A SIMPLE DESIGN APPROACH FOR HELICOIDAL STAIR SLABS

A THESIS BY

ZIA WADUD

Submitted in partial fulfillment of the requirements for the degree of

Master of Science in Civil & Structural Engineering

DEPARTMENT OF CIVIL ENGINEERING
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

June 2002

A SIMPLE DESIGN APPROACH FOR HELICOIDAL STAIR SLABS

A THESIS BY

ZIA WADUD

Approved as to the style and content on ______________

Dr. Sohrabuddin Ahmad — Chairman
Professor
Department of Civil Engineering
BUET, Dhaka

Dr. M. Shamim Z. Bosunia — Member
Professor
Department of Civil Engineering
BUET, Dhaka

Dr. Md. Abdur Rouf — Member
Professor and Head
Department of Civil Engineering
BUET, Dhaka

Dr. A. F. M Saiful Amin — Member
Assistant Professor
Department of Civil Engineering
BUET, Dhaka

Dr. Md. Wahhaj Uddin — Member
Professor
Department of Mechanical Engineering
BUET, Dhaka

DECLARATION

It is hereby declared that, except where specific references are made to other investigators, the work embodied in this thesis is the result of investigation carried out by the author under the supervision of Dr. Sohrabuddin Ahmad, Professor of Civil Engineering, BUET.

Neither this thesis, nor any part of it, has been or is being concurrently submitted to any other institution for any degree.

(Zia Wadud)

ACKNOWLEDGEMENTS

The author feels extremely privileged to work under his most revered teacher, Dr. Sohrabuddin Ahmad, Professor, Department of Civil Engineering, BUET. He is delighted to have the opportunity to express his gratitude to Dr. Ahmad, for his overall supervision, invaluable suggestions and ardent encouragement in every aspect of this work. Prof. Ahmad's keen interest and encouragement helped the author understand the subject that he is presenting now. He is also thankful to his supervisor for all the time, he has graciously provided.

The author is particularly grateful to Dr. A. F. M. Saiful Amin, Assistant Professor, Department of Civil Engineering, BUET, for his benevolent cooperation. He has always guided the author through his thoughtful suggestions. Heartiest thanks go to Ms. Charisma F. Choudhury, Lecturer of Civil Engineering, BUET, for her cooperation during the preparation of the dissertation.

Last, but not the least, the author is thankful to his mother and his sister, who have always been beside him through thick and thin.

Table of Contents

Title			Page

ABSTRACT

Stair is an important functional element of a building. Presently, helicoidal stairs are gaining popularity because of their attractive appearance. However, design of the helicoidal stair is quite difficult as the exact method of its analysis is very cumbersome. Due to the complex geometric configuration of this structure, the present methods of analysis are based on various idealizations and assumptions. Under this background, finite element approach has been applied to study the validity of the current methods in use. The study has been extended further to determine the stress resultants of the helicoidal stair slab including an intermediate landing for the development of a simplified design process.

The investigation has lead to a number of findings. Firstly, the existing methods of helicoidal stair slab analysis have been found to vary a little with the finite element analysis. The possibility of suggesting temperature and shrinkage steel for the design of the RCC helicoidal slab has been explored. The study resulted in a direct design approach to suggest the steel area based on geometric parameters. It is expected that the use of such direct design charts will gain popularity among the designers because of its ease of use.

In addition, behaviour of helicoidal stair slabs with landing has been investigated with a view to proposing a design chart. Strain energy method has been used to analyse the structure. Behaviour of the stair with a landing could be generally outlined as the outcome of the analysis. A simple design approach has been suggested in the end. It is important to note that no design charts are currently available for the analysis of helicoidal stair slabs with landing and the charts developed as a part of the proposed design method will be immensely helpful to the designers. Also, a parametric study with a limited scope has been carried out. The study suggests that the behaviour of the stair with and without landing is similar, with the effect of landing being prominent locally, in the vicinity of the landing. The maximums of all stress resultants, excepting lateral shear, show some variations because of the introduction of an intermediate landing. The effect of landing is most prominent in torsion. The deflection of a helicodial stair with landing has been found to be larger than that without a landing.

NOTATIONS

θ'	=	Angle measured from x-axis towards y-axis on a horizontal plane
θ	=	Angular distance from mid span (Fig. 3.4)
ν	=	Poisson's ratio
α	=	Slope of the helix contained within the helicoid at radius R (Fig. 2.2)
γ	=	Unit weight of concrete
ϕ	=	Angle subtended at the centre by half landing (Fig. 6.1)
δ_{rr}	=	Relative angular displacement about x-axis due to X_r=1
δ_{rw}	=	Relative angular displacement of the two ends of the girder at the mid span cut about x-axis due to a uniform load of 1 lb. per linear foot of horizontal projection with the redundants equal to zero
δ_{rx}	=	Relative angular displacement about x-axis due X_x=1
δ_{xr}	=	Relative linear displacement in the direction of x-axis due to X_r=1
δ_{xw}	=	Relative linear displacements of the two ends of the girder at the mid span in the direction of x-axis due to a uniform load of 1 lb. per linear foot of horizontal projection with the redundants equal to zero
δ_{xx}	=	Relative linear displacement at the same location in the direction of x-axis due to X_x=1
2β	=	Total central angle subtended on horizontal projection (Fig. 3.4)
b	=	Width of the stair slab
E	=	Young's modulus
e	=	Eccentricity of loading with respect to the girder centreline
EI	=	Flexural rigidity
EI_r	=	Flexural stiffness about r-axis
EI_s	=	Flexural stiffness about s-axis
F	=	Radial horizontal shear force
H	=	Radial horizontal shear force at mid span (redundant)
f'_c	=	28-day compressive strength of concrete
GJ	=	Torsional rigidity
Ht	=	Height of the helicoid
h	=	Waist thickness of stair slab
K	=	Ratio of flexural to torsional rigidity

M_v	=	Vertical moment
$\dot{M}$	=	Vertical moment at mid span (redundant)
M_h	=	Lateral moment
M_{sup}	=	Vertical moment at support
N	=	Thrust
R_2, R	=	Centreline radii on horizontal projection
R_1	=	Radius of centreline of load
Ri	=	Inner radius on horizontal projection
R_o	=	Outer radius on horizontal projection
T	=	Torsion
V	=	Lateral shear force
w	=	Dead load and live load per unit length of span, measured along the longitudinal centreline of the plan projection
X_r	=	A moment about x-axis at the mid span section
X_x	=	A horizontal force in the direction of x-axis at the mid span section

CHAPTER 1

INTRODUCTION

1.1 GENERAL

One of the most important functional elements of a building, be it residential or commercial, high or low rise, is its stair. It is a series of steps connecting adjacent floors of a building for transportation of men and goods from floor to floor. At the time of an emergency like an earthquake or a fire accident the stair loading becomes maximum. At the peak hour in a commercial building, business centre or market place, a stair plays a vital role. In a high rise building, a stair appears to be substituted significantly by the elevators, but during emergency, the stair is the only option for transports between floors in these buildings as well.

A stair is not only important from functional point of view but it provides a wide scope for the use of architect's creativity in this field. Depending on the architectural forms, there may be different types of stairs, such as (Fig. 1.1):

(i) Simple straight stair,
(ii) Saw-tooth/slabless stair,
(iii) Free standing stair, and
(iv) Helicoidal stair.

Among these types, the helicoidal stair has a grand and fascinating appearance from architectural point of view. For this reason, helicoidal stair slabs are increasingly used in many important buildings in Bangladesh and other countries of the world. This attractive structure can also be visualised as being a circular bow girder with one end displaced vertically out of plane of the other (Fig. 1.2).

Compared with other structural components of a building, stairs have some unique characteristics. Stairs are an assemblage of interconnected plates in a space

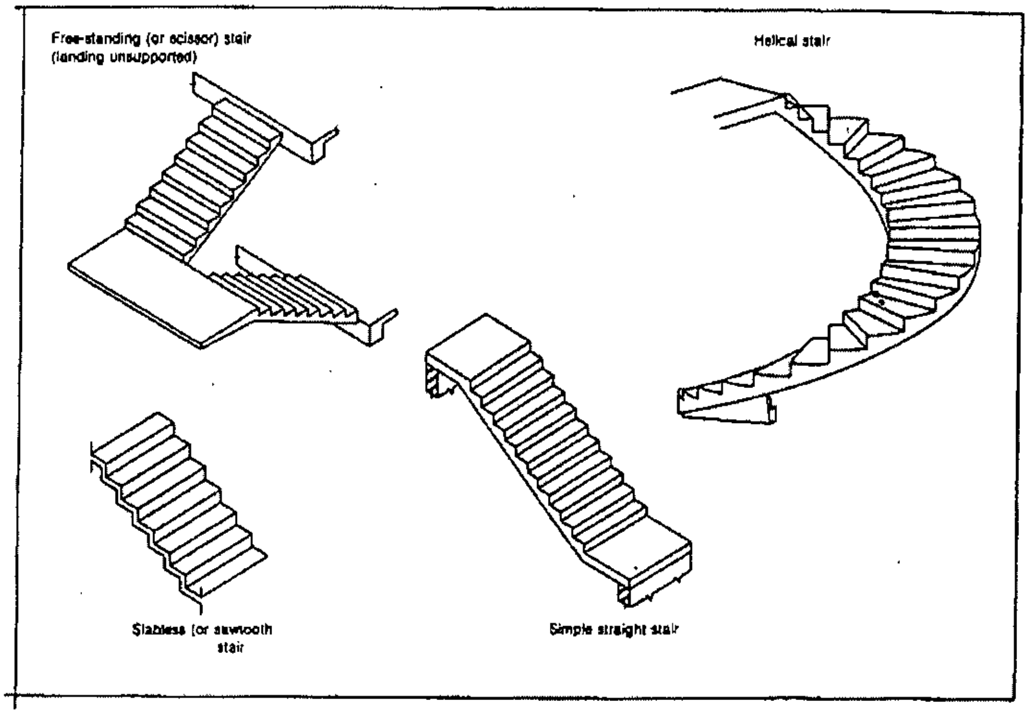


Fig. 1.1 Stairs of Different Types

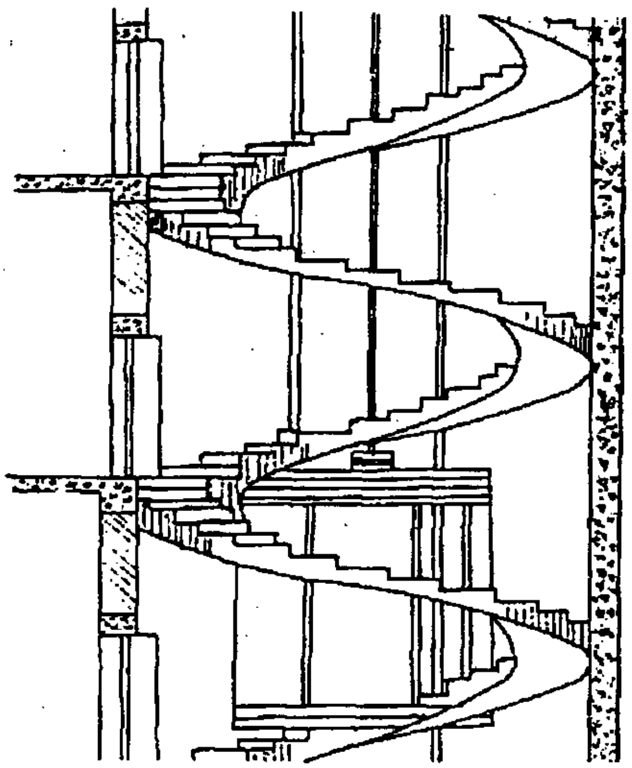


Fig. 1.2 Typical Sectional View of Helicoidal Stair Slab

space and are generally supported at the edges of the plates. Both, in-plane and out-of-plane forces may be present in the stair depending upon the arrangement of the supports and type of stairs. However, due to the complex geometrical configuration, the analysis and design of helicoidal stair slabs are more difficult than simple type of stairs. Therefore, the approaches are based on different idealisations and assumptions, which results in approximate and conservative design. They also fail to take full advantage of the beneficial structural behaviour of helicoids.

So in the practical field, increasing use of helicoidal stair slabs has necessitated the development of a thorough and 'exact' analysis and a simple design procedure for this type of stairs.

1.2 BACKGROUND OF RESEARCH

History reveals that the first helicoidal stair slab was constructed in 1908. It was designed as an open coiled helical spring, primarily as a torsion member. However, further research has indicated that this type of structure also carries bending, direct and shear forces besides torsion.

Helicoidal stair slabs, so far, have been analysed and designed on the basis of two different basic approaches.

In one approach, Bergman (1956) and Engles (1955) considered it as a fixed ended curved beam. In this approach, the simplest solution is produced by reducing the helicoid to its horizontal projection and resolving the problem into that of a fixed ended curved beam. Thus the structure is idealised as a two dimensional structure.

In the second approach, Fuchssteiner (1954), Gedizli (1955), Holmes (1957), Scordelis (1960), Morgan (1960) and Cohen (1964) considered the helicoid as a three-dimensional helical girder. Here, the helicoid is reduced to its elastic line having the same stiffness as that of the original structure. But this simplification neglects the slab action of helicoid and also assumes that the bending stiffness and torsional stiffness of a warped girder are the same as those of a straight beam.

A comparative assessment of the two approaches showed that the curved beam solution leads to a very conservative estimation of forces.

The efforts on the development of an 'exact' procedure for the analysis of helicoidal stair reached its culmination through the works of Santathadaporn and Cusens (1966), where the stair was assumed as a helical girder. The work presented thirty-six design charts for helical stairs with a wide range of geometric parameters. Based on this work, four design charts were compiled in somewhat modified form in the current design hand book by Reynolds *et. al* (1988). These design charts now stand as 'helical girder solution' for helicoidal stairs.

Both the curved beam and the helical girder solution fail to take into account the three dimensional characteristics of helicoid and its inherent structural efficiency. With a view to developing an 'exact' and general solution, Menn (1956), outlined an analytical method of solving helicoidal shell problems including edge perturbations or edge conditions. It was observed that the analysis of a helicoidal shell for certain boundary conditions is possible through highly complex mathematical calculations. Menn realised the fact and concluded finally to go for 'girder solution'.

An exact analysis of the problem by directly solving the differential equations of equilibrium and compatibility in accordance with the theory of elasticity is beyond the scope of existing rigorous mathematical methods, which is why this approach was discarded by Menn. The only alternative to this is to employ numerical methods. Among the available numerical methods, finite element approach is the most powerful one. The development of different curved shell elements in the field of finite element techniques and the availability of high speed digital computers at design engineers' desk have ushered in a new hope for the shell solution of this problem in a more logical and convenient way.

In this context, Modak (1991) adopted the "General Thick Shell Finite Element Program" of Ahmad (1969) to investigate the behaviour of fixed ended helicoidal stair slab under uniformly distributed loads. This investigation paved the way for developing a design rationale for helicoidal stair slabs. Morshed (1993) worked further on this problem to get more reliable and logical values of shear forces and torsion using gauss point stresses with necessary modification in the program. Amin (1998) worked further on it and carried out an extensive parametric study of the geometric parameters governing the design forces of the helicoidal stair slab. He also performed a comparative study of different methods through the analysis of a prototype stair employing different methods currently available in the literature for the design of helicoidal stairs.

Through the efforts at BUET, it was possible to model the behaviour of helicoidal stair slabs without any geometric idealisation. Recently, Choudhury (2002) has worked to develop a design guideline, based on finite element analysis. However no efforts have been made to study the behaviour of helicoidal slabs with an intermediate landing.

1.3 OBJECTIVES

The methods of structural analysis of helicoidal stairs available in the literatures are based on different idealisations and assumptions. These methods fail to consider the three-dimensional characteristics of helicoid and its inherent structural efficiency. In contrast, the recent advances at BUET have indicated that the introduction of thick shell finite elements can successfully tackle this problem more rationally. However, these developments were not convenient enough for practical application. The possibility of incorporating an intermediate landing was still waiting. In this context, the main objective of the present investigation is to cover the following aspects.

I. To study the behaviour of the fixed ended helicoidal stair slab using finite element approach, without any geometric idealisation.

II. To study the structural behaviour of the helicoidal stair slab over an extensive variation of parameters with the aim to extend Amin's studies.

III. To develop a simple but general design approach for the fixed ended helicoidal stair slabs with the least possible computation.

IV. To study the behaviour of helicoidal stair slabs with intermediate landing.

V. To develop a simplified design approach for helicoidal stair slabs with intermediate landing. No design charts are presently available for such cases and it is expected that a simple design approach can efficiently work as a guideline for designing helicoidal stair slabs with intermediate landings.

1.4 METHODOLOGY

A thorough survey of related literatures has been made to understand the current methods available to design the helicoidal stairs. It has also covered the recent studies made in this field at BUET using finite element method.

To begin with, the helicoidal stair slab has been analysed using both finite element method and helical girder method. The comparison generally justifies the existing methods for analysing helicoidal stair slab.

Because of the low steel requirement in the helicoidal stair slabs, often temperature and shrinkage reinforcement may govern the design at critical locations. Therefore, the possibility of suggesting a steel ratio for different dimensions of the helicoidal stair has been explored.

Next, the helicoidal stair slab with an intermediate landing has been analysed through the strain energy method. The behaviour of the stair slab with landing has led to the suggestion of a design chart.

Then, the helicoidal stair slab with intermediate landing has been modelled by the finite element method to compare deflection of stair slabs with and without landing.

At the end, guidelines for future research have been suggested.

1.5 SCOPE OF RESEARCH

This work is kept limited to:

- The helicoidal stair slabs with fixed boundaries at top and bottom only.
- Live load is assumed to be uniformly distributed. No influence line analysis has been carried out.
- Only one landing at the mid span is considered in both the classical and the finite element approach.

- The length of the landing is kept constant at 0.2 times the span of the helicodial slab, in the finite element analysis. However, the classical approach is capable of providing a solution for any length of the landing.

CHAPTER 2

HELICOIDAL STAIR SLAB PARAMETERS

2.1 INTRODUCTION

This chapter deals with the geometry of helicoid, its axes systems with sign conventions, loading and boundary condition along with stress resultants of the helicoidal stair slab in present study.

2.2 GEOMETRY OF A HELICOIDAL SURFACE

Geometrically, a helicoidal surface is a warped surface generated by moving a straight line touching a helix so that the moving line is always perpendicular to the axis of the helix. Although, in most cases, the generating line intersects the axis of the helix, the definition does not require it. In an oblique helicoid, the generating line always maintains a fixed angle with the helix. A helicoid may be right handed or left handed (Fig. 2.1). Mid surface of the helicoidal stair slab can be defined by the following equations :

$$x = R \cos \theta' \qquad (2.1)$$

$$y = R \sin \theta' \qquad (2.2)$$

$$z = \theta' R \tan \alpha \qquad (2.3)$$

where, $Ri \le R \le R_O$ and $0 \le \theta' \le 2\beta$

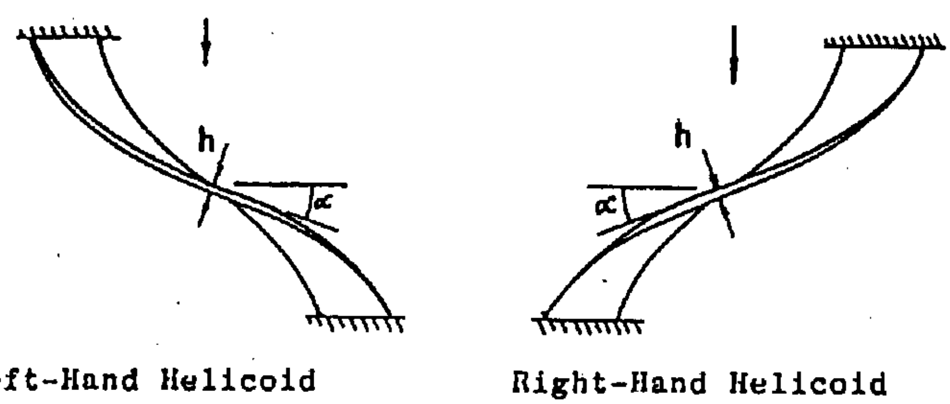


Fig. 2.1 Right handed and left handed helicoid

2.3 COORDINATE SYSTEM AND SIGN CONVENTION

Two types of co-ordinate system can be used in the analysis of helicoidal stair slab, namely:

- Global coordinate system
- Local coordinate system

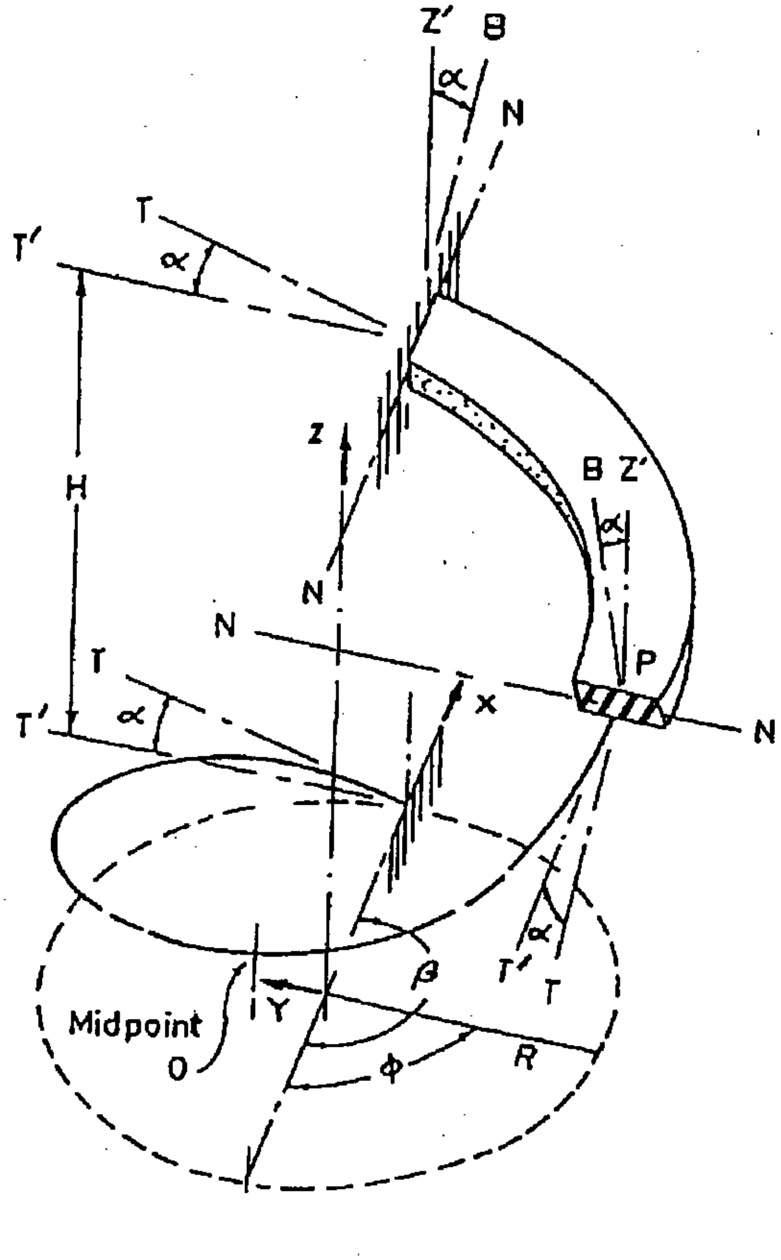


Fig. 2.2 Perspective sketch of helical beam

2.3.1 Global Coordinate System

In representing the mid surface of helicoid (Eqs. 2.1-2.3) the following global axes system is adopted:

X axis – radial and horizontal along the bottom boundary

Y axis - perpendicular to X axis on horizontal projection

Z axis - perpendicular to XY plane following right hand screw rule and along the axis of revolution of the helix as well as parallel to the direction of gravitational loading.

The helicoidal stair slab has a definite thickness. If the uniform thickness perpendicular to the helicoidal surface is h, then co-ordinates of points on top and bottom surface of the helicoidal stair slab can be found out easily from trigonometry and can be represented as follows:

$$X_{top} = R\cos\theta' + h\sin\alpha\sin\theta'/2 \qquad 2.4$$

$$X_{bot} = R\cos\theta' - h\sin\alpha\sin\theta'/2 \qquad 2.5$$

$$Y_{top} = R\sin\theta' - h\sin\alpha\cos\theta'/2 \qquad 2.6$$

$$X_{bot} = R\sin\theta' + h\sin\alpha\cos\theta'/2 \qquad 2.7$$

$$Z_{top} = H\,\theta'/2\beta + h\cos\alpha/2 \qquad 2.8$$

$$Z_{bot} = H\theta'/2\beta - h\cos\alpha/2 \qquad 2.9$$

These equations are valid when generator line is always parallel to XY plane.

2.3.2 Local Coordinate System

Local coordinate system is shown in Fig. 2.3. Displacements and stresses are defined with respect to the global axes system. However, the stresses are subsequently transferred to local axes system. Stress resultants are defined with respect to the local axes system. And these stress resultants with respect to the local axes system are the governing parameter in the design of the stair slab. The relationship between the two types of coordinate systems and their positive directions are shown in Fig. 2.4.

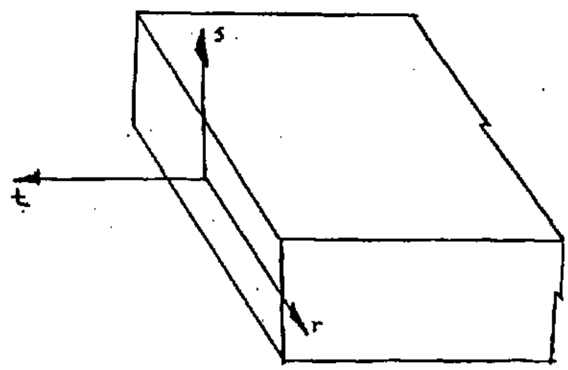


Fig. 2.3 Local axes system

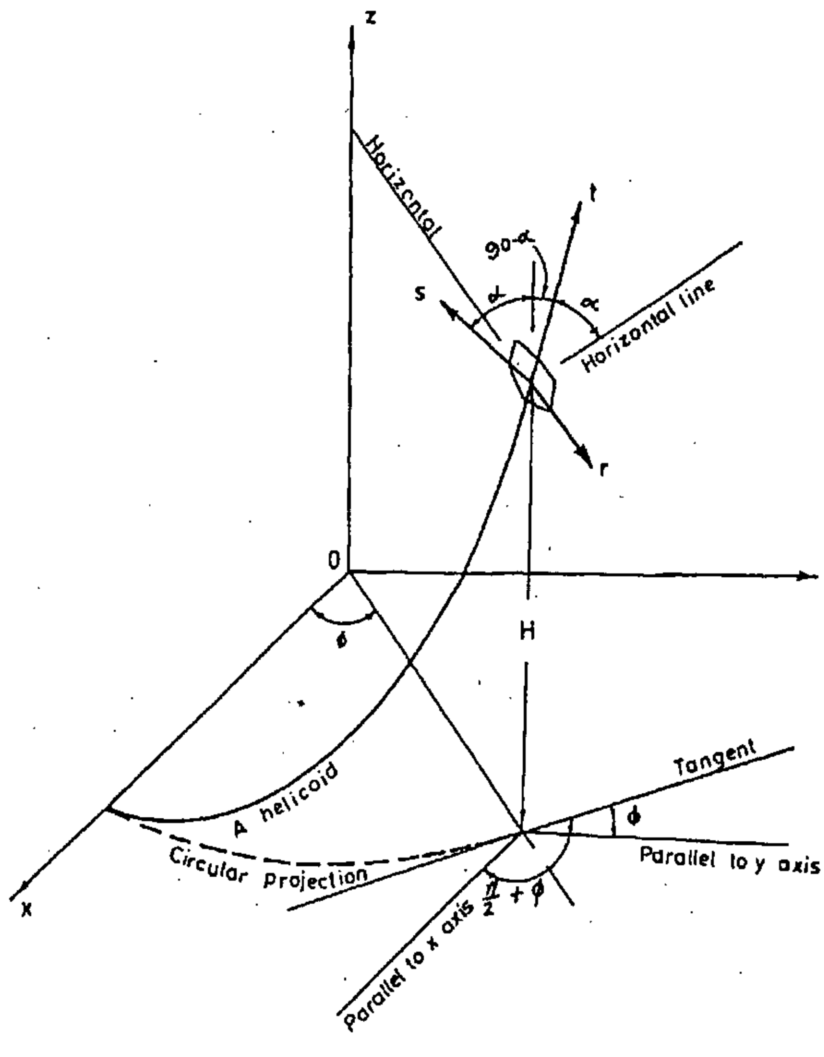


Fig. 2.4 Local and global coordinates and relationship between them

2.4 RELATIONSHIP BETWEEN GLOBAL AND LOCAL COORDINATE SYSTEMS

Relationship between the local and global axes system can be interpreted in terms of a direction cosine matrix (a_{ij})

$$a_{ij} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad 2.10$$

where,

(a_{11}, a_{12}, a_{13}) = direction cosines of t axis with respect to (w.r.t) X,Y,Z axes respectively

(a_{21}, a_{22}, a_{23}) = direction cosines of r axis w.r.t. X,Y,Z axes respectively

(a_{31}, a_{32}, a_{33}) = direction cosines of s axis w.r.t. X,Y,Z axes respectively

$$a_{ij} = \begin{vmatrix} -\sin\phi\cos\alpha & \cos\phi\cos\alpha & \sin\alpha \\ -\cos\phi & -\sin\phi & 0 \\ \sin\phi\cos\alpha & \cos\phi\sin\alpha & \cos\alpha \end{vmatrix} \qquad 2.11$$

Any displacement or force in the positive direction of axis is considered to be positive and any moment or rotation having the vector in positive direction of axis is taken to be positive.

2.5 LOADING AND BOUNDARY CONDITION

The helicoidal stair slab has its self weight. This dead load (self weight) is assumed to be uniformly distributed. In addition, the slab is subjected to live load. The live load could be uniformly distributed over the surface or point loads or line loads or symmetrical loads about the central axis of the slab. However, the live load is also considered to be uniformly distributed over the entire span and width of stair. Thus in the present study, the helicoidal stair slab is assumed to be subjected to a surface load distributed uniformly over the entire horizontal projection.

The ends of the slab may be fixed, partially fixed or hinged. The slab fixed at both ends is six degree indeterminate; there are six equilibrium equations and twelve unknown reactions. Helical slab with one end fixed and one end hinged is indeterminate of third degree. Three reaction moments at the hinged support are equal to zero.

In the current analysis, helicoidal stair slab, fixed at its ends in all directions, has been considered.

2.6 STRESS RESULTANTS

Generally, six stress resultants are available in any section of a space structure. Helicoidal staircase, being a space structure is no exception. The six stress resultants as found in any cross section of a helicoidal slab are:

- Vertical moment (M_v),
- Lateral moment (M_h),
- Torsion (T),
- Thrust (N),
- Lateral shear force (V), and
- Radial horizontal shear force (F).

The positive directions of these stress resultants have been illustrated in Fig. 2.5.

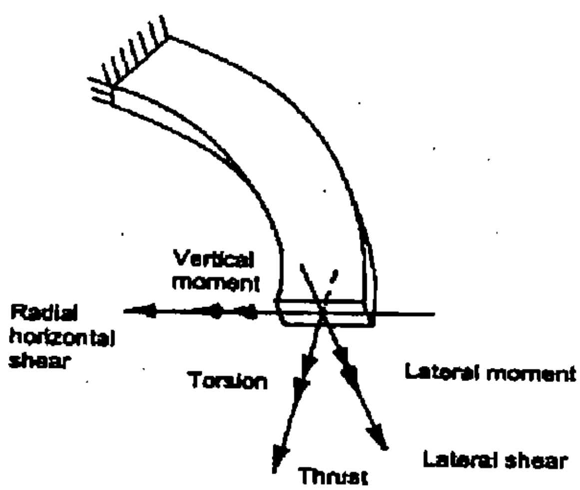


Fig. 2.5 Stress Resultants

CHAPTER 3

LITERATURE REVIEW

3.1 INTRODUCTION

At the beginning of this century, helicoidal stair slab received attention of a number of researchers. They worked on this problem of developing a simple procedure for analysing and designing of this attractive structure in a logical and rational way.

The analysis of a helicoidal stair slab involves torsional moments as well as flexural moments and shears. The analysis is consequently more difficult than that of a straight stair slab.

The analyses of helicoidal stair done were so far based on two basic approaches. In the first approach, the stair was considered as a fixed ended curved beam in a horizontal plane. In the second approach, it was considered as a helical girder in space. Both approaches were approximate and failed to utilise the inherent structural efficiency of the helicoid and thereby lead to a conservative design. However, with a view to obtaining a more realistic analysis, a shell solution was obtained. But due to its extreme mathematical complexity, this approach failed to be popular.

In the subsequent articles, some of these analytical approaches have been critically looked at along with their special assumptions. Afterward, the efforts of different researchers to substantiate the analytical findings through experimental studies have been presented.

Another stream of efforts led to the development of design charts based on 'helical girder approach'. The outcome of some of these efforts has been incorporated in the recently published design handbooks, i.e., Reynolds (1988). An effort on the formation of stiffness matrix with a view to opening up the possibility of getting numerical solution of this structure has been recorded in the literature.

At the end, the recent advancements at BUET in the analysis and development of a design procedure for helicoidal stair slabs have been recorded.

3.2 ASSUMPTIONS IN DIFFERENT APPROACHES

In any analysis, idealisation is an essential part. The actual structure can never be analysed without some simplifications. The accuracy of the analysis depends on the accuracy of the idealisations and on how closely the actual structure represents the ideal structure.

In both the approaches of analysis of helicoidal stairs, the following assumptions are made:

i. The material is linearly elastic and homogeneous.
ii. The bending and torsional stiffness of a helicoidal surface is defined by the straight prismatic member, as if the helicoidal surface was a prismatic bar.
iii. The unit load is uniformly distributed over the width of the girder.
iv. The structure is studied neglecting any slab effect.
v. The cross-section is considered to be symmetrical about two principal axes.
vi. Deformations due to shear and direct forces are negligible since these are small compared to the deformations caused by twisting and bending moments.

3.3 ANALYSIS OF HELICOIDAL GIRDER AS A PLANE CURVED BEAM

3.3.1 Bergman's Approach

Victor R. Bergman (1956) was one of the pioneers in analysing helicoidal stair slabs and in proposing design methods and charts to facilitate the design. In his approach, Bergman reduced the problem to a fixed ended circular bow girder/ring beam on the horizontal projection, that is, he assumed the space structure of helicoid to be projected on a horizontal plane as a curved beam (Fig. 3.1). The structure is thus simplified into a planar one. This idealised structure is then assumed to be loaded symmetrically under uniformly distributed load over entire span normal to the plane of curvature.

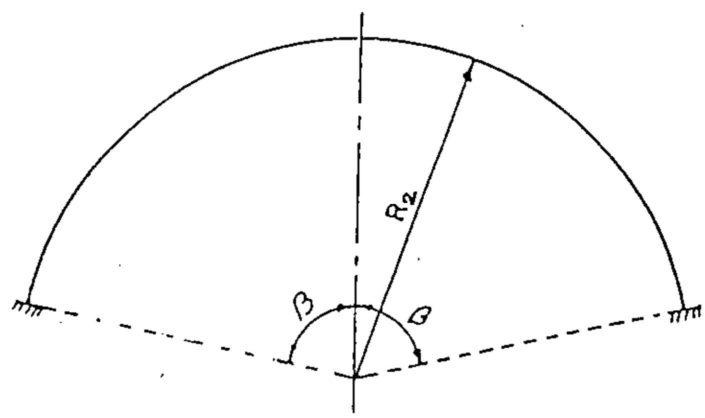


Fig. 3.1 Bergman's idealisation

At any cross-section of this curved beam, there exist, in general, a bending moment, a twisting moment and a lateral shear. The customary design condition of uniform loading over the entire span, symmetry of the loading and structure as well, dictate that torsion and lateral shear cannot exist at the mid span cross-section of the simplified structure. Therefore only bending moment at mid span remains to be determined analytically. This makes the rest of the structure statically determinate as far as the calculation of bending and torsional moments and shears is concerned. Bergman applied the principle of least work to a fixed ended, curved beam of a constant centerline radius R_2 to determine the expression for the mid span bending moment, M_v:

$$M_v = w R_2^2 (U-1) \qquad 3.1$$

$$U = f_1 (K, \beta) = \{2(K+1)\sin\beta - 2K\beta\cos\beta\}/\{(K+1) - (K-1)\sin\beta \cos\beta\} \qquad 3.2$$

$$K = f_2 (\nu, b, h) = EI/GJ \qquad 3.3$$

For a particular slab cross section, Bergman proposed the value of K. He also provided a chart which gives the value of U using half-central angle β and K as the inputs. Thus midspan bending moment M can easily be obtained.

The vertical moment (M_v), torsion (T) and lateral shear force (V) at any cross section located at an angular distance θ from the mid span can be obtained from the following expressions, as suggested by Bergman:

$$M_v = w R_2^2 (U \cos\theta - 1) \tag{3.4}$$

$$T = w R_2^2 (U \sin\theta - \theta) \tag{3.5}$$

$$V = w R_2 \theta \tag{3.6}$$

The above expressions, however, give the values of T and V in a vertical plane. So in planes of normal cross section the expressions will be:

$$M_v = w R_2^2 (U \cos\theta - 1) \tag{3.7}$$

$$T = w R_2^2 (U \sin\theta - \theta) \tag{3.8}$$

$$V = w R_2 \theta \cos\alpha \tag{3.9}$$

Bergman's approach is extremely simplified and is very convenient to use. However it leads to a very conservative design. His approach also fails to consider the inherent structural strength of helicoidal slab. The formula provided for mid span bending moment M_v and the curves are limited to the common case of uniform loading covering the entire span. It should be mentioned here that the flexural and twisting moments for concentrated loads were not developed by him but by Michalos (1953).

Bergman's approach may be applied for medium sized helicoidal stair slabs. However, Bergman had recommended to use a more accurate method for large structures.

3.3.2 Engle's Approach

In Engle's (1955) approach, the analysis was made on the structure idealised as a curved beam fixed at both ends, eliminating its third dimension. Both symmetric and antisymmetric loading with respect to plane of curvature were considered and the method of consistent deformation was followed during analysis. The following additional assumptions were made:

9

0

i. The cross section of beam is uniform and small compared to the radius of curvature.
ii. Out of plane deformation is small.

Based on these assumptions twelve loading conditions, for example, uniform torsion, uniform load, concentrated load, concentrated torsion, symmetrically and partially applied uniform torsion, etc. were considered.

In the process of solution, the number of unknowns came to three. For the case of symmetrically applied uniform load, vertical moment (M_v) and torsion (T) at an angular distance θ from mid span are given as,

$$M_v = w R_2^2 [1 - A (B - K\beta\cos\beta) \sin 2\beta] \tag{3.10}$$

$$T = w R_2^2 [\theta - A (B - K\beta\cos\beta) \sin\theta] \tag{3.11}$$

where, $A = 4 / [2K/\beta - (K - 1) \sin 2\beta]$

$B = K + 1$

3.4 ANALYSIS OF HELICOIDAL GIRDER AS A SPACE STRUCTURE

3.4.1 Scordelis' Approach

A. C. Scordelis (1960) was the pioneer, among other researchers, to consider helicoidal stair slab as a space structure. The stair slab is assumed to behave like a helicoidal girder neglecting any slab effect. The girder is reduced to its elastic line having stiffness same as that of original structure. Thus for a both-end-fixed structure, the problem becomes statically indeterminate to the sixth degree. However, considering symmetry of loading and structure only two redundants remain to be solved at the midspan section as others attain a zero value because of symmetry. This leads to a great simplification to the problem. Scordelis had given the general equations to determine the redundants at midspan of a uniformly loaded helicoidal girder fixed at both ends. He has also tabulated the values for these redundants for 510 different cases using horizontal angle (2β), with angle of slope and cross-sectional properties as the variables.

The helicoidal girder was analysed for a uniform vertical load of 1 lb per linear ft of horizontal projection of the girder longitudinal axis. The unknown two redundants (Fig. 3.2) at a mid span section are:

X_x : a horizontal force in the direction of X axis, and

X_r : a moment acting about X axis

Scordelis used the principle of superposition to get the displacements in the direction of the redundants:

$$\delta_{xw} + X_x\,\delta_{xx} + X_r\,\delta_{rr} = 0 \qquad 3.12$$

$$\delta_{rw} + X_x\,\delta_{rx} + X_r\,\delta_{rr} = 0 \qquad 3.13$$

From Maxwell's law of reciprocal displacements, $\delta_{rx} = \delta_{xr}$

Thus to solve the above two equations for redundant, five displacements must be determined. These are:

$$\delta_{xw},\ \delta_{rw},\ \delta_{xx},\ \delta_{xr} = \delta_{rx},\ \delta_{rr} \qquad 3.14$$

By conventional virtual work method the above displacements can be determined. For example,

$$\delta_{xw} = \int_{-\beta}^{+\beta}\frac{m_{rx}m_{rw}}{EI_r}R_2 d\beta + \int_{-\beta}^{+\beta}\frac{m_{sx}m_{sw}}{EI_s}R_2 d\beta + \int_{+\beta}^{-\beta}\frac{m_{tx}m_{tw}}{GJ_t}R_2 d\beta \qquad 3.15$$

The moments are denoted by special notations, first subscript denotes the axis about which the moment acts and the second subscript denotes the load causing moment (Fig. 3.3).

Now, due to uniform load of 1 plf with the redundant equal to zero and referring to Fig. 3.2 and Fig. 3.3, at any location θ from mid-span, these moments can be expressed by statics as follows:

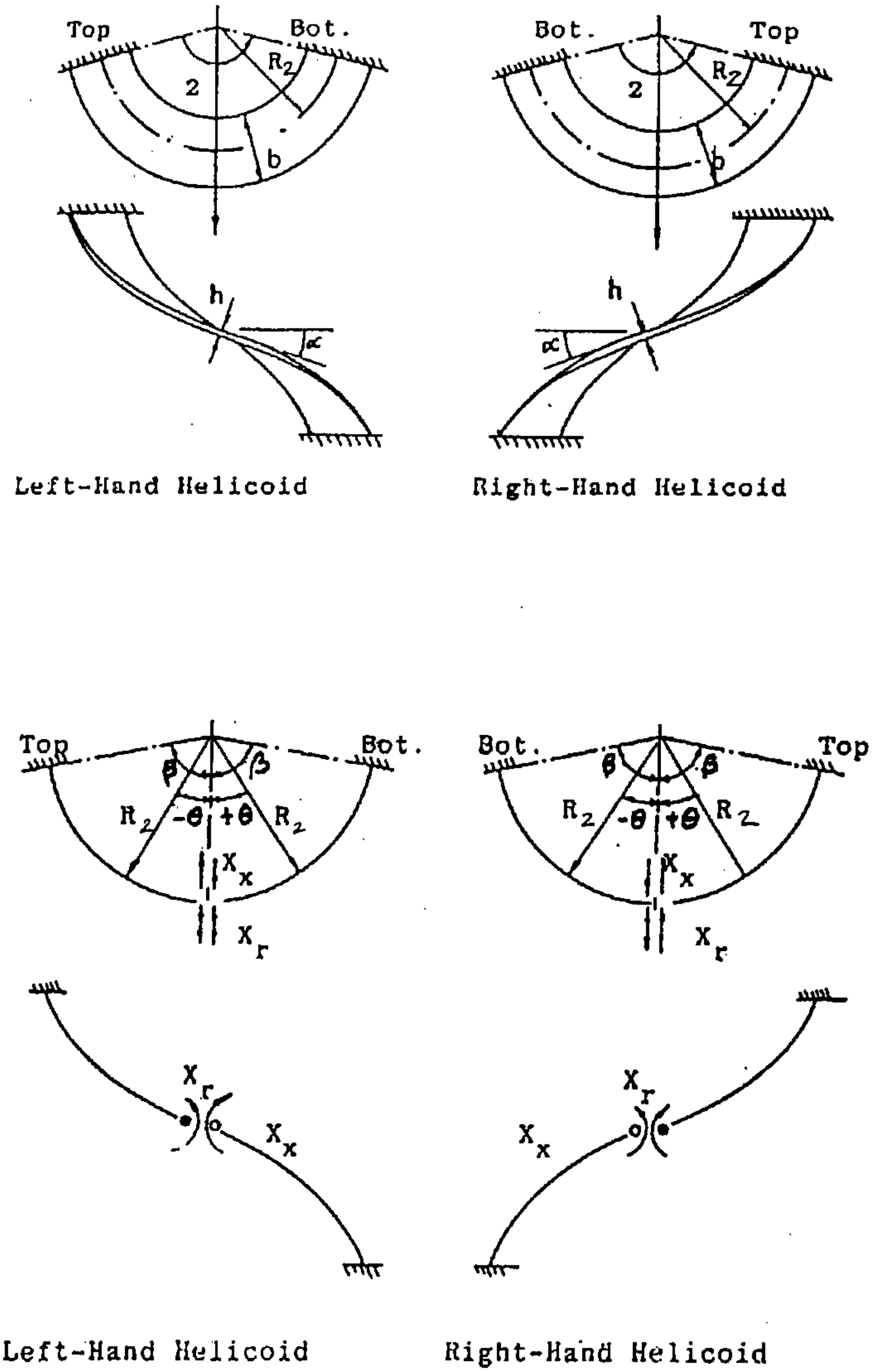


Fig. 3.2 Positive direction of redundants (Scordelis approach)

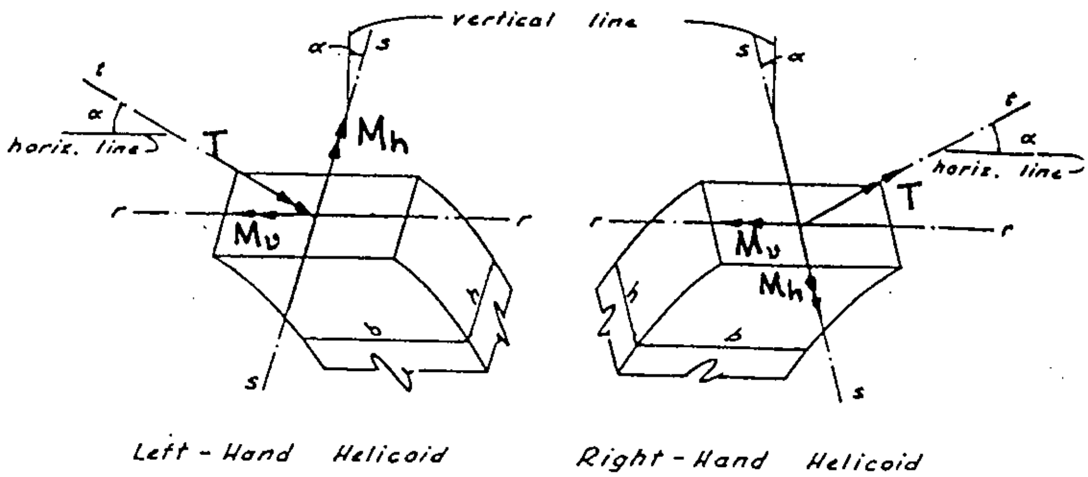


Fig. 3.3 Positive direction of bending and torsional moments

$$m_{rw} = -R_2^2\,(1 - \cos\theta) \qquad 3.16$$

$$m_{sw} = -R_2^2\,(\theta - \sin\theta)\sin\alpha \qquad 3.17$$

$$m_{tw} = -R_2^2\,(\theta - \sin\theta)\cos\alpha \qquad 3.18$$

For $X_x = 1$,

$$m_{rx} = -R_2\,(\theta\sin\theta)\tan\alpha \qquad 3.19$$

$$m_{sx} = R_2\,(\sin\theta)\cos\alpha + R\,(\theta\cos\theta)\sin\alpha\tan\alpha \qquad 3.20$$

$$m_{tx} = -R_2\,(\sin\theta)\sin\alpha + R\,(\theta\cos\theta)\sin\alpha \qquad 3.21$$

For, $X_r = 1$,

$$m_{rr} = \cos\theta \qquad 3.22$$

$$m_{sr} = \sin\theta\sin\alpha \qquad 3.23$$

$$m_{tr} = \sin\theta\cos\alpha \qquad 3.24$$

To get displacement terms, tables have been constructed to reduce the complexity in calculation. Now final bending and torsional moment at any section can be computed as:

$$M_v = (m_{rw} + X_x m_{rx} + X_r m_{rr})\, w \qquad 3.24$$

$$M_h = (m_{sw} + X_x m_{sx} + X_r m_{sr})\, w \qquad 3.26$$

$$T = (m_{tw} + X_x m_{tx} + X_r m_{tr})\, w \qquad 3.27$$

However, in practical cases, the uniform vertical load is actually distributed over the width of the girder. As a consequence of this, there exists an eccentricity with respect to the centreline of the girder, where the equivalent helix is considered. This eccentricity is equal to,

$$e = b^2 / 12\, R_2 \qquad 3.28$$

The eccentric loading produces a uniform torque on the structure. Scordelis and Lee investigated the matter and suggested that the torque loading is negligible for b/ R_2 less than 3. Otherwise they suggested to consider the torque loading during the calculation and have provided another chart for the torque loading.

