L-2/T-2/CSE **Date : 15/01/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2015-2016

Sub : **MATH 243** (Matrices, Vectors, Fourier Analysis and Laplace Transforms)

Full Marks: 280 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Obtain the canonical matrix C row equivalent to the given matrix A: **(14)**

$$A=\begin{bmatrix}2 & 4 & -3 & 6 & 4\\ 1 & 2 & -2 & 3 & 1\\ 1 & 3 & -2 & 3 & 0\\ 1 & 1 & -1 & 4 & 6\end{bmatrix}$$

(b) For the matrix $A=\begin{bmatrix}1 & 3 & -1 & 3\\ 3 & 4 & -3 & 4\\ 1 & 3 & 1 & 2\end{bmatrix}$ **($16\frac{2}{3}$)**

Find the non-singular matrices P and Q such that PAQ is in the normal form B.

(c) Using only elementary row transformations to reduce A to I, find the inverse of A, when **(16)**

$$A=\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 6\\ 3 & 4 & 5 & 7\\ 4 & 5 & 5 & 7\end{bmatrix}$$

2. (a) Determine the value of k such that the following system of equations has: (i) a unique solution (ii) no solution (iii) more than one solution: **(15)**

$$kx+y+z=1$$
$$x+ky+z=1$$
$$x+y+kz=1$$

(b) Find all eigen values, eigen vectors and the corresponding eigen spaces of the matrix **($16\frac{2}{3}$)**

$$A=\begin{bmatrix}1 & 2 & -1\\ 0 & -2 & 0\\ 0 & -5 & 2\end{bmatrix}$$

(c) State Cayley-Hamilton theorem and using this theorem find the inverse of the matrix A when **(15)**

$$A=\begin{bmatrix}1 & 2 & 2\\ 3 & 1 & 0\\ 1 & 1 & 1\end{bmatrix}.$$

Also find A^{-2}, A^{3}.

Contd P/2

MATH 243

3. (a) Consider the basis $S = \{\underline{u}_1, \underline{u}_2, \underline{u}_3\}$ for $\Re^3$, where $\underline{u}_1 = (1,1,1)$, $\underline{u}_2 = (1,1,0)$ and $\underline{u}_3 = (1,0,0)$ and let $T: \Re^3 \to \Re^3$ be the linear transformation such that $T(\underline{u}_1) = (2,-1,4)$, $T(\underline{u}_2) = (3,0,1)$, $T(\underline{u}_3) = (-1,5,1)$. Find a formula for $T(x, y, z)$ and use that formula to find $T(2,4,-1)$. **(16⅔)**

(b) Find the standard matrix for the stated composition of linear operator on $\Re^3$: A rotation of 30^0 about the y-axis, followed by a reflection about the xy-plane, followed by an orthogonal projection on the yz-plane. **(15)**
Hence find the image of $(-1,2,3)$ with respect to the stated composition of linear operators on $\Re^3$.

(c) Let $T: \Re^3 \to \Re^3$ be the linear operator defined by $T(x, y, z) = (x + 2y, y - z, x + 2z)$. Find the rank and nullity of T. Hence verify the Dimension Theorem for T. **(15)**

4. (a) Show that the three vectors $\underline{a} = 2\underline{i} + \underline{j} - 3\underline{k}$, $\underline{b} = \underline{i} - 4\underline{k}$, and $\underline{c} = 4\underline{i} + 3\underline{j} - \underline{k}$ are linearly dependent. Determine a relation among them and hence show that the terminal points are collinear. **(16⅔)**

(b) Prove that $[\underline{a}\ \underline{b}\ \underline{c}]^2 = \begin{bmatrix} \underline{a}\cdot\underline{a} & \underline{a}\cdot\underline{b} & \underline{a}\cdot\underline{c} \\ \underline{b}\cdot\underline{a} & \underline{b}\cdot\underline{b} & \underline{b}\cdot\underline{c} \\ \underline{c}\cdot\underline{a} & \underline{c}\cdot\underline{b} & \underline{c}\cdot\underline{c} \end{bmatrix}$. **(15)**

(c) Determine the constant a, b, $\underline{c}$ that the vector $\underline{F} = (x + 2y + az)\underline{i} + (bx - 3y - z)\underline{j} + (4x + cy + 2z)\underline{k}$ is irrotational. **(15)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Evaluate $\int_C (3x^2 + y^2)dx + 2xydy$ along the circular arc C given by $x = \cos t$, $y = \sin t, \left(0 \le t \le \frac{\pi}{2}\right)$. **(15)**

(b) Evaluate the surface integral $\iint_S x^2 ds$ over the sphere $x^2 + y^2 + z^2 = 1$. **(15)**

(c) State Gauss Divergence theorem. Verify the theorem for the vector function $F = 4xi - 2y^2 j + z^2 k$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **(16⅔)**

Contd P/3

MATH 243

6. (a) If $\mathcal{L}\{F(t)\} = f(s)$ then show that $\mathcal{L}\left\{\int_0^t (u)du\right\} = \frac{f(s)}{s}$. **(10⅔)**

(b) Evaluate $\mathcal{L}^{-1}\left\{\frac{8}{(s^2+1)^3}\right\}$ by using convolution theorem. **(16)**

(c) Solve the following differential equation using Laplace transformation $tY''+Y'+4tY=0$, $Y(0)=3, Y'(0)=0$. **(20)**

7. (a) Use Laplace transformation to evaluate $\int_0^\infty e^{-x^2}dx$. **(14)**

(b) Solve the following partial differential equation using Laplace transformation **(16)**

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, U(x,0) = 3\sin 2\pi x,$$

$$U(0,t)=0, U(1,t)=0, \text{ where, } 0<x<1, t>0.$$

(c) Draw a graph of the function $f(x)$ defined as **(16⅔)**

$$f(x) = \begin{cases} 1-x; & -1 \le x \le 0 \\ 0; & 0 \le x \le 1 \end{cases}$$

Having period 2 on the interval $[-3,3]$, then find a Fourier series of $f(x)$ and hence evaluate the value $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$.

8. (a) Find the Fourier integral of $f(x) = \begin{cases} x; & |x| \le \pi \\ 0; & |x| > \pi \end{cases}$ **(20)**

and hence evaluate the integral $\int_0^\infty \frac{(\sin \pi\omega - \pi\omega \cos \pi\omega)\sin \pi\omega}{\omega^2} d\omega$.

(b) Use Fourier transformation to solve $\frac{\partial^2 y}{\partial t^2} = C^2\frac{\partial^2 y}{\partial x^2}; -\infty < x < \infty, t > 0$ **(26⅔)**

$$y(x,0) = f(x), \left(\frac{\partial y}{\partial t}\right)_{t=0} = g(x); -\infty < x < \infty$$

Where, $f(x)=0$, $g(x) = \begin{cases} \sin x; & |x| \le \pi \\ 0; & |x| > \pi \end{cases}$ and $c = 2$.

