THE
SYMMETRICAL COMPONENTS THEIR MEASUREMENTS AND APPLICATIONS.

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Performed by ....

Date ....

PREFACE.

"However high we climb in the pursuit of knowledge we shall still see heights above us, and the more we extend our view, the more conscious we shall be of the immensity which lies beyond."

This work is, therefore, never a complete guide to the methods of "Symmetrical Components" and its applications in various fields of polyphase technique. The fundamental principles of the method, however, have been explained. Next follows the main "problem" viz. the complete design, assembly and use of the "Symmetrical Component Meter" for measuring the positive- and negative-sequence components and their relative phase-angles. This is based particularly upon R. L. Russells' Discussions in Proc. I. E. E. Feb. '50. Rest of the portion is devoted to various applications. In the end a little discussion about balancing (symmetrizing) the unsymmetrical systems has also been included.

The paragraphs, equations and the figures in the text are numbered chapterwise on the decimal system i.e. the integer of the number of a para. or an equation indicates the chapter in which it is inserted while the numerals after the decimal point indicate the serial order of the para. or equation in that chapter; for instance Fig 3.4 would mean the 4th Fig in chapter 3. Also, three colours have been used for 3 diff. phases.

The work has been carried out under the able guidance of Dr. W. Ahmad, Prof. E.E. Deptt, and without his help at every stage it would never have been complete. Mr. M. N. Azam, Asstt. Prof. E.E. Deptt, also ~~extended~~ his very kind help in changing the coils of the dynamometer instrument to make the experiment a success.

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LIST OF SYMBOLS.

Voltage Symbols: E = Vector Expression for voltage (RMS value)

$\bar{E}$ = Absolute value of vector E

e = Instantaneous voltage

Current Symbols: I = Vector Expression for current (RMS value)

$\bar{I}$ = Absolute value of vector I

i = Instantaneous current.

Impedance Symbols: Z = $(R+jX)$ for total impedance

$\bar{Z}$ = Absolute value of vector Z

z = $(r+jx)$ for impedance of element

Phase Symbols: Subscripts a, b & c indicate phase 'a' to neutral, phase 'b' to neutral & phase 'c' to neutral respectively & subscripts A, B & C indicate phase between conductors bc, phase between conductors ca & phase between conductors ab respectively.

Sequence Symbols: Subscripts 0, 1 & 2 indicate 0, +ve, & -ve seq respectively

1.

INTRODUCTORY.

A polyphase system of voltages (or currents) may be either balanced or unbalanced. A balanced system is that in which the vectors are equal in magnitude, and are displaced from one another by equal angles, while an unbalanced system is that in which the vectors are neither equal in magnitude nor displaced by equal angles.

An unbalanced polyphase system can be resolved into a set of balanced systems. For example, an unbalanced group of three related vectors can be resolved into three sets of vectors; the three sets have three vectors each, equal in magnitude and spaced either 0° or 120° apart. Then each set is a symmetrical component of the original unbalanced vectors. Or, to be more general "An unbalanced group of n associated vectors, all of the same type, can be resolved into n sets of unbalanced vectors, n vectors of each set are of equal length and symmetrically located with respect to each other."

. The method of "Symmetrical components" is analogous in some respects to the resolution of a periodic function into its fundamentals and higher harmonics by Fourier series. By the method of symmetrical components a set of unbalanced voltages, or currents, may be resolved into systems of balanced voltages, or currents, equal in number to the number of phases involved. By this method the unsymmetrical problems may be reduced to symmetrical problems, the solutions of which are well-known.

Obviously, a three phase unbalanced system of currents can be resolved into three component systems, each of which is balanced and symmetrical. The first is a system of three phase currents having normal or positive phase-sequence components. By this is meant that the currents in the lines (a, b, c) rise to the maximum in the order (a, b, c). The second symmetrical group of components has a phase-sequence opposite to the normal, and these are called the negative phase-sequence components. The third group consists of three line currents all equal and in phase, these being called the zero phase-sequence components. The magnitudes and phases of the various components, of course, depend upon the magnitude and the phases of the currents flowing. Referring to fig 1.1, therefore, I_{a_1}, I_{b_1} and I_{c_1} form the positive phase-sequence components; I_{a_2}, I_{b_2} and I_{c_2} the negative phase-sequence components; and I_{a_0}, I_{b_0} and I_{c_0} the zero phase-

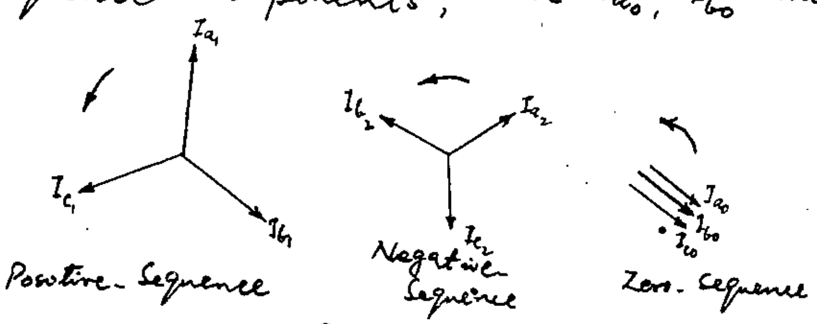

Fig 1.1

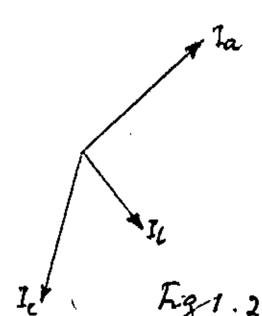

Fig 1.2.

sequence components. They combine together to form the unbalanced system I_a, I_b, I_c (Fig 1.2). Or, in other words, they form the symmetrical components of an unsymmetrical system.

In symmetrical circuits, currents and voltages of different sequences do not react upon each other; currents of the positive-sequence produce only voltage drops of positive sequence; currents of negative-sequence only voltage drops of negative-sequence; and, similarly, currents of zero-sequence only zero-sequence voltage drops. This results in considerable simplification of all kinds of problems involving asymmetry such as that introduced by short-circuiting conductors of a system, or also by open-circuiting of a conductor.

The three phase system has been considered exclusively as it is the most commonly used for all purposes — transmission as well as utilization for industrial purposes, though the fundamental conceptions are always applicable to systems having any number of phases.

2.

HISTORICAL DEVELOPMENT.

Before going into the details of "Symmetrical Components" it will be interesting to consider the historical background and development of its theory.

Analysis of single-phase induction motors by Ferraris and others as early as 1895 may be said to have led to the idea of symmetrical components. It was shown that the set-up in a single-phase induction motor could be resolved into two fields rotating in opposite directions. Later the idea of resolving the three-phase unbalanced currents into three sets of components (viz, positive-, negative- and zero-sequences) was developed in connection with the performance of three-phase machines.

Alexanderson, in 1913, proposed a qualitative treatment in connection with his work on phase balancers. Stokvis in France in 1915, published a mathematical analysis from the machine point of view. He assumed three sets of vectors viz

1. A set which produced positively rotating field,
2. A set which produced negatively rotating field, and
3. A set which produced a pulsating field.

Stokvis came very near to the modern concept, except that he failed to recognise the zero-sequence component which did not produce either rotating or pulsating field.

The modern concept of Symmetrical Components was developed by C. L. Fortescue and his associates R.E. Gilman, J. F. Peters and others while studying the problems of unbalanced circuits, and analyzing the characteristics of single-phase motors, poly-phase motors with unbalanced voltages, and synchronous MG sets, phase balancers for single-phase railway electrification. Fortescue writes in one of his famous papers "In my early investigation of phase-balancers I observed that certain symmetrical relations between phase currents and also between phase voltages recurred frequently, which led me to the investigation of general problem of unbalance. This investigation finally led to the discovery of the fundamental principles of the "Method of Symmetrical Co-ordinates," which was published in 1918. In this way was developed a new simple and complete method for handling the problem of unbalanced circuits.

The method as suggested by Fortescue in his classical paper obviously involves mathematical analysis. Various methods have also been suggested to determine the three sequence components by actual measurements. Works worth mentioning have been done by Evans, Wagner, Takatsku, Stubbings, Allcutt, Genkin and Fortescue, in this connection. A very simple and convenient method for the measurements of the positive and negative phase-sequence components and their

their relative phase angles, has, however, been suggested recently (1950) by Russell. This method involves a dynamometer type instrument with specially designed networks.

However, work is still being done to simplify the analysis of unsymmetrical poly-phase system as far as possible.

3.

BASIC CONSIDERATIONS

3.1 SYMMETRICAL AND UNSYMMETRICAL 3-PHASE SYSTEMS:

In a three-phase system, the currents and voltages (when represented vectorially) will appear as three-phase vector systems in which their lengths may be equal or unequal and may or may not have a phase-displacement of 120° according as the system is symmetrical or unsymmetrical. However, in almost every practical instance the three-phase system is unsymmetrical.

The calculation of a symmetrical system is, of course, simpler than that of an unsymmetrical system, as in the first case it is sufficient in general to investigate the conditions in one phase only with due consideration of possible reactions from the other phases. Due to the symmetry, the conditions prevailing in the other phases can also be determined.

If the asymmetries be of a small amount, it will frequently be permissible to neglect them altogether, or to make an allowance for them by slightly over-dimensioning the machinery, apparatus and lines.

Yet there are numerous cases in which the approximation is not permissible. For instance, asymmetries prevailing during a limited time only (eg. disturbances of normal working conditions caused by faults in the line); also there may exist permanent asymmetries which are encountered where single-phase loads (eg. lamps, cooking, and heating appliances etc.) are connected to three-phase supply system; or where three-phase motors are to be fed by a single-phase supply system.

The investigation of unsymmetrical system has, therefore, great practical importance. Even here it is possible to investigate the conditions prevailing in the different phases seperately by using the method of Symmetrical Components. This kind of procedure is as convenient as the normal procedure of solving the symmetrical (or balanced) systems. The method of symmetrical components allows of a better insight into the physics of a given problem than the older method of computation. And frequently this method may solve problems the solutions of which are difficult or sometimes impossible by the usual methods.

3.2 THE VECTOR OPERATOR "a"

By considering the equation

$$x^3 - 1 = 0 \qquad \text{———} \quad 3.1$$

$$\text{or } (x-1)(x^2+x+1) = 0$$

we have $x = 1$ or $-\frac{1}{2} \pm \frac{1}{2}\sqrt{-3}$ as its solutions. Calling the particular root $x = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$ (where $j = \sqrt{-1}$) as "a" we see that

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \qquad \text{———} \quad 3.2$$

$$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} \qquad \text{———} \quad 3.3$$

$$\text{and } a^3 = 1 \qquad \text{———} \quad 3.4$$

which correspond to the three solutions of (1). Thus $1, a, a^2$

are the three roots of equation (3.1) Or in the other words, 1, a and a^2 are the "cubic roots of unity."

As $a^3 = 1$ the higher powers of 'a' will have periodically the values a, a^2 and 1 i.e.

$$a^4 = a \;;\; a^5 = a^2 \;;\; a^6 = 1$$

$$a^7 = a \;;\; a^8 = a^2 \;;\; a^9 = 1 \text{ ---}$$

$$\text{-----} a^{3n-2} = a \;;\; a^{3n-1} = a^2 \;;\; a^{3n} = 1$$

Representing the values a, a^2 and $a^3 = 1$ on a unit circle by equidistant points (Fig 3.1) we find that a multiplication by 'a' corresponds in each case to a displacement of the point in question by one-third of the circumference of the circle i.e. by $\frac{2\pi}{3}$; multiplication by 'a^2' corresponds to a displacement by $\frac{4\pi}{3}$; and a multiplication by 'a^3' to a displacement by the whole circumference; and so on.

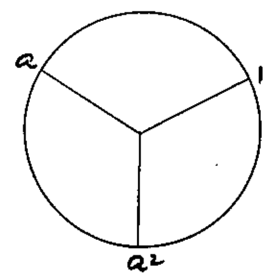

Fig 3.1

Thus a very important conclusion is

arrived at that multiplication of a vector by "a" corresponds to a rotation of the vector through 120°.

The term "a" is commonly known as rotational vector or vector operator (like the operator "j" which when operates on a vector turns it through 90°).

3.3 PROPERTIES OF THE OPERATOR "a".

From the fact that operation on a vector by "a" corresponds to a rotation of the vector through 120° a number of other important properties may be derived as given in table 3.1 and Fig 3.2.

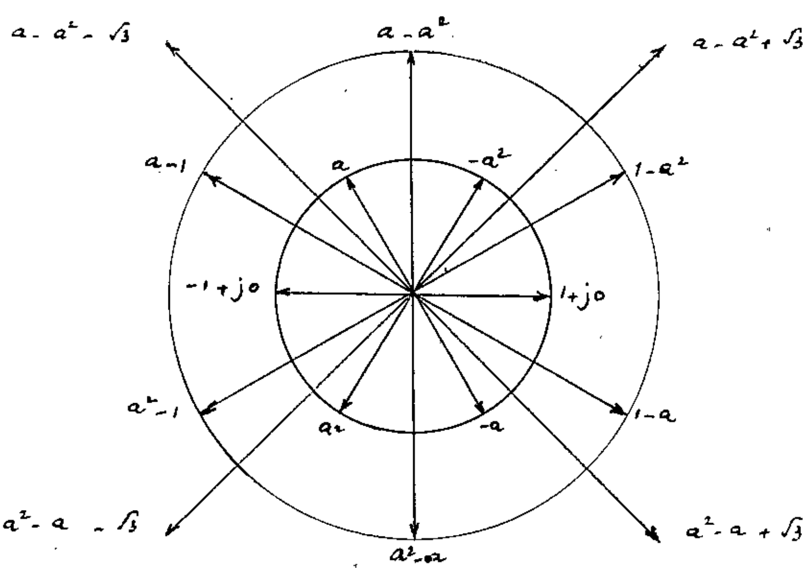

Fig 3.2

Table 3.1

Properties of the vector operator "a".

$1 = 1 + j0 = e^{j0}$	$a + a^2 + 1 = 0$
$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j120}$	$a + a^2 = -1 + j0 = e^{j180}$
$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{j240}$	$a - a^2 = 0 + j\sqrt{3} = \sqrt{3}\,e^{j90}$
$a^3 = 1 + j0 = e^{j0}$	$a^2 - a = 0 - j\sqrt{3} = \sqrt{3}\,e^{j270}$
$a^4 = a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j120}$	$1 - a = \frac{3}{2} - j\frac{\sqrt{3}}{2} = ja^2\sqrt{3} = \sqrt{3}\,e^{j330}$
$a^5 = a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{j240}$	$1 - a^2 = \frac{3}{2} + j\frac{\sqrt{3}}{2} = -ja\sqrt{3} = \sqrt{3}\,e^{j30}$
$a - 1 = -\frac{3}{2} + j\frac{\sqrt{3}}{2} = -ja^2\sqrt{3} = \sqrt{3}\,e^{j150}$	$a^2 - 1 = -\frac{3}{2} - j\frac{\sqrt{3}}{2} = ja\sqrt{3} = \sqrt{3}\,e^{j210}$
$1 + a^2 = \cancel{-a^2 =} \frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j60}$	$1 + a^2 = -a = +\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{j300}$
$(1+a)(1+a^2) = 1 + j0 = e^{j0}$	$(1-a)(1-a^2) = 3 + j0 = 3e^{j0}$
$\frac{1+a}{1+a^2} = a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j120}$	$\frac{1-a}{1-a^2} = -a = \frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{j300}$
$(1+a)^2 = a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j120}$	$(1+a^2)^2 = a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{j240}$

3.4 POSITIVE SEQUENCE SYSTEM.

From the basic properties of the "a" operator, it is obvious that a symmetrical system of vectors

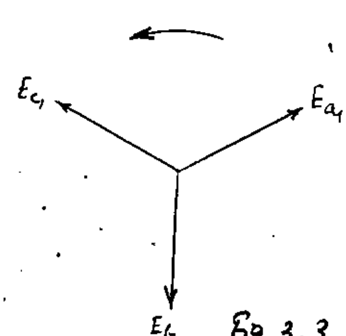

Fig 3.3

E_{a_1}, E_{b_1} and E_{c_1} (Fig 3.3) are relative to each other as:

$$\left.\begin{aligned} E_{a_1} &= E_{a_1} = E_{a_1} \\ E_{b_1} &= e^{j240} E_{a_1} = a^2 E_{a_1} \\ E_{c_1} &= e^{j120} E_{a_1} = a\, E_{a_1} \end{aligned}\right\} \text{———} \quad 3.5$$

This system of vectors is known as <u>positive-sequence system</u>, because the maxima occur in the sequence abc. Or, a system of three-phase currents of similar phase-sequence as that of the voltage is known as positive-sequence system. From the equations 3.5, by fixing the position and magnitude of E_{a_1}, the corresponding positions and magnitudes of E_{b_1} and E_{c_1} may be immediately determined.

3.5 NEGATIVE SEQUENCE SYSTEM.

Referring to Fig 3.4 we have as before:

$$\left.\begin{aligned} E_{a_2} &= E_{a_2} = E_{a_2} \\ E_{b_2} &= e^{j120} E_{a_2} = a E_{a_2} \\ E_{c_2} &= e^{j240} E_{a_2} = a^2 E_{a_2} \end{aligned}\right\} \text{———} \quad 3.6$$

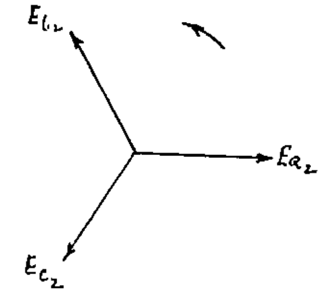

Fig 3.4

This system of vectors is called the negative-sequence system. In this case the maxima occur in the sequence c b a, and as for positive-sequence system, by knowing one vector E_{a_2} in magnitude and position others can be found out. The negative-sequence system is that system of three-phase currents with opposite phase-sequence to that of the voltages.

3.6 ZERO SEQUENCE SYSTEM

From fig 3.5 we have

$$\left.\begin{aligned} E_{a_0} &= E_{a_0} \\ E_{b_0} &= E_{a_0} \\ E_{c_0} &= E_{a_0} \end{aligned}\right\} \qquad (3.7)$$

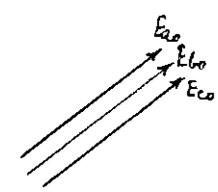

Fig 3.5

This is the zero-sequence system in which all the vectors are in the same phase, and, of course, equal in magnitude. Thus, a system representing the resultant or residue is the zero-sequence system, the physical significance of which arises when a path is established between any phase and neutral or the earth, so that the sum of the instantaneous line currents in the three-phase system is no longer zero; as it is a normal case, actually leaving a resultant which can be conceived of three vectors all in phase with each other and equal to one-third of the residue.

3.7 PHYSICAL SIGNIFICANCE OF THE COMPONENT SYSTEMS.

A physical picture of these vectors may be had by considering the fields which result when voltages of positive- and negative-sequence systems are applied to a three-phase machine like induction motor. If a, b and c phases of the positive-sequence voltages be applied to the terminals a, b and c respectively, a magnetic field will be produced which will revolve in a certain direction. If now the voltages to the terminals b and c are changed by interchanging the leads to the terminals b and c a magnetic field will be produced which will rotate in the opposite direction according to the theory of induction motor. It follows, therefore, that the negative-sequence set of voltages produce a field rotating in an opposite direction to that of the positive-sequence system.

3.8 COMBINATION OF SEQUENCE QUANTITIES TO FORM PHASE QUANTITIES.

The total voltage of any phase is equal to the sum of the corresponding components of the different sequences in that phase. Thus any three vectors E_a, E_b and E_c representing a three-phase unbalanced system may be written as:

$$\left.\begin{aligned} E_a &= E_{a_0} + E_{a_1} + E_{a_2} \\ E_b &= E_{b_0} + E_{b_1} + E_{b_2} \\ E_c &= E_{c_0} + E_{c_1} + E_{c_2} \end{aligned}\right\} \text{——— } 3.8$$

Or, from equations (3.5), (3.6) and (3.7) we have:

$$E_a = E_{a_0} + E_{a_1} + E_{a_2} \quad \text{———} \quad 3.9$$

$$E_b = E_{a_0} + a^2 E_{a_1} + a E_{a_2} \quad \text{———} \quad 3.10$$

$$E_c = E_{a_0} + a E_{a_1} + a^2 E_{a_2} \quad \text{———} \quad 3.11$$

where the suffixes denote the components in the different phases.

Thus the unbalanced system is defined in terms of three balanced systems.

3.9 RESOLUTION OF VECTORS INTO THEIR SYMMETRICAL COMPONENTS.

Zero-sequence Component: Adding the three equations (3.9), (3.10) and (3.11) we have:

$$E_a + E_b + E_c = 3E_{a_0} + (1+a^2+a)E_{a_1} + (1+a+a^2)E_{a_2}$$

But $(1+a^2+a) = (1+a+a^2) = 0$

$$\therefore \quad E_a + E_b + E_c = 3E_{a_0}$$

$$\text{Or,} \quad E_{a_0} = \tfrac{1}{3}(E_a + E_b + E_c) \quad \text{———} \quad 3.12$$

Positive-sequence Component: Multiplying the equation (3.10) by 'a' and (3.11) by a^2 and adding these to (3.9) we have:

$$E_a + aE_b + a^2E_c = (1+a+a^2)E_{a_0} + 3E_{a_1} + (1+a^2+a)E_{a_2}$$

Again $(1+a+a^2) = 0$

$$\therefore \quad E_{a_1} = \frac{1}{3}(E_a + aE_b + a^2E_c) \text{ ——— } 3.13$$

<u>Negative-sequence Component</u>: Multiplying (3.10) by a^2 and (3.11) by 'a' and adding these to (3.9) we have:

$$E_a + a^2E_b + aE_c = (1+a^2+a)E_{a_0} + (1+a+a^2)E_{a_1} + 3E_{a_2}$$

$$\therefore \quad E_{a_2} = \frac{1}{3}(E_a + a^2E_b + aE_c) \text{ ——— } 3.14$$

Graphical Method: <u>Zero-sequence Component</u>: This can be obtained by adding the vectors and trisecting this geometric sum as shown in fig 3.6

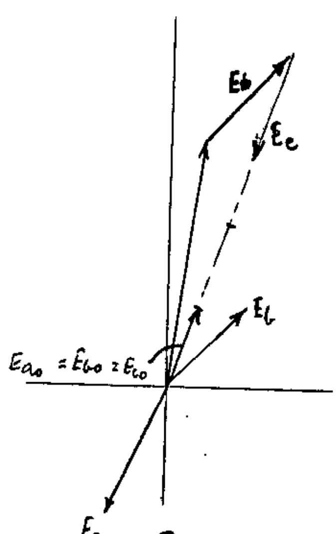

Fig 3.6

<u>Positive-sequence Component</u>:

The positive-sequence component E_{a_1} is found by adding the vector E_a

to the vector E_c rotated through $120°$, and the vector E_b rotated through $240°$, and then trisecting the sum as shown in fig (3.7).

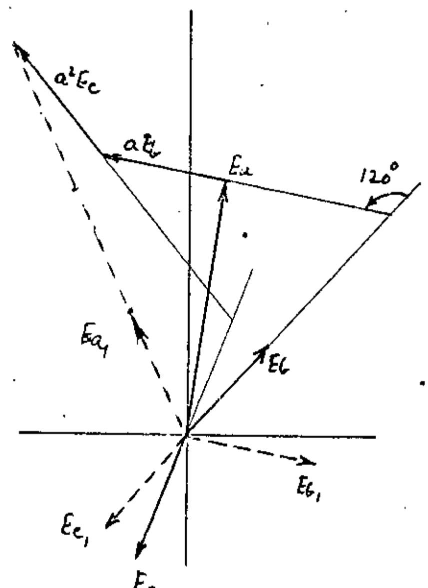

Fig 3.7

Negative-sequence Component: The negative-sequence component E_{a_2} is found by trisecting the sum of the vector E_a, the vector E_b turned through $240°$, and the vector E_c rotated through $120°$ as shown in fig (3.8).