For convenience, usually a separate analysis is made for torque loading and redundants found are algebraically summed to those obtained for vertical load to get actual value of X_x and X_r. Finally he proposed,

$$M_v = (m_{rw} + e m_{rt} + X_x\, m_{rx} + X_r m_{rr})\, w \qquad 3.29$$

$$M_h = (m_{sw} + e m_{st} + X_x\, m_{sx} + X_r\, m_{sr})\, w \qquad 3.30$$

$$T = (m_{tw} + e m_{tt} + X_x\, m_{tx} + X_r\, m_{tr})\, w \qquad 3.31$$

where,

$$m_{rt} = -\, wR\,(1 - \cos\theta)$$

$$m_{st} = w\, R_2 \sin\theta\, \sin\alpha$$

$$m_{tt} = w\, R_2 \sin\theta\, \cos\alpha$$

Scordelis has drawn some interesting conclusions:

I. A change in width-depth ratio (b/h) has an appreciable effect on maximum values of M_h and T in helicoidal girders, whereas it has little effect on a horizontal bow girder.

II. For high b/h ratios, the angle of slope of the girder has a relatively small effect on maximum values of midspan bending and torsional moments.

III. The slope of the girder becomes increasingly important as the b/h ratio decreases.

He also concluded that cross section with high width to depth ratio should be used in designing helicoidal girders since they carry the load most efficiently.

Obviously, the Scordelis method produces a more realistic analysis than the Bergman's approach. But this method has got some shortcomings.

Firstly, the helicoidal stair case is a sharply curved member. But the bending and torsional rigidities are computed assuming the member as straight and it is concentrated with the longitudinal axis.

Secondly, minimum slab dimension was used by Scordelis to determine the siffnesses. In the construction of almost all stairs, the steps are cast integrally with the slab, producing a 'sawtooth profile'. The actually greater average depth of slab due to presence of the steps must inevitably modify the stiffness in both bending and torsion.

Thirdly, the slab action of helicoidal surface is neglected in this approach, which necessarily underestimates the capacity of the stair slab.

Fourthly, the elastic line is taken as the centreline helix of the helicoidal stair slab. But actually due to skewness of the helicoidal section, the elastic line is likely to move nearer to the axis of helicoid.

However, in spite of all these shortcomings, the Scordelis method is the more logical and convenient method for use in analytical design purpose based on hand computation than other methods.

3.4.2 Morgan's Approach

In Morgan's (1960) approach, the helical stair is analysed with the ends being fixed. Out of six selected redundants at mid span only two non-zero redundant remain to be calculated. These two redundants are radial horizontal force (H) and vertical moment

(M) acting in a tangential plane. Morgan assumed the centreline load to be parallel to, but not coincident with the centreline of the stair slab and load at any section acts through a point which is on neither of these lines. This assumption leads to the following equations valid for any section at an angular distance θ from the mid span section (Fig. 3.4):

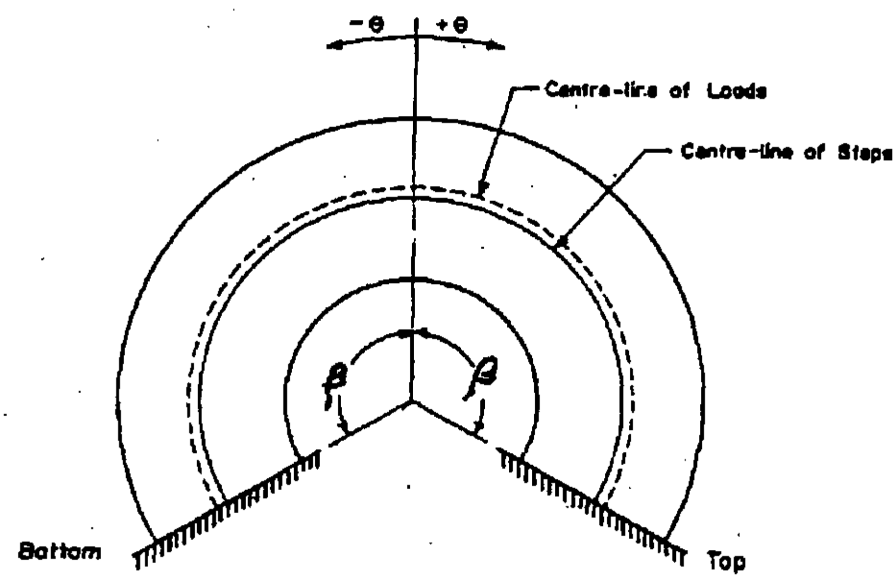


Fig. 3.4 Plan of stair

- Vertical moment:

$$M_v = M\cos\theta + HR_2\theta\tan\alpha\sin\theta - wR_1^2(1-\cos\theta) \tag{3.32}$$

- Lateral moment:

$$M_h = M\sin\theta\sin\alpha - HR_2\theta\tan\alpha\cos\theta\sin\alpha - HR_2\sin\theta\cos\alpha + (wR_1^2\sin\theta - wR_1R\theta)\sin\alpha \tag{3.33}$$

- Torsion:

$$T = (M\sin\theta - HR_2\theta\tan\alpha\cos\theta + wR_1^2\sin\theta + HR_2\sin\theta\sin\alpha - wR_1R_2\theta)\cos\alpha + HR_2\sin\theta\sin\alpha \tag{3.34}$$

- Thrust:

$$N = -H\sin\theta\cos\alpha - wR_1\theta\sin\alpha \tag{3.35}$$

- Lateral Shear:

$$V = w\theta\cos\alpha - H\sin\theta\sin\alpha \tag{3.36}$$

- Radial horizontal shear:

$$F = H\cos\theta \tag{3.37}$$

where, radius of centreline of load,

$$R_1 = \frac{2}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right) \qquad 3.38$$

The strain energy theorem was employed to determine the mid span redundant. But final expressions obtained by him were very lengthy and complicated.

3.4.3 Holme's Approach

Alan M. C. Holmes (1957) was another pioneer to consider helicoidal stair slab as a space structure and he analytically derived expressions for moments and deflections of such a structure. Holmes' excellent paper is probably the most comprehensive treatment of the subject. In a similar approach like Scordelis, Holmes idealised the structure as a helical girder with both ends fixed. The redundants are selected at the mid span cross section and as already stated, due to symmetry, all but two redundants are zero. These two redundants, namely, vertical moment and radial horizontal shear force remain to be solved. Holmes further assumed that the centre of gravity of the loads acts along the centre of the stair slab, thus the eccentricity considered in Scordelis analysis was not considered by Homes. Based on this assumption Holmes reached the following expressions:

$$M_v = wR_2^2\,(1 - C_1\cos\theta + C_2\,\theta\sin\theta + C_2\cos\theta)\,/\,\cos\alpha \qquad 3.39$$

$$T = w\,R_2^2\,(\theta - C_1\sin\theta + C_2\,\theta\cos\theta) \qquad 3.40$$

$$M_h = w\,R_2^2\tan\theta\,(\theta - C_1\sin\theta + C_2\,\theta\cos\theta + C_2\sin\theta/\sin^2\alpha) \qquad 3.41$$

And

$$M = C_1'w\,R_2^2/\cos\alpha \qquad 3.42$$

$$H = C_2w\,R_2/\sin\alpha \qquad 3.43$$

$$C_1 = (1 + C_1' + C_2) \qquad 3.44$$

Expressions for C_1, C_2 and C_1' were also prescribed by Holmes. These values depend on various geometric parameters.

Holmes concluded that for a constant uniform load, radius and cross sectional area mid span deflection of a helical girder decreases with the thickness of the cross section.

In addition, Holmes also reported that for helicoidal slabs with shallow-wide cross section and greater central angle, the maximum deflection does not occur at mid point. It should be added further that for a 360 degree helix of shallow-wide cross section, the deflection of the quarter points can be three times that of the mid point. However, in that case it was suggested to go for model tests for substantiation of this findings.

Holmes analysis is analytically more accurate than the Bergman approach and also less conservative. Holmes approach is similar to Scordelis approach. It also utilises the beneficial structural behaviour of the helicoid.

Holmes study concluded that thinner helicoidal slabs are more efficient due to the beneficial 'Bent Action'. For shallow-wide cross sections this 'bent action' reduces the normal and torsional moments. As the member becomes weak about a horizontal-radial axis, the upper and lower halves tend to 'lean' on each other. The forces set up by this leaning are carried by each half acting as a canted bent which is relatively stiff in bending about an axis roughly perpendicular. Thus the slab action of the helicoidal stair slab was apparent from his studies.

The occurrence of pseudo-fixity at the mid span, as measured by Holmes, was later substantiated by the finite element analysis also.

3.5 INTRODUCTION OF INTERMEDIATE LANDINGS

Intermediate landings are often used in stair slabs with long vertical elevation. In some cases code provides regulations on the inclusion of a landing. All the analysis to incorporate an intermediate landing assumes the helicoidal structures to be spatial.

3.5.1 Arya and Prakash's Approach

A. S. Arya and Anand Prakash (1973) attempted to analyse the case of the helicoidal stairs with intermediate landing. They used flexibility approach to analyse internal forces due to dead and live loads in fixed ended circular stairs having an intermediate landing. Like Scordeilis, they treated the structure as a linearly elastic member in space defined by its longitudinal centroidal axis. Influecnce lines were drawn at various cross sections for all the six stress resultants found at such sections for unit vertical load and unit moment about the axis of the structure. Critical positions of loads were determined to obtain the maximum values of the internal forces. From this analysis they drew the following conclusions:

i. Torsional moment has the maximum value at the ends for both concentrated and distributed loads. In order to obtain the maximum value of torsional moments, the live load should be placed on the outer half of the stair slab throughout the whole span.
ii. Radial horizontal shear force is maximum either at the ends or at the mid span of the stair slab in all cases.
iii. Lateral shear force is maximum at the ends for vertical loads and at a cross section lying in the end quarter span.
iv. The vertical moment is always maximum at the ends and the sagging vertical moment is maximum at a cross section lying in the central half portion of the stair slab.
v. The lateral moment is maximum at a cross section lying in the end quarter span of the stair slab. The nature of bending moment is such that it causes tension outside near the lower end and tension inside near the upper end of the stair slab.
vi. The thrust is maximum at a cross section lying in the end quarter span. The portion near the lower end is subjected to axial compression and the portion near the upper end to axial tension.

3.5.2 Solanki's Approach

Himat T. Solanki (1986) analysed the problem of intermediate landing using energy method to find the two unknown redundants at the mid span section. Other redundants at mid span have zero values because of symmetry of geometry and loads. He gave two equations, the simultaneous solution of which gives the values of the redundants. Solanki's findings were similar to those observed by Arya and Prakash. He further commented that this method could be used to analyse the structure with more than one landing in the stair slab. Moreover, simply supported or pin jointed helicoids could also be analysed using this method. The method followed in this study incepts from Solanki's approach.

3.6 EXPERIMENTAL FINDINGS

Researchers have not only attempted to analyse the helicoidal stair slabs from purely mathematical point of view, but also conducted extensive model tests to substantiate the analytical findings. Various materials were used to simulate the helicoidal stair slab along with half scale models of RCC structures. The conscious and extensive investigation of a number of researchers has unveiled many interesting and exclusive information about the behaviours of this complex but attractive structure. The following articles highlight some of these facts.

Magnel made some experiments on prestressed concrete helicoidal stair slabs but the results were inconclusive. Young and Scordelis (1960) tested a model helical slab of plexiglass. Holmes, in his paper mentions the use of models but gave no details. Mattock has made deflection measurements on a full sized reinforced concrete helicoidal stair slab under self weight when the formwork was removed. Cusens and Trirojna (1964) then tested half scale reinforced cement concrete models of helicoidal stair slab.

Findings of tests by Young and Scordelis, and Cusens and Trirojna are be presented here in brief.

3.6.1 Findings of Young and Scordelis

Y. F. Young and S. C. Scordelis (1960) made the earliest attempts to construct a model of the helicoidal slab for experimental purpose. They used plexiglass (a clear plastic possessing certain model qualities like flexibility and workability) as the material of the models. They tested four girders having b/h ratios of 1:1, 4:1, 8:1 and 16:1. The models were all ¼" thick with widths of ¼", 1", 2" and 4". All girders had the same helix angle (inclination with the horizontal) of 30 degree and were of rectangular cross section with central angle of 180 degree. These models were used to substantiate the influence lines for end reactions due to vertical load derived from analytical approaches. There was good agreement between the analytical results and experimental findings. The following conclusions proved the validity of Scordelis' earlier theoretical analysis:

i. For girders with b/h ratio within 1 to 16 and a centreline radius to width between 3 and infinity, sufficiently accurate results are obtained by analysing only the elastic line defined by the longitudinal centroidal axis of the girder, neglecting slab effect.
ii. The bending and torsional stiffnesses can be taken as those defined by for a straight prismatic member of same cross section.
iii. The influence ordinates across the width of the girder have a linear relationship.

3.6.2 AIT Model Study

A. R. Cusens and Supachai Trirojna (1964) were among the firsts to test reinforced concrete helicoidal stair slabs to failure. The tests were carried out at the Asian Institute of Technology, Bangkok.

In total, three models were tested at the AIT, all models constructed to half scale. Two models having 80 degree central angle were tested under uniform load. The models were scaled down from a prototype, which was designed for the dining hall project of Chulalongkorn University, Thailand. Following this study Morgan's method of analysis were used for analysis and design. However, Bergman, Holmes and Scordelis methods were also used for comparison purpose between the different approaches. The first one of these two models was constructed as per the analysis while in the second model the reinforcement required for resisting computed lateral moment was arbitrarily reduced by 50%. Later on, another model having a central angle of 180 degree was also tested under concentrated load and uniform load.

Cusens and Trirojna, and later Cusens alone have concluded the following from the model study for these stairs:

i. The assumption that the cross section of the stair behaves as a beam concentrated at the centerline is not correct.
ii. The cross-section of the stair slabs had a curved profile under load. In other words, under uniformly distributed load, there was evidence of slab action in the stair slab.
iii. Bergman's analysis is extremely conservative. In some cases, load factors as high as 20 have been achieved.
iv. The torsional moment by Morgan's method is considerably higher.

v. Under ideal construction for a stair slab subtending a small horizontal angle up to 80 degree and designed by elastic analysis of Morgan, Holmes and Scordelis, a load factor of 5 may be expected. However, in case of 180-degree model, a load factor of 9.83 was recorded.

vi. The use of ultimate strength design based on an equivalent straight fixed ended beam gave a simple and safe solution for vertical moment. Since reinforcement was not found necessary, nominal reinforcement against lateral direction is recommended.

vii. There was reasonable agreement between the analytical and experimental strains at design load at mid span of the stair slab. The correlation was not that good at the top and bottom of the stair slab.

viii. For stairs subtending small angles in plan, the moment about the horizontal axis is of greater importance than that about the vertical axis; for larger angles, approaching 180 degree, the reverse is the case. However, failure of the structure was due to combination of the bending moments in all the cases.

3.7 PROPOSAL FOR DESIGN CHARTS

3.7.1 First Attempts of Santathadaporn and Cusens

Morgan and Scordelis have given formulas to find the two unknown redundants at the mid span section of a helicoidal stair slab, the redundants being horizontal force and vertical moment.. However both these equations contained a relatively large number of terms and required tedious computations to determine the design bending moments and shear, as the design moments and shears were to be found from further equations. This is why these equations were not as attractive as they should have been to the practising engineers. Santathadaporn and Cusens (1966) worked further on helicoidal stair slabs to simplify the design process. They proposed some simple equations for design purpose with the provision of using coefficients in the equations. They used computer program to solve those complicated equations and constructed a series of design charts to provide the values of the coefficients. The simplified equations had the form:

$$M = a_1 \, w \, R_2^2 \qquad 3.45$$

$$H = a_2 \, w \, R_2 \qquad 3.46$$

$$M_{vsup} = a_3 \, w \, R_2^2 \qquad 3.47$$

The values of a_1, a_2 and a_3 are to be taken from the 36 design charts proposed. The input parameters used to find these coefficients were:

- Central half angle (30 to 360 degree)
- Slope made by the tangent to the helix centre-line with respect to horizontal plane (20 to 50 degree)
- The width/depth ratio of the stair section (0.5 to 16)
- The ratio of the centre-line of the load to the radius of the centre-line of the steps (1.0 to 1.1)

A typical distribution of forces and moments over the span of the stair is shown in the Fig. 3.5.

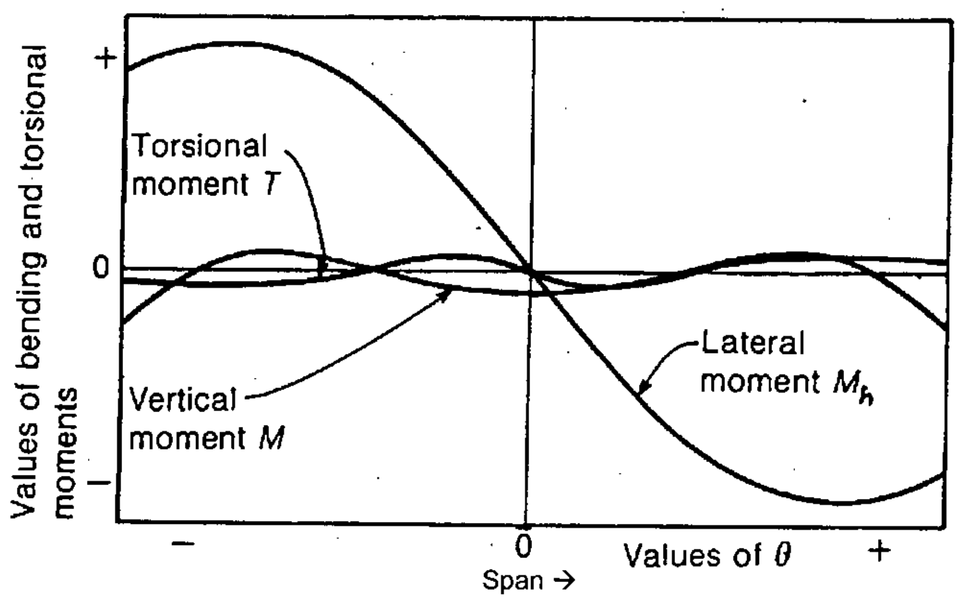


Fig. 3.5 Variation of Moments Along Stair

3.7.2 Reynold's Modification of the Design Charts

C. E. Reynolds and J. C. Steedman (1988) attempted further modifications of the charts provided by Santathadaporn and Cusens. The analytical approach they chose was the one made by Morgan and Scordelis. They incorporated CP110 to the design charts of Santathadaporn and Cusens. These typical design charts developed earlier were recalculated for a ratio of G/E of 0.4 as recommended in CP 110 and by taking C to be one half of the St. Vennant value for plain concrete. Four design charts, covering the range of the most frequently met helicoidal stair slabs, were proposed by Reynolds and Steedman. The coefficients a_1, a_2 and a_3, as suggested by Santathadaporn and Cusens were restated as k_1, k_2 and k_3. Therefore the final form of he equations become:

Redundant vertical moment at mid span, $M = k_1\, w\, R_2^{\,2}$ 3.48

Horizontal redundant force at mid span, $H = k_2\, w\, R_2$ 3.49

Vertical moment at supports, $M_{vsup} = k_3\, w\, R_2^{2}$ 3.50

Using these values it must be ensured that the slab is sufficiently thick to resist the final design moments. Values of other parameters at different points along the stair slab are found by using Morgan's equations and accordingly a detailed reinforcement can be designed. The moments were found to vary with horizontal angle from mid span in a way similar to Fig. 3.5.

3.8 FORMATION OF THE STIFFNESS MATRIX

After the analytical and experimental approaches proposed by various researchers, the first known attempt in the numerical analysis of helicoidal stair slabs was made by Fardis, Skouteropoulou and Bousias (1987). They constructed a full 12 x 12 stiffness matrix of the helicoidal stair slab in terms of its geometric characteristics. This formulation allowed the incorporation of the stair as a single element into a 3-dimensional model of a reinforced concrete building structure for lateral load analysis. The elements of the stiffness matrix that affect most the lateral stiffness of the structural system were presented in terms of the geometrical parameters like story height, axis inclination, central angle on plan, width and depth of the stair slab. For a given story height, the magnitude of these elements was found to depend strongly on the depth and central angle of the stair slab. The lateral stiffness of the stair in any horizontal direction was found almost negligible for central angle between 270 degree and 360 degree but became quite significant for angles less than 180 degree. The reduction of stiffness due to axial and shear deformations of the stair was generally found to be very small. However, apart from these analyses, this study failed to provide any idea regarding the design forces and moments of this stair slab. The equations to determine the internal forces suggested in the paper were complicated and present no interest to the designer.

3.9 THE RECENT FINDINGS AT BUET

3.9.1 The Background

Earlier studies of helicoidal stair slabs indicated the necessity of thorough but practicable shell solution to get. In one of his discussion papers on helicoidal stair slab, Gedizli called for the shell solution with reference to Menn's results.

With this end in view the *"General Thick Shell Finite Element Program"* of Ahmad was adopted at BUET for investigating helicoidal stairs with a view to developing a rational procedure for its analysis and design. This thick shell finite element program is quite general and versatile in nature. It is generalised to a large degree and can handle as many as five loading conditions. This program is capable of analysing the singly or doubly curved thick shell structures. It is also valid for moderately thin shells. Because of its general nature, it requires a large number of data each time when the analysis is to be performed.

3.9.2 Modak's Works

With a view to making an efficient and automated data feeding process, Modak (1991) in his graduate research work made modifications in the program to facilitate the analysis of this structure as a particular case of thick shell problem through an interactive mode of data entry. However, while making such modifications, provisions were made to preserve the general nature of the program by feeding data in file mode. The features of modifications made in the main program by Modak are summarised below:

- Automatic division of the structures into elements.
- Preparation of a simplified view of the nodal representation of the structure.

- Automatic generation of nodal co-ordinates.
- Automatic fixation of boundary co-ordinates.
- Automatic determination of adjusted unit weight to handle gravity and uniformly distributed load at a time as gravity load only.
- Automatic definition of element topology.
- Automatic transformation of global stresses to local stresses.
- Automatic determination of moments and forces.
- Automatic design and comments on whether further revision in design is necessary or not; if revision is to be done then probable thickness is also indicated.

Using the modified *"Thick Shell Finite Element Program"* of Ahmad, the actual behaviour of the helicoidal stair slabs was investigated by Modak. A tentative design procedure was also suggested. But in this work, some anomalies regarding the estimation of shear and torsion, obtained from this program, remained unsolved.

3.9.3 Morshed's Modifications

After Modak, Morshed (1993) modified the program to calculate the shear forces and torsion using Gauss point stresses. This led to the successful elimination of the anomaly of Modak's results.

3.9.4 Amin's Contributions

However, even after Morshed's modification, there were some anomalies, which eventually were corrected by Amin (1998) during his graduate research works. Amin worked on the development of a finite element software to optimise its mesh system and to assured the applicability of the thick shell element to model the helicoidal stair slab.

However, Amin's greatest success was to model the deflection behaviour of the helicoidal stair slab, which no one could do previously. He compared the finite element deflection behaviour with the experimental findings of previous researchers. The appearance of a pseudo fixity at mid span for higher central angles, as found by Holmes, was also established by Amin's study.

In addition, Amin collected the geometric data of some existing helicoidal stair slabs in Bangladesh and redesigned them using the finite element technique and helical girder solution. A comparative study was also made to check the design economy attainable through the finite element analysis. His study hinted that an economy in reinforcement may be possible, if designed by the finite element technique. The comparison of helical girder and finite element solutions for three different prototype stairs were also presented by Amin in a pictorial way.

Amin also carried out an extensive parametric study of the variables governing the values of the stress resultants and presented them in his M.Sc.(Engg.) thesis. His works proved that the finite element method using "General Thick Shell Finite Element Program" of Ahmad can be used to analyse the helicoidal slab in a rational way. His studies opened up a gate for a new breed of researchers at BUET to study the behaviour of helicoidal stair slabs through the finite element methods.

3.9.5 Works of Wadud, Khan and Choudhury

In the light of Amin's work, Z. Wadud (1999), R. A. Khan (2000), and C.F. Choudhury (2002) carried out further studies at BUET on helicoidal stair slabs without landings. Wadud, and Khan principally worked on parametric study of the helicodial stair cases to generalize the behaviour. Finally, Choudhury proposed new design charts on the basis of finite element results and compared them with Reynold's charts.

CHAPTER 4

VERIFICATION OF REYNOLDS' COEFFICIENTS

4.1 GENERAL

Among the conventional methods available for helicoidal staircase design, the helical girder solution produces closest approximation of stress resultants. The girder approach deals with the three degrees of indeterminacy in the form of three dimensionless coefficients k_1, k_2, and k_3. The traditional helical girder approach incorporates the evaluation of these three coefficients from Reynolds's design charts, which are based on design charts proposed by Santathadaporn and Cusens. This chapter aims at verifying these coefficients using the finite element methods. The thick shell finite element program developed by Ahmad (1969), and later modified at various stages at BUET has been used to model the helicoidal stair slab. The design charts available are discussed in article 3.7.

4.2 THE FINITE ELEMENT ANALYSIS

The finite element program uses Ahmad's thick shell element as the basic element. To accommodate the modeling of a helicoidal staircase, it was modified at BUET at various stages. The program currently in use takes various geometric parameters of the stair and automatically models the staircase. Stair with only fixed boundary conditions can be modeled. The FE program takes the following data as its input:

- Inner radius (R_i)
- Outer radius (R_o)
- Height of the stair (Ht)
- Total subtended angle on the horizontal projection (2β)
- Thickness of the slab (h)

In addition to the geometric data, load data are to be input as well. For the current analysis, uniformly distributed live load of 100 psf and concrete unit weight of 150 pcf have been used throughout the analysis.

The program gives the following output:

- Nodal coordinates
- Node and element number, connectivity
- Displacement along centreline, inner edge and outer edge along the span
- Vertical moment along the span
- Lateral moment along the span
- Torsion along the span
- Thrust along the span
- Lateral shear along the span
- Radial Horizontal shear along the span

Depending upon the user requirements, the stair can be divided into different combination of elements and nodes. For the present study, the stair has been modeled with 149 nodes and 40 elements. A pictorial representation of the stair divided into element with node numbers is given in Fig. 4.1.

4.3 METHODOLOGY

The values of selected geometric parameters are entered into the finite element program. The resultant output files provide different stress resultants. Mid span vertical moment, mid span radial horizontal shear and support vertical moment values are recorded. For the given stair, load is separately calculated. The value of mean radius can also be calculated directly. Thus the stress resultants, w and R are known. Back calculations provide the values of k_1, k_2 and k_3.

$$k_1 = w\,R^2/\,M_{cFE} \qquad 4.1$$

$$k_2 = w\,R/\,F_{cFE} \qquad 4.2$$

$$k_3 = w\,R^2/\,M_{vsFE} \qquad 4.3$$

The subscript FE refers to the stress resultants found from the finite element analysis. Detail of the method is described in terms of the flow chart in Fig. 4.2.

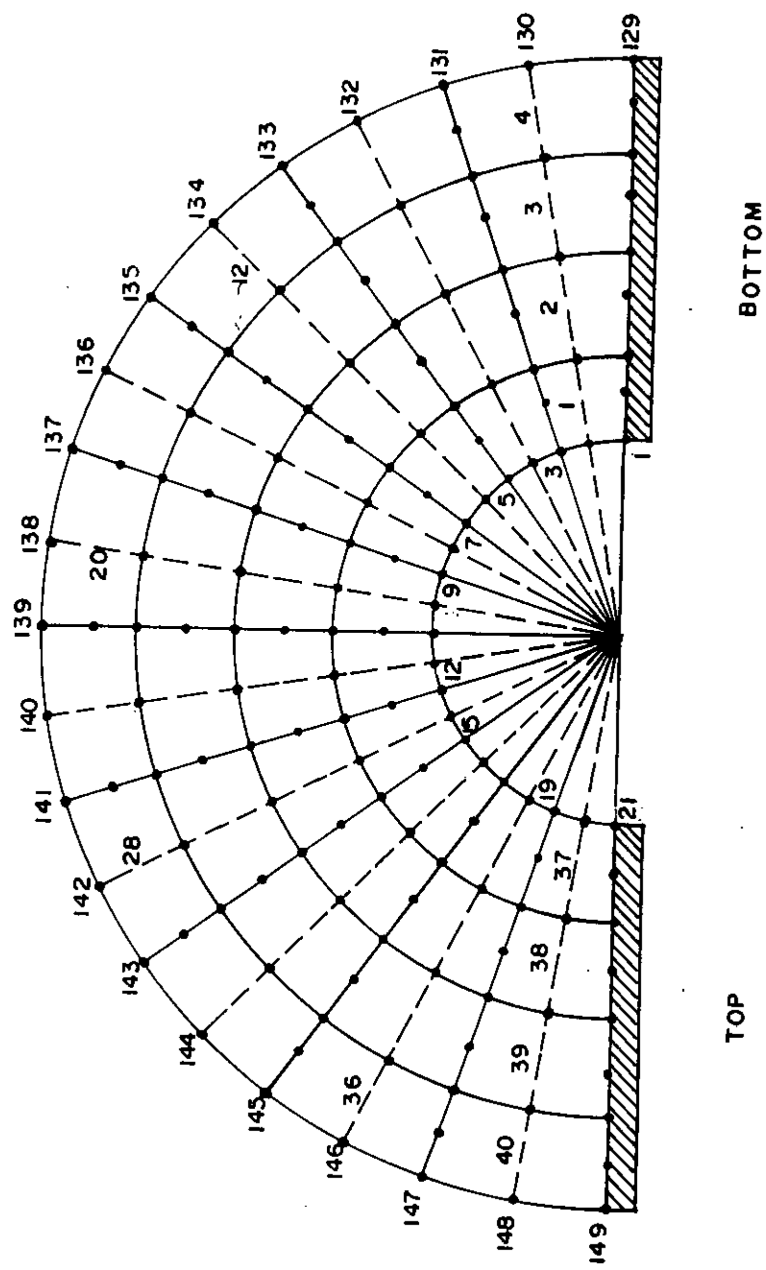


Fig. 4.1 Division of stair into elements and node numbering scheme

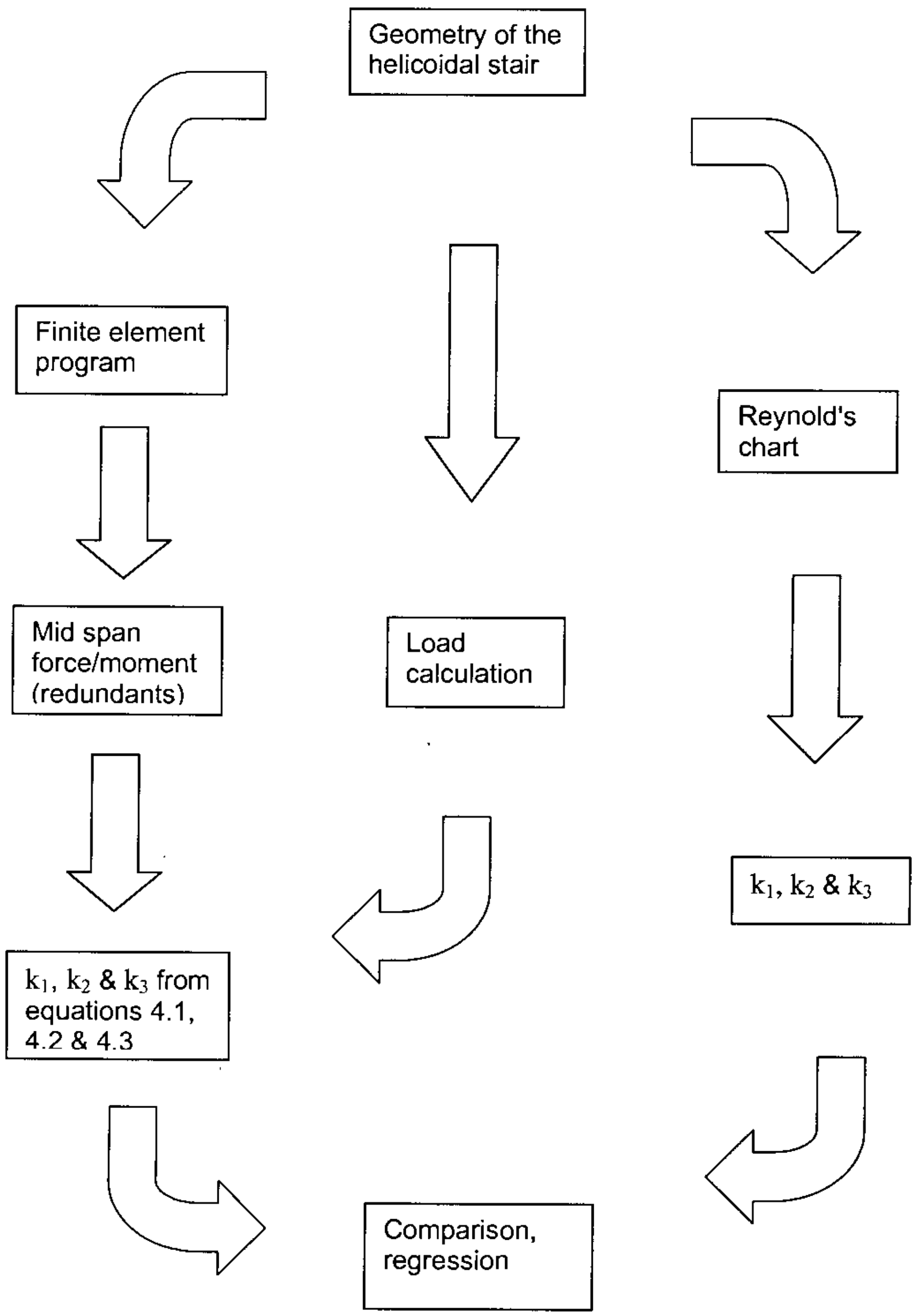


Fig. 4.2 Flow chart of the methodology

4.4 FINDINGS

The results of the finite element analysis are presented pictorially where the finite element coefficients and the Reynold's coefficients are plotted. Also a linear regression has been carried out with the data. Figs. 4.3-4.5 represent the individual regression results between the Reynold's coefficients and Finite Element coefficients for k_1, k_2 and k_3 for a given R_1/R_2 and b/h ratios. Figs. 4.6-4.8 give the regression curves for k_1, k_2 and k_3 respectively over the entire range of other variables. Fig. 4.9 is the plot of all the k_1, k_2 and k_3 values, together.

In the regression curve, a correlation value of 1 represents that the variables are perfectly related. The correlation values have been found above 0.95 for each of k_1, k_2, k_3 and all taken together. A slope of 1.0 and intercept of zero at the x axis is expected for an exact equality between Reynold's coefficients and Finite Element coefficients. The slope for k_2 has been found to be almost 1. This accompanied by the fact that the degree of correlation is also above 0.96, indicates that the Reynolds coefficients have almost the same value as the FE coefficients. The slope of k_1 and k_3 regression is not a perfect 1.0. However their correlation is still good enough. A greater than 1 slope for the k_1, indicates that the FE coefficients are greater than the Reynold's coefficients, whereas, a slope less than 1 for k_3 indicates that the Reynold's coefficients are larger.

While each of k_1, k_2, or k_3 taken together for a wide range gives a good correlation, for individual cases (specific central angle) only k_2 gives a good correlation. The values of k_1 and k_3 for individual case did not show a good relationship. This could be due to the following reasons:

- The Reynold's method actually does not give exact values
- Statistical errors, like
    - The number of data points is smaller for k_1 and k_3, than for k_2
    - The percent error in picking the value of k_1 and k_3 from the Reynold's chart is far higher than that for k_2, because k_1 and k_3 have very small values.

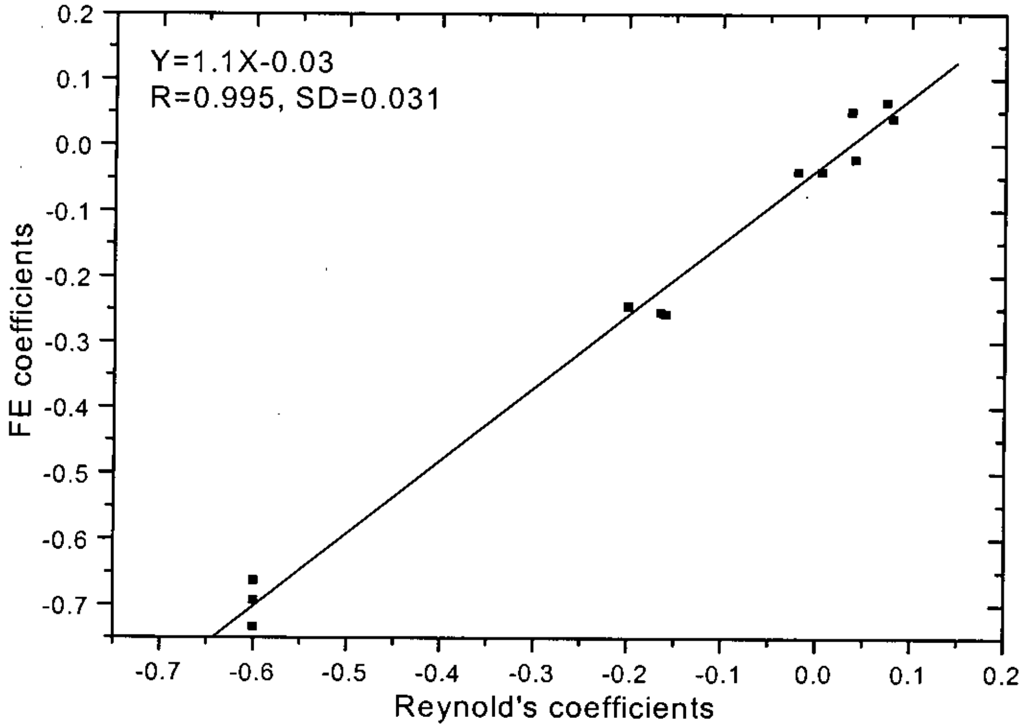


Fig. 4.3 Regression between FE and Reynold's coefficients (k_1)
[b/h=5, R_1/R_2=1.1]

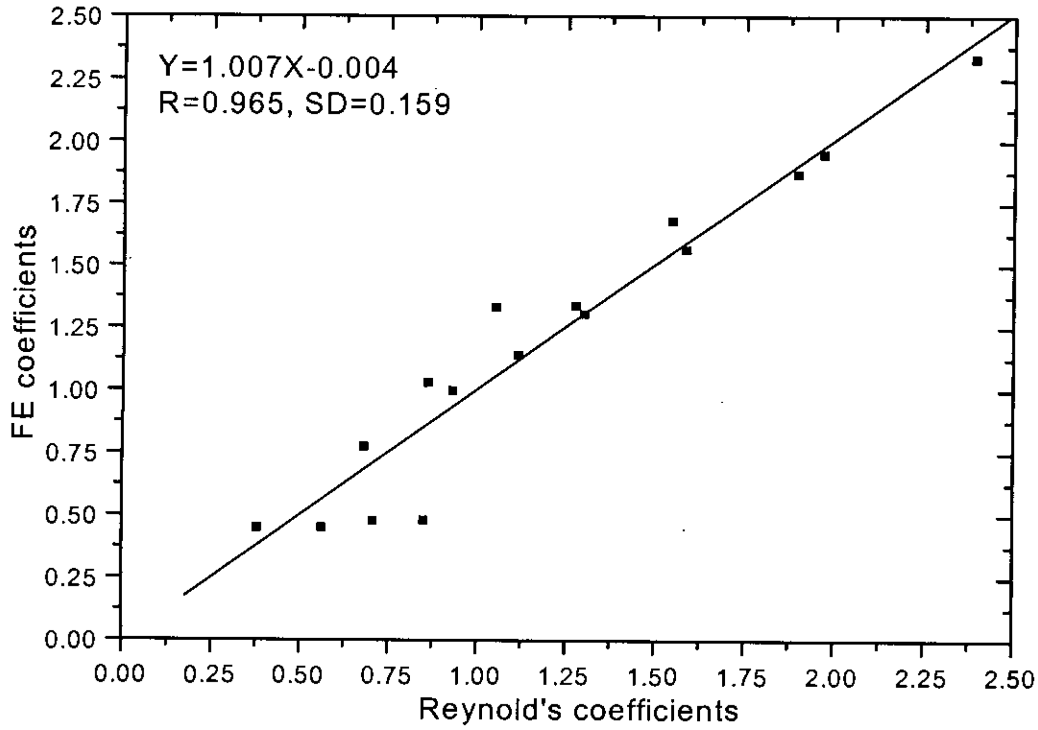


Fig. 4.4 Regression between FE and Reynold's coefficients (k_2)
[b/h=5, R_1/R_2=1.05]

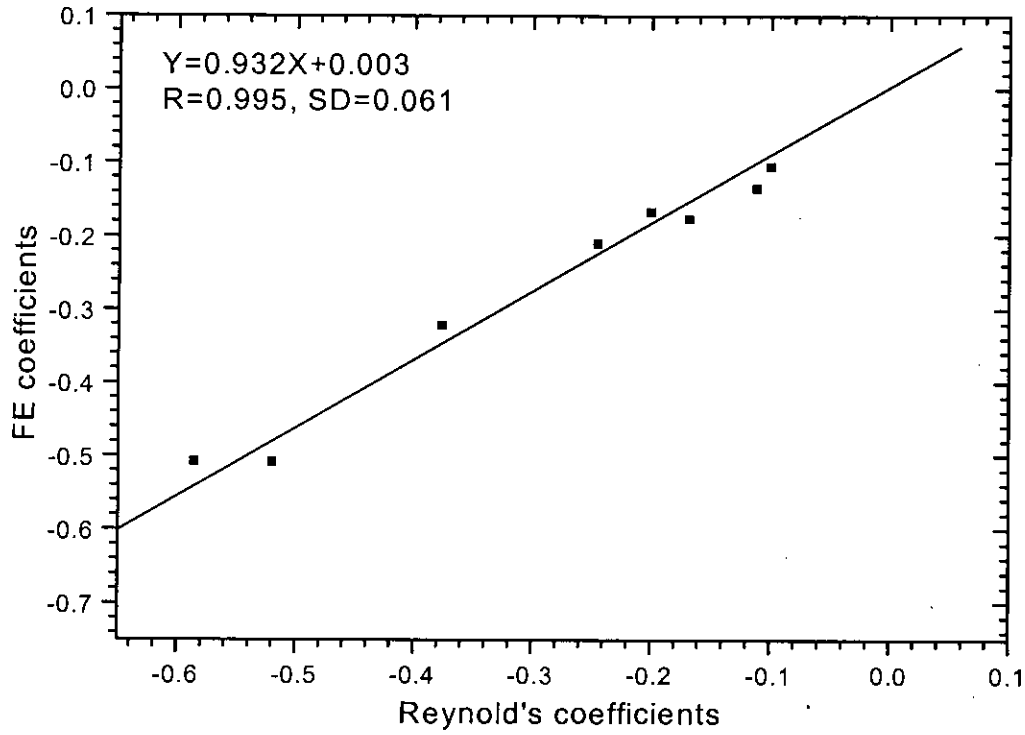


Fig. 4.5 Regression between FE and Reynold's coefficients (k_1)
[$b/h=5$, $R_1/R_2=1.05$]

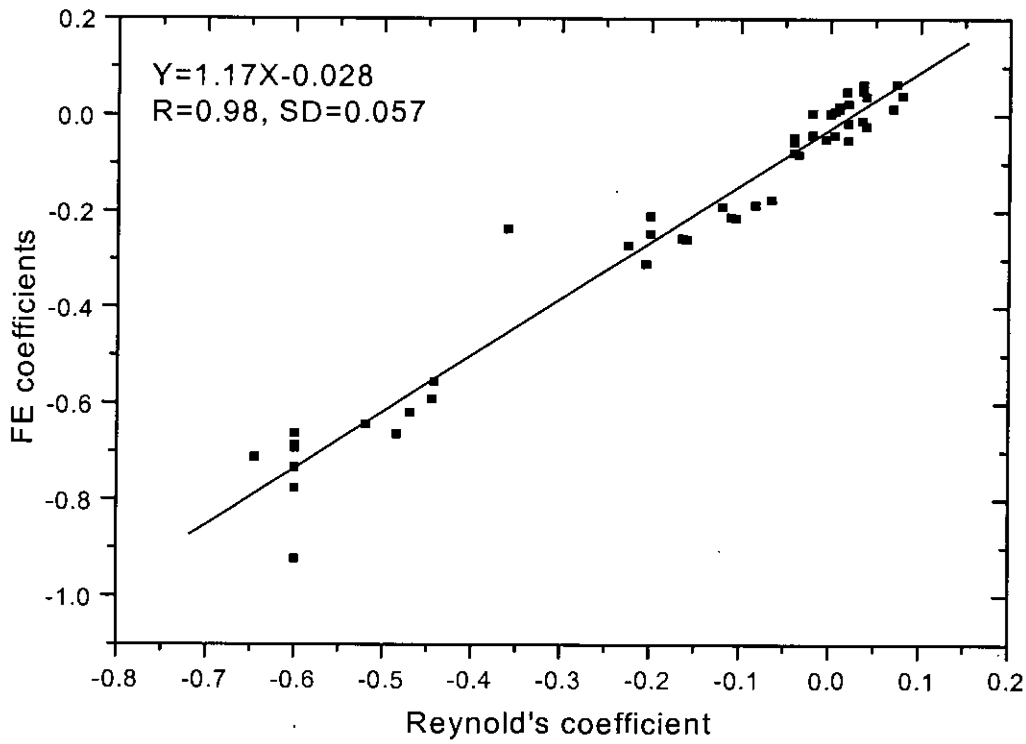


Fig. 4.6 Regression between FE and Reynold's coefficients (k_1)

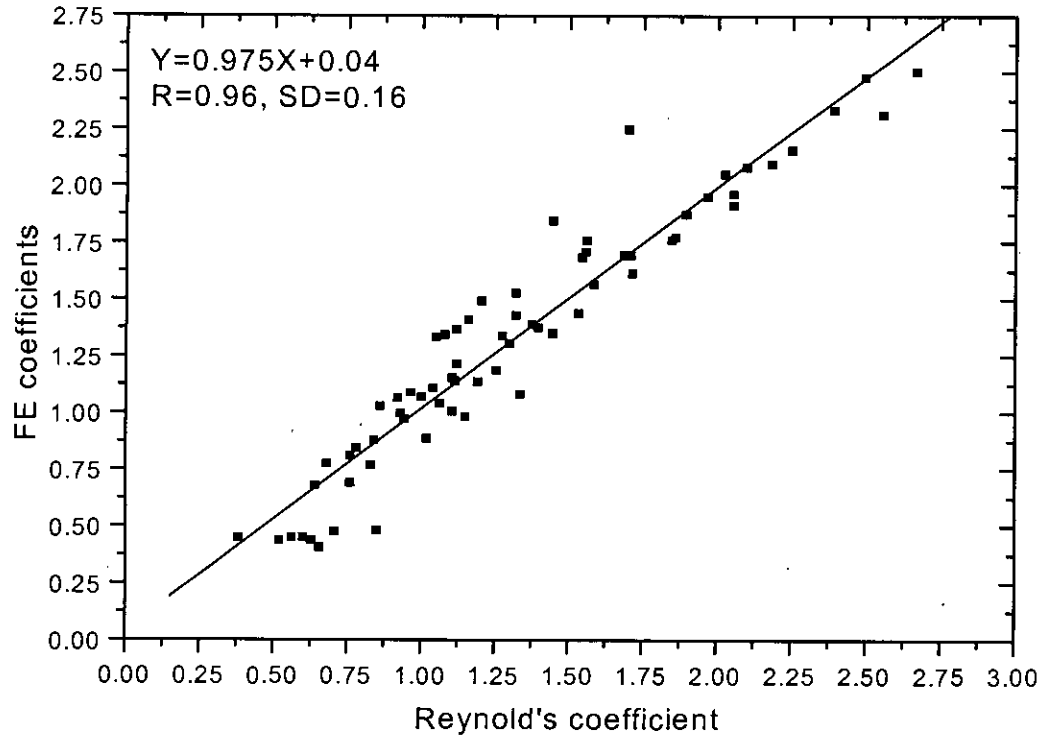


Fig. 4.7 Regression between FE and Reynold's coefficients (k_2)

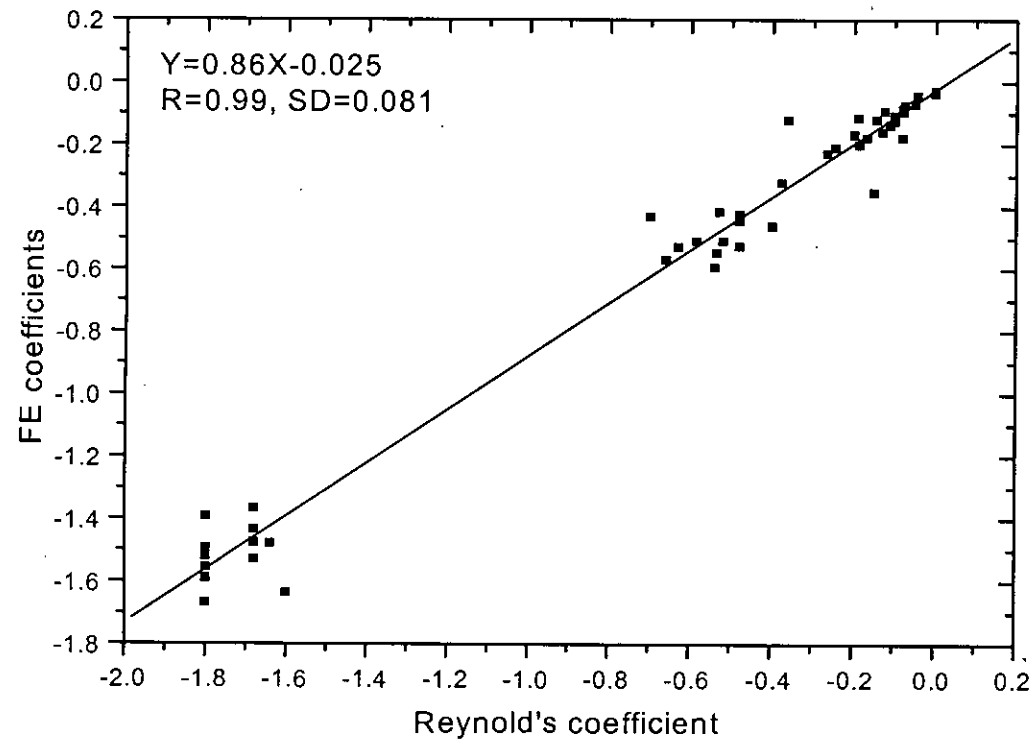


Fig. 4.8 Regression between FE and Reynold's coefficients (k_3)

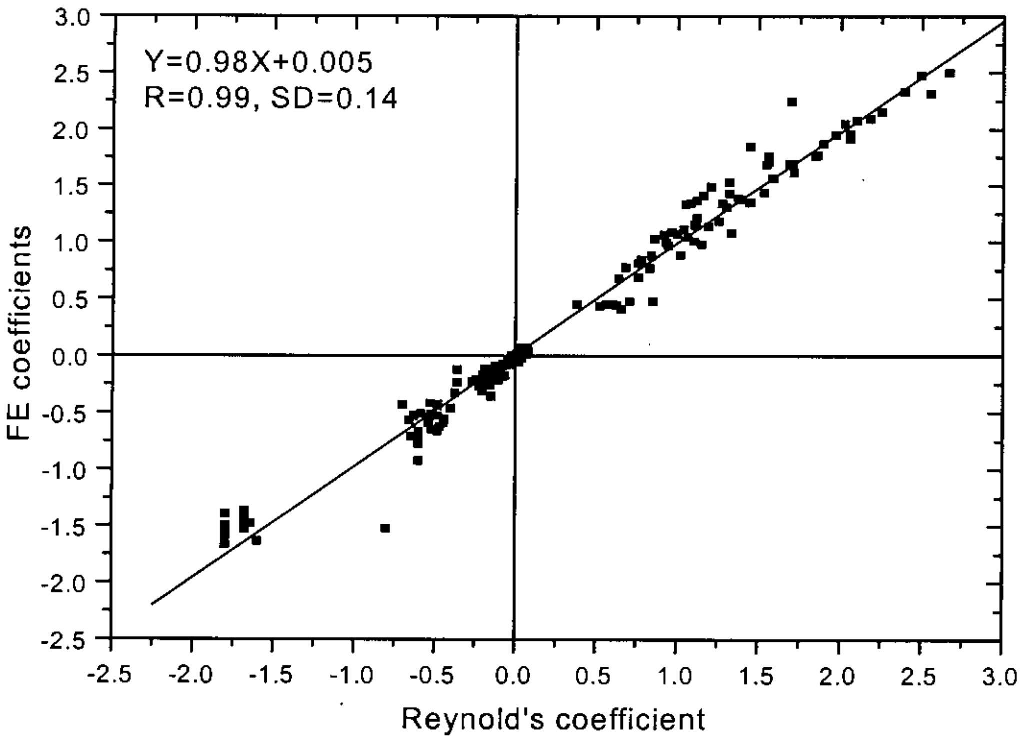


Fig. 4.9 Regression between FE and Reynold's coefficients (All)

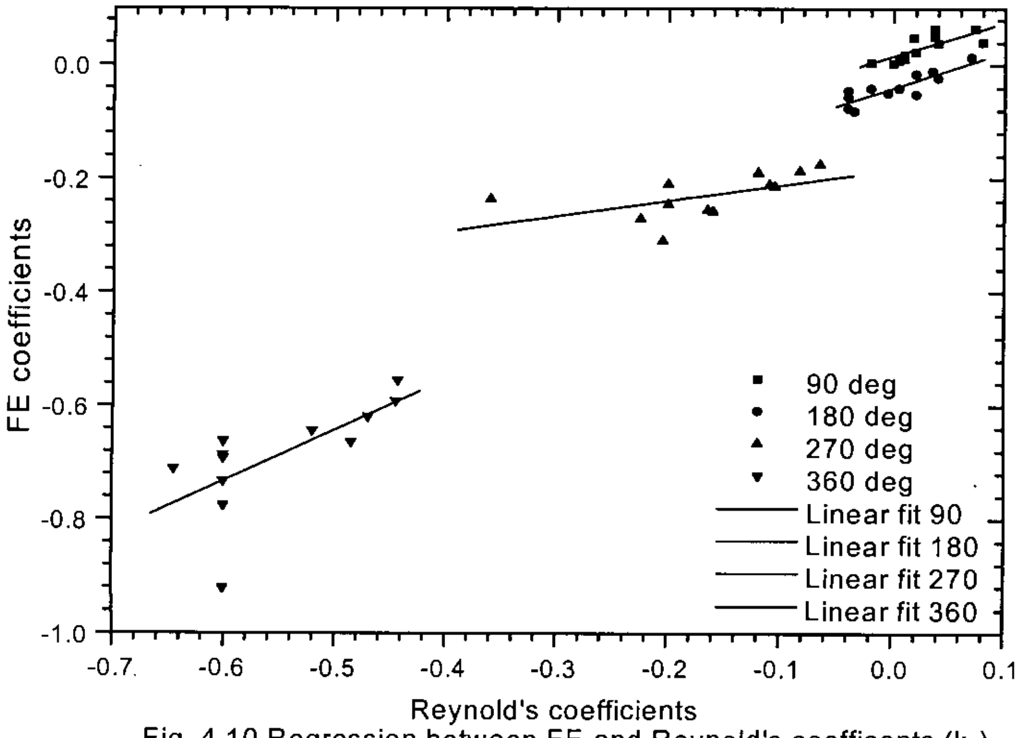


Fig. 4.10 Regression between FE and Reynold's coefficents (k_1)
Grouped according to central angle

The regression results for k_1 and k_2, their values being grouped according to central angles are presented in Figs. 4.10 to 4.12.

