HEAT TRANSFER AT CASTING/MOLD INTERFACE DURING SOLIDIFICATION OF CASTINGS IN METALLIC MOLDS

A THESIS
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DOCTOR OF PHILOSOPHY

by

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I, MYTHELY KRISHNAN, hereby declare that the work reported in this thesis is entirely original and was carried out by me under the guidance of Dr. GANPATH RAM SHARMA, Assistant Professor, Department of Metallurgical Engineering, Indian Institute of Technology, Madras-36, India. I further declare that this has not formed the basis for the award of any degree, diploma, fellowship, associateship or a similar title of any other university or institution.

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Dedicated to Mahima and Vyshakh
CONTENTS

PAGE NO.

viii

xii

xiv

xv

1

4

6

7

9

10

12

13

15

17

18

22

26

31

1

4

6

6

7

9

10

12

13

15

17

18

22

26

31

CHAPTER 1

INTRODUCTION

CHAPTER 2

LITERATURE SURVEY

2.1

Introduction

2.2

Studies on air gap formation

2.2.1

Early theories

2.2.2

Determination of time for air gap initiation

2.2.3

Measurement of air gap

2.2.4

Factors affecting air gap formation

2.3

Freezing time of castings in metallic molds

2.3.1

Factors affecting freezing time

2.3.2

Relation between and casting parameters

2.4

Prediction of by mathematical methods

2.4.1

The interfacial heat transfer coefficient

2.4.2

Analytical methods

2.4.3

Numerical methods

2.5

Determination of h
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5.1</td>
<td>Experimental studies</td>
</tr>
<tr>
<td>2.5.2</td>
<td>The inverse heat conduction problem (IHCP)</td>
</tr>
<tr>
<td>2.6</td>
<td>Factors affecting the mean value of $h$</td>
</tr>
<tr>
<td>2.7</td>
<td>Causes for variation of $h$ with time</td>
</tr>
<tr>
<td>2.8</td>
<td>Models for $h$</td>
</tr>
<tr>
<td>2.8.1</td>
<td>Models for $h$ between solids in contact</td>
</tr>
<tr>
<td>2.8.2</td>
<td>Models for $h$ at casting/mold interface</td>
</tr>
<tr>
<td>2.8.3</td>
<td>Empirical models</td>
</tr>
<tr>
<td>2.9</td>
<td>Summary</td>
</tr>
</tbody>
</table>

**CHAPTER 3**

**SCOPE OF THE PRESENT WORK**

**CHAPTER 4**

**A MODEL FOR HEAT FLOW ACROSS CASTING/MOLD INTERFACE**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>4.2</td>
<td>Variation of $h$ with time</td>
</tr>
<tr>
<td>4.3</td>
<td>Analysis of geometrically uniform grooves</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Analysis of heat flow across the interface</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Assumptions</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Analysis of heat flow in the groove</td>
</tr>
<tr>
<td>4.4</td>
<td>Results</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Effect of $y_f$ on $h$</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Effect of $\alpha$ on $h$</td>
</tr>
<tr>
<td>4.4.3</td>
<td>Effect of $\beta$ on $h$</td>
</tr>
<tr>
<td>4.4.4</td>
<td>Effect of $a$ on $h$</td>
</tr>
<tr>
<td>4.4.5</td>
<td>Effect of $\sigma$ on $h$</td>
</tr>
</tbody>
</table>
Summary

Analysis of combinations of uniform grooves

Grooves with thermal resistances in parallel

Grooves with thermal resistances in series

Random combination of grooves

Verification of the analytical model

Comparison of calculated and experimental liquid metal profiles

Comparison of calculated and experimental values of h

Analysis of random mold profiles

Generation of rough surfaces - fractals

Generation of liquid metal profiles

Determination of h

Verification of random mold profile model

Summary

CHAPTER 5

EXPERIMENTAL DETAILS

Experiments with constant metallostatic head

Materials used

Experimental set-up

Experimental procedure

Experiments with a receding casting interface

Mold details

Effect of insulating coatings on chill surface
5.3.1 Coatings used
5.3.2 Coating procedure
5.3.3 Measurement of coating thickness
5.4 Effect of chill roughness on a
5.4.1 Surface roughness measurement
5.5 Measurement of air gap
5.5.1 Mechanical arrangement
5.5.2 Measurement of displacement
5.5.3 Experimental procedure
5.6 Confirmatory experiments
5.6.1 Draining experiments
5.6.2 Chill temperature studies
5.7 Repeatability studies
5.8 Radiographic examination of test castings

CHAPTER 6 ANALYSIS OF EXPERIMENTAL DATA

6.1 Introduction
6.2 The direct and the inverse problems
6.3 Inverse heat conduction- an ill-posed problem
6.4 Solution of the inverse heat conduction problem
6.4.1 Exact solutions
6.4.2 Statistical estimation
6.5 Solution procedure used in the present work
6.5.1 Statement of the problem
6.5.2 Assumptions
6.5.3 Procedure
Chapter 7

Results and Discussion

7.1 Introduction

7.2 Results

7.2.1 Constant metallostatic head arrangement

7.2.1.1 Uncoated smooth chill

7.2.1.2 Coated smooth chill

7.2.1.3 Uncoated rough chill

7.2.1.4 Aluminium-Lithium alloy

7.2.2 Receding casting interface arrangement

7.2.2.1 Uncoated smooth chill

7.2.2.2 Coated smooth chill

7.2.2.3 Uncoated rough chill

7.2.2.4 Aluminium-Lithium alloy

7.2.3 Air Gap Measurement

7.3 Discussion

7.3.1 General observations from experimental results

7.3.1.1 Variation in chill temperatures $T_1$ and $T_2$
CHAPTER 8

7.3.1.2 Variation in casting temperatures $T_3$ and $T_4$

7.3.1.3 Variation in surface temperatures $T_c$ and $T_m$

7.3.1.4 Variations in $g$

7.3.1.5 Variation of $h$ with time

7.3.2 Duration of stage I, $t_I$

7.3.3 Value of $h$ in stage I

7.3.4 Value of $h$ in stage II

7.3.5 Time for end of stage II, $t_{II}$

7.3.6 $dh/dt$ in stage III

7.3.7 Air gap measurements

7.4 Effect of insulating mold coats

7.5 Effect of chill roughness

7.6 Aluminium-lithium alloy

7.7 Validation of the results

7.7.1 Validation of experimental results using representative values of $h$

7.7.2 Validation of the results from the model for constant metallostatic head arrangement

7.8 Causes for the variation in the values of $h$

7.8.1 Experimental disturbances

7.8.2 Physical causes

7.9 Sensitivity analysis

CHAPTER 8

CONCLUSIONS 222

SUGGESTIONS FOR FUTURE WORK 227

REFERENCES 228
ABSTRACT

In numerical simulation of casting solidification, the thermal behavior of the casting/mold interface can be represented by the interfacial heat transfer coefficient $h$. A survey of the literature shows that a number of investigators tried different methods to determine the value of $h$ experimentally. Experimental determination of $h$ is difficult because it requires the solution of the Inverse Heat Conduction Problem. Many factors affect the value of $h$ but the quantitative and empirical models available for determining the value of $h$ take into account only a few of the variables like metallostatic pressure, presence of insulating mold coats etc. at the interface. A comprehensive model to determine $h$ would be valuable for numerical simulation of casting solidification.

In this work, the variation of $h$ with time at the casting/mold interface in one dimensional heat flow is discussed. The total duration of the variation of $h$ with time at the casting/mold interface is divided into 3 stages- stage I, with a high fluctuating value of $h$, stage II with a steady value and stage III with either an increasing value of $h$ due to increase in contact pressure or decrease in $h$ due to increase in the air gap between the casting and the mold. When the metallostatic head acting at the casting surface is constant, stage III may be absent. In some cases, stage II may not be observed at all (i.e the duration of stage II is negligible).
In this work, a model for predicting the value of the interfacial heat transfer coefficient between the liquid metal and its mold in steady state one dimensional heat flow is developed and is used for determining the mean value of h in stage I and the value of h in stage II. The analysis is carried out for two types of mold profiles: 1. a mold profile consisting of a combination of uniform V grooves, and 2. a mold profile generated by the method of fractals. Analysis of the interface generated by the method of fractals is carried out on a computer using numerical techniques.

The casting surface profile which is formed when the liquid metal comes into contact with the mold surface consisting of parallel, uniform V grooves is modeled from a consideration of the surface tension of the liquid metal $\sigma$, metallocstatic pressure $P$, advancing contact angle $\alpha$ and the groove parameters, namely, the half-mouth width of the groove $a$ and the semi-apex angle $\beta$ of the groove. The presence of oxide and other films on the casting and mold surfaces is included in the model in the form of an additional air gap of width $y_f$ between the casting and the mold. The case of real mold surfaces is modeled by assuming that the mold surface consists of a random combination of uniform grooves. The overall thermal resistance of a random combination of grooves is obtained as a series/parallel combination of the thermal resistances of the constituent grooves.

The analysis of heat flow across a mold surface generated by the method of fractals and its corresponding liquid metal profile is
The calculations however, are carried out by numerical methods.

To validate the results from the model, the unidirectional solidification of three aluminium alloys - LM 6, LM 24 and Al-2.7% Li alloy against a smooth uncoated cast iron chill is studied. These experiments were carried out for two cases (i) with a constant metallostatic head and (ii) with a receding casting interface. The size of the air gap formed between the casting and mold in the case of an experiment with a receding casting interface is also measured. Using the time-temperature data from casting and chill, the values of $h$ are calculated by non-linear estimation procedure and are compared with the values of $h$ obtained from measurements of the air gap at the interface for a receding casting interface experiment. A satisfactory agreement is found between the two values.

The accuracy of the experimental values of $h$ is verified by calculating the temperature distribution in the casting and chill using representative values of $h$ obtained from the experimental results. The model is validated for the case of the constant metallostatic head by comparing the experimentally measured temperatures with those obtained by calculations using the values of $h$ obtained from the model. A sensitivity analysis is carried out to study the effect of changes in $h$ on the changes in the calculated temperature values for both LM 6 and LM 24 alloys.

(iii)
from the results of the calculations using the model to determine the value of $h$ in steady state unidirectional heat flow, the following conclusions can be drawn.

1. A satisfactory model for obtaining the mean value of $h$ in stage I ($h_I$) and the value of $h$ in stage II ($h_{II}$) at the mold/metal interface is developed for the case of steady state one dimensional heat flow. An analytical method for evaluating $h$ for geometrically regular grooves, grooves in parallel, grooves in series and random combination of grooves is also presented.

2. The model shows that the heat flow across the casting/mold interface depends on the topography of the mold surface and at least two parameters are necessary for representing the surface topography for heat flow predictions. In the model developed in the present study for parallel, uniform V grooves, the surface is characterized by the semi-apex angle $\theta$ and the half mouth width of the groove $a$ for predicting heat flow across the interface.

3. For a mold surface containing parallel, uniform V grooves, the parameters which influence the value of $h$ are, the half mouth width of the groove $a$, semi-apex angle of the groove $\theta$, surface tension of the liquid metal $\sigma$, advancing contact angle $\alpha$, net metallostatic pressure $P$, thermal conductivity $k$ of air at the interface and air gap equivalent $y_I$ of oxide and other films at the interface.

4. $h$ increases with (i) decrease in $a$ (ii) increase in $\theta$ (iii) decrease in $\sigma$ (iv) decrease in $\alpha$ and (v) increase in $k$. 

(iv)
5. The interfacial heat transfer coefficient has a low value at low pressures. Above a critical pressure determined by the groove configuration, \( h \) rises steeply and then with further increase in pressure, reaches a steady value determined by \( \gamma_f \).

6. The mold surface profile can be satisfactorily represented by treating it as (i) a random combination of uniform V grooves in series or in parallel and (ii) a fractal surface.

7. The variation of \( h \) with \( P \) shows a square root relationship i.e., \( h \propto \sqrt{P} \) when the value of \( h \) is calculated using the model for a fractal surface when the range of pressures employed is large i.e. from \( 1 \times 10^5 \) to \( 100 \times 10^5 \) Pa.

From the experimental results and the model of the present study, the following conclusions can be drawn.

8. The general variation of \( h \) with time in unidirectional heat flow shows three distinct stages- I, II and III. Stage I shows a high fluctuating value, stage II shows a steady value and stage III shows a decrease in \( h \) with time. Specific cases may show absence of stage III as in the case of constant metallostatic head experiments or absence of stage II as in some experiments with receding casting interface.

9. The experimental method developed in the present work is satisfactory for the determination of \( h \) in the case of constant metallostatic head and receding casting interface arrangement for unidirectional heat flow.
10. Beck's non-linear estimation procedure is a satisfactory solution for determining the value of $h$ from the temperature measurements obtained in the present experiments. For the experimental conditions used here, a time interval of 0.5 seconds for temperature measurements, 1 second for the determination of heat flow rate q and the use of one set of future temperatures is found to be satisfactory.

11. The variation of $h$ with change in metallostatic head is satisfactorily explained by the model for stages I and II. The experimental results validate the model developed in the present study.

12. The model developed in the present work gives a better fit between the value of $h$ and the metallostatic head, compared to the linear regression values. However, considering the sensitivity studies, the linear regression results are satisfactory.

13. The variation of $h$ with time is affected by casting height, sprue height and ingate size in the case of experiments with a receding casting interface. Increase in casting height $L$ and sprue height $S$ affects the value of $h_1$ and $dh/dt$ whereas increase in the size of the ingate affects the time $t_{II}$ for the onset of stage III.

14. In experiments with receding casting interface, the slope of $h$ with time $(dh/dt)$ is found to remain a constant i.e. $h$ decreases linearly with time.
15. Insulating mold coats decrease the value of the interfacial heat transfer coefficient $h$.

16. Increase in chill surface roughness decreases the value of $h$.

17. The type of alloy affects the value of $h$ considerably. Al-Li alloy has the highest value of $h$, followed by LM 24 and LM 6 alloys.

18. The value of $h$ obtained by air gap measurements shows a satisfactory agreement with that obtained from temperature measurements.

19. From the studies on the validation of the model, it is seen that the temperature values obtained by calculations using the values of $h$ from the various studies agree quite well on the chill side and reasonably well on the casting side with the temperatures obtained experimentally.

20. The calculated temperature values in the casting and chill are found to be sensitive to $h_{II}$ and $dh/dt$ in the case of both the alloys. Variation in $h_I$, $t_I$ and $t_{II}$ (by a factor of two) does not affect the temperature distribution significantly. The freezing time is affected by the values of $h$ in the initial stages.
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**NOMENCLATURE**

A - Area across which heat flow occurs
a - Half mouth width of the groove
c_l - Specific heat of liquid
c_m - Wavelength of surface roughness for the rougher surface
c_m - Specific heat of the mold
c_s - Specific heat of the casting
d - Depth of the groove
e_1 - Thermal diffusivity on mold surface
e_2 - Thermal diffusivity on metal surface
h - Interfacial heat transfer coefficient
h_I - Mean value of h in stage I
h_II - Mean value of h in stage II
h_III - Mean value of h in stage III
h_C - Heat transfer coefficient due to gas conduction
h_s - Heat transfer coefficient due to solid conduction
h_r - Heat transfer coefficient due to radiation
h_max - Maximum attainable value of h
K - Chvorinov's constant
k - Thermal conductivity of the gas film in the interface
k_m - Thermal conductivity of the mold
k_M - Harmonic mean of thermal conductivity of the contacting surfaces
k_s - Thermal conductivity of the casting
L - Latent heat
M_C - Modulus coefficient
P - Interfacial contact pressure

(xi)
**$P_c$** - Critical pressure

**$P_H$** - Metallostatic pressure

**$P_g$** - Gas or air pressure

**$P_V$** - Vapor pressure of the metal

**q** - Rate of heat flow across the interface

**R** - Radius of the liquid metal

**RA** - Surface roughness

**s** - Thickness of metal solidified

**SA** - Surface area of casting

**t** - Time

**$t_f$** - Duration of stage I

**$t_{II}$** - Duration of stage II

**$\Delta t$** - Time interval

**$t_f$** - Freezing time of the casting

**$T_a$** - Ambient temperature

**$T_c$** - Temperature of the casting surface

**$T_f$, $T_L$** - Freezing temperature

**$T_m$** - Temperature of the mold surface

**$T_M$** - Mean interfacial temperature

**$T_p$** - Pouring temperature

**$T_S$** - Solidus temperature

**$\Delta T$** - Interfacial temperature drop

**$V_c$** - Volume of casting

**$V_m$** - Volume of mold

**$VR$** - Volume ratio ($= V_m / V_c$)

**$\chi_x$** - Mold wall thickness

**$\Delta X$** - Element thickness

(xii)
\( \gamma \) - Thickness of air gap at the interface

\( \gamma_{eq} \) - Equivalent gap width at the interface

\( \gamma_f \) - Air gap width representing oxide and other films

\( \gamma_v \) - Varying width of air gap

\( \tilde{\gamma} \) - Equivalent gap thickness in the gap region

\( \alpha \) - Advancing contact angle

\( \alpha_1 \) - Numerical constant

\( \beta \) - Semi-apex angle of the groove

\( \phi \) - Sensitivity coefficient

\( f \) - Density

\( f_m \) - Density of the mold

\( f_s \) - Density of the casting

\( \sigma \) - Surface tension

\( \sigma_1 \) - Stefan-Boltzmann constant

\( \theta \) - Contact angle

(Xiii)
LIST OF TABLES

4.1 Data used for the analytical estimation of h
4.2 Groove parameters for analysis of random combination of grooves
4.3 Comparison of R_calculated and R_measured values
4.4 Data used for the estimation of h with a fractal surface

5.1 Composition of alloys used in the experiments
5.2 Specifications of the data acquisition system
5.3 Melt treatment temperatures
5.4 Constant metallostatic head experiments
5.5 Receding casting interface experiments
5.6 Draining experiments

6.1 A portion of time temperature data from a typical experiment
6.2 Thermal property values used in the calculation of h

7.1 Parameters used for the calculation of h_I using the model
7.2 Parameters used for the calculation of h_{II} using the model
7.3 Results of sensitivity analysis
LIST OF FIGURES

4.1 Heat flow across casting/mold interface
4.2 Variation of h with time (schematic)
4.3 Mold surface with uniform V grooves
4.4 Effect of pressure on liquid metal profile
4.5 Heat flow across casting/mold interface
4.6 Analysis of heat flow in the groove
4.7 Flow chart for the determination of h - analytical model
4.8 Effect of yg, \( \sigma, \beta, a \) and \( \sigma \) on the variation of h with pressure
4.9 Typical machined surface
4.10 Combinations of uniform grooves with thermal resistances in parallel/series
4.11 Variation of h with pressure for grooves with thermal resistances in parallel
4.12 Variation of h with pressure for grooves with thermal resistances in series
4.13 Variation of h with pressure for random combination of grooves
4.14 Comparison of mold and casting surface profiles
4.15 Variation of h with pressure
4.16 Flow chart for the determination of h - random mold profile model
4.17 Generation of fractal surface
4.18 Comparison between fractal surface and real mold surface
4.19 Liquid metal profile for complex mold surface
4.20 Variation of liquid metal profile with pressure
4.21 Variation of h with mold roughness and pressure
4.22 Comparison between simulated and experimental results
5.1 Set-up for the determination of \( h \) (schematic)
5.2 Dimensions of chill
5.3 Chill assembly
5.4 Mold assembly for constant metallostatic head experiments
5.5 Data acquisition system
5.6 Furnace used for melting
5.7 Mold assembly for receding casting interface experiments
5.8 Roughness profiles of chill and corresponding casting
5.9 Chill arrangement for air gap measurement
5.10 Set-up for air gap measurement
5.11 Casting surface after draining
5.12 Typical radiograph showing thermocouples

6.1 Comparison of direct and inverse problems
6.2 Inverse heat conduction problem in unidirectional heat flow
6.3 Approximation of heat flow by discrete values of \( q \)
6.4 Flow chart for the determination of \( h \) using Beck's non-linear estimation procedure
6.5 One dimensional grid for FDM calculations
6.6 Temperature record from a typical experiment
6.7 Comparison of calculated and measured temperatures and estimated values of \( q \)
6.8 Estimated values of \( q, T_c, T_m \) and \( h \)

7.1 Effect of casting height on the variation of \( h \) with time for LM 6 alloy with uncoated smooth chill and constant metallostatic head
7.2 Effect of casting height on the variation of \( h \) with time for LM 24 alloy with uncoated smooth chill and constant metallostatic head
7.3 Effect of coating material on the variation of \( h \) with time for LM 6 alloy with smooth chill and constant metallostatic head

7.4 Effect of coating material on the variation of \( h \) with time for LM 24 alloy with smooth chill and constant metallostatic head

7.5 Effect of chill roughness on the variation of \( h \) with time for LM 6 alloy with uncoated chill and constant metallostatic head

7.6 Variation of \( h \) with time for Al-Li alloy with uncoated smooth chill and constant metallostatic head

7.7 Effect of ingate size on the variation of \( h \) with time for LM 6 alloy with uncoated smooth chill and receding casting interface

7.8 Effect of casting height on the variation of \( h \) with time for LM 6 alloy with uncoated smooth chill and receding casting interface

7.9 Effect of casting height on the variation of \( h \) with time for LM 24 alloy with uncoated smooth chill and receding casting interface

7.10 Effect of sprue height on the variation of \( h \) with time for LM 6 alloy with uncoated smooth chill and receding casting interface

7.11 Effect of sprue height on the variation of \( h \) with time for LM 24 alloy with uncoated smooth chill and receding casting interface

7.12 Effect of coating material on the variation of \( h \) with time for LM 6 alloy with smooth chill and receding casting interface

7.13 Effect of coating material on the variation of \( h \) with time for LM 24 alloy with smooth chill and receding casting interface

7.14 Effect of chill roughness on the variation of \( h \) with time for LM 6 alloy with uncoated chill and receding casting interface

7.15 The variation of \( h \) with time for Al-Li alloy with uncoated smooth chill and receding casting interface

7.16 Comparison of \( h \) values from air gap measurements and solution of IHCP
7.17 Variation of \( h \) with metallostatic head (H) for LM 6 alloy in constant metallostatic experiments

7.18 Variation of \( h \) with metallostatic head (H) for LM 24 alloy in constant metallostatic experiments

7.19 Temperature distribution in the casting and chill using experimentally observed values of \( h \) for LM 6 alloy with a receding casting interface

7.20 Temperature distribution in the casting and chill using experimentally observed values of \( h \) for LM 24 alloy with a receding casting interface

7.21 Temperature distribution in the casting and chill using values of \( h \) obtained from the model for LM 6 alloy with a constant metallostatic head

7.22 Temperature distribution in the casting and chill using values of \( h \) obtained from the model for LM 24 alloy with a constant metallostatic head
The soundness of a casting depends on the thermal conditions during its solidification [1]. Castings made in metal molds have better strength compared to those cast in sand due to the faster rate of cooling in metal molds and they also have better surface finish compared to sand castings. Permanent mold castings form a significant portion of the castings produced in non-ferrous alloys.

The effect of various factors on the soundness of permanent mold castings has been studied by experimental and numerical methods. In the case of castings solidifying in metallic molds, the heat extraction from the casting is controlled by the casting/mold interface [2]. This behavior of the interface can be attributed to imperfect contact between the casting and the mold, and in some cases, the formation of an air gap at the interface. The thermal behavior of the interface is characterized by the 'interfacial heat transfer coefficient', $h$. The value of $h$ affects the freezing time $t_f$ of the casting and also the thermal profile inside the solidifying casting. For obtaining reliable prediction of the temperature field from numerical simulation of casting solidification in metallic molds, knowledge of the heat flow at the casting/mold interface is necessary.

The interfacial heat transfer coefficient $h$ is known to vary with time and location on the interface. $h$ depends on a number of
factors like metallostatic head, mold roughness, mold coatings, casting geometry etc. Determination of \( h \) by experimental methods is difficult since it involves the solution of 'The Inverse Heat conduction Problem' (IHCP) which is an ill-posed mathematical problem. IHCP implies the estimation of the surface heat flux history or surface temperature of a body given one or more measured temperature histories inside the heat conducting body, unlike in the case of a direct problem where the internal temperatures are determined from a knowledge of the boundary conditions.

The value of \( h \) at the interface during solidification of castings in metallic molds varies over a wide range. Considering the number of factors affecting \( h \), it is clear that a model for the determination of \( h \) would be extremely useful for numerical simulation of solidification in metallic molds. A model which can give an initial value of \( h \) would be adequate since information about the subsequent variation in \( h \) can be obtained during casting solidification by numerical simulation. In this work, a quantitative model for calculating the value of the interfacial heat transfer coefficient in steady one dimensional heat flow conditions is developed. The model takes into account the metallostatic pressure, mold surface microgeometry, surface tension of the liquid metal, the presence of oxide films, air gap and mold coats at the interface.

To verify this model, two types of experiments i.e. i) with a constant metallostatic head and ii) with a receding casting
interface are performed to determine the interfacial heat transfer coefficient in one dimensional heat flow for LM 5, LM 24 and AI-2.7% Lithium alloys freezing against a cast iron chill. $h$ is determined using temperatures measured in the casting and chill and solving the inverse heat conduction problem using the nonlinear estimation procedure [3]. The value of $h$ obtained from temperature measurements from a receding casting interface experiment is compared with those obtained from a direct measurement of the air gap at the interface. Representative values of $h$ are obtained from the experiments and the model and these values are used for obtaining the temperature field in the casting and chill. The calculated temperatures thus obtained are compared with the measured values. A sensitivity analysis is also carried out to determine the variation in the calculated temperatures with variation in $h$. 
CHAPTER 2
LITERATURE SURVEY

2.1 INTRODUCTION

The soundness of a casting depends on the thermal conditions during its freezing [1] and these conditions are in turn determined by the rate of heat extraction from the casting by the mold. The freezing time \( t_f \) of a casting is one of the important thermal parameters which is dependent on the rate of heat extraction and quantitative relationships have been developed between casting soundness and freezing time \( t_f \) [4].

Solidification of castings in metallic molds differs from that in sand molds because of two main reasons. 1. The sand mold thickness is large enough to justify the assumption of semi-infinite size of mold [2]. This assumption is not valid for metal molds due to the high thermal diffusivity of the mold material. 2. The heat flow from the casting into the mold is limited by the thermal resistance offered by the casting/mold interface in the case of metal molds [2] whereas in sand molds, thermal resistance of the interface is insignificant compared to the thermal resistance of the insulating sand mold. The thermal resistance at the mold/metal interface in metal molds arises due to imperfect contact between the casting and mold [5] and in some cases, the formation of an air gap between the casting and mold [6]. The formation of the air gap at the casting/mold interface was studied by the early investigators [6-13]. Factors relating to the size and material of the mold and cast material are found to
The influence of air gap. The main effect of the air gap is to increase the freezing time, $t_f$, which in turn affects the casting soundness. Many investigators tried to relate the freezing time of the casting with the factors relating to the mold, casting and the behavior of the interface [7-10, 14-24].

For simulation of casting solidification which is of great importance for Computer Aided Design (CAD) of castings, it is found that the behavior of the interface can be incorporated during simulation by introducing an appropriate value for the interfacial heat transfer coefficient $h$ [25-28]. Many attempts were made to determine $h$ [29-36] and it was realized that the determination of $h$ leads to the Inverse Heat Conduction Problem (IHCP) [35]. It was also found that the value of $h$ depends on the time after pouring and location on the interface.

These studies led to a better understanding of the solidification of castings in metal molds and particularly the behavior of the casting/metal mold interface. A number of factors affecting $h$ have been identified and a few models have also been proposed to explain its behavior.

This chapter begins with a survey of the literature on the formation of an air gap between a casting and its metal mold which influences the freezing time $t_f$ and hence the soundness of the casting. Literature on the prediction of $t_f$ by mathematical methods is then presented. The importance of the interfacial
heat transfer coefficient $h$ in predicting $t_f$, methods of determining $h$ and solutions for the related IHCP are discussed.

Literature relating to the factors affecting $h$ and models for predicting $h$ are included.

2.2 STUDIES ON AIR GAP FORMATION

The early investigators of solidification in metallic molds found that an air gap forms between a casting and its mold in the initial stages of solidification and this gap continues to increase in size with time [6]. Their efforts were directed towards finding the time, location and other factors relevant to this gap formation [7,9,11,12]. The variation in the size of the air gap during the solidification of the casting was monitored by some investigators [10,13]. These aspects are discussed below.

