L-2/T-1/IPE **Date: 20/08/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2016-2017

Sub: **EEE 271** (Electrical Machines and Electronics)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What is leakage flux? Define the components of magnetization current. Why is iron core used in a transformer? **(7)**

(b) In which side of a transformer open circuit test and short circuit tests are carried out? Why? **(8)**

(c) A 1-kVA. 230/115-V, 60-Hz single phase distribution transformer is tested with the following results: **(20)**

Open-circuit test	Short-circuit test
$I_{OC} = 0.11$ A	$V_{SC} = 17.1$ V, $I_{SC} = 8.7$ A
$P_{OC} = 3.9$ W	$P_{SC} = 38.1$ W

Find-

(i) the equivalent circuit for this transformer referred to the low voltage side,

(ii) the voltage regulation at the rated conditions and 0.8 PF lagging, and

(iii) the efficiency at the rated conditions and 0.8 PF lagging.

2. (a) 'An induction motor can speed up to near-synchronous speed, but it can never really reach synchronous speed' — explain. **(6)**

(b) Draw the torque-speed characteristic of an induction motor, and explain it. Identify- starting torque, pullout torque, full load torque. What is plugging? What is the effect of changing rotor resistance in torque-speed characteristics? Show in figure and explain in brief. Hint: **(12)**

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}$$

$$S_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

(c) A 208-V, two-pole, 60-Hz Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are: **(17)**

$R_1 = 0.200\ \Omega$ $R_2 = 0.120\ \Omega$

$X_1 = 0.410\ \Omega$ $X_2 = 0.410\ \Omega$

$P_{mech} = 250$ W $P_{misc} = 0$

$X_M = 15\ \Omega$ $P_{core} = 180$ W

Contd P/2

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Contd... Q. No. 2(c)

For a slip of 0.05, find -

(i) line current

(ii) stator copper losses P_{SCL}

(iii) air-gap power P_{AG}

(iv) power converted from electrical to mechanical form P_{conv}, and

(v) overall machine efficiency.

3. (a) Show that, $P_{CONV} : P_{AG} = (1-s):1$. **(13)**

(b) A DC test is performed on a 460V, Δ-connected, 100-hp induction motor. If V_{DC} = 21V and I_{DC} = 72A, what is the stator resistance? **(6)**

(c) A 208-V, 60-Hz six-pole, Y-connected, 25-hp design class B induction motor is tested in the laboratory, with the following results: **(16)**

No load: 208 V, 22.0 A, 1200 W, 60 Hz

Locked rotor: 24.6 V, 64.5 A, 2200 W, 15 Hz

DC test: 13.5 V, 64 A

Find the equivalent circuit of this motor.

(For Design Class B, $X_1 = 0.4X_{LR}$ and $X_2 = 0.6X_{LR}$)

4. (a) Derive **(10)**

$$\tau_{ind} = \frac{3V_\phi E_A}{w_m X_s} \sin\delta$$

(b) Draw the power flow diagram of a generator. With the phasor diagrams explain the effect of load changes in a synchronous generator. **(12)**

(c) Draw the phasor diagram of transformer for lagging, leading and unity PF loads. Also, comment on the voltage regulation in each case. Show that **(13)**

$$\frac{Vp}{a} \approx V_s + R_{eq} I_s \cos\theta + X_{eq} I_s \sin\theta .$$

SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.

Make necessary assumptions. Symbols have their usual meanings.

5. (a) Derive the equation that describes the terminal characteristics of a shunt DC motor. Explain why the characteristics changes if armature reaction is considered. **(13)**

(b) A 50 hp, 250 V, 1200 r/min dc shunt motor with compensating windings has an armature resistance (including the brushes, compensating windings and interpoles) of 0.06 Ω. Its field circuit has a total resistance $R_{adj} + R_F$ of 50 Ω, which produces a no load speed of 1200 r/min. There are 1200 turns per pole on the shunt field winding. Find the speed of the motor when its input current is 100 A. **(8)**

(c) Describe the methods for speed control of a shunt DC motor. **(14)**

Contd P/3

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6. (a) What happens when the load is changed in a synchronous motor? Explain with phasor diagram and necessary equations. **(13)**

(b) A 208 V, 45 kVA, 0.8 PF leading, Δ-connected 60 Hz synchronous machine has a synchronous reactance of 2.5 Ω and a negligible armature resistance. Its friction and windage loss is 1.5 kW and core loss 1.0 kW. Initially the shaft is supplying a 15 hp load, and the motor's pF is 0.8 leading. **(22)**

(i) Sketch the phasor diagram of this motor for the load supplied and find the values of I_A, I_L, and E_A.

