THE DIAGONAL COMPRESSION FIELD THEORY FOR
REINFORCED CONCRETE BEAMS SUBJECTED TO COMBINED
TORSION, FLEXURE AND AXIAL LOAD

by

WINSTON M. ONSONGO

Department of Civil Engineering

A Thesis submitted in conformity with the requirements
for the Degree of Doctor of Philosophy in the
University of Toronto

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THE DIAGONAL COMPRESSION FIELD THEORY FOR REINFORCED CONCRETE BEAMS
SUBJECTED TO COMBINED TORSION, FLEXURE AND AXIAL LOAD

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ABSTRACT

This thesis presents a theoretical model capable of predicting the post-cracking response of reinforced concrete beams subjected to combined torsion, flexure and axial load. The model extends the scope of the recently presented Diagonal Compression Field Theory which in its original form was only capable of predicting the behaviour of symmetrically reinforced sections under pure torsion.

The Diagonal Compression Field Theory is formulated by considering equilibrium conditions, compatibility of geometric deformations and the stress-strain characteristics of the steel and the concrete. It is assumed that plane sections remain plane and that after cracking the concrete can not resist tension. The concrete is assumed to resist the torsional shear flow by means of diagonal compressive stresses. The direction of these principal stresses at any particular point on the section is assumed to coincide with the direction of principal compressive strains at this point.

Also presented in this thesis are the test results of fourteen large, heavily reinforced rectangular beams tested in combined torsion and bending. The detailed measurements of deformation made during these tests (over 250 strain readings were taken on each beam at each load stage) verified the detailed predictions of the compression field theory.

Numerical techniques which enable a solution to be obtained for rectangular beams subjected to torsion and flexure
have been incorporated into a computer program which is listed and explained in the thesis.

The thesis restricts its attention to beams in which torsion is resisted by shear flow (St. Venant torsion) and in which the reinforcement is properly detailed and consists of closed hoops perpendicular to the axis of the member and longitudinal reinforcing steel (which may be prestressed) parallel to the axis of the member.
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TO MY PARENTS
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CHAPTER 1 - INTRODUCTION

Many modern structures such as elevated guideways are subjected to combined flexure, torsion, shear and axial load. Designing such structures requires an understanding of the behaviour of structural concrete under such combined actions. A long-term research program is in progress in the Department of Civil Engineering of the University of Toronto whose overall objective is to develop simple, general and rational models capable of predicting the response of structural concrete members under combined loading. Eventually it is hoped to replace the complex, restricted, empirical, design rules now used for shear and torsion by procedures comparable in rationality with those available for flexure and axial loads.

As part of this program, a rational model called the Diagonal Compression Field Theory capable of predicting the behaviour of symmetrically reinforced structural concrete members subjected to pure torsion, was developed. This theory like the well known theory for flexure was developed by considering equilibrium conditions, compatibility of geometric deformations and the stress-strain characteristics of the steel and the concrete. Further, after cracking, it was assumed that the concrete could not resist tensile stresses and that the concrete cover would spall off down to the hoop centreline.

This thesis extends the scope of the Diagonal Compression Field Theory so that it is capable of predicting the post-cracking behaviour of reinforced concrete members subjected to combined torsion,
The important difference between the Compression Field Theory as presented in this thesis and variable angle space truss models for beams in torsion and flexure such as that presented by Rabbat is that in these space truss models empirical assumptions are made as to the effective geometry of the section. In the Compression Field Theory the thickness of the concrete diagonals, the position of the resultant steel and concrete forces and the path of the shear flow are all determined from equilibrium and compatibility requirements.

In addition to presenting the theoretical model this thesis will describe an experimental program in which fourteen heavily reinforced, intensively instrumented rectangular beams were loaded in combined torsion and flexure. Heavily reinforced beams were chosen as they would provide the most sensitive test of the theory. Detailed deformation measurements were made during the tests so that the detailed predictions of the theory could be checked. While the detailed solutions described in this thesis will be for rectangular sections subjected to combined torsion and flexure the general theory presented can be applied to other cross-sectional shapes. The theory is however applicable only to St. Venant torsion (i.e., torsion resisted by shear flow) and hence cannot predict the response of sections where warping torsion dominates.

The thesis restricts its attention to beams in which the
reinforcement is properly detailed and consists of closed hoops perpendicular to the axis of the member and longitudinal steel (which may be prestressed) parallel to the axis of the member.

In the available English literature the only other attempt to develop a theory capable of predicting the complete post-cracking behaviour of under-reinforced and over-reinforced concrete beams subjected to combined torsion and flexure is that by Below, Rangan and Hall. However, their theory, which is based on the skew-bending model, contains a number of arbitrary assumptions and does not predict well the behaviour of over-reinforced beams nor the behaviour of beams loaded predominantly in torsion. Further, their theory is not capable of treating torsion and biaxial flexure nor torsion, bending and axial load.

It is emphasized that the objective of this thesis is to develop a model for structural concrete beams in combined torsion, flexure and axial load which is comparable in rationality and generality to the well known theory for flexure and axial load.
2.1 ASSUMPTIONS

The following basic assumptions are made in the development of the theoretical model.

1. The longitudinal strains along any line in a section vary linearly (Bernoulli-Navier hypothesis).

2. All strains at a point in a plane are compatible.

3. The principal direction of diagonal compression strain evaluated from a set of compatible strains at a point in a plane of loading is coincident with the direction of principal compression stress at that point in the plane.

4. In the post-cracking loading range the concrete can carry no tension and the applied actions will be resisted by a field of diagonal compression in the concrete and the reinforcing steel will resist the tension.

5. The stress-strain relationships for the concrete and the reinforcing steel in a structural concrete beam under loading are assumed the same as the stress-strain relationships obtained from a standard cylinder test for concrete and the stress-strain relationship obtained from a standard tension test for the steel.
2.2 EQUILIBRIUM CONDITIONS

Figure 2.1 shows the assumed cracked model for a rectangular beam section subjected to uniform flexural moments $M_y$ and $M_z$, a torque, $T$, and an axial force $N_x$. The shear flow set up by the torque is resisted in a field of diagonal compression in the concrete after cracking. The inclination of the concrete diagonals and their effective depths in the beam section are determined by the equilibrium requirements and requirements of compatibility of deformations.

**FIG. 2.1 CRACKED REINFORCED CONCRETE BEAM MODEL.**
2.2.1 Concrete Stresses

Consider the state of stress in a concrete element A in the cracked model (Fig. 2.1). Such an element which is assumed to be in the plane of shear flow will be subjected to a general set of stresses in the plane shown in Fig. 2.2 (a). If it is assumed that concrete has no tensile strength, the stresses on the element can be considered equivalent to a principal compression stress $f_d$ as shown in Fig. 2.2 (b). This state of stress can be represented by Mohr's circle shown in Figure 2.2 (c) and the following relationships are deduced:

\[
\begin{align*}
\tau &= f_d \frac{\tan \theta}{1 + \tan^2 \theta} \quad \cdots \quad (2.1) \\
\sigma_{ch} &= \tau \tan \theta \quad \cdots \quad (2.2) \\
\sigma_{cl} &= \tau / \tan \theta \quad \cdots \quad (2.3)
\end{align*}
\]

FIG. 2.2 STRESSES ON CONCRETE ELEMENT
The shear stress $\tau$ when integrated over the wall thickness, $t$, at a location on the section, will result in the shear flow $q$ which is constant around the section and balances the applied torque, $T$:

$$q = \int \tau \, dt \quad \text{(2.4)}$$

and $T = 2qA_0 \quad \text{(2.5)}$

where $A_0$ is the area enclosed by the effective path of shear flow around the section.

2.2.2 Steel Stresses

The diagonal concrete stresses, $f_d$, give rise to forces which are equilibrated by the tension developed in the reinforcing steel. The concrete and the steel stresses can be related by considering the equilibrium of appropriate free body diagrams. For simplicity a rectangular section shall be used to derive these relationships.

2.2.2.1 Longitudinal Steel Stresses

![Diagram of longitudinal steel reinforcement in a beam section](image)

**FIG. 2.4 LONGITUDINAL STEEL IN SECTION**
Consider section A-A of an unsymmetrically reinforced rectangular section shown in Fig. 2.4. The longitudinal stresses can be evaluated by considering the equilibrium of longitudinal forces at the section.

If given the longitudinal strain $\varepsilon_{\text{L}0}$ at the origin $(0,0)$, then using assumption (1) the strains of the steel element at position $(z_i,y_i)$ can be evaluated as:

$$\varepsilon_{\text{Li}} = \varepsilon_{\text{L}0} + \phi_{xy} y_i + \phi_{xz} z_i$$

Where:

$\phi_{xy} = \text{curvature in } Y \text{- direction}$

$\phi_{xz} = \text{curvature in } Z \text{- direction}$

With the longitudinal strains the corresponding steel stresses are evaluated using the stress-strain relationships for the steel obtained from standard tension tests of representative specimens of the steel. Thus the resultant tension force in the reinforcing steel in the longitudinal direction can be evaluated as:

$$F_{sl} = \sum f_{li} A_{li}$$

where $f_{li}$ in the stress of the steel element of area $A_{li}$ at $(z_i,y_i)$ and is a function of $\varepsilon_{li}$.

The position at which the resultant steel tension force, $F_{sl}$, acts with respect to the origin chosen is $(\bar{z}_s, \bar{y}_s)$ where

$$\bar{y}_s = \frac{\sum f_{li} A_{li} y_i / F_{sl}}{\sum f_{li} A_{li} / F_{sl}}$$

$$\bar{z}_s = \frac{\sum f_{li} A_{li} z_i / F_{sl}}{\sum f_{li} A_{li} / F_{sl}}$$

The resultant compression force, $F_{cl}$, in the concrete in the longitudinal direction at section A-A is evaluated by
integrating the compression concrete stresses $\sigma_{ci}$ over the cross section:

$$F_{ci} = \int_0^{t} \int_0^{p} f_{ci} \, dt \, dp$$

(2.9)

where the stresses are summed over the wall thickness, $t$, and around the entire section perimeter, $p$.

The position at which the force $F_{ci}$ acts with respect to the origin chosen is $(\bar{z}_{c}, \bar{y}_{c})$, where

$$\bar{y}_{c} = \int_0^{t} \int_0^{p} y_{ci} \, dt \, dp / F_{ci}$$

$$\bar{z}_{c} = \int_0^{t} \int_0^{p} z_{ci} \, dt \, dp / F_{ci}$$

(2.10)

The internal forces $F_{sl}$ and $F_{cl}$ and the moments they induce will be in equilibrium with the applied actions. If moments are evaluated about the section centroid $(Z_o, Y_o)$ and if it is assumed that the applied axial load $N_X$ is tensile and applied along the longitudinal centroidal axis then equilibrium requires:

$$N_X = F_{sl} - F_{cl}$$

$$M_Y = F_{sl} (\bar{y}_{c} - \bar{y}_{s}) + N_X (Y_o - \bar{y}_{c})$$

$$M_Z = F_{sl} (\bar{z}_{c} - \bar{z}_{s}) + N_X (Z_o - \bar{z}_{c})$$

(2.11)

If $F_{sl} = F_{cl}$, $\bar{y}_{c} = \bar{y}_{s}$ and $\bar{z}_{c} = \bar{z}_{s}$, a state of pure torsion exists.

2.2.2.2. Hoop Steel Stresses

The hoop steel stresses can be related to the concrete diagonal stresses, $f_{dl}$, by considering the equilibrium of forces in the transverse direction for free bodies obtained by taking sections parallel to the longitudinal axis as in Figure 2.5.
For simplicity assume the hoop steel spacing, \( s_h \), is constant over the entire beam span for all the beam faces. Then in equilibrium considerations in the direction of hoop steel, only the stresses over a typical length, \( s_h \), of the span are considered. With reference to Fig. 2.5 section B-B:

\[
F_{Hi} = A_{hf}f_{hi} = \text{resultant tension force in hoop steel at position '}i'\]

\( A_{hf} = \text{area of hoop steel section for face '}f' \) where 'i' is positioned

\( f_{hi} = \text{hoop steel stress at position '}i' \)

\[
F_{chi} = s_h \int \sigma_{chi} dt \quad \ldots \ldots \ldots \ldots \ldots (2.12)
\]

= resultant compression force in concrete in the direction of hoop steel at position 'i' which effectively acts at a distance \( a_{ij}/2 \) from the external surface.

\( \sigma_{chi} \) is given by equation (2.2) and the integral
in equation 2.12 is evaluated over the wall thickness of the section at position 'i'.

Applying the conditions for equilibrium (\(\Sigma F = 0\) and \(\Sigma M = 0\)) of the forces shown in Section B-B, deduce:

\[
F_{Hi} = F_{chi}[1-(a_i/2 - c_{hi})/d_{ij}] + F_{chj}(a_j/2 - c_{hj})/d_{ij} \ldots (2.13)
\]

\[
F_{Hj} = F_{chi}(a_i/2 - c_{hi})/d_{ij} + F_{chj}[1-(a_j/2 - c_{hj})/d_{ij}] 
\]

The evaluation of \(a_i\) and \(a_j\) is described in section 2.3.4.

2.3 GEOMETRIC CONDITIONS

2.3.1 Compatibility requirements for strains

While the local strains of the diagonally cracked beam may exhibit local discontinuities the average strains in the various directions are related to one another by the requirements of compatibility. Thus, if at a point in a plane the following average strains are measured (Fig. 2.6 (a)):

- \(\varepsilon_l\) in the longitudinal direction \(l-l\);
- \(\varepsilon_t\) in the transverse direction \(t-t\);
- \(\varepsilon_d\) in the diagonal direction \(d-d\) at angle \(\theta\) to \(l-l\);

and \(\gamma_{lt}\) is the shear strain between \(l-l\) and \(t-t\);

then these set of strains will be compatible. Mohr's circle of strain, Fig. 2.6 (b), can be used to obtain a relationship between these strains.
FIG. 2.6 GEOMETRY OF STRAINS

With reference to Figure 2.6(b):

\[ 1\Gamma_{G1} = \frac{1}{2} (\varepsilon_t + \varepsilon_l) \]
\[ 1\Gamma_{G1} = \frac{1}{2} (\varepsilon_t - \varepsilon_l) \]

By constructing \( \triangle GFC'' \) congruent to \( \triangle GAA' \) such that \( 1\Gamma_{G1} = 1\Gamma_{G1} \) and \( 1\Gamma_{C''1} = 1\Gamma_{AA'1} = \frac{1}{2} \gamma_l \), we deduce

\[ GC = OG + GF \cos (2\pi - 2\theta) + FC'' \cos (2\theta - \pi/2) \]

hence \( \varepsilon_d = (\varepsilon_t + \varepsilon_l)/2 - (\varepsilon_t - \varepsilon_l)\cos2\theta/2 + \gamma_l \sin2\theta/2 \)

or \( \varepsilon_d = \varepsilon_t \sin^2\theta + \varepsilon_l \cos^2\theta - \gamma_l \sin2\theta/2 \) \[ \ldots \ldots \ldots (2.14) \]

Equation (2.14) gives the relationship for compatible strains at a point in a plane.

SIGN CONVENTION FOR STRAINS

The longitudinal strain \( \varepsilon_l \) and the transverse strain
\( \varepsilon_t \) shall be considered positive when in tension. The concrete strains of interest are the principal diagonal compression strains denoted by \( \varepsilon_{dp} \). Because these strains will always be compressive it is convenient to consider them positive when compressive.

From Mohr's circle of strain (Fig. 2.6 (b)) it is deduced that the principal diagonal compressive strain, \( \varepsilon_{dp} \), will occur at an angle \( \theta_p \) with respect to the direction \( l-l \). If this average principal diagonal strain \( (\varepsilon_{dp}) \) is used to construct Mohr's circle instead of \( \varepsilon_d \), the point \( C'' \) in Fig. 2.6 (b) will coincide with \( C \) and with reference to Figure 2.6(c) the following simple relationships are deduced:

\[
\text{From } \triangle BBC: \tan \theta_p = \frac{\gamma_{lt}}{2(\varepsilon_t + \varepsilon_{dp})} \quad (a)
\]

\[
\text{From } \triangle ACC: \tan \theta_p = \frac{2(\varepsilon_l + \varepsilon_{dp})}{\gamma_{lt}} \quad (b)
\]

(a) \( \times \) (b) gives: \( \tan^2 \theta_p = \frac{\varepsilon_l + \varepsilon_{dp}}{\varepsilon_t + \varepsilon_{dp}} \quad ................ (2.15) \)

(a) \( \div \) (b) gives: \( \gamma_{lt} = 2\sqrt{(\varepsilon_l + \varepsilon_{dp})(\varepsilon_t + \varepsilon_{dp})} \quad ........ (2.16) \)

Thus for given values of the three strains \( \varepsilon_l, \varepsilon_t \) and \( \varepsilon_{dp} \) the direction of average principal compression strain can be evaluated using equation (2.15). It will be assumed that for the diagonally cracked beam the direction of average principal compressive stress will coincide with the direction of the average principal diagonal strain defined by equation (2.15). It is of interest to note that equation (2.15) is essentially identical to the expression for the direction of the principal tensile stresses in thin walled elastic metal beams derived from energy
considerations by Wagner in 1929.

2.3.2. EVALUATION OF TWIST

Figure 2.7(a) shows a portion of a prismatic beam of length $\delta x$ subjected to a torque $T$ which causes a change in the torsional rotation of section B relative to section A of $\delta \phi$ radians. From the beam section following any contour's' around the beam longitudinal axis, isolate a thin walled tube element (Fig. 2.7(b)) with a perimeter equal to the length of the contour 's' and length $\delta x$. For compatibility of displacement the thin walled tube element will also have a total change of rotation of $\delta \phi$ radians over the length $\delta x$.

The compatibility condition for the thin walled tube element is that the warping around its perimeter is zero. Suppose the tube element is cut along $A_1 B_1$. The resulting open section will warp under the change in rotation of section B relative to A. Consider the displacement of the 'slice' element $A_1 B_1 A_2 B_2$ which is cut out of the tube element. Under the change in rotation of $\delta \phi$ the slice element if free to displace will do so as indicated in Fig. 2.7(c), and an out of plane displacement $dW_1$ will result. If this element is also subjected to a shearing strain, $\gamma$, as indicated in Fig. 2.7(d), the shearing strain will also cause an out of plane displacement, $dW_2$. The compatibility condition requires that for the closed tube element the sum of the outofplane displacements for all slice elements around the section will be zero. Hence,
\[ \delta \phi_{1} + \delta \phi_{2} = 0 \]

Therefore, \( -\phi \int \frac{\delta \phi}{\delta x} ds + \phi \gamma ds = 0 \)

Recognizing that \( \frac{\delta \phi}{\delta x} \) is the twist \( \psi_x \) and is constant for all the slice elements, deduce

\[ \psi_x = \frac{1}{2A_s} \phi \gamma ds \] \[ (2.17) \]

where \( 2A_s = \phi r ds \)

\( A_s = \text{area enclosed by the contour 's'} \)

Thus, if the shear strain, at every point along any contour 's' in any section where St Venant torsion dominates, is known, equation (2.17) can be used to evaluate the twist.

FIG. 2.7 EVALUATION OF TWIST
2.3.3 Geometry of Curvature and Twist

When a concrete beam is twisted and bent it is observed that the beam surfaces do not remain plane. The twist, $\psi_x$, and the curvature, $\phi_l$, of the beam's longitudinal axis and the curvature, $\phi_d$, of the beam's transverse axis induce curvature, $\phi_d$, in a diagonal direction in a given beam face. For small deformations $\psi_x$, $\phi_l$, $\phi_t$ and $\phi_d$ will be compatible and they can be represented by Mohr's circle (Fig. 2.8). Curvature shall be considered positive when it tends to induce compression in the surface fibre. It is convenient to measure the curvatures in the same directions as those used for measurement of strains in section 2.3.1.

FIG. 2.8 GEOMETRY OF CURVATURE AND TWIST
From Mohr's circle in Figure 2.8 deduce:

\[
\overline{OC} = \overline{OG} + \overline{GF} \cos 2\theta' + \overline{FC}'' \cos (\pi/2 - 2\theta')/
\]

Hence \( \phi_d = \frac{1}{2}(\phi_t + \phi_i) + \frac{1}{2}(\phi_t - \phi_i) \cos 2\theta' + \psi_x \sin 2\theta' \)

or \( \phi_d = \phi_t \sin^2 \theta' + \phi_i \cos^2 \theta' + \psi_x \sin 2\theta' \)

Of particular interest is the diagonal curvature in the direction of principal diagonal strain which is at an angle \( \theta_p \) to the longitudinal direction \( l-l \). This curvature can be evaluated by setting \( \theta' = \theta_p \) and hence.

\[ \phi_{dp} = \phi_t \sin^2 \theta_p + \phi_i \cos^2 \theta_p + \psi_x \sin 2\theta_p \ldots \ldots \ldots (2.18) \]

Due to the curvature \( \phi_{dp} \), the concrete compression strains will vary through the thickness, \( t_d \), of the concrete diagonal. This results in the maximum average compressive strain, \( \varepsilon_{ds} \), occurring on the surface. Experimental evidence (1) suggests it is reasonable to assume a linear compressive strain variation down the concrete diagonal depth with \( \varepsilon_{ds} \) on the surface of the diagonal and zero at the depth \( t_d \) which is the effective diagonal depth. Hence, with reference to Figure 2.9, the effective depth can be evaluated from the curvature \( \phi_{dp} \) and the principal compression strain on the surface \( \varepsilon_{ds} \) as:

\[ t_d = \varepsilon_{ds}/\phi_{dp} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.19) \]

2.3.4. Path of shear flow

If the concrete strain distribution is known over the effective depth of the compression diagonal, the magnitude and line of action of the resultant compression force can be evaluated using the stress strain characteristics of the concrete.
A convenient approach is to replace the concrete stress distribution with an equivalent rectangular stress block having an equivalent uniform stress over a depth 'a' where depth 'a/2' from the surface defines the position of the line of action of the resultant compression force as illustrated in Figure 2.9.

\[
f_d = f_c \left(2 \frac{\Omega_d}{\Omega_d^2} \right) \quad \Omega_d = \frac{\varepsilon_d}{\varepsilon_o}
\]

For a simple and accurate stress-strain relationship for concrete, a parabola which has been found to provide a good fit for the stress-strain relationship obtained from a standard concrete cylinder compression test, will be used. This assumes that the concrete diagonals in a cracked concrete beam are virtually loaded in compression is the principal direction and this state of loading is closely approximated by the loading on concrete in the standard compression test. Thus the compression stress \( f_d \) can be evaluated from a given strain, \( \varepsilon_d \), as:

\[
f_d = f_c \left(2 \frac{\Omega_d}{\Omega_d^2} \right) \quad \Omega_d = \frac{\varepsilon_d}{\varepsilon_o}
\]
\[ e_o = \text{compression strain in concrete corresponding to} \]
\[ \text{the maximum stress achieved } f'_c \text{ for a standard } \]
\[ \text{cylinder test.} \]

Using equation (2.20) the factors \( \alpha \) and \( \beta \) which define the equivalent stress block in Figure 2.9 can be evaluated as follows:

\[ \alpha = \frac{6\Omega_{ds} (1-\Omega_{ds}/3)^2}{4 - \Omega_{ds}} \quad \text{(2.21)} \]

\[ \beta = \frac{4 - \Omega_{ds}}{6 - 2\Omega_{ds}} \quad \text{(2.22)} \]

\[ \alpha \beta = \Omega_{ds} - \Omega_{ds}^2/3 \quad \text{(2.23)} \]

where \( \Omega_{ds} = \frac{\varepsilon_{ds}}{e_o} \)

With the factor, \( \beta \), the depth of the equivalent stress block, \( a = \beta t_d' \), can be evaluated. The effective path of shear flow can be taken to be at \( a/2 \) from the surface along the plane of action of the resultant compression force (Fig. 2.9).

In the evaluation of the resultant diagonal compression force as presented above, it is assumed that the effective depth of compression at a given point, \( t_d \), is less than the section wall thickness, \( t \). If the depth \( t_d' \) is greater than the wall thickness, \( t \), correction terms (see Appendix A and B) can be introduced in the derived equations to obtain \( \alpha \) and \( \beta \) factors. It has also been assumed that the compression strains in the concrete diagonals are those introduced by the applied loads.
CHAPTER 3
APPLICATION TO RECTANGULAR CROSS-SECTION
WITH ONE AXIS OF SYMMETRY

The theory developed in chapter two will now be applied to analyse a rectangular structural concrete section with one axis of symmetry shown in Figure 3.1. The section will be analysed for the particular loading of a torque $T$ in the $x$-axis and a moment $M$ causing curvature of the $x-z$ plane. Hoop steel content is the same in all faces with the spacing of $s_h$ along the entire span of the beam.

FIG. 3.1 RECTANGULAR SECTION FOR ANALYSIS
3.1 EQUILIBRIUM REQUIREMENTS

3.1.1 Equilibrium in the Longitudinal Direction

With a longitudinal strain profile at a cross section and given the stress-strain relationships for the longitudinal steel the resultant tensile force in the steel, $F_{Cl}$, and the position at which it acts defined by $Y_s$ can be evaluated easily and exactly using equations (2.7) and (2.8). On the other hand the resultant compression force, $F_{C_\ell}$, given by equation (2.9) is not so easy to evaluate as it requires the evaluation of the principal compression angles, $\Theta_p$, which change from point to point on the perimeter along the path of shear flow.

Using equation (2.3) rewrite equation (2.9) as

$$F_{C_\ell} = \int \rho (\cot \Theta_p) \, dp \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.1)$$

If it is assumed that $\Theta_p$ remains constant down the depth of a diagonal at a particular point on the perimeter, equation (3.1) can be rewritten as:

$$F_{C_\ell} = \rho \cot \Theta_p \int \delta \, dt \, dp$$

and using equation (2.4), this becomes

$$F_{C_\ell} = \rho q \cot \Theta_p \, dp \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.2)$$

Because of symmetry of the section shown in Figure 3.1 and for the loading under consideration the angle of principal compression, $\Theta_p$, will be constant for the top face and for the bottom face but will vary over the side faces with the variation of longitudinal strains. For the side faces the $\Theta_p$-angles can be evaluated at a finite number of points and hence obtain

$$F_{C_\ell} = \Sigma q \cot \Theta_p \, \Delta p_i \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.3)$$
Hence

\[ F_{cl} = q \cot \theta_{top} b_{top} + q \cot \theta_{bot} b_{bot} + 2q \sum_{i=1}^{5} \cot \theta_{pi} \Delta p_{i} \ldots \] (3.3a)

where

- \( \theta_{t} \) = angle of principal compression strain in top face
- \( b_{top} \) = length of shear flow perimeter in top face
- \( \theta_{b} \) = angle of principal compression strain in bottom face
- \( b_{bot} \) = length of shear flow perimeter in bottom face

To evaluate the resultant compression force in the longitudinal direction for a side face sufficient accuracy is obtained if the angles \( \theta_{pi} \) are evaluated over five points \((i = 1 \text{ to } 5)\) equally spaced along the side, then, using Simpson's integration formula for the sides:

\[ 2q \sum_{i=1}^{5} \cot \theta_{pi} \Delta p_{i} = \frac{qh}{6} \left[ \cot \theta_{s1} + 4\cot \theta_{s2} + 2\cot \theta_{s3} + 4\cot \theta_{s4} + \cot \theta_{s5} \right] \] (3.3b)

where

- \( h_{o} \) = height of shear flow perimeter in the side faces
- \( \theta_{si} \) = angle of principal compression strain at point 'i' in side face.

