

INTERPRETATION OF PUMPING FROM AQUIFERS OVERLAIN BY A LOW PERMEABILITY ZONE USING A NUMERICAL MODEL

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ABSTRACT

An understanding of flow mechanism near an abstraction well is important for successful exploitation of groundwater resources. Properly planned and carefully conducted pumping tests provide basic information for the solution of many groundwater flow problems. Pumping test data are normally analysed using conventional curve-matching techniques. Although little information about the aquifer behaviour can be gained from this analysis, yet a far greater understanding can be achieved using numerical models.

The two zone numerical model described by Rathod and Rushton (1991) has been further developed and applied to interpret three pumping tests conducted in an alluvial aquifer in central Bangladesh. The model could reproduce the significant features of the aquifer behaviour and the overall agreement between the observed drawdowns and model results was very good. The important features of the flow mechanisms include: vertical components of flow in the vicinity of the well, leakage from overlying layer with fall of water table, well storage and well losses. The values of transmissivity and storage coefficient, as derived from numerical analysis, for the three test sites ranged from 940 to 1510 m²/day and 0.0006 to 0.001 respectively. The vertical hydraulic conductivity of the overlying layer was found to vary from 0.005 to 0.007 m/day.

The model has also been used to predict the response of the aquifer to pumping over a typical growing season. The results show significant drawdowns both at the pumped well and at outer boundary. Unless sufficient recharge takes place during the monsoon, the groundwater head will fall year by year. Since these predictions were made based on a model which had been verified only for a short period the results should be treated as an indication of the likely trend. Nevertheless, such information are quite useful for successful development of groundwater resources in the study area.

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Chapter 1

INTRODUCTION



1.1 Background

Groundwater constitutes an important source of domestic, industrial and irrigation water supplies in Bangladesh. The late Pleistocene to Recent age alluvial sediments deposited by three major rivers namely the Ganges, the Brahmaputra and the Meghna form some of the most productive aquifers in the world. Because of high rainfall and widespread flooding, these aquifers get good recharge during monsoon. The aquifer system which underlies most of the plain areas of Bangladesh consists of three lithological units: an upper silty clay layer; a middle composite aquifer of fine to very fine sands; and the main aquifer consisting of medium, medium to fine or medium to coarse sand with layers of clay and silt. The upper silty clay layer regulates vertical recharge to underlying aquifers. The groundwater is mainly of good quality except in the coastal area where most of the shallow aquifers are saline due to the influence of tides.

Although groundwater has been used in this country for hundreds of years particularly for rural water supplies, abstraction of this resource by modern technology started in 1960s. The principal modes of groundwater resource development for irrigation are: suction mode pump of nominal discharge capacity of 14 l/sec, known as shallow tubewell and force mode pumps with nominal capacity of 56 l/sec, referred to as deep tubewell. At present about 25700 deep tubewells and 350000 shallow tubewells are in operation. These tubewells along with numerous manually operated pumps abstract about 9 billion cubic meter of water annually to irrigate about 2 million hectares. According to National Water Plan an additional area of 1.5 Mha would be developed by 2010 from groundwater resources (MPO, 1990).

Although hundreds of deep tubewells and shallow tubewells have been installed in central part of Bangladesh, the present groundwater development in Kapasia Thana is quite limited. Until 1985, no deep tubewell had been sunk in this area. This was because the hydrogeological conditions of this area were believed to be poor. Consequently, Mott

MacDonald International Limited (MMIL) carried out a study to ascertain the hydrogeology and feasibility of installation of deep tubewells in this area. Ten exploratory boreholes and seven experimental wells were drilled and constant discharge pumping tests were carried out at three sites during the period 1985-89. The pumping test data were analyzed by MMIL to estimate aquifer properties using conventional curve matching techniques. This may lead to erroneous results as these methods have been developed based on certain assumptions and idealizations which deviate considerably from actual field conditions. These methods fail to consider certain important features such as vertical components of flow and well storage. Alternatively, a numerical model provides a greater flexibility; all the important features can be included in a single numerical solution. It is therefore, advisable to interpret these pumping tests using a suitably adapted numerical model.

1.2 Objectives of the Study

An understanding of flow mechanism near an abstraction well is important for successful exploitation of groundwater resources. A great deal of information can be gained about an aquifer from the radial flow due to pumping tests (Rushton, 1986). Pumping test data are normally analyzed using conventional curve matching techniques. Although little knowledge about the aquifer behaviour can be gained from this analysis, yet a far greater understanding can be achieved using numerical models (Rushton and Both, 1976).

The two zone numerical model described by Rathod and Rushton (1991) has proved to be useful in the study of wide range of groundwater flow problems including weathered - fractured aquifers, multi-piezometer tests and injection well tests in an alluvial aquifer. The model includes all the important features of flow to an abstraction well in a leaky layered aquifer. However, both the flow through the main aquifer and the response of the overlying layer should be considered in analysing pumping from aquifer systems having low permeability zone above the main aquifer. The two zone model can be used to represent the flow through the main aquifer; but it needs to be further developed to include important flow mechanisms in the overlying layer.

The main purpose of this study is to gain an insight into the flow mechanism near an abstraction well in a leaky layered aquifer in Bangladesh. This can be achieved through

interpretation of pumping tests carried out in this aquifer. The specific objectives of this study are:

- (a) to gain an in-depth understanding of the formulation of two zone numerical model and to make necessary modification in the program to incorporate the flow mechanisms in the overlying layer;
- (b) to interpret the pumping tests carried out at three sites in the study area; and
- (c) to predict the response of the aquifer to pumping over a typical growing season using the model.

Chapter 2

PUMPING TEST ANALYSIS TECHNIQUES

2.1 Introduction

The most common and reliable method of estimating the properties of an aquifer is by a pumping test. Properly planned and carefully conducted pumping tests may provide basic information for the solution of many regional and local groundwater flow problems. It is in effect a controlled experiment in which water is pumped from a well during a certain time at a certain rate and the changes in groundwater head both at the abstraction well and at some observation piezometers are recorded. The flow towards the abstraction well is assumed to be radial, hence the equation governing this flow can be expressed as:

$$\frac{\partial}{\partial r} \left(K_r \frac{\partial s}{\partial r} \right) + \frac{K_r}{r} \frac{\partial s}{\partial r} + K_z \frac{\partial^2 s}{\partial z^2} = S_s \frac{\partial s}{\partial t} \quad (2.1)$$

where K_r and K_z are radial and vertical hydraulic conductivities, s is drawdown, t is time, r is radial distance measured from the abstraction well and S_s is specific storage coefficient of the aquifer.

Analytical solutions to this equation have been derived for a wide range of conditions and, by comparing the field data with these analytical solutions the properties of the aquifer can be determined.

Alternatively, discrete space - discrete time numerical models have been developed for the analysis of pumping test data. Numerical methods of pumping test analysis also solve the governing equation subject to certain boundary conditions. A brief review of some of the widely used analytical methods and numerical models for the analysis of pumping test data is presented in the following sections.

2.2 Analytical Methods

Since the introduction of Thies (1935) method of analysis of pumping test results in a confined aquifer, many additional analytical solutions and curve fitting techniques have been developed. Notable contributions have been made by Cooper and Jacob (1946), Hantush (1956, 1960, 1964), Boulton (1963) and Neuman (1972, 1975). Some of the widely used analytical methods are discussed in this section. An exhaustive review of the analytical methods can be found in Walton (1970) and Kruseman and Ridder (1994).

2.2.1 Flow to a Well in a Confined Aquifer

The most widely used equation in well hydraulics is based on flow to a well abstracting water from a confined aquifer. The aquifer is assumed to be homogeneous, isotropic, infinite in areal extent and is of same thickness throughout. The well completely penetrates the aquifer.

Thies (1935) first developed a unsteady-state formula for this situation. He noted that when the production well is pumped at a constant rate, Q , the influence of pumping extends outward with time and the flow is radial throughout. Water is released from storage by compaction of the aquifer and expansion of water itself. Assuming that water is released instantaneously with decline in head, the differential equation governing the flow can be written as:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} = \frac{S}{T} \frac{\partial s}{\partial t} \quad (2.2)$$

where S is storage coefficient and T is transmissivity of the aquifer.

To obtain a simple mathematical solution, Thies (1935) assumed that the well is replaced by a mathematical sink of constant strength and of an infinitesimal diameter. For the boundary conditions

$$s \rightarrow 0 \text{ as } r \rightarrow \infty \text{ for } t \geq 0$$

$$\lim_{r \rightarrow 0} \left(r \frac{\partial s}{\partial r} \right) = -\frac{Q}{2\pi T}$$

and initial condition $s = 0$ for $t = 0$, the solution is as follows:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} du \quad (2.3)$$

$$\text{where } u = \frac{r^2 s}{4Tt} \quad (2.4)$$

Expanding the exponential integral in a convergent series, the drawdown, s can be expressed as:

$$s = \frac{Q}{4\pi T} \left[-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \dots \right] \quad (2.5)$$

The exponential integral in Equation (2.3) can be symbolically expressed as:

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du$$

Thus Equation (2.3) becomes

$$s = \frac{Q}{4\pi T} W(u) \quad (2.6)$$

$W(u)$ is termed as 'Thies well-function'. Wenzel (1942) tabulated the values of $W(u)$ in terms of practical range of u . Thies suggested a curve matching technique to determine the aquifer properties using Equations (2.4) and (2.6).

Cooper and Jacob (1946) noted that for small values of u , a much simpler form of expression can be used for the well function. For $u \leq 0.01$, $W(u)$ can be approximated as:

$$W(u) = -0.5772 - \ln u \quad (2.7)$$

The resulting expression for the drawdown, s is:

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25 Tt}{r^2 S} \quad (2.8)$$

In considering the drawdown at a particular radial distance, the only variable on the right hand side of Equation (2.8) is time, t . A graph of s against $\log t$ should be a straight line; the slope and intercept on the time axis allow the properties of the aquifer to be determined. The equations for calculating S and T can be expressed as:

$$S = \frac{2.25 Tt_0}{r^2} \quad (2.9)$$

$$T = \frac{2.3Q}{4\pi \Delta s} \quad (2.10)$$

where t_0 is the time corresponding to zero drawdown and Δs is the drawdown difference per log cycle of t .

2.2.2 Flow to a Well in a Leaky Aquifer

Figure 2.1 shows the schematic diagram of a leaky aquifer which is overlain by an aquitard and underlain by an aquiclude. Overlying the aquitard is the source bed in which there is a water table. The aquifer is homogeneous, isotropic, infinite in areal extent and is of same thickness throughout. The abstraction well completely penetrates the aquifer and the flow in the aquifer is radial. Jacob (1946) developed a partial differential equation for unsteady-state flow to a fully penetrating well in such an aquifer. The discharge of the well is derived from storage within the aquifer and leakage through the aquitard. He assumed that the rate of vertical leakage is proportional to the difference in head between the water table and the piezometric surface. Furthermore, it was assumed that water released from storage in the aquitard is negligible and the water table is not influenced appreciably by pumping. The resulting partial differential equation is:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} - \frac{s}{B^2} = \frac{S}{T} \frac{\partial s}{\partial t} \quad (2.11)$$

where B is leakage factor. The leakage factor is defined as:

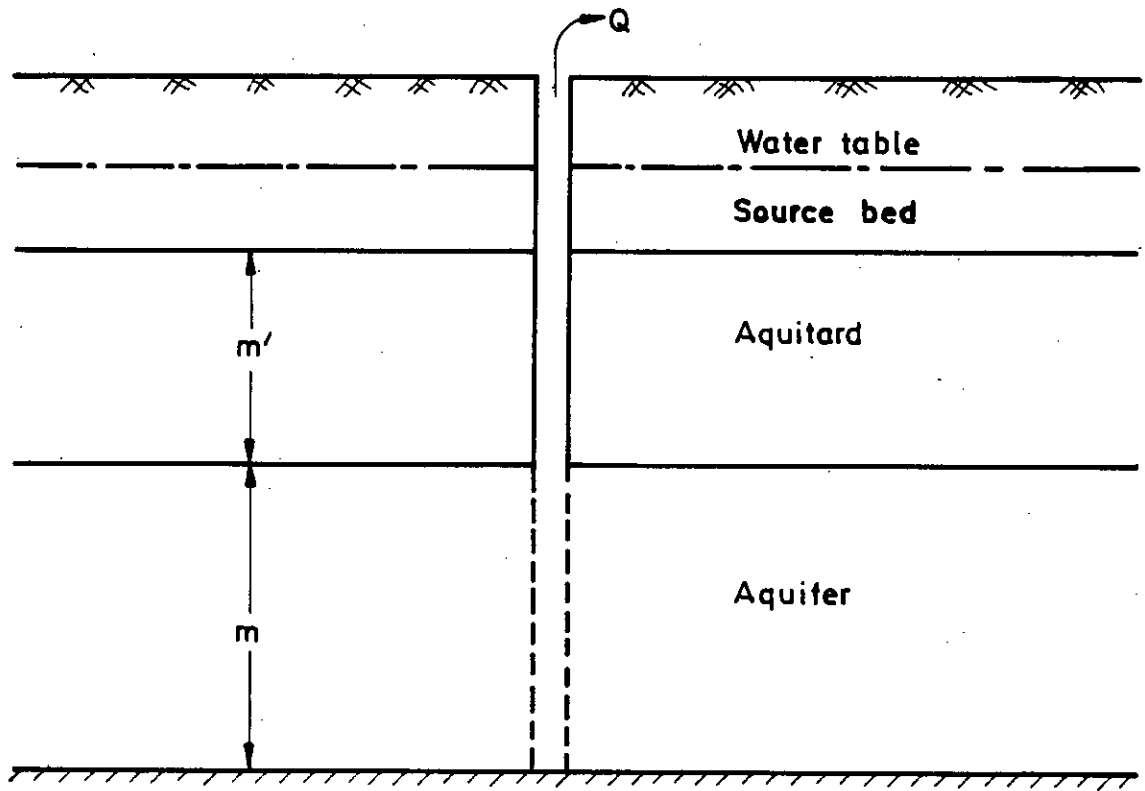


Fig. 2.1 Schematic diagram of a leaky aquifer

$$B = \sqrt{\frac{T}{K_v' m'}} \quad (2.12)$$

where K_v' and m' are respectively the vertical hydraulic conductivity and thickness of the aquitard. With boundary conditions

$$s \rightarrow 0 \text{ as } r \rightarrow \infty \text{ for } t > 0$$

$$\lim_{r \rightarrow 0} \left(r \frac{\partial s}{\partial r} \right) = -\frac{Q}{2\pi T}$$

and initial condition $s = 0$ for $t = 0$, Hantush and Jacob (1955) derived a solution to Equation (2.11) as follows:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{1}{y} \exp\left(-y - \frac{r^2}{4B^2y}\right) dy \quad (2.13)$$

With the integral in Equation (2.13) expressed symbolically as $\mathcal{W}\left(u, \frac{r}{B}\right)$, that is,

$$\int_u^\infty \frac{1}{y} \exp\left(-y - \frac{r^2}{4B^2y}\right) dy = \mathcal{W}\left(u, \frac{r}{B}\right)$$

Equation (2.13) may be written as:

$$s = \frac{Q}{4\pi T} \mathcal{W}\left(u, \frac{r}{B}\right) \quad (2.14)$$

where $u = \frac{r^2 s}{4Tt}$

Values of $\mathcal{W}\left(u, \frac{r}{B}\right)$ for certain values of u are tabulated by Hantush (1956).

On the basis of Equation (2.14), Walton (1962) developed a curve fitting method to determine the aquifer properties. He used the values of $\mathcal{W}\left(u, \frac{r}{B}\right)$ as published by Hantush

(1956) to draw a family of type curves:

When water released from storage within the aquitard is appreciable the equation governing the flow to a well in a leaky aquifer can be expressed as:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} + \frac{K_v'}{T} \frac{\partial s'}{\partial z} = \frac{S}{T} \frac{\partial s}{\partial t} \quad (2.15)$$

where s' is the drawdown in the aquitard.

With a set of equations representing the initial and boundary conditions, Hantush (1960) derived a solution to Equation (2.15) as follows:

$$s = \frac{Q}{4\pi T} W(u, \beta)$$

where $u = \frac{r^2 s}{4Tt}$

$$\beta = \frac{r}{4} \sqrt{\frac{S' K_v'}{T S m'}}$$

S' = storage coefficient of the aquitard

$$W(u, \beta) = \int_u^\infty \frac{e^{-y}}{y} \operatorname{erfc} \frac{\beta \sqrt{y}}{\sqrt{y(y-u)}} dy$$

Values of the function $W(u, \beta)$ are extensively tabulated by Hantush (1964).

This equation is applicable only for small values of pumping time (t) as given by:

$$t < \frac{S' m'}{10 K_v'}$$

If the ratio of the storage coefficient of the aquitard and the storage coefficient of the leaky aquifer is small, the effect of any storage changes in the aquitard on the drawdown in the aquifer is very small. In that case, and for small values of pumping time, the Thies formula (Equation 2.6) can be used.

2.2.3 Flow to a Well in an Unconfined Aquifer

Flow to a well in an unconfined aquifer is rather complicated. In this aquifer, the water is released from storage by gravity drainage, compaction of aquifer and expansion of water itself. The release of water from storage due to compaction of aquifer and expansion of water takes place instantaneously. However, the gravity drainage is often not immediate. Consequently, the aquifer shows the phenomenon of delayed yield. Boulton (1963) first derived an equation governing the unsteady flow to a well in an unconfined aquifer taking into consideration the phenomenon of delayed yield. The equation is:

$$T \left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} \right) = S \frac{\partial s}{\partial t} + \alpha S_y \int_0^t \frac{\partial s}{\partial t} e^{-\alpha(t-\tau)} d\tau \quad (2.16)$$

where S_y is specific yield of the aquifer and α is an empirical constant (reciprocal of delay index). The underlying assumption is that the aquifer is homogeneous, isotropic, infinite in areal extent and is of the same thickness throughout.

With boundary conditions $s \rightarrow 0$ as $r \rightarrow \infty$ and initial condition $s = 0$ for $t = 0$, the solution to the problem given by Equation (2.16) is (Boulton, 1963):

$$s = \frac{Q}{4Tt} W(u_{ay}, \frac{r}{D_t}) \quad (2.17)$$

where $W(u_{ay}, \frac{r}{D_t})$ = well function of Boulton

$$u_a = \frac{r^2 S}{4Tt}$$

$$u_y = \frac{r^2 S_y}{4Tt}$$

$$D_t = \sqrt{\frac{T}{\alpha S_y}}$$

Values of $W(u_{ay}, \frac{r}{D_t})$ in terms of the practical range of u_a , u_y and $\frac{r}{D_t}$ are tabulated by Boulton (1963).

Boulton's solution matches better with measured drawdowns than that of Theis. However, it is open to criticism due to introduction of delay index which lacks physical basis (Krosznki and Dagan, 1975). Streltsova (1973) demonstrated the subordinate role that unsaturated flow plays in the delay yield process and concluded that it is the free surface and flux that change with time but the specific yield remains constant.

Neuman (1972) developed another solution to the problem of flow to a well in an unconfined aquifer neglecting the unsaturated flow. He treated the aquifer as a compressible system and water table as a moving material boundary. He considered both radial and vertical flow components. The equation governing the flow is:

$$K_r \frac{\partial^2 s}{\partial r^2} + \frac{K_r}{r} \frac{\partial s}{\partial r} + K_z \frac{\partial^2 s}{\partial z^2} = S_s \frac{\partial s}{\partial t} \quad (2.18)$$

Where K_z is vertical hydraulic conductivity and S_s is specific storage coefficient of the aquifer.

Neuman (1972) gave a solution to Equation (2.18) based on certain initial and boundary conditions and his general solution is a function of both r and z . When considering an average drawdown, he was able to reduce his solution to one that is function of r alone. Mathematically, he simulated the delayed water table response by treating S and S_y as constants.

Neuman's drawdown equation can be written as (Neuman, 1975):

$$s = \frac{Q}{4\pi T} W(u_A, u_B, \beta) \quad (2.19)$$

For early-time condition, this equation reduces to

$$s = \frac{Q}{4\pi T} W(u_A, \beta)$$

$$\text{where } u_A = \frac{r^2 s}{4Tt}$$

For later stage Equation (2.19) reduces to

$$s = \frac{Q}{4\pi T} w(u_B, \beta)$$

$$\text{where } u_B = \frac{r^2 S_y}{4Tt}$$

$$\beta = \frac{r^2 K_r}{m^2 K_z}$$

Values of functions $W(u_A, \beta)$ and $W(u_B, \beta)$ are tabulated by Neuman (1975).

All the methods described in this section were derived on the basis of certain assumptions and idealisations. Actual field conditions may not conform to these theoretical conditions. Any deviation from the theoretical conditions will lead to an error in the computation. Although analytical solutions have been developed for a wide range of conditions, yet there are combinations of these conditions that occur in practice for which no analytical solutions are available. Sometimes a portion of the field data does not match with the analytical solution and it is valuable to investigate the features which cause this deviation. This can often be achieved by means of a numerical model of radial flow.

2.3 Numerical Models

Numerical models of radial flow to a pumped well have proved to be useful tools for interpretation of pumping tests under a wide range of aquifer conditions. Rushton and Chan (1976a) first developed an one-dimensional radial flow model for pumping test analysis using a discrete space - discrete time approach. This radial flow model became the basis of many other radial flow models (Rushton and Redshaw, 1979; Rathod and Rushton, 1984; Walton, 1987, Grout 1988; Rathod and Rushton, 1991). A great advantage of these numerical models is that as many conditions as necessary can be included in a single solution. The important

numerical radial flow models developed at the School of Civil Engineering, University of Birmingham, are briefly described in the following sections.

2.3.1 One-dimensional Radial Flow Model

When the vertical components of flow are sufficiently small to be neglected, the unsteady radial flow to an abstraction well can be represented as:

$$\frac{\partial}{\partial r} \left(mK_r \frac{\partial s}{\partial r} \right) + \frac{mK_r}{r} \frac{\partial s}{\partial r} = S \frac{\partial s}{\partial t} + q \quad (2.20)$$

where m is thickness of the aquifer and q is inflow rate per unit area.