L-2/T-2/CSE Date : 21/01/2017

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2015-2016

Sub : **CSE 209** (Digital Electronics and Pulse Techniques)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A DTL 3-input NAND gate is shown in Figure 1(a). Assume that, the drop across a conducting diode is 0.7 V, V_γ(diode) = 0.6 V, for transistor: V_{BE}(cutin) = 0.5 V, V_{BE}(sat) = 0.8 V, V_{CE}(sat) = 0.2 V, h_{FE} = 30. **(25)**
   (i) Calculate noise margins (NM(0) and NM(1) for this gate.
   (ii) Find the fanout for this gate.
   (iii) Implement the following logic function using minimum number of DTL NAND gates only:
   $$f = \left(\overline{AB}\right)\cdot\left(\overline{CDE}\right)$$
   (b) A DCTL NOR gate is shown in Figure 1(b). Discuss the current logging problem in this gate with appropriate figures. **(10)**

2. (a) An ECL OR/NOR gate is shown in Figure 2(a). Assume that, V_{BE}(cutin) = 0.5 V, V_{BE}(active) = 0.7 V. **(25)**
   (i) Verify that all the transistors (Q_1, Q_2, Q_3, Q_4, Q_5) remain in active region while conducting.
   (ii) Calculate average power dissipated by this gate.
   (iii) Prove with appropriate figures that, if the outputs of two ECL gates are tied together the OR function is satisfied.
   (b) An RTL positive NOR gate is shown in Figure 2(b). Using the NOR gate at Figure 2(b) mathematically prove that, high-level output voltage is a decreasing function of fanout. You must provide appropriate figures. **(10)**

3. (a) A TTL NAND gate with totem-pole output is shown in Figure 3(a). **(20)**
   Assume that the drop across a conducting diode is 0.7 V, V_γ(diode) = 0.6 V, for all transistors V_{BE}(cutin) = 0.5 V, V_{BE}(sat) = 0.8 V, V_{CE}(sat) = 0.2, h_{FE} = 20.
   (i) Calculate the peak current drawn by this gate from the power supply when the output makes a transition from low to high.
   (ii) Suppose we need to connect the outputs of several TTL gates of Figure 3(a) to transfer data using a shared bus.
   We want to avoid the huge power dissipation when any gate output is low. We also want to keep the high-speed operation of totem-pole arrangement.
   Now discuss how you can fulfill all the requirements with appropriate figures.
   (b) An inverter is shown in Figure 3(b) where both Q_1 and Q_2 are n-channel enhancement type MOSFETs. Let Q_1 and Q_2 are of identical characteristics and their drain characteristics is shown in Figure 3(c). **(15)**
   (i) Draw the load curve (I_D vs V_0 curve) for this inverter.
   (ii) Draw the transfer curve (V_0 vs V_i curve) for this inverter.

Contd P/2

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4. (a) Implement the following logic functions using minimum number of CMOS devices only: **(10)**

   (i) $f = \overline{a + b \cdot (c + d)}$

   (ii) $f = \overline{a \cdot (b + c) + de}$

   (b) Implement XOR and XNOR gates using minimum number of CMOS transmission gates only. **(10)**

   (c) Draw a 4-MOSFET dynamic RAM cell with Read-Write lines and refreshing circuitry. Let, the state of the cell is 0. Prove that, this state is reinforced during the refresh interval. **(15)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (b) Find out the transfer function for the given circuit and input (Fig. 5(A)) and draw the transfer function with output curve. Please use constant voltage drop model for diodes ($V_\gamma = 0.7$ V). **(10)**

   (b) Draw the output wave shape for the given circuit and input (Fig. 5(B)) with a detailed analysis. Please use constant voltage drop model for diodes ($V\gamma = 0.7$ V). **(10)**

   (c) Design a circuit that will take a sine wave as the input (Fig. 5(Ci)) and produce the output given in Fig. 5(Cii)). Please use constant voltage drop model for diodes ($V\gamma = 0.7$ V) if necessary. **(15)**

6. (a) Show that $V1 = V/(1+e^{-T/2RC})$ for a symmetric square input with period T in steady state in a high pass RC circuit. The point V1 is indicated in Fig. 6(A) and V is the peak to peak voltage of the input wave shape. **(15)**

   (b) The signal shown in Fig. 6(Bi) is fed to the circuit of Fig. 6(Bii) as input. Derive output equation and draw the output wave shape. Use R = 1 KΩ and C = 1 μF. You can assume no initial capacitor voltage. **(10)**

   (c) Briefly explain time shifting, time scaling and time reversal operations of the signal. **(10)**

7. (a) The output for a ramp input ($v_i(t) = \alpha t$) in a low pass RC circuit is, $v_o(t) = \alpha t - \alpha RC(1 - e^{-t/RC})$. Show that for large RC value $v_0(t) = 1/RC * \int v_i(t)dt$. **(10)**

   (b) Show that the rise time of the output signal is $t_r = 2.2$ RC for the low pass RC circuit. **(10)**

   (c) Design a square wave generator where the width of the square wave can be controlled independently for both positive half cycle and negative half cycle. **(15)**

8. (a) Deign a 4-bit weighted resistance Digital to Analog Converter (DAC). Clearly draw the circuit diagram and derive the output equation. **(15)**

   (b) Design a circuit diagram that will produce a triangular wave shape. **(10)**

   (c) What can we do if we want to generate following output for the input given in Table 8(C)? **(10)**

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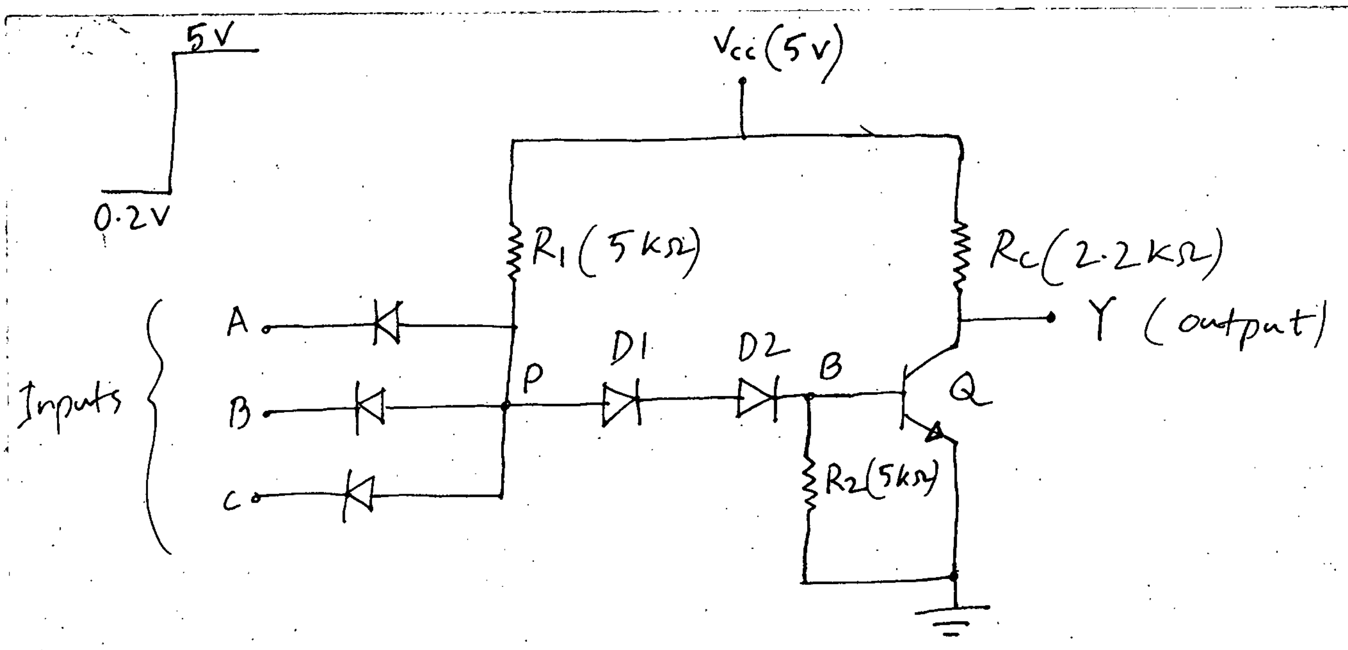

