THE

SYMMENTRICAL COMPONENTS
THEIR MEASUREMENTS
AND APPLICATIONS.

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PREFACE.

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"However high we climb in the pursuit of knowledge we shall still see heights above us, and the more we extend our view, the more conscious we shall be of the immensity which lies beyond."

This work is, therefore, never a complete guide to the methods of "Symmetrical Components" and its applications in various fields of polyphase technique. The fundamental principles of the method, however, have been explained. Next follows the main problem viz. the complete design, assembly and use of the "Symmetrical Component Meters" for measuring the positive and negative sequence components and their relative phase angles. This is based particularly upon R. L. Russell's discussions in Proc. I. E. E. Feb. '30. Rest of the portion is devoted to various applications in the
end a little discussion about balancing (symmetrizing).
The unCanonical systems has also been included.
The paragraphs, equations and the figures in
the text are numbered chapterwise on the decimal
system i.e. the integer of the number of a para or
an equation indicates the chapter in which it is
inserted while the numerals after the decimal point
indicate the serial order of the para or equation in that
chapter; for instance Eq 3.4 would mean the 4th Eq
in chapter 3. Also, three colours have been used for 3 diff. phases.

The work has been carried out under
the able guidance of Dr. H. Ahmed, Prof. E.E. Dept. and
without his help at every stage it would never
have been complete. Mr. M. N. Azam, Asst. Prof. E.E. Dept.
also extended his very kind help in changing
the coils of the dynamometer instrument to make
the experiment a success.

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LIST OF SYMBOLS.

Voltage Symbols: 
- \( E \): Vector Expression for voltage (RMS value)
- \( \bar{E} \): Absolute value of vector \( E \)
- \( e \): Instantaneous voltage

Current Symbols: 
- \( I \): Vector Expression for current (RMS value)
- \( \bar{I} \): Absolute value of vector \( I \)
- \( i \): Instantaneous current

Impedance Symbols: 
- \( Z \): \((R+jX)\) for total impedance
- \( \bar{Z} \): Absolute value of vector \( Z \)
- \( z \): \((R+jX)\) for impedance of element

Phase Symbols: 
- Subscripts \( a, b, c \) indicate phase \( a \) to neutral, phase \( b \) to neutral or phase \( c \) to neutral respectively. Subscripts \( A, B, C \) indicate phase between conductors \( A \) to \( B \), phase between conductors \( A \) to \( C \), and phase between conductors \( B \) to \( C \) respectively.

Sequence Symbols: 
- Subscripts \( 0, 1, 2 \) indicate \( 0 \), \(+ve\), \(-ve \) sequence.

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1. INTRODUCTORY
A polyphase system of voltages (or currents) may be either balanced or unbalanced. A balanced system is that in which the vectors are equal in magnitude and are displaced from one another by equal angles, while an unbalanced system is that in which the vectors are neither equal in magnitude nor displaced by equal angles.

An unbalanced polyphase system can be resolved into a set of balanced systems. For example, an unbalanced group of three related vectors can be resolved into three sets of vectors; the three sets have three vectors each, equal in magnitude and spaced either 0° or 120° apart. Then each set is a symmetrical component of the original unbalanced vectors. Or, to be more general, "An unbalanced group of n associated vectors, all of the same type, can be resolved into..."
n sets of unbalanced vectors, n vectors of each set are of equal length and symmetrically located with respect to each other.

The method of "symmetrical components" is analogous in some respects to the resolution of a periodic function into its fundamentals and higher harmonics by Fourier series. By the method of symmetrical components a set of unbalanced voltages, or currents, may be resolved into systems of balanced voltages, or currents, equal in number to the number of phases involved. By this method the unsymmetrical problems may be reduced to symmetrical problems, the solutions of which are well-known.

Obviously, a three phase unbalanced system of currents can be resolved into three component systems, each of which is balanced and symmetrical.

The first is a system of three phase currents having
normal or positive phase-sequence components. By this is meant that the currents in the lines (a, b, c) rise to the maximum in the order (a, b, c). The second symmetrical group of components has a phase-sequence opposite to the normal, and these are called the negative phase-sequence components. The third group consists of the line currents all equal and in phase, these being called the zero phase-sequence components. The magnitudes and phases of the various components, of course, depend upon the magnitudes and the phases of the currents flowing. Referring to Fig. 1.1, therefore, $I_a \ L_b$ and $I_c$ form the positive phase-sequence components; $I_{a2}, I_{b2}$ and $I_{c2}$ the negative phase-sequence components; and $I_{a0}, I_{b0}$ and $I_{c0}$ the zero phase-sequence components.

![Diagram of phase-sequence components](image)
Sequence components. They combine together to form the unbalanced system $I_a, I_b, I_c$ (Fig. 1.2). Or, in other words, they form the symmetrical components of an unsymmetrical system.

In symmetrical circuits, currents and voltages of different sequences do not react upon each other; currents of the positive sequence produce only voltage drops of positive sequence; currents of negative sequence only voltage drops of negative sequence, and, similarly, currents of zero-sequence only zero-sequence voltage drops. This results in considerable simplification of all kinds of problems involving symmetry such as that introduced by short-circuiting conductors of a system, or also by open-circuiting of a conductor.

The three-phase system has been considered exclusively as it is the most commonly used for all purposes - transmission as well as utilization for
industrial purposes, though the fundamental conceptions are always applicable to systems having any number of phases.

\[ x(t) \]

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HISTORICAL DEVELOPMENT.

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Before going into the details of 'gymnetrical components,' it will be interesting to consider the historical background and development of its theory.

Analysis of single-phase induction motors by Tenmani and others as early as 1895 may be said to have led to the idea of 'gymnetrical components.' It was shown that the set-up in a single-phase induction motor could be resolved into two fields rotating in opposite directions. Later, the idea of resolving the three-phase unbalanced currents into three sets of components (viz., positive, negative, and zero sequences) was developed in connection with the performance of three-phase machines.

Alexanderson, in 1913, proposed a qualitative treatment in connection with his work on phase balance. Stokvis, in France in 1915, published a mathematical analysis.
from the machine point of view. He assumed three sets of vectors viz.

1. A set which produced positively rotating field,
2. A set which produced negatively rotating field, and
3. A set which produced a pulsating field.

Stokes came very near to the modern concept, except that he failed to recognize the zero-sequence component which did not produce either rotating or pulsating field.

The modern concept of symmetrical components was developed by C.L. Fortescue and his associates R.E. Gilman, J.F. Peters and others while studying the problems of unbalanced circuits, and analyzing the characteristics of single-phase motors, poly-phase motors with unbalanced voltages, and synchronous MG sets, these balancers for single-phase railway electrification. Fortescue writes in one of his famous papers "In my early investigation of phase balancers I observed that..."
certain symmetrical relations between these currents and also between these voltages occurred frequently, which led me to the investigation of a general problem of imbalance. This investigation finally led to the discovery of the fundamental principles of the Method of Symmetrical Co-ordinates, which was published in 1918. In this way was developed a new simple and complete method for handling the problem of unbalanced circuits.

The method as suggested by Fortescue in his classical paper obviously involves mathematical analysis. Various methods have also been suggested to determine the time sequence components by physical measurements. Works worth mentioning have been done by Evans, Wagner, Takatsuka, Stiblings, Allcutt, Jenkins and Fortescue, in this connection. A very simple and convenient method for the measurements of the positive and negative phase sequence components and their
Their relative phase angles, however, have been suggested recently (1950) by Hunt. This method involves a dynamometer type instrument with specially designed networks.

However, work is still being done to simplify the analysis of unsymmetrical polyphase systems as far as possible.
3.

**BASIC CONSIDERATIONS**

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3.1 SYMMETRICAL AND UNSYMMETRICAL 3-PHASE SYSTEMS.

In a three-phase system, the currents and voltages (when represented vectorially) will appear as three-phase vector systems in which their lengths may be equal or unequal and may or may not have a phase displacement of 120°, according as the system is symmetrical or unsymmetrical. However, in almost every practical instance, the three-phase system is unsymmetrical.

The calculation of a symmetrical system is, of course, simpler than that of an unsymmetrical system, as in the first case it is sufficient in general to investigate the conditions in one phase only with the consideration of possible reactions from the other phase due to the symmetry, the conditions prevailing in the other phases can also be determined.

If the asymmetry be of a small amount, it
will frequently be permissible to neglect them altogether, or to make an allowance for them by slightly over-dimensioning the machinery, apparatus and lines.

Yet there are numerous cases in which this approximation is not permissible. For instance, asymmetries prevailing during a limited time (e.g. disturbances of normal working conditions caused by faults in the line); also there may exist permanent asymmetries which are encountered where single-phase loads (e.g. lamps, cooking, and heating appliances etc.) are connected to three-phase supply system; or where three-phase motors are to be fed by a single-phase supply system.

The investigation of unsymmetrical systems has therefore great practical importance. Even here it is possible to investigate the conditions prevailing in the different phases separately by using the method of symmetrical components. This kind of procedure is as
convenient as the normal procedure of solving the symmetrical (or balanced) systems. The method of symmetrical components allows for a better insight into the physics of a given problem than the old method of computation. And frequently this method may solve problems the solutions of which are difficult or sometimes impossible by the usual methods.

3.2 THE VECTOR OPERATOR "a"

By considering the equation

\[ x^3 - 1 = 0 \quad \text{(3.1)} \]

or \[ (x-1)(x^2 + x + 1) = 0 \]

we have \( x = 1, x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} \) as its solutions. Calling the particular root \( x = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \) (where \( j = \sqrt{-1} \)) as "a" we see that

\[ a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \quad \text{(3.2)} \]

\[ a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} \quad \text{(3.3)} \]

and \[ a^3 = 1 \quad \text{(3.4)} \]

which correspond to the three solutions of (3). Thus \( 1, a, a^2 \)
are the three roots of equation (3.1). Or in the other words, 1, \( a \) and \( a^2 \) are the "cubic roots of unity."

As \( a^3 = 1 \), the higher powers of \( a \) will have periodically. The values \( a, a^2 \) and \( 1 = a^3 \).

\[
\begin{align*}
a^4 &= a; & a^5 &= a^2; & a^6 &= 1 \\
a^7 &= a; & a^8 &= a^2; & a^9 &= 1 \\
3n &= a; & 3n &= a^2; & 3n &= 1
\end{align*}
\]

Representing the values \( a, a^2 \) and \( a^3 = 1 \) on a unit circle by equidistant points (Fig. 3.1) we find that a multiplication by \( a \) corresponds in each case to a displacement of the point in question by one-third of the circumference of the circle, i.e., by \( \frac{2\pi}{3} \); multiplication by \( a^2 \) corresponds to a displacement by \( \frac{4\pi}{3} \); and a multiplication by \( a^3 \) to a displacement by the whole circumference, and so on.

Thus a very important conclusion is
arrived at that multiplication of a vector by "a" corresponds to a rotation of the vector through 120°.

The term "a" is commonly known as rotational vector or vector operator (like the operator "j" which when operates on a vector turns it through 90°).

3.3 Properties of the Operator "a":

From the fact that operation on a vector by "a" corresponds to a rotation of the vector through 120°, a number of other important properties may be derived as given in Table 3.1 and Fig. 3.2.

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Table 3.1

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<thead>
<tr>
<th>Properties of the vector operator “a”</th>
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<tr>
<td>$1 = 1 + jo = e$</td>
<td>$a + a^2 + 1 = 0$</td>
</tr>
<tr>
<td>$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e$</td>
<td>$a + a^4 = -1 + jo = e$</td>
</tr>
<tr>
<td>$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e$</td>
<td>$a - a^2 = 0 + j\sqrt{3} = \sqrt{3} e$</td>
</tr>
<tr>
<td>$a^3 = 1 + jo = e$</td>
<td>$a^2 - a = 0 - j\sqrt{3} = -\sqrt{3} e$</td>
</tr>
<tr>
<td>$a^4 = a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e$</td>
<td>$1 - a = \frac{3}{2} - j\frac{\sqrt{3}}{2} = \frac{3}{2} e$</td>
</tr>
<tr>
<td>$a^5 = a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e$</td>
<td>$1 - a^2 = \frac{3}{2} + j\frac{\sqrt{3}}{2} = \frac{3}{2} e$</td>
</tr>
<tr>
<td>$a - 1 = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = -j\sqrt{3} = -\sqrt{3} e$</td>
<td>$a^2 - 1 = -\frac{3}{2} - j\frac{\sqrt{3}}{2} = -\frac{3}{2} e$</td>
</tr>
<tr>
<td>$1 + a^2 = -a^2 - j\frac{\sqrt{3}}{2} = e$</td>
<td>$1 + a^2 = -a - \frac{1}{2} - j\frac{\sqrt{3}}{2} = -e$</td>
</tr>
<tr>
<td>$(1+a)(1+a^2) = 1 + jo = e^0$</td>
<td>$(1-a)(1-a^2) = 3 + jo = 3 e^0$</td>
</tr>
<tr>
<td>$1 + a = a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e$</td>
<td>$1 - a = a = \frac{1}{2} - j\frac{\sqrt{3}}{2} = e$</td>
</tr>
<tr>
<td>$1 + a^2 = -a^2 - j\frac{\sqrt{3}}{2} = e$</td>
<td>$(1+a^2)^2 = a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = -e$</td>
</tr>
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3.4 POSITIVE SEQUENCE SYSTEM

From the basic properties of the “a” operator, it is obvious that a symmetrical system of vectors...
Eq., E6, and E9 (Fig. 3.3) are relative to each other as:

\[
\begin{align*}
E_a &= E_a = E_a \\
E_b &= e^{j240^\circ} E_a = a^2 E_a \\
E_c &= e^{-j120^\circ} E_a = a E_a
\end{align*}
\]

3.5

This system of vectors is known as positive-sequence system, because the maxima occur in the sequence a-b-c.

Or, a system of three-phase currents of similar phase- sequence as that of the voltage is known as positive- sequence system. From the equations 3.5, by fixing the position and magnitude of Eq., the corresponding positions and magnitudes of E6, and E9 may be immediately determined.

3.5 NEGATIVE SEQUENCE SYSTEM.

Referring to Fig. 3.4 we have as before:

\[
\begin{align*}
E_a &= E_a = E_a \\
E_b &= e^{-j240^\circ} E_a = a^2 E_a \\
E_c &= e^{j120^\circ} E_a = a E_a
\end{align*}
\]

Fig. 3.4

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This system of vectors is called the negative-sequence system. In this case the maximum occurs in the sequence cba, and as for positive-sequence system, by knowing one vector $E_a$ in magnitude and position others can be found out. The negative-sequence system is that system of three-phase currents with opposite phase sequence to that of the voltages.