Assuming that the product of the saturated depth and radial hydraulic conductivity is constant for all radii and introducing an alternative co-ordinate in the radial direction:

$$a = \log_e r$$

equation (2.20) becomes

$$mK_r \frac{\partial^2 s}{\partial a^2} = Sr_n^2 \frac{\partial s}{\partial t} + qr_n^2 \quad (2.21)$$

Solution to this equation with appropriate boundary and initial conditions can be obtained using finite-difference techniques. Using a regular mesh of constant interval Δa , the backward-difference approximation of Equation (2.21) can be written as:

$$\frac{mK_r}{\Delta a^2} [s_{n-1} - 2s_n + s_{n+1}]_{t+\Delta t} = \frac{Sr_n^2}{\Delta t} [s_{n,t+\Delta t} - s_{n,t}] + q_{t+\frac{\Delta t}{2}} r_n^2 \quad (2.22)$$

Use of a constant value of Δa allows the radial dimension to be divided into discrete intervals, increasing logarithmically from small values near the well to large values towards the outer boundary. The time is also divided into discrete steps which increase logarithmically. It is convenient to express Equation (2.22) in terms of resistances as shown in Figure 2.2. The resistances are defined as:

$$H_n = \frac{\Delta a^2}{mK_r}, \quad T_n = \frac{\Delta t}{Sr_n^2} \quad (2.23)$$

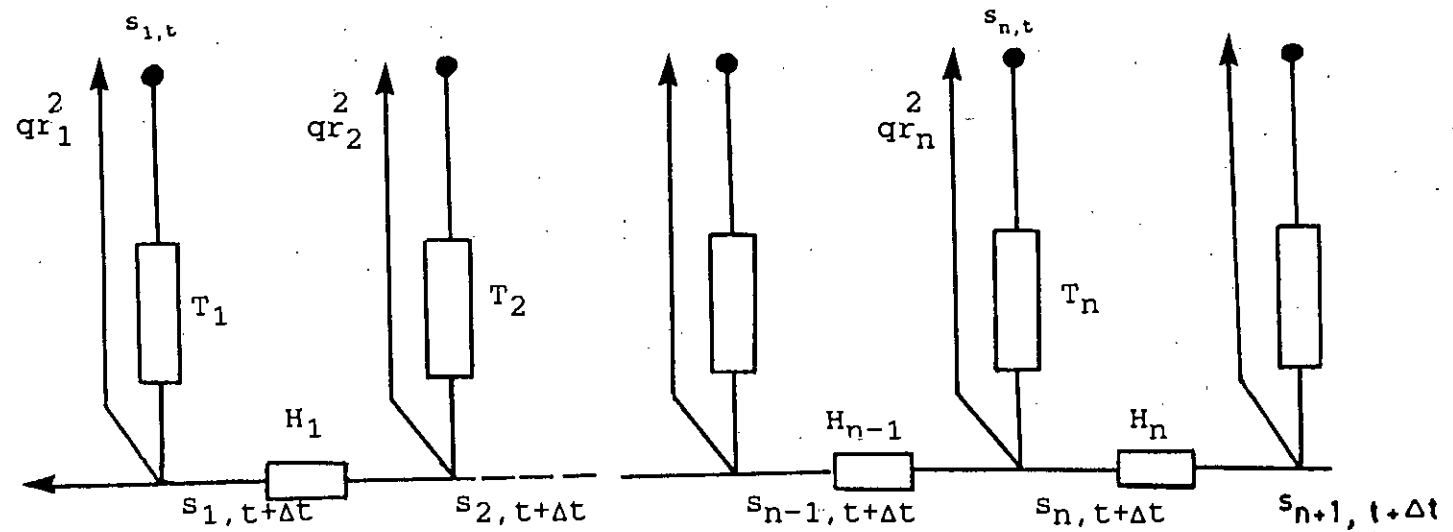


Fig. 2.2 Equivalent hydraulic resistances (after Rushton and Redshaw, 1979)

Substituting these values in Equation (2.22) we get

$$\frac{[s_{n-1} - 2s_n + s_{n+1}]_{t+\Delta t}}{H_n} = \frac{s_{n,t+\Delta t} - s_{n,t}}{T_n} + q_n r_n^2 \quad (2.24)$$

where $q_n r_n^2$ represents the outflow from node, n (Figure 2.2). If the aquifer is leaky, the leakage can be represented as recharge which depends on the drawdown (Rushton and Chan, 1976a); thus at node n

$$q_n = (s_{n,t+\Delta t} - s_{n,0}) K_r m / B^2 \quad (2.25)$$

where B is the leakage factor and $s_{n,0}$ is the initial drawdown which is maintained in the leaky strata throughout the test.

In this model special consideration is given to the inner and outer nodes to represent the well and outer boundary. The abstraction rate Q is represented as a negative recharge at node 1. This is given by

$$q = -\frac{Q}{2\pi r_1 \Delta r}$$

Since, $\Delta a = \frac{\Delta r}{r}$

$$q = -\frac{Q}{2\pi r_1^2 \Delta a}$$

where r_1 is the radial distance to node 1. At outer node the boundary condition may be specified either as fixed head or no-flow.

With these boundary conditions, Equation (2.24) is solved using Gaussian elimination technique. Features that can be included in this model are: well storage, various boundary conditions, variable saturated thickness, delayed yield, variation in hydraulic conductivity and storage coefficient with radial distance, change from confined to unconfined condition and leakage.

This model is capable of reproducing the traditional analytical solution for different aquifer situations as demonstrated by Rushton and Chan (1976a). They considered the Thies (1935) solution for confined and the Hantush and Jacob (1955) solution for leaky aquifers. The agreement between the analytical and the numerical model solutions was quite satisfactory.

2.3.2 Two Zone Models

In the derivation of one-dimensional radial flow model, the assumption was made that vertical components of flow could be neglected. In many practical situations this condition does not hold. Vertical flow components can be of great importance for unconfined aquifers or layered aquifers. Vertical flow components also arise from partial penetration of wells or from alternating slotted and solid casing of the well. The combined radial and vertical flow towards a pumped well can be expressed as:

$$\frac{\partial}{\partial r} \left(mK_r \frac{\partial s}{\partial r} \right) + \frac{mK_r}{r} \frac{\partial s}{\partial r} + mK_z \frac{\partial^2 s}{\partial z^2} = S \frac{\partial s}{\partial t} + c \quad (2.26)$$

It is possible to represent the vertical components of flow by using a detailed mesh in both radial and vertical directions. But this approach requires considerable computational efforts. There may not be sufficient field data to justify the use of these detailed models for pumping test analyses.

Alternatively, a simple idealization has been introduced to model approximately the vertical components of flow (Rushton and Redshaw, 1979). The drawdown is defined at two levels; at the free surface with drawdown s_f , and at one-quarter of the saturated depth above the base of the aquifer, where the drawdown is s_b . The flow is represented as taking place in an upper and a lower region. The equivalent horizontal hydraulic resistances are:

$$HU_n = HL_n = \frac{2\Delta a^2}{mK}$$

where U and L signify upper and lower zones.

Assuming that the vertical velocity of flow reduces linearly from a maximum value at the free surface to zero at the base of the aquifer, the vertical flow term becomes:

$$mK_z \frac{\partial^2 s}{\partial z^2} = \frac{K_z (s_{fn} - s_{bn})}{0.46875}$$

Thus the equivalent vertical hydraulic resistance in this discrete model is:

$$V_n = \frac{0.46875 m}{K_z r_n^2}$$

Consequently the discrete model takes the form shown in Figure 2.3. It is important to note that the figure shows the equivalent hydraulic network for an unconfined aquifer. The time resistances as defined in Equation (2.23) are connected to the upper line of nodes only. For confined aquifer, however, the time resistances of double the value of Equation (2.23) should be connected to both the upper and lower lines of nodes. Equations for continuity of flow at the upper and lower nodes are as follows:

$$\left[\frac{s_{fn+1} - s_{fn}}{HU_h} + \frac{s_{fn-1} - s_{fn}}{HU_{h-1}} + \frac{s_{bn} - s_{fn}}{v_n} \right]_{t+\Delta t} = \frac{s_{fn,t+\Delta t} - s_{fn,t}}{TU_n} + qr_n^2 \quad (2.27)$$

$$\left[\frac{s_{bn+1} - s_{bn}}{HL_h} + \frac{s_{bn-1} - s_{bn}}{HL_{h-1}} + \frac{s_{fn} - s_{bn}}{v_n} \right]_{t+\Delta t} = \frac{s_{fn,t+\Delta t} - s_{bn,t}}{TL_n} \quad (2.28)$$

Solutions of these equations for a particular problem can be obtained using a specially written elimination routine.

The greater flexibility of this model to represent different features made it possible to use it successfully to study a number of complex aquifer problems. Rushton and Booth (1976) applied the model to interpret a pumping test in a shallow gravel aquifer in which the drawdown in the pumped well was a significant proportion of the saturated thickness. An important aspect of the investigation was a sensitivity analysis which identified the most important parameters affecting the aquifer response. The ability of the model to allow for heterogeneity in the aquifer parameters, a variation in the discharge distribution with depth and non-Darcian flows close to the well, was noted.

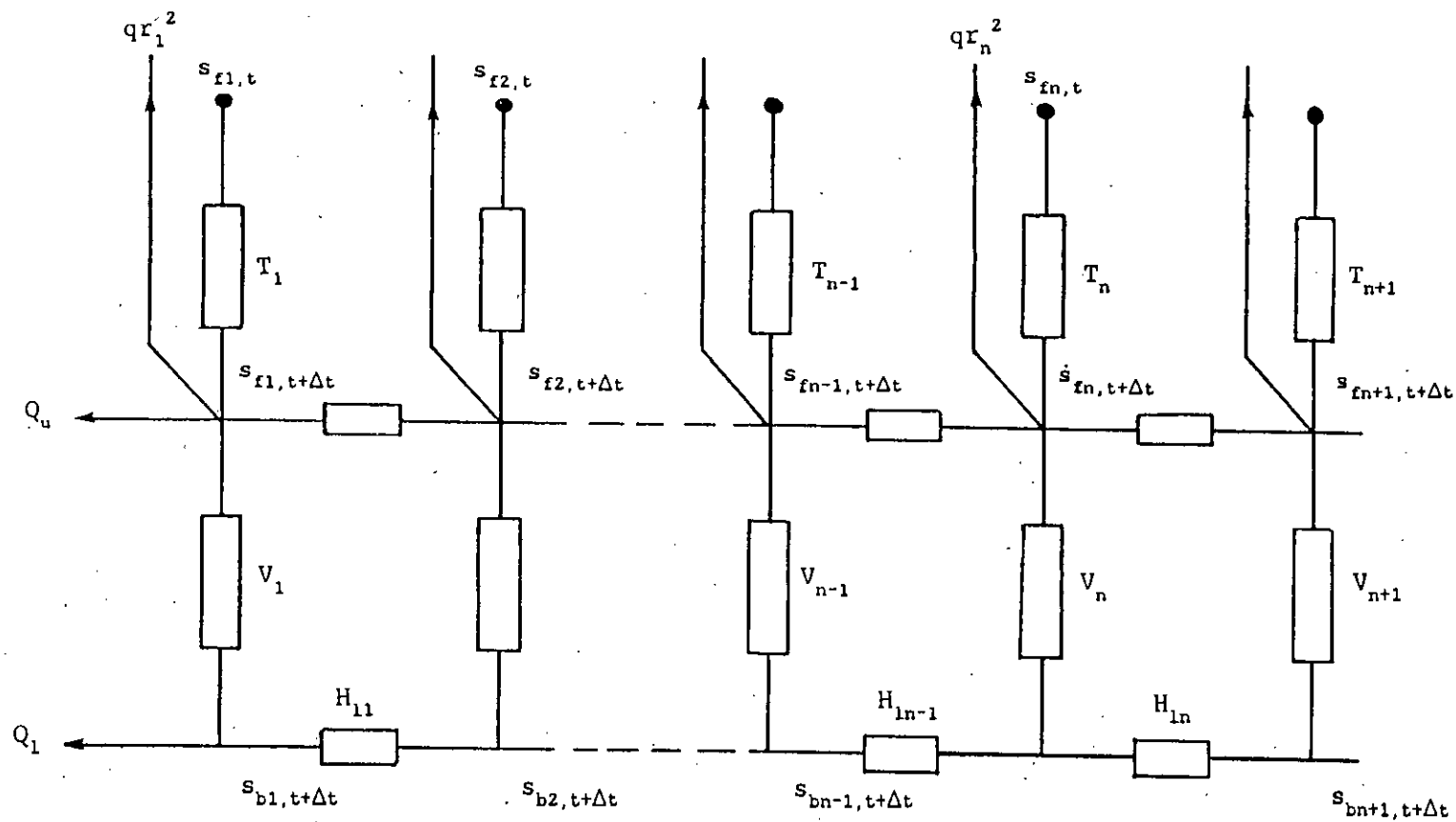


Fig. 2.3 Discrete model allowing for vertical flow components
(after Rushton and Redshaw, 1979)

Rushton and Chan (1976b) interpreted the results of two pumping tests carried out at different times of the year in a chalk aquifer using the two zone model. They demonstrated how the model could be modified to allow for aquifer parameters to vary with drawdown. Other examples of application of two zone model include: multi-piezometer test in an unconfined aquifer (Rushton and Howard, 1982) and weathered-fractured granite aquifer (Rushton and Weller, 1985).

Pumping tests have also been investigated using the modified form of the radial flow model in which the two flow domains are separated by a layer of low hydraulic conductivity. Examples of this application are pumping tests undertaken in the alluvial aquifer alongside the Wadi Bana in South Yemen (Rushton, 1983) and pumping and injection tests in an alluvial aquifer in Western India (Rushton and Srivastava, 1988).

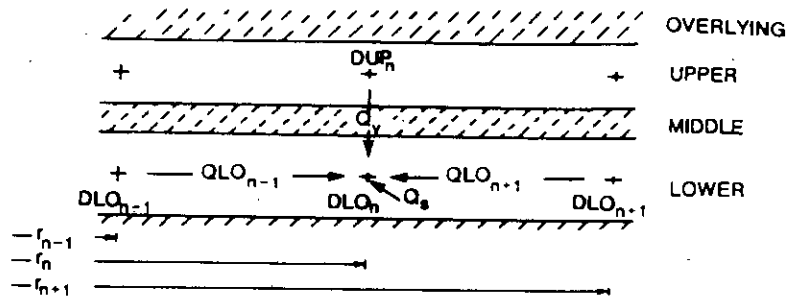
Rathod and Rushton (1991) described a numerical radial flow model for interpretation of pumping tests in two-zone layered aquifer. The conceptual aquifer system consists of two horizontal permeable zones with a less permeable intermediate layer. The drawdowns are defined at two levels: at the middle of lower zone and the free surface or top of the upper zone. They derived the discrete space - discrete time equations using lumping approach. Figure 2.4 shows the components of flow balance in lower zone and equivalent hydraulic resistances. The hydraulic and time resistance are defined as:

$$HUP () = \frac{\Delta a^2}{K_{ru} m_u}, \quad TSUP () = \frac{\Delta t}{S_u r^2} \quad (2.29)$$

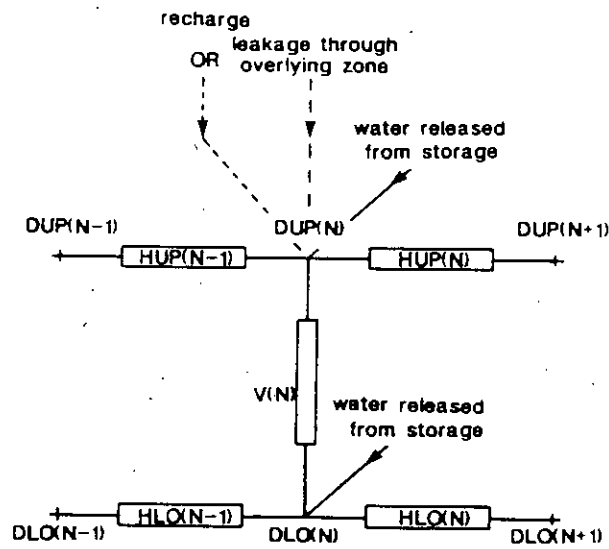
$$HLO () = \frac{\Delta a^2}{K_{rl} m_l}, \quad TSLO () = \frac{\Delta t}{S_l r^2} \quad (2.30)$$

$$V () = \frac{\Delta z}{K_v r^2} \quad (2.31)$$

Where $HUP ()$ and $HLO ()$ = horizontal hydraulic resistances for the upper and lower zones respectively; $TSUP ()$ and $TSLO ()$ = time resistances for the upper and lower zones respectively; $V ()$ = vertical hydraulic resistance; Δa = radial mesh interval; K_{ru} and K_{rl} = radial hydraulic conductivities of the upper and lower zones respectively; m_u



(a)



(b)

Fig. 2.4 Features of two zone numerical model: (a) components of flow balance in lower zone, (b) equivalent hydraulic resistances (after Rathod and Rushton, 1991)

and m_1 = saturated thicknesses of the upper and lower zones respectively; r = radial distance; Δt = length of the time step; S_u and S_l = confined storage coefficients of the upper and lower zones respectively; Δz = vertical distance from the centre of the lower zone to the free surface or the top of the aquifer if it is confined; and K_v = vertical hydraulic conductivity between the two zones.

The resulting flow balance equations together with appropriate boundary conditions are solved by using Gaussian elimination routine. The model includes important features such as delayed yield, leakage, well losses and different conditions at the well face and outer boundary.

Application of this model to interpret pumping tests in different types of aquifer has been described by Rathod and Rushton (1991). The model was also used to predict the long-term response of a weathered-fractured aquifer based on the parameters derived from a short-term pumping test.

2.3.3 Three Flow Domain Model

Grout (1988) developed a numerical model to simulate flow to a pumping well based on depth-averaging principles. The aquifer is divided into three flow domains. These can represent individual geological units or simply different flow horizons within a single homogeneous aquifer. The lower and middle zones of this model are assumed to be confined while the upper zone is considered as unconfined. The continuous vertical distribution of drawdown is approximated by three average values.

The continuity equations for each flow domain in their finite difference forms are given below:

Lower Flow Domain:

$$\left[\frac{s_l(n+1) - s_l(n)}{h_l(n)} - \frac{s_l(n) - s_l(n-1)}{h_l(n-1)} - \frac{s_l(n) - s_m(n)}{V_{ml}(n)} \right]_{t+\Delta t}$$

$$= \frac{s_l(n)_{t+\Delta t} - s_l(n)_t}{t_l(n)} \quad (2.32)$$

where:

s_l and s_m = average drawdowns in the lower and middle flow domains respectively,

$$h_l(n) = \frac{\Delta a^2}{K_l m_l},$$

$$t_l(n) = \frac{\Delta t}{S_l r_n^2},$$

$$V_{ml}(n) = \frac{0.5 m_m}{K_{mv} r_n^2} + \frac{0.5 m_l}{K_{lv} r_n^2},$$

K_l = radial hydraulic conductivity of the lower flow domain,

S_l = storage coefficient of the lower flow domain,

m_l and m_m = thicknesses of the lower and middle flow domains respectively, and

K_{lv} and K_{mv} = vertical hydraulic conductivities of the lower and middle flow domains respectively.

Middle Flow Domain:

$$\left[\frac{s_m(n+1) - s_m(n)}{h_m(n)} - \frac{s_m(n) - s_m(n-1)}{h_m(n-1)} - \frac{s_m(n) - s_u(n)}{V(n)} + \frac{s_l(n) - s_m(n)}{V_{ml}(n)} \right]_{t+\Delta t}$$

$$= \frac{s_m(n)_{t+\Delta t} - s_m(n)_t}{t_m(n)} \quad (2.33)$$

where:

S_u = average drawdown in the upper flow domain,

$$h_m(n) = \frac{\Delta a^2}{K_m m_m},$$

$$t_m(n) = \frac{\Delta t}{S_m r_n^2},$$

$$V_{mu} = \frac{0.5m_m}{K_m r_n^2} + \frac{0.5m_u}{K_{uv} r_n^2}$$

- K_m = radial hydraulic conductivity of the middle flow domain,
 m_u = saturated thickness of the upper flow domain,
 K_{uv} = vertical hydraulic conductivity of the upper flow domain, and
 S_m = storage coefficient of the middle flow domain.

Upper Flow Domain:

$$\begin{aligned}
 & \left[\frac{s_u(n+1) - s_u(n)}{h_u(n)} - \frac{s_u(n) - s_u(n-1)}{h_u(n-1)} + \frac{s_m(n) - s_u(n)}{V_{mu}(n)} \right]_{t+\Delta t} \\
 & = \frac{s_u(n)_{t+\Delta t} - s_u(n)_t}{t_u(n)} + \frac{[s_u(n) - s_f(n)]_{t+\Delta t} + qt_f}{v_{fu}(n) + t_f}
 \end{aligned} \tag{2.34}$$

Where:

$$h_u(n) = \frac{\Delta a^2}{K_u m_u}$$

$$t_u = \frac{\Delta t}{S_u r_n^2}$$

$$V_{fu}(n) = \frac{0.5m_u}{K_{fv} r_n^2}$$

$$t_f = \frac{\Delta t}{S_y r_n^2}$$

- K_u = radial hydraulic conductivity of the upper flow domain,
 S_u = storage coefficient of the upper flow domain,
 K_{fv} = vertical hydraulic conductivity of the upper flow domain,
 S_y = specific yield of the upper flow domain,

s_f = drawdown at the free surface, and
 q = vertical recharge to the water table.

The scheme of hydraulic resistances for this model is shown in Figure 2.5. The free surface drawdown is calculated, after solving the average drawdown equations for time $t + \Delta t$, using the interpolation function:

$$s_{f,t+\Delta t} = \frac{[s_{u,t+\Delta t} - s_{f,t} + qt_f] v_{fu}}{t_f + v_{fu}} + s_{u,t+\Delta t} \quad (2.35)$$

The application of Three Flow Domain model to interpret pumping tests in an alluvial aquifer in South Yemen has been described by Grout (1988).

The numerical models described in this section have been developed using depth - averaging techniques to represent vertical components of flow to an abstraction well. In two zone model the drawdowns are defined at two levels and the vertical flow between these levels is assumed to take place across a single resistance. Therefore, a detailed distribution of drawdowns in vertical direction can not be obtained. Nevertheless, the model has proved to be quite useful to analyse pumping tests in a wide range of aquifers. It includes all the important features of flow to an abstraction well in a leaky layered aquifer. However, the model calculates leakage assuming that the water table in the source bed remains unchanged throughout the period of pumping. This assumption may not be valid when the source bed is a low permeability zone overlying the main aquifer. Hence the model needs to be further developed to include important flow mechanisms in the overlying zone.