(ii) Assume that the shaft load is now increased to 30 hp. Sketch the behavior of phasor diagram in response to this change.

(iii) Find I_A, I_L, and E_A after the load changes. What is the new power factor?

7. (a) Explain the synchronous 'V' curve. **(7)**

(b) A 208 V, 45 kVA, 0.8 pF leading, Δ-connected, 60 Hz synchronous motor has a synchronous reactance of 2.5 Ω and a negligible armature resistance. Its friction and windage loss is 2.0 kW and its core losses are 1.0 kW. Initially the shaft is supplying a 15 hp load with a pF of 0.85 lagging. The field current I_F at these conditions is 4.0 A. If the motor's flux is increased by 25%, sketch the new phasor diagrams of this motor before and after the increase? What are the values of E_A, I_A and power factor after the increase? **(20)**

(c) Define holding current, latching current, and breakover voltage from the I-V characteristics of an SCR. **(8)**

8. (a) Explain natural commutation and forced commutation of SCR with example. **(8)**

(b) Describe the operation of an SCR using the 'Two-Transistor Model'. **(10)**

(c) Draw the circuit diagram and waveshapes of input current and output voltage of a single-phase full-wave controlled rectifier. Find input power factor for a firing angle of 'α'. **(17)**

L-2/T-1/IPE **Date: 16/08/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2016-2017

Sub: **CSE 295** (Computer Programming Techniques)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Briefly describe "call by value" and "call by reference" with example code fragments. **(10)**

   (b) Briefly describe two ways to return more than one value from a function. **(10)**

   (c) Briefly describe the prototype declaration procedure of a function. **(8)**

   (d) Can a local variable and a global variable have the same name? Explain your answer. **(7)**

2. (a) **Using pointer arithmetic,** implement the following function which will return the number of occurrences of the character c in the character string pointed to by the character pointer p. **(10)**

```
int numOfOccurrences(char c, char *p);
```

Constraints: you must have to use pointer arithmetic. You can't use any additional array. You can't use any library function.

(b)See the following C code fragment: **(14)**

```
#include<stdio.h>
int main()
{
    int n, i, a, arr[50];
    scanf("%d",&n);
    for(i=0;i<n;i++)
        scanf("%d",&arr[i]);
    a=largestPrime(arr,n);
    if(a==-1)
        printf("No Prime in the array\n");
    else
        Printf("Largest Prime: %d\n",a)
    Return 0;
}
```

Implement the `largestPrime` function which takes the base address of an integer array (the array will always contain positive integer numbers) and the number of elements in the array, and returns the largest prime in that array. If there is no prime number in that array, it returns-1;

Contd P/2

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Contd... Q. No. 2(a)

Constraints: you can't use any additional array. You can't use any library function but you can write and use your own defined function(s).

N.B.: Just write the full function definition. You don't have to write the prototype.

(c) Implement the following function which takes two integer numbers `num1` and `num2`, and returns the gcd (greatest common divisor) of these two numbers. **(11)**

```
int gcd(int num1, int num2);
```

Constraint: You can't use any library function.

3. (a) Implement the following function which determines whether the character string pointed to by the character pointer p is the reverse of the character string pointed to by the character pointer q. If it is, then the function returns 1 and if otherwise, then the function returns 0. **(12)**

```
int isReverse(char *p, char *q);
```

For example, if the first character string is "abc d" and the second character string is "d cba", then the function returns 1.

Constraint: You can't use any library function.

(b) See the following C code fragment. **(12)**

```
#include<stdio.h>
int main()
{
    int x,y;
    scanf("%d",&x);
    y=DigitCount(x);
    pringf("Number of Digits: %d\n",y);
    return 0;
}
```

Using recursive function, Implement the `DigitCount` function where the task of this function is to take a positive integer number and calculate the number of digits in that number. For example, if it takes 3305, then it returns 4.

Constraint: You must have to use recursive function. You can't use any global variable. You can't use any library function.

(c) Implement the following function which takes three double number `num1`, `num2` and `num3`, and returns the maximum of these three numbers. **(11)**

```
double getMax(double num, double num2, double num3);
```

Constraint: You can't use any library function.

