

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define Skewness and Kurtosis with classifications. Using moments calculate the coefficients of Skewness and Kurtosis from the following distribution given below and comment on the result obtained. (18)

Profits (in Taka)	10-20	20-30	30-40	40-50	50-60
No. of companies	18	20	30	22	10

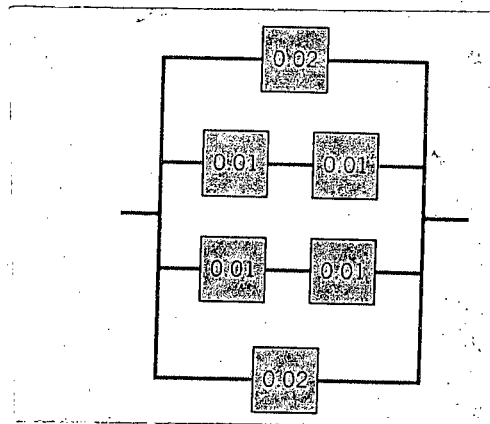
- (b) If the values of a series are in geometric progression $a, ar, ar^2, \dots, ar^{n-1}$, each with frequency unity, obtain the arithmetic mean (AM), geometric mean (GM) and harmonic mean (HM) and hence show that $AM \times HM = GM^2$. Also prove that $HM < GM < AM$ unless $n = 1$. (17)

2. (a) a survey, data on daily wages paid to workers of two factories A and B are as follows: (15)

Daily Wages	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Factory A	15	30	44	60	30	14	7
Factory B	25	40	60	35	20	15	5

Find out:

- (i) Which factory pays higher average wages?
 (ii) Which factory has greater variability about paying wages?
- (b) The following circuit operates if and only if there is a path of functional devices from left to right. Assume that devices fail independently and that the probability of failure of each device is as shown below. What is the probability that the circuit does not operate? (10)



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(c) If X and Y are independent random variable then prove that

(10)

(i) $\sigma_{aX+bY}^2 = a^2\sigma_X^2 + b^2\sigma_Y^2$

(ii) $\sigma_{aX-bY}^2 = a^2\sigma_X^2 + b^2\sigma_Y^2$

3. (a) The proportion of the budget for a certain type of industrial company that is allotted to environmental and pollution control is coming under scrutiny. A data collection project determines that the distribution of these proportions is given by

(8)

$$f(x) = \begin{cases} 5(1-x)^4, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Verify that the above is a valid density function. (ii) What is the probability that a company chosen at random expends less than 10% of its budget on environmental and pollution controls? (iii) What is the mean proportion of the budget allocated to environmental and pollution control? (iv) What is the probability that a company selected at random will have allocated to environmental and pollution control a proportion that exceeds the population mean given in (iii)?

(b) A computer system uses passwords that are exactly six characters and each character is one of the 26 letters (a-z) or 10 integers (0-9). Suppose there are 10,000 users of the system with unique passwords. A hacker randomly selects (with replacement) one billion passwords from the potential set, and a match to a user's password is called a hit. (i) What is the distribution of the number of hits? (ii) What is the probability of no hits? (iii) What are the mean and variance of the number of hits?

(7)

(c) If X is a binomial random variable with probability distribution $b(x; n, p)$. When $n \rightarrow \infty$, $p \rightarrow 0$, and $np \rightarrow \mu$ remains constant then prove that $b(x; n, p) \rightarrow p(x, \mu)$. In a certain industrial facility accidents occur infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other. What is the probability that

(20)

- (i) in any given period of 400 days there will be an accident on one day and
- (ii) there are at most three days with an accident.

4. (a) Find the mean and variance of the normal distribution.

(10)

(b) An article in Knee Surgery showed a mean time of 129 minutes and a standard deviation of 14 minutes for ACL reconstruction surgery at high-volume hospitals (with more than 300 such surgeries per year). (i) What is the probability that your ACL surgery at a high volume hospital requires a time more than two standard deviations above the mean? (ii) What is the probability that your ACL surgery at a high volume hospital is completed in less than 100 minutes? (iii) The probability of a completed ACL surgery at a high volume hospital is equal to 95% at what time? (iv) If your surgery requires 199 minutes, what do you conclude about the volume of such surgeries at your hospital? Explain. (Necessary table attached).

(15)

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(c) A U-shaped component is to be formed from the three parts A, B, and C. The picture is shown below. The length of A is normally distributed with a mean of 10 millimeters and a standard deviation of 0.1 millimeter. The thickness of parts B and C is normally distributed with a mean of 2 millimeters and a standard deviation of 0.05 millimeter. Assume all dimensions are independent.

(10)

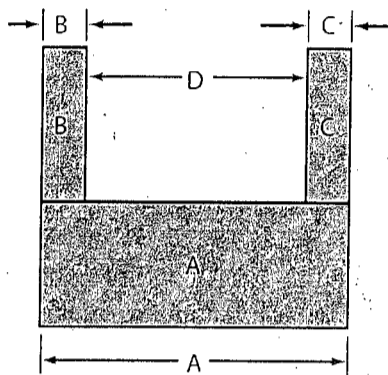


Figure: The U-shaped component.

- (i) Determine the mean and standard deviation of the length of the gap D.
- (ii) What is the probability that the gap D is less than 5.9 millimeters? (Necessary table attached).

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) The following table gives the heights of 100 students at XYZ University. Show how to select 15 random samples of 2 students each (with and without replacement) from the following table by using random numbers. (Necessary table attached)

(17)

Height (in)	Frequency
60 – 62	5
63 – 65	18
66 – 68	42
69 – 71	27
72 – 74	8

(b) Find the mean and standard deviation of the sampling distribution of means in part (a) for both the cases (with and without replacement). Compare the results with theoretical values, explaining any discrepancies.

(18)

6. (a) Certain tubes manufactured by a company have a mean life time of 400 h and standard deviation of 30 h. Find the probability that a random sample of 8 tubes taken from the group will have a mean life time (i) between 395 and 405 h (ii) less than 392 h. (iii) more than 405 h.

(9)

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(b) The coefficient of rank correlation of the marks obtained by 10 students in statistics and mathematics was found to be 0.8. It was later found that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 7 instead 9. Find the correct coefficient of rank correlation. (10)

(c) For certain X and Y series, the lines of regression of Y on X and X on Y are respectively $6Y=5X+90$ and $15X=8Y+130$. The standard deviation of X values is 4. Find (i) Mean of X values (ii) Mean of Y values (iii) Coefficient of correlation between X and Y (iv) Standard deviation of Y values. (16)

7. (a) sample of 100 electric light bulbs produced by manufacturer A showed a mean life time of 1190 h and a standard deviation of 90 h. A sample of 75 bulbs produced by manufacturer B showed a mean life time of 1230 h and a standard deviation of 120 h. Test the hypothesis that the bulbs of manufacturer B are superior to those of manufacturer A by using significance level of 0.01. (17)

(b) Using brand A gasoline, the mean number of kilometers per litre traveled by five similar automobiles under identical conditions was 11.3 with a standard deviation of 0.24. Using brand B, the mean number was 10.7 with a standard deviation of 0.27. Is there a significant difference between the mean numbers of kilometers of the two brands of gasoline? Use 5% level of significance. (Necessary table attached) (18)