2.2.1 Early theories

Henzel and Keverian [6] reported that the formation of an air gap between a casting and its metallic mold was investigated systematically by Matuschka [37] who concluded that the formation of an air gap between a casting and its mold is a result of the expansion of the mold due to absorption of heat and the contraction of the casting due to solidification and subsequent cooling.

Paschkis [38] explained that as a result of air gap formation, the casting skin may get reheated and this can cause the metal to break through the skin resulting in fresh metal freezing against the mold wall leading to a cyclic process referred to as "breath-
It is reported [6] that this view, however, was not accepted by other investigators [39]. The effect of the ferrostatic head on the deformation of the plastic skin of solidified metal and the resultant delay in gap formation was pointed out by Linacre [39].

Summarizing the results of the earlier investigators, Henzel and Keverton [6] concluded that the time and magnitude of gap formation is determined by casting size, mold wall thickness, mold surface roughness, mold wall temperature, superheat, rate of pouring and coefficient of expansion, density, conductivity and specific heat of mold and casting materials. Air gap formation induces sharp changes in the temporal variation of casting and mold surface temperatures.

1.2.2 Determination of time for air gap initiation

Some of the methods employed by investigators to determine the time of initiation of air gap at the interface are given below.

1. Temperature measurement

Air gap formation is associated with abrupt changes in the temperatures at the mold and casting surfaces [6]. Hence, by monitoring the interface temperatures, the time of formation of air gap can be determined [40,41].

Bishop et al [40] studied the solidification of steel in sand and metallic molds. Thermocouples of the sheathed type having a small mass were used for measuring the surface temperatures.
they showed that the sharp change observed in the surface temperature of the permanent mold can be associated with the formation of a gap between the mold and the casting.

Atterton and Houseman [42] used bare wire thermocouples with low thermal lag for accurate measurement of the casting interface temperature in sand castings. They showed that by using bare wire intermediate metal thermocouples, the surface temperatures measured were above the temperature of solidification, thereby indicating the presence of liquid metal at the interface. This aspect could not be detected by sheathed thermocouples. This shows that detection of the time for air gap formation by temperature measurement requires a proper technique for measurement. It was also concluded by some investigators [9,10] that measurement of the mold surface temperature is not a reliable method for determining the time for air gap formation especially in the case of thin molds with insulating mold coats since sharp temperature changes are not always noticed in these cases.

2. Heat flow method

The heat flow across the casting/mold interface can be determined from temperature measurements in the casting and the mold. The results of Mackenzie and Donald [43] show that the heat transfer rate across the casting/mold interface increases from an initially low value to a maximum and then decreases subsequently. The time of occurrence of the peak value in the heat transfer rate is associated with the onset of air gap formation. Henzel and Keverian [6] calculated the rate of heat transfer using the data.
and arrived at similar conclusions. In some cases, instead of a well-defined peak, wide fluctuations in the heat transfer rate are noticed [45].

3. Miscellaneous methods

Some of the early studies on the formation of an air gap at the interface were carried out by inserting electric probes which broke an electrical contact causing lights to be switched off, thus indicating the formation of an air gap [6]. Desars [11] made a casting of steel in a big end down ingot mold fitted with a loose plug at the bottom. The plug and the mold were supported on weight monitoring systems and the extent of gap formation could be determined from the percentage of ingot weight supported by the plug. Time for complete separation of the ingot from the mold could be determined by noting the time at which the entire weight of the ingot was supported by the plug.

2.2.3 Measurement of air gap

Direct measurement of air gap between the casting and mold was performed by some investigators [10,13]. The growth, distribution and size of air gap was obtained experimentally using inductive gauges which measured the movement of the casting surface relative to the mold surface [10].

The size of the air gap was monitored continuously by Winter et al. [13] using two inductive gauges, one connected to the mold surface and the other to the casting surface. The values of the
gap width were found to be in the range of 0.05-0.30 mm. Isaac et al [10] used a single inductive gauge for measuring the thickness of air gap at a given location in the case of aluminium solidifying in a cast iron mold. They monitored the variation in gap dimensions at the center of the face and corner of a bar casting of square cross section. In their experiments, the measured values of the air gap was in the range of 0.027 to 0.247 mm.

Hou and Pehlke [46] measured the interfacial gap during solidification of an Al-13% Si alloy casting solidifying in a sand mold. The gap formed during solidification was found to vary from near zero to 0.066 mm during solidification. Based on the above values, heat transfer coefficients at the interface were determined.

2.2.4 Factors affecting air gap formation

A number of factors have been found to influence gap formation namely: casting size, volume ratio, mold preheat, mold coating, pressure etc., and investigations pertaining to these factors are given below.

Panchanathan et al [41] and Srinivasan et al [7] found that in a metallic mold of a given material and thickness, the time for air gap formation increases with increase in casting size. The time for air gap formation is found to decrease with increase in volume ratio VR (i.e., the ratio of mold volume to casting volume) [7,9]. Beyond a ratio of 5.0, there is no appreciable
Differences in the location of gap initiation were also observed between top pouring and bottom pouring [6]. The formation of an air gap between the casting and the die in the case of continuous casting was studied by Fredriksson and Thegerstrom [50] and Grill et al [51]. These investigators found that the size of the air gap varies widely in continuous casting. The presence of an air gap is known to affect the freezing time of the casting and this aspect is discussed below.
2.1. Freezing time $t_f$ of castings in metallic molds

The solidification of a casting begins near the mold wall and proceeds towards the center. The time at which the last solidifying point in the casting crosses the freezing or solidus temperature is taken as the freezing time, $t_f$. The presence of an air gap between the solidifying casting and its metallic mold reduces the heat extraction rate owing to the insulating nature of the gases present in this gap. This, in turn, affects the freezing time of the casting. Literature relating to the time of freezing of castings solidifying in metallic molds is presented below.

2.3.1 Factors affecting freezing time

As a result of experimental investigations, it was found that the freezing time of a casting depends on the thermal properties of the mold and casting, interface conditions and application of pressure on the solidifying casting [7,8,10,14-21].

1. Mold material

Mold materials of higher thermal conductivity extract heat faster and this results in a lower time of freezing. It was found that $t_f$ is lesser in copper molds compared to cast iron and anodized aluminium molds [7].

2. Volume ratio and modulus coefficient

Freezing time $t_f$ is influenced by volume ratio (VR) if VR is below 5.0 [7]. Similar results were observed by Nehru [9] for
numeral castings. According to Mohan [12], volume ratio is not a suitable parameter for predicting $t_f$ in the case of cast iron castings produced in permanent molds. It was observed [9, 24] that $t_f$ is proportional to the modulus coefficient given by

$$(V_C/SA)^{1.5}(VR)^{-0.5}$$

where $SA =$ Surface area of casting

$V_C =$ Volume of casting

$VR =$ Volume ratio

3. Mold preheat

Ayyamperumal [23] found that $t_f$ increases with increase in mold preheat temperature in the case of aluminium alloy castings. $t_f$ increases marginally with increase in mold preheat in the case of cast iron solidifying in a cast iron mold [12].

4. Mold surface roughness

Prates and Biloni [52] and Morales et al [27] found that with an increase in the roughness of the mold surface, there is an increase in the freezing time. The faster rate of heat extraction by a smooth mold surface was demonstrated by a shift in the center line of the casting, poured in a mold with opposite walls of different roughness [27].

5. Mold coatings

The effect of mold coatings on $t_f$ was studied by many investigators [9, 12, 53] and it was found that insulating mold coats Ge-
increase the mold surface temperature and increase the freezing time. The effect of different types of coatings on freezing time was also investigated. Nehru [9] found that silica mold coats increase $t_f$ but there is an optimum coating thickness for which the freezing time is maximum.

4. Type of alloy

Srinivasan et al [7] found that in aluminium alloys, the value of $t_f$ was higher for aluminium - 12% silicon alloy compared to pure aluminium and long freezing range alloys like aluminium - 4.5% copper and aluminium - 10% magnesium for similar casting conditions. Nehru [9] found that the freezing time of gunmetal plate castings is more than that of pure copper. According to Patterson et al [54], the freezing time of copper-tin alloys increases with an increase in the solidification range of the alloy.

7. Pouring temperature

The freezing time of a casting increases with an increase in the pouring temperature [31,55]. This can be attributed to the increase in the average mold temperature during heat extraction with increased pouring temperature, resulting in a lower heat extraction rate.

8. Casting geometry

The freezing time of a casting is influenced by its shape. Monan [12] found that in the case of cast iron solidifying in a metal mold, for a given casting section thickness, coating material,
present temperature and mold wall thickness, the following relationship is observed:

\[ t_f(\text{cylinder}) < t_f(\text{square}) < t_f(\text{plate}) \]

Srinivasan et al. [56] found that in a given metallic mold, for a particular volume/surface area ratio for zinc and zinc-5% aluminium alloy the following relationship is observed:

\[ t_f(\text{square}) < t_f(\text{rectangle}) < t_f(\text{cylinder}) < t_f(\text{plate}) \]

9. Pressure

The freezing time of a casting decreases with increase in the pressure applied on the molten metal. This can be attributed to the increase in heat extraction caused by better contact between the casting and the mold [48, 49]. Davies [47] reported that freezing time decreases with increase in pressure up to 0.2 bar but increase in pressure above 0.2 bar does not cause further reduction in freezing time.

2.3.2 Relation between \( t_f \) and casting parameters

A number of relationships between \( t_f \) and some of the casting parameters have been put forward by various investigators [7, 9, 12, 57] and these are discussed below.

1. Chvorinov's rule [57]:

\[ t_f = K (V_c/SA)^2 \]

where \( t_f \) = freezing time

\[ K = \text{Chvorinov's constant} \]

\[ V_c = \text{volume of casting} \]

\[ SA = \text{surface area of casting} \]
This equation is valid for castings solidifying in insulating sand molds. Chvorinov's equation was found to be valid for a given sand mold/alloy combination irrespective of shape and size of the casting. For metallic molds, however, this relation was of limited validity [7,9] and the value of K changes with shape, volume ratio and mold coating.

2. An empirical relationship was obtained by Srinivasan et al [7] between two dimensionless parameters as follows:

\[ \frac{t_f}{(K \cdot SA \cdot VR)} = \frac{(V_c/SA)^3}{V_m} \quad \ldots \quad (2.2) \]

where \( t_f \) = freezing time of casting
\( V_c \) = volume of casting
\( V_m \) = volume of mold
\( SA \) = surface area of casting
\( VR \) = volume ratio \((V_m/V_c)\)
K = Chvorinov's constant

J. Nehru [9], Chinnathambi [24] and Ayyamperumal [23] found that a general relation connecting the casting modulus, volume ratio and freezing time is:

\[ t_f = K \cdot M_c = K \cdot (V_c/SA)^{1.5} \cdot (VR)^{-0.5} \quad \ldots \quad (2.3) \]

Where \( M_c \) = Modulus Coefficient = \((V_c/SA)^{1.5} \cdot (VR)^{-0.5}\)
K = Chvorinov's constant

4. The solidification of cast iron in metallic molds was investigated by Mohan [12] and in the case of uncoated molds, he found the following relationship:
There is an increase in $t_f$ with a decrease in freezing time, second::;

- pouring temperature, °C
- initial mold temperature, °C
- ambient temperature, °C
- $s_1 = $ solidification constant I, s/cm²
- $s_2 = $ solidification constant II, s.

J. Sehru [9] developed a nomogram to obtain the freezing time for plate shaped castings of copper and gunmetal whose casting volume, mold wall thickness, mold coat material and mold coat thickness are known. There is an increase in $t_f$ with a decrease in mold wall thickness [55]. This result is ascribed to the reduction in the thermal capacity of the mold walls with reduction in wall thickness.

2.4. PREDICTION OF $t_f$ BY MATHEMATICAL METHODS

The freezing time of a casting solidifying in a metallic mold can also be determined by mathematical methods. For determining the freezing time of a casting $t_f$, the heat conduction equation:

$$\frac{\partial^2 }{\partial x^2}(k_x \frac{\partial T}{\partial x}) + \frac{\partial^2 }{\partial y^2}(k_y \frac{\partial T}{\partial y}) + \frac{\partial^2 }{\partial z^2}(k_z \frac{\partial T}{\partial z}) = \rho c \frac{\partial T}{\partial t} \ldots (2.5)$$

should be solved with appropriate boundary conditions [5, 32, 58-62]. In the above equation, $X$, $Y$ and $Z$ represent the Cartesian coordinates of the point under consideration, $k_x$, $k_y$ and $k_z$.
represent the thermal conductivity in the X, Y and Z directions, \( \tau \) represents the temperature, \( t \) the time, \( \rho \) the density and \( c \) the specific heat of the material. The solution should take into account the liberation of latent heat during freezing. It should also incorporate the thermal behavior of the casting/mold interface as a boundary condition.

If perfect contact is assumed between a casting and its metallic mold, then the calculated time of freezing would be much lower than the value obtained experimentally. The difference between the calculated and experimental freezing times can be attributed to imperfect contact between the casting and the mold [58]. The calculation procedure can be modified to take the imperfect contact into account by using a finite value of the interfacial heat transfer coefficient \( h \) as a boundary condition of the casting surface [5].

Two broad methods of solution are available - analytical methods and numerical methods. These two methods for solving the heat conduction equation are discussed in the following paragraphs after presenting a brief account of the interfacial heat transfer coefficient, \( h \).

2.4.1. The interfacial heat transfer coefficient

The heat flow across the casting/mold interface shown in figure 2.1 can be obtained using the expression [2]:

\[
Q = h A (T_c - T_m)
\]  

... (2.6)
Heat flow across casting/mold interface

\[ q = hA(T_c - T_m) \]

\[ h = \frac{k}{\gamma} \]

Fig 2.1 Heat flow across casting/mold interface
where $q$ = rate of heat flow across the interface

$A$ = area across which heat flow occurs

$h$ = interfacial heat transfer coefficient

$T_C$ = temperature of the casting surface

$T_m$ = temperature of the mold surface

$h$ can be determined if the values of all the other terms in equation 2.6 are known. Sometimes, the inverse of $h$, i.e., $1/h$, referred to as the 'thermal resistance' of the interface is used to indicate the behavior of the interface [63].

The assumption that heat flow across the interface is by conduction through the gas layer of thickness $y$ in the interface, gives the relation [64]:

$$ q = \frac{k}{y} A (T_C - T_m) \quad \ldots (2.7) $$

where $k$ = thermal conductivity of the gas film in the interface

$y$ = thickness of the gas film

Comparison of equations 2.6 and 2.7 yields the relation:

$$ h = \frac{k}{y} \quad \ldots (2.8) $$

In the initial stages of solidification, the cast metal would be in contact with the rough mold surface and heat transfer across the interface occurs by solid conduction, gas conduction and radiation and the overall value of $h$ can be given by [35]:

$$ h = h_s + h_c + h_r \quad \ldots (2.9) $$
where \( h_g \) = heat transfer coefficient due to solid conduction
\( h_c \) = heat transfer coefficient due to gas conduction
\( h_r \) = heat transfer coefficient due to radiation.

\( h_g \) can be calculated from the following formula proposed by Rapier et al [65]:

\[
h_g \propto \frac{k_M}{c_M} \left( \frac{P}{H} \right)^{1/2}
\]

... (2.10)

where \( P \) = interfacial contact pressure

\( H \) = hardness of the softer solid in the contacting interface

\( k_M \) = harmonic mean of thermal conductivity of the contacting surfaces

\( c_M \) = wavelength of surface roughness for the rougher surface

\( h_c \) is given by:

\[
h_c = \frac{k}{\gamma}
\]

... (2.11)

If the gap is sufficiently small and the mold surface is smooth, Ho. and Penkove [35] suggest that \( h_c \) should take into account the effects of the 'temperature jump distance'.

\( h_r \) is given by:

\[
h_r = \left( \frac{c_1 T_M (4 T_M^2 + \Delta T^2)}{(1/e_1) + (1/e_2) - 1} \right)
\]

... (2.12)

where

\( T_M \) = mean interfacial temperature

\( \Delta T \) = interfacial temperature drop

\( c_1 \) = Stefan-Boltzmann constant

\( e_1 \) \& \( e_2 \) = thermal diffusivity on mold and metal surfaces respectively
When an air gap is formed, there is no solid conduction and in this case the value of $h$ can be obtained from the expression:

$$h = h_C + h_T$$  \hspace{1cm} (2.12)

In the case of aluminium and other alloys which solidify at low temperatures, heat transfer by radiation is very low compared to that by conduction through the gas film and hence may be neglected [9]. The effects of heat storage in the gap can also be neglected [35] and in this case, $h$ is given by equation 2.8.

The values of $T_o$ and $T_m$ cannot be measured directly because the introduction of thermocouples of finite mass at the interface can cause distortion in the temperature field. Further, the region close to the interface may not have a heat flow pattern predicted by theory due to the non-uniform surface conditions. Hence determination of $h$ using equation 2.6 by measurement of $T_o$, $T_m$ and $q$ is difficult [3] and indirect methods should be employed. As mentioned earlier, two methods are available for the determination of $h$ - analytical methods and numerical methods.

2.4.2. Analytical methods

To study the solidification of castings in metallic molds, the heat conduction equation 2.5 was solved by analytical methods for domains with simple geometric shapes and boundary conditions. The plane front solidification of liquid metal at its melting temperature in contact with a mold at the ambient temperature at
time \( t = 0 \) was considered by Schwarz [66]. The thickness of metal solidified \( S \) is given by the expression:

\[
S = 2s/t
\]

... (2.14)

where

\( s \) is given by the expression:

\[
s = \frac{\pi}{4} \left( k_s \rho_s c_s / k_m \rho_m c_m + \text{erf} \alpha l \right) - (T_f - T_a)/L
\]

... (2.15)

where

- \( T_f \) = melting temperature
- \( T_a \) = ambient temperature
- \( c \) = specific heat
- \( L \) = latent heat
- \( \alpha l \) = numerical constant
- \( k_s, \rho_s \) and \( c_s \) = thermal conductivity, density and specific heat of the casting
- \( k_m, \rho_m \) and \( c_m \) = thermal conductivity, density and specific heat of mold.

It is reported [67] that a similar expression was developed by Lyubov but this solution does not admit any superheat in the liquid metal.

Chvorinov’s rule [57] given in equation 2.1 was derived analytically for castings solidifying in insulating sand molds. As mentioned earlier, this relation is of limited validity in the case of permanent mold castings [7,12] and the value of Chvorinov’s constant \( K \) changed with shape, volume ratio and mold coating.
A modified form of the Schwarz solution was used to predict the thickness of metal solidified, $S$, at any given time [67]:

$$S = A_1 \sqrt{t} - B_1$$ ... (2.16)

where $A_1$ and $B_1$ are constants which have to be determined for a given system. Prates et al [67] found that the constant $A_1$ depends on mold properties and $h$, while $B_1$ is proportional to the square root of the superheat.

The solution of Schwarz indicated that the interface attained a constant temperature $T_i$. The case of imperfect contact was solved by assuming an interface at temperature $T_i$ with two heat conductances $h_1$ and $h_2$ on either side of this imaginary interface [58,60,61]. The overall value of $h$ is obtained by the relation:

$$\frac{1}{h} = \frac{1}{h_1} + \frac{1}{h_2}$$ ... (2.17)

Approximate solutions for the case of imperfect contact are developed by some investigators [58,60,61,68] and some of these methods are reviewed by Jones [5]. These solutions, however, are limited to the case of plane front freezing with simple boundary conditions. Adams [58] employed an iterative method to build a power series solution for simple casting shapes solidifying in metallic molds with imperfect contact. For the solution of this problem, Hills [69] developed the Integral Profile Method. Robertson and Fascetta [68] used the method of successive Approximations for obtaining the solution.
Garcia and Prates [60] developed the Virtual Adjunct Method assuming a constant value of $h$. The interface conductance was replaced by a solidified metal of appropriate thickness $S$. For thin molds this gives the solution:

$$t = A_2 S^2 + B_2 S$$  \hspace{1cm} (2.18)$$

where $A_2$, $B_2$ = constants to be determined

$t$ = time

$S$ = thickness solidified

This model could be used to predict the solidification rate and the temperature distribution during the unidirectional solidification of metals in molds cooled by fluids such as air or water. Assumption of a fixed value of $S$ in the above solution was found to be incorrect [70].

The value of $h$ in the case of thick molds was solved by the Virtual Adjunct Method by Garcia et al [61]. The form of the solution in this case is similar to equation 2.18. Once the value of $h$ is determined for a given situation, the value of $t_f$ can be determined for similar situations.

A review of the analytical methods reveals that exact or approximate solutions are available only for castings with simple shapes with severe limitations on the boundary conditions. They are of limited applicability for studying complex castings encountered in practice. Further, the solution assumes that the value of $h$ remains constant and variation of $h$ with time cannot be easily
incorporated in the solution. For solving the complex cases, numerical methods find greater application.

1.4.3 Numerical methods

The heat conduction equation 2.5 can be solved by numerical methods and from this, the freezing time of a case can be determined. For obtaining the temperature distribution in the casting and predicting the value of \( t_f \), the value of the latent heat, interracial heat transfer coefficient and variation of mold and casting properties with temperature [60]. The calculated value of \( t_f \) is found to be sensitive to the value of \( h \). If the value of \( h \) is known, then the value of \( t_f \) can be determined by the Finite Differences Method (FDM) or the Finite Element Method (FEM). These numerical methods are used to calculate the temperature at predetermined points referred to as nodes in the domain. A brief account of FDM and FEM and the method of incorporating \( h \) in these calculations is given below:

1. Finite Differences Method (FDM) [71]

In this method, the partial derivatives in the heat conduction equation 2.5 are replaced by their corresponding finite differences. This results in a set of algebraic equations. If the partial derivatives are evaluated at the beginning of the interval \( \Delta t \), it is referred to as the explicit finite differences method and the solution is quite straightforward. This, however, requires that the time steps \( \Delta t \) be below a certain value to avoid
effort to the end of the interval $\Delta t$ leads to the implicit method which does not impose any restrictions on the computational effort required. However, the temperatures are obtained by solving a set of simultaneous equations, which requires more computational effort than the explicit method. Other variants of FDM are the Crank-Nicholson method and the Alternate-Direction-Implicit method.

For incorporating $h$ in the solution, a method was developed by Jayarajan and Pahlke [72] where two nodes, one on the casting surface and the other on the mold surface are used with a slight separation to represent the interface. The heat flow between the two surfaces is represented by equation 2.6.

2. Finite Element Method (FEM) [73]

In the finite element method, the domain of interest is divided into elements and the solution is sought at the corners of the elements. Sometimes, the solution is sought at specific points on the boundaries and interior of the elements in addition to the corners. An approximate solution in the element which minimizes the error is sought by the method of weighted residuals like the Galerkin's method. This leads to a set of simultaneous equations which are then solved by using matrix methods for obtaining the temperature field.

Two methods are available for incorporating the interfacial heat transfer coefficient in FEM calculations: 1. Coincident node method and 2. Thin element method.
1. Coincident node method

In this method, a principle similar to that explained in the FDM procedure is used. Two nodes, one each on the casting and mold surfaces are given the same co-ordinates. To include the heat flow between these nodes, the conductivity matrix of the element is modified taking into account the value of $h$ at that location [74].

2. Thin element method

More, thin elements having very small thickness $\Delta x$ are associated with the interface. The longer side of the element may be about 100 $\Delta x$. Temperature distribution along the thin side is carried out by linear interpolation. The value of $h$ is introduced by using an appropriate value of $k$ ($k = h \Delta x$) where $k$ is the equivalent thermal conductivity of the material of the interfacial element. Variable $h$ can be obtained by varying the value of $k$. The heat capacity of the material in the interface region does not significantly affect the calculation if the length to width ratio of the element is large (i.e., about 100 or more) [74].

Methods for incorporating the liberation of latent heat for the simulation of casting solidification are available in literature [39, 75, 76]. Procedures for incorporating variable thermal properties of casting and mold materials have also been developed [33, 59]. Solidification simulation of castings in metallic molds incorporating the interfacial heat transfer coefficient $h$ in...
Numerical calculations were carried out by many investigators [10, 20, 30, 41, 51, 62, 76-79] and these investigations are presented below.

Sargent and Slack [76] calculated the thermal history of steel ingots from casting to rolling by numerical integration using the finite differences method (FDM) for the solution of the heat flow equation on a two-dimensional mesh representing the ingot midheight section.

Durham and Berry [30] developed a numerical model for predicting the solidification of pure lead and Al-4.5% Cu using FDM. Latent heat of fusion and variable specific heat were included in the calculations. Long and short freezing range alloys could be simulated. It was also possible to include various values of air gap conductance and superheat. From their studies, the rate of solidification was determined. The interaction between superheat and interfacial heat transfer coefficient was studied by Durham et al [31].

Srinivasan [77] studied the effect of variation in $h$ on the freezing time of plate and cylindrical cast iron castings solidifying in cast iron molds. He used the explicit FDM to solve the heat flow equations. By using appropriate values for $h$, good agreement between experimental and calculated values of $t_f$ were obtained [20]. The effect of $t_f$ on tensile strength and microstructure was also investigated [20].
rectangular elements to predict the heat flow and thermal stresses. Morgan et al. (62) used the finite element method with a nodal
breakout could be predicted from their work.
also included in the calculations. The conditions which led to
flows present at the interface and the effect of radiation was
casting/mold interface was determined from the physical condit-
and pent for determining the stresses. The heat flow across the
continuous casting using PPM for solving the thermal equations
performed to actual castings are reported.
examples solved by the explicit finite difference method. Two examples
and two-dimensional unsteady state heat transfer problem which was
of)
y and low pressure Al-alloy die castings by treating it as a
changes at al. (70) studied the solidification process for Al-
where was used to represent the interfacial conditions.
transfer on freezing time was studied by them. A fixed value of
problem and solved by the explicit PPM. The effect of read metal
problem was treated as a two dimensional heat flow
and Al-3% Cu-4.5% Si alloy plate castings pointed in cast iron
sizing and shrinkage (78) studied the solidification of Al-11.8%.
and the calculated values of the
and gap formation were used in the calculation and discussed agreement
values of the one before the gap formation and the other after it
for the gap formation in their simulation studies. Two
infinite and bounded castings (71) used the experimental values of the

The importance of incorporating the interfacial heat transfer coefficient in the numerical calculations for accurate prediction of the freezing conditions of a casting in a metallic mold is presented above. This leads to the question of determination of the value of \( h \) and many investigators \([29,33,35,36,52,63]\) attempted the determination of \( h \) for a variety of casting configurations. The experimental details used by these investigators are given below.

2.5. DETERMINATION OF \( h \)

It is seen from the earlier section that in the simulation of casting solidification, close agreement between the experimental and calculated values of \( t_f \) using analytical and numerical methods could be achieved by a proper selection of the value for \( h \). The value of \( h \) thus obtained is only an average value and does not give the variation of \( h \) with time and location on the interface. Comparison of actual and predicted temperature profiles in a solidifying casting, however, indicates that \( h \) is not a constant but varies with time and location on the interface. In this section, the attempts made by various investigators to determine \( h \) from experimentally measured temperatures are described. The methods reported for calculating the value of \( h \) from the measured temperatures are presented. From a knowledge of the internal temperatures in the casting, \( h \) could be calculat-
ed by treating the heat transfer problem as an Inverse Heat Conduction Problem (IHCP). Procedures for solving the IHCP are also discussed in this section.

2.5.1 Experimental studies

Sully [39], in his experimental work, used three different shapes of castings to obtain the value of $h$ between the casting and the mold by measuring the temperatures from appropriate locations in the casting and mold. The first case of a casting solidifying around a water cooled copper or steel pipe was analyzed by a simple analytical procedure. In the second case of steel solidifying against a horizontal chill, the implicit finite difference method was used to find the value of $h$. The variation of $h$ as a function of time could be obtained over the entire duration of solidification. In the third case, a permanent mold plate casting solidification problem was studied by FDUM. Using the known values of temperatures, the transient heat transfer coefficient could be obtained.