Figure 1(a)

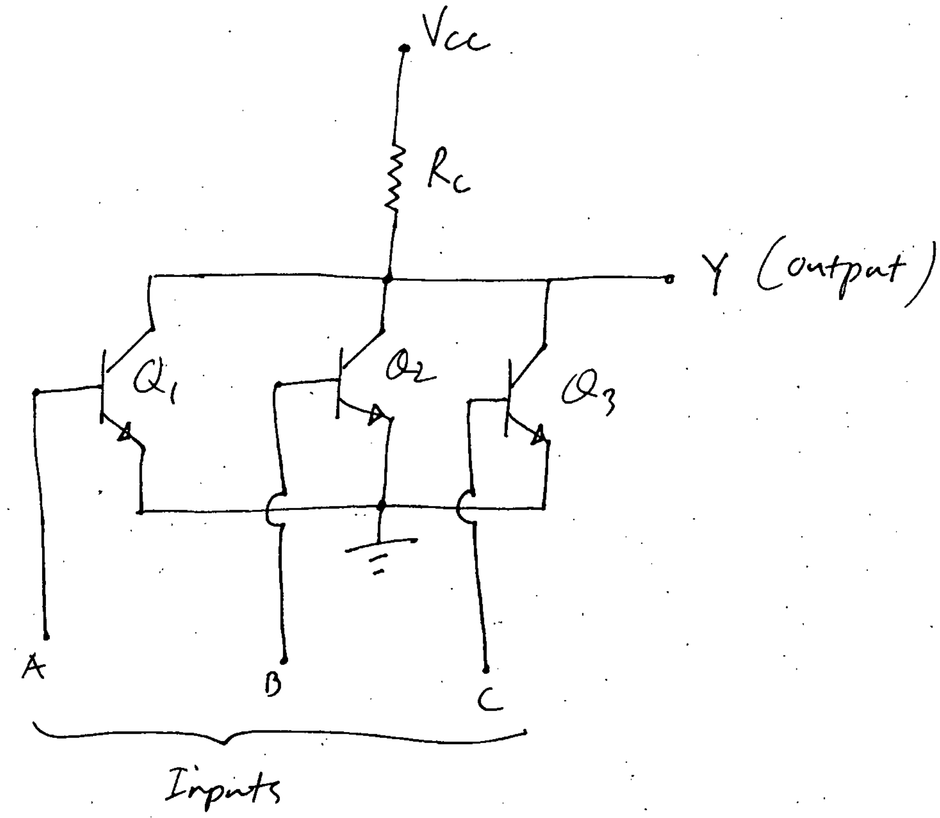

Figure 1 (b)

Contd . . . P/4

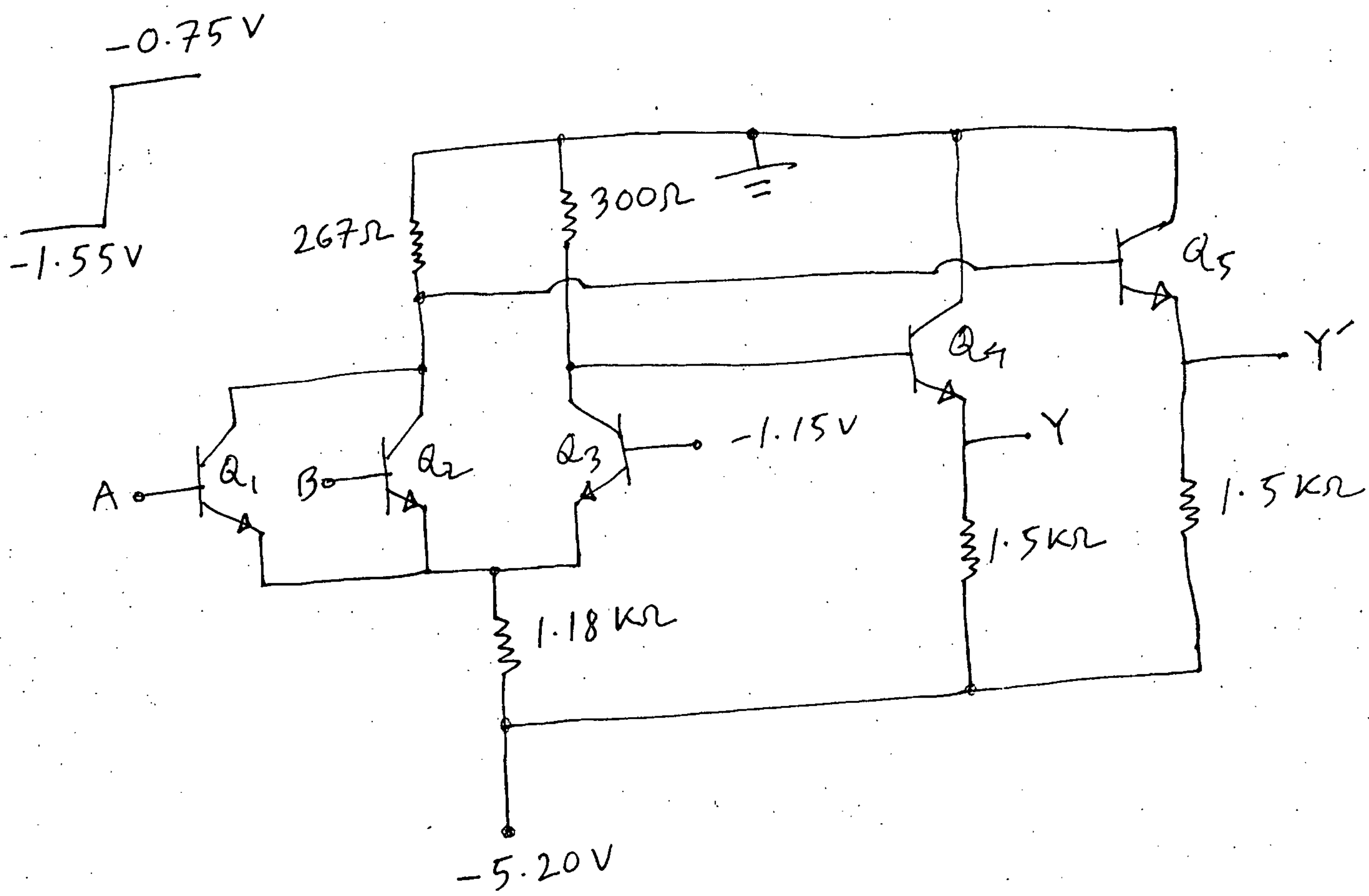

Figure 2(a)