In addition to the regression analysis, both the Finite Element coefficients and the Reynold's coefficients have been plotted on the same graph in the similar form as Reynold's (Figs 4.13 through 4.24). Although this does not give a mathematical relation between the two, the plots are convincing that these coefficients conform each other in many cases, specially for the k_2 values.

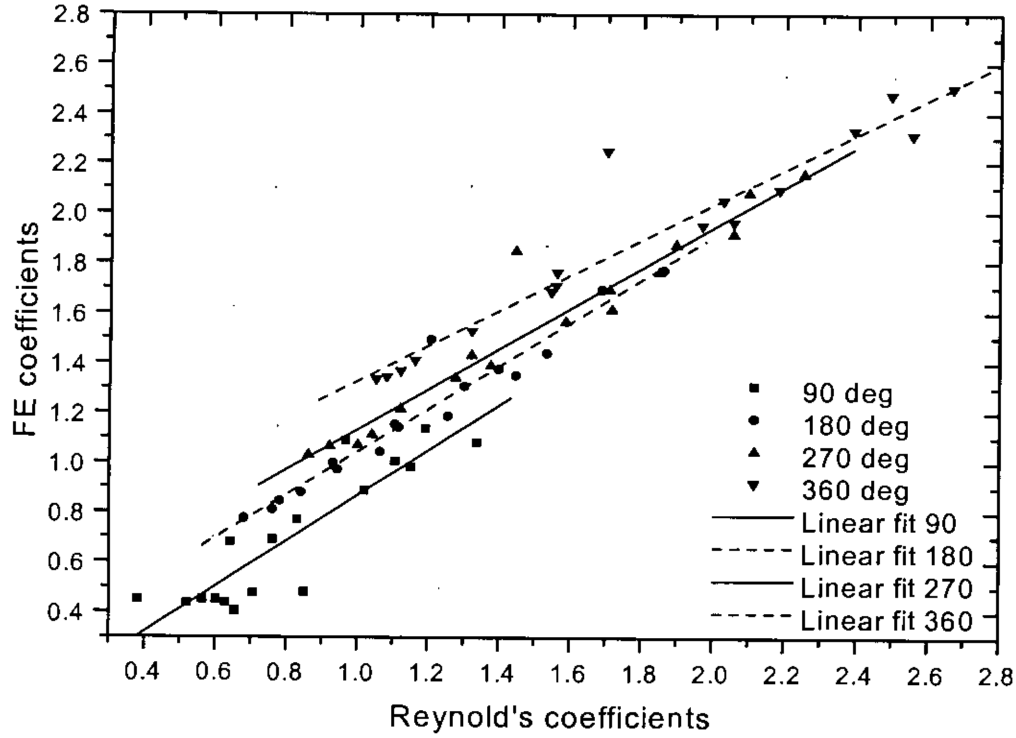


Fig. 4.11 Regression between FE and Reynold's coefficents (k_2)
Grouped according to central angle

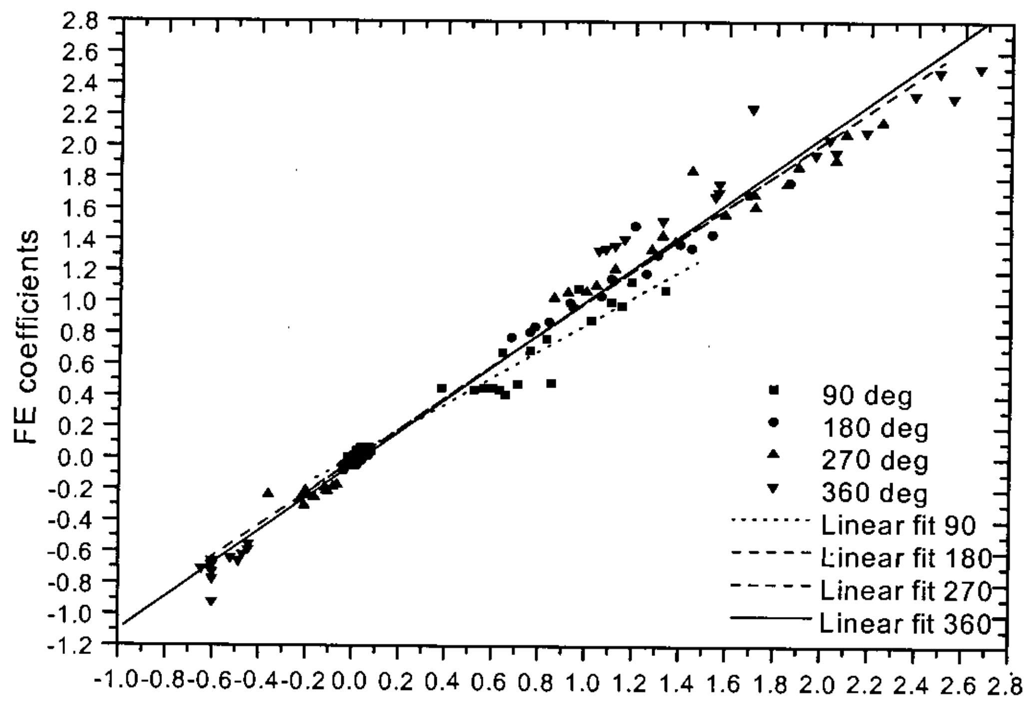


Fig. 4.12 Regression between FE and Reynold's coefficents (k_1 & k_2)
Grouped according to central angle

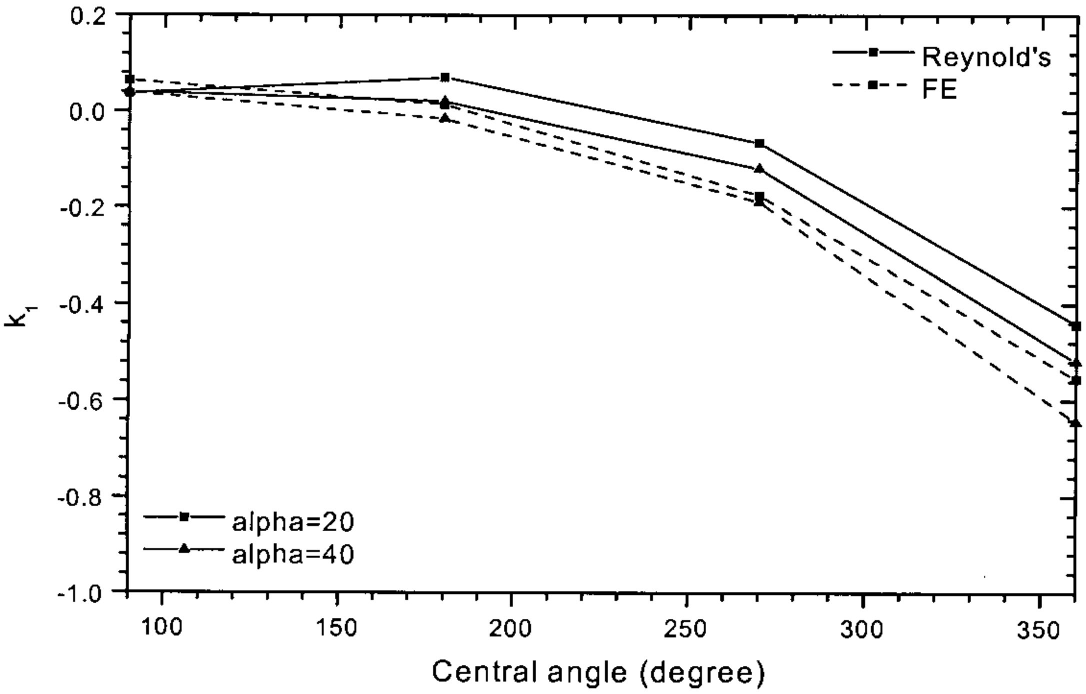


Fig 4.13 Comparison between Reynold's and FE coefficients: k_1
[b/h=5, R_1/R_2=1.05]

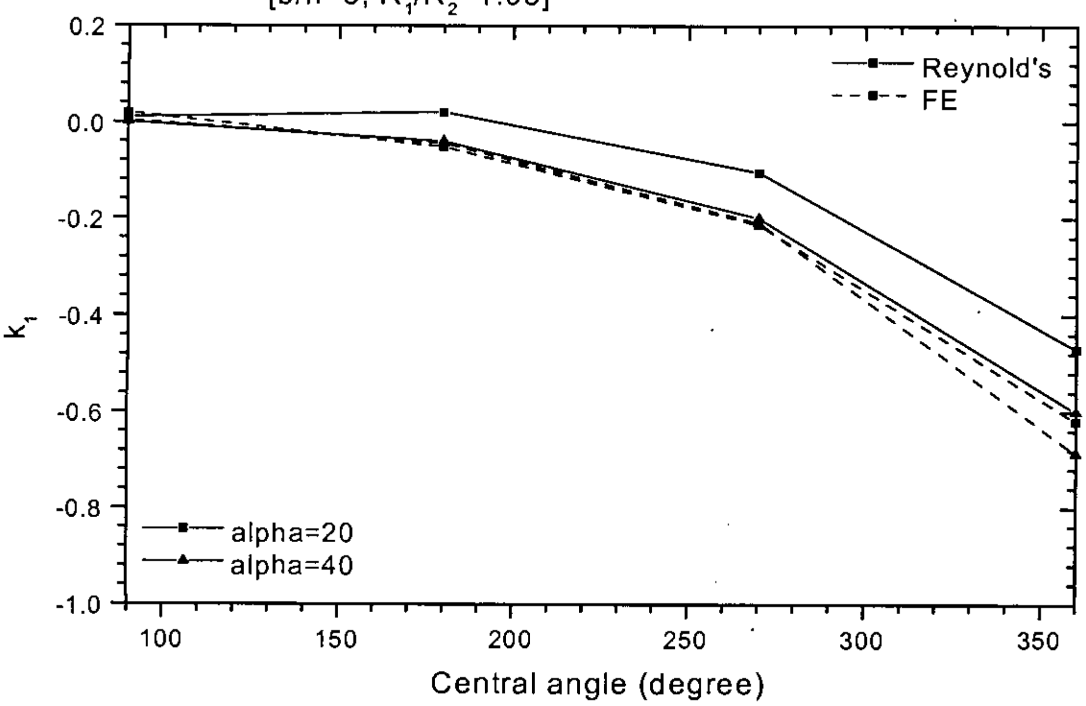


Fig 4.14 Comparison between Reynold's and FE coefficients: k_1
[b/h=13, R_1/R_2=1.05]

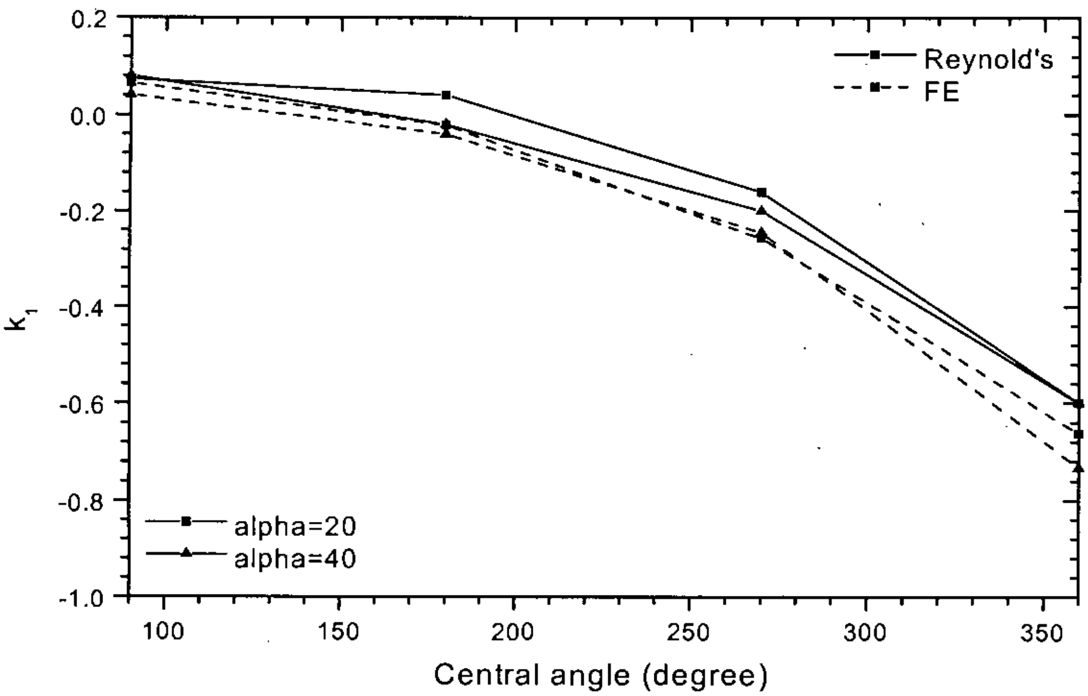


Fig 4.15 Comparison between Reynold's and FE coefficients: k_1 [b/h=5, R_1/R_2=1.1]

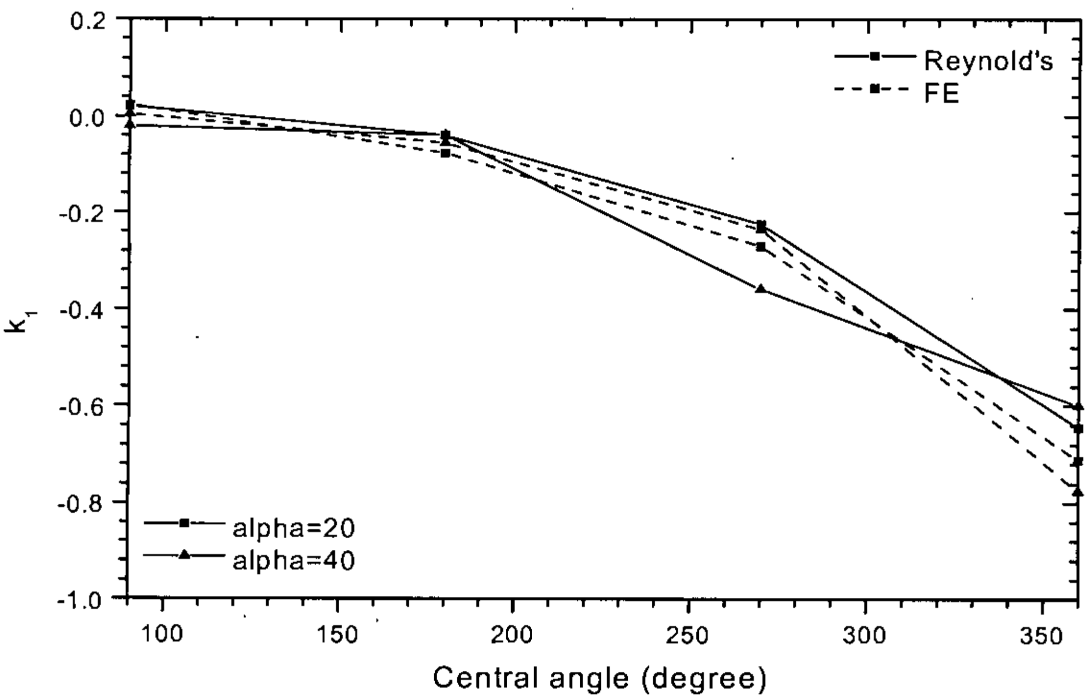


Fig 4.16 Comparison between Reynold's and FE coefficients: k_1 [b/h=13, R_1/R_2=1.1]

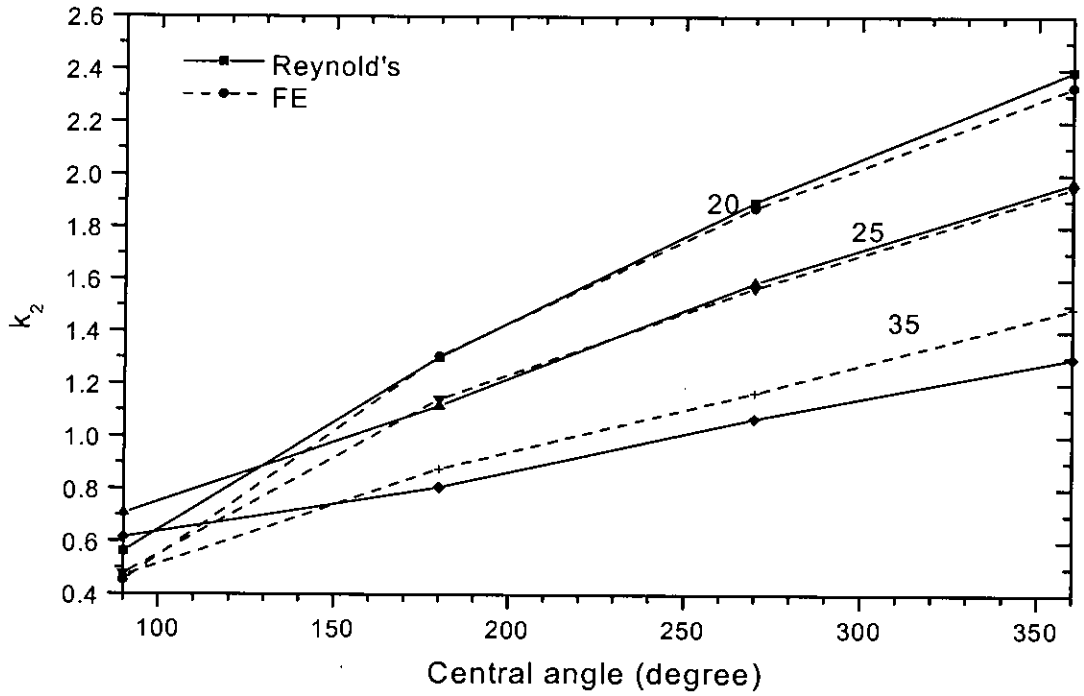


Fig 4.17 Comparison between Reynold's and FE coefficients: k_2
[b/h=5, R_1/R_2=1.05]

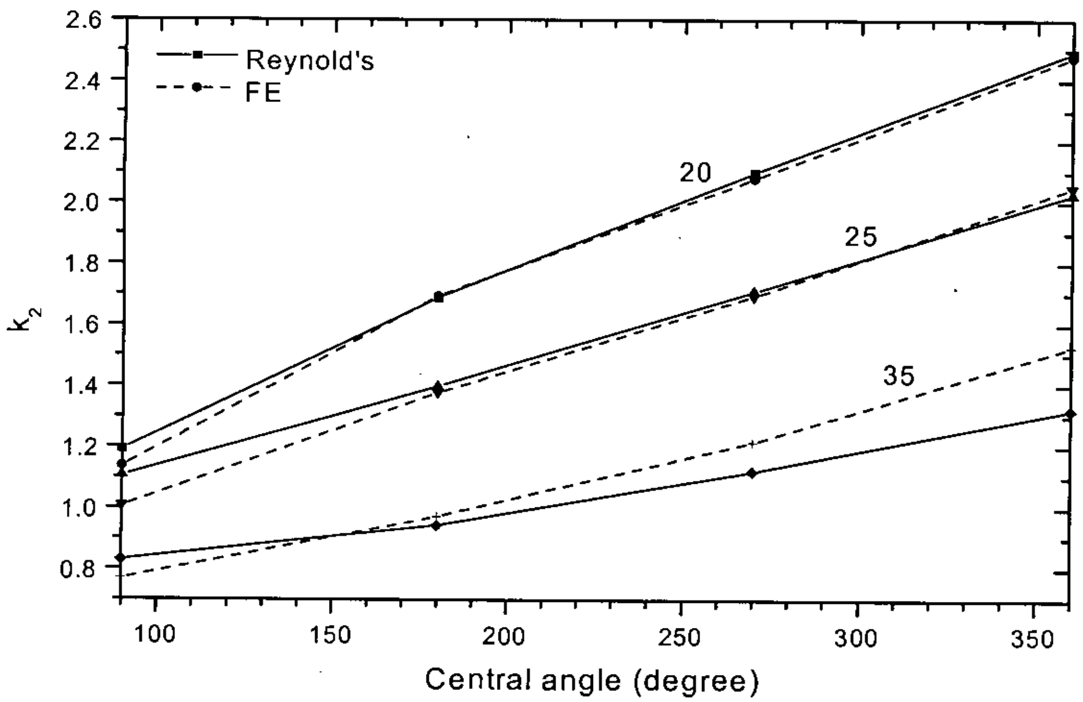


Fig 4.18 Comparison between Reynold's and FE coefficients: k_2
[b/h=13, R_1/R_2=1.05]

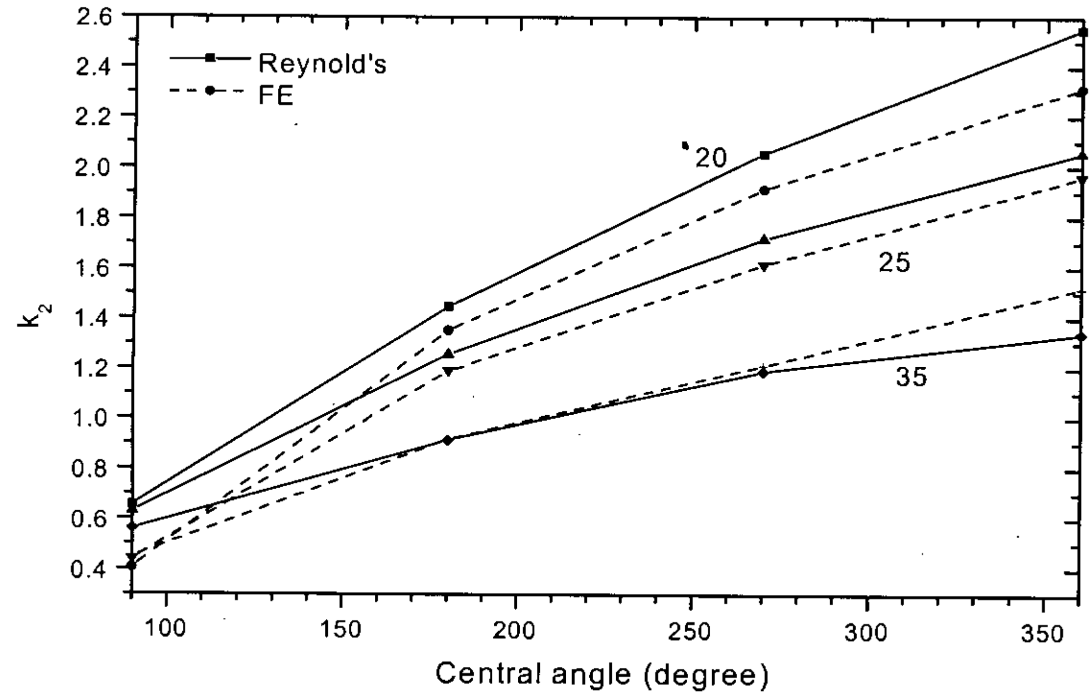


Fig 4.19 Comparison between Reynold's and FE coefficients: k_2
[$b/h=5$, $R_1/R_2=1.1$]

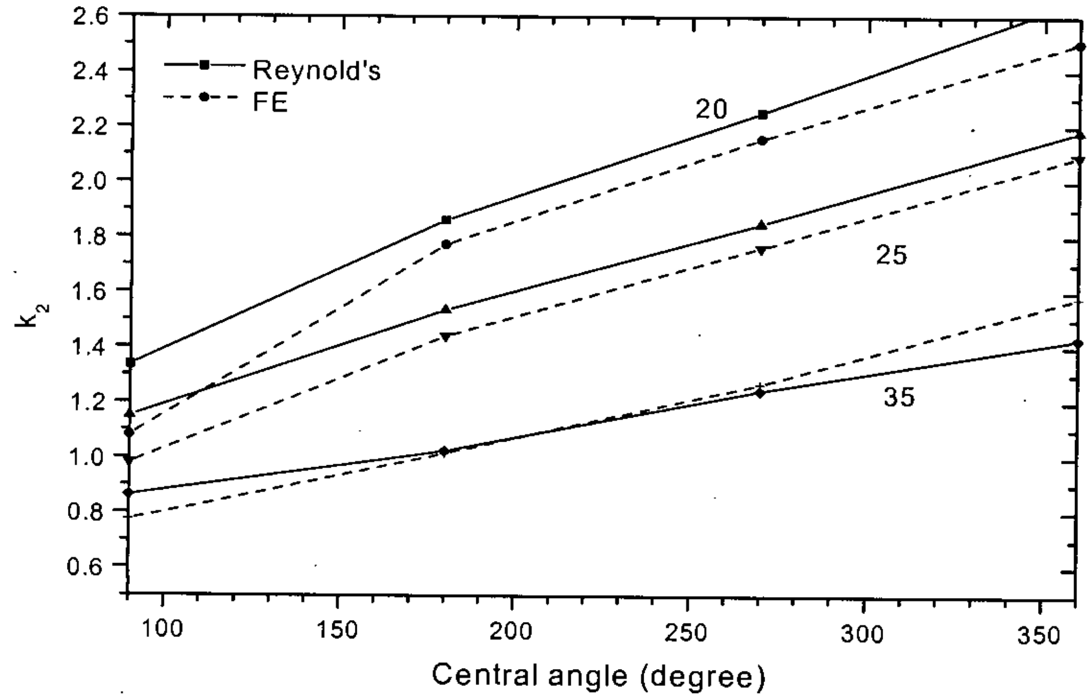


Fig 4.20 Comparison between Reynold's and FE coefficients: k_2
[$b/h=13$, $R_1/R_2=1.1$]

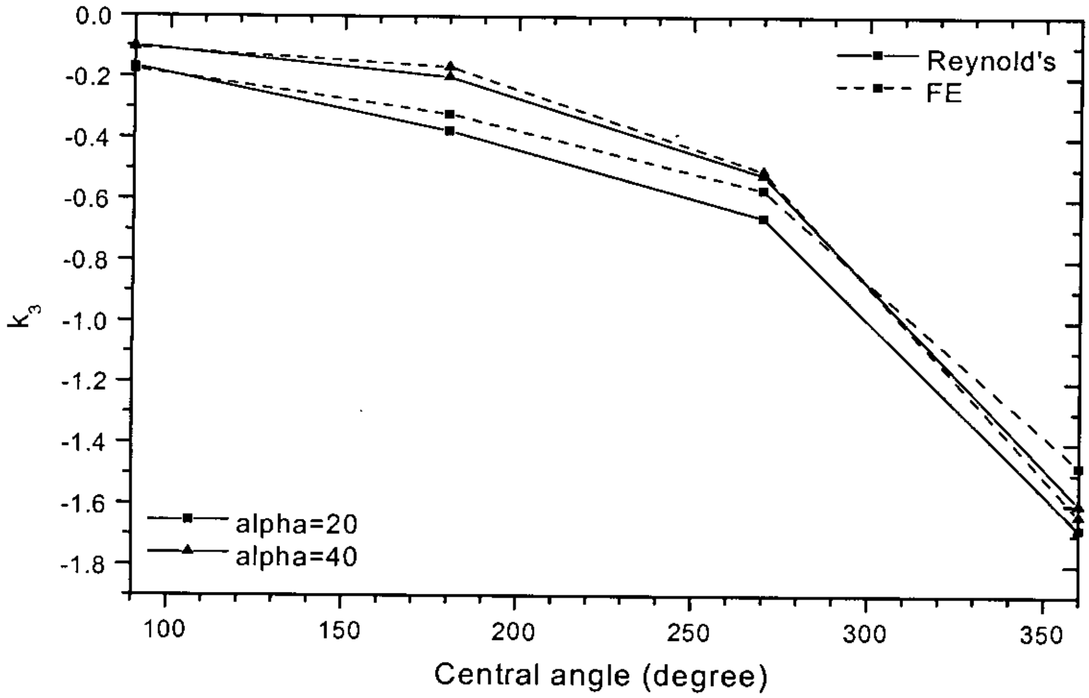


Fig 4.21 Comparison between Reynold's and FE coefficients: k_3
[$b/h=5$, $R_1/R_2=1.05$]

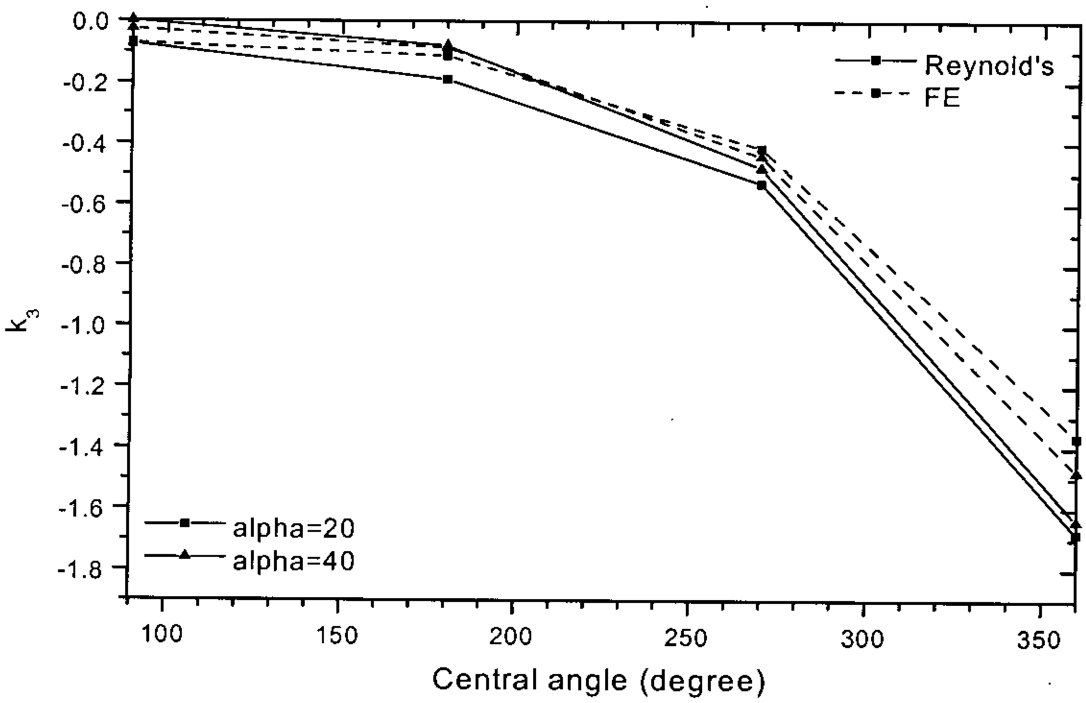


Fig 4.22 Comparison between Reynold's and FE coefficients: k_3
[$b/h=13$, $R_1/R_2=1.05$]

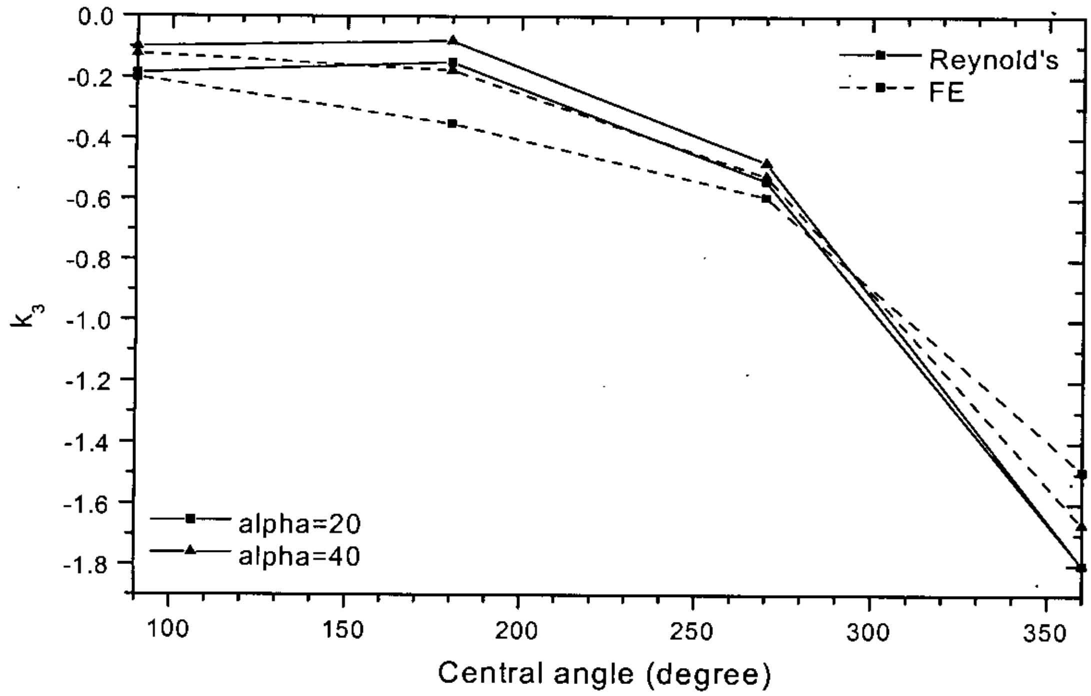


Fig 4.23 Comparison between Reynold's and FE coefficients: k_3
[b/h=5, R_1/R_2=1.1]

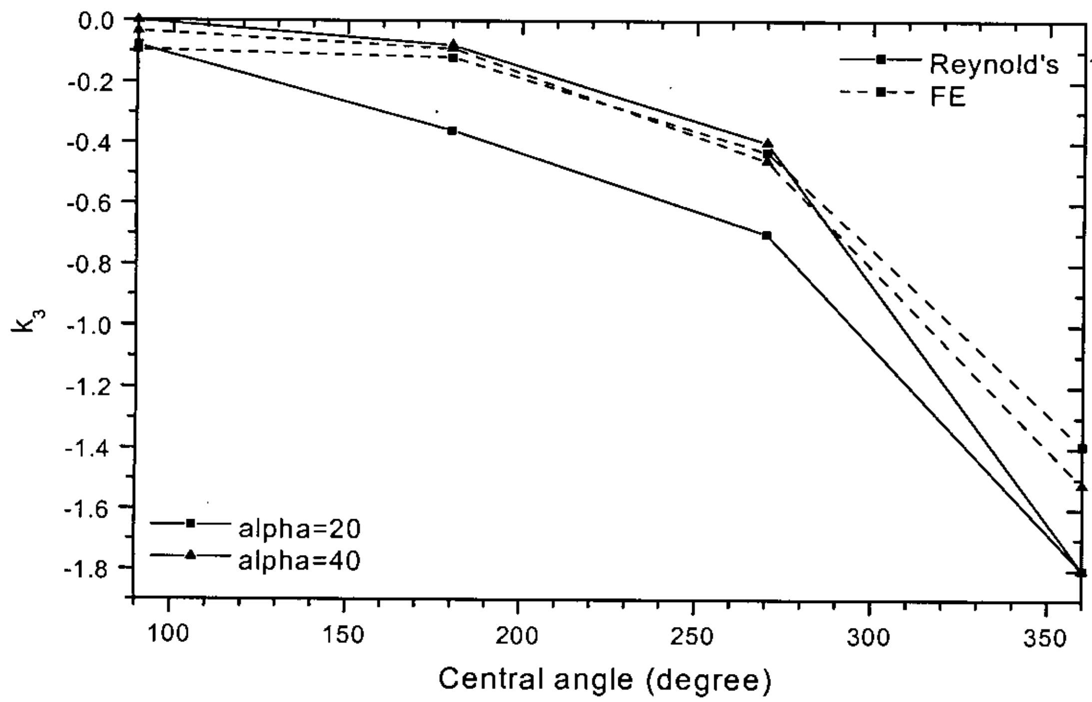


Fig 4.24 Comparison between Reynold's and FE coefficients: k_3
[b/h=13, R_1/R_2=1.1]

CHAPTER 5

DESIGN OF RCC HELICOIDAL STAIRS

5.1 INTRODUCTION

Shell structures often only require the minimum reinforcement for temperature and shrinkage, as specified by the ACI. It has been inferred from past studies at BUET that the helicoidal slab may also show substantial shell behaviour and may require only the temperature and shrinkage steel. In this light, a study has been undertaken to compare the temperature steel requirement and total steel requirement at the support in a helicodial stair slab. The work has been enhanced further to propose a design chart to calculate steel directly. It is expected that the use of the charts will make it easier for the designer to calculate reinforcement for a RCC helicodial stair slab.

5.2 METHODOLOGY

A number of prototype stairs have been analysed by the finite element methods to check if the temperature and shrinkage reinforcements govern or not. A live load of 100 psf has been considered, as most design codes recommend the use of 100 psf live load. The results of the design process suggest the temperature and shrinkage reinforcement may or may not govern at the support.

Focus is then switched to determine the governing force/moment parameter. Reynold's chart has been used to determine stress resultants and then steel requirements. It has been found that the lateral moment has a much higher value as compared to torsion or vertical moment. Because the available moment arm is much higher to resist the lateral moment, it cannot be readily concluded as to which type of moment governs the design. However the placement of steel is different for lateral and vertical moment. While reinforcement for lateral moment is to be placed concentrated nearer the inner or outer edge, that required for vertical moment is to be placed throughout the width of stair slab.

Because the vertical moment is maximum at the support, the steel requirement at support for vertical moment has been calculated. Also, the steel required for tension is placed in the similar manner as that for vertical moment. To expedite the design process, an investigation has been taken up to find the total steel requirement at the support. An Excel worksheet has been prepared in this connection.

Next, the possibility of developing a direct design chart to determine the steel area has been investigated. Accordingly a chart has been proposed.

5.3 THE DESIGN PHILOSOPHY

5.3.1 Slab Thickness:

The charts provided by Santathadaporn and Cusens (1966), and Reynolds (1988) provide the user with the design forces at different sections. Study reveals that the critical parameters are vertical moment and thrust at support, lateral moment at the end quarter span, maximum torsion (critical location varies widely), maximum radial horizontal shear either at mid span or end quarter span and lateral shear at support. Because the maximum radial horizontal shear has always been found to be greater than the maximum lateral shear for all practical cases, only maximum radial horizontal shear can be considered. Also no shear reinforcement is expected to be placed in the slab. Therefore, the radial horizontal shear and torsion requirements will govern the thickness of the slab. Deflection of the slab should not be neglected as well. Having determined the slab thickness, the steel area determination comes into action.

5.3.2 Axial Force and Vertical Moment:

The upper half of a helicodial stair is subject to axial tension. Therefore reinforcements are to be provided to carry this tension. As concrete is weak in tension, the entire tensile force is to be taken by the steel. The axial force at different sections can be found from Eq. 3.35.

Mid span radial horizontal shear H can be found from the charts provided by Santathadaporn, Cusens and Reynolds. Substituting the value of H ($=k_2wR_2$) in Eq. 3.35,

$$N = (-k_2\sin\theta\cos\alpha - R_1\theta\sin\alpha/R_2)wR_2$$

$$\Rightarrow \quad N = k_4wR_2 \qquad 5.1$$

Where k_4 is a function of k_2, slope of the slab, central angle subtended by the stair on plan and the ratio R_1/R_2. Once the axial force is known, the steel area becomes a function of this force and steel yield strength. Because the entire tension is to be carried by the reinforcement, concrete crushing strength or cross sectional area of the slab bears no significance. The USD method proposes that the steel required for a given tension N can be expressed as

$$A_s = \frac{N}{0.9.f_y} \qquad 5.2$$

where, 0.9 is the strength reduction factor as suggested by the ACI. Eqs. 5.1 and 5.2 combined together give rise to:

$$A_{sT} = s_1wR_2 \qquad 5.3$$

Here, s_1 is a function of k_4 and steel yield strength. Because 60,000 psi steel is now widely used, only this type of steel has been considered. Thus steel yield strength is eliminated as a variable. And s_1 becomes a direct function of k_4 only. An excel worksheet has been prepared in order to develop a chart that would give the values of s_1 directly, considering all the factors that affect its values. The result of the calculations is presented in a chart from (Figs. 5.1 & 5.2). As stated before, USD design method has been followed, and the strength reduction factor is incorporated within the value of s_1. The steel area required for tension can then be directly found out from the chart, when the unit load w and mean radius R_2 is known. Because this is an empirical equation, the units must be strictly followed. The unit of R_2 is ft. Unit of w could be psf, if steel area per ft width of the stair is to be determined, or it could be plf is the total steel requirement for the entire cross section is to be determined. The area of steel is given in in^2 or in^2/ft, depending on the w value used.