Sun [36] determined the interfacial heat transfer coefficient between cylindrical slugs and the molten metal solidifying around them. Slugs of cast iron, graphite, copper and molybdenum were used with insulating end covers to ensure radial heat flow. These slugs were given different surface coatings or treatments and then immersed into molten aluminum or Hastelloy X. The temperature rise of the slugs was monitored and from these temperature values, the interfacial heat transfer coefficient was
determined by appropriate calculations based on explicit FDM. Variation in the properties of the slug with temperature was incorporated in the computations. From his results, \( h \) is found to increase linearly with time, i.e.

\[
h = a_0 + b \cdot t
\]

(2.19)

where

- \( a_0 \) = initial value of heat transfer coefficient
- \( b \) = rate of increase in heat transfer coefficient with time

The increase in \( h \) is explained in terms of increased contact pressure due to expansion of the slug, contraction of the solidified metal and increase in the thermal conductivity of the coating layer at the elevated temperature.

Nishida and Matsubara [63], in their experiments, used a cylindrical casting solidifying in a metallic die under pressure. Assuming a radial heat flow in their calculations, they found the interfacial heat transfer coefficient by an iterative procedure by matching the temperature at a point closest to the interface in the mold. The difference between calculated and measured values of temperature at other points in the set up was attributed to axial heat flow.

Ko and Pehlke [33,35] determined the interfacial heat transfer coefficient by solving the inverse heat conduction problem using the non-linear estimation procedure proposed by Beck [3]. The interfacial heat transfer coefficient was also determined by calculating the conductive and radiative heat transfer coefficients based on the measurement of the gap width. Good correla-
The air gap size was measured using an inductive gauge. Winter et al. [81] determined the value of \( h \) in sand molds by measuring the air gap size during the solidification of white, gray, and ductile iron in sand molds. The sizes of the air gap were found to be 0.3 mm, 0.19 mm and 0.05 mm for white, gray, and ductile iron respectively.

The interfacial heat transfer coefficient could also be determined from fluidity tests. Using a fluidity test, Prates and Biloni [52] determined the heat transfer coefficient between a flowing metal stream and its metal mold. The value of \( h \) was assumed to be a constant during the test.

2.5.2 The Inverse Heat Conduction Problem (IHCP)

The inverse heat conduction problem refers to the determination of the boundary condition in a conducting body whose internal temperatures are known at a few locations [82]. This differs from the direct problem where the internal temperature field is determined by solving the governing differential equation 2.5 using appropriate boundary conditions. The solutions available for IHCP can be divided into two categories i.e.

1. analytical solutions and
2. numerical solutions
1. Analytical solutions

A few analytical solutions to the inverse conduction problem in which a temperature sensor is placed at an interior temperature sensor is placed at an interior location in the conducting body are available in the literature [83-85]. Burgraff [83] developed an exact solution for calculating surface heat flux and temperature from an interior temperature by considering a series solution of the IHCP. However, Beck [3] pointed out that the exact solution does not take into account variable time in the measured data. Krzysztof et al [84] developed an approximate analytical solution for the IHCP by using the Laplace transform technique. An effective analytical method is suggested for the solution of direct and inverse problems of heat conduction, thermoelastic stresses and heat transfer [85].

2. Numerical solutions

For the numerical solution of the IHCP an approximate form of the variation of the boundary condition with time is assumed. Using this form of the boundary condition with unknown coefficient, the interior temperature field is determined in the domain by numerical procedures like FDM or FEM. An objective function based on the values of measured and calculated temperatures at various internal points is then determined. The objective function is minimized (or maximized as the case may be) by correcting the values chosen for coefficients used in the boundary conditions. This is carried out iteratively till a stationary value of the objective function is obtained. The IHCP is said to be linear.
When the thermal properties of the material are independent of temperature or else the problem is nonlinear [82]. Both linear and non-linear IHCPs can be solved by using numerical methods. The two basic numerical procedures available are the 'function specification' and 'regularization methods' [82,86].

Function specification method [82,86]

In this method a functional form for the unknown heat flux variation with time is assumed. The functional form contains a number of unknown parameters that are estimated utilizing the method of least squares. In the whole domain function specification method, all the parameters for the complete time history are simultaneously estimated. In the sequential procedure, the parameters are estimated at each point of time successively. The sequential method is computationally more efficient than the whole domain procedure for IHCP.

Regularization method [82,86]

In this method, the unknown value of heat flux at the surface can be estimated as a single discrete value at each location of interest on the boundary at each point of time. A combined method of function specification and regularization was reported by Tu [86] by adding regularization terms to the functional form of the boundary values.

The paper by Stolz [87] in 1960 was one of the earliest on the inverse heat conduction problem. He used a numerical inversion
method to solve the inverse heat conduction problem. Though the
analysis developed by him was specifically for spheres it could
also be applied to other simple shapes. However, the solution
becomes unstable if the time steps are small. The method of
dynamic programming and its use for the formulation and solution
of the IHCP are presented in matrix form by Trujillo [88].

In the work of Nicholas et al [89] the two-dimensional linear
inverse heat transfer problem is investigated using the Boundary
Element Method (BEM) in conjunction with Beck's sensitivity
analysis [3]. A deforming FEM analysis of one-dimensional in-
verse Stefan problems is presented by Nicholas et al [90].

Beck [3] developed the non-linear estimation procedure for esti-
nating the best value of heat flow based on the method of least
squares. This method has the advantage of taking into account
inaccuracies in temperature measurement and sensor locations
besides uncertainties in thermal properties. This procedure is
described chapter 6 of this thesis.

From the above, it is seen that the determination of $h$ requires
the solution of the IHCP and many investigators have attempted to
determine $h$. It is also seen that $h$ is affected by many factors
and these are discussed below.

2.6 FACTORS AFFECTING THE MEAN VALUE OF $h$

In this section, the factors which affect the mean value of $h$ and
the causes for its variation with time are discussed.
1. Surface roughness of the mold

Prates and Biloni [52] related the value of \( h \) to the roughness of the mold surface. In the initial stages, the liquid metal contacts the mold only at the asperities and the rapid cooling at these points initiates nucleation. The surface tension of the liquid metal prevents the wetting of the valleys on the rough mold surfaces. Thus, mold roughness affects the value of \( h \) directly. Using a fluidity test, Prates and Biloni [52] found that in the case of aluminum - 5% copper alloy flowing over bare copper molds, the relationship between the number of predendritic nuclei \( N \) and the roughness \( h \) is:

\[
N = 150 \times RA^{-0.26} \quad \text{... (2.20)}
\]

where \( RA \) = roughness in Microns RMS.

The relation between \( h \) and \( N \) was found to be:

\[
N = C h^2 \quad \text{... (2.21)}
\]
where \( Q \) is a constant which is dependent on the composition of the alloy.

Morales et al. [27] found that the mold wall roughness as influenced by machining, polishing, and coating, has a profound influence on the grain structure of the casting. Studying the heat transfer between a solidifying casting and a metallic mold, they found that smooth surfaces extract heat faster, implying an increase in \( h \). This was indicated by a shift in the center line of the casting.

2. Presence of mold coatings

The presence of mold coatings decreases the interfacial heat transfer coefficient by introducing additional thermal barrier [80, 91]. Similar observations were made by Nehru [9] in the case of insulating mold coats while studying the solidification of copper base alloys in cast iron molds. Flemings et al. [92] pointed out that the effect of coatings in reducing the heat transfer is significant only during the first few seconds and later their effect on heat transfer is negligible. They showed that the coatings slow the heat transfer during the filling of the mold, but do not prevent the rapid heat extraction during the subsequent period.

3. Nature of cast surface

At the elevated temperature at which solidification occurs, the initially contacting mold/metal surfaces separate from each other and oxidation of the casting surface is likely to occur [35].

19
This affects the rate of heat flow. Isaac et al. [34] treated this factor as an additional thermal resistance which affects the overall heat transfer coefficient. The surface film is also known to affect radiative heat transfer from the casting surface to the mold by altering the emissivity of the surface [35].

3. Casting size

Sully [29] pointed out that casting size affects the value of $h$. Thin castings solidify in a short time during which the value of $h$ is high. Thick castings do not solidify completely until $h$ reaches a low steady state value which implies a low overall value of $h$. Similar results were observed by Sharma et al. [73]. The delay in gap formation due to plastic deformation in thick castings can lead to a high initial value of $h$ which persists for a long duration [19]. Thamban and Panchanathan [16] on the other hand state that for a given volume ratio, $h$ remains nearly the same for different casting thicknesses for long freezing range alloys.

5. Volume Ratio

Thamban and Panchanathan [16] found that the heat transfer coefficient is influenced by the volume ratio and is independent of the casting thickness. The variation of $h$ with volume ratio at the interface can be obtained from the work of Isaac et al. [34]. They found that there is a reduction in the value of heat transfer coefficient with decrease in volume ratio.
According to Sully [22], geometry of the casting is an overriding factor in controlling the interracial heat transfer coefficient. In cases where the casting surface moves away from the mold surface, the value of \( h \) decreases with time. However, under special conditions where the contraction of the casting causes a more intimate contact between the casting and the mold (as in the case of a core), \( h \) increases with time [36].

7. Properties of cast metal

The properties of the cast metal affects the value of \( h \) directly [35,93]. In the case of ductile iron, expansion caused by graphitization results in better contact and hence an increase in \( h \) with time [93]. The nature of the metal also determines the surface oxidation characteristics which in turn affects \( h \) [35].

8. Contact pressure

It was observed that \( h \) increases with an increase in the contact pressure between the casting and mold [36,47,63]. Nishida and Matsubara [63] studied the effect of pressure on \( h \) using a cylindrical casting. The maximum value of \( h \) as a function of pressure was found to obey a power law. From their results, it is seen that \( h \) varies as the square root of pressure over the wide range of pressures studied.
It was also suggested that it is a function of the factors which cause variation in the casting surface temperature [29] and this aspect was used in numerical simulation by Jeyarajan and Pehlke [95] and Tadayon and Lewis [96].

2.7 CAUSES FOR VARIATION OF \( h \) WITH TIME

It was observed that the value of \( h \) is not a constant but varies with time [29,33,35,36,63]. Factors which cause variation in the interfacial heat transfer coefficient with time are:

1. Mold expansion/casting contraction [6] and
2. Deformation of the casting [38,97]

1. Mold Expansion and casting contraction

It was suggested that a gap is formed at the casting/mold interface as a result of the expansion of the mold due to heat absorption and contraction of the cast metal due to loss of heat, by the earlier investigators [6]. It was also suggested that the mold may expand initially inwardly just after pouring because of the localized temperature rise.
1. Deformation of the casting

Plastic deformation of the casting delays separation between the casting and the mold. This accounts for the high initial value of $h$. This aspect was used to explain the variation of $h$ along the vertical wall of a plate casting by Sai Kumar [97]. Paschkis [65] suggested a mechanism for cyclic variation in $h$ due to brooding. The dynamic nature of $h$ was also pointed out by Muralidharan [45].

2.8 MODELS FOR $h$

A survey of the literature shows that some models are available for predicting the value of $h$ between liquid metal and its mold. Models are also available for explaining the variation of $h$ between two solids in contact with each other.

2.8.1 Models for $h$ between solids in contact

The heat transfer between contacting solids is of interest in many engineering situations [82]. Thomas and Sayles [98, 99] developed techniques for predicting $h$ based on the elastic deformation of the asperities on the contacting surfaces. Over a realistic range of contact loads between the surfaces, $h$ is found to be proportional to the load. Rapier et al [65] studied the thermal conductance between uranium dioxide fuel elements and their stainless steel casings and proposed the relationship given in equation 2.10.
1.3.2. Models for $h$ at casting/mold interface

Models available for predicting $h$ between a casting and its metallic mold can be divided into two categories, namely

1. qualitative models and
2. quantitative models.

The qualitative models explain the physical phenomena occurring at the mold/metal interface without giving a method for calculating the values of $h$. Some of the earlier investigators provided such models [35,47].

Davies [47] provided a qualitative explanation for the dynamic variation of $h$ between a casting and its metallic mold by dividing the events which occur at the interface into 3 stages:

1. period of liquid metal contact which is associated with a high value of $h$.
2. period of contact between solidified metal and die with adequate contact implying high $h$.
3. period of formation of air gap with drastic reduction in $h$.

Ho and Pehlke [35] developed a quantitative model for calculating the value of $h$ by considering the heat flow across the interfacial gap by gas conduction, convection and radiation. They found good agreement between the values obtained from gap measurement and the solution of IHCP.
2.3.1. Empirical Models

Sun [16], gave the following equation for $h$ between a chill and the metal solidifying around it given in equation 2.19.

According to Tiller [100], the value of $h$ can be predicted by the expression:

$$h = h'/2/ t$$  \hspace{1cm} (2.12)

where $h'$ = reference value obtained experimentally  
$t$ = time.

2.9 SUMMARY

The literature survey on the solidification of castings in metallic molds shows that the early investigators recognized the existence of imperfect contact and sometimes an air gap between a casting and its metallic mold. This imperfect contact leads to an increase in the freezing time $t_f$ of the casting. Since the soundness of the casting could be related to its freezing time, considerable attention was devoted to the determination of $t_f$ by mathematical and experimental methods. Experimental investigations were carried out to relate the freezing time of castings with factors like freezing ratio, volume ratio, mold coat thickness etc. In the mathematical methods, it was found necessary to introduce an interfacial heat transfer coefficient $h$ to account for the imperfect contact. For simulation of the solidification of castings with complex geometry, numerical methods are found to be more suitable. While thermal factors relating to the casting and mold material can be incorporated in the simulation.
procedure using established techniques, a knowledge of the value of \( h \) is necessary for the accurate prediction of the thermal conditions of the casting. Many investigators tried different methods to determine the value of \( h \) experimentally. Experiments showed that the value of \( h \) varies with time on the interface. A number of factors are found to influence the value of \( h \). Experimental determination is difficult because it requires the solution of the inverse heat conduction problem. Attempts were also made to determine the value of \( h \) from quantitative and empirical models. While the number of factors which influence \( h \) are large, the available models take into account only a few of the factors like metallostatic pressure, presence of insulating mold coats etc. From this it can be seen that a comprehensive model to determine \( h \) would be quite valuable for numerical simulation of casting solidification and such a model is not reported in literature so far.
A survey of the available literature shows that an satisfactory quantitative model for calculating the value of the interfacial heat transfer coefficient $h$ between a casting and its metallic mold would be useful for casting simulation. The present work is carried out with the following objectives:

1. To develop a quantitative model for determining the value of $h$ in steady state one-dimensional heat flow between a metallic chill and the liquid metal in contact with it.

2. To develop an experimental method to determine the value of $h$ between a casting and a metal chill for the following two cases, which involve one-dimensional heat flow:
   
   i) constant metallostatic head arrangement
   
   ii) receding casting interface arrangement

3. To study the effect of metallostatic head on $h$ in the case of constant metallostatic head experiments.

4. To study the effect casting height, sprue height and ingate size on $h$ in the case of experiments with a receding casting interface arrangement.

5. To relate the experimental value of $h$ in steady state one-dimensional heat flow with that obtained by the model in the case of a smooth uncoated chill for the case of constant metallostatic head arrangement.
To study qualitatively, the effect of insulating mold coatings on the chill surface and chill surface roughness on h for the two experimental arrangements.

7. To measure the dimensions of the air gap between the casting and chill at the interface and to relate it to h.

8. To validate the model by comparing the experimental results with the values predicted by the model.

9. To determine the sensitivity of the calculated temperature values with a change in the value of h.
CHAPTER 4

A MODEL FOR HEAT FLOW ACROSS CASTING/MOLD INTERFACE

4.1. INTRODUCTION

The survey of literature given in chapter 2 indicates that the interfacial heat transfer coefficient $h$ between a casting and its metallic mold varies with time and location on the interface. A number of factors influence the value of $h$. For simulation of casting solidification in metallic molds, the value of $h$ should be known as a function of time and a model for determining the value of $h$ would be valuable for this purpose.

In this chapter, the variation of $h$ with time in one dimensional heat flow is discussed. A model is developed for determining the value of $h$ between the liquid metal and its metal mold in steady state one dimensional heat flow. The model takes into account the effect of metallostatic pressure, mold surface profile, surface tension of the liquid metal and thermal conductivity of the gases in the interface. It also includes the effect of the oxide and other adherent films which are normally present on the metal and mold surfaces. The stationary value of $h$ thus obtained can then be used as a starting value for predicting the subsequent variation of $h$ with time by numerical simulation. The model developed here is used for the determination of $h$ for two types of mold surfaces:

1. Mold surfaces with uniform V grooves or a combination of uniform V grooves
The value of $h$ is determined for these two cases for a wide range of metallodielectric pressure. The results obtained by simulation using the model are presented and discussed in this chapter. Validation of the model using experimental results reported in literature are discussed here. Further validation of the model using results obtained by experiments carried out in the present work are discussed in chapter 7.

### 4.2 VARIATION OF $h$ WITH TIME:

A typical graph of the variation of $h$ with time in unidirectional heat flow is presented schematically in figure 4.1. The variation of $h$ can be divided into 3 stages, I, II and III as shown. Stage I shows a steep rise, followed by fluctuations and then a steady value. The duration of stage I, $t_I$, and the mean value of $h$ in this stage $h_I$ are marked in the figure. At the end of stage I, $h$ attains a steady value, $h_{II}$, and remains constant till time $t_{II}$. This is followed by stage III in which $h$ either decreases with time as shown by line 1 due to the movement of the casting surface away from the mold surface or increases with time due to increase in contact pressure as shown by line 3. A situation in which there is no stage III is shown by line 2 in the figure. A situation where the interface condition changes from stage I to stage III directly is shown as line 4 in the figure.
Fig 4.1 Variation of $h$ with time (Schematic)

1, 4 - Receding interface
2 - Constant pressure
3 - Increasing pressure
When molten metal is poured into the mold, there is turbulence which causes \( h \) to increase from zero to a high fluctuating value in stage I. The fluctuations can be associated with variations in pressure due to turbulence. Any skin of solid metal formed is ruptured by the turbulence in the melt. The mean pressure acting on the mold during this period depends on the pouring head and the casting height. When pouring is stopped, the fluctuations in pressure die out. The mold extracts heat from the molten metal and a thin stable skin of solidified metal is formed which is pressed against the mold by the metallostatic head. This marks the end of stage I. During stage II which lasts from time \( t_I \) to \( t_{II} \), the value of \( h \) i.e., \( h_{II} \) remains constant. When an adequate thickness of metal is solidified, it becomes self-supporting and the conditions of solidification may cause \( h \) to increase or decrease with time which marks the beginning of stage III. If the geometry of the casting is such that the casting surface recedes away from the mold surface, \( h \) decreases and is represented by line 1 in stage III. Increase in contact pressure may arise in situations like the solidification of the casting around a metal core resulting in an increase in \( h \) and this situation is shown by line 3 in the figure. If the casting geometry causes a constant contact pressure as in the case of the bottom of an open casting of uniform cross-section, there is no change in the value of \( h \) and hence no distinct stage III as shown by line 2 in the figure. The actual values of \( t_I \) and \( t_{II} \) depend on factors like the casting size and geometry, pouring conditions etc.
some cases where the metallostatic pressure is not enough to press the liquid metal surface formed against the chill surface in stage I, there is no distinct stage II. Examples of this behavior have been reported in literature (6) and the variation of $h$ with time in such a situation is shown by line 4 in the figure.

The mean value of $h$ in stage I is affected by the effective metallostatic head which acts during pouring. The value of $h$ in stage II can be determined from the metallostatic head acting after pouring is completed. Using the value of $h$ in stage II as the starting value, the variation of $h$ in stage III can be determined by simulating either the relative displacements between the casting and mold surfaces or the increase in contact pressure as the case may be. Even in cases where there is no distinct stage II, a notional value of $h_{III}$ can be determined for subsequent usage for calculations in stage III.

The present model for $h$ in stages I and II is developed by considering a mold surface having uniform $V$ grooves with its corresponding liquid metal profile. This work is divided into two parts:

1. A mold surface having uniform $V$ grooves and its corresponding liquid metal surface is studied analytically. Actual mold surfaces are analyzed by considering the irregularities on the surface as combinations of uniform grooves.
A casting/mold interface is studied numerically and the value of \( h \) is determined at the interface. This involves the following three steps:

i) Generation of a mold surface using the method of fractals;

ii) Generation of the liquid metal profile using the model developed in the present work.

iii) Determination of \( h \)

4.3 ANALYSIS OF GEOMETRICALLY UNIFORM GROOVES:

Metal molds are manufactured by machining processes like planning, milling, turning, shaping etc. These machining operations leave a series of V grooves on the machined surface. For simplifying the analysis, a mold surface consisting of a series of uniform parallel V grooves as shown in figure 4.2 is considered. The profile of the liquid metal which comes into contact with this mold surface is determined by the surface tension of the liquid metal and the net pressure acting on the liquid surface [101].

A cross section ABCD of the mold along a plane perpendicular to the mold surface is shown in figure 4.2. The line AB is perpendicular to the direction of the grooves and a portion of AB is shown in figure 4.3. A gap between the sloping walls of the mold and the liquid metal is included in the figure to incorporate the effect of oxide and other films. The figure also shows the liquid metal profiles that would be obtained by varying the net
FIG. 4.2 MOLD SURFACE WITH UNIFORM V GROOVES.

FIG 4.3 EFFECT OF PRESSURE ON LIQUID METAL PROFILE
pressure \( P \) on the liquid metal surface. The apex angle of the groove is \( 2\theta \) and the total width of the groove at the mouth is \( L_a \). The liquid metal is acted on by metallostatic pressure \( P_H \) and the back pressure in the groove is given by \( P_g + P_v \) where \( P_g \) is the gas or air pressure and \( P_v \) is the vapor pressure of the metal. \( P_v \) is negligible in the present case since the boiling points of most metals are much higher than the pouring temperature. The net pressure acting on the liquid metal surface \( P \) is the difference between the metallostatic pressure and the back pressure i.e.

\[
P = P_H - (P_g + P_v)
\]  \hspace{1cm} (4.1)

From principles governing the behavior of surface tension, the pressure \( P \) and the surface tension \( \sigma \) are connected by the relation:

\[
P = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]  \hspace{1cm} (4.2)

where \( R_1 \) and \( R_2 \) are the radii of the liquid metal in two perpendicular directions. From figure 4.3, it can be seen that the liquid surface is cylindrical i.e. \( R_2 = \infty \). Taking \( R_1 = \infty \), this leads to the equation (101):

\[
P = \frac{\sigma}{P} = \frac{\sigma}{P_H - (P_g + P_v)}
\]  \hspace{1cm} (4.3)

It is well known that when liquid metal rests on a non-wettable substrate, it maintains a certain equilibrium contact angle which is determined by the interfacial tensions. When the liquid metal attempts to flow on the substrate, the contact angle changes its value depending on the direction of flow. When it
flows so as to increase the area of contact, it does so only when the contact angle reaches the 'advancing contact angle' \( \alpha \). Similarly, when the liquid metal flows so as to decrease the area of contact, flow will commence only when the contact angle decreases to the 'receding contact angle' \( \theta \). The receding contact angle is not required in the present analysis.

In the present case when the contact angle (the angle between the sloping side of the groove and the tangent to the liquid metal surface at the point of contact with the mold indicated as \( \theta \) in figure 4.3) reaches the advancing contact angle \( \alpha \), the liquid metal advances into the groove. This implies that when the pressure \( P \) is less than a critical value \( P_c \), the liquid metal will not enter the groove implying the existence of an air gap between the mold and liquid metal surfaces. When the pressure exceeds \( P_c \), the liquid metal advances into the groove maintaining the advancing contact angle \( \alpha \). To maintain the same value of \( \alpha \) when the liquid metal advances further into the groove, the radius \( R \) should become smaller implying the need for a higher pressure \( P \). This phenomenon can be used to develop a quantitative relationship between \( h \) and \( P \).

The liquid metal profiles for three cases (i.e. \( P < P_c \), \( P = P_c \) and \( P > P_c \)) are shown in figure 4.3. It can also be seen from equation 4.3 that for completely filling the groove \( (R = 0) \), the pressure required is infinite. This implies the existence of a partial air gap between the casting and mold surfaces at all pressures.
When liquid metal touches an ideal smooth mold surface and establishes perfect contact, then the casting and mold surfaces would give the same temperature i.e., \( T_c = T_m \) as shown in figure 4.4 a. The heat flow in this case is controlled by the lower value of the thermal conductivities of the mold and casting. However, this is an ideal situation, not realizable in practice. The contact between a real mold and a casting is imperfect and the liquid metal does not touch all points on the surface of the mold as shown in figure 4.4 b. The casting and mold surface temperatures vary from point to point depending on the contact conditions. Representative temperatures of the casting and mold surfaces are indicated as \( T_c \) and \( T_m \) and the variation in the distance between the casting and the mold is shown as an equivalent air gap. The heat flow across the interface in such a situation is presented in figure 2.1 and the interfacial heat transfer coefficient is given by [6]:

\[
\frac{q}{A} = \frac{1}{h} (T_c - T_m) \quad \ldots (4.4)
\]

If the interface is considered as an air gap of thickness \( y \) having thermal conductivity \( k \), then it can be shown that [35]:

\[
h = \frac{k}{y} \quad \ldots (4.5)
\]

The inverse of interfacial heat transfer coefficient i.e. \( 1/h \) is referred to as the interfacial thermal resistance [63].
a. PERFECT CONTACT

b. IMPERFECT CONTACT

FIG. 4.4 CASTING / MOLD INTERFACE.
The temperatures at various points on the surface of the mold are not constant but vary considerably. The points on the mold having contact with liquid metal would have a higher temperature than those which are not in contact. Similarly, the points on the liquid metal surface which are in contact with the mold have a lower temperature compared to the other points. However, a representative temperature for $T_c$ and $T_m$ can be found such that their use yields the actual or measured value of heat flow [102].

4.3.1. Analysis of heat flow across the interface

The typical configuration at the casting/mold interface is shown in figure 4.5a. Since all grooves are similar and each groove is symmetric about a vertical axis through the root of the groove, the region indicated for analysis is representative of the entire configuration.

The heat flow in the casting and the mold can be considered to be one dimensional at locations away from the interface. However, close to the interface, more heat flows in the regions where there is contact between the liquid metal and the asperities of the mold and the flow lines would be as indicated in figure 4.5b. This shows that there is a region in the vicinity of the interface where heat flow is not unidirectional and this region is marked as the "disturbed region" in figure 4.5b. The thickness of the disturbed region is small (of the same order as the surface roughness) compared to the thickness of the mold. The analysis of this region can be simplified by considering the heat flow to be across a uniform gap whose thickness is such that it allows
a TYPICAL GROOVE CONFIGURATION FOR ANALYSIS

Region of unidirectional heat flow (liquid metal)

b HEAT FLOW LINES (ACTUAL)

Region of unidirectional heat flow (mold)

c HEAT FLOW LINES IN THE EQUIVALENT GAP

FIG. 4.5 HEAT FLOW ACROSS CASTING / MOULD INTERFACE
the same amount of heat flow as the series of grooves as shown in figure 4.5c. The width of this equivalent gap \( y_{eq} \) can be obtained by assuming that the mold and casting surface temperatures \( T_m \) and \( T_c \) remain constant i.e. steady state heat flow conditions are assumed.

4.3.2. Assumptions

The following assumptions are made in developing the model:

1. Heat flow across the interface is by conduction through the gases in the gap between the casting and mold surfaces. The thermal conductivity \( k \) of this mixture of gases remains constant.

2. Heat flow occurs along parallel paths which are normal to the plane of the interface. The effect of deviation from unidirectional flow in the vicinity of the interface is negligible.

3. The casting and mold surfaces are at constant temperatures \( T_c \) and \( T_m \) respectively.

4. The liquid metal does not wet the mold surface. The surface tension of the liquid metal \( \sigma \) and the advancing contact angle \( \alpha \) remain constant. The profile of the liquid metal in contact with the mold surface is determined by \( \alpha \) and \( \sigma \) of the liquid metal.

5. The thickness of oxide and other films adhering to the mold and casting surfaces is uniform and this can be represented...
by an equivalent air gap of width $y_t$. The presence of a mold coat can be included by choosing an appropriate value for $y_t$. The chosen value of $y_t$ gives the same thermal resistance as the combination of oxide films and mold coats present at the interface. $y_t$ is given by:

$$y_t = \frac{k}{h_{\text{max}}}.$$  

where $y_t$ is the air gap width representing oxide and other films, $k$ is the thermal conductivity of gases in the gap and $h_{\text{max}}$ is the maximum attainable value of $h$.