3.6 Zero-Sequence System

From fig. 3.5 we have

$$E_a = E_a$$

$$E_b = E_b$$

$$E_c = E_c$$

This is the zero-sequence system in which all the vectors are in the same phase, and of course, equal in magnitude. Thus, a system representing the resultant or residue is the zero-sequence system, the physical significance of which arises when a path...
is established between any phase and neutral or the earth, so that the sum of the instantaneous line currents in the three-phase system is no longer zero; as it is a normal case, actually leaving a resultant which can be conceived of three vectors all in phase with each other and equal to one-third of the residue.

**3.7 Physical Significance of the Component Systems.**

A physical picture of these vectors may be had by considering the fields which result when voltages of positive- and negative-sequence systems are applied to a three-phase machine like induction motor. If a, b and c phases of the positive-sequence voltages be applied to the terminals a, b and c respectively, a magnetic field will be produced which will revolve in a certain direction. If now the voltages to the terminals b and c are changed
by interchanging the leads to the terminals b and c, a magnetic field will be produced which will rotate in the opposite direction according to the theory of induction motor. It follows, therefore, that the negative sequence set of voltages produce a field rotating in an opposite direction to that of the positive sequence system.

3.8 COMBINATION OF SEQUENCE QUANTITIES TO FORM PHASE QUANTITIES.

The total voltage of any phase is equal to the sum of the corresponding components of the different sequences in that phase. Thus any three vectors $E_a$, $E_b$ and $E_c$ representing a three-phase unbalanced system may be written as:

$$E_a = E_{a0} + E_{a1} + E_{a2}$$
$$E_b = E_{b0} + E_{b1} + E_{b2}$$
$$E_c = E_{c0} + E_{c1} + E_{c2}$$

3.8
Or, from equations (3.5), (3.6) and (3.7) we have:

\[ E_a = E_{a0} + E_{a1} + E_{a2} \quad \text{(3.9)} \]

\[ E_b = E_{b0} + a^2 E_{b1} + a E_{b2} \quad \text{(3.10)} \]

\[ E_c = E_{c0} + a E_{c1} + a^2 E_{c2} \quad \text{(3.11)} \]

where the suffixes denote the components in the different phases.

Thus, the unbalanced system is defined in terms of three balanced systems.

3.9 RESOLUTION OF VECTORS INTO THEIR SYMMETRICAL COMPONENTS.

Zero-sequence Component: Adding the three equations (3.9), (3.10) and (3.11) we have:

\[ E_a + E_b + E_c = 3 E_{a0} + (1 + a^2 + a) E_{a1} + (1 + a + a^2) E_{a2} \]

But \( (1 + a^2 + a) = (1 + a + a^2) = 0 \)

\[ : E_a + E_b + E_c = 3 E_{a0} \]

\[ E_{a0} = \frac{1}{3} (E_a + E_b + E_c) \quad \text{(3.12)} \]

Positive-sequence Component: Multiplying the 

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equation (3.10) by \( a \) and (3.11) by \( a^2 \) and adding these to (3.9) we have:

\[
E_a + aE_b + a^2E_c = (1 + a + a^2)E_0 + 3E_a + (1 + a^2 + a)E_a
\]

Again \( 1 + a + a^2 = 0 \)

\[
E_a = \frac{1}{3} (E_a + aE_b + a^2E_c)
\]

3.13

**Negative sequence Component:** Multiplying (3.10) by \( a^2 \) and (3.11) by \( a \) and adding these to (3.9) we have:

\[
E_a + a^2E_b + aE_c = (1 + a^2 + a)E_0 + (1 + a^2 + a)E_a + 3E_a
\]

\[
E_a = \frac{1}{3} (E_a + a^2E_b + aE_c)
\]

3.14

**Graphical Method:** Zero sequence Component: This can be obtained by adding the vectors and trisecting this geometric sum as shown in fig 3.6

**Positive sequence Component:**

The positive sequence component \( E_a \) is found by adding the vector \( E_a \)

\[
E_a = E_b + E_c
\]

Fig 3.6

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to the vector $E_c$, rotated through $120^\circ$, and the vector $E_i$, rotated through $240^\circ$, and then bisecting the sum as shown in Fig. 3.7.

**Negative sequence component**

The negative sequence component $E_{a2}$ is found by bisecting the sum of the vector $E_a$, the vector $E_i$, turned through $240^\circ$, and the vector $E_c$ rotated through $120^\circ$ as shown in Fig. 3.8.

Obviously, these graphical constructions are based upon the equations (3.12), (3.13) and (3.16). The analytical method of resolution of the vectors into their symmetrical components is more accurate than the graphical method.

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3.10 STAR-DELTA TRANSFORMATIONS.

In the case of a mesh-connected system, the calculations are simpler, than for star-connected system, as the voltage across each phase of the load and the phase currents can then be calculated independently of one another. For this reason, it is often desirable to reduce an unbalanced star-connected system to the equivalent mesh-connected system, or alternatively, a mesh-connected system can be converted to the equivalent star. The results established in this connection are particularly due to Wagner and Evans.

Current Relations: Let \( I_a, I_b, \) and \( I_c \)

and \( I_a, I_b, \) and \( I_c \) be the currents as shown in fig 3.9 (p. 27) for a delta-connected load.

Then, vectorially we have the following relations, as also given in the vector diagrams.
\[ I_{A0} = \text{indeterminate} \]
\[ I_{A1} = j \frac{I_a}{\sqrt{3}} \]
\[ I_{A2} = -j \frac{I_a}{\sqrt{3}} \]
\[ I_0 = 0 \]
\[ I_A = j \sqrt{3} I_a \]
\[ I_A^* = +j \sqrt{3} I_a \]

\[ E_{A0} = 0 \]
\[ E_{A1} = j \sqrt{3} E_a \]
\[ E_{A2} = -j \sqrt{3} E_a \]
\[ E_0 = \text{indeterminate} \]
\[ E_A = -j \frac{E_A}{\sqrt{3}} \]
\[ E_A^* = +j \frac{E_A}{\sqrt{3}} \]

\[ Y - \Delta \text{ Transformation for currents} \]
\[ \Delta - Y \text{ Transformation for voltages} \]
The zero-sequence component of the star currents $I_{0}$ is given by equation (3.12):

$$I_{0} = \frac{1}{2} (I_{a} + I_{b} + I_{c})$$

$$= \frac{1}{2} [(I_{a} + I_{c} + I_{A}) - (I_{c} + I_{A} + I_{b})] = 0 - 3.16$$

This shows that the zero-sequence current of a polyphase circuit feeding into a delta connection is always zero. The current of zero-sequence may circulate within the delta without getting out into the line. The converse of determining $I_{0}$ in terms of $I_{0}$ is indeterminate.

For positive-sequence component, by equation 3.13

$$I_{a} = \frac{1}{3} (I_{a} + a I_{b} + a^{2} I_{c})$$

$$= \frac{1}{3} [(I_{a} + a I_{c} + a^{2} I_{b}) - (I_{c} + a I_{A} + a^{2} I_{B})]$$

$$= \frac{1}{3} [(a^{2} I_{A} + I_{B} + a I_{c}) - (a I_{A} + a^{2} I_{b} + I_{c})]$$
\[ i_a = \frac{a^2}{b} \left( i_A + a^2 i_A + a^2 i_C \right) - \frac{a^2}{b} \left( i_A + a i_A + a^2 i_C \right) \]
\[ = \frac{1}{3} \left( i_A + a i_A + a^2 i_C \right) (a^2 - a) \]
\[ = (a^2 - a) i_A, \quad i_A = -j \sqrt{3} i_A, \]

Q. \[ i_A = \frac{j}{\sqrt{3}} i_a \]  

For negative sequence component, similarly \[ i_A = -j \sqrt{3} i_a \]  

These results are shown in Fig. 3.9

Voltage Relations: Let \( E_A, E_B \) and \( E_C \) (Eq. 3.16) be the delta voltages, and \( E_a, E_b \) and \( E_c \) the star voltages. By definition, therefore:

\[
\begin{align*}
E_A &= E_c - E_b \\
E_B &= E_a - E_c \\
E_C &= E_b - E_a
\end{align*}
\]

Since delta voltages form a closed triangle, therefore, \( E_A, E_B \) and \( E_C \) must always be zero and \( E_{BA} = 0 \).

The delta voltages, therefore, can never contain a zero sequence component. The star voltages, on
The other hand, may contain a zero-sequence component. As before, it follows that $E_{ca}$ cannot be found out from $E_{bc}$.

The relations between star- and delta-voltages of positive- and negative-sequence voltages are shown in fig. 3.10.

### 3.11 Unbalanced Impedances

Unbalanced impedances

\[
Z_{0} = \frac{1}{3} \left( Z_{a} + Z_{b} + Z_{c} \right) \\
Z_{a} = \frac{1}{3} \left( Z_{a} + a^2 Z_{c} + a Z_{b} \right) \\
Z_{c} = \frac{1}{3} \left( Z_{a} + a^2 Z_{b} + a Z_{c} \right)
\]

Fig. 3.11

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The sequence components of currents through the impedances, and the sequence components of the line voltages impressed across them are interrelated by the following equations:

\[
\begin{align*}
E_0 &= \frac{1}{3} \left( E_a + E_b + E_c \right) = Z_0 (I_0 + I_1 Z_1 + I_2 Z_2) \\
E_1 &= \frac{1}{3} \left( E_a + \omega E_b + \omega^2 E_c \right) = Z_1 (I_0 + I_1 Z_0 + I_2 Z_2) \\
E_2 &= \frac{1}{3} \left( E_a + \omega^2 E_b + \omega E_c \right) = Z_2 (I_0 + I_1 Z_1 + I_2 Z_2)
\end{align*}
\]

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4.

POWER IN

UNBALANCED SYSTEMS.
It is evident that no power can be associated with either the positive or negative sequence components of voltages and the zero sequence currents (or vice versa), since the sum of the components in either a positive or negative sequence is zero, and all the components of the zero sequence are equal. Again, no resultant power is associated with voltages and currents of opposite sequence. In fact, if
\[ e_1 = \sqrt{2} E \cos \omega t \]
\[ e_2 = \sqrt{2} E \cos (\omega t - \phi) \]
\[ e_3 = \sqrt{2} E \cos (\omega t - \frac{2\pi}{3}) \]
\[ e_4 = \sqrt{2} E \cos (\omega t - \frac{4\pi}{3}) \]

Then we have:
\[
e_1 i_1 + e_2 i_2 + e_3 i_3 + e_4 i_4 = \frac{E I}{2} \left[ \cos (2\omega t - \phi) + \cos \phi + \cos (2\omega t - \phi) + \cos \phi \right]
\]
\[
+ \cos (\phi - \frac{2\pi}{3}) + \cos (2\omega t - \phi) + \cos (\phi - \frac{2\pi}{3}) \]
\[
= 3 EI \cos (2\omega t - \phi) = 4.1 \]

Thus there is a considerable oscillating power, but no
resultant. It therefore follows that the total average power can be considered as being due to the sum of the powers in the positive, negative, and zero phase sequences.

4.1 True Power and Apparent Power.

Writing the unsymmetrical voltages and currents in terms of their symmetrical components, we have the expressions for the apparent power in volts x amps. for the three phases as:

\[
(E_a + E_q + E_r)(I_a + I_q + I_r); \\
(E_a + a^2E_q + aE_r)(I_a + aI_q + a^2I_r); \\
(E_a + aE_q + qE_r)(I_a + a^2I_q + aI_r).
\]

The results of multiplication are:

\[
\begin{align*}
E_a I_a + E_a I_q + E_a I_r + E_q I_a + E_q I_q + E_q I_r + E_r I_a + E_r I_q + E_r I_r; \\
E_a I_a + aE_a I_q + a^2E_a I_r + a^2E_q I_a + aE_q I_q + a^2E_q I_r + aE_r I_a + a^2E_r I_q + aE_r I_r; \\
E_a I_a + aE_a I_q + a^2E_a I_r + a^2E_q I_a + aE_q I_q + a^2E_q I_r + a^2E_r I_a + a^2E_r I_q + a^2E_r I_r; \\
= 3E_a I_a + 3E_q I_q + 3E_r I_r.
\end{align*}
\]

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Thus, by adding these the total apparent power is obtained. As is obvious from the fundamental equations, all the terms containing expressions of different systems (i.e., except those which are marked `v`) will disappear in summing up.

Thus, \( VA = 3 (E_o I_o + E_i I_i + E_\delta I_\delta) \) - 4.2

If the apparent power of the zero-, positive-, and negative-systems are, say, \( VA_0, VA, \) and \( VA_\delta \), then equation (4.2) may be written as

\[ VA = 3 (VA_0 + VA_1 + VA_\delta) \]

i.e., the total apparent power is three times the sum of the apparent power of the zero-, positive-, and negative-systems.

The apparent power may be resolved into its real and imaginary parts i.e., true and reactive power as:

\[
\begin{align*}
P_0 &= 3 E_o I_o \cos \phi_0 \\
P_1 &= 3 E_i I_1 \cos \phi \quad \text{and} \quad Q_1 = 3 E_i I_1 \sin \phi \\
P_\delta &= 3 E_\delta I_\delta \cos \phi_\delta \\
Q_\delta &= 3 E_\delta I_\delta \sin \phi_\delta
\end{align*}
\]
In these expressions \( \phi_1 \), \( \phi \), and \( \phi_e \) represent the respective phase angles between the voltages and currents of the three systems, and the latter are expressed in effective values. It should, however, be remembered that the expressions for power are mean values.

Summing up, therefore, we may say that:

\[
\begin{align*}
P_T &= P_a + P_b + P_c \\
Q_T &= Q_a + Q_b + Q_c \\
\Rightarrow \quad P_T + jQ_T &= (E_a I_a + E_b I_b + E_c I_c)
\end{align*}
\]

\[
= 3[(P_a + P_b + P_c) + (Q_a + Q_b + Q_c)]
\]

As shown in Fig. 4.1.

4.2. Power Factor.

It has been shown by Savarage that his definition of power factor for unsymmetrical poly-phase systems may lead to certain inconveniences, as is the case with non-sinusoidal voltages and currents. In his investigations, Savarage carried out his experiments...
with a large slow speed induction motor, the coils of which were unsymmetrical in the three phases. In trying to verify the power factor guaranteed by the manufacturer, different values were obtained according to the measurements made in the different phases. At that time Stalvits proposed to resolve the system into its symmetrical components and to investigate the power factor of the symmetrical system. In a discussion about the definitions of A.C. quantities before the A.I.E.E. Stepien referred to a similar case in connection with a synchronous machine. The manufacturer had guaranteed a unity p.f. of a synchronous converter. The connecting leads between the transformers and the slip rings were laid unsymmetrical by the customer and the fulfilling of guarantee would have required an excitation which the machine was unable to provide.
In connection with these discussions in the American Standards the following value of p.f. was fixed:

\[ \text{p.f.} = \frac{P}{\sqrt{P^2 + Q^2}} \]

where \( P \): True power = \( P_a + P_e + P_c \) and
\[ Q \]: Reactive power = \( Q_a + Q_e + Q_c \)

The same definition was recommended by Sekering.

