*(I-1)
      J1=NDF*(N1-1)
      DO 50 J=1,NNE
      L2=L1+J
      N2=CON(L2)
      I2=NDF*(J-1)
      J2=NDF*(N2-1)
      DO 50 K=1,NDF
      KI=1
      IF(N1-N2) 20,10,30
 10   KI=K
 20   KR=J1+K
      IC=J2-KR+1
      K1=I1+K
      GO TO 40
 30   KR=J2+K
      IC=J1-KR+1
      K2=I2+K
 40   DO 50 L=KI,NDF
      KC=IC+L
      IF(N1-N2) 45,45,46
 45   K2=I2+L
      GO TO 50
 46   K1=I1+L
 50   TM(KR,KC)=TM(KR,KC)+ELMAT(K1,K2)
C
      RETURN
      END
```

```
      SUBROUTINE  STIFF(NEL,X,Y,PROP,CON,S,AL)
      COMMON/CDAT/NCN,NED,IBC,NE3,NN3
      COMMON/CDATA/NRMX,NCMX,NDFEL,NN,NE,NLN,NBN,NDF,NNE,N,MS,E,G
      DIMENSION X(NED),Y(NED),CON(NCN),PROP(NED),S(16,16)
      DX=X(NEL)
      DY=Y(NEL)
      ET=E*PROP(NEL)/(1-G*G)
      AB=DX*DY
      AD=DX/DY
      BD=DY/DX
      X2=DX*DX
      Y2=DY*DY
      XY=(1.-G)/2.
      S(1,1)=11.*(Y2+XY*X2)/AB/18.
      S(1,2)=17.*(XY+G)/36.
      S(1,3)=-8.*BD/9.
      S(1,4)=(G-5.*XY)/9.
      S(1,5)=(5.*Y2+XY*4.*X2)/AB/18.
      S(1,6)=(XY-G)/12.
      S(1,7)=-4./9.*AD*XY
      S(1,8)=-(G+XY)/9.
      S(1,9)=2./9.*(Y2+XY*X2)/AB
      S(1,10)=7./36.*(G+XY)
      S(1,11)=-4./9.*BD
      S(1,12)=-(G+XY)/9.
      S(1,13)=(4.*Y2+5.*X2*XY)/18./AB
      S(1,14)=-(XY-G)/12.
      S(1,15)=-8./9.*AD*XY
      S(1,16)=    (XY-5.*G)/9.
      S(2,2)=11./18./AB*(X2+XY*Y2)
      S(2,3)=    (XY-5.*G)/9.
      S(2,4)=-8./9.*BD*XY
      S(2,5)=(G-XY)/12.
      S(2,6)=(5.*Y2*XY+4.*X2)/18./AB
      S(2,7)=S(1,8)
      S(2,8)=-4.*AD/9.
      S(2,9)=S(1,10)
      S(2,10)=2.*(XY*Y2+X2)/9./AB
      S(2,11)=S(1,8)
      S(2,12)=-4.*BD/9.*XY
      S(2,13)=(XY-G)/12.
      S(2,14)=(XY*4.*Y2+5.*X2)/18./AB
      S(2,15)=S(1,4)
      S(2,16)=-8./9.*AD
      S(3,3)=(16.*Y2+XY*6.*X2)/9./AB
      S(3,4)=0.0
      S(3,5)=S(1,3)
      S(3,6)=(5.*G-XY)/9.
      S(3,7)=0.0
      S(3,8)=-4./9.*(G+XY)
      S(3,9)=S(1,11)
      S(3,10)=S(2,11)
      S(3,11)=(8.*Y2-6.*X2*XY)/9./AB
      S(3,12)=0.0
      S(3,13)=S(1,11)
      S(3,14)=-S(2,7)
      S(3,15)=0.0
      S(3,16)=-S(3,8)
      S(4,4)=(6.*X2+XY*16.*Y2)/9./AB
      S(4,5)=-S(1,4)
      S(4,6)=S(2,4)
      S(4,7)=S(3,8)
      S(4,8)=0.0
      S(4,9)=S(2,7)
      S(4,10)=S(2,12)
      S(4,11)=0.0
      S(4,12)=(XY*8.*Y2-6.*X2)/9./AB
      S(4,13)=S(3,14)
      S(4,14)=S(2,12)
      S(4,15)=S(3,16)
      S(4,16)=0.0
      S(5,5)=S(1,1)
      S(5,6)=-S(1,2)
      S(6,6)=11.*(X2+XY*Y2)/18./AB
C
      TI=0
      K=4
      DO 300 II=1,2
```

```
      TI=TI+1
      I=TI
      K=K+1
      IF(II.EQ.2) GO TO 500
      JJ=15
      DO 200 J=7,15,2
      S(K,JJ)=S(I,J)
      S(K,(JJ+1))=-S(I,(J+1))
      JJ=JJ-2
  200 CONTINUE
      GO TO 300
  500 JJ=16
      DO 400 J=8,16,2
      S(K,JJ)=S(I,J)
      S(K,(JJ-1))=-S(I,(J-1))
      JJ=JJ-2
  400 CONTINUE
  300 CONTINUE
C
C
      S(7,7)=(6.*Y2+XY*16.*X2)/AB/9.