The procedure for obtaining the equivalent gap thickness $y_{\text{eq}}$, and the interfacial heat transfer coefficient $h$ for the configuration shown in figure 4.5a is discussed below.

4.3.3. Analysis of heat flow in the groove

Case (1): $P > P_c$

The region marked for analysis in figure 4.5a is representative of the entire mold surface and an enlarged view of this region is given in figure 4.6a. For convenience of calculations, Cartesian co-ordinate axes are introduced with origin $O$ at the root of the groove. The $X$-axis lies in the plane of the mold surface in a direction perpendicular to the grooves and the $Y$-axis is chosen along the normal to the plane of the mold as shown in figure 4.6a. $OL$ represents the sloping groove wall and this makes an angle $\beta$ with the $Y$-axis where $\beta$ is the semi apex angle of the groove. The half width $a$ of the groove is divided into two re-
a. Details of region under analysis.

b. Air gap size (Schematic)

FIG. 4.6 ANALYSIS OF HEAT FLOW IN THE GROOVE
gions (i) gap region and (ii) contact region as shown in the figure. In the gap region, the liquid metal profile JK has a radius \( r \) with the center of curvature located at a point \( H \) on the \( Y \)-axis. Angle \( KHJ \) is marked as \( \Gamma \) in the figure. An air gap of varying width \( \gamma' \) exists in the gap region between the profile JK and the sloping groove wall OL. In the contact region, the liquid metal profile IJ is shown to be parallel to the groove wall OL separated by a vertical distance \( y_f \) representing the equivalent air gap corresponding to the oxide films and mold coats.

The center of curvature \( H \) of the profile JK lies on the \( Y \)-axis such that:

\[
\hat{KHJ} = \Gamma = \alpha - \beta - 90^\circ
\] ... (4.7)

Generally, the mold materials and the mold coats used are such that the liquid metal does not wet them. The value of \( \alpha \) in equation 4.7 for molten aluminium resting against a cast iron mold generally lies in the range of 150 to 170°. This gives positive values for \( \Gamma \) for the values of \( \beta \) found on normal machined surfaces and the present analysis would be satisfactory. On the other hand, if \( \Gamma \) is negative or zero, due to a large value of \( \beta \), it would result in instability and lead to complete filling of the groove giving a value of \( h_{\text{max}} \) determined only by \( y_f \).

Considering a point \( Q \) at a distance \( x \) from the origin on the \( X \)-axis, the air gap width at this location is the vertical dis-
The distance $y'$ between the mold and liquid metal surfaces as shown in figure 4.6a which can be determined from the configuration. The total heat flow in the groove is considered as a sum of the heat flow in the gap region and contact region as shown in figure 4.6a. The heat flow $dq$ in the gap region through a strip of width $dx$ and unit depth in the direction perpendicular to the plane of the paper is given by:

$$dq = \frac{x}{y'} (T_C - T_m) dx \quad \ldots (4.9)$$

where $k = \text{thermal conductivity of gases in the gap}$

$T_C = \text{casting surface temperature}$

$T_m = \text{mold surface temperature}$

Hence, the overall heat flow in the gap region, $q_1$, is given by:

$$q_1 = \int dq = \int_{0}^{y'} \frac{k}{y'} (T_C - T_m) dx = \frac{k}{y'} (T_C - T_m) R \sin \Gamma \quad \ldots (4.9)$$

where $y'$ is the equivalent gap thickness in the gap region 0J.

The value of $y'$ is obtained by integration as follows:

This gives

$$\frac{1}{\bar{y}} = \frac{1}{R \sin \Gamma} \int_{0}^{y'} \frac{R \sin \Gamma}{y'} \quad \ldots (4.10)$$

We have

$$R = \sigma/P$$

$$x = R \sin \theta$$

$$dx = R \cos \theta \, d\theta$$

$$M = R \cot \beta \sin \Gamma \cos \Gamma + R \cos \Gamma \quad \ldots (4.10)$$

$y_f = \text{air gap equivalent of oxide and thin films}$. 

66
Also \( y' = M - R \cos \theta - 2 \sin \theta \cot \theta \)

\[
\frac{1}{y} = \frac{1}{R \sin \Gamma} \int_{0}^{\Gamma} \frac{R \cos \theta \, d\theta}{M - R \cos \theta - R \sin \theta \cot \theta}
\]

On integration, this yields the relation

\[
\frac{1}{y} = \frac{2}{(M + R) \sin \Gamma} \left( I_1 - I_2 - I_3 \right)
\]

where

\[
I_1 = \frac{\Lambda}{2} \ln \left( 1 + \tan^2 \frac{\Gamma}{2} \right)
\]

\[
I_2 = \frac{\Gamma}{2}
\]

\[
I_3 = C \left[ \ln \left( \frac{\tan^2 \frac{\Gamma}{2} - k_1 \tan \frac{\Gamma}{2} + k_2}{k_2} \right) + \left( \frac{4 \tan \frac{\Gamma}{2}}{k_1 (k_1 - 2 \tan \frac{\Gamma}{2})} \right) \right]
\]

If \( k_4 < 0 \)

\[
I_3 = \frac{C}{2} \ln \left( \frac{\tan^2 \frac{\Gamma}{2} - k_1 \tan \frac{\Gamma}{2} + k_2}{k_2} \right) + \left( \frac{4 \tan \frac{\Gamma}{2}}{k_1 (k_1 - 2 \tan \frac{\Gamma}{2})} \right)
\]

If \( k_4 > 0 \)
\[ r_i = \frac{c}{2} \left[ \ln \left| \frac{\tan^2 \frac{\pi}{2} - k_1 \tan \frac{\Gamma}{2} + k_2}{\frac{k_2}{2}} \right| \right] \]

\[ + \left( D + \frac{c}{2} \right) k_2 \frac{1}{j/k_4} \left( \tan^{-1} \left( \frac{-k_1}{2j/k_4} \right) \right) \]

where \[ k_1 = \frac{2 \cot \theta}{M + R} \]

\[ k_2 = \frac{M - R}{M + R} \]

\[ k_3 = k_1^2 + (k_2 - 1)^2 \]

\[ k_4 = k_2 - \frac{k_1^2}{4} \]

\[ A = \frac{2k_1}{k_3} \quad \quad \quad C = -A = \frac{-2k_1}{k_3} \]

\[ B = \frac{2(k_2 - 1)}{k_3} \quad \quad \quad D = 1 - B \frac{k_2}{k_3} = \frac{1 - 2(k_2 - 1)}{k_3} k_2 \]

In the contact region 1J with a uniform air gap of width \( y_f \), the heat flow \( q_2 \) is given by:

\[ q_2 = \frac{k}{y_f} (T_c - T_m) (a - R \sin \gamma) \quad \quad \quad \ldots (4.11) \]
The overall heat flow $q$ is given by:

$$q = q_1 + q_2 = k(T_C - T_m) \left( \frac{R \sin \Gamma}{\bar{y}} + \frac{a - R \sin \Gamma}{Y_f} \right) \quad (4.12)$$

$$= h a (T_C - T_m)$$

Hence, the overall heat transfer coefficient for the region under analysis is given by:

$$h = \frac{k}{a} \left( \frac{R \sin \Gamma}{\bar{y}} + \frac{a - R \sin \Gamma}{Y_f} \right) = \frac{k}{Y_{eq}} \quad (4.13)$$

where $Y_{eq}$ is the equivalent air gap size for the entire groove.

Figure 4.6b shows the variation of $y'$, in addition to the value of $\bar{y}$ and $Y_{eq}$.

Case(2): $P < P_c$

If $P < P_c$, then there is no contact region and the angle $KHI$ would be given by:

$$\sin(KHI) = \sin \Gamma = \frac{a}{R} \quad (4.14)$$

In this case $\bar{y}$ is obtained by substituting $\Gamma$ for $\Gamma$ in equation 4.9. $h$ is given by:

$$h = k / \bar{y} \quad (4.15)$$

A flow chart for the calculation of $h$ using the analytical model is presented in figure 4.7.
FIG 4.7 FLOW CHART FOR THE DETERMINATION OF h-
ANALYTICAL MODEL
4.4 RESULTS:

The results obtained by calculations using the analytical model for mold surfaces with uniform V grooves are presented in this section. The effect of \( y_f, \alpha, \beta, a \) and \( \sigma \) on the variation of \( h \) with pressure are presented in figure 4.8. Figure 4.8a gives the variation of \( h \) with pressure for two values of \( y_f \) (3 x 10^{-4} \text{ mm} \) and 120 x 10^{-4} \text{ mm}). In figure 4.8b the variation of \( h \) with pressure for two values of the advancing contact angle \( \alpha \) (160° and 170°), is given. Figure 4.8c gives the variation of \( h \) with pressure for three values of \( \beta \) (30°, 45° and 60°). Figure 4.8d gives the variation of \( h \) with pressure for four values of the half mouth width \( a \) (0.01, 0.03, 0.12 and 0.2 mm). The effect of \( \sigma \) on the variation of \( h \) with pressure is given in figure 4.8e, for two values of \( \sigma \) i.e. 0.6 and 0.840 N/m. The values used for the calculation of \( h \) in the analytical model are given in Table 4.1.

Table 4.1

Data used for the analytical estimation of \( h \).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity of air</td>
<td>( k )</td>
<td>0.024117 \text{ W/m K}</td>
</tr>
<tr>
<td>Max interfacial heat transfer</td>
<td>( h_{\text{max}} )</td>
<td>29309 \text{ W/m}^2 K</td>
</tr>
<tr>
<td>coefficient</td>
<td></td>
<td>2093.5 \text{ W/m}^2 K</td>
</tr>
<tr>
<td>Surface tension of aluminium</td>
<td>( \sigma )</td>
<td>0.6, 0.84 \text{ N/m}</td>
</tr>
<tr>
<td>Semi apex angle of groove</td>
<td>( \beta )</td>
<td>30°, 45°, 60°</td>
</tr>
<tr>
<td>Half width of groove</td>
<td>( a )</td>
<td>0.01, 0.03, 0.12, 0.2 \text{ mm}</td>
</tr>
<tr>
<td>Advancing contact angle</td>
<td>( \alpha )</td>
<td>160°, 170°</td>
</tr>
<tr>
<td>Pressure ( P_h=(P_g+P_y) )</td>
<td>( P )</td>
<td>1x10^5 - 101x10^5 \text{ Pa}</td>
</tr>
</tbody>
</table>

71
a) EFFECT OF $\gamma$

b) EFFECT OF $\alpha$

72
e) EFFECT of $\beta$.

\[ h, W/\text{sq.m K} \]

\[ P_c = 0.007 \text{ atm} \]
\[ P_c = 0.057 \text{ mm} \]
\[ P_c = 0.027 \text{ atm} \]

\[ \beta = 60^\circ \]
\[ \alpha = 160^\circ \]
\[ \gamma = 0.84 \text{ N/m} \]
e) EFFECT OF $\sigma$.

FIG. 48 EFFECT OF $y$, $\alpha$, $\beta$, $a$ AND $\sigma$ ON THE VARIATION OF $h$ WITH PRESSURE
The results obtained in the form of $h$ as a function of pressure show two distinct regions. The region $\Delta P > P_C$ corresponds to the non-entry of liquid metal into the groove. When $\Delta P > P_C$ the liquid metal enters into the groove as shown in figure 4.3.

The general trend in the variation of $h$ with increase in pressure in all the cases studied is the same i.e., $h$ rises slowly till $P_C$ is reached. The increase in $h$ is rapid when $P$ is just above $P_C$ and with further increase in pressure it becomes asymptotic with the value of $h_{\text{max}}$. This limiting value $h_{\text{max}}$ is dependent on the value of $\gamma_f$.

At low values of pressures ($P < P_C$), the liquid metal clings to the top corners of the grooves. A large gap exists between the liquid metal and the mold surface and this results in low values of $h$. The small increase in $h$ with increase in $P$ when $P < P_C$ is only due to the changes in the curvature of the liquid metal profile associated with the changes in pressure. Once the pressure reaches $P_C$, the metal starts advancing into the groove, resulting in a reduction in the gap and hence, an increase in the values of $h$. With further increase in pressure well above $P_C$, $h$ approaches the limiting value $h_{\text{max}}$ which is determined by $\gamma_f$.

4.4.1. Effect of $\gamma_f$ on $h$:

Figure 4.8a shows the variation of $h$ with $P$ for two values of $\gamma_f$ i.e., $8 \times 10^{-4}$ mm and $120 \times 10^{-4}$ mm. A decrease in the value of $\gamma_f$ results in a higher value of $h$ at all pressures. Since $P_C$ de-
pends only on the geometry of the groove. $\gamma_f$ does not influence $P_c$. The equivalent air gap is given by:

\[
Y_{eq} = Y_f \pm Y \quad \text{(when } P<P_c) \quad \ldots \quad (4.16)
\]
\[
Y_{eq} \approx Y_f \quad \text{(when } P>P_c) \quad \ldots \quad (4.17)
\]

From the equations 4.16 and 4.17 it can be seen that for a given change in $Y_f$, the change in $Y_{eq}$ is less when $P<P_c$ compared to the change when $P>P_c$.

### 4.4.2. Effect of $\alpha$ on $h$:

The variation of $h$ with $P$ for two values of $\alpha$ i.e. $150^\circ$, $170^\circ$ are shown in figure 4.8b. From these results, it is seen that with an increase in advancing contact angle $\alpha$, the critical pressure required by the metal to enter into the groove also increases. For maintaining the higher value of $\alpha$, the radius of the liquid metal has to decrease and this requires a higher pressure. This explains the increase in critical pressure with increase in $\alpha$.

### 4.4.3. Effect of $\beta$ on $h$:

The variation of $h$ with $P$ for 3 values of $\beta$ ($30^\circ$, $45^\circ$ and $60^\circ$) is shown in figure 4.8c. It is seen from the figure that with an increase in $\beta$, the critical pressures required for the metal to enter into the groove decreases. This is due to the fact that with a larger value of $\beta$, the radius of the liquid metal required for attaining the advancing contact angle is also larger. From
equation 4.4. it is seen that a larger radius of liquid metal profile occurs at a lower value of pressure.

From figure 4.8c, it is also seen that with an increase in $\beta$, the value of $h$ below the critical pressure increases. This is because for a constant value of $a$, increase in $\beta$ causes a decrease in the depth $d$ of the groove as shown in the insert in figure 4.8c.

4.4.4. Effect of $a$ on $h$:

The variation of $h$ with $P$ for four values of $a$ (0.01, 0.03, 0.12 and 0.2 mm) is given in figure 4.8d. From the figure it is seen that $P_c$ decreases with an increase in $a$. With increase in $a$, the metal reaches the advancing contact angle at a lower value of pressure i.e., the critical pressure is decreased. With an increase in $a$ for a constant value of $\beta$, there is a wider and deeper gap as shown in the insert in figure 4.8d and hence the value of $h$ becomes lower.

4.4.5. Effect of $\sigma$ on $h$

Figure 4.8e shows the variation of $h$ with $P$ for two values of $\sigma$ i.e., 0.6 and 0.840 N/m. An increase in $\sigma$ causes an increase in the value of $P_c$. The radius of the liquid metal profile just before entering into the groove is determined by the value of $\sigma$. When the value of $\sigma$ is fixed, the required radius is attained at a higher pressure when $\sigma$ is higher as seen from equation 4.3. This explains the increase of $P_c$ with $\sigma$. It is also seen from figure 4.8e that the value of $h$ at $P_c$ is the same for both
values of $\sigma$. This occurs because the liquid metal profiles at the critical pressures are identical in both the cases with different values of $\sigma$.

In alloys, $\sigma$ varies with temperature and the addition of alloying elements. The change in $\sigma$ with temperature is also associated with a change in the value of $\sigma$. Determination of $h$ in actual calculations should take these aspects into account as in the case of determination of $h$ in stages I and II. Determination of $h$ in stage I should also take into account the increase in $P$ associated with pouring in the initial stages.

The thermal conductivity $k$ of the gas mixture in the gap does not have any effect on the profile of the liquid metal or on the value of $\gamma_{eq}$. The value of $h$ varies in direct proportion to the change in $k$ since $h = k / \gamma_f$. From the above discussions, it can be seen that the actual value of $h$ depends on the equivalent air gap thickness corresponding to oxide films $\gamma_f$, the advancing contact angle $\sigma$, the two groove configuration parameters namely, half mouth width $a$ and semi apex angle $\beta$, surface tension $\sigma$ of the liquid metal and the thermal conductivity $k$ of the gas mixture in the air gap.

4.5. ANALYSIS OF COMBINATIONS OF UNIFORM GROOVES:

The results obtained using the analytical model show a sharp change in $h$ when $P$ lies just above $P_c$ but in practice the variation of $h$ with $P$ is more gradual [47,63]. The reason for the sharp variation in $h$ with $P$ obtained in the calculations is due
to the assumption of geometrically regular and uniform grooves on the mold surface. Real machined surfaces do not have this geometric regularity. A typical machined surface is shown in figure 4.9. The actual profile consists of undulations on the surface and this can be treated as a combination of primary texture, secondary texture and error of form [103]. The difference between these textures is the wavelength of the undulations as seen in the figure. The primary texture is constituted by coarse grooves and the secondary texture is composed of fine grooves. The individual grooves on the machined surface have a range of values for $a$ and $\beta$ and the individual grooves have different values of $h$. Hence, mold surfaces can be analyzed by treating them as combinations of uniform grooves with thermal resistances in parallel or in series as shown in figure 4.10. Actual mold surfaces can be treated as a random combination of coarse grooves with finer grooves superimposed on them, neglecting the error of form. The effect of error of form with a large wavelength can be neglected in heat flow studies as the value of $\beta$ associated with it is large. In such a situation, the heat flow is controlled only by the primary and secondary textures.

4.5.1. Grooves with thermal resistances in parallel:

The actual grooves on the mold surface can be grouped together based on the values of the groove parameters $a$ and $\beta$. At a given pressure, if grooves having mouth width $2a_1$ and apex angle $2\beta_1$ have a value $h_1$ and cover an area $11$, grooves having mouth width
FIG. 4.9 TYPICAL MACHINED SURFACE [1031].
\[ h = \frac{h_1 l_1 + h_2 l_2}{l_1 + l_2} \]

**a. Thermal Resistances in Parallel (Schematic)**

- \( h_1 \) (Fine grooves)
- \( h_2 \) (Coarse grooves)
- \( \frac{1}{h} = \frac{1}{h_1} + \frac{1}{h_2} \)

**b. Thermal Resistances in Series (Schematic)**

**Fig 410 Combinations of Uniform Grooves with Thermal Resistances in Parallel / Series**
and apex angle $2\beta_2$ have a value $h_2$ and cover an area $l_2$ and so on, then the overall value of $h$ is the weighted average of the $h$ values of these individual sets of grooves. Figure 4.10a shows two sets of grooves where $h_1$ is the heat transfer coefficient over an area $l_1$, $h_2$ is the heat transfer coefficient over an area $l_2$ etc. If there are $n$ types of grooves having groove parameters $a_1$, $a_2$, ... $a_n$, $\beta_1$, $\beta_2$, ... $\beta_n$ etc., then the overall value of $h$ can be given by the weighted average value of $h$ i.e.,

\[
h = \frac{h_1 l_1 + h_2 l_2 + \cdots + h_n l_n}{l_1 + l_2 + \cdots + l_n} = \frac{\sum h_i l_i}{\sum l_i} \quad (4.18)
\]

Figure 4.11 gives the variation of $h$ with $P$ for two sets of grooves with thermal resistances in parallel. The variation of $h$ with pressure for individual grooves is also included in the figure. The variation of $h$ with $P$ shows two steps corresponding to the values of $P_{c_1}$ and $P_{c_2}$, the critical pressures of the two constituent grooves. When $P_{c_1}$ is reached, metal enters the first set of grooves. When $P$ increases further and reaches $P_{c_2}$, metal enters grooves of type 2 as well. Thus the net value of $h$ is the combined effect of the two types of grooves. The result is an additional step in the variation of $h$ with $P$ as shown in figure 4.11.

4.5.2. Grooves with thermal resistances in series:

The machined surface of a mold can be represented as a set of coarse grooves with fine grooves superimposed on them as shown in
FIG. 4.11 VARIATION OF \( h \) WITH PRESSURE FOR GROOVES WITH THERMAL RESISTANCES IN PARALLEL
In this case, the interfacial heat transfer coefficient can be determined as:

\[ \frac{1}{h} = \frac{1}{h_1} + \frac{1}{h_2} \]

... (4.19)

where \( h_1' = h_1 \left( \frac{\gamma_f}{2} \right) \) and \( h_2' = h_2 \left( \frac{\gamma_f}{2} \right) \)

\( h_1 \left( \frac{\gamma_f}{2} \right) \) represents the value of \( h_1 \) obtained by using a value \( \gamma_f/2 \) for the air gap equivalent of oxide and other films. The parameter \( h_2 \left( \frac{\gamma_f}{2} \right) \) is defined similarly. The value of \( \gamma_f \) chosen for each type of groove is taken as half of the nominal value of \( \gamma_f \) because the resistance of fine and coarse grooves are in series and the sum of the individual values of \( \gamma_f/2 \) associated with each type of groove would yield a net value \( \gamma_f \). Figure 4.12 gives the variation of \( h \) with pressure when two types of grooves are present on the mold surface with thermal resistances in series. The variation of \( h \) with pressure for the individual grooves are also included. The variation of the overall value of \( h \) with \( P \) shows two steps at the two critical pressures \( P_{c1} \) and \( P_{c2} \) of the constituent grooves. The step arises because initially the liquid metal enters only the coarser grooves. When the critical pressure corresponding to the finer grooves is reached, liquid metal starts entering these finer grooves as well resulting in the second step in the variation of \( h \) with \( P \). The presence of steps in the variation of \( h \) with \( P \) in figures 4.11 and 4.12 indicates that by considering more types of grooves in the calculations, the variation can be expected to become more gradual without sharp changes in \( h \).
4.5.3. Random combination of grooves:

The results of a random combination of grooves on the variation of h with P is presented in this section. A set of grooves with different parameters chosen for analysis is given in Table 4.2 and variation of h with P for this combination is given in figure 4.13.

Table 4.2

<table>
<thead>
<tr>
<th>$\theta$ (°)</th>
<th>a (mm)</th>
<th>$\rho_0 \times 10^5$ Pa</th>
<th>Series(S)</th>
<th>Weightage</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>0.35</td>
<td>0.010</td>
<td>P</td>
<td>0.2</td>
</tr>
<tr>
<td>60</td>
<td>0.0287</td>
<td>0.051</td>
<td>P</td>
<td>0.2</td>
</tr>
<tr>
<td>60</td>
<td>0.014</td>
<td>0.104</td>
<td>P</td>
<td>0.2</td>
</tr>
<tr>
<td>50</td>
<td>0.0028</td>
<td>0.520</td>
<td>P</td>
<td>0.2</td>
</tr>
<tr>
<td>45</td>
<td>0.0035</td>
<td>1.014</td>
<td>P</td>
<td>0.2</td>
</tr>
<tr>
<td>60</td>
<td>0.000287</td>
<td>5.081</td>
<td>S</td>
<td>0.5</td>
</tr>
<tr>
<td>45</td>
<td>0.00035</td>
<td>10.145</td>
<td>S</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The calculations are carried out with $\alpha = 160°$ and $\sigma = 0.34$ N/m.

From figure 4.13 it can be seen that the value of h does not change significantly with pressure until the lowest value of critical pressure of the chosen grooves is reached. With further increase in P, h rises slowly and then reaches the limiting value.
FIG. 4.12 VARIATION OF h WITH PRESSURE FOR GROOVES
WITH THERMAL RESISTANCES IN SERIES

FIG. 4.13 VARIATION OF h WITH PRESSURE FOR RANDOM
COMBINATION OF GROOVES
The variation of \( h \) with \( P \) is gradual in figure 4.13 as expected. The rate of increase of \( h \) with \( P \) depends on the actual set of grooves selected. In figure 4.13, the variation of \( h \) with \( P \) for a groove having \( a = 0.025 \) mm and \( \beta = 62^\circ \) is included for comparison which shows the same range of values of \( h \) with \( P \) as the random combination of grooves. The agreement between the two curves is not satisfactory. The range of groove parameters used for the random grooves is quite wide and a single equivalent groove showing the same variation of \( h \) with \( P \) cannot be obtained. Further, the range of pressures used in the calculations is also quite wide. When the range of groove parameters is not wide and when the range of pressures used in the calculations is not quite large, an equivalent groove showing a more satisfactory agreement can be expected.

4.6. VERIFICATION OF THE ANALYTICAL MODEL

The analytical model is verified by i) comparison of the calculated and experimental liquid metal profiles obtained by pouring liquid metal against a chill with a known surface profile and ii) comparison of calculated variation of \( h \) with pressure with the experimental values published in literature [47].

4.6.1 Comparison of calculated and experimental liquid metal profiles

The results of the analytical model are verified by pouring castings of LM 6 alloy against flat chills having parallel \( V \) grooves. The mold profile and the liquid metal profile obtained along two corresponding lines on the mold and casting are shown.
in figure 4.14. The two profiles show a satisfactory match, and the liquid metal profile shows a radius as expected. The groove parameters $a$ and $\beta$ for a few grooves on the chill surface and the radius of the casting surface corresponding to these grooves are measured from such profiles. The radius of the casting surface corresponding to the groove parameters is also obtained by calculation. The calculated and measured radii on the casting surface are given in Table 4.3. The agreement between the calculated and measured values of radii is satisfactory and this validates the assumption regarding the shape of the liquid metal profile.

4.6.2 Comparison of calculated and experimental values of $h$

The variation of $h$ for aluminium reported by Davies [47] for values of pressure in the range of 0 to $0.5 \times 10^5$ Pa agrees closely with the shape of the curves obtained by the model as shown in figure 4.15. The maximum deviation between the calculated and measured values of $h$ is less than 5%. Values of the parameters used in these calculations are:

\[
a = 0.05 \text{ mm}, \quad \alpha = 170^\circ, \quad \beta = 60^\circ, \quad \sigma = 0.840 \text{ N/m},
\]

\[
h_{\text{max}} = 2009 \text{ W/m}^2 \text{ K} \quad \text{and} \quad k = 0.0241171 \text{ W/m K}.
\]

This shows that an appropriate choice of the above parameters can give satisfactory agreement for the variation of $h$ with pressure when the range of applied pressures is not large.
FIG. 4.14 COMPARISON OF MOLD AND CASTING SURFACE PROFILES

FIG. 4.15 VARIATION OF h WITH PRESSURE (47).
### Table 4.1

Comparison of $R_{\text{calculated}}$ and $R_{\text{measured}}$ values

($\theta = 170^\circ, \sigma = 0.840 \text{ N/m}$)

<table>
<thead>
<tr>
<th>$a$ (mm)</th>
<th>$\phi$ (°)</th>
<th>$R_{\text{cal}}$ (mm)</th>
<th>$R_{\text{measured}}$ (mm)</th>
<th>% deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1041</td>
<td>54</td>
<td>0.727</td>
<td>0.75</td>
<td>2.57</td>
</tr>
<tr>
<td>0.22</td>
<td>68</td>
<td>0.586</td>
<td>0.66</td>
<td>11.23</td>
</tr>
<tr>
<td>0.66</td>
<td>65</td>
<td>0.660</td>
<td>0.76</td>
<td>10.52</td>
</tr>
<tr>
<td>0.166</td>
<td>70</td>
<td>0.624</td>
<td>0.56</td>
<td>-11.43</td>
</tr>
</tbody>
</table>

### 4.7. ANALYSIS OF RANDOM MOLD PROFILES

The work in this section consists of three parts:

a. Generation of mold roughness profiles

b. Generation of liquid metal profile over the rough surfaces

c. Determination of $h$

#### 4.7.1. Generation of Rough Surfaces—Fractals

The word "fractal" was coined by the French Mathematician Benoit Mandelbrot [104]. A fractal is an entity which is invariant under a scale transformation. A typical example is a coast line which presents a similar appearance from a wide range of altitudes. Another example is a mountain range which shows the same type of undulations when viewed from various distances.