If the asymmetries are comparatively small the power factor of the positive sequence system is practically equal to the total power factor. With large asymmetries, however, this is not the case. If the unsymmetrical voltages and currents are known in magnitude and phase displacement, the power factor of the positive, negative, and zero - sequence systems may be easily computed. These results may be checked by graphical methods.

Note: For the definition of p.f. \( VA \) should not be used instead of reactive power, which would arise confusion.

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4.3: UNBALANCE FACTOR.

Unbalance factor is used to designate the amount of asymmetry present in an unbalanced system. It is defined as the ratio of the negative to positive sequence component of voltage or current in a system with zero resultant. The German Electrical Standards consider a poly-phase system to be symmetrical if the unbalance factor is not greater than 0.05 i.e. if the amplitude of the negative sequence system amounts to not more than 5% of the amplitude of the positive.

Charts for determining the positive and negative sequence components, or the unbalance factor, if the unsymmetrical voltages or currents are given, have been developed by Dubose and also by Hanff Wagner and Evans give similar charts generalized in so far as the curves given by Dubose are developed.
Determination of imbalance factor.

Fig 4.2

For low values of the imbalance factor also, while those given by Haufe are extended to higher values, one given by Wagner and Evans is reproduced in Fig. 4.2. In this, the ratio \( \frac{E_b}{E_a} \) and \( \frac{E_c}{E_a} \) are used as coordinates and the point thus determined gives the imbalance factor by interpolation of a family of
curves. The angle plotted in the diagram gives the angle by which the vector $E_1$ leads the vector $E_2$.

4.4 PULSATING POWER AND UNBALANCE FACTOR OF POWER.

The "pulsating power" $P^*$ is given for a system without zero conductor by the formula

$$P^* = 2 \sqrt[3]{(E_1^2)^2 + (E_2^2)^2 + 2E_1E_2 \cos(\theta_2 - \theta_1)}$$

and the ratio $P^* / VA$ is known as the unbalance factor of power, where $VA$ is the volt-amper of the system.

4.5 FLOW OF POWER DUE TO UNBALANCE

In any symmetrical system with symmetrical loads only the positive-sequence power $(P + j Q_1)$ can flow. But in case of unbalanced load negative- and zero-sequence component power may also be produced. A symmetrical machine can produce only positive-sequence voltages. Therefore, in a sense positive-sequence components of power are supplied by the

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generator, and the asymmetry converts part to negative and zero sequence components of power, which are fed into the system at the point of fault.

Consider an unbalanced load from line-to-neutral on a system supplied through a grounded generator with positive, negative, and zero sequence impedances of $Z_1$, $Z_2$, and $Z_0$ respectively, and with a generated positive sequence line-to-neutral voltage of $E_1$. Assume that the system is supplying a load on phase 'a' whose impedance is $Z_L$ as shown in Fig. 4.3. For sequence voltages, we have, therefore:

\[
E_1 = (Z_1 + Z_0 + 3Z_L) \frac{E_g}{Z_L} \tag{4.1}
\]
\[
E_2 = -Z_2 \frac{E_g}{Z_L}
\]
\[
E_0 = -Z_0 \frac{E_g}{Z_L}
\]
\[
Z_1 = Z_2 = Z_0 = \frac{E_g}{Z_L}
\]
where \( Z_t = Z_1 + Z_2 + Z_0 + 3Z_L \).

The total phase values at point of load may be obtained from the previous equations, so that:

\[
E_a = 3Z_L \frac{E_{ar{f}}}{Z_t} \quad \text{and} \quad I_a = \frac{3E_{ar{f}}}{Z_t}
\]

The sequence power per phase at the load are:

\[
P_1 + jQ_1 = (Z_1 + Z_0 + 3Z_L) \left( \frac{E_{ar{f}}}{Z_t} \right)^2 \quad \text{(4.4)}
\]

\[
P_2 + jQ_2 = -Z_2 \left( \frac{E_{ar{f}}}{Z_t} \right)^2 \quad \text{(4.5)}
\]

\[
P_0 + jQ_0 = -Z_0 \left( \frac{E_{ar{f}}}{Z_t} \right)^2 \quad \text{(4.6)}
\]

From equation (4.3) total power

\[
P_T + jQ_T = 3[(P_1 + jQ_1) + (P_2 + jQ_2) + (P_0 + jQ_0)] = 9Z \left( \frac{E_{ar{f}}}{Z_t} \right)^2
\]

The significance of negative sign of the equations (4.5) and (4.6) is that flow of \((P_2 + jQ_2)\) and \((P_0 + jQ_0)\) is in a direction opposite to that of \((P_1 + jQ_1)\). Thus an unbalanced load not only draws from a symmetrical, the total power required for the

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load, but also the negative and zero sequence components of the power, which are fed back into the system at the point of unbalance and which are consumed in producing negative and zero sequence losses, and in absorbing inductive reactance voltages.

4.6 COMBINATION OF UNBALANCED LOAD AND SYMMETRICAL MACHINE LOAD.

In the combination of an unbalanced load and a balanced load due to a rotating machinery, the unbalanced load produces negative sequence voltage at the point of unbalance, and the negative sequence voltage impressed on the system causes negative sequence current flow not only through the source but also in all the short branches including the symmetrical rotating machinery load. The power due to the flow of negative sequence...
currents produces losses in the rotating machinery, and increases the total amount of power absorbed by the symmetrical load. Rotating machinery on a system tends to balance the load on a system and to restore the voltages to their normal values. This action takes place with the flow of negative-sequence currents in the rotating machinery, and with an increased amount of loss and a reduction in the maximum load capacity of the apparatus.

Under some conditions the negative-sequence current flowing through machines will produce excessive heating in the rotor so that unbalanced operation may require special considerations.

In a number of respects, as suggested by Fourier, positive-sequence components of power represent more accurately than the total power,
The requirements of power supply systems for properly handling the unbalanced loads.

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DEPARTMENT OF ELECTRICAL ENGINEERING
5.

THE

SYMМETRICAL COMPONENT METER.
S.1. GENERAL REMARKS.

It has already been shown that it is useful in investigating unsymmetrical three-phase systems to resolve into their symmetrical components, since these afford a clear insight into the kind and size of the asymmetry. It has also been shown that the symmetrical components can be found by graphical and analytical methods if the unsymmetrical quantities are known. In this section is considered the practical methods for the measurements of symmetrical components and their relative phase angles.

The sequence components may be measured by means of instruments or meters, operating on well-known principles, alongside specially designed external windings, or special networks in some cases. In certain cases, it may be sufficient to get only

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an indication whether or not an asymmetry exists, in other cases it will be required actually to measure the value of the components, and finally, there is the problem of using such measurements for the operation of circuit breakers or the like by means of a relay in order to prevent, or render harmless the effects of asymmetry.

Sequence quantities that can be measured include for each sequence the same quantities that can be measured in a single phase circuit viz., voltage, current, kW, kVAR, power factor, VA, etc. In addition the sequence quantities may be combined in various ways for measurements.

The first proposal for measuring the sequence quantities was the result of a discussion between Toresean, Chubb and Slippman. They proposed the use of rotating machines to eliminate the

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sequence to permit the measurement of another sequence quantity. The first measurements, however, were actually carried out by Evans who suggested static networks to segregate a desired sequence quantity in the presence of other sequence quantities. Eventually, various types of networks have been worked out by Evans, Allcutt, Fortescue, Russell and others.

In a three-phase four-wire system there may exist three sequence voltages. Consequently, in order to measure all of these sequence components, it is necessary to have available positive, negative, and zero-sequence voltmeters. In a three-phase three-wire system, the zero-sequence voltage is indeterminate, and the corresponding voltmeter is unnecessary and inapplicable. Similar relations hold for the sequence currents. The sequence meters may then be classified as the positive, negative (which are similar), and zero-sequence meters.
5.1 THE SYMMETRICAL COMPONENT METER.

Introduction: The unbalanced three-phase systems may or may not contain the zero-sequence component. For instance, the three-phase three-wire system does not contain the zero-sequence component under normal circumstances.

In our symmetrical component meter, the zero-sequence component has been assumed to be absent or eliminated, while measuring the positive and negative-sequence components. This instrument is based upon the discussions by R. L. Russell (Proc. I.E.E. Feb '50). A number of circuits have been devised for measuring the magnitude of the positive- and negative-sequence components of an unbalanced system. In our use, the particular network suggested by Russell has been selected because of its simplicity and because it enables the measurement of both...
relative phase displacements. As will be seen later, the only equipments required for the complete set-up are a few resistances and capacitances, a dynamometer-type instrument (preferably a voltmeter), two double-throw switches, a 2-pole switch and isolating transformers.

The principles of measurements are equally applicable to either phase or line values of voltage. But a phase-voltage system has been chosen because of the ease with which imbalanced voltages to neutral, containing no zero-sequence component, can be synthesized for test and calibration purposes.

Theory: Let \( E_a \) and \( E_b \) be the positive-and negative-sequence components. From equations 3.9 and 3.10 (p. 23) we have:

\[
E_a = E_{a0} + E_{a1} + E_{a2}
\]

\[
E_b = E_{b0} + a^2 E_{a1} + a E_{a2}
\]

In absence of zero-sequence component, i.e., when \( E_{a0} = 0 \)
we have

$$E_a = E_4 + E_1$$

$$E_6 = a^2 E_4 + a E_1$$

Multiplying (5.2) by $a^2$ we have:

$$a^2 E_6 = a E_4 + E_1$$

Subtracting (5.3) from (5.1), therefore:

$$E_a - a^2 E_6 = E_4 (1 - a) = E_3 = \left(\frac{E_4 - a^2 E_6}{1 - a}\right)$$

Multiplying the numerator and denominator by $(1 - a)$ and referring to Table 3.1 (p. 103):

$$E_4 = \frac{(1 - a^2) (E_4 - a^2 E_6)}{(1 - a^2)(1 - a)} = \frac{(1 - a)(E_4 - a^2 E_6)}{3}$$

$$E_4 = \left(\frac{1 - a^2}{\sqrt{3}}\right) \left(\frac{E_4 - a^2 E_6}{\sqrt{3}}\right)$$

Similarly, for negative sequence component, we have, by multiplying (5.2) by $a^2$ and subtracting from (5.1), and finally multiplying the numerator and denominator by $(1 - a)$:

$$E_{a} = \frac{(1 - a)(E_4 - a^2 E_6)}{(1 - a)(1 - a^2)} = \frac{(1 - a)(E_4 - a^2 E_6)}{3}$$

$$E_{a} = \left(\frac{1 - a}{\sqrt{3}}\right) \left(\frac{E_4 - a^2 E_6}{\sqrt{3}}\right)$$

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which means that in the absence of zero-sequence component, the positive-sequence component of a particular vector of an unbalanced delta system is given correctly in magnitude and phase by adding to it the succeeding vector advanced in phase by 60°, and advancing the resultant reduced in the ratio 1:1.53 a further 30°. A corresponding procedure, as given by equation (5.5), also holds for the negative-sequence component.

The instrument designed is primarily based upon equations (5.4) and (5.5), which clearly shows that for an instrument to respond to positive- or negative-sequence component separately the essential operation is the 60° phase shift. Based upon this, circuits can be designed to give a phase shift of 60° by using either inductance or capacitance. Capacitance has been chosen for on
purpose due to convenience, and the basic set-up is
shown in Fig. 5.1. In the absence of a capacitive branch the meter
responds to \( E_a \) while in the absence of the resistance \( R \), the
meter reading is proportional to
\( E_b \). Now applying Kirchhoff's laws to the two branches
we have:

\[
\begin{align*}
E_a &= L_2 I_2 + L_3 I_3 \\
E_b &= I_1 Z_a + I_2 Z_2
\end{align*}
\]

\( Z_a \) and \( Z_2 \) are so chosen that they are equal in
magnitude but differ in phase by \( 60^\circ \) so \( Z_2 = -\frac{Z_a}{3} \).
Eliminating \( I_2 \) from (5.6) and comparing the results
with (5.4), the positive-sequence component can be expressed
as:

\[
E_a = \left[ \frac{R_2 + 2 (1-a^2)}{\sqrt{3}} \right] I_1, \left( \frac{1-a^2}{\sqrt{3}} \right)
\]

Interchanging the reactive and capacitive branches with (5.5),

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The corresponding equation for the negative sequence component is:

\[ E_a = \left[ \frac{r + z(1-z^2)}{\sqrt{3}} \right] I_m \left( -\frac{a^2}{\sqrt{3}} \right) \quad (S.8) \]

We have therefore:

\[ E_a = I_m \left[ \frac{r + z(1-z^2)}{\sqrt{3}} \right] \left( -\frac{a^2}{\sqrt{3}} \right) \quad (S.9) \]

\[ E_a = I_m \left[ \frac{r + z(1-z^2)}{\sqrt{3}} \right] \left( \frac{2(a-1)}{\sqrt{3}} \right) \]

The numerical factor \( \left[ \frac{r + z(1-z^2)}{\sqrt{3}} \right] \), which is absorbed as a scale factor, is the same in both cases. The operator terms \( \left( -\frac{a^2}{\sqrt{3}} \right) \) and \( \frac{2(a-1)}{\sqrt{3}} \) show that the two currents \( I_m \) and \( I_m' \) differ in phase by 120° in addition to the phase difference that exists between \( E_a \) and \( E_a' \). (S.8) can be rewritten as:

\[ aE_a = E_a' = I_m \left[ \frac{r + z(1-z^2)}{\sqrt{3}} \right] \left( \frac{a^2}{\sqrt{3}} \right) \quad (S.10) \]

The result of interchanging the branches is therefore to measure \( E_a' \) and not the negative sequence component corresponding to the reference vector \( E_a \).
To develop the circuit further, for the measurement of phase angles, it is proposed to have similar networks of the type discussed, one for the positive sequence component and the other for the negative sequence component. The fixed and moving coils, respectively, of a dynamometer instrument are connected in the positions normally occupied by the meters. With given currents in the two coils, the deflections will be proportional to the cosine of their phase difference in the usual way. To convert indication the currents should correspond to the same reference vector.