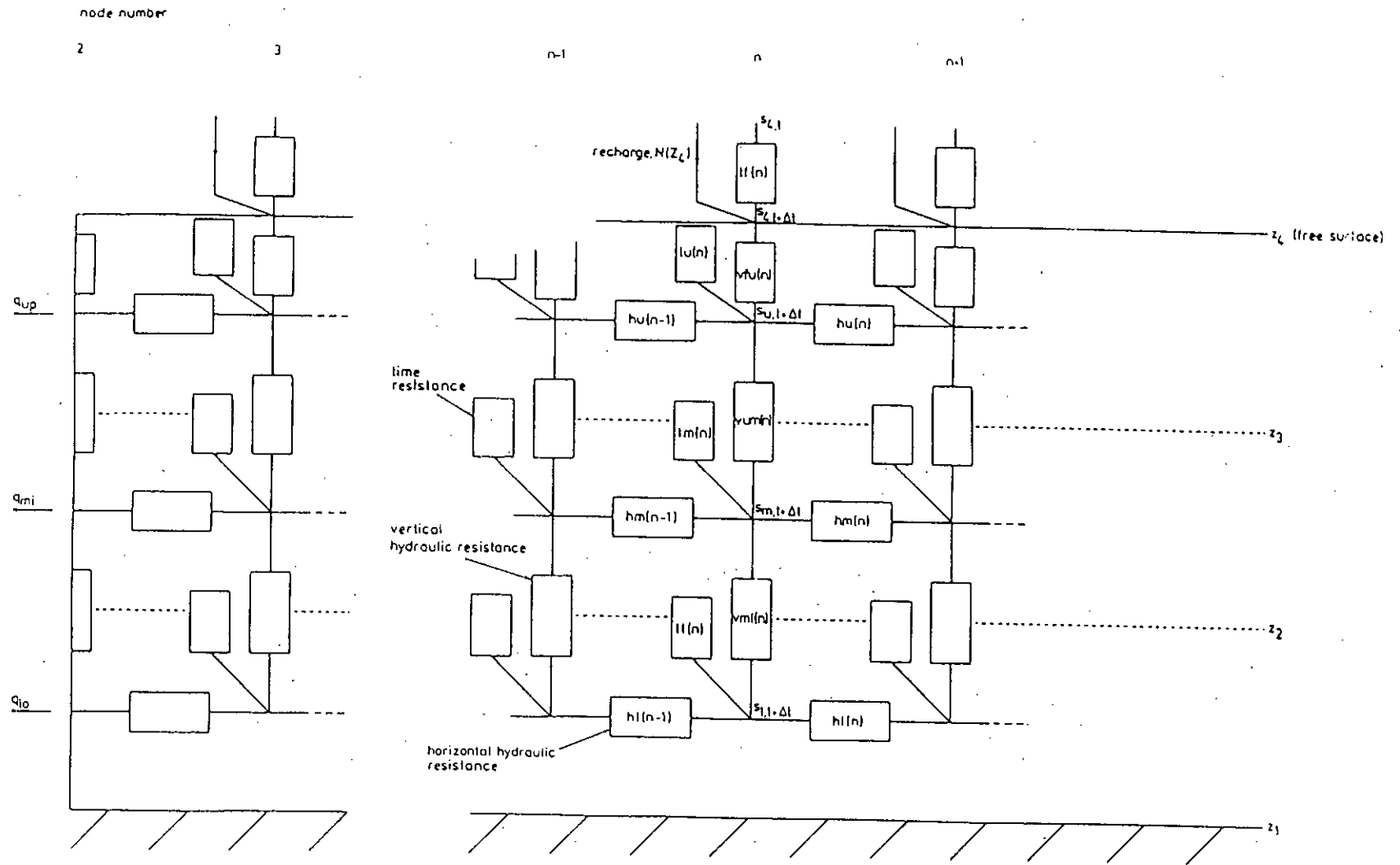


Fig. 2.5 Resistance scheme for the Three Flow Domain Model (after Grout, 1988)

Chapter 3

STUDY AREA AND MODEL DESCRIPTION

3.1 Study Area

The study area is located in the central part of Bangladesh. The aquifer underlying this area is known as Madhupur aquifer. It consists of Pleistocene alluvial sediments derived from the erosion of the Himalayas. These sediments are primarily composed of sands, gravels and silts derived from plutonic igneous and high grade metamorphic rocks rich in quartz, feldspar and mica (Devis and Exley, 1992). This aquifer is overlain by the Madhupur clay, a residual soil horizon composed of red-brown silty clays.

A large number of 15 l/sec capacity shallow tubewells have been installed in the area to abstract groundwater for irrigation during the dry season. But until 1985 no deep tubewell had been sunk in this area for irrigation because the area was believed to have poor hydrogeological condition. Ten exploratory boreholes were drilled in the study area during 1985-1987 under the framework of Deep Tubewell II Project (Mott MacDonald International Limited, 1990). Figure 3.1 shows the locations of these boreholes and their logs are presented in Figure 3.2. An examination of these logs reveals that the aquifer is a complex mixture of sands, silt and clay. A surface clay layer overlies the aquifer; the thickness of this layer varies from about 5 m in the northeast to about 30 m in the southwest. The thickness of screenable formation in the top 122 m varies from about 40 m in the southwest to about 90 m in the northeast part of the study area.

The average annual rainfall in the study area is about 2370 mm; but about 80% of the rainfall occurs during the monsoon months of July through October. There is little information on groundwater levels in the area. Mott MacDonald International Limited (1990) reported that the minimum and maximum static water levels were 4.3 and 7.5 m respectively during 1988-89. The fluctuation of the water level is mainly due to abstraction during the dry season and recharge from rainfall during the monsoon.

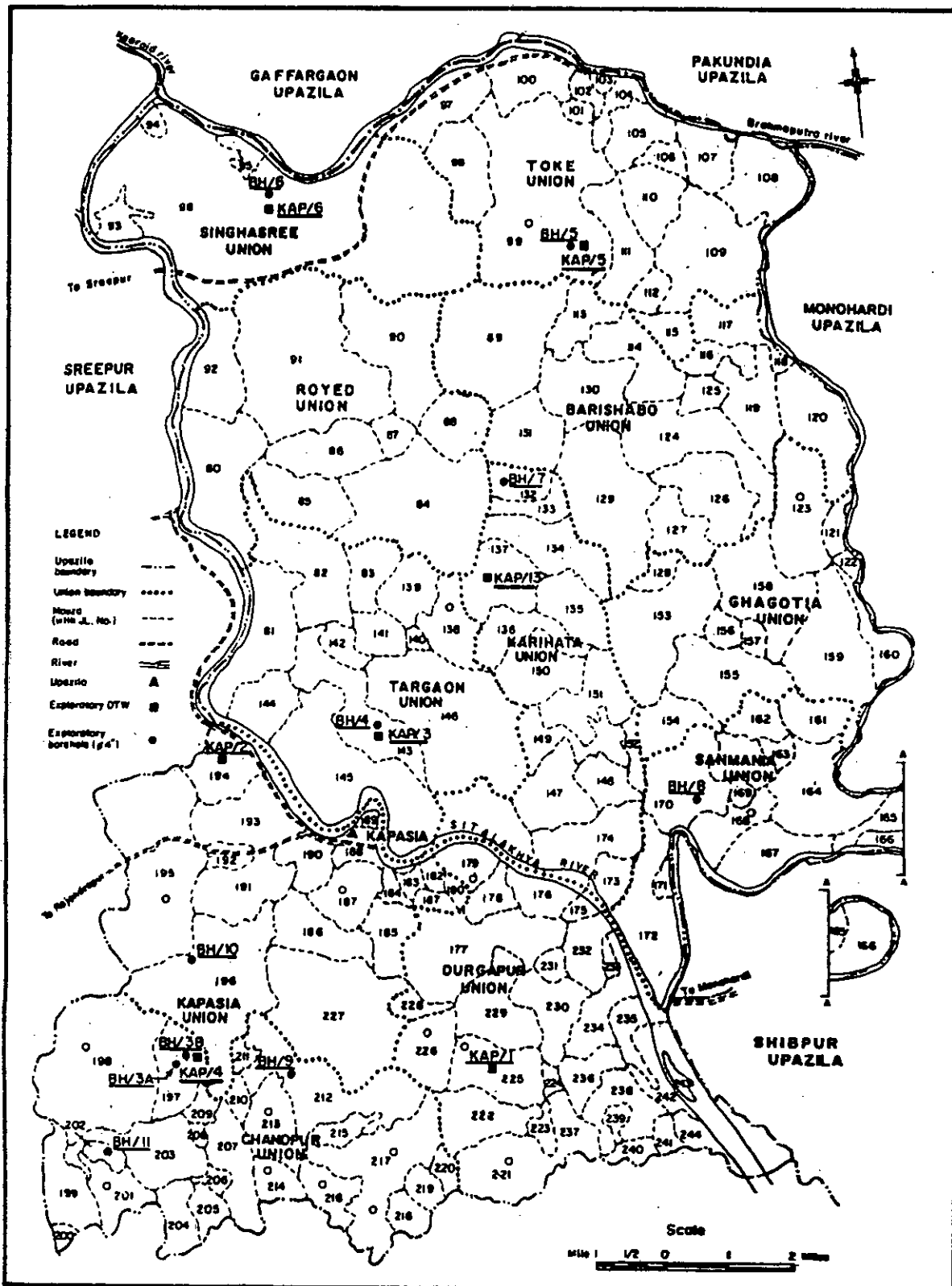


Fig. 3.1 Map of Kapsasia Thana showing the locations of the exploratory boreholes and test wells (after Mott Macdonald International, 1990)

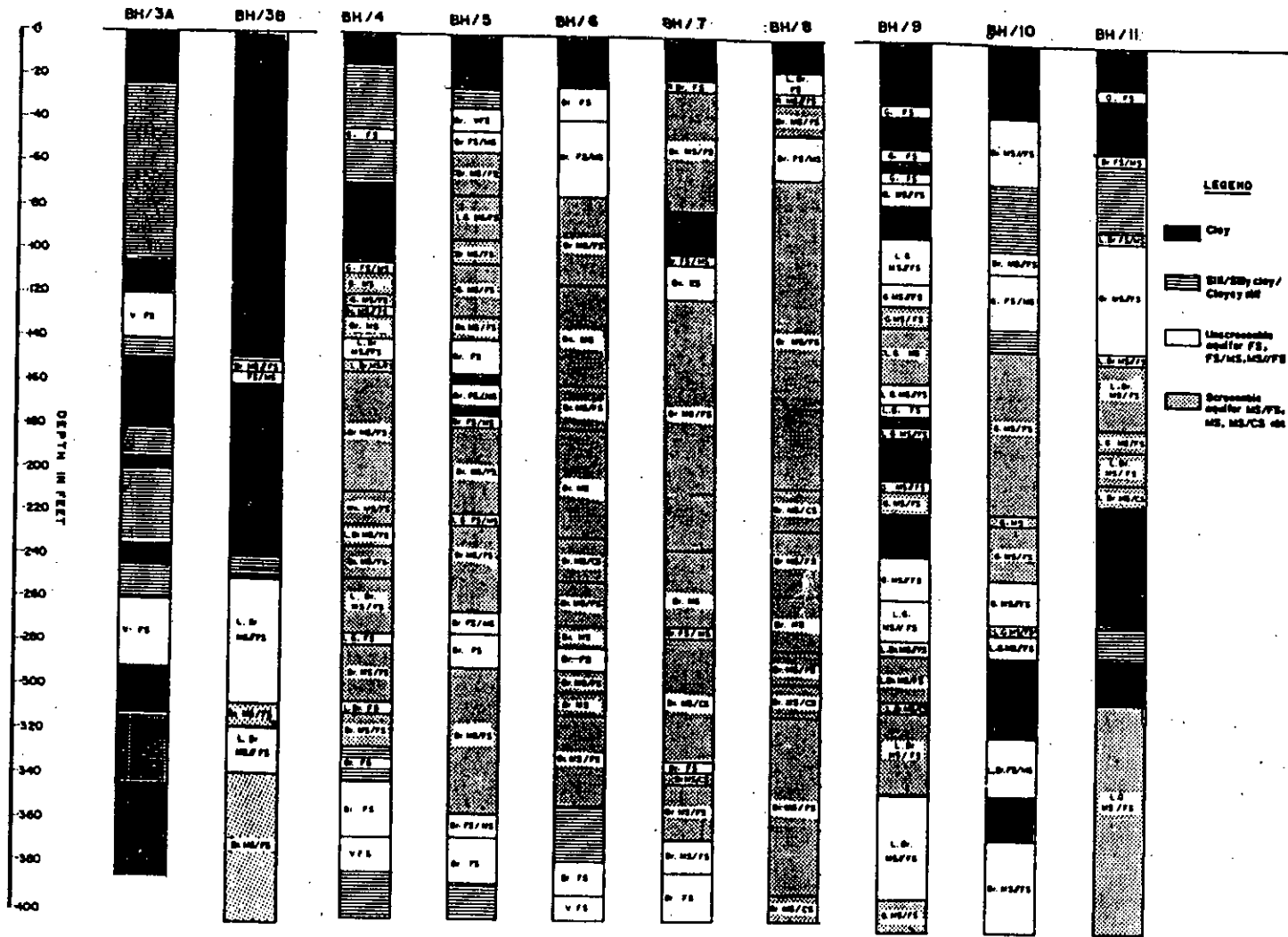


Fig. 3.2 Logs of the exploratory boreholes
 Source: Mott Macdonald International, 1990

3.2 Groundwater Flow Mechanism During Pumping

An understanding of flow mechanism near a pumped well is the key to successful prediction of long-term response of the aquifer. The aquifer underlying the study area is of semi-confined or leaky type. The important features of the flow mechanisms which operate under the influence of pumping from this type of aquifer are briefly described as follows.

Following the start of pumping, the water level in the well drops relatively quickly because most of the water pumped during the first few minutes is derived from the well storage. As the well water level falls, pressure is released within the aquifer and water flows towards the well. This water is mainly derived from the confined storage.

As discussed in the proceeding section the aquifer consists of a complex system of zones of higher and lower hydraulic conductivities. The tubewells pumping from this aquifers are normally screened only against the more permeable zones and are in effect partially penetrating. Therefore, both radial and vertical components of flow occur within the aquifer system. Furthermore, the aquifer is overlain by a low permeability zone. As the groundwater head in the aquifer falls considerably, a sufficient vertical hydraulic gradient is established to initiate leakage from the overlying zone. Leakage from this zone results in fall in water table and the important flow mechanism in this zone is the balance between any recharge reaching the water table from above and the water released from storage as the water table falls. The rate of leakage from the overlying layer depends on the position of water table in the overlying layer and groundwater head in the main aquifer.

3.3 Description of the Model

The aquifer system underlying the study area can be adequately represented by a two zone model as illustrated in Figure 3.3. Rathod and Rushton (1991) developed a numerical model for interpretation of pumping from two zone layered aquifers. The model considers a full range of important features including vertical components flow and leakage. However, in the formulation of the model it was assumed that the water level in the overlying leaky

layer remains constant. This may not be valid when the source bed is a low permeability zone overlying the main aquifer as shown in Figure 3.3. Consequently, a modification has been made in the program to calculate the leakage taking into consideration the position of water level in the leaky layer and groundwater head in the upper aquifer. Three possible conditions may occur. These are:

- (a) As long as the water level in the overlying layer and groundwater head in the main aquifer both lie above the base of the covering layer, Figure 3.4 (a), the main aquifer remains confined and the leakage is proportional to the difference between the phreatic and piezometric heads and inversely proportional to the hydraulic resistance of the saturated part of the covering layer, or

$$L = \frac{2\pi r_n^2 K_v' [DUP(N) - WL(N)]}{TPUP - WL(N)} \quad (3.1)$$

where L = leakage at node N ; r_n = radial distance of the node N ; Δa = radial mesh interval; K_v' = vertical hydraulic conductivity of overlying layer; $DUP(N)$ = drawdown at node N in the upper aquifer; $WL(N)$ = water level at node N in the overlying layer and $TPUP$ = level of top of upper aquifer.

- (b) When the water level in the overlying layer remains above $TPUP$, but the piezometric head in the upper aquifer falls below $TPUP$, the unconfined condition applies to the main aquifer whilst perched water table condition occurs in the overlying zone (Figure 3.4(b)). The vertical hydraulic gradient is unity because atmospheric conditions apply at the base of the overlying zone. As such the leakage can be expressed as:

$$L = 2\pi r_n^2 \Delta a K_v' \quad (3.2)$$

- (c) When the water level in the overlying layer reaches the base of this layer as a result of drainage it becomes unsaturated and leakage tends to be zero, ie

$$L = 0 \quad (3.3)$$

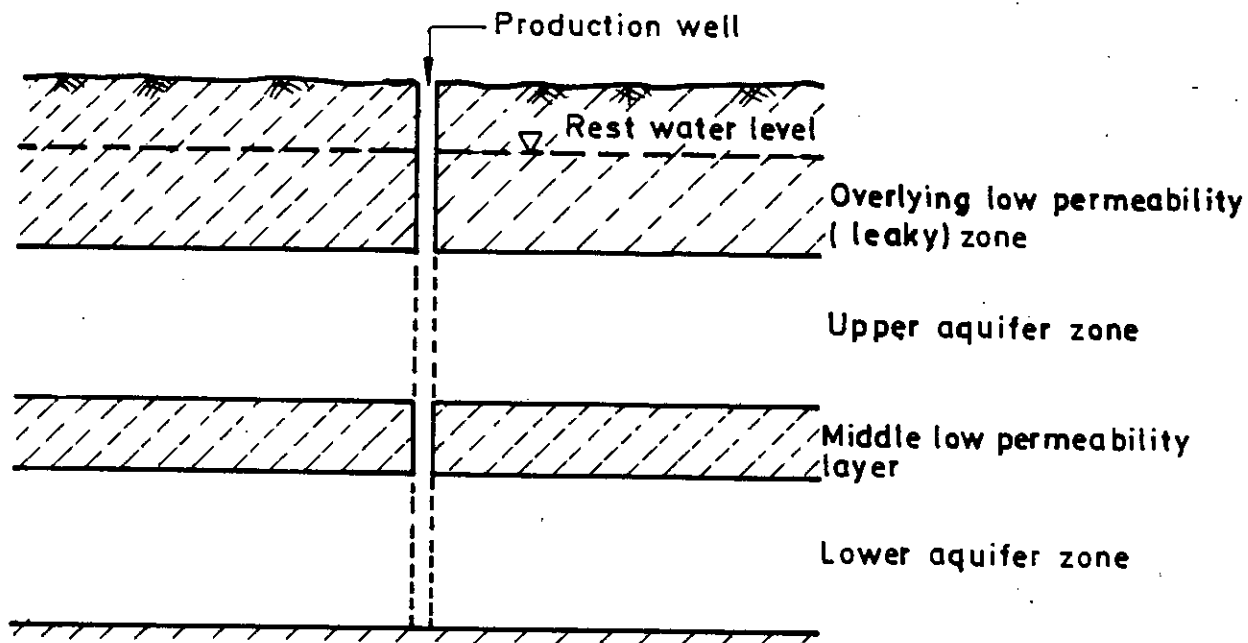
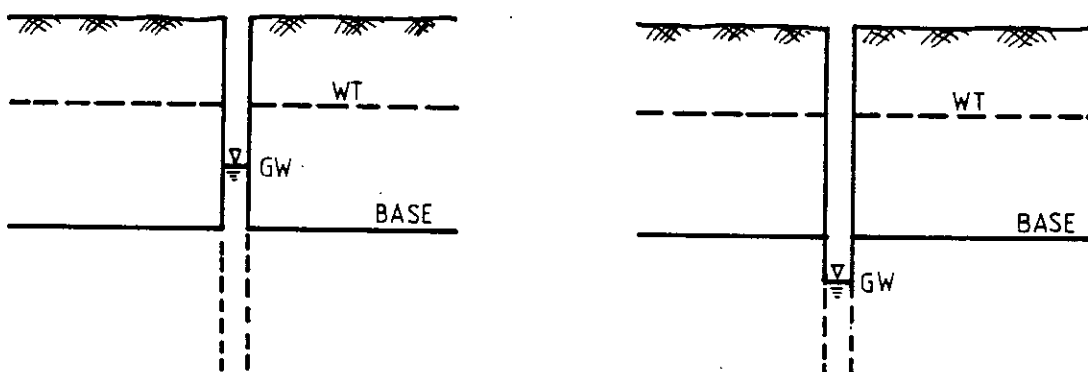
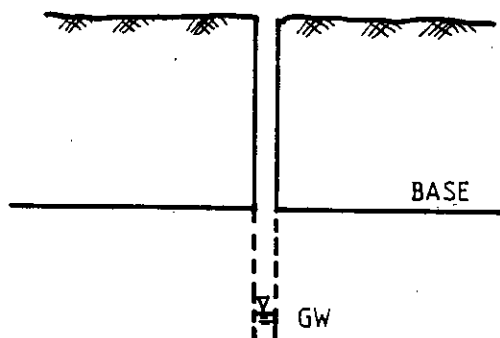


Fig. 3.3 Conceptual two zone model



(a) Water table (WT) and groundwater head (GW) both lie above the base of the overlying layer (BASE)
 (b) WT lies above BASE but GW is below BASE



(c) GW is below BASE and no WT exists in the overlying layer

Fig. 3.4 Possible conditions in the overlying low permeability zone

Another important feature of response of overlying layer is the fall of water table. In each time step the elevation of water table in the overlying layer is determined from the balance between the recharge reaching the water table from above and the water released from the storage as the water table falls. The balance at the water table can be expressed as:

$$\frac{2\pi r_n^2 \Delta a K_v [DUP(N) - WL(N)]}{TPUP - WL(N)} - q 2\pi r_n^2 \Delta a = \frac{S_y 2\pi r_n^2 \Delta a [WL(N) - OLDWL(N)]}{\Delta t} \quad (3.4)$$

Where q = recharge reaching the water table; S_y = specific yield of the overlying layer; $OLDWL(N)$ = water level in the overlying layer at the end of previous time step and Δt = length of time step. The other variables were defined earlier.

Dividing equation (3.4) throughout by $2\pi r_n^2 \Delta a$, we get

$$\frac{K_v [DUP(N) - WL(N)]}{TPUP - WL(N)} - q = \frac{S_y [WL(N) - OLDWL(N)]}{\Delta t} \quad (3.5)$$

Equation (3.5) has been incorporated in the program. The modified version of the program has been used in this study. The following paragraphs describe the salient features of the model.

The model uses discrete space - discrete time approximations. The radial dimension is divided into mesh intervals which increase logarithmically away from the well. Six mesh subdivisions were used for a tenfold increase in radial distance. A logarithmic increase in time was also used with ten time steps for a tenfold increase in elapsed time. The flow is represented as taking place in the upper and lower aquifer zones. The drawdowns are defined at two levels: at the middle of lower zone and at the free surface or top of the upper zone if it is confined. The hydraulic resistances for the upper and lower aquifer zones are:

$$HUP(N) = \frac{\Delta a^2}{m_u(N) K_u(N)}, \quad TSUP(N) = \frac{\Delta t}{S_u(N) r_n^2} \quad (3.6)$$

$$HLO(N) = \frac{\Delta a^2}{m_1(N)K_1(N)}, \quad TSLO(N) = \frac{\Delta t}{S_1(N)r_n^2} \quad (3.7)$$

Where HUP () and HLO () = radial hydraulic resistances for the upper and lower zones respectively; TSUP () and TSLO () = time resistances for the upper and lower zones respectively; Δa = radial mesh interval = $\ln \left(\frac{r_{n+1}}{r_n} \right)$; K_u and K_l = radial hydraulic conductivities of the upper and lower zones respectively; m_u and m_l = saturated thicknesses of the upper and lower zones respectively; r_n = radial distance to node N ; S_l = confined storage coefficient of the lower zone and S_u = confined or unconfined storage coefficient of the upper zone depending upon the magnitude of drawdown in this zone. Confined storage coefficient is used as long as this zone remains confined. However, when the drawdown in the upper zone reaches the base of the overlying layer, unconfined conditions apply and the unconfined storage coefficient and, if delayed drainage occurs, a delayed yield index are used.

The vertical hydraulic resistance, V_n can be expressed as:

$$V(N) = \frac{\Delta z}{K_v r_n^2} \quad (3.8)$$

where Δz = vertical distance through which the flow takes place and K_v = vertical hydraulic conductivity between the two zones.