8. (a) In his experiments with peas, Gregor Mendel observed that 315 were round and yellow, 108 were round and green, 101 were wrinkled and yellow and 32 were wrinkled and green. According to his theory of heredity the numbers should be in proportion 9:3:3:1. Is there any evidence to doubt his theory at the 5% significance level? (Necessary table attached) (15)

(b) A company wishes to purchase one of five different machines: A, B, C, D or E. In an experiment designed to test whether there is a difference in the machines' performance, each of five experienced operators works on each of the machines for equal times. Table below shows the number of units produced per machine. Test the hypothesis that there is no difference between the machines at significance level of 0.05. (Necessary table attached) (20)

A	58	62	67	32	43
B	62	43	53	43	38
C	50	72	54	65	62
D	38	51	47	54	40
E	54	55	60	58	43

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

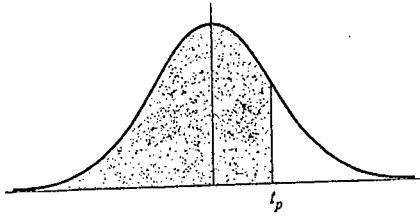
Table for QN.4

Random Numbers

51772	74640	42331	29044	46621	62898	93582	04186	19640	87056
24033	23491	83587	06568	21960	21387	76105	10863	97453	90581
45939	60173	52078	25424	11645	55870	56974	37428	93507	94271
30586	02133	75797	45406	31041	86707	12973	17169	88116	42187
03585	79353	81938	82322	96799	85659	36081	50884	14070	74950
64937	03355	95863	20790	65304	55189	00745	65253	11822	15804
15630	64759	51135	98527	62586	41889	25439	88036	24034	67283
09448	56301	57683	30277	94623	85418	68829	06652	41982	49159
21631	91157	77331	60710	52290	16835	48653	71590	16159	14676
91097	17480	29414	06829	87843	28195	27279	47152	35683	47280
50532	25496	95652	42457	73547	76552	50020	24819	52984	76168
07136	40876	79971	54195	25708	51817	36732	72484	94923	75936
27989	64728	10744	08396	56242	90985	28868	99431	50995	20507
85184	73949	36601	46253	00477	25234	09908	36574	72139	70185
54398	21154	97810	36764	32869	11785	55261	59009	38714	38723
65544	34371	09591	07839	58892	92843	72828	91341	84821	63886
08263	65952	85762	64236	39238	18776	84303	99247	46149	03229
39817	67906	48236	16057	81812	15815	63700	85915	19219	45943
62257	04077	79443	95203	02479	30763	92486	54083	23631	05825
53298	90276	62545	21944	16530	03878	07516	95715	02526	33537

table for question no 5(a)

Percentile Values (t_p) for Student's t Distribution with ν Degrees of Freedom (shaded area = p)

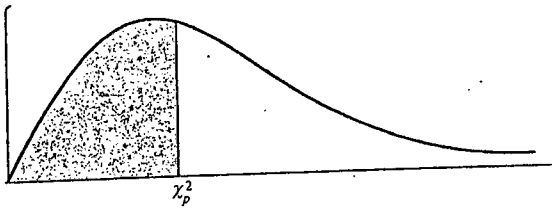


ν	$t_{.995}$	$t_{.99}$	$t_{.975}$	$t_{.95}$	$t_{.90}$	$t_{.80}$	$t_{.75}$	$t_{.70}$	$t_{.60}$	$t_{.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	.727	.325	.158
2	9.92	6.96	4.30	2.92	1.89	1.061	.816	.617	.289	.142
3	5.84	4.54	3.18	2.35	1.64	.978	.765	.584	.277	.137
4	4.60	3.75	2.78	2.13	1.53	.941	.741	.569	.271	.134
5	4.03	3.36	2.57	2.02	1.48	.920	.727	.559	.267	.132
6	3.71	3.14	2.45	1.94	1.44	.906	.718	.553	.265	.131
7	3.50	3.00	2.36	1.90	1.42	.896	.711	.549	.263	.130
8	3.36	2.90	2.31	1.86	1.40	.889	.706	.546	.262	.130
9	3.25	2.82	2.26	1.83	1.38	.883	.703	.543	.261	.129
10	3.17	2.76	2.23	1.81	1.37	.879	.700	.542	.260	.129
11	3.11	2.72	2.20	1.80	1.36	.876	.697	.540	.260	.129
12	3.06	2.68	2.18	1.78	1.36	.873	.695	.539	.259	.128
13	3.01	2.65	2.16	1.77	1.35	.870	.694	.538	.259	.128
14	2.98	2.62	2.14	1.76	1.34	.868	.692	.537	.258	.128
15	2.95	2.60	2.13	1.75	1.34	.866	.691	.536	.258	.128
16	2.92	2.58	2.12	1.75	1.34	.865	.690	.535	.258	.128
17	2.90	2.57	2.11	1.74	1.33	.863	.689	.534	.257	.128
18	2.88	2.55	2.10	1.73	1.33	.862	.688	.534	.257	.127
19	2.86	2.54	2.09	1.73	1.33	.861	.688	.533	.257	.127
20	2.84	2.53	2.09	1.72	1.32	.860	.687	.533	.257	.127
21	2.83	2.52	2.08	1.72	1.32	.859	.686	.532	.257	.127

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table for question no 7(b)

Percentile Values (χ_p^2) for the Chi-Square Distribution with ν Degrees of Freedom (shaded area = p)