A machined surface, strictly speaking, is not a fractal surface because it is not scale invariant under all magnifications. The method of fractal generation can still be used to generate sur-
faces that resemble machined surfaces. A simplified method for generating fractal surfaces [104] to represent the geometry of a machined surface is described below and a flow chart for the determination of $h$ using the random mold profile model is given in figure 4.16.

A straight line $AB$ of length $x_1$ with two fixed end points $A$ and $B$ is chosen as shown in figure 4.17 (stage 1). This line lies along the $X$-axis and the $Y$-axis is taken perpendicular to it. The midpoint $D$ of the line $AB$ is displaced in a direction along the $Y$-axis by a random value in the range $-cx_1$ to $+cx_1$, where $c$ is a constant. The value of this constant determines the roughness of the profile generated. By the above procedure, two line segments $AD$ and $DB$ are generated (stage 2 in figure 4.17). The midpoints $E$ and $F$ of these segments $AD$ and $DB$ are again similarly displaced along the $Y$-axis, by random values in the range $-cx_1/2$ to $+cx_1/2$ thus generating 4 segments $AE, ED, DF$ and $FB$ (stage 3 in figure 4.17). Typical results for 4 stages are also shown in figure 4.17. The profile shown in stage 4 represents a fourth generation fractal. An eighth generation fractal is obtained by repeating the operation 8 times and the resultant fractal is shown in figure 4.18.

A fractal is an abstract entity. Therefore, scale factors can be chosen for the $X$ and $Y$ axes as desired. The selection can be done in such a way that the fractal resembles a real machined surface. Using these scales, the Root Mean Square (RMS) value of the roughness of the fractal surface generated can be determined.
FIG. 4.16 FLOW CHART FOR THE DETERMINATION OF h-RANDOM MOLD PROFILE MODEL
y1 - Random number in the range -c x1; b+c x1

c - Constant which determines roughness

Fig. 4.17 Generation of Fractal Surface.
FIG. 4.18 COMPARISON BETWEEN FRACTAL SURFACE AND REAL MOLD SURFACE.
For comparison, an actual surface profile obtained using a stylus instrument is also given in figure 4.18. The profiles show no significant difference and may be treated as similar. The actual fractal surfaces generated can have a wide range of roughness values for a given value of c and a few trials have to be performed to obtain a surface profile with the desired roughness.

4.7.2. Generation of Liquid Metal Profiles

The liquid metal profile in contact with the mold surface has a radius given by equation 4.3. If the liquid metal profile is drawn with a radius touching all the sharp upper corners of the mold, then this would lead to unstable configurations. Figure 4.19 shows such a situation. The configuration shown in figure 4.19a shows the liquid metal profile obtained by drawing two arcs AB and BC of radius R corresponding to a pressure P touching the mold profile at points A, B and C. This configuration is unstable since it does not have the least liquid metal surface area.

A more stable configuration can be obtained by drawing an arc of the same radius R touching the points A and C as shown in figure 4.19b. In generating liquid metal profiles in contact with the mold surface profile obtained by the method of fractals, it is necessary to take into account the stability of the liquid metal profile. Further, the contact angle should be maintained at the value $\alpha$. The procedure for generating the liquid metal profile is as follows:
(a) Unstable configuration.

(b) Stable configuration

FIG. 4.19 LIQUID METAL PROFILE FOR COMPLEX MOLD SURFACE.
The radius of the liquid metal is calculated using equation 4.3.

2.a) The farthest points on the mold surface profile are connected by an arc of radius $R$ such that this arc does not cut across any other point on the mold surface.

b) The arc is drawn such that the sloping wall of the groove makes an angle $\alpha$ (the advancing contact angle) with the tangent to the arc at the point of contact.

3. If the condition of the contact angle is not satisfied, then the arc of appropriate radius is drawn joining the farthest asperity points on the mold surface such that the arc does not intercept any other points on the mold surface. This situation would occur for the grooves for which the applied pressure is less than the critical pressure.

A computer program is developed for determining the liquid metal profile and this program automatically checks the stability of the configuration. A typical set of liquid metal profiles for a given mold profile at various applied pressures is shown in figure 4.20. The values used for the generation of mold and liquid metal profiles are presented in Table 4.4.
FIG. 4.20 VARIATION OF LIQUID METAL PROFILE WITH PRESSURE.
Table 4.4. Data used for the estimation of h with a fractal surface.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface tension of Aluminium</td>
<td>$c$</td>
<td>0.84 N/m</td>
</tr>
<tr>
<td>Advancing Contact Angle</td>
<td>$\alpha$</td>
<td>160$^\circ$</td>
</tr>
<tr>
<td>Fractal generation</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>R.M.S. Value of Mold Roughness</td>
<td></td>
<td>10 $\mu$m</td>
</tr>
<tr>
<td>Pressure difference</td>
<td>$p$</td>
<td>$(1-101) \times 10^2$ Pa</td>
</tr>
<tr>
<td>Constant</td>
<td>$c$</td>
<td>1.5</td>
</tr>
</tbody>
</table>

4.7.3. Determination of h

The value of the gap thickness $y_i$ is measured at each location $i$ along the X-axis. This is determined by counting the number of pixels between the mold and metal profiles on a computer graphic monitor and multiplying it by the scale factor. The equivalent gap thickness due to oxide films $y_f$ is added to each measurement and the equivalent gap thickness $y_{eq}$ is determined over the entire profile using the relation:

$$y_{eq} = \frac{1}{n} \sum_{i=1}^{n} y_i + y_f$$

where $n$ is the number of points measured.

The variation of $h$ with pressure for a fixed mold profile having a roughness of 10 $\mu$m RMS is shown in figure 4.21. A linear rela-
FIG. 4. 21 VARIATION OF h WITH MOLD ROUGHNESS AND PRESSURE.
tion can be obtained between log h vs log P as shown in the figure.

4.7.4. Verification of Random Mold Profile Model

The results of Nishida and Matsubara [63] showing the variation of h with P are replotted using units chosen in the present work in figure 4.22. These results are superimposed on the simulation results obtained using a roughness profile with an RMS value of 10 microns. The agreement is satisfactory considering the extent of scatter in the experimental and simulated results.

The variation of h with pressure in the results of Nishida and Matsubara [63] show a square root relationship while the results of Davies [47] show a saturation value. The differences in these two behaviors can be explained as follows:

The results of Davies are based on low values of pressure while the results of Nishida and Matsubara are based on very high pressures. From the model, it is seen that the combination of grooves on the surface affects the variation of h with pressure and that at low pressures, the secondary texture will not affect the value of h. In other words, the secondary textures can be treated as an increase in the value of $\gamma_f$ with higher pressures, however, the primary texture becomes unimportant in affecting the variation of h with pressure since these grooves with large mouth widths are easily filled due to the low critical pressure. Here, smooth surfaces have very fine grooves with low value of $a$ and they alone are responsible for the variation of h.
FIG. 4.22 COMPARISON BETWEEN SIMULATED AND EXPERIMENTAL RESULTS [63]
With pressure. Since the type of mold surface preparation is not clearly explained by Davies or by Nishida and Matsubara, it is presumed that the former has a normal rough mold surface and the latter has used a smooth polished machined surface for the mold.

4.8 SUMMARY

In this chapter, the variation of $h$ with time at the casting/mold interface in one dimensional heat flow is discussed. The total duration of the variation of $h$ with time at the casting/mold interface is divided into 3 stages—stage I, with a high fluctuating value of $h$, stage II with a steady value and stage III with either an increasing value of $h$ due to increase in contact pressure or decrease in $h$ due to increase in the air gap between the casting and the mold. The variation of $h$ in stage III is determined by the casting geometry. Though there are fluctuations in the value of $h$ in stage I, a mean value of $h$ can be found based on the metallostatic head associated with pouring. The value of $\sigma$ and $\alpha$ based on a mean temperature in this stage should be used. The determination of the mean value of $h$ in stage I and the steady value of $h$ in stage II is determined by the procedure described in this chapter. The analysis is carried out for two types of mold profiles: 1. a mold profile consisting of a combination of uniform V grooves, and 2. a mold profile generated by the method of fractals. The casting surface profile which is formed when the liquid metal comes into contact with the mold surface is modeled in this chapter from a consideration of the surface tension of the liquid metal, metallostatic pressure,
The contact angle and parameters $a$ and $b$ relating to the groove. The presence of oxide and other films on the casting and mold surfaces is included in the model in the form of an additional air gap of width $y_f$ between the casting and the mold. Expressions are developed for calculating the value of $h$ at a given pressure for a given set of uniform V grooves. The variation of $h$ with $P$ in this case shows a low value of $h$ at low pressures until a critical pressure $P_c$ is reached. There is a step in the value of $h$ corresponding to $P_c$ and with increase in pressure beyond $P_c$, $h$ rises steeply, and then reaches the value of $h_{max}$ asymptotically.

$h$ varies with $P$ gradually in the case of real mold surfaces. This is due to the presence of a large number of grooves having different parameters $a$ and $b$. This situation is modeled by assuming that the mold surface consists of a random combination of uniform grooves. The overall thermal resistance of a random combination of grooves is obtained as a series/parallel combination of the thermal resistances of the constituent grooves.

Analysis of the interface generated by the method of fractals is carried out on a computer using numerical techniques.

The analytical model is verified by comparing the predicted and actual liquid metal profiles formed against a known mold profile. Satisfactory agreement is obtained in regard to the calculated and measured radii. By selecting a set of suitable values for the various parameters used in the model, the variation of $h$ with $P$ very close to that reported by Davies [47] could be obtained in
the case of the analytical model. By using a wide range of pressures with a mold surface of roughness 10 μm RMS and using the method of fractals, the variation of h with P close to that reported by Nishida and Matsubara [63] could be obtained. These results indicate that the model developed in this chapter is satisfactory.
CHAPTER 5

EXPERIMENTAL DETAILS

The details of the experiments carried out to determine the interfacial heat transfer coefficient $h$ in one-dimensional heat flow are described in this chapter.

The experiments carried out are categorized as follows:

a) Experiments with a constant metallostatic head
b) Experiments with a receding casting interface

The effect of metallostatic head on $h$ is investigated in the case of experiments with a constant metallostatic head. The effect of metallostatic head, sprue height and ingate size on $h$ are investigated in the case of experiments with a receding casting interface. In these experiments, the effects of chill roughness and insulating mold coats i.e., fireclay and alumina on $h$ are also studied. The details of the experimental arrangements are discussed below. The arrangement to measure the change in the size of the air gap in a receding casting interface experiment is also described.

5.1 EXPERIMENTS WITH CONSTANT METALLOSTATIC HEAD:

5.1.1 Materials used:

Alloys: Three alloys are used in this investigation namely LM 6, LM 24, and binary Al-2.7% Li alloy. The composition of the alloys are given in Table 5.1.

The alloys are obtained in ingot form and used directly for melting.
Clay/graphite crucibles of size A20 are used for melting the alloys.

### Fluxes and Degasser

1. Coverall-II, a proprietary flux supplied by M/s Greaves Foseco is used as a cover flux during melting of LM 6 and LM 24 alloys.

2. Coverall-36 A, a proprietary flux supplied by M/s Greaves Foseco is used as the modifying flux for LM 6 alloy.

3. LiCl+LiF in the ratio 1:1 is used as a cover flux in the melting of Al-2.7% Li alloy.

### Degasser

Degasser -190, a proprietary degasser supplied by M/S Greaves Foseco in the form of tablets weighing 50 gms each is used for degassing LM 6 and LM 24 alloys.

### Gas

Argon gas is used for providing an inert atmosphere during the melting of Al-2.7% Li alloy.

### Molding Sand

Synthetic green molding sand containing silica sand of AFS grain fineness number 55 is used for preparing the molds. 6% Sodium based bentonite is used as binder with %
moisture. The sand is milled in a laboratory muller before the preparation of the molds.

5.1.2 Experimental set-up:

The experimental set-up consists of the chill and mold assemblies, and molten metal is poured into the asbestos lined mold cavity. The time-temperature data from the set-up is obtained from thermocouples which are inserted into the chill and mold assembly at appropriate locations. The output of the thermocouples is fed into a data acquisition system. The digitized temperature data from the data acquisition system are fed to a personal computer for storage on the hard disk. A schematic arrangement of the set-up for the determination of $h$ is shown in figure 5.1. The procedure for assembling the experimental set-up is described below.

5.1.3 EXPERIMENTAL PROCEDURE:

The sequence of operations for performing an experiment are given below:

a. Preparation of thermocouples
b. Preparation of the chill assembly
c. Preparation of the mold assembly
d. Assembling the experimental set-up
e. Melting and pouring
f. Temperature measurement.
FIG. 5.1 SET-UP FOR THE DETERMINATION OF h (SCHEMATIC)
1. Preparation of thermocouples:

Chromel-Alumel wires of size 28 SWG and of adequate length are used for making the thermocouples. A thermocouple bead is formed by twisting the ends of the wires with a pair of pliers and welding this end by an electric arc in a gas atmosphere consisting of a mixture of hydrogen and nitrogen. Twin bore porcelain beads having an outside diameter of 3 mm and a length of 25 mm are used for covering the full length of the thermocouple wires.

b. Preparation of the chill assembly:

A cylindrical cast iron chill with dimensions as shown in figure 5.2 is used. The composition of cast iron used for the chill is:

\[ \text{C} - 3.45\%, \text{Si} - 2.45\%, \text{Mn} - 0.5\%, \text{P} - 0.1\%, \text{S} - 0.08\% \]

Two thermocouples are inserted into two holes at locations A and B marked in the figure. These holes are drilled to a depth of 2 mm on the side of the cast iron chill. Polarity of each thermocouple wire is checked and marked before insertion into the chill. The thermocouple bead is pressed against the end of the hole and held in place by self setting cement placed on the outside. This prevents the thermocouples from getting loosened from the chill during the course of the experiment.

The procedure for preparing the smooth uncoated chill is as follows. The surface of the chill which comes into contact with molten metal is first obtained by fine turning on a lathe. Before each experiment, the chill surface is cleaned and polished.
A, B = Thermocouple locations.

All dimensions are in mm.

FIG. 5.2 DIMENSIONS OF CHILL.
with 4/0 emery paper before each experiment. Air is blown to
remove the dust particles sticking on the surface. It is wiped
clean with cotton an then cleaned in a solution of dilute HCl.
It is then washed with water, dried and placed in a mild steel
ring. This ring has a slot for allowing the thermocouple leads.
The annular space between the chill and the ring is filled with
ceramic wool taking care to see that the flat surface of the
chill is left open. The chill assembly appears as shown in
figure 5.1.

Preparation of the mold assembly:

A cylindrical wooden pattern of 50 mm dia is used for forming the
mold cavity. This pattern is wrapped with an asbestos fiber
board (3 mm thickness) of required height. It is held tightly in
place by using a short length of binding wire. The pattern with
the asbestos sheet around it is located in the mold box such
that it is aligned with the holes drilled in the mold box for the
thermocouples. Cylindrical pipes of 150 mm diameter cut to the
required lengths are used as mold boxes. Green molding sand is
then rammed around the pattern. The pattern is then removed
leaving the asbestos fiber board in the mold. Holes are then
pierced in the asbestos board with a sharp tool for inserting the
thermocouples. The details of the mold assembly for constant
metallostatic head experiments are shown in figure 5.4a and a
photograph of the mold assembly ready for pouring the test cast-
ing is shown in figure 5.4b. Three casting heights—100, 150, &
200 mm are used in general experiments. In addition, two experi-
FIG. 5.3 CHILL ASSEMBLY.

FIG. 5.4 MOLD ASSEMBLY (SCHEMATIC)

b MOLD ASSEMBLY.

1 Asbestos fiber board 2 Thermocouples

FIG. 5.4 MOLD ASSEMBLY FOR CONSTANT METALLOSTATIC HEAD EXPERIMENTS.
ments are performed with LM 6 and LM 24 alloys and with a casting height of 500 mm to find the limiting value of \( h \), i.e., \( h_{\text{max}} \). The chill assembly is placed at the bottom of the mold and the top end is kept open to the atmosphere. The mold assembly has two thermocouples at locations C and D at distances of 10 and 35 mm respectively from the chill end of the mold.

d. Assembling the experimental set-up:

The thermocouple beads are inserted in the mold and the free ends of the wires of the thermocouples are fixed to a connector block which in turn is electrically connected to the data acquisition system with a 14 core shielded cable. After checking the polarity of the thermocouples by heating the bead, they are inserted into the holes in the mold. Care is taken to ensure that the bead is located at the center of the mold cavity and that it does not hinder the casting movement due to contraction. The mold assembly is aligned properly on top of the chill assembly. The thermocouples from the chill are also connected to a second connector block which, in turn, is connected to the input terminal of the data acquisition system using another 14 core shielded cable.

The data acquisition system 570 manufactured by M/s Advanced Micronic Devices Pvt. Ltd., is used for recording the time temperature data from the chill and the solidifying casting. It has a 12-bit A-D converter and hence has a resolution of \( 1/4096 \) of the full range of input. The data acquisition system is shown in
The data acquisition system is set for an input in the range of 0-50 mV which is suitable for use with chromel-alumel thermocouples. The data acquisition system has provisions for connecting inputs in the differential mode or single ended mode i.e., 16 channels for input in differential mode or 32 channels for input in single ended mode. The differential mode is used in the present set of experiments because it has a common mode rejection by which stray signals can be eliminated. The system has a built-in multiplexer which chooses one channel at a time for digitizing the input. The data acquisition is controlled by software developed in the present work, which is run on the personal computer. The output from the A-D converter is stored on the hard disk of the personal computer. With these preparations, the set-up is ready for pouring.

Table 5.2

Specifications of the data acquisition system

<table>
<thead>
<tr>
<th>Interface</th>
<th>Custom parallel bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog input capacity</td>
<td>32 single ended or 16 differential</td>
</tr>
<tr>
<td></td>
<td>input channels</td>
</tr>
<tr>
<td>Input impedance</td>
<td>&gt; 100 MΩ in parallel with 1000 pF</td>
</tr>
<tr>
<td>Common mode rejection</td>
<td>&gt; 80 dB at 60 Hz</td>
</tr>
<tr>
<td>Full scale input</td>
<td>12 ranges, ±2.5 mV to ±10V</td>
</tr>
<tr>
<td>Gain selection</td>
<td>X1, X10, X100 switch selectable</td>
</tr>
<tr>
<td></td>
<td>X1, X2, X5, X10 Software selectable</td>
</tr>
<tr>
<td>Input ranges</td>
<td>± 10V, ±5V, ±2.5V, 0 to +10V, 0 to +5V</td>
</tr>
<tr>
<td></td>
<td>switch selectable</td>
</tr>
<tr>
<td>Resolution</td>
<td>12 bits (1 part in 4096)</td>
</tr>
<tr>
<td>Maximum sampling rate</td>
<td>31.4 k samples per second</td>
</tr>
<tr>
<td>Input noise</td>
<td>30 µV p-p typical, 0.1 to 10Hz</td>
</tr>
<tr>
<td>Non-linearity</td>
<td>± 0.025% FSR</td>
</tr>
<tr>
<td>Accuracy</td>
<td>adjustable to ±1 lsb/range</td>
</tr>
</tbody>
</table>
2. Melting and pouring:

The alloys are melted in a 7KW, 1000 °C electrical resistance furnace shown in figure 5.6. In the case of LM 6 and LM 24 alloys, the alloy ingots are cut into smaller pieces and placed in a clay/graphite crucible of size A 20 located in the furnace and the cover flux, Coverall 11A, is added. Coverall 11A is also added during the time of fresh metal addition to ensure that the metal always remains under flux cover. When the metal is melted completely and the required temperature is attained, the crucible is removed from the furnace and is placed in a suitable holder for pouring. The dross on the surface of the liquid metal is skimmed off with a preheated tool. The temperature of the liquid metal is determined using an immersion pyrometer. When the metal reaches the required temperature of degassing, it is skimmed and then degassed by plunging crushed and dried tablets of Degasser 190. In the case of LM 6 alloy, the modifying flux, Coverall 36A, is added after degassing. When the temperature of the molten metal reaches the required pouring temperature, the data acquisition system is switched on and the metal is poured into the mold. The data is recorded for a period of 6 minutes for each experiment. The degassing and pouring temperatures for the alloys are given in Table 5.3.
FIG. 5.5 DATA ACQUISITION SYSTEM.
1. A/D Converter

FIG. 5.6 FURNACE USED FOR MELTING.
Table 5.3. Melt treatment temperatures

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Degassing temp. °C</th>
<th>Pouring temp. °C</th>
<th>Modification temp. °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM 6</td>
<td>750</td>
<td>700</td>
<td>730</td>
</tr>
<tr>
<td>LM 24</td>
<td>810</td>
<td>760</td>
<td>no modification</td>
</tr>
<tr>
<td>Al-Li</td>
<td>no degassing</td>
<td>750</td>
<td>no modification</td>
</tr>
</tbody>
</table>

For the Al-2.7% Li alloy, melting is carried out under a flux cover consisting of a mixture of lithium fluoride and lithium chloride in the ratio 1:1. When the melt attains a temperature of 750 °C, it is taken out and held for a period of 6-8 minutes and then poured into the mold under an argon gas cover maintained using a metal hood over the crucible. The plunger used for treating the alloy is preheated to a temperature of about 120 °C before using.

f. Temperature measurement:

Temperature measurements from the chill and the solidifying casting form the basis for the determination of the interfacial heat transfer coefficient $h$. The temperature data from the thermocouples is recorded using the data acquisition system. To ensure reliable data acquisition, the thermocouples have to be properly calibrated. Further, the output from the thermocouples is obtained in the form of counts from the A/D converter by using a software developed for this purpose.
1) Calibration of the thermocouples:

Calibration of the thermocouples is carried out at two temperatures i.e., 100 °C corresponding to the boiling point of water and 660 °C corresponding to the melting point of pure aluminium. The millivolt readings measured using the thermocouples are compared with the standard values reported in thermocouple data sheets and converted to temperature values. The difference between the temperatures obtained by measurement and the standard value is found to be less than 2°C.

11) Data acquisition software:

Data acquisition is carried out using a program written in PASCAL and is controlled by pressing a key on the key board of the PC/AT. When the necessary command is issued for data acquisition by the software, the channel number and the counts for that channel are obtained. The software is written such that once the time for data acquisition is reached, all the four thermocouples at locations A, B, C and D are read sequentially. The next set of readings are taken after 0.5 seconds and the values of the counts corresponding to the mV output of the thermocouples are obtained. The time, channel number and the counts are then stored in a file in the hard disk.

The data file containing information on time, channel number and counts is then fed as an input file to another program to convert the counts into temperatures. This conversion is performed off line to ensure that there is no time delay due to conversion
during data acquisition. A look-up table with temperature intervals of 50 °C from 0 to 1000 °C is prepared using standard thermocouple data sheet and is fed into the computer program. This program reads the counts obtained from a channel, converts it into millivolts and the temperature corresponding to this voltage is obtained by linear interpolation using the look-up table.

The details of the various experiments performed with a constant metallostatic head arrangement are given in Table 5.4 below. Details of coating the chills with refractory materials and details of the rough uncoated chill are described in later sections.

Table 5.4. Constant metallostatic head experiments

<table>
<thead>
<tr>
<th>Alloys</th>
<th>Chill condition</th>
<th>Casting heights L(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>200</td>
</tr>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>150</td>
</tr>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>100</td>
</tr>
<tr>
<td>LM24</td>
<td>Smooth uncoated</td>
<td>200</td>
</tr>
<tr>
<td>LM24</td>
<td>Smooth uncoated</td>
<td>150</td>
</tr>
<tr>
<td>LM24</td>
<td>Smooth uncoated</td>
<td>100</td>
</tr>
<tr>
<td>LM6</td>
<td>Alumina coating</td>
<td>200</td>
</tr>
<tr>
<td>LM6</td>
<td>Fireclay coating</td>
<td>200</td>
</tr>
<tr>
<td>LM24</td>
<td>Alumina coating</td>
<td>200</td>
</tr>
<tr>
<td>LM24</td>
<td>Fireclay coating</td>
<td>200</td>
</tr>
<tr>
<td>LM6</td>
<td>Rough uncoated</td>
<td>200</td>
</tr>
<tr>
<td>LM6</td>
<td>Rough uncoated</td>
<td>100</td>
</tr>
<tr>
<td>Al-Li</td>
<td>Smooth uncoated</td>
<td>100</td>
</tr>
</tbody>
</table>
5.2 EXPERIMENTS WITH A RECEDING CASTING INTERFACE:

The materials used in these experiments, chill assembly and the data acquisition system are the same as those described in section 5.2. However, the molds used and the arrangement of the mold/chill assembly are different from those described earlier. Further, an additional thermocouple is used at a location \( z \) in the ingate (figure 5.7a) to monitor the freezing time of the ingate, in these experiments. The mold details and the details of the mold/chill assembly are described below:

5.2.1. Mold details:

Figure 5.7a gives the details of the experimental set-up used in this study. Figure 5.7b shows the mold without the chill in position to show the interior of the mold cavity and figure 5.7c shows the mold assembly ready for pouring. As seen in figure 5.7, the mold is assembled using two rectangular mold boxes and a sprue cup made of a short length of cylindrical pipe of required height. The upper mold box has two cylindrical cavities connected to each other by an ingate of appropriate size, while the lower mold box has a plane surface. The main casting cavity is provided with an asbestos lining. Holes are provided at distances of 10 and 35 mm from the free face of the chill by piercing through the asbestos board for inserting the thermocouples. The second cavity acts as a down sprue. The required head of metal is provided by the sprue cup made of a cylindrical pipe of appropriate height which is rammed with molding sand so that a sprue of 50 mm dia and required height is obtained.
Sprue height (S): 50, 100, 150 mm
Casting height (C): 100, 150, 200 mm
Ingate sizes:
- 46x30mm (large)
- 20x20mm (medium)
- 18x12mm (small)

FIG 5.7a EXPERIMENTAL SET-UP
b. VIEW OF MOLD CAVITY.

c. COMPLETE ASSEMBLY

FIG. 5.7 MOLD ASSEMBLY FOR RECEIVING CASTING INTERFACE EXPERIMENTS.
The mold-chill assembly is placed such that the liquid metal rises through the mold cavity from the bottom and touches the chill which is placed on top of the mold. To vary the time for which the pressure of the liquid metal acts on the chill, three ingate sizes 46 mm x 30 mm (large), 20 mm x 20 mm (medium) and 16 mm x 11 mm (small) are used. The time of solidification of the ingate is monitored by an additional thermocouple placed at location E in the center of the ingate. From a study of the freezing time of the ingates, it is found that the smallest ingate size gives a reasonable time for studying stage III, while the large ingate did not freeze within the duration of the experiment. The time available for observing stage III with medium ingate is quite small. Hence, in all the subsequent experiments, the small ingate is used for the receding casting interface experiments. To vary the pressure acting at the interface, three different space heights ($S$) 50, 100, and 150mm are used. To study the effect of the length of the casting on $h$, three casting heights ($L$) 150, 150 and 200mm are used. The procedure for melting and pouring the castings and acquiring time-temperature data is same as that described in the previous section. The only difference in this case is the monitoring of the ingate temperature on an additional channel.