The equivalent system in terms of hydraulic resistances is shown in Figure 3.5. The flow balance equations for the upper and lower zones can be written in the follow forms:

$$\begin{aligned} &AUP(N).DUP(N-1) + BUP(N).DUP(N) + CUP(N).DUP(N+1) + E(N).DLO(N) \\ &= FUP(N) \end{aligned} \quad (3.9)$$

$$\begin{aligned} &ALO(N).DLO(N-1) + BLO(N).DLO(N) + CLO(N).DLO(N+1) + E(N).DUP(N) \\ &= FLO(N) \end{aligned} \quad (3.10)$$

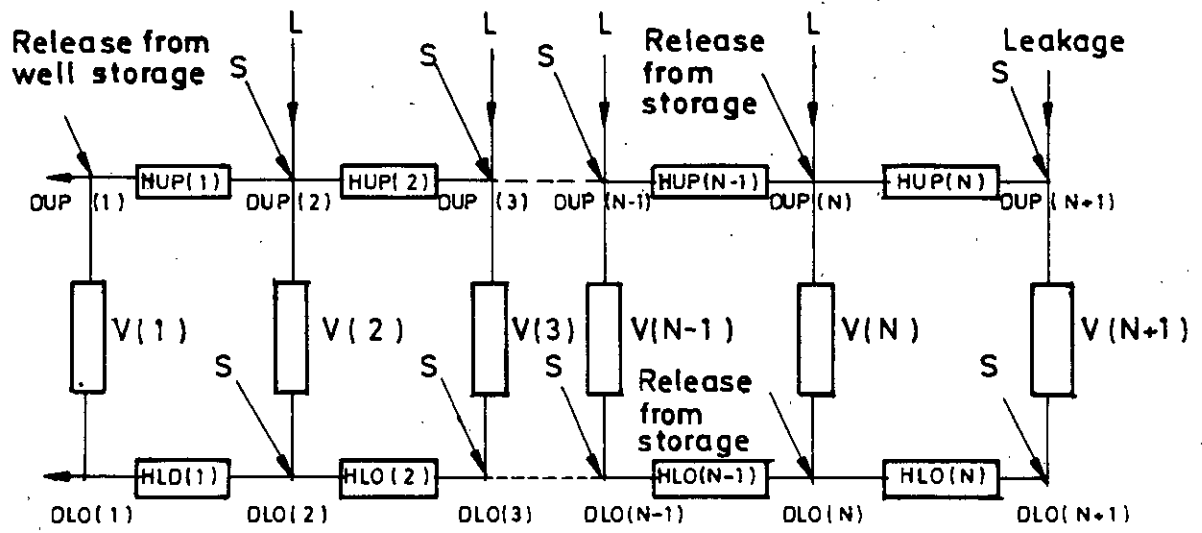


Fig. 3.5 Discrete model allowing for vertical flow components and leakage

Where:

$$AUP(N) = \frac{1}{HUP(N-1)} \quad (3.11)$$

$$CUP(N) = \frac{1}{HUP(N)} \quad (3.12)$$

$$E(N) = \frac{1}{V(N)} \quad (3.13)$$

$$BUP(N) = - \left[AUP(N) + CUP(N) + E(N) + \frac{1}{TSUP(N)} + \frac{K_v \cdot r_n^2}{(TPUP - WL(N))} \right]$$

if $DUP(N) \leq TPUP$ and $WL(N) < TPUP$ (3.14)

$$= - \left[AUP(N) + CUP(N) + E(N) + \frac{1}{TSUP(N)} \right],$$

if $DUP > TPUP$ and $WL(N) \geq TPUP$ (3.15)

$$FUP(N) = - \frac{OLDDUP(N)}{TSUP(N)} - \frac{K_v \cdot r_n^2 \cdot WL(N)}{TPUP - WL(N)},$$

if $DUP(N) \leq TPUP$ and $WL(N) < TPUP$ (3.16)

$$= - \frac{OLDDUP(N)}{TSUP(N)} + K_v \cdot r_n^2, \quad \text{if } DUP(N) > TPUP \text{ and } WL(N) < TPUP \quad (3.17)$$

$$= - \frac{OLDDUP(N)}{TSUP(N)}, \quad \text{if } WL(N) \geq TPUP \quad (3.18)$$

$$ALO(N) = \frac{1}{HLO(N-1)} \quad (3.19)$$

$$CLO(N) = \frac{1}{HLO(N)} \quad (3.20)$$

$$BLO(N) = -[ALO(N) + CLO(N) + E(N) + \frac{1}{TSLO(N)}] \quad (3.21)$$

$$FLO(N) = -\frac{OLDDLO(N)}{TSLO(N)} \quad (3.22)$$

DUP(N) and DLO(N) = drawdowns in the upper and lower zones respectively;
 OLDDUP(N) and OLDDLO(N) = drawdowns in the upper and lower zones respectively
 at the end of previous time step.

Equations (3.9) and (3.10) are solved using a Gaussian elimination routine with appropriate boundary conditions. At the outer boundary of each permeable zone the condition may be specified either as a zero - flux or as a zero - drawdown. Well storage effects are included by extending the mesh into the well and setting the storage coefficient within the well equal to unity. Well losses may occur at the face of the well. These losses may be incorporated by including a well loss factor which alters the hydraulic resistance for the mesh interval adjacent to the well face.

Parameter values must be assigned to each of the zones. These are as follows:

Overlying and middle layer:	thickness and vertical hydraulic conductivity
Upper zone	: thickness, radial and vertical hydraulic conductivities and confined and unconfined storage coefficients
Lower zone	: thickness, radial and vertical hydraulic conductivities and confined storage coefficient.

Figure 3.6 shows a flow chart of the program and the source codes for the program are given in Appendix-A.

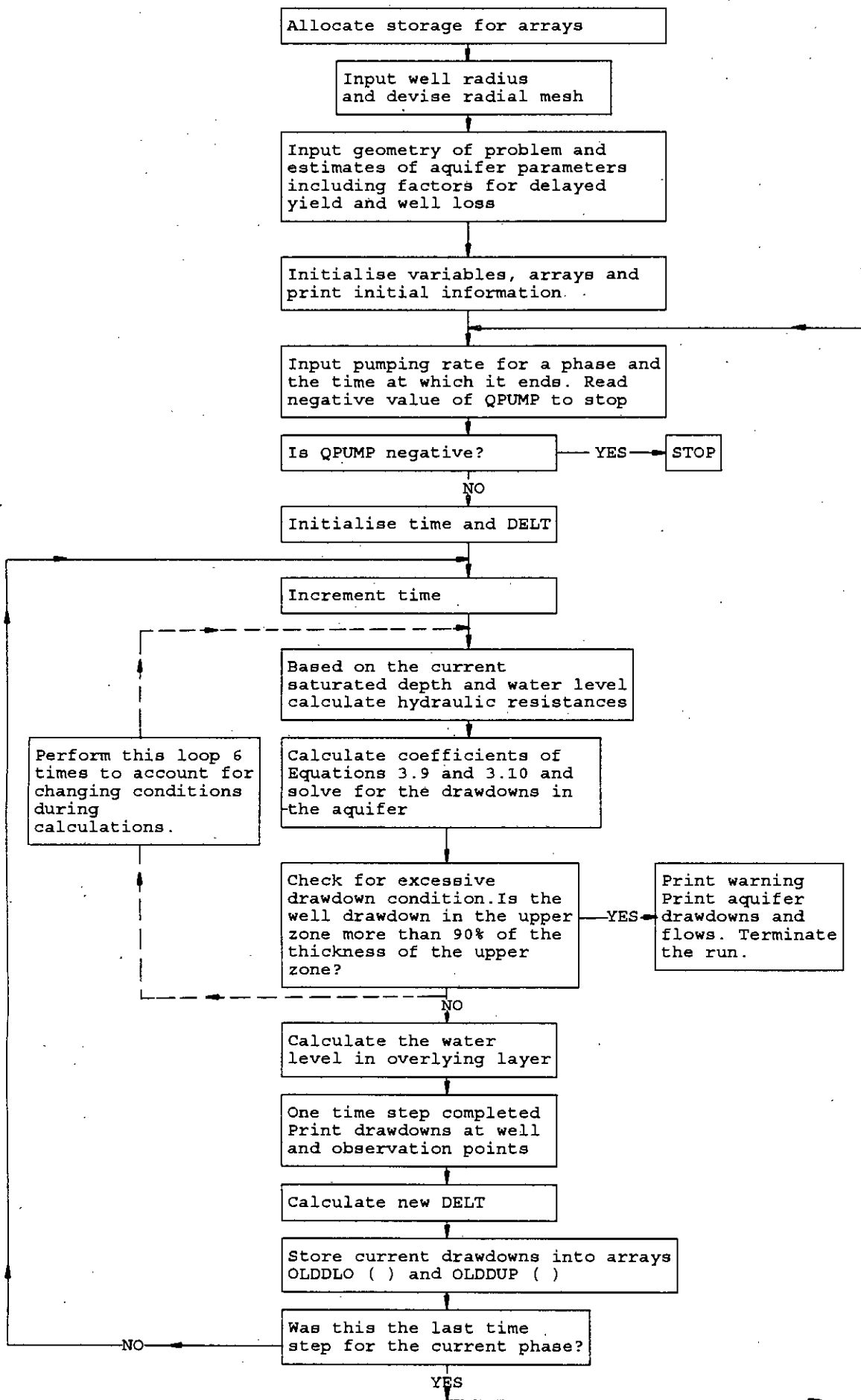


Fig. 3.6 Flow chart of computer program for the two-zone model

Chapter 4

RESULTS AND DISCUSSION

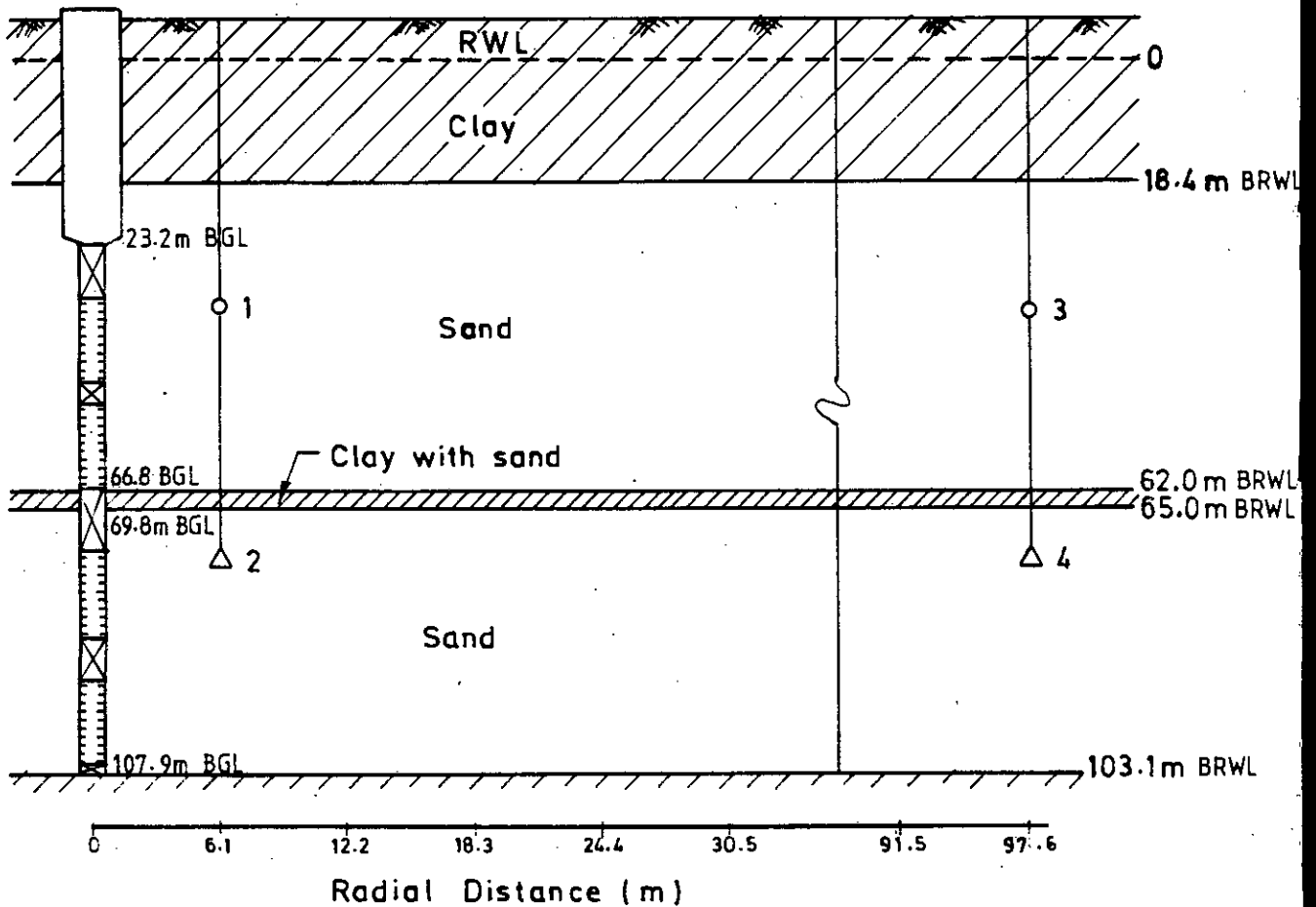
4.1 Interpretation of Pumping Test Data

Seven test wells were drilled in the study area and constant discharge pumping tests were conducted at three sites namely KAP/2, KAP/3 and KAP/13 under the framework of Deep Tubewell II Project. Figure 3.1 shows the locations of these sites. The field data were analysed to determine aquifer characteristics using Cooper and Jacob (1946) and Walton (1962) methods. This may lead to erroneous results as the actual field conditions deviate considerably from the assumptions and idealizations made in the derivation of these methods. Alternatively, a numerical model can provide a greater flexibility and it is possible to represent all the important features of flow mechanism in the model. Therefore, the modified two zone model as described in chapter 3 has been used to interpret these pumping tests. The results of the analysis are summarized in the following sections.

4.1.1 Pumping Test : KAP/2

This well was drilled in May 1986 and the test was conducted in March 1987. The details of the well and observation piezometers used during the test are shown in Figure 4.1. The well was drilled to a depth of 107.9 m BGL. The log of the well shows that the aquifer consists of sand with a thin layer of sandy clay lying between 66.8 and 69.8 m BGL. The aquifer is overlain by a clay layer which is 23.2 m thick. The diameters of the upper and lower casings of the well are 350 and 150 mm respectively and the later consists of alternating slotted and solid sections. Shallow and deep piezometers have been installed at radial distances of 6.1 and 97.6 m from the well, Figure 4.1.

The rate of abstraction during the test was 4737 m³/day and the durations of the pumping and recovery phases were 3.0 and 2.02 days respectively. Water levels were monitored in the well and piezometers during both the pumping and recovery phases. Prior



BGL : Below Ground Level

BRWL: Below Rest Water Level

Fig. 4.1 Details of well and observation piezometers used for pumping test: KAP/2

to the test the rest water level (RWL) was 4.8 m below the reference level. The field data for the pumped well and piezometers are plotted in Figures 4.2 to 4.4.

The two zone model has been used to simulate the response recorded in the pumped well and piezometers both during pumping and recovery phases. The initial estimates of the aquifer parameters were made from the study report (Mott MacDonald International Limited, 1990) and general guidelines (Walton, 1987). A zero-flux boundary condition has been enforced at a distance of 3 km. The program was run several times with trial values of parameters. After each run, the parameter values were changed until the calculated drawdowns in the well and piezometers during both the pumping and recovery phases were close to the observed values. The model results are also plotted in the same figures for comparison.

Figure 4.2 indicates that as soon as the pumping started, the water level in the pumped well fell by about 4.5 m and thereafter the rate of fall declined as approximated to a straight line on a log-normal plot. The recovery curve is a mirror image of the drawdown curve. The model results match very well with the field values. However, well loss factors of 11.0 and 8.0 respectively for upper and lower zones had to be used to obtain such matching. This relatively high values of well loss factor could be due to partial penetration effects.

The general shape of the drawdown curves for piezometers 1 and 2 is quite similar to that of the pumped well (Fig. 4.3). However, the drawdowns in piezometer 1 remained higher than that in piezometer 2 throughout the period of pumping. This could be due to convergence of flow near the well as a result of partial penetration. A good match between model results and field values could be achieved for the recovery phase. But the model results did not match well with the field data recorded in these piezometers during the pumping phase. This might be because significant vertical flow occurred near the well and the model gave drawdowns at two specified depths which did not possibly coincide with the locations of these piezometers.

Figure 4.4 indicates that the responses in piezometers 3 and 4 are rather slow showing little change in groundwater head during the first few minutes. This is mainly because of the

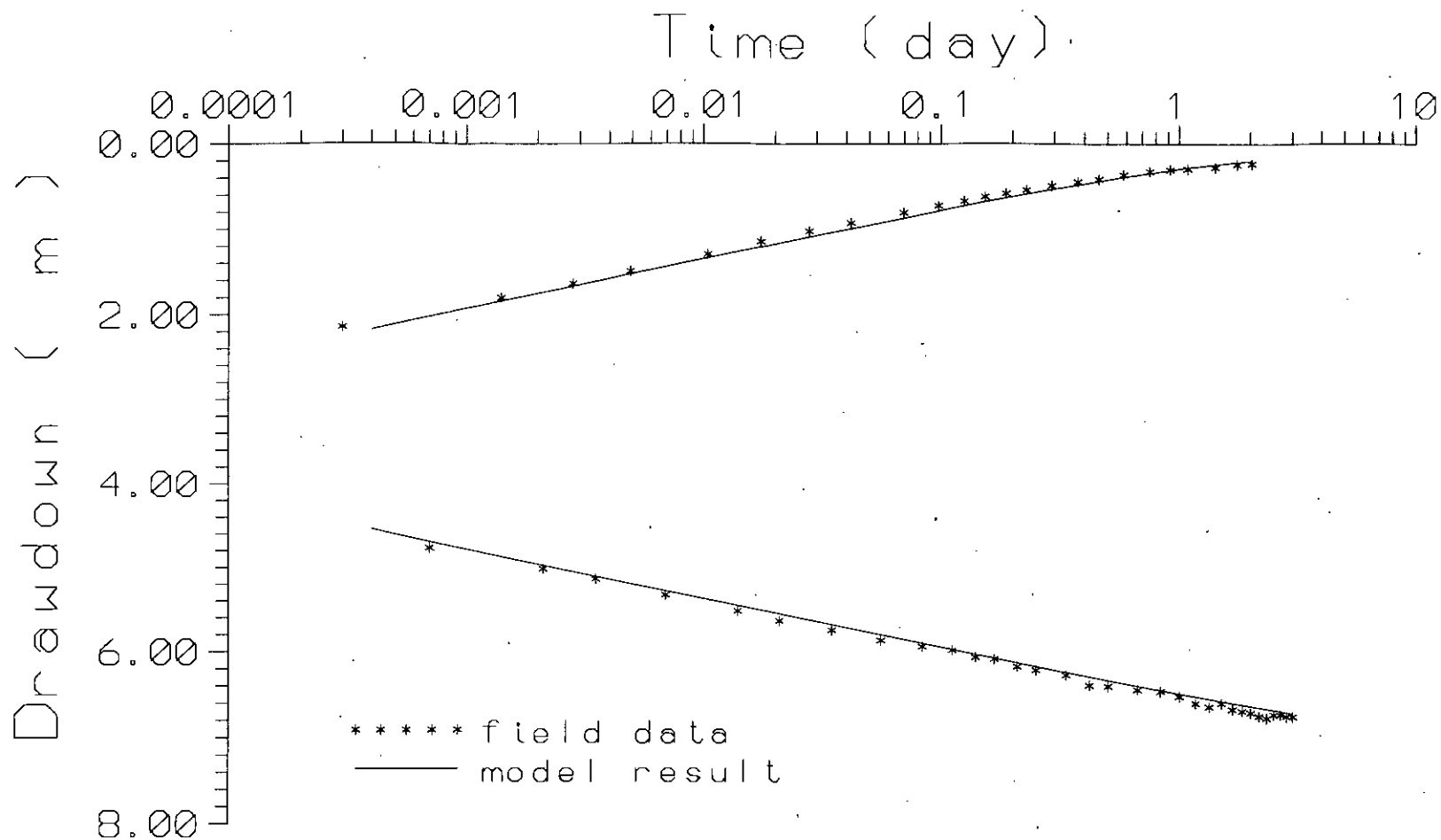


Fig. 4.2 Comparison between field and modelled drawdowns in the pumped well for pumping test: KAP/2

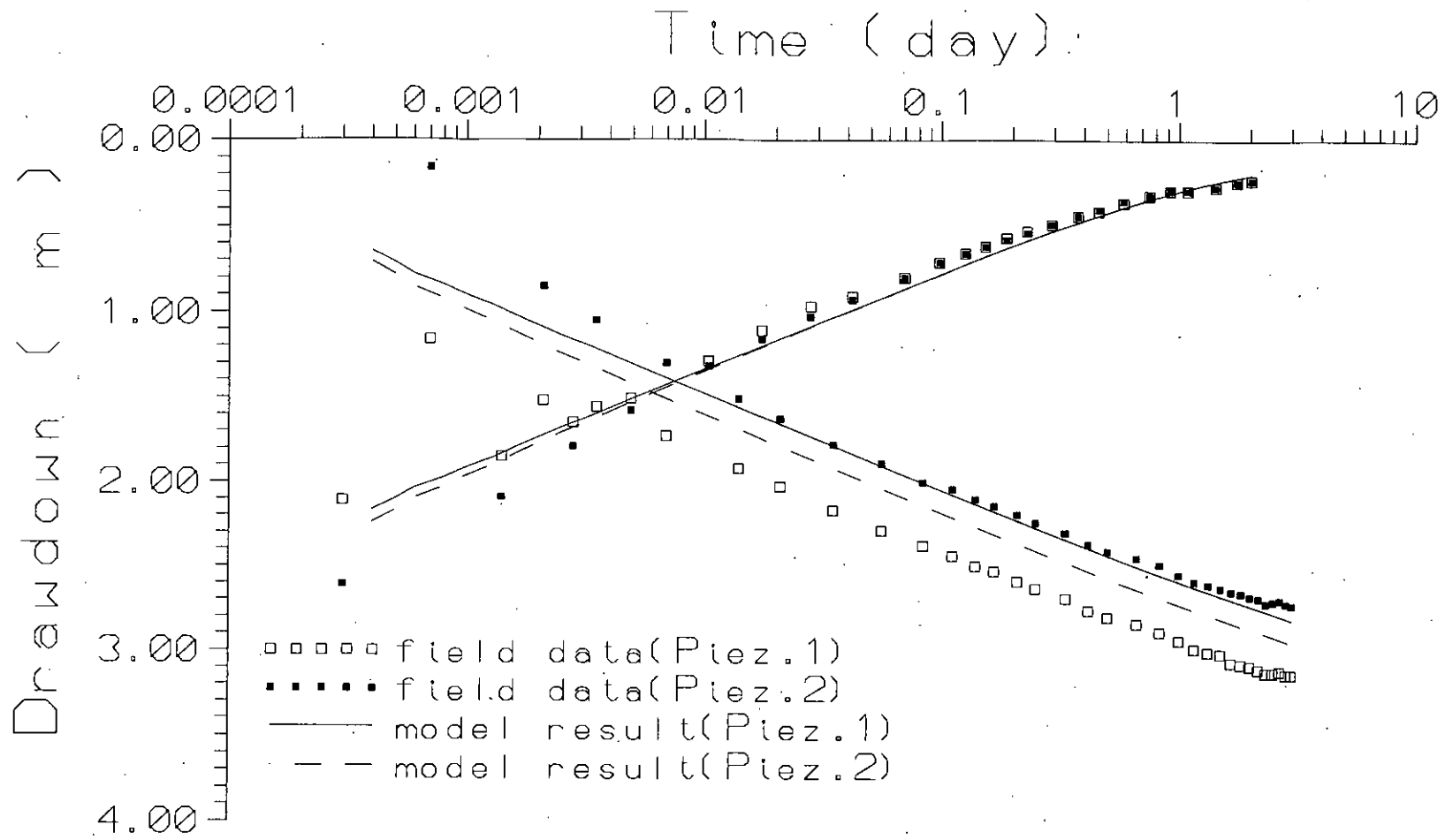


Fig. 4.3 Comparison between field and modelled drawdowns in piezometers 1 and 2 for pumping test: KAP/2