Equations (3.3a) and (3.3b) can thus be used to evaluate the resultant compression force, \( F_{cl} \), in the longitudinal direction.

The position at which this force acts denoted by \( y_{c} \) can be evaluated for the section in Figure 3.1 as

\[ y_{c} = \frac{\sum_{i=1}^{5} \cot \theta_{pi} \Delta p_{i} y_{i}}{F_{cl}} \] (3.4)

For flexure and torsion only the equilibrium requirements for forces in the longitudinal direction is that the resultant axial force is zero. This requires \( F_{s\ell} = F_{cl} \).
3.1.2. **Equilibrium in the Transverse Direction**

With the assumption that \( \Theta \) will remain constant over the effective depth of a compression diagonal at a particular point of the section perimeter, equations (2.2) and (2.4) can be used to rewrite equation (2.12) as

\[
F_{\text{chi}} = s_h q \tan \Theta \pi
\]

Due to section symmetry (Figure 3.1) the hoop stresses at corresponding points in the side face will be the same; hence for the sides, the equilibrium equations (2.13) give

\[
F_{Hi} = A_h f_{hi} = F_{\text{chi}} = s_h q \tan \Theta \pi
\]

or

\[
t_h f_{hi} = q \tan \Theta \pi
\]

where

\[
t_h = A_h / s_h
\]

For the top and bottom faces the equilibrium equation (2.13) give

\[
t_h f_{ht} = q \tan \Theta_t \left[1 - \left( \frac{1}{2} a_t - c_{ht}\right)/h_1 \right] + q \tan \Theta_b \left( \frac{1}{2} a_b - c_{hb}\right)/h_1
\]

(3.7)

\[
t_h f_{hb} = q \tan \Theta_t \left[1 - \left( \frac{1}{2} a_t - c_{ht}\right)/h_1 \right] + q \tan \Theta_b \left( \frac{1}{2} a_t - c_{hb}\right)/h_1
\]

(3.8)

where

- \( a_t \) = depth of the equivalent rectangular stress block for the top face compression diagonal (Fig. 2.8)
- \( a_b \) = depth of the equivalent rectangular stress block for the bottom face compression diagonal
- \( f_{ht}, f_{hb} \) = hoop stress in the top and bottom faces, respectively
- \( c_{ht}, c_{hb} \) = cover to centreline of hoop steel in the top and bottom faces, respectively
3.2 COMPATIBILITY REQUIREMENTS

The strain compatibility equations (2.15) and (2.16) can be used to evaluate the principal angles, δp, and the shearing strains. It is convenient to evaluate the strains in the plane defined by the centrelines of hoop steel so that the transverse strain, ε₂, will be equal to the hoop strain and the shearing strain evaluated will be in the plane defined by the hoop steel centrelines.

3.2.1 Evaluation of Twist

As indicated in section 2.3.2 the twist can be evaluated using equation (2.17) evaluated over any contour's within the wall thickness of the section. For the section shown in Figure 3.1 it is convenient to choose the contour's to coincide with the path of the hoop centreline around the section. Due to section symmetry and for the loading under consideration, the shear strains γ₉ and γ₈, for the top and bottom faces respectively, will be constant over the width b₁ in the faces. Suppose γ₁, γ₂, γ₃, γ₄, and γ₅, are the shear strains evaluated at the five points selected over the height h₁ of a side face along the hoop centreline. Then by rewriting equation 2.17 as:

\[ \psi_x = \frac{1}{2A_s} \sum_{l=1}^{s} \gamma_{ls} \Delta s_l \] .................................(3.9)

the twist can be evaluated to sufficient accuracy using Simpson's integration rule:

\[ \psi_x = \frac{1}{2A_1} [b_1 (γ_b + γ_t) + \frac{h_1}{6} (γ_1 + 4γ_2 + 2γ_3 + 4γ_4 + γ_5) ] \] ....(3.10)
where \( A_s = A_1 = b_1 h_1 \)

\[ b_1 = \text{width of hoop centreline (for top and bottom faces)} \]

\[ h_1 = \text{height of hoop centreline (for side faces)} \]

3.2.2 Evaluation of Diagonal Curvature and Effective Diagonal Depth

Equation (2.18) is used to evaluate the diagonal curvature \( \phi_{dp} \) and equation (2.19) to evaluate the effective depth \( t_d \) of the concrete compression diagonal. The longitudinal and transverse curvatures together with the twist are required for the evaluation of the diagonal curvature at a given position using equation (2.18). Due to section symmetry there will be no longitudinal curvature nor transverse curvature for the side surfaces. Hence for the side faces only, at any given position, the diagonal curvature can be evaluated as:

\[ \phi_{dp} = \psi_x \sin 2\theta_p \]

on the other hand, the top and bottom surfaces will be subjected to longitudinal curvature, \( \phi_L \), and transverse curvature, \( \phi_T \), together with the twist, \( \psi_x \), where

\[ \phi_L = \frac{\varepsilon_{cb} - \varepsilon_{ct}}{h} \] ............................(3.11)

where \( \varepsilon_{ct} \), \( \varepsilon_{cb} \) are longitudinal strains in the top and bottom surfaces, respectively, for the section shown in Figure 3.1,

and

\[ \phi_T = \frac{\varepsilon_{hb} - \varepsilon_{ht}}{h_1} \] ............................(3.12)

where \( \varepsilon_{ht} \) and \( \varepsilon_{hb} \) are the hoop steel strains in the top and bottom faces, respectively.
3.3 **SOLUTION TECHNIQUE**

The equilibrium and geometric equations which have been derived are adequate for a solution to predict the complete post-cracking response of a structural concrete beam subjected to flexure and torsion, given the geometry of the beam cross-section and the stress-strain characteristics of the concrete and of the reinforcing steel.

### 3.3.1 Pure Flexure Case

A convenient solution technique for the complete response of a given structural concrete beam in pure flexure using the well known theory for flexure can be summarized as follows:

1. Select a trial strain profile over the cross section.
2. With the strains use the given stress-strain relationships for the concrete and steel to evaluate the stress in every fibre and hence obtain resultant forces.
3. Check that the forces evaluated in step (2) satisfy equilibrium. If equilibrium is not satisfied, revise the strain profile.
4. Repeat steps (2) and (3) until equilibrium is satisfied.

The basic principle in the outlined technique is that one start off with strains then get stresses from the strains and finally forces from the stresses. The forces are then checked for equilibrium. This basic approach will be used for the case of pure torsion and for the combined loading cases. It is also of interest to show that the theory for pure flexure is a special case of the more general theory presented.
In the equations developed, the equilibrium equations (2.7) and (2.8) are directly applicable for the pure flexure case for evaluation of the resultant tension force in the steel and its line of action. With reference to the section in Figure 3.1, the concrete compression stresses will be distributed over the top face down to a depth $t_M$ if the moment $M$ alone were acting. Then using the rectangular stress block theory and assuming that $t_M$ is less then $t_t$ the resultant compression force for Malone acting is given by

$$C = \alpha \beta \frac{f_c}{t_M} b \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.13a)$$

where $\alpha$ and $\beta$ are functions of the top longitudinal strain $\varepsilon_{ct}$ which will be compressive.

If besides the moment a torque is also applied then the concrete compression stresses are distributed around the section perimeter as has been discussed. For this case the contribution of the top face to the total resultant compression force in the section is (see equation 3.3a)

$$C_T = q \cot \theta_t b_{ot} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.13b)$$

Imagine reducing the applied torque, $T$, until only the moment $M$ is left acting on the section. In the limit state of pure flexure the theoretical model suggests that the total resultant compression force will be given by equation (3.13b). Using equation (2.4) and (2.1), $q$ can be expressed as:

$$q = \int_0^{t_d} f_d \frac{\tan \theta}{1 + \tan^2 \theta} dt$$
With reference to the top face and assuming

\[ \theta = \theta_t = \text{constant over the effective depth } t_{dt} \]

and

\[ q = \frac{\tan \theta_t}{1 + \tan^2 \theta_t} \int_0^t \theta_t \frac{d \theta}{d t} = \frac{\tan \theta_t}{1 + \tan^2 \theta_t} \alpha \beta f_c t_{dt} \ldots (3.14) \]

where \( \alpha \beta f_c t_{dt} \) is equivalent to \( \int_0^t f_d d \) (Fig. 2.9)

Hence equation (3.13b) can be rewritten as:

\[ C_T = \frac{\alpha \beta f_c t_{dt} b_{ot}}{1 + \tan^2 \theta_t} \]

In the limit \( T = 0 \), the angle of principal compression is the top face \( \theta_t = 0 \), hence for pure flexure the theoretical model predicts:

\[ C_T = \alpha \beta f_c t_{dt} b_{ot} \]

This equation is identical with equation (3.13a) for \( t_{dt} = t_M \) and \( b_{ot} = b \). Thus equation (3.13a) can be considered a special case of equation (3.3) used in the evaluation of the resultant compression force in the section for the combined loading case.

It is also of interest to note that equation (2.15) applies for the case of pure flexure for the special case when

(a) for \( \epsilon_\theta \leq 0 \) : \( \epsilon_t = 0 \) and \( \epsilon_\theta + \epsilon_{dp} = 0 \),

(b) for \( \epsilon_\theta > 0 \) : \( \epsilon_t = 0 \) and \( \epsilon_{dp} = 0 \).

Then equation (2.15) gives:

for case (a) \( \tan^2 \theta_p = 0 \), and hence \( \theta_p = 0 \); and for case (b) \( \tan^2 \theta_p = \infty \), and hence \( \theta_p = 90^\circ \).
3.3.2 Pure Torsion Case

As in the case of pure flexure, a convenient solution technique for pure torsion on the section shown in Figure 3.1 is to fix the longitudinal strain in the top face, $\varepsilon_{ct}$, then for any chosen curvature $\phi$ the steps for solution can be as summarized below. It is assumed that the stress-strain relationships for the concrete and steel are given. The solution is basically an iterative procedure and to initiate the iteration process:

(a) set all principal diagonal angles at all points to be 45°
(b) set all values of the effective diagonal thicknesses for all points to be equal to $0.375 \frac{bh}{(b+h)}$ where $b$ and $h$ are the effective outside dimensions for the section.
(c) set $b_{ot} = b_{ob} = 0.9b$ and $h_{o} = 0.9h$

**STEP 1**

With the chosen curvature, $\phi$, evaluate strains at each fibre and using the given stress-strain relationships for the steel evaluate the resultant tension force in the longitudinal steel ($F_{sl}$) using equation (2.7) and the position at which this force acts ($\bar{y}_s$) using equation (2.8).

**STEP 2**

For unit shear flow ($q=1.0$), using equations (3.3a)
and (3.3b) evaluate the resultant compression force \( F_{c_1}\) in the longitudinal direction. The shear flow required for equilibrium of forces in the longitudinal direction can be evaluated as \( q = \frac{F_{s_1}}{F_{c_1}}\).

**STEP 3**

With the shear flow, \( q \), from Step 2 and the longitudinal strain, \( \varepsilon_{x_1} \), at a given point evaluate the compatible strains \( \varepsilon_{t_1}, \varepsilon_{dp}, \text{ and } \gamma_{lt} \) and determine the angle of principal compression, \( \theta_p \). The procedure outlined in Appendix B can be used in this step.

**STEP 4**

With the shear strains from Step 3, evaluate the twist \( \psi_x \) using equation (3.10).

**STEP 5**

Evaluate the diagonal curvatures using equation (2.18) and with equation (2.19) obtain new values for the effective depths \( (t_d) \) of the concrete compression diagonals.

**STEP 6**

Iterate with steps 2 to 5 until convergence is obtained in the \( t_d \)-values.

**STEP 7**

Using equation (2.22) evaluate \( B \) factors and hence evaluate the values of \( a = \beta t_d \) which determine the shear flow path and hence obtain the area \( A_o \) enclosed by the path of shear flow.
**STEP 8**

Using equation 3.4 evaluate $\bar{y}_C$. If $\bar{y}_C = \bar{y}_S$ the solution for pure torsion has been obtained, otherwise a new value of $\phi_{lt}$ (with $\varepsilon_{ct}$ still fixed) is chosen and steps 1 to 8 repeated. Note that for $\bar{y}_C \neq \bar{y}_S$ a moment $M = F_s \lambda (\bar{y}_C - \bar{y}_S)$ is also acting.

**STEP 9**

Evaluate the torque $T = 2qA_o$

By repeating steps 1 to 9 for a number of values of $\varepsilon_{ct}$ a complete response of a beam section in pure torsion is obtained.

### 3.3.3 Combined Torsion and Flexure

In the solution for pure flexure and pure torsion it was only necessary to fix the longitudinal strain profile for a section in order to make the problem statically determinate. In the combined loading case the problem becomes statically determinate if besides specifying the longitudinal strain profile at a section the shear flow, $q$, is also fixed. The shear flow can be fixed by either specifying the applied torque, $T$, or by fixing the principal compression diagonal strain in concrete at some point in the section. A procedure which can be used to evaluate the shear flow for the latter case is outline in Appendix A.

Assuming that $q$ is determined and the longitudinal strain profile is chosen by fixing the longitudinal strain in
the top face, $c_{ct'}$, with a chosen curvature $\phi'_c$, then with the initial values of the variables as given for the pure torsion case, the steps to obtain a solution are as follows:

**STEP 1**

Same as for the Pure Torsion Case

**STEP 2**

With the specified shear flow $q$ this step is same as Step 3 of the Pure Torsion Case

**STEP 3**

Same as Step 4 of the Pure Torsion Case

**STEP 4**

Same as Step 5 of the Pure Torsion Case

**STEP 5**

Same as Step 7 of the Pure Torsion Case. With the value of $A_o$ found in this step a re-evaluation of $q$ may be necessary if the torque $T$ is specified so that $q = T/2A_o$

**STEP 6**

Iterate with Steps 2 to 5 until convergence is obtained in the $t_d$-values.

**STEP 7**

Using equations (3.3a) and (3.3b) evaluate the resultant compression force, $F_{c'l'}$ in the longitudinal direction

**STEP 8**

Check equilibrium of forces in the longitudinal direction.
If $F_{s'l'} = F_{c'l'}$, then the equilibrium of forces is satisfied.
If $F_{s'l'} \neq F_{c'l'}$, then there is an axial force acting, $N_x = F_{s'l'} - F_{c'l'}$. For the case of flexure and torsion
loading only, new curvatures, $\phi_l$, have to be selected and iterate with steps 1 to 8 until $F_s l = F_c l$

**STEP 9**

Using equation (3.4) evaluate $y_c$ and hence evaluate the applied moment $M$ using equation (2.11). If the torque was not specified, then it can be evaluated from the shear flow using equation (2.5).

By selecting new values of $q$ and $\varepsilon_{ct}$ and repeating Steps 1 to 9 provides the complete response prediction of the structural concrete section subjected to combined torsion and bending.

3.4 RESPONSE PREDICTION FOR A RECTANGULAR SECTION HAVING ONE AXIS OF SYMMETRY

The solution technique outlined in section 3.3.3 has been programmed to enable a computer aided solution and the program is given in Appendix D. Moreover in Appendix C an example is worked out to show how the solution technique is applied for a specified case. In this section the capability of the theoretical model presented is briefly shown by looking at the response prediction for the section shown in Figure 3.2 subjected to flexure and torsion. This beam section was one of the fourteen beam sections tested to verify the validity of the predictions of the theoretical model.
The properties for steel are given in table 4.2.

Dimensions are in mm.

**FIG. 3.2 CROSS SECTION OF BEAM TBU3**

**MATERIAL PROPERTIES**

**CONCRETE**

- $f'_c = 34.8$ N/mm²
- $E_o = 0.0031$

**STEEL**

The properties for steel are given in Table 4.2.
FIG. 3.3 RESPONSE PREDICTIONS FOR TBU3
FIG. 3.4 PREDICTED STRAIN VARIATIONS ON SIDE FACES FOR TBU3

FIG. 3.5 PREDICTED FAILURE INTERACTION CURVE FOR SPALLED SECTION DIMENSIONS
Figure 3.3 shows a computer plot of the results of the model prediction for TBU3. The initial flexural moment and torque of the actual test specimen as tested is taken into account. In addition to these initial values the loading for analysis was increased in small steps at a fixed ratio of torque to moment of 0.696 as seen in Figure 3.3(a). For each load increment a complete analysis using the solution technique outlined in section 3.3.3 was done and the result of the analysis is plotted in Fig. 3.3(b) - (h). The analysis was terminated when the maximum diagonal strain in the section was $\geq 1.5\varepsilon_0$.

The prediction results shown are based on spalled* section dimensions whereby it is assumed that the effective external section dimensions are those described by the centreline of hoop steel. It should also be noted that in Fig. 3.3 (b) - (g), the values plotted are for points at the middle of the faces.

A closer look at Fig. 3.3 (b)-(h) reveals the following:

1) Longitudinal strains (Fig. 3.3(b))

In (b) it is seen that for all load levels the bottom face longitudinal strains at the level of the centreline of hoop steel are tensile whereas the corresponding strains in the top face are compressive. It is also seen that the bottom face strains exceeded the yield strain of bottom face longitudinal steel bars of 0.00225 before the attainment of capacity loads. After the yield of bottom face

* The phenomenon of spalling is discussed in Chapter 5
longitudinal steel there was little load carrying capacity left in the beam.

2) Hoop strains (Fig. 3.3(c))

In (c) the transverse strains which are also equal to hoop steel strains are plotted. The model predicts that at any load level the bottom face will have the largest hoop steel strains and the top face the minimum. Comparing (a) and (b) it is seen that the model predicts that the hoop steel in the bottom face will yield before the bottom face longitudinal steel bars yield. It is also seen from (b) that at capacity load the hoop strains at the midheight of the side faces will also be in the post yield range but the model predicts that the yielding of hoop steel will not extend to the top face.

3) Principal Compression diagonal strains (Fig. 3.3(d)) and Diagonal curvatures (Fig. 3.3(e)).

The predicted principal diagonal strains and the curvatures in the corresponding directions are plotted in (d) and (e) respectively. As expected from the longitudinal strain profile, at any load level, the highest principal diagonal strains and the corresponding curvatures occur in the top face and the lowest in the bottom face. It is observed that after the yield of the reinforcing steel the principal diagonal strains and the curvatures increase rapidly with little gain in the load carrying capacity of the beam.

4) Principal Diagonal Angles (Fig. 3.3(f))

In (f) is shown the pattern of variation of the angles
defining the direction of principal diagonal strain with respect to the beam's longitudinal axis. It is seen that the angles are much more acute in the top face than the angles in the bottom face for the same load level. Notice that because of the effects of the initial dead weights which were predominantly flexural, the angles at the low load levels approached 90° in the bottom face and 0° at the top face. With more load added at a fixed ratio of torque to moment, it is seen that the angles quickly stabilised and changed little until the onset of steel yielding.

5) Shear Strains (Fig. 3.3(g))

In (h) the shear strains predicted are plotted. At any load level the bottom face had the greatest shear strains and the top face the least. A rapid increase in the shear strains is noticed when the reinforcing steel gets into the post-yield range.

6) Angular Deformations (Fig. 3.3(h))

In (h) the twist, the longitudinal curvature and the transverse curvature are plotted against the torque. The model predicts greater twist than there is longitudinal curvature. It also predicts that the transverse curvatures will be small until the bottom face hoop steel yields after which all the angular deformation increase rapidly with little gain in load carrying capacity of the section.

It is of interest to look at the variation of the pattern of
deformations on the side faces where a linear variation of longitudinal strains is assumed. Figure 3.4 shows a plot of the strains $\varepsilon_x$, $\varepsilon_h$, $\varepsilon_d$, and $\gamma$ and the principal angles, $\theta_p$, and the effective depths of the compression diagonals, $t_d$, for the side faces for the torque and bending moment values shown. The plots in (a) are at a load level approximately half the capacity load with all the reinforcing steel in the elastic range. The plots in (b) are for the capacity loading level as predicted by the model, and, at this load level, some of the reinforcing steel has yielded and the yield strains are indicated. From these plots the following is observed:

a) If the $\varepsilon_x$-strains vary over a given face of a section, there will be a corresponding variation in the $\varepsilon_h$, $\varepsilon_d$, and $\gamma$ strains, and in the $\theta$ angles and the $t_d$ values. The variation in the $\varepsilon_h$, $\varepsilon_d$, $\gamma$, $\theta$, and $t_d$ values is generally non-linear. However, in those regions where the $\varepsilon_x$-strains are tensile the $\varepsilon_d$ strains and the $t_d$ values vary little. The $\varepsilon_d$ strains and $t_d$ strains show significant variation when the varying $\varepsilon_x$ strains are compressive.

b) For those regions in the section where the hoop steel strain is within the plateau (if present) of the stress-strain curve of the hoop steel, the $\theta$-angles assume a constant value despite the variation in $\varepsilon_x$ strains for the given load level.
c) For the bottom face and in those regions where the longitudinal strains are tensile the concrete is active in compression as indicated by the principal compressive strains, $\varepsilon_d$, in these regions.

d) The plot of $t_d$ values in Figure 3.4 indicates that the spalled section wall thickness was smaller than the effective wall thickness as predicted by the theoretical model. This is taken into account in the analysis using the theoretical model.

Figure 3.5 shows the failure interaction curve for the section shown in Fig. 3.2 as predicted by the model. The curve is obtained by fixing the maximum principal diagonal compression strain in the section at approximately $1.5\varepsilon_0$ and systematically varying the curvature $\theta_\lambda$ and evaluating the applied torque and flexural moment at each step. It is important to note that the interaction curve shown contains no empirical fitting factors.

The trend of behaviour illustrated in Figures 3.3 and 3.4 with the observations noted and the failure interaction prediction shown in Figure 3.5 need experimental verification. This is the purpose of the experimental programme described in chapter 4.
3.5 PREDICTION OF THE BALANCED FAILURE MODE

FOR COMBINED TORSION AND BENDING

Unlike pure flexure which induces compression in one region of the section and tension in the rest of the section, combined flexure and torsion will induce diagonal compression in concrete in all faces with tensile strains at right angles to the direction of the compression diagonals. Moreover, flexure alone will only stress the longitudinal reinforcing steel but combined flexure and torsion will stress both longitudinal steel and the hoop steel. In pure flexure, the balanced failure is normally defined when the longitudinal steel in the farthest face from the compression face attains yield strains simultaneously with the attainment of the capacity load for the beam section. This will occur, say, when the extreme compression fibre strain is $\varepsilon_{cu}$. In the case of combined flexure and torsion, a balanced failure can be considered defined when both the longitudinal and hoop steel reach their respective yield strains simultaneously with the attainment of capacity loads. It is assumed that this will occur when the diagonal compression strain in the surface fibres of all the sides is $\varepsilon_{cu}$. Assuming that longitudinal steel yield strain is $\varepsilon_{ly}$ at a yield stress of $f_{ly}$ and that the hoop steel content is the same in all faces with a yield strain of $\varepsilon_{hy}$ at a yield stress of $f_{hy}$, then balanced failure will require uniform beam elongation ($\phi_x = 0$). Then, using the equations already derived and assuming adequate concrete wall thickness is provided in the section the following is deduced:
Equation (2.16) \[ \gamma = 2 \sqrt{\left( \varepsilon_{uy} + \varepsilon_{cu} \right) \left( \varepsilon_{hy} + \varepsilon_{cu} \right)} \] For all points

Equation (3.9) \[ \psi_x = \frac{p}{2A} \gamma \]

where \( p = 2(b + h) \) = surface perimeter of section
\( A = bh \) = Area enclosed by surface perimeter.

Equation (2.15) \[ \tan^2 \theta_p = \frac{\varepsilon_{uy} + \varepsilon_{cu}}{\varepsilon_{hy} + \varepsilon_{cu}} \]

for all points

Equation (2.18) \[ \phi_{dp} = \psi_x \sin 2\theta_p \]

Equation (2.19) \[ t_d = \frac{\varepsilon_{cu}}{\phi_{dp}} = \frac{A}{2p} \frac{\varepsilon_{cu} \left( \varepsilon_{uy} + \varepsilon_{hy} + 2\varepsilon_{cu} \right)}{\left( \varepsilon_{uy} + \varepsilon_{cu} \right) \left( \varepsilon_{hy} + \varepsilon_{cu} \right)} \quad (3.16) \]

Equation (3.14) \[ q = \alpha \beta f'_c \sin 2\alpha \cos \theta \]

where \( \alpha \beta = \frac{\Omega_{cu} - \frac{1}{3} \Omega_{cu}^2}{\Omega_{cu}} \) and \( \Omega_{cu} = \frac{\varepsilon_{cu}}{\varepsilon_{o}} \)

Equation (3.6) \[ \frac{A_h f_{hy}}{s_h} = q \tan \theta_p = \frac{A}{2p} \alpha \beta f'_c \frac{\varepsilon_{cu}}{\varepsilon_{hy} + \varepsilon_{cu}} \quad (3.17) \]

Equation (3.3) \[ \frac{A_T f_{ly}}{p_o} = q \cot \theta_p = \frac{A}{2p} \alpha \beta f'_c \frac{\varepsilon_{cu}}{\varepsilon_{ly} + \varepsilon_{cu}} \quad (3.18) \]

where \( p_o = p - \frac{10}{3} t_d \) for a rectangular section, and \( \Omega_{cu} = 1.5 \)

By defining
\[ t_\alpha = A_T / p_o, \quad t_h = A_h / s_h, \quad \text{and} \quad t_c = A / p, \text{then:} \]

From (3.17) \[ \frac{t_h}{t_c} = \rho_{hb} = \frac{f'_c \alpha \beta}{f_{hy} \varepsilon_{hy} + \varepsilon_{cu}} \quad (3.19) \]

= balanced hoop steel ratio.
From (3.18) \( \frac{t_c}{t} = \frac{\rho_{lb}}{t} = \frac{f_y}{f_{cy}} \cdot \frac{\alpha \beta}{2} \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cy} + \varepsilon_{cu}} \) \hspace{1cm} (3.20)

(3.20) = balanced longitudinal steel ratio.

The shear flow can be evaluated as \( q = \sqrt{\tau_n f_{cy} t_c f_y} \) and for balanced failure the shear flow can also be evaluated as

\[
q_b = t_c f_y \frac{\alpha \beta}{2} \frac{\varepsilon_{cu}}{\sqrt{(\varepsilon_{cy} + \varepsilon_{cu})(\varepsilon_{hy} + \varepsilon_{cu})}} \hspace{1cm} (3.21)
\]

The torque, \( T_b \), and the bending moments \( M_{bz} \) and \( M_{by} \) for the balanced failure condition can be evaluated for a section for which the resultant compression force in concrete can be considered to act at \((\bar{z}_c, \bar{y}_c)\) as follows:

\[
T_b = 2q_b A_o
\]

\[
M_{bz} = \rho_{lb} t_c f_y P_0 \bar{z}_b \hspace{1cm} (3.22)
\]

\[
M_{by} = \rho_{lb} t_c f_y P_0 \bar{y}_b
\]

where

\[
\bar{z}_b = \bar{z}_c - \bar{z}_s
\]

\[
\bar{y}_b = \bar{y}_c - \bar{y}_s
\]

and \( \bar{z}_s \) and \( \bar{y}_s \) are evaluated using equations (2.8).

It should be noted that for a symmetrically reinforced case \( \bar{z}_b = \bar{y}_b = 0 \); hence a pure torsion case will result for the balanced failure condition. The moments \( M_{bz} \) and \( M_{by} \) are greatest when the longitudinal steel is positioned so that the position \((\bar{z}_s, \bar{y}_s)\) of the resultant steel force is furthest
from the position at which the resultant concrete compression force acts.