The details of the various experiments with a receding casting interface are given in Table 5.5.
### Table 5.5. Receding casting interface experiments

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Chill condition</th>
<th>Casting height (L) mm</th>
<th>Sprue height (S) mm</th>
<th>Ingate Size (mm x mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>100</td>
<td>100</td>
<td>46x30 (large)</td>
</tr>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>100</td>
<td>100</td>
<td>20x20 (medium)</td>
</tr>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>100</td>
<td>100</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>100</td>
<td>150</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>150</td>
<td>150</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>200</td>
<td>150</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM24</td>
<td>Smooth uncoated</td>
<td>100</td>
<td>150</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM24</td>
<td>Smooth uncoated</td>
<td>150</td>
<td>150</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM24</td>
<td>Smooth uncoated</td>
<td>200</td>
<td>150</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM5</td>
<td>Smooth uncoated</td>
<td>200</td>
<td>50</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM6</td>
<td>Smooth uncoated</td>
<td>200</td>
<td>100</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM5</td>
<td>Smooth uncoated</td>
<td>200</td>
<td>150</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM24</td>
<td>Smooth uncoated</td>
<td>200</td>
<td>50</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM24</td>
<td>Smooth uncoated</td>
<td>200</td>
<td>100</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM24</td>
<td>Smooth uncoated</td>
<td>200</td>
<td>150</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LX6</td>
<td>Alumina coating</td>
<td>150</td>
<td>100</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>L124</td>
<td>Alumina coating</td>
<td>150</td>
<td>100</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM6</td>
<td>Fireclay coating</td>
<td>150</td>
<td>100</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM24</td>
<td>Fireclay coating</td>
<td>150</td>
<td>100</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>LM6</td>
<td>Rough uncoated</td>
<td>150</td>
<td>100</td>
<td>18x12 (small)</td>
</tr>
<tr>
<td>Al-Li</td>
<td>Smooth uncoated</td>
<td>100</td>
<td>50</td>
<td>13x12 (small)</td>
</tr>
</tbody>
</table>

### 5.3 EFFECT OF INSULATING COATINGS ON CHILL SURFACE:

The constant metallostatic head arrangement (L = 200 mm) and the receding casting interface arrangement with casting heights 150 mm, sprue heights 100 mm and ingate size 18x12 mm are used in this study. The molding, assembling, melting, pouring and data acquisition are as described earlier. Coating materials and method of coating the chill surface are described below.
5.3.1 Coatings Used:

Two coating materials namely fireclay and alumina are used. The compositions of the coating slurry used are given below:

<table>
<thead>
<tr>
<th>Fireclay coating</th>
<th>Alumina coating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fireclay powder (300 mesh)</td>
<td>Alumina powder (300 mesh)</td>
</tr>
<tr>
<td>Sodium silicate</td>
<td>5 %</td>
</tr>
<tr>
<td>Water</td>
<td>45 %</td>
</tr>
</tbody>
</table>

5.3.2 Coating procedure:

The chill is heated to a temperature of 110 °C. The coating is sprayed on to the heated chill surface by a spray gun operated by compressed air. The coating obtained is of uniform thickness and surface finish. Maintenance of this temperature is essential for obtaining necessary strength, uniformity of thickness and surface finish of the coating.

5.3.3 Measurement of coating thickness:

The thickness of the coating is measured by an eddy current test instrument. The response of the instrument for the coating thickness is compared with the responses for different values of lift off (obtained by inserting layers of insulating sheets of known thickness) on an uncoated reference chill and selecting the matching value. A coating thickness of 0.24 mm is obtained in the case of fireclay and a coating thickness of 0.46 mm is obtained in the case of alumina. The time-temperature data obtained from the experiments using the coated chills are recorded for subsequent analysis.
5.4. EFFECT OF CHILL ROUGHNESS ON H:

For this study, experiments are carried out using both constant mettallostatic head and receding casting interface using LM 6 alloy. A casting height of 200 mm is used for the case of the constant mettallostatic head. For the receding casting interface experiment, the casting height (L) is 150 mm, sprue height (S) is 120 mm, and the size of ingate is 18 mm x 12 mm. Two chills having different values of roughness are used to evaluate the effect of change in the chill roughness on interfacial heat transfer coefficient. A normal smooth chill surface is produced by fine turning on a lathe and then polishing this surface with a 600 emery paper. This smooth chill is the same as that used in the previous experiments. A rough chill surface is produced by machining uniform grooves on a shaping machine maintaining uniform depth and distance between the grooves. The time-temperature data are obtained from the casting experiments using the rough chill and stored for subsequent analysis.

5.4.1 Surface roughness measurement:

The surface roughness of the chill and the casting surface made against this chill are obtained using the Perhrometer, a stylus instrument. A run of about 5 mm is used for the roughness measurement. The instrument directly gives the roughness profile as well as the values of surface roughness in terms of $R_A$, $R_{max}$ etc. The surface roughness measurements on the chill and the casting produced against the chill at predetermined locations are made and compared with each other. Roughness profiles for a smooth
The changes in the dimensions of the air gap with time formed between the casting and the chill in the case of a receding casting interface experiment are measured using an inductive gauge in this study. The mechanical arrangement, and the details of the displacement measuring set-up are described here. The procedure for molding, melting, pouring and temperature measurement are as described earlier.

5.5.1. Mechanical arrangement:

The arrangement of the chill with the inductive gauge is shown in figure 5.9. The method is based on the procedure described by Isaac et al [34]. The movement of the casting surface is followed by an assembly of an aluminium freeze-on plate and a stainless steel pin riveted on to this plate. The molten metal engulfs the freeze-on plate and solidifies around it. Any subsequent movement of the casting surface causes the freeze-on plate - stainless steel pin assembly to move with it. The mechanical details of the complete arrangement are given below:

An aluminium plate of size 10 mm x 14 mm x 0.9 mm bent into a U shape with each arm 2 mm long is used as a freeze-on plate. One end of a stainless steel pin of 2.5 mm diameter and length 120 mm is riveted to the center of the freeze-on plate. This stainless
Fig. 5.5 Roughness Profiles of Chill and Corresponding Casting.
b. OVERALL VIEW OF CHILL.
1 Freeze-on plate 2 Chill
3 Inductive gauge plunger

c. CLOSE VIEW OF INDUCTIVE GAUGE.
1 Inductive gauge 2 Light spring
3 Tube retaining spring

FIG. 5 9 CHILL ARRANGEMENT FOR AIR GAP MEASUREMENT.
In the tube is such that it allows free movement of the central due to differential thermal expansion. The diameter of the hole is the same material for the tube and the pin reduces possible errors fixing the inductive gauge after initial adjustments. Using the aluminum block could be assisted by a locking screw provided for the casting surface. The location of the inductive gauge is with the movement of thereece-on place due to the movement of metal. The force offered by the spring is negligible to interference that the pin does not move freely due to touch in the right used as a counter. A slight spring shown in figure 4.0 ensures inductive gauge with a thicker block of size 12 mm x 9 mm x 9 mm to the plunger of an

The central stainless steel pin is connected to the plunger of an

With proper alignment.

With a hole of diameter 12.5 mm for holding the inductive gauge and which carries an aluminum block of size 25 mm x 25 mm x 25 mm is threaded so as to carry a bent aluminum bracket of length 90


to remain flush with the surface of the chill which faces the centrally in the chill. One end of the tube is squared so that it passes through a stainless steel tube of 4.5 mm diameter, 2.9 mm inside diameter and 92 mm length which is threaded stainless pin passes through a stainless steel tube of 4.5 mm diameter,...
5.5.2 Measurement of displacement:

An inductive gauge with range = 1mm is used for measurement of displacement. The inductive gauge is connected to NWS/13-5 single channel Carrier Frequency Amplifier (CFA). The inductive gauge and CFA are supplied by Hottinger Baldwin Messtechnik GmbH, West Germany.

The CFA is balanced after removing the plunger of the inductive gauge. The amplifier is set to measure a total displacement of 0.5 mm of the plunger. The calibration of the inductive gauge is carried out using a micrometer depth gauge with a least count of 0.01 mm. Known displacements are given to the plunger of the inductive gauge and the indications on the CFA meter are noted to ensure accuracy in the full range. The DC output of the CFA is fed into one of the channels of the data acquisition system through a potential divider so that the maximum input voltage to the data acquisition system is less than 50 mV. Figure 5.10a gives a schematic of the set-up used for air gap measurement. The experimental set-up for air gap measurements showing the chill with inductive gauge and CFA are shown in figures 5.10 b and c.

5.5.3 Experimental procedure:

The freeze-on plate is riveted to one end of the stainless steel pin which is then inserted into the stainless steel tube fixed to the chill. The plunger of the inductive gauge with the fiber block at its end is then fixed to the other end of the central
b. MOLD ASSEMBLY.

1. Chill arrangement for air gap measurement.

C. DATA ACQUISITION SYSTEM

1 A/D Converter 2 Chill assembly 3 Carrier frequency amplifier.

FIG. 5.10 SET-UP FOR AIR GAP MEASUREMENT.
The inductive gauge is mounted in position and the electrical connector is fixed to it. The chill is placed on the mold assembly for receding casting interface studies. The position of the inductive gauge is adjusted such that adequate movement of the plunger can be recorded during the movement of the interface. Molten metal is then poured and the displacement data is recorded along with the time temperature data.

5.6 CONFIRMATORY EXPERIMENTS:

Two sets of experiments are carried out to confirm that the heat flow is one dimensional in the present investigation. The first set of experiments on the molten metal side consists of draining the liquid metal from the mold in the receding casting interface experiment and studying the liquid/solid interface. The second set of experiments are aimed at finding the difference between the center and surface temperature of the chill in a plane perpendicular to the axis of the chill.

5.6.1. Draining experiments:

The experimental set-up is the same as that used for the determination of $h$ for the receding casting interface studies but in this case, the thermocouples are not used. The dimensions of the casting are: casting height (L) 150 mm, sprue height (S) 100 mm and ingate size 46 mm x 30 mm. The green sand mold with the asbestos lining is lifted away from the bottom, box one minute after end of pouring. The casting obtained in the case of LM 6 is shown in figure 5.11 which shows a plane front solidification.
thus validating the assumption of unidirectional heat flow. Similar results are obtained with LM 24 alloy. The details of the experiments are given in Table 5.6.

5.6.2 Chill temperature studies:

The cast iron chill with two thermocouples, one inserted at the center and the other inserted to a depth of 2 mm from the surface is heated on a hot plate. The surface of the chill is insulated by ceramic wool. The temperatures from the thermocouples are measured and the difference is noted. The difference between the two measured temperatures is found to be less than 3 °C. This validates the assumption that the heat flow in the chill is unidirectional.

Table 5.6. Draining experiments

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Casting height (L) (mm)</th>
<th>Sprue height (S) (mm)</th>
<th>Ingate size (mm x mm)</th>
<th>Time for draining (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM 6</td>
<td>150</td>
<td>100</td>
<td>46 x 30</td>
<td>60</td>
</tr>
<tr>
<td>LM 24</td>
<td>150</td>
<td>100</td>
<td>46 x 30</td>
<td>60</td>
</tr>
</tbody>
</table>

5.7 REPEATABILITY STUDIES:

To check the repeatability of the values of \( q \) and \( h \) obtained in these experiments, trials are carried out using a casting height of 150 mm, sprue height of 100 mm and small ingate of size 18 mm x 12 mm in the receding casting interface experiment. The results
of the repeatability studies yield a variation in $h$ which is
$\pm 4$% and may be considered to be quite satisfactory.

3.3 RADIOPHIC EXAMINATION OF TEST CASTINGS:

The castings are radiographed to measure the exact locations of
the thermocouples as this is necessary for computation of the
value of $h$. The details of the radiographic work are given
below:

- **X-ray unit**: Macrotank®L 200 kv Portable
  Industrial X-Ray unit.
- **Tube voltage**: 110 kV
- **Tube current**: 2 mA
- **Exposure time**: 4 minutes
- **Film**: NDT 65, Manufactured by M/s Hindustan
  Photo Films Mfg Ltd., India.

The developing procedure recommended by the film manufacturer is
used for developing the films. A positive print taken from a
typical radiograph is shown in figure 5.12 which gives the location
of the thermocouples at C and D.

The results of the experiments in the form of time temperature
data are analyzed for determining the value of $h$. Determination
of the heat flow rate $q$, interfacial heat transfer coefficient $h$
and the surface temperatures $T_C$ and $T_m$, using the measured
temperatures $T_1$, $T_2$, $T_3$ and $T_4$ from locations A, B, C and D
respectively form an Inverse Heat Conduction Problem (IHCP).
The procedure for the analysis is given in the next chapter.
FIG. 5.11 CASTING SURFACE AFTER DRAINING.

FIG. 5.12 TYPICAL RADIOGRAPH SHOWING THERMOCOUPLES.
CHAPTER 6
ANALYSIS OF EXPERIMENTAL DATA

6.1. INTRODUCTION

The determination of the heat flow rate per unit area, $q$, from the casting into the chill across the casting/mold interface as a function of time can be carried out using the time temperature data obtained from the experiments described in chapter 5. Along with $q$, the values of the interface temperatures $T_c$ and $T_m$ on the casting and chill can be obtained. This value of $q$ along with the surface temperatures $T_c$ and $T_m$ can be used for calculating $h$ as a function of time. The determination of $q$ at the surface from temperatures measured inside a body forms an 'Inverse Heat Conduction Problem' (IHCP). The inverse heat conduction problem is difficult to solve owing to the ill-posed nature of the problem. One of the satisfactory solution procedures available is the Nonlinear Estimation Procedure proposed by Beck [3]. The implementation of this procedure for determining the value of $q$ and hence $h$ in the present case is described in this chapter.

6.2 THE DIRECT AND THE INVERSE PROBLEMS

The direct problem in heat flow refers to the situation in which the boundary conditions in the form of temperatures or heat fluxes are known and the temperature distribution inside the body is to be obtained from the known initial and boundary conditions. Analytical and numerical methods are available for solving the partial differential equation 2.5 relating to the direct
problem satisfactorily. However, the direct problem has many experimental impediments due to the difficulties in determining the actual boundary conditions. For example, it may not be possible to attach a sensor and measure the surface temperature accurately due to the physical limitation imposed by the configuration.

The IHCP on the other hand, implies the estimation of the surface heat flux history or surface temperature history of a body given one or more measured temperature histories inside the heat conducting body. The word 'estimation' is used because in measuring the internal temperatures, errors are always present to some extent. Besides, the IHCP is an ill-posed problem and hence the heat flux cannot be calculated accurately. A comparison of the direct and the inverse heat conduction problems is shown in figure 6.1.

6.3. INVERSE HEAT CONDUCTION- AN ILL-POSED PROBLEM:

The solution of a well posed problem should satisfy the conditions of existence, uniqueness and stability. The solution of the inverse heat conduction problem does not satisfy these conditions \[82\] and hence it is an ill-posed problem. The IHCP is difficult to solve analytically because the temperature response at an interior location in a body due to a given stimulus at the surface is both delayed and diminished in amplitude. The presence of measurement errors further complicates the solution. Measurement of temperatures at discrete locations and at discrete
FIG. 6.1 COMPARISON OF DIRECT AND INVERSE PROBLEMS.
time intervals provides incomplete information for obtaining an accurate solution.

For the one-dimensional IHCP, when discrete values of $q_i$ are estimated at various points of time $i$, maximizing the amount of information implies small time steps between $q_i$ values. However, the use of small time steps frequently introduces instabilities in the solution of the IHCP [3].

The direct problem suffers from inaccuracies in the measurement of boundary temperatures or heat fluxes while the IHCP poses a difficult analytical problem. According to Beck [32] an accurate and tractable solution to the inverse problem would minimize both the disadvantages simultaneously.

6.4 SOLUTION OF THE INVERSE HEAT CONDUCTION PROBLEM:

The solutions available for the inverse heat conduction problem can be divided into two broad categories: 1. Exact solutions and 2. Statistical estimation.

6.4.1. Exact solutions

The exact solutions are analytical procedures which develop expressions for the boundary condition for a given temperature history in the casting. Exact solutions were developed by several investigators [31, 87, 105-108]. The analytical procedures are, however, limited [32] due to:
6.4.2. Statistical estimation:

Statistical estimation procedures solve the IHCP by maximizing (or minimizing) an objective function. The objective function SS is given as (86):

\[
SS = \sum_{i=1}^{n_s} \sum_{j=1}^{n_f} (Y_i, j - T_i, j)^2 w_i \sum_{k=1}^{n_f} \beta^2_k + w_0 \sum_{k=1}^{n_f-1} (\beta_{k+1} - \beta_k)^2
\]  \quad (6.1)

where

- \(SS\) = objective function
- \(n_s\) = number of sensors
- \(w_j\) = weighting constant for the temperature measurements from the jth sensor
- \(n_f\) = number of future measurements used
- \(Y\) = measurement (K)
- \(T\) = temperature (K)
- \(w_0\) = the zeroth order regularization constant
- \(n_p\) = number of parameters
- \(\beta\) = parameter (\text{W/cm}^2)
- \(w_1\) = the first order regularization constant

By putting \(w_j = 1\) and \(w_0, w_1 = 0\), a simplified form of the objective function is obtained which is the least square estimator based on the minimization of:

\[
F(q) = \sum_{i=1}^{1} (T_{est} - T_{mea})^2
\]  \quad (6.2)
The objective function is used by some investigators [3, 86]. The estimate for \( q \) can be a single discrete value at each location of the boundary at each point of time. Estimation of these sets of values is called a regularization method. If on the other hand, \( q \) is defined as a polynomial function on the boundary surface, then it is referred to as function specification method [85]. A brief literature survey on the solution of the IHCP is already presented in chapter 2.

6.5 SOLUTION PROCEDURE USED IN THE PRESENT WORK:

The experimental data in the form of time temperature values at four locations, two in the casting and two in the chill are analyzed for the determination of \( h \) by using the method proposed by Beck [3] in the present study. The configuration under analysis is shown in figure 6.2. It is necessary to estimate \( q \) at the surface \( S \), with known temperature \( T_2 \) at point \( P \) and known boundary temperature \( T_1 \) at the point \( Q \). The analysis in the region \( P \) to \( Q \) is a direct problem but in the region \( S \) to \( P \), the problem is an inverse one. Beck's solution procedure consists in minimizing
a function \( F(q) \) defined by equation 6.2 and the details of the solution procedure are given below.

### 6.5.1 Statement of the problem

Referring to figure 6.2, the heat flux \( q \) and the surface temperature \( T_0 \) at \( S \) have to be determined from the measured temperature \( T_2 \) at \( x_2 \). The boundary temperature \( T_1 \) at \( Q \) is specified. The solution must satisfy the one-dimensional heat conduction equation:

\[
\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = \rho c \frac{\partial T}{\partial t} \quad \ldots \ (6.3)
\]

### 6.5.2 Assumptions

1. The discretized values of temperature at time intervals \( \Delta t \) are known.

2. The heat flux across the interface is constant during any time interval \( \Delta \theta \) where \( \Delta \theta \) is an integral multiple of \( \Delta t \) i.e.,

\[
\Delta \theta = n \Delta t \quad \ldots \ (6.4)
\]

where \( n \) is a small integer. If \( t_m \) and \( t_n \) refer to the same point of time, then the subscripts \( M \) and \( n \) are related by the relation:

\[
mM = n \quad \ldots \ (6.5)
\]

The relation between \( M, n \) and the discretization of the values of \( q \) are shown in figure 6.3 [3]. This means that the regularization method is used in the present case.
FIG. 6.2 INVERSE HEAT CONDUCTION PROBLEM IN UNIDIRECTIONAL HEAT FLOW.

- $q$ - Heat flux per unit area at surface $S$ (Unknown)
- $T_0$ - The temperature at the surface $S$ (Unknown)
- $T_1$ - Known boundary temperature
- $T_2$ - Known internal temperature

FIG. 6.3 APPROXIMATION OF HEAT FLUX BY DISCRETE VALUES OF $q$ [3].
To take into account the delayed temperature response of an interior location, 'future temperatures' are used in estimating \( q \). Thus if \( r \) intervals of \( \Delta t \) are used in estimating \( q \), it is assumed that the same heat flux \( q \) flows during these \((r-1)\) future intervals. That is,

\[
q_{M+2} = q_{M+3} = \ldots = q_{M-r} = q_{M-1} \quad \ldots (6.6)
\]

where \( q_{M-1} \) is to be estimated.

The nonlinear estimation procedure determines the value of \( q \) which minimizes \( F(q) \) given in equation 6.2.

6.5.3 Procedure

The initial temperature distribution in the chill and casting is obtained from the measured temperatures and known boundary conditions. A suitable initial value for \( q \) (1 cal/cm\(^2\).s) is assumed and with this value, the temperature distribution at the end of each interval \( \Delta t \) for \( r \) number of intervals is determined using a suitable numerical method like FDM or FEM. This gives \( T_{\text{est}}(q) \) at \( x_1 \) at all points of time where measured temperatures are available. \( q \) is increased by a small value \( q \epsilon \) where \( \epsilon \) is a small value and the new value of \( T_{\text{est}}(q(1+\epsilon)) \) is determined. With these values, 'sensitivity coefficients' are calculated at each iteration during the estimation. The sensitivity coefficient \( \phi \) is defined as the first derivative of a dependent variable, such as temperature, with respect to an unknown parameter, such as a
heat flux component [62]. The sensitivity coefficient is determined using the following relation:

\[ \phi = \frac{\Delta T}{\phi_q} = \frac{T_{\text{meas}} - T_{\text{est}}(q)}{q} \quad \ldots (6.7) \]

The sensitivity coefficients are determined for each thermocouple location and if the properties of the material vary with temperature, then these sensitivity coefficients have to be evaluated at each iteration in the estimation procedure.

The assumed value of \( q \) is corrected by using the relation:

\[ q = \frac{\sum_{i=1}^{I} (T_{\text{meas}} - T_{\text{est}}) \phi_i}{\sum_{i=1}^{I} (\phi_i)^2} \quad \ldots (6.8) \]

i.e.,

\[ q(\text{corrected}) = q(\text{old}) + \Delta q \quad \ldots (6.9) \]

The procedure is repeated with \( q(\text{corrected}) \) in place of \( q(\text{old}) \) until:

\[ \Delta q < 0.005 \quad \ldots (6.10) \]

The above procedure depends on the value of the sensitivity coefficients \( \phi \) for arriving at the value of \( q \). Higher value of \( \phi \) is preferred in IMCP for faster convergence of the value of \( q_{n+1} \). The importance of sensitivity coefficients and the factors which influence it are discussed below.
An examination of the sensitivity coefficients reveals the following: (see figure 6.2)

1. If the sensor is located at a suitable distance from the surface \( S \) where the heat flux is to be estimated, then the increase in \( T_2 \) at location \( P \) with a unit increase in \( q \) depends on:

1. Nearness of \( P \) to the surface \( S \)
2. The time interval over which \( q \) flows at the surface.

A decrease in the distance between the sensor location and the surface results in an increase in \( \phi \). Increasing \( \phi \) by keeping a sensor close to the interface has a limit since keeping sensors in the 'disturbed region' shown in figure 4.5b can result in errors in the measured temperature values due to the absence of one-dimensional heat flow conditions.

An increase in the time interval over which \( q \) is estimated causes an increase in \( \phi \). This time interval cannot be increased indefinitely since this would lead to an averaged value of \( q \) over a longer duration and this does not reflect the variation in \( q \) satisfactorily. On the other hand, as explained earlier, the effect of a change in \( q \) on the surface is felt at a later instant of time at an interior location, and this delayed response depends on the distance between the surface and the sensor location. This problem can be solved by assuming that the same value of \( q \) would be flowing for a few \( (r-1) \) more intervals of time \( (\Delta \theta) \) and estimating the effect of this flow at an interior location. In other words, \( q \) in the present interval can be
estimated accurately by taking 'future temperatures' at an interior location. The number of future temperatures to be considered depends on the depth of the sensor from the surface at which \( q \) flows. Hence a satisfactory procedure for the accurate estimation of \( q \) would be to select a sensor location away from the disturbed region and to use appropriate time steps for calculating \( q \) using the required number of future temperatures.

6.6 IMPLEMENTATION IN THE PRESENT STUDY:

In the present study, the inverse heat conduction problem on the chill side is solved using the known temperatures \( T_1 \) and \( T_2 \). \( T_1 \) is used as the known boundary condition. The calculations are performed using the explicit finite differences method. This yields the value of heat flow rate \( q \) per unit area and also \( T_m \), the chill surface temperature. Using this estimated value of \( q \) and \( T_4 \) as the boundary conditions, the temperature distribution in the casting side is obtained by the explicit finite differences method. These calculations yield \( T_c \), the casting surface temperature. Knowing \( q \), \( T_c \) and \( T_m \), \( h \) is calculated using the equation:

\[
h = \frac{q}{(T_c - T_m)} \tag{6.11}
\]

where \( q \) = heat flow rate per unit area of casting/mold interface

\( T_c \) = casting surface temperature and

\( T_m \) = chill surface temperature.
The flow chart for the determination of \( h \) in the present studies using Beck's method [3] is given in figure 6.4. Comparison of the measured temperature \( T_3 \) with the calculated temperature at the same location shows good agreement between the two values. The details of the procedure employed for the estimation of \( h \) are discussed in the following steps:

1. Defining the one dimensional grid for FDM calculations
2. Determining the initial temperature distribution
3. Determination of \( q \) and \( T_m \) on chill surface by Beck's method
4. Determination of temperature distribution in the casting by FDM using the value of \( q \) obtained in step 2.
5. Calculation of \( h \)

6.6.1. Grid for FDM calculations:

For the calculation of heat flow and temperature distribution using the explicit finite differences method, the one dimensional grid shown in figure 6.5 is used. For calculations, a unit cross-sectional area is considered. The distances between the points A, B, interface, C and D i.e., \( x_1, x_2, x_3 \) and \( x_4 \) are divided into \( n_1, n_2, n_3 \) and \( n_4 \) intervals respectively. The nodes are numbered starting from 1 corresponding to the point A to \( I_{\text{max}} \) corresponding to the point D and the node numbers at the points of interest i.e., B, C and the chill interface i.e., IB, IC and \( I_{\text{int}} \) are included in the figure. The values of \( x_3 \) and \( x_4 \) used are those measured from the radiographs of the respective cast-
FIG. 6.6 FLOW CHART FOR THE DETERMINATION OF h USING BECK'S NON-LINEAR ESTIMATION PROCEDURE.
FIG 6.5 ONE DIMENSIONAL GRID FOR FDM CALCULATIONS
ings. This is to account for small changes in the locations of the thermocouples which arise during pouring of liquid metal.

A general FEM program is written such that desired values can be assigned to the numbers of nodes $n_1$, $n_2$, $n_3$, $n_4$ and distances $x_1$, $x_2$, $x_3$, $x_4$. Fine grid spacings to improve accuracy is employed in the regions closer to the casting/mold interface. This is necessary because rapid temperature changes occur in this region at the start of solidification. A typical set of values for these variables is:

- $n_1 = 2$, $x_1 = 8.5$ mm
- $n_2 = 1$, $x_2 = 3.5$ mm
- $n_3 = 2$, $x_3 = 10$ mm
- $n_4 = 4$, $x_4 = 26$ mm

6.6.2. Initial temperature distribution:

The temperature readings $T_3$ and $T_4$ in the casting are examined for determining the starting point of the calculations. This is taken as the time at which the maximum temperature is reached by the thermocouples. The typical temperature values recorded in an experiment for LM 24 alloy for a casting height of 100 mm, sprue height of 150 mm and ingate size 20x20 mm are given in Figure 6.6 and a portion of the temperature readings obtained in a typical experiment are given in Table 6.1. A linear variation of temperature is assumed in the liquid metal and the chill for calculating the initial temperature distribution. The nodal temperatures in the chill side are calculated using $T_1$ and $T_2$ by
FIG 6.6 TEMPERATURE RECORD FROM A TYPICAL EXPERIMENT.
Linear extrapolation and on the casting side using temperatures $T_2$ and $T_4$. The initial temperature values are obtained from the measured values of $T_1$, $T_2$, $T_3$ and $T_4$ at the chosen point of time $(t = 0)$ for starting the calculations.

6.6.3. Determination of $q$ in the chill by Beck's method:

The material properties of the chill used in the calculations are [109]:

- Density = 7100 Kg/m$^3$
- Specific heat = $0.465T + 543.66 \ (J/Kg \ K)$
- Thermal conductivity = $3.14025T + 5191.88 \ (W/m \ K)$

where $T$ is the temperature of the chill in °C.