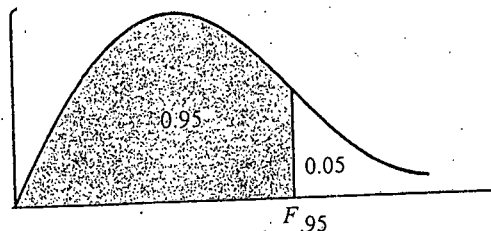


ν	$\chi_{.995}^2$	$\chi_{.99}^2$	$\chi_{.975}^2$	$\chi_{.95}^2$	$\chi_{.90}^2$	$\chi_{.75}^2$	$\chi_{.50}^2$	$\chi_{.25}^2$	$\chi_{.10}^2$	$\chi_{.05}^2$	$\chi_{.025}^2$	$\chi_{.01}^2$	$\chi_{.005}^2$
1	7.88	6.63	5.02	3.84	2.71	1.32	.455	.102	.0158	.0039	.0010	.0002	.0000
2	10.6	9.21	7.38	5.99	4.61	2.77	1.39	.575	.211	.103	.0506	.0201	.0100
3	12.8	11.3	9.35	7.81	6.25	4.11	2.37	1.21	.584	.352	.216	.115	.072
4	14.9	13.3	11.1	9.49	7.78	5.39	3.36	1.92	1.06	.711	.484	.297	.207
5	16.7	15.1	12.8	11.1	9.24	6.63	4.35	2.67	1.61	1.15	.831	.554	.412
6	18.5	16.8	14.4	12.6	10.6	7.84	5.35	3.45	2.20	1.64	1.24	.872	.676
7	20.3	18.5	16.0	14.1	12.0	9.04	6.35	4.25	2.83	2.17	1.69	1.24	.989
8	22.0	20.1	17.5	15.5	13.4	10.2	7.34	5.07	3.49	2.73	2.18	1.65	1.34
9	23.6	21.7	19.0	16.9	14.7	11.4	8.34	5.90	4.17	3.33	2.70	2.09	1.73
10	25.2	23.2	20.5	18.3	16.0	12.5	9.34	6.74	4.87	3.94	3.25	2.56	2.16
11	26.8	24.7	21.9	19.7	17.3	13.7	10.3	7.58	5.58	4.57	3.82	3.05	2.60
12	28.3	26.2	23.3	21.0	18.5	14.8	11.3	8.44	6.30	5.23	4.40	3.57	3.07
13	29.8	27.7	24.7	22.4	19.8	16.0	12.3	9.30	7.04	5.89	5.01	4.11	3.57
14	31.3	29.1	26.1	23.7	21.1	17.1	13.3	10.2	7.79	6.57	5.63	4.66	4.07
15	32.8	30.6	27.5	25.0	22.3	18.2	14.3	11.0	8.55	7.26	6.26	5.23	4.60
16	34.3	32.0	28.8	26.3	23.5	19.4	15.3	11.9	9.31	7.96	6.91	5.81	5.14
17	35.7	33.4	30.2	27.6	24.8	20.5	16.3	12.8	10.1	8.67	7.56	6.41	5.70
18	37.2	34.8	31.5	28.9	26.0	21.6	17.3	13.7	10.9	9.39	8.23	7.01	6.26
19	38.6	36.2	32.9	30.1	27.2	22.7	18.3	14.6	11.7	10.1	8.91	7.63	6.84
20	40.0	37.6	34.2	31.4	28.4	23.8	19.3	15.5	12.4	10.9	9.59	8.26	7.43
21	41.4	38.9	35.5	32.7	29.6	24.9	20.3	16.3	13.2	11.6	10.3	8.90	8.03

(Contd)

table for question no 8(a)

95th Percentile Values for the F Distribution



ν_1 degrees of freedom in numerator
 ν_2 degrees of freedom in denominator

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.10	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.02	1.97
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.03	1.98	1.93
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.99	1.95	1.90
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73

(Contd)

Table for question no 8(b)

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Answer in brief and to the point.

1. (a) Define electric field intensity \vec{E} . Using appropriate fundamental relation, determine \vec{E} for a point charge q_1 at a distance R from it. **(5+6=11)**
 Also, determine the force that \vec{E} will exert on a test charge q_2 at the field point. State the resulting equation in words and identify its popular name.
- (b) Define electric potential V and explain its physical significance. **(4+2+5=11)**
 With appropriate relation define potential difference, $V_{21} = V_2 - V_1$.
 Define absolute potential and determine the absolute potential due to a point charge q at a distance R from it.
- (c) "A polarized dielectric gives rise to an equivalent volume charge density, ρ_p " — in the light of this statement, modify the divergence relation, $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$, and define an electric flux density vector, \vec{D} . **(5+8=13)**
 A positive point charge Q is located at the centre of a spherical dielectric shell ($\epsilon_r = 2$) of an inner radius R_i and an outer radius R_o . For the region, $R_i < R < R_o$, determine \vec{E} , V , \vec{D} and \vec{P} as a function of the radial distance, R .
2. (a) From appropriate fundamental postulates of electrostatics, derive the Poisson's equation for a simple medium. What is Laplace equation? **(5+8=13)**
 The two plates of a parallel-plate capacitor are separated by a distance d and maintained at potentials 0 and V_0 volts. Using Laplace's equation, determine (i) the potential, V , at any point between the plates, and (ii) the surface charge densities on the plates.
- (b) What is method of images? When can we use this method to solve electrostatic problems? **(3+10=13)**
 For a point charge Q located at distances d_1 and d_2 , respectively, from two grounded perpendicular conducting half-planes, find the expressions for (i) the potential and the electric field intensity at an arbitrary point $P(x, y)$ in the first quadrant, and (ii) the surface charge densities on the two half-planes.
- (c) Define capacitance of a capacitor. Does it depend on Q and/or V ? If not, what does it depend on? **(4+5=9)**
 A cylindrical capacitor consists of an inner conductor of radius a and an outer conductor whose inner radius is b . The space between the conductors is filled with a dielectric of permittivity ϵ , and the length of the capacitor is L . Determine the capacitance of this capacitor.

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3. (a) State Ampere's circuital law. Using this law determine the magnetic flux density both inside and outside of an infinitely long, straight conductor with a circular cross-section of radius **b** and carrying a steady current **I**. (2+10=12)

(b) Show that the energy stored in a magnetic field can be given by (9+4=13)

$$W_m = \int_{V'} \bar{H} \cdot \bar{B} dv'$$

Define self inductance of a current loop and mutual inductance between two circuits.

(c) Relate W_m with L and I , where a current I flows in a single inductor with inductance L . (2+8=10)

Determine the inductance per unit length of an air coaxial transmission line that has a solid inner conductor of radius **a** and a very thin outer conductor of inner radius **b**.

4. (a) Write down the divergence relation of the magnetic flux density vector \bar{B} and explain its physical significance. (2+8=10)

(b) How can you define vector magnetic potential \bar{A} from the relation in part (a)? How can you determine \bar{A} from \bar{J} ? Also, explain how \bar{A} is related to the total flux ϕ . (8+6=14)

(c) Write short notes on any TWO of the following: (5 1/2 + 5 1/2 = 11)
(i) Electric and magnetic dipoles
(ii) Hall effect
(iii) Magnetic screening.

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

All symbols have their usual meanings.

5. (a) Show that $\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$ and also show that the volume charge density (ρ_v) decays exponentially with time. What is Relaxation time? (15)

(b) A certain amount of charge is placed within an isolated conductor. The current through a closed surface bounding the charge is observed to be $i(t) = 0.125 e^{-25t}$ A. Determine (i) the Relaxation time, (ii) the initial charge and (iii) the charge transported through the surface in time $t = 5\tau$, where τ is the time constant. (10)

(c) A d.c voltage V_0 is applied across a cylindrical capacitor of length L . The radii of the inner and outer conductors are **a** and **b** respectively. The space between the conductors is filled with two different lossy dielectrics having respectively, permittivity ϵ_1 and conductivity σ_1 in the region $a < r < c$, and permittivity ϵ_2 and conductivity σ_2 in the region $c < r < b$. Determine (i) the equivalent R-C circuit between the inner and outer conductors (ii) the current density in each region. (10)

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6. (a) For a linear, homogeneous, isotropic, source-free conductive region show that, (13)

$$\begin{aligned}\nabla^2 \tilde{E} - \gamma^2 \tilde{E} &= 0 \\ \nabla^2 \tilde{H} - \gamma^2 \tilde{H} &= 0\end{aligned}$$

$$\text{Where, } \gamma^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$$

- (b) Starting with Maxwell's equation from Faraday's law and the definition of vector magnetic potential \vec{A} , show that the line integral of $\left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right)$ around a closed path is zero. (10)

- (c) A uniform plane wave propagates in the +z-(downward) direction into the ocean ($\epsilon_r = 72, \mu_r = 1.0, \sigma = 4 \text{ S/m}$). The magnetic field at the ocean surface ($z = 0$) is $\vec{H}(0, t) = \hat{a}_y 0.3 \cos 10^8 t \text{ A/m}$. Determine (12)

- (i) the skin depth and the intrinsic impedance of the ocean water.
- (ii) Expressions of $\vec{E}(z, t)$ and $\vec{H}(z, t)$.