The evaluation of the balanced failure mode as developed above is not restricted to a rectangular section but is applicable to any section where St. Venant torsion dominates. The loading ratio for balanced failure is evaluated as:

\[
\frac{T_b}{M_b} = \frac{2A_o}{\rho_o j_d} \sqrt{\frac{\epsilon_{ly} + \epsilon_{cu}}{\epsilon_{hy} + \epsilon_{cu}}} \tag{3.23}
\]

where \( j_d = \) moment lever arm.

For the special case when \( \epsilon_{ly} = \epsilon_{hy} = \epsilon_y \) and \( f_{ly} = f_{hy} = f_y \) equations (3.19) and (3.20) give

\[
\rho_{hb} = \rho_{lb} = \frac{f'}{f_y} \cdot \alpha \beta \cdot \frac{\epsilon_{cu}}{\epsilon_y + \epsilon_{cu}} \tag{3.24}
\]
CHAPTER 4

EXPERIMENTAL STUDY

4.1 OBJECTIVES AND CHOICE OF TEST SPECIMENS

The theoretical model presented is closely related to and similar in rationality and generality to the well known model for flexural analysis. Indeed it has been shown that the pure flexure case can be treated as a special case of the model presented. It is therefore relevant to note the comment made by Rüsch in his discussion of the various flexural theories: "Only tests of over-reinforced members can furnish a true measure of the validity of a flexural theory." In accordance with Rüsch's comment a test programme consisting of 14 heavily reinforced beams was designed to test the validity of the theoretical model.

All the specimens had identical steel cages and the same outside section dimensions. Four of the specimens were solid in contrast to the rest which were hollow. A typical section of the hollow beams as designed is shown in Figure 4.1. It should be noted that English units were used in designing the specimen sections but in the testing of the specimens the loads were measured in metric units. The actual measured dimensions of the specimens are given in metric units in Table E-1 of Appendix E,
and Figure 3.2 shows the metric dimensions for one of the specimens.

**NOTE:**
1. Section dimensions are given in inches
2. Cover to hoop steel is 1/4" 

**FIG. 4.1 CROSS-SECTION OF HOLLOW SPECIMENS AS DESIGNED**

Heavy instrumentation of the specimens was necessary for the purpose of obtaining enough strain measurements to give us reasonable representative strain patterns to enable a comparison with the corresponding patterns predicted by the model, described in section 3.4. Furthermore, basic to the development of the theoretical model is the Bernoulli-Navier hypothesis concerning the variation of longitudinal strains at a section. This assumption, which is also basic to the development of the pure flexure theory, was to be verified for the case of combined flexure and torsion.
Besides the need to have experimental verification for the complete response prediction of a given beam section loaded in a specific manner, it was also the object of the test programme to verify how well the model prediction of the failure interaction curve for a given concrete section (such as that given in Fig. 3.5) was in agreement with test results. This necessitated testing the specimens of the same concrete strength under widely varying torsion to bending moment ratios. The effect of the variation of concrete strength was also to be experimentally investigated.

In section 3.5 of the theoretical development it was shown that the theoretical model can predict the steel ratios for a given section which will lead to balanced failure for the special conditions outlined in the section. This prediction needs experimental verification and the test programme was also aimed at doing this.
4.2 TEST SPECIMENS

4.2.1 Manufacture of Test Specimens

The hollow beams were cast in two groups of five so that for each group all the specimens were not only cast at the same time but also were made of the same concrete mix and they were cured together under the same conditions. One of these groups of beams made up the TBO series and the other group made up the TBU series. Figure 4.2 shows a complete steel cage for TBO4. Figure 4.3 shows the forms with five steel cages in position during the casting of the TBO series.

The hollow section was created by using styrofoam six feet (1.829m) long with a cross section of 10" X 14" (254mm x 356mm). The bottom wall thickness of the section shown in Figure 4.1 was concreted before the styrofoam was placed in position. The styrofoam was positioned by sliding it into the steel cage through one end of the cage with appropriate steel end plates designed for this purpose. The styrofoam was guided into and held in position by five equally spaced welded steel collars made out of ¼" steel bars. These collars were designed to be anchored to the bottom cage steel and were to restrain the styrofoam from the tendency to float upwards under the pressure of the almost fluid concrete during casting. Loss of this anchorage for some of the collars for TBU4 and TBU2 led to upward displacement of the styrofoam with a reduced wall thickness for the top face of these specimens.
The four solid specimens constituted the TBS-series. Three of these specimens were also cast at the same time with the same concrete mix. The fourth was cast alone at a later time with a different concrete mix.

The capacity of the MTS actuators, used in loading during the testing, dictated the loading spans and consequently the total length of a specimen particularly for the pure flexure loading case. It was found that on the basis of the load carrying capacity of the chosen test section, as predicted by the model the total length of the specimen required which would be failed by the available MTS actuators was not convenient in terms of handling and the available casting space in the laboratory. To solve this problem it was decided to cast shorter concrete beams basically to provide the necessary test span length and to provide appropriate extensions to the concrete beams using structural steel sections. An available hollow steel sections 12" x 12" x ½" (305mm x 305mm x 13mm) was used for this purpose. The moment and shear connection between the steel beams and the concrete beam which involved the welding of reinforcing bars to steel end plates turned out to be the trouble spot in the testing programme.

Figure 4.4 shows the end plate with nuts and anchor bars welded in position. This plate enabled a successful moment connection between the steel beams and the concrete beam for each specimen of the TBO and TBS series. Figure 4.5 shows the moment connection used in the testing of the TBU series specimens which were cast and tested before the
TBO and TBS series specimens were fabricated. The connection in Fig. 4.5 had to be replaced by that in Fig. 4.6 for the testing of the TBO and TBS series specimens. This was because the connection shown in Fig. 4.5 failed before the failure load was reached for the pure flexure specimen TBU1.

FIG. 4.2 REINFORCING STEEL CAGE FOR TB04

FIG. 4.3 CASTING OF TBO SERIES
FIG. 4.4
END PLATE FOR CONCRETE BEAM

FIG. 4.5
CONNECTION FOR TBU SERIES

FIG. 4.6
CONNECTION FOR TBO AND TBS SERIES
4.2.2 MATERIALS

4.2.2.1 Concrete

In Table 4.1 are summarized, the results of standard compression cylinder tests for the concrete used in casting the test specimens. The loading rate used was a stroke movement of 20mm in 3600 seconds corresponding to a strain of .064mm/mm per second. A continuous load stroke plot was automatically plotted during the test which lasted about 5 minutes. More accurate load deformation characteristics were obtained from some of the cylinder tests by plotting the applied compression load against the movement of the two LVDT's shown in Figure 4.7. The LVDT's measured the deformation of the middle part of the cylinder between the two collars shown.

During the testing of the beam specimens it was necessary to stop the loading by holding the stroke of the MTS actuators at several stages to enable the readings of deformations before proceeding with the loading. The stops generally lasted 20 to 30 minutes. During these stops there was a drop in the loads due to the creep in the concrete. These stops were simulated in some cylinder tests by holding the stroke of the cylinder testing machine for either 20 minutes for some tests or 30 minutes for others. This was done at several load levels and the results of these tests are shown in Figure 4.7 and 4.8. Also shown in these Figures are the curves for the standard cylinder tests. It was found that the maximum compression cylinder stress as obtained in the standard cylinder tests was always attained in the short term 'creep' tests as
well. However, with the more accurate load deformation characteristics obtained using the LVDT's as shown in Fig. 4.7, it was found that the strains at peak stress for the 'creep' tests was always greater than the corresponding strain for the standard test.

The parabolic stress-strain relationships used in the theoretical model to approximate the actual stress-strain relationship of the concrete are also plotted in Figure 4.7 and 4.8. It is noted that the representation is excellent up to the peak stress, at a strain of \( \varepsilon_0 \); however for strains much greater then \( \varepsilon_0 \) the parabola tends to under-estimate the stress especially for low concrete strengths.

**TABLE 4.1 CONCRETE PROPERTIES**

<table>
<thead>
<tr>
<th>BEAM</th>
<th>AGE DAYS</th>
<th>( f'_c ) N/mm²</th>
<th>( \varepsilon_0 \times 10^{-3} )</th>
<th>TOTAL NO. OF CYLINDERS TESTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBO1</td>
<td>25</td>
<td>19.5</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>TBO2</td>
<td>28</td>
<td>19.7</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>TBO3</td>
<td>20</td>
<td>19.1</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>TBO4</td>
<td>33</td>
<td>20.4</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>TBO5</td>
<td>36</td>
<td>20.5</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>TBU1</td>
<td>102</td>
<td>14.3</td>
<td>3.1</td>
<td>23</td>
</tr>
<tr>
<td>TBU2</td>
<td>113</td>
<td>14.3</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>TBU3</td>
<td>109</td>
<td>14.3</td>
<td>3.1</td>
<td>34</td>
</tr>
<tr>
<td>TBU4</td>
<td>92</td>
<td>14.3</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>TBU5</td>
<td>33</td>
<td>14.3</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>TBS1</td>
<td>3</td>
<td>23.0</td>
<td>2.5</td>
<td>4</td>
</tr>
<tr>
<td>TBS2</td>
<td>7</td>
<td>22.9</td>
<td>2.5</td>
<td>4</td>
</tr>
<tr>
<td>TBS3</td>
<td>45</td>
<td>45.3</td>
<td>2.5</td>
<td>10</td>
</tr>
<tr>
<td>TBS4</td>
<td>31</td>
<td>15.3</td>
<td>2.0</td>
<td>19</td>
</tr>
</tbody>
</table>
TESTING OF CONCRETE CYLINDERS USING THE STIFF MTS COMPRESSION TEST MACHINE

FIG. 4.7 STRESS-STRAIN CURVES OF CONCRETE FOR TBO AND TBU SERIES

FIG. 4.8 STRESS-STRAIN CURVES OF CONCRETE FOR TBS SERIES
4.2.2.2 **Steel**

In Table 4.2 are summarized the steel properties for all the reinforcing steel used in all the specimens. The strain rate was 0.125 mm/mm per 9000 seconds from zero load up to the onset of strain hardening. This strain rate was doubled after the onset of strain hardening in order to cut down the total test time. The load was automatically plotted against the strain and the average moduli of elasticity in the elastic range and after the onset of strain hardening were evaluated from the plots.

Figure 4.9 shows a plot of the representative stress-strain curves, for the reinforcing steel up to a strain of 0.015. The strain signal was obtained by a sensitive extensometer attached to the test specimen and calibrated so as to give a plot of direct strain on an X-Y plotter. The test specimens were representative pieces cut from the actual reinforcing bars used to make the steel cages for the specimens.

<table>
<thead>
<tr>
<th>BAR TYPE</th>
<th>BAR SIZE</th>
<th>NO.</th>
<th>AREA mm$^2$</th>
<th>fy N/mm$^2$</th>
<th>fu N/mm$^2$</th>
<th>$E_s \times 10^3$ N/mm$^2$</th>
<th>$E_{sh} \times 10^3$ N/mm$^2$</th>
<th>$\varepsilon_{sh} \times 10^{-3}$ mm/mm</th>
<th>WHERE USED</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LONGITUDINAL</strong></td>
<td>#8</td>
<td>16</td>
<td>510</td>
<td>436</td>
<td>758</td>
<td>194.2</td>
<td>9.0</td>
<td>6.13</td>
<td>ALL BEAMS.</td>
</tr>
<tr>
<td></td>
<td>#4</td>
<td>8</td>
<td>129</td>
<td>393</td>
<td>620</td>
<td>201.9</td>
<td>4.9</td>
<td>15.18</td>
<td>TBO&amp;TBU SERIES</td>
</tr>
<tr>
<td></td>
<td>#4</td>
<td>5</td>
<td>129</td>
<td>433</td>
<td>723</td>
<td>185.7</td>
<td>11.2</td>
<td>2.00</td>
<td>TBS SERIES</td>
</tr>
<tr>
<td></td>
<td>#3</td>
<td>7</td>
<td>71</td>
<td>552</td>
<td>763</td>
<td>205.1</td>
<td>9.0</td>
<td>2.00</td>
<td>TBO SERIES</td>
</tr>
<tr>
<td></td>
<td>#3</td>
<td>5</td>
<td>71</td>
<td>401</td>
<td>646</td>
<td>201.7</td>
<td>7.5</td>
<td>2.00</td>
<td>TBS SERIES</td>
</tr>
<tr>
<td></td>
<td>#4</td>
<td>3</td>
<td>71</td>
<td>406</td>
<td>606</td>
<td>200.0</td>
<td>3.9</td>
<td>15.67</td>
<td>TBS SERIES</td>
</tr>
<tr>
<td><strong>HOOP</strong></td>
<td>#4</td>
<td>9</td>
<td>129</td>
<td>379</td>
<td>605</td>
<td>198.5</td>
<td>5.2</td>
<td>15.4</td>
<td>TBO&amp;TBU SERIES</td>
</tr>
<tr>
<td></td>
<td>#4</td>
<td>6</td>
<td>129</td>
<td>443</td>
<td>717</td>
<td>190.4</td>
<td>9.6</td>
<td>7.96</td>
<td>TBS SERIES</td>
</tr>
</tbody>
</table>

* Bar sizes are in eighths of an inch (3.2 mm)
FIG. 4.9 STRESS-STRAIN CURVES FOR REINFORCING STEEL

NOTE: The yield indicated in Table 4.2 for those bars which did not have a distinct yield point was taken as the proof stress using the 0.2% set method.
4.3 PARAMETERS OF THE TEST PROGRAM

The concrete compression strength, $f'_c$, and the ratio of torque to flexural moment, $R$, were the basic parameters varied in the tests. The concrete strength for the TBO series was selected so as to obtain failure of the section without yielding any of the reinforcing steel for both the special loading case that produces uniform beam elongation and for pure flexure. Thus according to the criteria for balanced failure given in section 3.5, the TBO series specimens were designed to be over-reinforced. On the other hand the concrete strength for the TBU series was chosen so that at least some of the reinforcing steel would be in the post-yield range at failure. The TBS series was designed to investigate the effect of the variation of $f'_c$ for a fixed loading ratio, $R$, and to test the validity of the theoretical model for solid sections.

The loading ratios, $R$, for the TBO and TBU series were chosen so as to span out the torsion-flexural interaction domain in such a way as to reveal important representative response. The schematic loading arrangement is shown in Figure 4.10. Statical analysis of the beam AB under the loads shown in Figure 4.10 reveals the applied actions shown in Figure 4.11. The lever arm lengths $x_T$ and $x_M$ shown in Figure 4.10 were chosen so as to achieve the desired $R$ ratios which are summarized in Table 4.3.
FIG. 4.10 SCHEMATIC LOADING ARRANGEMENT

FIG. 4.11 APPLIED ACTIONS ON TEST BEAM

TABLE 4.3 LOADING RATIOS

<table>
<thead>
<tr>
<th></th>
<th>TBO SERIES</th>
<th>TBU SERIES</th>
<th>TBS SERIES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TBO1</td>
<td>TBO2</td>
<td>TBO3</td>
</tr>
<tr>
<td>$X_T (m)$</td>
<td>0.000</td>
<td>0.610</td>
<td>1.200</td>
</tr>
<tr>
<td>$X_M (m)$</td>
<td>2.321</td>
<td>2.321</td>
<td>1.711</td>
</tr>
<tr>
<td>$R$</td>
<td>0.000</td>
<td>0.261</td>
<td>0.701</td>
</tr>
</tbody>
</table>

4.4 THE TEST RIG

Figure 4.12 shows a view of the test rig and Fig. 4.12 shows a corresponding schematic representation. Except for specimens TB05 and TBU5 all the specimens tested in the test rig shown had the steel extension beams connected to the ends of the concrete specimens by a carefully designed moment.
and shear connection. At the ends of the steel extension beams frictionless universal bearings were attached. These bearings, shown in Fig. 4.15, were designed and constructed specifically for the tests to enable free rotation at the supports due to the applied torque as well as free rotation at the supports due to the curvature of the specimens under flexure. For specimens TB05 and TBU5 these bearings were attached directly to the ends of the specimens. Which had specially designed end plates for the purpose.

The torsion arms were provided by the heavy welded steel section which were attached 90° (2.286 m) apart on the central part of the concrete beam. They were clamped to the specimens as seen in Fig. 4.14 using 1 1/3" (34 mm) dwidag bars secured with the appropriate nuts and plates.

The test beam with the attached extension beams and the attached torsion arms was supported overhead at its ends. A load cell was positioned at each support end, as indicated in Figure 4.13 to monitor the support reaction. These load cells were specially made for this purpose. The height of the specimen above the floor was chosen to enable convenient and easy attachment of the MTS actuators which were used to apply the loads and also to enable strain readings in all the faces particularly the bottom face.

The torque and flexural moment were applied to the test specimen by the two MTS actuators synchronized and programmed to apply equal displacements downwards at the ends of the torsion arms. The actuators were anchored in position
to the floor by a system of floor beams shown in Figure 4.16. Enough hinges enabling free rotation in all directions were provided so that the actuators were self-aligning in application of the load to the torsion arms. So well were the friction effects eliminated in the entire test set up that at any load level an additional load applied by the actuators and monitored by the special MTS load cells attached to the actuators were equal and also reflected by the overhead support load cells with a discrepancy of less than ±4% between the support loads and actuator loads.

FIG. 4.12 TEST RIG
Universal spherical bearing
LVDT for measuring deflection

Overhead support load cell
Metrisite for measuring twist

Steel extension beam
Flexible chain
Suspended rod
Self aligning bearing

Inclinometer targets
Brass target attached to hoop steel for measuring strains

Load cell
MTS actuator

FIG. 4.13 SCHEMATIC REPRESENTATION OF TESTING RIG SHOWING INSTRUMENTATION
FIG. 4.14 TORSION ARMS

FIG. 4.15 UNIVERSAL BEARING SUPPORT ATTACHED TO END OF EXTENSION BEAM

FIG. 4.16 MTS ACTUATOR ANCHORING ARRANGEMENT.
4.5 INSTRUMENTATION

The loads applied by the MTS actuators and the support reactions were monitored by the load cells indicated in Fig. 4.13. A continuous plot of load versus rotation due to the applied torque was obtained on an X-Y plotter driven by a load signal from one of the actuators and a rotation signal from a rotational transducer attached to the specimen to measure relative rotation over the central 1.75 metres of the specimen. As a check on the torsional rotation obtained using the metrisite three inclinometer readings were taken over three positions 12 inches (305mm) apart in the central part of the top face. A measure of the amount of torsional rotation was also given by the continuous plot of load versus the stroke of the synchronized actuators.

A continuous plot of load versus the midspan deflection of the specimen was also recorded on another X-Y plotter driven by a load signal from one of the actuators and the deflection signal was obtained from an LVDT positioned at the beam midspan in an arrangement enabling the relative deflection of the midspan section with respect to sections 0.875 metres away on either side of the midspan. This arrangement is shown in Figure 4.13. From the deflection measured the overall curvature over the central 1.75 metres of the specimen was evaluated and checked against the curvature obtained from the longitudinal strain profile.

Most of the time during the testing was taken up in reading the longitudinal, transverse and diagonal strains in
the top, bottom and side faces of a specimen at each load stage. The strains were measured from targets attached on to the hoop steel in regular patterns using rapid hardening epoxy glue. The pattern of targets used for the top and bottom faces is shown in Figure 4.17 and that for the north and south faces is shown in Figure 4.18. Mechanical strain measuring gauges (1 div = 0.01 mm) were used in the measurement of strains. The longitudinal strains were measured over targets 6 inches (152 mm) apart, the hoop strains over 3 inches (76 mm), and the diagonal strains over $6\sqrt{2}$ inches (216 mm) apart.

With the target patterns shown in Figures 4.17 and 4.18, 258 strain reading were taken at every load stage. This large number of strain readings was necessary to establish experimentally the strain variation pattern in the faces so as to enable a reasonable comparison with the corresponding strain patterns predicted by the theoretical model. It should be noted that due to the actual orientation of the specimens in the testing position, the side faces were appropriately designated North face and South face.
FIG. 4.17 TARGET PATTERN FOR TOP & BOTTOM FACES

FIG. 4.18 TARGET PATTERN FOR NORTH AND SOUTH FACES
4.6 TESTING PROCEDURE

Before the start of each test, the theory developed in Chapter 3 was used to predict the expected response using the known or estimated material properties for the specimen, with the theoretical model prediction it was possible to decide at approximately what load levels or deformation levels at which it would be appropriate to have load stages for the purpose of strain readings.

All the strain readings were taken before the steel extension beams and the torsion arms were attached. The strains were also read again when the specimen was in position in the test rig before the first load increment. This ensured that the initial readings taken were correct.

The specimen was then loaded in steps using the two MTS actuators whose stroke movements were synchronized and controlled by the same MTS control unit. The actuators were programmed to pull down the torsion arms (Fig. 4.13) by equal amounts. With the friction-free universal joints provided in the test set up, the loads applied by the two actuators were always exactly equal. It is of interest to note that, in loading the specimens using stroke control, a consistent loading procedure was achieved in the testing of the materials (concrete and reinforcing steel) used in making the test specimen and in the testing the specimen itself.

In a typical loading stage the load was steadily increased by actuator stroke movement at rate of 127 mm per
3600 seconds to a desired load value or deformation value and the stroke was then held until all the strain readings and other readings were done. The crack patterns were marked and photographed during this time interval. The actuator loads and the support reactions as given by the load cells were recorded at the beginning and at the end of each load stage. During a load stage the loads were observed to drop. The rotation due to torsion and the deflection due to flexure remained reasonably constant during each load stage but with a tendency to increase as the load dropped. This tendency would be eliminated with a stiffer testing rig. Because the torsional rotation and the deflection due to flexure remained close to constant, it is reasonable to assume that the strains in the beam remained close to constant for the load stage.

In the testing of the pure flexure specimens it was not necessary to take the transverse and diagonal strains. However, to ensure that there were no torsional effects these strains were taken at almost half capacity load and at a load stage close to capacity loading and no significant transverse and diagonal strains were found. Thus the load stages for these specimens lasted a comparatively short time.

In the case of pure flexure the test set up shown in Figure 4.13 with zero level arm for torsion did not produce uniform flexure in the test region as planned. This was due to the fact that unequal settlements and unequal deformations at the ends of the specimens resulted in unequal loads in the
two actuators which were programmed to apply equal stroke movements. TB01 which was tested to failure using this arrangement failed with one actuator exerting a load 28% higher than the other actuator. Specimens TB01 was not failed with the test arrangement because the connection between the concrete specimens and the extension beam failed prematurely. The beam was tested to failure under regular knife-edge end-supports with two equal point loads applied at 1/3 span point and 2/3 span point from one end. The total span between the supports was 12 feet (3.048 mm).
CHAPTER 5

EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS

5.1 INTRODUCTION

In Chapter 4 a comprehensive test programme has been described which was designed to provide test data for comparison with the predictions of the theoretical model presented in Chapter 2 and Chapter 3. The comparison is made in this Chapter. The Chapter first reviews the assumed validity of the Bernoulli-Navier hypothesis with the test data. Also discussed is the assumption that concrete has no tensile strength and the phenomenon of the spalling of concrete cover. It is then shown that the model predicts the deformations well. The failure loads for the specimens tested as well as the failure loads for other tests reported in the literature are shown to be also well predicted. The failure modes of the specimens tested are then discussed and the criteria for balanced failure based on the model is shown to be in good agreement with the test results. The chapter closes with a discussion of torque-twist relationships and moment curvature relationships with reference to the parameters varied in the tests and good agreement of test data with the model prediction is noted.
5.2 REVIEW OF THE BERNOULLI-NAVIER HYPOTHESIS WITH EXPERIMENTAL RESULTS

The assumption that longitudinal strains vary linearly across the section is basic in the development of the theoretical model. Fig. 5.1 shows the measured average longitudinal strains for each face of the section for the TBO series specimens at approximately half capacity loading level (Fig. 5.1(a)) and at approximately capacity loading level (Fig. 5.1(b)). Similar plots are shown in Appendix E.4 for the specimens in the TBU and TBS series.

These figures indicate a linear variation of the measured longitudinal strains in the North and South faces for all the specimens. It is further observed that the longitudinal strains remained virtually constant over the top and bottom faces and the strain measurements in the North face are in good agreement with the corresponding measurements in the South face. However, comparatively noticeable oscillations of the strains measured in any given face are noted in those specimens loaded at the ratio R (Table 4.3) of about 1.5.

The strains plotted in Fig. 5.1 are averaged for the section and plotted in Fig. 5.2 for the indicated load levels. The corresponding average strains for the TBU series are also plotted in Fig. 5.2 for comparison. Also shown in Fig. 5.2 are the correlation coefficients, r, obtained from a linear regression analysis, using all the measured strains on each section at the indicated load levels.
Fig. 5.1(a)
LONGITUDINAL STRAINS
AT ~ HALF CAPACITY LOADS
FOR TSO SERIES

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Fig. 5.1(b)
LONGITUDINAL STRAINS
AT APPROX. CAPACITY LOADS
FOR TBO SERIES

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<tr>
<td>05</td>
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</table>
FIG. 5.2 LONGITUDINAL STRAIN PROFILES FOR TBO & TBU SERIES
(r=Linear Correlation Coefficient)
Except for the specimens TBO4 and TBU4 the linearity of the longitudinal strain profiles is unquestionable. For specimens TBO4 and TBU4 the strains, particularly at capacity loads, were practically the same in all faces and statistical analysis using the students t-test confirmed the noted uniform beam elongation at a confidence level of 99%. This was also the case with the variation of the longitudinal strains for the TBS series particularly at approximately capacity loading levels.

Thus on the basis of experimental evidence it is valid to assume a linear longitudinal strain distribution across the section for the combined loading case.

5.3 DISCUSSION OF THE ASSUMPTION OF ZERO TENSILE STRENGTH OF CONCRETE AND SPALLING OF CONCRETE COVER

A simplifying assumption used in the development of the theoretical model is that concrete has no tensile strength after cracking. Thus it is assumed that diagonal cracks will form in all the faces of a beam subjected to combined torsion and bending as soon as the load level causing the first cracking as evaluated by elastic analysis is reached. From the test results it was observed that the specimens subjected to a predominantly torsion loading did indeed have diagonal cracks in all faces quite early in their loading history, whereas those specimens with predominantly flexural loading had diagonal cracking, if any, forming at much higher
load levels. For example beams TB05 and TBU5, which were
loaded with a high torsion to moment ratio, had noticeable
diagonal cracks forming in all faces by the second load stage,
whereas beams TB02 and TBU2, which had a predominantly
flexural loading, did not have any diagonal cracks forming in
their top faces and noticeable diagonal cracks did not form
in the bottom and side faces till the third load stage. It
should be noted that in the testing procedure much greater
loading increments were used in the early load stages than
for those loads stages close to capacity loading.

A further consequence of ignoring the tensile
strength of concrete is the spalling of the concrete cover to
hoop steel. Reference (1) discusses in detail the phenomenon
of spalling of concrete cover to hoop steel and its effects
on the response of a structural concrete beam subjected to
pure torsion. It is suggested that the spalling can be assumed
to occur to the level of hoop steel centrelines. On the other
hand, for the case of beams subjected to pure flexure, spalling
as discussed in reference (1) does not occur and is not con-
sidered in the well established theory of flexure. It is
therefore logical to expect that in the case of combined torsion
and flexure the effects of spalling will become increasingly
significant as the torsion to moment ratio is increased in the
loading of a beam.