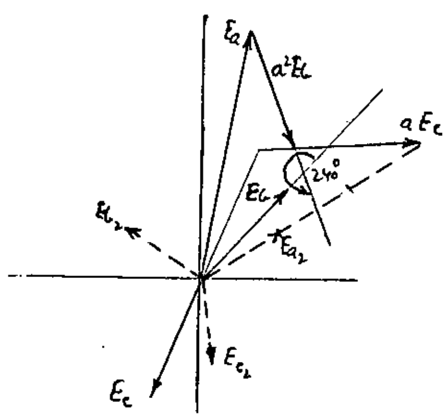

Fig 3.8

Obviously, these graphical constructions are based upon the equation (3.12), (3.13) and (3.14). The analytical method of resolution of the vectors into their symmetrical components is more accurate than the graphical methods

3.10 STAR-DELTA TRANSFORMATIONS.

In the case of a mesh-connected system the calculations are simpler, than for star-connected system, as the voltage across each phase of the load and the phase currents can then be calculated independently of one another. For this reason, it is often desirable to reduce an unbalanced star-connected system to the equivalent mesh-connected system, or alternatively, a mesh-connected system can be converted to the equivalent star. The results established in this connection are particularly due to Wagner and Evans.

<u>Current Relations</u>: Let I_A, I_B, and I_C, and I_a, I_b and I_c be the currents as shown in fig 3.9 (p. 27) for a delta-connected load. Then, vectorially we have the following relations, as also given in the vector diagrams.

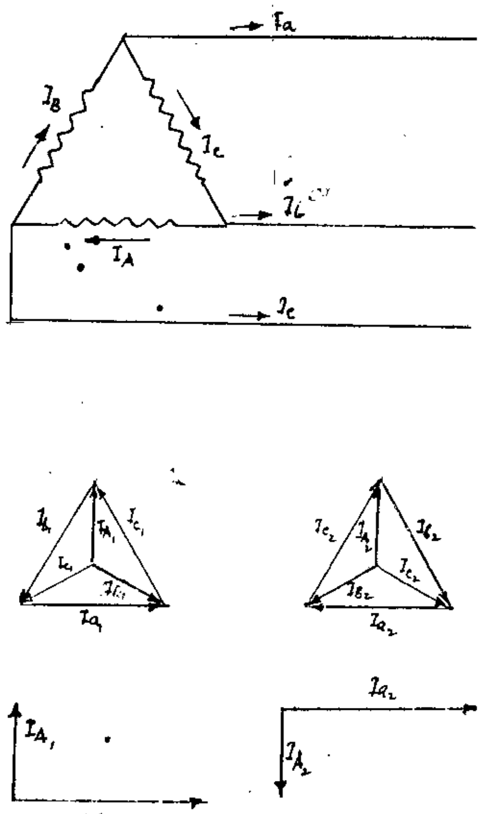

Fig 3.9

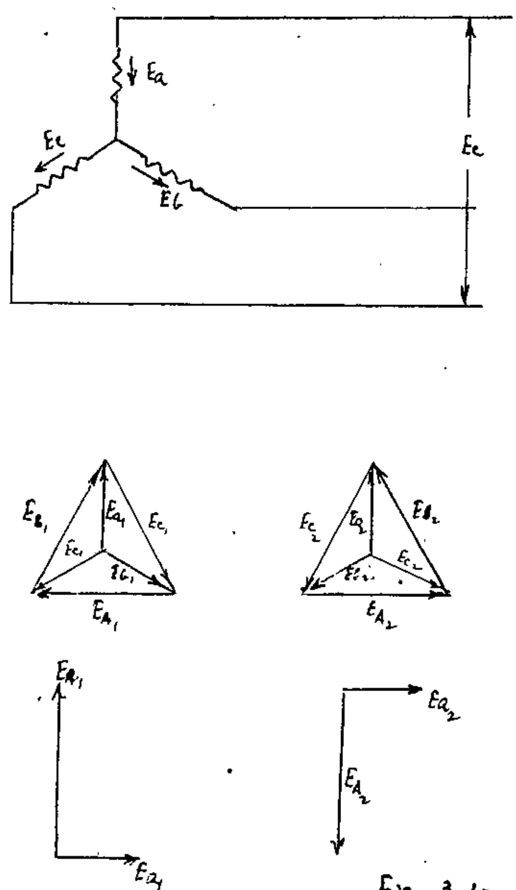

Fig 3.10

I_{A0} = indeterminate

$I_{A_1} = j \dfrac{I_{a_1}}{\sqrt{3}}$

$I_{A_2} = -j \dfrac{I_{a_2}}{\sqrt{3}}$

$I_{a0} = 0$

$I_{a_1} = -j\sqrt{3}\, I_{A_1}$

$I_{a_2} = +j\sqrt{3}\, I_{A_2}$

Y - Δ Transformation for currents.

$E_{A0} = 0$

$E_{A_1} = j\sqrt{3}\, E_{a_1}$

$E_{A_2} = -j\sqrt{3}\, E_{a_2}$

E_{a0} = indeterminate

$E_{a_1} = -j \dfrac{E_{A_1}}{\sqrt{3}}$

$E_{a_2} = +j \dfrac{E_{A_2}}{\sqrt{3}}$

Y - Δ Transformation for voltages.

$$\left.\begin{aligned} I_a &= I_B - I_C \\ I_b &= I_C - I_A \\ I_c &= I_A - I_B \end{aligned}\right\} \text{———— } 3.15$$

The zero-sequence component of the star-currents I_{a0} is given by equation (3.12):

i.e. $I_{a0} = \frac{1}{3}(I_a + I_b + I_c)$

$$= \frac{1}{3}\left[(I_B + I_C + I_A) - (I_C + I_A + I_B)\right] = 0 \text{ — } 3.16$$

This shows that the <u>zero-sequence current of a poly-phase circuit feeding into a delta-connection is always zero</u>. The current of zero-sequence may circulate within the delta without getting out into the line. The converse of determining I_{A0} in terms of I_{a0} is indeterminate.

For positive-sequence component, by equation 3.13

$$I_{a1} = \frac{1}{3}(I_a + aI_b + a^2 I_c)$$

$$= \frac{1}{3}\left[(I_B + aI_C + a^2 I_A) - (I_C + aI_A + a^2 I_B)\right]$$

$$= \frac{1}{3}\left[(a^2 I_A + I_B + aI_C) - (aI_A + a^2 I_B + I_C)\right]$$

Or, $I_{a_1} = \frac{a^2}{3}(I_A + aI_B + a^2 I_C) - \frac{a}{3}(I_A + aI_B + a^2 I_C)$

$= \frac{1}{3}(I_A + aI_B + a^2 I_C)(a^2 - a)$

$= (a^2 - a) I_{A_1} = -j\sqrt{3}\, I_{A_1}$

$\therefore$ Or, $I_{A_1} = \frac{j}{\sqrt{3}} I_{a_1}$ ——— 3.17

For negative-sequence component, similarly

$I_{A_2} = -\frac{j}{\sqrt{3}} I_{a_2}$ ——— 3.18

These results are shown in Fig 3.9

<u>Voltage Relations</u>: Let E_A, E_B and E_C (Fig 3.10) be the delta-voltages, and E_a, E_b and E_c the star-voltages. By definition, therefore:

$$\left.\begin{aligned} E_A &= E_c - E_b \\ E_B &= E_a - E_c \\ E_C &= E_b - E_a \end{aligned}\right\} \quad \text{——— 3.19}$$

Since delta voltages form a closed triangle, therefore, E_A, E_B and E_C must always be zero and $E_{A_0} = 0$. The delta-voltages, therefore, can never contain a zero-sequence component. The star voltages, on the other hand, may contain a zero-sequence component. As before, it follows that E_{ao} cannot be found out from E_{Ao}.

The relations between star- and delta-voltages of positive- and negative-sequence voltages are shown in fig 3.10.

3.11 UNBALANCED IMPEDANCES.

Unbalanced impedances can be resolved into symmetrical components, although the impedances are vector operators, and not rotating vectors as are three-phase voltages and currents. From fig 3.11 we have the symmetrical components as:

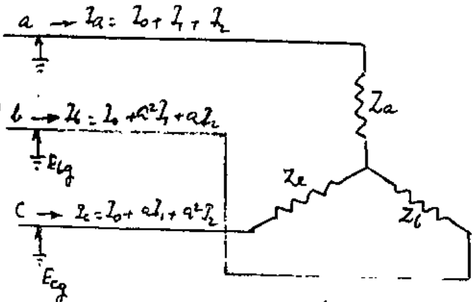

Fig 3.11

$$
\left.\begin{aligned}
Z_0 &= \tfrac{1}{3}(Z_a + Z_b + Z_c) \\
Z_a &= \tfrac{1}{3}(Z_a + a^2 Z_b + a Z_c) \\
Z_c &= \tfrac{1}{3}\cdot(Z_a + a Z_b + a^2 Z_c)
\end{aligned}\right\} \qquad 3.20
$$

The sequence-components of currents through the impedances, and the sequence-components of the line voltages impressed across them are interrelated by the following equations:

$$\left.\begin{aligned} E_0 &= \tfrac{1}{3}(E_{ag} + E_{bg} + E_{cg}) = I_0 Z_0 + I_1 Z_2 + I_2 Z_1 \\ E_1 &= \tfrac{1}{3}(E_{ag} + aE_{bg} + a^2 E_{cg}) = I_0 Z_1 + I_1 Z_0 + I_2 Z_2 \\ E_2 &= \tfrac{1}{3}(E_{ag} + a^2 E_{bg} + aE_{cg}) = I_0 Z_2 + I_1 Z_1 + I_2 Z_0 \end{aligned}\right\} \quad 3.21$$

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4.

POWER IN UNBALANCED SYSTEMS.

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It is evident that no power can be associated with either the positive- or negative-sequence components of voltages and the zero-sequence currents (or vice verse), since the sum of the components in either a positive- or negative-sequence is zero, and all the components of the zero-sequence are equal. Again, no resultant power is associated with voltages and currents of opposite sequence. For instance, if

$$e_{a_1} = \sqrt{2}\,E\cos\omega t \qquad\qquad i_{a_2} = \sqrt{2}\,I\cos(\omega t - \phi)$$

$$e_{b_1} = \sqrt{2}\,E\cos\left(\omega t - \tfrac{2\pi}{3}\right) \quad \text{and} \quad i_{b_2} = \sqrt{2}\,I\cos\left(\omega t - \phi + \tfrac{2\pi}{3}\right)$$

$$e_{c_1} = \sqrt{2}\,E\cos\left(\omega t - \tfrac{4\pi}{3}\right) \qquad\qquad i_{c_2} = \sqrt{2}\,I\cos\left(\omega t - \phi + \tfrac{4\pi}{3}\right)$$

then we have:

$$e_{a_1} i_{a_2} + e_{b_1} i_{b_2} + e_{c_1} i_{c_2} = EI\left[\cos(2\omega t - \phi) + \cos\phi + \cos(2\omega t - \phi) + \cos\left(\phi - \tfrac{4\pi}{3}\right) + \cos(2\omega t - \phi) + \cos\left(\phi - \tfrac{8\pi}{3}\right)\right]$$

$$= 3EI\cos(2\omega t - \phi) \qquad 4.1$$

Thus there is a considerable oscillating power, but no resultant. It therefore follows the total average power can be considered as being due to the sum of the powers in the positive-, negative- and the zero-phase sequences.

4.1 TRUE POWER AND APPARENT POWER.

Writing the unsymmetrical voltages and currents in terms of their symmetrical components, we have the expressions for the apparent power in volt-amps. for the three phases as:

$$(E_{a0} + E_{a1} + E_{a2})(I_{a0} + I_{a1} + I_{a2});$$
$$(E_{a0} + a^2E_{a1} + aE_{a2})(I_{a0} + aI_{a1} + a^2I_{a2});$$
$$(E_{a0} + aE_{a1} + a^2E_{a2})(I_{a0} + a^2I_{a1} + aI_{a2}).$$

The results of multiplication are:

$$E_{a0}I_{a0} + E_{a0}I_{a1} + E_{a0}I_{a2} + E_{a1}I_{a0} + E_{a1}I_{a1} + E_{a1}I_{a2} + E_{a2}I_{a0} + E_{a2}I_{a1} + E_{a2}I_{a2},$$
$$E_{a0}I_{a0} + aE_{a0}I_{a1} + a^2E_{a0}I_{a2} + a^2E_{a1}I_{a0} + E_{a1}I_{a1} + aE_{a1}I_{a2} + aE_{a2}I_{a0} + a^2E_{a2}I_{a1} + E_{a2}I_{a2},$$
$$E_{a0}I_{a0} + a^2E_{a0}I_{a1} + aE_{a0}I_{a2} + aE_{a1}I_{a0} + E_{a1}I_{a1} + a^2E_{a1}I_{a2} + a^2E_{a2}I_{a0} + aE_{a2}I_{a1} + E_{a2}I_{a2}$$

$$= 3E_{a0}I_{a0} + 3E_{a1}I_{a1} + 3E_{a2}I_{a2}$$

Thus, by adding these the total apparent power is obtained. As is obvious from the fundamental equation all the terms containing expressions of different systems (i.e. except those which are marked "✓") will disappear in summing up.

$$\text{Thus, } VA = 3\left(E_{a0} I_{a0} + E_{a1} I_{a1} + E_{a2} I_{a2}\right) \quad - 4.2$$

If the apparent power of the zero-, positive- and negative- systems are, say, VA_0, VA_1 and VA_2, then equation (4.2) may be written as

$$VA = 3\left(VA_0 + VA_1 + VA_2\right)$$

i.e. the total apparent power is thrice the sum of the apparent power of the zero-, positive- and negative- systems.

The apparent power may be resolved into its real and imaginary parts i.e. true and reactive power as:

$$\left.\begin{aligned} P_0 &= 3 E_0 I_0 \cos\phi_0 \\ P_1 &= 3 E_1 I_1 \cos\phi_1 \\ P_2 &= 3 E_2 I_2 \cos\phi_2 \end{aligned}\right] \quad \text{and} \quad \left.\begin{aligned} Q_0 &= 3 E_0 I_0 \sin\phi_0 \\ Q_1 &= 3 E_1 I_1 \sin\phi_1 \\ Q_2 &= 3 E_2 I_2 \sin\phi_2 \end{aligned}\right]$$

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In these expressions ϕ_0, ϕ_1 and ϕ_2 represent the respective phase angles between the voltages and currents of the three systems, and the latter are expressed in effective values. It should, however, be remembered that the expressions for power are mean values.

Summing up, therefore, we may say that:

$$P_T = P_a + P_b + P_c$$

and

$$Q_T = Q_a + Q_b + Q_c$$

or

$$P_T + jQ_T = E_a I_a + E_b I_b + E_c I_c$$

$$= 3[(P_0 + P_1 + P_2) + j(Q_0 + Q_1 + Q_2)] \quad \text{—— 4.3}$$

as shown in fig 4.1.

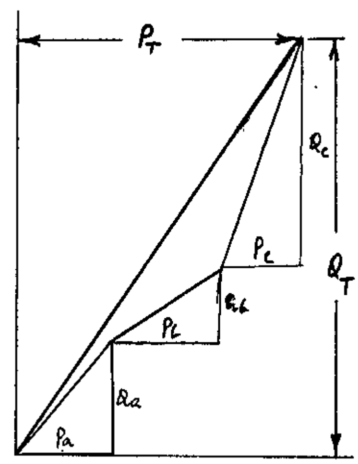

Fig 4.1

4.2. POWER FACTOR.

It has been shown by Saurage that the definition of power factor for unsymmetrical poly-phase systems may lead to certain inconveniences, as is the case with non-sinusoidal voltages and currents. In his investigations, Saurage carried out the experiments

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with a large slow speed induction motor, the coils of which were unsymmetrical in the three phases. In trying to verify the power factor guaranteed by the manufacturer, different values were obtained according to the measurements made in the different phases. At that time Stakvis proposed to resolve the system into its symmetrical components and to investigate the power factor of the symmetrical system. In a discussion about the definitions of A.C. quantities before the A.I.E.E. Slepian referred to a similar case in connection with a synchronous machine. The manufacturer had guaranteed a unity p.f. of a synchronous convertor. The connecting leads between the transformers and the slip rings were laid unsymmetrical by the customer and the fulfilling of guarantee would have required an excitation which the machine was unable to provide.

In connection with these discussions in the American Standards the following value of p.f was fixed.

$$p.f = \frac{P}{\sqrt{P^2 + Q^2}}$$

where P = True power = $P_a + P_b + P_c$ and

Q = Reactive power = $Q_a + Q_b + Q_c$

The same definition was recommended by Schering.

If the asymmetries are comparatively small the power factor of the positive-sequence system is practically equal to the total power-factor. With large asymmetries, however, this is not the case. If the unsymmetrical voltages and currents are known in magnitude and phase displacement, the power factor of the positive-, negative- and zero-sequence systems may be easily computed. These results may be checked by graphical methods.

Note: For the definition of p.f. VA should not be used instead of reactive power, which would arise confusion.

4.3: UNBALANCE FACTOR.

Unbalance factor is used to designate the amount of asymmetry present in an unbalanced system. It is defined as the ratio of the negative to positive sequence component of voltage, or current, in a system with zero resultant. The German Electrical Standards consider a poly-phase system to be symmetrical if the unbalance factor is not greater than 0.05 i.e. if the amplitude of the negative sequence system amounts to not more than 5% of the amplitude of the positive.

Charts for determining the positive and negative sequence components, or the unbalance factor if the unsymmetrical voltages or currents are given, have been developed by Dubusc and also by Hauffe. Wagner and Evans give similar charts generalised in so far as the curves given by Dubuse are developed

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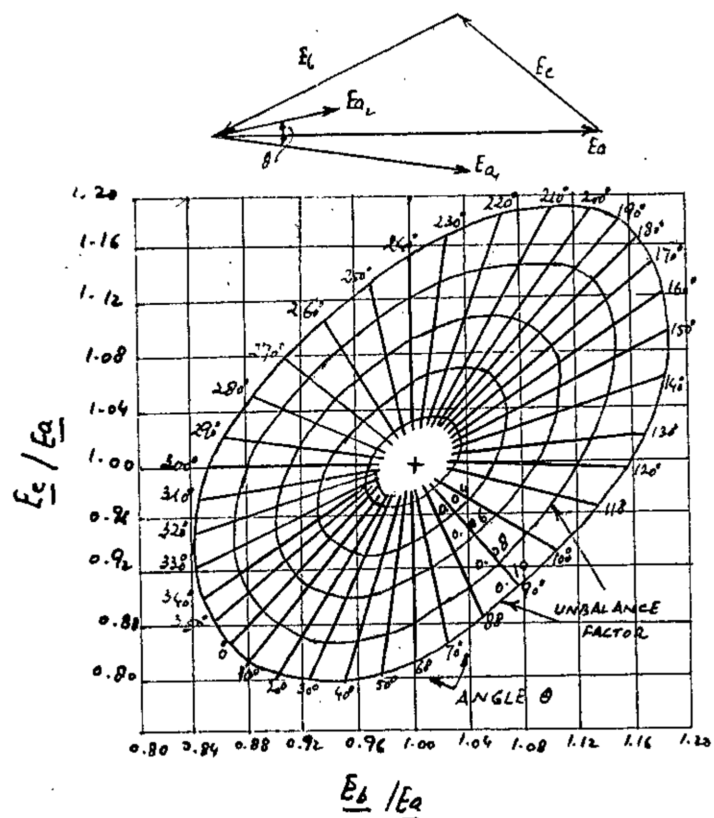

Determination of unbalance factor.

Fig 4.2

for low values of the unbalance factor also, while those given by Hauffe are extended to higher values. One given by Wagner and Evans is reproduced in fig. 4.2. In this the ratio $\frac{\bar{E_b}}{\bar{E_a}}$ and $\frac{\bar{E_c}}{\bar{E_a}}$ are used as coordinates and the point thus determined gives the unbalance factor. by interpolation of a family of

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curves. The angle plotted in the diagram gives the angle by which the vector E_{a_2} leads the vector E_{a_1}.

4.4 PULSATING POWER AND UNBALANCE FACTOR OF POWER.

The "pulsating power" P_p is given, for a system without zero conductor, by the formula

$$P_p = 3\sqrt{(E_1 I_2)^2 + (E_2 I_1)^2 + 2E_1 I_1 E_2 I_2 \cos(\phi_2 - \phi_1)}$$

and the ratio $\frac{P_p}{VA}$ is known as the <u>unbalance factor of power</u>, where VA is the volt-amp of the system.

4.5 FLOW OF POWER DUE TO UNBALANCE

In any symmetrical system with symmetrical loads only the positive-sequence power ($P_1 + jQ_1$) can flow. But in case of unbalanced load negative- and zero-sequence component power may also be produced. A symmetrical machine can produce only positive-sequence voltages. Therefore, in a sense positive-sequence components of power are supplied by the generators, and the dissymmetry converts part to negative and zero-sequence components of power, which are fed into the system at the point of fault.

Consider an unbalanced load from line-to-neutral on a system supplied through a grounded generator with the positive-, negative-, and zero-sequence impedances of Z_1, Z_2, and Z_0 respectively, and with

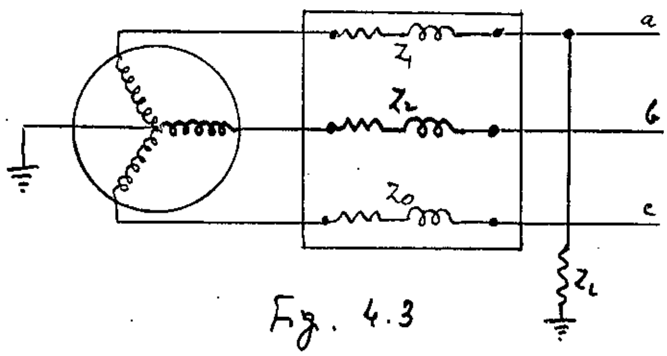

Fig. 4.3

a generated positive-sequence line-to-neutral voltage of E_{g_1}. Assume that the system is supplying a load on phase 'a' whose impedance is Z_L as shown in fig. 4.3. For sequence voltages, we have, therefore:

$$E_1 = (Z_2 + Z_0 + 3Z_L)\frac{E_{g_1}}{Z_t}$$

$$E_2 = -Z_2\frac{E_{g_1}}{Z_t}$$

$$E_0 = -Z_0\frac{E_{g_1}}{Z_t}$$

$$I_1 = I_2 = I_0 = \frac{E_{g_1}}{Z_t}$$

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where $Z_t = Z_1 + Z_2 + Z_0 + 3Z_L$.