      S(7,8)=0.0
      S(7,9)=-8.*AD*XY/9.
      S(7,10)=S(1,4)
      S(7,11)=0.0
      S(7,12)=S(3,16)
      S(7,13)=S(1,7)
      S(7,14)=-S(2,7)
      S(7,15)=-(6.*Y2-XY*8.*X2)/9./AB
      S(7,16)=0.0
      S(8,8)=(16.*X2+XY*6.*Y2)/9./AB
      S(8,9)=S(2,3)
      S(8,10)=S(2,16)
      S(8,11)=4.*(G+XY)/9.
      S(8,12)=0.0
      S(8,13)=-S(2,7)
      S(8,14)=-4.*AD/9.
      S(8,15)=0.0
      S(8,16)=-(XY*6.*Y2-8.*X2)/9./AB
C
C
      I=8
      TK=0
      DO 100 III=1,8
      I=I+1
      TK=TK+1
      K=TK
      JJ=K
      DO 100 J=1,16
      S(I,J)=S(K,JJ)
      JJ=JJ+1
  100 CONTINUE
C
      DO 600 I1=1,16
      DO 600 J1=1,I1
  600 S(I1,J1)=S(J1,I1)
      DO 50 I3=1,16
      DO 50 J3=1,16
   50 S(I3,J3)=ET*S(I3,J3)
      RETURN
      END
```

```
      SUBROUTINE BOUND(TK,AL,REAC,IB,REA)
C
      COMMON/CDAT/NCN,NED,IBC,NE3,NN3
      COMMON/CDATA/NRMX,NCMX,NDFEL,NN,NE,NLN,NBN,NDF,NNE,N,MS,E,G
      DIMENSION AL(NRMX),      REAC(NN3),TK(NRMX,NCMX),IB(IBC),REA(NN3)
      DO 100 L=1,NBN
C
      L1=(NDF+1)*(L-1)+1
      NO=IB(L1)
      K1=NDF*(NO-1)
      DO 100 I=1,NDF
      L2=L1+I
      IF(IB(L2))100,10,100
   10 KR=K1+I
      DO 50 J=2,MS
      KV=KR+J-1
      IF(N-KV)30,20,20
   20 AL(KV)=AL(KV)-TK(KR,J)*REA(KR)
      TK(KR,J)=0.
   30 KV=KR-J+1
      IF(KV)50,50,40
   40 AL(KV)=AL(KV)-TK(KV,J)*REA(KR)
      TK(KV,J)=0.
   50 CONTINUE
      TK(KR,1)=1.
      AL(KR)=REA(KR)
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SLBSI(A,B,D,N,MS,NRMX,NCMX)
C
      DIMENSION A(NRMX,NCMX),B(NRMX),D(NCMX)
      N1=N-1
      DO 100 K=1,N1
      C=A(K,1)
      K1=K+1
      IF(ABS(C)-0.000001) 1,1,3
    1 WRITE(3,2) K
    2 FORMAT('***** SINGULARITY IN ROW',I5)
      GO TO 300
C
    3 NI=K1+MS-2
      L=MIN0(NI,N)
      DO 11 J=2,MS
   11 D(J)=A(K,J)
      DO 4 J=K1,L
      K2=J-K+1
    4 A(K,K2)=A(K,K2)/C
      B(K)=B(K)/C
C
      DO 10 I=K1,L
      K2=I-K1+2
      C=D(K2)
      DO 5 J=I,L
      K2=J-I+1
      K3=J-K+1
    5 A(I,K2)=A(I,K2)-C*A(K,K3)
   10 B(I)=B(I)-C*B(K)
  100 CONTINUE
C
      IF(ABS(A(N,1))-0.000001)1,1,101
  101 B(N)=B(N)/A(N,1)
C
      DO 200 I=1,N1
      K=N-I
      K1=K+1
      NI=K1+MS-2
      L=MIN0(NI,N)
      DO 200 J=K1,L
      K2=J-K+1
  200 B(K)=B(K)-A(K,K2)*B(J)
  300 RETURN
      END
```

```
      SUBROUTINE FORCE( CON,PROP,FORC,REAC,X,Y,AL,NODES)
C
      COMMON/CDAT/NCN,NED,IBC,NE3,NN3
      COMMON/CDATA/NRMX,NCMX,NDFEL,NN,NE,NLN,NBN,NDF,NNE,N,MS,E,G
      DIMENSION CON(NCN),PROP(NED),FORC(NE3),X(NED),Y(NED),REAC(NN3),
     +NODES(NCN),AL(NRMX),R(3,16),KC(16),NC(8)
      DO 89 II=1,NN3
   89 REAC(II)=0.0
      DO 99 III=1,NE3
   99 FORC(III)=0.0
      DO 100 NEL=1,NE
      L=NNE*(NEL-1)
      B=Y(NEL)
      D=X(NEL)
      XY=(1.-G)/2.