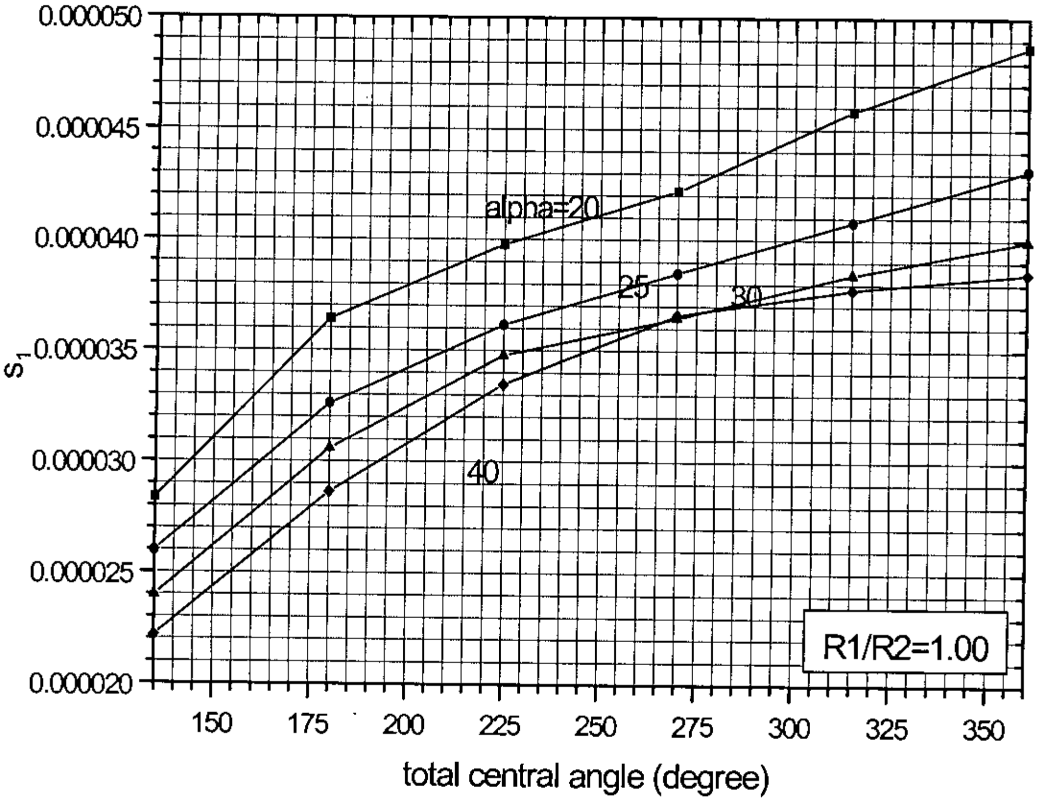


Fig. 5.1 Coefficient s_1 for R_1/R_2=1.00, f_y=60 ksi

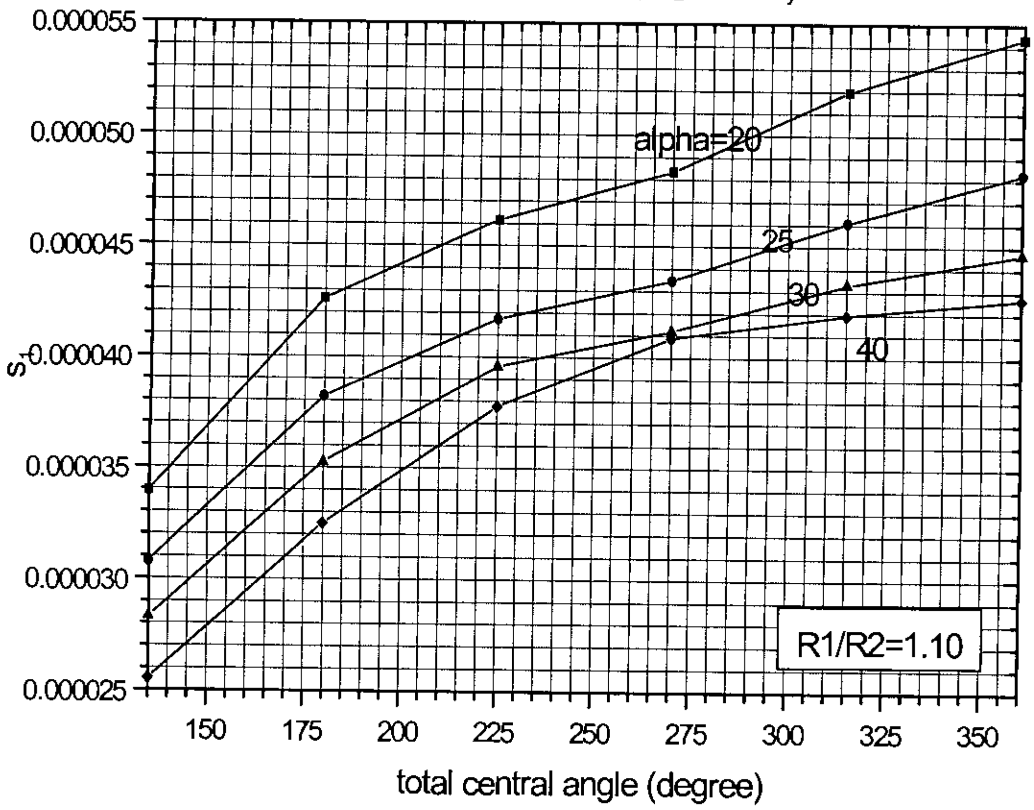


Fig. 5.2 Coefficient s_1 for R_1/R_2=1.10, f_y=60 ksi

The vertical moment at support as expressed by Reynolds is given in Eq. 3.50. The steel area required for moment is a function of the vertical moment at support and therefore a function of w and R_2^2. The steel area is also a function of steel yield strength and concrete crushing strength. With 0.9 as the strength reduction factor,

$$A_{sVM} = \frac{M_v}{0.9 f_y (d - a/2)} \qquad 5.4$$

where,

d = effective depth

a= depth of equivalent rectangular compression zone = $A_{sVM} f_y / 0.85 f'_c b$

Because Eq. 5.4 cannot be solved at one step, to simplify the derivation of chart, some assumptions have been necessary. It has been found that for 3,000 psi concrete and 60,000 psi steel, the term (d-a/2) varies from 0.97d to 0.88d. An average value of 0.92d has been assumed to simplify Eq. 5.4. Taking strength reduction factor bo be equal to 0.9 and steel yield strength to be 60,000 psi, Eq. 5.4 becomes

$$A_{sVM} = \frac{M_v}{4.15d} \qquad 5.5$$

Here, M_v is in k-ft and d in inches. Combining Eqs. 5.5 and 3.50, along with the simplifications, a formula can be derived as follows:

$$A_{sVM} = s_2 w R_2^2 \qquad 5.6$$

Where s_2 will be a function of k_3, concrete crushing strength and steel yield strength and obviously the effective depth of slab. As seen from the Reynold's charts, k_3 is a function of R_1/R_2, slope of helix α, central angle and b/h ratio.

It has been found that the variation of k_3 with b/h ratio is not very high. Moreover, the slab b/h ratio is expected to be nearer to 10 than 5 for all practical cases. This is why only one b/h ratio has been considered and thus, s_2 becomes independent of b/h ratio for the present study.

For the reason mentioned previously, only 60,000 psi steel was considered. Taking the cover for reinforcement to be equal to 1", the effective depth of the slab can be expressed in terms of slab thickness. Therefore in the end, the coefficient s_2 becomes a function of:

I. R_1/R_2 ratio
II. Slab thickness
III. Slope of helix centreline, α
IV. Central angle, and
V. Concrete strength

An excel worksheet has been prepared to develop a chart to find the values of s_2. In the worksheet the coefficient s_2 has been determined following the USD method as suggested by the ACI. The strength reduction factor is incorporated within the s_2 values. Figs. 5.3 and 5.4 have been initially developed. A closer look suggests that the effect of slope of the stair slab is very small and can be neglected for all practical purposes. This reduces the number of variables further by one. Figs. 5.5 and 5.6 are the final forms of the charts: these can be readily used to determine steel area (in in^2), for a given load of w and mean radius of R_2. Once again, the units must be strictly followed. The unit of R_2 is ft. Unit of w could be psf, if steel area per ft width of the stair is to be determined, or it could be plf is the total steel requirement for the entire cross section is to be determined.

Once s_1 and s_2 are calculated from the appropriate charts, the total steel requirement at the support can be expressed as:

$$A_{stot} = s_1 w R_2 + s_2 w R_2^2 \qquad 5.5$$

This steel has to be compared with the minimum temperature and shrinkage reinforcement. Whichever is larger, will govern the design. The maximum limit of reinforcement must also be kept in mind for stairs with larger mean radius or central angle. In these cases, the steel area from the charts may exceed the ACI code requirement of 3/4 times the balanced steel ratio. Therefore a check must be made against this criteria. In case the steel requirement becomes larger, it is suggested to

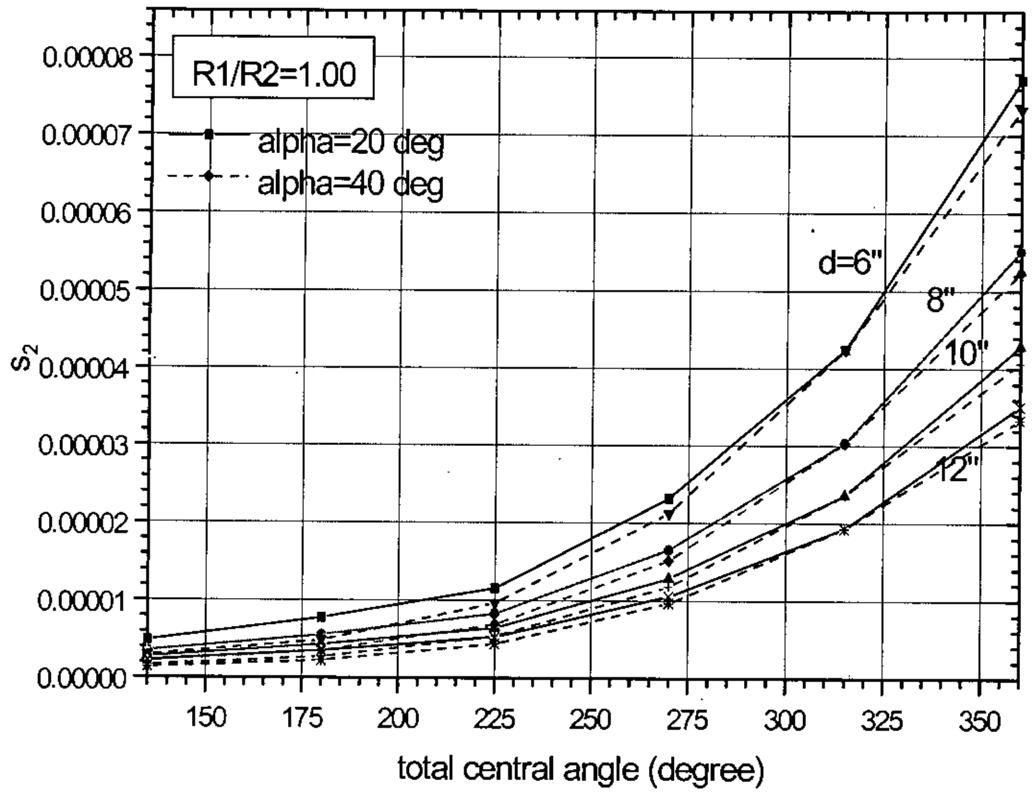


Fig 5.3 Coefficient s_2 for R_1/R_2=1.00, f_y=60 ksi, f_c'=3 ksi

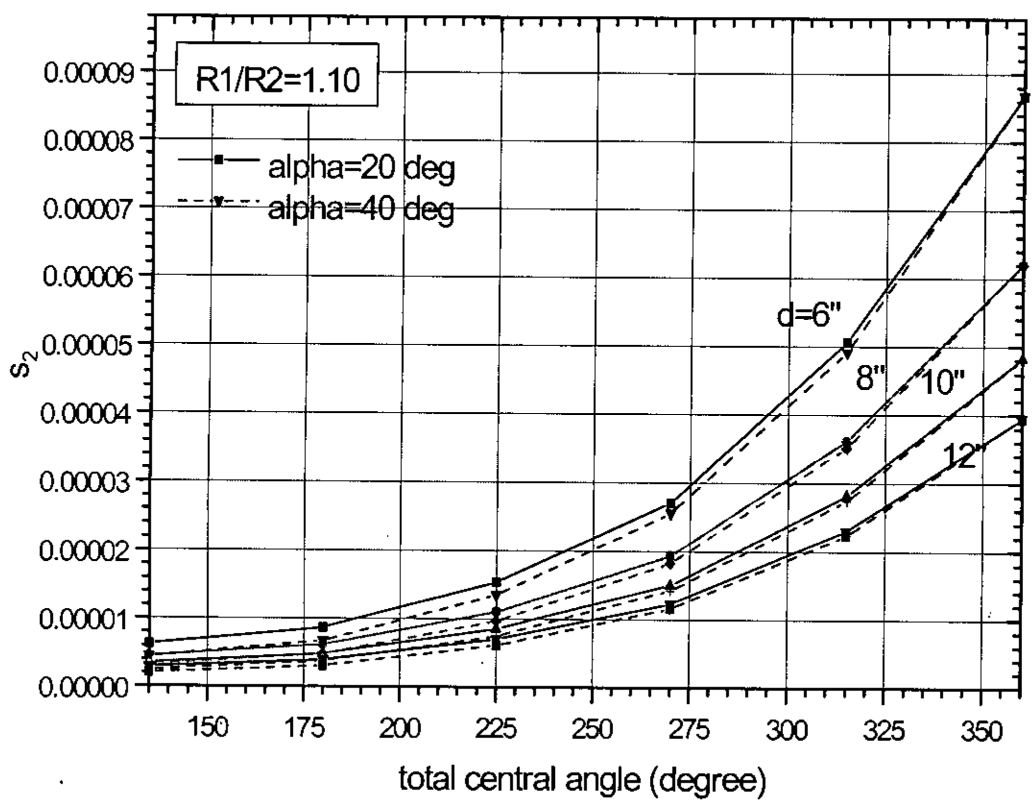


Fig 5.4 Coefficient s_2 for R_1/R_2=1.10, f_y=60 ksi, f_c'=3 ksi

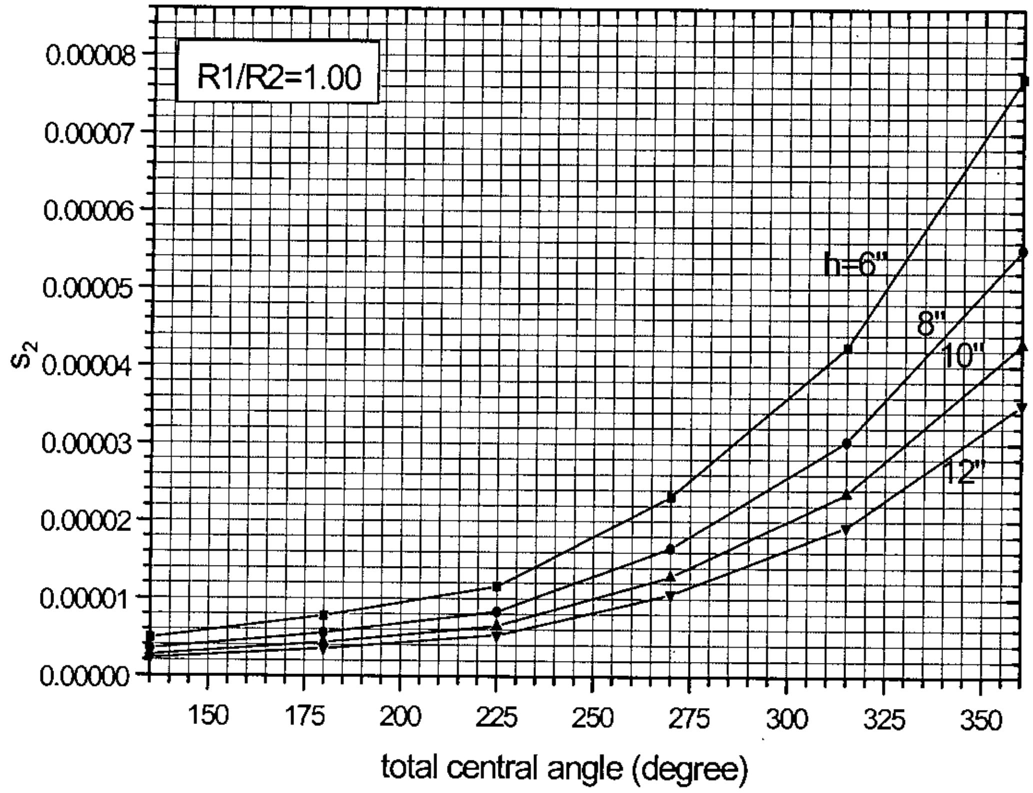


Fig. 5.5 Coefficient s_2 for R_1/R_2=1.0, f_y=60 ksi, f_c'=3 ksi

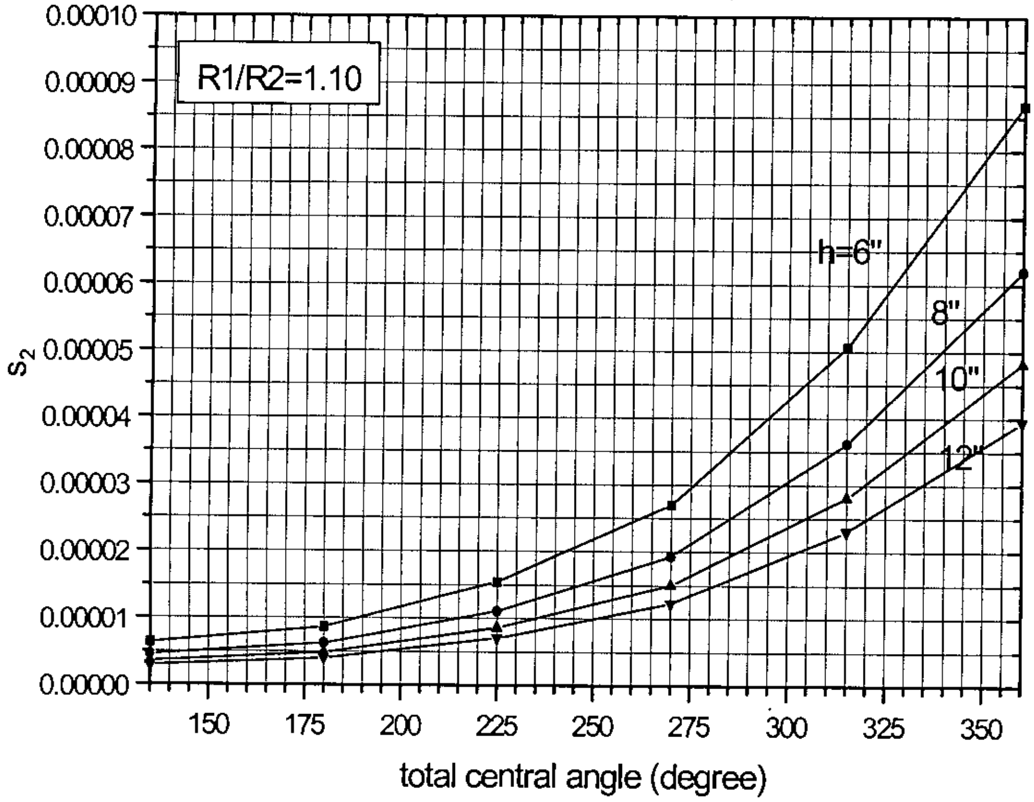


Fig. 5.6 Coefficient s_2 for R_1/R_2=1.10, f_y=60 ksi, f_c'=3 ksi

opt for a higher thickness, as slabs will not be designed as doubly reinforced RCC elements.

In addition to these charts, Tables 5.1 and 5.2 are provided. These two tables give the designer directly the steel area required per ft width of the slab to accommodate axial force and vertical moment for a live load of 100 psf and concrete unit weight of 150 pcf. Steel yield strength is 60,000 psi, concrete crushing strength 3,000 psi. The tables have been derived considering the lower limit of temperature and shrinkage requirement and upper limit of ½ balanced steel ratio.

5.3.3 Lateral Moment:

Having decided on the steel requirement for axial force and vertical moment, now the steel requirement for the lateral moment should be determined. In keeping with the previous philosophy, maximum lateral moment can also be expressed as:

$$M_h = k_5 w R_2^2 \tag{5.6}$$

Here, k_5 is a function of R_1/R_2, slope of helix α, central angle and b/h ratio. A chart for k_5 has been proposed in Figs. 5.7 and 5.8. The maximum lateral moment occurs at the region between support and end quarter span. There the reinforcement is to be provided up to quarter span from the support. Past the point of quarter span, the lateral moment distribution can be taken as linear and curtailment can be made accordingly. For ease of use, the k_5 values have been determined from the finite element analysis.

5.4 SUMMARY

The design of the RCC helicoidal stair slab may follow the following steps:

I. To find the geometric parameters: R_1/R_2, slope of helix α, central angle.
II. To determine the thickness of the slab (h) on the basis of torsion and shear requirements. These values can be found from Reynold's charts. Deflection criteria should also be checked.

Table 5.1: Steel area (sq. in.) recommended per ft width of slab (USD)

Steel yield strength 60 ksi, concrete crushing strength 3 ksi

Live load 100 psf, concrete unit weight 150 pcf, R1/R2 = 1.00

Thicker slab recommended for the shaded values

Italics indicate temperature & shrinkage governs

Slope	Central angle	R2= 60" 6" slab	R2= 60" 8" slab	R2= 60" 10" slab	R2= 60" 12" slab	R2= 90" 6" slab	R2= 90" 8" slab	R2= 90" 10" slab	R2= 90" 12" slab
20	135	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.15844	*0.1728*	*0.216*	*0.2592*
20	180	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.23145	0.21127	*0.216*	*0.2592*
20	225	0.15982	*0.1728*	*0.216*	*0.2592*	0.31075	0.27658	0.26223	*0.2592*
20	270	0.2585	0.22623	*0.216*	*0.2592*	0.5298	0.45165	0.41316	0.39269
20	315	0.42225	0.35757	0.3252	0.30751	0.8938	0.74225	0.6634	0.6176
20	360	0.71104	0.58737	0.52246	0.48424	1.54005	1.25541	1.10296	1.01059
25	135	*0.1296*	*0.1728*	*0.216*	0.2592	0.15256	*0.1728*	*0.216*	*0.2592*
25	180	*0.1296*	*0.1728*	*0.216*	0.2592	0.22213	0.20094	*0.216*	*0.2592*
25	225	0.15389	*0.1728*	*0.216*	0.2592	0.30185	0.26673	0.25143	*0.2592*
25	270	0.25242	0.21949	*0.216*	0.2592	0.52067	0.44155	0.40208	0.38064
25	315	0.41406	0.34849	0.31525	0.29669	0.8815	0.72864	0.64847	0.60135
25	360	0.70196	0.57732	0.51143	0.47225	1.52643	1.24034	1.08643	0.9926
30	135	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.1476	*0.1728*	*0.216*	*0.2592*
30	180	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.21717	0.19545	*0.216*	*0.2592*
30	225	0.15164	*0.1728*	*0.216*	*0.2592*	0.29849	0.263	0.24735	*0.2592*
30	270	0.24913	0.21585	*0.216*	*0.2592*	0.51574	0.43609	0.39609	0.37412
30	315	0.41014	0.34416	0.3105	0.29152	0.87563	0.72214	0.64135	0.5936
30	360	0.69674	0.57155	0.5051	0.46536	1.5186	1.23167	1.07693	0.98227
35	135	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.14533	*0.1728*	*0.216*	*0.2592*
35	180	0.1296	*0.1728*	*0.216*	*0.2592*	0.21396	0.1919	*0.216*	*0.2592*
35	225	0.14996	*0.1728*	*0.216*	*0.2592*	0.29597	0.26022	0.24429	*0.2592*
35	270	0.24777	0.21435	*0.216*	*0.2592*	0.5137	0.43383	0.39362	0.37143
35	315	0.40797	0.34175	0.30786	0.28864	0.87237	0.71853	0.63739	0.58929
35	360	0.69443	0.56899	0.50229	0.46231	1.51514	1.22783	1.07272	0.97769
40	135	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.14318	*0.1728*	*0.216*	*0.2592*
40	180	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.21237	0.19014	*0.216*	*0.2592*
40	225	0.1495	*0.1728*	*0.216*	*0.2592*	0.29527	0.25944	0.24344	*0.2592*
40	270	0.24929	0.21603	*0.216*	*0.2592*	0.51598	0.43635	0.39638	0.37444
40	315	0.40906	0.34297	0.30918	0.29009	0.87401	0.72034	0.63938	0.59146
40	360	0.69433	0.56888	0.50218	0.46218	1.51499	1.22768	1.07254	0.9775

Table 5.1 (contd): Steel area (sq. in.) recommended per ft width of slab (USD)

Steel yield strength 60 ksi, concrete crushing strength 3 ksi

Live load 100 psf, concrete unit weight 150 pcf, R1/R2 = 1.00

Thicker slab recommended for the shaded values

Italics indicate temperature & shrinkage governs

Slope	Central	R2=120"				R2=150"			
	angle	6" slab	8" slab	10" slab	12" slab	6" slab	8" slab	10" slab	12" slab
20	135	0.25072	0.2276	0.21916	*0.2592*	0.36272	0.32349	0.30721	0.30137
20	180	0.37174	0.3316	0.31497	0.30902	0.54359	0.4769	0.44692	0.43364
20	225	0.50903	0.44364	0.4135	0.39938	0.75466	0.64814	0.59669	0.57028
20	270	0.8958	0.75194	0.67859	0.63728	1.35649	1.12711	1.00788	0.93871
20	315	1.53897	1.26419	1.11867	1.03189	2.35774	1.9234	1.69101	1.55041
20	360	2.68473	2.17302	1.89632	1.72643	4.14507	3.34021	2.90252	2.63174
25	135	0.24287	0.21891	*0.216*	*0.2592*	0.35291	0.31264	0.29531	0.28842
25	180	0.3593	0.31784	0.29987	0.29259	0.52804	0.45969	0.42805	0.41311
25	225	0.49717	0.43051	0.3991	0.38371	0.73984	0.63172	0.57869	0.55069
25	270	0.88363	0.73847	0.66382	0.62121	1.34128	1.11027	0.98941	0.91863
25	315	1.52257	1.24604	1.09877	1.01024	2.33725	1.90071	1.66613	1.52334
25	360	2.66657	2.15292	1.87428	1.70244	4.12237	3.31508	2.87497	2.60177
30	135	0.23626	0.21159	*0.216*	*0.2592*	0.34464	0.30349	0.28528	0.2775
30	180	0.35269	0.31052	0.29184	0.28385	0.51977	0.45054	0.41802	0.40219
30	225	0.49268	0.42554	0.39365	0.37779	0.73423	0.62552	0.57189	0.54329
30	270	0.87705	0.73119	0.65583	0.61252	1.33306	1.10117	0.97943	0.90777
30	315	1.51474	1.23738	1.08927	0.9999	2.32746	1.88988	1.65425	1.51042
30	360	2.65613	2.14137	1.8616	1.68866	4.10932	3.30064	2.85913	2.58453
35	135	0.23324	0.20825	*0.216*	*0.2592*	0.34087	0.29931	0.28069	0.27251
35	180	0.34841	0.30578	0.28665	0.2782	0.51442	0.44462	0.41153	0.39513
35	225	0.48932	0.42183	0.38958	0.37335	0.73003	0.62087	0.56679	0.53774
35	270	0.87433	0.72818	0.65254	0.60893	1.32966	1.09741	0.97531	0.90328
35	315	1.51039	1.23256	1.08398	0.99415	2.32202	1.88386	1.64765	1.50324
35	360	2.65151	2.13625	1.85599	1.68255	4.10354	3.29424	2.85212	2.5769
40	135	0.23037	0.20508	*0.216*	*0.2592*	0.33729	0.29534	0.27634	0.26778
40	180	0.34629	0.30343	0.28408	0.2754	0.51177	0.44168	0.40831	0.39163
40	225	0.48839	0.42079	0.38844	0.37211	0.72886	0.61958	0.56537	0.5362
40	270	0.87737	0.73155	0.65622	0.61294	1.33346	1.10161	0.97992	0.90829
40	315	1.51258	1.23499	1.08664	0.99705	2.32476	1.88689	1.65097	1.50685
40	360	2.65132	2.13604	1.85576	1.6823	4.1033	3.29397	2.85183	2.57659

Table 5.2: Steel area (sq. in.) recommended per ft width of slab (USD)

Steel yield strength 60 ksi, concrete crushing strength 3 ksi

Live load 100 psf, concrete unit weight 150 pcf, R1/R2 = 1.10

Thicker slab recommended for the shaded values

Italics indicate temperature & shrinkage governs

Slope	Central angle	R2= 60"				R2= 90"			
		6" slab	8" slab	10" slab	12" slab	6" slab	8" slab	10" slab	12" slab
20	135	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.19876	0.1835	*0.216*	*0.2592*
20	180	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.26448	0.2422	0.2348	*0.2592*
20	225	0.20184	0.18348	*0.216*	*0.2592*	0.39747	0.3501	0.32917	0.32025
20	270	0.30014	0.26233	0.24509	*0.2592*	0.61593	0.52452	0.47938	0.45527
20	315	0.4994	0.42175	0.38265	0.36108	1.05983	0.87829	0.7835	0.72814
20	360	0.79928	0.66009	0.58699	0.54393	1.73161	1.41129	1.23967	1.13566
25	135	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.19098	0.17489	*0.216*	*0.2592*
25	180	0.13357	*0.1728*	*0.216*	*0.2592*	0.25362	0.23019	0.22162	*0.2592*
25	225	0.19457	0.17543	*0.216*	*0.2592*	0.38655	0.33802	0.31592	0.30584
25	270	0.29214	0.25348	0.23539	0.2592	0.60393	0.51125	0.46483	0.43943
25	315	0.48977	0.41109	0.37096	0.34835	1.04538	0.8623	0.76596	0.70905
25	360	0.78927	0.64901	0.57484	0.53071	1.71659	1.39466	1.22144	1.11582
30	135	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.18492	*0.1728*	*0.216*	*0.2592*
30	180	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.24637	0.22216	*0.216*	*0.2592*
30	225	0.19104	*0.1728*	*0.216*	*0.2592*	0.38126	0.33216	0.30949	0.29885
30	270	0.28842	0.24936	0.23087	*0.2592*	0.59835	0.50507	0.45805	0.43206
30	315	0.48517	0.406	0.36538	0.34228	1.03849	0.85467	0.75759	0.69995
30	360	0.78338	0.64249	0.56769	0.52293	1.70775	1.38488	1.21072	1.10415
35	135	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.18089	*0.1728*	*0.216*	*0.2592*
35	180	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.24214	*0.21748*	*0.216*	*0.2592*
35	225	0.18862	*0.1728*	*0.216*	*0.2592*	0.37762	0.32814	0.30508	0.29405
35	270	0.28642	0.24715	0.22844	*0.2592*	0.59535	0.50175	0.45441	0.4281
35	315	0.48191	0.40239	0.36142	0.33798	1.0336	0.84926	0.75166	0.6935
35	360	0.78043	0.63922	0.56411	0.51903	1.70332	1.37998	1.20535	1.09831
40	135	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.17809	*0.1728*	*0.216*	*0.2592*
40	180	*0.1296*	*0.1728*	*0.216*	*0.2592*	0.23959	0.21466	*0.216*	*0.2592*
40	225	0.18811	*0.1728*	*0.216*	*0.2592*	0.37686	0.32729	0.30416	0.29304
40	270	0.28791	0.2488	0.23025	*0.2592*	0.59759	0.50422	0.45712	0.43105
40	315	0.48298	0.40358	0.36272	0.33939	1.03521	0.85103	0.75361	0.69562
40	360	0.7801	0.63886	0.56371	0.5186	1.70283	1.37944	1.20475	1.09766

Table 5.2 (contd): Steel area (sq. in.) recommended per ft width of slab (USD)

Steel yield strength 60 ksi, concrete crushing strength 3 ksi

Live load 100 psf, concrete unit weight 150 pcf, R1/R2 = 1.10

Thicker slab recommended for the shaded values

Italics indicate temperature & shrinkage governs

Slope	Central	R2=120"				R2=150"			
	angle	6" slab	8" slab	10" slab	12" slab	6" slab	8" slab	10" slab	12" slab
20	135	0.31631	0.28522	0.27323	0.26992	0.4595	0.40722	0.38477	0.37589
20	180	0.42366	0.37909	0.36096	0.35485	0.61836	0.54406	0.51107	0.49685
20	225	0.65622	0.56663	0.52403	0.5028	0.97811	0.83307	0.76146	0.72324
20	270	1.0422	0.87406	0.78817	0.73967	1.57895	1.31095	1.17146	1.09039
20	315	1.82742	1.49862	1.32404	1.21955	2.80216	2.28272	2.00425	1.83531
20	360	3.01905	2.44325	2.13181	1.94055	4.66161	3.75597	3.26341	2.95861
25	135	0.30594	0.27374	0.26064	0.2592	0.44654	0.39287	0.36904	0.35877
25	180	0.40918	0.36307	0.34339	0.33573	0.60026	0.52403	0.4891	0.47295
25	225	0.64167	0.55052	0.50637	0.48358	0.95992	0.81294	0.73939	0.69922
25	270	1.02621	0.85636	0.76876	0.71855	1.55897	1.28883	1.1472	1.06399
25	315	1.80815	1.47729	1.30065	1.19411	2.77807	2.25606	1.97502	1.80351
25	360	2.99903	2.42108	2.10751	1.9141	4.63658	3.72827	3.23303	2.92556
30	135	0.29786	0.2648	0.25084	0.2592	0.43644	0.38169	0.35678	0.34543
30	180	0.39952	0.35237	0.33166	0.32297	0.58818	0.51066	0.47444	0.457
30	225	0.63461	0.5427	0.4978	0.47426	0.95109	0.80317	0.72868	0.68757
30	270	1.01876	0.84812	0.75973	0.70872	1.54966	1.27852	1.1359	1.0517
30	315	1.79896	1.46712	1.28949	1.18197	2.76658	2.24335	1.96108	1.78833
30	360	2.98724	2.40804	2.09321	1.89854	4.62185	3.71196	3.21515	2.90611
35	135	0.29249	0.25886	0.24432	0.2592	0.42973	0.37427	0.34864	0.33657
35	180	0.39388	0.34613	0.32482	0.31552	0.58113	0.50286	0.46589	0.4477
35	225	0.62976	0.53734	0.49192	0.46786	0.94504	0.79646	0.72133	0.67957
35	270	1.01477	0.8437	0.75488	0.70344	1.54466	1.27299	1.12984	1.0451
35	315	1.79244	1.4599	1.28158	1.17336	2.75844	2.23433	1.95119	1.77758
35	360	2.98134	2.40151	2.08604	1.89075	4.61448	3.7038	3.2062	2.89636
40	135	0.28875	0.25472	0.23979	0.2592	0.42506	0.3691	0.34297	0.3304
40	180	0.39048	0.34237	0.32069	0.31103	0.57688	0.49815	0.46073	0.44208
40	225	0.62875	0.53622	0.49069	0.46652	0.94377	0.79506	0.71978	0.67789
40	270	1.01775	0.84699	0.75849	0.70738	1.54839	1.27712	1.13436	1.05002
40	315	1.79458	1.46227	1.28418	1.17619	2.76111	2.23729	1.95444	1.78111
40	360	2.98069	2.40078	2.08525	1.88988	4.61366	3.70289	3.2052	2.89528

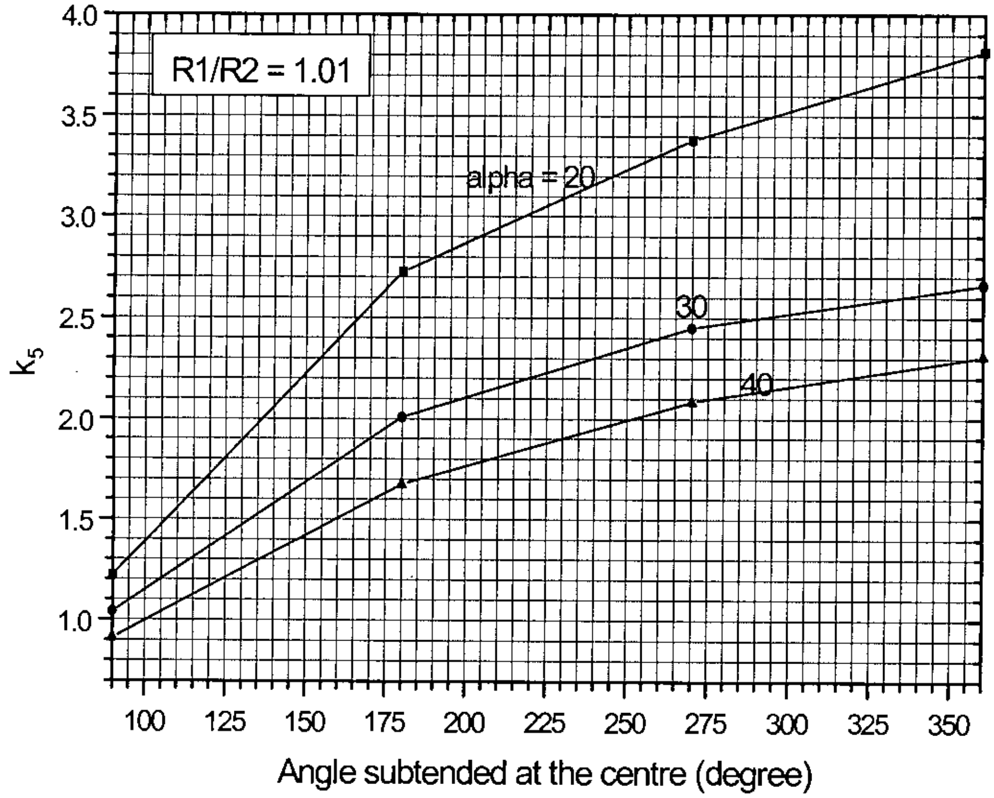


Fig. 5.7 Coefficient k_5 for maximum lateral moment, R_1/R_2=1.01

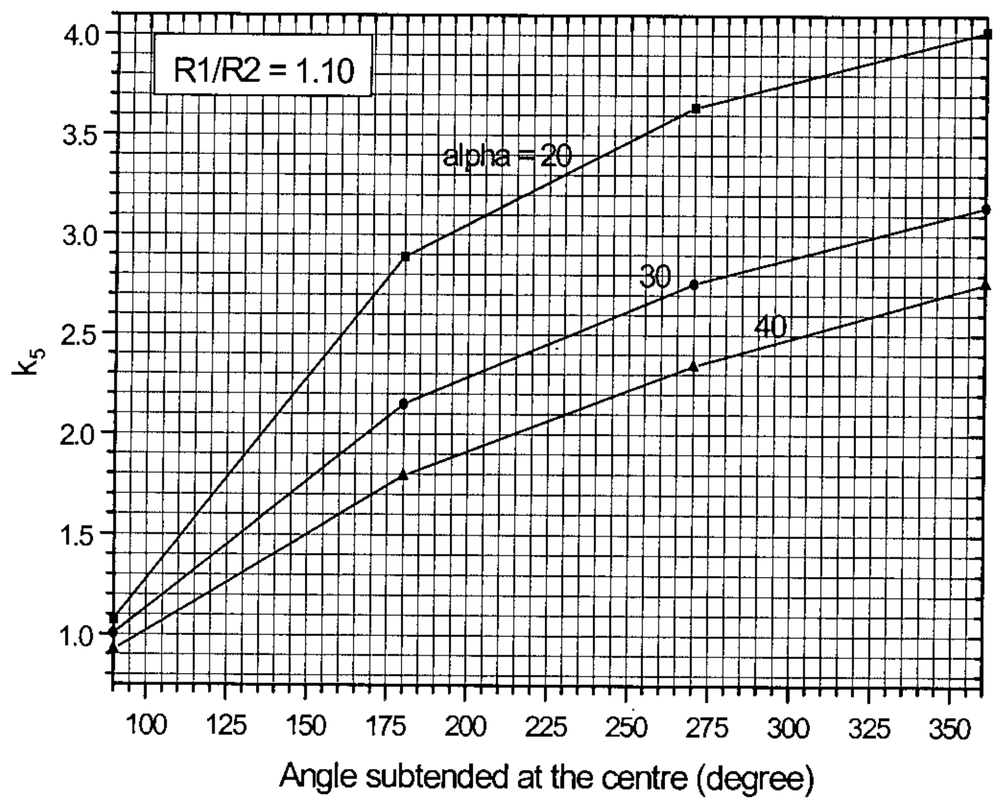


Fig 5.8 Coefficient k_5 for maximum lateral moment, R_1/R_2=1.10

III. To find the distributed steel required across the width of the stair at the support from Figs. 5.1, 5.2, 5.5 & 5.6 and to check with upper and lower limits of steel as specified by the code. Or, for 100 psf live load Tables 5.1 and 5.2 can be used directly. The charts and tables have been derived for 60,000 psi steel and 3,000 psi concrete.

IV. The maximum lateral moment can be found by using Figs. 5.7 and 5.8. Steel is to be provided at the end quarter span region to take care of this moment. After, that the lateral moment can be assumed to be linearly varying to zero at the mid span.

A qualitative reinforcement layout for a helicoidal stair slab is given in Fig. 5.9.

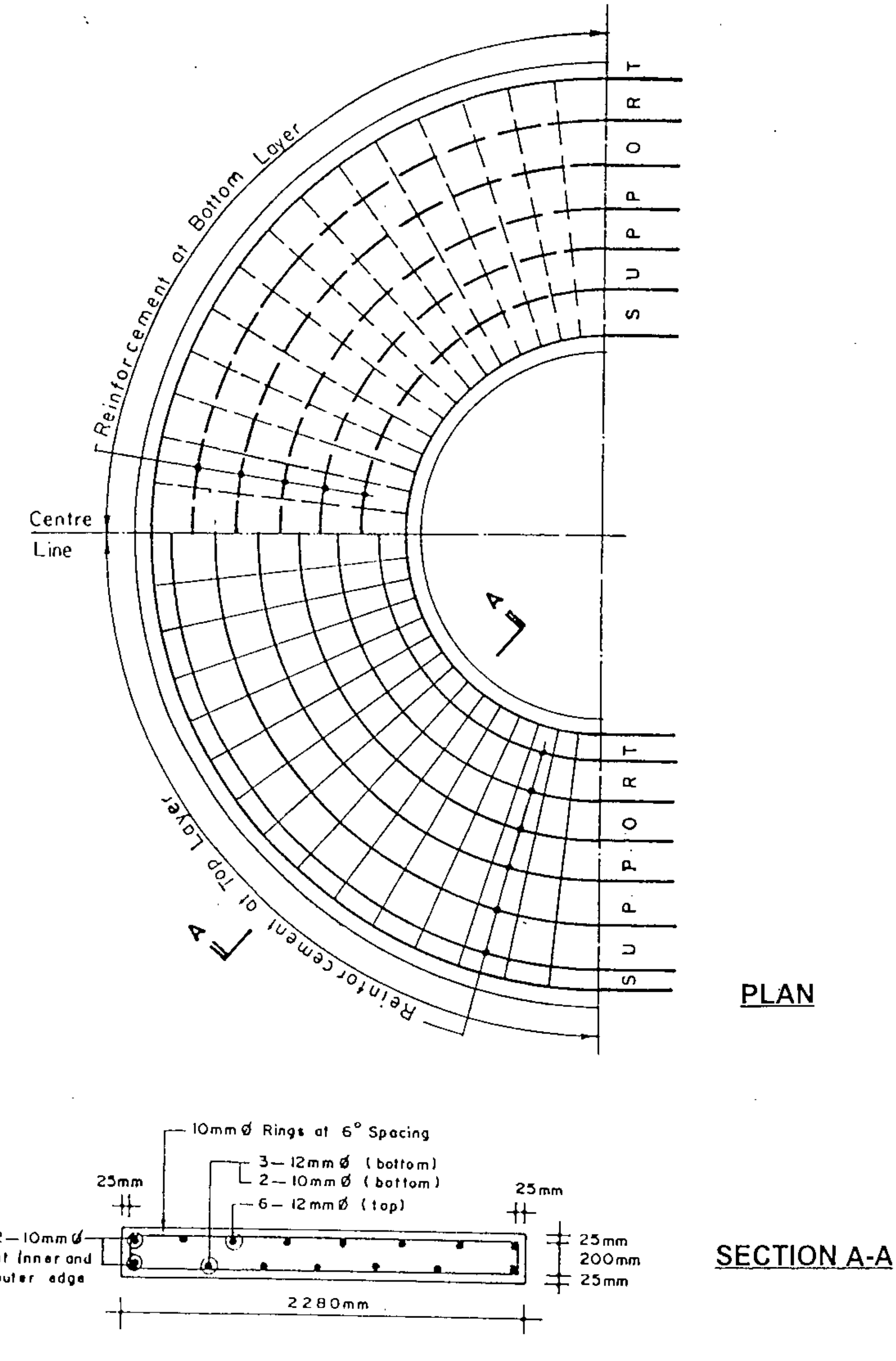


Fig. 5.9 Qualitative reinforcement details in a helicoidal stair slab

CHAPTER 6

INCORPORATION OF INTERMEDIATE LANDING

6.1 INTRODUCTION

The complex geometry of a regular helicoidal stair slab has made the analysis quite difficult. The introduction of an intermediate landing further complicates the situation. However, the addition of an intermediate landing is often desired specially because of functional reasons. In this chapter, the analysis of the helicoidal stair with an intermediate landing has been carried out following the method proposed by Solanki (1986). In the end, a simple procedure to analyse a fixed ended helicoidal slab with landing at the mid span has been proposed.

6.2 GEOMETRY OF THE HELICOIDAL SLAB WITH INTERMEDIATE LANDING

The geometry of a circular helicoidal stair slab with an intermediate landing can be defined as follows:

$$\alpha' = \tan^{-1} \frac{Ht}{R \ 2\beta} \qquad 6.1$$

where,

α' = slope made by tangent helix with a radius R with respect to horizontal plane

Ht = height of stair, from one support to another

β = Angle subtended in the centre measured on plan by half the flight

The mid surface coordinates can be expressed as:

$$x = R\cos\theta' \qquad 6.2$$

$$y = R\sin\theta' \qquad 6.3$$

$$z = \theta' R\tan\alpha' \qquad 0 \le \theta' \le \beta \qquad 6.4$$

$$z = \beta R\tan\alpha' \qquad \beta \le \theta' \le \beta + 2\phi \qquad 6.5$$

$$z = (\theta' - 2\phi) R\tan\alpha' \qquad \beta + 2\phi \le \theta' \le 2(\beta+\phi) \qquad 6.6$$

where,

θ' = angle subtended at the centre measured on plan from the bottom support

ϕ = angle subtended at the centre measured on plan by half the landing

6.3 ANALYSIS

6.3.1 Assumptions:

To facilitate the analysis procedure, the following assumptions have been made during the analysis:

1. Deformation due to shear and direct forces, being small in comparison to the deformations caused by twisting and bending moments, are neglected.
2. The cross section is symmetric about the two principal axis of the section.
3. The angle subtended at centre by the landing is small compared to the total angle subtended by the stair at the centre.
4. The moment of inertia of the helicoidal slab section with respect to a horizontal radial axis is negligible as compared to the moment of inertia with respect to the axis perpendicular to it.

6.3.2 Stress Resultants in the Helicoidal Stair with Intermediate Landings

The analysis method followed in this chapter incepts from Solanki's approach. However, to ensure a correct analysis, instead of starting from where Solanki had stopped, efforts have been made to start from the same point where Solanki had started. Strain energy method has then been applied to find the mid span redundants. In the end, using these equations a simplified method has been proposed following the Santathadaporn and Cussen's (1966) approach.

Because of symmetry in geometry and loading, four of the six redundants at the mid span of a helicoidal stair slab with intermediate landing, becomes zero. Only two, vertical moment M and radial horizontal shear H remain to be calculated (Figs. 6.1 & 6.2).

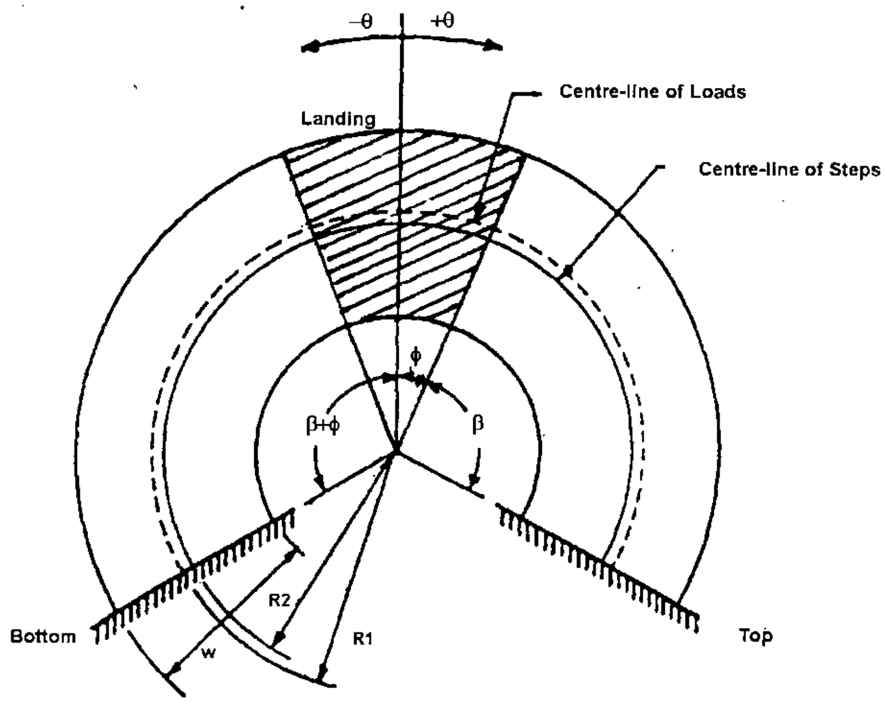


Fig. 6.1 Plan of a Helicoidal Staircase with Intermediate Landing

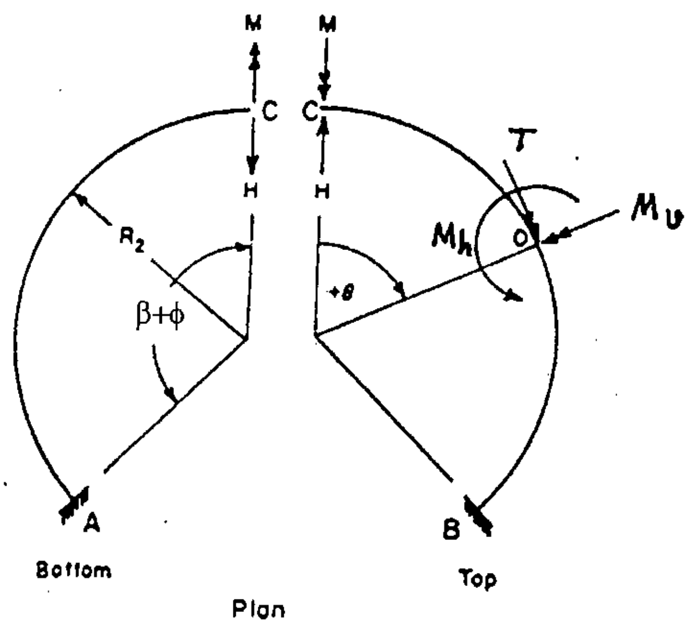


Fig. 6.2: Mid Span Redundant Moment, Horizontal Force and Other Resultants

Solanki assumed that at the landing level the bending and torsional moments along the upper half of the stair slab to be:

- Vertical Moment:

$$M_v = M\cos\Psi - wR_1^2(1-\cos\Psi) \tag{6.7}$$

- Lateral Moment:

$$M_h = -HR_2\sin\Psi \tag{6.8}$$

- Torsion:

$$T = M\sin\Psi + wR_1^2\sin\Psi - wR_1R_2\Psi \tag{6.9}$$

where,

ψ = angle subtended in plan measured from mid point of landing ($0 \le \psi \le \phi$)

For the flight the moments and other forces were assumed to follow Morgans (1960) derivations (Eqs. 3.32 to 3.37)

It should be mentioned that these equations are valid when the angle subtended by the landing at the centre (ϕ) is small as compared to angle subtended at the centre by the whole stair plus the landing. It should be also be noted that the equations for the landing are nothing but the expressions for the flight, with the slope of the helix (α) put to zero.

6.3.3 The Strain Energy Method

If a force is gradually applied to a body, there will essentially be only static deformation and displacement of the body. In such cases, due to deformation of the body, the work done by the force is stored up in the body as a particular form of potential energy, known as the strain energy.