When connected in the manner shown, the instrument will respond to \( E_a \), \( E_b \), \( E_c \), and \( E_{ab} \) of the networks in turn. \( E_a \) and \( E_{ab} \) are

![Diagram](image_url)

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can be found separately, and hence the phase difference can be found out by calculation.

For readability and convenience a nomogram can be constructed for this purpose.

In the completed instrument, the junction S shown in Figs. 5.1 and 5.2 is internally connected to the secondary input of an isolating voltage transformer. The primaries of the transformer are connected to the external source.

In order that the currents shall be related to the voltages measured by the same factor in all cases, it is essential that the resistance in the meter branch in each network should be the same for different settings of the selector switch. Compensating resistances are, therefore, introduced when measuring phase angles, making the resistance of each meter coil in the instrument equal to the total meter resistance when the coils are in series.
Sign of the Phase Angle: The method of measuring phase differences makes no distinction between positive and negative phase angles. Without this information, therefore, two unbalanced systems can be constructed from the test results. In order to select the correct one, it is sufficient to observe that, if the negative-sequence component of the reference vector \( E_a \) lags behind the positive-sequence component (i.e., for negative angles), \( E_6 \) is less than \( E_0 \), whereas for leading or positive angles \( E_6 \) is the larger of the two as has been verified below.

Referring the angular position of a negative-sequence component to the corresponding positive-sequence component, with the usual sign convention for positive rotations, we have in Fig. 5.3, the reference vectors \( E_a \) and \( E_0 \), displaced by an angle \( \theta \) where \( \theta \) is negative.
The magnitudes of unbalanced voltages $E_1$ and $E_2$ are then given by:

$$E_1 = E_1^2 + E_2^2 - 2E_1E_2 \cos (60^\circ + \phi)$$

$$E_2 = E_1^2 + E_2^2 - 2E_1E_2 \cos (60^\circ - \phi)$$

$E_1 + E_2$ being the magnitudes of the positive and negative sequence components respectively.

The sign of the difference of the expression of $E_1^2$ and $E_2^2$ depends only on the sign of $\phi$.

$$E_1^2 - E_2^2 = 2E_1E_2 (\cos (60^\circ - \phi) - \cos (60^\circ + \phi))$$

$$= 2E_1E_2 (\cos (60^\circ - \phi) + 2 \sin \phi \sin \phi - \cos (60^\circ + \phi))$$

$$= 2E_1E_2 (2 \sin \phi - \phi) = 2E_1E_2 \cdot 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$\therefore (E_1^2 - E_2^2) = 2\sqrt{3} E_1E_2 \cos \phi \quad 5.11$$

Thus for:

- $0 < \phi < -180^\circ$, $E_1 < E_2$
- $0 < \phi < +180^\circ$, $E_1 > E_2$
- $\phi = 0$ or $180^\circ$, $E_1 = E_2$

It is possible, therefore, by measuring $E_1$ and $E_2$ separately, to deduce the sign of $\phi$ from the results (5.12)

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Instead of using separate measurements for comparing $E_b$ and $E_c$, the dynamometer instrument itself may be employed. For this purpose, the switches $P$ and $Q$ (Fig. 5.7) have been provided, both the circuit broken at these points the readings will be proportional to $E_b$ or $E_c$ in the positive and negative sequence settings respectively of the select switch.

The Meter Current: The range of the instrument is evidently, limited by the meter current. Ordinarily, in the sequence measuring networks, the meter current is given approximately by

$$I_m = \frac{\sqrt{3} E_b}{R_1} \quad \text{or} \quad I_m = \frac{\sqrt{3} E_c}{R_2} \quad \text{5.13}$$

whereas, when measuring $E_b$ or $E_c$, the largest value which may be encountered, that is when $\theta = 120^\circ$, is

$$I = \frac{E_1 + E_2}{R_1} \quad \text{or} \quad \frac{E_1 + E_2}{R_2} \quad \text{5.11}$$

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Scale: The dynamoscope instrument used in the normal way, with the coils in series, responds to the square of the voltage, or current (the impedance of the circuit being constant), so that the scale for the measurement of the sequence voltages is of the square-law type. Used for phase measurements, it responds directly to the quantity being measured, so that the scale and naturally be a linear one in this case.

If \( r_1 \) is the positive-sequence component, \( r_2 \) the negative-sequence component, and \( r_{12} \) the reading on the same scale for the phase-measuring setting, then the phase angle \( \phi \) is given by the expression:

\[
\cos \phi = \frac{r_{12}}{r_1 r_2} \quad 5.15
\]

This equation shows that \( \cos \phi \) can be computed in terms of meter readings on any arbitrary scale and the right form of \( \phi \) is independent of the particular scale.
5.3 SETTING UP THE CIRCUIT.

Kits required: The essential equipments required for the symmetrical component method are:

1. A dynamometer instrument (a helix wattmeter)

2. Resistances: 9300 Ω (2) (R, L, B); 4650 Ω (2) (Y, ø, B)
   2520 Ω (1) (x)*; 1115 Ω (1) (y)*

3. Capacitances: 0.3950 µF (2) (G + Q)*

4. Isolating transformers: 3 1-φ 50V (a, b + c)*
   Connected in Δ/Y voltage 220/185 (L-L)

5. Double throw switches: 2 (L + M)*

6. A 2-pole switch or 2 1-pole switches (P and Q)*

7. Connecting wires (flexible) about 6 ft.

8. Terminal strips: 3

9. Wood panels, screws etc.

* These symbols refer to Fig 5.7 (p.70)

* Calculations of these values are given on following page p. 64.

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Calculation of suitable circuit elements:

Referring to equation (5.13) (6.61), taking the maximum allowable meter current as 23.5 mA, the resistance $R_1$ (Fig. 5.4) can be taken as 9.300 ohms for a voltage of 220 V (line-to-line), as calculated below:

\[ I_m = \frac{\sqrt{3} E_0}{3 I_m} \quad \Rightarrow \quad R = \frac{3 E_0}{I_m} \]

\[ R_1 = \frac{220 \times 10^3}{23.5} = 9.300 \text{ ohms} \]

\[ R_1 = R_2 = 9.300 \text{ ohms} \]

From the triangle (Fig. 5.4), since the phase shift is 60°, $R_1 = \frac{1}{2} R_4$ and $R_2 = \frac{1}{2} R_4$

\[ R_1 = R_2 = \frac{1}{2} \times 9.300 = 4.650 \text{ ohms} \]

Again:

\[ \frac{1}{I_m} = \sqrt{3} \quad \Rightarrow \quad I_m = \sqrt{3} \times 4.650 \]

At a frequency of 50 Hertz:

\[ C = \frac{1}{100 \pi \times \sqrt{3} \times 4.650} = 0.3950 \text{ pF} \]

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\[ C_1 = C_2 = 0.395 \text{ \mu F} \]

The respective resistances of the current and pressure cells were found to be:

\[ x = 2520 \text{ ohm} \]
\[ y = 111.5 \text{ ohm} \]

Hence, the values chosen were:

\[ R_1 = R_2 = 9300 \text{ ohm} \]
\[ r_1 = r_2 = 4650 \text{ ohm} \]
\[ C_1 = C_2 = 0.395 \text{ \mu F} \]
\[ x = 2620 \text{ ohm} \text{ and } y = 111.5 \text{ ohm} \]

Measurements of the values of circuit elements:

Resistances: The resistances were all measured by a sensitive wheatstone bridge using a ballistic galvanometer.

The measured values were:

\[ R_1 = 9325 \text{ ohm, Error} = 0.268\% \]
\[ R_2 = 9319 \text{ ohm, Error} = 0.204\% \]
\[ x = 2518 \text{ ohm, Error} = 0.008\% \]
\[ y = 111.4 \text{ ohm, Error} = 0.089\% \]

These errors are allowable as better values could not be obtained.

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\[
\begin{align*}
x_1 &= 4645 \text{ ohm} \quad \text{Error: 0.107}\% \\
x_2 &= 4654 \text{ ohm} \quad \text{Error: 0.107}\%
\end{align*}
\]

Capacitance: The capacitances \( C_1 \) and \( C_2 \) were measured by C.R. Test capacitance bridge which was found to be the most suitable for the purpose. First \( C_1 \) was measured, and was found to be 0.3950 \( \mu \)F. The bridge was left undisturbed and \( C_2 \) was taken out. \( C_2 \) was then connected in place of \( C_1 \) and its value was so adjusted that the balance was again obtained. This meant that \( C_2 = C_1 = 0.3950 \mu \)F.

The measured values of capacitances were:

\[
C_1 = C_2 = 0.3950 \mu \text{F}
\]

Note: Actually the capacitances were chosen (0.3950 \( \mu \)F) first and the resistances were then calculated from it (Eq. 54), as it was easy to adjust the resistances to the desired value.

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The Dynamometer Instrument: The instrument that was used was a \textit{Welsh} dynamometer type wattmeter (rated up to 1000 watts). The wattmeter had to be used due to the unavailability of the dynamometer wattmeter. The fixed coil of the instrument (which had very few turns of thick wire) was changed, and instead two new coils, up to 8\textsection 6 wire, were wound on a former that was made of the same size as the original coils, so that the new wound coils just filled in the space provided in the instrument. However, after winding the two coils, they were properly insulated and fitted into the instrument. Care was taken in joining the two coils together so that they produced the lines of force (and therefore the torque) in the same direction. These new coils, obviously, had very large number of turns, so that the meter gave the...
full-scale deflection even with very low current (40mA).

After winding the coils and fixing them, the two leads of each of the pressure and current coils were brought out and soldered to the binding screws. The coils were then connected in series and the instrument was then tested as a voltmeter. The circuit that was used for testing is shown in Fig. 5.5. The varac was used to vary the applied voltage and shunting resistor was connected to limit the current in the pressure coil.

![Circuit Diagram]

V: VOLTMETER
mA: MILLI-AMMETER
CC: CURRNT COIL
P.C.: PRESS. COIL

Fig. 5.5

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Date ______________________
Isolating Transformers: Three old single-phase transformers were taken out from the tank and properly insulated and tested. Windings 3-4 of each were connected to form delta primary and 1-2 were connected to form Y secondary as shown in Fig 5.6. The secondary was connected to the meter circuit (Fig. 7). The voltage ratio as found by testing is indicated in Fig. 5.6. It is figure (220: 185V). Each line gave the same output.

Circuit Connections and General Set-up: The meter, 2-pole switch, double throw switches were all mounted on a panel as shown in the plan (Fig. 5.8). The prepared network was then connected as shown in Fig. 5.7, and the conventional phase sequence A, B, C marked on the appropriate terminals.

The double from two-pole switches L and M

* See also Appendix 2 p. 146.
SYMMETRICAL COMPONENT METER.

PLAN OF SYMMETRICAL-COMPONENT METER.

Performs by

Date
From the selector switch to sequence measurements, the instrument coils whose resistances are shown as $x$ and $y$ ohms, are drawn in lines in each network in turn. In phase measuring setting, one coil only appears in each of the networks and additional reactors are therefore introduced so that the value of the resistance in the meter branch in both networks is kept constant at (20) ohms.

All the reactors and capacitors used were clamped to the top inside the box (8 x 5 x 8).

All the connections were properly soldered and insulated to avoid short-circuiting.

5.4 Calibration of the Meter.

A variable three-phase balanced voltage supply was applied to the instrument in the proper phase sequence, and the meter reading was taken down for various voltages. Also, every time the voltage
Neutral was measured by a sensitive voltmeter. A calibration curve was thus obtained for the positive-sequence component. Identical results were obtained when the phase-sequence of the applied voltage was reversed i.e., for the negative-sequence component, as shown in the graph (p. 74) fig. 5.9.

In order to test the accuracy of the instrument, an unbalanced three-phase voltage system was applied and the negative, positive-sequence components and their phase angles were measured. Also, the applied voltages were measured by an ordinary voltmeter. The latter values were synchronized to give the sequence values. The unbalanced supply system was obtained by using three transformers placed with phase shifts.

* The method of measurements is given on next page.

Performed by _____________________________

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### 5.5 Procedure for Measurements

The instrument has been designed to work up to 220/230 volts 50 Hz supply. The switching operations for various measurements are as given below:

<table>
<thead>
<tr>
<th>Position of P x A.</th>
<th>L + M Left</th>
<th>L + M Right</th>
<th>L Left</th>
<th>M Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>P x A closed</td>
<td>Positive Sequence</td>
<td>Negative Sequence</td>
<td>E, E&lt;sub&gt;c&lt;/sub&gt; Conp</td>
<td></td>
</tr>
<tr>
<td>P x A open</td>
<td>E&lt;sup&gt;+&lt;/sup&gt;</td>
<td>E&lt;sup&gt;-&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Instrument readings for different switch settings.

The usual procedure for the measurement of sequence components are as follows:

1. Adjust the zero error of the instrument, after placing it on a plane horizontal surface.
2. Connect the terminals a, b, c to the circuit under test. (S is to be connected to grounded neutral if the transformers are not being used.)

*Note: The meter, for this setting, responds to E<sub>c</sub> and E<sub>c</sub>, but the same calibration (as for the sequence components) does not hold well.*

---

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(1) Set the selector switch (L.S.S.) to read the positive and negative sequence components in turn. (See Table 1.75) Interchange any two leads, if required, to obtain larger readings with the former setting. Note the scale readings \( r \), \( r' \), and \( r'' \). The order \( a, b, c \) and \( c \) indicates the phase sequence of the system under test.

(4) Set the selector switch to the phase measuring position, reversing one meter coil if necessary, to give a positive deflection. Note the scale reading \( r_1 \).

(5) Transform \( r_1 \) and \( r_2 \) to sequence voltages by the calibration curve (Table 1.73). Compute \( \Phi \) from the expression

\[
\cos \Phi = \frac{r_1}{r_2}
\]

If the coil was reversed, subtract the values obtained from 180° to give the true value of \( \Phi \).

(6) Open \( P \) and \( Q \) in the positive and negative sequence settings respectively of the selector switch in turn. If the former deflection is larger, \( \Phi \) is positive.
If smaller \( \Phi \) is negative (see p. 59).

It is an important fact that, for practical purposes, the meter cannot be damaged by operating the switches incorrectly, and no special precautions are necessary in this connection.

Note: While taking the readings, try to keep the glass on the meter very clean so that the moving part of the meter does not remain sticking.

5.6 MEASUREMENTS OF SEQUENCE COMPONENTS.

Unbalanced voltages were obtained on the secondaries of three transformers (combined to make a three-phase bank) provided with tips. Three unbalanced voltages were measured by the symmetrical component meter, in terms of the positive and negative sequence components, and the results were verified by measuring the phase voltages by a
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Valtimeter and then girtsved to give the sequence components. Some of the results, their verification and percentage errors are tabulated below. 