Contd P/3

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4. (a) Mr. Mashrafee wants to encrypt a message (a character string). He wants to replace each alphabetic character (any character which is a letter) by its next alphabetic character of the same case (uppercase/lowercase) that is, 'a' should be replaced by 'b', 'b' should be replaced by 'c' and so on, and 'A' should be replaced by 'B', 'B' should be replaced by 'C' and so on. The last alphabetic character should be replaced by the first alphabetic character of the same case ('z' should be replaced by 'a' and 'Z' should be replaced by 'A'). Mr. Mashrafee doesn't care about the non-alphabetic character (any character which is not a letter) and keeps as it is. **(20)**

Using pointer arithmetic, you have to write the full C code which will serve the above-mentioned purpose of Mr. Mashrafee.

Constraints: you must have to use pointer arithmetic. You can't use any library function except the library functions used for input/output purpose (printf, scanf, puts, gets etc.).

Sample Input	Corresponding Output(s)
Hello! Zurith!!	Ifmmp! Avsjdi!!
I took 10$	J uppl 10$

(b) Write a C code to find out the number of distinct numeric characters (any character which is a digit) in a character string **using pointer arithmetic**. Suppose, the character string is "ab0283! N207#$". Here, the number of distinct numeric characters is 5 ('0', '2', '8', '3' and '7'). So, the output should be 5. **(15)**

Constraints: You must have to use pointer arithmetic. You can't use any library function except the library functions used for input/output purpose (printf, scanf, puts, gets etc.)

Sample Input	Corresponding Output(s)
Hello123! Zurith 30!!	4
I took 10$	2

SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Write a C program to calculate the following series up to n terms. **(14)**

$$1+{}^{2!}\!/_{2^2}+{}^{3!}\!/_{3^3}+\cdots$$

(b) A number is called an "even_sum" number if the sum of its digits is an even number. For an example, 431 is an even_sum number as $8(4+3+1)$ is an even number. Write C program to determine whether a given integer n is an even_sum number. **(14)**

Contd P/4

CSE 295/IPE

Contd... Q. No. 5

(c) What is the output of the following code segment? **(7)**

```
int numbers[] = {2,3,5,7,9,8,1};
int i=0, sum=0;
while (numbers[i] < 6)
    {
        sum += numbers[i];
         i+=2;
    }
        printf("Sum: %d", sum);
```

6. (a) Given an integer n, write a C program to create a pyramid as shown in the examples. Here n indicates the number of rows you need to print. **(15)**

Examples:

n = 2	n = 3
1 1 2 1	1 1 2 1 1 2 3 2 1

(b) Given an integer n, write a C program to find the reverse of n (store the value to an integer variable named reverse_n). For an example, if n = 432, then the reverse_n = 234. **(14)**

(c) Given three integers, write a C program using if-else to find the second maximum of these three numbers. **(6)**

7. (a) Given a string str with small case letters, write a C program to sort the characters in str in alphabetical order. For an example, if str = "programming", sorted str = "aggimmnprro". **(12)**

(b) Given two strings str1 and str2 (with lengths len1 and len2, respectively), and an integer m, write a C program to insert str2 into str1 at index m. Assume that m < len1. You cannot use any string other than the given two strings. **(18)**

`Str1 = "abcdefg"`
`Str2 = "Hello"`
`m = 2`
`str1` after insertion:
`"abHellocdefg"`
	`Str1 = "Rocks"`
	`Str2 = "IPE"`
	`m = 0`
	`Str` after insertion:
	`"IPERocks"`

(c) Show an example of a scenario where you may prefer do-while loop over while loop. **(5)**

Contd P/5

CSE 295/IPE

8. (a) An Identity matrix is a square matrix in which all the elements of the principal diagonal are ones and all other elements are zeros. Write a C program to determine whether a given square matrix M (a two dimensional array of integers) with n rows and n columns is an identity matrix. **(12)**

(b) Given two string str1 and str2, write a C program to determine whether str2 is a substring of str1. **(16)**

(c) Given the marks of a subject, write a C program using if-else to calculate the grade as follows: **(7)**

Marks >=80: Grade A

Marks >=70: Grade B

Marks >=60: Grade C

Marks <=60: Grade D

L-2/T-1/IPE **Date : 16/07/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2016-2017

Sub : **ME 245** (Engineering Mechanics and Theory of Machines)

Full Marks: 280 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols indicate their usual meanings. Assume any missing data.