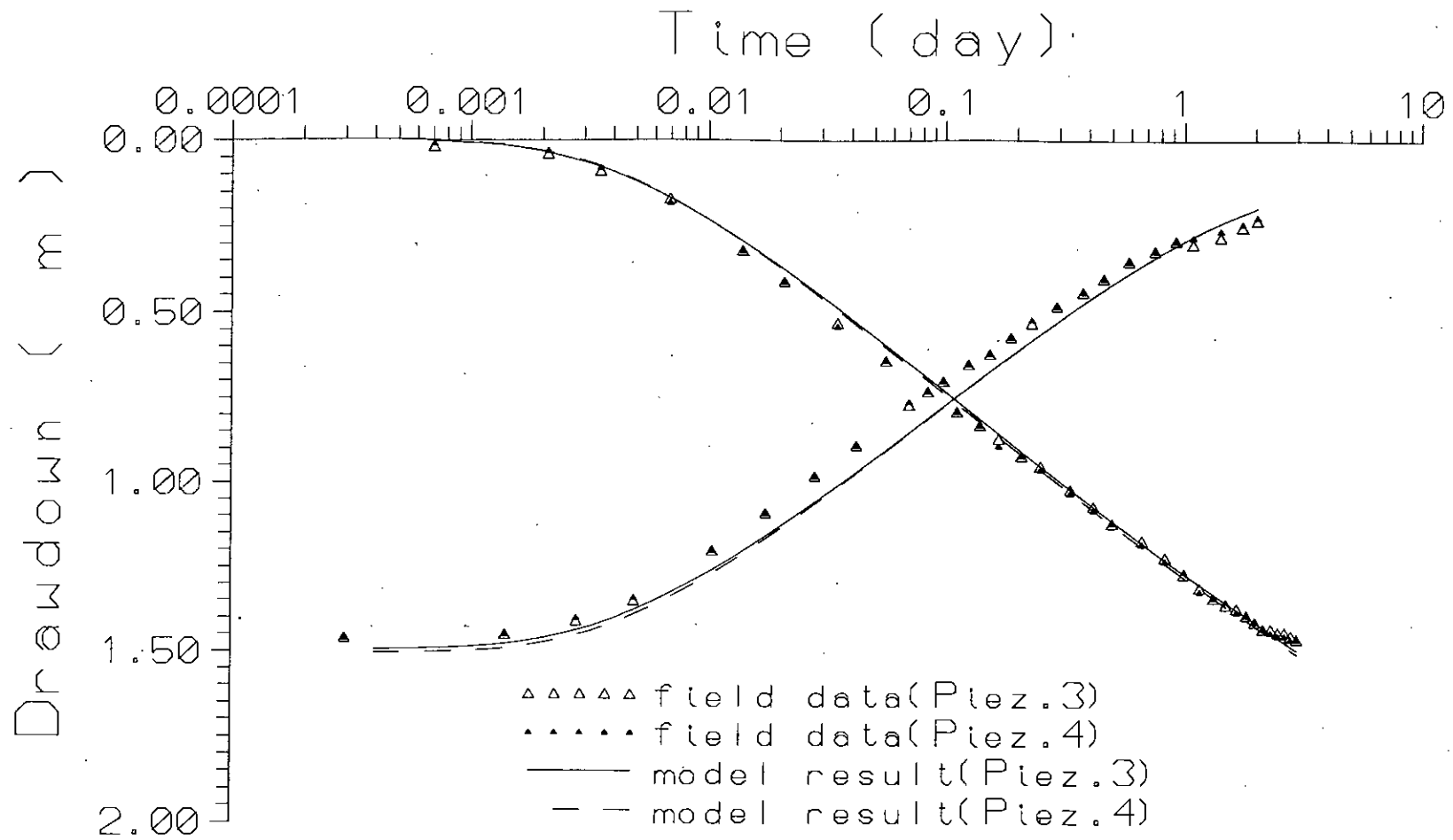


Fig. 4.4 Comparison between field and modelled drawdowns in piezometers 3 and 4 for pumping test: KAP/2

effect of well storage. Afterwards, however, the drawdown curves are similar to those of the pumped well and piezometers 1 and 2. It is noted from the figure that piezometers 3 and 4 registered almost equal drawdowns during the test indicating that the flow was essentially horizontal at this location. The model results match very well with the field data for these piezometers both for pumping and recovery phases. The aquifer parameters deduced from this simulation are summarized in Table 4.1.

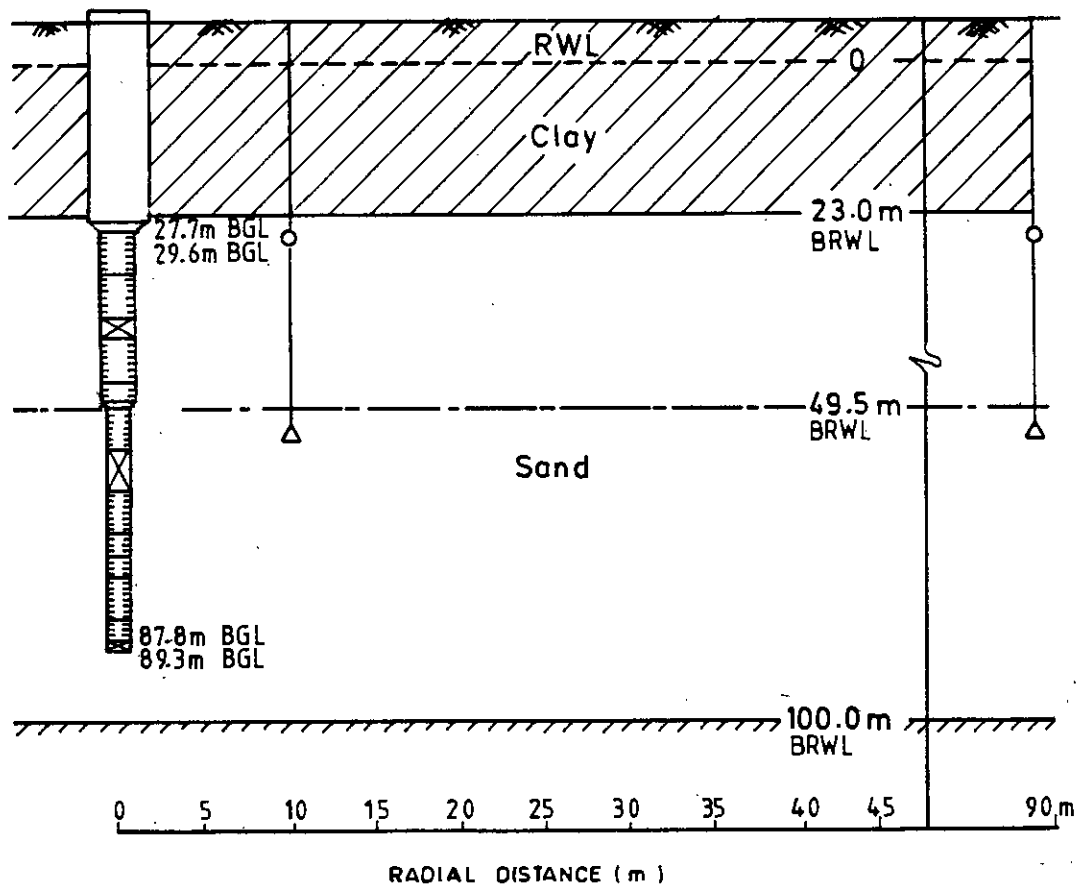
Table 4.1 Parameters deduced from numerical analysis of pumping test : KAP/2

Zone or Layer	Depth, m below RWL	Hydraulic conductivity, m/d		Storage coefficient	Specific yield
		Radial	Vertical		
Overlying	0-18.4		0.005		0.03
Upper	18.4-62.0	18.5	12.0	0.0009	0.15
Middle	62.0-65.0		1.0		
Lower	65.0-103.1	18.5	12.0	0.0009	0.15

4.1.2 Pumping Test : KAP/3

This test was conducted in November 1988 using the experimental well KAP/3 drilled in May 1988. Figure 4.5 shows the arrangement of the well and piezometers used for the test. The experimental well was drilled to a depth of 89.3 m BGL. The log of the well indicates that the aquifer consisting of sand is overlain by a 27.7 m thick clay layer. The diameter of the upper solid casing is 350 mm and the slotted casing has been placed between 29.6 and 87.8 m. The slotted casing has been designed to have two diameters being 200 and 150 mm and it contains two solid sections. Two pairs of shallow and deep piezometers were installed at radial distances of 10 and 90 m from the well, Figure 4.5.

The test was carried out with a constant discharge of 4882 m³/day and the water levels were measured in the pumped well and piezometers 1 to 4 both during the pumping and recovery phases. The durations of the pumping and recovery phases were 1.83 and 1.92 days respectively. The rest water level prior to pumping was 4.77 m below the reference level. The drawdowns recorded in the pumped well and piezometers were plotted in Figures 4.6 to 4.8.



BGL : Below Ground Level

BRWL: Below Rest Water Level

Fig. 4.5 Details of well and observation piezometers used for pumping test: KAP/3

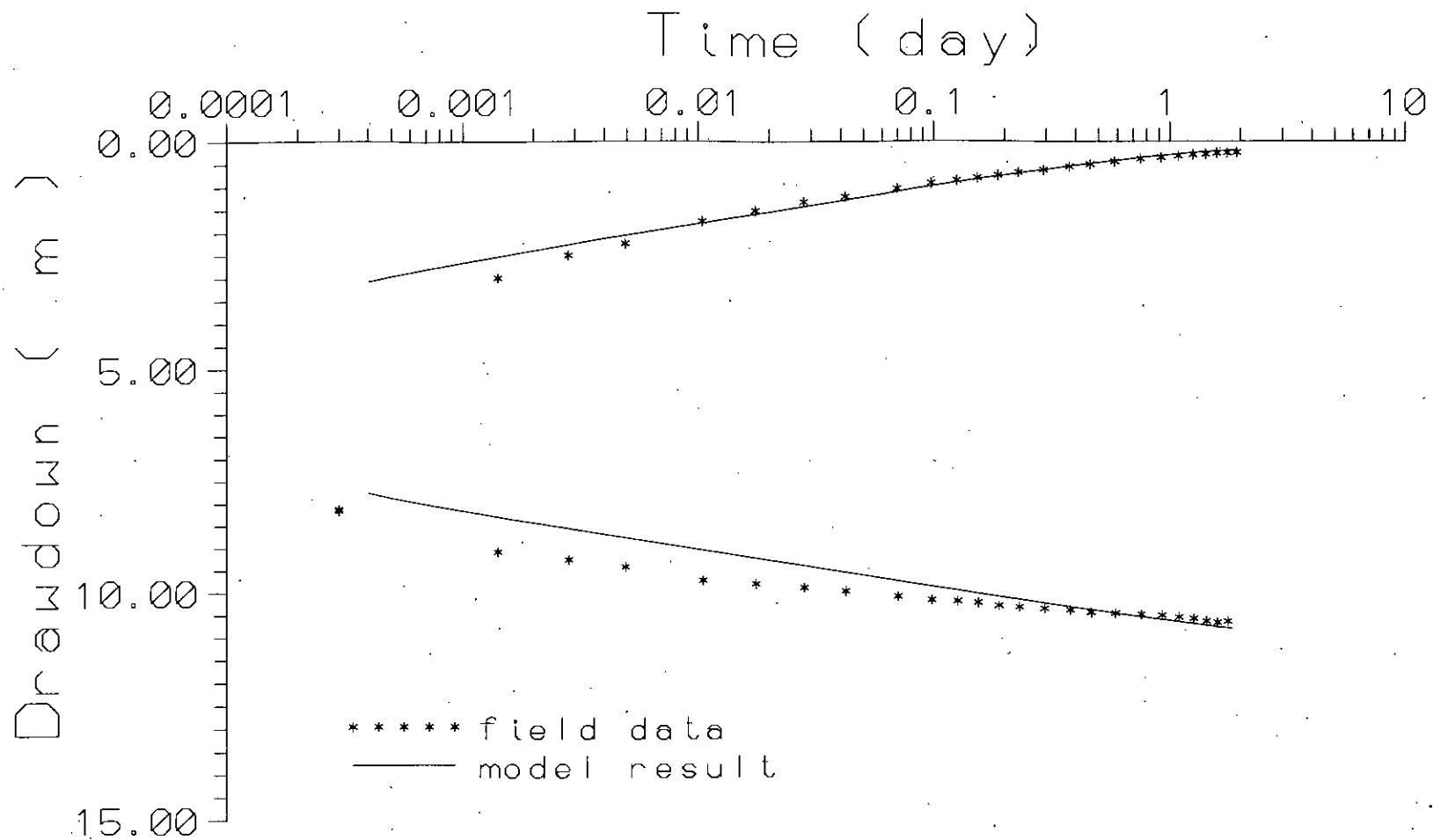


Fig. 4.6 Comparison between field and modelled drawdowns in the pumped well for pumping test: KAP/3

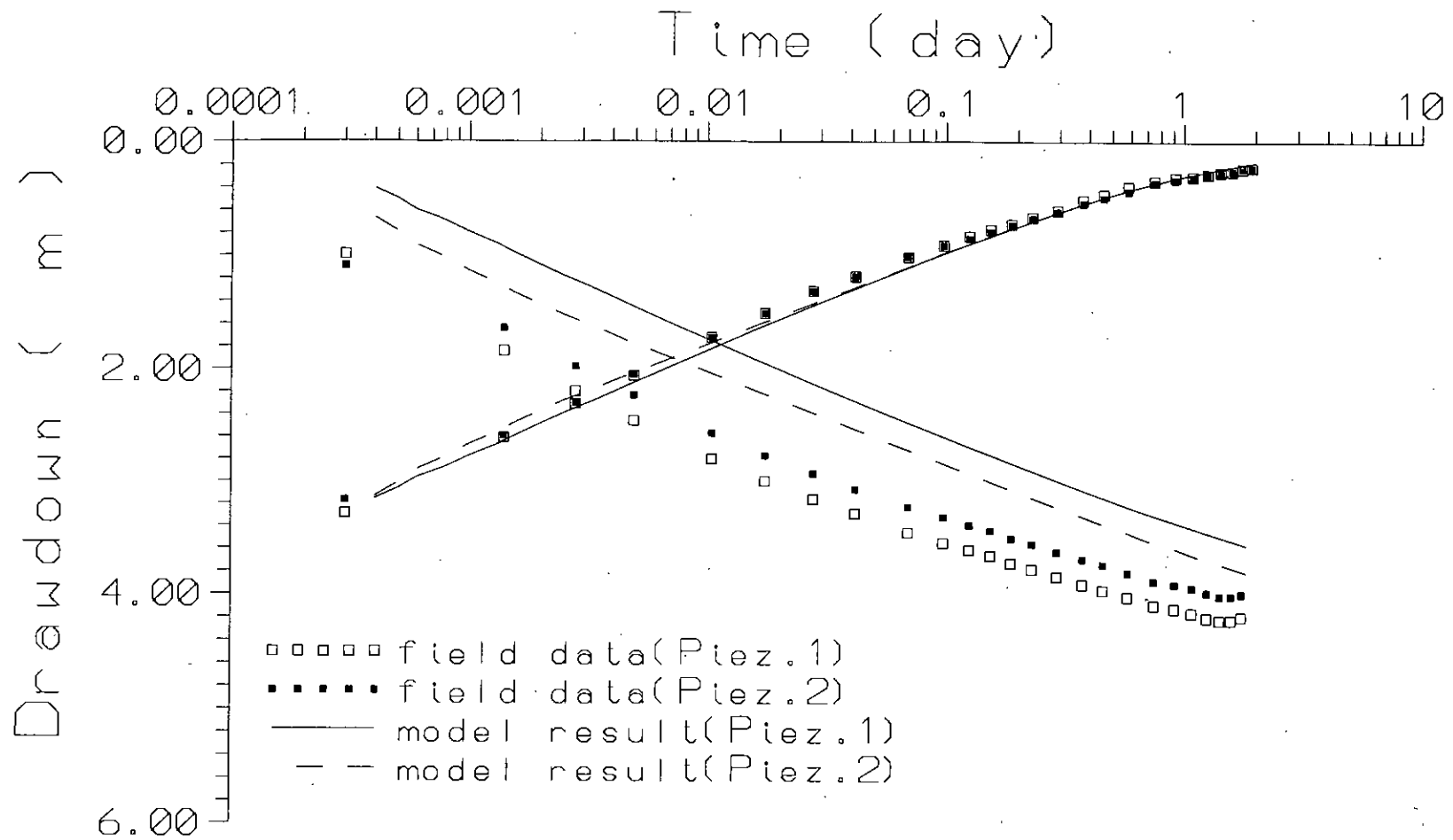


Fig. 4.7 Comparison between field and modelled drawdowns in piezometers 1 and 2 for pumping test: KAP/3

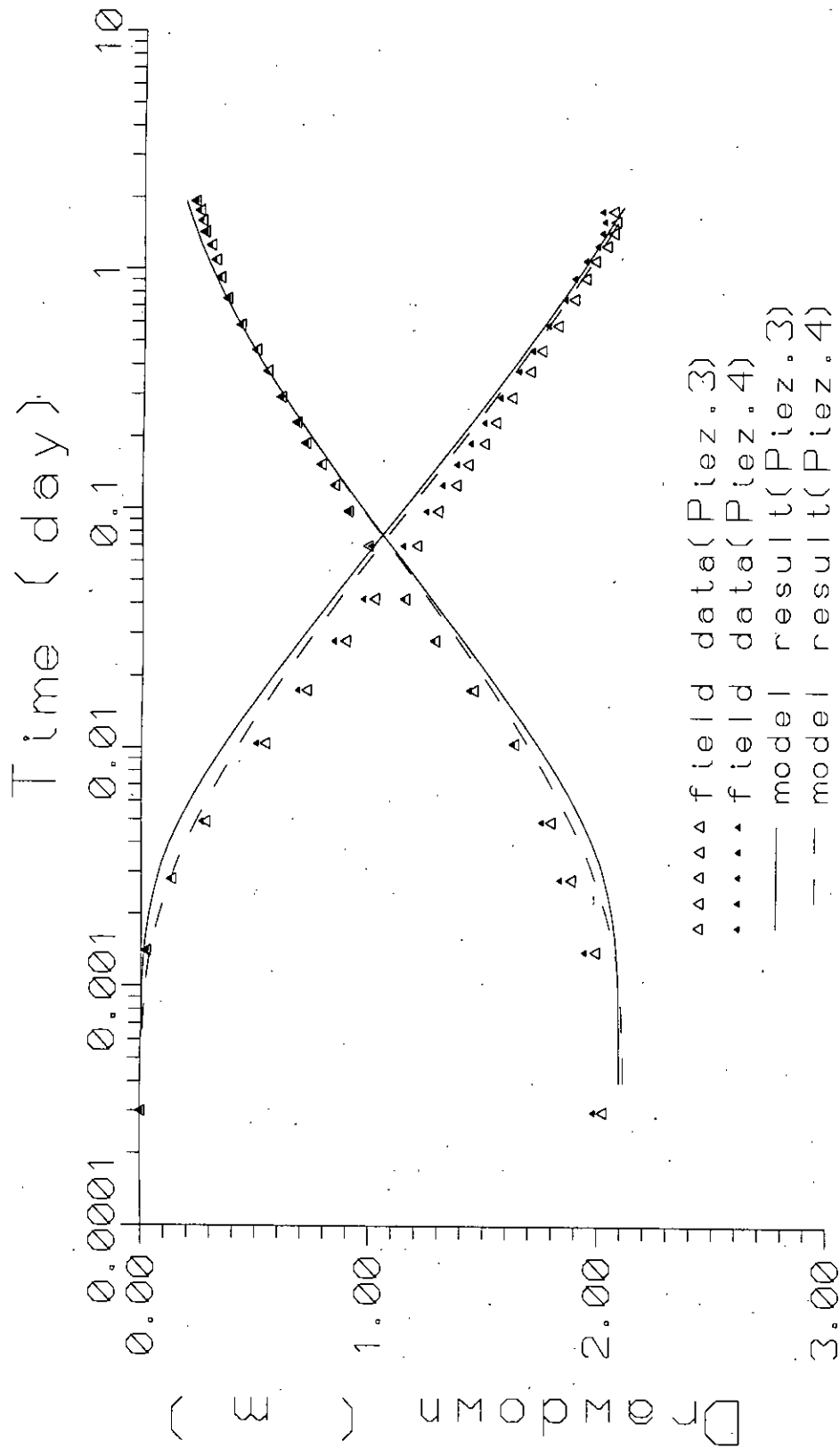


Fig. 4.8 Comparison between field and modelled drawdowns in piezometers 3 and 4 for pumping test: KAP/3

The two zone model has been used to simulate the drawdowns recorded in the well and piezometers. There is no distinctive division within the aquifer unit, and therefore an arbitrary horizontal division has been defined at a depth of 49.5 m BGL. The well radius was taken as 0.09 m and a no flow boundary condition has been specified at a radial distance of 3 km. The computer program was run several times with trial values of parameters of the aquifer until a good match between the modelled drawdowns and field data was obtained. The model results were also presented in Figures 4.6 to 4.8.

Figure 4.6 shows that a drawdown of about 8.2 m was recorded in the pumped well after 0.5 minute since the start of pumping. Following this, the drawdown in the well increased steadily during the pumping phase. The recovery curve seems to be the mirror image of the drawdown curve. A good match between the modelled drawdowns and field values could be obtained for the recovery phase. However, the agreement between the model results and field data is less satisfactory for the first few hours during the pumping phase. This might be because of changing conditions in the vicinity of the well. Nevertheless, the matching is quite well after about 0.25 day of pumping. Well loss factors were found to be 18.0 and 12.0 for the upper and lower zones.

Figure 4.7 indicates that the water levels in piezometers 1 and 2 dropped by about 1 m immediately after the start of pumping. The rate of fall of water levels in these piezometers decreased with time and tended to level off towards the end of the pumping phase. The water levels in piezometer 2 remained higher than that in piezometer 1 indicating a vertical flow at this location. The model could simulate the drawdowns in these piezometers very well for the recovery phase. However, the match between the model results and field values is less satisfactory for the pumping phase. The possible reason for such disagreement has been discussed in the previous section.

Figure 4.8 shows that responses in piezometers 3 and 4 are rather slow registering no measurable drawdowns during the first few minutes of pumping. Following this, the drawdowns increased steadily except near the end of the pumping phase when the water levels tended to level off. Both these features are reproduced in the model results. The agreement between the model results and field data is also very good for the recovery phase. The aquifer parameters derived from this simulation are given in Table 4.2.