<table>
<thead>
<tr>
<th>Position of</th>
<th>Position of</th>
<th>Scale</th>
<th>Positive</th>
<th>Negative</th>
<th>Phase</th>
<th>Sign of</th>
</tr>
</thead>
<tbody>
<tr>
<td>L as M</td>
<td>P and R</td>
<td>feet</td>
<td>k. 700</td>
<td>100 volts</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Both left</td>
<td>Open</td>
<td>340</td>
<td>460</td>
<td>38 volts</td>
<td>34.5°</td>
<td>+ve</td>
</tr>
<tr>
<td>Both right</td>
<td>Open</td>
<td>200</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L left right</td>
<td></td>
<td></td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Therefore, positive sequence component = 100 volts Error 1.5%

negative sequence component = 38 volts Error 1.5%

Phase angle = 34.5°

Verification (as calculated from measured phase voltages by ordinary multimeter). The measured phase voltages were \( E_a = 130 \text{ volts} \), \( E_b = 6.8 \text{ volts} \), \( E_c = 114 \text{ volts} \).

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Verification of Results

Phase angle \( \phi = 34.5^\circ \)

Positive sequence component: 260 volts

Negative sequence component: 30 volts

Measured Phase Voltages

\[ E_a = 130 \text{ volts} \]
\[ E_b = 168 \text{ volts} \]
\[ E_c = 114 \text{ volts} \]

Calculated Phase Voltages

\[ E_a = 134 \text{ volts} \]
\[ E_b = 165 \text{ volts} \]
\[ E_c = 115 \text{ volts} \]

Combination of sequence quantities to form phase quantities

In more results see Appendix 1, p. 139.

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3.7 ERRORS AND PRECAUTIONS.

Any deviation from the normal (specified) frequency will evidently result in an erroneous response of the meter for both the positive and negative sequence components.

By expressing the frequency as a fraction of the calibrated frequency and neglecting the resistance of the meter, the relation \( R_1 = -a^2 Z_0 \) what amounts to the same as \( Z_0 = -a R_1 = \frac{1}{2} \frac{b}{a} (1-jb) \) will no longer be true, and a modified form must be employed. Now

\[ Z_0 = \frac{1}{2} \frac{b}{a} (1-j \frac{\sqrt{3}}{3}) \]  

so that we have from (5.6), (5.5), and (5.16), by eliminating \( Z_0 \):

\[ E_0 + \frac{2E_1}{1-j \frac{\sqrt{3}}{3}} = P \left[ R + Z \left( 1 + \frac{2}{1-j \frac{\sqrt{3}}{3}} \right) \right] \]  

Equation (5.17), expressed in terms of sequence components.

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can be simplified by assuming that \( z \) is relatively small, and

impedance can be neglected in comparison with \( R \), and

by converting to the \( j \) notation, by putting \( z = j \frac{z}{2} \).

\[
E_a (1 + x) + E_a (1 - x) = \frac{E_a}{2} \left( \frac{R}{T} \right) (\frac{J}{2} + j x) \quad \text{(5.18)}
\]

The instrument reading will be interpreted

incorrectly as a positive sequence component \( E_a' \),

whereas it depends on both sequence components,

as shown by (5.18). The apparent value is obtained

from the equation (5.7):

\[
E_a' = \frac{E_a}{2} \left( \frac{R}{T} \right) (\frac{J}{2} + j x) \quad \text{(5.19)}
\]

It follows from (5.18) and (5.19) that \( E_a \) and

\( E_a' \) can be expressed in terms of \( E_a' \):

\[
E_a (1 + x) + E_a (1 - x) = E_a' \left[ 2 + (1 - x) x^2 \right] \quad \text{(5.20)}
\]

A similar expression for the apparent negative-sequence component is:

\[
E_a (1 - x) + E_a (1 + x) = E_a' \left[ 2 + (1 + x) x^2 \right] \quad \text{(5.21)}
\]

Solving (5.20) and (5.21), corrections can be made.

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The observed readings for observed values of $\alpha$.

The effect of frequency changes may be more clearly visualized by referring to the vector diagram (Fig. 5.11). Given $E_q$ and $E_i$ at an angle $\phi$, and for values of $\alpha$ less than unity, $OE$ and $OF$ are vectors proportional to $E_q'$ and $E_i'$ respectively. The errors $\theta_1$ and $\theta_2$, in this case, are negative, but for values of $\alpha$ larger than unity they would be positive. As $\phi$ varies, $SF$ will describe a circle with centre $S$ such that a maximum when $0 \leq E_0 \leq 90^\circ$ for values of $\alpha$ close to unity, and with $\theta_1$ measured in degrees, approximate values are obtained for the maximum error as:

\[
\theta_1 = -\left(\frac{180}{\pi}\right) \left(\frac{E_i}{E_q}\right) \left(\frac{1-\alpha}{1+\alpha}\right)
\]

\[
\theta_2 = -\left(\frac{180}{\pi}\right) \left(\frac{E_q}{E_i}\right) \left(\frac{1-\alpha}{1+\alpha}\right)
\]
These maximum values will not in general occur at the same time. The largest possible error \( \theta \) measured in degrees, will not therefore exceed the sum of the two:

\[
\theta = - \left( \frac{180}{\pi} \right) \left( \frac{E_i E_r}{E_i E_r} \right) \left( \frac{1-x}{1+x} \right) \]

5.22

The values of \( \theta \) in (5.22) will vary with the ratio \( E_i : E_r \) but will be greatest when \( E_i = E_r \), so that

\[
\theta^2 = - \left( \frac{360}{\pi} \right) \left( \frac{1-x}{1+x} \right) \]

5.23

A more convenient expression is obtained by putting \( \alpha^2 (1+\beta^2) \), where \( \beta \) is the percentage variation for the calibration frequency. The error in the measurement of phase angles, for small values of \( \theta \), say \(-5 \leq \beta \leq 5\), will not exceed the value given by (5.24)

\[
\theta = 0.6 \beta \]

5.24

The largest error occurs in this particular...
case when \( \theta = 90^\circ - \beta \), \( \frac{90^\circ - \Theta}{2} = \theta - 0.2\beta \). A general expression of this nature can not be readily obtained.

Other important sources of error in the meter response are the transient errors and the errors due to the presence of harmonics. Summing up, the three important errors are:

1. Frequency errors
2. Transient errors and
3. Errors due to harmonics.

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6.

APPLICATIONS

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At first consideration his theory of symmetrical components may appear to introduce unnecessary complication into the study of unbalanced three-phase currents and voltages. A single-phase earth fault is, according to this theory, to be replaced by no less than nine components, two sets of three cancelling and leaving no resultant single-phase currents flowing alone in a three-phase circuit; however, represent limiting currents. The general case of an unbalanced three-phase system of currents comprises three currents of the magnitude of two currents and their phase difference. Although according to the theory of symmetrical components, these three unbalanced currents are replaced by six components and completely specified by two magnitudes and a phase difference, since a symmetrical system is completely defined by one of its members. The
complication introduced by symmetrical component conception is
least only apparent. Similarly, if a three-phase
three-wire system becomes four-wire due to an
incidence of earth fault, the three line constants
are completely independent, and the system can
only be defined by three magnitudes and two phase
angles. The alternative definition in terms of
symmetrical components also involves three magnitudes
and two phase angles, so that in this case also
no real complication is introduced.

To enumerate a few, the following involve the
application of symmetrical components:

(i) Simple cases of faults
(ii) Automatic protection
(iii) Distribution networks
(iv) Induction motor, transformers and
(v) Synchronous machines and phase balancers.
6.1 Simple Cases of Faults

Fundamental Principles: The most important application of the symmetrical components is the calculation of faults in power systems. Two assumptions are made to solve such problems by this method:

1. It is assumed a symmetrical system of voltages and their influences of asymmetrical voltages are dealt with.

A further assumption is that the values of the impedances concerned are known, and their determination by calculation or measurement is considered later. Most important works in this connection have been carried out by Zeller and Oberdofer.

The basic equations which characterize the symmetrical system of voltages are:

\[ \begin{align*}
E_a &= E \\
E_b &= \alpha E \\
E_c &= \alpha^2 E \\
E_0 &= 0
\end{align*} \]
Let

\[ \begin{align*}
Z_0, Z_1, Z_2 & \text{ be impedances of zero, positive, and negative system} \\
I_0, I_1, & \text{ be current components at point of fault} \\
E_0', E_1', & \text{ be voltages at the same point. Then}
\end{align*} \]

considering each system separately, we have by Kirchhoff's laws

\[ \begin{align*}
E_1' + I_1Z_1 &= E_1 \\
E_0' + I_0Z_0 &= 0 \\
E_0' + I_0Z_0 &= 0 \end{align*} \]

Types of Faults: The types of faults that are likely to occur in a three-phase system are:

1. Three phase (3P):
   - Either line-to-line-to-line (L-L-L), or
   - Three lines to ground (L-L-G)

2. Single line-to-ground (L-G)

3. Double line-to-ground (2L-G)

4. Line-to-line (L-L)

These are represented schematically in Fig. 6.1.
Fig. 6.1 Types of faults on 3-Phase Systems.
3. Single faults: The voltage at the point of fault is zero in all three phases. Therefore, the symmetrical components of the voltage system are also zero, i.e.,

\[ E_1 = E_2 = E_3 = 0 \]

This result was to be expected as the voltages and current systems are symmetrical and therefore no negative- or zero-sequence components can exist.

Line to line fault: Let the fault be between \( b \) and \( c \), assuming the neutral to be free as shown in Fig. 8.1, so let a zero component cannot exist in the current system. \( E_b \), \( E_c \), and \( E_n \) in the system of phase voltages before the fault takes place. After the fault has occurred, the voltage

...
between the far point and the point of fault would be given by the equations:

\[ E_0 = a^2E - a^2Z_2 - a^2Z_1 \]

\[ E_c = aE - aZ_2 - a^2Z_1 \]

The voltage drops being expressed by their symmetrical components.

But \[ E_0 = E_c \]

\[ a^2E - a^2Z_2 - a^2Z_1 = aE - aZ_2 - a^2Z_1 \]

Also \[ I_2 = I + I_2 = 0 \]

since the current in the open place is zero.

\[ 0 = I + I_2 \]

Substituting into (6.8) \[ I_2 = I \], we have:

\[ a^2E - a^2Z_2 + a^2Z_1 = aE - aZ_2 + a^2Z_1 \]

\[ 0 = (a^2-a)E = (a^2-a)Z_2 \]

\[ 0 = E = Z_2 + Z_1 \]

Therefore \[ I_1 = \frac{E}{Z_2 + Z_1} \]

and \[ I_2 = -\frac{E}{Z_2 + Z_1} \]
Now considering the current relations in phase \( E \) and \\
we have:

\[
L_c = aL_1 + aL_2 = aL_1 - qL_1
\]

\[
L_c = aL_1 + aL_2 = aL_1 - qL_1
\]

\[
Q_c = L_c - L_e = (a^2 - a)I_z - (a^2 - a) \frac{E}{Z + Z_L}
\]

which is the fault current.

Now, \((a^2 - a) = -j \sqrt{3} \) (Table 3, p. 18)

\[
L_c - L_e = -j \sqrt{3} \frac{E}{Z + Z_L}
\]

\[
|L_c| = |L_e| = \sqrt{3} \frac{|E|}{|Z + Z_L|}
\]

For voltage relations we have:

\[
E_c = E_c = aL_1 - aL_2 - qL_2
\]

\[
= aL_1 - aL_2 \frac{E}{Z + Z_L} + qL_2 \frac{E}{Z + Z_L}
\]

\[
= \frac{qL_2}{Z + Z_L} \left[(Z + Z_L) - Z_L + qL_2 \right]
\]

\[
= \frac{qL_2}{Z + Z_L} \frac{1 + qL_2}{Z + Z_L} = \frac{qL_2}{Z + Z_L} \frac{(1 + qL_2)}{Z_L}
\]

\[
= (q^2 + q) \frac{E}{Z + Z_L}
\]

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$$\text{But } 1 + a + a^2 = 0$$

$$E_6 = E_0 = \frac{E}{Z + jx}$$

Further, from equation (6.4) we have $$\frac{E}{Z + jx} = \frac{Z_c}{(a^2 - 1)} \Rightarrow \frac{Z_c}{j\sqrt{3}}$$

The equation $$E_0 = \frac{1}{3} (E_a + E_b + E_c)$$ furnishes the value of the voltage at the free phase viz.

$$E_a = -2E_b = \frac{2Ez_c}{Z + jx} = j\frac{2z_e z_c}{\sqrt{3}}$$

$$|E_a| = \frac{2|z_c|}{\sqrt{3}}$$

The line voltage between the free phase and the phases containing the fault is

$$E_{ab} = E_a - E_c = \frac{2Ez_c}{Z + jx} - j\frac{z_c}{\sqrt{3}}$$

Line-to-ground fault: Let the phase a be grounded (Fig. 6.3). The phases b and c be unloaded. The currents are zero. In this case, obviously, a
zero component will appear in the current system, so

\[ I_0 + a \cdot I_1 + a^2 \cdot I_2 = 0 \quad (6.5) \]

\[ I_0 + a \cdot I_1 + a^2 \cdot I_2 = 0 \quad (6.6) \]

Subtracting,

\[ (a^2 - 1) I_1 + (a - a^2) I_2 = 0 \]

So, \( I_1 = I_2 \)

From (6.5) and (6.6), therefore by addition

\[ 2I_0 + (a + a^2) I_1 + (a + a^2) I_2 = 0 \]

So, \( 2I_0 + (a + a^2)(I_1 + I_2) = 0 \)

But \( 1 + a + a^2 = 0 \)

Therefore \( I_1 = I_2 = I_0 \)

Also, as before,

\[ E_0 = E - (L_0 Z_0 + L_1 Z_1 + L_2 Z_2) \]

So, \( I_0 = \frac{E}{Z_1 + Z_2 + Z_0} \quad (6.7) \)

and \( I_0 = \frac{3E}{Z_1 + Z_2 + Z_0} \quad (6.8) \)

The voltages on the free phases are obtained by subtracting from the voltages existing...
before the fault occurs there is no voltage drop which
which exist after the fault has developed is

\[ E_b = a \frac{Z_2 - Z_L}{Z_0 + Z_L + Z_L} \]

\[ E_c = a \frac{Z_2 - Z_L}{Z_0 + Z_L + Z_L} \]