Using the initial temperature distribution on the chill side and a starting value of 1 Cal/ cm$^2$.s for $q$, the computer program is run. The sensitivity coefficients and corrections in $q$ are obtained with this starting value using the procedure by Beck [10]. $q$ is estimated at intervals of one second with the temperature values available at intervals of 0.5 s. $(m = 2)$. One set of future temperatures is used in the calculations i.e., $r = 2$. The boundary temperature for the chill is obtained by linear interpolation using measured values at the beginning and at the end of the time interval.
<table>
<thead>
<tr>
<th>Time (s)</th>
<th>$T_1$ (°C)</th>
<th>$T_2$ (°C)</th>
<th>$T_3$ (°C)</th>
<th>$T_4$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0</td>
<td>87.7</td>
<td>131.7</td>
<td>599.3</td>
<td>614.8</td>
</tr>
<tr>
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<td>89.7</td>
<td>136.8</td>
<td>594.4</td>
<td>614.0</td>
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<td>91.2</td>
<td>141.3</td>
<td>592.4</td>
<td>613.1</td>
</tr>
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<td>147.3</td>
<td>590.7</td>
<td>611.1</td>
</tr>
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<td>594.4</td>
<td>610.0</td>
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<td>161.5</td>
<td>598.2</td>
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</tr>
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<td>100.6</td>
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<td>603.9</td>
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<td>602.5</td>
<td>605.1</td>
</tr>
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<td>598.7</td>
<td>604.2</td>
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<td>184.4</td>
<td>597.3</td>
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<tr>
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<td>114.1</td>
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<td>601.0</td>
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<td>594.1</td>
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<td>205.7</td>
<td>589.8</td>
<td>594.4</td>
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<td>129.0</td>
<td>214.2</td>
<td>587.3</td>
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<td>131.7</td>
<td>221.5</td>
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<td>592.1</td>
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<tr>
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<td>134.7</td>
<td>227.0</td>
<td>581.3</td>
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<td>231.5</td>
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<td>588.7</td>
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<td>240.6</td>
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<td>242.7</td>
<td>574.4</td>
<td>587.3</td>
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<td>246.1</td>
<td>574.9</td>
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<td>156.3</td>
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<td>568.1</td>
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<td>259.5</td>
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<td>179.5</td>
<td>263.6</td>
<td>564.3</td>
<td>577.5</td>
</tr>
</tbody>
</table>

Table 6.1 A portion of time temperature data from a typical experiment
The FDM calculations are performed using time intervals of \( \frac{1}{24} \) seconds which satisfies the stability criteria for the explicit FDM used in the calculations. The resultant temperature distribution and the value of \( q \) are used as starting values for the next cycle of calculations. These calculations also give the value of \( T_m \) at the chill surface. The value of \( q \) obtained by solving the IHCP on chill side is given in figure 6.7.

6.6.4. Temperature distribution in the casting:

Using the value of \( q \) obtained from the calculations in the chill side and knowing the boundary temperature \( T_4 \), the heat conduction problem in the casting side is treated as a direct problem and solved using explicit FDM. The casting properties used in the calculations are given in Table 6.2 below.

<table>
<thead>
<tr>
<th>Alloys</th>
<th>( \rho ) (Kg/m(^3))</th>
<th>( c ) (J/Kg K)</th>
<th>( k ) (W/m K)</th>
<th>( L \times 10^3 )</th>
<th>( T_L ) (°C)</th>
<th>( T_S ) (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM 6</td>
<td>2660</td>
<td>1045.5</td>
<td>161.1995</td>
<td>460</td>
<td>577</td>
<td>-</td>
</tr>
<tr>
<td>[110]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM 24</td>
<td>2700</td>
<td>1045.5</td>
<td>125.61</td>
<td>489.294</td>
<td>615</td>
<td>520</td>
</tr>
<tr>
<td>[78,110]</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Al-Li</td>
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<td>1045.5</td>
<td>108.862</td>
<td>393.108</td>
<td>652</td>
<td>645</td>
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<td>[111,112,113]</td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

* Units for \( \rho \) in Kg/m\(^3\), \( c \) in J/Kg K, \( k \) in W/m K and \( L \) in J/Kg
FIG 6.7 COMPARISON OF CALCULATED AND MEASURED TEMPERATURES AND ESTIMATED VALUES OF $q$
Liberation of latent heat $L$

For the LM 6 alloy, the liberation of latent heat in the casting is taken into account using the method of Eyers et al. [114]. It is a post-iterative method in which the latent heat of fusion $L$ is converted into an equivalent number of degrees $\Delta T$ by dividing the latent heat of fusion $L$ by the specific heat $c_s$ ($\Delta T = L/c_s$). After each time step, the temperature of the nodes that fall below the eutectic temperature during the calculations are reset to eutectic temperature and the drop in temperatures is recorded for each node in a separate account. This procedure is continued till the required number of degrees ($\Delta T = L/c_s$) have been accumulated for a particular node. When the accumulated drop at a node equals $L/c_s$, the temperature at that node is allowed to drop as per FDM calculations. This procedure is well suited for congruent melting alloys like LM 6.

The LM 24 alloy has a wide freezing range (615-520°C) and the method of Eyers et al. [114] would not be appropriate to account for the liberation of latent heat. A more suitable method is the enthalpy method [76] in which the heat flow equation 2.5 is modified as:

$$ k \frac{\partial^2 T}{\partial x^2} = \frac{\partial H}{\partial t} \quad \ldots \ (6.12) $$

where $H$ is the enthalpy. The value of temperature as a function of enthalpy is obtained by the equations:
\text{T < T_S,} \quad H = c_s \times T \\
\text{T_S < T < T_L,} \quad H = c_s \times T_S + L \times (T - T_S) / (T_L - T_S) \\
\text{T > T_L,} \quad H = c_s \times T_S + L + c_l \times (T - T_L) \quad \ldots \quad (6.13)

where \( T \) = calculated temperature \\
\( T_S \) = solidus temperature \\
\( T_L \) = liquidus temperature \\
\( H \) = enthalpy \\
\( c_s \) = specific heat of solid \\
\( c_l \) = specific heat of liquid \\
\( L \) = latent heat

The initial enthalpy at various points in the casting is determined from the known initial temperature distribution and the change in enthalpy is obtained from the above equations. This gives the new enthalpy value at the end of each temperature measurement interval. The temperatures at the end of the interval are obtained from the enthalpy values.

The computer program developed here for the present calculations can use a non-linear relationship between \( H \) and \( T \) by substituting appropriate relationships but in the present work, a linear relationship was found to be adequate. This is seen by a comparison of the calculated and measured temperatures at the point \( C \) (\( T_3 \)) which show a good agreement. From the calculated value of \( T_3 \), the value of the casting surface temperature \( T_c \) is also obtained. Figure 6.7 shows the measured temperature \( T_2 \) in the chill and the corresponding calculated temperature at the same location. The calculated and measured values of \( T_3 \) in the cast-
ing are also included in the figure. The temperature values obtained by explicit FDM using the value of \( q \) determined by solving the IMCP on the chill side agrees very closely with the measured temperature values, while the measured and calculated temperatures in the casting agree reasonably well as shown in figure 6.7 thus validating the calculation procedure.

6.6.5. Calculation of \( h \):

From a knowledge of the temperatures \( T_n \) and \( q \) obtained from step 2 and \( T_C \) obtained from step 3, values of \( h \) can be obtained using equation 6.11.

The value of \( h \) obtained from a typical experiment is presented in figure 6.8. The results from various experiments are presented and discussed in the following chapter.
FIG 6.8 ESTIMATED VALUES OF q, Tc, Trm AND h
CHAPTER 7

RESULTS AND DISCUSSION

7.1. INTRODUCTION:

The variation of the interfacial heat transfer coefficient $h$ with time in one-dimensional heat flow obtained from the experiments carried out are presented in this chapter. The values of $h$ obtained by measuring the air gap dimensions are compared with those obtained from calculations using the experimental values of temperature. The results of experiments with a smooth uncoated chill are discussed in relation to the model. The effect of insulating coatings and chill roughness on the variation of $h$ with time are discussed qualitatively. A sensitivity analysis is carried out to study the effect of changes in $h_I$, $h_{II}$, $t_I$, $t_{II}$ and $\Delta h/\Delta t$ on the changes in temperature.

7.2 RESULTS:

From the experiments, time-temperature data from the casting and chill are obtained which are analyzed for obtaining the variation of $h$ with time. The results are presented in the following sequence.

1. Experiments with constant metallostatic head
2. Experiments with a receding casting interface
3. Air gap measurements

The effect of the presence of alumina and fireclay coatings on the chill surface on the variation of $h$ with time and the effect
of a change in the roughness of the chill on the variation of \( h \) with time are also included.

7.2.1 Constant metallostatic head arrangement:

7.2.1.1 Uncoated smooth chill:

The variation of \( h \) with time for LM 6 alloy for three values of casting height \( (L) \) namely 100, 150 and 200 mm for an uncoated smooth chill are presented in figure 7.1. The results show that in all the three cases, \( h \) rises to an initially high value which decreases rapidly and then remains constant for the remaining duration of the experiment. This constant value of \( h \) increases with increase in casting height in all these cases. Similar observations for LM 24 alloy are presented in figure 7.2.

7.2.1.2 Coated smooth chill:

The variation of \( h \) with time for LM 6 alloy for a casting height of 200 mm with two types of coatings on the chill surface namely alumina (of thickness 0.46 mm) and fireclay (of thickness 0.24 mm) are presented in figure 7.3. The variation of \( h \) with time for an uncoated smooth chill for the same casting conditions is shown in the figure for comparison. Similar results for LM 24 alloy are presented in figure 7.4.

7.2.1.3 Uncoated rough chill:

The variation of \( h \) with time in the case of LM 6 alloy poured against an uncoated rough chill for a casting height of 200 mm is given in figure 7.5. The results for an uncoated smooth chill
FIG. 7.1 EFFECT OF CASTING HEIGHT ON THE VARIATION OF h WITH TIME FOR LM 6 ALLOY WITH UNCOATED SMOOTH CHILL AND CONSTANT METALLOSTATIC HEAD.
FIG 7.2 EFFECT OF CASTING HEIGHT ON THE VARIATION OF \( h \) WITH TIME FOR LM 24 ALLOY WITH UNCOATED SMOOTH CHILL AND CONSTANT METALLOSTATIC HEAD
FIG 7.3 EFFECT OF COATING MATERIAL ON THE VARIATION OF $h$ WITH TIME FOR LM 6 ALLOY WITH SMOOTH CHILL AND CONSTANT METALLOSTATIC HEAD
FIG 7.4 EFFECT OF COATING MATERIAL ON THE VARIATION OF h WITH TIME FOR LM 24 ALLOY WITH SMOOTH CHILL AND CONSTANT METALLOSTATIC HEAD
FIG 7.5 EFFECT OF CHILL ROUGHNESS ON THE VARIATION OF $h$ WITH TIME FOR LM 6 ALLOY WITH UNCOATED CHILL AND CONSTANT METALLOSTATIC HEAD
for the same casting conditions is included in this figure for comparison.

7.2.1.4 Aluminium-Lithium alloy:

The variation of \( h \) with time in the case of aluminium-lithium alloy for a casting height of 100 mm solidifying against an uncoated smooth chill is given in figure 7.6.

7.2.2 Receding casting interface arrangement:

Results of the experiments with a receding casting interface are presented as follows:

7.2.2.1 Uncoated smooth chill:

The effect of the three ingate sizes 46 mm x 30 mm (large), 20 mm x 20 mm (medium) and 18 mm x 12 mm (small) on the variation of \( h \) with time for LM 6 alloy for a casting height of 100 mm and sprue height of 100 mm is presented in figure 7.7. The time at which \( h \) starts decreasing, indicating the freezing of the ingate is shown as \( t_l \), \( t_m \) and \( t_s \) in the figure for the large, medium and small ingates respectively. \( t_l \) lies outside the limit of the graph. The graphs follow a trend similar to that shown by line 1 in figure 4.1. As mentioned earlier, all subsequent castings are poured with an ingate size of 18x12 mm to get the effect of a receding casting interface within the duration of the experiment.

Results showing the variation of \( h \) with time for the experiments with a receding casting interface with a smooth uncoated chill
FIG 7.6 VARIATION OF $h$ WITH TIME FOR AI-LI ALLOY WITH UNCOATED SMOOTH CHILL AND CONSTANT METALLOSTATIC HEAD ($L = 100$ mm)
Fig 7.7 EFFECT OF INGATE SIZE ON THE VARIATION OF $h$ WITH TIME FOR LM 6 ALLOY WITH UNCOATED SMOOTH CHILL AND RECEDING CASTING INTERFACE

**INGATE SIZE**

1. 46x50 mm (large)
2. 20x20 mm (medium)
3. 18x12 mm (small)

$ts$: time of freezing of small ingate
$tm$: time of freezing of medium ingate
$tl$: time of freezing of large ingate
for LM 6 alloy for a sprue height of 150 mm with three casting heights 100, 150, and 200 mm are shown in figure 7.8 and results for LM 24 alloy are shown in figure 7.9. The results for three different sprue heights 50, 100 and 150 mm and for a casting height of 200 mm for LM 6 and LM 24 alloys are presented in figures 7.10 and 7.11 respectively.

7.2.2.2 Coated smooth chill:

The effect of fireclay and alumina coatings on the chill surface on the variation of $h$ with time for LM 6 and LM 24 alloy castings with a casting height of 150 mm and sprue height of 100 mm with a small ingate are presented in figures 7.12 and 7.13 respectively.

7.2.2.3 Uncoated rough chill:

The variation of $h$ with time for LM 6 alloy for a casting height of 150 mm and sprue height of 100 mm using a rough uncoated chill is given in figure 7.14. The variation of $h$ with time for the corresponding case using a smooth uncoated chill is included in the figure for comparison.

7.2.2.4 Aluminium-Lithium alloy:

The variation of $h$ with time for aluminium-lithium alloy with a casting height of 100 mm and sprue height of 50 mm is presented in figure 7.15.
FIG 7.8 EFFECT OF CASTING HEIGHT ON THE VARIATION OF $h$ WITH TIME FOR LM 6 ALLOY WITH UNCOATED SMOOTH CHILL AND RECEDING CASTING INTERFACE
FIG 7.9 EFFECT OF CASTING HEIGHT ON THE VARIATION OF $h$ WITH TIME FOR LM 24 ALLOY WITH UNCOATED SMOOTH CHILL AND RECEIVING CASTING INTERFACE
FIG 7.10 EFFECT OF SPRUE HEIGHT ON THE VARIATION OF $h$ WITH TIME FOR LM 6 ALLOY WITH UNCOATED SMOOTH CHILL AND RECEIVING CASTING INTERFACE
FIG 7.11 EFFECT OF SPRUE HEIGHT ON THE VARIATION OF h WITH TIME FOR LM 24 ALLOY WITH UNCOATED SMOOTH CHILL AND RECEDING CASTING INTERFACE
Fig 7.12 EFFECT OF COATING MATERIAL ON THE VARIATION OF h WITH TIME FOR LM 6 ALLOY WITH SMOOTH CHILL AND RECEDING CASTING INTERFACE.
FIG 7.13 EFFECT OF COATING MATERIAL ON THE VARIATION OF $h$ WITH TIME FOR LM 24 ALLOY WITH SMOOTH CHILL AND RECEDING CASTING INTERFACE
FIG 7.14 EFFECT OF CHILL ROUGHNESS ON THE VARIATION OF h WITH TIME FOR LM 6 ALLOY WITH UNCOATED CHILL AND RECEIVING CASTING INTERFACE
FIG 7.15  THE VARIATION OF \( h \) WITH TIME FOR AI–LI ALLOY WITH UNCOATED SMOOTH CHILL AND RECEDING CASTING INTERFACE
7.2.3 Air gap measurement:

The variation of air gap dimensions at the interface and the variation in the calculated values of h with time obtained from the air gap dimensions for LM 6 alloy in the case of experiments with a receding casting interface for a casting height of 150 mm, sprue height of 100 mm and small ingate size are presented in figure 7.16. The value of h obtained by solving the IHCP is included in the figure for comparison with the value of h obtained from air gap measurements.

7.3 DISCUSSION:

The experimental results of the constant metallostatic head arrangement, receding casting interface arrangement and air gap measurements for the case of smooth uncoated chill are discussed in this section. The model developed in chapter 4 is used for explaining the trends observed in the variation of h with time in the case of experiments with a constant metallostatic head. The effect of coatings and chill surface roughness on the variation of h with time are discussed qualitatively. The discussion of these results is presented in the following sequence:

1. General observations from experimental results
2. Duration of stage I, \( t_1 \)
3. Value of h in stage I, \( h_1 \)
4. Value of h in stage II, \( h_{II} \)
5. Time for end of stage II, \( t_{II} \)
6. \( dh/dt \) in stage III
7. Air gap measurement
FIG. 7.16 COMPARISON OF h VALUES FROM AIR GAP MEASUREMENTS AND SOLUTION OF IHCP.
7.3.1 General observations from experimental results:

From the experiments carried out, data are obtained in the form of temperatures $T_1$ and $T_2$ from the chill and $T_3$ and $T_4$ from the casting as a function of time. A study of these temperatures shows some common features and these are discussed here. In addition to these, features observed on the variation of the calculated values of the casting and mold surface temperatures $T_c$ and $T_m$ respectively, heat flow rate $q$ and interfacial heat transfer coefficient $h$ are presented.

7.3.1.1 Variation in chill temperatures $T_1$ and $T_2$:

The temperatures $T_1$ and $T_2$ in the chill show a continuously increasing value, $T_2$ being higher than $T_1$ since $T_2$ is measured at a point closer to the casting surface. In the initial stages, the temperatures $T_1$ and $T_2$ show a kink and this is marked as $t_1$ and $t_2$ respectively in the typical result shown in figure 6.6. The early investigators [6, 40, 41] associated this kink to the formation of an air gap.

In the model presented in chapter 4, it is shown that a partial air gap always exists between the casting and mold surfaces due to the presence of grooves on the mold surface. Hence these kinks observed in $T_1$ and $T_2$ must be associated with the rapid changes in the size of the existing air gap rather than the formation of a new air gap as proposed by the early investigators [6, 40, 41]. The presence of a partial air gap is demonstrated by the work of Frates and Bilcini [52]. Hence, the time of occurr-
rence of the kink in the temperatures \( T_1 \) and \( T_2 \) in Figure 5.6 depends on the distance of the thermocouple from the interface. A realistic estimation of the time of change in air gap size can be obtained from the kink observed in the mold surface temperature. These observations point to the need for a closer look at the mold surface temperature \( T_m \).

7.3.1.2 Variation in casting temperatures \( T_3 \) and \( T_4 \):

The variation of \( T_3 \) and \( T_4 \) with time represent typical cooling curves of the alloy under study. The variation consists of an initial steep rise in temperature followed by fluctuations at the high temperature. This is followed by a rapid drop in temperature due to loss of superheat with a reduced slope in the case of LM 24 alloy or a constant temperature in the case of LM 6 alloy, associated with the liberation of latent heat. Finally, a drop in temperature associated with the cooling of the solidified metal is observed. The initial fluctuations in these temperatures can be associated with turbulence arising due to the pouring of molten metal into the mold cavity. It is observed that the fluctuations are less severe in the experiments with a receding casting interface compared to those in the case of experiments with a constant metallostatic head. This can be attributed to the absorption of the fluctuations in pressure in the two columns of liquid metal connected by the ingate (Figure 5.8). The temperature \( T_3 \) is lower than \( T_4 \) and this is to be expected since \( T_3 \) is measured at a point closer to the interface.
than $T_4$. It is reported that the behavior of the interface can be related to the casting surface temperature $T_C$ [9, 29, 40, 41, 96].

### 7.3.1.3 Variation in surface temperatures $T_C$ and $T_m$:

The casting and mold surface temperatures $T_C$ and $T_m$ are obtained from the FDM calculations for the determination of $q$ and $h$ and a typical result for LM 24 alloy for $L = 100$ mm, $S = 150$ mm and ingate size $20$ mm x $20$ mm is shown in figure 6.8. The casting surface temperature $T_C$ shows the same trend as a typical cooling curve of an alloy solidifying rapidly and this is to be expected since the heat extraction is rapid at the interface. The variation of $T_C$ with time also shows that liquid metal is present at the interface during the initial stages.

The chill surface temperature $T_m$ shows sharp changes thereby indicating changes in the rate of heat flow at the interface. The kink observed in $T_m$ is also sharp and well defined. In addition to the kink shown in figure 6.8, $T_m$ also shows sharp changes in slope and two such points are marked in the figure. It can be seen from the figure that every sharp change in $T_m$ is associated with changes in the heat flow rate $q$ and the interfacial heat transfer coefficient $h$.

### 7.3.1.4 Variations in $q$:

The heat flow rate per unit area $q$, calculated by solving the IHCP on the chill side using temperatures $T_1$ and $T_2$ shows a sharp increase initially, followed by fluctuations. A typical graph showing the variation in $q$ with time is given in figure 6.8. The
value then falls rapidly and is followed by a slow and steady decrease for the rest of the experiment. At the time of pouring the liquid metal into the mold cavity, the molten metal is at a high temperature and the chill is at a low temperature. Immediately after pouring, the contact between the liquid metal and the chill is also quite good [47]. This results in an initial high value of heat flow rate/ unit area, $q$ at the interface. Fluctuations in $q$ observed in the experiments can be attributed to turbulence in the metal. Subsequently, an increase in the size of the air gap at the interface decreases the heat flow rate significantly. Marginal increase in the air gap size and increase in the chill temperature with decrease in the casting temperature contribute to the decrease in the value of $q$ with further passage of time. These changes in $q$ along with the changes in $T_c$ and $T_p$ are reflected in the changes in $h$. A knowledge of the variation of $h$ with time is adequate to understand the heat flow behavior at the interface and hence its variation with time is studied.

7.3.1.5 Variation of $h$ with time:

The variation in $h$ with time in the present experiments shows a high fluctuating value in the initial stages. This is followed by an almost steady value in the case of experiments with constant metallostatic head. In the experiments with receding interface arrangement, the steady value of $h$ starts decreasing more or less linearly after sometime, around the freezing time of the ingate. In some cases, the steady value of $h$ is not ob-
served. Further, in some experiments, the value of $h$ is found to show wider scatter than those in other experiments and this scatter is always associated with small fluctuations in measured temperatures. The general trend in the variation of $h$ is discussed below in relation to the schematic variation presented in chapter 4.

The variation of $h$ with time in one dimensional heat flow is presented in figure 4.1 and described in section 4.2. The variation of $h$ with time is divided into 3 stages - I, II and III, stage I lasting upto time $t_I$ followed by stage II from time $t_I$ to time $t_{II}$. This is followed by stage III. In stage I, $h$ rises from zero to a high value with fluctuations and this falls to a lower value. In stage II, $h$ remains constant and in stage III two kinds of variations are possible- (i) a decrease in $h$ with time associated with a receding casting surface from the mold surface (line 1) in figure 4.1 and (ii) an increase in $h$ with time due to increase in contact pressure at the interface (line 3). A situation with no distinct stage III as observed in the experiments with constant metallostatic head is shown by line 2 in figure 4.1. A situation having no distinct stage II is indicated by line 4 in this figure. These variations can be explained as follows:

When hot molten metal at temperatures well above the freezing temperature is poured into the mold cavity the metal in contact with the chill surface also remains at a temperature above the solidification temperature for a short duration as seen from the
The liquid metal at the high temperature has a low value of surface tension \[115\] and a low value of the advancing contact angle \[116\] compared to the values at the freezing temperature. A low value of surface tension leads to a smaller radius of the liquid metal profile (compared to the value at the freezing temperature) as shown by equation 4.3 and hence a higher value of \( h \) at a given metallostatic pressure as seen from figure 4.7e. The variation in \( h \) can be expected to affect the value of \( h \) similarly as shown in figure 4.7b.

In the case of castings poured with a constant metallostatic head, the pressure head acting on the chill surface is the height to which liquid metal is poured in the mold cavity. This height varies from zero at the start of pouring to the maximum casting height at the end of pouring. In addition to this metallostatic head, the overall value of pressure at the interface includes the pressure acting on the casting/chill interface due to the falling liquid metal stream. The head associated with pouring is only a fraction of the height from which it is poured due to losses by turbulence and the value of this fraction is difficult to determine. The presence of turbulence causes fluctuations in the actual pressure at the casting/mold interface.

In the case of experiments with a receding casting interface, the metal gradually rises in the two limbs of the mold cavity. The
pressure of the liquid metal in the sprue along with a fraction of the metallostatic head associated with pouring acts on the chill surface only after the liquid metal surface establishes contact with the mold surface. Further, a part of the fluctuations in pressure is dissipated away in the two columns of the liquid metal and the net fluctuation at the casting/mold interface is much lower in these experiments.

During stage I in the constant metallostatic head and receding casting interface experiments, a stable liquid metal profile cannot form adjacent to the chill surface since any skin that is formed undergoes continuous deformation and rupturing due to turbulence in the liquid metal. This condition of high turbulence also helps to maintain the metal in a molten condition adjacent to the chill surface. The presence of molten metal in the initial stages of pouring is also borne out by the observations in the casting surface temperature $T_C$ as shown in Figure 6.8. The formation of a stable liquid metal profile adjacent to the chill surface marks the end of stage I.

When pouring is stopped, the liquid metal surface forms a stable profile, depending on the metallostatic pressure $P$ and the other factors like advancing contact angle $\alpha$, surface tension of the liquid metal $\sigma$ and mold surface profile. The metal surface originally in the liquid state, starts solidifying from the asperity points on the mold and finally forms a fully solidified skin. During this period, changes in the liquid metal profile occur due to changes in $\sigma$ and $\alpha$. In the case of the mold surface
consisting of uniform V grooves, the liquid metal surface in the groove is cylindrical with a small radius (depending on σ at the elevated temperature) in the initial stages. Due to solidification starting at the asperities, there is a change in temperature along the surface of the liquid metal. There is a change in surface tension along the profile of the liquid metal associated with the change in temperature. This results in a change in the liquid metal profile from its original cylindrical shape. When freezing of the skin is completed, the surface tension at all points on the surface would have attained the value corresponding to that at the freezing temperature and the surface once again would become cylindrical but with a larger radius related to the increased values of surface tension at the freezing point. To accommodate this increase in radius, the surface of the liquid metal lifts itself away from the grooves maintaining contact at the asperities at all times. Similar changes occur on actual mold profiles.

Thus, it is clear that during stage I, i) there is a change in the liquid metal profile due to variation in σ and α with temperature and ii) changes in metallostatic head due to pouring and turbulence. Though it is possible to calculate the continuous variation in h due to variations in temperature and effective head, it is difficult to incorporate the effect of turbulence in estimating the value of h using the model. Usually, the entire duration of this stage is quite small and a mean value of h during this stage i.e., h₁ can be calculated using the model by
using the values $\sigma$ and $\alpha$ corresponding to a representative temperature and by using a mean metallostatic pressure head which includes the effect of pouring head in the initial period. Small errors in the estimated value of $h_I$ would not affect the computed values of temperature distribution significantly since $t_I$ is quite small.

During stage II, a constant pressure acts on the chill surface and no changes occur in the stable casting skin which forms the casting surface profile. Thus a constant value of $h$ exists in stage II which can be obtained by using equation 4.10 of the model developed in chapter 4. The value of $h$ in stage II, $h_{II}$ acts from time $t_I$ to $t_{II}$ as shown by lines 1 and 3 in figure 4.1. This value of $h$ can be determined using the model by using appropriate values of $\sigma$, $\alpha$, $P$ and groove profile parameters $\phi$ and $\alpha$. $t_{II}$ is determined experimentally in the present case. This can also be obtained by simulation by calculating the change in contact pressure or air gap formed with time in an actual casting.

In the experiments with a constant metallostatic head, the geometry of the casting causes the liquid metal to apply a constant pressure against the chill surface after pouring is completed. The formation of a skin of solidified metal does not cause any change in the contact pressure (neglecting frictional effects between mold and casting) resulting in a steady value of $h$.

In the experiments with a receding casting interface, the tendency of the casting surface to move away from the chill surface is
higher due to the effect of gravity and this results in lower values of \( h \) in this case compared to the values of \( h \) in the constant metallostatic head experiments. The forces which oppose the movement of the casting surface are the metallostatic head due to the sprue height \( S \) and the friction between the casting and lateral surfaces of the mold. When the ingate freezes, the metallostatic head due to the sprue is not transmitted to the casting resulting in a reduction in the contact pressure at the interface resulting in a decrease in the values of \( h \). In this case, it is also possible for the casting surface at the interface to remain in a partially molten condition, when the movement of the casting surface away from the chill surface begins. Such a situation does not have a distinct stage II in the variation of \( h \) with time. When the casting surface containing incompletely solidified skin recedes from the chill surface, there is a theoretical possibility for the reheating of the skin and associated changes in the casting surface profile. The situation changes directly from stage I to stage III. Even in stage I, a stable skin might not have formed and such a situation is shown by line 4 in figure 4.1.