Table 4.2 Parameters deduced from numerical analysis of pumping test : KAP/3

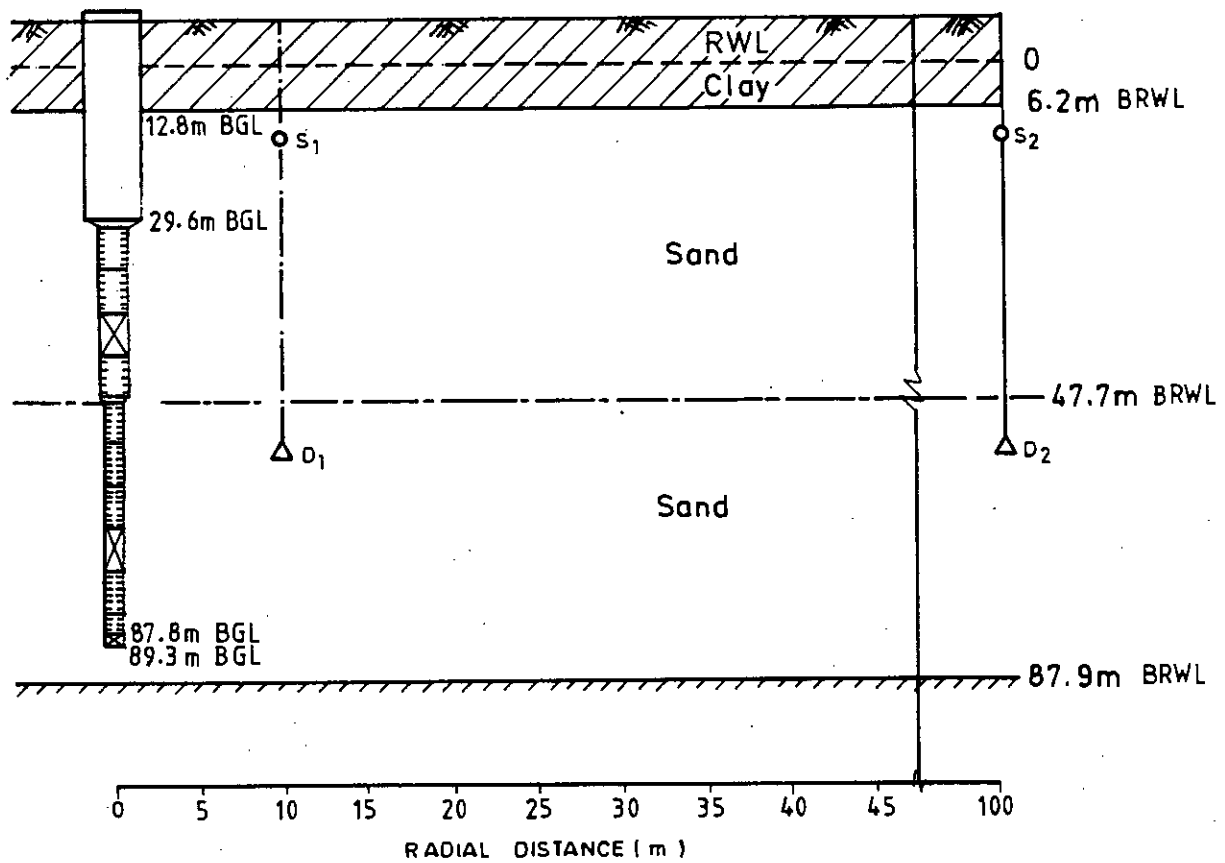
Zone or Layer	Depth, m below RWL	Hydraulic conductivity, m/d		Storage coefficient	Specific yield
		Radial	Vertical		
Overlying	0-23.0		0.006		0.03
Upper	23.0-49.5	13.7	6.0	0.0006	0.15
Lower	49.5-100.0	13.7	6.0	0.0006	0.15

4.1.3 Pumping Test : KAP/13

This test was conducted on 22 February 1989 and the test well was drilled on 8 February 1989. Figure 4.9 shows the positions of the well and piezometers used for the test. The well was drilled to a depth of 89.3 m BGL. The log of the well indicates that the aquifer consists of sand which is overlain by a 12.8 m thick clay layer. The upper solid casing is of 350 mm in diameter and the lower slotted casing lies between 29.6 and 87.8 m BGL. The slotted casing is of two different sizes - 200 and 150 mm in diameters and contains two solid sections. Two pairs of shallow and deep piezometers have been installed at radial distances of 10 and 100 m from the well, Figure 4.9.

The rate of abstraction during the test was 5141 m³/day and the water levels were monitored in the pumped well and piezometers both during the pumping and recovery phases. The durations of pumping and recovery phases were 3.0 and 2.0 days respectively. The rest water level before the commencement of the test was 6.6 m below the reference level. Figures 4.10 to 4.12 show the responses recorded in the pumped well and piezometers.

The two zone model has been applied to analyse the pumping test data. As evident from Figure 4.9, there is no distinct division within the aquifer unit. Therefore, an arbitrary division has been made at a depth of 47.7 m BGL. A no flow boundary condition has been defined at a distance of 3 km; the well radius was taken as 0.09 m. After several runs of the program with trial values of the parameters a close agreement was reached between the model results and field data recorded in the well and piezometers during the pumping and recovery phases. Figure 4.10 to 4.12 also contain the model results.



BGL : Below Ground Level .

BRWL: Below Rest Water Level

Fig. 4.9 Details of well and observation piezometers used for pumping test: KAP/13

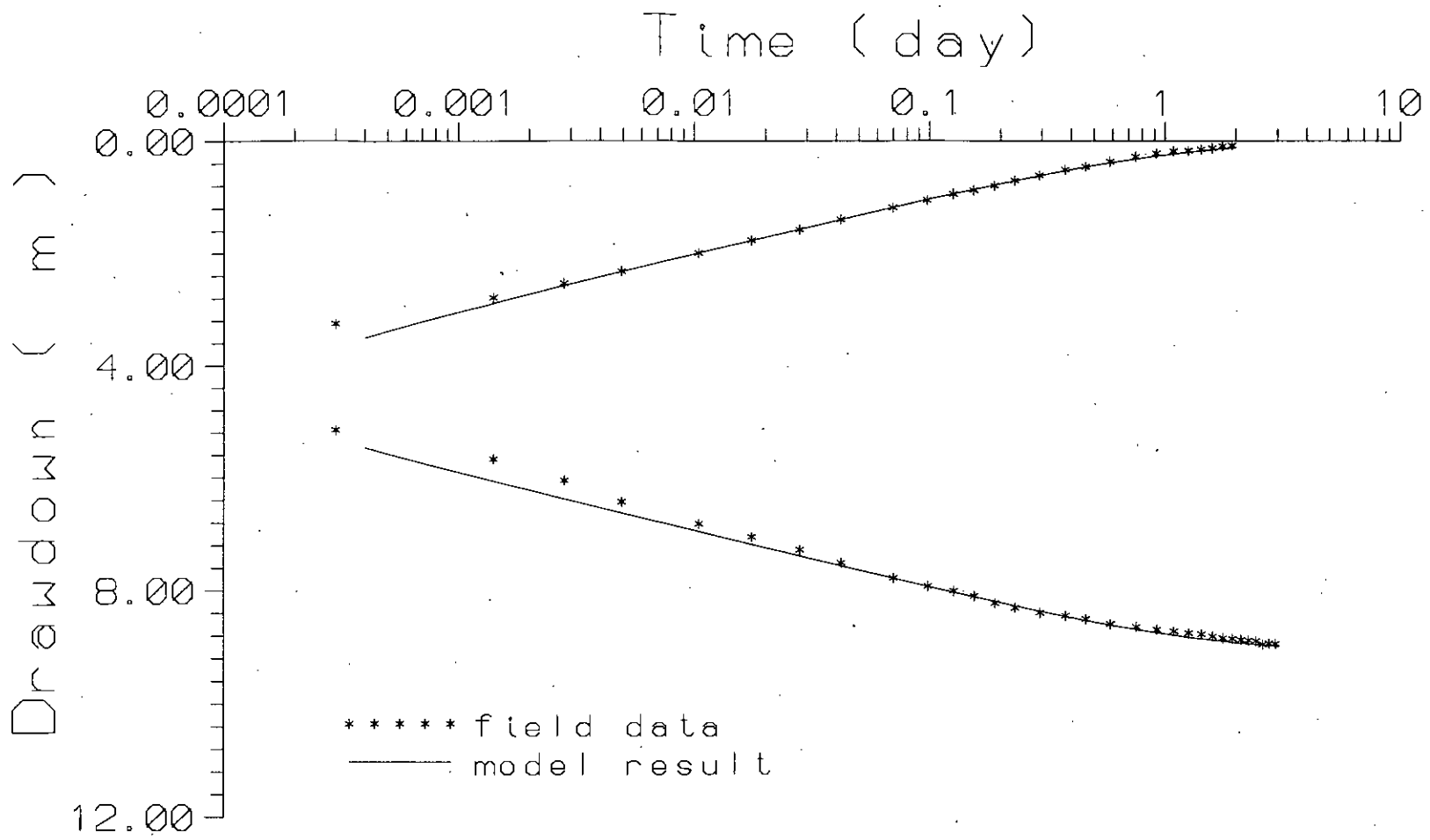


Fig.4.10 Comparison between field and modelled drawdowns in the pumped well for pumping test: KAP/13

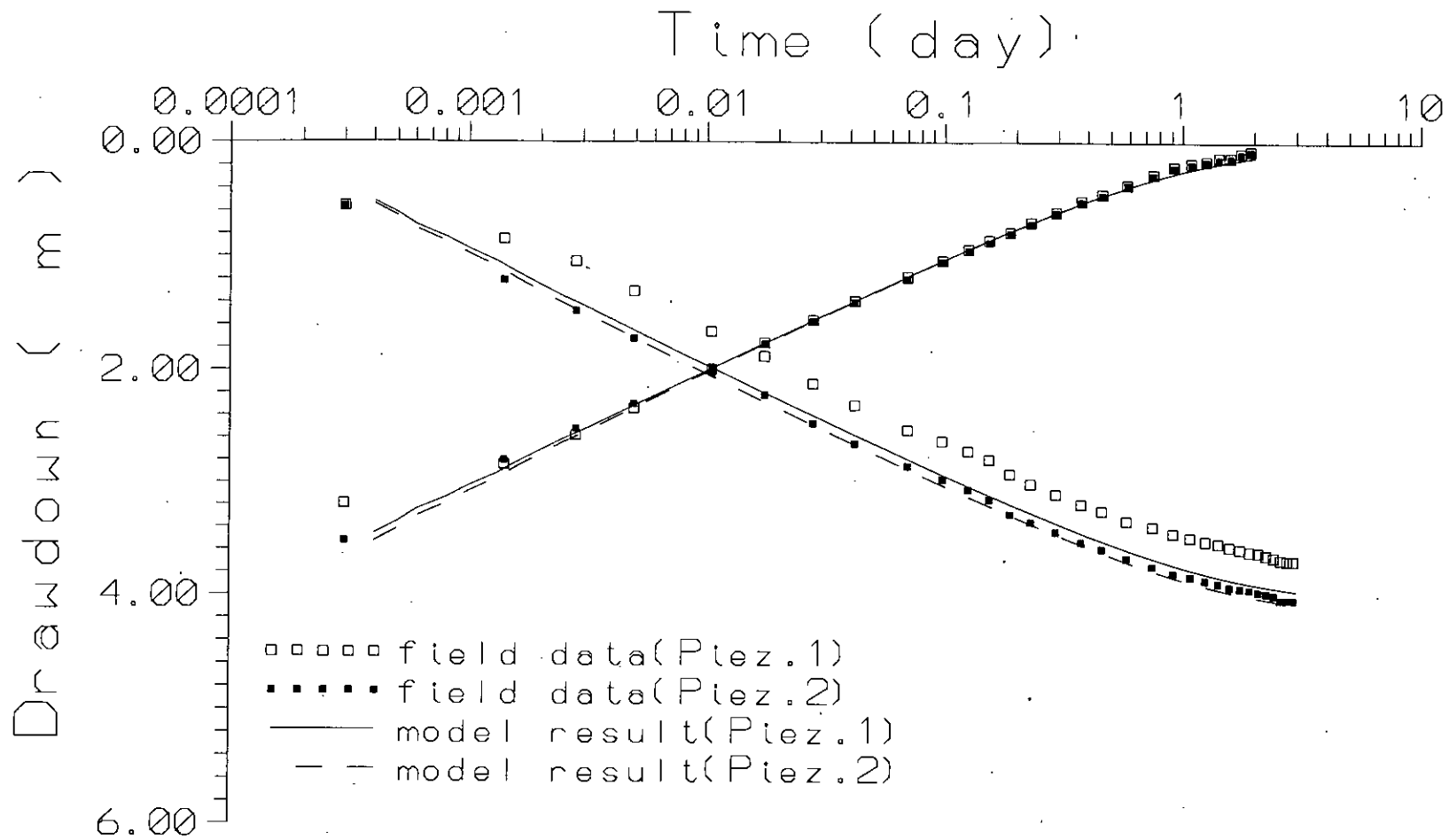


Fig. 4.11 Comparison between field and modelled drawdowns in piezometers 1 and 2 for pumping test: KAP/13

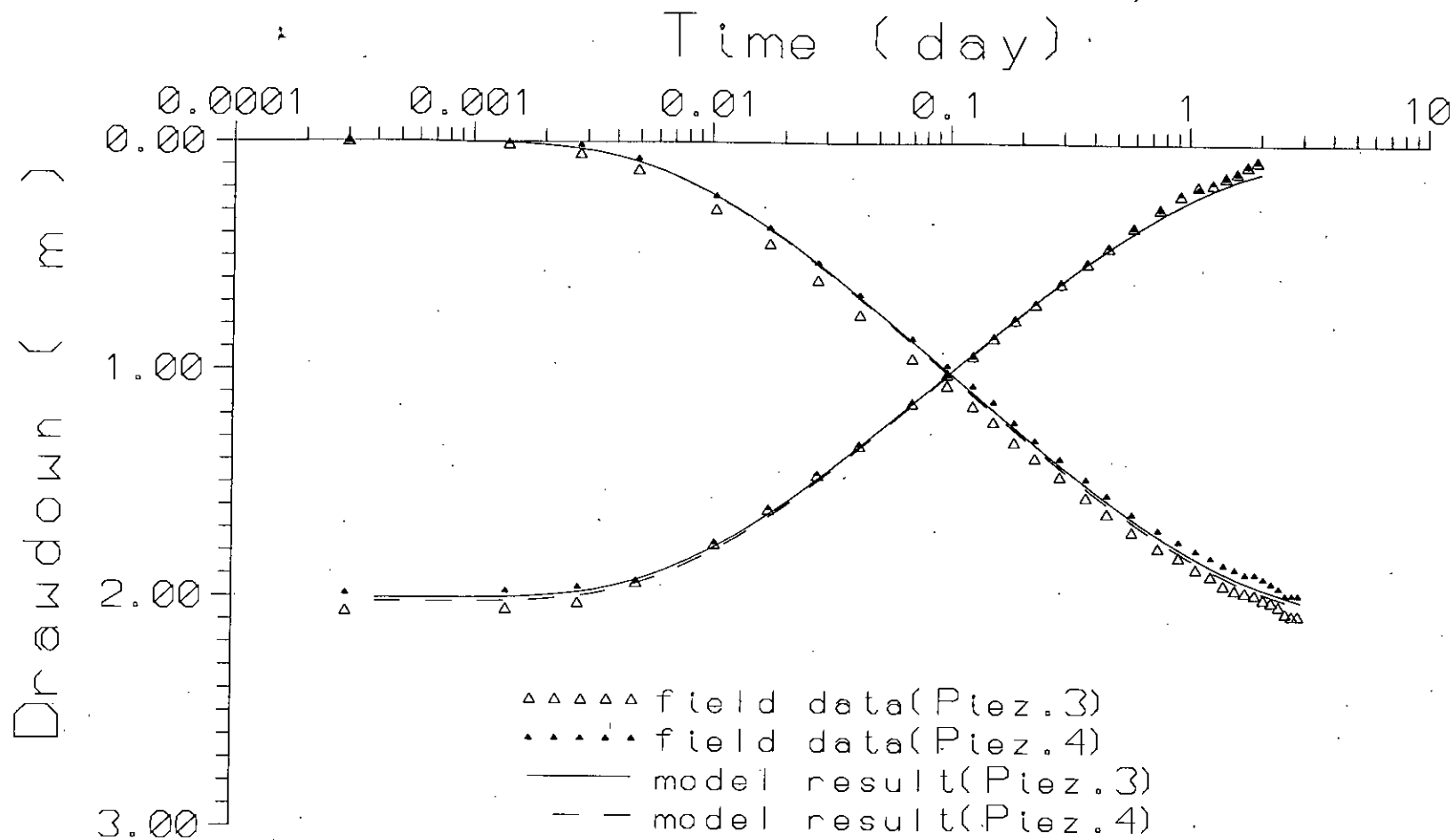


Fig. 4.12 Comparison between field and modelled drawdowns in piezometers 3 and 4 for pumping test: KAP/13

As soon as pumping started, water level in the well dropped by more than 5 m (Figure 4.10). After this, it declined steadily and the water level tended to level off towards the end of the pumping phase. The water level recovered to almost rest level within 2 days. The recovery curve is a mirror image of the drawdown curve. The model results fit very well with the field data both for the pumping and recovery phases. However, well loss factors of 4.0 and 3.0 were required for the upper and lower zones respectively to achieve this matching.

The responses recorded in piezometers 1 and 2 are quite similar to that of the pumped well. Both the piezometers showed a drawdown of about 0.6 m after 0.5 minute since the start of pumping. Afterwards, however, the groundwater head in piezometer 1 remained higher than that in piezometer 2. This indicates that vertical flow occurred near the well. The model could reproduced the responses very well except for the drawdowns recorded in piezometer 1 during the pumping phase. A possible reason for this discrepancy has been discussed earlier.

The responses in piezometers 3 and 4 are rather slow showing little change in groundwater head during the first few minutes. This is because of the effect of well storage. After this, however, the shape of the drawdown curve is similar to that of the pumped well. The model results match very well with the field data recorded in these piezometers during the pumping and recovery phases. The parameter values derived from this analysis are given in Table 4.3.

Table 4.3 Parameters deduced from numerical analysis of pumping test: KAP/13

Zone or Layer	Depth, m below RWL	Hydraulic conductivity, m/d		Storage coefficient	Specific yield
		Radial	Vertical		
Overlying	0-6.2		0.007		0.03
Upper	6.2-47.7 or DUP) -47.7	11.5	6.0	0.001	0.15
Lower	47.7-87.9	11.5	6.0	0.001	0.15

4.2 Comparison of Results with Analytical Solutions

The values of the aquifer properties derived from the analysis using the two zone numerical model were compared with those obtained by using Jacob and Walton methods. Table 4.4 contains the values of transmissivity and storage coefficient for the three test sites ascertained by using different methods. It is seen from the table that the values of the transmissivity derived from the numerical analysis are quite close to those obtained by using Walton method. But the value of storage coefficient ascertained by using numerical method are considerably less than those calculated by using Walton method. Both Jacob and Walton methods ignore some of the important features such as well storage, vertical flow, etc. which operate under the influence of pumping. The numerical model, on the other hand, takes into consideration all these features. Furthermore, in Jacob and Walton methods of analysis, only the data recorded in piezometer 4 during the pumping phase have been used. However, in the application of numerical model all the data recorded in the well and piezometers both during pumping and recovery phases have been used. Therefore, the results deduced by using the numerical method seem to be superior to those obtained by analytical methods.

Table 4.4 Transmissivity and storage coefficient for test sites KAP/2, KAP/3 and KAP/13 calculated by different methods

Method of analysis	KAP/2		KAP/3		KAP/13	
	Transmissivity (m ² /day)	Storage coeff.	Transmissivity (m ² /day)	Storage coeff.	Transmissivity (m ² /day)	Storage coeff.
Jacob	1740	0.0012	1238	0.0007	1203	0.0015
Walton	1623	0.0018	1034	0.0011	1083	0.0021
Two zone model	1510	0.0009	1055	0.0006	940	0.001

4.3 Long-term Pumping Effect

Having obtained a reasonable match for pumping test, the model was used to predict the response at each test site over a typical growing season of 120 days. Each day the pumping cycle is 4892 m³/day for 12 hours and recovery for 12 hours. A well spacing of 1 km x 1 km was used; this was represented as an impermeable boundary at a distance of 564 m. Figures 4.13 to 4.15 show the predicted drawdowns in the pumped well. Significant drawdowns occur in the wells. After 120 days of pumping the predicted drawdowns in the pumped wells KAP/2, KAP/3 and KAP/13 are 19.8, 24.8 and 13.6 m respectively. This variation in predicted drawdowns may be attributable to difference in aquifer properties and well losses. Figure 4.15 shows that there is a change in the shape of drawdown curve after 73 days. This is because the leakage becomes zero as the water level in the overlying layer reaches the base of this layer after about 73 days of pumping. Since these long-term predictions are based on a model which has been verified only for a relatively short period they should be treated with caution. However the results provide an indication of the likely trends which are quite useful in planning a sustainable groundwater development project.