Q. by substituting the current values already found

\[ E_b = a \frac{Z_2 - Z_L}{Z_0 + Z_L + Z_L} \]

\[ = \frac{E}{Z_0 + Z_L + Z_L} \left( \frac{Z_0^2 - Z_L^2}{Z_0 + Z_L + Z_L} \right) \]

\[ = E \cdot a (q-1) \frac{Z_0 - Z_L}{Z_0 + Z_L + Z_L} - 6.9 \]

Similarly

\[ E_c = a \frac{Z_2 - Z_L}{Z_0 + Z_L + Z_L} \]

\[ = \frac{(q-1) Z_0 + (q-q_2) Z_L}{Z_0 + Z_L + Z_L} \]

\[ E = E (q-1) \frac{Z_0 - Z_L}{Z_0 + Z_L + Z_L} - 6.10 \]

The line voltage between the phases is

\[ E_{bc} = E_c - E_b = E (q-1) \left( \frac{Z_2 - Z_L}{Z_0 + Z_L + Z_L} \right) \]

\[ = E (q-1) \left( \frac{Z_2 - Z_0}{Z_0 + Z_L + Z_L} \right) \]

\[ = E (q-1) \frac{Z_2 - Z_0}{Z_0 + Z_L + Z_L} \]

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Substituting $I_0$ from (6.8) and

$$a(a-1) = -j\sqrt{3}$$ we have

$$E_1 = -\frac{j}{\sqrt{3}}(z_1 + z_2) I_0$$

$$0, \quad |E_0| = |z_2 + z_0| \frac{|I_0|}{\sqrt{3}}$$

Also $E_0 = 0$, the sum of the voltages $E_6$ and $E_c$ must be three times the real component of the voltage $E_0$.

$$E_6 + E_c = E (a-1) \frac{z_0 + z_2}{z_0 + z_2 + z_e} + E (a-1) \frac{z_0 - q z_e}{z_0 + z_2 + z_e}$$

$$= E (a-1) \frac{z_0 - q z_e}{z_0 + z_2 + z_e}$$

$$= E (a-1) (1 - q^2) \frac{z_0}{z_0 + z_2 + z_e}$$

$$= -3E \frac{z_0}{z_0 + z_2 + z_e} \quad \text{since} \quad (a-1)(1-q^2) = -3$$

Also $\frac{E}{z_0 + z_2 + z_e} = Z_0$ from equation (6.7). Therefore

$$E_6 + E_c = -3Z_0 I_0 = -3E_0$$

$$= -I_0 Z_0 \quad \text{from (6.9)}$$

The negative sign is due to the fact that the calculation was based upon voltage drops.
6.2 Automatic Protection.

The resolution of an unsymmetrical system of voltages or currents, into its symmetrical components, found a practical application a number of years ago in several proposals and designs actually executed for regulating and protecting gear for the control of generator, transformers, motors etc.

A balanced three-phase load absorbs power at a uniform rate. In practice, however, all loads are not balanced and as a matter of fact, balanced load is an ideal to which the actual industrial loads or large power stations tend to approximate. The normal current output of a power station contains a very small fraction of either negative or zero sequence components. Negative sequence currents can and do arise locally due to unbalance of single phase demands of domestic consumers.
But usually the diversity of their amplitude is such that the resultant negative sequence current is a very small fraction of the positive sequence current, while the zero sequence component is almost absent.

However, a considerable negative sequence component arises on the occurrence of phase to phase short-circuit fault on a three-phase system, and the zero sequence component on the occurrence of an earth fault. Thus the appreciable presence of the negative sequence currents is an indication of phase to phase short-circuit and fault of the zero sequence current of an earth fault.

In the early methods of protecting electrical circuits against the destructive effects of short-circuits, the operation of circuit breakers and fuses essentially depended upon the currents carried by them, exceeding a fixed adjusted value. The modern
practice of protection is to design such devices that the relays controlling the operation of circuit breakers carry no current except in such fault conditions in which they are required to respond. Hence relays have been designed to be responsive to the presence of the negative sequence current, thereby indicating the presence of a phase to phase short circuit or an earth fault indirectly.

The core balance system of protection essentially embodies the principle of zero sequence protection and is one of the most common methods for protection against earth faults. It is based on the fact that under healthy conditions the muf's due to the three line currents balance as well as in the phase to phase short circuit as the positive sequence and also the negative sequence currents balance. But on the occurrence of an
earth fault on any line, the zero sequence currents being in the same phase in the three phases do not balance, thus energizing the relay.

This system of protection is very suitable for circuits subjected to normal earth faults.

Zero sequence Protection: This is in fact a new name for "leakage protector." Fig 6.4 shows a typical circuit for leakage protection relays. There are three current transformers of similar ratios, their primaries of which carry the line current, connected in parallel. The common return lead containing the relay would carry a current corresponding to the vector sum of the line currents, which should normally be zero. Hence the relay will carry only the zero sequence component and will operate accordingly, i.e., the...
relay will be responsible to zero-sequence currents indicating the occurrence of an earth-fault. If the relay is replaced by an ammeter multiplied by the ratio of the CTs, it would give the residual current as three times the zero-sequence component.

However, in this simple circuit there is one difficulty normally experienced viz. the relay carries the residual currents of the secondaries of the CTs and if the errors in the CTs are not negligible (as in the case of large currents), when a considerable error is introduced in CT1, a false residual current also appears in the relay, which makes the protection scheme unstable.

The circuit in Fig. 6.5 (next page) with a relay of the wattmeter pattern, with its secondary element connected to a potential transformer (Y-D) with primary short-circuited as
shown, makes the relay responsive only to zero-sequence currents arising due to an earth fault, because of the relay indicator being controlled by the zero-sequence voltage through the P.T. connected to the secondary element.

Zero-sequence relays are finding wider field of application to the method in which the time lag of over-current relays is adjusted to correspond to their electrical distances from the source of supply. The over-current relays of inverse time pattern energized by residual current could be controlled by sensitive directional relays energized by zero-sequence component. It is also possible to control the relays carrying...
The residual currents are the residual voltages because the residual voltages to an earth fault is greatest at the fault position and diminishes as the distance from the point of fault.

Zero sequence components in Y-D connected system. In case of Y connection the vector sum of the phase voltages is zero; hence the zero sequence component of the voltages cannot exist across the line, though they can exist in individual phase conductors. Zero sequence currents also cannot exist in such a system as they do not get any path to flow. However, if the system is earthed at Y point, the zero sequence currents get a path to flow and so can exist when an earth fault occurs.

In case of D connection as well, the zero sequence voltages cannot exist for the same reason, but the zero sequence currents can exist.
but cannot flow into the line. They act merely as single-phase currents circulating round the phase.

The impedances of a three-phase transformer connected between two sections of a transmission network is the same for the positive and the negative-sequence currents; but the impedances to the zero-sequence current depends entirely upon the interconnection of its windings and upon the earth connections. Zero-sequence currents can only flow from one point in a three-wire system of conductors, which is connected to the earth. Hence, if the primary winding of a three-phase transformer are Δ-connected or Y-connected with a free neutral, no zero-sequence current can flow. But if the transformer is connected Y/Y and supplied from a source with an earthed neutral, zero-sequence current will have a normal path.
Negative sequence Protection. The network shown in Fig. 6.6 is used to negative sequence relays. The impedances $\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_3$, and $\bar{Z}_4$ are all of equal ohmic value, but the phase angle of $\bar{Z}_4$ and $\bar{Z}_2$ being equal and 60° greater than the phase angles of $\bar{Z}_3$ and $\bar{Z}_4$. The impedance of the relay itself is negligible. The current from the phase $a$ of the C.T. will divide itself equally into two paths at $A$, each being one-third in magnitude and differing in phase with each other by 60°. The current from the phase $c$ will also divide itself in a like manner. The positive sequence components of these currents will obviously be in phase opposition with each other at $B$ and $D$, and so cancel each other, but the negative sequence component does not cancel out as there is a phase difference.

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between them in the two sections and hence can flow into the relay. Thus the relay in response to negative sequence currents when acting in the network.

However, if there is a residual current composed of zero sequence component, one-third of its value will flow in the branches AB and BC. These differing in phase by 60° will give rise to a resultant equal to twice the zero sequence component, which will also flow in the relay. Thus the relay will be responsive to the negative as well as zero sequence currents.

To obtain unique response to negative sequence currents only, the network can be arranged from a current transformer with delta-connected secondary, whereby the zero sequence currents are eliminated.

Another relay shown in Fig 6.7 (next page)

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in of the induction pattern having separate windings. A small transformer with special design with an air-gap in its magnetic circuit, is also incorporated as shown. The coil C of the relay carries the residual current, while coil B being supplied from the phase 'b' of the CT, while the current in the coil A, supplied from an auxiliary transformer, is arranged to lead 60° on the current in the phase of the CT. The two windings A and C are so designed that the components due to the currents I_A and I_B (x-form) are equal in magnitude and separated by an additional 60°. Hence the torque set up by the two cells will be proportional to negative sequence component only.

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If a three-phase alternator of normal design supplies an unbalanced load, local heating will be set up in the rotor due to the oscillating component of the induced reaction, which is equivalent to that of a rotating field moving at synchronous speed in an opposite direction to that of the rotor. If for any reason, a single-phase short circuit is sustained on any part of the transmission system, dangerous heating is liable to be set up in the alternator, not only due to excessive currents but also due to the negative-sequence current, as already explained. Hence a special relay is usually incorporated with the alternator protective equipment.

Zero-sequence currents, like negative-sequence currents, also give rise to abnormal heating.
and hence instead of providing a zero-sequence relay, the connections of the negative-sequence relay (energised with a C.T. having secondary as Y connected) are so arranged that they are also responsive to the zero-sequence currents.

Such a relay will operate and trip the circuit breaker of the alternator at the occurrence of an earth fault or phase-to-phase short circuit for a prolonged time greater than the time-setting of the relay.

Negative-sequence relays have also been employed for protection of large A.C. motors against earth fault, short circuit, single phasing. The operation of the relay for earth fault or short circuit is as explained earlier. The conditions of single phasing results in case when one of the lines supplying a time-phase motor is made dead.
Somehow, after the motor has been run up to normal speed, the motor will continue to run, but the current input to the two lines will be materially increased. It is evident that a negative-sequence relay will operate as a condition as above is developed. As a matter of fact, a suitably adjusted negative-sequence relay will afford protection to the motor in all three conditions viz., single phasing, short circuit and earth fault.

Sometimes when a medium-pressure line is connected to a transmission network by a step-down transformer, a phase-to-phase short circuit may occur, but a short circuit current in a section of the line may not be greater than the full load current; in which case it is not liable to be detected by any other relay.
except a negative sequence relay, and its use is the best method of obtaining all not satisfactory protection.

6.3 AUTOMATIC VOLTAGE REGULATORS

The control relay of an automatic voltage regulator of a three phase alternator is excited by two of the line wires as shown in Fig 6.8.

Fig 6.8 M.V. REGULATOR

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Under abnormal conditions this voltage may be higher or lower than the normal value, and if the control relay were controlled solely by this voltage, the regulator may act in an undesirable fashion. If, however, the relay were controlled by the positive-sequence voltage, the control of the voltage would be unaffected by unbalanced load or fault conditions, since the positive-sequence voltage forms a symmetrical system.

Fig. 6.9 shows the connections employed for the control relay of the Metropolitan Vickers voltage regulator. The current in the coil of this relay is the vector sum of the currents in the circuit XR and R connected to secondary.
windings of the two voltage transformers, the primary windings of which are connected in spin - delta to the bus bars. Circuit $x_2$ is inductive and is adjusted to give a phase shift of 60°. Circuit $r$ is now inductive and its resistance is so adjusted that the current is equal to that in circuit $x_2$. This current being in phase with the voltage $E_b$.

For positive sequence voltages, the current in circuit $r$ is represented, in Fig. 6.10, by $I_r$ and that in circuit $x_2$, by $I_x$. The vector sum of these currents is given by $I_1$.

For negative sequence voltages, the current in circuit $r$ is represented, in Fig. 6.11, by $I_{r}'$ and...
that the current in circuit X by 5%. Since the current in circuit X lags 60° from $E_x$, this is in opposition to the current in circuit $E$, when the phase sequence is negative. Hence the control relay is operative only to positive-sequence voltages.

6.4 DISTRIBUTION NETWORKS.

Positive, negative, and zero-sequence impedances of distribution conductors can be found out by knowing the self- and mutual-inductances of the conductors. The following are the expressions for the positive-sequence impedance of conductors of a three-phase line:

$$Z_a = R_a + jw (L_a + a^2 M_{ab} + aM_{ac})$$
$$Z_b = R_b + jw (L_b + a^2 M_{bc} + aM_{ba})$$
$$Z_c = R_c + jw (L_c + a^2 M_{ca} + aM_{cb})$$

where $R_a$, $R_b$, and $R_c$ are the line resistances, $L_a$, $L_b$, and $L_c$ the self-inductances, and $M_{ab}$, $M_{bc}$, and $M_{ca}$ the mutual-inductances of the three conductors. The
Self- and mutual inductances may be found out from the dimensions and spacing of the conductors. Similarly, the negative-sequence impedances may also be determined.

In the case of zero-sequence impedance, the problem of the earth conductance and its inductance comes into play. According to Mayer, the impedance per unit length of a loop consisting of a wire and the earth return is:

\[ Z = R + \omega L e + j \omega C e \]  

where

- \( L e \) = wire inductance in \( \text{H/m} \)
- \( C e \) = earth capacitance

\[ \omega = 2\pi f \]  

an empirical value lying between 0.6 \( \times 10^{-5} \) and 1.03 \( \times 10^{-5} \) cgs units at 50 cycles.

\( f \) = specific resistance of a fictitious thin plate replacing the earth.

\( \mu \) = Permeability of wire material or air, radius

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and self inductance of the loop is \( L = 2 \times 10^{-7} \log \frac{2.38 k}{\gamma} \) in henry/kilometer, where \( k = \sqrt{\frac{B}{2 \pi}} \) and known as \( \gamma \\)

depth of penetration'.

Therefore, on this basis derived that

\[ L_e = L_a + M_e + M_a + 0.345 \times 10^3 A/km. \]

Thus, with this value and the values of \( L \) and \( M \) taken from the tables, we have for a three-phase line without earth:

\[ Z_0 = R_e + jX_e \quad \text{Ohm/km} \]

\[ Z_0 = R_e + jX_e + jL_e \quad \text{Ohm/km} \]

\[ Z_0 = R_e + jX_e \quad \text{Ohm/km} \]

Other methods have been suggested by Rekha, Hackermus and Haberland.