1. (a) A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB* and is pulled at a constant speed by cable *CD* as shown in Fig. for Q. No. 1(a). Knowing that $\alpha = 30°$, $\beta = 10°$ and that the combined weight of the boatswain's chair and the sailor is 900 N, determine the tension (i) in the support cable *ACB*, (ii) in the traction cable *CD*. **(20)**

   (b) Three cables are connected at *A*, where the forces *P* and *Q* are applied as shown in Fig. for Q. No. 1(b). Knowing that $P = 1200$ N, determine the range of values of *Q* for which cable *AD* is taut. **(26⅔)**

2. (a) For the beam and loading as shown in Fig. for Q. No. 2(a), determine the range of the distance *a*, for which the reaction at *B* does not exceed 100 N downward or 200 N upward. **(26⅔)**

   (b) Determine the force in member *BD*, *CE* and *DG* of the Fink roof truss shown in Fig. for Q. No. 2(b). State whether the members are in tension or compression. **(20)**

3. (a) The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force *P* required to start the 30-kg block moving if **(20)**

   (i) cable *AB* is attached as shown in Fig. for Q. No. 3(a) and

   (ii) cable *AB* is removed.

   (b) Determine the capacity, shown in Fig. for Q. No. 3(b), of the punch bowl if $R = 250$ mm using Pappus-Guldinus theorem. **(26⅔)**

4. (a) Determine the moments of inertia I_x and I_y of the area shown in Fig. for Q. No. 4(a) with respect to centroidal axes respectively parallel and perpendicular to side *AB*. **(28)**

   (b) Automobile *A* is traveling east at the constant speed of 36 km/h as shown in Fig. for Q. No. 4(b). As automobile *A* crosses the intersection shown, automobile *B* starts from rest 35 m north of the intersection and moves south with a constant acceleration of 1.2 m/s^2. Determine the position, velocity, and acceleration of *B* relative to *A*, 5 s after *A* crosses the intersection. **(18⅔)**

Contd P/2

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. In the toggle mechanism, shown in Fig. for Q. No. 5, the slider D is constrained to move on a horizontal path. The crank OA is rotating in the counter-clockwise direction at a speed of 180 rpm, which is increasing at the rate of 50 rad/s^2. The dimensions of the various links are as follows: **(46⅔)**

OA = 180 mm; CB = 240 mm; AB = 360 mm; and BD = 540 mm.

For the given configuration, find-

(a) Velocity of slider D and angular velocity of link BD;

(b) Acceleration of slider D and angular acceleration of link BD.

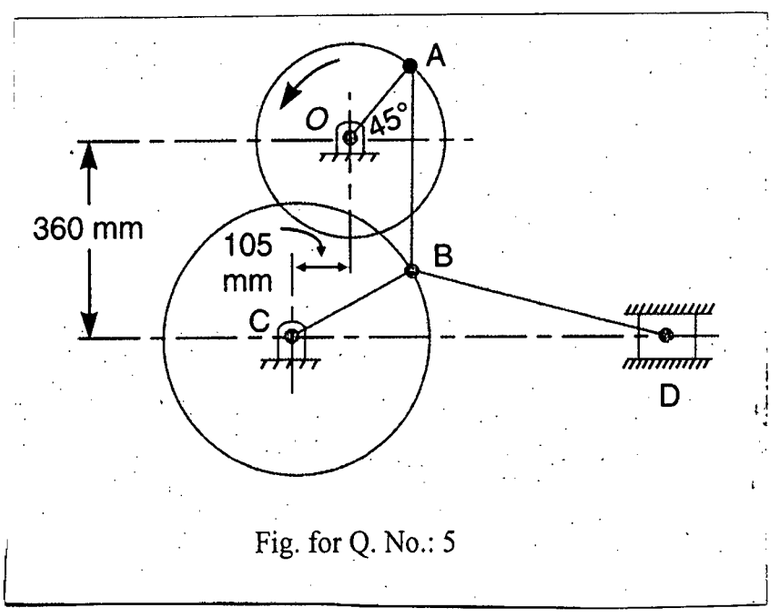

Fig. for Q. No.: 5

6. (a) A rear engine automobile is travelling along a track of 100 meters mean radius. Each of the four road wheels has a moment of inertia of 2.5 kg-m^2 and an effective diameter of 0.6 m. The rotating parts of the engine have a moment of inertia of 1.2 kg-m^2. The engine axis is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The ratio of engine speed to back axle speed is 3:1. The automobile has a mass of 1600 kg and has its centre of gravity 0.5 m above road level. The width of the track of the vehicle is 1.5 m. Determine the limiting speed of the vehicle around the curve for all four wheels to maintain contact with the road surface. Assume that the road surface is not cambered. **(30⅔)**