The widely used Castigliano's Second theorem states that in any structure the material of which is elastic and follows Hooke's law and in which the temperature is constant and the supports are unyielding, the first partial derivative of the strain energy with respect to any particular force is equal to the displacement of the point of application of that force in the direction of its line of action. Mathematically expressing,

$$\frac{\partial U}{\partial P} = \delta \qquad 6.10$$

where U is the strain energy, P is the force, and δ is the deflection in the direction of force.

The strain energy due to shear stress and axial force is neglected, because they are small. The strain energy stored by the bending moment is given by:

$$U = \sum \int \frac{M^2}{2EI} dL \qquad 6.11$$

And that by the twisting moment is:

$$U = \sum \int \frac{T^2}{2GJ} dL \qquad 6.12$$

Where, E, I, G, J all bear their usual meanings in mechanics of solids.

6.3.4 The Strain Energy Method Applied to the Helicoidal Stair Slabs

The strain energy method has previously been successfully employed by Morgan and Holmes to analyse helicodial stair slabs. Because of symmetry in loading and geometry, in a helicodial stair slab with a landing at the middle, the slope at the midspan is zero and so is the horizontal deflection. This is why, according to the Castigliano's second theorem, the partial derivatives of the strain energy function with respect the vertical moment (M) and radial horizontal force (H) is equal to zero. That is,

$$\frac{\partial U}{\partial M} = 0 \qquad 6.13$$

And,

$$\frac{\partial U}{\partial H} = 0 \qquad 6.14$$

The strain energy function U, is given by,

$$U = \int_0^{\phi} \frac{M_v^2}{2EI} ds + \int_0^{\phi} \frac{M_h^2}{2EI_h} ds + \int_0^{\phi} \frac{T^2}{2GJ} ds + \int_0^{\beta} \frac{M_v^2}{2EI_h} ds + \int_0^{\beta} \frac{M_h^2}{2EI} ds + \int_0^{\beta} \frac{T^2}{2GJ} ds \quad 6.15$$

Where, for the landing

$$ds = R_2\, d\theta \quad 6.16$$

and, for the flight

$$ds = R_2 \sec\alpha\, d\theta \quad 6.17$$

The partial derivative of the strain energy function with respect to H is,

$$\frac{\partial U}{\partial H} = \int_0^{\phi} \frac{2M_v}{2EI} . \frac{\partial M_v}{\partial H} .ds + \int_0^{\phi} \frac{2M_h}{2EI_h} . \frac{\partial M_h}{\partial H} .ds + \int_0^{\phi} \frac{2T}{2GJ} . \frac{\partial T}{\partial H} .ds + \int_0^{\beta} \frac{2M_v}{2EI} . \frac{\partial M_v}{\partial H} .ds + \int_0^{\beta} \frac{2M_h}{2EI_h} . \frac{\partial M_h}{\partial H} .ds + \int_0^{\beta} \frac{2T}{2GJ} . \frac{\partial T}{\partial H} .ds \quad 6.18$$

Because the stair width is large as compared to its thickness, the moment of inertia with respect to a vertical axis I_h is much greater than the moment of inertia about the horizontal axis I. The ratio I/I_h can therefore be neglected, i.e.,

$$\frac{I}{I_h} \approx 0 \quad 6.19$$

As per Solanki, the torsional rigidity can be taken as:

$$GJ = \frac{2\, EII_h}{I + I_h} \quad 6.20$$

Then,

$$\frac{EI}{GJ} = \frac{1}{2}\left[\frac{I}{I_h} + 1\right] \approx \frac{1}{2} \quad 6.21$$

Eq. 6.14 can be rewritten with the help of Eqs. 6.18, 6.19 and 6.21 as:

$$\frac{\partial U}{\partial H} = \int_0^{\phi} M_v . \frac{\partial M_v}{\partial H} .ds + \frac{1}{2} \int_0^{\phi} T . \frac{\partial T}{\partial H} .ds + \int_0^{\beta} M_v . \frac{\partial M_v}{\partial H} .ds + \frac{1}{2} \int_0^{\beta} T . \frac{\partial T}{\partial H} .ds = 0 \quad 6.22$$

Similarly Eq. 6.13 stands as:

$$\frac{\partial U}{\partial H} = \int_0^{\phi} M_v . \frac{\partial M_v}{\partial M} .ds + \frac{1}{2} \int_0^{\phi} T . \frac{\partial T}{\partial M} .ds + \int_0^{\beta} M_v . \frac{\partial M_v}{\partial M} .ds + \frac{1}{2} \int_0^{\beta} T . \frac{\partial T}{\partial M} .ds = 0 \qquad 6.23$$

Eqs. 6.22 and 6.23 expands to:

$MR_2{}^2 tan\alpha sec\alpha.A + HR_2{}^3 tan^2\alpha sec\alpha.B - wR_1{}^2 R_2{}^2 tan\alpha sec\alpha.(C-A) + ½[MR_2{}^2 sin\alpha(D-A) + HR_2{}^3 sin^2\alpha sec\alpha\ (D-2A+E) + wR_1{}^2 R_2{}^2 sin\alpha.(D-A) - wR_1 R_2{}^3 sin\alpha.(C-F)] = 0$

$\Rightarrow M\ [tan\alpha sec\alpha.A + ½\ sin\alpha.(D-A)] + HR_2\ [tan^2\alpha sec\alpha.B + ½\ sin^2\alpha sec\alpha\ (D-2A+E)] - wR_1{}^2 [tan\alpha sec\alpha.(C-A) - ½\ sin\alpha.(D-A) + ½\ R_2 sin\alpha.(C-F)/R_1] = 0$

$\Rightarrow A_1 M + A_2 HR_2 = A_3 wR_1{}^2$

$$\Rightarrow A_1 M + A_2 HR_2 = A_4 wR_2{}^2 \qquad 6.24$$

and

$MR_2 sec\alpha.G_1 + HR_2{}^2 tan\alpha sec\alpha.A - wR_1{}^2 R_2 sec\alpha(H_1-G_1) + ½[MR_2 cos\alpha.D + HR_2{}^2 sin\alpha(D-A) + wR_1{}^2 R_2 cos\alpha.D - wR_1 R_2{}^2 cos\alpha.C] + [MR_2.G' - wR_1{}^2 R_2(H' - G')] + ½[MR_2.\ D' + wR_1{}^2 R_2.\ D' - wR_1 R_2{}^2.C'] = 0$

$\Rightarrow M\ [sec\alpha.G_1 + ½\ cos\alpha.D + G' + ½\ D'] + HR_2\ [tan\alpha sec\alpha.A + ½\ sin\alpha(D-A)] - wR_1{}^2 [sec\alpha(H_1-G_1) - ½\ cos\alpha.D + ½\ R_2 cos\alpha.C/R_1 + H' - G' - ½\ D' + ½\ R_2 C'/R_1] = 0$

$\Rightarrow B_1 M + B_2 HR_2 = B_3 wR_1{}^2$

$$\Rightarrow B_1 M + B_2 HR_2 = B_4 wR_2{}^2 \qquad 6.25$$

where,

$A = (1/8)sin2\beta - (1/4)\beta cos2\beta$

$B = \beta^3/6 - (\beta^2/4 - 1/8)\ sin2\beta - (1/4)\ \beta cos2\beta$

$C = sin\beta - \beta cos\beta$

$C' = sin\phi - \phi cos\phi$

$D = (1/2)\ \beta - (1/4)sin2\beta$

$D' = (1/2)\,\phi - (1/4)\sin 2\phi$

$E = \beta^3/6 + (\beta^2/4 - 1/8)\sin 2\beta + (1/4)\,\beta\cos 2\beta$

$F = 2\beta\cos\beta + (\beta^2 - 2)\sin\beta$

$G_1 = (1/2)\,\beta + (1/4)\sin 2\beta$

$G' = (1/2)\,\phi + (1/4)\sin 2\phi$

$H_1 = \sin\beta$

$H' = \sin\phi$

A1, A2, A3, A4, B1, B2, B3, B4 are constants, their value being evident from Eqs. 6.24 and 6.25.

The simultaneous solution of Eqs. 6.24 and 6.25 yields the values of M and H, mid span redundant moment and radial horizontal force:

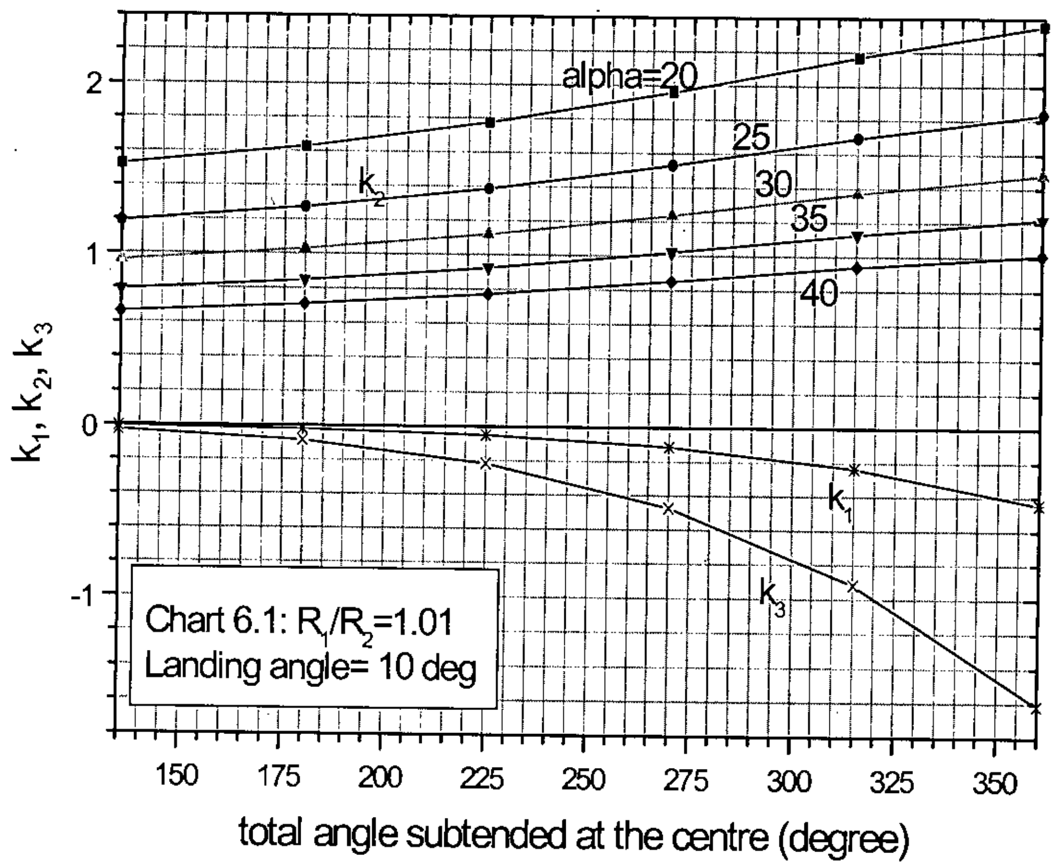
$$M = \frac{A_4B_2 - A_2B_4}{A_1B_2 - A_2B_1}\, wR_2^2 = k_1 wR_2^2 \qquad 6.26$$

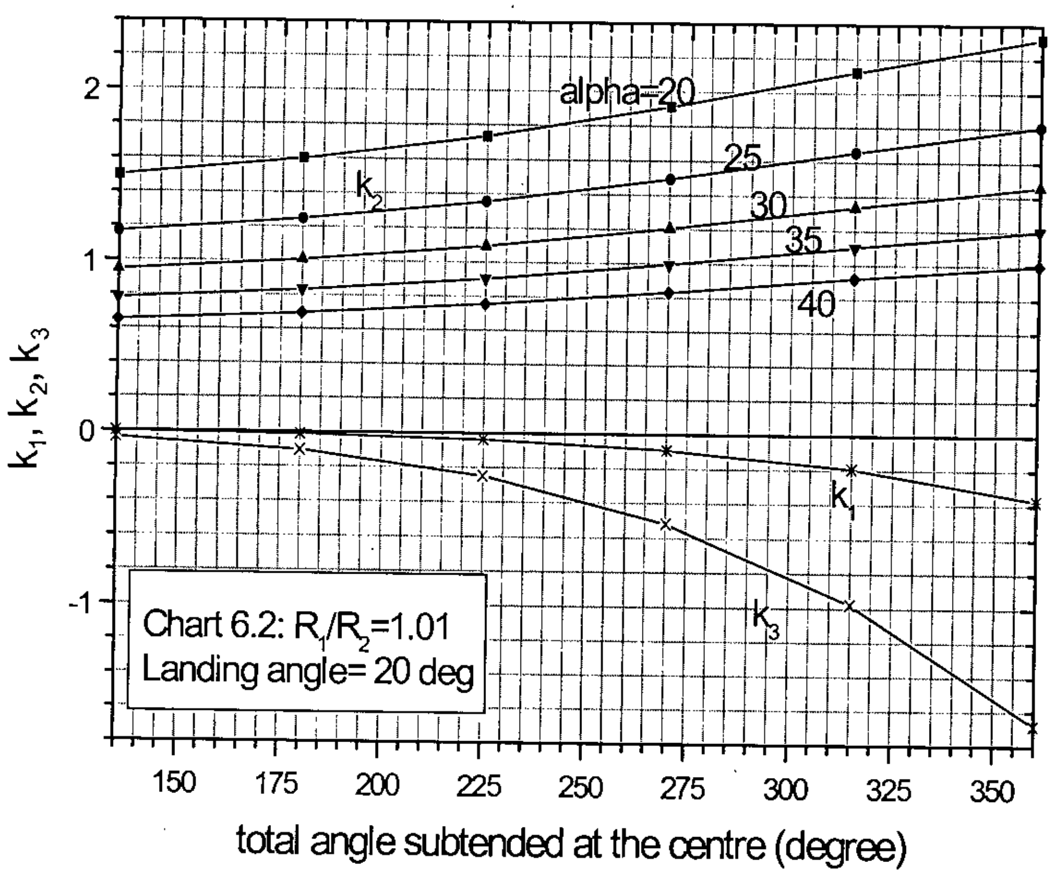
$$H = \frac{A_4B_1 - A_1B_4}{A_2B_1 - A_1B_2}\, wR_2 = k_2 wR_2 \qquad 6.27$$

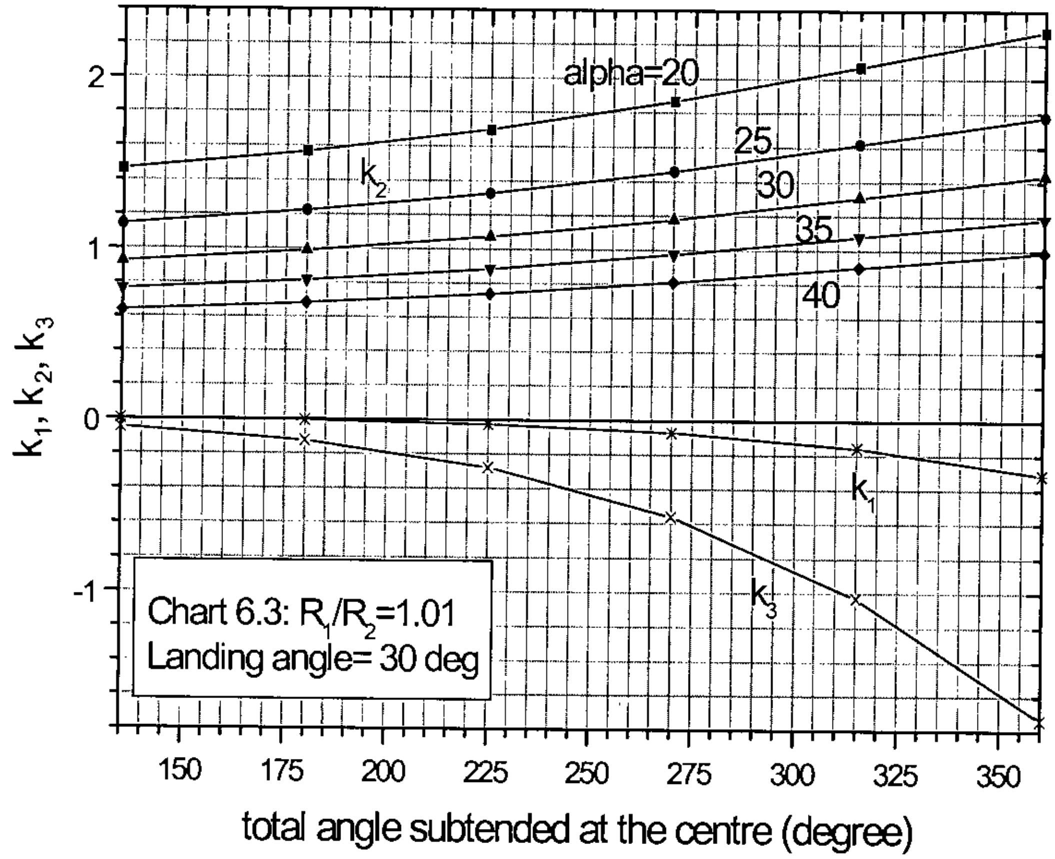
6.4 SUGGESTION FOR A CHART

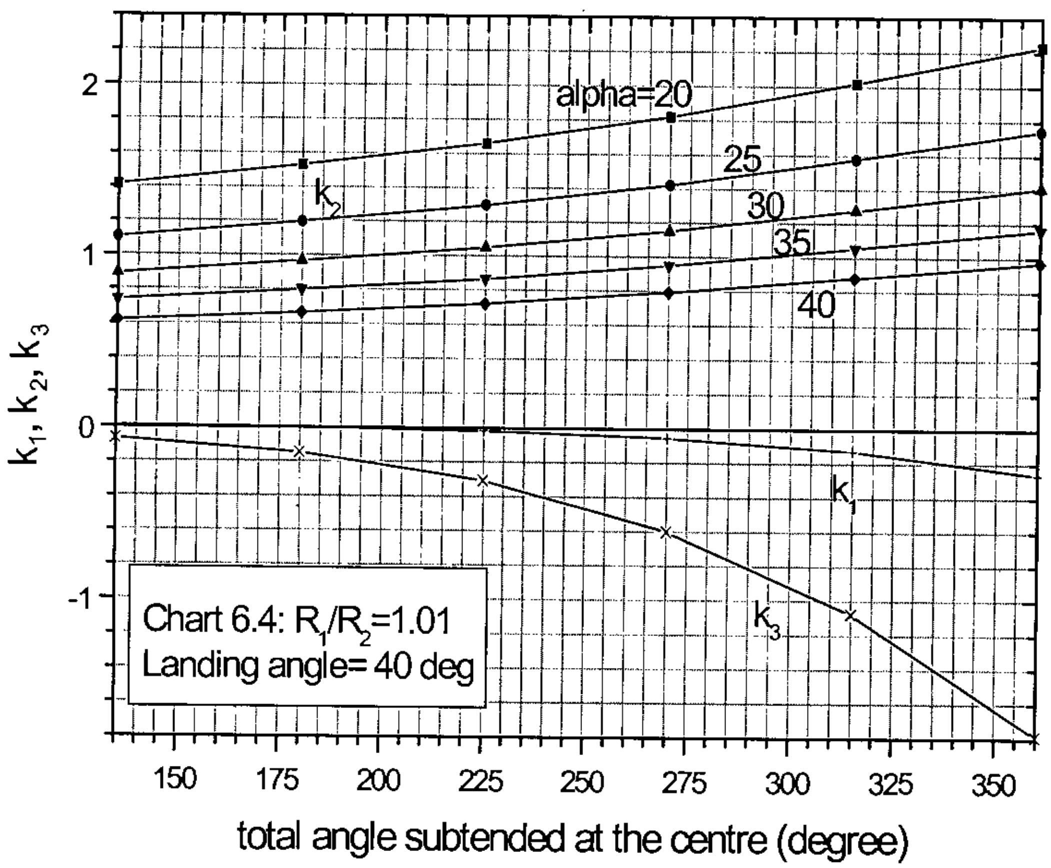
Once M and H are determined from Eqs. 6.26 and 6.27, Eqs. 6.7 to 6.9 and 3.32 through 3.37 can be used to determine the six stress resultants at any section of the helicoidal stair slab. However, the derivation of Eqs. 6.26 and 6.27 requires tedious mathematical computations. To facilitate the design procedure a series of design charts have been proposed here (Charts 6.1 to 6.21). These charts provide the values of k_1 and k_2 for a wide range of parameters:

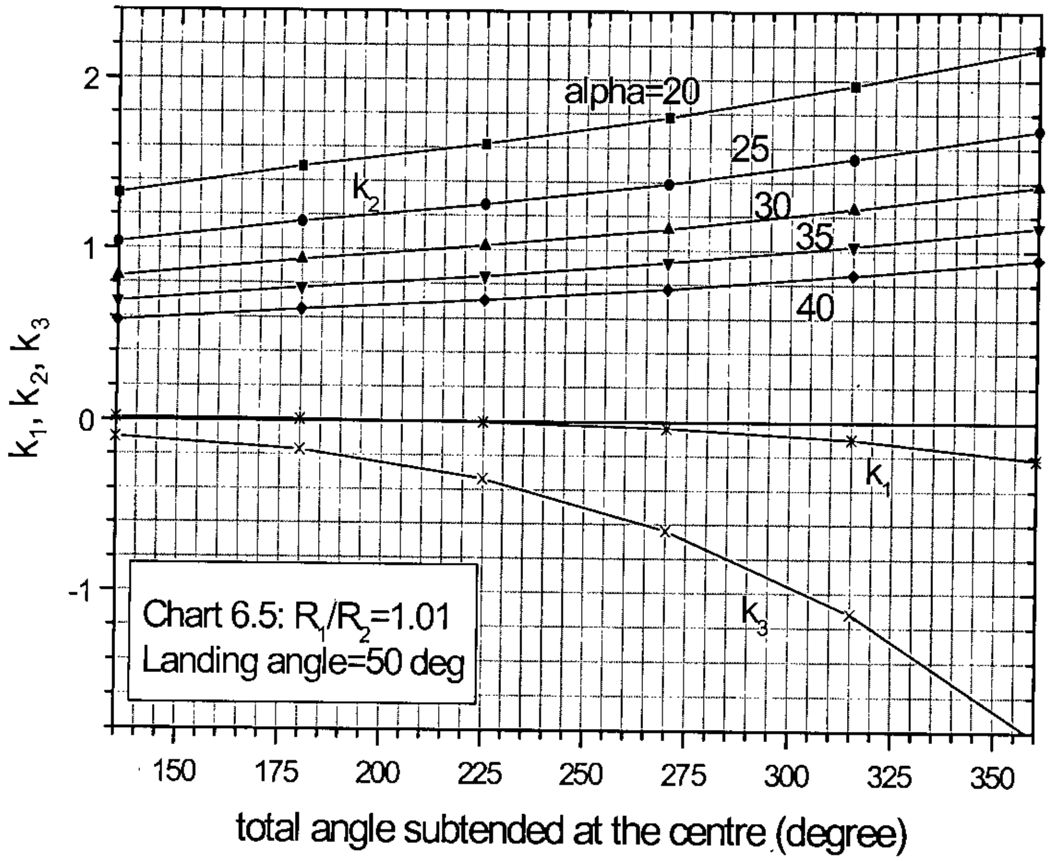
- Total central angle subtended by the stair (range 135 degree to 360 degree)
- Slope of the tangent helix centre line with respect to the horizontal plane, α (20 degree to 40 degree)
- Ratio of radius of the centre line of load to the mean radius of the stair, R_1/R_2 (1.01 to 1.10)
- Total angle subtended by the landing (10 degree to 70 degree)

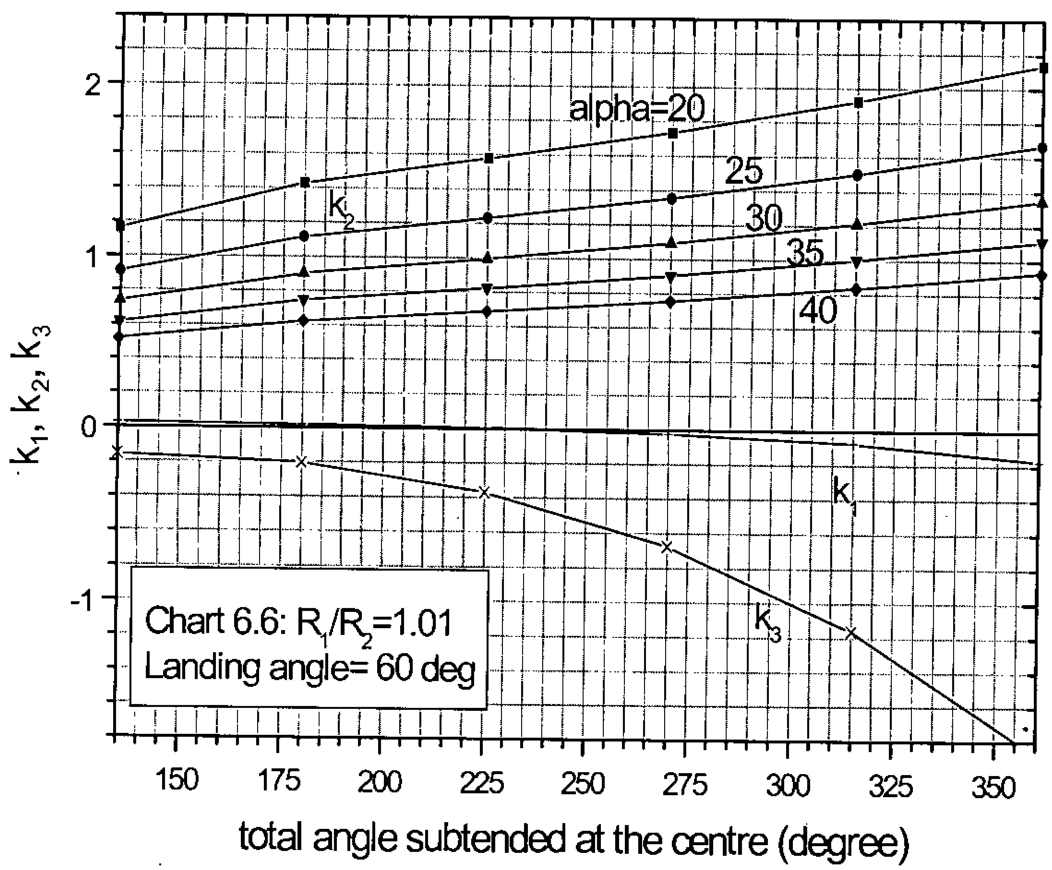

alpha=20
25
30
35
40
k2
k1
k3
Chart 6.1: R1/R2=1.01
Landing angle= 10 deg
k1, k2, k3
total angle subtended at the centre (degree)

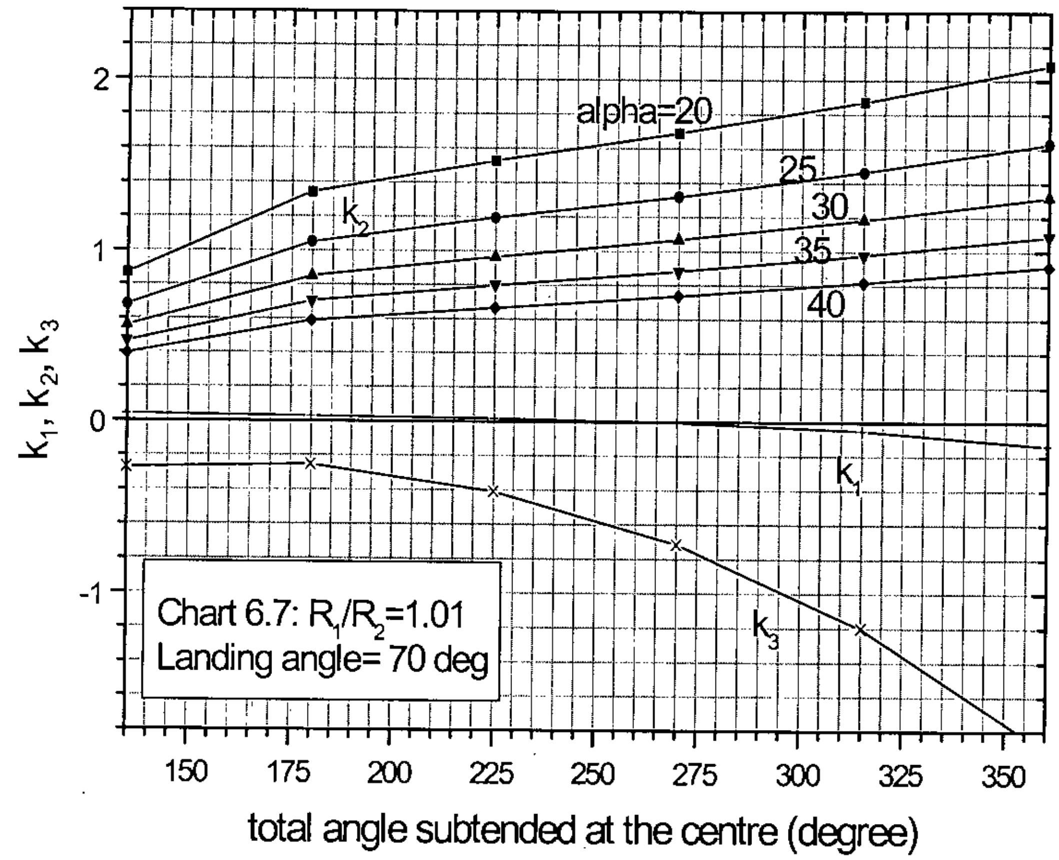

alpha=20
25
30
35
40
k2
k1
k3
Chart 6.2: R1/R2=1.01
Landing angle= 20 deg
k1, k2, k3
total angle subtended at the centre (degree)

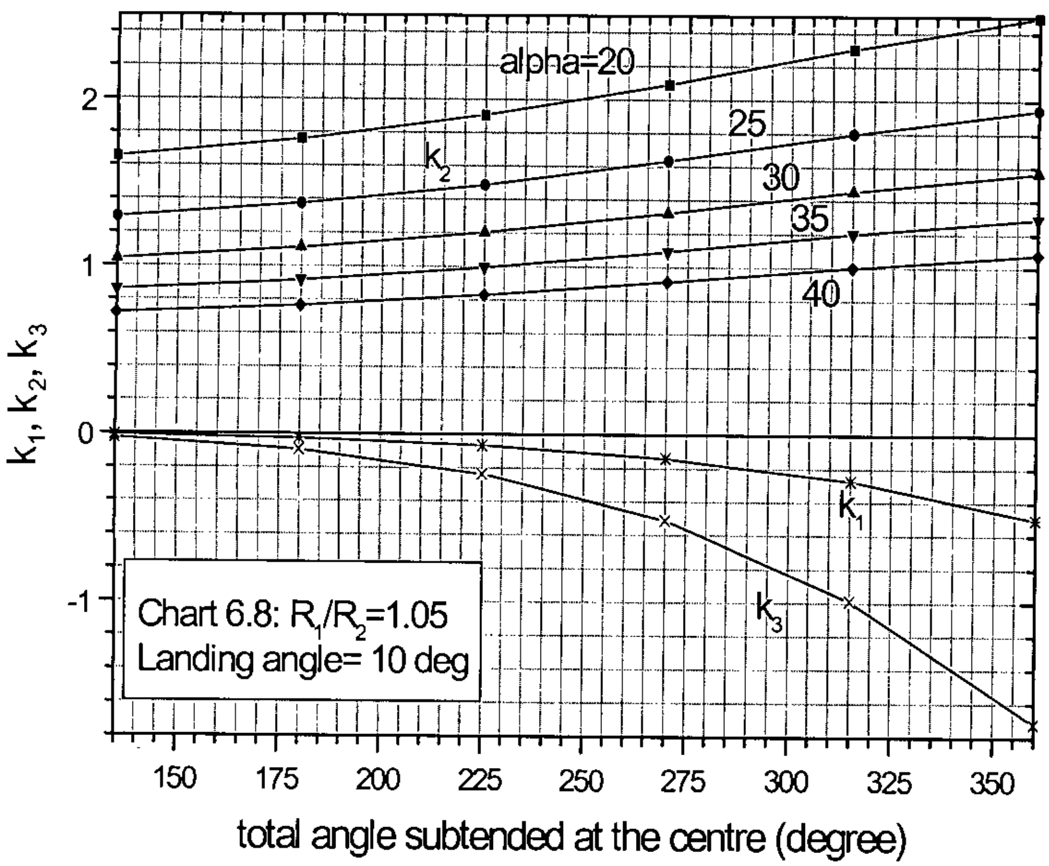

alpha=20
25
30
35
40
k_2
k_1
k_3
Chart 6.3: R_1/R_2=1.01
Landing angle= 30 deg
k_1, k_2, k_3
total angle subtended at the centre (degree)


alpha=20
25
30
35
40
k_2
k_1
k_3
Chart 6.4: R_1/R_2=1.01
Landing angle= 40 deg
k_1, k_2, k_3
total angle subtended at the centre (degree)


alpha=20
25
30
35
40
k_2
k_1
k_3
Chart 6.5: R_1/R_2=1.01
Landing angle=50 deg
k_1, k_2, k_3
2
1
0
-1
150
175
200
225
250
275
300
325
350
total angle subtended at the centre (degree)


alpha=20
25
30
35
40
k_2
k_1
k_3
Chart 6.6: R_1/R_2=1.01
Landing angle= 60 deg
k_1, k_2, k_3
2
1
0
-1
150
175
200
225
250
275
300
325
350
total angle subtended at the centre (degree)


alpha=20
25
30
35
40
k2
k1
k3
k1, k2, k3
Chart 6.7: R1/R2=1.01
Landing angle= 70 deg
150 175 200 225 250 275 300 325 350
total angle subtended at the centre (degree)


alpha=20
25
30
35
40
k2
k1
k3
k1, k2, k3
Chart 6.8: R1/R2=1.05
Landing angle= 10 deg
150 175 200 225 250 275 300 325 350
total angle subtended at the centre (degree)

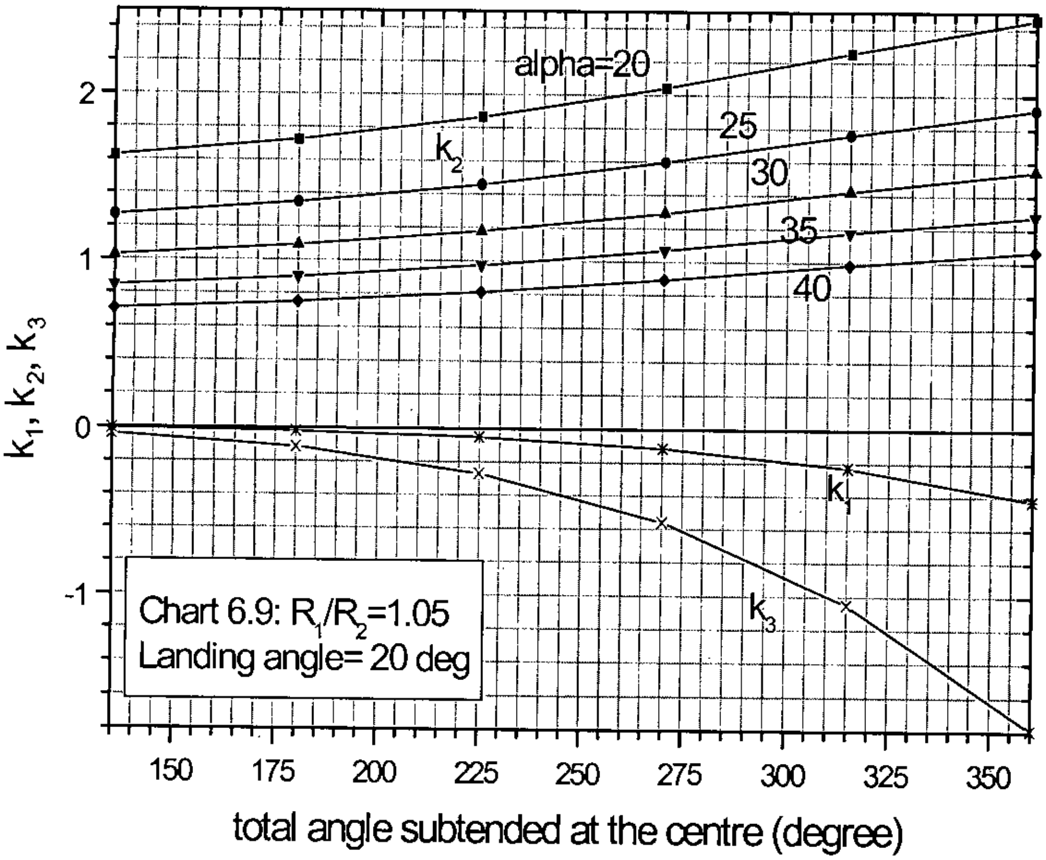


Chart 6.9: R_1/R_2=1.05
Landing angle= 20 deg

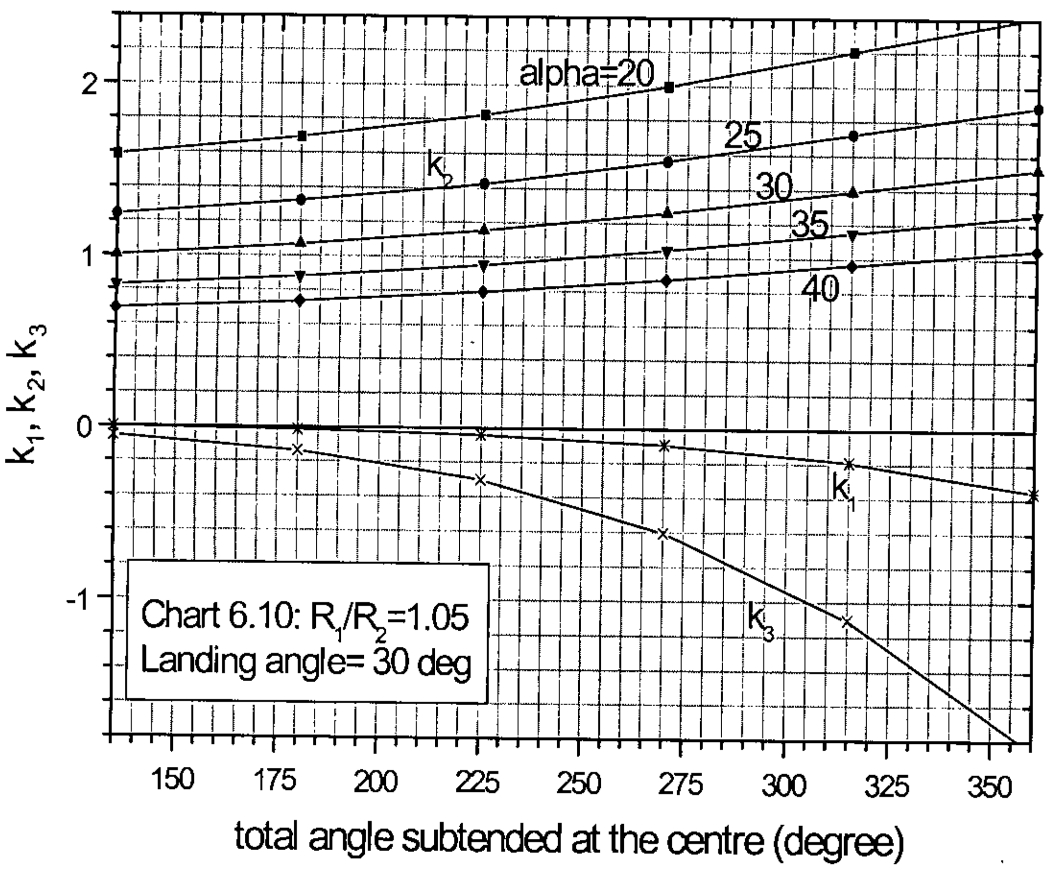
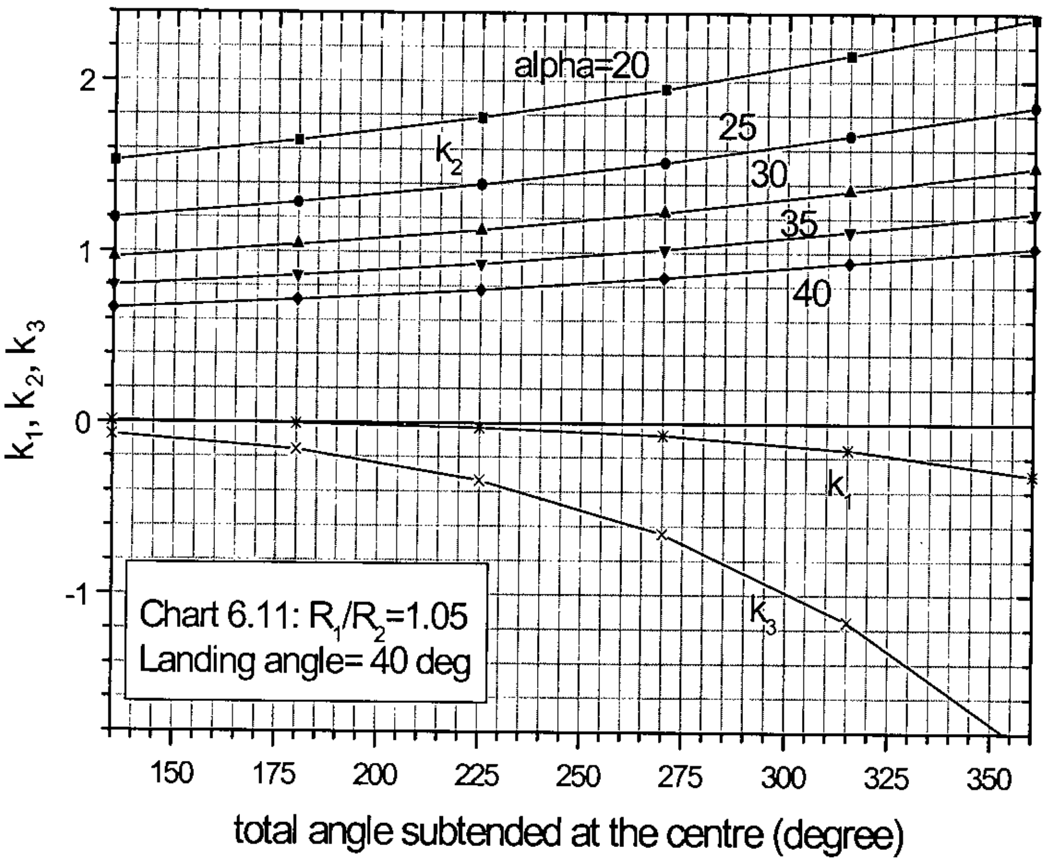
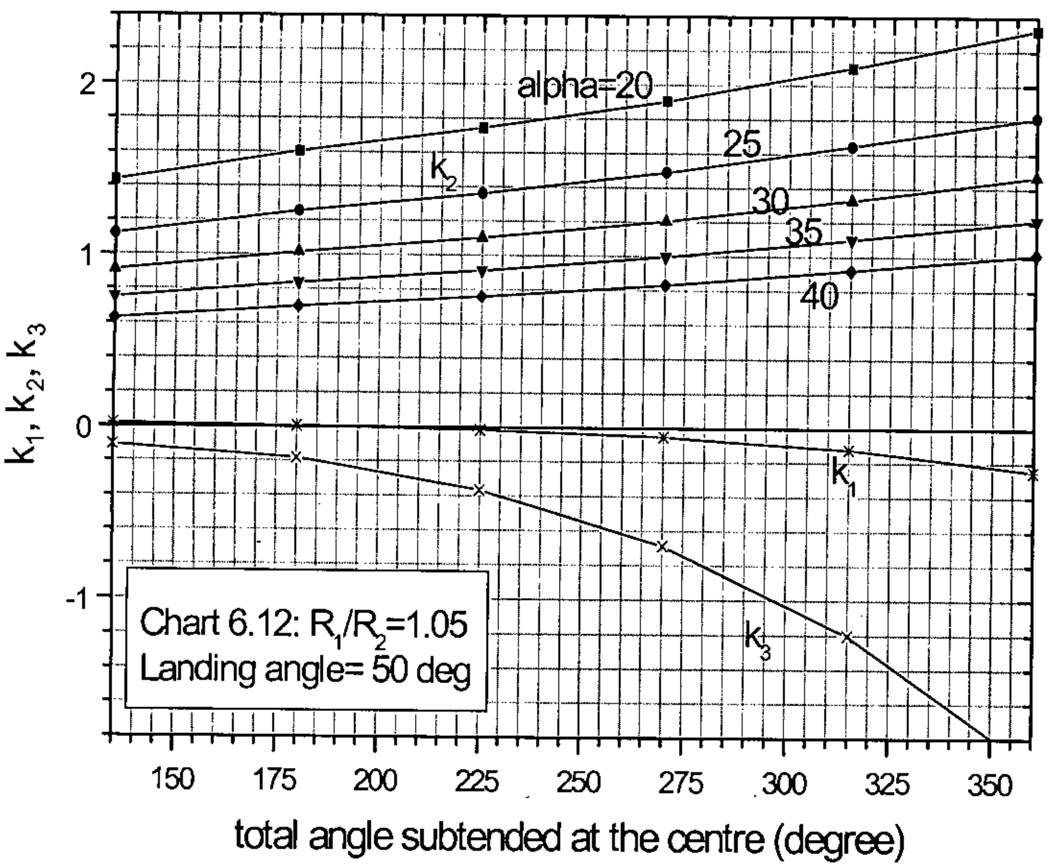


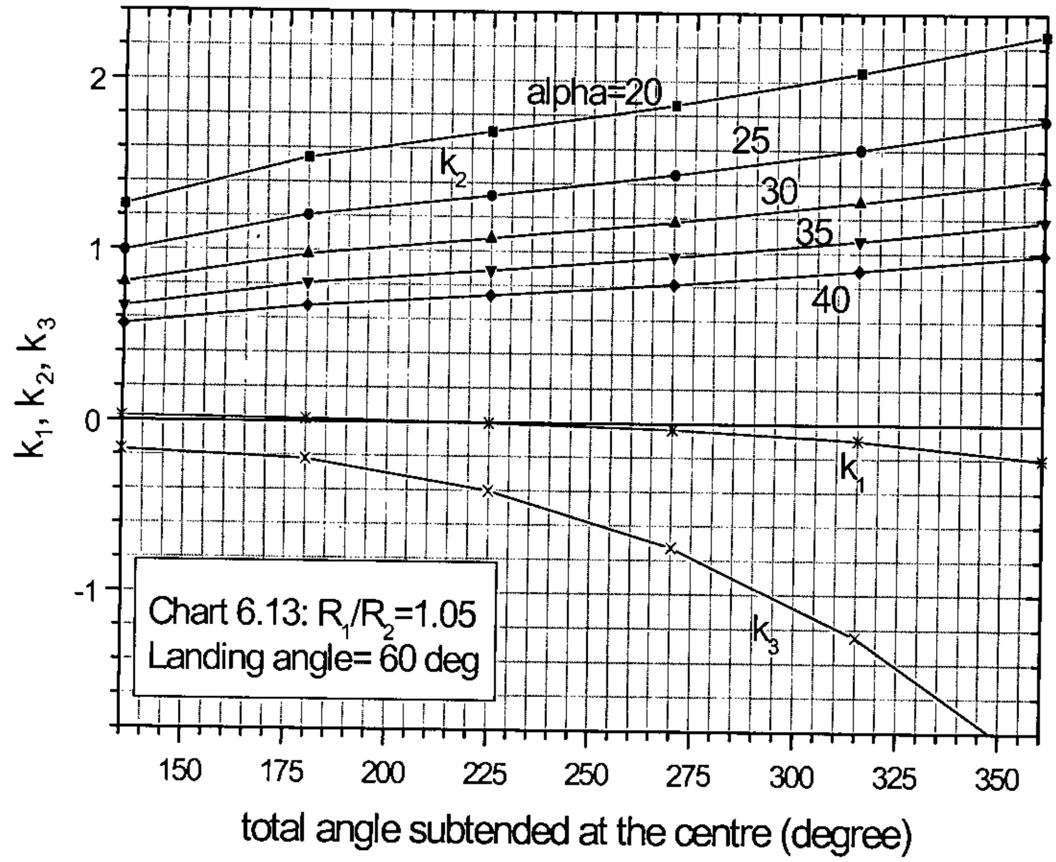
Chart 6.10: R_1/R_2=1.05
Landing angle= 30 deg