7. (a) What is meant by the polarization of a wave? With a suitable example explain the different types of polarization. (13)

- (b) Determine the polarization of the wave if the electric field intensity in a region is given by $\vec{E} = (3\hat{a}_x + j4\hat{a}_y)e^{-0.2z}e^{-j0.5z} \text{ V/m}$. (10)

- (c) A y-polarized uniform plane wave (E_i, H_i) with a frequency of 100 (MHz) propagates in air in the +x-direction and impinges normally on a perfectly conducting plane at $x = 0$. Assuming the amplitude of E_i to be 10 (mV/m). Write the phasor and instantaneous expression for E_t and H_t of the total wave in air. (12)

8. (a) State and explain Poynting Theorem. Derive the instantaneous and average power densities of a uniform plane wave propagating through a good conductor, in the positive z-direction. Assume that the plane wave has only the x-component of the electric field intensity. (18)

- (b) In a nonmagnetic medium $\vec{E} = 4 \sin(2\pi \times 10^7 t - 0.8x) \hat{a}_z \text{ V/m}$ (17)

Find (i) ϵ_r, η (ii) the time average power carried by the wave (iii) the total power crossing 100 cm^2 of plane $2x + y = 5$.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

The questions are of equal value.

1. (a) Describe with necessary diagrams the brushless systems of excitation for synchronous generator. Compare these systems from the viewpoint of their advantages and disadvantages.
 (b) A 100 kVA, 3000 V, 50 Hz, 3-phase, Y-connected synchronous generator has effective per phase armature resistance of 0.2Ω . A field current of 40 A produces short circuit current of 200 A and open-circuit e.m.f of 1040 V (line value). Calculate full-load percentage regulation at a power factor of 0.8 lagging. How will the regulation be affected if the generator delivers its full-load output at a power factor of 0.8 leading.
2. (a) Explain why the terminal voltage of a synchronous generator is different from the internally generated voltage at loaded condition. Derive the equivalent circuit of a synchronous generator. Draw the equivalent circuits when the armature windings are Y and Δ connected.
 (b) Using necessary House diagrams and phasor diagram explain the load sharing of a synchronous generator working in parallel with infinite bus system. Explain also the load sharing of two synchronous generators of comparable power working in parallel.
3. (a) With necessary phasor diagram explain the effect of load changes on synchronous motor. With the increase of load how the power factor of the motor changes? What is a synchronous condenser? Draw its phasor diagram and explain how it operates.
 (b) A synchronous machine has a synchronous reactance of 2.0Ω per phase and an armature resistance of 0.4Ω per phase. If $E_A = 460 \angle -8^\circ$ V and $V_\phi = 480 \angle 0^\circ$ V, is this machine a motor or a generator? How much power p is this machine consuming from or supplying to the electrical system? How much reactive power Q is this machine consuming from or supplying to the electrical system?
4. (a) Explain how synchronous motor can be used for power factor correction of the supply system. Explain the methods of starting of synchronous motor by changing frequency and by using Amortisseur Windings.
 (b) Explain the photo voltaic effect, which is used to produce voltage in a solar cell. Derive the equivalent circuit of a solar cell. Derive the expression of open circuit voltage of a solar cell. Explain how the voltage and current density of solar cell is found at maximum power density condition.

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SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.

The figures in the margin indicate full marks.

All the symbols and notations have their usual meanings.

5. (a) What is commutation? What will happen if commutator is not used in a DC motor? (7)
- (b) What is the $L di/dt$ voltage problem of commutation? Briefly explain how this problem can be solved? (13)
- (c) What are the purposes of starter circuits used in a DC motor? Draw a DC motor starter circuit using time-delay relays and describe its operation. What are the limitations of this circuit? (15)
6. (a) Derive the terminal characteristic of a DC series motor. Explain with necessary figure, why these types of motors are suitable for starting high-inertia loads. What will happen, if the shaft of a DC series motor suddenly gets unloaded? (15)
- (b) Explain the voltage build-up process in a DC shunt generator. (8)
- (c) A family of magnetization curves (E_A vs I_F) of a DC shunt generator for various prime mover speed is shown in Fig. for Q. 6(c). The machine has compensating windings to offset the effect of armature reaction. When the prime mover speed is 1200 rpm, no-load terminal voltage is 150 V. The armature resistance is negligible. (12)
- (i) Determine the field circuit resistance of the DC shunt generator.
- (ii) If the prime mover speed increases to 1500 rpm, what will be the terminal voltage?
- (iii) If the prime mover speed cannot exceed 1500 rpm, what will be the critical resistance?
- (iv) If the field circuit resistance cannot exceed 60Ω , what will be its critical speed?
7. (a) A separately excited DC generator rated at 6 kW, 120 V, 50 A, and 1800 r/min is shown in the Fig. for Q. 7(a). The magnetization curve for the machine at 1800 r/min is also given in Fig. for Q. 7(a). (15)
- (i) If the armature current of the generator is 50 A, the speed of the generator is 1700 r/min. and the terminal voltage is 106 V, how much field current must be flowing in the generator?
- (ii) Assuming that the generator has an armature reaction at full load equivalent to 400 Ampere-turns of magnetomotive force, what will the terminal voltage of the generator be when $I_F = 5 \text{ A}$, $n_m = 1700 \text{ r/min}$, $I_A = 50 \text{ A}$?

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Contd... Q. No. 7

- (b) What are the advantages and disadvantages of a PMDC motor over a DC shunt motor? What are the desirable characteristics of the permanent magnets in PMDC machines? (8)
- (c) Derive the expression of internal generated voltage of a real DC machine. (12)
8. (a) Briefly describe the methods for controlling speed of a DC shunt motor. Also show the change in torque-speed characteristics for each method. (17)
- (b) A 240-V, 100-A shunt dc motor has following parameters: (18)
 $R_A = 0.14 \Omega$, $R_F = 200 \Omega$, $R_{adj} = 0$ to 300Ω , currently set to 120Ω , $N_F = 1500$ turns, $n_n = 1200$ r/min.
The magnetization curve for the dc motor is given in Fig. for Q. 8(b)
- (i) What is the no-load speed of this motor when $R_{adj} = 120 \Omega$?
 - (ii) What is its full-load speed?
 - (iii) Under no-load conditions, what range of possible speeds can be achieved by adjusting R_{adj} ?
-