Contd P/3

ME 245/IPE

Contd... Q. No. 6

(b) A mass of 12 kg is attached to two springs each of stiffness 4000 N/m as shown in Fig. for Q. No. 6(b). At the equilibrium position, there is no force in the spring. The mass is then displaced by a small distance, x to the right of its equilibrium position and released. Find the natural frequency of the system for small oscillations about the equilibrium position? **(16)**

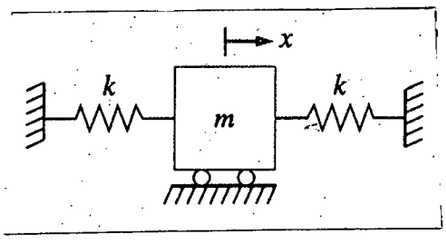

Fig. for Q. No. 6(b)

7. (a) A single cylinder oil engine drives directly a centrifugal pump. The rotating mass of the engine, flywheel and pump with the shaft is equivalent to a three rotor system as shown in Fig. for Q. No. 7(a). Mass moment of inertia of the rotors A, B, C are 0.15, 0.3 and 0.09 kg-m^2 respectively. Find the natural frequency of the torsional vibration of the system. The modulus of rigidity for the Shaft material is 84 GN/m^2. **(26⅔)**

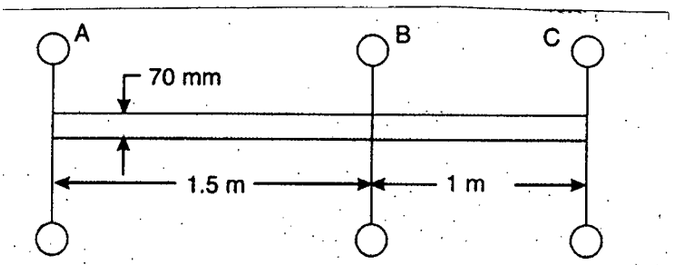

Fig. for Q. No.: 7(a)

(b) Two uniform identical slender rods AB and CD of mass m is maintained in the horizontal position by two springs and a rigid massless link as shown in Fig. for Q. No. 7(b). The end B of rod AB is then depressed by a small distance and released. Derive the expression of the natural frequency of free vibration for the system (neglect the effect of the mass of the spring). **(20)**

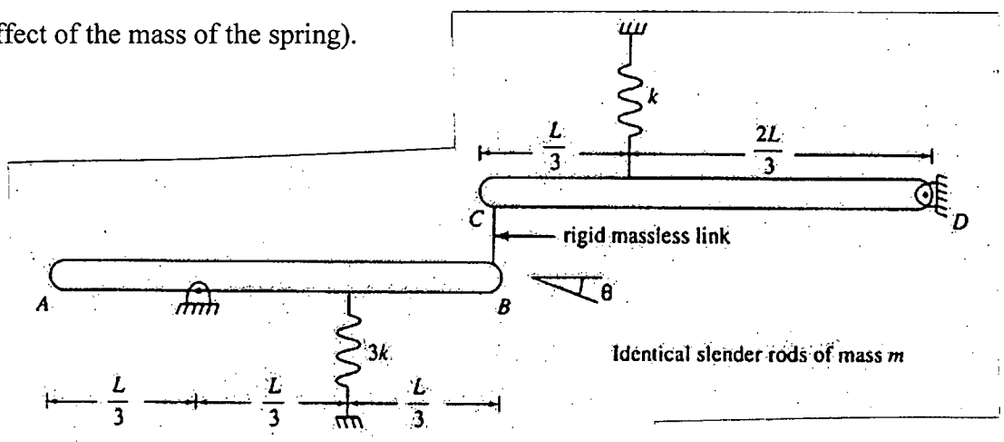

Fig. for Q. No.: 7(b)

Contd P/4

ME 245/IPE

8. Construct the profile of a cam to suit the following specifications: **(46⅔)**

- Least radius of cam = 25 mm;
- Diameter of roller = 25 mm;
- Angle of lift = 120°;
- Angle of fall = 150°;
- Lift of the follower = 40 mm;
- Number of pauses are two of equal interval between motions.

During the lift, the motion is S.H.M. During the fall, the motion is uniform acceleration and deceleration with the magnitude of acceleration being same as the deceleration. The speed of the cam shaft is uniform. The line of stroke of the roller type follower passes through the centre of the cam.