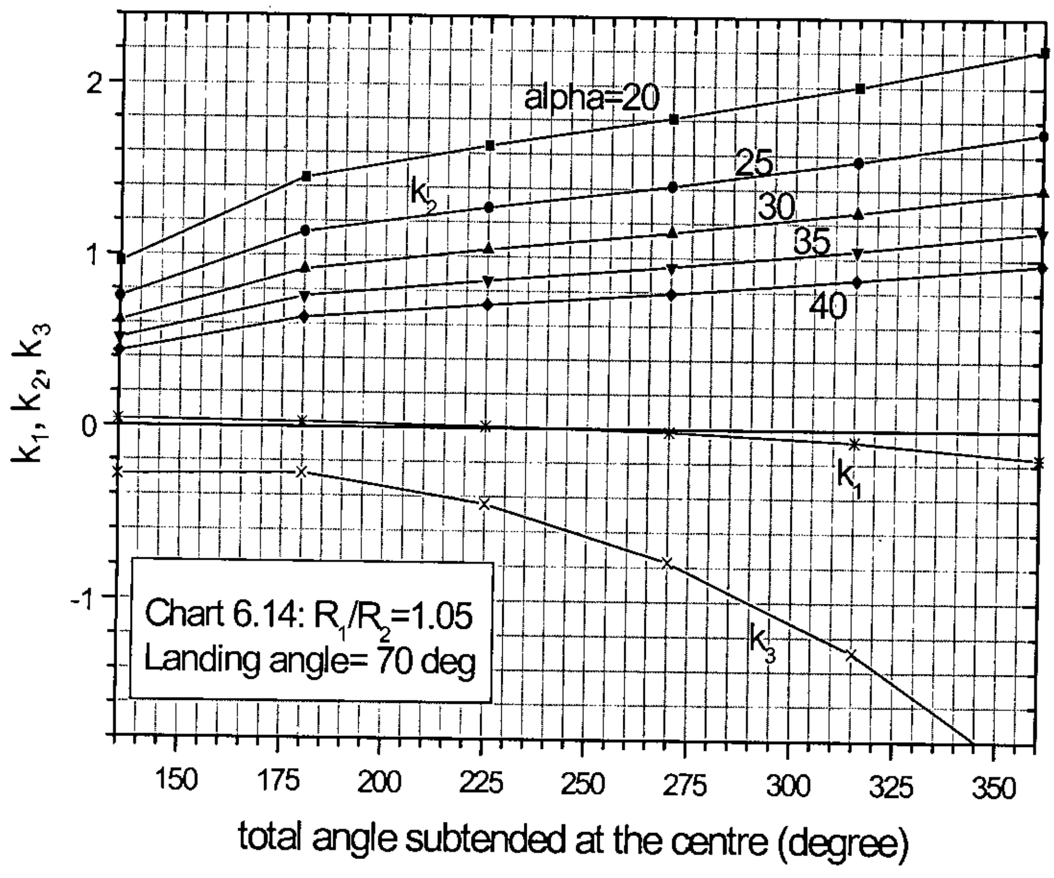


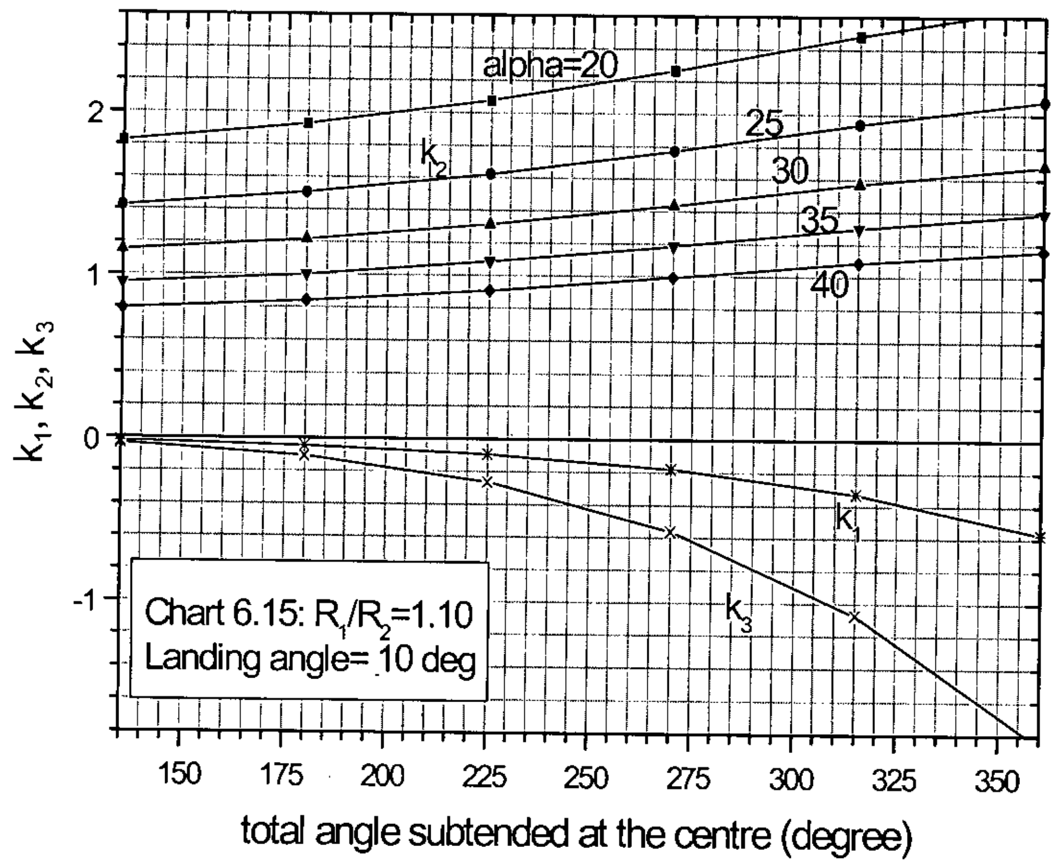
alpha=20
25
30
35
40
k_2
k_1
k_3
Chart 6.11: R_1/R_2=1.05
Landing angle= 40 deg
k_1, k_2, k_3
total angle subtended at the centre (degree)

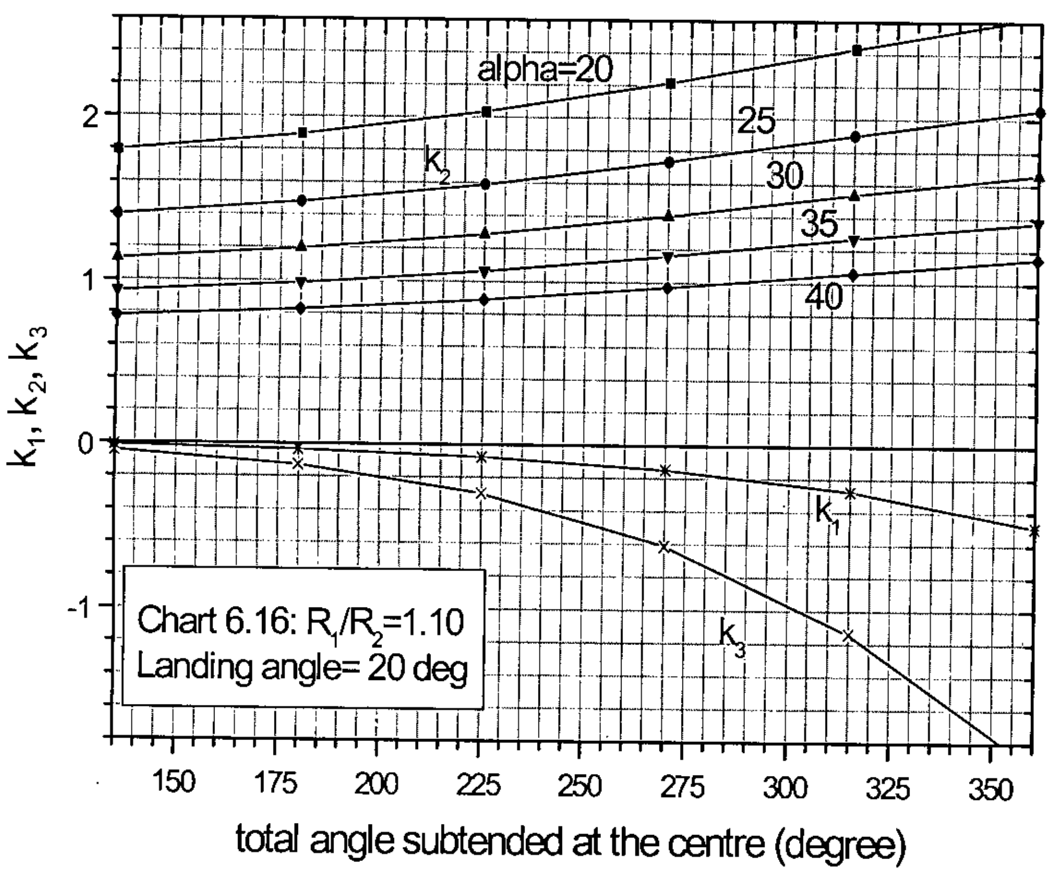


alpha=20
25
30
35
40
k_2
k_1
k_3
Chart 6.12: R_1/R_2=1.05
Landing angle= 50 deg
k_1, k_2, k_3
total angle subtended at the centre (degree)


alpha=20
25
30
35
40
k_2
k_1
k_3
k_1, k_2, k_3
2
1
0
-1
150
175
200
225
250
275
300
325
350
total angle subtended at the centre (degree)
Chart 6.13: R_1/R_2=1.05
Landing angle= 60 deg


alpha=20
25
30
35
40
k_2
k_1
k_3
k_1, k_2, k_3
2
1
0
-1
150
175
200
225
250
275
300
325
350
total angle subtended at the centre (degree)
Chart 6.14: R_1/R_2=1.05
Landing angle= 70 deg


alpha=20
25
30
35
40
k_2
k_1
k_3
k_1, k_2, k_3
total angle subtended at the centre (degree)
Chart 6.15: R_1/R_2=1.10
Landing angle= 10 deg


alpha=20
25
30
35
40
k_2
k_1
k_3
k_1, k_2, k_3
total angle subtended at the centre (degree)
Chart 6.16: R_1/R_2=1.10
Landing angle= 20 deg

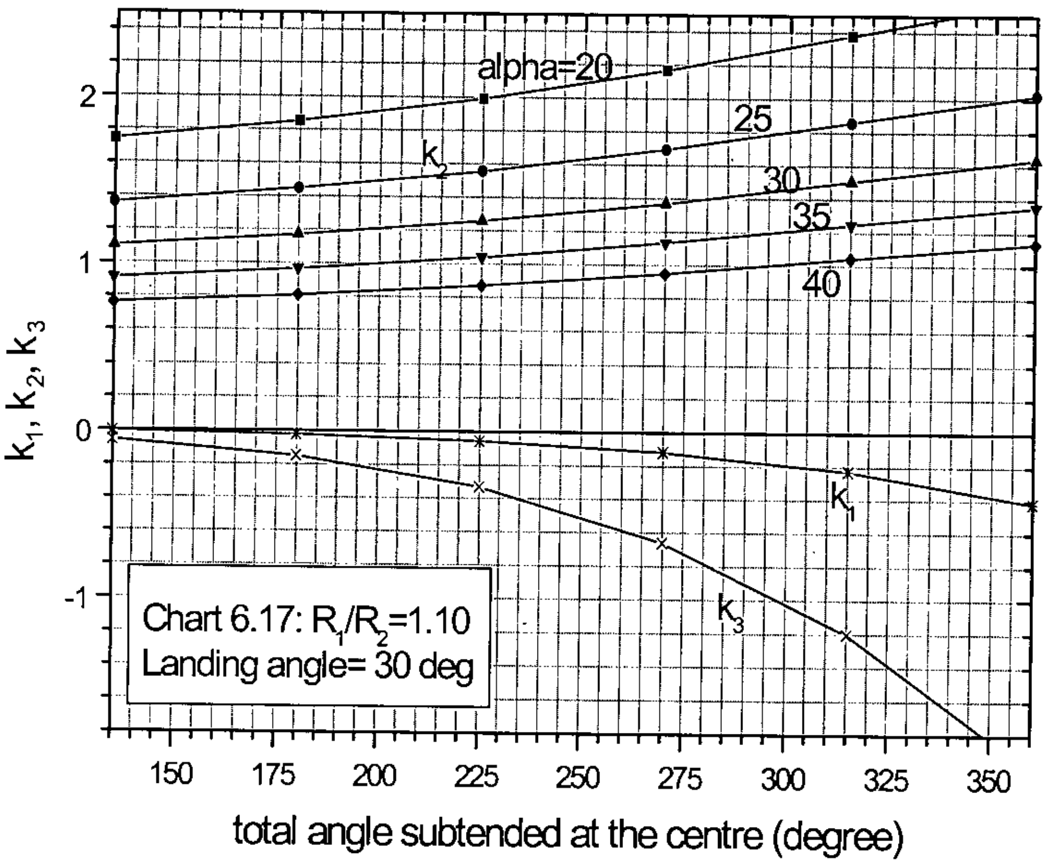


Chart 6.17: R_1/R_2=1.10
Landing angle= 30 deg

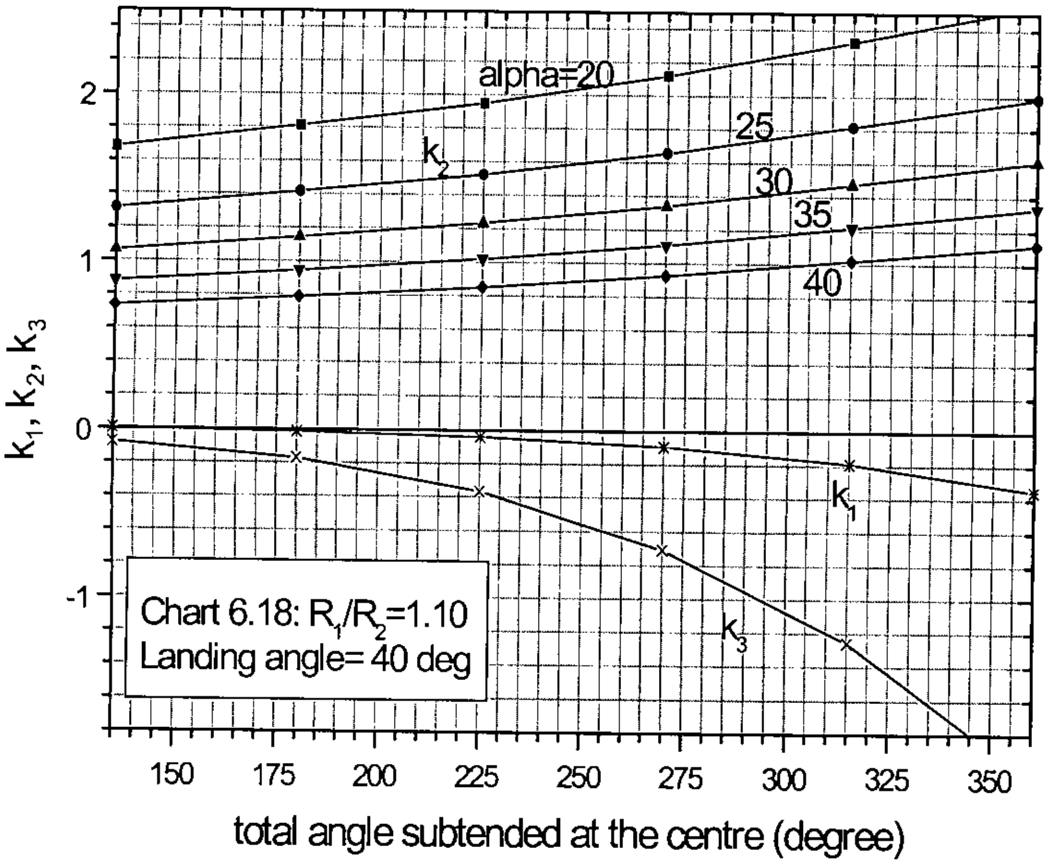
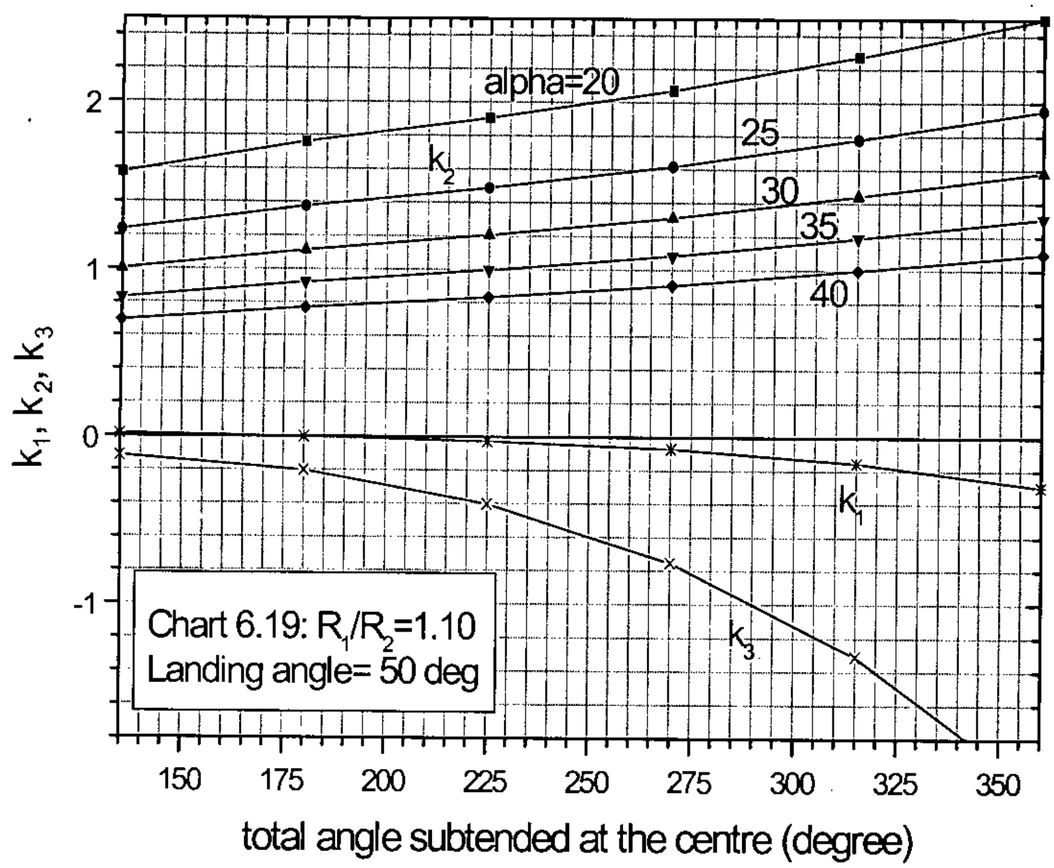
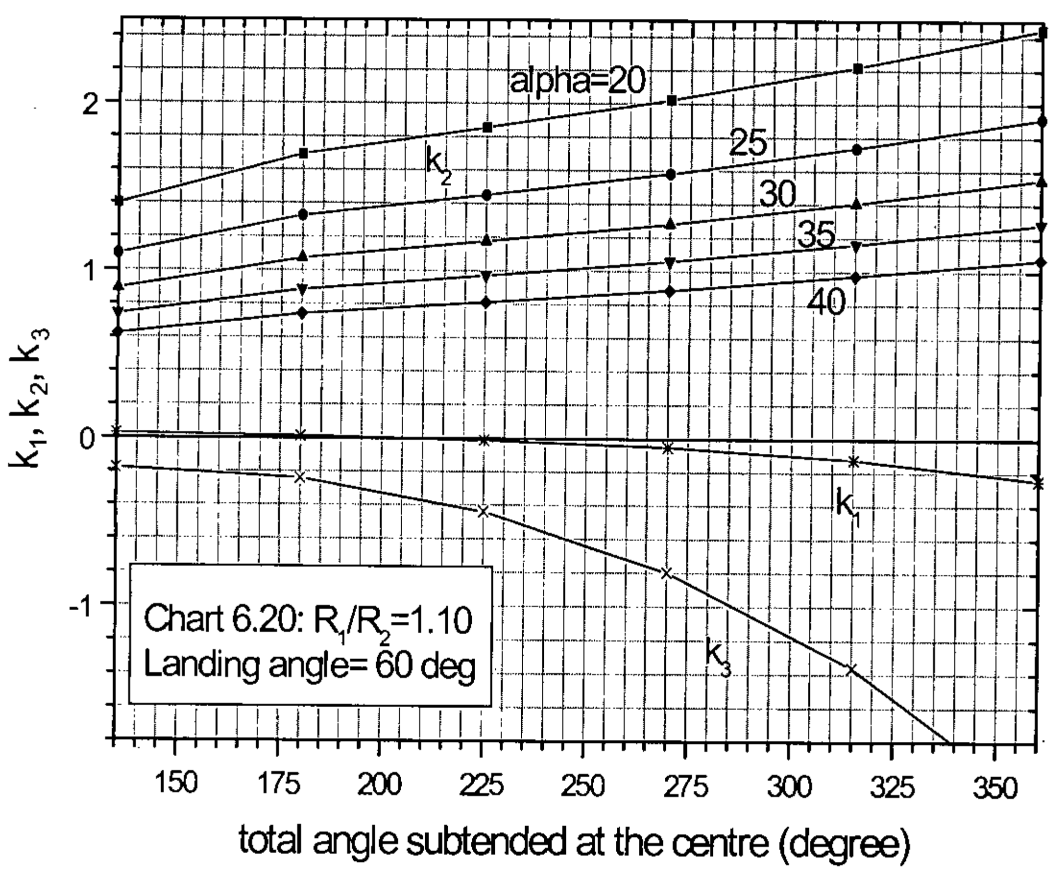
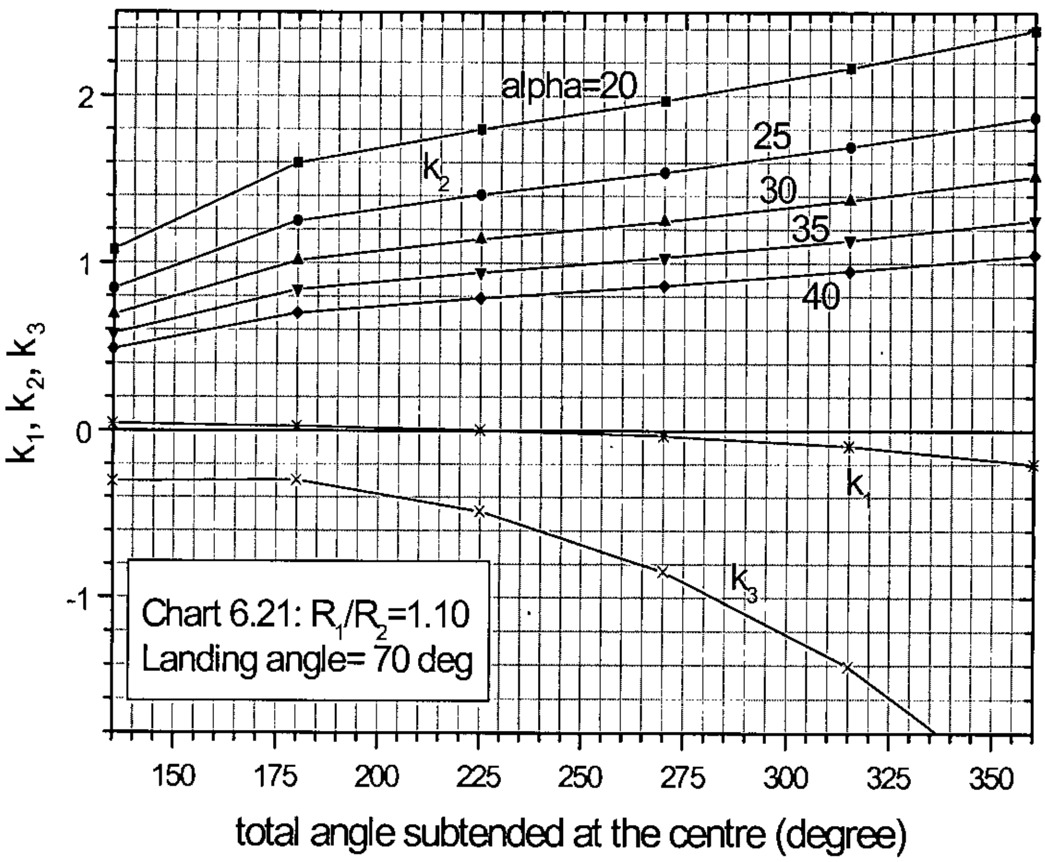


Chart 6.18: R_1/R_2=1.10
Landing angle= 40 deg


alpha=20
25
30
35
40
k_2
k_1
k_3
k_1, k_2, k_3
2
1
0
-1
150
175
200
225
250
275
300
325
350
total angle subtended at the centre (degree)
Chart 6.19: R_1/R_2=1.10
Landing angle= 50 deg


alpha=20
25
30
35
40
k_2
k_1
k_3
k_1, k_2, k_3
2
1
0
-1
150
175
200
225
250
275
300
325
350
total angle subtended at the centre (degree)
Chart 6.20: R_1/R_2=1.10
Landing angle= 60 deg



alpha=20
25
30
35
40
k_2
k_1
k_3
2
1
0
-1
k_1, k_2, k_3
150
175
200
225
250
275
300
325
350
total angle subtended at the centre (degree)
Chart 6.21: R_1/R_2=1.10
Landing angle= 70 deg

The vertical moment at the support often becomes the most critical design force. Another factor k_3 has been introduced in order to expedite the design process, where,

$$M_{sup} = k_3.wR_2^2 \tag{6.28}$$

The factor k_3 has also been depicted in the proposed design charts.

6.5 VARIATION OF STRESS RESULTANTS ALONG THE SPAN

Figs. 6.3 through 6.6 are presented to depict the variation of the stress resultants for two different stairs. The mid span radial horizontal force and vertical moment have been found using the charts and the rest are calculated using Morgan's equations.

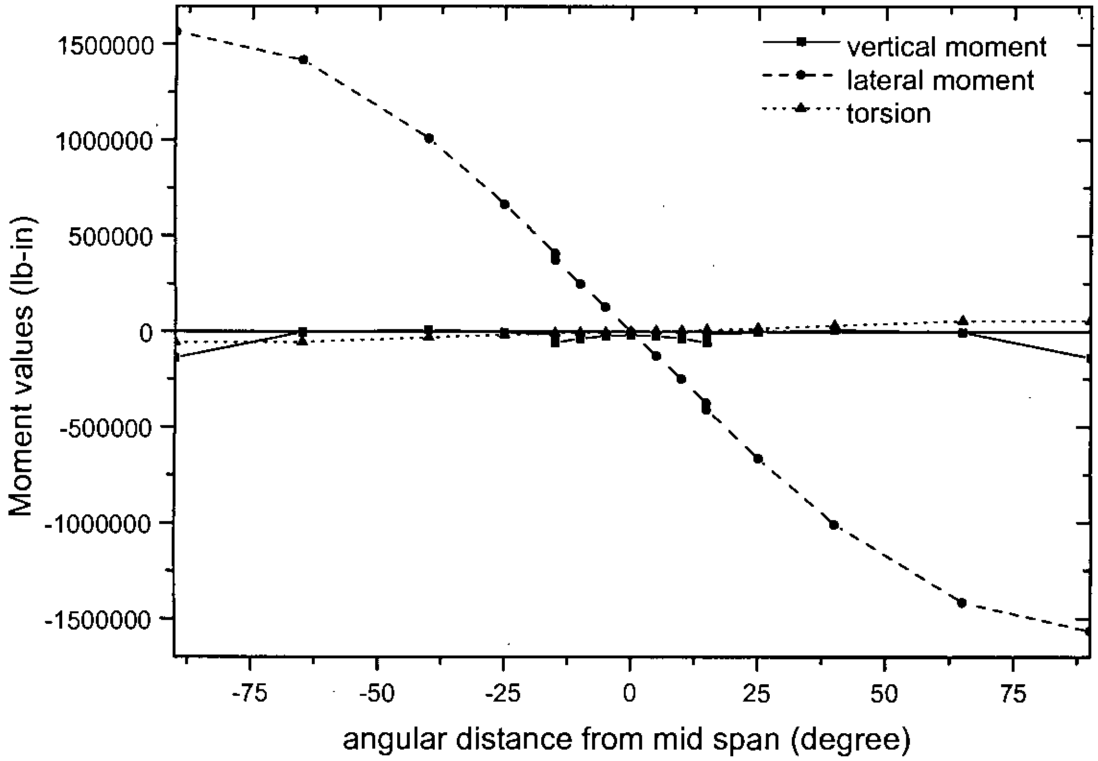


Fig 6.3 Variation of moment along the span for a 180 degree stair

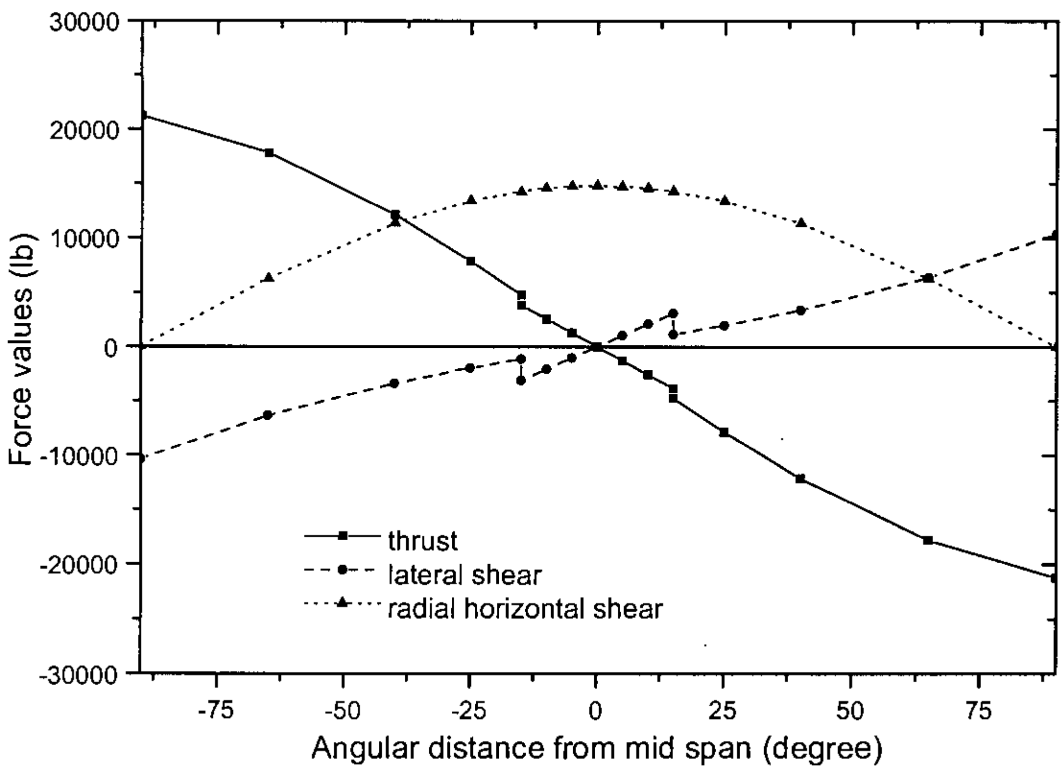


Fig 6.4 Variation of forces along the span for a 180 degree stair

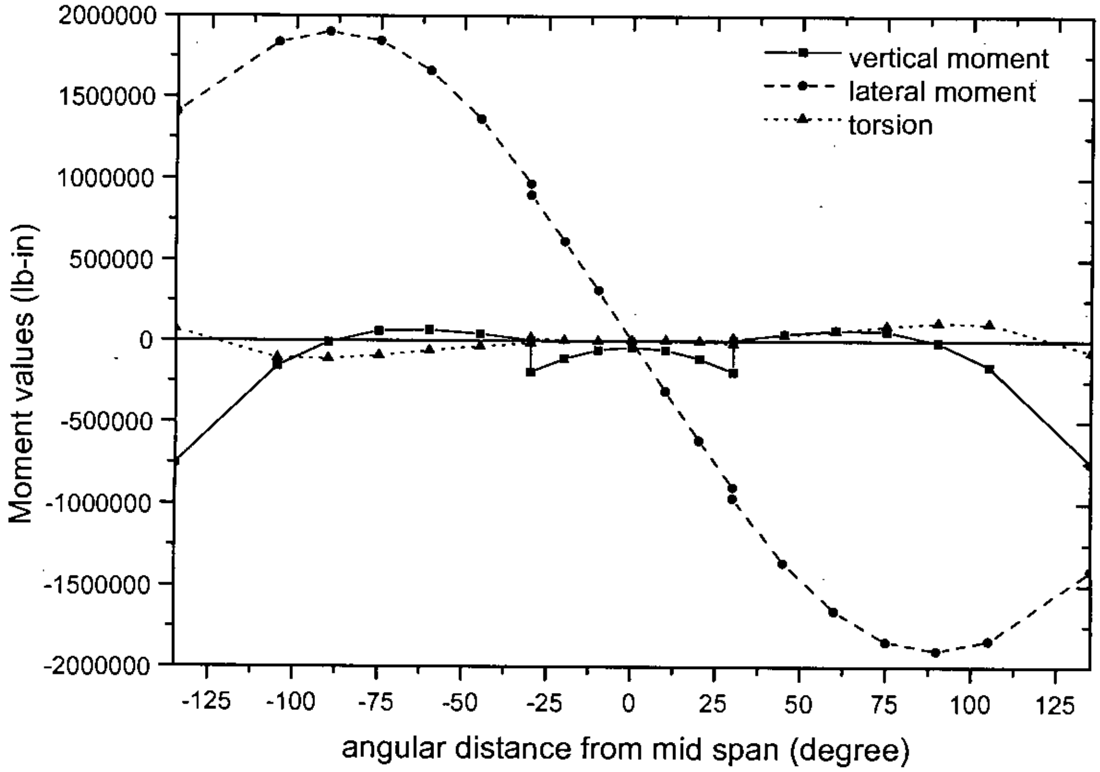


Fig 6.5 Variation of moment along the span for a 270 degree stair

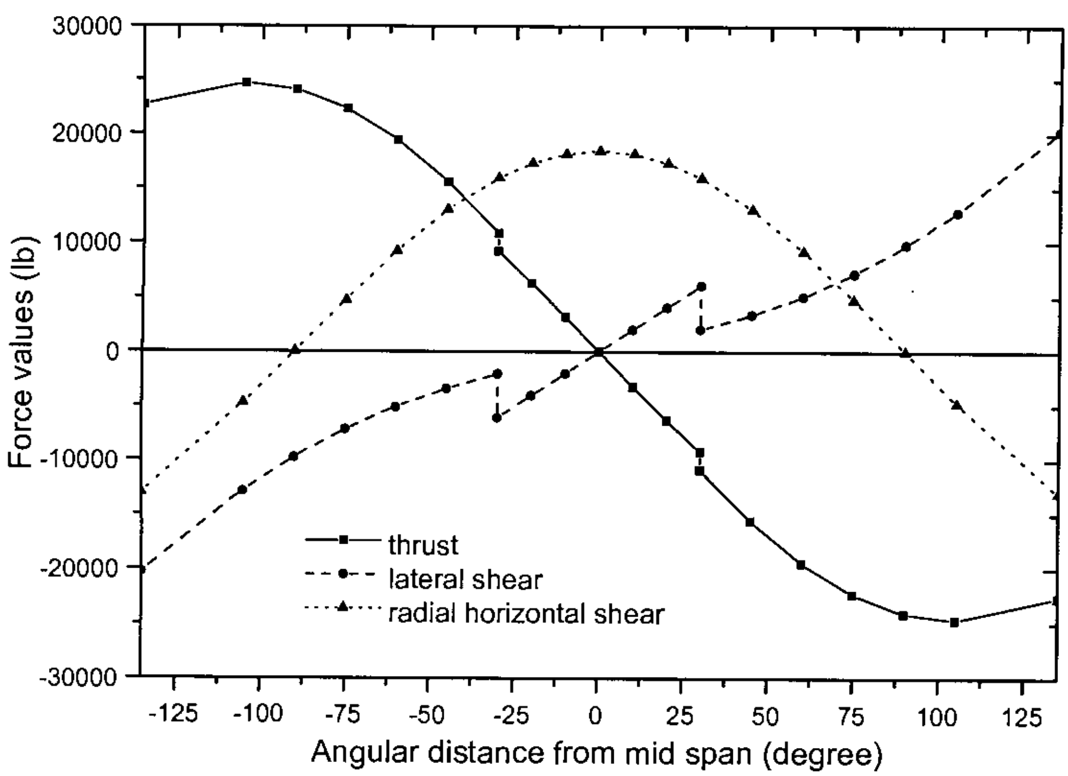


Fig 6.6 Variation of forces along the span for a 270 degree stair

CHAPTER 7

EFFECTS OF LANDING

7.1 INTRODUCTION

The critical design forces in the helical structure depend upon the geometric parameters of the stair slab and, of course the loading conditions. It is obvious that the design forces and moments are critical at different locations for different types of forces and moments. The critical locations also depend upon the geometric parameters.

During the past few years, a detailed study on helicoidal stair slabs without landings has been carried out at BUET with a view to generalize their behaviour. The variation of the design forces and moments with respect to various geometric parameters has been studied. Also, the critical locations for different stress resultants have been determined. It is known from the earlier studies that, for helicoidal stair slabs with no landings:

- Maximum vertical moment occurs at or near the support, depending on the central angle
- Maximum lateral moment occurs at or near the support, depending on the central angle
- Location of maximum torsion varies greatly with the central angle
- Maximum thrust occurs at or near the support, being dependent upon the central angle
- Maximum radial horizontal shear occurs at midspan
- Maximum lateral shear occurs at the support

While all these features are expected to show up in a helicoidal stair with an intermediate landing, the exact effect of the landing length has not yet been characterized. To understand the effect of landing on the behaviour of the helicoidal stair slab a study with a limited scope has been taken up.

7.2 THE PARAMETERS

The parameters which have significant impact on the stress resultants are:

I. Total angle subtended by the whole stair slab (θ_f)
II. Slope of the helicodial slab with respect to horizontal plane (α)
III. Landing length, expressed as the angle subtended at the centre (2ϕ)
IV. Ratio of the radius of centreline of loading to the mean radius (R_1/R_2)
V. Mean radius (R_2)

The variation of the stress resultants with respect to the mean radius is clearly evident through the 2nd degree equation for mid span redundant moment ($M = k_1wR_2^2$) and 1st degree equation for the mid span horizontal shear force ($H = k_2wR_2$). The effect of central angle, slope of the slab and the ratio R_1/R_2 has been extensively studied before in one form or other during the study of helicoidal stairs without landings and are expected to be valid for stairs with landing as well. Also the charts provide an idea as to the variation of the design forces with respect to these parameters. Emphasis will be placed here on the effect of the landing length on the force and moment values.

7.3 EFFECT ON FORCES AND MOMENTS

Figs. 7.1 through 7.12 depict the effect of the landing length on design forces for two stairs with 180 degrees and 270 degrees central angles. The geometric dimensions and loading of the stairs are given in Table 7.1. The results from the two stairs are summarized in the following paragraphs:

Table 7.1: Geometric parameters of the two stairs

Parameter	Stair 1	Stair 2
Central angle	180	270
Inner radius	60 inches	60 inches
Outer radius	120 inches	150 inches
Height	120 inches	135 inches
Uniformly distributed surface live load	100 psf	100 psf
Variation of landing angle	20-60 degrees	20-60 degrees

Vertical Moment (Figs. 7.1 & 7.2)

The effect of landing is more prominent for stairs with a lower central angle. This is expected, because for a given fixed length of the landing, the smaller the total central angle, the larger the ratio of landing to the flight, and more dominant will be the action of the landing.

The values of the maximum vertical moment show some change, specially for longer spans. The location of maximum vertical moment remains unchanged. The variation of vertical moment around the landing section however is significantly affected. As expected, the moment in the landing is higher for a larger landing length.

The vertical moment diagram of the landing section is interesting. It shows that the landing section acts as a slab supported at two ends.

Lateral Moment (Figs. 7.3 & 7.4)

The effect of the landing is moderate. However, the lateral moment remains unaffected due to the presence of the landing, even at the landing region. It is seen that the presence of a larger landing reduces the overall lateral moment, to a small extent.

Torsion (Figs. 7.5 & 7.6)

The effect of the landing has been most prominent in torsion. A longer landing results in a higher torsion. A landing angle as small as 20 degrees has almost no effect on the variation of torsion in the landing region.

Thrust (Figs. 7.7 & 7.8)

The effect of landing is small. Near the landing region some effect can be observed. For a smaller landing angle, the local effect is virtually nil. A longer landing produces a slightly smaller thrust.

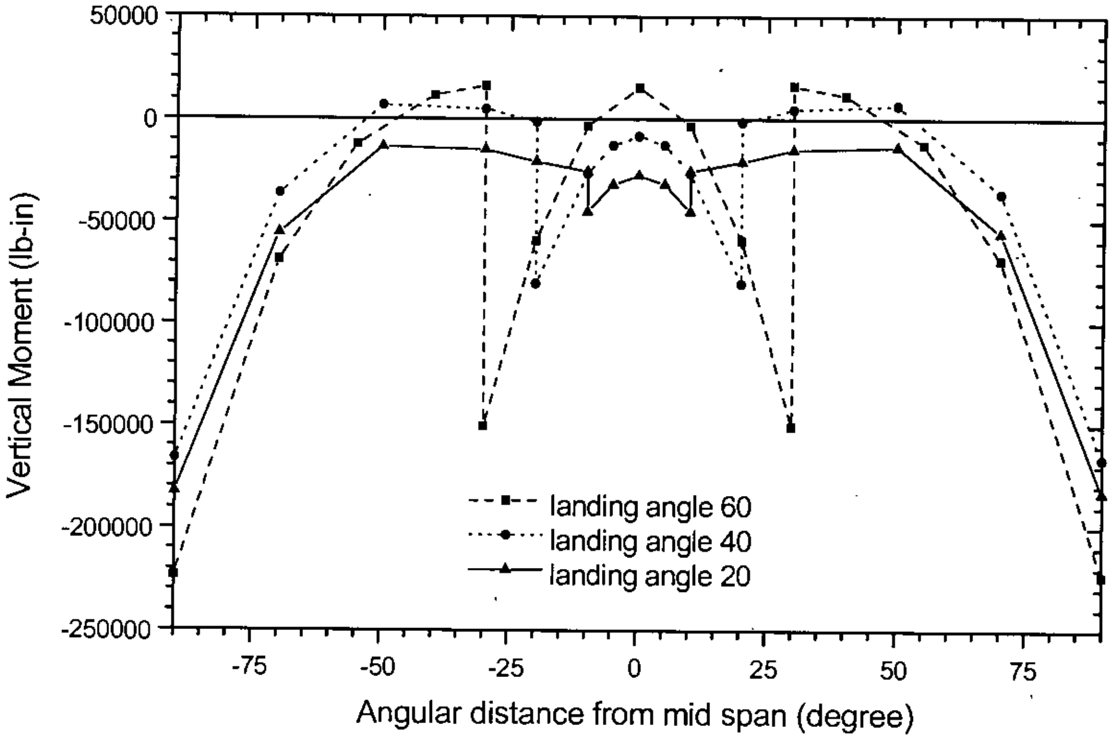


Fig 7.1 Variation of vertical moment along the span (270 degree)

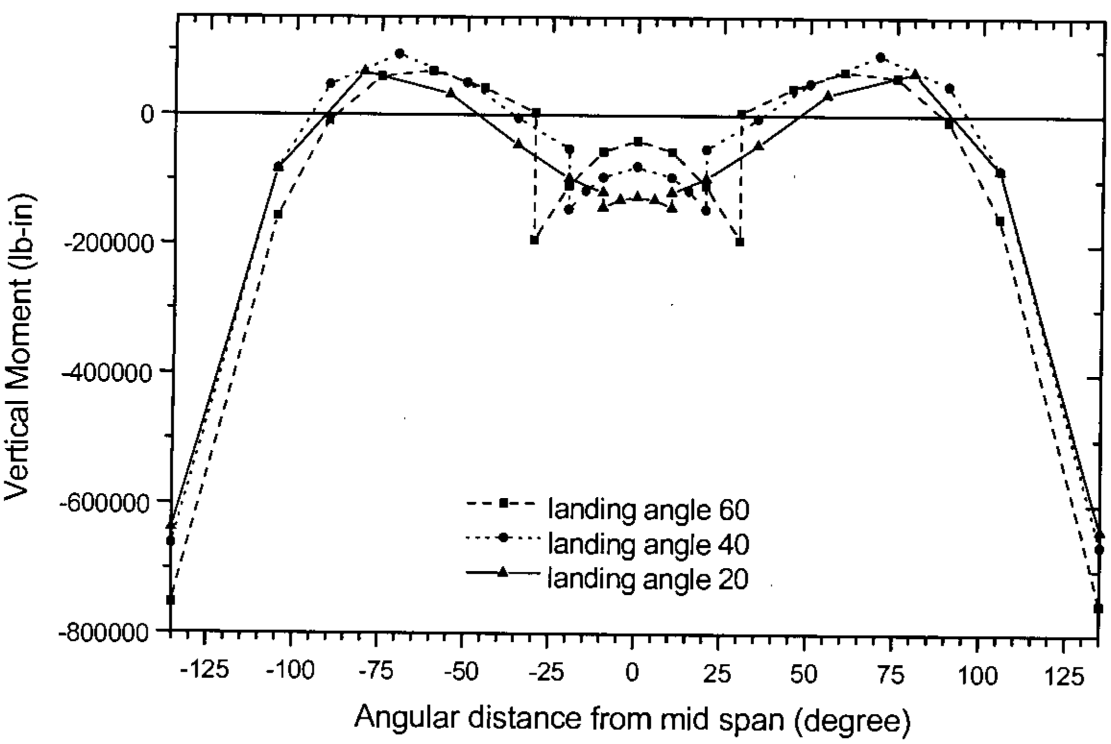


Fig 7.2 Variation of vertical moment along the span (270 degree)

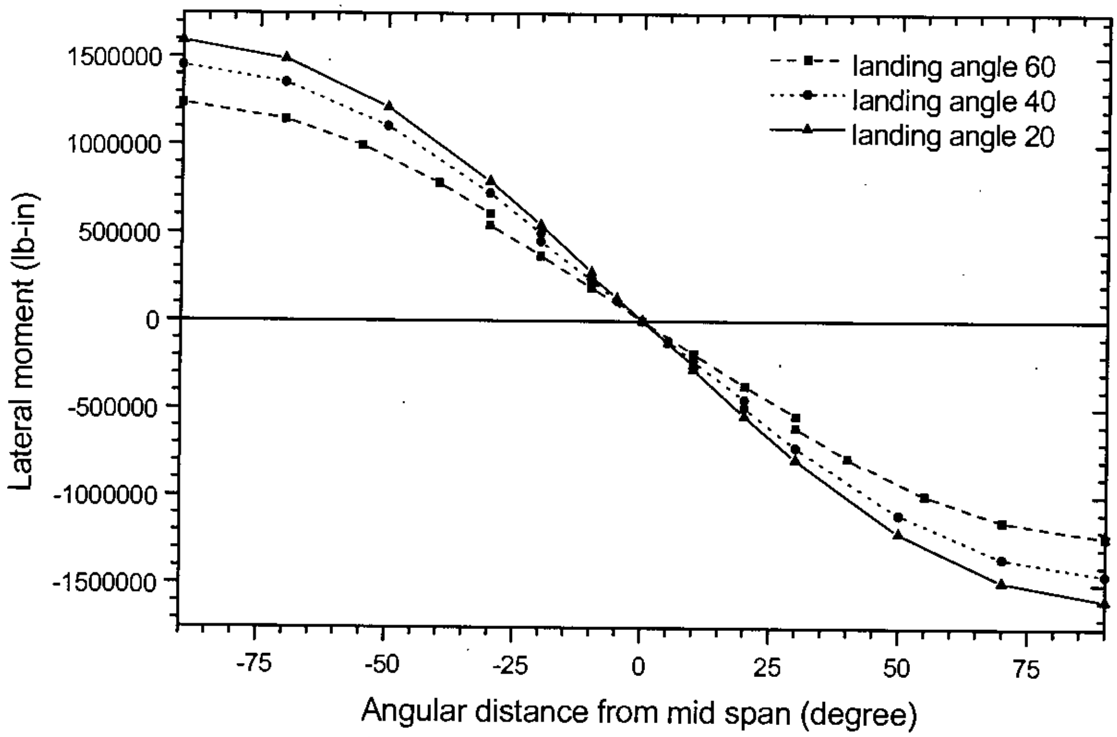


Fig 7.3 Variation of lateral moment along the span (270 degree)

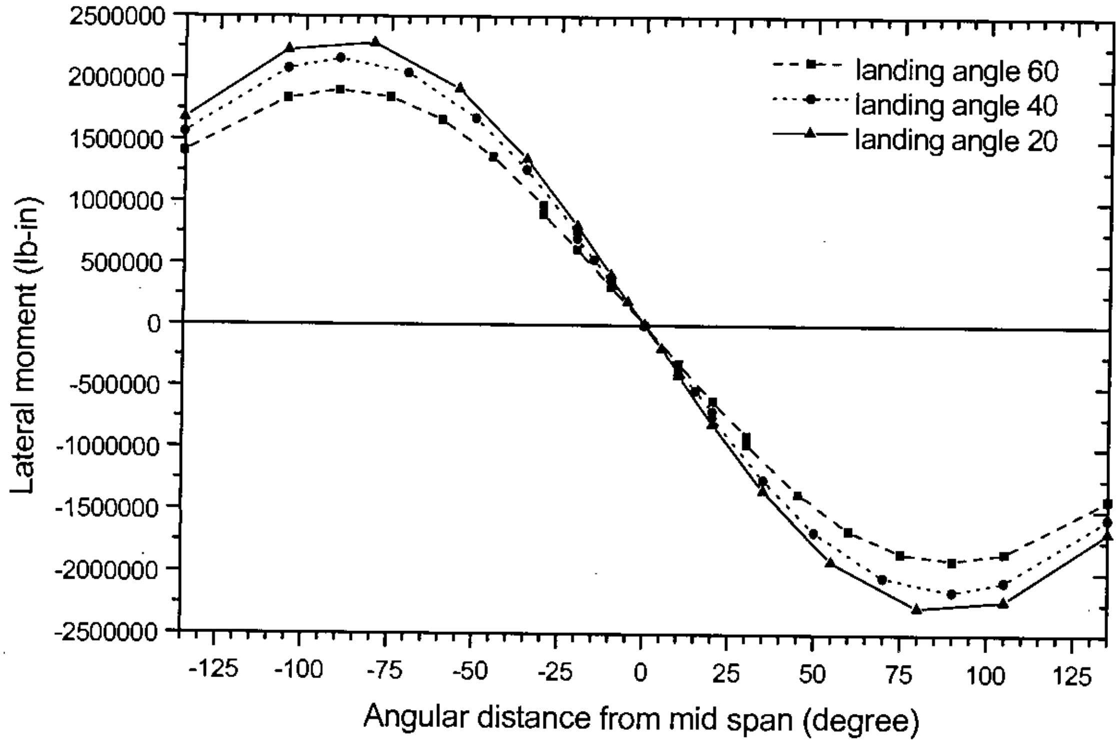


Fig 7.4 Variation of lateral moment along the span (270 degree)