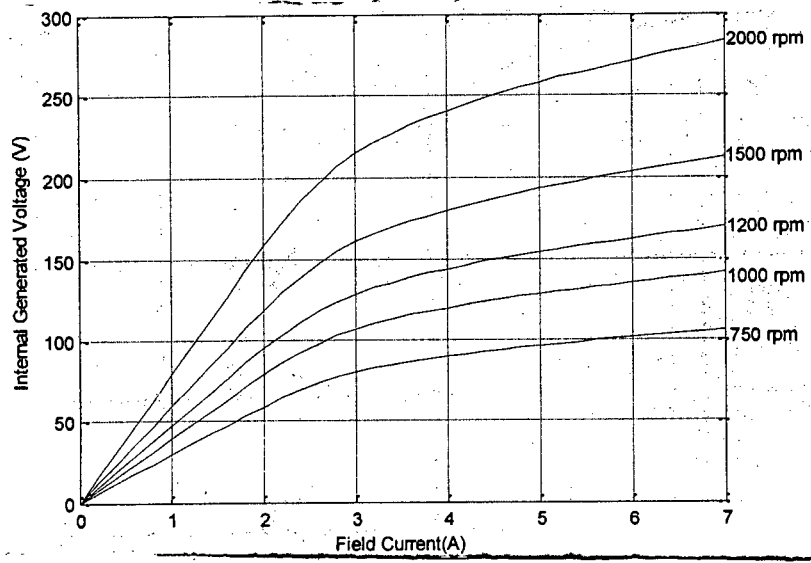
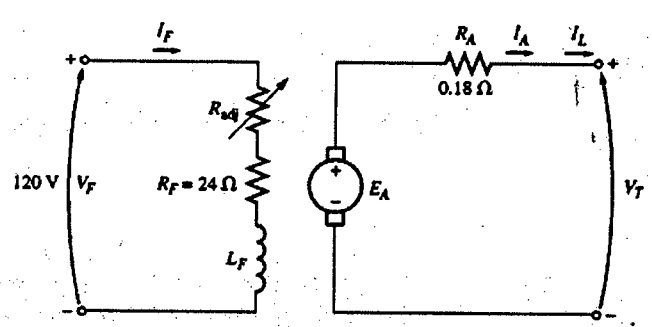
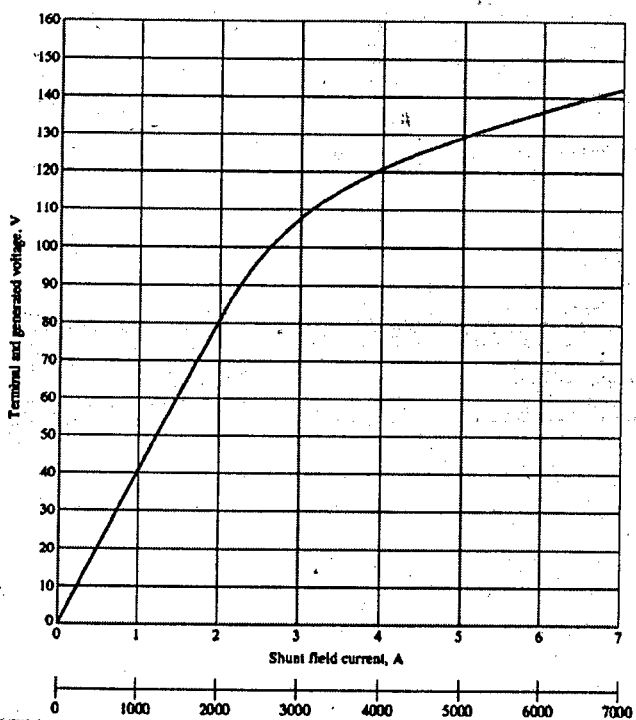


Fig. for Q. 6(c)



$R_A = 0.18 \Omega$
 $R_{adj} = 0 \text{ to } 30 \Omega$
 $N_F = 1000 \text{ turns per pole}$
 $V_F = 120 \text{ V}$
 $R_F = 24 \Omega$

Fig. for Q. 7(a)

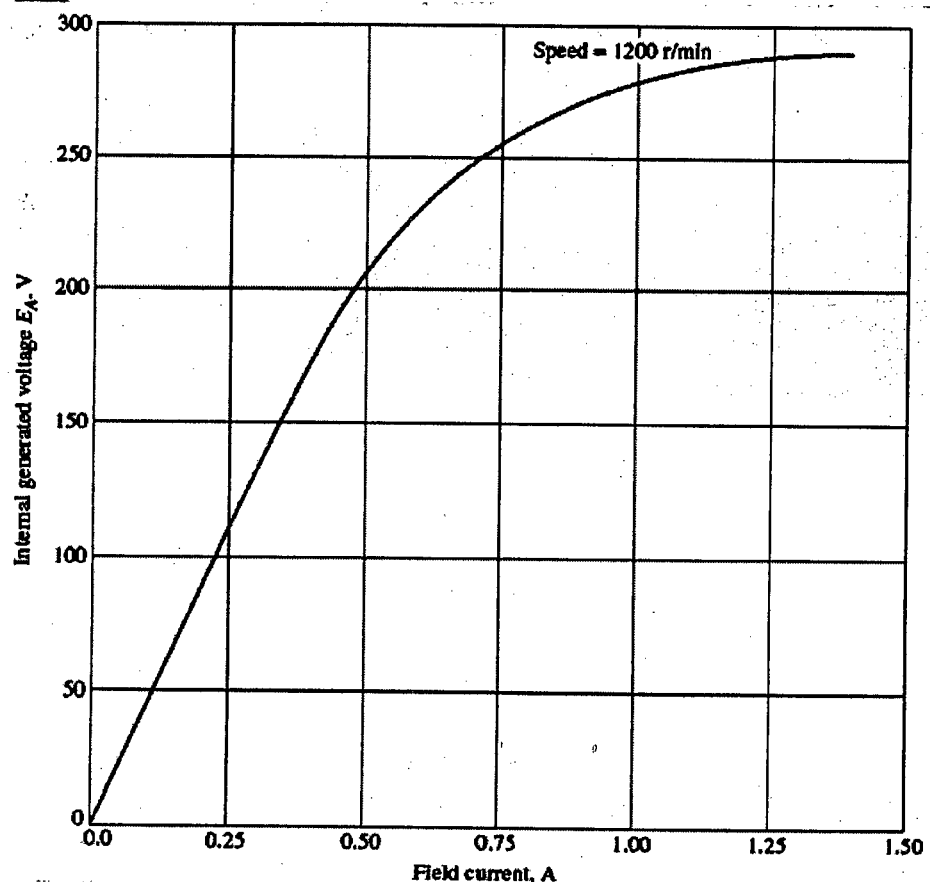


Fig. for Q. 8(b)

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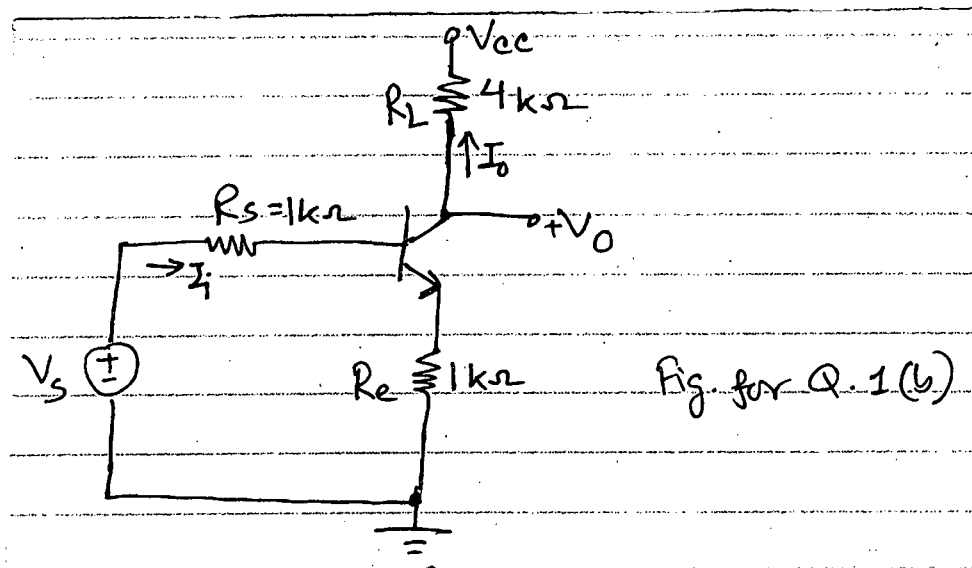
USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION - A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) With necessary diagrams, derive the expressions of input resistance (R_{if}) and output resistance (R_{of}) of a voltage shunt feedback amplifier. (18)