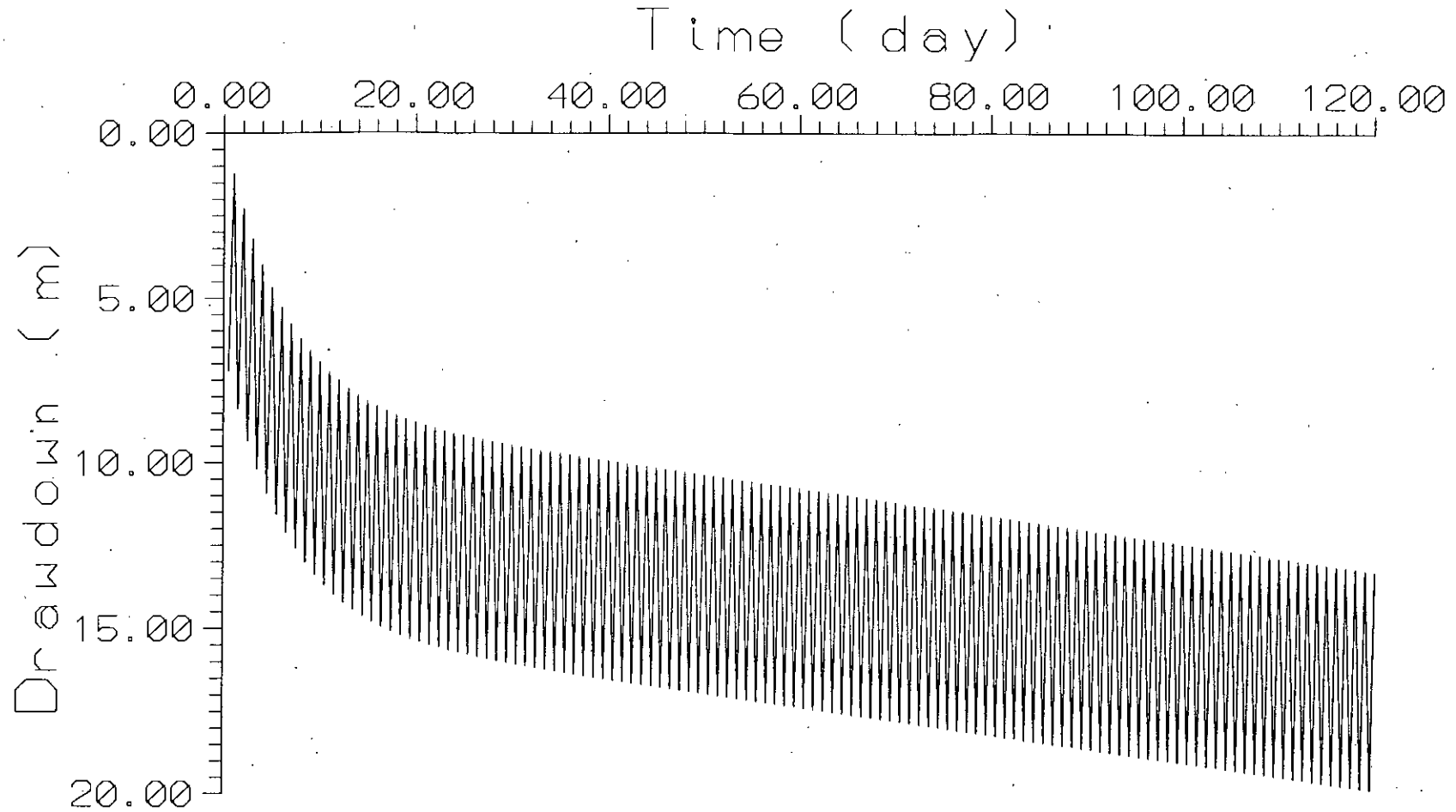


Fig. 4.13. Predicted drawdowns in pumped well KAP/2 for pumping cycle repeated for 120 days

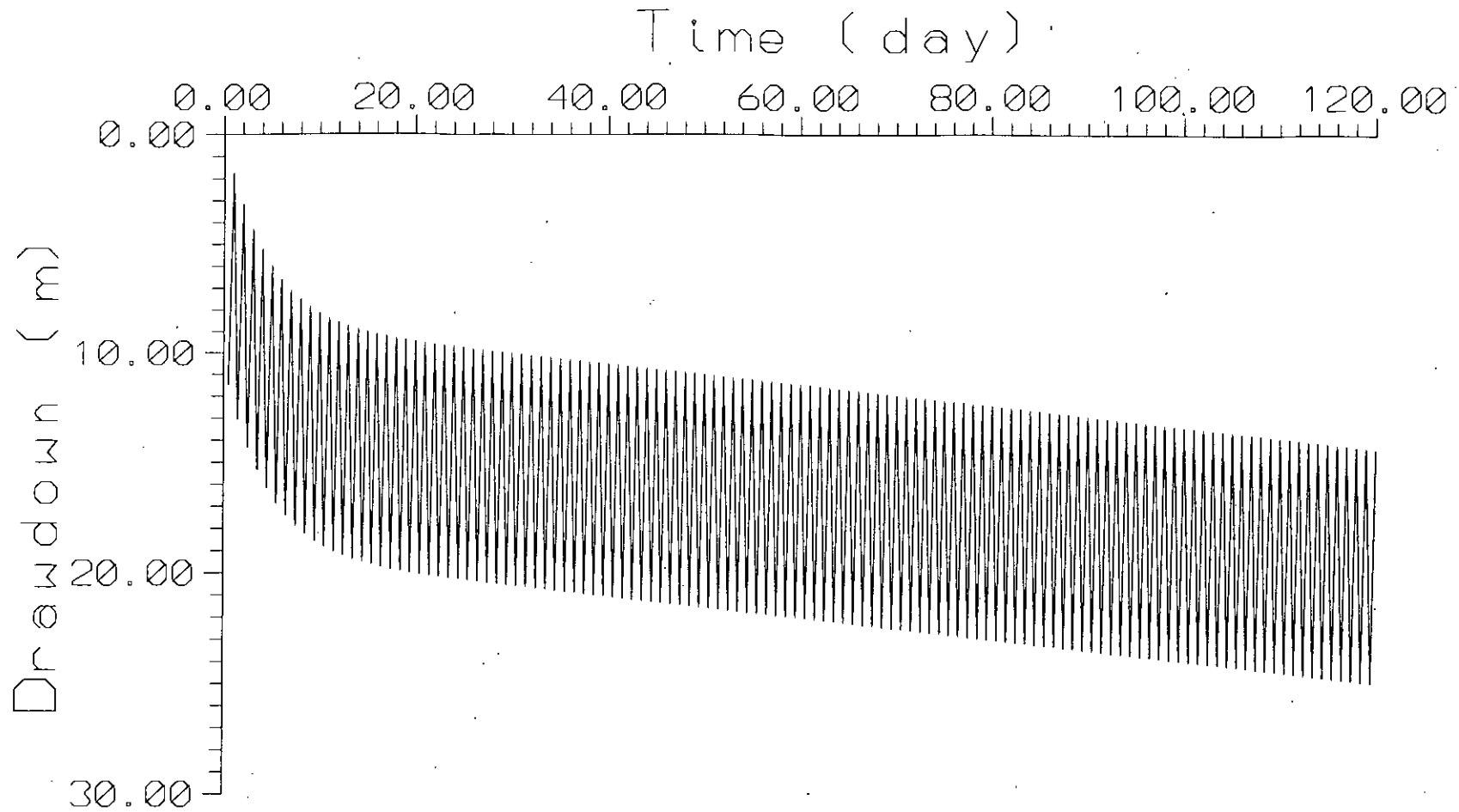


Fig. 4.14. Predicted drawdowns in pumped well KAP/3 for pumping cycle repeated for 120 days

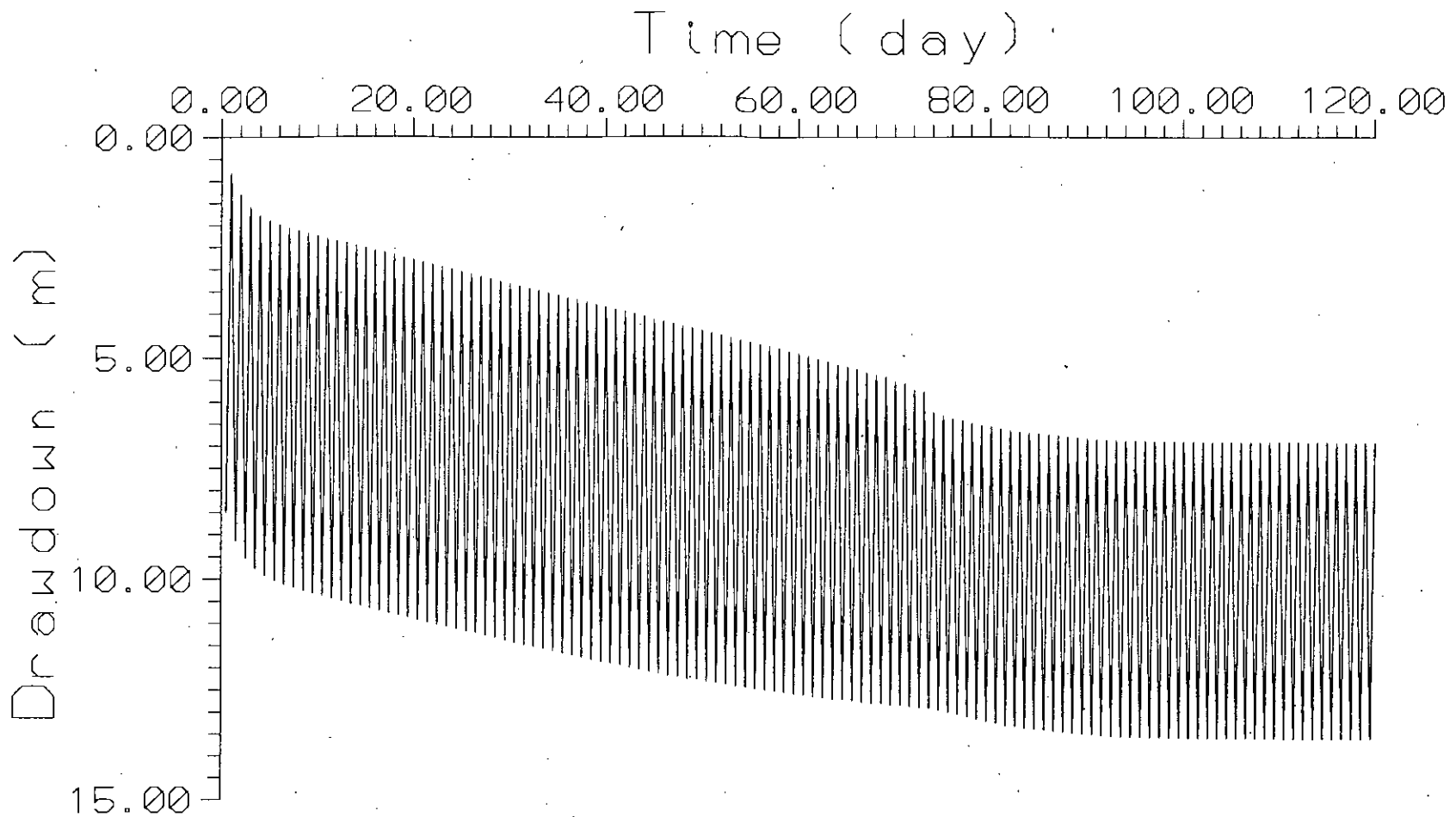


Fig. 4.15. Predicted drawdowns in pumped well KAP/13 for pumping cycle repeated for 120 days

Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The two zone numerical model developed by Rathod and Rushton (1991) has been modified and applied to interpret three pumping tests carried out in a leaky layered aquifer in Bangladesh. The model could reproduce the features of the response and the overall agreement between the observed drawdowns and model results was very good.

The leakage from the overlying layer constitutes a significant part of the discharge during pumping. This results in an appreciable fall in water table in the overlying layer. Therefore, negligence of this fall in water table in predicting the response of the aquifer to pumping over a long period may lead to erroneous results.

Significant vertical components of flow occurs in the vicinity of the abstraction well. This is mainly because of layering of the aquifer, partial penetration and alternating slotted and solid casing of the well. Most standard analytical methods of pumping test analysis do not take these vertical components of flow into consideration. However, the two zone model includes these effects.

High values of well loss factors required to calibrate the model for two out of three tests suggest that considerable head loss occurs near the well. This may be because of combined effects of deterioration of aquifer condition near the well and partial penetration. Since the well losses constitute a significant proportion of pumping head, efforts should be made to improve the design and installation of well to minimize these losses.

Field results show that well storage has significant effect on response during the first few minutes of the pumping period. This feature has been reproduced in the numerical model quite satisfactorily.

The values of transmissivity and storage coefficient, as deduced from the numerical analysis, for the three test sites ranged from 940 to 1510 m²/day and 0.6 x 10⁻³ to 1.0 x 10⁻³ respectively. The vertical hydraulic conductivity of the overlying layer was found to vary from 0.005 to 0.007 m/day.

The model has been used to predict the response of the aquifer to pumping over a typical growing season. The results indicated significant drawdowns both at the pumped well and outer boundary. Unless there is sufficient recharge during the monsoon the groundwater heads will fall year by year. Since these predictions were made based on a model which had been verified only for a short period, the results should be treated only as an indication of the likely trend.

The two zone model gives drawdowns at two levels and the vertical flow between these levels is lumped across a single vertical resistance. A detailed description of the vertical flow mechanism is, therefore, not possible. A model having detailed mesh in both radial and vertical directions may be used for this purpose. However, there may not be sufficient field data to justify the use of these detailed models for pumping test analysis.

5.2 Recommendations

Model results show that there is appreciable fall in water table in the overlying layer during pumping; but these results could not be verified due to lack of field data. Therefore, pumping test should be carried out with a few piezometers installed in the overlying layer in order to check the effect of the pumping on the water table.

Results of the analysis indicate that considerable head loss occur near the face of the test wells; the cause of such high well losses should be investigated.

Groundwater heads should be monitored during the pumping season at the pumped wells and at few observation piezometers. This will provide a basis for verification of the model results.

The consequences of several annual cycles of pumping and recharge should be examined for sustainable groundwater resources development in the study area. This will require to estimate the rate of recharge to the water table.

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REFERENCES

- Boulton, N.S., 1963. Analysis of data from non-equilibrium pumping tests allowing for delayed yield from storage. *Proc. Inst. Civ. Eng.*, 26, pp 469-482.
- Copper, H.H. and C.E. Jacob, 1946. A generalized graphical method for evaluating formation constants and summarizing well field history. *Trans. Amer. Geophys. Union*, 27, pp 526-534.
- Devies, J. and C. Exley, 1992. Short-term BGS pilot project to assess the hydrochemical character of the main aquifer units of central and northeastern Bangladesh and possible toxicity of groundwater to fish and humans. Final Report, British Geological Survey Technical Report WD/92/43R.
- Grout, M.W., 1988. Numerical model for interpreting complex pumping test response. Ph.D. Thesis, University of Birmingham, UK. (unpublished).
- Hantush, M.S., 1956. Analysis of data from pumping tests in leaky aquifers. *Trans. Amer. Geophys. Union*, 37, pp 702-714.
- Hantush, M.S., 1960. Modification of the theory of leaky aquifers. *J. Geophys. Res.*, 65, pp 3713-3725.
- Hantush, M.S., 1964. *Hydraulics of wells*, *Advances in Hydroscience*, 1, pp 281-432, Academic Press, New York.
- Hantush, M.S. and C.E. Jacob, 1955. Non-steady radial flow in an infinite leaky aquifer. *Trans. Amer. Geophys. Union*, 36(1), pp 95-100.
- Jacob, C.E., 1946. Radial flow in a leaky artesian aquifer. *Trans. Amer. Geophys. Union*, 27(2), pp 198-208.

- Krosznski, U.I. and G. Dagan, 1975. Well pumping in unconfined aquifers: The influence of unsaturated zone. *Wat. Resour. Res.*, 11(3), pp 479-490.
- Kruseman, G.P. and N.A. Ridder, 1994. *Analysis and Evaluation of Pumping Test Data*. Intern. Inst. Land Reclam. and Improv. (ILRI), Wageningen, the Netherlands, Publication 47, 377 pp.
- Mott MacDonald International Limited, 1990. Report on exploratory drilling in Kapasia Upazila, Working Paper 53, IDA Deep Tubewell II Project, Bangladesh Agricultural Development Corporation, Ministry of Agriculture, Government of Bangladesh.
- MPO, 1990. National water plan, Phase II. Final Report, Master Plan Organization, Ministry of Irrigation, Water Development and Flood Control, Government of Bangladesh.
- Neuman, S.P., 1972. Theory of flow in unconfined aquifers considering delayed response of the water table. *Wat. Resour. Res.* 8(4), pp 1031-1045.
- Neuman, S.P., 1975. Analysis of pumping test data from anisotropic unconfined aquifers considering delayed gravity response *Wat. Resour. Res.* 11(2), pp 329-342.
- Rathod, K.S. and K.R. Rushton, 1984. Numerical method of pumping test analysis using microcomputers. *Ground Water*, 22(5), pp 602-608.
- Rathod, K.S. and K.R. Rushton, 1991. Interpretation of pumping from two-zone layered aquifers using a numerical model. *Ground Water*, 29(4) pp 499-509.
- Rushton, K.R., 1983. Aquifer properties in the Wadi Bana well field. Report to John Taylor and Sons.
- Rushton, K.R., 1986, *Groundwater Models*, in *Groundwater: Occurrence, Development and Protection*, The Inst. Wat. Eng. Scient., London, pp 189-228.

- Rushton, K.R. and S.J. Booth, 1976, Pumping test analysis using a discrete time-discrete space numerical method. *J. Hydrol.*, 28, pp 13-27.
- Rushton, K.R. and Y.K. Chan, 1976a. A numerical model for pumping test analysis. *Proc. Inst. Civ. Eng.*, 61, pp 281-296.
- Rushton, K.R. and Y.K. Chan, 1976b, Pumping test analysis when parameters vary with depth. *Ground Water*, 14(2), pp 82-87.
- Rushton, K.R. and K.W.F. Howard, 1982. The unreliability of open observation boreholes in unconfined aquifer pumping tests. *Ground Water*, 20(5), pp 546-550.
- Rushton, K.R. and S.C. Redshaw, 1979. *Seepage and Groundwater Flow*, John Wiley, Chichester, 332 pp.
- Rushton, K.R. and N.K. Srivastava, 1988. Interpreting injection well tests in an alluvial aquifer. *J. Hydrol.*, 99, pp 49-60.
- Rushton, K.R. and J. Weller, 1985. Response to pumping of a weathered-fractured granite aquifer. *J. Hydrol.*, 80, pp 299-309.
- Streltsova, T.D., 1973. Flow near a pumped well in an unconfined aquifer under nonsteady conditions. *Wat. Resour. Res.*, 9(1), pp 227-235.
- Theis, C.V., 1935. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage. *Trans. Amer. Geophys. Union*, 16, pp 519-524.
- Walton, W.C., 1962. Selected analytical methods for well and aquifer evaluation. *Illinois State Water Survey Bull.*, 49, 81 pp.
- Walton, W.C., 1970. *Groundwater Resources Evaluation*, McGraw-Hill, New York, 664 pp.

Walton, W.C., 1987. Groundwater Pumping Tests, Design and Analysis, Lewis Publishers, Michigan, 201 pp.

Wenzel, L.K., 1942. Methods of determining permeability of water bearing materials, with special reference to discharge-well methods, U.S. Geol. Surv. Water Supply Paper 887.

Appendix-A Listing of Computer Program

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PROGRAM TWZNPORG
C *****
C * INTERPRETATION OF PUMPING FROM TWO-ZONE LAYERED AQUIFERS USING A *
C * NUMERICAL MODEL. by K. S. Rathod and K. R. Rushton *
C * BIRMINGHAM UNIVERSITY, BIRMINGHAM. B15 2TT (UNITED KINGDOM). *
C * TWO ZONE RADIAL FLOW MODEL WITH VERTICAL FLOW AND LEAKAGE *
C * FORTRAN VERSION *
C * ----- *
C * *
C * THE ORIGINAL PROGRAM HAS BEEN MODIFIED TO ALLOW FOR MOVEMENT *
C * OF WATER TABLE IN THE OVERLYING LAYER AND CALCULATION OF *
C * LEAKAGE CONSIDERING THE POSITION OF WATER TABLE AND GROUND *
C * WATER HEAD. M. Mirjahan Miah , May 1995. *
C *****

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PARAMETER (NA=40,NB=5)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON R(NA), RR(NA), DLO(NA), DUP(NA), OLDDLO(NA), OLDDUP(NA),
1 RECH(NA), PERMRUP(NA), PERMVUP(NA), SCONUP(NA), SUNCNUP(NA),
2 PERMVD(NA), TSUP( NA), PERMRLO(NA), PERMVLO(NA), SCONLO(NA),
3 VL(NA), HUP(NA), HLO(NA), V(NA), TSLO(NA), AUP(NA), BUP(NA),
4 CUP(NA), E(NA), FUP(NA), ALO(NA), BLO(NA), CLO(NA), FLO(NA),
5 U1(NA), U2(NA), V1(NA), V2(NA), RJD(NA), RLN(NA), X(NA), Y(NA),
6 DOBS(NB), DFAC(NB), ARAY(NB), ARRAY(NB), TP(NB), C1, RWL, TME, TIMIN,
7 NMAX, NOB(NB), NMONE, NOBS, OUT1, ESCM, CHAR15, CHAR18, ESC, WL(NA),
8 OLDWL(NA), RECHW(NA), THLEAK(NA), TPUP
CHARACTER ESC*1, CHAR15*1, CHAR18*1, ESCM*2
INTEGER OUT1
IN1=5
OUT1=6
C CALL XUFLOW(0)
OPEN (IN1, FILE='EXAMPLE.DAT')
OPEN (OUT1, FILE='EXAMPLE.OUT')
OPEN (7, FILE='[.GRAPHS]DDNM.DAT')
C WIDTH #2, 160
C INPUT WELL RAD, DIST. TO OUTER BOUND. AND SET LOG RADIAL MESH $$$$$
READ(IN1,*) RWELL, RMAX !<<<<<<<< INPUT
MM=6 ! MM IS NO. OF MESH INTERVALS PER TENFOLD INCREASE IN RAD$$
DO 320 N=1,40
AN = FLOAT(N - 2) / FLOAT(MM)
R(N) = RWELL * (1.0D+01 ** AN)
IF (R(N).LT. RMAX ) GOTO 310
300 R(N) = RMAX
RR(N) = RMAX * RMAX
NMAX = N
NMONE = N - 1
GOTO 330
310 RR(N) = R(N) * R(N)
320 CONTINUE
330 AN = 1.0 / FLOAT(MM) ! CALCULATING(= Da) AND DELA SQUARED $$$$$
DELA = AN * DLOG(1.0D+01)
DELA2 = DELA * DELA

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C
C INITIALISE VARIOUS VARIABLES AND ARRAYS
C KT IS NUMBER OF TIME STEPS PER DECADE
  KT = 10
  AKT = 1 / FLOAT(KT)
  DT = (1.0D+01** AKT) - 1.0
  TME=0.0
  IPRINT = 0
  ITP = 1 ! INITIALIZE COUNTERS FOR PRINTING TIME          $$$$$
  ESC=CHAR(27)
  CHAR15=CHAR(15) !BEGIN CONDENSED MODE
  CHAR18=CHAR(18) !CANCEL CONDENSED MODE
  ESCM= ESC/'M' !PRESTIGE ELITE MODE
  FACA = 0.0
  FACB = 0.0
  FACC = 0.0 !INITIALIZE DELAYED YIELD FACTOR          $$$$$
  FLEAK = 1.0 !LEAKAGE FACTOR
  TLEAK=TPUP-TPOL
  IF(TLEAK .LT. 1.0D-10 .OR. PERMVOL .LE. 0.0) FLEAK=0.0
  RCH=RCH1
  IF (TPUP.GT.RWL) RCH=0.0
  PI=4.0*DATAN(1.0D+00)
  C1=2.0*PI*DELA !CONST. USED FOR CALCULATION OF FLOW          $$$$$
  DRAWMX = 0.9 * BSUP + 0.1 * TPUP
  IF (IWELL .EQ. 1) RATIO=1.0
  IF (IWELL .EQ. 2) RATIO=0.0
  IF (IWELL .EQ. 3) RATIO=0.5 ! INITIALIZE ARRAYS
  DO 2640 N = 1, NMAX
  RECH(N)=RCH
  RECHW(N)=RCH2
  WL(N)=RWL
  DLO(N)=RWL
  DUP(N)=RWL
  THLEAK(N)=TLEAK
  VL(N)=0.0
  OLDWL(N)=WL(N)
  OLDDLO(N)=DLO(N)
  OLDDUP(N)=DUP(N)
2640 CONTINUE
C          OUTPUT INITIAL INFORMATION
C          WRITE(OUT1,*) ESCM ! SET ELITE CHARACTER MODE
  WRITE(OUT1,3020)
3020 FORMAT(1X,'RADIAL FLOW MODEL WITH VERTICAL COMPONENTS OF FLOW')
  WRITE(OUT1,3030) MM
3030 FORMAT(1X,I2, ' MESH INTERVALS PER DECADE')
  WRITE(OUT1,3040) KT
3040 FORMAT(1X,I3, ' TIME STEPS PER DECADE')
  WRITE(OUT1,3050) RWELL,RMAX
3050 FORMAT(1X,'RWELL=',F5.3,' RMAX=',F8.2)
  WRITE(OUT1,3060) RWL
3060 FORMAT(1X,'INITIAL PIEZOMETRIC LEVEL = ',F6.2)
  IF(FLEAK .GT. 0.0) WRITE(OUT1,3065)TPOL
3065 FORMAT(1X,'TOP OF OVERLYING LEAKY ZONE = ',F6.2)
  WRITE(OUT1,3070) TPUP,BSUP
3070 FORMAT(1X,'TOP OF UPPER ZONE=',F6.2,' BASE OF UPPER ZONE=',F6.2)
  WRITE(OUT1,3080) TPLO,BSLO
3080 FORMAT(1X,'TOP OF LOWER ZONE=',F6.2,' BASE OF LOWER ZONE=',F6.2)