This was indeed found to be the case in the tests.
Signs of spalling were very evident close to capacity load
for these specimens loaded with a high torsion to moment ratio. For example the spalled concrete cover could be easily picked off any of the faces of beams TBO4 (Fig. 5.3), TBO5, TBU4 and TBU5 at failure. On the other hand TBO2 (Fig. 5.4) and TBU2 showed no such clearly marked signs of spalling although some signs of spalling were evident at the corners in the bottom face. No signs of spalling were seen in the pure flexure specimens TBO1 and TBU1.

Evidently torsion induces spalling. Whereas pure flexure will induce a principal compression field in a localized region of a beam section that is aligned to the beam longitudinal axis, torsion when applied alone or in combination with flexure will align the direction of principal compression to create a field of diagonal compression in all the beam faces. The orientation with respect to the beam longitudinal axis of the principal compression is governed by the requirements of equilibrium and compatibility of deformations. The resulting compression diagonals meeting at the section corner will initiate the spalling of the concrete cover to hoop steel at the corners in the manner described in reference (1).

Complete spalling of the concrete cover to hoop steel is prevented by the tensile stresses in concrete. However, if the tensile strength of concrete is assumed to be zero then it is consistent to assume that the spalling initiated at the corners will spread around the section and the concrete outside the hoop centreline perimeter can be considered ineffective in
carrying load.

Finally, ignoring the tensile strength of concrete also means ignoring the stiffening of the reinforcing steel bars by the uncracked concrete between the crack spacing. This tension stiffening effect will be particularly significant in the low levels of loadings after the formation of the first cracks.

It is therefore evident that by assuming that concrete has zero tensile strength and using the spalled section dimensions in the model prediction, the predicted deformations should be larger than those measured for a given loading level. The test results to be discussed do indeed confirm this. It is possible to get predictions of deformations a little closer to those measured by averaging the spalled predictions and the unspalled predictions. Such a procedure is used in reference (1). However, this still fails to take into account the tension stiffening effects of the concrete on the reinforcement.

It is considered reasonable to use the spalled prediction as the basis of evaluation because it is conservative and it best predicts the behaviour at failure for a section loaded with a significant amount of torsion. For loading cases which induce diagonal cracks in all the beam faces and at high load levels, the conditions assumed in the spalled prediction will be closely approximated and the measured load deformation response will then approach that predicted by the model.
FIG. 5.3 TB04 AT FAILURE LOAD

FIG. 5.4 TB02 AT FAILURE LOAD
5.4 STRAIN PATTERNS

Fig. 5.5 shows the computer plot of the results of the model prediction for beam TBO3 and the corresponding plot of the measured values obtained from the test of the specimen. The model predictions for the spalled and unspalled section dimensions are plotted and these plots are similar to the plots in Fig. 3.3 described in details in section 3.4. A complete set of the plots similar to that shown in Fig. 5.5 for all the beams tested is given in Appendix E.5.

These plots show that for all the specimens tested covering widely varying loading ratios \( R = 0.0 \) to 5.06) and for the ranging concrete strengths used (\( f'_c = 15.5 - 45.8 \text{ N/mm}^2 \)) the model predicts the load-deformation responses for the rectangular section tested well. With reference to Fig. 5.5 it is seen that the model predicted deformations are higher than those measured but at close to capacity loads the measured strains show a rapid increase approaching the values of the model prediction for the spalled section condition. It is also of interest to note that for TBO3 the spalled model prediction shows that at capacity loading the longitudinal and hoop steel will not yield and this is confirmed by the test results. For those specimens for which either hoop steel or some longitudinal steel was yielded, the load level at which the yielding was initiated is well predicted by the model for spalled section conditions as indicated by the plots in Appendix E.5.
FIG. 5.5 COMPARISON OF THEORETICAL PREDICTION WITH TEST RESULTS
FIG. 5.5 COMPARISON OF THEORETICAL PREDICTION WITH TEST RESULTS (cont'd)
Besides the overall load-deformation responses such as those shown in Fig. 5.5 it is of interest to investigate how well the strain patterns across a face of a section as predicted by the model agree with those obtained from the test data.

5.4.1 Variation of Longitudinal Strains

Fig. 5.6 shows a plot of the longitudinal strain profiles at failure as predicted by the model for the TBO series beams assuming spalled section dimensions. The strains measured at the last load stage before failure are also plotted for comparison. It is seen that beams TBO4 and TBO5, whose condition at failure closely approximated the assumed model conditions, have their measured longitudinal strain profiles in very good agreement with the predicted profiles. On the other hand TBO2 which did not have any diagonal cracks in the top face at failure did not show such a good agreement. Notice however, that the model predicts greater curvature for TBO2 than that measured and this is as expected.

The measured longitudinal strains were used to evaluate flexural curvatures at the various load stages. The curvatures thus evaluated were compared with those evaluated from the continuously monitored midspan beam deflection and very good agreement was revealed.
FIG. 5.6 PREDICTED & MEASURED LONGITUDINAL STRAIN PROFILES FOR TBO SERIES
5.4.2 Variation of Transverse Strains

Fig. 5.7 shows the measured average transverse strains for each face of the section for the TBO series specimens at approximately half capacity loading level (Fig. 5.7(a)) and at approximately capacity loading level (Fig. 5.7(b)). Similar plots are shown in Appendix E.4 for the TBU and TBS series beams.

These figures indicate that there is some significant variation in the transverse strains in the side faces if there is longitudinal strain variation in the faces. Fig. 5.8 shows a plot of the model predictions of the transverse strains at load levels corresponding to those of the average measured transverse strains which are also plotted for comparison. It is seen that the model predicts greater transverse strains than those measured in Fig. 5.8(a). However at approximately capacity loading level (Fig. 5.8(b)) when the condition of the test specimens closely approximates the conditions assumed for the model prediction the agreement between the test values and the predicted values is very good.

5.4.3 Variation of Diagonal Strains

Fig. 5.9 shows the average principal compression strains evaluated from the test data and plotted for each face of the section for the TBO series at approximately half capacity loading level (Fig. 5.9(a)) and at approximately capacity load-
Fig. 5.7(a)
TRANSVERSE STRAINS
AT APPROX. HALF CAPACITY LOADS
FOR TBO SERIES

LEGEND

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Fig. 5.7(b)
TRANSVERSE STRAINS
AT APPROX. CAPACITY LOADS
FOR TBO SERIES

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- $\varepsilon_{hy}$: YIELD STRAIN FOR HOOP STEEL
- DOTTED LINES USED FOR CLARITY
FIG. 5.8 TRANSVERSE STRAIN PROFILES FOR TBO SERIES
Fig. 5.9(a)  
PRINCIPLE DIAGONAL STRAINS  
AT APPROX. HALF CAPACITY LOADS  
FOR TBO SERIES  

LEGEND  
SYMBOL | BEAM | LOAD STAGE  
--- | --- | ---  
02 | TBO2 | 4  
03 | TBO3 | 4  
04 | TBO4 | 3  
05 | TBO5 | 3  

DOTTED LINES USED FOR CLARITY  

Fig. 5.9(b)  
PRINCIPLE DIAGONAL STRAINS  
AT APPROX. CAPACITY LOADS  
FOR TBO SERIES  

LEGEND  
SYMBOL | BEAM | LOAD STAGE  
--- | --- | ---  
02 | TBO2 | 8  
03 | TBO3 | 7  
04 | TBO4 | 10  
05 | TBO5 | 8  

DOTTED LINES USED FOR CLARITY
FIG. 5.10 PRINCIPAL COMPRESSION STRAIN PROFILES FOR TBO SERIES
level (Fig. 5.9(b)). Similar plots are shown in Appendix E.4 for the TBU and TBS series specimens.

These strains were evaluated from strains measured over a grid of targets making 6" (152 mm) squares. Since to evaluate the principal compression strain from a given square needed not only the measurement of the longitudinal and transverse strains but also the diagonal strains (see Appendix E.2) it was only feasible for the available test time to use the 6" squares with the mechanical measuring gauges. Smaller squares would enable a more realistic profile of the principal compression strain to be obtained for any given face, however, considerably more time would be required per load stage.

Fig. 5.10 shows a plot of the principal compression strains predicted by the model at load levels corresponding to those of the average measured principal compression strains which are also plotted for comparison. Again we notice in Fig. 5.10 (a) that the model predicts greater compression strains than those measured. However at approximately capacity loading level (Fig. 5.10(b)) the agreement between the predicted and measured is much better. With reference to Fig. 5.10(b), it should be noted that the measured principal compression strains were for a load stage close to capacity loading whereas the model prediction is for capacity loading condition. Recognizing that close to capacity loading the principal compression strains increase rapidly with little gain in load carrying capacity (see in Fig. 5.5), it is
concluded that agreement between the test data and the theory as seen in Fig. 5.10(b) is very good.

5.4.4 Variation of Angles of Principal Diagonal Compression and Crack Patterns

In Fig. 5.11 are plotted the average angles of principal compression evaluated from the strains of the TBO series beams for the load stages indicated. Appendix E.2 shows how these angles were evaluated from the measured strains. Also shown in Fig. 5.11 is the model predictions for the angles for the specimens at the corresponding load levels.

These plots indicate that the model predicts the angles of principal compression well. Moreover, both test data and model predictions indicate that principal angles vary little for a wide range of loading levels particularly for those regions of the beam where the longitudinal strains are tensile. The principal compression angles vary relatively rapidly down the depth of the section when the varying longitudinal strains are compressive and particularly for large curvatures. However when the longitudinal strains in a section are all tensile, the variation is relatively slow even for large curvatures.

Fig. 5.12 shows crack patterns for the beams in the TBO series for the same load stages used in the plotting of the principal compression angles in Fig. 5.11. Although the
inclination of cracks at any load stage do not necessarily indicate the exact direction of principal compression the crack directions and the pattern of variation of their angles of inclination give an indication of the change in the variation of the principal compression directions. For example the pattern of the cracks in the North face of TB02 is seen to be quite different from that for TB05 for the same face as shown in Fig. 5.12. The trend of the variation of the crack patterns as the torsion to moment ratio increases from 0.0 for TB01 to over 4.0 for TB05 is in good agreement with the trend of variation of the predicted angles of principal compression shown in Fig. 5.11.
FIG. 5.11 VARIATION OF PRINCIPAL COMPRESSION ANGLES FOR TBO SERIES
FIG. 5.12 CRACK PATTERNS FOR TBO SERIES
5.5 PREDICTION OF CAPACITY LOADS

5.5.1 Capacity Loads for Heavily Reinforced Beams

The TBO series specimens were designed such that under pure flexure and for the loading ratio causing uniform beam elongation, the capacity loads would be achieved before yielding of the reinforcing steel. Thus the TBO series section was overreinforced. The TBU series specimens were designed such that, at capacity loads, some of the reinforcing steel would be yielded. Thus they were partially overreinforced. In any case, both these series of specimens were relatively heavily reinforced in comparison to the content of reinforcement used in sections normally used in practise.

Fig. 5.13 shows the theoretical predictions of the failure interaction curves for the TBO and TBU series for both the spalled and the unspalled section dimensions. The capacity loads of the specimens obtained from the tests are also plotted in Fig. 5.13.

For the pure flexure case it is evident that the model predictions for the unspalled section conditions agree well with the test results whereas the spalled predictions underestimate the beam flexural capacities. However, the unspalled prediction quickly becomes unconservative with the application of torsion and the capacity loads are best predicted by the spalled prediction for the test specimen failing
at a ratio of \( \frac{Tu}{Mu} \) greater than 0.6.

Specimen TBU4 failed prematurely because of the problem in casting mentioned in Section 4.2.1. The interaction prediction is based on a top face spalled thickness of 64 mm whereas TBU4 had a local minimum top wall thickness of 39 mm. It was in this local region where failure suddenly occurred. Specimen TBU2 suffered from the same problem but not as severely. TBU2 had a minimum wall thickness of 44 mm compared to the 64 mm used in the spalled prediction. Fig. 5.13 shows that the model predicts well the capacity loads for the heavily reinforced specimens for the widely varying loading ratios and for the concrete strengths shown.
FLEXURE - TORSION INTERACTION FOR TBU AND TBO SERIES

FIG. 5.13 FAILURE CURVES FOR HEAVILY REINFORCED BEAMS UNDER FLEXURE AND TORSION
5.5.2 Capacity Loads for Under-reinforced Beams

It has been demonstrated that the theoretical model predicts well the capacity loads for heavily reinforced specimens. The model is now used to predict the failure interaction lines for under-reinforced beams tested in combined torsion and bending in Zurich. Fig. 5.14 shows the section used in the series and the average values of the properties of the materials used are indicated.

The predicted failure interaction curves are shown for the spalled and unspalled section dimensions and the failure loads for the specimens tested are plotted for comparison. As before, it is again noted that as the ratio, $T_u/M_u$, (at failure) increases, the capacity of the beams tends towards the spalled prediction value. The peak point in the interaction curve with the maximum attainable torsion capacity is characteristic of under-reinforced specimens which have an unsymmetrical longitudinal steel reinforcement pattern such as that shown for the section. Such a peak is not seen for similar over-reinforced specimens.

It should be noted that specimen TB4, unlike the other specimens indicated, was not hollow. It was a solid section with the same outside dimensions and reinforcement layout as for the section shown. The theoretical model indicates that this specimen will have the same load carrying capacity as a hollow one with a wall thickness of 80 mm.
because the effective depths of the compression diagonals at failure are less than the provided wall thickness. As indicated in the failure interaction diagram, the capacity of TB4 and the capacity of the other specimens are well predicted.

FIG. 5.14 FAILURE INTERACTION CURVES FOR UNDER-REINFORCED BEAM TESTED IN ZURICH UNDER COMBINED FLEXURE AND TORSION
5.5.3 Capacity Loads for Prestressed Beams

The theoretical model can also be applied to predict the complete response of prestressed beams loaded in flexure and torsion. In the case of the presence of prestressed steel in a section the resultant force in the reinforcing steel in the longitudinal direction \( F_{sL} \) is evaluated for a given longitudinal strain profile as

\[
F_{sL} = \sum \sigma_{Li} A_{Li} + \sum \sigma_{pj} A_{pj} \tag{5.1}
\]

where

\[
f_{pj} = \text{stress in prestressing steel area, } A_{pj}, \text{ at position 'j' with a strain of } \varepsilon_{pj}
\]

\[
\varepsilon_{pj} = \varepsilon_{Lj} + \Delta \varepsilon_{pj} \tag{5.2}
\]

\[
\varepsilon_{Lj} = \text{longitudinal strain in the concrete at the position 'i' obtained from given strain profile}
\]

\[
\Delta \varepsilon_{pj} = \text{initial prestrain in the prestressed steel area } A_{pj}
\]

If given the initial prestress \( f_{pI} \), then the prestress \( \Delta \varepsilon_{p} \) for a symmetrically prestressed section can be evaluated as:

\[
\Delta \varepsilon_{p} = f_{pI} \left[ \frac{1}{E_{s}} + \frac{A_{p}}{A_{Tl} E_{s} + A_{c} E_{c}} \right] \tag{5.3}
\]

With these expressions the theoretical model was used to predict the failure interaction curve for a series of prestressed beams tested in torsion and bending in Toronto.
FIG. 5.15 FAILURE INTERACTION CURVES FOR PRESTRESSED BEAMS UNDER COMBINED FLEXURE AND TORSION TESTED IN TORONTO
The cross section details and the material properties of the specimens tested are shown in Fig. 5.15. The failure interaction lines predicted are again shown for the spalled and unspalled section dimensions. The failure loads of the beams are plotted and it is seen that the model predicts the capacities well.

It is of interest to note that for the pure torsion case the unspalled prediction overestimates the torsional capacity by about 39%. This is because of the large cover to hoop steel in the top and bottom faces. The spalled prediction for the pure torsion case is in excellent agreement with the test failure loads, thus confirming that the cover to hoop steel can be considered ineffective in carrying loads at failure for this case.

5.6 PREDICTION OF FAILURE MODE

As discussed in Section 3.5 the loading ratio which will lead to uniform stress in concrete in all faces at capacity loads will also induce uniform longitudinal strains in the section. Using the equations developed in Section 3.5 and for spalled section dimensions the amounts of steel reinforcement in the transverse and longitudinal directions for balanced failure have been evaluated and the results are summarized in Table 5.1.
**Table 5.1**

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<th>( t_{min} )</th>
<th>( \rho_{hb} )</th>
<th>( \rho_{tb} )</th>
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<td>2.19</td>
<td>73.1</td>
<td>39.0</td>
<td>.0244</td>
<td>.0198</td>
<td>.0160</td>
<td>.0296</td>
<td>1.328</td>
</tr>
<tr>
<td>TBS1</td>
<td>28.0</td>
<td>2.5</td>
<td>443</td>
<td>2.33</td>
<td>433</td>
<td>2.23</td>
<td>66.2</td>
<td>206.5</td>
<td>.0146</td>
<td>.0152</td>
<td>.0159</td>
<td>.0289</td>
<td>1.271</td>
</tr>
<tr>
<td>TBS2</td>
<td>32.9</td>
<td>2.5</td>
<td>443</td>
<td>2.33</td>
<td>433</td>
<td>2.23</td>
<td>66.2</td>
<td>206.5</td>
<td>.0172</td>
<td>.0179</td>
<td>.0159</td>
<td>.0289</td>
<td>1.277</td>
</tr>
<tr>
<td>TBS3</td>
<td>45.8</td>
<td>2.5</td>
<td>443</td>
<td>2.33</td>
<td>433</td>
<td>2.23</td>
<td>66.2</td>
<td>206.5</td>
<td>.0239</td>
<td>.0249</td>
<td>.0159</td>
<td>.0289</td>
<td>1.312</td>
</tr>
<tr>
<td>TBS4</td>
<td>15.5</td>
<td>2.0</td>
<td>443</td>
<td>2.33</td>
<td>433</td>
<td>2.23</td>
<td>66.2</td>
<td>206.5</td>
<td>.0074</td>
<td>.0077</td>
<td>.0159</td>
<td>.0268</td>
<td>1.157</td>
</tr>
</tbody>
</table>

**Notes:**

- \( t_{min} \) = Minimum concrete wall thickness in section
- \( \epsilon_{cu} = 1.5 \epsilon_0 \) used in evaluation of \( \rho_{hb} \), \( \rho_{tb} \) and \( (T/M) \)**
- \( \rho_h = t_h/t_c \) = supplied hoop steel ratio
- \( \rho_L = t_L/t_c \) = supplied longitudinal steel ratio
- \( f'_{ty} \) = Equivalent yield stress for all longitudinal steel
- \( \epsilon'_{ty} \) = Equivalent yield strain for all longitudinal steel
- \( T/M \) = Failure torsion to moment ratio in test
- \( (T/M) \)** = Ratio of loading for zero flexural curvature
As is evident from Table 5.1 all the specimens tested were over-reinforced with respect to the longitudinal steel content. With respect to hoop steel content, beams TBO4, TBS1 and TBS4 were over-reinforced whereas TBU4, TBS2 and TBS3 were under-reinforced. It is also noted that beams TBS1 and TBS2 had hoop steel content close to that of balanced failure.

The actual ratio of torque to flexural moment \( (T_u/M_u) \) at failure of the beams is also indicated in Table 5.1. Due to the presence of dead loads the overall T/M ratios in a test varied during the loading even though the increments in the loads were at the constant ratios, R, shown in Table 4.3. Shown beside the \( T_u/M_u \) ratios in Table 5.1 are the loading ratios determined using expressions given in Section 3.5 for uniform beam elongation and it is seen that these ratios agree closely with the test loading ratios. It was indeed observed from the continuously plotted midspan deflection which gave a measure of the average flexural curvature over the central 1.75 metres of the specimen that there was negligible curvature up to capacity loads.

5.6.1 Steel and Concrete Strains

Fig. 5.16 shows a plot of the longitudinal strains for the beams shown in Table 5.1 at the last load stage for which strains were measured before attainment of capacity
loads. The yield strain of the main longitudinal reinforcing bars is also plotted in Figure 5.16. The figure shows that for all the specimens, the average longitudinal strains at capacity loads were less than the steel yield strain. The strain profiles also indicate that the beam sections had almost uniform strains at the capacity loads. These observations are in good agreement with the model predictions.

Fig. 5.17 shows a plot of hoop strains for the specimens at the load stage closest to capacity loading. The uniformity of the hoop strains profiles is comparable to that of the corresponding longitudinal strain profiles in Fig. 5.16.

The theory predicts that for TB04 and TBS4 the average transverse strains at capacity load will be less than the yield strain of the hoop steel. A close look at the transverse strains given in Appendix E.3 reveals that strains in excess of yield strain were measured locally. However the overall average transverse strains measured at close to capacity loads for beams TB04 and TBS4 are indeed less than the yield strain of the hoop steel used. This is in agreement with the model prediction.

Fig. 5.18 shows the average principal compression strains evaluated from the test data for the specimens listed in Table 5.1. The premature failure of TBU4 is indicated by the rather low principal compression strains at capacity loading. The average principal compression strains for the top faces of TBS1, TBS2, TBS3 and TBO4 are close to
the $1.5\varepsilon_0$ values used in the theoretical prediction for failure. However, TBS4 shows principal compression strain in excess of $3\varepsilon_0$ in the top face at capacity loading. Large principal compression strains were also indicated for TBS2 and TBS3 for decreasing loads after attainment of capacity loading.

![FIG. 5.16](image1.png)
LONGITUDINAL STRAINS AT ≈ CAPACITY LOADS

![FIG. 5.17](image2.png)
TRANSVERSE STRAINS AT ≈ CAPACITY LOADS

![FIG. 5.18](image3.png)
PRINCIPAL COMPRESSION STRAINS AT ≈ CAPACITY LOADS
5.6.2 Effect of Variation of Concrete Strength on Failure Mode

The TBS series was specifically designed to investigate the effect of varying concrete strength on the failure loads for a fixed loading ratio, $R$, of 1.524 (see Table 4.3). This loading ratio was chosen because at failure the specimen had nearly zero flexural curvature. For uniform beam elongation expressions for equilibrium and compatibility requirements at failure can be derived as follows:

(a) Equilibrium requirements

$$q_u \tan \theta = t_h f_h$$ ..................................................(5.4)
$$q_u \cot \theta = t_l f_l$$ ..................................................(5.5)

$$q_u = \alpha f'_c d \sin \theta \cos \theta$$ ...............................(5.6)

and (5.4) and (5.5) also give

$$q_u = \sqrt{t_h t_l f_h f_l}$$ ..................................................(5.7)

(b) Geometric Requirements

$$\tan^2 \theta = \frac{\varepsilon_l + \varepsilon_{cu}}{\varepsilon_h + \varepsilon_{cu}}$$ ..................................................(5.8)

$$\gamma = 2 \sqrt{(\varepsilon_l + \varepsilon_{cu})(\varepsilon_h + \varepsilon_{cu})}$$ ..................................................(5.9)

$$\psi_x = \frac{p}{Z \gamma} = \gamma / 2 t_c$$ ..................................................(5.10)

$$\phi_d = \psi_x \sin 2 \theta = \varepsilon_{cu} / t_d$$ ..................................................(5.11)
From the above equations of equilibrium and compatibility
the following expressions are derived:

\[ t_d = \frac{t_c \epsilon_{cu}(\epsilon_L + \epsilon_H + 2\epsilon_{cu})}{2(\epsilon_L + \epsilon_{cu})(\epsilon_H + \epsilon_{cu})} \] ................................(5.12)

\[ q_u = \frac{a\beta \frac{t_c f'}{2 e_{cu}}}{\sqrt{(\epsilon_L + \epsilon_{cu})(\epsilon_H + \epsilon_{cu})}} \] ..........................(5.13)

\[ \epsilon_H = \frac{a\beta \frac{t_c f'}{2 e_{cu}} - 1}{\frac{t_c f'}{h_H}} \epsilon_{cu} \] ..........................(5.14)

\[ \epsilon_L = \frac{a\beta \frac{t_c f'}{2 e_{cu}} - 1}{\frac{t_c f'}{h_L}} \epsilon_{cu} \] ..........................(5.15)

If the parabolic stress-strain relationships is
used for concrete then:

\[ a\beta = \Omega_u - \Omega^2_u/3 \]

where \[ \Omega_u = \epsilon_{cu}/\epsilon_0 \]

Expressions developed in section 3.5 enable the de-
termination in advance of whether hoop steel or longitudinal
steel will yield at capacity loading for a given section
and for the given material properties. Whichever is the case,
equations (5.14) and (5.15) together with the given stress-
strain relationships for the reinforcing steel can be used to
evaluate the strains \( \epsilon_H \) and \( \epsilon_L \). It should be noted that
the longitudinal strain \( \epsilon_L \) will be evaluated using an itera-
tive procedure since \( t_L = A_{TL}/p_o \) requires knowledge of \( p_o \)
which in turn requires knowledge of \( t_d \). To initiate the
iteration procedure \( p_0 \) can be guessed and the iterations are carried out to obtain convergence in \( p_0 \).

With the strains \( \epsilon_x', \epsilon_h \) and \( \epsilon_{cu}' \), the equations given above can be used to obtain \( \theta, \gamma, \psi_x', f_h', f_x' \) and \( q_h \). The area \( A_o' \), enclosed by the path of shear flow can be evaluated for the spalled section dimensions, and hence the torque \( T_{up} = 2q_u A_o' \) can be evaluated. The moment acting together with this torque, \( M_{up} \), is evaluated as:

\[
M_{up} = A_T f_x j_d \tag{5.16}
\]

where \( j_d \) is the moment lever arm.

The procedure outlined above was used to obtain failure torques and moments for uniform beam elongation of the section used in the TBS series for \( \epsilon_{cu} = 0.00375 \) and for a series of concrete strengths. The torques and moments obtained are plotted versus the concrete strengths in Figure 5.19. Also plotted in Fig. 5.19 are the failure torques and moments for the TBS series specimens which had different concrete strengths as shown. The lines joining the failure points predicted by the model for each individual specimen are also shown.

The theory indicates that if sufficiently low concrete strength is used the longitudinal steel and hoop steel will not yield at the attainment of capacity loads. This is considered an over-reinforced case, and beams TBS4 and TBS1 were therefore over-reinforced.
There is a range of concrete strengths over which the model predicts only hoop steel will yield for the section chosen. For the TBS series beams the hoop steel content was less than the longitudinal steel content ($\rho_h < \rho_l$). For such a case hoop steel will yield before the longitudinal steel does for a loading that produces zero flexural curvature. Note that if $\rho_l < \rho_h$ then the longitudinal steel will yield before the hoop steel yields in this range of concrete strengths. This can be considered to be a partially over-reinforced range and beams TBS2 and TBS3 were in this range.

If sufficiently high concrete strength is used both longitudinal steel and hoop steel will be yielded at the attainment of capacity loads. This constitutes the under-reinforced case.

It is emphasized here that the failure mode as discussed here pertains to a unique loading case producing zero flexural curvature and zero transverse curvature. Unsymmetry in the reinforcing pattern of longitudinal and/or hoop steel and differences in properties of steel in the various faces may lead to yield of some of the steel in some local area of the section for some loading ratio with some curvature. Note that the unique loading case of interest results in the maximum possible torque to be resisted by the section at failure.

Figure 5.19 indicates that the model predicts well the effect of the variation of concrete on the capacity loads for the specimens tested.
5.6.3 Observed Behaviour of Beams Tested at Failure

Table 5.2 summarizes the experimental failure loads and the predicted failure loads for all the specimens tested. Except for TBU4 the agreement between theory and experiment is good.