When no change in pressure occurs on the surface of the chill, there is no distinct stage III (as shown by line 2 in figure 4.1) but if the casting geometry is such as to cause an increase in the contact pressure, as in the case of castings solidifying around cores, the value of \( h \) increases as shown by line 3 in figure 4.1. According to Sun [36] the variation of \( h \) with time
in the case of castings solidifying around a core can be represented as a linear relationship (equation 4.19). On the other hand, a movement of the casting surface away from the chill surface causes a decrease in $h$ (line 1). This decrease in $h$ is caused by contraction of the length of the casting and this contraction is opposed by friction between the lateral surface of the casting and mold as mentioned earlier. The dimensions of the cast metal changes due to liquid-liquid contraction due to loss of superheat, volumetric contraction due to phase change and solid state contraction. These changes result in the movement of the casting surface away from the chill surface. Friction between the casting and mold walls prevents the relative movement between these surfaces and in the present set of experiments, this would prevent an increase in the size of the air gap at the casting/chill interface. Thus the net air gap size is a combined result of the change in casting volume and the friction between the casting and mold surfaces. From the present results of experiments with a receding casting interface, it is seen that the decrease in $h$ with time in stage III can be treated as linear.

According to Ho and Pehlke, [35,47] lateral contraction of the casting (in a direction in the plane of the chill surface) can cause the nature of contact at the asperities on the interface to change from conforming to nonconforming and this results in a decrease in $h$. In the present case however, the lateral contraction was observed to be negligible since the mold and casting profiles match reasonably well after complete solidification of
the casting (figure 4.13). Similar results were observed by Morales et al [27]. In castings with a large surface area, the large contraction stresses cause shrinking of the casting in the plane of the interface with the resultant change in the contact pattern at the interface from the conforming type to the nonconforming type.

7.3.2. Duration of stage I, $t_I$:

Stage I lasts from the time of pouring the metal till a stable casting skin is formed adjacent to the chill surface. In the case of experiments with constant metallostatic head $t_I$ is found to vary within a narrow range from 35 to 50 seconds for both LM 6 and LM 24 alloys using chills with and without coatings. Hence a mean value of 46 seconds with a standard deviation of 9 seconds can be used as a representative value of $t_I$. In the case of Al-Li alloy, $t_I$ is found to be higher.

In experiments with a receding casting interface, $t_I$ is found to vary from 50 to 80 seconds and a representative value for both LM 6 and LM 24 alloys is 67 seconds with a standard deviation of 11 seconds. For the Al-Li alloys, the value of $t_I$ falls within the same range as that of LM 6 and LM 24 alloys.

7.3.3. Value of $h$ in stage I:

The value of $h$ in stage I, $h_I$ obtained from the experiments with constant metallostatic head are given in figure 7.17 for LM 6 alloy and in figure 7.18 for LM 24 alloy. Along with the experi-
FIG. 7.17 VARIATION OF h WITH METALLOSTATIC HEAD (H) FOR LM 6 ALLOY IN CONSTANT METALLOSTATIC EXPERIMENTS.
FIG. 7.18 VARIATION OF h WITH METALSTATIC HEAD (H) FOR LM 24 ALLOY IN CONSTANT METALSTATIC EXPERIMENTS.
mental values, the variation of $h$ with pressure obtained by calculations using the model are also included in the figure. The value of $h$ in stage I for a casting height of 500 mm was found to be 1910 W/m$^2$K for LM 6 alloy and 2307 W/m$^2$K for LM 24 alloy. The values of the material properties and the groove parameter values used in these calculations for the two alloys are given below in Table 7.1. The values of these groove parameters are obtained from the calculations for the value of $h$ in stage II.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>LM 6</th>
<th>LM 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>153'</td>
<td>154'</td>
</tr>
<tr>
<td>$\sigma$ (N/m)</td>
<td>0.8</td>
<td>0.795</td>
</tr>
<tr>
<td>$\eta_{\text{max}}$ (W/m$^2$K)</td>
<td>1910</td>
<td>2307</td>
</tr>
<tr>
<td>$a$ (mm)</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>38'</td>
<td>38'</td>
</tr>
<tr>
<td>$k$ (W/m K)</td>
<td>0.0241</td>
<td>0.0241</td>
</tr>
</tbody>
</table>

In these calculations, the head of metal associated with pouring is taken as 20 mm for the two alloys while the actual height of pouring was around 40 mm more than the casting height (i.e., loss of metal due to pouring was assumed to be 50%).

In the case of experiments with a receding casting interface, the value of $h_I$ is found to be affected by the casting height in
addition to the sprue height. This can be attributed to the higher temperature of the molten metal with the increased casting volume in the case of longer castings. The mean value of $h$ in stage I is found to increase by 100 W/m²K for an increase of casting height from 100 mm to 200 mm in the case of LM 6 alloy and about 500 W/m²K in the case of LM 24 alloy for an increase of casting height from 100 mm to 200 mm.

7.3.4. Value of $h$ in stage II:

The value of pressure acting at the casting/mold interface in stage II is determined only by the metallostatic head $H$. For castings with constant metallostatic head, this is represented by the casting height $L$. In the experiments with receding casting interface, the value of $h$ is determined by the metallostatic head represented by the height of the sprue $S$. The value of $h$ in stage II from the experiments with constant metallostatic head are given in figure 7.17 for LM 6 alloy and figure 7.18 for LM 24 alloy. The value of $h$ for a casting height of 500 mm was obtained experimentally as 1710 and 1908 W/m²K for LM 6 and LM 24 alloys respectively. The results showing higher $h$ for LM 24 alloy are in agreement with the results of Srinivasan et al. [7] who found that $t_f$ of LM 6 alloy casting is higher than that of Al-4.5 % Cu alloy castings. The variation of $h$ calculated by using the model gives a satisfactory fit when $a$ is 0.2 mm and $p$ is chosen as 38°. The maximum deviation between the values of $h$ obtained using the model and the experimental values is 10% for LM 6 alloy. A linear regression line was also fitted using the
experimental data of $h_{II}$ and the metallostatic head $H$. The linear regression equation for LM 6 alloy is:

$$h = 2.898 H - 301.271$$

and this does not give a fit as close as that obtained from the model (as seen from figure 7.17). The corresponding results from the model in the case of LM 24 alloy also gives a better fit with the experimental values compared to the linear regression results. The regression equation for LM 24 alloy is:

$$h = 3.3 H - 302.813$$

Thus, in both the cases, the model gives a better fit with experimental values. However, this deviation in $h$ would not affect the calculated temperature values significantly as seen from the sensitivity analysis presented in section 7.9.

The values of the material properties and $h_{max}$ used for the two cases are given in Table 7.2.

In the case of experiments with a receding casting interface, $h$ is affected by the length of the casting $L$ in addition to the metallostatic head which is given by the sprue height $S$. Increase in casting height $L$ gives a higher value of $h$.

The measured values of $a$ and $\beta$ for the smooth chill from the chill roughness profiles are given below and it can be seen that the values used in the calculations are within the range of measured values.
### Table 7.2 Parameters used for the calculation of $h_{II}$ using the model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range of measured values</th>
<th>Values used in calculation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$ (mm)</td>
<td>0.05 - 0.225</td>
<td>0.2</td>
</tr>
<tr>
<td>$\beta$ (°)</td>
<td>36 - 60</td>
<td>38</td>
</tr>
<tr>
<td>$\alpha$ (°)</td>
<td>150 - 175</td>
<td>170-172</td>
</tr>
</tbody>
</table>

7.3.5 Time for end of stage II, $t_{II}$:

In the experiments with constant metallostatic head, stage III does not occur within the duration of the experiment. In the case of experiments with a receding casting interface, two types of variations in $h$ are seen—(i) A variation showing 3 distinct stages I, II and III with $t_I$ and $t_{II}$ showing end of stage I and stage II respectively; (ii) The second type of variation is represented by line 4 in figure 4.1 in which the behavior changes from stage I to stage III directly without a distinct stage II. Longer casting heights with considerable volume of
metal show this trend. In the cases where a distinct stage II is seen, this is found to be related to the freezing time of the ingate and occurs around 75 seconds for small ingates. Castings with the medium ingate show a freezing time of approximately 235 seconds. Castings with a large ingate show a value of \( t_{II} \) much larger than 300 seconds and the experimental result is similar to that of the constant metallostatic head experiment within the duration of the experiment.

7.3.6 \( dh/dt \) in stage III:

In all castings with a receding casting interface, a distinct stage III within the duration of the experiment is seen and \( h \) is seen to decrease linearly with time in this stage within the duration of the experiment. The value of the slope of this line i.e., \( -dh/dt \) is found to be affected by casting height \( L \) and the sprue height \( S \).

The contraction of the casting depends on its length, longer castings having a higher amount of contraction. The actual extent of contraction also depends on the solidification contraction of the type of alloy used. This contraction causes the receding of the casting surface from the chill surface as described earlier.

The effect of sprue height is to increase the metallostatic pressure and hence the value of \( h \) in the initial stages of casting solidification. A high value of \( h \) implies faster heat extraction and hence a faster decrease in the length of the casting
Due to contraction. This means that \( \frac{dh}{dt} \) also increases with increase in the sprue height \( S \). Depending on the initial value of \( h \) and the subsequent value of \( \frac{dh}{dt} \), the lines showing the variation of \( h \) with time for two castings may cross over under suitable conditions i.e., the casting with a high initial value of \( h \) may have a very low value in the later stages as observed in figures 7.7, 7.9 and 7.10. The movement of the casting is hindered by friction between the casting and the chill surfaces as described earlier. The effect of friction cannot be determined accurately since its variation is not always predictable. In the case of LM 6 alloy for a sprue height of 150 mm, with a small ingate, the value of \(-\frac{dh}{dt}\) is 2.65 W/m\(^2\) K.s for a casting height of 200 mm, 0.16 W/m\(^2\) K.s for a casting height of 150 mm and almost zero for a casting height of 100 mm. The corresponding results for the same configuration with LM 24 alloy are 4.36, 3.5 and 1.74 W/m\(^2\) K.s for casting heights of 200, 150 and 100 mm respectively.

From the above discussion, it is seen that a number of factors influence the value of \( h \) in stages I and II and the value of \( \frac{dh}{dt} \) in stage III.

### 7.3.7 Air gap measurements:

The variation of air gap size and the value of \( h \) obtained using the air gap size are presented in figure 7.16. The variation of \( h \) with time obtained from the solution of the inverse heat conduction problem and from the air gap measurements agree reasona-
bly well considering the variability in \( h \). The difference between the two values of \( h \) can be attributed to:

1. Neglecting the oxide and other films on the casting and mold surfaces in the calculation of \( h \) using air gap. 11) In the later stages, effects like a change in the thermal conductivity of the gases in the gap and oxide film with temperature are neglected and this can account for the deviation between the two values.

### 7.4 Effect of Insulating Mold Coats

The effect of insulating mold coats is to reduce the value of \( h \). The reduction in \( h \) is due to the insulating nature of the coating material and the extent of reduction in \( h \) depends on the thickness and the thermal conductivity of the coating at the elevated temperature. Further, the coatings are porous in nature and this alters their effective thickness and thermal conductivity. Menru [9] found that silica mold coats increase the freezing time and there is an optimum coating thickness for which the freezing time is maximum. Flemings et al [92] showed that coatings affect the heat transfer rate initially but do not prevent the subsequent rapid heat extraction.

In the present studies also, the value of \( h \) is found to be lower than the corresponding uncoated chills in both constant metallo-static head experiments (as shown in figures 7.3 for LM 6 and 7.4 for LM 24) and in receding interface experiments (as shown in figures 7.13 for LM 6 and 7.14 for LM 24). It is also seen that the coating thickness of 0.46 mm of alumina and 0.24 mm of fire-
clay causes a marginal decrease in $h$. This is in agreement with the observations of Nehru [9] and Flemings et al [92].

The value of $h$ with 0.24 mm fireclay coating is only marginally higher than that with 0.46 mm of alumina coating in the case of castings poured with constant metallostatic head. With a receding interface experiment, the differences in the $h$ value between alumina and fireclay coatings is more prominent. This can be attributed to the larger difference in the thermal conductivity values between alumina and fireclay [109] at the lower chill surface temperature in the case of experiments with a receding casting interface. Another important effect of the mold coats is that they give a smoother surface finish to the casting. This can be attributed to the fact that while applying the coating, the powdery material fills the valleys on the mold surface and presents a smoother surface on the mold. This would also result in a smoother surface finish of the castings. Quantitative estimation of the effect of mold coats on $h$ is not undertaken in this work.

7.5 Effect of chill roughness

It is generally recognized that the case of a rough chill results in a lower value of $h$ compared to that with a smooth chill [52,27]. The present model developed also shows that surface roughness influences $h$ and that for heat transfer calculations, the roughness is characterized by the two parameters $a$ and $\beta$ and that a single measure of roughness like the RMS value cannot be satisfactorily used to explain the variation of $h$ with
pressure. Generally, the semi-apex angle $\beta$ falls within a narrow range for machined surfaces produced by conventional cutting tools with standard geometry of cutting edge. Hence, for these standard grooves, $h$ is affected by the value of $a$ alone. Increase in $a$, results in a decrease in $h$ as seen from the results in figure 4.7 d. The results obtained in the present experiments also show a trend as expected i.e., $h$ decreases with increase in roughness. Quantitative estimation of change in $h$ with change in roughness parameters is not undertaken as this is not within the scope of the present work.

7.6 Aluminium- lithium alloy:

The variation of $h$ with time for the Al-Li alloy are presented in figures 7.6 and 7.15 for constant metallostatic head and receding casting interface arrangement respectively. The constant metallostatic head experiment shows that the values of $h$ are much higher for the Al-Li alloy than the corresponding values for LM 6 and LM 24 alloys. This high value of $h$ can be attributed to the very low surface tension (0.56 N/m) of this alloy [115]. As seen from the results in figure 4.7 e, decrease in $\sigma$ results in a higher value of $h$ due to the lower radius of the liquid metal profile in the groove resulting in reduced air gap width. In contrast to the constant metallostatic head arrangement, the value of $h$ in the receding interface arrangement is much lower for this alloy and falls within the same range as LM 6 and LM 24 alloys.
7.7 Validation of the results:

In the present study, the model developed for estimating the values of \( h \) in steady state one dimensional heat flow for constant metallostatic head experiments, is found to give values which agree satisfactorily with the experimental values. This is to be expected since the variables affecting the value of \( h \) have been incorporated in the model. On the other hand, the number of variables which affect the variation of \( h \) with time in the case of receding casting interface experiments are more and to incorporate these variables which vary with time, a more elaborate, dynamic model is needed. The development of such a model is not within the scope of this work. In the present case, the representative values for the variation of \( h \) with time in receding casting interface experiments have been obtained and these values have been obtained for validating the experimental results. The values of \( h \) obtained from the model are validated by the experimental results of constant metallostatic head arrangement.

7.7.1 Validation of experimental results using representative values of \( h \):

The variation in the value of \( h \) with time is obtained from the various experiments using Beck's non-linear estimation procedure [3]. The values of \( h \) obtained in receding casting interface experiments are found to give a very close match between experimental and calculated values (obtained by FDM). For these calculations, the boundary conditions and the initial temperature distribution are maintained at the same values as that used for
IHCP. As discussed earlier, this variation of \( n \) with time can be represented by values of \( h_I, h_{II}, t_I, t_{II} \) and \( dh/dt \). Using these representative values, the time temperature distribution obtained from these calculations in the casting and the chill are given in figure 7.19 for LM 6 alloy and in figure 7.20 for LM 24 alloy. These calculations are performed for the case of \( L = 200 \) mm, \( S = 150 \) mm, using a small ingate for LM 6 alloy and for \( L = 100 \) mm and \( S = 150 \) mm with a small ingate for LM 24 alloy.

From these results, it is seen that the agreement on the chill side is excellent and on the casting side, the agreement is satisfactory considering the fact that factors like turbulence, variation of properties with temperature etc. have not been considered in the calculations.

7.7.2 Validation of the results from the model for constant metallostatic head arrangement:

The model has been used to calculate the values of \( h_I \) and \( h_{II} \) for the experiments for the constant metallostatic head. These values are used for calculating the temperature distribution in the case of both the alloys. The calculated and measured temperatures in the casting and chill for LM 6 alloy is shown in figure 7.21 and for LM 24 alloy in figure 7.22. From these results, it is seen that the values obtained using the model are quite satisfactory for predicting the value of \( n \) in the case of constant metallostatic head experiments.
FIG 7.19 TEMPERATURE DISTRIBUTION IN THE CASTING AND CHILL USING EXPERIMENTALLY OBSERVED VALUES OF $h$ FOR LM 6 ALLOY WITH A RECEDING CASTING INTERFACE
7.20 TEMPERATURE DISTRIBUTION IN THE CASTING AND CHILL USING EXPERIMENTALLY OBSERVED VALUES OF h FOR LM 24 ALLOY WITH A RECEDING CASTING INTERFACE.
TEMPERATURE DISTRIBUTION IN THE CASTING AND CHILL USING VALUES OF $h$ OBTAINED FROM THE MODEL FOR LM 6 ALLOY WITH A CONSTANT METALLOSTATIC HEAD
7.22 TEMPERATURE DISTRIBUTION IN THE CASTING AND CHILL USING VALUES OF $h$ OBTAINED FROM THE MODEL FOR LM 24 ALLOY WITH A CONSTANT METALLOSTATIC HEAD.
7.8 Causes for the variation in the values of $h$:

The experimentally measured values of $h$ under identical experimental conditions shows a variation of less than $2\%$. However, in some experiments though $h$ shows a definite mean value or definite trend, scatter is observed around the experimental value. The causes for the variation in the values of $h$ are discussed below.

7.8.1 Experimental disturbances:

One of the experimental causes for the fluctuations in $h$ is the turbulence during pouring and this has been discussed in the earlier section. Another important observation is that in some experiments, the electrical noise affects the thermocouple readings giving rise to slight fluctuations in the temperature values and whenever this condition occurs the value of $h$ shows a greater scatter.

7.8.2 Physical causes:

Variation in $h$ can occur due to the change in the equivalent air gap thickness $\gamma_f$ along various locations on the casting/mold interface. This variation is possible since the thickness of oxide and other adherent films at all points on the casting and mold surfaces cannot be expected to remain constant.

A second cause for the variation in the value of $h$ is the change in the back pressure $P_s + P_v$. This can occur if there are blockages for the flow of gases in the grooves. Local regions in the
grooves may have different values or the back pressure and this can lead to variation in \( h \). Further, the grooves which have been assumed to be made up of uniform \( V \) grooves or combinations of uniform \( V \) grooves may have complicated profiles and this may affect the variation of \( h \) with pressure \( P \) to a certain extent. Finally, changes in surface tension due to the presence of surface contaminants can cause changes in the values of \( h \) to a limited extent.

7.9 Sensitivity Analysis:

A sensitivity analysis is performed to determine the variation in the calculated temperature with a change in the value of \( h \) for the receding interface arrangement for the two alloys, LM 6 and LM 24. From these experiments, it is seen that the overall value of \( h \) vary over more than one order of magnitude and in individual experiments, the variation is more than a factor of 2. A large amount of this variation occurs only in stage I due to the turbulence and it is recommended that these fluctuations are smoothed out by taking the mean value of \( h \) in this stage. If this fact is taken into account, then it is observed that the maximum variation is less than a factor of 2. Hence, the sensitivity analysis is carried out in all cases with a factor of 2 and one example is provided for each of the alloys LM 6 and LM 24. The change in the calculated temperatures with a doubling of \( h_I, t_I, h_{II}, t_{II} \) and \( dh/dt \) in the case of constant and receding interface arrangement for LM 6 and LM 24 alloys are given in Table 7.1. These changes are calculated corresponding to the temperatures \( T_2 \)
and $T_1$ in the chill and the casting respectively at 100, 200 and 300 seconds.

The calculated temperature values in the casting and chill are found to be sensitive to $h_{II}$ and $dh/dt$ in the case of both the alloys. Variations in $h$, $t_I$ and $t_{II}$ do not affect the temperature distribution significantly. The freezing time $t_f$ is affected by changes in $h_I$ in the case of LM 6 and LM 24 alloys. In the case of LM 24 alloy, it is found that changes in $h_{II}$ and $dh/dt$ also affect the freezing time significantly and that changes in $t_{II}$ affect the freezing time marginally. These results are to be expected in this case since $t_f$ for LM 24 alloy is more than the duration of stage I.
Table 7.1 Results of sensitivity analysis

a. Alloy LM 6

[Receding casting interface expt., small ingate, L=200 mm, S=150 mm]

<table>
<thead>
<tr>
<th>Standard values*</th>
<th>Freezing time $t_f$, s</th>
<th>Casting Temperatures $T_3$ at 100s, 200s, 300s (°C)</th>
<th>Chill Temperatures $T_3$ at 100s, 200s, 300s (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_I = 1468$</td>
<td>86</td>
<td>563, 546, 536</td>
<td>317, 365, 391</td>
</tr>
<tr>
<td>$t_I = 64$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{II} = 1062$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_{II} = 64$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{dh}{dt} = 2.65$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_I = 2936$</td>
<td>60</td>
<td>552, 546, 536</td>
<td>315, 365, 391</td>
</tr>
<tr>
<td>$t_I = 128$</td>
<td>86</td>
<td>563, 546, 536</td>
<td>317, 365, 391</td>
</tr>
<tr>
<td>$h_{II} = 2124$</td>
<td>85</td>
<td>550, 532, 523</td>
<td>340, 383, 408</td>
</tr>
<tr>
<td>$t_{II} = 128$</td>
<td>86</td>
<td>562, 543, 533</td>
<td>320, 369, 395</td>
</tr>
<tr>
<td>$\frac{dh}{dt} = 5.3$</td>
<td>86</td>
<td>565, 554, 551</td>
<td>315, 355, 372</td>
</tr>
</tbody>
</table>

* Units for $h$ is W/m² K, $t$ is s and $\frac{dh}{dt}$ is W/m² K s
### b. Alloy LM24

(Receding casting interface expt., small ingate, L=100 mm, S=150mm)

<table>
<thead>
<tr>
<th>Standard values*</th>
<th>Freezing time $t_F$, s</th>
<th>Casting Temperatures $T_3$ at 100s, 200s, 300s (°C)</th>
<th>Chill Temperatures $T_3$ at 100s, 200s, 300s (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_I$ = 1919</td>
<td>170</td>
<td>532, 512, 492</td>
<td>304, 340, 360</td>
</tr>
<tr>
<td>$t_I$ = 50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{II}$ = 1340</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_{II}$ = 50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{dh}{dt}$ = 1.74</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $h_I$ = 3838     | 157                     | 524, 511, 492                                    | 302, 339, 360                                   |
| $t_I$ = 100      | 169                     | 532, 512, 492                                    | 304, 340, 360                                   |
| $h_{II}$ = 2680  | 94                      | 515, 494, 460                                    | 327, 356, 374                                   |
| $t_{II}$ = 100   | 161                     | 532, 510, 491                                    | 306, 341, 361                                   |
| $\frac{dh}{dt}$ = 3.48 | 189                     | 533, 517, 500                                    | 302, 334, 351                                   |

* Units for $h$ is W/m² K, $t$ is s and $\frac{dh}{dt}$ is W/m² K s
CHAPTER 8
CONCLUSIONS

From the results of the calculations using the model to determine the value of \( h \) in steady state unidirectional heat flow, the following conclusions can be drawn.

1. A satisfactory model for obtaining the mean value of \( h \) in stage I \( (h_I) \) and the value of \( h \) in stage II \( (h_{II}) \) at the mold/metal interface is developed for the case of steady state one-dimensional heat flow. An analytical method for evaluating \( h \) for geometrically regular grooves, grooves in parallel, grooves in series and random combination of grooves is also presented.

2. The model shows that the heat flow across the casting/mold interface depends on the topography of the mold surface and at least two parameters are necessary for representing the surface topography for heat flow predictions. In the model developed in the present study for parallel, uniform V grooves, the surface is characterized by the semi-apex angle \( \beta \) and the half mouth width of the groove \( a \) for predicting heat flow across the interface.

3. For a mold surface containing parallel uniform V grooves, the parameters which influence the value of \( h \) are, the half mouth width of the groove \( a \), semi-apex angle of the groove \( \beta \), surface tension of the liquid metal \( \sigma \), advancing contact angle \( \alpha \), net metallostatic pressure \( P \), thermal conductivity \( k \)
of air at the interface and air gap equivalent \( y_f \) of oxide and other films at the interface.

4. \( h \) increases with (i) decrease in \( \alpha \) (ii) increase in \( \beta \) (iii) decrease in \( \sigma \) (iv) decrease in \( \alpha \) and (v) increase in \( k \).

5. The interfacial heat transfer coefficient has a low value at low pressures. Above a critical pressure determined by the groove configuration, \( h \) rises steeply and then with further increase in pressure, reaches a steady value determined by \( y_f \).

6. The mold surface profile can be satisfactorily represented by treating it as (i) a random combination of uniform \( \nu \) grooves in series or in parallel and (ii) a fractal surface.

7. The variation of \( h \) with \( P \) shows a square root relationship i.e., \( h \propto \sqrt{P} \) when the value of \( h \) is calculated using the model for a fractal surface when the range of pressures employed is large i.e. from \( 1 \times 10^5 \) to \( 100 \times 10^5 \) Pa.

From the experimental results and the model of the present study, the following conclusions can be drawn.

8. The general variation of \( h \) with time in unidirectional heat flow shows three distinct stages- I, II and III. Stage I shows a high fluctuating value, stage II shows a steady value and stage III shows a decrease in \( h \) with time. Specific cases may show absence of stage III as in the case of con-
stant metallostatic head experiments or absence of stage II as in some experiments with receding casting interface.

9. The experimental method developed in the present work is satisfactory for the determination of h in the case of constant metallostatic head and receding casting interface arrangement for unidirectional heat flow.

10. Beck's non-linear estimation procedure is a satisfactory solution for determining the value of h from the temperature measurements obtained in the present experiments. For the experimental conditions used here, a time interval of 0.5 seconds for temperature measurements, 1 second for the determination of heat flow rate q and the use of one set of future temperatures is found to be satisfactory.

11. The variation of h with change in metallostatic head is satisfactorily explained by the model for stages I and II. The experimental results validate the model developed in the present study.

12. The model developed in the present work gives a better fit between the value of h and the metallostatic head, compared to the linear regression values. However, considering the sensitivity studies, the linear regression results are satisfactory.

13. The variation of h with time is affected by casting height, sprue height and ingate size in the case of experiments with a receding casting interface. Increase in casting height L
and sprue height $S$ affects the value of $h_1$ and $dh/dt$ whereas increase in the size of the ingate affects the time $t_{II}$ for the onset of stage III.

14. In experiments with receding casting interface, the slope of $h$ with time $(dh/dt)$ is found to remain a constant i.e. $h$ decreases linearly with time.

15. Insulating mold coats decrease the value of the interfacial heat transfer coefficient $h$.

16. Increase in chill surface roughness decreases the value of $h$.

17. The type of alloy affects the value of $h$ considerably. Al-Li alloy has the highest value of $h$, followed by LM 24 and LM 6 alloys.

18. The value of $h$ obtained by air gap measurements shows a satisfactory agreement with that obtained from temperature measurements.

19. From the studies on the validation of the model, it is seen that the temperature values obtained by calculation using the values of $h$ from the various studies agree quite well on the chill side and reasonably well on the casting side with the temperatures obtained experimentally.

20. The calculated temperature values in the casting and chill are found to be sensitive to $h_{II}$ and $dh/dt$ in the case of both the alloys. Variation in $h_1$, $t_I$ and $t_{II}$ by a factor of
two does not affect the temperature distribution significantly. The freezing time is affected by the values of \( h \) in the initial stages.
SUGGESTIONS FOR FUTURE WORK

1. The variation of $h$ with surface roughness parameters have to be determined using mold surfaces with well defined roughness parameters $a$ and $\beta$. This requires preparation of mold surfaces by precision machining techniques.

2. The effect of mold coats of different materials and thickness should be evaluated quantitatively and this has to be related to the model. The variation in thermal conductivity of the coating materials should be incorporated in the calculations.

3. The effect of chill materials and of preheating the chill on the variation of $h$ with time should be evaluated quantitatively and related to the model.

4. The effect of water cooling of the chill on the variation of $h$ with time should be studied.

5. A model for the dynamic variation of $h$ with time incorporating effects of changes in casting dimensions and friction between coating and mold could be developed.
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