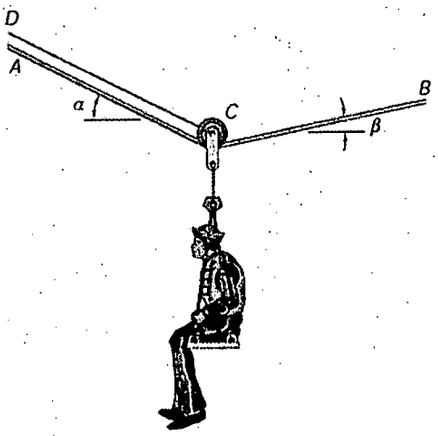

Fig. for Q. No. 1(a)

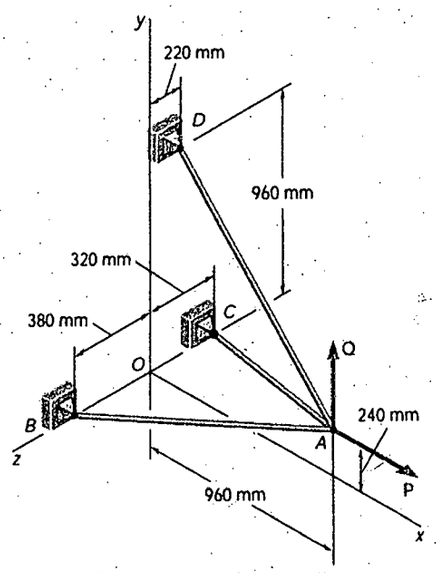

Fig. for Q. No. 1(b)

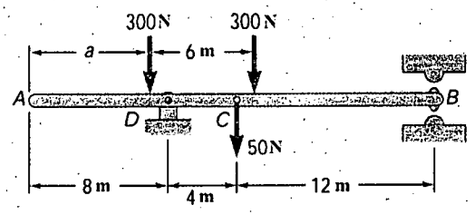

Fig. for Q. No. 2(a)

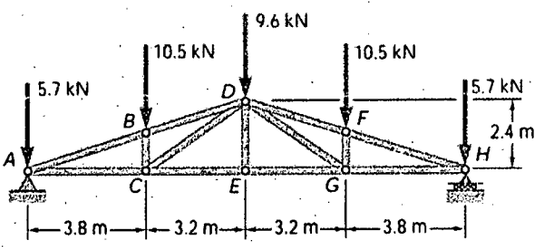

Fig. for Q. No. 2(b)

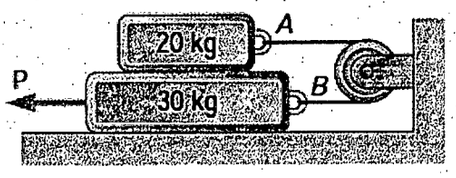

Fig. for Q. No. 3(a)

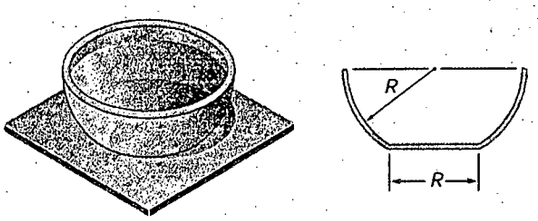

Fig. for Q. No. 3(b)

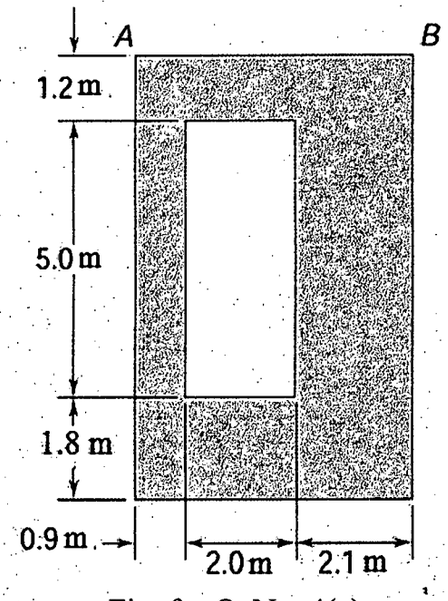

Fig. for Q. No. 4(a)