- (b) Calculate the voltage gain, A_{vf} , input resistance R_{if} and output resistance (including load) R'_{of} for the circuit shown in Fig. for Q. 1(b). Given that $h_{fe} = 150$, $h_{ie} = 1 \text{ k}\Omega$, while h_{re} and h_{oe} are negligible. (17)



2. (a) Design a diode-compensated class-B power amplifier with low frequency cut-off at 60 Hz, output power of 1.0 W and load resistance of 8.0 Ω . Given that, the supply voltage is 12 V, transistors have $\beta = 60$, and the diode forward resistance is 8.0 Ω . Also calculate the input resistance and current gain. (18)

(Use necessary diagram with proper derivations)

- (b) Showing Q-point on load line, classify BJT power amplifier. Also mention how long the collector current remains non-zero for each class. (8)

- (c) Using schematic diagram of a single loop negative feedback amplifier, show that $|A_f| < |A|$. (9)

where, symbols have their usual meaning.

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3. (a) Derive the approximate equation for the lower 3 dB frequency of an amplifier (4+6+2=12)
- (i) if a dominant pole exists.
 - (ii) if a dominant pole does not exist.
 - (iii) Also, show that the second equation is also valid when a dominant pole exists.

(b) Derive the expression of upper 3 dB frequency if the poles and zeroes of the amplifier transfer function cannot be determined easily. (8)

(c) Sketch Bode plot for the magnitude of the following transfer function. (15)

$$T(s) = \frac{10^7 s (s + 10^3)}{(s + 10)(s + 10^2)(s + 10^5)}$$

From the plot, estimate the approximate magnitude at $\omega = 10^6$ rad/s. What is the exact magnitude at $\omega = 10^6$ rad/s?

(Use graph paper if necessary).

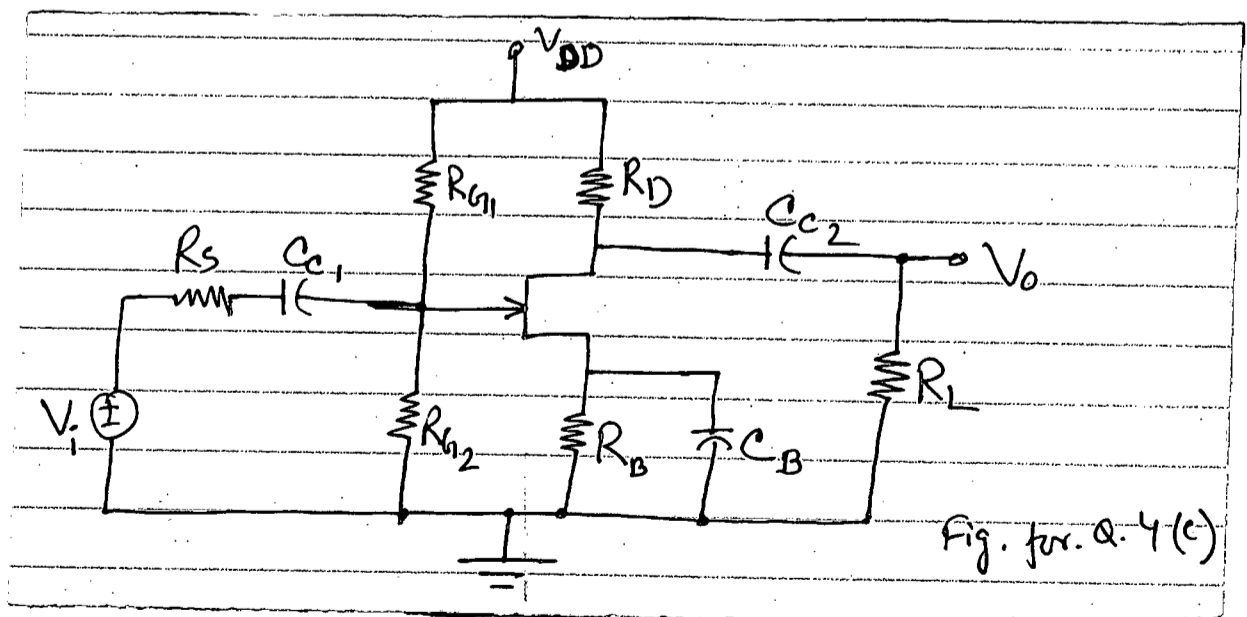
4. (a) For class-A operation of an amplifier, show that, (10)

$$I_{CQ} = \frac{V_{cc}}{R_{dc} + R_{ac}}$$

where, symbols have their usual meaning.

(b) Briefly explain Miller's theorem. (7)

(c) Find the expression of low frequency amplifier gain $A_L(s)$ for the amplifier given in Fig. for Q. 4(c). Also mention the poles and zeroes for this derived transfer function. (18)

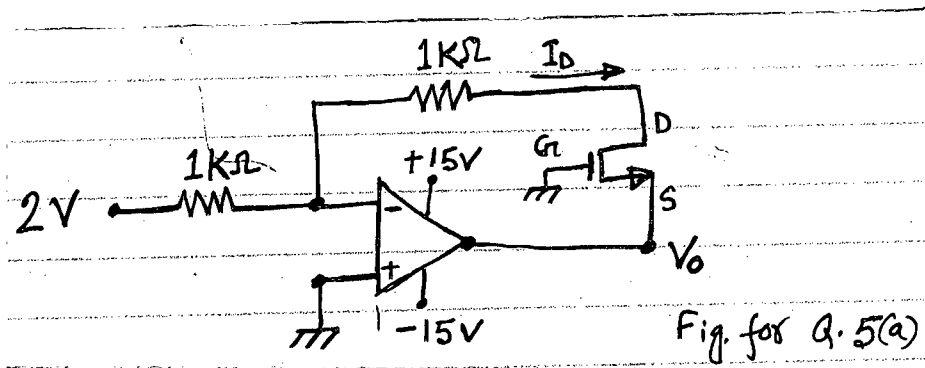


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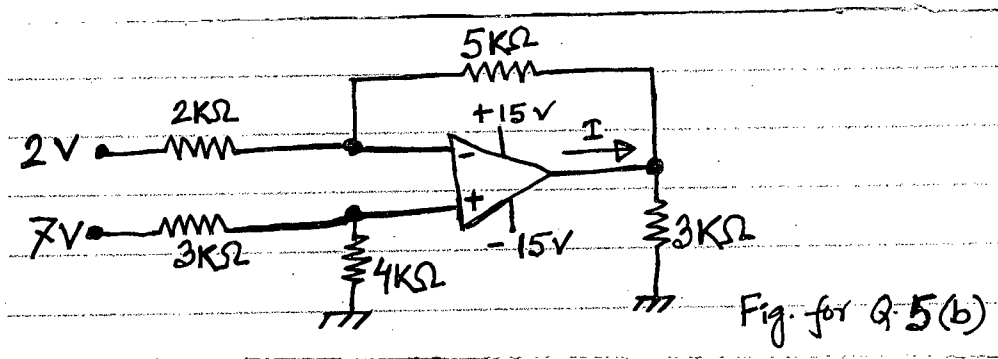
SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.
 Symbols have their usual meanings. Make necessary assumptions.