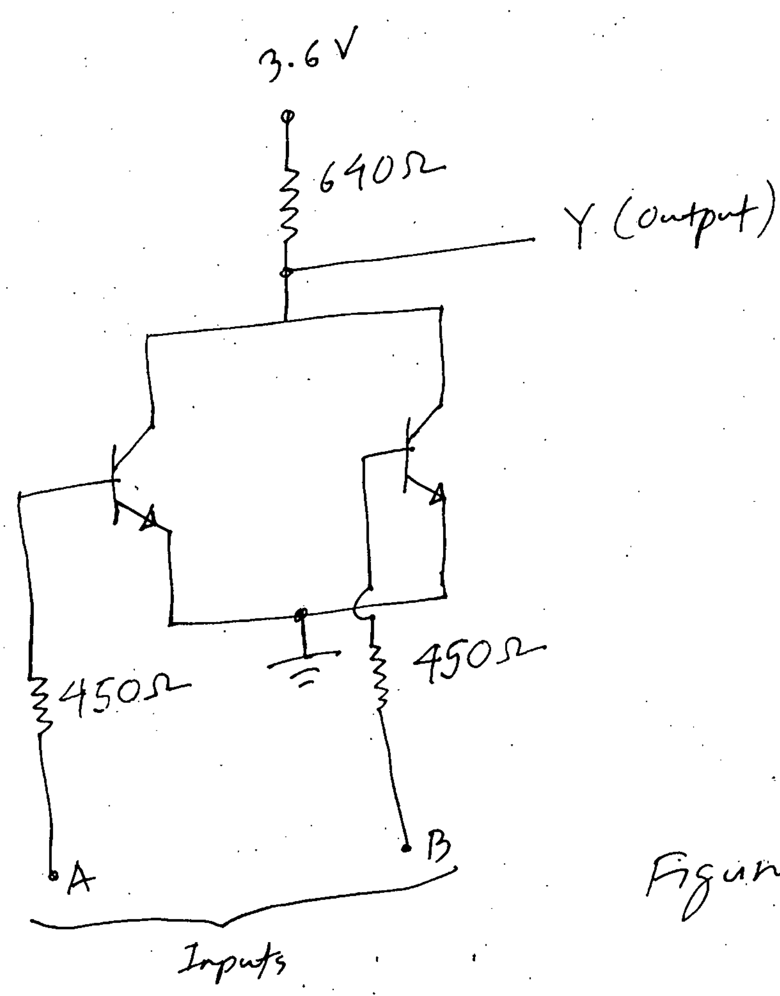

Figure 2(b)

Contd — P/5

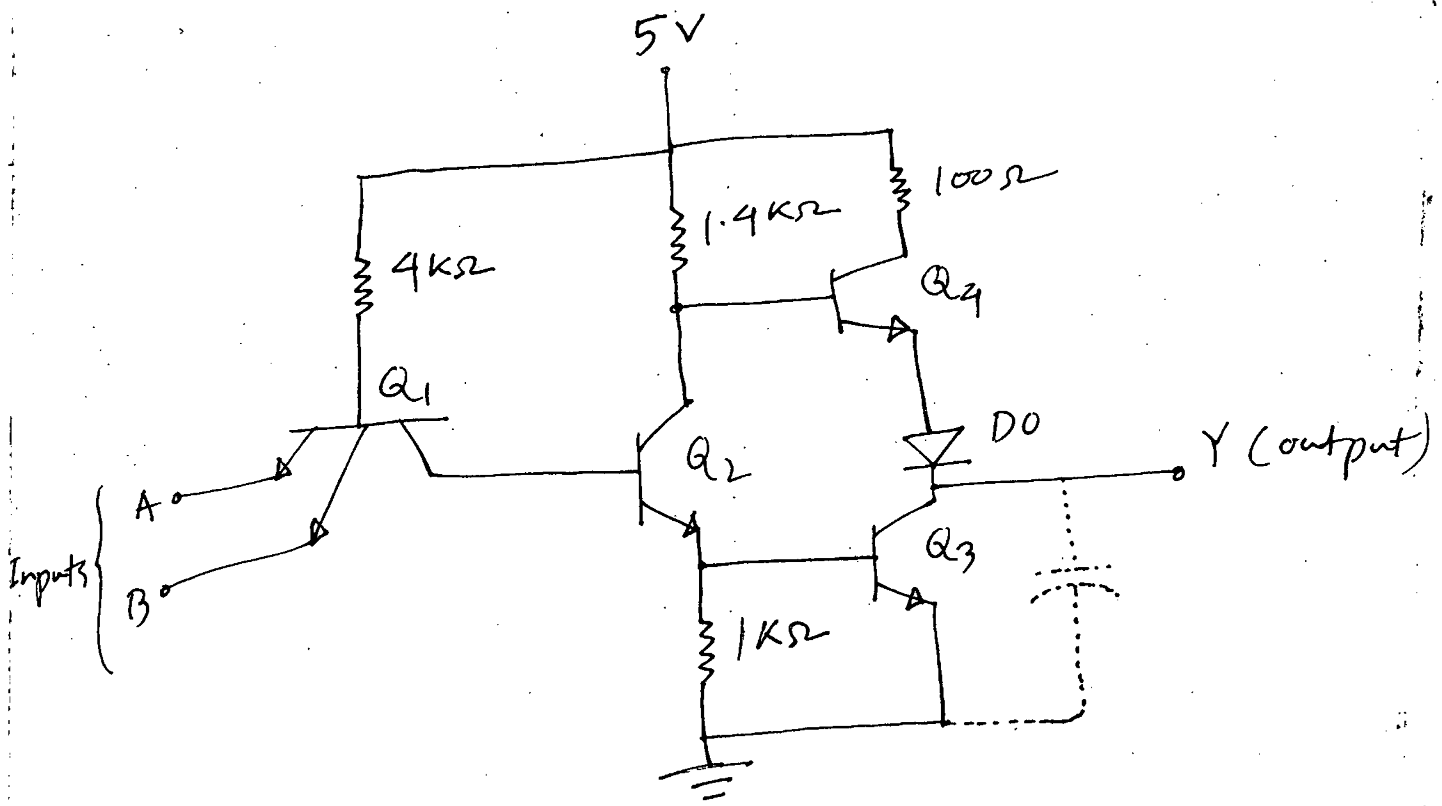

Figure 3 (a)

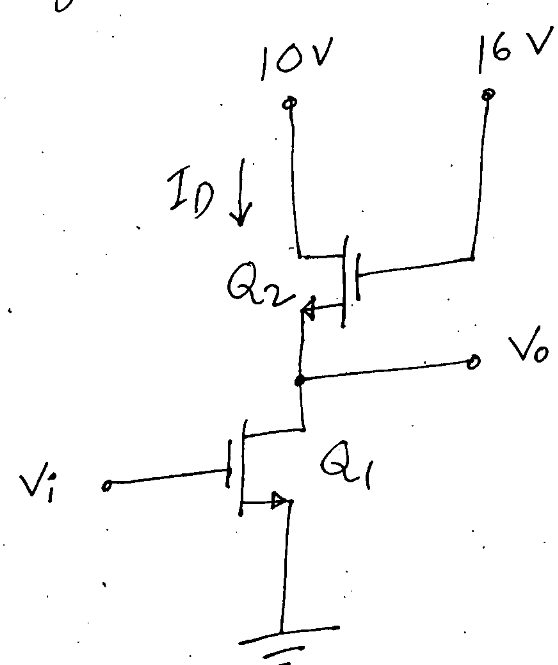

Figure 3(b)