      C=E/(1-G*G)
      DO 300 I=1,8
  300 NC(I)= CON(L+I)
      DO 400 II=1,8
      L=NDF*(II-1)
      KC(L+1)=NC(II)*NDF-1
  400 KC(L+2)=NC(II)*NDF
C
      L=(NC(1)-1)*3
      BY=-B/2.
      DX=-D/2.
      CALL STRESS(R,B,D,DX,BY,C,XY,REAC,L)
      DO 500 I=1,16
      REAC(L+1)=REAC(L+1)+R(1,I)*AL(KC(I))
      REAC(L+2)=REAC(L+2)+R(2,I)*AL(KC(I))
  500 REAC(L+3)=REAC(L+3)+R(3,I)*AL(KC(I))
      L=(NC(2)-1)*3
      DX=0.0
      CALL STRESS(R,B,D,DX,BY,C,XY,REAC,L)
      DO 600 I=1,16
      REAC(L+1)=REAC(L+1)+R(1,I)*AL(KC(I))
      REAC(L+2)=REAC(L+2)+R(2,I)*AL(KC(I))
  600 REAC(L+3)=REAC(L+3)+R(3,I)*AL(KC(I))
      L=(NC(3)-1)*3
      DX=D/2.
      CALL STRESS(R,B,D,DX,BY,C,XY,REAC,L)
      DO 700 I=1,16
      REAC(L+1)=REAC(L+1)+R(1,I)*AL(KC(I))
      REAC(L+2)=REAC(L+2)+R(2,I)*AL(KC(I))
  700 REAC(L+3)=REAC(L+3)+R(3,I)*AL(KC(I))
      L=(NC(4)-1)*3
      BY=0.0
      CALL STRESS(R,B,D,DX,BY,C,XY,REAC,L)
      DO 800 I=1,16
      REAC(L+1)=REAC(L+1)+R(1,I)*AL(KC(I))
      REAC(L+2)=REAC(L+2)+R(2,I)*AL(KC(I))
  800 REAC(L+3)=REAC(L+3)+R(3,I)*AL(KC(I))
      L=(NC(5)-1)*3
      BY=B/2.
      CALL STRESS(R,B,D,DX,BY,C,XY,REAC,L)
      DO 900 I=1,16
      REAC(L+1)=REAC(L+1)+R(1,I)*AL(KC(I))
      REAC(L+2)=REAC(L+2)+R(2,I)*AL(KC(I))
  900 REAC(L+3)=REAC(L+3)+R(3,I)*AL(KC(I))
      L=(NC(6)-1)*3
      DX=0.0
      CALL STRESS(R,B,D,DX,BY,C,XY,REAC,L)
      DO 1000 I=1,16
      REAC(L+1)=REAC(L+1)+R(1,I)*AL(KC(I))
      REAC(L+2)=REAC(L+2)+R(2,I)*AL(KC(I))
 1000 REAC(L+3)=REAC(L+3)+R(3,I)*AL(KC(I))
      L=(NC(7)-1)*3
      DX=-D/2.
      CALL STRESS(R,B,D,DX,BY,C,XY,REAC,L)
      DO 1100 I=1,16
      REAC(L+1)=REAC(L+1)+R(1,I)*AL(KC(I))
      REAC(L+2)=REAC(L+2)+R(2,I)*AL(KC(I))
 1100 REAC(L+3)=REAC(L+3)+R(3,I)*AL(KC(I))
      L=(NC(8)-1)*3
      BY=0.0
      CALL STRESS(R,B,D,DX,BY,C,XY,REAC,L)
      DO 1200 I=1,16
      REAC(L+1)=REAC(L+1)+R(1,I)*AL(KC(I))
```

```
      REAC(L+2)=REAC(L+2)+R(2,I)*AL(KC(I))
 1200 REAC(L+3)=REAC(L+3)+R(3,I)*AL(KC(I))
      L=(NEL-1)*3
      DX=0.0
      CALL STRESS(R,B,D,DX,BY,C,XY,FORC,L)
      DO 1300 I=1,16
      FORC(L+1)=FORC(L+1)+R(1,I)*AL(KC(I))
      FORC(L+2)=FORC(L+2)+R(2,I)*AL(KC(I))
 1300 FORC(L+3)=FORC(L+3)+R(3,I)*AL(KC(I))
 100  CONTINUE
C
      RETURN
      END
```

```fortran
      SUBROUTINE STRESS(SM,B,D,DX,BY,C,XY,REAC,L)
      COMMON/CDAT/NCN,NED,IBC,NE3,NN3
      COMMON/CDATA/NRMX,NCMX,NDFEL,NN,NE,NLN,NBN,NDF,NNE,N,MS,E,G
C
      DIMENSION SM(3,16),REAC(NN3)
C
      REAC(L+1)=0.0
      REAC(L+2)=0.0
      REAC(L+3)=0.0
C
      Y2=2.*BY
      X2=2.*DX
      D2=2.*D
      B2=2.*B
      DX1=(Y2/B-1.)/D2+X2/D/D*(1.-Y2/B)+(1.-Y2*Y2/B/B)/D2
      DY1=(X2/D-1.)/B2+(1.-X2*X2/D/D)/B2+Y2/B/B*(1.-X2/D)
      DX2=(Y2/B-1.)*2.*X2/D/D
      DY2=(X2*X2/D/D-1.)/B