Beam TBU4 failed prematurely due to a casting defect explained in Chapter 4. The beam failed suddenly...
during a loading stage and the failure was located at the region with the least wall thickness in the top face. Failure occurred after the yielding of hoop steel.

All the specimens with the hollow section failed in a brittle manner with a sudden drop in load. TBU2 and TBU3 failed suddenly as the main longitudinal steel in the bottom face reached yield strain. On the other hand the solid section of the TBS series showed remarkable ductility even for TBS4 which had the lowest concrete strength in the series. However, the ductility displayed was not at sustained loads but at decreasing lower load levels than the capacity loading. The confining effect of the closely spaced hoops on the solid concrete core of these specimen is largely responsible for the ductility displayed and the very large concrete strains measured. The hollow sections did not have such a concrete core for effective confinement. Moreover, the thickness of the spalled sections was smaller than the effective depths predicted by the model for the compression diagonals at failure. Thus the thin concrete walls were heavily stressed over their entire depth and at failure they crushed leading to the sudden drop in load observed for the hollow sections.

TBS2 and TBS3 failed at slightly lower loads than those predicted. Both of these specimens failed after the yield of hoop steel in all faces. As the capacity loads were approached there was a rapid increase in the measured diagonal
strains and the transverse strains, and this was accompanied by a relatively little increase in load carrying capacity. The predicted principal compression strains at the onset of the yield of hoop steel were greater than $\epsilon_o$ for TBS2 and greater than $0.85\epsilon_o$ for TBS3. The large shearing strains at the high levels of compression in the concrete diagonals is believed to have led to the deterioration in the effective concrete strength leading to an early failure.

It should also be noted that for all the specimens tested the sections had a large amount of longitudinal steel in the bottom face. The tendency of this longitudinal steel was to weaken the concrete in the compression diagonals in the bottom face particularly for high levels of tensile strain in the bars such as was the case for TBS2 and TBS3 at close to their attainment of capacity loads. All TBS series specimens showed a marked upward deflection after the attainment of capacity loads. This upward deflection increased rapidly with further loading.
**TABLE 5.2**

<table>
<thead>
<tr>
<th>BEAM</th>
<th>$f_c^*$ N/mm²</th>
<th>$T_u$ kN-m</th>
<th>$M_u$ kN-m</th>
<th>$T_u$ kN-m</th>
<th>$M_u$ kN-m</th>
<th>FAILURE MODE**</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBO1</td>
<td>19.5</td>
<td>-</td>
<td>401</td>
<td>-</td>
<td>379</td>
<td>over-reinforced: brittle failure</td>
</tr>
<tr>
<td>TBO2</td>
<td>19.7</td>
<td>78</td>
<td>334</td>
<td>72</td>
<td>309</td>
<td>&quot;</td>
</tr>
<tr>
<td>TBO3</td>
<td>19.1</td>
<td>143</td>
<td>232</td>
<td>139</td>
<td>227</td>
<td>&quot;</td>
</tr>
<tr>
<td>TBO4</td>
<td>20.4</td>
<td>149</td>
<td>117</td>
<td>147</td>
<td>114</td>
<td>&quot;</td>
</tr>
<tr>
<td>TBO5</td>
<td>20.6</td>
<td>143</td>
<td>35</td>
<td>140</td>
<td>34</td>
<td>&quot;</td>
</tr>
<tr>
<td>TBU1</td>
<td>34.8</td>
<td>0</td>
<td>551</td>
<td>-</td>
<td>519</td>
<td>under-reinforced: rig not able to fail it</td>
</tr>
<tr>
<td>TBU2</td>
<td>34.8</td>
<td>104</td>
<td>439</td>
<td>104</td>
<td>440</td>
<td>Partially over-reinforced: brittle failure</td>
</tr>
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<td>TBU3</td>
<td>34.8</td>
<td>207</td>
<td>327</td>
<td>207</td>
<td>326</td>
<td>&quot;</td>
</tr>
<tr>
<td>TBU4</td>
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<td>221</td>
<td>164</td>
<td>&quot;</td>
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<td>TBU5</td>
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<td>41</td>
<td>181</td>
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<td>&quot;</td>
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<td>164</td>
<td>202</td>
<td>159</td>
<td>over-reinforced: ductile failure</td>
</tr>
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<td>TBS2</td>
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<td>169</td>
<td>229</td>
<td>176</td>
<td>Partially over-reinforced: ductile failure</td>
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<td>245</td>
<td>186</td>
<td>269</td>
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<td>&quot;</td>
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<td>TBS4</td>
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<td>125</td>
<td>108</td>
<td>119</td>
<td>104</td>
<td>over-reinforced-ductile failure</td>
</tr>
</tbody>
</table>

* Except for pure flexure specimen the theoretical prediction given are those based on spalled section dimensions

**The decision as to whether a beam was over-reinforced or under reinforced is based on the theoretical consideration given in Section 3.5.
5.7 TORQUE-TWIST RELATIONSHIPS

5.7.1 Effect of Variation of Concrete Strength on Torque-Twist Curves

Fig. 5.20(a) shows the test torque-twist curves for the TBS series beams for which the varied parameter in the tests was the concrete strength. Fig. 5.20(b) shows the predicted torque-twist curves for the same specimens for spalled section dimensions. The predicted curves were obtained using the computer program in Appendix D in which the termination of analysis is done when the maximum principal compression strain was approximately $1.5\varepsilon_o$ according to the criteria described in Appendix B-5.

The predicted plots in Fig. 5.20(b) indicate that the effect of increasing the concrete strength is to increase the capacity torque as well as the twist at capacity torque for a given section. The load levels at which the hoop steel yields are indicated.

The test curves in Fig. 5.20(a) confirm the increase in torsional capacity with the increase in concrete strength as predicted. In regard to the yielding of hoop steel, Table 5.3 shows good agreement between the model prediction and test results at failure.
(a) TEST TORQUE-TWIST CURVES FOR TBS SERIES

(b) PREDICTED TORQUE-TWIST CURVES FOR TBS SERIES

FIG. 5.20 TORQUE-TWIST CURVES FOR TBS SERIES
TABLE 5.3
YIELD OF HOOP STEEL AT FAILURE*

<table>
<thead>
<tr>
<th>BEAM</th>
<th>SPALLED</th>
<th>UNSPALLED</th>
<th>TEST</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS1</td>
<td>-</td>
<td>HBS</td>
<td>HBS</td>
</tr>
<tr>
<td>TBS2</td>
<td>HBS</td>
<td>HBS</td>
<td>HBS</td>
</tr>
<tr>
<td>TBS3</td>
<td>HAF</td>
<td>HAF</td>
<td>HAF</td>
</tr>
<tr>
<td>TBS4</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* see legend in Fig. 5.20

The test torque-twist curves for TBS1 and TBS2 indicate much greater twists at close to capacity loading than those predicted. Also TBS1 and TBS2 showed greater twist at failure than TBS3 contrary to theoretical prediction. However, it is of interest to note with reference to Table 4.1 that TBS1 and TBS2 were tested at 3 and 7 days respectively after casting whereas TBS3 and TBS4 were tested 45 and 31 days respectively after casting. Thus TBS1 and TBS2 were tested when the concrete was still curing and the strength gain per day was substantial. On the other hand TBS3 and TBS4 were tested when the curing process was practically complete and the strength gain per day was negligible. The creep effects on the young concrete of TBS1 and TBS2 due to the sustained high load levels as capacity loading was approached in the loading programme which lasted over 3.5 hours is believed to be largely responsible for the greater torsional
ductility for TBS1 and TBS2. Furthermore, the effective stress-strain curve for the concrete for TBS1 and TBS2 with the effects of creep was no longer well represented by the assumed parabola used in the model analysis.

A comparatively rapid load drop was observed soon after the yield of hoop steel in all the faces for TBS3. The greater capacity load predicted for TBS3 was not achieved because of the large shearing strains generated after the yielding of hoop steel at the high levels of principal compression in the concrete. This tended to reduce the effective concrete strength to much lower values than those obtained from the standard compression cylinder test.
5.7.2 Effect of the Variation of Loading Ratio on Torque-Twist Curves

Fig. 5.21 and Fig. 5.22 show plots of the test and predicted torque-twist curves for the TBO series beams and TBU series beams, respectively. For low torque levels these plots indicate, as expected, that the measured twist values are less than the corresponding predicted twist values. However, as capacity torque is approached there is an increase in the measured twists tending to the values predicted for the spalled dimensions of the sections especially for those specimens when diagonal cracks formed in all faces. The predicted torque-twist curves for the pure torsion cases are also shown in Fig. 5.21 and Fig. 5.22 for comparison.

These plots show that as the torque to moment ratio, \( R \), in the loading of a beam is increased from zero to the ratio causing uniform beam elongation, the torque capacity is steadily increased to its maximum at the balanced failure point. A further increase in the torque to moment ratio results in a decrease in the torque capacity. With regard to the twist at failure, the effect of increasing \( R \) is always to increase the twist.
FIG. 5.21 TORQUE-TWIST CURVES FOR TBO-SERIES

FIG. 5.22 TORQUE-TWIST CURVES FOR TBU-SERIES
5.8 MOMENT-CURVATURE RELATIONS

5.8.1 Effect of the Variation of Loading Ratio on Moment-Curvature Curves

Fig. 5.23 and Fig. 5.24 show plots of the test and predicted moment-curvature curves for the TBO series beams and the TBU series beams respectively. The well known theory of pure flexure is used for the prediction curves of the pure flexure specimens TBO1 and TBU1. The accuracy of the flexural theory in predicting the moment-curvature response is shown by the plots for TBO1 and TBU1. The theoretical prediction for the rest of the specimens is made using the model for spalled section dimensions. It is seen that the model does predict the moment-curvature responses for the specimens tested well.

These plots show that the effect of increasing the loading torque to moment ratio, R, is always to reduce the flexural capacity. With regard to curvature, if curvature under pure flexure is taken to be positive, then as R increases from zero to the value at balanced loading the curvature is steadily reduced to zero. A further increase in R results in negative curvature which increases as R is increased. It is important to note that we the discussion is restricted to applied flexural moments that tend to cause sagging in a beam.
$f_c = 19.8 \text{ N/mm}^2$

** Legend **

<table>
<thead>
<tr>
<th>SYMBOLS</th>
<th>BEAM</th>
<th>$\frac{T_y}{M_y}$</th>
<th>THEORY</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>TB01</td>
<td>0.00</td>
<td>UNSPALED</td>
</tr>
<tr>
<td>A O2</td>
<td>TB02</td>
<td>0.233</td>
<td>SPALLED</td>
</tr>
<tr>
<td>O3</td>
<td>TB03</td>
<td>0.614</td>
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</tr>
<tr>
<td>A O4</td>
<td>TB04</td>
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<td></td>
</tr>
<tr>
<td>X O5</td>
<td>TB05</td>
<td>4.154</td>
<td></td>
</tr>
</tbody>
</table>

--- THEORY

LT YIELD OF LONG STEEL IN TOP FACE

** FIG. 5.23 MOMENT-CURVATURE RELATIONSHIPS FOR TBO SERIES **
FIG. 5.24 MOMENT-CURVATURE RELATIONSHIPS FOR TBU SERIES
5.8.2 Effect of the Variation of Concrete Strength on Moment-Curvature Curve

A comparison of the moment-curvature curves in Fig. 5.23 and Fig. 5.24 reveals that the change in the concrete strength from 19.8 N/mm² for TBO series to 34.8 N/mm² for TBU series resulted in an increase in the flexural capacities and curvatures of the corresponding beams with a common loading ratio. The trend of variation as predicted by the model is in good agreement with the test results.

5.9 REVIEW OF BASIC ASSUMPTIONS WITH EXPERIMENTAL RESULTS

To develop the theoretical model five basic assumptions were made. These assumptions were necessary to enable the formulation of the equilibrium and geometric conditions needed in the solution for the problem of a structural concrete beam loaded in combined torsion, bending and axial load. The first basic assumption regarding the distribution of longitudinal strains at a section has been shown to be well verified by test results. The rest of the basic assumptions can be considered valid because the model predictions have been shown to be in good agreement with experimental results in the preceding sections of this chapter for the widely varying changes in the parameters examined. In particular the simple parabolic stress-strain curve used to approximate the behaviour of concrete in a loaded beam has been shown to give good predictions for the heavily reinforced sections tested.
CHAPTER 6

CONCLUSIONS

In this thesis a theoretical model has been presented which has the capability of predicting the response of a structural concrete section subjected to combined torsion, flexure and axial load. The model is comparable in rationality to the well known model for flexural analysis and it has been shown that pure flexure and pure torsion can be treated as special cases of the more general formulation presented.

The capability of the model to predict the complete response of reinforced concrete sections under combined torsion and flexure for the post-cracking loading range of the sections has been demonstrated for rectangular beam sections. However the basic equilibrium and geometric conditions can be applied to any cross-section shape for which the torsion resisted is predominantly St. Venant torsion. Moreover, since the actual stress-strain characteristics of the steel and the concrete are accounted for in the theory, the theory can be extended to cover such factors as repeated loading, reversed loading, and sustained loading which affect the stress-strain characteristics of the materials.

The theory has been verified by comparing its detailed predictions with the results of a test programme involving fourteen heavily reinforced, heavily instrumented beams reported in the thesis. The model has also been shown to predict well the behaviour of prestressed concrete beams tested under combined
torsion and flexure at the University of Toronto. Accurate predictions were also obtained for a series of beams tested under combined torsion and flexure at the Swiss Federal Institute of Technology in Zurich.

In the theoretical model presented in this thesis loading which produces a change of moment in the span of a beam has not been considered. The next development of the model should be to include the effects of such shearing loads. However, results of beams tested in shear suggest that large shearing strains affect the stress-strain characteristics of diagonally cracked concrete. Thus, a realistic stress-strain relationship for concrete which takes into account the effect of the shearing strains generated by torsion and shear will be required before accurate predictions of behaviour can be made.

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Finally, I am thankful to my mother and my father who sacrificed much to send me to school. To them I humbly dedicate this thesis.
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13. Lampert, P. "Post-cracking stiffness of Reinforced Concrete Beams in Torsion and Bending", Publication 71-20, University of Toronto, Department of Civil Engineering.

<table>
<thead>
<tr>
<th>NOTATION</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>Area enclosed by external surface of section</td>
</tr>
<tr>
<td>A_C</td>
<td>Cross sectional area of concrete</td>
</tr>
<tr>
<td>A_h</td>
<td>Cross sectional area of one hoop leg</td>
</tr>
<tr>
<td>A_i</td>
<td>Cross section area of longitudinal reinforcing bar 'i' located at ((z_i', y_i'))</td>
</tr>
<tr>
<td>A_pj</td>
<td>Cross section area of prestressing steel 'j' located at ((z_j', y_j'))</td>
</tr>
<tr>
<td>A_T</td>
<td>Total area of longitudinal rebars in the section</td>
</tr>
<tr>
<td>A_o</td>
<td>Area enclosed by shear flow</td>
</tr>
<tr>
<td>A_l</td>
<td>Area enclosed by the perimeter described by the hoop centrelines</td>
</tr>
<tr>
<td>a_i</td>
<td>Depth of equivalent rectangular stress block at 'i'</td>
</tr>
<tr>
<td>a_b</td>
<td>Depth of equivalent rectangular stress block for bottom face</td>
</tr>
<tr>
<td>a_t</td>
<td>Depth of equivalent rectangular stress block for top face</td>
</tr>
<tr>
<td>b, h</td>
<td>External dimensions of rectangular section</td>
</tr>
<tr>
<td>b_1, h_1</td>
<td>Centreline dimensions of rectangular hoop</td>
</tr>
<tr>
<td>b_o, h_o</td>
<td>Dimensions defining the area enclosed by shear flow</td>
</tr>
<tr>
<td>C, C_T</td>
<td>Resultant compression force in concrete in top face</td>
</tr>
<tr>
<td>C_h_i</td>
<td>Concrete cover to hoop steel centreline at 'i'</td>
</tr>
<tr>
<td>d_i_j</td>
<td>Distance between two points 'i' and 'j' on the perimeter defined by the hoop centrelines</td>
</tr>
<tr>
<td>E_c</td>
<td>Modulus of elasticity of concrete</td>
</tr>
</tbody>
</table>
Resultant compression force in concrete in the direction of hoop steel at 'i'

Resultant Compression Force in the concrete in the longitudinal direction at a section

Resultant tension force in hoop steel at 'i'

Resultant tension force in reinforcing steel in the longitudinal direction

Compressive strength of concrete (cylinder strength)

Average compressive stress in concrete diagonals

Stress in hoops at 'i' (f_{hy} = yield stress)

Stress in longitudinal reinforcing steel at 'i' (f_{ly} = yield stress)

Stress in prestressing steel at 'j' (f_{py} = yield stress f_{pI} = initial stress)

Dimensional parameters defined with b, b_1, b_0

Lever arm for flexural moment

Moment causing curvature to a plane perpendicular to the y-axis (M_{by} = moment at balanced failure)

Moment causing curvature to a plane perpendicular to the z-axis (M_{bz} = moment at balanced failure)

Axial force in the x-axis

Perimeter of external surface of section

Perimeter of shear flow path

Shear flow (q_u = shear flow at failure)

Shear flow for balanced loading

Ratio of applied torque to applied moment
Linear correlation coefficient; or magnitude of a vector defining position of an element.

**S_h**
Hoop spacing

**T**
Torque ($T_b = Torque$ at balanced failure)

**t**
Section wall thickness ($t_t, t_s, t_b$ for top, side and bottom faces, respectively)

**t_c**
Ratio of section area to surface perimeter ($t_c = A/p$)

**t_d**
Depth of compression strains in a concrete diagonal ($t_d = t_M$ for pure flexure)

**t_h**
Hoop steel area in beam face per unit length of beam ($t_h = A_h/s_h$)

**t_L**
Longitudinal steel area per unit length of perimeter ($t_L = A_{TL}/P_o$)

**w**
Measure of warping displacement for open section

**x**
Longitudinal axis of the beam

$$(z_o, y_o)$$
Point of action of $N_x$ in section

$$(z_c, y_c)$$
Point of action of $F_{cl}$ in section

$$(z_s, y_s)$$
Point of action of $F_{sl}$ in section

**a**
An equivalent stress block factor (Equ. 2.21)

**β**
An equivalent stress block factor (Equ. 2.22)

**γ_i**
Shearing strain at 'i'

**ε_o**
Strain in concrete corresponding to maximum stress

**ε_{cb}, ε_{ct}**
Longitudinal strains in bottom surface and top surface

**ε_{cu}**
Maximum compression strain in concrete at capacity loads

**ε_{dp}**
Principal compression diagonal strain in concrete (compressive positive) ($ε_{ds} = Principal compression strain at surface$)

**ε_{hi}**
Strain in hoop steel at 'i' (tensile positive)
\( \epsilon_{nb}, \epsilon_{nt} \): Strain in hoop steel in bottom face and top face for rectangular section

\( \epsilon_{li} \): Strain in longitudinal direction at 'i' (tensile positive)

\( \epsilon_{pj} \): Strain in prestressing steel at 'j' (tensile positive)

\( \epsilon_t \): Strain in transverse direction (tensile positive)

\( \Delta \epsilon_p \): Difference in strain between the bonded tendon and the surrounding concrete

\( \theta \): Angle of diagonal compression with respect to longitudinal axis (\( \theta_p \) for principal compression direction)

\( \rho_h \): Hoop reinforcement ratio (\( \rho_h = t_h/t_c \))

\( \rho_l \): Longitudinal reinforcement ratio (\( \rho_l = t_l/t_c \))

\( \rho_{hb} \): Balanced value of \( \rho_h \)

\( \rho_{lb} \): Balanced value of \( \rho_l \)

\( \sigma_{ch} \): Compressive stress on concrete element in direction of hoop steel

\( \sigma_{cl} \): Compressive stress on concrete element in direction of longitudinal steel

\( \tau \): Shearing stress

\( \phi \): Angular deformation over a given length

\( \phi_{dp} \): Curvature of concrete diagonal in direction of principal compression

\( \phi_L \): Longitudinal axis curvature (Eqn. 3.11)

\( \phi_T \): Transverse axis curvature (Eqn. 3.12)

\( \phi_{xy} \): Longitudinal axis curvature in y direction

\( \phi_{xz} \): Longitudinal axis curvature in z direction

\( \psi_x \): Twist in longitudinal axis (torsional rotation/length)

\( \Omega_d \): Ratio of principal compression strain to the \( \epsilon_o \) value of cylinder test (\( \Omega_d = \epsilon_d/\epsilon_o \))
APPENDIX A

Evaluation of Shear Flow Given $\varepsilon_{ds}$

The equilibrium equations (3.7) and (3.8) can be written as

\[(t^h_n)^b = q\tan \theta_b r^b + qr^b_t\]
\[(t^h_n)^t = q\tan \theta_t r^t + qr^t_b\]

where

\[r^b = 1 - \left(\frac{1}{2}a_b - c_{hb}\right)/h_l\]
\[r^t = \tan \theta_t \left(\frac{1}{2}a_t - c_{ht}\right)/h_l\]
\[r^t = 1 - \left(\frac{1}{2}a_t - c_{ht}\right)/h_l\]
\[r^b = \tan \theta_b \left(\frac{1}{2}a_b - c_{hb}\right)/h_l\]

In general

\[(t^h_n)_i = q(\tan \theta_i r_{ii} + r_{ij})\]

where $i$ and $j$ are the two walls cut by an appropriate section, e.g. Fig. 2.5. Rewrite simply as

\[t^h_n = q(\tan \theta_i r_i + r_j)\] \hspace{1cm} \text{(A1)}

Therefore $q\tan \theta_i = \frac{t^h_n - qr_j}{r_i}$ \hspace{1cm} \text{(A2)}

The shear flow $q$ is related to the principal concrete compression strain $\varepsilon_{ds}$ by equation (3.14) which can be rewritten as

\[k_{ij} = a\theta = \frac{\sigma}{c_i^t} \sin \theta \cos \theta \] \hspace{1cm} \text{(A3)}
where \[ a^2 = (1 - \mu^2) \Omega_{ds} - \frac{1}{3}(1 - \mu^3) \Omega_{ds}^2 \]

\[ \Omega_{ds} = \frac{\varepsilon_{ds}}{\varepsilon_0} \]

\[ \mu = \frac{t_d - t}{t_d} \geq 0 \text{ and } \mu < 1.0, \mu = 0 \text{ for } t_d \leq t \]

\[ t_d = \text{effective diagonal thickness} \]

\[ t = \text{actual section wall thickness} \]

Compatibility of strains requires that

\[ \tan^2 \theta_i = \frac{\varepsilon_{lh} + \varepsilon_{dh}}{\varepsilon_h + \varepsilon_{dh}} \ldots (A4) \]

where \[ \varepsilon_{dh} = f \varepsilon_{ds} \]

- concrete principal diagonal strain at level of hoop steel

\[ \varepsilon_{lh} = \text{longitudinal steel strain at level of hoop steel} \]

\[ f = \frac{(t_d - c_h)}{t_d} \leq 1.0 \]

\[ c_h = \text{cover to centreline of hoop steel} \]

Recognizing that \[ \sin \theta \cos \theta = \tan \theta / (1 + \tan^2 \theta) \], equation A3 becomes

\[ k_1 = \frac{q \tan \theta_i (1 + \tan^2 \theta_i)}{f' r_i c_d} \]

Then using equations (A1) and (A4) gives

\[ k_1 = \frac{(t_h f_h - qr)(\varepsilon_h + \varepsilon_{lh} + 2\varepsilon_{dh})}{(\varepsilon_{lh} + \varepsilon_{dh})^2} \ldots \ldots \ldots \ldots (A5) \]

In equation (A5) the unknowns are \( \varepsilon_h, f_h \) and \( q \), but the stress strain relationship between \( \varepsilon_h \) and \( f_h \) is known. Assuming that \( \varepsilon_h = \varepsilon_{hy} \), then \( f_h = f_{hy} \). The
terms $r_i$ and $r_j$ are also unknown initially but in general
$r_i$ is close to 1.0 and $r_j$ is close to zero. Thus by
initially ignoring the term $qr_j$ and setting $r_i = 1$, the RHS of
equation A5 can be evaluated. It is assumed that the
quantities $\epsilon_{lh}$, $\epsilon_{ds}$, $t_d$ and $t$ are given, hence $k_l$ of equation
A5 can be determined.

If the RHS $< k_l$ then hoop steel is in the poor
yield range.

If the RHS $> k_l$ then hoop steel is in the elastic
range.

**Case A: Hoop Steel in Elastic Range**

If hoop steel is in elastic range then $f_h = E_s \epsilon_h$,
hence equation A5 becomes

$$k_l = \frac{(t_h E_s \epsilon_h - qr_j)(\epsilon_h + \epsilon_{lh} + 2 \epsilon_{dh})}{t_c t_d f_i (\epsilon_l + \epsilon_{dh})}$$

This equation can be rewritten as a quadratic in $\epsilon_h$ as

$$\epsilon_h^2 + \frac{\{t_h E_s \epsilon_{lh} + 2 \epsilon_{dh}\} - qr_j}{t_h E_s} \epsilon_h - \frac{[k_l f_i t_d i (\epsilon_l + \epsilon_{dh}) + qr_j (\epsilon_{lh} + 2 \epsilon_{dh})]}{t_h E_s} = 0 \ldots (A6)$$

This equation can be written in the form

$$\epsilon_h^2 + b \epsilon_h - c = 0$$

and hence obtain a solution for the hoop strain as

$$\epsilon_h = \frac{1}{2} (-b \pm \sqrt{b^2 + 4c})$$

Clearly the positive root is the correct one, hence

*RHS = Right Hand Side*
\[
\varepsilon_h = \frac{b}{2} \left[ \sqrt{1 + \frac{4c}{b^2}} - 1 \right] \quad \text{(A7)}
\]

With \( \varepsilon_h \) determined, \( \tan \theta_i \) can be evaluated using (A4) and then using A2 the shear flow \( q \) can be evaluated. Clearly the value of \( q \) has to be guessed initially if \( r_j \neq 0 \) so as to solve equation (A6). The final solution is obtained iteratively.

**Case B:** Hoop Steel in the Post Yield Range

For this case rewrite equation A5 as

\[
\varepsilon_h = \frac{k_1 f'_c t_d r_i (\varepsilon_{\varepsilon_{h}} + \varepsilon_{dh})}{(t_{h} f_{h} - q r_j)} - (\varepsilon_{\varepsilon_{h}} + 2 \varepsilon_{dh}) \quad \text{(A8)}
\]

Knowing that \( \varepsilon_h \geq \varepsilon_{hy} \), equation (A8) is solved iteratively using the given stress strain relationship for hoop steel.
APPENDIX B

Evaluation of Compatible Strains Given q

The equilibrium and compatibility equations required for a solution are:

I. Equilibrium Equations

From Appendix A: \[ q \tan \theta_i = \frac{t_i f_i - qr_j}{r_i} \] \hspace{1cm} (A2)

\[ (1 - \mu^2) \Omega_d s - \frac{1}{3} (1 - \mu^3) \Omega_d s^2 = q/(f_i t_d \sin \theta \cos \theta) \quad (\theta \geq 0) \] \hspace{1cm} (A3)

II. Compatibility Requirement

\[ \tan^2 \theta_i = \frac{\varepsilon_{i h} + \varepsilon_{d h}}{\varepsilon_h + \varepsilon_d} \] \hspace{1cm} (A4)

There are thus 3 equations A2, A3 and A4 to solve for three unknowns \( \theta, \varepsilon_h \) and \( \varepsilon_d \). It is assumed that values are given for all the other variables.