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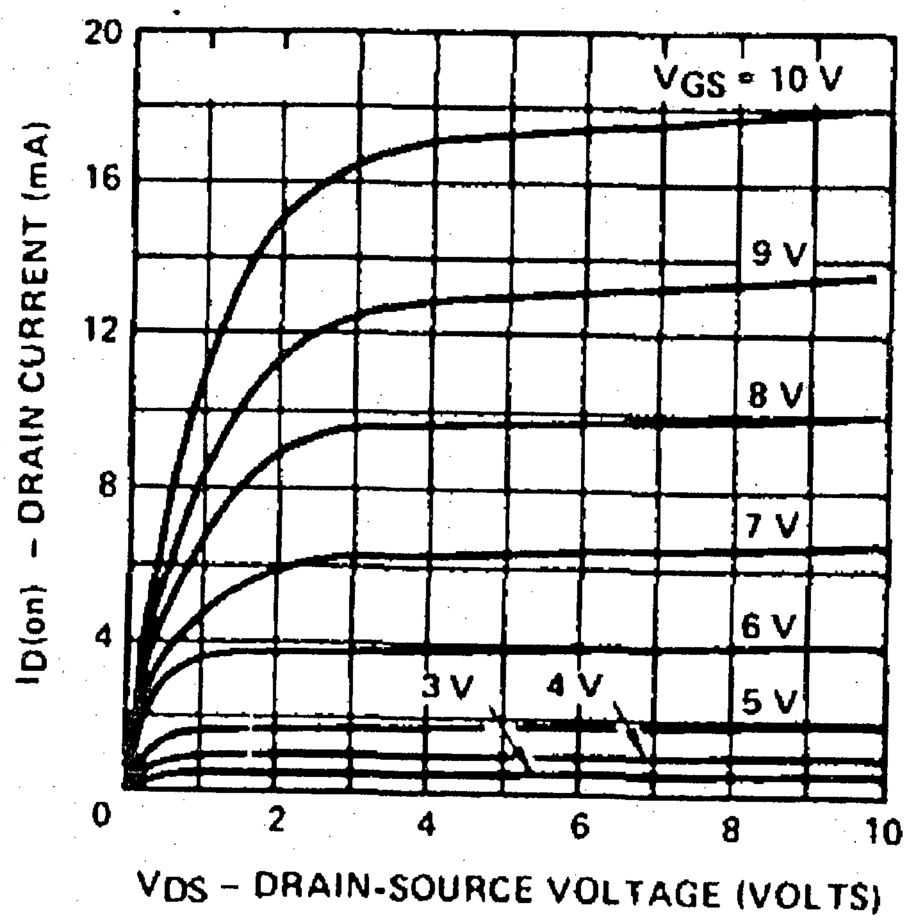

Figure 3(c)

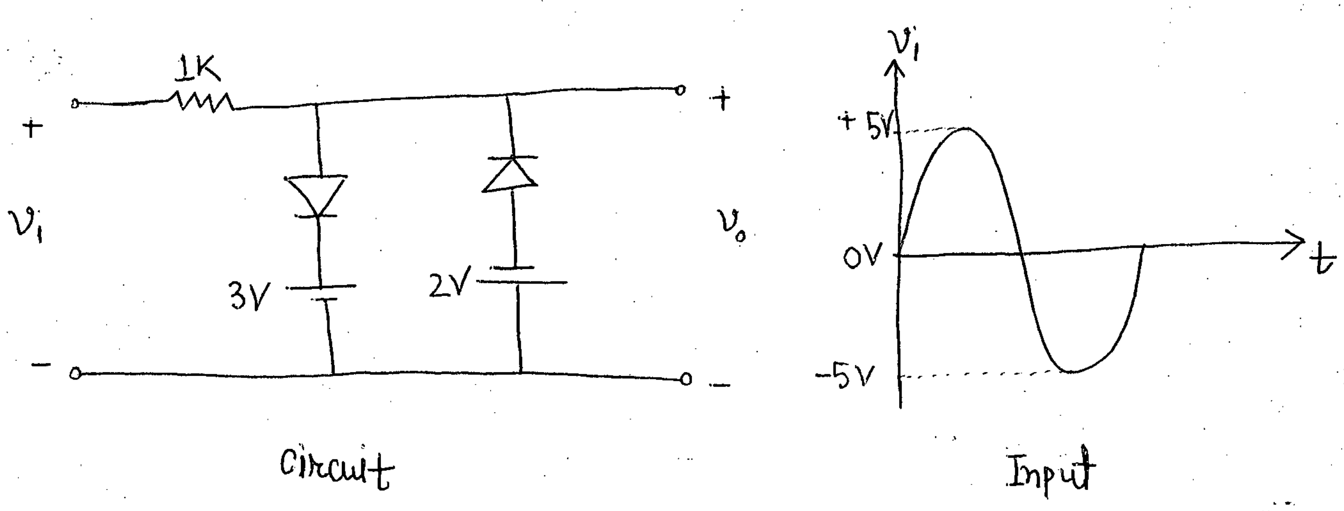

Fig. 5(A)

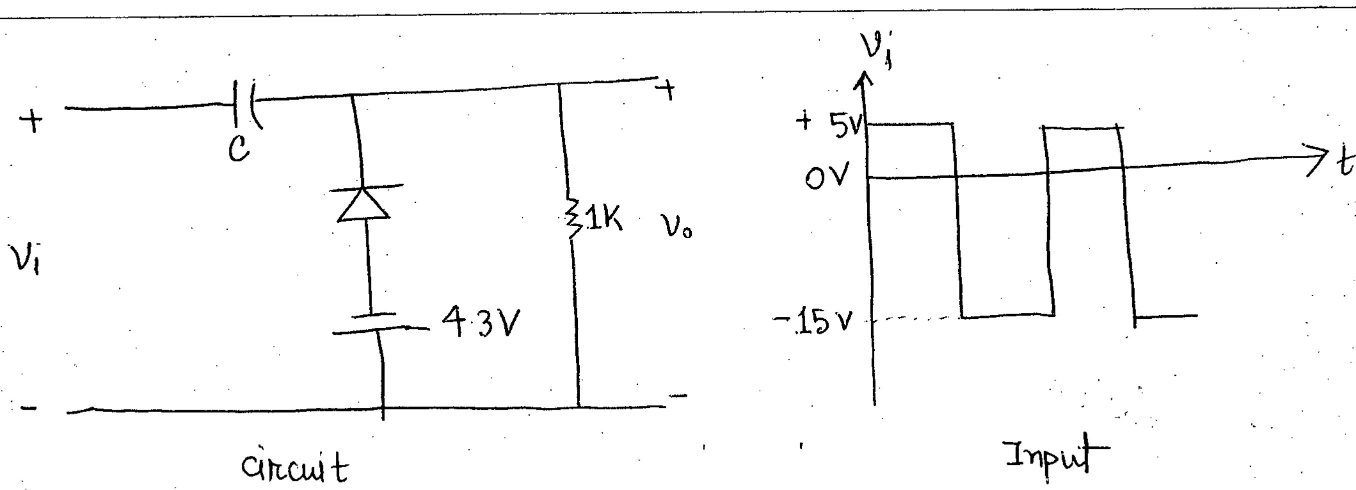

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Fig. 5(B)