      DX3=(1.-Y2/B)/D2+X2/D/D*(1.-Y2/B)-(1.-Y2*Y2/B/B)/D2
      DY3=(1.-X2*X2/D/D)/B2-(1.+X2/D)/B2+Y2/B/B*(1.+X2/D)
      DX4=(1.-Y2*Y2/B/B)/D
      DY4=-(1.+X2/D)*2.*Y2/B/B
      DX5=(1.+Y2/B)/D2-(1.-Y2*Y2/B/B)/D2+X2/D/D*(1.+Y2/B)
      DY5=(1.+X2/D)/B2+Y2/B/B*(1.+X2/D)-(1.-X2*X2/D/D)/B2
      DX6=-(1.+Y2/B)*2.*X2/D/D
      DY6=(1.-X2*X2/D/D)/B
      DX7=X2/D/D*(1.+Y2/B)-(1.+Y2/B)/D2+(1.-Y2*Y2/B/B)/D2
      DY7=(1.-X2/D)/B2-(1.-X2*X2/D/D)/B2+Y2/B/B*(1.-X2/D)
      DX8=-(1.-Y2*Y2/B/B)/D
      DY8=-(1.-X2/D)*2.*Y2/B/B
C
      P=XY*C
C
      SM(1,1)=DX1*C
      SM(1,2)=G*DY1*C
      SM(1,3)=DX2*C
      SM(1,4)=G*DY2*C
      SM(1,5)=DX3*C
      SM(1,6)=G*DY3*C
      SM(1,7)=DX4*C
      SM(1,8)=G*DY4*C
      SM(1,9)=DX5*C
      SM(1,10)=G*DY5*C
      SM(1,11)=DX6*C
      SM(1,12)=G*DY6*C
      SM(1,13)=DX7*C
      SM(1,14)=G*DY7*C
      SM(1,15)=DX8*C
      SM(1,16)=G*DY8*C
C
      SM(2,1)=G*DX1*C
      SM(2,2)=DY1*C
      SM(2,3)=DX2*G*C
      SM(2,4)=DY2*C
      SM(2,5)=G*DX3*C
      SM(2,6)=DY3*C
      SM(2,7)=DX4*G*C
      SM(2,8)=DY4*C
      SM(2,9)=G*C*DX5
      SM(2,10)=DY5*C
      SM(2,11)=G*C*DX6
      SM(2,12)=DY6*C
      SM(2,13)=DX7*G*C
      SM(2,14)=DY7*C
      SM(2,15)=DX8*G*C
      SM(2,16)=DY8*C
C
      SM(3,1)=P*DY1
      SM(3,2)=P*DX1
      SM(3,3)=P*DY2
      SM(3,4)=P*DX2
      SM(3,5)=P*DY3
      SM(3,6)=P*DX3
      SM(3,7)=P*DY4
      SM(3,8)=P*DX4
      SM(3,9)=P*DY5
      SM(3,10)=P*DX5
      SM(3,11)=P*DY6
```

```
      SM(3,12)=P*DX6
      SM(3,13)=P*DY7
      SM(3,14)=P*DX7
      SM(3,15)=P*DY8
      SM(3,16)=P*DX8
C
      RETURN
      END
```

```
      SUBROUTINE OUTPT(AL,FORC,REAC)
C
      COMMON/CDAT/NCN,NED,IBC,NE3,NN3
      COMMON/CDATA/NRMX,NCMX,NDFEL,NN,NE,NLN,NBN,NDF,NNE,N,MS,E,G
      DIMENSION AL(NRMX),FORC(NE3),REAC(NN3)
C
      WRITE(3,1)
 1    FORMAT(//1X,130('*')//' RESULTS'//' NODAL DISPLACMENTS'/7X,'NODE'
     +,11X,'U',14X,'V')
      DO 10 I=1,NN
      K1=NDF*(I-1)+1
      K2=K1+NDF-1
 10   WRITE(3,2) I,(AL(J),J=K1,K2)
 2    FORMAT(I10,2E15.4)
C
      WRITE(3,4)
 4    FORMAT(//' ELEMENT STRESSES AT MID POINT',/5X,'ELEMENT',8X,'S11',
     +12X,'S22',12X,'S12')
      DO 20 I=1,NE
      K1=3*(I-1)+1
      K2=K1+2
 20   WRITE(3,7) I,(FORC(J),J=K1,K2)
 7    FORMAT(I10,3E15.4)
C
      WRITE(3,6)
 6    FORMAT(//' NODAL AVERAGE STRESSES'/7X,'NODE',9X,'S11',12X,'S22',
     +12X,'S12')
      DO 30 I=1,NN
      K1=3*(I-1)+1
      K2=K1+2
 30   WRITE(3,7) I,(REAC(J),J=K1,K2)
      WRITE(3,5)
 5    FORMAT(//1X,130('*'))
      RETURN
      END
```

APPENDIX B-1

PROGRAM B-1 FINITE ELEMENT PLANE STRESS PROGRAMME

SCOPE The computer programme was used to analyse the 'wall-modules' and 'wall-frame' structure. However, it can be used to analyse any type of plane continuum.