The total ~~phase~~ values at point of load may be obtained, from the previous equations, so that,

$$E_a = 3Z_L \frac{E_{g_1}}{Z_t} \quad \text{and} \quad I_a = \frac{3E_{g_1}}{Z_t}$$

The sequence power per phase at the load are:

$$P_1 + jQ_1 = (Z_2 + Z_0 + 3Z_L)\left(\frac{\overline{E_{g_1}}}{Z_t}\right)^2 \qquad 4.4$$

$$P_2 + jQ_2 = -Z_2\left(\frac{\overline{E_{g_1}}}{Z_t}\right)^2 \qquad 4.5$$

$$P_0 + jQ_0 = -Z_0\left(\frac{\overline{E_{g_1}}}{Z_t}\right)^2 \qquad 4.6$$

From equation (4.3) total power

$$P_T + jQ_T = 3[(P_1 + jQ_1) + (P_2 + jQ_2) + (P_3 + jQ_3)] = 9Z_1\left(\frac{\overline{E_{g_1}}}{Z_t}\right)^2$$

The significance of negative sign of the equations (4.5) and (4.6) is that flow of $(P_2 + jQ_2)$ and $(P_0 + jQ_0)$ is in a direction opposite to that of $(P_1 + jQ_1)$. Thus an unbalanced load not only draws from a symmetrical. the total power required for the load, but also the negative- and zero-sequence components of the power, which are fed back into the system at the point of unbalance and which are consumed in producing negative- and zero-sequence losses, and in absorbing inductive-reactance voltamps.

4.6 COMBINATION OF UNBALANCED LOAD AND SYMMETRICAL MACHINE LOAD.

In the combination of an unbalanced load and a balanced load due to a rotating machinery, the unbalanced load produces negative-sequence voltage at the point of unbalance, and the negative-sequence voltage impressed on the system causes negative-sequence current flow not only through the source, but also in all the shunt branches including the symmetrical rotating machinery load. The power due to the flow of negative-sequence currents produces losses in the rotating machinery load, and increases the total amount of power absorbed by the symmetrical load. Rotating machinery on a system tends to balance the load on a system and to restore the voltages to their normal values. This action takes place with the flow of negative-sequence currents in the rotating machinery, and with an increased amount of loss and a reduction in the maximum load capacity of the apparatus.

Under some conditions the negative-sequence current flowing through machines will produce excessive heating in the rotor so that unbalanced operation may require special considerations.

In a number of respects, as suggested by Fortescue, positive-sequence components of power represent more accurately, than the total power, the requirements of power supply systems for properly handling the unbalanced loads.

———— x(x)x ————

5.

THE SYMMETRICAL COMPONENT-METER.

5.1. GENERAL REMARKS.

It has already been shown that it is useful in investigating unsymmetrical three-phase systems to resolve into their symmetrical components, since these afford a clear insight into the kind and size of the asymmetry. It has also been shown that the symmetrical components can be found by graphical and analytical methods if the unsymmetrical quantities are known. In this section is considered the practical methods for the measurements of symmetrical components and their relative phase angles.

The sequence components may be measured by means of instruments or meters, operating on well known principles, alongwith specially designed external windings, or special networks in some cases. In certain cases. it may be sufficient to get only an indication whether or not an asymmetry exists, in other cases it will be required actually to measure the value of the components, and finally there is the problem of using such measurements for the operation of circuit breakers or the like by means of a relay in order to prevent, or render harmless the effects of asymmetry.

Sequence quantities that can be measured include for each sequence the same quantities that can be measured in a single phase circuit viz. voltage, current, KW, KVAr, power factor, VA, etc. In addition the sequence quantities may be combined in various ways for measurements.

The first proposal for measuring the sequence-quantities was the result of a discussion between Fortescue, Chubb and Slepian. They proposed the use of rotating machines to eliminate one sequence to permit the measurement of another sequence-quantity. The first measurements, however, were actually carried out by Evans who suggested static networks to segregate a desired sequence-quantity in the presence of other sequence quantities. Eventually, various types of networks have been worked out by Evans, Allcutt, Fortescue, Russell and others.

On a three-phase four-wire system there may exist three sequence-voltages. Consequently, in order to measure all of these sequence components, it is necessary to have available positive-, negative- and zero-sequence voltmeters. On a three-phase three-wire system the zero-sequence voltage is indeterminate and the corresponding voltmeter is unnecessary and inapplicable. Similar relations hold for the sequence-currents. The sequence-meters may thus be classified as the positive- or negative- (which are similar) and zero-sequence meters

5.2 THE SYMMETRICAL-COMPONENT METER.

Introduction: The unbalanced three-phase systems may or may not contain the zero-sequence component. For instance, the three-phase three-wire system does not contain the zero-sequence component under normal circumstances.

In our symmetrical-component meter the zero-sequence component has been assumed to be absent or eliminated, while measuring the positive- and negative-sequence components. This instrument is based upon the discussions by R. L. Russell (Proc. I.E.E. Feb '50). A number of circuits have been devised for measuring the magnitude of the positive- and negative-sequence components of an unbalanced system. For our use, the particular network suggested by Russell has been selected because of its simplicity and because it enables the measurement of their relative phase displacements. As will be seen later, the only equipments required for the complete set-up are a few resistances and capacitances, a dynamometer-type instrument (preferably a voltmeter), two double-throw switches, a 2-pole switch and isolating transformers.

The principles of measurements are equally applicable to either phase or line values of voltage. But a phase-voltage system has been chosen because of the ease with which unbalanced voltages to neutral, containing no zero-sequence component, can be synthesized for test and calibration purposes.

Theory: Let E_{a_1} and E_{a_2} be the positive- and negative-sequence components. From equations 3.9 & 3.10 (p. 23) we have:

$$E_a = E_{a_0} + E_{a_1} + E_{a_2}$$

$$E_b = E_{a_0} + a^2 E_{a_1} + a E_{a_2}$$

In absence of zero-sequence component, i.e. when $E_{a_0} = 0$

we have $$E_a = E_{a_1} + E_{a_2} \qquad \text{——— } 5.1$$

$$E_b = a^2 E_{a_1} + a E_{a_2} \qquad \text{——— } 5.2$$

Multiplying (5.2) by a^2 we have:

$$a^2 E_b = a E_{a_1} + E_{a_2} \qquad \text{——— } 5.3$$

Subtracting (5.3) from (5.1), therefore:

$$E_a - a^2 E_b = E_{a_1}(1-a) \quad \text{or} \quad E_{a_1} = \left(\frac{E_a - a^2 E_b}{1-a}\right)$$

multiplying the numerator and denominator by $(1-a^2)$ and referring to table 3.1 (p.18):

$$E_{a_1} = \frac{(1-a^2)(E_a - a^2 E_b)}{(1-a^2)(1-a)} = \frac{(1-a^2)(E_a - a^2 E_b)}{3}$$

$$\therefore \quad E_{a_1} = \left(\frac{1-a^2}{\sqrt{3}}\right)\left(\frac{E_a - a^2 E_b}{\sqrt{3}}\right) \qquad \text{——— } 5.4$$

Similarly, for negative sequence component, we have, by multiplying (5.2) by 'a' and subtracting from (5.1), and finally multiplying the numerator and denominator by $(1-a)$

$$E_{a_2} = \frac{(1-a)}{(1-a)(1-a^2)}\left(E_a - a E_b\right) = \frac{(1-a)(E_a - a E_b)}{3}$$

$$\therefore \quad E_{a_2} = \left(\frac{1-a}{\sqrt{3}}\right)\left(\frac{E_a - a E_b}{\sqrt{3}}\right) \qquad \text{——— } 5.5$$

which means that in the absence of zero-sequence component, the positive-sequence component of a particular vector of an unbalanced star system is given correctly in magnitude and phase by adding to it the succeeding vector advanced in phase by 60°, and advancing the resultant reduced in the ratio $1:\sqrt{3}$ a further 30°. A corresponding procedure, as given by equation (5.5), also holds for the negative-sequence component.

The instrument designed is primarily based upon equations (5.4) and (5.5), which clearly shows that for an instrument to respond to positive- or negative-sequence component seperately the essential operation is the 60° phase-shift. Based upon this, circuits can be designed to give a phase-shift of 60° by using either inductance or capacitance. Capacitance was ~~been~~ chosen for our purpose due to convenience, and the basic set-up is shown in Fig 5.1. In the absence of capacitive branch the meter responds to E_a while in the absence of the resistance R_1, the meter reading is proportional to E_b.

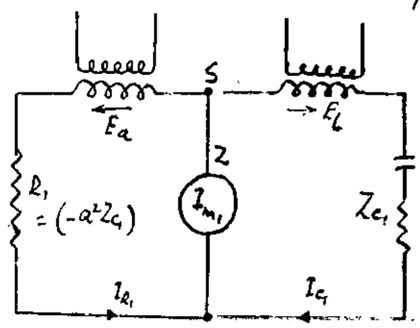


Fig 5.1, Positive-sequence component network:

Now applying Kirchhoff's laws to the two branches we have:

$$\left.\begin{aligned} E_a &= I_{R_1} R_1 + I_{m_1} Z \\ E_b &= I_{C_1} Z_{C_1} + I_m Z \end{aligned}\right\} \quad \text{---}$$

R_1 and Z_{C_1} are so chosen that they are equal in magnitude but differ in phase by 60° i.e. $R_1 = -a^2 Z_{C_1}$. Eliminating Z_{C_1} from (5.6) and comparing the results with (5.4), the positive-sequence component can be expressed as:

$$E_{a_1} = \left[\frac{R_1 + Z(1-a^2)}{\sqrt{3}}\right] I_{m_1} \left(\frac{1-a^2}{\sqrt{3}}\right) \text{ ——— } 5.7$$

Interchanging the resistive and capacitive branches in Fig 5.1

The corresponding equation for the negative sequence component is:

$$E_{a_2} = \left[\frac{R_1 + Z(1-a^2)}{\sqrt{3}}\right] I_{m_2} \left(\frac{1-a^2}{\sqrt{3}}\right) a^2 \quad \text{—— 5.8}$$

We have therefore:

$$\left.\begin{aligned} E_{a_1} &= I_{m_1}\left[\frac{R_1 + Z(1-a^2)}{\sqrt{3}}\right]\left(\frac{1-a^2}{\sqrt{3}}\right) \\ E_{a_2} &= I_{m_2}\left[\frac{R_1 + Z(1-a^2)}{\sqrt{3}}\right]\left[\frac{a(a-1)}{\sqrt{3}}\right] \end{aligned}\right\} \quad \text{—— 5.9}$$

The numerical factor $\left[\frac{R_1 + Z(1-a^2)}{\sqrt{3}}\right]$, which is absorbed as a scale factor, is the same in both cases. The operator terms $\left(\frac{1-a^2}{\sqrt{3}}\right)$ and $\left[\frac{a(a-1)}{\sqrt{3}}\right]$ show that the two currents I_{m_1} and I_{m_2} differ in phase by 120° in addition to the phase difference that exists between E_{a_1} and E_{a_2}. (5.8) can be rewritten as:

$$a E_{a_2} = E_{b_2} = I_{m_2}\left[\frac{R_1 + (1-a^2)Z}{\sqrt{3}}\right]\left(\frac{1-a^2}{\sqrt{3}}\right) \quad \text{—— 5.10}$$

The result of interchanging the branches is therefore to measure E_{b_2} and not the negative-sequence component corresponding to the reference vector E_a.

To develop the circuit further, for the measurement of phase angles, it is proposed to have similar networks of the type discussed, one for the positive sequence component and the other for the negative-sequence component. The fixed and moving coils, respectively, of a dynamometer instrument are connected in the positions normally occupied by the meter. With given currents in the two coils, the deflection will be proportional to the cosine of their phase difference in the usual way. For correct indication the currents should correspond to the same reference vector.

When connected in the manner shown, the instrument will respond to $E_{a_1} E_{a_2} \cos\phi$. By putting the coils in series in each of the networks in turn, E_{a_1} and E_{a_2}

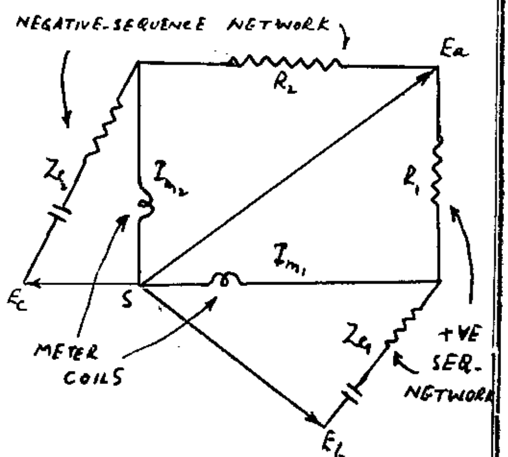


Fig 5.2

can be found seperately, and hence ~~+~~ the phase-difference can be found out by calculation. For rapidity and convenience a nomogram can be constructed for this purpose.

In the completed instrument, the junction S shown in Figs 5.1 and 5.2 is internally connected to the secondary starpoint of an isolating voltage transformer. The primaries of the transformer are connected to the external source.

In order that the currents shall be related to the voltages measured by the same factor in all cases, it is essential that the resistance in the meter branch in each network should be the same for different settings of the selector switch. Compensating resistances are, therefore, introduced when measuring phase angles, making the resistance of each meter coil in the instrument equal to the total meter-resistance when the coils are in series.

Sign of the Phase Angle: The method of measuring phase differences makes no distinction between positive and negative phase angles. Without this information, therefore, two unbalanced systems can be constructed from the test results. In order to select the correct one it is sufficient to observe that, if the negative-sequence component of the reference vector E_a lags behind the positive-sequence component (i.e. for negative angles), E_b is less than E_c, whereas for leading or positive angles E_b is the larger of the two as has been verified below:

Referring the angular position of a negative-sequence component to the corresponding positive-sequence component, with the usual sign convention for

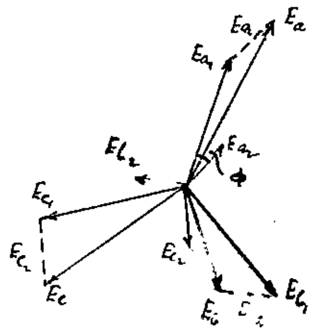


Fig 5.3

positive rotations, we have in Fig 5.3 the reference vectors E_{a_1} and E_{a_2} displaced by an angle ϕ where ϕ is negative

The magnitudes of unbalanced voltages E_b and E_c are then given by:

$$E_b^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(60^\circ+\phi)$$

$$E_c^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(60^\circ-\phi)$$

E_1 & E_2 being the magnitudes of the positive- and negative-sequence components respectively.

The sign of the difference of the expression of E_b^2 and E_c^2 depends only on the sign of ϕ.

Or, $$E_b^2 - E_c^2 = 2E_1E_2\left[\cos(60^\circ-\phi) - \cos(60^\circ+\phi)\right]$$

$$= 2E_1E_2\left(\cos 60^\circ\cos\phi + \sin 60^\circ\sin\phi - \cos 60\cos\phi + \sin 60\sin\phi\right)$$

$$= 2E_1E_2\left(2\sin 60\sin\phi\right) = 2E_1E_2 \cdot 2 \cdot \frac{\sqrt{3}}{2} \cdot \sin\phi$$

i.e. $$\left(E_b^2 - E_c^2\right) = 2\sqrt{3}\,E_1E_2\sin\phi \quad\text{———}\quad 5.11$$

Thus for
$$\left.\begin{array}{ll} 0 < \phi < -180^\circ, & E_b < E_c \\ 0 < \phi < +180^\circ, & E_b > E_c \\ \phi = 0 \text{ or } 180^\circ, & E_b = E_c \end{array}\right\} \quad\text{———}\quad 5.12$$

It is possible, therefore, by measuring E_b and E_c separately, to deduce the sign of ϕ from the results (5.12)

Instead of using separate measurements for comparing E_b and E_c the dynamometer instrument itself may be employed. For this purpose, the switches P and Q (Fig 5.7) have been provided. With the circuit broken at these points the readings will be proportional to E_b or E_c in the positive- and negative-sequence settings respectively of the selector switch.

<u>The Meter Current</u>: The range of the instrument is, evidently, limited by the meter current. Ordinarily, in the sequence-measuring networks, the meter current is given approximately by

$$I_{m_1} = \frac{\sqrt{3}\,E_{a_1}}{R_1} \quad \text{or} \quad I_{m_2} = \frac{\sqrt{3}\,E_{a_2}}{R_2} \qquad 5.13$$

whereas, when measuring E_b or E_c, the largest value which may be encountered, that is when $\phi = \pm 120^\circ$, is

$$I = \frac{E_1 + E_2}{R_1} \quad \text{or} \quad \frac{E_1 + E_2}{R_2} \qquad 5.14$$

Scale: The dynamometer instrument used in the normal way, with the coils in series, responds to the square of the voltage, or current (the impedance of the circuit being constant), so that the scale for the measurement of the sequence voltages is of the square-law type. Used for phase measurements, it responds directly to the quantity being measured, so that the scale would naturally be a linear one in this case.

If r_1 is the positive-sequence component, r_2 the negative-sequence component, and r_{12} the reading on the same scale for the phase-measuring setting, then the phase angle ϕ is given by the expression

$$\cos\phi = \frac{r_{12}^2}{r_1 . r_2} \qquad \text{——— } 5.15$$

This equation shows that $\cos\phi$ can be computed in terms of meter readings to any arbitrary scale of the right form & is independent of the particular calibration.

5.3 SETTING UP THE CIRCUIT.

Kits required: The essential equipments required for the symmetrical component meter are:

1. A dynamometer instrument (a Welch wattmeter)
*2. Resistances: 9300 Ω (2) (R_1 & R_2)+; 4650 Ω (2) (r_1 & r_2)+
   2520 Ω (1) (x)+; 111.5 Ω (1) (y)+
*3. Capacitances: 0.3950 μf (2) (C_1 & C_2)+
4. Isolating transformers: - 3 1-φ 50~ (a, b & c)+
   connected in Δ/Y voltage 220/185. (L-L)
5. Double throw switches - 2. (L & M)+
6. A 2-pole switch, or, 2 1-pole switches. (P and Q)+
7. Connecting wires (flexible) about 6 ft.
8. Terminal Strips. - 3
9. Wooden panels, screws etc.

+. These symbols refer to Fig 5.7 (p. 70)

*. Calculations of these values are given on following page p. 64

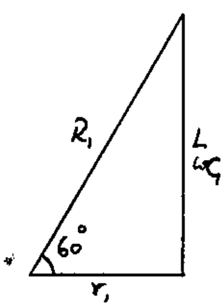
Calculation of Suitable Circuit elements:

Referring to equation (5.13) (p.61), taking the maximum allowable meter current as 23.5 mA, the resistance R_1 (Fig. 5.4) can be taken as 9300 ohms for a voltage of 220 V (line-to-line), as calculated below,

$$I_{m_1} = \frac{\sqrt{3}\,E_{a_1}}{R_1} \quad \text{or} \quad R_1 = \frac{\sqrt{3}\,E_{a_1}}{I_{m_1}}$$

$$\therefore R_1 = \frac{220}{23.5} \times 10^3 \simeq 9{,}300 \text{ ohm.}$$

$$\therefore \underline{R_1 = R_2 = 9300 \text{ ohm.}}$$



From the triangle (Fig. 5.4), since the phase-shift is $60°$, $r_1 = \frac{1}{2} R_1$ and $r_2 = \frac{1}{2} R_2$

Fig 5.4.

$$\therefore r_1 = r_2 = \frac{1}{2} \times 9{,}300 = 4650 \text{ ohm.}$$

$$\therefore \underline{r_1 = r_2 = 4650 \text{ ohm.}}$$

Again $\frac{1}{\omega C_1} = \sqrt{3} \cdot r_1 = \sqrt{3} \cdot 4650$

At a frequency of 50 cycles/sec

$$\therefore C_1 = \frac{1}{100\pi \times \sqrt{3} \times 4650} = 0.3950 \ \mu f.$$

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$$\therefore \; C_1 = C_2 = 0.3950 \; \mu f$$

The respective resistances of the current- and pressure coils were found to be

$$x = 2520 \text{ ohm}$$

$$y = 111.5 \text{ ohm}$$

Hence the values chosen were:

$$\left.\begin{aligned} R_1 &= R_2 = 9300 \text{ ohm} \\ r_1 &= r_2 = 4650 \;\; " \\ C_1 &= C_2 = 0.3950 \; \mu f \\ x &= 2620 \text{ ohm and } y = 111.5 \text{ ohm.} \end{aligned}\right\}$$

Measurements of the values of Circuit elements:

Resistances: The resistances were all measured by a sensitive wheatstone bridge using a ~~[illegible]~~ galvo.

The measured values were:

$R_1 = 9325$ ohm, Error $= 0.268\%$
$R_2 = 9319$ ohm, Error $= 0.204\%$
$x = 2518$ ohm, Error $= 0.008\%$
$y = 111.4$ ohm, Error $= 0.089\%$

These errors are allowable as better values could not be obtained.

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$r_1 = 4645$ ohm Error: 0.107 %
$r_2 = 4654$ ohm Error: 0.107 %

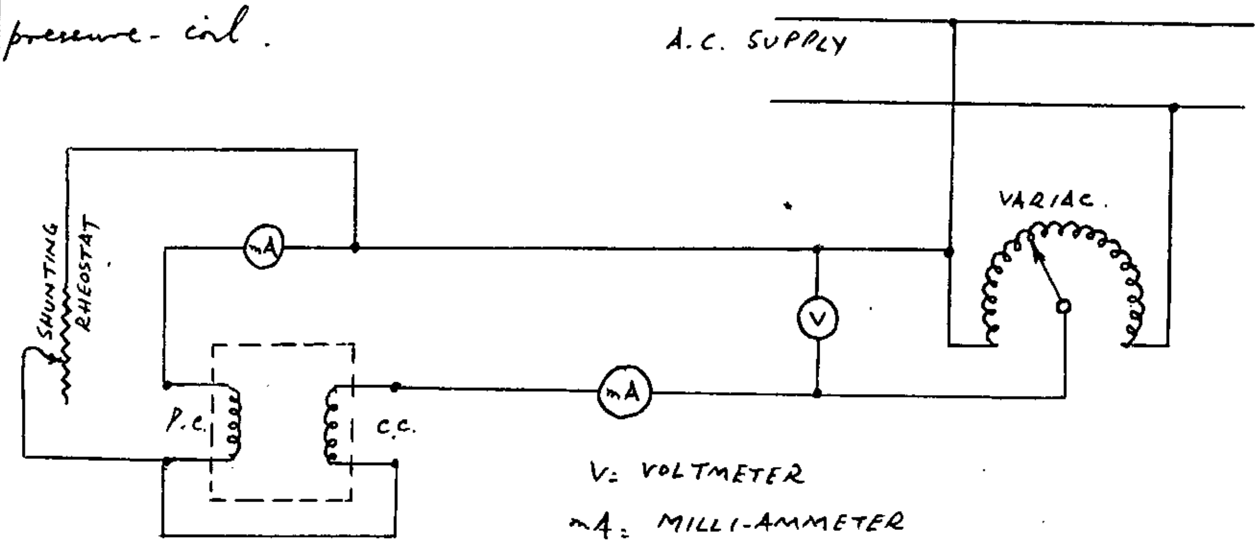
Capacitances: The capacitances C_1 and C_2 were measured by G.R. Test capacitance bridge which was found to be the most suitable for the purpose. First C_1 was measured, and was found to be 0.3950 μf. The bridge was left undisturbed and C_1 was taken out. C_2 was then connected in place of C_1 and its value was so adjusted that the balance was again obtained. This meant that $C_1 = C_2 = 0.3950$ μf.

The measured values of capacitances were $C_1 = C_2 = 0.3950$ μF.