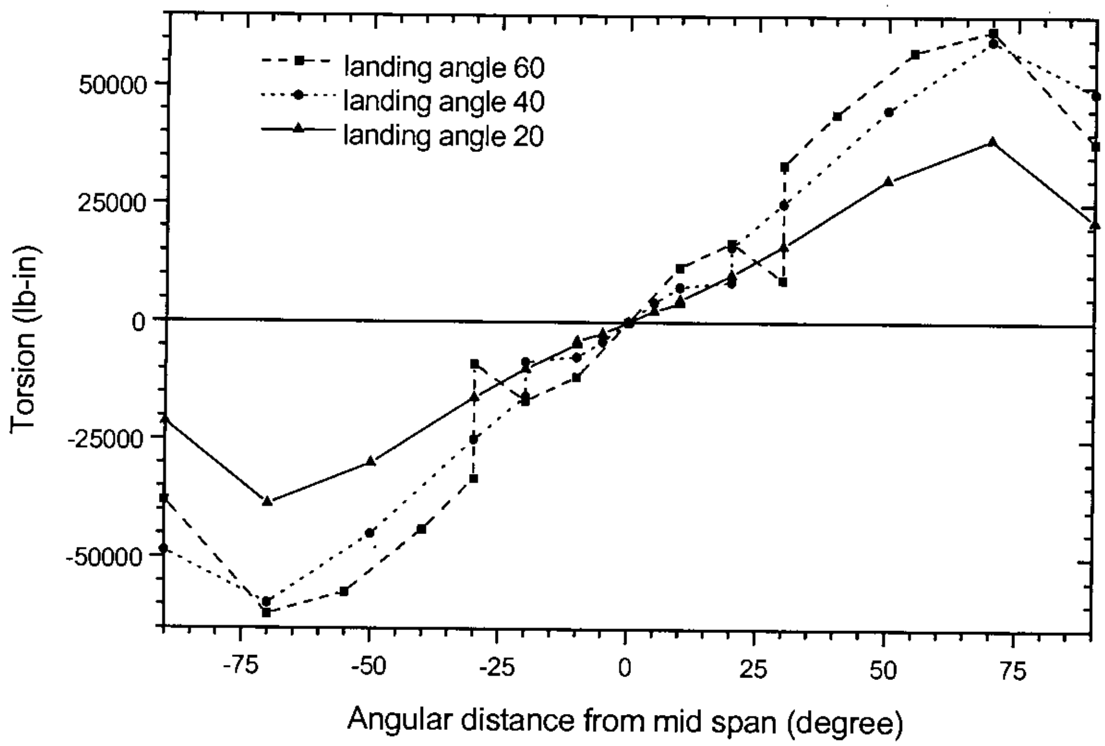


Fig 7.5 Variation of torsion along the span (270 degree)

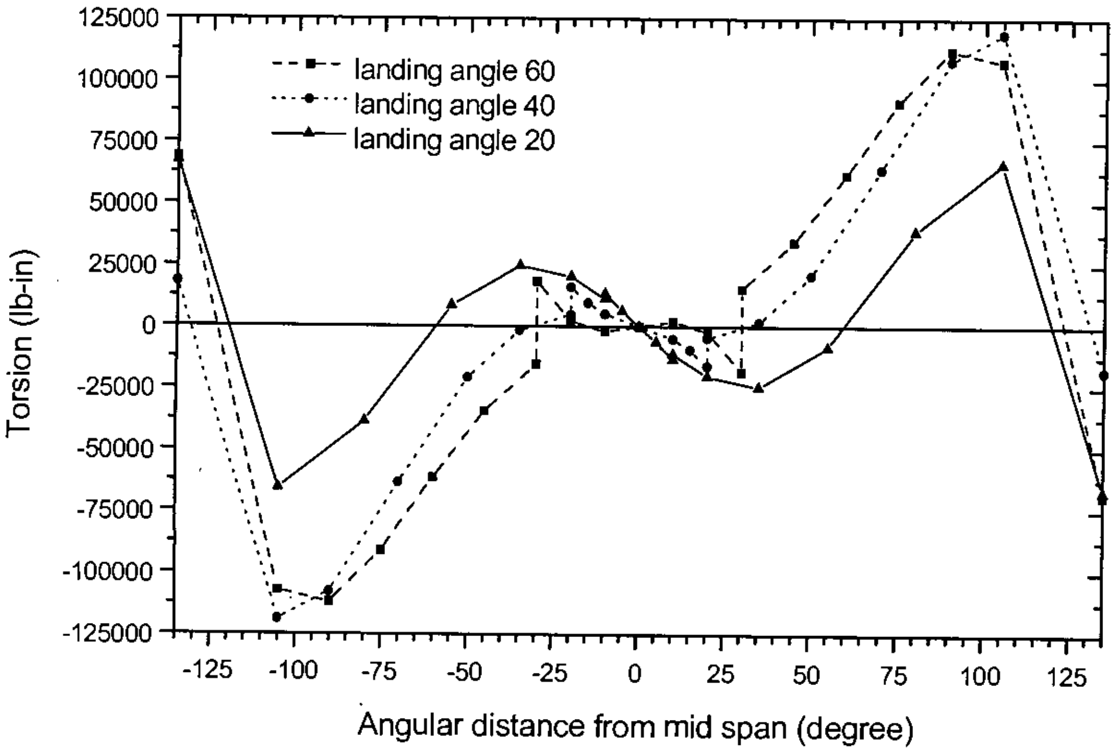


Fig 7.6 Variation of torsion along the span (270 degree)

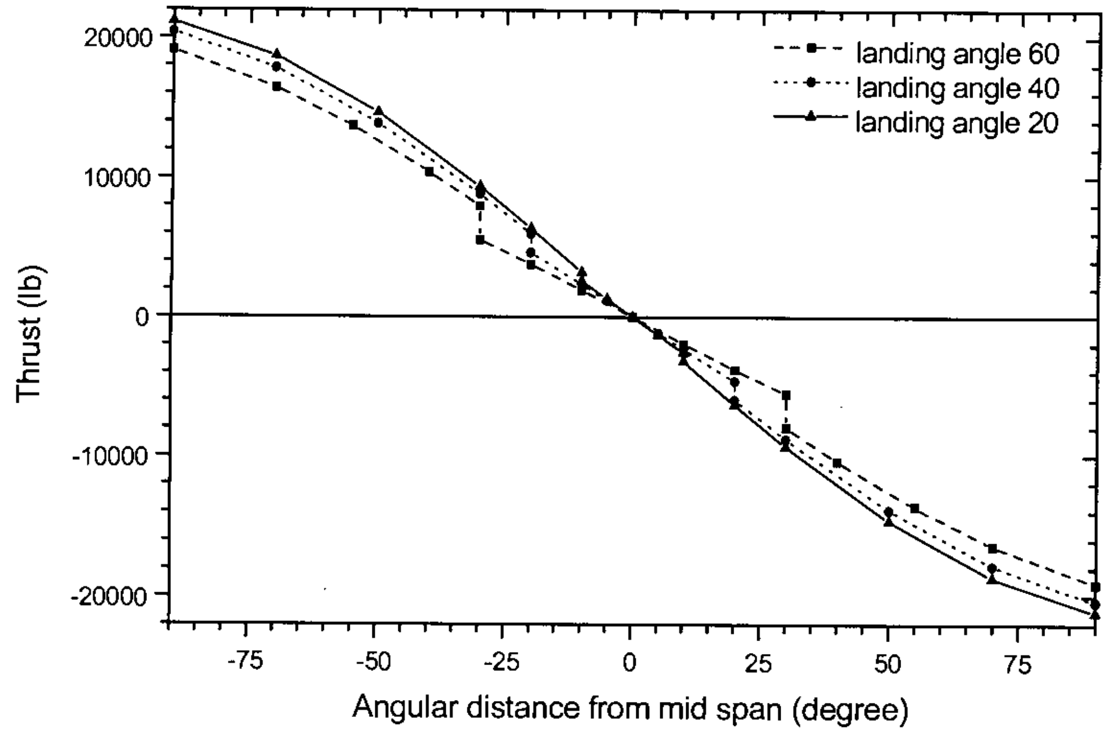


Fig 7.7 Variation of thrust along the span (270 degree)

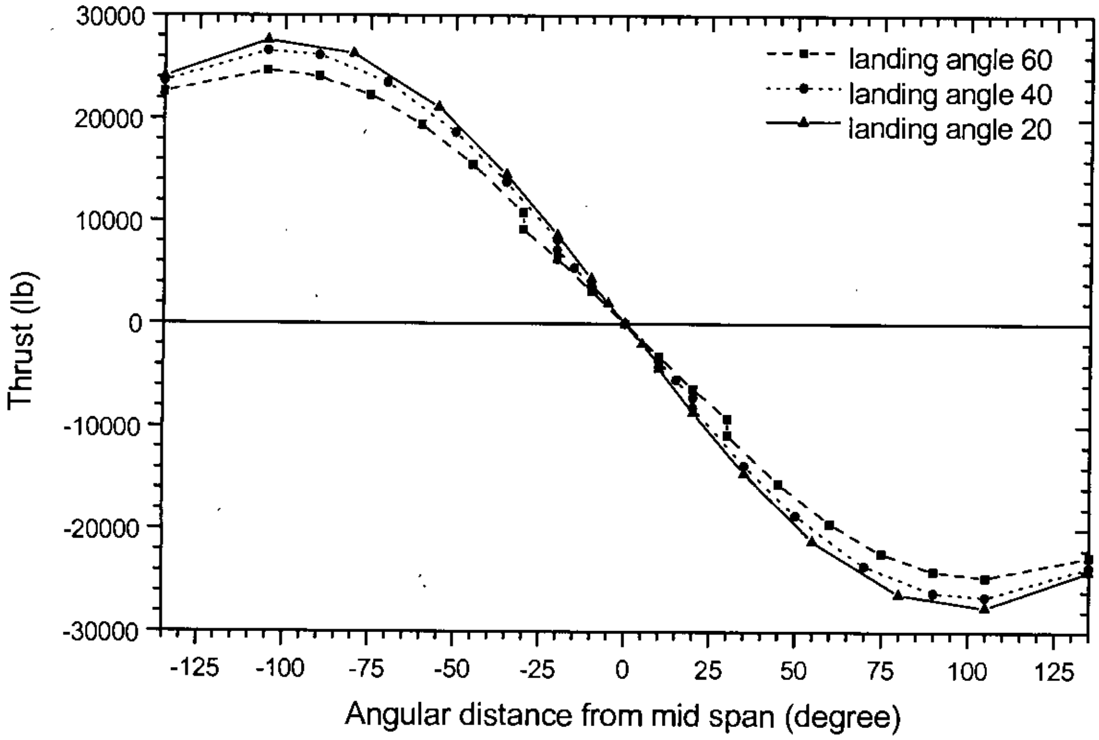


Fig 7.8 Variation of thrust along the span (270 degree)

Lateral Shear (Figs. 7.9 & 7.10)

There is absolutely no effect of the landing on the flight section. The lateral shear diagram at the landing level shows that the landing section acts as a pure slab supported at the ends by the flights.

Radial Horizontal Shear (Figs. 7.11 & 7.12)

The effect of the landing is not at all evident locally in the landing section. However, the radial horizontal shear is much reduced due the presence of a larger landing. This reduction is more prominent in the stair with smaller total central angle.

7.4 DEFLECTION COMPARISON

The serviceability criteria of deflection may be an important parameter while designing a helicoidal stair. The thickness of the stair, specially for stairs with higher mean radius and central angle higher than around 200 degrees may be governed by the maximum deflection. This calls for the determination of deflection of the helicodial stair, specially with an intermediate landing.

In order to determine the deflection, finite element approach has been employed. The computer program developed by Amin, which uses the Ahmad's thick shell finite element program, has been modified to calculate the deflections specifically, for only one case of landing length, 0.2 times the span on plan. The total angle, i.e., span can be varied, but the landing will also vary proportionally. The details of the modifications in the finite element program are not presented here as they bear no consequence.

The comparison of deflection of helicoidal stairs with and without landing shows that the stairs with landings deflect more than those without landings. The comparative deflection profile is depicted in Figs 7.13 through 7.18. The effect of thickness on the deflection profile is presented in Figs. 7.19 to 7.24. Fig. 7.25 provides the deflection pattern for stairs with different central angles.

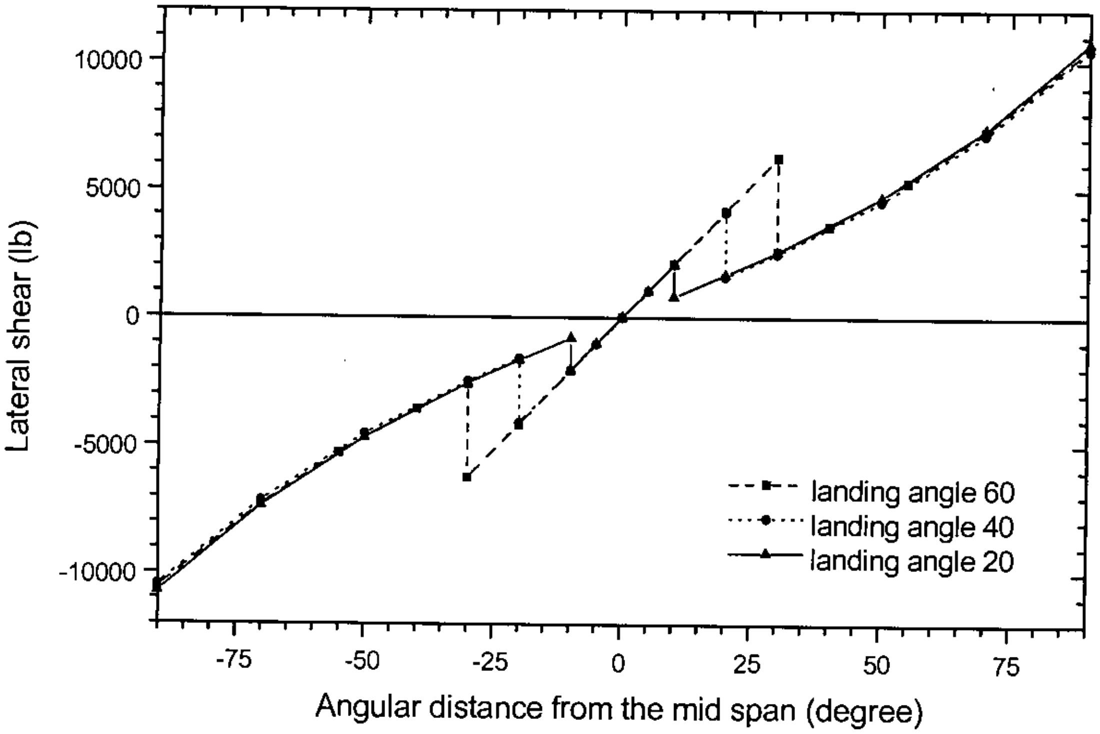


Fig 7.9 Variation of lateral shear along the span (270 degree)

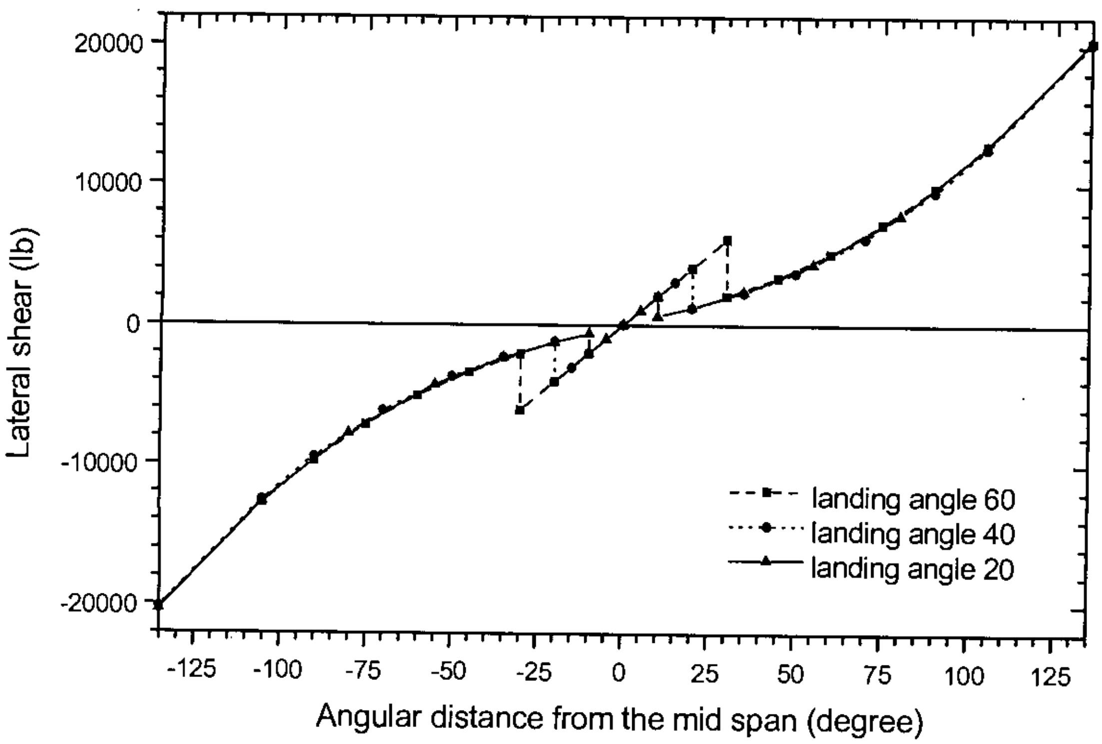


Fig 7.10 Variation of lateral shear along the span (270 degree)

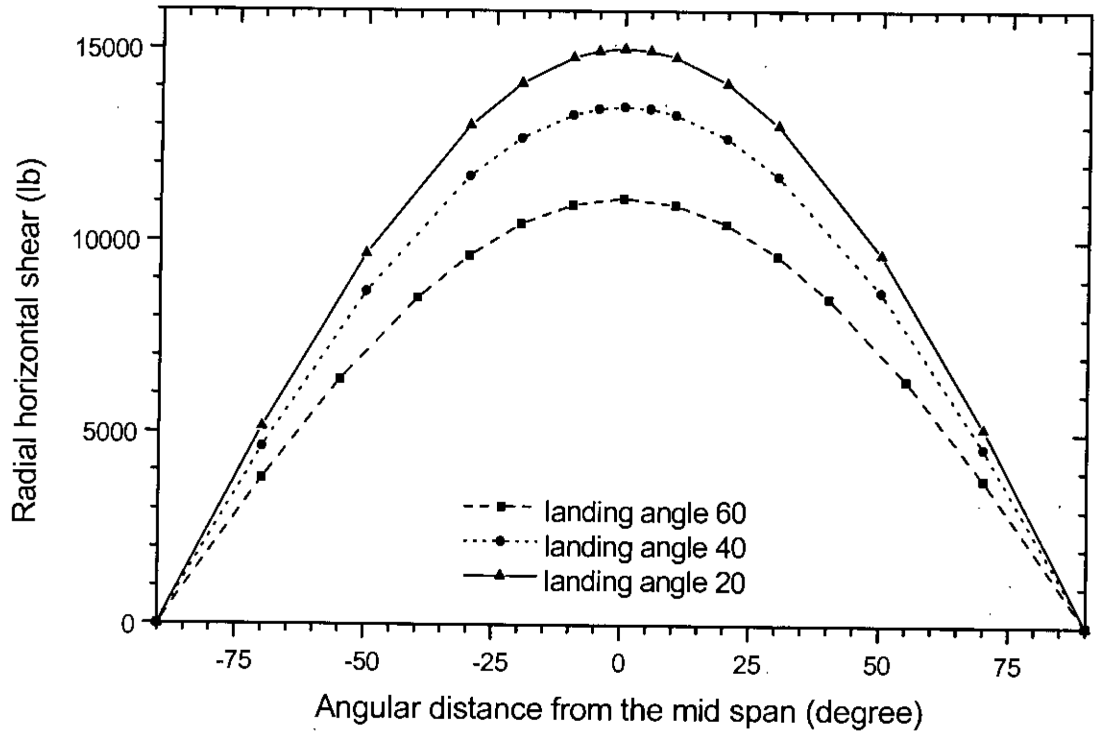


Fig 7.11 Variation of lateral shear along the span (270 degree)

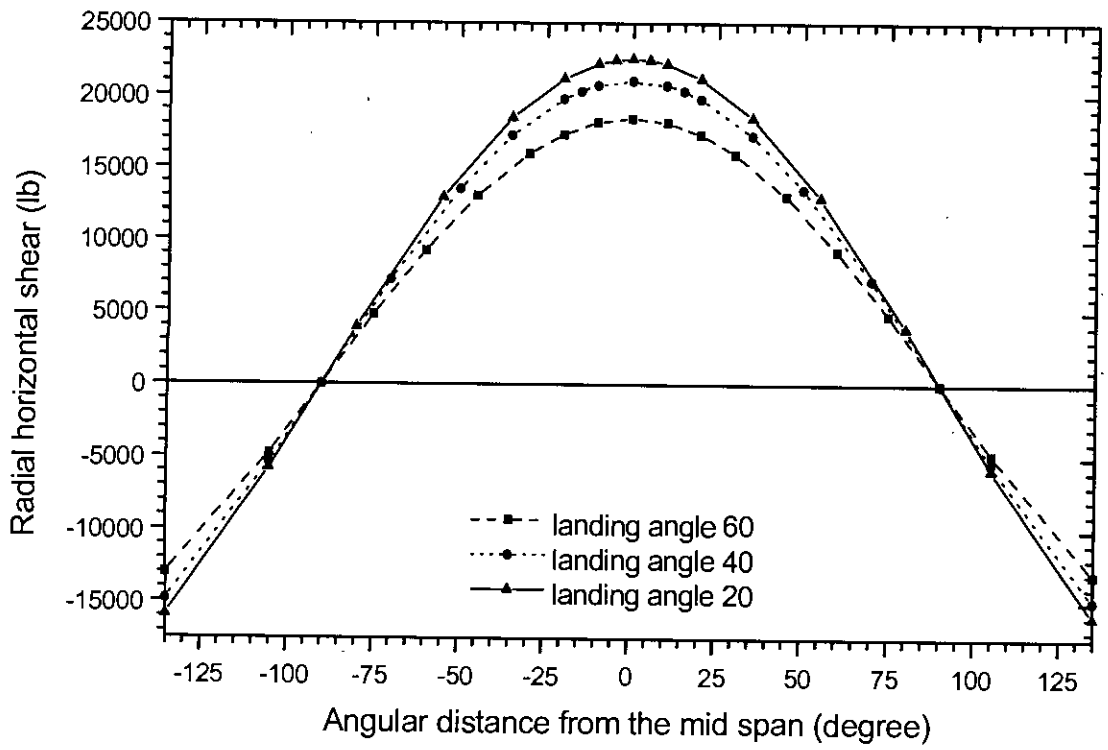


Fig 7.12 Variation of radial horizontal shear along the span (270 degree)

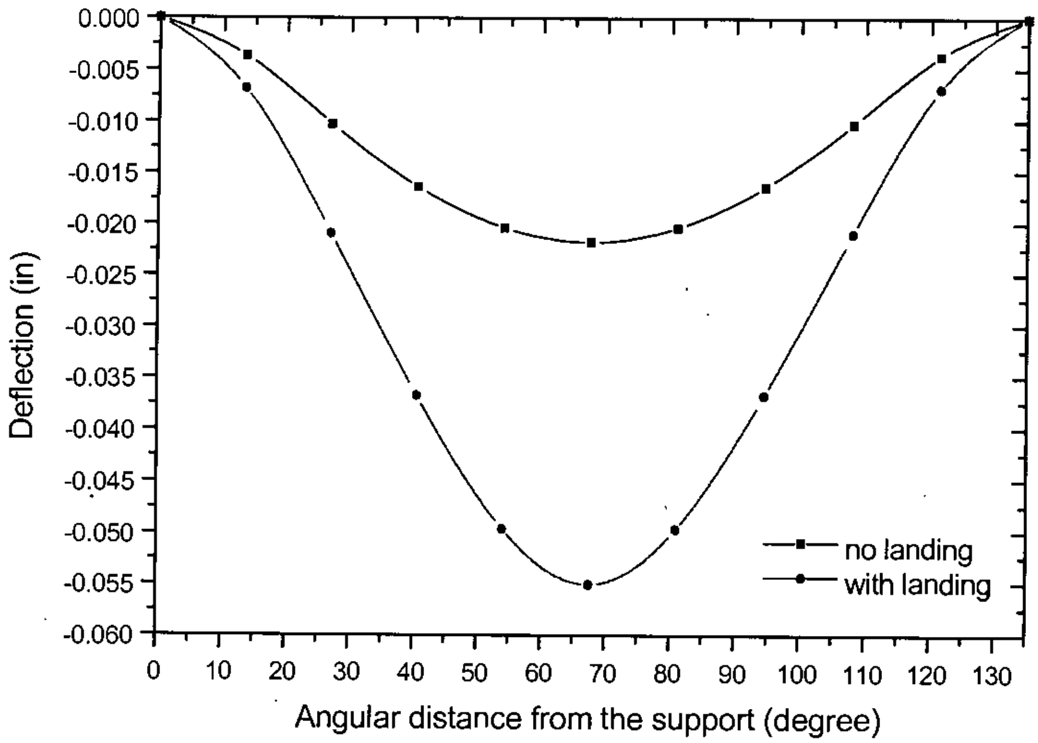


Fig. 7.13: Deflection pattern of helicoidal stair slab with and without landing (135 deg)

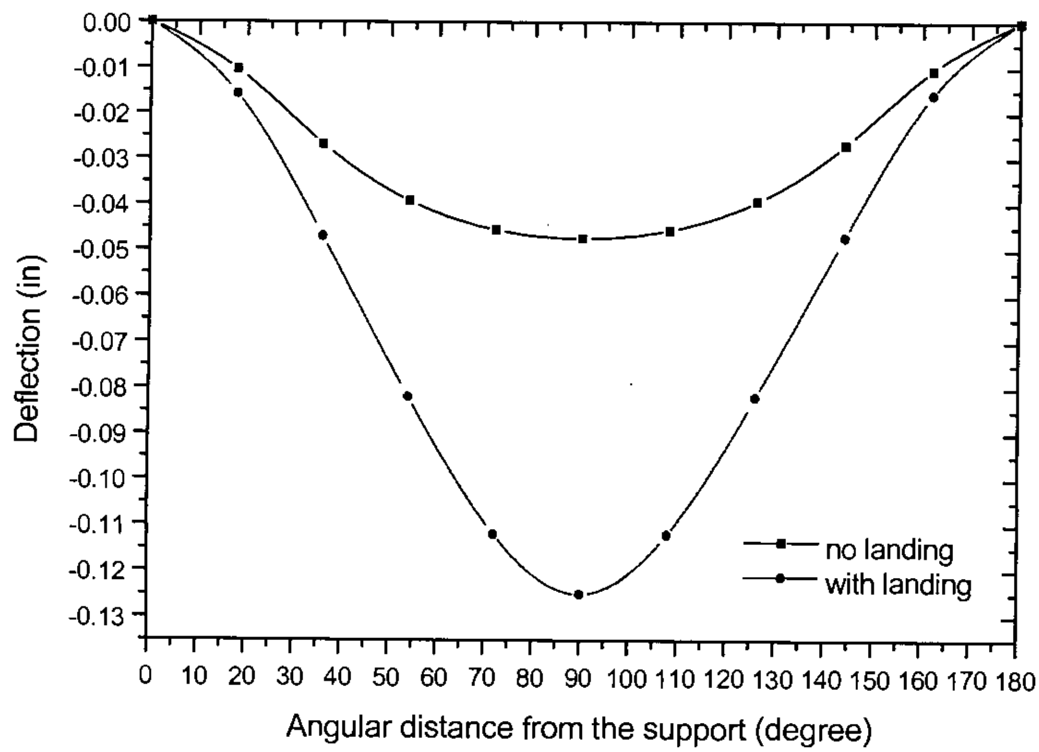


Fig. 7.14: Deflection pattern of helicoidal stair slab with and without landing (180 deg)

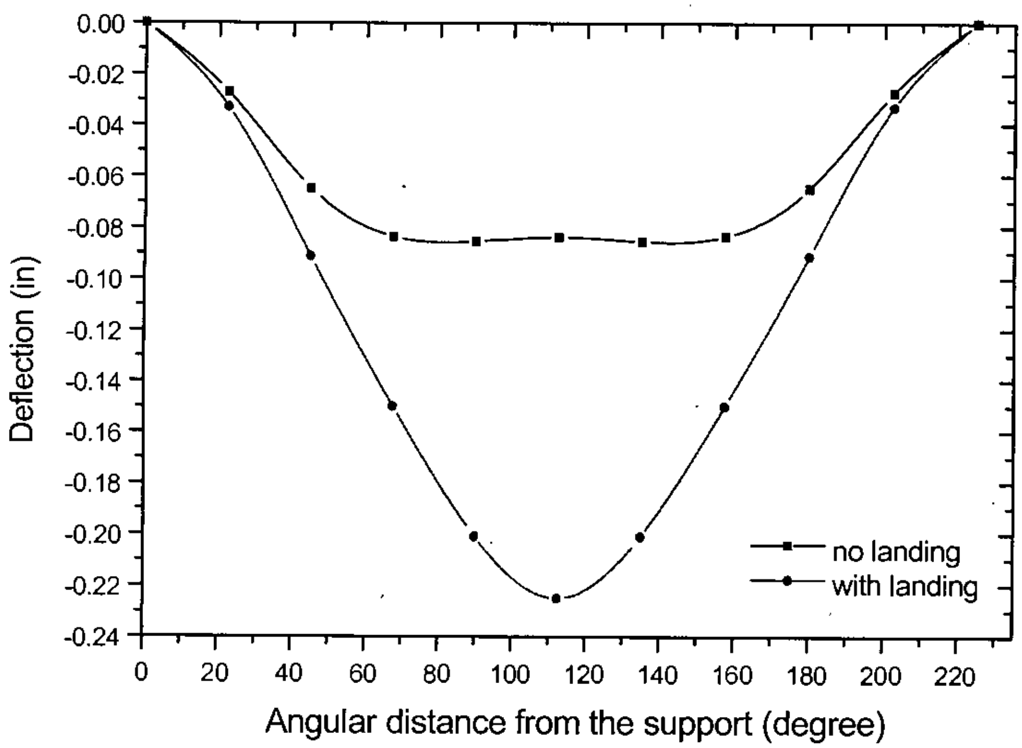


Fig. 7.15: Deflection pattern of helicoidal stair slab with and without landing (225 deg)

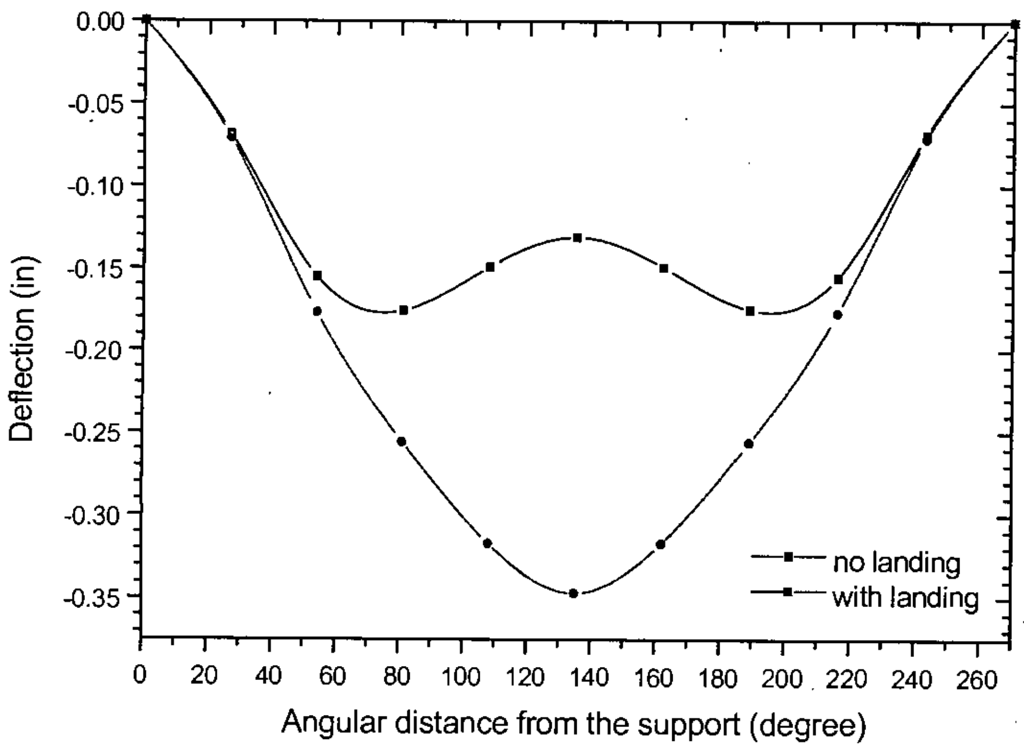


Fig. 7.16: Deflection pattern of helicoidal stair slab with and without landing (270 deg)

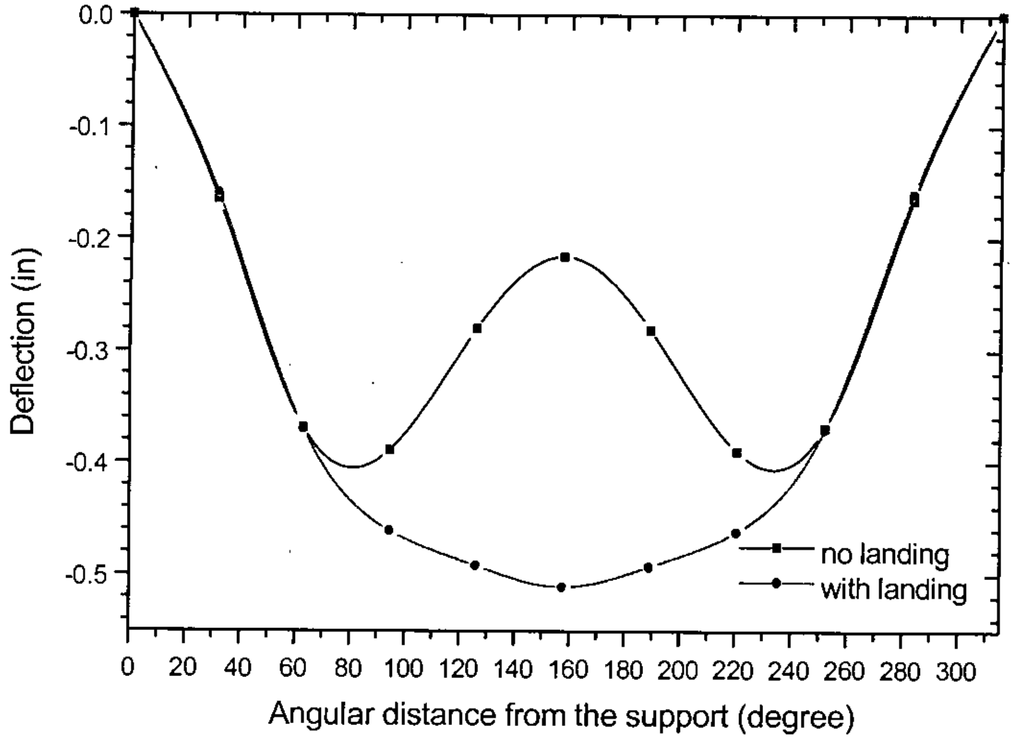


Fig. 7.17: Deflection pattern of helicoidal stair slab with and without landing (315 deg)

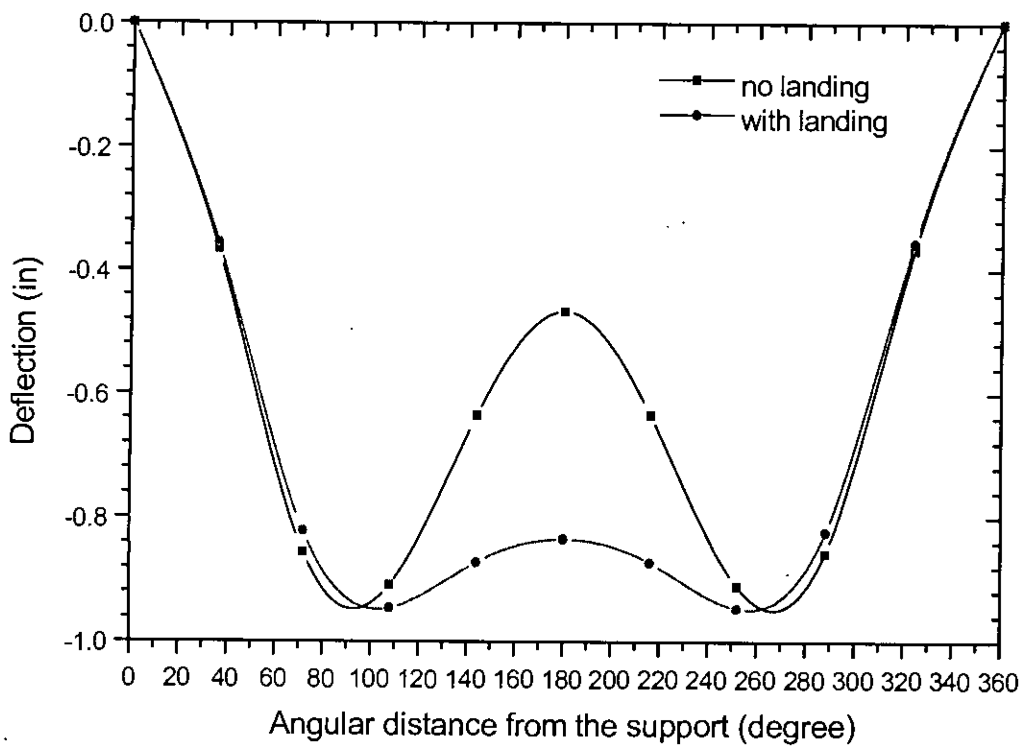


Fig. 7.18: Deflection pattern of helicoidal stair slab with and without landing (360 deg)

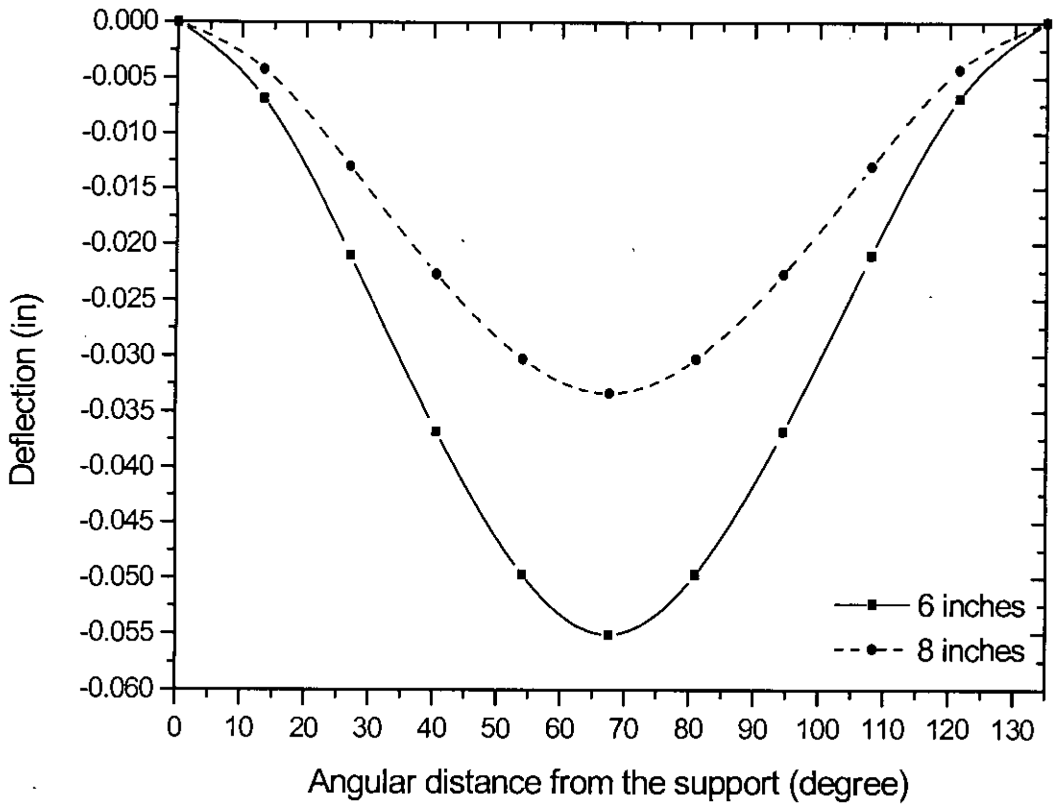


Fig. 7.19 Effect of thickness on deflection pattern (135 deg)

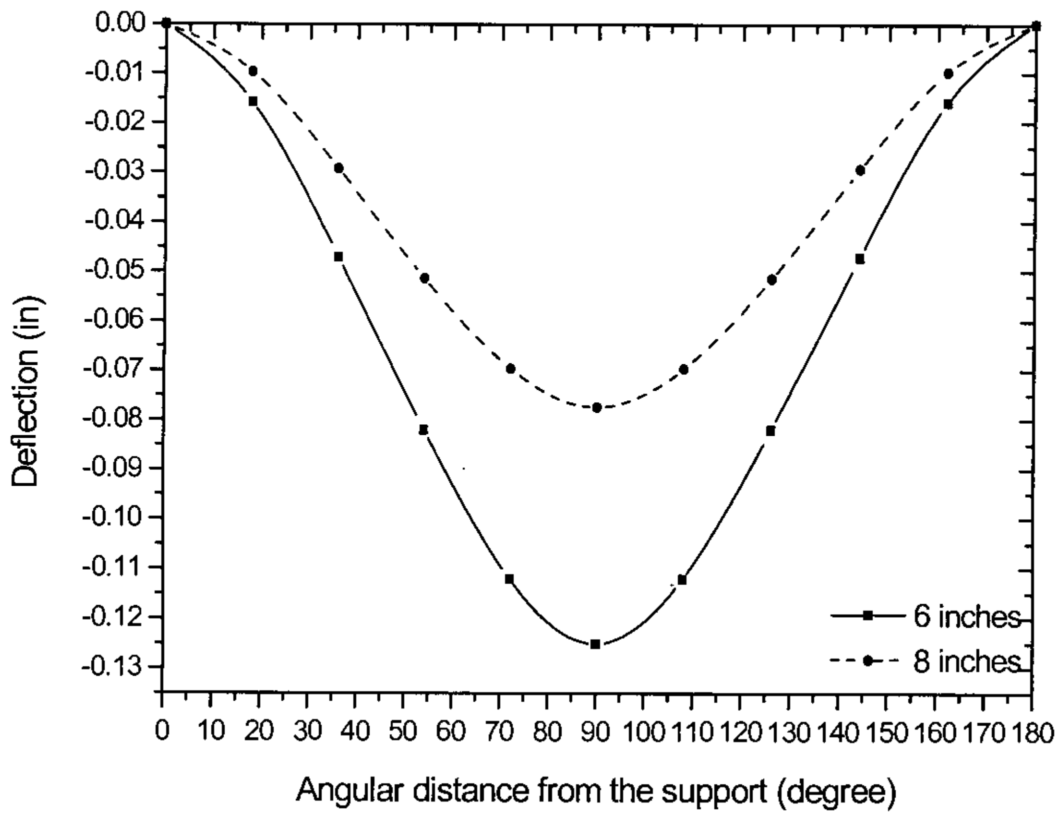


Fig. 7.20 Effect of thickness on deflection pattern (180 deg)

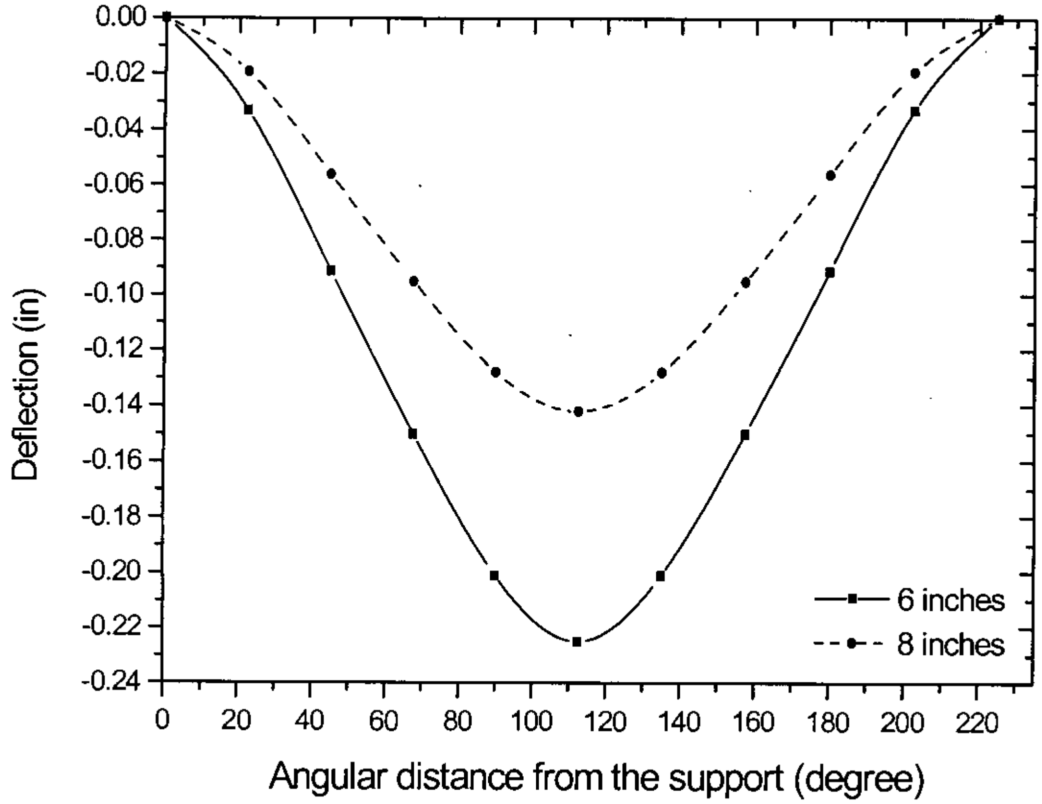


Fig. 7.21 Effect of thickness on deflection pattern (225 deg)

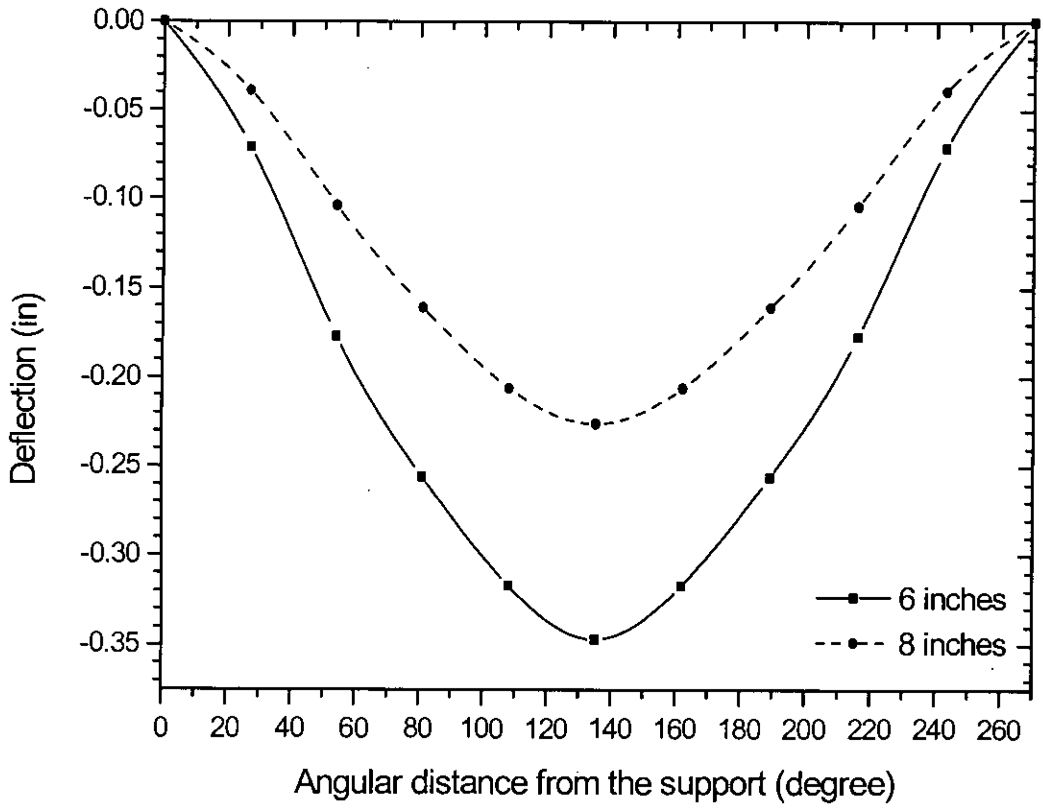


Fig. 7.22 Effect of thickness on deflection pattern (270 deg)