B-1 Determination of Limits for Strains \( \varepsilon_h \) and \( \varepsilon_d \)

It is convenient to evaluate \( \varepsilon_h \) in terms of the hoop yield strain \( \varepsilon_{h y} \) so that

\[ \varepsilon_h = n \varepsilon_{h y} \] \hspace{1cm} (B1)

If the hoop steel is in the elastic range then rewrite equation (A2) as

\[ \tan \theta_i = \frac{n t_i f_i - qr_j}{qr_i} \quad \text{for } 0 \leq n \leq 1.0; \text{ and} \]

define \( C = \frac{t_i f_i}{q} \) \hspace{1cm} (B2)
Then

\[ \tan \theta_i = \frac{nC - r_j}{r_i} \] ..........................(B3)

Substituting (B3) into (A4), obtain

\[ \tan^2 \theta_i = \left( \frac{nC - r_j}{r_i} \right)^2 = \frac{\Omega_{zh} + \Omega_{dh}}{n\Omega_{hy} + \Omega_{dh}} \]

where \( \Omega_{zh} = \varepsilon_{zh}/\varepsilon_o, \Omega_{hy} = \varepsilon_{hy}/\varepsilon_o, \) and \( \Omega_{dh} = \varepsilon_{dh}/\varepsilon_o \)

Thus

\[ \Omega_{dh} = \left( \frac{nC - r_j}{r_i} \right)^2 \left( \frac{n\Omega_{hy} - \Omega_{zh}}{1 - \left( \frac{nC - r_j}{r_i} \right)^2} \right) \] ..........................(B4)

For a practical solution \( \Omega_{dh} \geq 0 \). This simple requirement can be used to define limits for \( n \) for use in an iteration scheme. For example for the case \( r_j = 0 \) and \( r_i = 1 \) which is the case for the side faces of the section in Fig. 3.1, equation (B4) becomes

\[ \Omega_{dh} = \left( n^3 C^2 \Omega_{hy} - \Omega_{zh} \right) / \left( 1 - n^2 C^2 \right) \] ..........................(B5)

For equation (B5) to give \( \Omega_{dh} \geq 0 \)

\[ \left[ \frac{\Omega_{zh}}{n\Omega_{hy} C^2} \right]^{1/3} \leq n \leq \frac{1}{C} \text{ or } \frac{1}{C} < n \leq \left[ \frac{\Omega_{zh}}{n\Omega_{hy} C^2} \right]^{1/3} \] ..........................(B6)

Thus for hoop steel in the elastic range \( n \) must always lie between \( 1/C \) and \( \left[ \frac{\Omega_{zh}}{n\Omega_{hy} C^2} \right]^{1/3} \).

These limits shall be referred to as \( \ell_1 \) and \( \ell_2 \) respectively.

B-2 Determination of Compatible Strains for Hoop Steel in Post Yield Range.

If both \( \ell_1 \) and \( \ell_2 \) are greater than 1.0 then it is certain that \( \varepsilon_h > \varepsilon_{hy} \). In this case for a
bilinear stress strain relationship for the hoop steel

a solution for $\Omega_{ds}$ is obtained as follows:

Equation (A2) gives $\tan \theta_i = (C - r_j)/r_i$

and rewrite equation (A3) as

$$\Omega_{ds}^2 - \frac{3(1 - \mu^2)}{1 - \mu^3} \Omega_{ds} + \frac{3\frac{q}{c'd's_i\cos \theta_i}}{1 - \mu^3} = 0 \quad (B7)$$

and let $z = \sin \theta_i \cos \theta_i = \frac{\tan \theta_i}{1 + \tan^2 \theta_i} = \frac{(C - r_j)/r_i}{1 + [(C - r_j)/r_i]^2}$

Equation (B7) can be written in the form

$$\Omega_{ds}^2 - b\Omega_{ds} + g = 0$$

For which the solution for $\Omega_{ds}$ is obtained as

$$\Omega_{ds} = \frac{1}{2} [b \pm \sqrt{b^2 - 4g}]$$

For minimum strain energy the correct root is the smaller of the two roots i.e.

$$\Omega_{ds} = \frac{1}{2} [b - \sqrt{b^2 - 4g}] \quad (B8)$$

and

$$\varepsilon_{dh} = f\Omega_{ds} \varepsilon_0$$

The value of hoop strain $\varepsilon_h$ can now be evaluated using equation (A4):

$$\varepsilon_h = [\varepsilon_{zh} + \varepsilon_{dh} (1 - \tan^2 \theta_i)] \tan^2 \theta_i \quad (B9)$$
B-3. Determination of Compatible strains for Hoop Steel in Elastic Range

If both $\ell_1$ and $\ell_2$ are less than 1.0 then it is certain that $\varepsilon_h < \varepsilon_{hy}$. To solve for this case rewrite equation (A3) as

$$f(\eta) = (1 - \mu^2) \Omega_{ds} z - \frac{1}{3}(1 - \mu^2) \Omega_{ds} z - \eta/(f' t_d) = 0$$

(for $\theta > 0$) ............ (B10)

where $z = \sin\theta_i \cos\theta_i = \frac{\tan\theta_i}{1 + \tan^2\theta_i} = \frac{(nC - r_j)/r_i}{1 + [(nC - r_j)/r_i]^2}$

$$\Omega_{ds} = \Omega_{dh}/f \text{ and } \Omega_{dh} \text{ is given by equation (B4)}$$

The problem here is to find $\eta$ to satisfy equations (B4) and (B10). The solution is to be sought using an iterative procedure.

Now $f'(\eta) = (1 - \mu^2)[\frac{d\Omega_{ds}}{d\eta} z + \Omega_{ds} \frac{dz}{d\eta}]$

$$- \frac{1}{3}(1 - \mu^2)[2\Omega_{ds} \frac{d\Omega_{ds}}{d\eta} z + \Omega_{ds}^2 \frac{dz}{d\eta}]$$

$$\frac{d\Omega_{ds}}{d\eta} = \frac{1}{f} \frac{d\Omega_{ds}}{d\eta} = \frac{2N}{r_i} \frac{n\Omega_{hy} + N^2 \Omega_{hy}}{f(1 - N^2)} + \frac{2N}{r_i} \frac{\Omega_{ds}}{(1 - N^2)}$$

where $N = \frac{nC - r_j}{r_i}$

$$\frac{dz}{d\eta} = \frac{c}{r_i}(1 - N^2)/(1 + N^2)^2$$

The iteration process to solve equation (B10) can be initiated by assuming $\eta_i = \frac{1}{2}(\ell_1 + \ell_2)$ for $i = 1$. 
For subsequent iteration steps, use the recursion formula:
\[ \eta_{i+1} = \eta_i - \frac{f(\eta_i)}{f'(\eta_i)} \] (B12)

This gives quadratic convergence close to the root.

**B-4 Limitation on \( t_d \) Value**

Equations (B7) and (B10) are identical if \( \sin \theta_i \cos \theta_i \) is replaced by \( z \) so that:
\[
\Omega_{ds}^2 - \frac{3(1 - \mu^2)}{1 - \mu^3} \Omega_{ds} + \frac{3}{1 - \mu^3} \frac{q}{f't'dz} = 0
\]

This equation is solved directly to give
\[
\Omega_{ds} = \frac{3}{2} \frac{1 - \mu^2}{1 - \mu^3} [1 - \sqrt{1 - \frac{4}{3} \frac{q}{f't'dz} \frac{1 - \mu^3}{(1 - \mu^2)^2}}] \] (B13)

For a real solution,
\[
\frac{q}{f't'dz} \leq 0.75z \frac{(1 - \mu^2)^2}{1 - \mu^3} \] (B14)

Taking \( \mu = 0 \) and recognizing that the maximum possible value of \( z \) is 0.5, obtain the limitation for \( t_d \) for a given \( q \) as
\[
t_d \geq \frac{q}{0.375f't'} \] (B15)

**B-5 Limitation on the Value of \( \Omega_{ds} \)**

Rewrite equation (B10) as
\[ f(n) = f^*(\Omega_d) - k \]
where \( f^*(\Omega_d) = (1 - \mu^2)\Omega_{ds}z - \frac{1}{3}(1 - \mu^3)\Omega_{ds}^2z \)

and \( k = q/f'_c t_d \)

For a given \( k \), \( f(n) \) will be a maximum when \( f^*(\Omega_d) \) is maximum. Now for a maximum value for \( f^*(\Omega_d), \frac{\partial}{\partial \Omega_d} f^*(\Omega_d) \)

will be zero. Now

\[
\frac{\partial}{\partial \Omega_d}(f^*(\Omega_d)) = (1 - \mu^2)z - \frac{2}{3}(1 - \mu^3)\Omega_{ds}z + \frac{dz}{d\Omega_d}(1 - \mu^2)\Omega_{ds} - \frac{1}{3}(1 - \mu^2)\Omega_{ds}^2 \]

\[
\Omega_{ds} = \frac{3}{2} \frac{1 - \mu^2}{1 - \mu^3} - \frac{dz}{d\Omega_d}(1 - \mu^2)\Omega_{ds} - \frac{1}{3}(1 - \mu^2)\Omega_{ds}^2 \]

When close to the maximum of \( f(n) \) it is found that \( dz/d\Omega_d \) is very small and hence can be neglected. In fact in the limit when \( \eta = \xi_1, \), \( dz/d\Omega_d = 0 \) and \( \theta_i = 45^\circ \). Thus for a maximum value of \( f(\Omega) \) the value of \( \Omega_{ds} \) is given by

\[
\Omega_{ds} = \frac{3}{2} \frac{1 - \mu^2}{1 - \mu^3} \]

\[
\text{(B16)}
\]

Figure B-1 shows the possible cases for which a solution is to be sought for equation (B10). It is indicated that there are two solutions of \( \Omega_{ds} \) for each case. The smaller solution value is chosen. The other solution value is generally found to be quite large often giving \( \varepsilon_{ds} \) values in excess of \( 2\varepsilon_0 \). Such large strain values will only occur after a failure mechanism has developed. By limiting the value of \( \Omega_{ds} \) as given by equation B16, the search for a solution
is restricted to be for compression strain up to capacity loading. It should however be noted that for some reinforced concrete sections it is possible at capacity loading to have $\Omega_{ds}$ values in excess of the value given by equation (B16) for the highly compressed zone, however the rest of the section will have $\Omega_{ds}$ values less than those given by (B16). Using the limitation on $\Omega_{ds}$ as given by B16 is convenient for the iteration scheme used and the capacity loading is always practically predicted with this limitation.
FIG. B.1

\[ \Omega_{ds} = \frac{1}{f} \Omega_{dh} = \frac{1}{f} \frac{(\eta^3 c^2 \Omega_{hy} - \Omega_{ih})}{(1-\eta^2 c^2)}, \quad \kappa = q/f_{td} \]

\[ z = \tan \theta_i /(1+\tan^2 \theta_i); \quad \tan \theta_i = (\eta c - r_i)/r_i; \quad \text{maximum value of } z \text{ is } 0.5 \]

\[ f^* = (1 - \mu^2) \Omega_{ds} z - \frac{1}{3}(1 - \mu^3) \Omega_{ds}^2 z; \quad \text{maximum value of } f^* \text{ with } z = 0.5 \text{ is } 0.375 \]

The circles show the two possible solutions for each case. The solution giving the smaller value of \( \Omega_{ds} \) is chosen.
APPENDIX C

EXAMPLE: Combined Flexure, Torsion and Axial Load

Given

Concrete

\[ f'_c = 34.8 \, \text{N/mm}^2 \]
\[ \varepsilon_o = 3.1 \times 10^{-3} \]

Steel - (a) Longitudinal

<table>
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<tr>
<th>I</th>
<th>( A_s )</th>
<th>( Y_s )</th>
<th>( f_{ly} )</th>
<th>( E_s \times 10^3 )</th>
<th>( \varepsilon_{ly} )</th>
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<td>3</td>
<td>142</td>
<td>190.5</td>
<td>552</td>
<td>205.1</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>142</td>
<td>110.5</td>
<td>552</td>
<td>205.1</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3570</td>
<td>2.30</td>
<td>436</td>
<td>194.2</td>
<td>2.25</td>
<td></td>
</tr>
</tbody>
</table>

(b) Hoop Steel

\[ A_h = 129 \, \text{mm}^2 \]
\[ S_h = 76.2 \, \text{mm} \]
\[ t_h = 1.693 \]
\[ f_{hy} = 379 \, \text{N/mm}^2 \]
\[ E_s = 198.5 \times 10^3 \, \text{N/mm}^2 \]
\[ \varepsilon_{hy} = 1.91 \times 10^{-3} \]

Assume all steel has a bilinear stress-strain relationship.

Problem

Assuming spalled section dimensions effective, evaluate the moment and axial load acting together with a torque \( T = 200 \, \text{kN-m} \) so as to induce the following longitudinal strain profile with a curvature of \( 5.94 \times 10^{-6} \, \text{rad/mm} \)
\( \varepsilon_{lth} = 0.0 \) = longitudinal strain at level of hoop steel centreline in top face

\( \varepsilon_{ls2} = 1.129 \times 10^{-3} \)

\( \varepsilon_{ls3} = 1.609 \times 10^{-3} \)

\( \varepsilon_{lb} = \varepsilon_{ly}(1) \) = longitudinal strain at level of centreline of longitudinal steel in bottom face

**Solution**

To illustrate how the theory presented is used in the solution of this problem the procedure outlined in section 5.3.3 shall be used. To simplify the problem we shall take three points on the side faces for the evaluation of concrete forces and strains. Two of these points have the same longitudinal strains as the centrelines of hoop steel at the top and bottom faces respectively. The third point is taken at the midheight of the hoop steel.

Using spalled section dimensions equations (3.7) and (3.8):

\[
\begin{align*}
\theta_{hsb} &= q\tan\theta_br_{bb} + qr_{bt} \\
\theta_{ht} &= q\tan\theta_tr_{tt} + qr_{tb}
\end{align*}
\]

are used in the first iteration with \( r_{bb} = r_{tt} = 1.0 \) and \( r_{ht} = r_{tb} = 0.0 \). This is a good approximation since \( h_1 \) is always very much larger than either \( a_b \) or \( a_t \). For subsequent iterations the values of \( r_{bb}, r_{bt}, r_{tt}, \) and \( r_{tb} \) are evaluated
using the appropriate values from the previous iteration.

**Initial Values**

With the given strain profile, evaluate the strains at the various points selected.

**Top**

3. \( \varepsilon_{th} = \varepsilon_{s3} = 0.0 \) (at level of top face hoop steel centreline)

**Side**

2. \( \varepsilon_{s2} = 1.129 \times 10^{-3} \)

**Bottom**

1. \( \varepsilon_{bh} = \varepsilon_{s1} = 2.259 \times 10^{-2} \) (at level of bottom face hoop steel centreline)

To start off the iteration process, assume the effective depths of the concrete compression diagonals to be:

\[
t_{db} = t_{dt} = t_{d1} = t_{d2} = t_{d3} = \frac{0.375b_{1}h_{1}}{b_{1} + h_{1}} = 79.84 \text{ mm}
\]

To evaluate the shear flow due to the torque of 200 kN-m, the area enclosed by the path of shear flow is required.

Assume

\[
A_{o} = b_{o}h_{o}
\]

where \( b_{o} = 0.9b_{1} \) and \( h_{o} = 0.9h_{1} \)

therefore \( A_{o} = 0.81b_{1}h_{1} = 148935 \text{ mm}^2 \)

and \( g = \frac{T}{2A_{o}} = 671.433 \text{ N/mm} \)
Step 1  Evaluation of Resultant Tension Force in Steel in Long Direction \( F_{st} \) and its Point of Action

<table>
<thead>
<tr>
<th>( A_s ) ( \text{mm}^2 )</th>
<th>( y_s ) mm</th>
<th>( \varepsilon_f ) mm/mm</th>
<th>( f_s ) N/mm(^2)</th>
<th>( T_s = A_s \varepsilon_f ) N</th>
<th>( T_s y_s ) N-mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>3570</td>
<td>2.30</td>
<td>2.245</td>
<td>436.00</td>
<td>1556520</td>
<td>3579996</td>
</tr>
<tr>
<td>142</td>
<td>110.5</td>
<td>1.604</td>
<td>328.91</td>
<td>46705</td>
<td>5160901</td>
</tr>
<tr>
<td>142</td>
<td>190.5</td>
<td>1.129</td>
<td>231.63</td>
<td>32892</td>
<td>6265936</td>
</tr>
<tr>
<td>142</td>
<td>270.5</td>
<td>0.655</td>
<td>134.36</td>
<td>19079</td>
<td>5160901</td>
</tr>
<tr>
<td>387</td>
<td>354.0</td>
<td>0.160</td>
<td>32.39</td>
<td>12507</td>
<td>4427477</td>
</tr>
</tbody>
</table>

\( F_{st} = 1,667,703 \text{ N} \)

\( y_s = 14.748 \text{ mm} \)

Step 2  Evaluation of Compatible Strains at a Point in a Plane Given \( q \) and \( \varepsilon_f \) Using Appendix B.

(i) Determine Limits \( l_1 \) and \( l_2 \)

\[
C = \frac{\frac{t_h}{f_{hy}}}{q} = \frac{1.693 \times 379}{671.433} = 0.956
\]

\[
l_1 = \frac{1}{C} = 1.046
\]

\[
l_2 = \left( \frac{\varepsilon_f}{\varepsilon_{hy} C^2} \right)^{1/3}
\]

<table>
<thead>
<tr>
<th>Point</th>
<th>( \varepsilon_f \times 10^{-3} )</th>
<th>( l_2 )</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>2.259</td>
<td>1.090</td>
<td>As both ( l_1 ) and ( l_2 &gt; 1.0 ), ( \varepsilon_{hb} &gt; \varepsilon_{hy} )</td>
</tr>
<tr>
<td>1</td>
<td>2.259</td>
<td>1.090</td>
<td>Hoop Steel &lt; ( \varepsilon_{hy} )</td>
</tr>
<tr>
<td>2</td>
<td>1.129</td>
<td>0.865</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>
(ii) Bottom Face (Use equation in Appendix B-2)

\[ \tan \theta_b = C = 0.956 \]

Hence \[ \theta_b = 43.7^\circ \]

and \[ z = \sin \theta \cos \theta = 0.499 \]

Now \[ k = q/(f_e t_d) = \frac{671.433}{348 \times 79.84} = 0.242 \]

\[ \Omega_{ds}^2 - 3 \Omega_{ds} + 3 \times 0.242 / 0.499 = 0 \]

Solving, obtain \[ \Omega_{ds} = 0.606 = \Omega_{dh} \]

\[ \varepsilon_{dhb} = \Omega_{dh} \varepsilon_0 = 1.88 \times 10^{-3} \]

\[ \varepsilon_{hb} = \frac{[2.259 + 1.88(1 - 0.956^2)] \times 10^{-3}}{0.956^2} = 2.652 \times 10^{-3} \]

\[ \gamma_b = 2\sqrt{(2.259 + 1.88)(2.652 + 1.88)} \times 10^{-3} = 8.662 \times 10^{-3} \]

(iii) Top Face - Hoop Steel in Elastic Range

Use iteration procedure outlined in Appendix B to obtain \[ \varepsilon_{dst} \] and \[ \varepsilon_{ht} \] with the following equation

\[ N = nC \]

\[ \Omega_{ds} = \frac{N^2 \Omega_{hy} - \Omega_{dh}}{1 - N^2} \]

\[ z = N/(1 + N^2) \]

\[ \frac{dz}{d\eta} = C(1 - N^2)/(1 + N^2)^2 \]

\[ \frac{d\Omega_{ds}}{d\eta} = \frac{2NC(\Omega_{hy} + \Omega_{ds}) + N^2 \Omega_{hy}}{(1 - N^2)} \]
\[ f(\eta) = \Omega_{ds} z - \frac{1}{3} \Omega_{ds}^2 z - c/f_0^t d \]

\[ f'(\eta) = z \frac{d\Omega_{ds}}{d\eta} + \Omega \frac{dz}{d\eta} - \frac{1}{3} [2\Omega_{ds}^2 \frac{d\Omega_{ds}}{d\eta} + \Omega_{ds}^2 \frac{dz}{d\eta}] \]

Iterate with

\[ \eta_{i+1} = \eta_i - f(\eta_i)/f'(\eta_i) \]

where

\[ \Omega_{hy} = \epsilon_{hy} / \epsilon_o = \frac{1.909}{3.1} = 0.616 \]

\[ \Omega_{h} = \epsilon_{h} / \epsilon_o = 0.0 \]

\[ C = 0.956 \]

<table>
<thead>
<tr>
<th>Iter.</th>
<th>( \eta )</th>
<th>( \Omega_{ds} )</th>
<th>( f(\eta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.523</td>
<td>0.107</td>
<td>-0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.829</td>
<td>0.863</td>
<td>0.058</td>
</tr>
<tr>
<td>3</td>
<td>0.790</td>
<td>0.644</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.789</td>
<td>0.639</td>
<td>2.77*10^{-6}</td>
</tr>
<tr>
<td>5</td>
<td>0.789</td>
<td>0.639</td>
<td>2.0*10^{-10}</td>
</tr>
</tbody>
</table>

\[ \epsilon_{ht} = \eta_c \epsilon_{hy} = 0.789*1.909*10^{-3} = 1.506*10^{-3} \]

\[ \epsilon_{dst} = \Omega_{ds} \epsilon_o = 0.639*3.1*10^{-3} = 1.980*10^{-3} \]

\[ \tan^2 \theta_t = \frac{0 + 1.980}{1.506 + 1.980} = 0.568; \text{ hence } \theta_t = 37.005^\circ \]

\[ \gamma_t = 2\sqrt{(0 + 1.980)(1.506 + 1.980)} *10^{-3} = 5.255*10^{-4} \]

(iv) Side Face

a) Level 1 - same as for Bottom Face
b) Level 3 - same as for Top Face
c) Level 2

Hoop steel will be in elastic range and start iteration with \( \eta = \frac{1}{2} (.865 + 1.046) \) i.e. \( \eta = 0.955 \).
Convergence in $\eta$ is obtained in two iterations with the following results.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$\varepsilon_{h2} \times 10^{-3}$</th>
<th>$\varepsilon_{l2} \times 10^{-3}$</th>
<th>$\varepsilon_{d2} \times 10^{-3}$</th>
<th>$\gamma_2 \times 10^{-3}$</th>
<th>$\theta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.946</td>
<td>1.806</td>
<td>1.127</td>
<td>1.890</td>
<td>6.681</td>
<td>42.11°</td>
</tr>
</tbody>
</table>

Step 3 - Evaluate Twist

$$\psi_x = \frac{1}{2A_1} [b_1(\gamma_b + \gamma_t) + \frac{h_1}{3} (\gamma_1 + 4\gamma_2 + \gamma_3)]$$

$$= \frac{1}{2 \times 381 \times 482.6} \left[ 482.6(8.662 + 5.255) + \frac{381}{3}(8.662 + 4 \times 6.681 + 5.255) \right] \times 10^{-3}$$

$$= 0.0323 \times 10^{-3} \text{ rad/mm}$$

Step 4: Evaluation of Diagonal Curvatures and New $t_d$ Values

$$\phi_d = \phi_t \sin^2 \theta + \phi_2 \cos^2 \theta + \psi \sin \theta$$

$$t_d = \varepsilon_{ds} / \phi_d$$

$$(t_d)_{new} = \frac{1}{2} (t_d + (t_d)_{old})$$

<table>
<thead>
<tr>
<th>Position</th>
<th>$\theta^\circ$</th>
<th>$\phi_t \times 10^{-3}$</th>
<th>$\phi_2 \times 10^{-3}$</th>
<th>$\psi \times 10^{-3}$</th>
<th>$\phi \times 10^{-3}$</th>
<th>$t_d$</th>
<th>$t_d\text{old}$</th>
<th>$t_d\text{new}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>43.7</td>
<td>-.00301</td>
<td>-.00594</td>
<td>.0323</td>
<td>.0277</td>
<td>67.81</td>
<td>79.84</td>
<td>73.82</td>
</tr>
<tr>
<td>Top</td>
<td>37.0</td>
<td>.00301</td>
<td>.00594</td>
<td>&quot;</td>
<td>.0359</td>
<td>55.11</td>
<td>&quot;</td>
<td>67.48</td>
</tr>
<tr>
<td>1</td>
<td>43.7</td>
<td>0</td>
<td>0</td>
<td>&quot;</td>
<td>.03227</td>
<td>58.26</td>
<td>&quot;</td>
<td>69.05</td>
</tr>
<tr>
<td>2</td>
<td>42.11</td>
<td>0</td>
<td>0</td>
<td>&quot;</td>
<td>.03214</td>
<td>58.82</td>
<td>&quot;</td>
<td>69.33</td>
</tr>
<tr>
<td>3</td>
<td>37.0</td>
<td>0</td>
<td>0</td>
<td>&quot;</td>
<td>.03105</td>
<td>60.55</td>
<td>&quot;</td>
<td>70.19</td>
</tr>
</tbody>
</table>

* $t_d$ values to be used in the next iteration
Step 5 Evaluation of Area $A_0$ Enclosed by Path of Shear Flow and Re-evaluation of $q$

<table>
<thead>
<tr>
<th>Position</th>
<th>$\Omega_{ds}$</th>
<th>$\beta$</th>
<th>$t_d$</th>
<th>$a=\beta t_d$</th>
<th>$h_o=h_1 \frac{1}{2}(a_b+a_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>.606</td>
<td>.709</td>
<td>79.84</td>
<td>56.60</td>
<td>$b_{01}=b_1-a_1=426.00 \text{ mm}$</td>
</tr>
<tr>
<td>Top</td>
<td>.639</td>
<td>.712</td>
<td>&quot;-&quot;</td>
<td>56.83</td>
<td>$b_{02}=b_1-a_2=425.98 \text{ mm}$</td>
</tr>
<tr>
<td>1</td>
<td>.606</td>
<td>.709</td>
<td>&quot;-&quot;</td>
<td>56.60</td>
<td>$b_{03}=b_1-a_3=425.77 \text{ mm}$</td>
</tr>
<tr>
<td>2</td>
<td>.610</td>
<td>.709</td>
<td>&quot;-&quot;</td>
<td>56.62</td>
<td>$A_o=\frac{1}{4}[b_{01}+b_{02}+b_{03}]=138126 \text{ mm}^2$</td>
</tr>
<tr>
<td>3</td>
<td>.639</td>
<td>.712</td>
<td>&quot;-&quot;</td>
<td>56.83</td>
<td>$r_{bb}=.926, r_{bt}=.075, r_{tt}=.925, r_{tb}=.074$</td>
</tr>
</tbody>
</table>

For evaluation of $q$ take $A_o = \frac{1}{2}(138126 + 148935) = 143531 \text{ mm}^2$

For next iteration $q = \frac{T}{2A_o} = 696.716 \text{ N/mm}$
### Step 6: Iterations with Steps 1-5 to get Convergence in td Values