Meshed Network with several sources: According to Hessenberg, with an unsymmetrical load part of the delivered positive-sequence power is returned or 'reflected' as negative- or zero-sequence power.
A total reflection of the power occurs with a line-to-line fault in an otherwise unloaded three-phase line. Thus the following equations apply:

\[
I_2 = -I_1, \quad E_2 = E_1, \quad \text{and} \quad E_{L1} = -E_{L2},
\]

i.e., the total positive sequence power delivered to the point of fault is converted into negative sequence power. By drawing equivalent diagrams a number of asymmetric problems can be solved.

Break in a line conductor: The conditions resulting from such a fault may be treated in a similar manner to the line-to-line fault.

If in a network consisting of a generator, a long transmission line, and a motor, the negative- and zero-sequence impedances on the motor side are denoted by \( Z_n \), \( Z_0 \), and \( Z_{00} \) the sides being taken from the point of fault, then a reversal of the rotating field will occur only when...
\[
\left| \frac{Z_b}{Z_a} \left( 1 - \frac{Z_{A} + Z_{B}}{Z_{A} + Z_{B}} \right) \right| < 1
\]

A further investigation shows that with an earthed reactor tuned exactly, the reversal of the rotating field may be avoided.

These results are due to Beekin.

In short, the application of the methods of symmetrical components is enormous, out of which only a few have been enumerated. For details, standard works on the subject may be referred to. (See bibliography)

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Induction Motors

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7.1 GENERAL CONSIDERATIONS.

The study of induction motors under unbalanced conditions is an important application of the theory of symmetrical components. The induction motor is investigated by the use of circle diagrams. In developing the circle diagram of a poly-phase induction motor, symmetry of phase voltages and currents is always assumed. The equivalent circuit is drawn for one phase only, and to determine the torque and power from the circle diagram, the values furnished for single phase are multiplied by the number of phases. But if the system is unsymmetrical, in order to know its currents etc., it will have to be resolved into the symmetrical components. Consequently, two circle diagrams may be developed — one for the positive, and the other for the negative-sequence system. A zero-sequence system is not likely to occur as the neutral of the motor is not generally earthed.
Evidently, for power values only corresponding components must be combined (as shown on page 14).

In Fig. 7.1 the voltages of the positive sequence system are denoted by \( E_1 \), \( E_2 \), and \( E_3 \), the currents of the negative sequence system by \( I_1 \), \( I_2 \), \( I_3 \), \( E \) and \( I \) being the R.H.S. values. If the phase difference between the current and voltage is \( \phi \), then

\[ \text{Power in phase } 1 = E_1 I_1 \cos \phi \]
\[ \text{Power in phase } 2 = E_2 I_2 \cos (\phi + 120^\circ) \]
\[ \text{Power in phase } 3 = E_3 I_3 \cos (\phi + 240^\circ) \]

\[ \text{Total power} = E_1 \left[ \cos \phi + \cos (\phi + 120^\circ) + \cos (\phi + 240^\circ) \right] = 0 \]

So, the power resulting from the combination of positive sequence voltage system and the negative sequence current system, or vice versa, is always zero.

Denoting the voltage and current for phase \( \alpha \) of the positive sequence system by \( E_\alpha \) and \( I_\alpha \), the voltage
and current per phase in the negative system by \( I_1 - I_2 \) and the respective phase angles by \( \theta_1 \) and \( \theta_2 \). Then the total power will be:

\[
3 E_1 I_1 \cos \theta_1 - 3 E_2 I_2 \cos \theta_2
\]

as the negative-sequence system tends to turn the motor in the opposite direction to the torque of the negative-sequence system is a breaking torque.

If the slip of the motor with respect to the positive-sequence system is \( s \), then \( s = \frac{1}{2} \). With regard to the negative-sequence system, the slip will be locked rotor, being unity.

7.2 EQUIVALENT CIRCUITS FOR SYMMETRICAL AND UNSYMMETRICAL SYSTEMS.

Let \( R_s \) = Stator resistance per phase

\( X_s \) = " leakage reactance " at supply frequency

\( R_r \) = Rotor resistance

\( X_r \) = " leakage reactance "

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From the equivalent circuit (Eq. 7.2) we have,

\[ E_s = E_g + (R_s + jX_s) I_s \]

\[ \sigma E_g = (R_r + jX_r) I_r \]

\[ E_g = jX_m (I_s - I_r) = \left( \frac{R_s}{\sigma} + jX_s \right) I_r \]

\[ X_m \] being the reactance value connecting the magnetizing sta\text{ }r current and the demagnetizing recta\text{ }r current with the voltage corresponding to the air-gap field.

Then \( \frac{R_r}{\sigma} \) may be further resolved into two parts \( R_r \) and \( \frac{R_s}{\sigma} = R_{r} = \frac{1 - \sigma}{\sigma} R_r \).

So that \( \frac{1 - \sigma}{\sigma} R_r \) represents that value which when multiplied by \( I_r^2 \) gives the mechanical power per phase. The total power, \( P \), at the shaft of a three-phase

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motor is, therefore, \[ P = 3. \frac{1 - \sigma}{\sigma} I_s^2, \]
and the torque is
\[ T = 0.974 \frac{P}{n} \text{ m.kg m} = \frac{7P}{n} \text{ ft-lb}, \]
if \( P \) is in watts and \( n \) the speed in rpm. As the synchronous speed \( n_s \) is related to the supply frequency \( f \) by the relation \( n_s = \frac{n}{1 - \sigma} = \frac{60f}{1 - \sigma} \) where \( f \) is the supply frequency.

The torque \( T = 0.974 \frac{38}{60f} \frac{L_u^2}{\sigma} = 0.0162 \frac{38}{f} \frac{L_u^2}{\sigma} \text{ m.kg m} = 0.117 \frac{38}{f} \frac{L_u^2}{\sigma} \text{ ft-lb}. \)

This derivation is made under the assumption of symmetrical voltage system. It may also be used, therefore, without any change for the positive sequence system of an unsymmetrical voltage system. It may also be used for a negative sequence system, if the slip \((1 - \sigma)\) is substituted for \(\sigma\), and the value \(I_{r_s}\) for the rotor current. The power of the inverse system is
\[ P_{r_s} = 3 \frac{1 - \sigma}{2 - \sigma} I_r l^2. \]
and the total power becomes

\[ P_{\text{total}} = 3 \left(1 - o\right) P_1 \left(\frac{I_{\text{r1}}^2}{o} - \frac{I_{\text{r2}}^2}{2 - o}\right) \]

and the total torque is

\[ T_{\text{total}} = 3 \times 0.0162 \left(\frac{R_1}{o} - \frac{R_2}{2 - o}\right) \text{ mkg} \]

The equivalent circuit diagrams for the positive- and negative-sequence systems are shown in Figs. 7.3 and 7.4, respectively.

**EQUIVALENT CIRCUITS**

![Equivalent Circuit Diagrams](image)

Fig 7.3  +ve. seq.

Fig 7.4  -ve. seq.

For instance, if a three-phase motor is running with one of its leads open-circuited, the equivalent circuit will reduce to

![Equivalent Circuit Diagram](image)

If the phase a is open-circuited, then:

![Equivalent Circuit Diagram](image)

**Fig 7.5**

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\[ I_x = 0 \]
\[ I_y = 1 \]
\[ I_z = 1 \]

Therefore,
\[ I_x = \frac{1}{3} (E_a - E_r + E_z) = \frac{E_r - E_z}{3} = -I_z \]
\[ I_y = \frac{1}{3} (E_a - E_r - E_z) = \frac{E_r + E_z}{3} = -I_z \]

Let the stator be \( Y \)-connected. The line voltage \( E_{ls} \) is known. The phase voltages are:
\[ E_{ls} = a^2 E_r + a E_z \]
\[ E_{ls} = a E_r + a^2 E_z \]

so that the line voltage may be written as:
\[ E_{ls} = E_r - E_z = (E_r - a^2 E_z) = j\sqrt{3} (E_r - E_z) \]

These relations may, therefore, be represented by the series connection of the equivalent circuit diagram of the positive sequence system with that of the negative, so that the condition \( I_z = I_z \) is fulfilled.

The voltage applied is \( (E_r - E_z) = \frac{E_{ls}}{\sqrt{3}} \).

Thus if the unknown impedances are known
When using the equivalent circuit diagram, the power and torque, depending on the slip, may be evaluated from the above relations. The resistance value that corresponds to mechanical power is

\[ (1 - \frac{1}{2}) R = \frac{2(1 - \frac{1}{2})}{\sigma^2} R \]

Assuming \( jX_0 \) of the positive sequence component to be acting at the terminals and neglecting \( jX_0 \) of the negative sequence component. Denoting \( I_r \) the rotor current with these approximations, we have the expression for torque as:

\[ T_{\text{total}} = 0.117 \frac{R_2}{R_1} \frac{C}{f} \cdot f \cdot I_r^2 \]

which shows that the torque is zero when \( \sigma = 1 \), i.e., the motor will not start from rest. Though it can keep running if the slip is small.

**Note:** These statements were experimentally verified by applying unbalanced voltages to a 1 HP induction motor. Also, the motor did not start when one of its phases was left open-circuited.
BALANCING OF UNSYMMETRICAL SYSTEMS
8.1 INTRODUCTION.

The three-phase unbalanced system may be symmetrical by many methods, e.g., by static balancers, rotary balancers. They will be considered here in a very broad outline. For a detailed study "Symmetrical Components" by Wagner & Evans may be referred.

8.2 STATIC BALANCERS.

(1) By balancing transformers with inductances and capacitances.

(2) By induction regulators.

(3) Hessenberg's method.

Hessenberg used zig-zag coil for symmetrical Y-connected load as shown in Fig. 6.1.

A = Switch B, C and D = Load.

C = Transformer D = Zig-zag coil.

Fig. 6.1

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8.3 ROTARY BALANCERS.

Rotating balancers tend to balance voltages and currents by periodically absorbing and restoring energy to the cycle in using in this process the energy by the inertia of rotating parts. Thus rotating machines have an inherent property of providing the balancing action and do not require the adjustable feature characteristic of static balancers.

Rotary balancers are of two types:

1. Negative sequence enf. generators
2. Impedance type balancers.

For details, the reader is referred to any standard book (see bibliography) as it is beyond the scope of this work.

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CONCLUSION

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It is pretty evident from the previous discussions that the use of the theory of symmetrical components is inevitable in the study of unbalanced three-phase or polyphase technique. It is worthwhile, therefore, to sum up the results arrived at.

The conditions which give rise to negative and zero-sequence components are:

1. Negative-sequence currents are produced either by unsymmetrical loading or by short-circuits.
2. Zero-sequence currents are produced by earth-faults.
3. Only positive-sequence voltages are produced by rotating machines.
4. Negative- and zero-sequence voltages are only produced respectively by negative- and zero-sequence currents in a system with symmetrical line impedances.
5. Positive- and negative-sequence currents can be
generated by an alternator, but no zero-sequence current, unless the neutral is earthed.

(6) Each earthed point on a system is a source of supply of zero-sequence currents.

(7) Each sequence current circulates through the system in a manner depending on its source of supply and independent of the circulation of other sequence currents.

(8) No zero-sequence current can flow into or away from an unearthed machine.

(9) The reactances of transmission lines are the same to positive- and negative-sequence currents but greater to zero-sequence currents.

(10) The impedance of rotating machines to negative-sequence currents produced by negative-sequence voltages is very much less than the impedance to positive-sequence currents. This is
because rotating machines tend to operate as phase balances, and when supplied by a system of unbalanced voltages, draw from the system heavy unbalanced currents which tend to correct the imbalance. Rotating machines also absorb negative-quence powers.

(x)
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Performed by ________________________

Date ________________________
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PROTECTIONS.

Performed by

Date
APPENDIX 1

Performed by ____________________________

Date ____________________________
A few of the measurements taken by the "Symmetrical Component Meters" are appended in the following pages. The measured phase voltages in unbalanced circuits have been compared with the phase voltages synthesized for the measured sequence voltages and their relative phase angles. It has been observed that the meter gives fairly accurate results (error ±1.2%), and when carefully used gives almost exact results, as could be seen from the figures given.

Performed by

Date
Phase angle $\phi = 42^\circ$

Positive sequence Component = 105 volts

Negative sequence Component = 65 volts

Measured Phase voltages
- $E_a = 150$ volts
- $E_b = 133$ volts
- $E_c = 50$ volts

Calculated Phase voltages
- $E_a = 150$ volts
- $E_b = 133$ volts
- $E_c = 50$ volts

Combination of sequence voltages to form phase voltages.

Performed by

Date
Positive Sequence
Component = 110 V

Measured Phase Voltages:

\( E_a = 147 \text{ V} \)
\( E_b = 61 \text{ V} \)
\( E_c = 131 \text{ V} \)

Calculated Phase Voltages:

\( E_a = 150 \text{ V} \)
\( E_b = 60 \text{ V} \)
\( E_c = 135 \text{ V} \)

Combination of sequence voltages to form phase voltages.

Phase angle \( \phi = +46^\circ \)
Negative Sequence
Component = 50 V

Performed by

Date
Phase angle $\phi = -110^\circ$

Positive sequence voltage = 100 volt

Reconstructed phase voltages

$E_a = 125$ volt
$E_b = 185$ volt
$E_c = 70$ volt

Calculated phase voltages

$E_a = 124$ volt
$E_c = 188$ volt
$E_c = 74$ volt

Combination of sequence voltages to form phase voltages.

Performed by _______________________

Date _______________________.

Positive sequence voltage = 85 volt
Phase angle $\phi = 90^\circ$

Positive Sequence
Component = 120 V

Negative Sequence
Component = 65 V

Measured Phase Voltages:
$E_a = 140 \text{ V}
E_b = 170 \text{ V}
E_c = 70 \text{ V}$

Calculated Phase Voltage:
$E_a = 140 \text{ V}
E_b = 172 \text{ V}
E_c = 75 \text{ V}$

Combination of Sequence Voltages to form Phase Voltage.

Performed by _______________________________

Date _______________________________
Phase angle $\phi = 129^\circ$

Positive sequence component = 112 volts

Negative sequence component = 100 volts

Measured phase voltages:

$E_a = 90$ volts
$E_b = 218$ volts
$E_c = 120$ volts

Calculated phase voltages:

$E_a = 90$ volts
$E_b = 215$ volts
$E_c = 121$ volts

Combination of sequence voltages to form phase voltages.

Performed by ____________________________

Date ____________________________
APPENDIX 2.
DESIGN OF ISOLATING TRANSFORMERS.