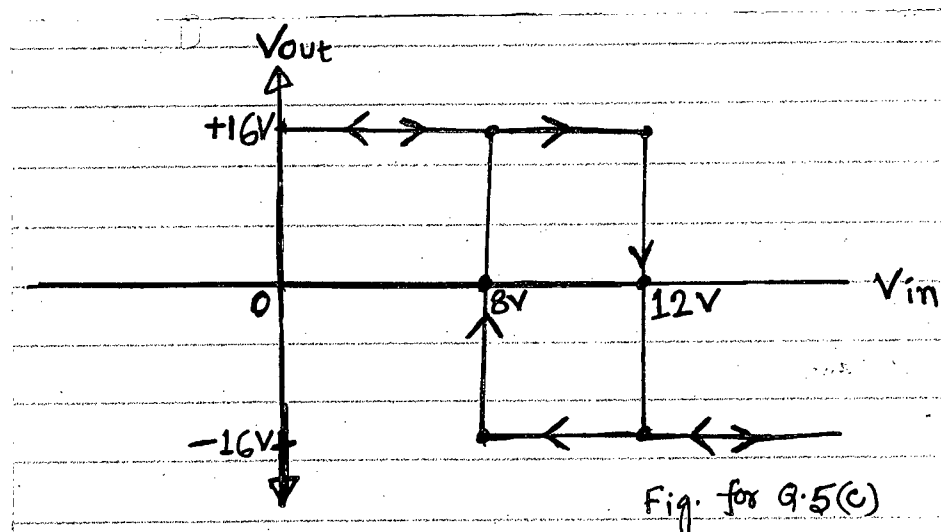
5. (a) For the circuit shown in Fig. for Q. 5(a) find V_o . Assume that the MOSFET is in saturation and $k'_n \frac{W}{L} = 1 \text{ mA/V}^2$. (8)



- (b) Calculate I for the circuit shown in Fig. for Q. 5(b). (8)



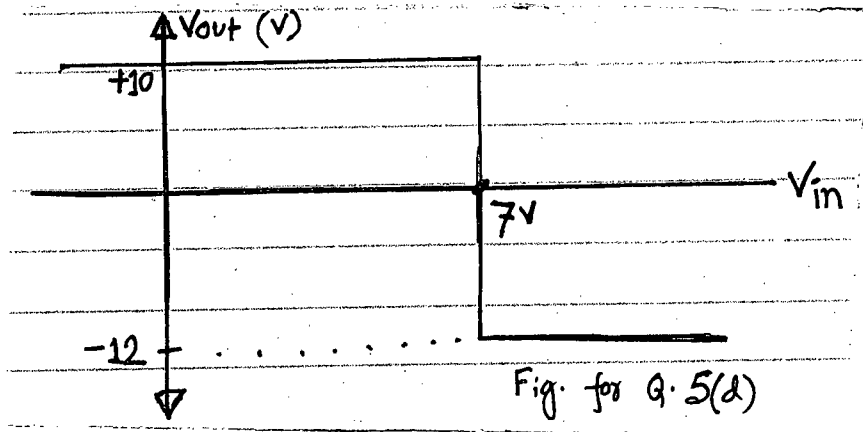
- (c) Design a Schmitt Trigger circuit using Op-Amp that has the transfer characteristics shown in Fig. for Q. 5(c). Indicate reference voltage, biasing voltage and all the resistors used in your design. (13)



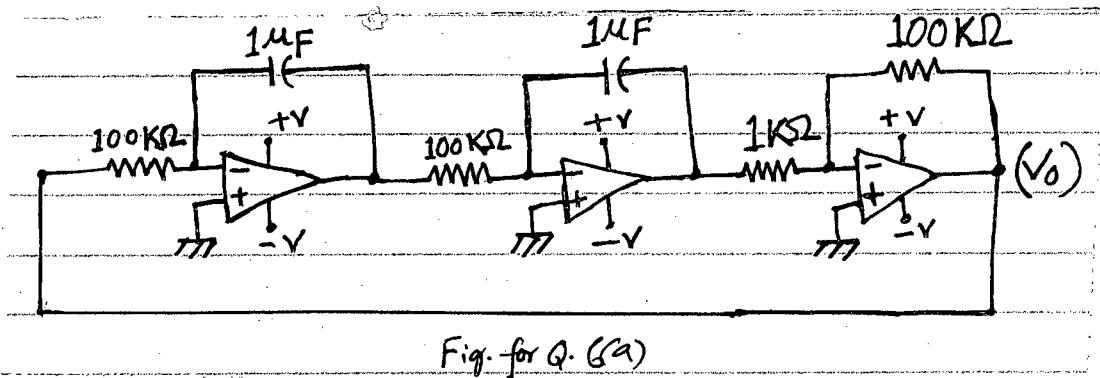
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Contd... Q. No. 5

(d) Design a circuit using Op-Amp that has the transfer characteristics shown in Fig. for Q. 5(d). (6)

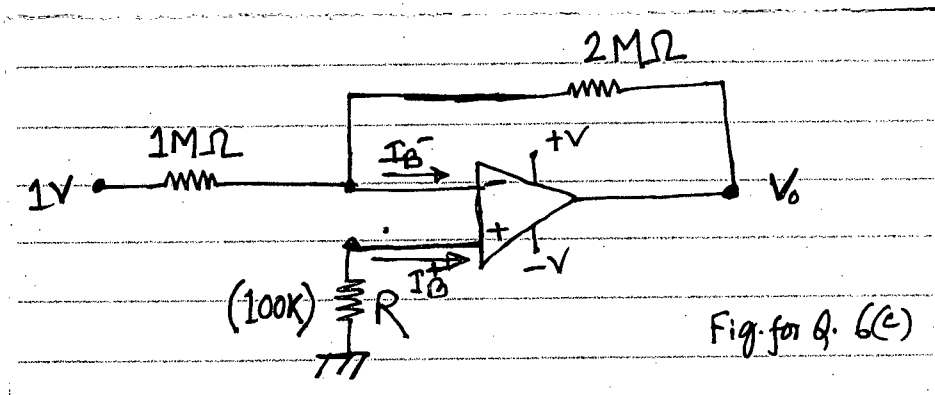


6. (a) For the oscillator circuit shown in Fig. for Q. 6(a) find oscillation frequency in Hz. (10)