Note: Actually the capacitances were chosen (0.3950 μf) first and the resistances were then calculated from it (Eq 5.4), as it was easy to adjust the resistances to the desired value

The Dynamometer Instrument: The instrument that was used was a "Welch" dynamometer type wattmeter (rated upto 1000 Watts) The wattmeter had to be used due to the unavailibility of the dynamometer voltmeter. The fixed-coil of the instrument (which had very few turns of thick wire) was changed, and instead two new coils with no. 40 SWG wire were wound on a former that was made of the size of the original coils, so that the new wound coils juste fitted into the space provided in the instrument. However, after winding the two coils they were properly insulated and fitted into the instrument. Care was taken in joining the two coils together so that they produced the lines of force (and therefore the torque) in the same direction. These new coils, obviously, had very large number of turns so that the meter gave the full-scale deflection even with very low current (40 mA).

After winding the coils and fixing them, the two leads of each of the pressure- and current coils were brought out and soldered to the binding screws. The coils were then connected in series and the instrument was then tested as a voltmeter. The circuit that was used for testing is shown in Fig 5.5. The variac was used to vary the applied voltage and shunting rheostat was connected to limit the current in the pressure-coil.



V: VOLTMETER
mA: MILLI-AMMETER
CC. : CURRENT COIL P.C.: PRESS. COIL

Fig 5.5.

*Isolating Transformer: Three old single-phase transformers were taken out from the junk and properly insulated and tested. Windings 3-4 of each were connected to form delta-primary and 1-2 were connected to form Y-secondary as shown in Fig 5.6. The secondary was connected to the meter circuit (Fig 5.7). The voltage ratio as found by testing is indicated in the figure (220 : 185V). Each line gave the same voltage.

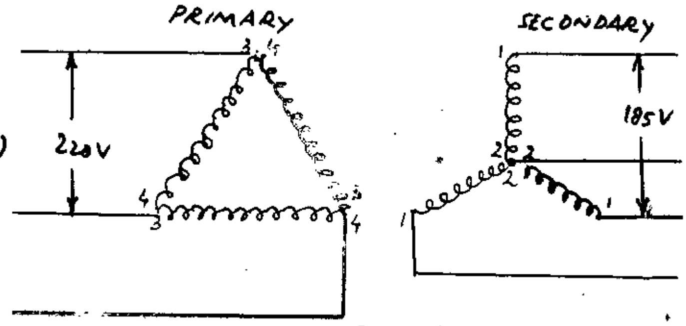


Fig 5.6

Circuit Connections and General Set-up: The meter, 2-pole switch, double-throw switches were all mounted on a panel as shown in the plan (Fig 5.8). The prepared network was then connected as shown in Fig 5.7, and the conventional phase sequence A, B, C marked on the appropriate terminals.

The double throw two-pole switches L and M

*See also appendix 2 p146.

CIRCUIT DIAGRAM OF THE SYMMETRICAL COMPONENT METER.

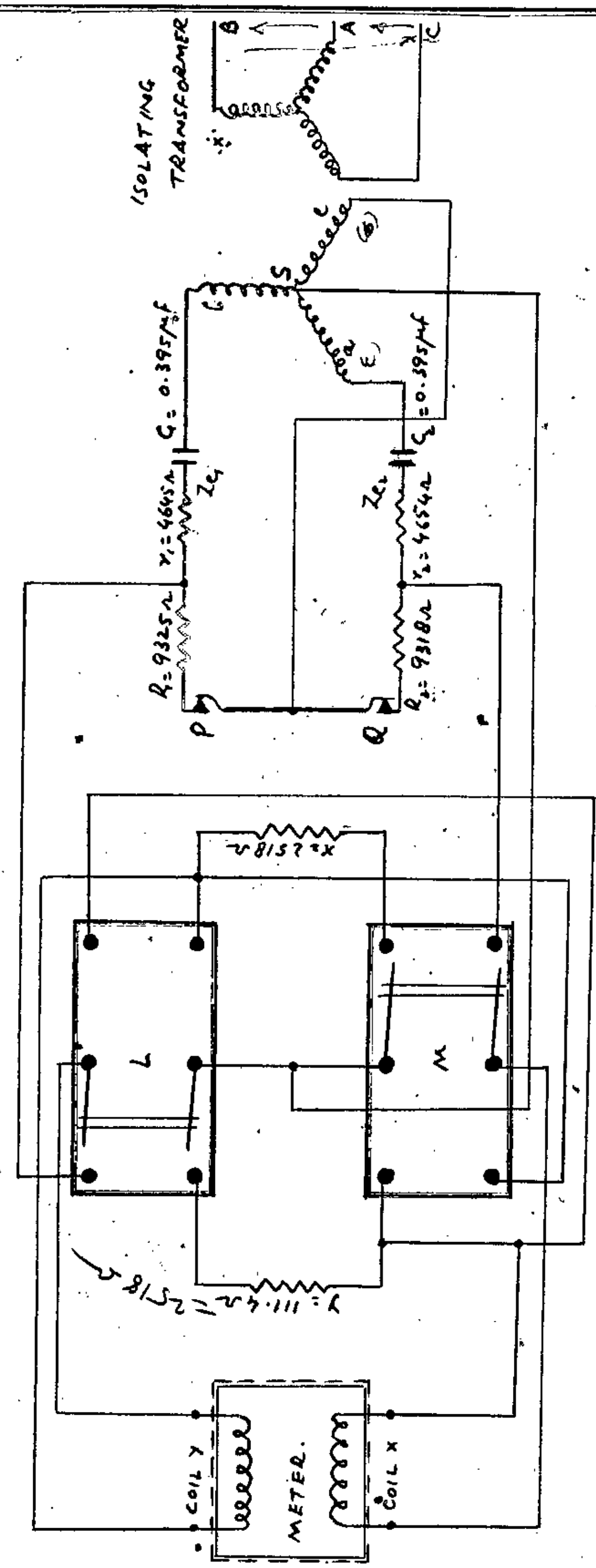


Fig 5.7

* Note: PRIMARY MAY BE CONNECTED IN Y. OR Δ.

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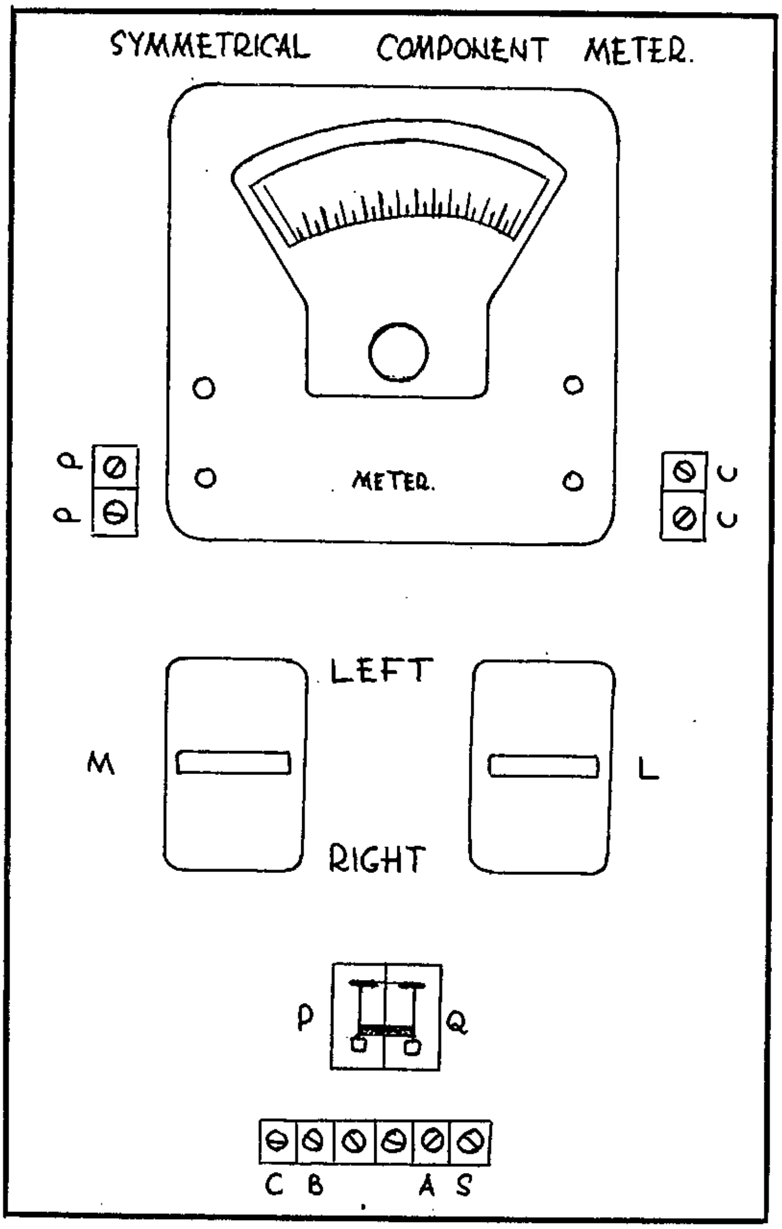

PLAN OF SYMMETRICAL - COMPONENT METER.

Fig 5.8.

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from the selector switch. For sequence measurements, the instrument coils, whose resistances are shown as x and y ohms, are thrown in series in each network in turn. In phase-measuring setting, one coil only appears in each of the networks and additional resistors are therefore introduced so that the value of the resistance in the meter branch in both networks is kept constant at (x+y) ohm.

All the resistors and capacitors used were clamped to the top inside the box (Fig 5.8). All the connections were properly soldered and insulated to avoid short-circuiting.

5.4 CALIBRATION OF THE METER.

A variable three-phase balanced voltage supply at 50c/s was applied to the instrument in the proper phase-sequence, and the meter reading was taken down for various voltages. Also, every time the voltage neutral was measured by a sensitive voltmeter. A calibration curve was thus obtained for the positive-sequence component. Identical results were obtained when the phase-sequence of the applied voltage was reversed i.e. for the negative-sequence component, as shown in the graph (p. 74) Fig 5.9

In order to test the accuracy of the instrument, an unbalanced three-phase voltage system was applied and the negative-, positive-sequence components* and their phase angle* were measured. Also the applied voltages were measured by an ordinary voltmeter. The latter values were synthesized to give the sequence values. The unbalanced supply system was obtained by using three transformers provided with taps.

* The method of measurements is given on next pages.

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Project/Design/Laboratory [illegible] Page No. 74

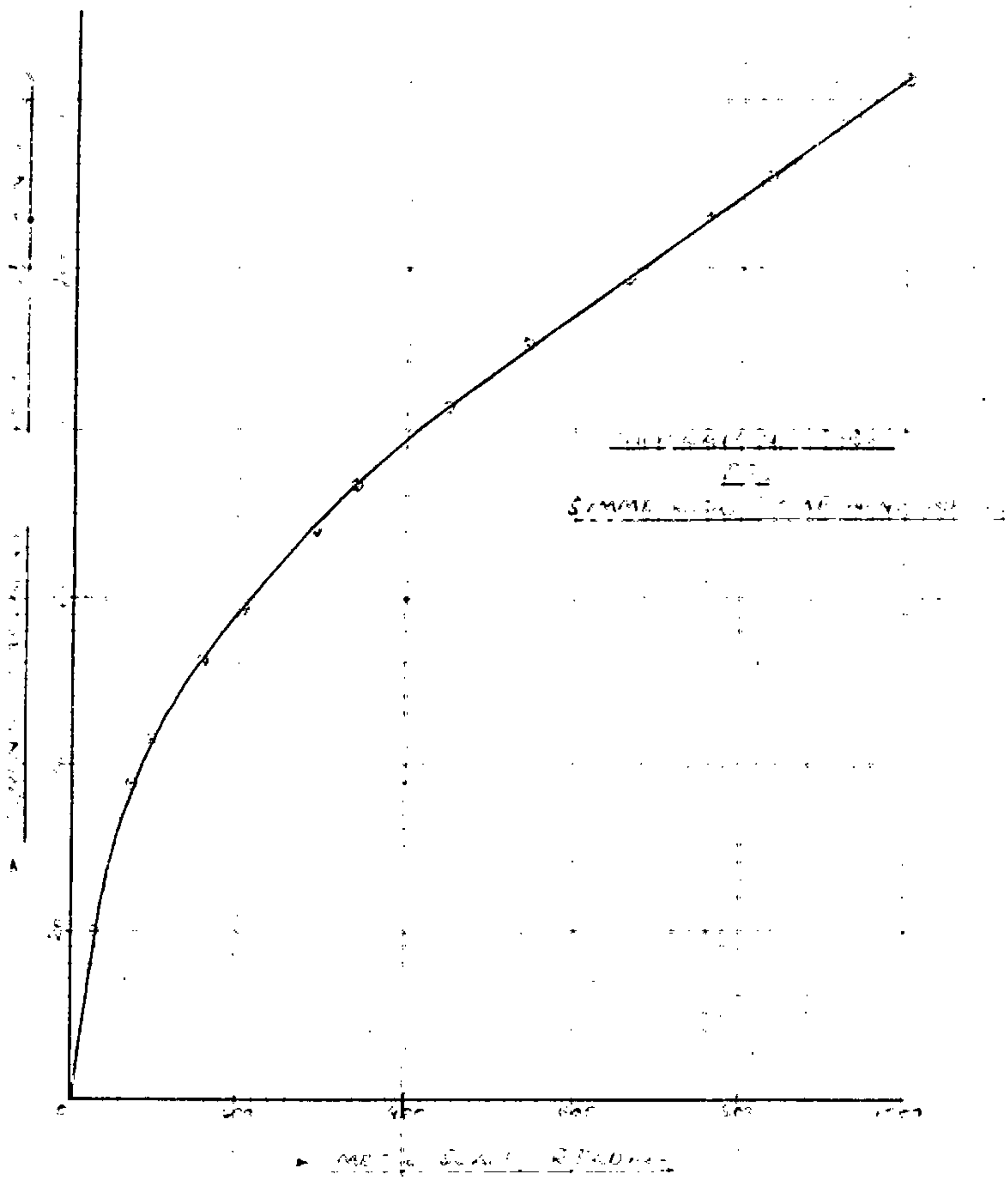

Fig. 5.9

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Project/Design/Laboratory Symmetrical Component Meter Page No. 75.

5.5 PROCEDURE FOR MEASUREMENTS.

The instrument has been designed to work upto 220/230 volts 50 c/s supply. The switching operations for various measurements are as given below:

Position of P & Q.	L & M Left	L & M Right	L Left M Right
P & Q closed	Positive Sequence E_1	Negative Sequence E_2	$E_1 E_2 \cos\phi$
P & Q Open	E_l *	E_c *	

Instrument Readings for Different Switch Settings.

The usual procedure for the measurement of the sequence components are as follows.

(1) Adjust the zero-error of the instrument, after placing it on a plane horizontal surface.

(2) Connect the terminals a, b, c to the circuit under test. (S is to be connected to grounded neutral if the transformers are not being used).

* The meter, for this setting, responds to E_l and E_c, but the same calibration (as for the sequence components) does not hold good.

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(3) Set the selector switch (L & M) to read the positive- and negative-sequence components in turn. (See table p. 75). Interchange any two leads, if required, to obtain larger reading with the former setting. Note the scale readings r_1 and r_2. The order a, b, and c indicates the phase sequence of the system under test.

(4) Set the selector switch to the phase-measuring position, reversing one meter-coil, if necessary, to give a positive deflection. Note the scale reading r_{12}

(5) Transform r_1 and r_2 to sequence voltages by the calibration curve (p. 74). Compute ϕ from the expression

$$\cos\phi = \frac{r_{12}^2}{r_1 r_2}.$$

If the coil was reversed, subtract the values obtained from 180° to give the true value of ϕ.

(6) Open P and Q in the positive- and negative-sequence settings respectively of the selector switch in turn. If the former deflection is larger, ϕ is positive,

if smaller ϕ is negative. (See p. 59)

It is an important fact that, for practical purposes, the meter cannot be damaged by operating the switches incorrectly, and no special precautions are necessary in this connection.

Note: While taking the readings tap the glass on the meter very slowly so that the moving part of the meter does not remain sticking.

5.6 MEASUREMENTS OF SEQUENCE COMPONENTS.

Unbalanced voltages were obtained on the secondaries of three transformers, (combined to make to make a three-phase bank) provided with taps. Three unbalanced voltages were measured by the symmetrical component meter in terms of the positive and negative-sequence components, and the results were verified by measuring the phase voltages by a voltmeter and then synthesized to give the sequence-components. Some of the results, their verification and percentage errors are tabulated below:

Sequence components as measured by the "meter":

Position of L and M	Position of P and Q	Scale Reading	positive seq. comp.	negative seq-comp	Phase angle ϕ	Sign of 'ϕ'
Both left	closed	x_1 700	200 volts	–	–	–
Both right	"	x_2 80	–	38 volts	–	–
L left M right	"	x_{12} 200	–	–	34.5°	–
Both left	Open	460	–	–	–	+ve
Both right	"	340	–	–	–	

Therefore, positive-sequence component = 200 volts Error 1.5%

negative-sequence component = 38 volts Error 1.5%

Phase angle = 34.5°.

Verification (as calculated from measured phase-voltages by ordinary voltmeter). The measured phase voltages were E_a = 230 volts, E_b = 68 volts, E_c = 224 volts.

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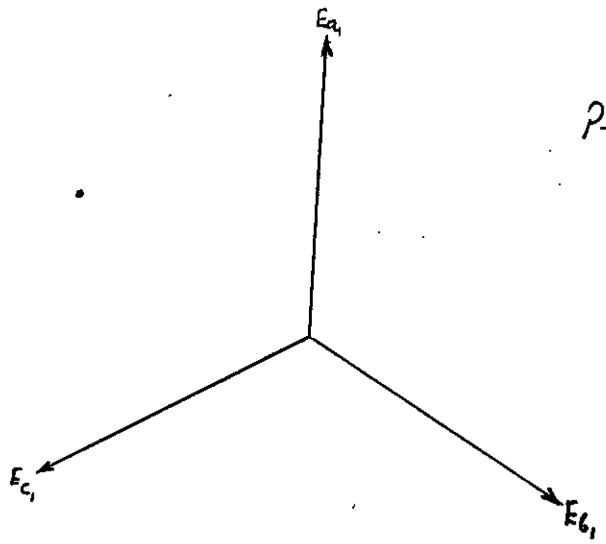
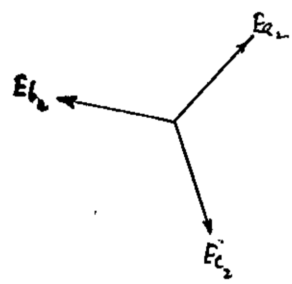
*Verification of Results.



Phase angle ϕ = +34.5°



Positive sequence component = 100 volts

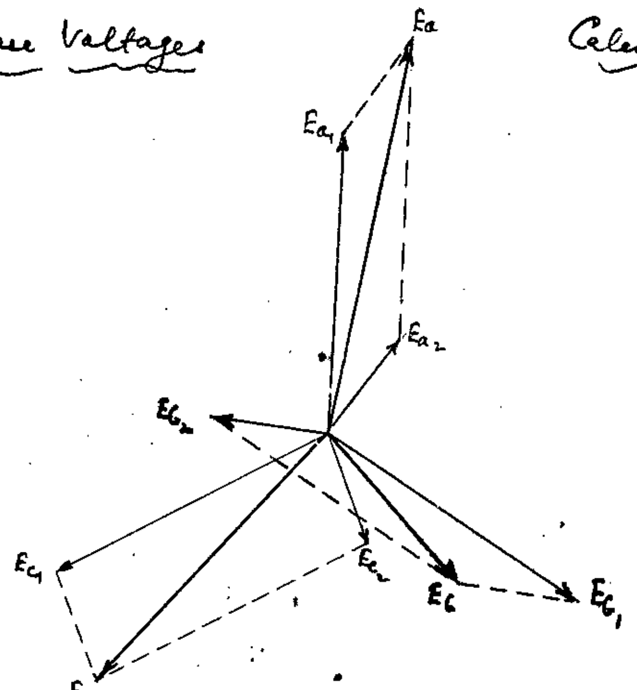
Negative sequence component = 38 volts.

Measured Phase Voltages

E_a = 130 volts

E_b = 68 volts

E_c = 114 volts.



Calculated Phase Voltages

E_a = 134 volts

E_b = 65 volts

E_c = 115 volts

Fig 5.10

Combination of sequence quantities to form phase quantities.

*: For more results see appendix 1 p 139.

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5.7 ERRORS AND PRECAUTIONS.

Any deviation from the normal (specified) frequency will evidently result in an erroneous response of the meter for both the positive and negative sequence components.

By expressing the frequency as a fraction α of the calibrating frequency, and neglecting the resistance of the meter, the relation $R_1 = -a^2 Z_1$, or what amounts to the same as $Z_1 = -aR_1 = \frac{1}{2}R_1(1-j\sqrt{3})$ will no longer be true, and a modified form must be employed. Now

$$Z_1 = \frac{1}{2} R_1 \left(1 - j\frac{\sqrt{3}}{\alpha}\right) \text{ ——— } 5.16$$

So that we have from (5.6) (5.55) and (5.16), by eliminating Z_1:

$$E_a + \frac{2E_c}{1-j\frac{\sqrt{3}}{\alpha}} = I_m\left[R_1 + Z\left(1 + \frac{2}{1-j\frac{\sqrt{3}}{\alpha}}\right)\right] \text{ ——— } 5.17$$

Equation (5.17), expressed in terms of sequence components can be simplified by assuming that Z the meter impedance can be neglected in comparison with R_1, and by converting to the j-notation, by putting $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

$$E_{a_1}(1+\alpha) + E_{a_2}(1-\alpha) = I_{m_1}\left(\frac{R_1}{\sqrt{3}}\right)(\sqrt{3} + j\alpha) \text{ —— } 5.18$$

The instrument reading will be interpreted incorrectly as a positive sequence component E_{a_1}', whereas it depends on both sequence components, as shown by (5.18). The apparent value is obtained from the equation (5.7):

$$E_{a_1}' = I_{m_1}\left(\frac{R_1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) \text{ —— } 5.19$$

It follows from (5.18) and (5.19) that E_{a_1} and E_{a_2} can be expressed in terms of E_{a_1}'

$$E_{a_1}(1+\alpha) + E_{a_2}(1-\alpha) = E_{a_1}'\left[2 + (1-\alpha)\alpha^2\right] \text{ —— } 5.20$$

A similar expression for the apparent negative-sequence component is

$$E_{a_1}(1-\alpha) + E_{a_2}(1+\alpha) = E_{a_1}'\left[2 + (1-\alpha)\alpha^2\right] \text{ —— } 5.21$$

Solving (5.20) and (5.21) corrections can be made to the observed readings for observed values of α.