FACILITIES
- i. Concentrated loads at nodal points can be applied.
- ii. Non-zero displacements can be applied to nodes.
- iii. Variable section can be analysed.

OUTPUT
- 1. Printing of input data
- ii. Displacement at nodal points
- iii. Stresses at the nodal points
- iv. Stresses at the centroid of elements

DATA INPUT INSTRUCTIONS: For users convenience, a detail data input instruction with a description of the variable names are given below. The sequence followed indicates the order of data to be input in the programme.

Total no. of data cards (records) = (2+2 NE+NLN+NBN)

1ST RECORD PROBLEM NO. (I2)

Column	Variable Name	Description
1-2	NPROB	Problem number

2ND RECORD PROBLEM DESCRIPTION AND MATERIAL PROPERTIES (4I10,2F10.2)

1-10	NN	Number of nodes
11-20	NE	Number of Elements
21-30	NLN	Number of loaded nodes
31-40	NBN	Number of boundary nodes
41-50	E	Young's Modulus of Elasticity
51-60	G	Poisson's ratio

3RD RECORD ELEMENT NUMBER AND DIMENSIONS OF THE ELEMENT (I10,3F10.3)

1-10	I	Element number
11-20	X	Horizontal dimension of an element
21-30	Y	Vertical dimension of an element
31-40	PROP	Thickness of an element

Repeat for each element (1 record for each element)

(NE+3)TH RECORD ELEMENT CONNECTIVITY(9I4)

1-4	I	Element number
5-8	IC(1)	Node number of node 1
9-12	IC(2)	Node number of node 2
13-16	IC(3)	Node number of node 3
17-20	IC(4)	Node number of node 4
21-24	IC(5)	Node number of node 5
25-28	IC(6)	Node number of node 6
29-32	IC(7)	Node number of node 7
33-36	IC(B)	Node number of node 8

Repeat for each element (1 record for each element)

(2 NE+3)TH RECORD DESCRIPTION OF APPLIED LOAD(I10,2F10.3)

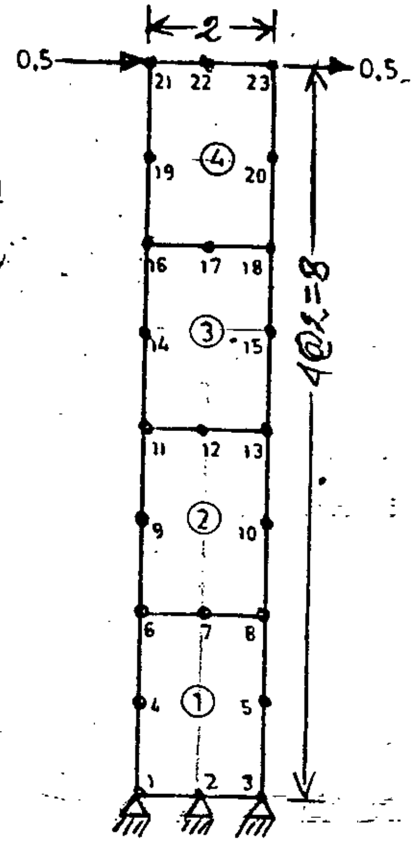
1-10	J	Node number
11-20	W(1)	Magnitude of applied load in X-direction
21-30	W(2)	Magnitude of applied load in Y-direction

Repeat for each load (one card per load)

(2 NE+NLN+3)TH RECORD DESCRIPTION OF BOUNDARY CONDITIONS (3I10,2F10.4)

1-10	J	Node number
11-20	IC(1)	Restraint code for X-direction IC(1)=0 if restrained IC(1)=1 if free
21-30	IC(2)	Restraint code for Y-direction IC(2)=0 if restrained IC(2)=1 if free
31-40	W(1)	Magnitude of applied displacement in X-direction
41-50	W(2)	Magnitude of applied displacement in Y-direction

Repeat for each boundary node (one card per boundary node)



DATA INPUTE INSTRUCTION

// EXEC

PROBLEM NUMBER 1

****

INTERNAL DATA

NUMBER OF NODES = 23
NUMBER OF ELEMENTS = 4
NUMBER OF LOADED NODES = 2
NUMBER OF SUPPORT NODES= 3
MODULUS OF ELASTISITY = 432000.