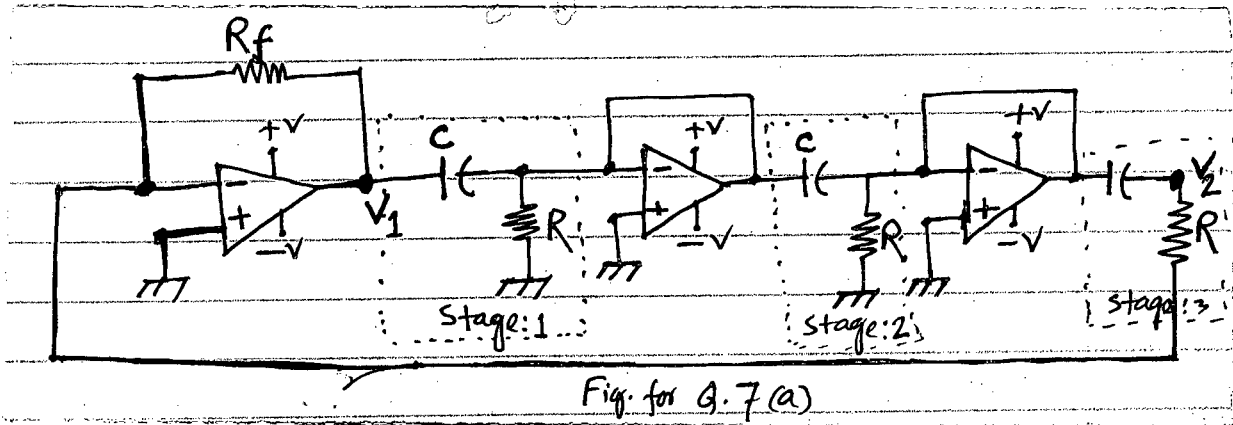
(b) Design a circuit using Op-Amps that takes a sinusoidal signal as input and gives a sinusoidal output which has frequency twice as input frequency. Note that, if $V_{in} = A \sin(\theta)$ then $V_{out} = B \sin(2\theta)$. You do not need to take care of the amplitude of the signals that means you do not need to indicate the values of resistors, capacitors and other circuit components used. (15)

(c) For the circuit shown in Fig. for Q. 6(c) assuming the Op-Amp used is non-ideal, find V_0 . Suggest a value of R to get the minimum error. Given input bias current components $I_B^+ = I_B^- = 0.1 \mu A$. (10)



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7. (a) For the oscillator circuit shown in Fig. for Q 7(a) find oscillation frequency (ω), β -network gain (β), and Amplifier gain (A). Also design this oscillator for an oscillation frequency of 1 kHz. (18)

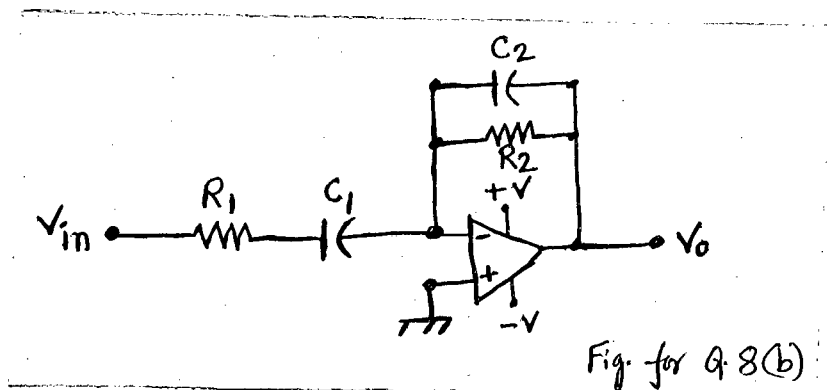


Note that, each stage is separated by a source follower circuit. Hence each stage will give a phase shift of 60° and 3 stages will give in total 180° phase shift. Here, $\beta = \frac{V_2}{V_1}$.

- (b) Design a colpitts oscillator to produce a 100 kHz output frequency. (12)
 (c) The slew rate for a 741 Op-Amp is $0.5 \text{ V}/\mu\text{s}$. For an inverting amplifier with a gain of -1 , find maximum peak of undistorted sine wave output at a frequency of 50 kHz. (5)

8. (a) While tuning an electric guitar in the 4th octave between C_4 (262 Hz) and C_5 (524 Hz), suppose you want to hear E_4 note only. Design the filter required with a quality factor 20. Given the 4th octave sequence $C_4, C\#, D, D\#, E, F, F\#, G, G\#, A, A\#, B, C_5$. where frequency of each note can be found out by the relation, $f = 2^{n/12} \times f(C_4)$. For example, for note A_4 , $n = 9$. (10)

- (b) Determine the type of filter that is shown in the circuit in Fig. for Q. 8(b) implement. For that purpose you need to calculate transfer function, $T(s) = \frac{V_o(s)}{V_{in}(s)}$, where $s = j\omega$. Find poles and zeros of $T(s)$. (15)

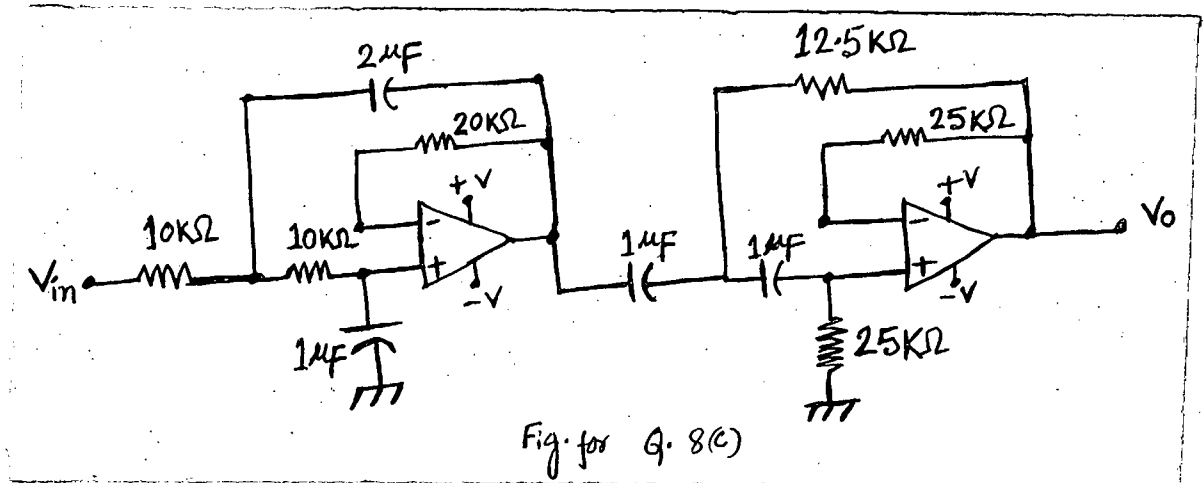


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Contd... Q. No. 8

(c) Determine which of the following signal(s) will pass through the filter shown in Fig. for Q. 8(c). (10)

- (i) $10 \sin(16000\pi t)$ (ii) $10 \sin(24000\pi t)$ (iii) $10 \sin(10000\pi t)$



Give your reason.