The effect of frequency changes may be more clearly visualised by referring to the vector diagram (Fig 5.11). Given E_{q_1} and E_{q_2} at an angle ϕ, and for values of α less than unity, OE and OF are vectors proportional to E_{q_1}' and E_{q_2}' respectively. The

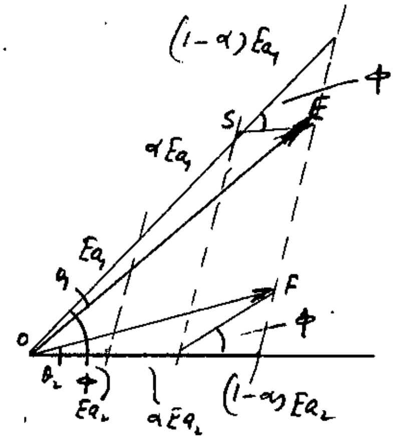

Fig 5.11

errors θ_1 and θ_2, in this case, are negative, but for values of α larger than unity they would be positive. As ϕ varies, SE will describe a circle with centre S such that a maximum when $\angle OES = 90°$. For values of α close to unity and with θ_1 measured in degrees, approximate values are obtained for the maximum error as:

$$\theta_1 = -\left(\frac{180}{\pi}\right)\left(\frac{E_2}{E_1}\right)\left(\frac{1-\alpha}{1+\alpha}\right)$$

$$\theta_2 = -\left(\frac{180}{\pi}\right)\left(\frac{E_1}{E_2}\right)\left(\frac{1-\alpha}{1+\alpha}\right)$$

These maximum values will not in general occur at the same time. The largest possible error θ measured in degrees, will not therefore exceed the sum of the two:

$$\theta = -\left(\frac{180}{\pi}\right)\left(\frac{E_1^2 + E_2^2}{E_1 E_2}\right)\left(\frac{1-\alpha}{1+\alpha}\right) \text{ ——— } 5.22$$

The values of θ in (5.22) will vary with the ratio $E_1 : E_2$ but will be greatest when $E_1 = E_2$, so that

$$\theta = -\left(\frac{360}{\pi}\right)\left(\frac{1-\alpha}{1+\alpha}\right) \text{ ——— } 5.23$$

A more convenient expression is obtained by putting $\alpha = (1 + \beta/100)$, where β is the percentage variation from the calibration frequency. The error in the measurement of phase angles, for small values of θ, say $-5 \le \beta \le +5$ will not exceed the value given by (5.24)

$$\theta = 0.6\beta \text{ ——— } 5.24$$

The largest error occurs in this particular case when $\phi = 90^\circ - \theta = 90^\circ - \frac{\theta}{2} = 90^\circ - 0.8\beta$. A general expression of this nature can not be readily obtained.

Other important sources of error in the meter response are the transient errors and the errors due to the presence of harmonics. Summing up, the three important errors are

(1) Frequency errors

(2) Transient errors and

(3) Errors due to harmonics.

——— (x) ———

6.

APPLICATIONS.

At first consideration the theory of symmetrical components may appear to introduce unnecessary complication into the study of unbalanced three-phase currents and voltages. A single phase earth fault is, according to this theory, to be replaced by no less than nine components, two sets of three cancelling and leaving no resultant. Single-phase currents flowing alone in a three-phase circuit, however, represent limiting currents. The general case of an unbalanced three-phase system of currents comprises three currents or the magnitude of two currents and their phase difference. Although according to the theory of symmetrical components, these three unbalanced currents are replaced by six components and completely specified by two magnitudes and a phase difference, since a symmetrical system is completely defined by one of its members. The complication introduced by symmetrical component conception is thus only apparent. Similarly, if a three-phase three-wire system becomes four-wire due to an incidence of earth-fault, the three line constants are completely independent, and the system can only be defined by three magnitudes and two phase-angles. The alternative definition in terms of symmetrical components also involves three magnitudes and two phase-angles, so that in this case also no real complication is introduced.

To enumerate a few, the following involve the application of symmetrical components:

(i) Simple cases of faults,
(ii) Automatic protection,
(iii) Distribution networks,
(iv) Induction motors, transformers and
(v) Synchronous machines and phase balancers.

6.1 SIMPLE CASES OF FAULTS

Fundamental Principles; The most important application of the symmetrical components is the calculation of faults in power systems. Two assumptions are made to solve such problems by this method.

First, it is assumed a symmetrical system of voltages and then the influences of unsymmetrical voltages are dealt with.

A further assumption is that the values of the impedances concerned are known, and their determination by calculation or measurement is considered later. Most important works in this connection have been carried out by Fallon and Oberdorfer.

The basic equations which characterize the symmetrical system of voltages are:

$$\left.\begin{array}{lll} E_a = E & E_b = aE & E_c = a^2E \\ E_1 = E & E_2 = 0 & E_0 = 0 \end{array}\right\} \quad -6.1$$

Let,

Z_0, Z_1 & Z_2 be impedances of zero-, positive- & negative system

I_0, I_1 & I_2 be current components at point of fault

E_0', E_1' & E_2' be voltages at the same point. Then considering each system seperately, we have by Kirchhoffs' laws

$$\left.\begin{aligned} E_1' + I_1 Z_1 &= E \\ E_2' + I_2 Z_2 &= 0 \\ E_0' + I_0 Z_0 &= 0 \end{aligned}\right\} \qquad 6.2$$

Types of Faults: The types of faults that are likely to occur in a Threephase system are:

1. Three phase (3ϕ):

   Either line-to-line-to-line (L-L-L), or

   Three lines to ground (3L-G)

2. Single line-to-ground (L-G)

3. Double line-to ground (2L-G)

4. Line-to-line (L-L)

These are represented schematically in fig 6.1.

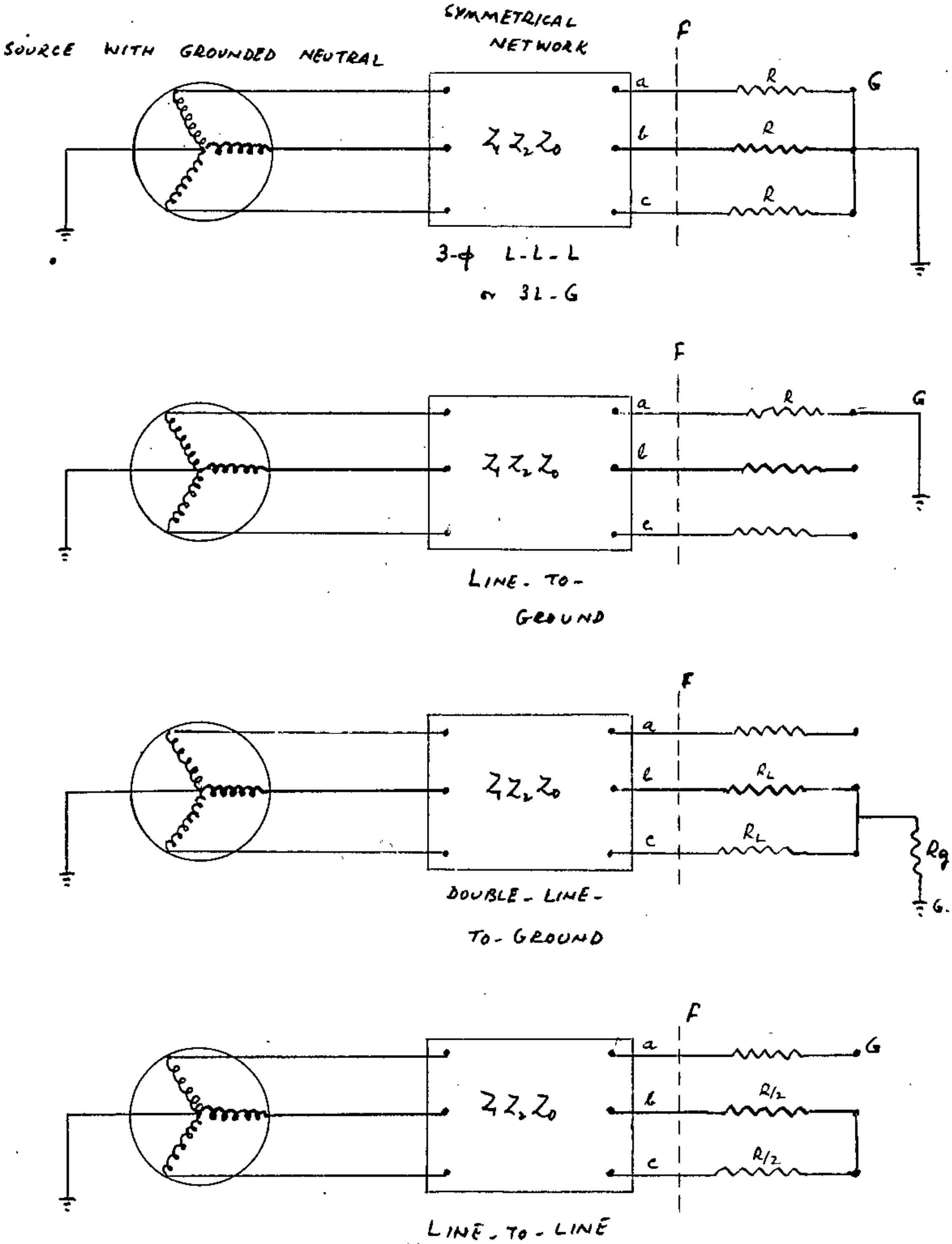

Fig. 6.1 . Types of Faults on 3-φ Systems.

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3-ϕ faults: The voltage at the point of fault is zero in all the three phases. Therefore, the symmetrical components of the voltage system are also zero i.e.

$$E_1' = E_2' = E_0' = 0$$

Or, $$I_1 = \frac{E}{Z_1}, \quad I_2 = 0 \quad \text{and} \quad I_0 = 0$$

This result was to be expected as the voltages and current systems are symmetrical and therefore no negative- or zero-sequence components can exist.

Line-to-line fault: Let the fault be between b and c, assuming the neutral to be free as shown in fig 6.2, so that a zero-component cannot exist in the current system. E, a^2E and aE is the system of phase voltages before the fault takes place. After the fault has occurred, the voltage

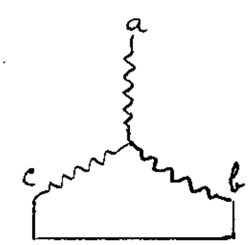

Fig 6.2

between the star-point and the point of fault would be given by the equations:

$$E_b = a^2E - a^2 I_1 Z_1 - a I_2 Z_2$$

$$E_c = aE - a I_1 Z_1 - a^2 I_2 Z_2 ,$$

the voltage drops being expressed by their symmetrical components.

$$\text{But} \quad E_b = E_c$$

$$\text{Or,} \quad a^2E - a^2 I_1 Z_1 - a I_2 Z_2 = aE - a I_1 Z_1 - a^2 I_2 Z_2 \quad \text{———} \; 6.3$$

$$\text{Also} \quad I_a = I_1 + I_2 = 0$$

since the current in the open phase is zero.

$$\text{Or,} \quad I_1 = - I_2$$

Substituting in (6.3) $I_2 = - I_1$, we have:

$$a^2E - a^2 I_1 Z_1 + a I_1 Z_2 = aE - a I_1 Z_1 + a^2 I_1 Z_2$$

$$\text{Or,} \quad (a^2 - a) E = (a^2 - a) I_1 Z_1 + (a^2 - a) I_1 Z_2$$

$$\text{Or,} \quad E = I_1 Z_1 + I_1 Z_2$$

Therefore $$I_1 = \frac{E}{Z_1 + Z_2}$$

and $$I_2 = \frac{-E}{Z_1 + Z_2}$$

Now considering the current relations in phases b and c, we have:

$$I_b = a^2 I_1 + a I_2 = a^2 I_1 - a I_1$$

$$I_c = a I_1 + a^2 I_2 = a I_1 - a^2 I_1$$

Or, $$I_b = - I_c = (a^2 - a) I_1 = (a^2 - a) \frac{E}{Z_1 + Z_2} \qquad \text{--- } 6.4$$

which is the fault current.

Now, $(a^2 - a) = -j\sqrt{3}$ (Table 3.1, p 18)

$$\therefore I_b = - I_c = -j\sqrt{3} \frac{E}{Z_1 + Z_2}$$

Or, $$|I_b| = |I_c| = \frac{\sqrt{3}\,|E|}{|Z_1 + Z_2|}$$

For voltage relations we have:

$$E_b = E_c = a^2 E - a^2 I_1 Z_1 - a I_2 Z_2$$

$$= a^2 E - a^2 Z_1 \frac{E}{Z_1 + Z_2} + a Z_2 \frac{E}{Z_1 + Z_2}$$

$$= \frac{a^2 E}{Z_1 + Z_2} \left[(Z_1 + Z_2) - Z_1 + a^2 Z_2 \right]$$

$$= \frac{a^2 E}{Z_1 + Z_2} (Z_2 + a^2 Z_2) = \frac{a^2 E}{Z_1 + Z_2} (1 + a^2) Z_2$$

$$= (a^2 + a) E \frac{Z_2}{Z_1 + Z_2}$$

But $1 + a + a^2 = 0$

$$\therefore \quad E_b = E_c = -E\,\frac{Z_2}{Z_1+Z_2}$$

Further, from equation (6.4) we have $\frac{E}{Z_1+Z_2} = \frac{I_c}{(a^2-a)}$ so that

$$E_b = E_c = -\frac{Z_2 I_c}{a^2-a} = \frac{Z_2 I_c}{j\sqrt{3}} = -j\,\frac{Z_2 I_c}{\sqrt{3}}$$

The equation $E_0 = \frac{1}{3}(E_a + E_b + E_c) = 0$ furnishes the value of the voltage at the free phase viz

$$E_a = -2E_b = \frac{2EZ_2}{Z_1+Z_2} = j\,\frac{2Z_2 I_c}{\sqrt{3}}$$

$$\therefore \quad |E_a| = 2|Z_2|\,\frac{|I_c|}{\sqrt{3}}$$

The line voltage between the free phase and the phases containing the fault is

$$E_{ab} = E_b - E_c = \frac{3EZ_2}{Z_1+Z_2} = j\sqrt{3}\,\frac{Z_2 I_c}{Z_1+Z_2}.$$

Line-to-Ground fault: Let the phase a be grounded (Fig 6.3). The phases b and c be unloaded i.e. they carry no current. In this case, obviously, a

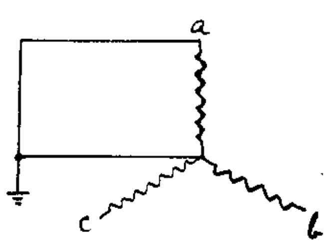

Fig 6.3

zero component will appear in the current system, so that

$$I_b = I_0 + a^2 I_1 + a I_2 = 0 \quad \text{——— } 6.5$$

$$I_c = I_0 + a I_1 + a^2 I_2 = 0 \quad \text{——— } 6.6$$

Subtracting, $(a^2 - a) I_1 + (a - a^2) I_2 = 0$

$$\text{Or,} \quad I_1 = I_2$$

from (6.5) and (6.6), therefore by addition

$$2I_0 + (a^2 + a) I_1 + (a^2 + a) I_2 = 0$$

Or, $2I_0 + (a^2 + a)(I_1 + I_2) = 0$

But $1 + a + a^2 = 0$

Therefore $I_1 = I_2 = I_0$

Also, as before, $E_a = E - (I_0 Z_0 + I_1 Z_1 + I_2 Z_2)$

Or $$I_0 = \frac{E}{Z_1 + Z_2 + Z_0} \quad \text{——— } 6.7$$

and $$I_a = I_0 + I_1 + I_2 = 3I_0 = \frac{3E}{Z_1 + Z_2 + Z_0} \quad \text{——— } 6.8$$

The voltages on the free phases are obtained by subtracting from the voltages existing before the fault occurs ~~these~~ fictitious voltage drops ~~which~~ which exist after the fault has developed is

$$E_b = a^2E - a^2 I_1 Z_1 - a I_2 Z_2 - I_0 Z_0$$

$$E_c = aE - a I_1 Z_1 - a^2 I_2 Z_2 - I_0 Z_0.$$

Or by substituting the current values already found

$$E_b = a^2E - E \cdot \frac{a^2 Z_1 + a Z_2 + Z_0}{Z_0 + Z_1 + Z_2}$$

$$= \frac{E}{Z_0 + Z_1 + Z_2}\left(a^2 Z_0 + a^2 Z_1 + a^2 Z_1 - a^2 Z_1 - a Z_2 - Z_0\right)$$

$$= \frac{E}{Z_0 + Z_1 + Z_2}\left[(a^2 - a) Z_2 + (a^2 - 1) Z_0\right] = E \cdot a(a-1) \frac{Z_2 - a Z_0}{Z_0 + Z_1 + Z_2} \quad - 6.9$$

Similarly

$$E_c = aE - E \cdot \frac{a Z_1 + a^2 Z_2 + Z_0}{Z_0 + Z_1 + Z_2}$$

$$= \frac{(a-1) Z_0 + (a - a^2) Z_2}{Z_0 + Z_1 + Z_2} \cdot E = E(a-1) \frac{Z_0 - a Z_2}{Z_0 + Z_1 + Z_2} \quad - 6.10$$

The line voltage between the phases is

$$E_{bc} = E_b - E_c = E(a-1)\left[a\left(\frac{Z_2 - a Z_0}{Z_0 + Z_1 + Z_2}\right) - \left(\frac{Z_0 - a Z_2}{Z_0 + Z_1 + Z_2}\right)\right]$$

$$= \frac{E(a-1)}{Z_0 + Z_1 + Z_2}\left(a Z_2 - a^2 Z_0 - Z_0 + a Z_2\right)$$

$$= E a(a-1) \frac{2 Z_2 + Z_0}{Z_0 + Z_1 + Z_2}$$

Substituting I_a from (6.8) and

$$a(a-1) = -j\sqrt{3} \quad \text{we have}$$

$$E_{cb} = -\frac{j}{\sqrt{3}}(2Z_2 + Z_0) I_a$$

$$\text{Or } |E_{a}| = |2Z_2 + Z_0| \left|\frac{I_a}{\sqrt{3}}\right|$$

Also $E_a = 0$, the sum of the voltages E_b and E_c must be three times the zero component of the voltage i.e.

$$E_b + E_c = E\, a(a-1)\frac{Z_2 - aZ_0}{Z_0 + Z_1 + Z_2} + E(a-1)\frac{Z_0 - aZ_2}{Z_0 + Z_1 + Z_2}$$

$$= E(a-1) \cdot \frac{aZ_2 - a^2 Z_0 + Z_0 - aZ_2}{Z_0 + Z_1 + Z_2}$$

$$= E(a-1)(1-a^2)\frac{Z_0}{Z_0 + Z_1 + Z_2}$$

$$= -3E\frac{Z_0}{Z_0 + Z_1 + Z_2} \quad \text{since } (a-1)(1-a^2) = -3$$

Also $\dfrac{E}{Z_0 + Z_1 + Z_2} = I_0$ from equation (6.7), Therefore

$$E_b + E_c = -3I_0 Z_0 = -3E_0$$

$$= -I_a Z_0 \quad \text{from (6.8)}$$

The negative sign is due to the fact that the calculation was based upon voltage drops.

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6.2 AUTOMATIC PROTECTION.

The resolution of an unsymmetrical system of voltages, or currents, into their symmetrical components found a practical application a number of years ago in several proposals and designs actually executed for regulating and protecting gear for the control of generators, transformers, motors etc.

A balanced three-phase load absorbs power at a uniform rate. In practice, however, all loads are not balanced and as a matter of fact balanced load is an ideal to which the actual industrial loads on large power stations tend to approximate. The normal current output of a power station contains a very small fraction of either negative- or zero-sequence components. Negative-sequence currents can and do arise locally due to unbalance of single-phase demands of domestic consumers

But usually the diversity of their unbalance is such that the resultant negative-sequence current is a very small fraction of the positive-sequence current, while the zero-sequence component is almost absent.

However, a considerable negative-sequence component arises on the occurrence of phase-to-phase short-circuit fault on a three-phase system, and the zero-sequence component on the occurrence of an earth-fault. Thus the appreciable presence of the negative-sequence currents is an indication of phase-to-phase short-circuit and that of the zero-sequence current of an earth-fault.

In the early methods of protecting electrical circuits against the destructive effects of short-circuits, the operation of circuit-breakers and fuses essentially depended upon the currents carried by them, exceeding a pre-adjusted value. The modern practice of protection is to design such devices, that the relays controlling the operation of circuit-breakers carry no current except in such fault conditions in which they are required to respond. Hence relays have been designed to be responsive to the presence of the negative-, or positive-sequence current, thereby indicating the presence of a phase-to-phase short-circuit or an earth-fault accordingly.

The core balance system of protection essentially embodies the principle of zero-sequence protection, and is one of the most common methods for protection against earth-faults. It is based on the fact that under healthy conditions the mmf's due to the three-line currents balance, as well as in the phase-to-phase short-circuit, as the positive-sequence and so also the negative-sequence currents balance. But on the occurrence of an earth-fault on any line, the zero-sequence currents being in the same phase in the three phases do not balance, thus energizing the relay.

This system of protection is very suitable for circuits subjected to usual earth faults.

Zero-sequence Protection: This is in fact a new name for "leakage protection". Fig 6.4 shows a typical circuit for leakage protection relays. There are three current transformers of similar ratios, the primaries of which carry the line current, connected in parallel. The common return lead containing the relay would carry a current corresponding to the ~~vector~~ sum of the line currents, which should normally be zero. Hence the relay will carry only the zero-sequence component and will operate accordingly i.e. the

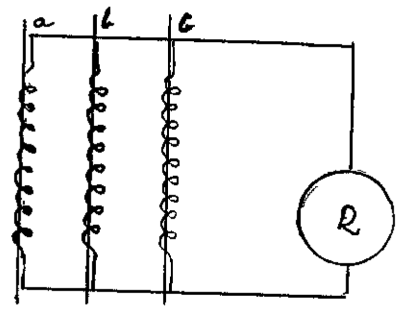

Fig 6.4

relay will be responsible to zero-sequence currents indicating the occurence of an earth-fault. If the relay is replaced by an ammeter, multiplied by the ratio of the CTs, would give the residual current or three times the zero sequence component.

However, in this simple circuit there is one difficulty normally experienced viz the relay carries the residual currents of the secondaries of the C.T's and if the errors in the CT's are not negligible (as in the case of larger currents, when a considerable error is introduced in CTs) a false residual current also appears in the relay which makes the protection scheme unstable.

The circuit in fig 6.5 (next page) with a relay of the wattmeter pattern, with its secondary element connected to a potential transformer (Y-Δ) with primary short-circuited as shown, makes the relay responsive only to zero-sequence currents arising due to an earth-fault; because of the relay indication being controlled by the zero-sequence voltage through the P.T. connected to the secondary element.

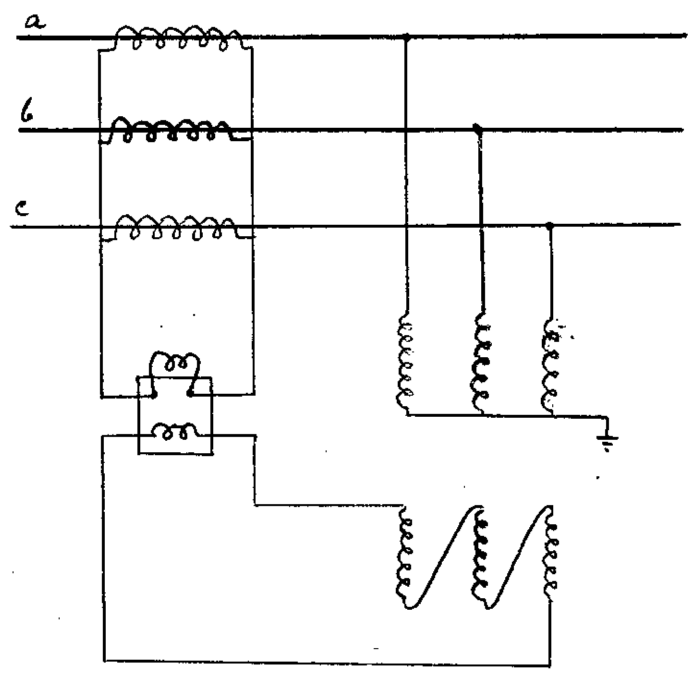

Fig 6.5

Zero-sequence relays are finding another field of application to the methods in which the time-lag of over-current relays are adjusted to correspond to their electrical distances from the source of supply. Thus over current relays of inverse time pattern energised by residual currents could be controlled by sensitive directional relays energised by zero-sequence component. It is also possible to control the relays carrying the residual currents by the residual voltages, because the residual voltages to an earth-fault is greatest at the fault position and diminishes as the distance from the point of fault.