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      IF (JFIX .EQ. 1) WRITE(OUT1,3090)
3090  FORMAT(1X, '***RECHARGE BOUNDARY AT RMAX**')
      IF (JFIX .EQ. 2) WRITE(OUT1,3100)
3100  FORMAT(1X, '***IMPERMEABLE BOUNDARY AT RMAX**')
      IF (ALPHA .LT. 0.0) WRITE(OUT1,3110)
3110  FORMAT(1X, '*** NO DELAYED YIELD ****')
      IF (ALPHA .GT. 0.0) WRITE(OUT1,3120)ALPHA
3120  FORMAT(1X, 'DELAYED YIELD INDEX=',D10.4)
      IF (PERMVOL .LE. 0.0) GOTO 3145
      WRITE(OUT1,3140) PERMVOL
3140  FORMAT(1X, 'PERMEABILITY OF LEAKY LAYER=',D9.3)
3145  IF (RCH .GT.0.0) WRITE(OUT1,3146)RCH
3146  FORMAT(1X, 'UNIFORM RECHARGE=',F6.4, ' M/DAY')
      IF (RCH .NE. RCH1) WRITE(OUT1,3149)
3149  FORMAT(1X, 'NOTE: NO RECHARGE WHEN THERE IS OVERLYING ZONE')
      WRITE(OUT1,3150)
3150  FORMAT(1X, 'WELL LOSS FACTORS ')
      WRITE(OUT1,3160) WLOSSUP, WLOSSLO
3160  FORMAT(1X, 'FOR UPPER ZONE ',F6.3, ' FOR LOWER ZONE ',F6.3)
      IF (IWELL.EQ.1) WRITE(OUT1,3180)
3180  FORMAT(1X, '***FULLY PENETRATING WELL WITH UPPER ZONE CASED**')
      IF (IWELL .EQ. 2) WRITE(OUT1,3190)
3190  FORMAT(1X, '***WELL PENETRATING ONLY THE UPPER ZONE***')
      IF (IWELL .EQ. 1 .OR. IWELL .EQ. 2) GOTO 3220
      WRITE(OUT1,3200)
3200  FORMAT(1X, '***FULLY PENETRATING WELL, ',
1 ' ABSTRACTION FROM BOTH THE ZONES**')
3220  IF (TP(1).LE.0.00001 .OR. LEVPRN .LT. 3) GOTO 3250
      WRITE(OUT1,3230) (TP(I),I=1,5)
3230  FORMAT(1X, 'FLOWS WILL BE PRINTED OUT AT TIMES CLOSEST',
1 ' TO -',5(1X,D10.3)//)
C ***** END OF OUTPUT OF INITIAL INFORMATION ***
3250  IF (LEVPRN .GE. 2) THEN
C***** PRINT AQUIFER PARAMETERS
C      WRITE(OUT1,*)CHAR15          !SET PRINTER TO CONDENSED MODE   $$$$
      WRITE(OUT1,3520)
3520  FORMAT(1X, 'NO. RADIUS ',12HO'LYING KV ', ' UPPER KH ',
1 'UPPER KV SP.YIELD UPPER SCON MIDDLE. KV LOWER KH',
2 ' LOWER KV LOWER SCON ')
      DO 3580 N=1, NMAX
      WRITE(OUT1,3550) N, R(N), PERMVOL, PERMRUP(N), PERMVUP(N),
1 SUNCNUP(N), SCONUP(N), PERMVMD(N), PERMRLO(N), PERMVLO(N),
2 SCONLO(N)
3550  FORMAT(1X,I2,1P,D10.3,1X,9D12.4)
3580  CONTINUE
C      WRITE(OUT1,*)CHAR18
      ENDIF
C FACTORS TO INTERPOLATE DRAWDOWNS AT OBHS.
      DO 1170 J=1,NOBS
      I=2
      DO WHILE (R(I) .LT. DOBS(J))
      I=I+1
      END DO
C THE NODE JUST BEFORE THE JTH OBH IS I-1
      NOB(J)=I-1
C FACTOR TO INTERPOLATE DRAWDOWN AT OBH
      DFAC(J)=(DLOG(DOBS(J))-DLOG(R(I-1))) / (DLOG(R(I))-DLOG(R(I-1)))
1170  CONTINUE

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C CALCULATE LEAKAGE COEFF. ONLY IF LEAKY LAYER IS PRESENT AND
C LEAKAGE PERMEABILITY IS NON-ZERO POSITIVE VALUE.
  IF(FLEAK .LE. 0.00) GOTO 1620
  THLEAK(N)=TPUP-WL(N)
  IF(WL(N).GE.TPUP)GOTO 1531
  VL(N)=(PERMVOL*RR(N))/THLEAK(N)          !LEAKAGE COEFFICIENT  $$$$
  GOTO 1620
1531 VL(N)=0.0
1620 CONTINUE
  HLO(1) = .00005 * HLO(1)
  HUP(1) = .00005 * HUP(1)
  HLO(2) = HLO(2) * WLOSSLO
  HUP(2) = HUP(2) * WLOSSUP          !MODIFY FOR WELLOSS  $$$
C SETTING UP CONDITIONS FOR WELL CASING
  IF(IWELL .NE. 1) GOTO 1700
C ABSTRACTION FROM LOWER LAYER ONLY
  TSLO(1) = 2.0 * DELT * DELA / RR(2)
  TSLO(2) = 2.0 * TSLO(2)
  TSUP(1) = TSUP(1) * 1.0D+10
  V(1) = V(1) * 1.0D+20
  GOTO 1760
1700 IF (IWELL .NE. 2) GOTO 1740
C ABSTRACTION FROM UPPER LAYER ONLY
  TSUP(1) = 2.0 * DELT * DELA / RR(2)
  V(1) = V(1) * 1.0D+20
  GOTO 1760
C ABSTRACTION FROM BOTH THE LAYERS
1740 TSLO(1) = 4.0 * DELT * DELA / RR(2)
  TSUP(1) = TSLO(1)
1750 TSLO(2) = 2.0 * TSLO(2)
  V(1) = 1.0D-10
  V(2) = V(1)
1760 TSUP(2) = 2.0 * TSUP(2)
C %%%% MODIFY RESISTANCES AT THE OUTER BOUNDARY %%%%/
VN1=0.0
VN2=0.0
VN3=0.0
HLO(NMAX) = 2.0D+10
HUP(NMAX) = 2.0D+10
DELAN = DLOG(R(NMAX) / R(NMONE))
DELAN2 = DELAN * DELAN
HUP(NMONE) = 2.0 * DELAN2 / (SD * PERMRUP(NMONE))
HLO(NMONE) = DELAN2 / ((BSLO - TPLO) * PERMRLO(NMONE))
ACONS = 2.0 * DELT * DELA
RN = R(NMONE) * (R(NMAX) - R(NMAX - 2))
TSLO(NMONE) = ACONS / (STORLO * RN)
TSUP(NMONE) = ACONS / (STORUP * RN)
RN1 = R(NMAX) * (R(NMAX) - R(NMONE))
TSLO(NMAX) = ACONS / (STORLO * RN1)
TSUP(NMAX) = ACONS / (STORUP * RN1)
IF (JFIX .NE. 1) GOTO 4610
TSUP(NMAX) = 1.0D-15 * TSUP(NMAX)
TSLO(NMAX) = 2.0D-15 * TSLO(NMAX)
4610 VN1 = 0.5 * (BSLO - TPLO) / (PERMVLO(NMONE))
  D = 0.5 * (DUP(NMONE) + DUP(NMAX))
  IF (D .LT. TPUP) D = TPUP
  VN2 = (BSUP - D) / (PERMVUP(NMONE))

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      IF(DABS(TPLO-BSUP).LT. 0.00001 .OR.
1 PERMVM(NMONE).LE.0.0) GOTO 4640
      VN3 = (TPLO - BSUP) / PERMVM(NMONE)
4640 V(NMONE) = (VN1 + VN2 + VN3) * 2.0 * DELA / RN
      VN1 = 0.5 * (BSLO - TPLO) / (PERMVLO(NMAX))
      D = DUP(NMAX)
      IF( D .LT. TPUP) D = TPUP
      VN2 = (BSUP - D) / (PERMVUP(NMAX))
      IF(DABS(TPLO-BSUP).LT.0.00001 .OR. PERMVM(NMAX).LE.0.0)GOTO 4680
      VN3 = (TPLO - BSUP) / PERMVM(NMAX)
4680 V(NMAX) = (VN1 + VN2 + VN3) * 2.0 * DELA / RN1
      IF(FLEAK .LE. 0.0) GOTO 4750
      IF(WL(NMONE).GE.TPUP)GOTO 4681
      VL(NMONE) = (PERMVOL * RN)/(THLEAK(NMONE)* 2.0 * DELA)
      GOTO 4682
4681 VL(NMONE)=0.0
4682 IF(WL(NMAX).GE.TPUP)GOTO 4683
      THLEAK(NMAX)=TPUP-WL(NMAX)
      VL(NMAX) = (PERMVOL *RN1)/(THLEAK(NMAX) * 2.0 * DELA)
      GOTO 4750
4683 VL(NMAX)=0.0
4750 IF(JFIX .EQ. 1) V(NMAX) = 1.0D-15 * V(NMAX)
      DO 4755 N=2,NMAX
      IF(DUP(N).LT. WL(N))VL(N)=0.0
4755 CONTINUE
C      %% SOLVE EQNS. USING GAUSSIAN ELIMINATION
      ALO(1) = 0.0
      AUP(1) = 0.0
      E(1) = 1.0 / V(1)
      CUP(1) = 1.0 / HUP(1)
      BUP(1) = -(1.0 / HUP(1) + 1.0 / TSUP(1) )
      FUP(1) = -OLDDUP(1) / TSUP(1) - (1.0 - RATIO) * QABST
      CLO(1) = 1.0 / HLO(1)
      BLO(1) = -(1.0 / HLO(1) + 1.0 / TSLO(1))
      FLO(1) = -OLDDLO(1) / TSLO(1) - RATIO * QABST
      DO 5120 N = 2, NMAX
      AUP(N) = 1.0 / HUP(N - 1)
      CUP(N) = 1.0 / HUP(N)
      E(N) = 1.0 / V(N )
      IF(DUP(N).LE.TPUP)GOTO 4756
      BUP(N) = -(AUP(N) + CUP(N) + E(N) + 1.0 / TSUP(N))
      FUP(N) = -OLDDUP(N)/TSUP(N) + RR(N) * RECH(N)+VL(N)*THLEAK(N)
      GOTO 4757
4756 BUP(N) = -(AUP(N) + CUP(N) + E(N) + 1.0 / TSUP(N) + VL(N))
      FUP(N) = -OLDDUP(N) / TSUP(N) + RR(N) * RECH(N) - VL(N)*WL(N)
4757 ALO(N) = 1.0 / HLO(N - 1)
      CLO(N) = 1.0 / HLO(N)
      BLO(N) = -(ALO(N) + CLO(N) + E(N) + 1.0 / TSLO(N))
      FLO(N) = -OLDDLO(N) / TSLO(N)
5120 CONTINUE
      IF(DUP(NMONE).LE.TPUP)GOTO 5121
      FUP(NMONE) = -OLDDUP(NMONE)/TSUP(NMONE) +
1 0.5*RECH(NMONE)*RN/DELA+VL(NMONE)*THLEAK(NMONE)
      GOTO 5122
5121 FUP(NMONE) = -OLDDUP(NMONE)/TSUP(NMONE) +
1 0.5*RECH(NMONE)*RN/DELA - VL(NMONE) *WL(NMONE)
5122 IF(DUP(NMAX).LE. TPUP)GOTO 5123

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      FUP(NMAX) = -OLDDUP(NMAX)/TSUP(NMAX) +
1 0.5*RECH(NMAX)*RN1/DELA+VL(NMAX)*THLEAK(NMAX)
      GOTO 5124
5123 FUP(NMAX) = -OLDDUP(NMAX)/TSUP(NMAX) +
1 0.5*RECH(NMAX)*RN1/DELA - VL(NMAX)*WL(NMAX)
C   GAUSSIAN ELIMINATION      --      FORWARD SOLUTION
5124 U1(1) = BLO(1)
      V1(1) = FLO(1)
      U2(1) = BUP(1)
      V2(1) = FUP(1)
      RJD(1) = 0.0
      RLN(1) = 0.0
      DO 5250 N = 2, NMAX
      M = N - 1
      U1(N) = BLO(N) - CLO(M) / U1(M) * ALO(N)
1 + RJD(M) / U1(M) * RLN(M) / U2(M) * ALO(N)
      RLN(N) = E(N) + CUP(M) / U1(M) * RLN(M) / U2(M) * ALO(N)
      V1(N) = FLO(N) - V1(M) / U1(M) * ALO(N)
1 + V2(M) / U1(M) * RLN(M) / U2(M) * ALO(N)
      U2(N) = BUP(N) - CUP(M) / U2(M) * AUP(N) - RLN(N) / U1(N) * E(N)
      U2(N) = U2(N) + RJD(M) / U1(N) * AUP(N) / U2(M) * RLN(N)
      RJD(N) = -CLO(N) / U1(N) * E(N)
1 + CLO(N) / U1(N) * RJD(M) / U2(M) * AUP(N)
      V2(N) = FUP(N) - AUP(N)/U2(M)*V2(M) - E(N)/U1(N)*V1(N)
1 + RJD(M)/U2(M)*AUP(N)/U1(N)*V1(N)
5250 CONTINUE
C   NOW BACK SUBSTITUTION
      DUP(NMAX) = V2(NMAX) / U2(NMAX)
      DLO(NMAX) = V1(NMAX)/U1(NMAX) - RLN(NMAX)/U1(NMAX)*DUP(NMAX)
      DO 5320 N = NMONE, 1, -1
      M = N + 1
      DUP(N) = V2(N) / U2(N) - RJD(N) / U2(N) * DLO(M)
1 -CUP(N) / U2(N) * DUP(M)
      DLO(N) = V1(N) / U1(N) - RLN(N) / U1(N) * DUP(N)
1 -CLO(N) / U1(N) * DLO(M)
5320 CONTINUE
C
C   IF(TIMIN .LT.0.00035)GOTO 5322
C   WRITE(7,5321)TME,NUM,DUP(20)
C 5321 FORMAT(1X,F12.5,I5,F12.5)
C
      IF (DUP(1) .LT. DRAWMX) GOTO 1830
      WRITE(OUT1,1800)
1800 FORMAT(1X,'**EXCESSIVE DRAWDOWN IN THE WELL**')
      CALL OUTPT          ! FOR PRINTING HYD.RES,D/D,ETC.   $$$$$
      STOP          ! STOP DUE TO EXCESSIVE DRAWDOWN IN THE WELL
1830 CONTINUE !***** END OF ITERATIVE LOOP FOR UNCONFINED AQUIFER
C   ADDITIONAL STATEMENTS FOR DELAYED YIELD
      DO 1860 N = 1, NMAX
      Y(N) = X(N) + FACC * (DUP(N) - OLDDUP(N))
1860 CONTINUE
C
C   CALCULATION OF NEW POSITION OF WATER LEVEL IN LEAKY LAYER
C
      DO 1865 N=1,NMAX
      IF(WL(N).GE.TPUP)GOTO 1861
      DLEAK=PERMVOL*(DUP(N)-WL(N))/THLEAK(N)
      IF(DUP(N).GT. TPUP)DLEAK=PERMVOL

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      IF(DLEAK .LT.0.0)DLEAK=0.0
      GOTO 1862
1861 DLEAK=0.0
1862 WL(N)=WL(N)+DELT*(DLEAK-RECHW(N))/SYOL
      IF(WL(N).GT.TPUP)WL(N)=TPUP
1865 CONTINUE
C
      IF(TIMIN .LT. 0.00035) GOTO 1900 ! SKIP PRINTING FOR EARLIER TIMES
C      CALL OUTFLD !TO CALCULATE AND PRINT FLOWS AND DRAWDOWNS AT OBHS.
1900 IF(LEVPRN .LT. 3) GOTO 2050
      IF(DABS(TME-TP(ITP)) .GT. .05*TIMIN*DT) GOTO 2050
      CALL OUTPT
      WRITE(OUT1,7500)R(2),(DOBS(J),J=1,NOBS),R(NMAX)
      ITP=ITP+1
2050 DELT = TIMIN * DT
C      IF(DELT .GT. 0.06) DELT=0.06
      IF (IND .EQ. 0) GOTO 2070          !NEW DELT
      CALL OUTFLD          !ONE PHASE COMPLETED, PRODUCE SUMMARY PRINTOUT
2070 DO 2075 N=1, NMAX
      OLDDLO(N) = DLO(N)
      OLDDUP(N) = DUP(N)
      OLDWL(N)=WL(N)
2075 CONTINUE
      IF(IND .NE. 0) GOTO 1250
      GOTO 1360
C ***** END OF TIME STEP LOOP *****
C TO 1360 FOR NEXT TIME STEP OR IF THIS WAS THE LAST TIME STEP FOR
C THE PHASE, THEN TO 1250 TO READ Q AND TIME FOR THE NEXT PHASE
C      WRITE(OUT1,*)CHAR18
2120 WRITE(OUT1,2130)
2130 FORMAT(1X,'END OF RUN')
      STOP
      END
      SUBROUTINE OUTFLD
      PARAMETER (NA=40,NB=5)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      COMMON R(NA), RR(NA), DLO(NA), DUP(NA), OLDDLO(NA), OLDDUP(NA),
      1 RECH(NA), PERMRUP(NA),PERMVUP(NA),SCONUP(NA),SUNCNUP(NA),
      2 PERMVMD(NA),TSUP(NA), PERMRLO(NA), PERMVLO(NA), SCONLO(NA),
      3 VL(NA), HUP(NA), HLO(NA), V(NA), TSLO(NA), AUP(NA), BUP(NA),
      4 CUP(NA), E(NA), FUP(NA), ALO(NA), BLO(NA), CLO(NA), FLO(NA),
      5 U1(NA), U2(NA), V1(NA), V2(NA), RJD(NA), RLN(NA), X(NA), Y(NA),
      6 DOBS(NB), DFAC(NB),ARAY(NB), ARRAY(NB), TP(NB),C1,RWL,TME,TIMIN,
      7 NMAX,NOB(NB), NMONE,NOBS,OUT1,ESCM,CHAR15,CHAR18,ESC,WL(NA),
      8 OLDWL(NA),RECHW(NA),THLEAK(NA),TPUP
C %%% SUBROUTINE TO INTERPOLATE AND PRINT D/D AND FLOWS %%
      CHARACTER ESC*1, CHAR15*1, CHAR18*1, ESCM*2
      INTEGER OUT1
      DO 6030 I = 1, NOBS
      I1 = NOB(I)
      ARAY(I) = DLO(I1) + DFAC(I) * (DLO(I1 + 1) - DLO(I1))
      ARRAY(I) = DUP(I1) + DFAC(I) * (DUP(I1 + 1) - DUP(I1))
6030 CONTINUE
      QUP = C1 * (DUP(2) - DUP(3)) / HUP(2)          ! UPPER RADIAL FLOW $$$$$
      QLO = C1 * (DLO(2) - DLO(3)) / HLO(2)          ! LOWER RADIAL FLOW $$$$$
C QST IS STORAGE CONTRIBUTION TO HALFWAY BET'N NODE 2 AND 3
      QST1 = C1 * (DLO(2)-OLDDLO(2)) * (1.0/TSLO(1) + 1.0/TSLO(2))
      QST2 = C1 * (DUP(2)-OLDDUP(2)) * (1.0/TSUP(1) + 1.0/TSUP(2))

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QST = (QST1 + QST2)
QL = 0.0
DO 6110 N = 2, NMAX
DQL = C1*(DUP(N)-OLDWL(N))*VL(N)
IF(DUP(N).GT.TPUP)DQL=C1*VL(N)*THLEAK(N)
IF(DQL .LT.0.0)DQL=0.0
QL = QL + DQL
6110 CONTINUE
WRITE(OUT1,6120) QUP, QLO, QST, QL
6120 FORMAT(1X,'QUP= ',F6.1,' QLO= ',F6.1,' WELL STORAGE= ',
1 F6.1,' TOTAL LEAKAGE= ',F6.1)
WRITE(OUT1,6160)TIMIN,DUP(1),(ARRAY(I),I=1,NOBS),DUP(NMAX)
6160 FORMAT(1X,'T=',F9.5,F9.3, 7(2X,F9.3))
WRITE(OUT1,6200)DLO(1),(ARAY(I),I=1,NOBS),DLO(NMAX)
6200 FORMAT(8X,F13.3,7F11.3)
WRITE(OUT1,*)
WRITE(7,6201)TME,DUP(1),DUP(NMAX)
6201 FORMAT(1X,3F12.5)
C WRITE(7,6202)DUP(8),DUP(14),DUP(20),DUP(24),DUP(NMAX)
C 6202 FORMAT(13X,5F12.5)
C WRITE(7,6203)VL(8),VL(14),VL(20),VL(24),VL(NMAX)
C 6203 FORMAT(13X,5F12.5)
RETURN
END
SUBROUTINE OUTPT
PARAMETER (NA=40,NB=5)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON R(NA),RR(NA), DLO(NA), DUP(NA), OLDDLO(NA), OLDDUP(NA),
1 RECH(NA), PERMRUP(NA),PERMVUP(NA),SCONUP(NA),SUNCNUP(NA),
2 PERMVMD(NA),TSUP( NA), PERMRLO(NA), PERMVLO(NA), SCONLO(NA),
3 VL(NA), HUP(NA), HLO(NA), V(NA), TSLO(NA), AUP(NA), BUP(NA),
4 CUP(NA), E(NA), FUP(NA), ALO(NA), BLO(NA), CLO(NA), FLO(NA),
5 U1(NA), U2(NA), V1(NA), V2(NA), RJD(NA), RLN(NA), X(NA), Y(NA),
6 DOBS(NB), DFAC(NB),ARAY(NB), ARRAY(NB), TP(NB),C1,RWL,TME,TIMIN,
7 NMAX,NOB(NB), NMONO,NOBS,OUT1,ESCM,CHAR15,CHAR18,ESC,WL(NA),
8 OLDWL(NA),RECHW(NA),THLEAK(NA),TPUP
CHARACTER ESC*1, CHAR15*1, CHAR18*1, ESCM*2
INTEGER OUT1
C TO PRINT D/D AND FLOW FOR THE WHOLE AQUIFER %%%
WRITE(OUT1,7010)TME,TIMIN
7010 FORMAT(1X,' SUMMARY AT TOTAL TIME',F12.5,' DAYS',
1 ' PHASE TIME ',F12.5,' DAYS'/)
C WRITE(OUT1,*)CHAR15 ! SET PRINTER TO CONDENSED MODE
WRITE(OUT1,7040)
7040 FORMAT(1X,'NO RADIUS UPP. RHR LOW. RHR VERT. R',
1 ' UPP. D/D LOW. D/D QLEAK QST-UPP.',
2 ' UPP.FLOW VERT.FLOW QST-LOWER LOW. FLOW '
3 ' WL '/)
DO 7150 I = 1, NMAX
QUP = C1 * (DUP(I) - DUP(I + 1)) / HUP(I)
QLO = C1 * (DLO(I) - DLO(I + 1)) / HLO(I)
QV = C1 * (DLO(I) - DUP(I)) / V(I)
IF(I .EQ. 1) QV=0.0
QL = C1 * (DUP(I) - OLDWL(I)) * VL(I)
IF(DUP(I).GT.TPUP)QL=C1*VL(I)*THLEAK(I)
IF(QL .LT.0.0)QL=0.0
QSTUP=C1*(DUP(I)-OLDDUP(I))/TSUP(I)
QSTLO=C1*(DLO(I)-OLDDLO(I))/TSLO(I)

```

WRITE(OUT1,7120) I, R(I),HUP(I),HLO(I),V(I),DUP(I),DLO(I),
1 QL, QSTUP, QUP, QV, QSTLO, QLO,WL(I)
7120 FORMAT(1X,12,F9.3,1P,1X,12(D9.2,1X))
7150 CONTINUE
C WRITE(OUT1,*)CHAR18
RETURN
END