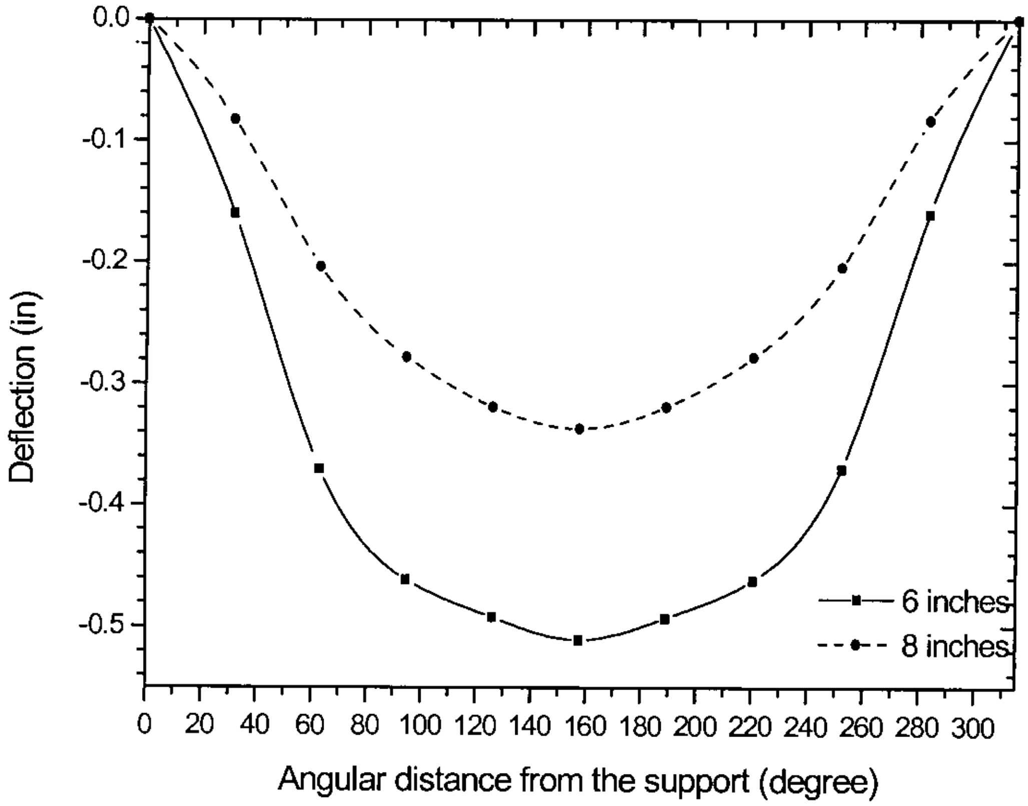


Fig. 7.23 Effect of thickness on deflection pattern (315 deg)

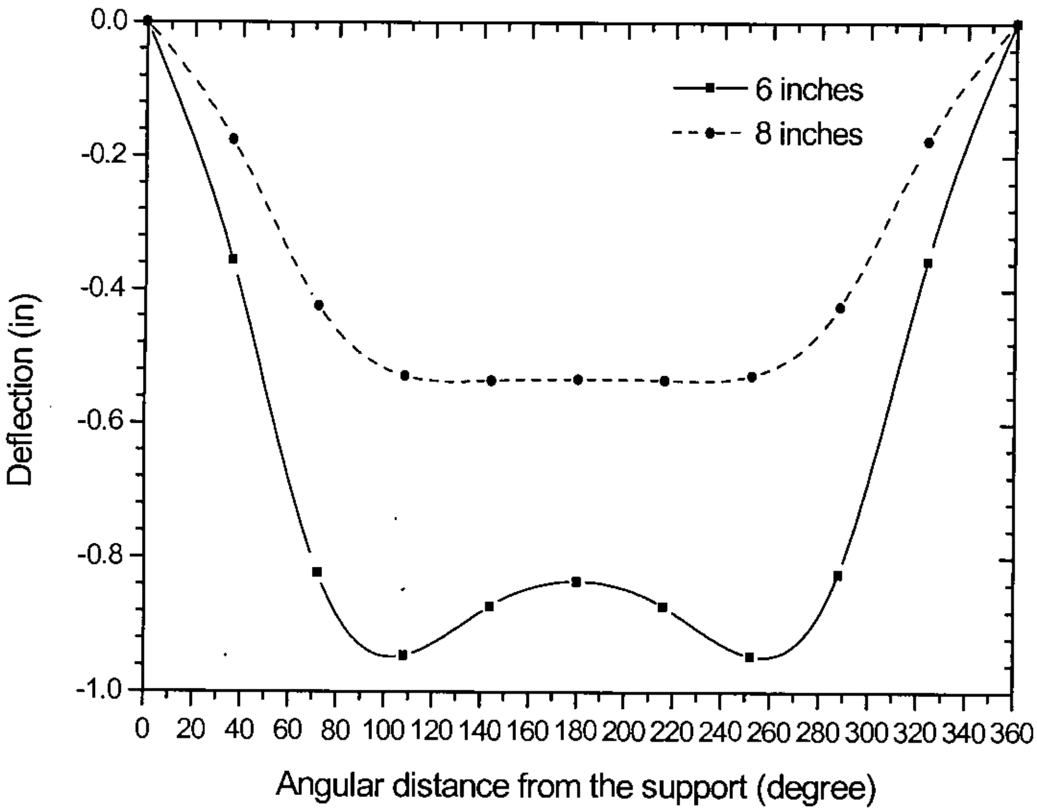


Fig. 7.24 Effect of thickness on deflection pattern (360 deg)

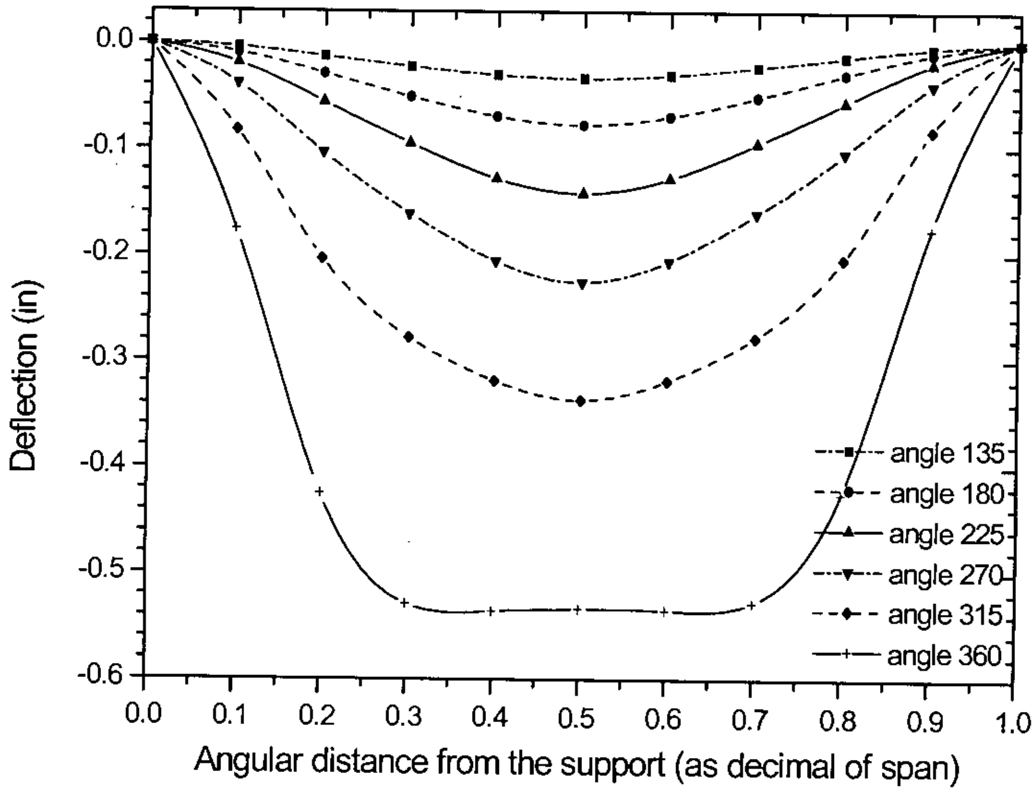


Fig. 7.25 Deflection pattern of helicoidal stair slab with an intermediate landing

Because the helicoidal stairs with landing undergo appreciable deflection, the thickness of the slab may be governed from limiting deflection criteria.

7.5 FINDINGS

Based on the behaviour of the helicoidal stair slab under varying landing length (Figs. 7.1 through 7.12) following conclusions have been drawn. Some of these observations contradict and many consolidate the conclusions by Arya and Prakash (1973) and Solanki (1973).

I. Both, Arya and Prakash, and Solanki concluded that torsional moment has the maximum value at the support. The present study concludes that torsion may have maximum value at or nearer the support. The critical location depends on the geometric parameters. The effect of central angle is most prominent.

II. The vertical bending moment is maximum at the support. However, for a stair with smaller total central angle and larger landing, the vertical moment at the landing level is appreciably high.

III. Maximum lateral moment occurs at the end quarter span. This supports the findings by Arya and Prakash, and Solanki.

IV. Radial horizontal shear force is maximum either at the supports or at the mid span depending on the geometric parameters, central angle being the dominant factor.

V. The thrust is maximum at a section lying at the end quarter span. The lower portion of the stair is in compression and upper portion is subject to tension.

VI. Lateral shear is maximum always at the ends.

VII. Maximum values of vertical moment increase, and lateral moment, thrust and radial horizontal shear decrease with the increase of the landing length. Landing length has virtually no effect on the lateral shear distribution.

VIII. The effect of landing is most prominent on torsion. Torsion increases appreciably with the increase in landing length.

IX. Local effect of landing is quite important, specially in the case of vertical moment, lateral shear and torsion.

CHAPTER 8

CONCLUSION

8.1 GENERAL

Helicoidal stair slabs are now being increasingly used throughout the world. This thesis looks into a number of aspects regarding the helicoidal stair slabs. These include:

- Verification of the Reynold's coefficients
- Investigation into the possibilities of temperature and shrinkage steel requirement governing the design
- Determination of the stress resultants in a helicoidal stair slab with intermediate landing
- Suggestion for a design chart for the design of helicoidal stair slab with intermediate landing
- Study on the behaviour of helicoidal stair slabs with intermediate landing

8.2 SPECIFIC FINDINGS

8.2.1 Verification of Reynolds Coefficients

Finite element methods have been used to verify the coefficients suggested by Reynolds, Santathadaporn and Cusens. A regression analysis has also been carried out in this regard. The correlation between the coefficients found from the Reynolds charts and the finite element analysis has been strong. The study revealed that the Reynolds coefficients for calculating mid span horizontal shear are in good agreement with the finite element results. The Reynolds coefficients for calculating moments, however differ slightly with the finite element findings.

8.2.2 Design of RCC Helicoidal Stair

The possibility of suggesting temperature and shrinkage reinforcements to take care of the axial force and vertical moment has been investigated. A direct design method has been proposed in this regard. Figs. 5.1 through 5.6 can be used to design the slab when 60000 psi steel and 3000 psi concrete is to be used. Tables 5.1 and 5.2 further simplifies the results of these figures. Steel requirement at the support for axial force and vertical moment per ft width of the stair can be determined using these tables, when live load is 100 psf. All these design charts and tables follow the USD method.

8.2.3 Introduction of Intermediate Landing

Detail analysis of a helicoidal stair slab with intermediate landing has been carried out. The assumptions in the analysis has been noted. The analysis followed Solanki's efforts using helical girder solution.

Further studies on the stair differ with the observations by Solanki, Arya and Prakash that the torsion has the maximum value at the support. Other findings supported Arya, Prakash and Solanki's conclusions.

8.2.4 Proposal for a Design Chart

The most important outcome of this research is the proposed design method for helicoidal stair slabs with intermediate landings. Various charts are proposed for different geometric combinations. These charts are presented in Chapter 6. The step by step procedure to be followed in designing a helicoidal stair slab with intermediate landing using these charts is presented in the Appendix C. It is expected that the use of these charts will simplify the design of such stair slabs and encourage the use of such beautiful structural components.

8.2.5 Effect of the Landing

The introduction of an intermediate landing does not radically change the critical design forces and, as a whole, the helicoidal stair slab with an intermediate landing behaves similarly to another helicoidal stair slab without any landing. The effect of landing is more prominent for stairs with a lower central angle.

Landing has the most prominent effect on torsion. Torsion increases appreciably with the increase in landing width.

Maximum values of vertical moment increase, and lateral moment, thrust and radial horizontal shear decrease with the increase of the landing length. Landing length has virtually no effect on the maximum lateral shear. The local effect of landing is however quite important, specially in the case of vertical moment, and lateral shear. However these conclusions are drawn on the basis of two prototype stairs only.

8.3 SCOPE AND GUIDELINES FOR FUTURE STUDIES

The present study has investigated into the validity of Reynold's method for designing a helicoidal stair slab without a landing. A simple design rationale has been proposed for helicoidal stairs with intermediate landings. However, the work does not end here. During the course of the research it has been felt that certain areas need further investigation. The future studies may encompass the following areas:

8.3.1 Development of a Direct Design Procedure

The charts provided for the direct steel design method is not comprehensive. It deals with only 3000 psi concrete and 60000 psi steel. Extensive studies should be carried out to expand the scope of these charts. Also only one width to thickness ratio (b/h=10) has been assumed. The use of one b/h ratio may be justified by another study.

8.3.2 Modification of the Program to Accommodate Intermediate Landing(s)

The finite element program modified by the author for analysing deflection behaviour has a very limited scope. Only one landing length (0.2 times the central angle) can be analysed. Also, efforts regarding determining the stress resultants could not have been completed. The program needs further modifications to analyse the forces and moments in a helicoidal stair slab with an intermediate landing. The option of providing the landing length as an input parameter may be introduced. This will encourage the use of the same program to analyse helicoidal stairs with no landing as well.

8.3.3 Study on Maximum Deflection

From the investigation and literature review, it is perceived that maximum deflection plays an important factor in designing helicoidal stair slabs. But in practical cases, due to architectural reasons, it may be more difficult to adjust different geometric parameters other than waist thickness for deflection control. So for maximising design economy, a thorough study to investigate the possibility of reduction of waist thickness in keeping with the deflection to an acceptable range is necessary. For design simplicity, this study may go for suggesting a minimum thickness requirement for different geometric configurations. The finite element methods may be incorporated for this purpose. Also, a direct thickness can be suggested on the basis of deflection control as well as shear and torsion requirement. This will be more user friendly.

8.3.4 Study on the Effect of Steps

The finite element software analyses helicoidal stair slab as a uniform thickness slab, whereupon the stair steps are assumed to contribute the dead load only. In practice stair steps are monolithically cast with the stair slab and their integrated action will be somewhat different from the slab assumption. The possibility of taking an average thickness considering the steps as the waist thickness could be investigated. This will have the advantage of using the same program for designing driving or loading ramps. Alternately, the finite element program may be modified as to incorporate the steps in the analysis. This will produce an accurate understanding of the behaviour of the stair slab.

8.3.5 Study of Different End Conditions

Present study includes stair slabs (both with and without landings) only with both end fixed. In actual field, a complete fixity of supports may not always be attainable.

Therefore, helicoidal stair slab hinged at one or both of the supports or having partial fixity at the support should also be studied. The program could be modified for this purpose.

8.3.6 Non-linear Analysis

Throughout the present study, a linear elastic analysis has been made. Concrete is not in the strictest sense a linear material. A finite element analysis with non-linear material properties may be attempted.

8.3.8 Influence Line Analysis

It is believed that the most severe loading condition in a helicoidal stair slab is the uniformly distributed load over the entire span. But an influence line analysis should be carried out to verify this.

REFERENCES

ACI Committee 318-319 (Revised 1992), 'Building Code Requirements for Reinforced Concrete', American Concrete Institute, Detroit, Michigan-48219, 1994.

Amin, A. F. M. S., 'Improved Design Rational for Helicoidal Stair Slabs Based on Finite Element Analysis', M.Sc. Engg. Thesis, Department of Civil Engineering, Bangladesh University of Engineering and Technology, Dhaka, 1998.

Amin, A. F. M. S., and Ahmad, S., 'An Economic Design Approach for Helicoidal Stair Slabs Based on Finite Element Analysis', Proceedings of the Fourth International Conference on Computational Structures and Technology, Edinburgh, UK, 18-20 August, 1998.

Arya, A. S. and Prakash, A., 'Analysis of Helicoidal Staircases with Intermediate Landing' in Analysis of Structural System for Torsion, SP 35, American Concrete Institute, Detroit, MI, 1973.

Bergman, V. R., 'Helicoidal Staircases of Reinforced Concrete', ACI Journal, Vol. 28, pp 403-412, 1956.

Choudhury, C. F., ' A Rational Design Approach for Helicoidal Stair Slabs Using Finite Element Method', B.Sc. Engg. Thesis, Department of Civil Engineering, Bangladesh University of Engineering and Technology, Dhaka, 2002.

Cohen, J. S., 'Design of Helicoidal Staircases-2. Statically Indeterminate Cases', Concrete and Construction Engineering, (London), Vol. 54, No. 7, July 1959, pp. 249-256.

Cusens, A. R. and Trirojna, S., 'Helicoidal Staircase Study', ACI Journal, Proceedings, Vol. 61, No. 1, January 1964, pp. 85-101.

Engles, A., 'Design of Helical Stresses', Concrete and Construction Engineering, (London), Vol. 50, No. 5, May 1955, pp 181-194.

Everado, N. J., and Tanner , J. L., 'Theory and Problems of Reinforced Concrete Design', McGraw Hill Book Company, New York, 1989.

Fardis, M. N., Skouteropoulou, A. O., and Bousias, S. N., 'Stiffness Matrix of Free-Standing Helical Stairs', Journal of Structural Engineering, ASCE 113, 74-87, 1987.

Holmes, A. M. C., 'Analysis of Helical Beams Under Symmetrical Loading', Proceedings, ASCE, Paper NO. ST 1437, Nov. 1957, pp. 1437-1 to 1437-37.

Michalos, J. P., 'Numerical Analysis of Frames with Curved Girders', Proceedings, ASCE, Vol. 79, Separate No. 250, August, 1953.

Modak, S., 'A Design rationale for Helicoidal Stair Slabs', M.Sc. Engg. Thesis, Department of Civil Engineering, Bangladesh University of Engineering and Technology, Dhaka, 1991.

Morgan, V. A., 'Comparison of Analysis of Helical Staircases', Concrete and Construction Engineering (London), Vol. 55, No. 3, March 1960, pp. 127-132.

Morshed, A. S. M. M., ' Computer Aided Design of Helicoidal Stair Slab Based on Finite Element Analysis', B.Sc. Engg. Thesis, Department of Civil Engineering, Bangladesh University of Engineering and Technology, Dhaka, 1993

Niyomvit, S., 'An Experimental Study of a 180-Degree Fixed-ended Helicoidal Staircase of Reinforced Concrete', M. Engg. Thesis, SEATO Graduate School of Engineering, Bangkok, Thailand, 1963.

Reynolds, C. E. and Steedman, J. C., 'Reinforced Concrete Designer's Handbook', Tenth Edition, E. & F. N. Spon, London, 1988.

Santathadaporn, S. and Cusens, A. R., ' Charts for the Design of Helical Stairs with Fixed Supports', Concrete and Construction Engineering, Feb. 1966, pp. 46-54.

Scordelis, A. C., 'Closure to Discussion of Internal Forces in Uniformly Loaded Helicoidal Girder', ACI Journal, Proceedings, Vol. 56, No. 6, Part-2, Dec. 1960, pp. 1491-1502.

Scordelis, A. C., 'Internal Forces in Uniformly Loaded Helicoidal Girder', ACI Journal, Proceedings, Vol. 56, No. 6, Part-2, Dec. 1960, pp. 1491-1502.

Solanki, H. T., 'Helicoidal Staircases with Intermediate Landing', Structural Engineering Practice, Vol. 3(2), pp. 133-140, 1986.

Trirojna, S., 'The Design and Behaviour Under Load of Fixed-ended Hellicoidal Staircases of Reinforced Concrete', M. Engg. Thesis, SEATO Graduate School of Engineering, Bangkok, Thailand, 1962.

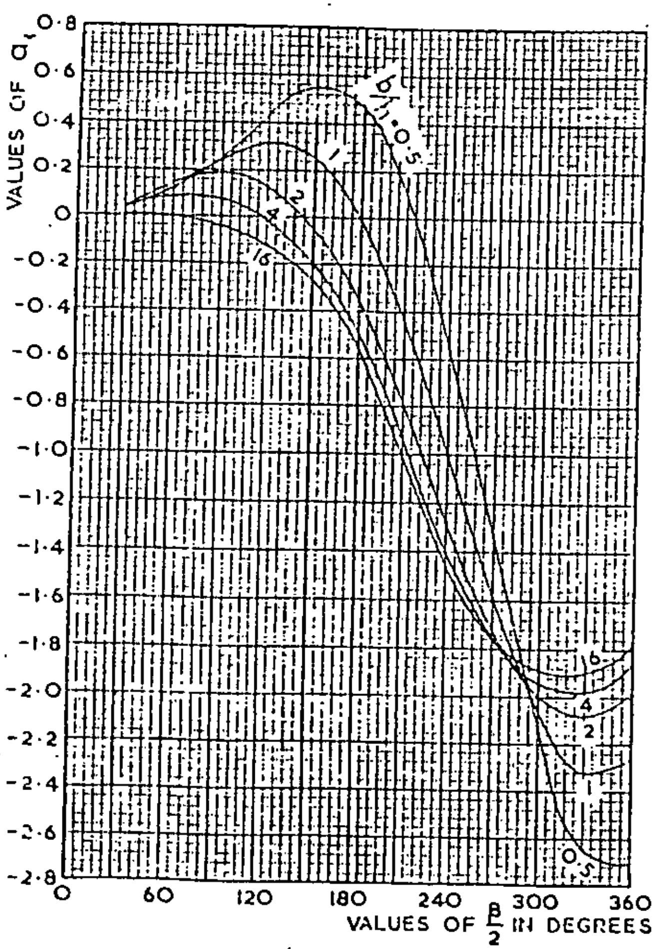
Wadud, Z. and Ahmad, S., 'Deflection Behaviour of Helicoidal Stair Slab and Its Effect on Design', Proceedings from the Eighth East Asia-Pacific Conference on Structural Engineering & Construction (EASEC-8), Singapore, 5-7 December, 2001.

Wadud, Z., 'Study on the Effect of Geometric Parameters on the Design Stresses of Helicodial Stair Slabs by Finite Element Analysis', B.Sc. Engg. Thesis, Department of Civil Engineering, Bangladesh University of Engineering and Technology, Dhaka, 1999.

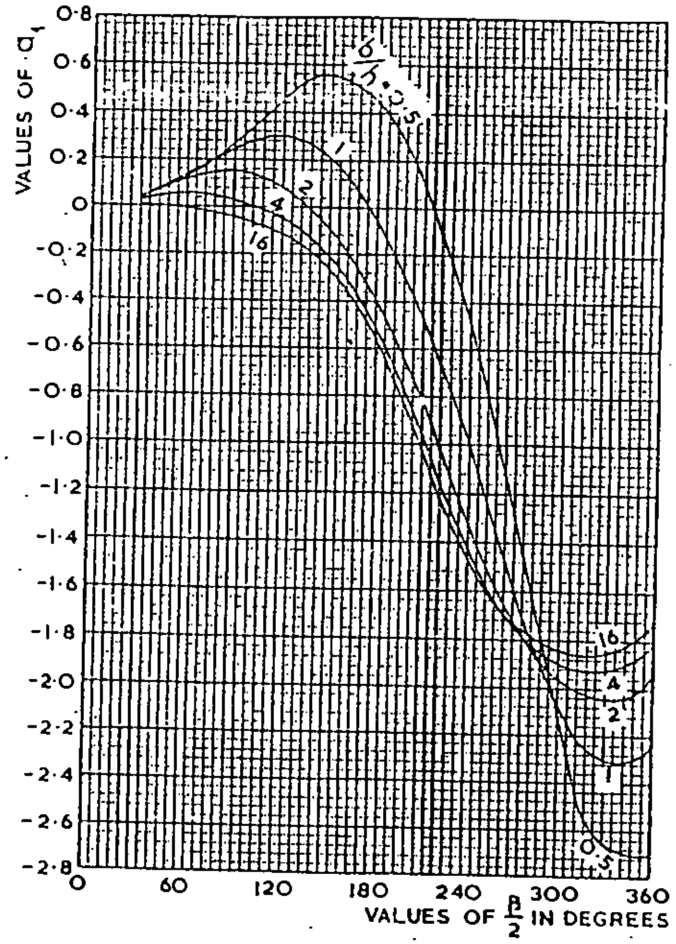
Young, Y. F. and Scordelis, A. C., 'An Analytical and Experimental Study of Helicoidal Girders', Transactions, ASCE, Vol. 125, Part-1, 1960, pp. 48-62.

Young, Y. F. and Scordelis, A. C., 'An Analytical and Experimental Study of Helicoidal Girders', Proceedings, ASCE, Paper No. ST 1756, Sept. 1958.

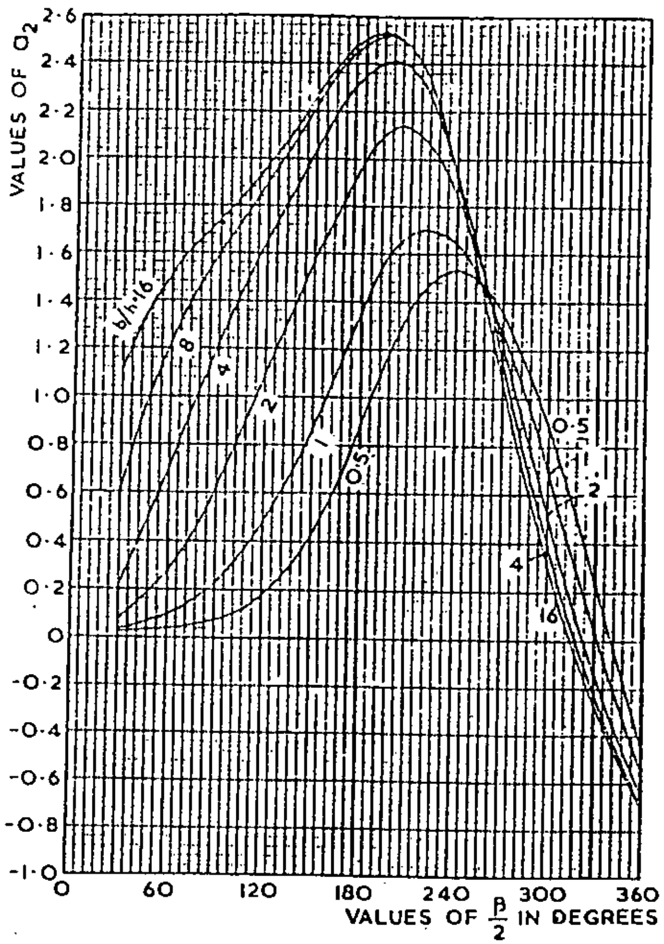
SANTATHADAPORN AND CUSENS CHARTS



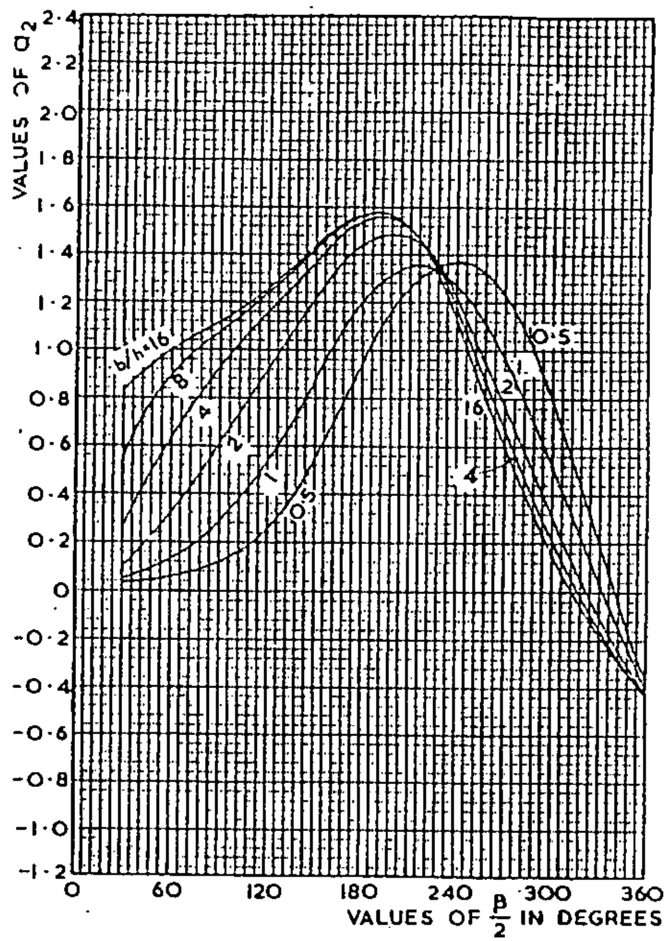
$R_1/R_1 = 1{\cdot}05$. $\phi = 20$ deg.



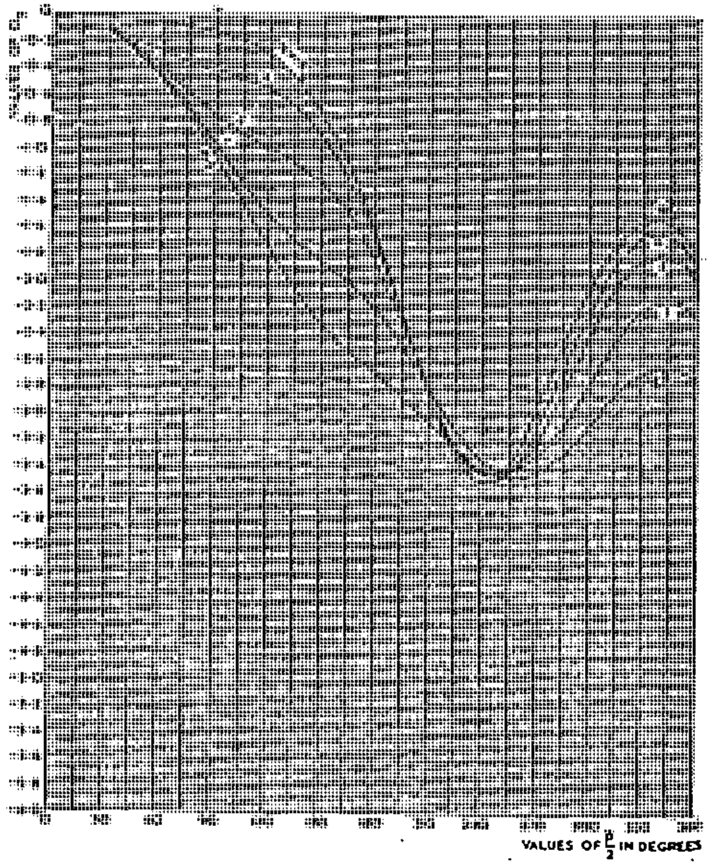
$R_1/R_2 = 1{\cdot}05$. $\phi = 30$ deg.



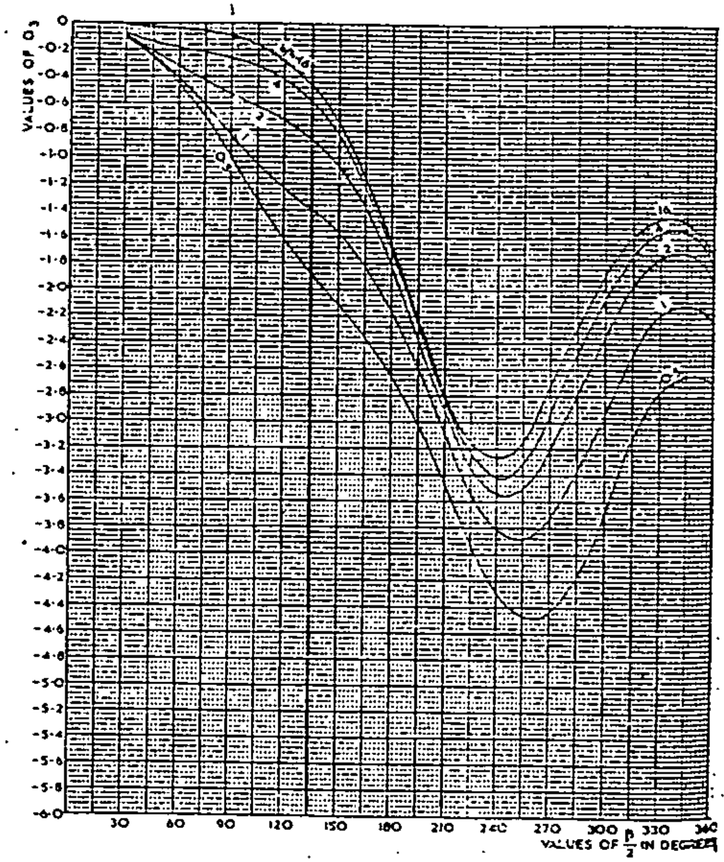
$R_1/R_2 = 1{\cdot}05$, $\phi = 20$ deg.



$R_1/R_2 = 1{\cdot}05$, $\phi = 30$ deg.



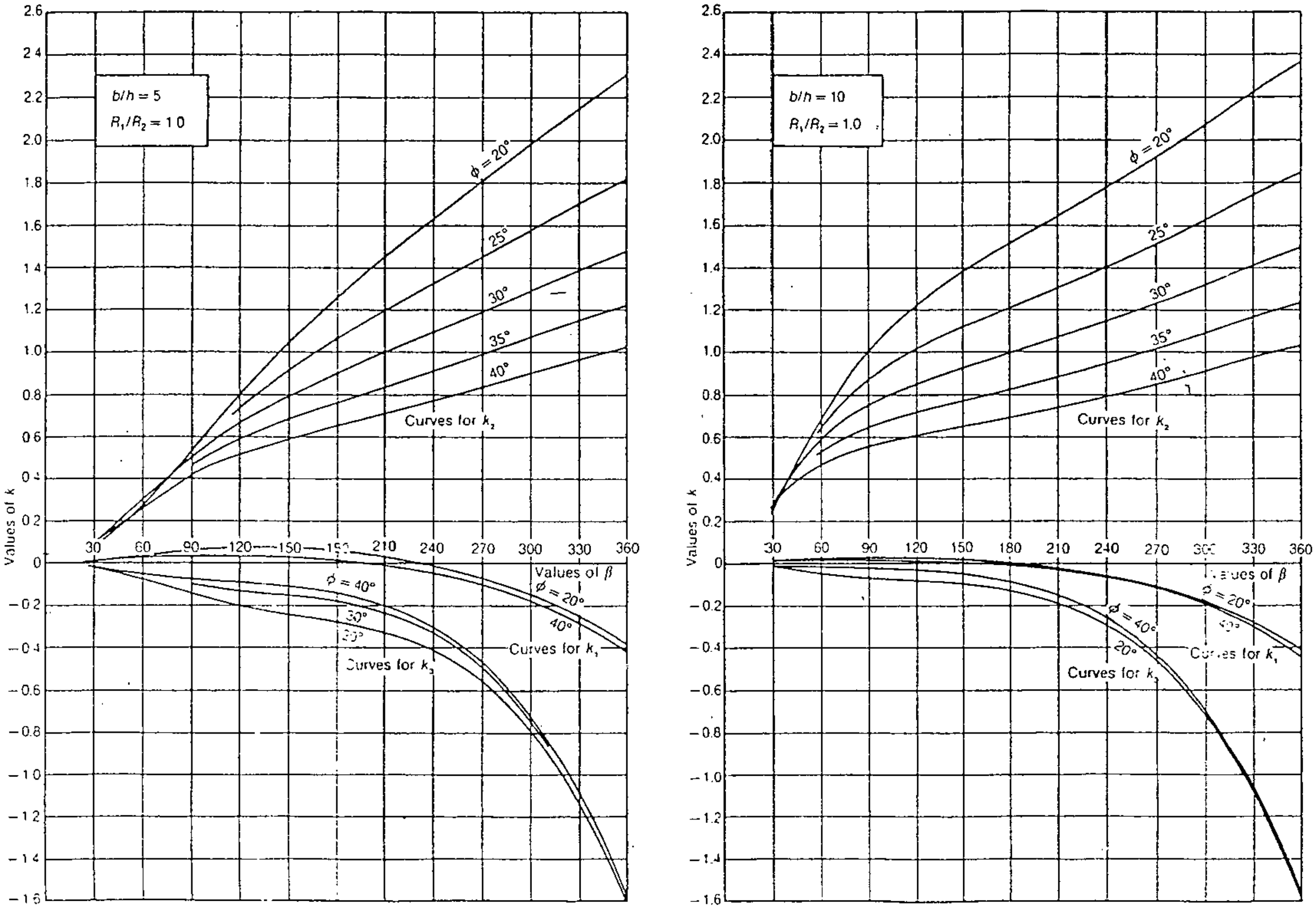
$R_1/R_2 = 1{\cdot}05. \quad \phi = 20$ deg.

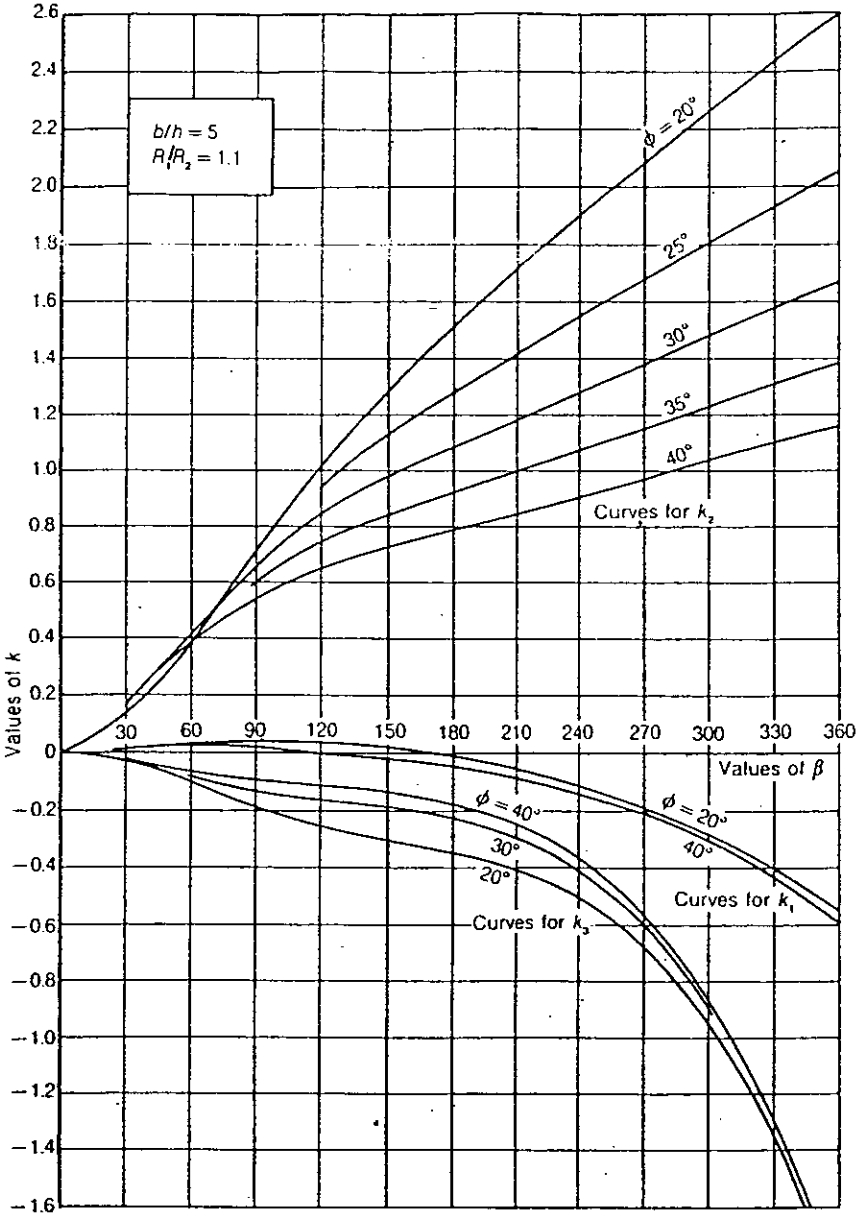


$R_1/R_2 = 1{\cdot}05. \quad \phi = 30$ deg.

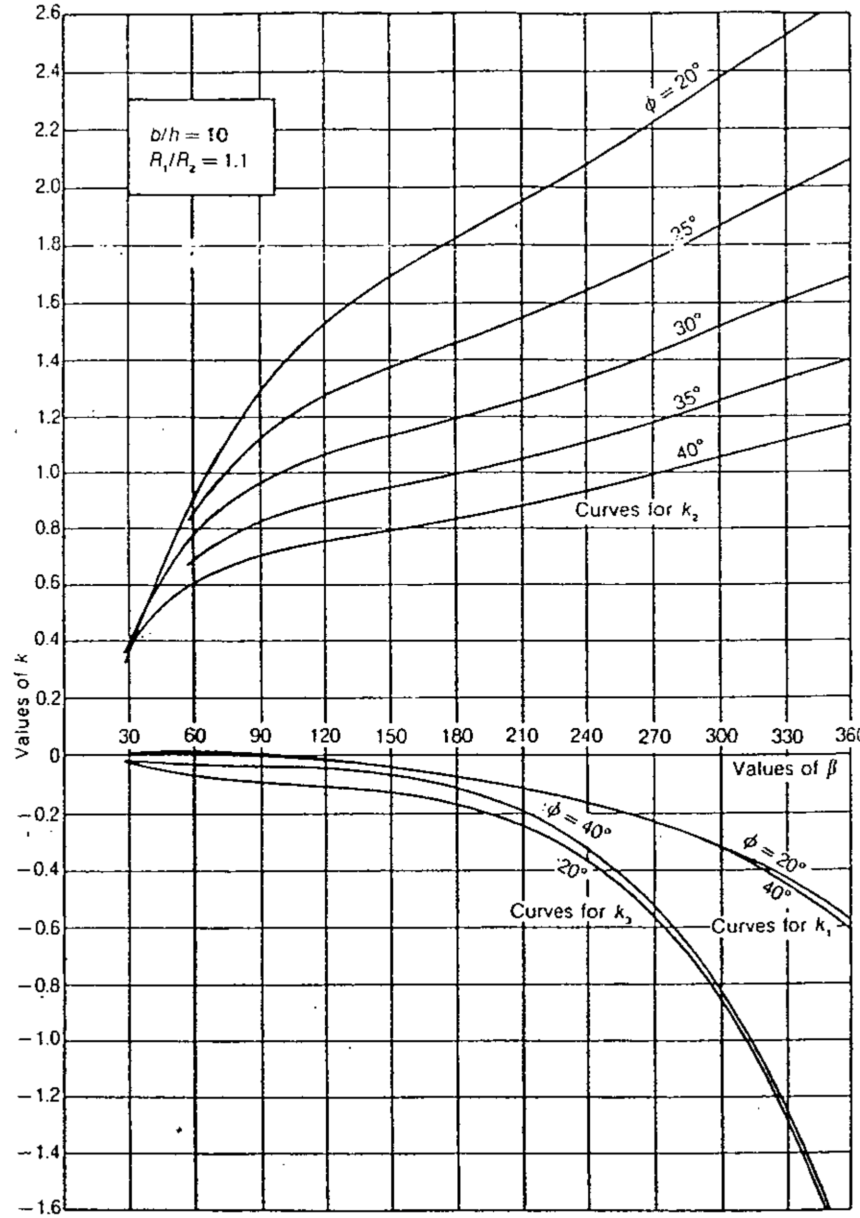
APPENDIX B

REYNOLDS CHART





b/h = 5
$R_1/R_2 = 1.1$
$\phi = 20°$
25°
30°
35°
40°
Curves for k_2
Values of k
Values of β
$\phi = 40°$
30°
20°
Curves for k_3
$\phi = 20°$
40°
Curves for k_1



b/h = 10
$R_1/R_2 = 1.1$
$\phi = 20°$
35°
30°
35°
40°
Curves for k_2
Values of k
Values of β
$\phi = 40°$
20°
Curves for k_3
$\phi = 20°$
40°
Curves for k_1

APPENDIX C

DESIGN PROCESS FOR A HELICOIDAL STAIRCASE WITH AN INTERMEDIATE LANDING

C-1 INTRODUCTION

This chapter functions as a user guide. The step by step procedure for analysis of a helicodal staircase with an intermediate landing using the proposed design charts is presented here. The geometric limitation must be kept in mind: the landing is located mid way of the stair.

C-2 ANALYSIS PROCEDURE

The height of the stair, inner radius, outer radius (or alternatively mean radius and width of stair), the total angle (θ_f) through which the stair is to rotate to reach its height, the length of landing (L) etc. are generally suggested by the architect. Having fixed the geometric parameters, a designer then has to determine the stress resultants. The introduction of the chart will substantially reduce the tedious computations required otherwise to find the design forces and moments. The analysis procedure using the charts consists of the following steps:

1. Determine mean radius R_2 from given inner radius R_i and outer radius R_o

$$R_2 = (R_o + R_i)/2$$

2. Determine the angle (2ϕ) subtended by the landing of length L at a distance R at the centre $L = 2\phi R$

3. Find the Total angle subtended at the centre by the flights (2β) from θ_f and 2ϕ

$$2\beta = \theta_f - 2\phi$$

4. From the height of stair (Ht), mean radius (R_2), and total angle subtended by flight at the centre (2β) calculate the slope of the tangent helix centreline (α) as

$$\alpha = \tan^{-1} \frac{Ht}{R_2 2\beta}$$

5. Determine the radius of centre line of loading (R_1) from

$$R_1 = \frac{2}{3} \cdot \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$$

6. Find w, total dead and live load per unit length along the centreline

7. With the values of R_1/R_2 and central angle subtended by the landing (2ϕ) go to the appropriate chart, find k_1, k_2, and k_3 for the given value of total angle subtended at the centre (θ_f). Determine mid span moment, M, mid span radial horizontal shear, H, and support moment M_{sup}, from

$$M = k_1 w R_2^2$$

$$H = k_2 w R_2$$

$$M_{sup} = k_3 w R_2^2$$

8. Determine other stress resultants at various distance, θ, from the mid span toward the top support. Keep in mind that $\alpha = 0$ at the landing.

- Vertical moment:

  $$M_v = M\cos\theta + HR_2\theta \tan\alpha \sin\theta - wR_1^2(1 - \cos\theta)$$

- Lateral moment:

  $$M_h = M\sin\theta \sin\alpha - HR_2\theta \tan\alpha \cos\theta \sin\alpha - HR_2 \sin\theta \cos\alpha + (wR_1^2\sin\theta - wR_1R_2\theta) \sin\alpha$$

- Torsion:

  $$T = (M\sin\theta - HR_2\theta \tan\alpha\cos\theta + wR_1^2\sin\theta + HR_2 \sin\theta \sin\alpha - wR_1R_2\theta)\cos\alpha + HR_2 \sin\theta\sin\alpha$$

- Thrust:

  $$N = -H\sin\theta \cos\alpha - wR_1\theta \sin\alpha$$

- Lateral Shear:

  $$V = w\,\theta \cos\alpha - H\sin\theta \sin\alpha$$

- Radial horizontal shear:

  $$F = H\cos\theta$$

C-3 EXAMPLE

It is required to analyse a reinforced concrete helicodial stair slab with a height of 12.5 ft, inner radius of 5 ft, and outer radius of 11.25 ft. The stair is to reach its full height within a 270° turn. The length of landing at the inner edge is 5.25 ft. Live load = 100 psf. Concrete unit weight = 150 pcf. The stair slab is 6 inches thick and the risers are 6 inches high.

Step 1: $P_2 = (5 + 11.25)/2 = 8.125$ ft

Step 2: $\phi = 5.25/(5*2) = 0.525$ radian = 30°

Step 3: $2\beta = 270 - 2*30 = 210° = 3.665$ radian

Step 4:

$$\alpha = \tan^{-1}\frac{12.5}{8.125 * 3.665} = 22.8°$$

Step 5:

$$R_1 = \frac{11.25^3 - 5^3}{11.25^2 - 5^2} = 8.526\ ft.$$

$R_1/R_2 = 8.526/8:125 = 1.05$

Step 6: Total thickness in vertical direction is approximately 9.5 inches.

Surface UDL = 12.5*9.5 + 100 = 218.75 psf

w = 218.75*(11.25 – 5) = 1367 plf

Step 7:

With the calculated values of R_1/R_2, α and ϕ, refer to chart 6.13. For a 270° stair,

$k_1 = 0.036 \Rightarrow M = 0.036*1367*8.125^2 = 3250$ lb-ft

$k_2 = 1.657 \Rightarrow H = 1.657*1367*8.125 = 18400$ lb

$k_3 = -0.73 \Rightarrow M_{sup} = -0.73*1367*8.215^2 = 65880$ lb-ft

Step 8:

The variation of stress resultants along the span, found using the previously stated equations, is depicted through Figs. 6.5 & 6.6.