<table>
<thead>
<tr>
<th>Iteration</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>q N/mm</td>
<td>671.433</td>
<td>696.433</td>
<td>699.330</td>
<td>697.821</td>
<td>696.370</td>
</tr>
<tr>
<td>td db mm</td>
<td>79.84</td>
<td>73.82</td>
<td>70.18</td>
<td>69.53</td>
<td>69.37</td>
</tr>
<tr>
<td>θb deg</td>
<td>43.70</td>
<td>43.19</td>
<td>43.09</td>
<td>43.08</td>
<td></td>
</tr>
<tr>
<td>k_b</td>
<td>.242</td>
<td>.271</td>
<td>.286</td>
<td>.2884</td>
<td>.2884</td>
</tr>
<tr>
<td>e db *10^-3</td>
<td>1.880</td>
<td>2.208</td>
<td>2.394</td>
<td>2.425</td>
<td></td>
</tr>
<tr>
<td>e hb *10^-3</td>
<td>2.652</td>
<td>2.893</td>
<td>2.966</td>
<td>2.972</td>
<td></td>
</tr>
<tr>
<td>b db</td>
<td>8.662</td>
<td>9.547</td>
<td>9.988</td>
<td>10.055</td>
<td></td>
</tr>
<tr>
<td>td db mm</td>
<td>79.84</td>
<td>69.05</td>
<td>66.90</td>
<td>66.28</td>
<td>66.17</td>
</tr>
<tr>
<td>θl deg</td>
<td>43.70</td>
<td>42.64</td>
<td>42.54</td>
<td>42.60</td>
<td></td>
</tr>
<tr>
<td>k_l</td>
<td>.242</td>
<td>.290</td>
<td>.300</td>
<td>.3025</td>
<td>.3024</td>
</tr>
<tr>
<td>e dl *10^-3</td>
<td>1.880</td>
<td>2.448</td>
<td>2.586</td>
<td>2.621</td>
<td></td>
</tr>
<tr>
<td>e hl *10^-3</td>
<td>2.652</td>
<td>3.102</td>
<td>3.700</td>
<td>3.151</td>
<td></td>
</tr>
<tr>
<td>Yl *10^-3</td>
<td>8.662</td>
<td>10.223</td>
<td>10.562</td>
<td>10.614</td>
<td></td>
</tr>
<tr>
<td>td 2 mm</td>
<td>79.84</td>
<td>69.33</td>
<td>66.92</td>
<td>66.26</td>
<td>66.22</td>
</tr>
<tr>
<td>θ 2 deg</td>
<td>42.11</td>
<td>42.26</td>
<td>42.32</td>
<td>42.35</td>
<td></td>
</tr>
<tr>
<td>k 2</td>
<td>.242</td>
<td>.289</td>
<td>.300</td>
<td>.3022</td>
<td>.3023</td>
</tr>
<tr>
<td>e d2 *10^-3</td>
<td>1.890</td>
<td>2.437</td>
<td>2.589</td>
<td>2.620</td>
<td></td>
</tr>
<tr>
<td>e h2 *10^-3</td>
<td>1.805</td>
<td>1.884</td>
<td>1.895</td>
<td>1.893</td>
<td></td>
</tr>
<tr>
<td>Y2 *10^-3</td>
<td>6.681</td>
<td>7.851</td>
<td>8.166</td>
<td>8.225</td>
<td></td>
</tr>
<tr>
<td>td 3 mm</td>
<td>79.84</td>
<td>70.19</td>
<td>58.95</td>
<td>67.94</td>
<td>67.90</td>
</tr>
<tr>
<td>θ 3 deg</td>
<td>37.00</td>
<td>37.92</td>
<td>38.04</td>
<td>38.14</td>
<td></td>
</tr>
<tr>
<td>k 3</td>
<td>.242</td>
<td>.285</td>
<td>.291</td>
<td>.2952</td>
<td>.2947</td>
</tr>
<tr>
<td>¥d3 *10^-3</td>
<td>1.980</td>
<td>2.491</td>
<td>2.563</td>
<td>2.625</td>
<td></td>
</tr>
<tr>
<td>e h3 *10^-3</td>
<td>1.506</td>
<td>1.615</td>
<td>1.628</td>
<td>1.531</td>
<td></td>
</tr>
<tr>
<td>Y3 *10^-3</td>
<td>5.255</td>
<td>6.397</td>
<td>6.565</td>
<td>6.684</td>
<td></td>
</tr>
<tr>
<td>e dt mm</td>
<td>79.84</td>
<td>67.48</td>
<td>65.45</td>
<td>64.97</td>
<td>64.77</td>
</tr>
<tr>
<td>θ 4 deg</td>
<td>37.00</td>
<td>38.12</td>
<td>38.35</td>
<td>38.40</td>
<td></td>
</tr>
<tr>
<td>k 4</td>
<td>.242</td>
<td>.297</td>
<td>.307</td>
<td>.3086</td>
<td>.3091</td>
</tr>
<tr>
<td>e dt *10^-3</td>
<td>1.980</td>
<td>2.647</td>
<td>2.798</td>
<td>2.821</td>
<td></td>
</tr>
<tr>
<td>e ht *10^-3</td>
<td>1.506</td>
<td>1.632</td>
<td>1.673</td>
<td>1.671</td>
<td></td>
</tr>
<tr>
<td>Y 4 *10^-3</td>
<td>5.255</td>
<td>6.748</td>
<td>7.073</td>
<td>7.119</td>
<td></td>
</tr>
<tr>
<td>δ t *10^-3 rad/mm</td>
<td>.00301</td>
<td>.00326</td>
<td>.00339</td>
<td>.00342</td>
<td></td>
</tr>
<tr>
<td>ν x *10^-3 rad/mm</td>
<td>.03230</td>
<td>.03797</td>
<td>.03959</td>
<td>.03987</td>
<td></td>
</tr>
<tr>
<td>λ 0 mm²</td>
<td>1489.15</td>
<td>1435.31</td>
<td>1429.94</td>
<td>1433.03</td>
<td>1436.02</td>
</tr>
</tbody>
</table>
It is seen that by the end of 4 iterations an accuracy better than .5% in \( t_d \) values have been achieved. With good initial guesses of the \( t_d \) values for the various points in the section selected an accuracy of .1% is achieved in 3 to 4 iterations. This will be realized, for example, if a new longitudinal strain profile is taken which only differs slightly from the strain profile for which an iteration process has been carried out and convergence obtained for the \( t_d \) values which are then used as initial values for the new strain profile.

Steps 7 and 8 Evaluation of Resultant Compression Force and its Point of Action

Use \( q = 696.37 \text{ N/mm} \)

<table>
<thead>
<tr>
<th>Position</th>
<th>( \theta_i )</th>
<th>( y_{ci} )</th>
<th>( \Delta p_i )</th>
<th>( \omega_i = q \cot \theta_i \Delta p_i )</th>
<th>( \omega_i y_{ci} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>43.08</td>
<td>25</td>
<td>434</td>
<td>323200</td>
<td>8080000</td>
</tr>
<tr>
<td>1</td>
<td>42.60</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>42.35</td>
<td>190.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>38.14</td>
<td>357</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>38.40</td>
<td>357^2</td>
<td>433</td>
<td>380400</td>
<td>135815000</td>
</tr>
</tbody>
</table>

\[ F_c = 1223800 \text{ N} \]
\[ \bar{y}_c = 245450000/1223800 = 200.57 \text{ mm} \]
Step 9 Evaluation of Applied Actions

\[ N_x = F_{st} - F_{cl} = 1667700 - 1223800 \text{ N} = 443.9 \text{ KN} \]

\[ N_x \text{ is assumed to act as the section centroid} \]

\[ M_y = F_{st} (\bar{y}_c - \bar{y}_s) + N_x (0.5h_1 - \bar{y}_c) \]

\[ = 1667.70 (200.57 - 14.75) + 443.9 (381/2 - 200.57) \text{ KN - mm} \]

\[ = 305.42 \text{ kN - m} \]

\[ \psi_x = .040 \text{ rad/m, } \phi_j = .006 \text{ rad/m, } \phi_t = .003 \text{ rad/m} \]

Thus for the given strain profile and for the section, internal resisting actions have been evaluated. If a strain profile, which will result in \( N_x = 0 \) for \( \varepsilon_{th} = 0.0 \), is required then \( \varepsilon_{bth} \) is varied until this is achieved.

Furthermore if the strains for a given T/M ratio with \( T = 200 \text{ kN - m} \) and \( N_x = 0 \) are required, then with the given moment the top longitudinal strain is varied to get the desired internal moment and the bottom longitudinal strain is varied to ensure \( N_x = 0 \) for any chosen top face longitudinal strain. This procedure is used in the computer program given in Appendix D to analyze the test specimens.
Computer program for analysis of beams under combined flexure and torsion by the diagonal compression field theory.

D.1 Program Documentation

Program Name: Diagonal Compression Field Analysis
Language: Fortran IV

Purpose and Limitations

To analyse the complete response of a rectangular structural concrete beam under combined flexure and torsion. The theory is applicable only to St. Venant torsion loading and the program is specifically for a section with one axis of symmetry.

Program Capabilities

1. For a given flexural moment and torque and for given section dimensions (spalled or unspalled) the program predicts the twist, the longitudinal and transverse strain profiles in the section, the concrete principal compression strains at chosen points, the angles of the principal compression strains, the thicknesses of the concrete compression diagonals at the chosen points, the shear flow and the area enclosed by the shear flow.

2. The effective thicknesses of the concrete compression diagonals, $t_d$, can be greater than that provided by the section wall thickness.

3. The reinforcing steel in the section analysed can be initially unstressed or prestressed or a combination of both.
(4) The stress-strain relationships for the concrete (parabolic relationship assumed) and the steel (option of bilinear or trilinear or continuous stress-strain curves for steel) are accounted for.

(5) The response of under-reinforced, partially over-reinforced and over-reinforced beams can be predicted.

Solution Technique

The solution technique is outlined in section 3.3.3. The sequence of operations in the program are as follows:

1. Input beam data
2. Print input data
3. Input the analysis control parameters which includes the flexural moment and torque for analysis and the initial guess of the longitudinal strain profile. Also, initialize analysis dimensions.
4. Calculate the resultant tension force in the reinforcing steel
5. Evaluate the shear flow and hence obtain the compatible strains at chosen points
6. Evaluate the twist and the diagonal curvatures at chosen points
7. Evaluate \( t_d \) - values
8. Determine path of shear flow and evaluate the area \( A_0 \)
9. Evaluate the resultant compression force in the longitudinal direction in concrete
(10) Evaluate strain in reinforcing steel in bottom face

(11) Evaluate the internal flexural moment and the torque

(12) Evaluate strain in top face surface

(13) Output results. A summarized output of results or a detailed output can be specified.

Commentary is given in the appropriate sections of the program giving in greater details the operations in the program.

D.2 NOTATION USED TO INPUT DATA

D.2.1 Section Geometry

B,H Outside section dimensions

B1,H1 Dimensions of hoop centrelines

CHB,CHS,CHT Cover to hoop steel centrelines in bottom, side and top faces respectively

CLB Cover to centroid of longitudinal steel in bottom face

TB,TS,TT Wall thickness in section for bottom, side and top faces respectively

TGH Level at which test strains are measured with respect to hoop centrelines.

D.2.2 Material Properties

Concrete

FC Concrete cylinder compression strength ($f'_c$)

EPSO Strain, $\varepsilon_0$, corresponding to the peak stress, $f'_c$, in a standard cylinder compression test.
Reinforcing Steel

(a) Longitudinal Steel

AS(I), YS(I)  Area of steel bar(s) positioned at distance YS(I) with respect to the bars in the bottom face

EPLY(I), EPSH(I)  Yield strain and strain at onset of strain hardening, respectively

ES(I), ESH(I)  Moduli of elasticity before yield and after onset of strain hardening, respectively

FLY(I), FU(I)  Yield and ultimate stresses, respectively

(b) Prestressing Steel

AP(I), YP(I)  Area of tendon(s) positioned at distance YP(I) with respect to the reinforcing steel bars in bottom face

EP  Modulus of elasticity

EPPO, EPPY  Prestrain and strain at .2% proof stress

FP, FPY, FPU  Prestress, .2% proof stress and ultimate stress respectively for the tendons.

(c) Hoop Steel

AH  Cross section area of one hoop leg

SH  Spacing of hoop steel

EPHY, EPSHH  Yield strain and strain at onset of strain hardening respectively

ESS, ESHH  Moduli of elasticity before yield and after onset of the strain hardening respectively

FHY, FUH  Yield and ultimate stresses respectively
### D.2.3 Control Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMNO</td>
<td>Beam Number</td>
</tr>
<tr>
<td>DEADMO</td>
<td>Initial moment acting due to dead weights</td>
</tr>
<tr>
<td>DEADTO</td>
<td>Initial torque acting due to dead weights</td>
</tr>
<tr>
<td>DMO</td>
<td>Amount by which the flexural moment is to be increased in the analysis</td>
</tr>
<tr>
<td>EPCCB, EPCCT</td>
<td>Longitudinal strains in bottom and top face defining the longitudinal strain profile</td>
</tr>
<tr>
<td>IOUT</td>
<td>Parameter controlling printout</td>
</tr>
<tr>
<td></td>
<td>If IOUT = 0 complete printout of the strains, angles, $t_d$-values and curvatures for all the points for each load increment as well as a table at the end summarizing the analysis results for mid-points of each side at the level of strain measurements in a test. If IOUT &gt; 0 only the final summary of results is given</td>
</tr>
<tr>
<td>INTV</td>
<td>Number of load increments required. If the loading is increased beyond the capacity of the section the analysis is terminated when the maximum principal compression strain is $\approx 1.5\varepsilon_0$</td>
</tr>
<tr>
<td>IPRE</td>
<td>Indicator for the presence of prestressed steel. If there is no prestressed steel IPRE is set to zero</td>
</tr>
<tr>
<td>LSIMP</td>
<td>Number of points on the side face at which evaluations of strains, $t_d$-values and angles are done. LSIMP takes the values $2n+1$ (n=1,2,3...)</td>
</tr>
</tbody>
</table>
MES  Indicator of uniformity of the reinforcing steel properties in the section. If all longitudinal steel bars have the same stress-strain characteristics then set MES=0 otherwise set MES=1

N  Number of layers of reinforcing bars along the side faces

NPRE  Number of layers of prestressed steel along the side faces

SOURCE  Identification of origin of beam BMNO being analysed

TAGMO  Specified initial target moment for analysis

TMRO  Fixed ratio at which the torque and flexural moment applied to the beam are increased

TTMM  Value of moment above which further increments in moments are reduced to DMO/4 in an attempt to approach the failure loads more closely
### D.3 DATA INPUT

<table>
<thead>
<tr>
<th>DATA GROUP</th>
<th>INPUT FORMAT</th>
<th>PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(I3,2I2,I3,1X,16A4)</td>
<td>N,MES,IPRE,NPRE,BMNO,SOURCE</td>
</tr>
<tr>
<td>2</td>
<td>(8F10.0)</td>
<td>B, H, TB, TS, TT, B1, H1, CLB, CHS, CHT, TGH</td>
</tr>
<tr>
<td>3</td>
<td>(8F10.0)</td>
<td>(AS(I),I=1,N), (YS(I),I=1,N)</td>
</tr>
<tr>
<td>4**</td>
<td>(8F10.0)</td>
<td>(AP(I),I=1,NPRE), (YP(I),I=1,NPRE), FPY, FPI, EP, FPU</td>
</tr>
<tr>
<td>5*</td>
<td>(8F10.0)</td>
<td>(FLY(I),I=1,N), (FU(I),I=1,N), (ES(I),I=1,N), (ESH(I),I=1,N), (EPSH(I),I=1,N)</td>
</tr>
<tr>
<td>6</td>
<td>(8F10.0)</td>
<td>AH, SH, FHY, FUH, ESS, ESHH, EPSHH</td>
</tr>
<tr>
<td>7</td>
<td>(8F10.0)</td>
<td>FC, EPSO, DEADMO, DEADTO</td>
</tr>
<tr>
<td>8</td>
<td>I5, 6F10.0, 215</td>
<td>INTV, TMRO, TAGMO, DMO, EPCCT, EPCCB, TTMM, IOUT, LSIMP</td>
</tr>
</tbody>
</table>

* If MES = 0 for group 5 we read FY, FULTM, EMOD, EMODH, EPSH where these variables denote the common values of the parameters in group 5 for any 'I' value.

**If IPRE = 0 group 4 cards are omitted.
D.4 NOTATION USED IN PRINTING RESULTS

The notation used in the input of data is also used in the printout of the input data. Besides, the following further symbols are used in the printout of analysis results. Note that the values of the indices IJK, IIM, ITR and KK which are printed in the detailed printout of results refer to the value at the end of the last iteration step.

AO Area enclosed by path of shear flow
CN Depth from top face to plane with zero longitudinal strains
EPLB, EPLT Strains in the longitudinal steel bars in the bottom face and the top face respectively
FCC Resultant compression force in concrete in the direction of the beam's longitudinal axis
FSS Resultant tension force in the reinforcing steel in the direction of the beam's longitudinal axis
HO Height of the path of shear flow
III General counter index
IJK Counter of number of load increments
IIM Counter of number of iterations done on EPCCT to get the internal flexural moment to converge to the applied moment (maximum value is 7)
ITR Counter of number of iterations done on EPCCB for a fixed EPCCT to get equilibrium of forces in the longitudinal axis direction (maximum value is 8)
KK Counter of number of iterations to obtain convergence in $t_d$-values for a fixed longitudinal strain profile
(maximum allowed is 7)

M  Evaluated internal flexural moment about the geometric centroid of the section

PHIX Curvature due to variation in longitudinal strains from top face to bottom face (rad/unit length)

PHIZ Curvature due to variation of hoop strains from top face to bottom face (rad/unit length)

PSIX Twist (rad/unit length)

Q  Shear flow

T  Evaluated internal torque

TAGMÖM Given applied moment

TAGTÖQ Given applied torque

TDMIN Minimum $t_d$-value for the given shear flow according to the theoretical model

YC Defines the position at which FCC acts with respect to the rebars in the bottom face

YSB Defines the position at which FSS acts with respect to the longitudinal steel bars in the bottom face

The indices indicated in the printout of the detailed output of results are as follows

INDEX HOOP = 0 for hoop steel strain $\varepsilon_h \leq \varepsilon_{hy}$

= 1 for hoop steel strain $\varepsilon_h < \varepsilon_{hy}$

= 2 for hoop steel strain in strain hardening range.

INDEX CONC = 0 indicates normal convergence on principal diagonal strain, $\varepsilon_{ds}$.

$\neq$ 0 indicates problem in convergence on principal diagonal strain, $\varepsilon_{ds}$. 
D-5 CONVERGENCE ACCURACY

In the program an iterative procedures are used to obtain convergence of \( t_d \)-values, at various points in the section, the shear flow \( q \), the resultant axial force which is required to be zero for torsion and flexure loading and convergence of the applied internal actions to the corresponding external ones. The iteration process is terminated when the following criteria are met:

(a) For \( t_d \)-values:
   - top face and bottom face - error \( \leq .1\% \)
   - side faces at selected points - error \( \leq .2\% \)

(b) For shear flow - error \( \leq .1\% \)

(c) Axial force \( (F_{ss} - F_{cc}) \) - error \( \leq .2\% \) for IIM \( \leq 1 \)
    - \( \leq .3\% \) for ITR \( > 5 \)
    - maximum number of iterations is ITR=8

(d) Flectural moment - error \( \leq .2\% \)
    - \( \leq .5\% \) for IIM \( > 5 \)
    - maximum number of iterations is IIM=7
D.6 PROGRAM LISTING

PROGRAM ANALYSES RECT. R.C. SECTION IN TORSION AND BENDING

REAL AS(10),YS(10),ESI(10),FLY(10),FU(10),00(10)
REAL AP(10),YP(10),EPN(10),EPP(10),PPX(10)
REAL THETA(10),ATH(10),DMS(10),GAM(10),PLT(10)
REAL EPSH(10),PLY(10),ELX(10),EPSH(10)
REAL EPS(10),PHI0(10),SOURCE(10),ECTI(10),DMSH(10)
REAL STI(10),EPCSH(10),PPF(10),JO,WH(10)
DIMENSION INDEX(10),IC(10)
REAL TH1(90),TH2(90)
REAL TEPL(90),SEP(90),REPL(90),RSEP(90),RSEP(90)

SECTION GEOMETRY

IF ALL LONG. REBARS HAVE SAME STRESS-STRAIN CHARACTERISTICS THEN
SET MESS =O otherwise SET MESS =1
IPRE =INDICATOR FOR PRESENCE OF PRESTRESSED STEEL
IF THERE IS PRESTRESSED STEEL SET IPRE =1
IF NOT, SET IPRE =0
OF LAYERS OF PRESTRESSED STEEL
IT IS ASSUMED THAT PROPERTIES OF ALL PRESTRESSED STEEL ARE SAME
CMB, CSH, CHT = COVER TO CENTRE-LINES IN BOTTOM SIDES AND TOP FACES
CBB = COVER TO CENTRE-LINE OF BOTTOM REBARS
FOR SMOOTHED CASE CMB, CHT, CMS ARE ALL ZERO
C = TGT = TARGET HEIGHT FROM HOOP CENTRE LINE

MATERIAL PROPERTIES

AMOUNT AND LOCATION OF STEEL IN SECTION
POSITION OF STEEL IS TAKEN W.R.T. BOTTOM CORNER LONG. REBARS

READS,5,1001,AS(10),Y(10),ESI(10),FLY(10),FU(10),00(10)
READS,5,1002,ATH(10),DMS(10),GAM(10),PLT(10)
READS,5,1003,PHI0(10),SOURCE(10),ECTI(10),DMSH(10)
READS,5,1004,STI(10),EPCSH(10),PPF(10),JO,WH(10)

PRINT INPUT DATA

COMMON /BLOCK/FC.EPSH,TH,ESI,EMOD,PHYS,PHYM,EPSh

1002 ISPLALLC
1003 READS,5,5000,IPRE,IPRE,NPRE,BNDF.SOURCE
1004 IF(IPRE.EQ.0) GO TO 999
1005 IF(IPRE.EQ.0) GO TO 1007
1006 IF(IPRE.EQ.0) GO TO 999
1007 IF(IPRE.EQ.0) GO TO 999
1008 IF(IPRE.EQ.0) GO TO 999
1009 IF(IPRE.EQ.0) GO TO 999
1010 IF(IPRE.EQ.0) GO TO 999
1011 IF(IPRE.EQ.0) GO TO 999
1012 IF(IPRE.EQ.0) GO TO 999
1013 IF(IPRE.EQ.0) GO TO 999
1014 IF(IPRE.EQ.0) GO TO 999
1015 IF(IPRE.EQ.0) GO TO 999
1016 IF(IPRE.EQ.0) GO TO 999
1017 IF(IPRE.EQ.0) GO TO 999
1018 IF(IPRE.EQ.0) GO TO 999
1019 IF(IPRE.EQ.0) GO TO 999
1020 IF(IPRE.EQ.0) GO TO 999
1021 IF(IPRE.EQ.0) GO TO 999
1022 IF(IPRE.EQ.0) GO TO 999
1023 IF(IPRE.EQ.0) GO TO 999
1024 IF(IPRE.EQ.0) GO TO 999
1025 IF(IPRE.EQ.0) GO TO 999
1026 IF(IPRE.EQ.0) GO TO 999
1027 IF(IPRE.EQ.0) GO TO 999
1028 IF(IPRE.EQ.0) GO TO 999
1029 IF(IPRE.EQ.0) GO TO 999
1030 IF(IPRE.EQ.0) GO TO 999
1031 IF(IPRE.EQ.0) GO TO 999
1032 IF(IPRE.EQ.0) GO TO 999
1033 IF(IPRE.EQ.0) GO TO 999
1034 IF(IPRE.EQ.0) GO TO 999
1035 IF(IPRE.EQ.0) GO TO 999
1036 IF(IPRE.EQ.0) GO TO 999
1037 IF(IPRE.EQ.0) GO TO 999
1038 IF(IPRE.EQ.0) GO TO 999
1039 IF(IPRE.EQ.0) GO TO 999
1040 IF(IPRE.EQ.0) GO TO 999
1041 IF(IPRE.EQ.0) GO TO 999
1042 IF(IPRE.EQ.0) GO TO 999
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1045 IF(IPRE.EQ.0) GO TO 999
1046 IF(IPRE.EQ.0) GO TO 999
1047 IF(IPRE.EQ.0) GO TO 999
1048 IF(IPRE.EQ.0) GO TO 999
1049 IF(IPRE.EQ.0) GO TO 999
1050 IF(IPRE.EQ.0) GO TO 999
1051 IF(IPRE.EQ.0) GO TO 999
1052 IF(IPRE.EQ.0) GO TO 999
1053 IF(IPRE.EQ.0) GO TO 999
1054 IF(IPRE.EQ.0) GO TO 999
1055 IF(IPRE.EQ.0) GO TO 999
1056 IF(IPRE.EQ.0) GO TO 999
1057 IF(IPRE.EQ.0) GO TO 999
1058 IF(IPRE.EQ.0) GO TO 999
1059 IF(IPRE.EQ.0) GO TO 999
1060 IF(IPRE.EQ.0) GO TO 999
1061 IF(IPRE.EQ.0) GO TO 999
1062 IF(IPRE.EQ.0) GO TO 999
1063 IF(IPRE.EQ.0) GO TO 999
1064 IF(IPRE.EQ.0) GO TO 999
1065 IF(IPRE.EQ.0) GO TO 999
1066 IF(IPRE.EQ.0) GO TO 999
1067 IF(IPRE.EQ.0) GO TO 999

1007 DOH = CLB
1008 DOH = CLB
1009 DOH = CLB
1010 DOH = CLB
1011 DOH = CLB
1012 DOH = CLB
1013 DOH = CLB
1014 DOH = CLB
1015 DOH = CLB
1016 DOH = CLB
1017 DOH = CLB
1018 DOH = CLB
1019 DOH = CLB
1020 DOH = CLB
1021 DOH = CLB
1022 DOH = CLB
1023 DOH = CLB
1024 DOH = CLB
1025 DOH = CLB
1026 DOH = CLB
1027 DOH = CLB
1028 DOH = CLB
1029 DOH = CLB
1030 DOH = CLB
1031 DOH = CLB
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1056 DOH = CLB
1057 DOH = CLB
1058 DOH = CLB
1059 DOH = CLB
1060 DOH = CLB
1061 DOH = CLB
1062 DOH = CLB
1063 DOH = CLB
1064 DOH = CLB
1065 DOH = CLB
1066 DOH = CLB
1067 DOH = CLB

FACTOR FOR CONVERTING DEGREES TO RADIANS

DEGREE = 45./ATANI(1.0)

PRINT INPUT DATA

WHITE(6,2000)
WHITE(6,6050)BNDF.SOURCE
WHITE(6,6100)
WRITE(6,5100)HT.TT.TS.BL.ML.CLB.D.TIN
WRITE(6,6102)
WRITE(6,6107)
EVALUATE RESULTANT FORCE DUE TO STRESSES IN LONG STEEL (FSS)

ESTABLISH THE POSITION AT WHICH THIS FORCE ACTS W.R.T. BOTTOM-CORNER LONG. REAARS (Y85)

CALL EPLB(0,EPLBH-PHIX*YP(K))

DO 64 EPL(K)=EPLBH-PHIX*YP(K)

DO 63 64 EPL(K)=PHIX*YP(K)

DO 62 63 EPL(K)=FP(K)*YP(K)

DO 61 62 EPL(K)=FP(K)*FP(K)

DO 60 61 EPL(K)=FP(K)