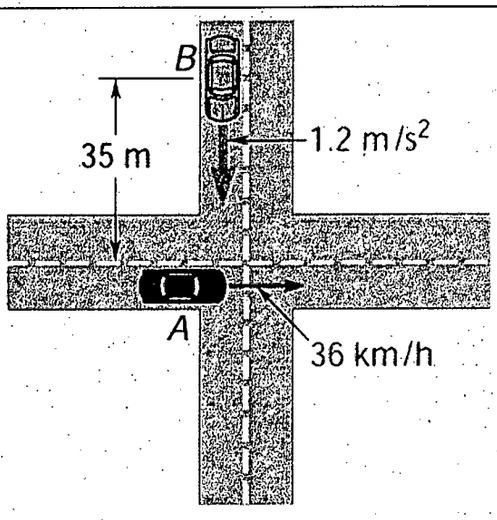

Fig. for Q. No. 4(b)

L-2/T-1/IPE **Date: 09/08/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2016-2017

Sub: **MME 295** (Engineering Materials - II)

Full Marks: 140 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Suppose your are given two Al_2O_3 (ceramic) samples. You tested one in tensile loading and the other in compressive loading. For which cases the obtained fracture load will be higher? With neat sketches explain your answer in terms of the mechanism of failure. **(2+6+4=12)**

   (b) Mention the characteristic features of high performance ceramics. **(5)**

   (c) Discuss some common methods of joining of ceramics. **(6⅓)**

2. (a) What is Superconductivity? **(2)**

   (b) Describe the working principle of ruby laser with necessary sketches. **(8)**

   (c) Discuss briefly the factors on which glass transition temperature (Tg) depends. **(6⅓)**

   (d) Give short descriptions on laminated glass and toughened glass. **(7)**

3. (a) Why tempering is done after heat treatment of glass? How is it done? Explain with necessary figures: **(3+5+2=10)**

   (b) What are piezoelectric, pyroelectric and ferroelectric ceramics? **(6)**

   (c) Explain the tape casting by doctor blade process with schematic diagram. **(7⅓)**

4. (a) Illustrate with figures the mechanism of hardening of Portland Cement. **(15)**

   (b) Which glass farming method would you select for the manufacturing of glass rod? Describe the method with figures. **(8⅓)**

SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) We have a polyethylene sample containing 4000 chains with molecular weights between 0 and 5000 g/mol, 8000 chains with molecular weights between 5000 and 10,000 g/mol, 7000 chains with molecular weights between 10,000 and 15,000 g/mol, and 2000 chains with molecular weights between 15,000 and 20,000 g/mol. Determine both the number and weight average molecular weights. **(7)**

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(b) How are thermosetting polymers formed? **(5)**

(c) Briefly discuss the addition polymerization reactions. **(6)**

(d) Describe the crystal structures of zinc blende and perovskite structures mentioning positions of atoms and coordination number. **(5⅓)**

6. (a) On the basis of crystal structure, compute the theoretical density of cesium chloride. Given, atomic radius: $r_{Cs} = 0.167$ nm, $r_{Cl} = 0.181$ and $M_{Cs} = 132.9$ g/mol, $M_{Cl} = 35.45$ g/mol. [Hint: atoms touch along the body diagonal] **(6)**

(b) What are the three types of materials required for traditional ceramic manufacturing? Describe their function as a raw material and give examples. **(7)**

(c) Briefly explain the dry/semi-dry pressing, hydroplastic forming and slip casting process of ceramic forming. **(10⅓)**

7. (a) "During drying only two types of water present in the formed ceramic body is evaporated; the other two types are not removed" — explain. **(6)**

(b) Why do ceramics need to be sintered? Briefly discuss the stages of sintering. **(8)**

(c) During drying of a ceramic body two periods are observed. The constant rate-period and the falling rate period. Give the drying curve and explain this occurrence. **(5)**

(d) Suppose you have two batches of ceramic powders. Batch A has mostly spherical particles about 2 microns in size and batch B contains irregular shaped particles of diameter 1 micron on average. If these powders are compacted to form two same ceramic bodies, which one will sinter more easily and why? **(4⅓)**

8. (a) Draw and discuss different types of molecular structures of polymers. **(5)**

(b) Write short notes on thermosets and elastomers. **(8)**

(c) Calculate the degree of polymerization if 6,6-nylon has a molecular weight of 80,000 g/mol. 1 mol hexamethylene diamine and 1 mol adipic acid react to form 1 mol 6,6 nylon and 2 mol of water. The molecular weights are 116 g/mol for hexamethylene diamine, 146 g/mol for adipic acid, and 18 g/mol for water. **(5)**