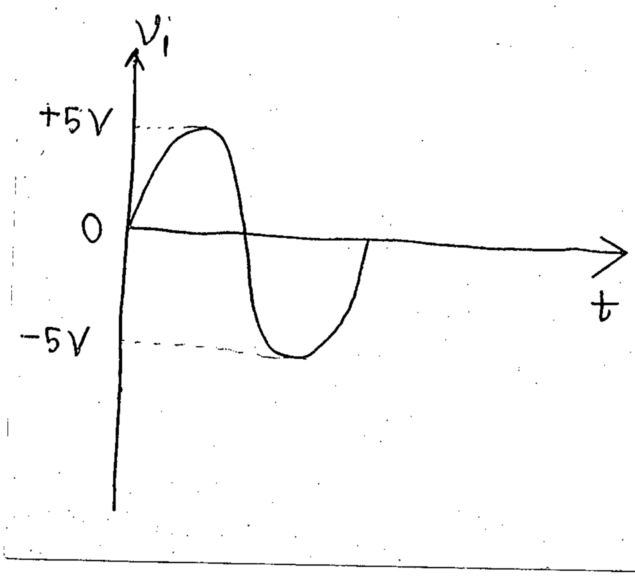

Fig. 5(ci)

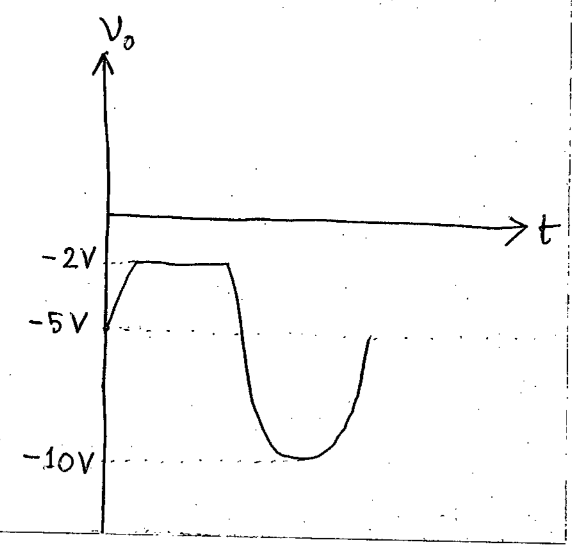

Fig. 5(cii)

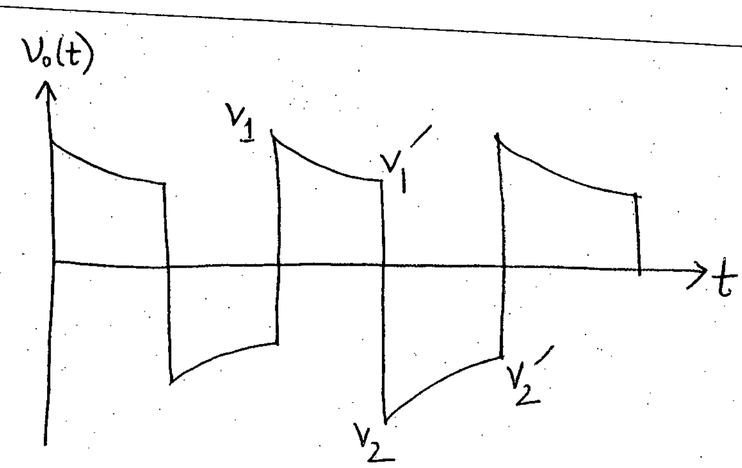

Fig. 6(A)

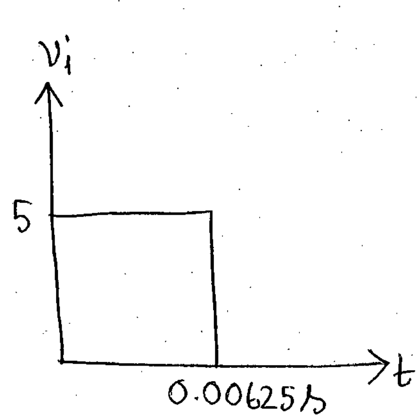

Fig. 6(Bi)

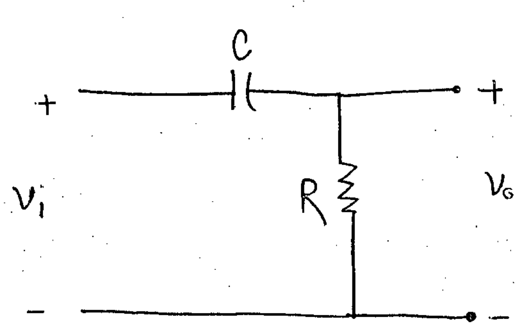

Fig. 6(Bii)

Contd..... P/8

Table – 8 (c)

Digital Binary Input			Output in volts
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	-5
1	0	1	-4
1	1	0	-3
1	1	1	-2

L-2/T-2/CSE **Date : 25/01/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2015-2016

Sub : **CSE 211** (Theory of Computation)

Full Marks: 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Formally define a Finite State Machine (FSM). Formally describe the FSM F1 of Figure 1.a. **($8\frac{1}{3}$)**

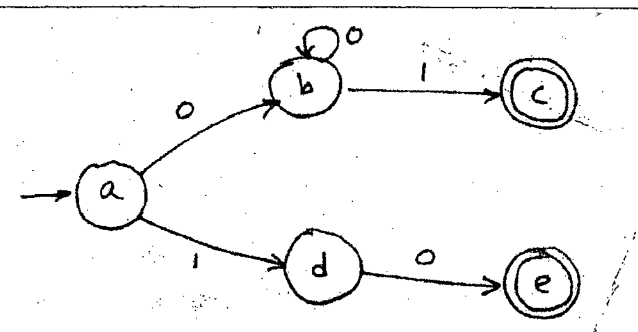

Figure 1.a: Finite State Machine, F1

(b) What language does F1 (Figure 1.a) recognize? **(4)**

(c) Suppose your friend slightly changes δ of F1 (Figure 1.a) by only adding the transition δ(d,1) = d. If the resulting FSM is F2, then what language does F2 recognize? **(4)**

(d) Another friend of yours slightly changes F1 (Figure 1.a) without changing the number of states and the number of transitions. Now, he calmly claims that the resulting FSM, F3, is such that $|L(F3)| = |L(F1)| + 1$, i.e., the cardinality of the language of F3 increases by exactly 1 (because of the change made on F1). Do you think this is possible? Justify your answer. **(4)**