References: 1. Design of Small Transformers - E. Holley  

Specifications: Three single phase transformers  
Connection: Star / Star  
Ratio: 1:1 (220V/220V across lines)  
Frequency: 50 cycles  
Current: 25 millamps.  
VA rating: 5 per transformer.

Core Dimensions:  
"SHELL TYPE" core  
of dimensions as labeled  
in the sketch. To be taken  
out from 220/110 auto-transformer. (not to scale)  
One such core is required for each phase. Therefore,  
three such cores are required.

Performed by ___________________________  
Date ___________________________
Number of turns: Approximate number of turns per volt (t) is given by the formula

\[ t = \frac{8.4}{\text{area of core in } \text{in}^2} \text{ at } 50 \text{ cycles/sec.} \]

\[ = \frac{8.4}{\frac{1}{4} \times 1} = 11.2 \text{ turns/volt} \]

Therefore, number of turns required for 220 volts = 2500 turns.

Hence:

- Number of turns on primary = 2500 turns
- Number of turns on secondary = 2500 turns.

Winding particulars: From wire table, it was found that No. 36 SWG Enamelled was best suited. Its particulars are:

- Diameter: 8.3 mill
- Turns/cord: 120.5

If 125 turns are wound in one layer, total number of layers required for each winding is 20.

Therefore, space required for both windings = \( 40 \times 0.3 \times \frac{1}{1000} \times \frac{1}{20} \) which is less than the window dimensions.

Performed by

Date
Hence, No 36SWG Enam. can be approved for both the primary and secondary windings. The current carrying capacity of this wire is 68 milliamps, while the maximum current to be handled by the transformers is only 25 to 30 milliamps.

Insulation: Enam. tape and Varn. paper can be used for insulation. After properly insulating the coils, they should be baked in an oven and dried up, and finally assembled with the core.

To complete the insulating transformer, three such transformers are to be made and connected in star/star.
(iii) Arm 1-2 should be purely resistive.
(iv) The circuit parameters be so chosen that the current-transformers are not over-burdened.

Design Considerations:

Bearing the above-mentioned points in mind, the design and assembly procedure are very simple. For example, for a 25-VA current transformer, the maximum permissible burden is 1 ohm at 5 amps. This dictates the values of the bridge parameters. At the first attempt, however, these values used not be very accurate. A purely inductive coil of approximate value may be taken and connected in series with a variable resistance to form the arm 3-4 of the bridge. Another variable resistance R may be used to form the arm 1-2. The remaining two arms consist of two identical ammeters. By adjusting the resistances r and R, a 3-phase balanced system, the meter Ap shall give a deflection, while Aq will read zero, the negative-sequence current being zero. Obviously, Ap will indicate the magnitude of the positive-sequence current. Under these conditions, the arm 3-4 shall have a phase-shift of exactly 60°. After obtaining such a balance, the resistances r and R should not be disturbed. A further check may be made at this stage by opening up the phase c of the 3-phase balanced system. In this case, if the bridge is properly balanced, the meters Ap and Aq should read equal, since the magnitudes of the positive- and negative-sequence currents are equal in the opened phase.

The Modified Circuit:

Based on the above discussion, using an electrodynamometer-type ammeter, the following is a circuit (Fig. 2) designed for the measurements of positive- and negative-sequence currents and their relative phase-angles. It is now in the sense that only one ammeter is needed to make all the measurements, and it can measure the magnitudes of the sequence currents as well as their relative phase-angles. In this circuit, use has been made of the fact that an electrodynamometer instrument responds not only to the product of the currents through its coils but also to the phase-angles between the currents. This would become more evident by referring to switching operations given below:

<table>
<thead>
<tr>
<th>Setting</th>
<th>Switch P</th>
<th>Switch Q</th>
<th>Switch R</th>
<th>Switch S</th>
<th>Meter Reads</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>left</td>
<td>left</td>
<td>left</td>
<td>open</td>
<td>Ic₁</td>
<td>3a.</td>
</tr>
<tr>
<td>II</td>
<td>right</td>
<td>right</td>
<td>left</td>
<td>open</td>
<td>Ic₂</td>
<td>3b.</td>
</tr>
<tr>
<td>III</td>
<td>open</td>
<td>right</td>
<td>left</td>
<td>closed</td>
<td>Ic₁, Ic₂</td>
<td>3c.</td>
</tr>
</tbody>
</table>

As shown in Fig. 2, to make a measurement, the two current transformers are connected to the lines a and b and the meter responds to the sequence currents of line c. In setting I and II (Figs. 3a. and 3b.) the meter acts as an ordinary ammeter and reads directly the positive- and negative-sequence currents Ic₁ and Ic₂ respectively. In setting III (Fig. 3c.), however, it can be seen that the two coils of the meter form the two arms (1-3 and 2-4) of the bridge. In this case the meter deflection is proportional to Ic₁ Ic₂ cos f Ic₁, Ic₂. Ic₁ and Ic₂ having been already determined from the settings I and II, the phase-angle between Ic₁ and Ic₂ can be computed.

To sum up, the circuit essentially requires an electrodynamometer type ammeter with the two coils having identical impedances and the leads brought out independently; each coil having an impedance equal to twice the impedance of either of the meter-coils; two current transformers; a pure resistance; a 60°-phase-shift coil; three double-pole double-throw switches; and a single-pole switch. It can also be seen from Fig. 2, that the circuiting switch of the current transformers should always be kept closed except the
switches P, Q, R and S occupy any of the positions indicated in setting I, II or III.

**THE SEQUENCE-VOLTAGE BRIDGE**

The bridge is shown in Fig. 4. It is similar to the current-bridge described in the preceding section. In the absence of zero-sequence voltages, the meters respond to the positive- and negative-sequence voltages, as given by the following equations:

\[
V_x = Vca_1 \\
V_y = Vca_2
\]

if \( R(Z+Z_m) = -Z(R+Z_m) \), where the symbols are shown in Fig. 4 and \( a \) is the phasor-operator, as defined in the nomenclature.

Following the same reasoning as that for the current-bridge, it may be readily seen that the following conditions should be fulfilled in order that the sequence-components of voltages may be measured:

(i) The voltmeters should have exactly the same impedance \( Z_m \).

(ii) Arm 3-4 of the bridge should have a phase-shift of exactly 60°.

(iii) Arm 1-2 should be purely resistive.

(iv) The circuit parameters should be so chosen that the potential-transformers are not over-burdened.

For the measurements of relative phase-angles between the sequence-voltages, the principles governing the current-bridge (Fig. 2) may be utilized. A circuit similar to that shown in Fig. 2 may be set up, using an electrodynamometer voltmeter having two exactly identical coils. With proper switching arrangements, the same meter can be used to respond to the positive- and negative-sequence voltages, and to their relative phase-angles, as in case of the sequence-current bridge.

**THE SEQUENCE-POWER BRIDGE**

After the sequence-components of currents and voltages have been segregated, the sequence-power measurements can be carried out using two ordinary wattmeters. The current coils of the wattmeters form the arms 1-3 and 2-4 of the sequence-current bridge, while the voltage-coils form the arms 1-3 and 2-4 of the sequence voltage bridge. With such a connection, the two identical wattmeters respond to the sequence-components of power. However, care should be taken to apply the necessary phase-angle and scale-factor corrections (8).

**SOME APPLICATIONS**

The applications of the methods of symmetrical components are too numerous to mention (2, 3, 5). These components can be determined either by analytical methods or by direct methods, that is, by measurements. Obviously, the direct methods are more rapid and less tedious. The methods suggested in this paper give complete information regarding the sequence quantities of an unbalanced system. These methods can be used to correlate the theory with experiments. Conventional methods described in textbooks and elsewhere, do not give a complete analysis as the relative phase angles cannot be measured by these methods. The method given here can also be used in usual laboratory teaching and research to analyze unbalanced systems, and for the measurements of sequence quantities in rotating machinery, rotary balancers and other special machines (4, 8). Being a bridge method, the method can also find applications in detecting unbalance and actuating control elements to
restore balance to the system. A further application could be in conjunction with protective-relays for sensing the flow of negative-sequence power.

**CONCLUSIONS**

(1) Detailed mathematical analysis is given to prove the validity of the methods discussed. As such, it is not available in textbooks.

(2) The bridge methods are extended to measure the sequence-components of power.

(3) Modifications are suggested in order that phase-angles between sequence-components of currents can be measured. This can be further extended for the measurements of phase-angles between sequence-components of voltages.

(4) Measurements of the sequence-components and their relative phase-angles give the complete information regarding the nature and degree of unbalance in a system. This is in contrast to the earlier methods which give only the magnitudes of sequence quantities.

(5) The method is useful in correlating theoretical predictions with experimental results. The method thus has an educational value. A further application is that it gives direct and rapid results while the numerical calculations by analytical methods may be too involved.

(6) It is agreed that the suggested circuits are relatively expensive and the design is somewhat critical.
Proof of Equations (1) and (2):

Let the secondary currents divide as shown in Fig. 1. Then, by Kirchhoff's laws:

\[ I_a = I_a' + I_a'' \]
\[ I_a'(Z+Z_m) = I_a''(R+Z_m) \]

where \( Z \) is the impedance of either of the transformers. From (5) and (6), therefore, writing in terms of \( I_a' \) and \( I_a'' \):

\[ I_a' = I_a \left( \frac{R+Z_m}{R+Z+2Z_m} \right) \]
\[ I_a'' = I_a \left( \frac{Z+Z_m}{R+Z+2Z_m} \right) \]

Similarly for the currents \( I_b' \) and \( I_b'' \):

\[ I_b' = I_b \left( \frac{Z+Z_m}{R+Z+2Z_m} \right) \]
\[ I_b'' = I_b \left( \frac{R+Z_m}{R+Z+2Z_m} \right) \]

From the superposition theorem it follows:

\[ A_x = I_a' + I_b' = I_a \left( \frac{R+Z_m}{R+2Z+2Z_m} \right) + I_b \left( \frac{Z+Z_m}{R+Z+2Z_m} \right) \]
\[ A_y = I_a'' + I_b'' = I_a \left( \frac{Z+Z_m}{R+Z+2Z_m} \right) + I_b \left( \frac{R+Z_m}{R+Z+2Z_m} \right) \]

Rewriting \( A_x \) and \( A_y \) in terms of sequence-components of \( Ic \), therefore:

\[ A_x = Ic_1 \left[ \frac{a^2(R+Z_m) + a(Z+Z_m)}{R+Z+2Z_m} \right] + Ic_2 \left[ \frac{a(R+Z_m) + a^2(Z+Z_m)}{R+Z+2Z_m} \right] \]
\[ A_y = Ic_1 \left[ \frac{a^2(Z+Z_m) + a(R+Z_m)}{R+Z+2Z_m} \right] + Ic_2 \left[ \frac{a(Z+Z_m) + a^2(R+Z_m)}{R+Z+2Z_m} \right] \]

If \( (R+Z_m) = -a(Z+Z_m) \), from Equations (13) and (14):

\[ A_x = Ic_1 \left[ \frac{(a-1)(Z+Z_m)}{R+Z+2Z_m} \right] = -Ic_1 \]

Similarly,
\[ A_y = \beta_2 \left[ \frac{(b-1)(Z_m)}{R_0 + \beta_2 Z_m} \right] = \beta_2 \tag{16} \]

Equations (15) and (16) are identical to Equations (1) and (2), except that in the latter the negative sign has been dropped.

Proof of Equations (3) and (4):

Let the voltages and currents be as shown in Fig. 4. Then, by Kirchoff's laws:

\[ V_{ab} = I_R Z_m + I_Z Z_m = I_Z Z_m + I_Z Z \tag{17} \]
\[ V_{bc} = I_y Z_m - I_Z Z_m = I_x Z_m - I_R \tag{18} \]
\[ V_{ab} + V_{bc} = I_Z Z_m + I_x Z_m = 2I_Z Z_m + I_Z Z - I_R \tag{19} \]
\[ I_x = I_Z = I_y = I_R \tag{20} \]

where \( Z_m \) is the impedance of either of the voltmeters. From Equations (19) and (20):

\[ V_{ab} + V_{bc} = 2I_Z Z_m + I_x Z_m - I_Z Z_m \tag{21} \]

But from Equations (17) and (18):

\[ I_R = \frac{-V_{bc} + I_x Z_m}{R} \tag{22} \]
\[ I_Z = \frac{V_{ab} - I_x Z_m}{Z} \tag{23} \]

Substituting Equations (22) and (23) in Equation (21) and simplifying:

\[ V_{ab}(1 + \frac{Z_m}{Z}) + V_{bc}(1 + \frac{Z_m}{R}) = V_Z Z_m (2 + \frac{Z_m}{R} + \frac{Z_m}{Z}) \tag{24} \]

Writing in terms of sequence-components:

\[ V_{ab} \left[ \frac{R(Z_m + Z) + Z(Z_m + Z)}{Z_m} \right] + V_{bc} \left[ \frac{R(Z_m + Z) + Z(Z_m + Z)}{Z_m} \right] = V_Z Z_m = V_x \tag{25} \]

A similar expression can be written for the voltage read by \( V_y \).

Now, if \( R(Z_m + Z) = -aZ(Z_m + Z) \)

\[ \text{Now, of } R(Z_m + Z) = -aZ(Z_m + Z) \tag{26} \]
\[
V_{ab} \left[ \frac{B(Z_e + Z_m)(1-a)}{R(Z_e + Z_m)(1-a^2)} \right] = X_{el} = V_x
\] (27)

or,

\[
V_x = V_{ab} \left( \frac{1}{1+a} \right) = V_{ab} / 0 = \frac{v}{a}
\] (28)

Similarly,

\[
V_y = V_{ca}
\] (29)

Equations (28) and (29) are identical to Equations (5) and (4) except for the difference in sign.

**SYMBOLS**

- **A** = Ammeter reading
- **I** = Current
- **V** = Voltage, voltmeter reading
- **R** = resistance
- **Z** = impedance
- **a** = phasor operator \(e^{j120^\circ}\)

**Subscripts:**

- a, b, c = Phases a, b and c
- 1, 2 = Positive- and negative-sequence components respectively
- m = meter (Voltmeter or Ammeter).

Symbols used locally are labelled on circuit diagrams.

**ACKNOWLEDGMENT**

The assistance given by the Electrical Engineering Department, University of California, Berkeley, in preparation of this paper is gratefully acknowledged.

**REFERENCES**


Fig. 1: Bridge for Measuring Sequence-Currents.

Fig. 2: Bridge for Measuring Sequence-Currents and their Phase-Angles.
Fig. 3: Simplified Circuit Diagrams.

(a) Setting I  
(b) Setting II  
(c) Setting III

Fig. 4: Bridge for Measuring Sequence Voltages.