Zero-sequence Components in Y-Δ Connected System:

In case of Y-connexion the vector sum of the phase-voltages is zero; hence the zero-sequence component of the voltages cannot exist across the line, though they can exist in individual phase conductors. Zero-sequence currents also cannot exist in such a system, as they do not get any path to flow. However, if the system is earthed at Y-point, the zero-sequence currents get a path to flow and so can exist when an earth fault occurs.

In case of Δ-connexion as well, the zero-sequence voltages cannot exist for the same reason, but the zero-sequence currents can exist, but cannot flow into the line. They act merely as single phase currents circulating round the phases.

The impedances of a three phase transformer connected between two sections of a transmission network is the same for the positive and the negative - sequence currents; but the impedances to the zero sequence current depends entirely upon the interconnection of its windings and upon the earth connexions. Zero-sequence currents can only flow to or from a point in a three-wire system of conductors, which is connected to the earth. Hence if the primary winding of a threephase transformer are Δ- connected or Y- connected with a free neutral, no zero sequence current can flow. But if the transformer is connected Y/Y and supplied from a source with an earthed neutral, zero-sequence current will have a normal path.

Negative-sequence Protection: The network shown in fig 6.6 is used in negative-sequence relays. The impedances Z_1, Z_2, Z_3 and Z_4 are all of equal ohmic value; but the phase angle of Z_1 and Z_2 being equal and 60° greater than the phase angles of Z_3 and Z_4. The impedance of the relay itself is negligible. The current from the phase 'a' of the C.T. will divide itself equally into two paths at A, each being one-third in magnitude and differing in phase with each other by 60°. The current from the phase 'c' will also divide itself in a like manner. The positive-sequence components of these currents will obviously be in phase opposition with each other at B and D, and so cancel each other. But the negative-sequence component does not cancel out as there is a phase difference

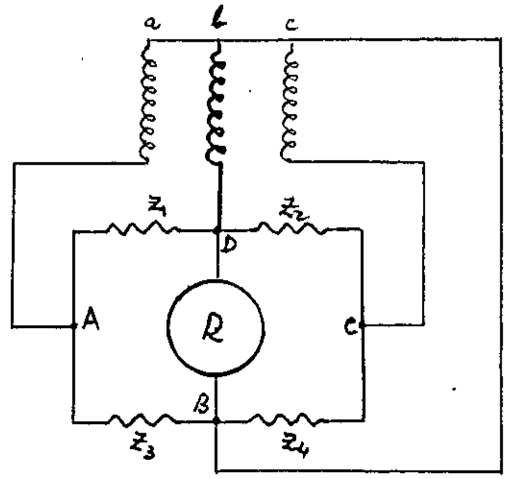

fig 6.6

between them in the two sections, and hence can flow into the relay. Thus the relay is responsive to negative-sequence currents when arising in the network.

However, if there is a residual current component of zero-sequence component, one-third of its value will flow in the branches AB and CB. These differing in phase by 60° will give to rise to a resultant equal to twice the zero-sequence component, which will also flow in the relay. Thus the relay will be responsive to the negative- as well as zero-sequence currents.

To obtain unique responce to negative-sequence currents only the network can be energized from a current transformer with delta-connected secondary, whereby the zero-sequence currents are eliminated.

Another relay, shown in Fig 6.7 (next page) is of the induction pattern having separate windings. A small transformer with special design with an air-gap in its magnetic circuit, is also incorporated as shown. The coil C of the relay carries the

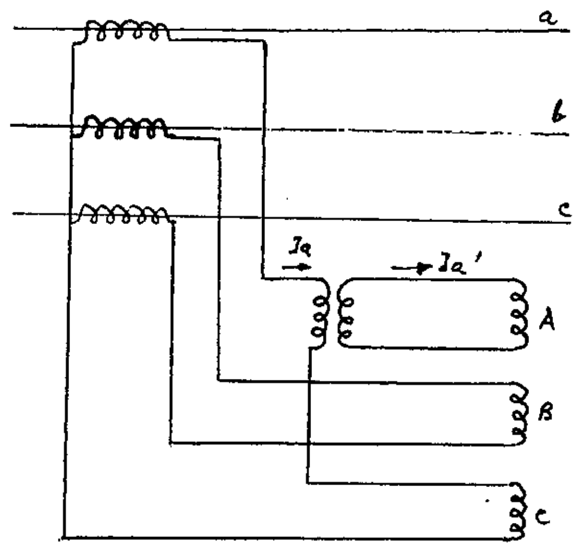

Fig 6.7

residual current, the coil B being supplied from the phase 'b' of the C.T. while the current in the coil A, supplied from an auxiliary transformer, is arranged to lead 60° on the current in the phase of the C.T. The two windings A and C are so designed that the ampere-turns due to the currents I_0 and I_B (x'former) are equal in magnitude and separated by an additional 60°. Hence the torque set-up by the two coils will be proportional to negative sequence component only.

If a Three-phase alternator of normal design supplies an unbalanced load, local heating will be set-up in the rotor due to the oscillating component of the armature reaction, which is equivalent to that of a rotating field moving at synchronous speed in an opposite direction to that of the rotor. If for any reason, a single-phase short-circuit is sustained on any part of the transmission system dangerous heating is liable to be set-up in the alternator, not only due to excessive currents, but also due to the negative-sequence current, as already explained. Hence a special relay is usually incorporated with the alternator-protective-equipment.

Zero-sequence currents, like negative-sequence currents, also give rise to abnormal heating and hence instead of providing a zero-sequence relay, the connexions of the negative-sequence relay (energised with a C.T. having secondary as Y-connected) are so arranged that they are also responsive to the zero-sequence currents.

Such a relay will operate and trip the circuit-breaker of the alternator at the occurrence of an earth-fault or phase-to-phase short-circuit for a prolonged time greater than the time-setting of the relay.

Negative-sequence relays have also been employed for protection of large A.C. motors against earth-faults, short-circuit, ~~single~~ phasing. The operation of the relay for earth fault or short-circuit is as explained earlier. The conditions of single-phasing results in case when one of the lines supplying a three-phase motor is made dead somehow, after the motor has been run up to normal speed. The motor will continue to run but the current input to the other two lines will be materially increased. It is evident that a negative-sequence relay will operate as a condition as above is developed. As a matter of fact, a suitably adjusted negative-sequence relay will afford protection to the motor in all the three conditions viz. single-phasing, short-circuit and earth fault.

Sometimes when a medium-pressure-line is coupled to a transmission network by a step-down transformer, a phase-to-phase short-circuit may occur, but a short-circuit current in a section of the line may not be greater than the full-load current; in which case it is not liable to be detected by any other relay except a negative-sequence relay, and its use is the best method of obtaining all but satisfactory protection.

6.3 AUTOMATIC VOLTAGE REGULATORS

The control relay of an automatic voltage regulator of a three-phase alternator is excited by two of the line wires as shown in Fig 6.8.

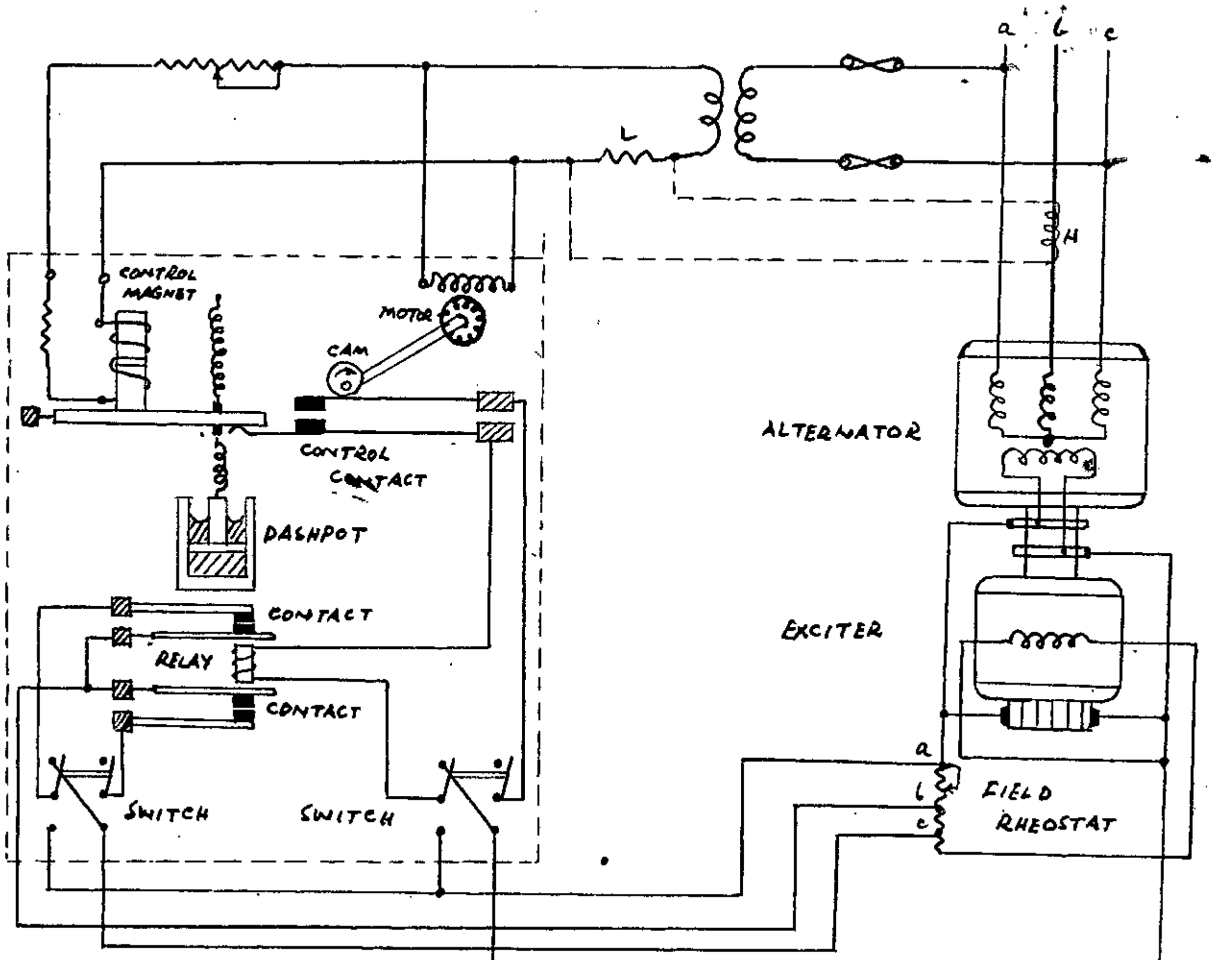

Fig 6.8 M.V. REGULATOR

Under abnormal conditions this voltage may be higher or lower than the normal value, and if the control relay were controlled solely by the voltage, the regulator may act in an undesirable fashion. If, however, the relay were controlled by the positive-sequence voltage, the control of the voltage would be unaffected by unbalanced load or fault conditions, since the positive-sequence voltage forms a symmetrical system.

Fig 6.9 shows the connexions employed for the control relay of the Metropolitan-Vickers' voltage regulator. The current in the coil of this relay is the vector sum of the currents in the circuits XR, and R connected to secondary

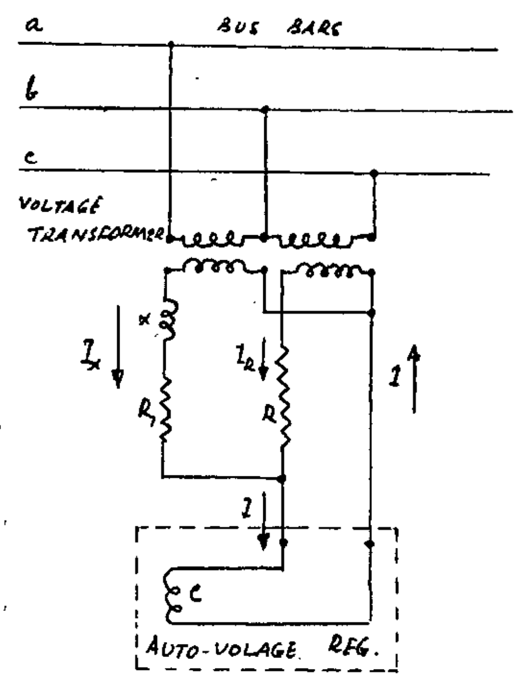

Fig 6.9.

windings of the two voltage transformers, the primary windings of which are connected in open-delta to the bus-bars. Circuit XR, is inductive and is adjusted to give a phase shift of 60°. Circuit R is non inductive and its resistance is so adjusted that the current is equal to that in circuit XR, this current being in phase with the voltage E_{bc}.

For positive-sequence voltages, the current in circuit R is represented, in fig 6.10., by OI_R and that in circuit XR, by OI_x. The vector sum of these currents is given by OI.

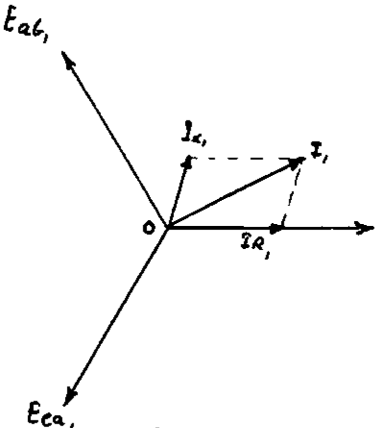

Fig 6.10

For negative-sequence voltages, the current in circuit R is represent, in fig 6.11, by OI_{R_2} and

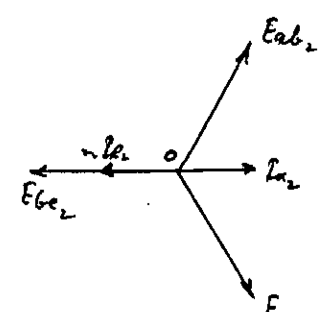

Fig 6.11

that in circuit X by δI_{x}. Since the current in circuit X lags 60° from E_{ab} this is in opposition to the current in circuit R, when the phase-sequence is negative. Hence the control relay is operative only to positive-sequence voltages.

6.4 DISTRIBUTION NETWORKS.

Positive-, negative- and zero-sequence impedances of distribution conductors can be found out by knowing the self- and mutual-inductances of the conductors. The following are the expressions for the positive-sequence impedance of conductors of a three-phase line:

$$Z_a = R_a + j\omega (L_a + a^2 M_{ab} + a M_{ac})$$

$$Z_b = R_b + j\omega (L_b + a^2 M_{bc} + a M_{ba})$$

$$Z_c = R_c + j\omega (L_c + a^2 M_{ca} + a M_{cb})$$

where R_a, R_b and R_c are ohmic resistances, L_a, L_b, and L_c the self-inductances, and M_{ab}, M_{bc}, and M_{ca} the mutual-inductances of the three conductors. The self- and mutual inductances may be found out from the dimensions and spacing of the conductors. Similarly, the negative-sequence impedances may also be determined.

In the case of zero-sequence impedance, the problem of the earth conductance and its inductance comes into play. According to Mayr, the impedance per unit length of a loop consisting of a wire and the earth return is:

$$Z = R + R_e + j\omega\left(2\log_e \frac{1}{Xr} - 1.154 + \frac{\mu}{2}\right)\times 10^{-4}\ \text{ohm/km}$$

where R = wire resistance in ohm / k.metre

R_e = earth " " "

$X = \frac{2\pi\omega}{\rho}$, an empirical value lying between 0.6×10^{-5} and 1.03×10^{-5} cgs. units at 50 cycles/sec

ρ = specific resistance of a fictitious thin plate replacing the earth.

μ = Permeability of wire material & r = radius of wire and self-inductance of the loop is $L = 2\times10^{-4}\log_e \frac{2.38k}{r}$ in henry/kilometre, where $k = \sqrt{\frac{\rho}{2\pi\omega}}$ and known as the "depth of peneteration".

Tocacia on this basis derived that

$$L_{ao} = L_a + M_{ab} + M_{ac} + 0.345\times10^{-3}\ H/km.$$

Thus, with this value and the values of L and M taken from the table, we have for a three-phase line without earth:—

$$Z_{ao} = R_a + 3R_e + j\omega L_{ao} \quad \text{ohm/km}$$

$$Z_{bo} = R_b + 3R_e + j\omega L_{bo} \quad \text{ohm/km}$$

$$Z_{co} = R_c + 3R_e + j\omega L_{co} \quad \text{ohm/km.}$$

Other methods have been suggested by Bekku, Mackerras and Haberland.

Meshed Network with Several Sources: According to Hessenberg, with an unsymmetrical load part of the delivered positive-sequence power is returned or "reflected" as negative- or zero-sequence power.

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A total reflection of the power occurs with a line-to-line fault in an otherwise unloaded three-phase line. Thus the following equations apply:

$I_2 = -I_1$, $E_2 = E_1$ and $E_1 I_1 = - E_2 I_2$

i.e. the total positive-sequence power delivered to the point of fault is converted into negative-sequence power. By drawing equivalent diagrams a number of asymmetry problems can be solved.

Break in a line Conductor: The conditions resulting from such a fault may be treated in a similar manner as the line-to-line fault. If in a network consisting of a generator, a long transmission line and a motor, the negative- and zero-sequence impedances on the motor side are denoted by Z_{B_1}, Z_{B_2} and Z_{B0} the sides being taken from the point of fault, then a reversal of the rotating field will occur only when

$$\left| \frac{Z_{B_1}}{Z_{B_2}} \left(1 - \frac{Z_{A_2} + Z_{B_2}}{Z_{A_0} + Z_{B_0}} \right) \right| < 1$$

A further investigation shows that with an earthing reactor tuned exactly, the reversal of the rotating-field may be avoided.

These results are due to Bekker.

In short, the application of the methods of symmetrical components is enormous, out of which only a few have been enumerated. For details, standard works on the subject may be referred to. (See Bibliography)

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INDUCTION MOTORS.

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7.1 GENERAL CONSIDERATIONS.

The study of induction motors under unbalanced conditions is an important application of the theory of symmetrical components. The induction motor is investigated by the use of circle diagrams. For developing the circle diagram of a poly-phase induction motor symmetry of phase-voltages and currents is always assumed. The equivalent circuit is drawn for one phase only, and to determine the torque and power from the circle diagram the values furnished for single phase are multiplied by the number of phases. But if the system is unsymmetrical, in order to know its currents etc, it will have to be resolved into the symmetrical components. Consequently, two circle diagrams may be developed — one for the positive-, and the other for the negative-sequence system. A zero-sequence system is not likely to occur as the neutral of the motor is not generally earthed.

Evidently, for power values only corresponding components must be combined (as shown on p 34)

In Fig 7.1 the voltages of the positive-sequence system are denoted by E_{a_1}, E_{b_1}, & E_{c_1}, the currents of the negative-sequence system by I_{a_2}, I_{b_2}, & I_{c_2}, E and I being the R.m.s. values. If the phase difference between the current and voltage is ϕ, then

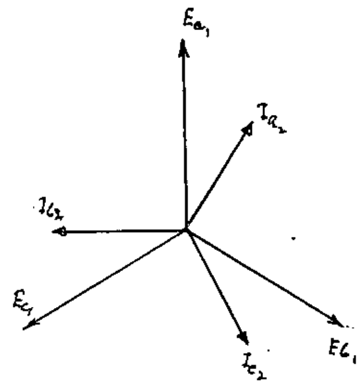

Fig 7.1

Power in phase 1 $= EI\cos\phi$

" " " 2 $= EI\cos(\phi+120°)$

" " " 3 $= EI\cos(\phi+240°)$

So, Total power $= EI[\cos\phi + \cos(\phi+120°) + \cos(\phi+240°)] = 0$

i.e. the power resulting from the combination of positive-sequence voltage system and the negative-sequence current system, or vice-versa, is always zero.

Denoting the voltage and current per phase in the positive-sequence system by E_1 & I_1, the voltage and current per phase in the negative system by E_2 & I_2 and the respective phase angles by ϕ_1 and ϕ_2 then the total power will be:

$$3 E_1 I_1 \cos\phi_1 - 3 E_2 I_2 \cos\phi_2$$

as the negative-sequence system tends to turn the motor in the opposite direction. i.e. the torque of the negative-sequence system is a braking torque.

If the slip of the motor with respect to the positive sequence system is σ, then it is $(2-\sigma)$ with regard to the negative-sequence system, the slip with locked rotor being unity.

7.2 EQUIVALENT CIRCUITS FOR SYMMETRICAL AND UNSYMMETRICAL SYSTEMS

Let R_s = Stator resistance per phase

X_s = " leakage reactance " " at supply frequency

R_r = Rotor resistance " "

X_r = " leakage reactance " " " "

I_s = Stator current per phase

I_r = Rotor " " "

E_g = Fictitious voltage corresponding to air-gap field.

σ = slip

Then from equivalent circuit (Fig 7.2) we have,

$E_s = E_g + (R_s + jX_s) I_s$

$\sigma E_g = (R_r + j\sigma X_r) I_r$, and

$E_g = jX_m (I_s - I_r) = \left(\frac{R_r}{\sigma} + jX_r\right) I_r$

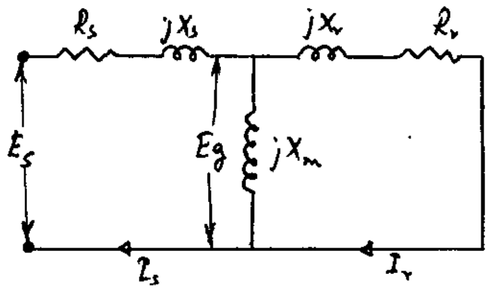

Fig 7.2

X_m being the reactance value connecting the magnetizing stator current and the demagnetizing rotor current with the voltage corresponding to the air-gap field.

Then $\frac{R_r}{\sigma}$ may be further resolved into two parts

R_r and $\frac{R_r}{\sigma} - R_r$ or $\left(\frac{1-\sigma}{\sigma}\right) R_r$

so that $\left(\frac{1-\sigma}{\sigma}\right) R_r$ represents that value which when multiplied by I_r^2 gives the mechanical power per phase. The total power, P, at the shaft of a three phase motor is, therefore, $P = 3 \cdot \frac{1-\sigma}{\sigma} I_r^2 R_r$.

and the torque is

$$T = \frac{0.974}{n} P \text{ mkg or } \frac{7}{n} P \text{ ft lb}$$

if P is in watts and n the speed in rpm. As the synchronous speed n_s is related to the supply frequency f by the relation $n_s = \frac{n}{1-\sigma} = \frac{60 f}{p}$

where 'p' is the pair of poles.,

the torque $T = \frac{0.974}{60 f} \cdot \frac{3 R_r}{\sigma} I_r^2 = 0.0162 \frac{p}{f} \cdot \frac{3 R_r}{\sigma} I_r^2$ mkg

$= 0.117 \frac{p}{f} \cdot \frac{3 R_r}{\sigma} I_r^2$ ft-lb.

This derivation is made under the assumption of symmetrical voltage system. It may also be used, therefore, without any change for the positive-sequence system of an unsymmetrical voltage system. It may also be used for a negative-sequence system, if the slip $(2-\sigma)$ is substituted for σ, and the value I_{r_2} for the rotor current. The power of the inverse system is thus $P_2 = -3 \frac{1-\sigma}{2-\sigma} R_r \cdot I_{r_2}^2$.

and the total power becomes

$$P_{total} = 3(1-\sigma) R_r \left(\frac{I_{r_1}^2}{\sigma} - \frac{I_{r_2}^2}{2-\sigma} \right)$$

and the total torque is

$$T_{total} = 3 \times 0.0162 \frac{p}{f} \cdot R_r \left(\frac{I_{r_1}^2}{\sigma} - \frac{I_{r_2}^2}{2-\sigma} \right) \text{ m kg.}$$

The equivalent circuit diagrams for the positive- and negative-sequence systems are shown in Figs 7.3 & 7.4 respectively.