(e) Suppose the cardinality of the language represented by the RE R1 is 5. Now, you have designed a DFA D1 equivalent to R1 and have written a C program implementing D1. Subsequently, you have tested the program against 1000 random different strings and for each of those strings your program returns "Accept" indicating that the DFA accepts that string. Assuming that your C implementation is correct your genius little brother claims that your DFA is wrong. Do you agree with him? Justify your answer. **(3)**

Contd P/2

CSE 211

2. (a) Design a DFA, M that takes as input a binary string and recognizes numbers that are NOT divisible by 3. For example, M rejects 1111 (because the corresponding numeric value is 15, which is divisible by 3), M accepts 111 (because the corresponding numeric value is 7, which is not divisible by 3) and so on. You need to discuss your intuition/idea behind the design and explain how it will work. Drawing the DFA only will not guarantee full marks. **(8⅓)**

(b) Prove that the class of regular languages is closed under the regular operations. **(15)**

3. (a) Compute DFA and Regular Expressions for the following languages: **(18)**

   (i) L = {w | w begins with a 1 and ends with a 0}.
   (ii) L = {w | w has length at least 3 and its third symbol is a 0}.
   (iii) L = {w | w contains an even number of 0's or exactly two 1's}.
   (iv) L = {w | w is any string except 11 and 111}.
   (v) L = {w | w contains at least 2 0's and at most one 1}.

(b) Your teacher has proved that if M is a DFA that recognizes language B, swapping the accept and non-accept states in M yields a new DFA recognizing the complement of B. Now your genius brother comes up with an example that if M1 is an NFA that recognizes language C, swapping the accept and non-accept states in M1 doesn't necessarily yield a new NFA that recognizes the complement of C. Can you find such an example? **(5⅓)**

4. (a) Consider the following context-free Grammar G: **(7⅓)**

$R \rightarrow XRX \mid S$

$S \rightarrow aTb \mid bTa$

$T \rightarrow XTX \mid X \mid \varepsilon$

$X \rightarrow a \mid b$

Now answer all the following questions:

   (i) What are the variables of G?
   (ii) What are the terminals of G?
   (iii) Which is the start variable of G?
   (iv) Give three strings in L(G).
   (v) Give three strings not in L(G).
   (vi) Give a description of L(G).

Contd P/3

CSE 211

Contd... Q. No. 4(a)

(vii) Now, write True or False for the following statements (S1-S9):

S1: $T \Rightarrow$ aba .

S2: $T \overset{*}{\Rightarrow}$ aba .

S3: $T \Rightarrow T$.

S4: $T \overset{*}{\Rightarrow} T$.

S5: $XXX \overset{*}{\Rightarrow}$ aba .

S6: $X \overset{*}{\Rightarrow}$ aba .

S7: $T \overset{*}{\Rightarrow} XX$.

S8: $T \overset{*}{\Rightarrow} XXX$.

S9: $S \overset{*}{\Rightarrow} \varepsilon$.

(b) Find context free grammars for the following languages: **(10)**

(i) The set of binary strings containing at least three 1's.

(ii) The set of odd-length binary strings having 0 at the middle.

(iii) The set of binary strings having more 0's than 1's.

(iv) $L = \{w\#x \mid w^R$ is a substring of x for w, x $\in \{0, 1\}^*\}$.

(c) Use pumping lemma to prove that the languages $A = \{0^n1^n2^n \mid n \geq 0\}$ and $B = \{0^i1^j \mid i > j\}$ are not regular. **(6)**

SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Suppose the PDA $P = (\{q, p\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$ has the following transition function: **(8⅓)**

(i) $\delta(q, 0, Z_0) = \{(q, X Z_0)\}$

(ii) $\delta(q, 0, X) = \{(q, X X)\}$

(iii) $\delta(q, 1, X) = \{(q, X)\}$

(iv) $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$

(v) $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$

(vi) $\delta(p, 1, X) = \{(p, X X)\}$

(vii) $\delta(p, 1, Z_0) = \{(p, \epsilon)\}$

Draw the transition diagram for this PDA.

Contd P/4

CSE 211

Contd... Q. No. 5

(b) For the PDA described in Ques. 5(a), starting from the initial ID (q, w, Z_0), show all the reachable ID's when the input w is: **(15)**

(i) 0011

(ii) 0101

(iii) 1010

6. (a) Define a PDA that accepts the language $\{w, \in \{a, b\}^* : w$ has the same number of a's and b's$\}$. **(13)**

(b) Consider a PDA P_F that accepts a language L by final state. Is it always possible to construct another PDA P_N that accepts the same language L by empty stack? If possible, show how the construction is done. Provide definition of both P_F and P_N. If its not possible, justify why. **(10⅓)**

7. (a) Design a Turing machine for the language $\{a^n b^n c^n \mid n \geq 1\}$. Also briefly describe its working principle. **(13)**

(b) Suppose, you know a particular problem P is undecidable. You want to prove another problem Q is undecidable too. Utilize P and the concept of reduction to prove Q is undecidable. **(10⅓)**

8. (a) Design a Turing machine that computes the function $a \dot{-} b = max(b - a, 0)$. Initially the tape contains a, followed by b. Both a and b are represented by consecutive 1's separated by the symbol 0. After computation, the tape will contain only the output $a \dot{-} b$, represented by consecutive 1's and surrounded by infinite blanks. Provide brief description of the working principle of your designed Turing machine. **(13)**

(b) Describe how multiple tracks and multiple tapes extend the basic Turing machine. Briefly discuss the difference between these extensions. **(10⅓)**

L-2/T-2/CSE **Date : 30/01/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2015-2016

Sub : **CSE 207** (Algorithms)

Full Marks: 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What is the matrix chain multiplication problem? Prove that the matrix chain multiplication problem has both the overlapping subproblems property and the optimal substructure property. **(10)**

(b) (i) Write the steps that are to be followed for solving a problem using dynamic programming method. **(4+6=10)**

(ii) Let $X = \langle x_1, x_2, \cdots, x_m \rangle$ and $Y = \langle y_1, y_2, \cdots, y_n \rangle$ be sequences, and let $Z = \langle z_1, z_2, \cdots, z_k \rangle$ be any LCS of X and Y. If $x_m \neq y_n$ and $z_k \neq x_m$, then prove that Z is an LCS of X_{m-1} and Y.

(c) Find all longest common subsequences of $X = \langle \text{AGGCTGAT} \rangle$ and $Y = \langle \text{GGGCATA} \rangle$ using dynamic programming method. **(15)**

2. (a) (i) Explain why one can solve the fractional knapsack problem by a greedy strategy, but one cannot solve the 0-1 knapsack problem by such a strategy. **(5+5=10)**

(ii) Find an optimal solution to the fractional knapsack instance of $n = 6$, $M = 40$, $(p_1, p_2, \cdots p_6) = (18, 5, 15, 30, 18, 35)$, and $(w_1, w_2, \cdots w_6) = (9, 5, 10, 10, 12, 10)$.

(b) (i) Show that the 0-1 knapsack problem has the overlapping subproblems property. **(4+6=10)**

(ii) Write an algorithm for solving the 0-1 knapsack problem. Analyze the time-complexity of the algorithm.