DO 59 60 EPL(K)=FPPK

DO 58 59 EPL(K)=FPPK

DO 57 58 EPL(K)=FPPK

DO 56 57 EPL(K)=FPPK

DO 55 56 EPL(K)=FPPK

DO 54 55 EPL(K)=FPPK

DO 53 54 EPL(K)=FPPK

DO 52 53 EPL(K)=FPPK

DO 51 52 EPL(K)=FPPK

DO 50 51 EPL(K)=FPPK

DO 49 50 EPL(K)=FPPK

DO 48 49 EPL(K)=FPPK

DO 47 48 EPL(K)=FPPK

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DO 45 46 EPL(K)=FPPK

DO 44 45 EPL(K)=FPPK

DO 43 44 EPL(K)=FPPK

DO 42 43 EPL(K)=FPPK

DO 41 42 EPL(K)=FPPK

DO 40 41 EPL(K)=FPPK

DO 39 40 EPL(K)=FPPK

DO 38 39 EPL(K)=FPPK

DO 37 38 EPL(K)=FPPK

DO 36 37 EPL(K)=FPPK

DO 35 36 EPL(K)=FPPK

DO 34 35 EPL(K)=FPPK

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DO 25 26 EPL(K)=FPPK

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DO 19 20 EPL(K)=FPPK

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DO 17 18 EPL(K)=FPPK

DO 16 17 EPL(K)=FPPK

DO 15 16 EPL(K)=FPPK

DO 14 15 EPL(K)=FPPK

DO 13 14 EPL(K)=FPPK

DO 12 13 EPL(K)=FPPK

DO 11 12 EPL(K)=FPPK

DO 10 11 EPL(K)=FPPK

DO 9 10 EPL(K)=FPPK

DO 8 9 EPL(K)=FPPK

DO 7 8 EPL(K)=FPPK

DO 6 7 EPL(K)=FPPK

DO 5 6 EPL(K)=FPPK

DO 4 5 EPL(K)=FPPK

DO 3 4 EPL(K)=FPPK

DO 2 3 EPL(K)=FPPK

DO 1 2 EPL(K)=FPPK

END
EVALUATE RESULTANT FORCE IN LONG. DIRECTION (FCCI) DUE TO CONCRETE STRESSES

EVALUATE NEW VALUE OF EPCC FOR NEXT ITERATION

ESTABLISH THE POSITION AT WHICH FCC ACTS W.R.T. BOTTOM FACE LONG. HEBARS

APPLIED ACTIONS EVALUATION OF TORQUE AND FLEXURAL MOMENT

EVALUATE NEW VALUE OF EPCC FOR NEXT ITERATION
CONVERT ANGLES FROM RADIANS TO DEGREES

** DETAILED PRINT OUT OF RESULTS **

** STCRE RESULTS FOR PLOTTING AND PRINTING OF FINAL RESULTS IN SUMMARY **

** FORMAT STATEMENTS **

5000 FORMAT(1x,k12,12,13,14,15)
FUNCTION STFAS, ES, EPS, FY, FULT, EPSH, EPSH

C FUNCTION TO COMPUTE FORCE IN STEEL BAR GIVEN STRAIN
IF(EPSH.GT.0.0.OR.FULT.EQ.0.0)GO TO 10
IF(EPS.LT.0.0) GO TO 5
R=ALOG10(12.0)/ALOG10(FULT/FY)
STFAS=AP.EPS.EPS/FULT
RETURN

5
STP=AS.EPS
RETURN

10
STFAS=AP.EPS
IF(EPS.EQ.0.0.AND.FPL.EQ.0.0) STFAS=AP.EPS
IF(EPS.FLT.EPSH) STFAS=AP.EPS
IF(EPS.EQ.EPSH) STFAS=AP.EPS
RETURN

FUNCTION STLON(L, EPS, EPSH, EPSH)

FUNCTION TO COMPUTE RESULTANT CONCRETE FORCE IN LONG. DIRECTION
REAL PL(10)
PXY=0.0
PTX=PTX
IF(L.EQ.0.0) GO TO 10
PSX=PSX
GO TO 20
10
PSX=PL(1)*X
20
FUNCTION FLP3(FX, FY, PSX)

FUNCTION AND SUBROUTINES

FUNCTION STFAS, ES, EPS, FY, FULT, EPSH, EPSH

FUNCTION TO COMPUTE FORCES IN STEEL BAR GIVEN STRAIN
IF(EPSH.GT.0.0.OR.FULT.EQ.0.0)GO TO 10
IF(EPS.LT.0.0) GO TO 5
R=ALOG10(12.0)/ALOG10(FULT/FY)
STFAS=AP.EPS.EPS/FULT
RETURN

5
STP=AS.EPS
RETURN

10
STFAS=AP.EPS
IF(EPS.EQ.0.0.AND.FPL.EQ.0.0) STFAS=AP.EPS
IF(EPS.FLT.EPSH) STFAS=AP.EPS
IF(EPS.EQ.EPSH) STFAS=AP.EPS
RETURN

FUNCTION STLON(L, EPS, EPSH, EPSH)

FUNCTION TO COMPUTE RESULTANT CONCRETE FORCE IN LONG. DIRECTION
REAL PL(10)
PXY=0.0
PTX=PTX
IF(L.EQ.0.0) GO TO 10
PSX=PSX
GO TO 20
10
PSX=PL(1)*X
20
FUNCTION FLP3(FX, FY, PSX)
SUBROUTINE STRAIN(O, THETA, THK, TD, EPL, EPH, OMS, EPCH, INDEX, IC, Y1, Y2, ANG, GAMA, ITR)

COMMON /BLOCK/ FC, EPSO, THL, ESS, ESHH, FHY, FUY, EPSH

DIMENSION X(20), FX(20), FYRME(20), FXY(20), FX(20)

SUBROUTINE STRAIN CALCULATES HOOP STRAIN, PRINCIPAL COMPRESSION STRAIN
AND ANGLE OF PRINCIPAL COMPRESSION GIVEN LONG. STRAIN, SHEAR FLOW AND
EFFECTIVE COMPRESSION DIAGONAL DEPTH

1 = 0

IF (INDEX) GO TO 1

R0 = 0

THETA = ANGANG

IF (THETA.EQ.0.0) GO TO 5

RETURN

1 = 0

INDEX = 0

RETURN

SUBROUTINE STRAIN(O, THETA, THK, TD, EPL, EPH, OMS, EPCH, INDEX, IC, Y1, Y2, ANG, GAMA, ITR)

COMMON /BLOCK/ FC, EPSO, THL, ESS, ESHH, FHY, FUY, EPSH

DIMENSION X(20), FX(20), FYRME(20), FXY(20), FX(20)

SUBROUTINE STRAIN CALCULATES HOOP STRAIN, PRINCIPAL COMPRESSION STRAIN
AND ANGLE OF PRINCIPAL COMPRESSION GIVEN LONG. STRAIN, SHEAR FLOW AND
EFFECTIVE COMPRESSION DIAGONAL DEPTH

1 = 0

IF (INDEX) GO TO 1

R0 = 0

THETA = ANGANG

IF (THETA.EQ.0.0) GO TO 5

RETURN

1 = 0

INDEX = 0

RETURN
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SUBROUTINE GAMNL(EPL, EPH, EPO, GAMA, THETA)
FUNCTION TO COMPUTE SHEAR STRAIN AND ANGLE OF PRINCIPAL COMPRESSION GIVEN LUNGS, AND PRINCIPAL COMPRESSION STRAINS AT A POINT IN A PLANE

EPL = EPL, EPH = PH, EPO = PO, GAMA = GAM, THETA = TETA
RETURN

FUNCTION ARB(OMOS, TDK, THK, FC)
FUNCTION TO COMPUTE DEPTH OF THE EQUIVALENT RECTANGULAR STRESS BLOCK FOR TO GREATER THAN THE WALL THICKNESS

TDK = TDK - THK
THK = THK - TDK
TR1 = TR1 - TDK
TR2 = TR2 - TDK
ALRT1 = OMOS - OMOSZ / 3.0
ALRTZ = TR1 * OMOS / (6.0 - 2.0 * TR1 / OMOS)
BETA1 = (4.0 - OMOS1) / (4.0 + TR1 / OMOS1)
BETAZ = (4.0 - TR1 / OMOS) / (4.0 + 2.0 * TR1 / OMOS)
RF1 = ALRT1 * F(R, TDK) / OMOS
RFZ = ALRTZ * F(R, TDK) / OMOS
ARUN = Z.0 * IO.5 * RF1 + RFZ
RETURN

FUNCTION ARB(OMOS, TDK, THK, FC)
FUNCTION TO COMPUTE DEPTH OF THE EQUIVALENT RECTANGULAR STRESS BLOCK FOR TO LESS THAN OR EQUAL THE WALL THICKNESS

OMOSZ = OMOS - 2.0 * TDK
OMOS = OMOS - 2.0 * TDK
ALRT = OMOS - OMOSZ / (6.0 - 2.0 * TDK)
BETA = (4.0 - OMOS1) / (6.0 - 2.0 * OMOS1)
ARUN = TR1 * OMOS / (6.0 - 2.0 * TDK)
RETURN

FUNCTION AR02(OMOS, TDK, THK, FC)
FUNCTION TO COMPUTE SHEAR STRAIN AND ANGLE OF PRINCIPAL COMPRESSION GIVEN LUNGS, AND PRINCIPAL COMPRESSION STRAINS AT A POINT IN A PLANE

EPL = EPL, EPH = PH, EPO = PO, GAMA = GAM, THETA = TETA
RETURN

FUNCTION SIMP(X, L, DIVH, H)
INTEGRATION USING SIMPSONS RULE

REAL X(10)
SUMX = 0.0
DO 350 K = 2, L, 2
K1 = K - 1
K2 = K + 1
SUMX = SUMX + X(K1) + X(K2) + X(K)
350 CONTINUE
RETURN

FUNCTION GUESS(F, UF, STR, EPV)

FRC = F
SIGN = FRC / ABS(FRC)
IF (ABS(FRC) .GT. 1.0) GO TO 10
IF (ABS(FRC) .LT. 0.1) GO TO 10
IF (ABS(FRC) .GT. 0.5) GO TO 10
IF (ABS(FRC) .LT. 0.5) GO TO 10
RETURN

FUNCTION PARAB(X3, X2, X1, Y3, Y2, Y1, F)

CONVERGENCE TECHNIQUES USED IN ITERATION ARE:
*** FITTING A PARABOLA THROUGH 3 POINTS
*** SECANT METHOD

IF (ABS(X3 - X1) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(X3 - X2) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(X2 - X1) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(Y3 - Y2) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(Y3 - Y1) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(Y2 - Y1) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(Y3 - Y2) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(Y1 - Y2) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(X3 - X2) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(X2 - X1) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(Y3 - Y1) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(Y2 - Y3) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(Y3 - Y2) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
IF (ABS(Y3 - Y3) .LE. 1.25 .AND. F .NE. 0.0) GO TO 460
RETURN
APPENDIX E

EXPERIMENTAL DATA AND PLOTS

E.1 TEST SPECIMENS

The stirrups used in making the steel cages were bent careful and their dimensions were checked after bending to ensure uniformity in the dimensions to within ±2 mm.

Three inches of overlap in the bent hoop were fillet welded as shown. In the fabrication of the cages the welded parts were in the top face and they were staggered on plan (see Fig. E.3).

To make the cages the hoops were tied to longitudinal steel, with tying wire so that the hoop legs were spaced at 76 mm centres. The central nine hoops were positioned with great care and using an accurately marked pattern plate positions were marked for placement of targets to form the grids
### TABLE E.1 MEASURED DIMENSIONS

<table>
<thead>
<tr>
<th>SERIES</th>
<th>b</th>
<th>h</th>
<th>b₁</th>
<th>h₁</th>
<th>tₜ</th>
<th>tₜ</th>
<th>tₜ</th>
<th>Cₜₜ</th>
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<th>Cₜₚ</th>
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<th>Cₜₚ</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBO</td>
<td>508</td>
<td>411</td>
<td>480</td>
<td>380</td>
<td>76</td>
<td>76</td>
<td>81</td>
<td>39</td>
<td>16</td>
<td>14</td>
<td>15</td>
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<tr>
<td>TBU</td>
<td>508</td>
<td>410</td>
<td>480</td>
<td>378</td>
<td>76</td>
<td>76</td>
<td>80*</td>
<td>39</td>
<td>16</td>
<td>14</td>
<td>16</td>
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</tr>
<tr>
<td>TBS**</td>
<td>508</td>
<td>413</td>
<td>482</td>
<td>381</td>
<td>206.5</td>
<td>254</td>
<td>206.5</td>
<td>39</td>
<td>16</td>
<td>13</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

* for TBU2 minimum tₜ = 60 mm
* for TBU4 minimum tₜ = 55 mm
** TBS series specimens were solid.

![Diagram](image)
shown in Figure 4.17 and 4.18. The hoop bar ribs on these positions were filed off and cleaned in readiness for the placing of the targets used in the measurement of strains.

After the cages were complete the final overall measurements over the central 1 metre of the span were taken. Also after the casting of the specimens the outside dimensions of the specimens were measured. The average values of these measurements for each series of specimens are recorded in Table E.1.

E.2 Evaluation of Strains from Experimental Data

Figure E.1 is part of the grid of targets for measuring strains shown in Figure 4.15. Indicated are the strains measured.

![Diagram of evaluation of average measured strains]

**FIG. E.1 EVALUATION OF AVERAGE MEASURED STRAINS**
A summary of all the strains measured for all the specimens in tabulated in Appendix E.3. With reference to Fig. E.1 the average strains were evaluated as follows:

The average longitudinal strains for row RL1 is \( \epsilon_{l1} = \frac{1}{4} \sum_{i=1}^{4} \epsilon_{ll} \). The average longitudinal strains for rows RL2 and RL3 were evaluated in a similar way.

The average transverse strain for row RH1 is \( \epsilon_{t1} = \frac{1}{9} \sum_{i=1}^{9} \epsilon_{tli} \). The average transverse strain for row RH2 was similarly evaluated.

The average diagonal strain for row RD1 is \( \epsilon_{dl} = \frac{1}{4} \sum_{i=1}^{4} \epsilon_{dl} \). The average diagonal strain for row RD2 was similarly obtained.

The average longitudinal strain \( \epsilon_{l} = \frac{1}{3} (\epsilon_{l1} + \epsilon_{l2} + \epsilon_{l3}) \);

The average transverse strain \( \epsilon_{t} = \frac{1}{2} (\epsilon_{t1} + \epsilon_{t2}) \); and the average diagonal strains \( \epsilon_{dl} \) and \( \epsilon_{d2} \) constitute a set of compatible strains which can be used to draw Mohr's circle of strain. Note that average diagonal strains \( \epsilon_{dl} \) and \( \epsilon_{d2} \) are measured in directions which are mutually perpendicular.

FIG. E.2 MOHR'S CIRCLE FOR MEASURED STRAINS
From the circle it can be seen that $\Delta A'B'B'$ is similar to $\Delta C'D'D'$ and hence $C'D'=A'A'=\gamma_{lt}$. If tension strains are taken to be positive then $\varepsilon_{dp}$ will be negative; and assuming $\varepsilon_{d2}$ is compressive, deduce: $\varepsilon_{d1} + \varepsilon_{d2} = \varepsilon_{t} + \varepsilon_{\ell}$ ....................... (E.1)
and the shear strain $\gamma_{lt}$ is evaluated as $\gamma_{lt} = \varepsilon_{d1} - \varepsilon_{d2}$ ....... (E.2)
Using equation (2.16) it follows that

$$\left(\frac{\gamma_{lt}}{2}\right)^2 = (\varepsilon_{\ell} - \varepsilon_{dp})(\varepsilon_{t} - \varepsilon_{dp})$$

Hence

$$\varepsilon_{dp}^2 - (\varepsilon_{t} + \varepsilon_{\ell})\varepsilon_{dp} + \varepsilon_{t}\varepsilon_{\ell} - \left(\frac{\gamma_{lt}}{2}\right)^2 = 0$$

$$\varepsilon_{dp} = \frac{1}{2}[(\varepsilon_{t} + \varepsilon_{\ell}) - \sqrt{(\varepsilon_{t} + \varepsilon_{\ell})^2 - 4[\varepsilon_{t}\varepsilon_{\ell} - \left(\frac{\gamma_{lt}}{2}\right)^2]}}$$

$$= \frac{1}{2} (\varepsilon_{t} + \varepsilon_{\ell}) - \sqrt{(\varepsilon_{t} - \varepsilon_{\ell})^2 + \gamma_{lt}^2}$$ ...............(E.3)

Thus from the measured diagonal strains the shear strain with respect to the transverse and longitudinal axes is directly obtained, using equation (E.2). With the shear strain determined and the measured average strains in the longitudinal and transverse directions, the principal compression strain is evaluated by equation (E.3). Finally the angle of principal compression is evaluated as

$$\tan 2\theta_p = \frac{\gamma_{lt}}{\varepsilon_{t} - \varepsilon_{\ell}} = \frac{\varepsilon_{d1} - \varepsilon_{d2}}{\varepsilon_{t} - \varepsilon_{\ell}}$$ ...............(E.4)
E.3 SUMMARY OF STRAINS AND LOADS OBTAINED FROM TEST DATA

E.3.1 MEASURED TORQUE, FLEXURAL MOMENT, TWIST AND CURVATURE

In the following tables are listed the torque, moment, twist and curvature values measured for all the specimens for all the load stages. The units used are kilonewtons and meters. The angular deformations are given in radians per meter x 10^{-3}. The twist was measured by the metrisite over 1.75 meters in the central part of the specimen; and by inclinometer readings on the top face over the central 0.61 meters of the beam. The curvature was measured over the central 1.75 meters and also the curvature was evaluated from the average longitudinal strains measured over the central 0.61 meters.
### DATA FOR TEAM TH01

<table>
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<th>L/S</th>
<th>0</th>
<th>1</th>
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<tr>
<td>M S</td>
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<td>91.01</td>
<td>4.01</td>
<td>7.35</td>
<td>11.29</td>
<td>11.29</td>
<td>11.29</td>
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<tr>
<td>END</td>
<td>110.72</td>
<td>104.58</td>
<td>101.01</td>
<td>7.35</td>
<td>11.29</td>
<td>16.13</td>
<td>16.13</td>
<td>16.13</td>
</tr>
</tbody>
</table>
E.3.2 MEASURED STRAINS

In the following tables are listed the strains measured from the target patterns shown in Figures 4.17 and 4.18 for all the section faces of all the beams at all the load stages. For any particular type of strain such as longitudinal strain in the top face, the strains at a given load stage for a particular row of targets, the number of strain readings in that row are listed below the title of the row. If it was not possible to obtain readings of strain over any two targets in a row, the listing will have a zero entry for that position. All the strains are listed in units of $10^{-3}$.

Figure E.3 shows the positions of the targets on the steel cage for a side face and the top face. Steel studs, inside plastic tubes with springs to enable easy extraction, were glued on to the hoops and waxed to keep moisture out. These studs were replaced after casting by brass targets from which strains were measured.
E.4 VARIATION OF STRAINS IN BEAM FACES

In this section the average strains for any beam face are plotted so as to reveal the variation trends of the measured strains on the sides. Plots are made for all the TBO and TBU series beams at approximately half capacity loading and at approximately capacity loading. For the TBS series plots are given for the near capacity loading case.
LONGITUDINAL STRAINS AT ≈ HALF CAPACITY LOADS FOR TB0 SERIES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM</th>
<th>LOAD STAGE</th>
</tr>
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<tbody>
<tr>
<td>02</td>
<td>TB02</td>
<td>4</td>
</tr>
<tr>
<td>03</td>
<td>TB03</td>
<td>4</td>
</tr>
<tr>
<td>04</td>
<td>TB04</td>
<td>5</td>
</tr>
<tr>
<td>05</td>
<td>TB05</td>
<td>5</td>
</tr>
</tbody>
</table>

LONGITUDINAL STRAINS AT APPROX. CAPACITY LOADS FOR TB0 SERIES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM LOAD STAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>02</td>
<td>TB02 6</td>
</tr>
<tr>
<td>03</td>
<td>TB03 7</td>
</tr>
<tr>
<td>04</td>
<td>TB04 10</td>
</tr>
<tr>
<td>05</td>
<td>TB05 8</td>
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</table>

LONGITUDINAL STRAINS AT APPROX. CAPACITY LOADS FOR TB0 SERIES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM</th>
<th>LOAD STAGE</th>
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</thead>
<tbody>
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<tr>
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<td>7</td>
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<tr>
<td>04</td>
<td>TB04</td>
<td>10</td>
</tr>
<tr>
<td>05</td>
<td>TB05</td>
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</tr>
</tbody>
</table>

LONGITUDINAL STRAINS AT APPROX. CAPACITY LOADS FOR TB0 SERIES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM</th>
<th>LOAD STAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>02</td>
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<td>TB04</td>
<td>10</td>
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<tr>
<td>05</td>
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</table>
TRANSVERSE STRAINS AT APPROX. HALF CAPACITY LOADS FOR TBO SERIES

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<tr>
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<td>5</td>
</tr>
<tr>
<td>05</td>
<td>TB05</td>
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</tbody>
</table>

LEGEND

- DOTTED LINES USED FOR CLARITY
- ENH = YIELD STRAIN FOR HOOP STEEL
**PRINCIPLE DIAGONAL STRAINS AT APPROX. HALF CAPACITY LOADS FOR TBO SERIES**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM</th>
<th>LOAD STAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>02</td>
<td>TB02</td>
<td>4</td>
</tr>
<tr>
<td>03</td>
<td>TB03</td>
<td>4</td>
</tr>
<tr>
<td>04</td>
<td>TB04</td>
<td>5</td>
</tr>
<tr>
<td>05</td>
<td>TB05</td>
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</tbody>
</table>

**LEGEND**

DOTTED LINES USED FOR CLARITY

**PRINCIPLE DIAGONAL STRAINS AT APPROX. CAPACITY LOADS FOR TBO SERIES**

<table>
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<tr>
<th>SYMBOL</th>
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<th>LOAD STAGE</th>
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</thead>
<tbody>
<tr>
<td>02</td>
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<td>04</td>
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<td>10</td>
</tr>
<tr>
<td>05</td>
<td>TB05</td>
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**LEGEND**

DOTTED LINES USED FOR CLARITY
LONGITUDINAL STRAINS AT APPROX. HALF CAPACITY LOADS FOR TBU SERIES

**LEGEND**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM LOAD STAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>U2</td>
<td>TBU2</td>
</tr>
<tr>
<td>U3</td>
<td>TBU3</td>
</tr>
<tr>
<td>U4</td>
<td>TBU4</td>
</tr>
<tr>
<td>U5</td>
<td>TBU5</td>
</tr>
</tbody>
</table>

*U1* = YIELD STRAIN FOR TOP FACE LONGITUDINAL STEEL

DOTTED LINES USED FOR CLARITY

LONGITUDINAL STRAINS AT APPROX. CAPACITY LOADS FOR TBU SERIES

**LEGEND**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM LOAD STAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>U2</td>
<td>TBU2</td>
</tr>
<tr>
<td>U3</td>
<td>TBU3</td>
</tr>
<tr>
<td>U4</td>
<td>TBU4</td>
</tr>
<tr>
<td>U5</td>
<td>TBU5</td>
</tr>
</tbody>
</table>

*U1* = YIELD STRAIN FOR TOP FACE LONGITUDINAL STEEL

DOTTED LINES USED FOR CLARITY
TRANSVERSE STRAINS AT APPROXIMATE HALF CAPACITY LOADS FOR TBU SERIES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM LOAD STAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>U2</td>
<td>TBU2</td>
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<td>U3</td>
<td>TBU3</td>
</tr>
<tr>
<td>U4</td>
<td>TBU4</td>
</tr>
<tr>
<td>U5</td>
<td>TBU5</td>
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</table>

DOTTED LINES USED FOR CLARITY

TRANSVERSE STRAINS AT APPROXIMATE CAPACITY LOADS FOR TBU SERIES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM LOAD STAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>U2</td>
<td>TBU2</td>
</tr>
<tr>
<td>U3</td>
<td>TBU3</td>
</tr>
<tr>
<td>U4</td>
<td>TBU4</td>
</tr>
<tr>
<td>U5</td>
<td>TBU5</td>
</tr>
</tbody>
</table>

DOTTED LINES USED FOR CLARITY

SYMBOL BEAM LOAD STAGE

U2 TBU2 5
U3 TBU3 6
U4 TBU4 4
U5 TBU5 4
PRINCIPAL DIAGONAL STRAINS AT APPROX. HALF/CAPACITY LOADS FOR TBU SERIES

**Legend**
- **Symbol**: Beam Load Stage
- **U2**: TBU2
- **U3**: TBU3
- **U4**: TBU4
- **U5**: TBU5

**Table**
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Beam</th>
<th>Load Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>U2</td>
<td>TBU2</td>
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<tr>
<td>U3</td>
<td>TBU3</td>
<td>6</td>
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<tr>
<td>U4</td>
<td>TBU4</td>
<td>4</td>
</tr>
<tr>
<td>U5</td>
<td>TBU5</td>
<td>4</td>
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</table>

**Graph 1**
- **Axes**: $E_d \times 10^{-3}$
- **Lines**: Dotted lines used for clarity

**Graph 2**
- **Axes**: $E_d \times 10^{-3}$
- **Lines**: Dotted lines used for clarity
LONGITUDINAL STRAINS AT APPROX. CAPACITY LOADS FOR TBS SERIES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM LOAD STAGE</th>
</tr>
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<tbody>
<tr>
<td>S2</td>
<td>TBS2</td>
</tr>
<tr>
<td>S3</td>
<td>TBS3</td>
</tr>
<tr>
<td>S4</td>
<td>TBS4</td>
</tr>
<tr>
<td>S5</td>
<td>TBS5</td>
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</tbody>
</table>

LEGEND

$\varepsilon_{yy}$ - YIELD STRAIN FOR LONG. STEEL

DOTTED LINES USED FOR CLARITY

TRANSVERSE STRAINS AT APPROX. CAPACITY LOADS FOR TBS SERIES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BEAM LOAD STAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2</td>
<td>TBS2</td>
</tr>
<tr>
<td>S3</td>
<td>TBS3</td>
</tr>
<tr>
<td>S4</td>
<td>TBS4</td>
</tr>
<tr>
<td>S5</td>
<td>TBS5</td>
</tr>
</tbody>
</table>

LEGEND

$\varepsilon_{yy}$ - YIELD STRAIN FOR HOOP STEEL

DOTTED LINES USED FOR CLARITY
Diagonal Strains at Approx. Capacity Loads for TBS Series

Legend

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Beam</th>
<th>Load Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1, S3, S5</td>
<td>TBS3</td>
<td>8</td>
</tr>
<tr>
<td>S2, S4</td>
<td>TBS4</td>
<td>6</td>
</tr>
<tr>
<td>S5</td>
<td>TBS5</td>
<td>9</td>
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</tbody>
</table>

Dotted lines used for clarity.
E.5 PLOTS OF MEASURED STRAINS AND PREDICTED STRAINS AT MID-POINTS OF SECTION FACES AGAINST APPLIED TORQUE

The following plots give the complete range of the response predictions for all the beams tested and the corresponding measured values for comparison. In the plots of test data separate lines are plotted for the North and South faces. These lines are evidently in close agreement. For the model predictions for the section with one axis of symmetry, the values for the mid-points of the North face and South face are identical and have been referred to as side face values. It should also be noted that response predictions were made for the spalled and unspalled section dimensions for each beam. The spalled predictions are plotted in thick lines and the unspalled in thin lines.
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS
THEORETICAL PREDICTION

TEST RESULTS