(d) Briefly mention the functions of different polymer additive. **(5⅓)**

L-2/T-1/IPE **Date: 05/08/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2016-2017

Sub: **MATH 291** (Differential Equation, Vector Calculus and Laplace Transform)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Form the differential equation of the family of curves $y = c(x-c)^2$ by eliminating the arbitrary constant c. **(11)**

   (b) Solve the differential equation $(3x-2y+1)dx-(6x-4y+1)dy=0$. **(12)**

   (c) Solve $\frac{dy}{dx} = x^3y^3 - xy$. **(12)**

2. (a) Solve the differential equation $\frac{dy}{dx}+\left(2x\tan^{-1}y-x^3\right)\left(1+y^2\right)=0$. **(11)**

   (b) Solve $\left(D^3-3D^2+3D-1\right)y = xe^x+e^x$. **(12)**

   (c) Apply the method of variation of parameters to solve the differential equation **(12)**

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-4x\frac{dy}{dx}-\left(1+x^2\right)y=x.$$

3. Using the method of Fröbenious find the series solution of the differential equation **(35)**

$$\left(x-x^2\right)\frac{d^2y}{dx^2}+(1-5x)\frac{dy}{dx}-4y=0.$$

4. (a) Show that the Legendre polynomial $P_n(x)$ is the coefficient of h^n in the expansion of $\left(1-2xh+h^2\right)^{-\frac{1}{2}}$. **(17)**

   (b) For the Bessels function $J_n(x)$ prove that **(18)**

$$J_n(x)=\frac{1}{\pi}\int_0^{\pi}\cos(n\phi-x\sin\phi)d\phi \text{ where } n \text{ is any integer.}$$

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Prove that $\int_0^{\infty}\frac{1-\cos t}{t^2}dt=\frac{\pi}{2}$ by Laplace transform. **(10)**

   (b) Evaluate $L^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\}$ by using the convolution theorem. **(10)**

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(c) Using Laplace transform solve the given system of differential equations (15)

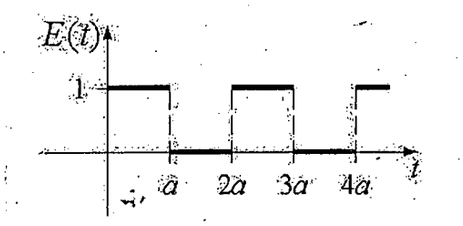
$$2x'(t)+y'(t)-2x(t)=1$$
$$x'(t)+y'(t)-3x(t)-3y(t)=2$$

Subject to x(0) = y(0) = 0.

6. (a) Use the Laplace transform to solve the given integral equation for $f(t)$: (15)

$$f(t)+2\int_0^t f(u)\cos(t-u)du = 4e^{-t}+\sin t$$

(b) Solve the differential equation $L\frac{di}{dt}+Ri(t)=E(t)$ subject to $i(0)=0$ with L = 1, R = 1 and $E(t)$ is given by the following square wave function with amplitude 1 and a = 1. **(20)**



7. (a) Find the values of the constants a,b,c such that the directional derivative of $\phi = axy^2+byz+cz^2x^3$ at (1,2,–1) has a maximum magnitude 64 in the direction parallel to z-axis. **(20)**

(b) Prove the following identities, assuming that all derivatives involved exist and are continuous. Here $\vec{F}=\vec{F}(x,y,z)$ and $\varphi=\varphi(x,y,z)$. **(10)**

(i) $div(curl\vec{F})=0$

(ii) $curl(\varphi\vec{F})=\varphi.curl(\vec{F})+div\varphi\times\vec{F}$

(c) If c_1 and c_2 are constant vectors and λ is a constant scalar, show that $H=e^{-\lambda x}(c_1\sin\lambda y+c_2\cos\lambda y)$ satisfies the partial differential equation $\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}=0$. **(5)**

8. (a) Given the vector field $\vec{G}=(16xy-z)\hat{i}+8x^2\hat{j}-x\hat{k}$ **(20)**

(i) Is $\vec{G}$ irrotational (or conservative)?

(ii) Find the net flux of $\vec{G}$ over the cube $0<x,y,z<1$.

(b) Consider the vector field given by $\vec{F}=(x-z)\hat{i}+(y-x)\hat{j}+(z-xy)\hat{k}$. Use Stokes' theorem to find the circulation around the triangle with vertices A(1,0,0), B(0,2,0), and C(0,0,1) oriented counterclockwise looking from the origin toward the first octant. **(15)**