EQUIVALENT CIRCUITS.

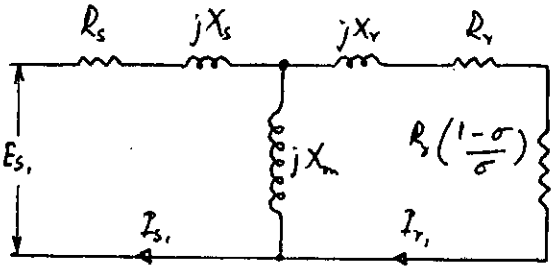

Fig 7.3 +ve-seq

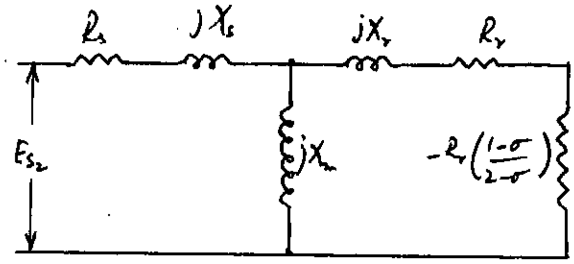

Fig 7.4 -ve-seq.

For instance, if a three-phase motor is running with one of the leads open-circuited, the equivalent circuit will reduce to that as shown in Fig 7.5 $E_{s_1} - E_{s_2} = \frac{E_{bc}}{\sqrt{3}}$

If the phase a is open-circuited, then:

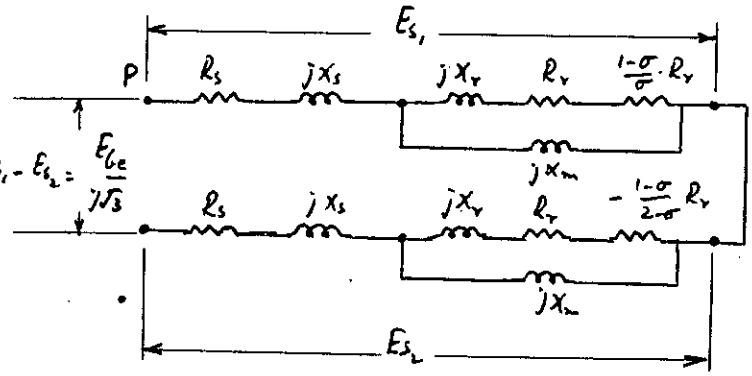

Fig 7.5

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$$I_a = 0$$

$$I_b = I$$

$$I_c = -I$$

Therefore, $I_{s_1} = \frac{1}{3}(0 + aI - a^2 I) = \frac{a-a^2}{3} I = j\frac{I}{\sqrt{3}} = -I_{s_2}$

$$I_{s_2} = \frac{1}{3}(0 + a^2 I - aI) = \frac{a^2-a}{3} I = -j\frac{I}{\sqrt{3}} = -I_{s_1}$$

Let the stator be Y-connected. The line voltage E_{bc_s} is known. The phase-voltages are:

$$E_{bs} = a^2 E_{s_1} + a E_{s_2}$$

$$E_{cs} = a E_{s_1} + a^2 E_{s_2}$$

so that the line-voltage may be written as:

$$E_{bc_s} = E_{bs} - E_{cs} = (a - a^2)(E_{s_1} - E_{s_2}) = j\sqrt{3}\,(E_{s_1} - E_{s_2})$$

These relations may, therefore, be represented by the series connection of the equivalent-circuit diagram of the positive-sequence system with that of the negative-, so that the condition $I_{s_1} = -I_{s_2}$ is fulfilled. The voltage applied is $(E_{s_1} - E_{s_2}) = \frac{E_{bc_s}}{\sqrt{3}}$.

Thus if the different impedances are known then using the equivalent-circuit diagram, the power and torque, depending on the slip, may be evaluated from the above relations. The resistance value that corresponds to mechanical power is

$$\left(\frac{1-\sigma}{\sigma} - \frac{1-\sigma}{2-\sigma}\right) R_r = \frac{2(1-\sigma)^2}{\sigma(2-\sigma)} \cdot R_r$$

assuming jX_m of the positive-sequence component to be acting at the terminals and neglecting jX_m of the negative-sequence component. Denoting I_r' the rotor current with these approximations, we have the expression for torque as

$$T'_{total} = 0.117 \frac{p}{f} \cdot \frac{6}{\sigma} \frac{(1-\sigma)}{(2-\sigma)} R_r \cdot I_r' \text{ ft-lb}$$

which shows that the torque is zero when $\sigma = 1$ i.e. the motor will not start from rest, though it can keep running if the slip is small.

Note: These statements were experimentally verified by applying unbalanced voltages to a 1 HP induction motor. Also, the motor did not start when one of its phases was left open-circuited.

(x)

(8)

BALANCING OF UNSYMMETRICAL SYSTEMS.

8.1 INTRODUCTION.

The three-phase unbalanced system may be symmetrized by many methods eg. by static-balancers, rotary-balancers etc. They will be considered here in a very broad outline. For a detailed study "Symmetrical Components" by Wagner & Evans may be referred to.

8.2 STATIC BALANCERS.

(1) By balancing transformers with inductances and capacitances.

(2) By induction regulator

(3) Hesenberg's method,

Hesenberg used zig-zag coil for symmetrizing Y-connected load as shown in Fig 6.1.

A = Switch B = 1-ϕ load

C = X'former D = Zig-zag coil.

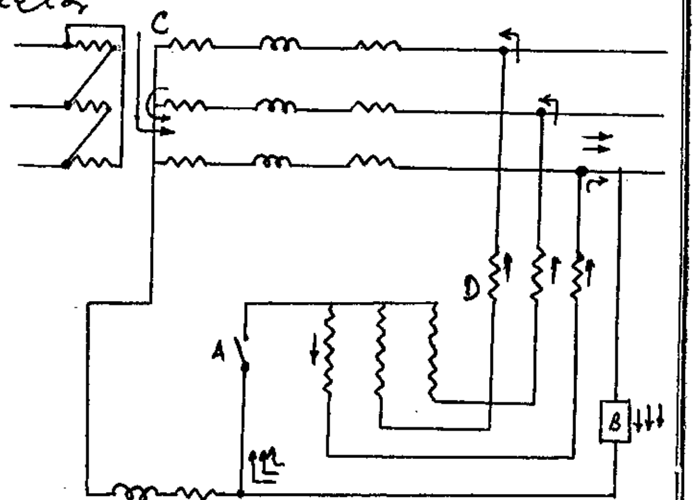

Fig 6.1

8.3 ROTARY BALANCERS.

Rotating balancers tend to balance voltages and currents by periodically absorbing and restoring energy to the system using in this process the energy by the inertia of rotating parts. Thus rotating machines have the inherent property of providing the balancing action and do not require the adjustable feature characteristic of static balancers.

Rotary balancers are of two type:

1. Negative-sequence emf generator
2. Impedance type balancer.

For details, the reader is referred to any standard book (See bibliography) as it is beyond the scope of this work.

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(9)

CONCLUSION.

It is pretty evident from the previous discussions that the use of the theory of symmetrical components is inevitable in the study of unbalanced three-phase or polyphase technique. It is worthwhile, therefore, to sum up the results arrived at.

The conditions which give rise to negative- and zero-sequence components are:

(1) Negative-sequence currents are produced either by unsymmetrical loading or by short-circuits.

(2) Zero-sequence currents are produced by earth-faults.

(3) Only positive-sequence voltages are produced by rotating machines.

(4) Negative- and zero-sequence voltages are only produced respectively by negative- and zero-sequence currents, in a system with symmetrical line impedances.

(5) Positive- and negative-sequence currents can be

generated by an alternator, but no zero-sequence currents unless the neutral is earthed.

(6) Each earthed-point on a system is a source of supply of zero-sequence currents.

(7) Each sequence-current circulates through the system in a manner depending on its source of supply and independent of the circulation of other sequence-currents.

(8) No zero-sequence current can flow into or away from an unearthed machine

(9) The reactances of transmission lines are the same to positive- and negative-sequence currents, but greater to zero-sequence currents.

(10) The impedance of rotating machines to negative-sequence currents produced by negative-sequence voltages is very much less than the impedance to positive-sequence currents. This is

because rotating machines tend to operate as phase-balancers, and when supplied by a system of unbalanced voltages, draw from the system heavy unbalanced currents which tend to correct the unbalance. Rotary machines then absorb negative-sequence power.

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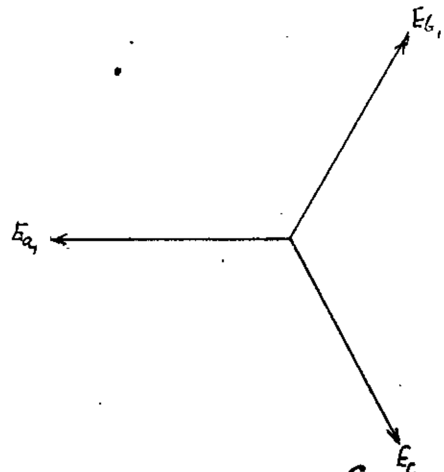
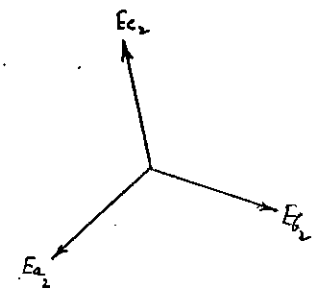
APPENDIX. I.

A few of the measurements taken by the "Symmetrical Component Meter" are appended in the following pages. The measured phase-voltages in unbalanced circuits have been compared with the phase-voltages synthesized from the measured sequence-voltages and their relative phase angles. ~~It~~ has been observed that the meter gives fairly accurate results (Error ± 1to2 %), and when carefully used gives almost exact results, as could be seen from the figures given.

Phase angle $\phi = -42^\circ$



Positive-sequence Component = 165 volt



Negative-sequence Component = 65 volt

Measured Phase voltages

$E_a = 150$ volts

$E_b = 133$ volts

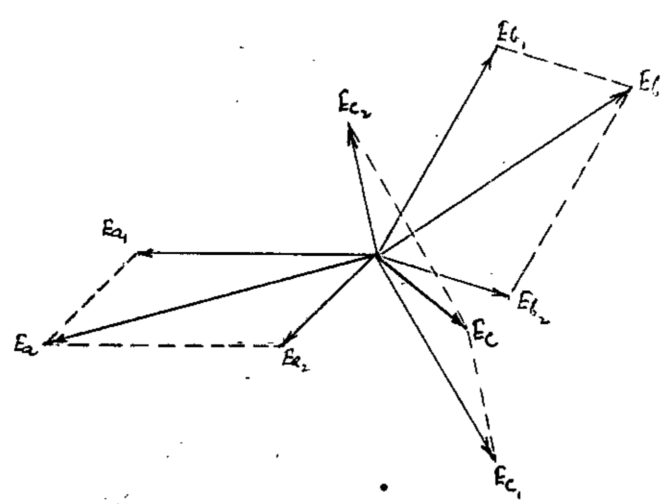
$E_c = 50$ volts

Calculated Phase voltages.

$E_a = 150$ volts

$E_b = 135$ volts

$E_c = 50$ volts

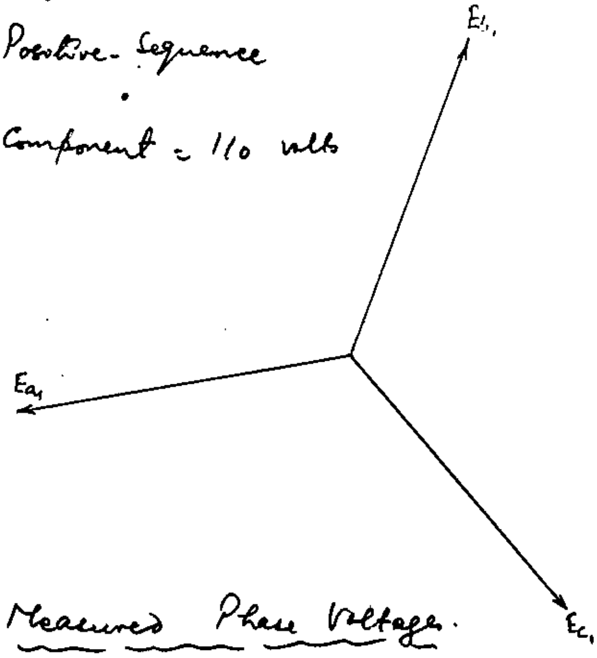


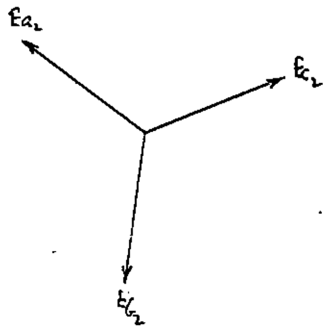
Combination of sequence voltages to form phase voltages.

Phase angle $\phi = +46°$

Positive-Sequence Component = 110 volts

Negative-sequence Component = 50 volts





Measured Phase Voltages.

E_a = 147 volts

E_b = 61 volts

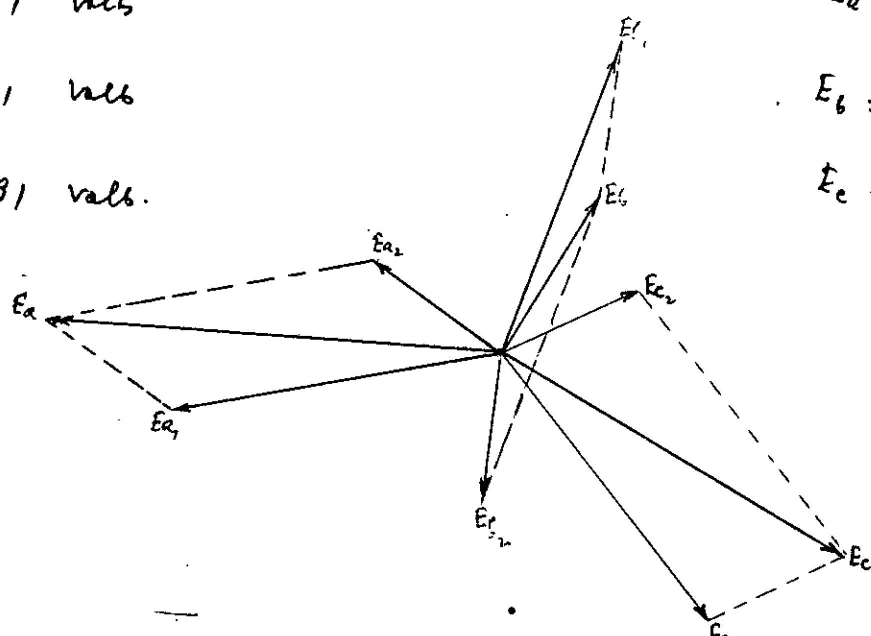
E_c = 131 volts.

Calculated Phase voltages.

E_a = 150 volts.

E_b = 60 volts.

E_c = 135 volts.



Combination of Sequence voltages to form phase voltages.

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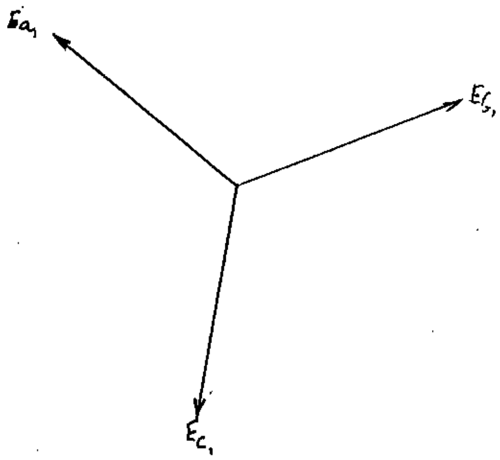
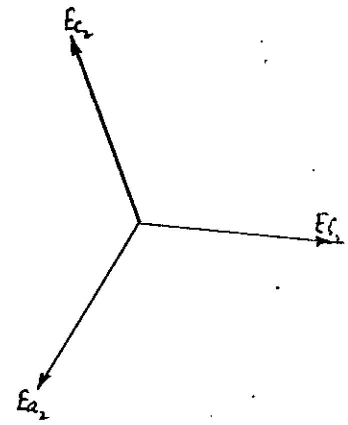
Phase angle $\phi = -110^\circ$





Positive-sequence voltage = 100 volts

Negative-sequence voltage = 85 volts

Measured Phase Voltages

E_a = 125 volts

E_b = 185 volts

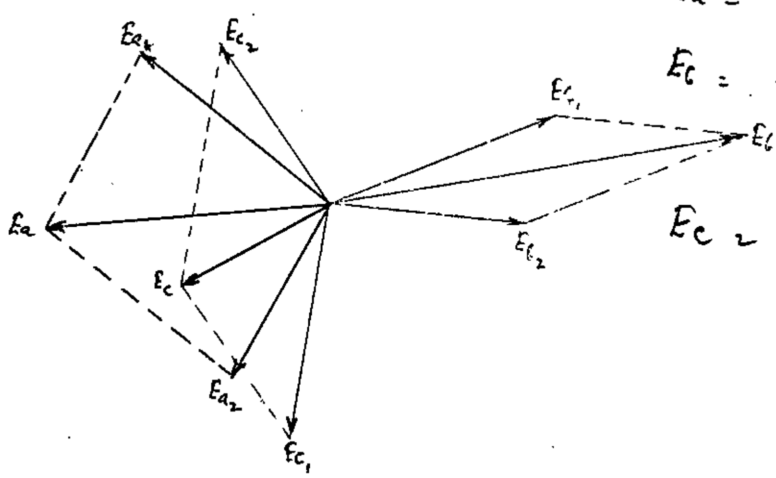
E_c = 70 volts

Calculated Phase Voltages

E_a = 124 volts.

E_b = 188 volts.

E_c = 74 volts.



Combination of sequence voltages to form Phase voltages.

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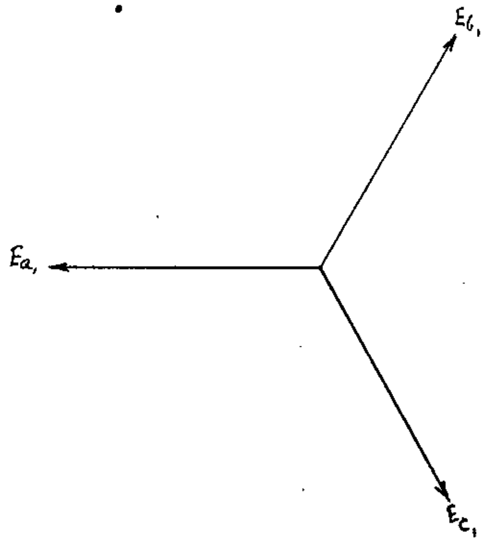
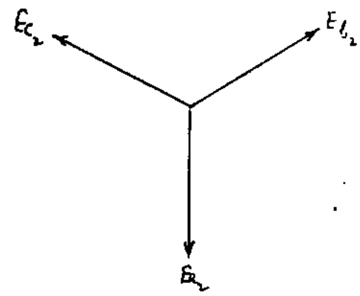
Phase angle $\phi = -90°$

Positive-sequence Component = 120 volts.



Negative-sequence Component = 65 volts.

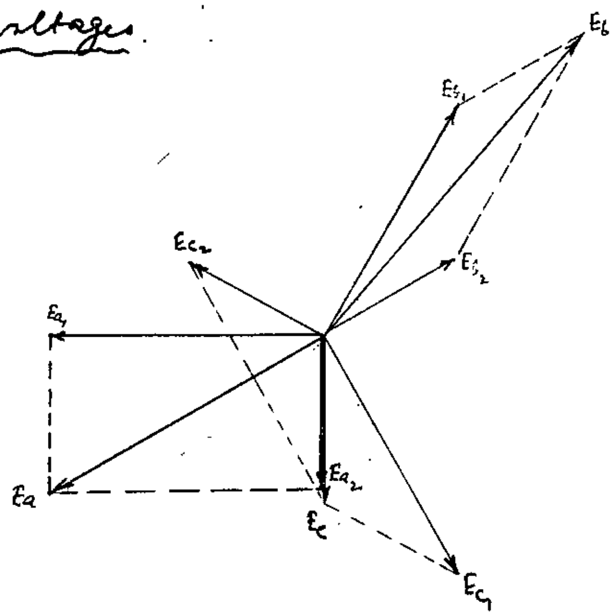


Measured Phase-voltages.

$E_a = 140$ volts

$E_b = 170$ volts

$E_c = 70$ volts.



Calculated Phase voltage

$E_a = 140$ volts

$E_b = 172$ volts

$E_c = 75$ volts

Combination of sequence voltages to form Phase voltages.

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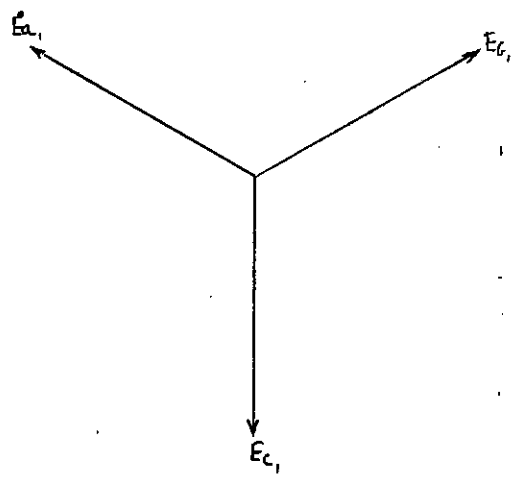
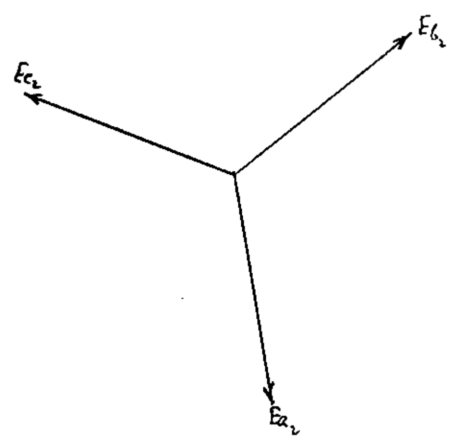
Phase angle $\phi = -129°$

Positive-sequence component = 112 volts



Negative-sequence component = 100 volts.

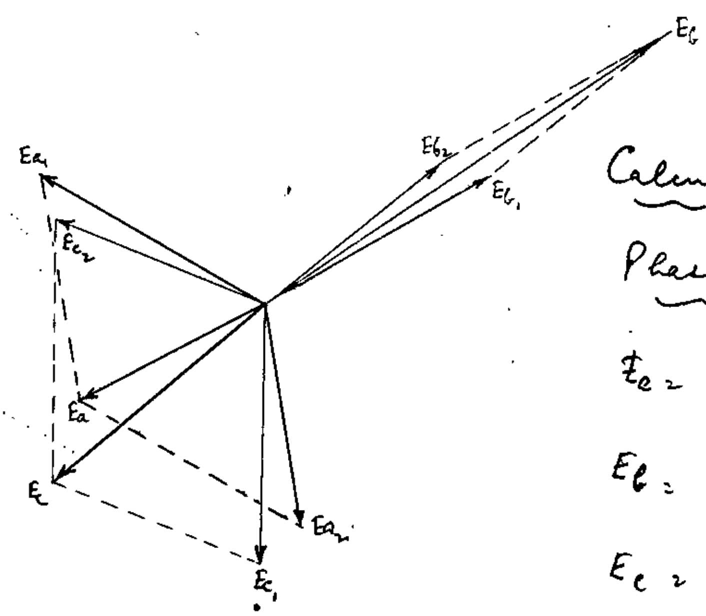


Measured Phase voltages.