POISSON-COEFFICIENT = 0.1500

ELEMENT DIMENSION

ELEMENT	X	Y	T
1	1.000	2.000	1.000
2	1.000	2.000	1.000
3	1.000	2.000	1.000
4	1.000	2.000	1.000

ELEMENT CONNECTIVITY

ELEMENT	NODES							
1	1	2	3	5	8	7	6	4
2	6	7	8	10	13	12	11	9
3	11	12	13	15	18	17	16	14
4	16	17	18	20	23	22	21	19

NODAL LOADS

NODES	PX	PY
21	0.500	0.0
23	0.500	0.0

BOUNDARY CONDITION DATA

NODE	STATAS (0 PRECRIBEED, 1 FREE) U	V	PRESCRIBED VALUES U	V
1	0	0	0.0	0.0
2	0	0	0.0	0.0
3	0	0	0.0	0.0

HALF BAND WIDTH= 16

NODAL DISPLACEMENTS

NODE	U	V
1	0.0	0.0
2	0.0	0.0
3	0.0	0.0
4	0.1109E-03	0.1028E-03
5	0.1109E-03	-0.1028E-03
6	0.4205E-03	0.1587E-03
7	0.4162E-03	0.9218E-08
8	0.4205E-03	-0.1587E-03
9	0.8998E-03	0.2746E-03
10	0.8998E-03	-0.2746E-03
11	0.1523E-02	0.3425E-03
12	0.1521E-02	0.1664E-07
13	0.1523E-02	-0.3425E-03
14	0.2261E-02	0.3893E-03
15	0.2261E-02	-0.3892E-03
16	0.3085E-02	0.4283E-03
17	0.3084E-02	0.1821E-07
18	0.3085E-02	-0.4283E-03
19	0.3967E-02	0.4467E-03
20	0.3967E-02	-0.4466E-03
21	0.4877E-02	0.4573E-03
22	0.4377E-02	0.1374E-07
23	0.4377E-02	-0.4573E-03

ELEMENT STRESSES AT MID POINT

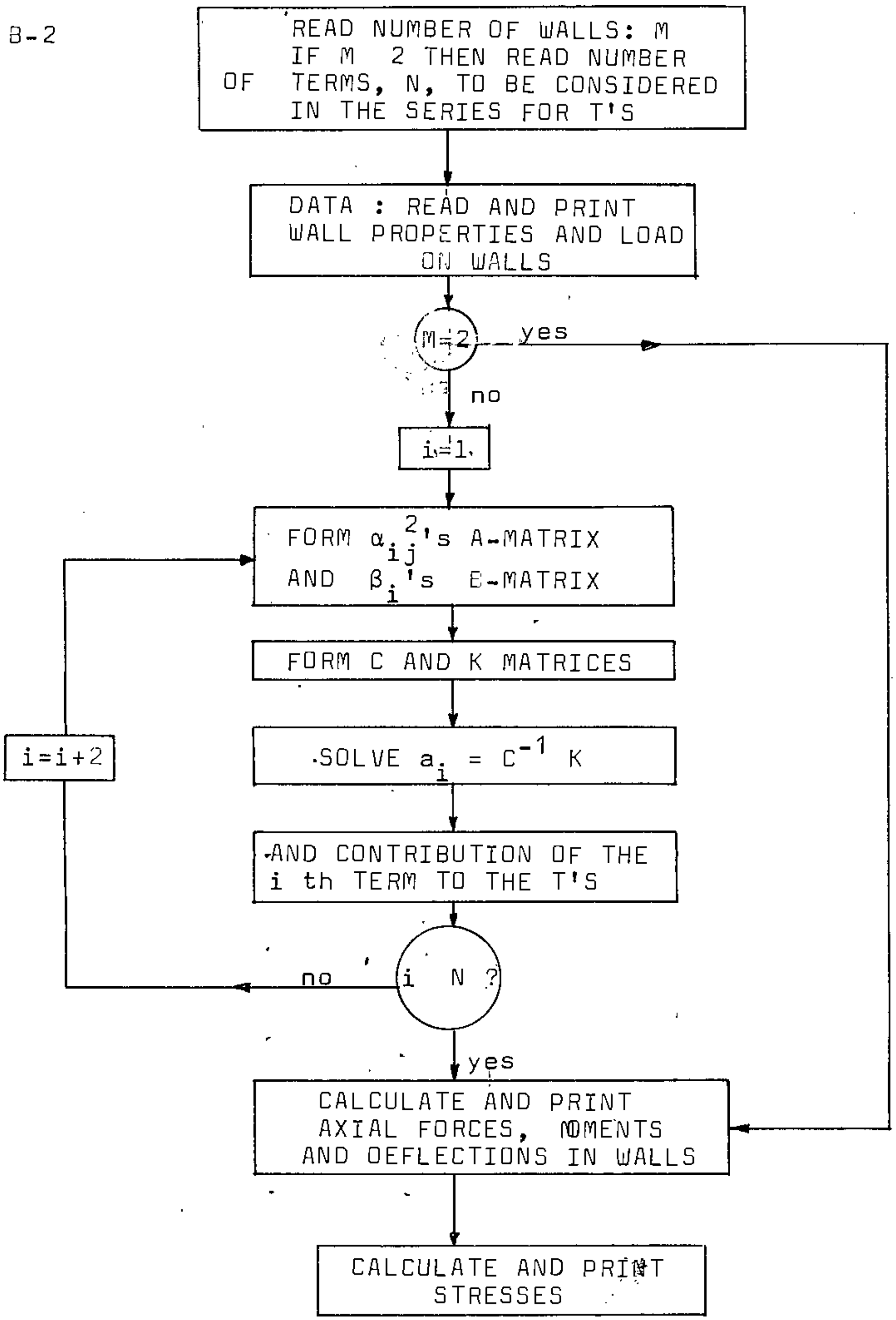
ELEMENT	S11	S22	S12
1	0.6104E-04	0.2000E-02	0.4680E 00
2	0.0	0.1633E-02	0.6086E 00
3	0.2441E-03	0.3967E-03	0.5766E 00
4	0.2930E-02	-0.7324E-03	0.5676E 00

NODAL AVERAGE STRESSES

NODE	S11	S22	S12
1	0.7043E 01	0.4696E 02	0.2167E 01
2	0.3233E-03	0.2167E-02	0.1762E 01
3	-0.7043E 01	-0.4695E 02	0.2166E 01
4	0.2783E 01	0.4334E 02	0.8720E 00
5	-0.2783E 01	-0.4334E 02	0.8723E 00
6	-0.2313E 01	0.3415E 02	0.1878E 01
7	0.9766E-03	0.1801E-02	0.2133E 01
8	0.2315E 01	-0.3414E 02	0.1876E 01
9	-0.7300E 00	0.3095E 02	0.3546E 00
10	0.7302E 00	-0.3094E 02	0.3840E 00
11	-0.3052E-01	0.2187E 02	0.1938E 01
12	-0.7477E-03	0.1663E-02	0.2024E 01
13	0.2872E-01	-0.2186E 02	0.1937E 01
14	0.2810E 00	0.1858E 02	0.4685E 00