(c) (i) Compare backtracking, and branch and bound techniques. **(3+12=15)**

(ii) Solve the following instance of the 0/1 knapsack problem using the branch-and-bound approach with a state-space-tree. Assume that the knapsack capacity is 15.

Item	Weight	Value
1	7	140
2	6	132
3	4	140
4	3	90

Contd P/2

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3. (a) Write the Edmonds-Karp algorithm that solves the maximum-flow problem. Analyze the time-complexity of the algorithm for integral capacities. **(10)**

(b) Explain how network flow algorithms allow us to find the maximum bipartite matching. **(10)**

(c) Write an application of prefix sums. What are prefix sums of an array A[i], $1 \le i \le n$? Write a parallel algorithm that finds prefix sums of the array A. **(15)**

Illustrate the prefix sums algorithm for the array [15, 35, 40, 25, 20, 50, 10, 30] of eight elements by showing the values of arrays used in bottom-up and top-down traversals.

4. (a) (i) Compare the techniques of brute-force algorithms, heuristics, and approximation algorithms for coping with hard problems. **(5+5=10)**

(ii) Write a backtracking algorithm to color a map with no more than four colors.

(b) Explain the term 'reducibility'. If L_1 and L_2 are two languages such that $L_1 \le_p L_2$, then show that $L_2 \in$ P. If any NP-complete problem is polynomial-time solvable, then prove that P = NP. **(10)**

(c) (i) What is the vertex-cover problem? Write a polynomial-time approximation algorithm for the vertex-cover problem. Prove the time-complexity and find the approximation ratio of the algorithm. **(7+8=15)**

(ii) For the general traveling-salesman problem, prove that one cannot find good approximate tours in polynomial time unless P = NP.

SECTION-B

There are **NINE** questions in this section. Answer any **SEVEN**.

5. (a) Define big O-notation, Ω-notation and Θ-notation. **(6+9=15)**

(b) For each of the following pairs indicate whether $f(n)$ is O(g(n)), Ω(g(n)) or both (i.e. Θ(g(n))). Provide brief justifications.

$f(n)$	$g(n)$
$\sqrt{n}$	$\log_2 n$
$n^{1.01}$	$n \log_2^2 n$
2^n	2^{n+1}
$n!$	2^n
n^{100}	1.01^n
$\sum_{i=1}^{n} i^k$	n^{k+1}

Contd P/3

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6. Suppose you are choosing among the following three (possibly hypothetical) algorithms for multiplying two $n \times n$ matrices: **(15)**

   - Algorithm A solves the problem by dividing each $n \times n$ matrix into four $n/2 \times n/2$ matrices, make 8 recursive calls to compute intermediate results and then combine them in $\Theta(n^2)$ time to get the result.
   - Algorithm B (Strassen's algorithm) solves the problem by dividing each $n \times n$ matrix into four $n/2 \times n/2$ matrices, make 7 recursive calls to compute intermediate results and then combine them in $\Theta(n^2)$ time to get the result.
   - Algorithm C solves the problem by dividing each $n \times n$ matrix into nine $n/3 \times n/3$ matrices, make 9 recursive calls to compute intermediate results and then combine them in $\Theta(n^2)$ time to get the result.

   Use the master theorem to determine asymptotic running times of the three algorithms. Which one would you choose?

7. Counting Inversions: This problem arises in the analysis of rankings. Consider comparing two rankings. One way is to label the elements (books, movies, etc.) from 1 to n according to one of the rankings then order these labels according to the other ranking, and see how many pairs are "out of order". **(15)**

   We are given a sequence of n distinct numbers $a_1, \cdots, a_n$. We say that two indices $i < j$ form an inversion if $a_i > a_j$ that is if the two elements a_i and a_j are "out of order". Provide a divide and conquer algorithm to determine the number of inversions in the sequence $a_1, \cdots, a_n$ in time $O(n \log n)$.

   (Hint: Modify merge sort to count during merging)

8. Given an undirected graph, $G = (V, E)$, provide a linear time algorithm to check whether there is a cycle of odd length in the graph. Briefly justify its correctness. **(15)**

9. Suppose a CS curriculum consists of n courses, all of them mandatory. The prerequisite directed graph G has a node for each course, and an edge from course v to course w if and only if v is a prerequisite for w. Find an algorithm that works directly with this graph representation, and computes the minimum number of semesters necessary to complete the curriculum (assume that a student can take any number of courses in one semester). The running time of your algorithm should be linear. **(15)**

Contd P/4

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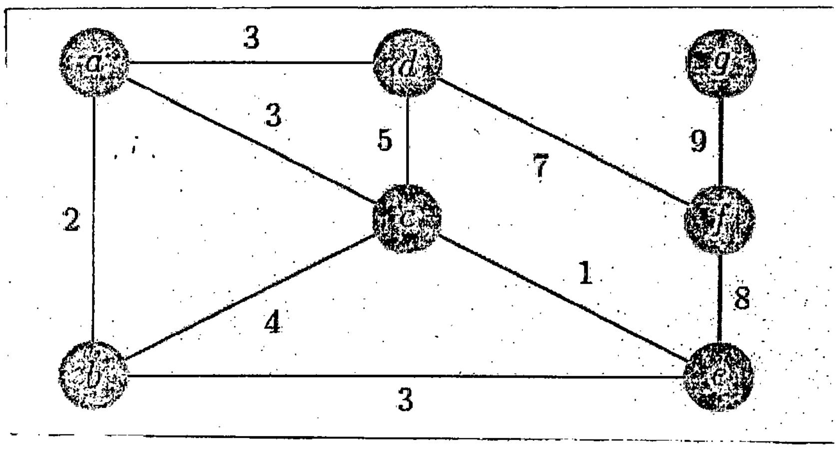
10. There is a network of roads $G = (V, E)$ connecting a set of cities V. Each road in E has an associated length l_e. There is a proposal to add one new road to this network, and there is a list E' of pairs of cities between which the new road can be built. Each such potential road $e' \in E'$ has an associated length. As a designer for the public works department you are asked to determine the road $e' \in E'$ whose addition to existing network G would result in the maximum decrease in the driving distance between two fixed cities s and t in the network. Give an algorithm for solving this problem. Your algorithm should run in $O((V + E + E') \log V)$ time. **(15)**

11. Find a minimum spanning tree (MST) in the following graph by running (i) Kruskal' algorithm, and (ii) Prim's algorithm. Show the order of the edges that are selected in each case (whenever there are choices of vertices, always use alphabetic ordering). **(15)**



12. Recall the Interval Scheduling Problem from class. You are given n intervals with the starting and finishing times of i-th interval are given by s_i and f_i respectively. A subset of the intervals is compatible if no two of them overlap in time. The goal is to find a compatible set of maximum size. Prove that the greedy strategy of picking an interval, which is compatible with intervals already picked, with smallest finishing time gives an optimal solution. **(15)**

13. (a) Under a Huffman encoding of n symbols with frequencies $f_1, f_2, \cdots, f_n$, what is the longest codeword could possibly be? Give an example set of frequencies that would produce this case. **(7+8=15)**

(b) Following a set of characters and corresponding frequencies:

Character	Frequency
A	45
B	13
C	12
D	16
E	5
F	9

Construct Huffman codes for the characters using the greedy algorithm.