$E_a = 90$ volts

$E_b = 218$ volts

$E_c = 120$ volts



Calculated Phase voltages.

$E_a = 90$ volts

$E_b = 215$ volts

$E_c = 121$ volts

Combination of sequence voltages to form phase voltages.

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APPENDIX 2.

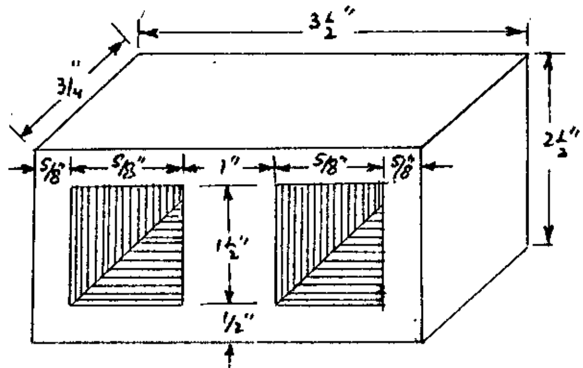
DESIGN OF ISOLATING TRANSFORMERS.

References: 1. Design of Small Transformers – E. Molloy
2. Radiotron Designer's Handbook.

Specifications: Three single-phase transformers
Connection – Star / Star
Ratio – 1 : 1 (220V / 220V across lines)
Frequency – 50 cycles
Current – 25 milli-amps.
VA rating – 5 per transformer

Core Dimensions:

"SHELL TYPE" Core of dimensions as labelled in the sketch. To be taken out from 220/110 auto-transformer.



(NOT TO SCALE)

One such core is required for each phase. Therefore, three such cores are required.

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Number of turns: Approximate number of turns per volt (t) is given by the formula

$$t = \frac{8.4}{\text{Area of core in } \square''} \quad \text{at 50 cycles/sec.}$$

$$= \frac{8.4}{\frac{3}{4} \times 1} = \underline{11.2 \text{ turns/volt}}$$

Therefore, number of turns required for 220 volts = 2500 turns.

Hence, number of turns on primary = 2500 turns

number of turns on secondary = 2500 turns.

Winding Particulars: From wire tables, it was found that No 36 SWG Enamelled was best suited. Its particulars are:

Diameter 8.3 mils

Turns/inch 120.5

If 125 turns are wound in one layer, total number of layers required for each winding is 20. Therefore, space required for both windings = $\frac{40 \times 8.3}{1000} = \frac{1}{3}$ inch which is less than the window dimensions.

Hence, No 36 SWG Enam. can be approved for both the primary and secondary windings. The current carrying capacity of this wire is 68 milliamps, while the maximum current to be handled by the transformers is only 25 to 30 milliamps.

Insulation: Empire tape and thick paper can be used for insulation. After properly insulating the coils, they should be baked in an oven and dried up, and finally assembled with the core.

To complete the isolating transformer, three such transformers are to be made and connected in star/star.

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(iii) Arm 1-2 should be purely resistive.

(iv) The circuit parameters be so chosen that the current-transformers are not over-burdened.

Design Considerations:

Bearing the above-mentioned points in mind, the design and assembly procedure are very simple. For example, for a 25-VA current transformer, the maximum permissible burden is 1 ohm at 5 amps. This dictates the values of the bridge parameters. At the first attempt, however, these values need not be very accurate. A purely inductive coil of approximate value may be taken and connected in series with a variable resistance r to form the arm 3-4 of the bridge. Another variable resistance R may be used to form the arm 1-2. The remaining two arms consist of two identical ammeters. By adjusting the resistances r and R, on a 3-phase balanced system, the meter A_x shall give a deflection, while A_y will read zero, the negative-sequence current being zero. Obviously, A_x will indicate the magnitude of the positive-sequence current. Under these conditions, the arm 3-4 shall have a phase-shift of exactly 60^0. After obtaining such a balance, the resistances r and R should not be disturbed. A further check may be made at this stage by opening up the phase c of the 3-phase balanced system. In this case, if the bridge is properly balanced, the meters A_x and A_y should read equal, since the magnitudes of the positive- and negative-sequence currents are equal in the opened phase.

The Modified Circuit:

Based on the above discussion, using an electrodynamometer-type ammeter, the following is a circuit (Fig. 2) designed for the measurements of positive- and negative-sequence currents and their relative phase-angles. It is new in the sense that only one ammeter is needed to make all the measurements, and it can measure the magnitudes of the sequence currents as well as their relative phase-angles. In this circuit, use has been made of the fact that an electrodynamometer instrument responds not only to the product of the currents through its coils but also to the phase-angles between the currents. This would become more evident by referring to switching operations given below:

Setting	Switch P	Switch Q	Switch R	Switch S	Meter Reads	Figure
I	left	left	left	open	Ic_1	3a.
II	right	right	left	open	Ic_2	3b.
III	open	right	right	closed	$Ic_1 . Ic_2 \cos \angle_{Ic_2}^{Ic_1}$	3c.

As shown in Fig. 2, to make a measurement, the two current transformers are connected to the lines a and b and the meter responds to the sequence currents of line c. In settings I and II (Figs. 3a. and 3b.) the meter acts as an ordinary ammeter and reads directly the positive- and negative-sequence currents Ic_1 and Ic_2 respectively. In setting III (Fig. 3c.) however, it can be seen that the two coils of the meter form the two arms (1-3 and 2-4) of the bridge. In this case the meter deflection is proportional to $Ic_1 . Ic_2 \cos \angle Ic_1, Ic_2$. Ic_1 and Ic_2 having been already determined from the settings I and II, the phase-angle between Ic_1 and Ic_2 can be computed.

To sum up, the circuit essentially requires an electrodynamometer type ammeter with the two coils having identical impedances and the leads brought out independently; another coil having an impedance equal to twice the impedance of either of the meter-coils; two current transformers; a pure resistance; a 60^0-phase-shift coil; three double-pole double-throw switches; and a single-pole switch. It can also be seen from Fig. 2, that the short-circuiting switch of the current transformers should always be kept closed except w switches P, Q, R and S occupy any of the positions indicated in setting I, II or III.

THE SEQUENCE-VOLTAGE BRIDGE

The bridge is shown in Fig. 4. It is similar to the current-bridge described in the preceding section. In the absence of zero-sequence voltages, the meters respond to the positive- and negative-sequence voltages, as given by the following equations:

$$Vx = Vca_1 \quad (3)$$

$$Vy = Vca_2 \quad (4)$$

if $R(Z+Z_m) = -aZ(R+Z_m)$, where the symbols are shown in Fig. 4 and a is the phasor-operator, as defined in the nomenclature.

Following the same reasoning as that for the current-bridge, it may be readily seen that the following conditions should be fulfilled in order that the sequence-components of voltages may be measured:

(i) The voltmeters should have exactly the same impedance Z_m.

(ii) Arm 3-4 of the bridge should have a phase-shift of exactly 60°.

(iii) Arm 1-2 should be purely resistive.

(iv) The circuit parameters should be so chosen that the potential-transformers are not over-burdened.

For the measurements of relative phase-angles between the sequence-voltages, the principles governing the current-bridge (Fig. 2) may be utilized. A circuit similar to that shown in Fig. 2 may be set up, using an electrodynamometer voltmeter having two exactly identical coils. With proper switching arrangements, the same meter can be used to respond to the positive- and negative-sequence voltages, and to their relative phase-angles, as in case of the sequence-current bridge.

THE SEQUENCE-POWER BRIDGE

After the sequence-components of currents and voltages have been segregated, the sequence-power measurements can be carried out using two ordinary wattmeters. The current coils of the wattmeters form the arms 1-3 and 2-4 of the sequence-current bridge, while the voltage-coils form the arms 1-3 and 2-4 of the sequence voltage bridge. With such a connection, the two identical wattmeters respond to the sequence-components of power. However, care should be taken to apply the necessary phase-angle and scale-factor corrections (8).

SOME APPLICATIONS

The applications of the methods of symmetrical components are too numerous to mention (2, 3, 5). These components can be determined either by analytical methods or by direct methods, that is, by measurements. Obviously, the direct methods are more rapid and less tedious. The methods suggested in this paper give complete information regarding the sequence quantities of an unbalanced system. These methods can be used to correlate the theory with experiments. Conventional methods described in text books and elsewhere, do not give a complete analysis as the relative phase angles cannot be measured by these methods. The method given here can also be used in usual laboratory teaching and research to analyze unbalanced systems, and for the measurements of sequence quantities in rotating machinery, rotary balancers and other special machines (4, 8). Being a bridge method, the method can also find applications in detecting unbalance and actuating control elements to restore balance to the system. A further application could be in conjunction with protective-relays for sensing the flow of negative-sequence power.

CONCLUSIONS

(1) Detailed mathematical analysis is given to prove the validity of the methods discussed. As such, it is not available in textbooks.

(2) The bridge methods are extended to measure the sequence-components of power.

(3) Modifications are suggested in order that phase-angles between sequence-components of currents can be measured. This can be further extended for the measurements of phase-angles between sequence-components of voltages.

(4) Measurements of the sequence-components and their relative phase-angles give the complete information regarding the nature and degree of unbalance in a system. This is in contrast to the earlier methods which give only the magnitudes of sequence quantities.

(5) The method is useful in correlating theoretical predictions with experimental results. The method thus has an educational value. A further application is that it gives direct and rapid results while the numerical calculations by analytical methods may be too involved.

(6) It is agreed that the suggested circuits are relatively expensive and the design is somewhat critical.

APPENDIXES

Proof of Equations (1) and (2):

Let the secondary currents divide as shown in Fig. 1. Then, by Kirchoff's laws:

$$I_a = I_a' + I_a'' \tag{5}$$

$$I_a'(Z+Z_m) = I_a''(R+Z_m) \tag{6}$$

where Z_m is the impedance of either of the ammeters. From (5) and (6), therefore, writing in terms of I_a' and I_a'':

$$I_a' = I_a\left(\frac{R+Z_m}{R+Z+2Z_m}\right) \tag{7}$$

$$I_a'' = I_a\left(\frac{Z+Z_m}{R+Z+2Z_m}\right) \tag{8}$$

Similarly for the currents I_b' and I_b'':

$$I_b' = I_b\left(\frac{Z+Z_m}{R+Z+2Z_m}\right) \tag{9}$$

$$I_b'' = I_b\left(\frac{R+Z_m}{R+Z+2Z_m}\right) \tag{10}$$

From the superposition theorem it follows:

$$A_x = I_a' + I_b' = I_a\left(\frac{R+Z_m}{R+Z+2Z_m}\right) + I_b\left(\frac{Z+Z_m}{R+Z+2Z_m}\right) \tag{11}$$

$$A_y = I_a'' + I_b'' = I_a\left(\frac{Z+Z_m}{R+Z+2Z_m}\right) + I_b\left(\frac{R+Z_m}{R+Z+2Z_m}\right) \tag{12}$$

Rewriting A_x and A_y in terms of sequence-components of Ic, therefore:

$$A_x = Ic_1\left[\frac{a^2(R+Z_m) + a(Z+Z_m)}{R+Z+2Z_m}\right] + Ic_2\left[\frac{a(R+Z_m) + a^2(Z+Z_m)}{R+Z+2Z_m}\right] \tag{13}$$

$$A_y = Ic_1\left[\frac{a^2(Z+Z_m) + a(R+Z_m)}{R+Z+2Z_m}\right] + Ic_2\left[\frac{a(Z+Z_m) + a^2(R+Z_m)}{R+Z+2Z_m}\right] \tag{14}$$

If $(R+Z_m) = -a(Z+Z_m)$, from Equations (13) and (14):

$$A_x = Ic_1\left[\frac{(a-1)(Z+Z_m)}{R+Z+2Z_m}\right] = -Ic_1 \tag{15}$$

Similarly,

$$A_y = Ic_2\left[\frac{(R-1)(Z+Z_m)}{R+Z+2Z_m}\right] = -Ic_2 \tag{16}$$

Equations (15) and (16) are identical to Equations (1) and (2), except that in the latter the negative sign has been dropped.

<u>Proof of Equations (3) and (4):</u>

Let the voltages and currents be as shown in Fig. 4. Then, by Kirchoff's laws:

$$V_{ab} = I_R R + I_y Z_m = I_x Z_m + I_Z Z \tag{17}$$

$$V_{bc} = I_y Z_m - I_Z Z = I_x Z_m - I_R R \tag{18}$$

$$V_{ab} + V_{bc} = I_x Z_m + I_y Z_m = 2I_x Z_m + I_Z Z - I_R R \tag{19}$$

$$I_x - I_Z = I_y - I_R \tag{20}$$

where Z_m is the impedance of either of the voltmeters. From Equations (19) and (20):

$$V_{ab} + V_{bc} = 2I_x Z_m + I_x Z_m - I_Z Z_m \tag{21}$$

But from Equations (17) and (18):

$$I_R = \frac{-V_{bc} + I_x Z_m}{R} \tag{22}$$

$$I_Z = \frac{V_{ab} - I_x Z_m}{Z} \tag{23}$$

Substituting Equations (22) and (23) in Equation (21) and simplifying:

$$V_{ab}\left(1 + \frac{Z_m}{Z}\right) + V_{bc}\left(1 + \frac{Z_m}{R}\right) = I_x Z_m\left(2 + \frac{Z_m}{R} + \frac{Z_m}{Z}\right) \tag{24}$$

Writing in terms of sequence-components:

$$V_{ab_1}\left[\frac{R(Z_m+Z) + a^2 Z(R+Z_m)}{R(Z_m+Z) + Z(R+Z_m)}\right] + V_{ab_2}\left[\frac{R(Z_m+Z) + aZ(R+Z_m)}{R(Z_m+Z) + Z(R+Z_m)}\right] = I_x Z_m = V_x \tag{25}$$

A similar expression can be written for the voltage read by V_y.

Now, of $R(Z+Z_m) = -aZ(R+Z_m)$ (26)

$$V_{ab_1}\left[\frac{R(Z+Z_m)(1-a)}{R(Z+Z_m)(1-a^2)}\right] = I_2 Z_m = V_x \tag{27}$$

or,

$$V_x = V_{ab_1}\left(\frac{1}{1+a}\right) = V_{ab_1}\angle -60^\circ = -V_{ca_1} \tag{28}$$

Similarly;

$$V_y = -V_{ca_2} \tag{29}$$

Equations (28) and (29) are identical to Equations (3) and (4) except for the difference in sign.

NOMENCLATURE

A = Ammeter reading

I = Current

V = Voltage, voltmeter reading

R = resistance

Z = impedance

a = phasor operator e^{j120°

Subscripts:

a, b, c = Phases a, b and c

1, 2 = Positive- and negative-sequence components respectively

m = meter (Voltmeter or Ammeter).

Symbols used locally are labelled on circuit diagrams.

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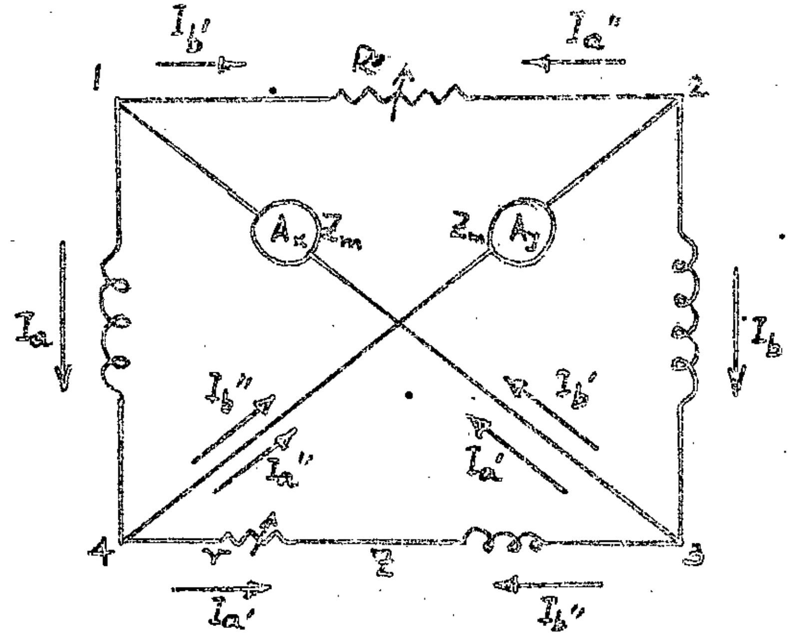


Fig. I: Bridge for Measuring Sequence-Currents.

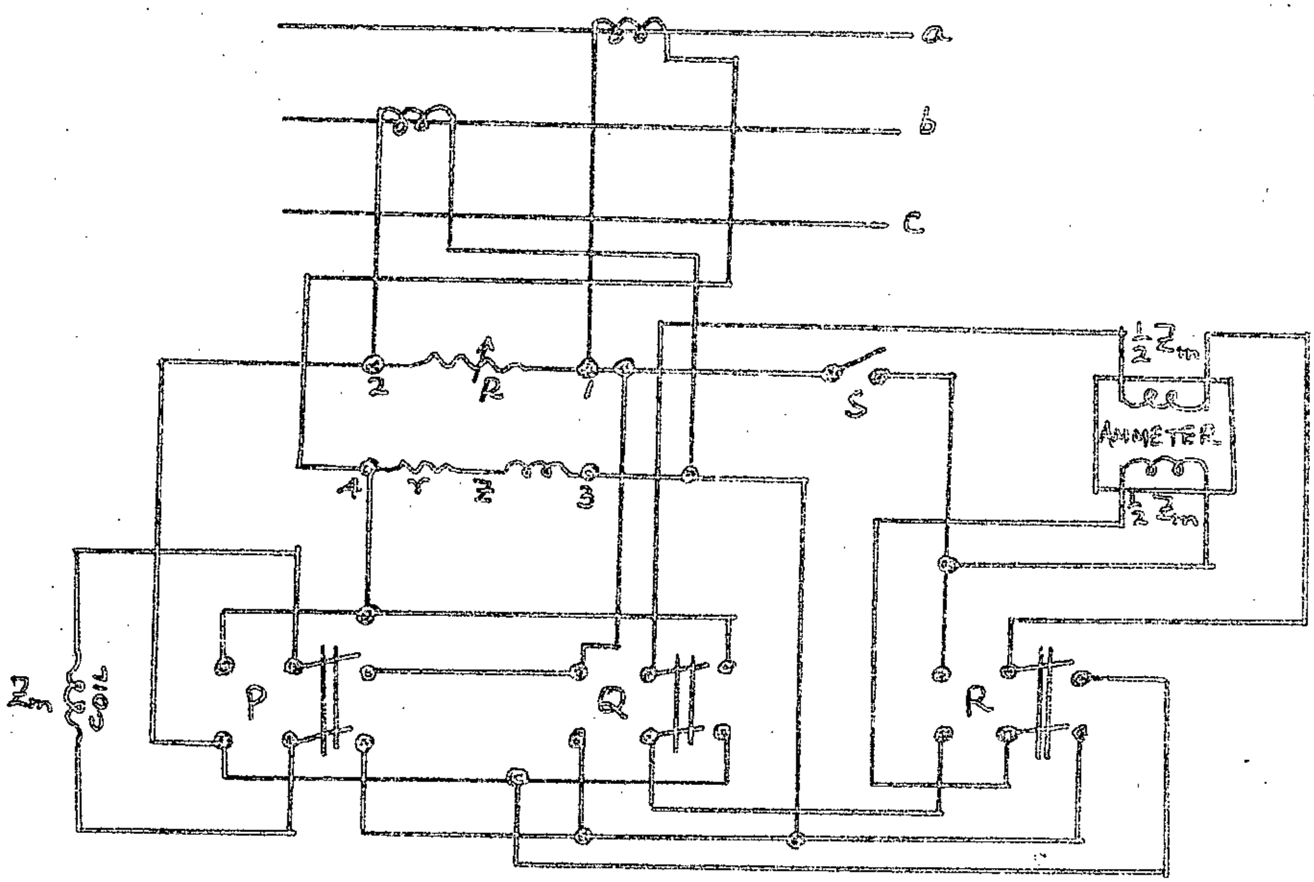
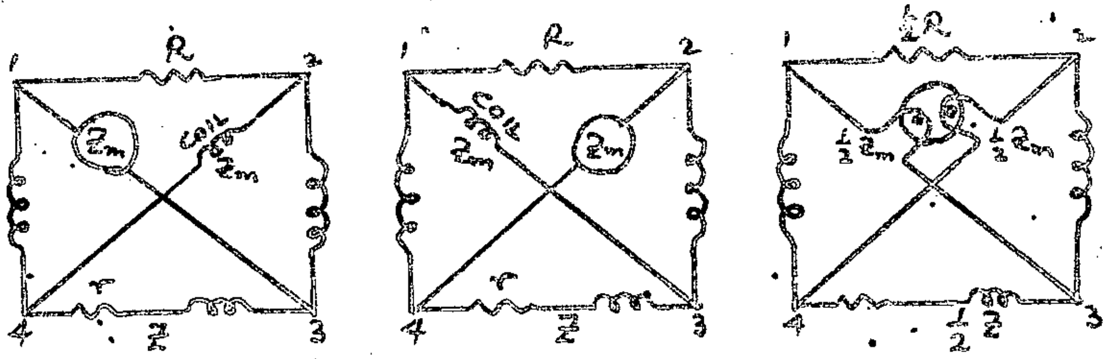


Fig. 2: Bridge for Measuring Sequence-Currents and their Phase-Angles.



(a) Setting I (b) Setting II (c) Setting III

Fig. 3: Simplified Circuit Diagrams.

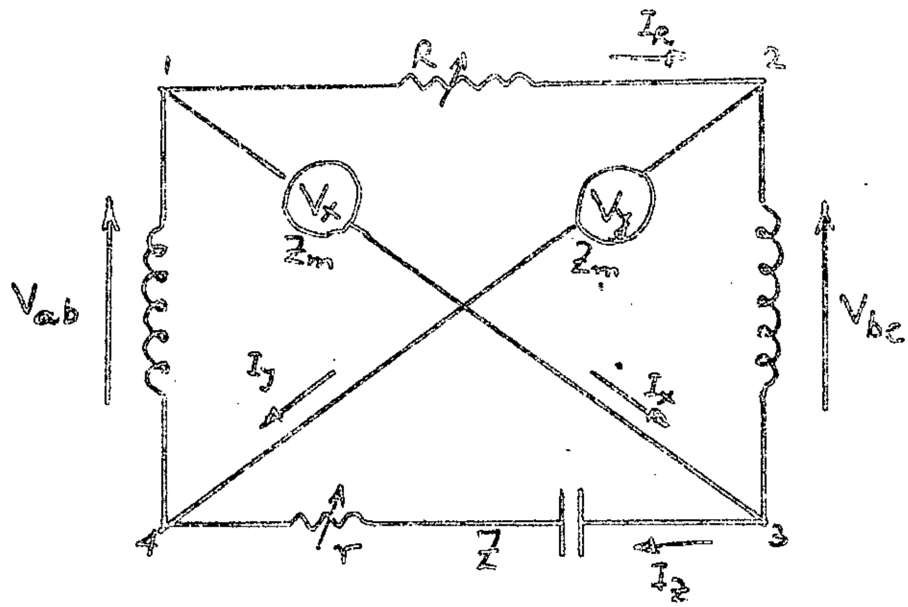


Fig. 4: Bridge for Measuring Sequence-Voltages.