15	-0.2800E 00	-0.1858E 02	0.4903E 00
16	-0.2664E 00	0.9531E 01	0.1967E 01
17	0.8774E-02	0.8392E-03	0.2001E 01
18	0.2874E 00	-0.9530E 01	0.1962E 01
19	-0.4382E 00	0.6203E 01	0.5327E 00
20	0.4443E 00	-0.6205E 01	0.5309E 00
21	-0.6049E 00	0.2876E 01	0.1966E 01
22	0.7721E-02	-0.4578E-03	0.2005E 01
23	0.6133E 00	-0.2878E 01	0.1970E 01

****

B-2



FLOW DIAGRAM OF THE COMPUTER PROGRAMME FOR THE ANALYSIS OF A UNIFORM SHEAR WALL WITH OPENINGS (USING CONTINUOUS MEDIUM METHOD)

```
      DIMENSION DW(7), B(6), DB(6 ),H(6 ),XL(6 ),XI(7 ),IP(6 ),IC(6 ),
     .ALPHA(6,6),   BETA(6 ),T(6 ),C(6,7),D(6,15), BK(6),A(7),CK(6,7),
     .F(7),BM(7),SL(7),SR(7),XX(50),BMW(50,7),TI(50,6),FF(50,7)
     +,YS(6),YL(50)
      READ(1,4) TW,TB,H1,E,NU,W,M
4     FORMAT(6F10.3,I2)
      N=M-1
      READ(1,5) (B(I),DB(I),H(I),I=1,N)
5     FORMAT(3F10.4)
      READ(1,7) (DW(I),I=1,M)
7     FORMAT(7F10.3)
C
      WRITE(3,4) TW,TB,H1,E,NU,W,M
      WRITE(3,5) (B(I),DB(I),H(I),I=1,N)
      WRITE(3,7) (DW(I),I=1,M)
C
      XIT=0
      DO 11 I=1,M
      A(I)=TW*DW(I)
      XI(I)=TE*DW(I)**3/12
      XIT=XIT+XI(I)
11    CONTINUE
      DO 9 I=1,N
      IP(I)=TE*DB(I)**3/12
      IC(I)=IP(I)/(1+2.4*(1+NU)*(DB(I)/B(I))**2)
      XL(I)=DW(I)/2+B(I)+DW(I+1)/2
C     BETA(I)=12*IC(I)/B(I)**3/H(I)*W/XIT*XL(I)/2
      BETA(I)=12*IC(I)/B(I)**3/H(I)*W/XIT*XL(I)
9     CONTINUE
      DO 8 I=1,N
      DO 8 J=1,N
      IF (I.NE.J) GO TO 22
      ALPHA(I,J)=12*IC(I)/H(I)/B(I)**3*(XL(I)**2/XIT+1/A(I)+1/A(I+1))
      GO TO 8
22    IF (J.EQ.(I-1)) GO TO 33
      IF (J.EQ.(I+1)) GO TO 44
      ALPHA(I,J)=12*IC(I)/H(I)/B(I)**3*XL(I)*XL(J)/XIT
      GO TO 8
33    ALPHA(I,J)=12*IC(I)/H(I)/B(I)**3*(XL(I)*XL(I-1)/XIT-1/A(I))
      GO TO 8
44    ALPHA(I,J)=12*IC(I)/H(I)/B(I)**3*(XL(I)*XL(I+1)/XIT-1/A(I+1))
8     CONTINUE
C
      PI=3.14159
      DO 100 KK=1,15
      K=KK*2-1.
      R=PI**2/4/H1**2
      DO 99 I=1,N
      DO 98 J=1,N
      IF (J.NE.I) GO TO 97
      C(I,J)=ALPHA(I,J)+R*K**2
      GO TO 98
97    C(I,J)=ALPHA(I,J)
98    CONTINUE
C     BK(I)=BETA(I)*16*H1**2/PI**3/K**3*(PI*K*SIN(K*PI/2)-2)
      BK(I)=BETA(I)*8*H1/PI/PI/K/K/SIN(PI*K/2)
      C(I,M)=BK(I)
 99   CONTINUE
C
      IF (N-1) 111,110,111
 110  D(1,KK)=C(1,2)/C(1,1)
      GO TO 100
C
 111  CALL SOLVE (C ,D,KK,N,M)
 100  CONTINUE
      DO 1700 L=1,21
      XX(L)=(L-1)*H(1)/2.
      X=XX(L)
      HT=H1-X
      DO 500 I=1,N
      YS(I)=0
      T(I)=0
      DO 700 KK=1,15
      K=KK*2-1.
      YS(I)=YS(I)+D(I,KK)*((SIN(K*PI/2)-SIN(K*PI*X/2/HI))/K**2)
      T(I)=T(I)+D(I,KK)*SIN(K*PI*X/HI/2.)
 700  CONTINUE
```

```
      YY=0
      DO 1760 I=1,N
      YY=YY+YS(I)*XL(I)
 1760 CONTINUE
C     Y=(W*(X**4/12-H1**3*X/3+H1**4/4)-4*H1**2/PI**2*YY)/E/XIT/144.
      Y=(W*(X**3/6-H1*H1*X/2+H1**3/3)-4*H1*H1/PI/PI*YY)/E/XIT
      YL(L)=Y
C
      DO 555 I=1,M
      IF(I.GT.N) GO TO 777
      IF(I.GT.1) GO TO 666
      F(I)=T(I)
      GO TO 555
  666 F(I)=T(I)-T(I-1)
      GO TO 555
  777 F(I)=-T(I-1)
  555 CONTINUE
C
      TSL=0
      DO 1000 I=1,N
      TSL=TSL+T(I)*XL(I)
 1000 CONTINUE
      DO 1100 I=1,M
C     BM(I)=(W*X**2/2-TSL)*XI(I)/XIT
      BM(I)=(W*X-TSL)*XI(I)/XIT
 1100 CONTINUE
C
      WRITE(3,2000) X,Y,(T(I),I=1,N)
      WRITE(3,2001) (F(I),I=1,M)
      WRITE(3,2001) (BM(I),I=1,M)
 2000 FORMAT(//,F10.4,7E15.5)
 2001 FORMAT(10X,7E15.5)
 1700 CONTINUE
      STOP
      END
```

```
      DIMENSION  C(6,7),D(6,15)
      DO 301 K2 = 1,N
      K3=K2+1
      DO 301 J = K3,M
      C(K2,J)=C(K2,J)/C(K2,K2)
      DO 301 I = 1,N
      IF (K2-I)302,301,302
302   C(I,J)=C(I,J)-C(I,K2)*C(K2,J)
301   CONTINUE
      DO 304 I=1,N
      D(I,K1)=C(I,M)
 304  CONTINUE
      RETURN
      END
```

PROGRAM B-2 CONTINUOUS MEDIUM METHOD

SCOPE: The computer programme was used to analyse the wall-frame structure. However it can be used to analyse any type of plane shear wall.

FACILITIES:
i. Concentrated load at the top can be applied
ii. Uniformly distributed load can be applied

OUTPUT:
i. Printing of input data
ii. Displacement at any height
iii. Axial force at the wall at any height
iv. Bending moment of the wall

DATA INPUT INSTRUCTIONS:

1ST RECORD(6F10.3,I2)

Column	Variable name	Description
1-10	TW	Thickness of wall
11-20	TB	Width of beam
21-30	H1	Total height of the structure
31-40	E	Young's Modulus of Elasticity
41-50	NU	Poisson's ratio
51-60	W	Magnitude of applied load
61-62	M	Number of wall

2ND RECORD (3F10.4)

1-10	B	Length of beam
11-20	DB	Depth of beam
21-30	H	Storey height

(One card per opening)

3RD RECORD(7F10.3)

1-10	DW	Width of wall-1
11-21	DW	Width of wall-2
21-30	DW	Width of wall-3
31-40	DW	Width of wall-4
41-60	DW	Width of wall-5
51-60	DW	Width of wall-6
61-70	DW	Width of wall-7