There are FOUR questions in this section. Answer any THREE.

1. (a) Consider the instruction: "call 0020: 00ABCDEF". Here, a call gate has been used. Deduce the segment and offset of the instruction that will be executed next. Use Table A for GDT. You may be use Table B and Table C to refresh your memory.

(b) Modify the appropriate entry of the GDT so that the next instruction to be executed is at offset 1234. You can change only one entry in the GDT (Table A); you cannot add any new entry. You need to explain your modification and write down the modified entry. You have to assume that a call gate is being used for the call. Would it be possible to make the offset FF12? Justify your answer.

(c) Your genius little brother looks at the GDT and the above two questions and comments with a crude smile that the call gate was not at all necessary! Do you agree with him? Justify your answer.
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2. (a) Describe the logical address to linear address translation in the microprocessor considering both segmentation and paging with appropriate illustrations. 

(b) What is identity mapping with respect to paging? What could be the use of such a mapping?

3. (a) What is a program-invisible register? Describe the program-invisible registers in advanced microprocessors.

(b) What is a translation Look-aside Buffer and how is it used?

(c) Describe the Flat Mode Memory System.

4. (a) Describe the Task State Segment with an appropriate illustration. Why does TSS have three different sets of stack related registers?

(b) Describe the task switch operation with appropriate illustrations.

SECTION-B

There are FOUR questions in this section. Answer any THREE questions.

05. (a) Suppose, Machine A is trying to send data to Machine B in double handshake mode. However, they are facing a problem. The timing diagram they are following is shown below.

(i) What are the problems of their timing diagram?

(ii) Correct the timing diagram. In your corrected timing diagram mark each transition of every signal with a number surrounded by a circle. For each transition you will have to write down in a separate table who initiates the transition (Machine A or B) and what the transitions signifies.

(b) Implement a C function void UART_send (unsigned char data) which receives a character as an argument and transmits it using UART by polling on the TXC bit of UCSRA.

(c) Suppose you want to use SPI for sending data to three different devices A, B, and C one after another. Briefly explain how you can achieve this with necessary block diagrams.

Contd ............ P/3
6. (a) Suppose one active low push switch A and two active high push switches, B and C, are connected to INTO, INT1, and INT2 pin of an ATmega32 MCU, respectively. Also, eight LEDs are connected to PORTB. Write a C code to implement an 8-bit ring counter which counts up when the push switch A is pressed and counts down when B is pressed. Pressing C will reset the counter to 0. The output is shown with the LEDs. Keep in mind that the buttons bounce a lot. Briefly describe how the relevant registers were set and how debouncing was achieved.

The codes for external interrupt events of INT0 and INT1 are as following:

<table>
<thead>
<tr>
<th>Code</th>
<th>Interrupt Triggering Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>Low Level</td>
</tr>
<tr>
<td>01</td>
<td>Any Logical Change</td>
</tr>
<tr>
<td>10</td>
<td>Falling Edge</td>
</tr>
<tr>
<td>11</td>
<td>Rising Edge</td>
</tr>
</tbody>
</table>

The code for external interrupt events of external INT2:

<table>
<thead>
<tr>
<th>Code</th>
<th>Interrupt Triggering Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Falling Edge</td>
</tr>
<tr>
<td>1</td>
<td>Rising Edge</td>
</tr>
</tbody>
</table>

(b) Consider you are using ADC with a reference voltage of 5V and you are reading the ADC value only from ADCH. Calculate the maximum precision error you will get when the ADC value is (i) left adjusted and (ii) right adjusted.

(c) Consider you are reading a tape recorder with an 8088 microcomputer system with a built-in 8255 PPI (e.g., MTS-88.C). The tape recorder is connected to PORTA and it is read using strobed I/O mode. You also have an output device connected to PORTB which works in simple I/O mode. You will have to design a flowchart to continuously read data from the tape and output it to PORTB. The 8255 is connected with the 8088 microprocessor in the address 010000xxh. Clearly specify the control word.

7. (a) Consider a buggy C code "Buggy.c" in Fig.3, which attempts to receive a byte every second using UART with the following connection parameters: 1200 bps baud rate, even parity, 1 stop bit, normal speed mode, and 8 data bits. Rewrite the code correcting all the mistakes. Clearly mark the portion of your code added or, modified and specify what was the mistake before.

Assume the clock speed of the ATmega32 MCU is set at 8MHz.

The code for parity is as follows: 00, 10, and 11 is for no parity, even parity, and odd parity, respectively.

The code for stop bit is as follows: 0 for 1 and 1 for 2 stop bits.

The code for 8 data bits is 011.

Ignore the time needed for polling and also the status of error bits.
(b) The digital output of ATmega32 is stored in two 8-bit registers: ADCL and ADCH. While reading the 16-bit digital output, does the order of reading from these two registers matter? Explain why or why not.

(c) Suppose two active high push switches are connected to PA0 and PA4 of an ATmega32 MCU. Also, eight active low LEDs are connected to PORT B. Write a C code to implement an 8-bit ring counter which counts up when the push switch connected to PA0 is pressed and counts down when the other one is pressed. The output is shown with the LEDs. Use polling approach. Keep in mind that the buttons bounce a lot. Also assume the push switches will not be pressed simultaneously.

8. (a) Suppose you are using a particular temperature sensor which produces an output of 0V to 3.3V for 0 degree to 330 degree Celsius linearly. Write a C code to use ATmega32 ADC in polling mode to read the sensor value and determine the temperature using (i) by reading only ADCH (ii) by reading ADCH and ADCL. Just store the temperature in a variable. Assume that you are using internal reference voltage of 5V (code: 0x1) and a prescaler of 2 (code: 0x1). The sensor is connected to the pin ADC0 (code: 0x0).

(10)

(b) Draw a timing diagram showing SPI data transfer format for SPI mode 1 (CPOL = 0 and CPHA = 1). Clearly show with respect to clock pulse when the data is sampled and when relevant pins are changed. In SPI mode 1 reading (rising) edge is setup edge and trailing (falling) edge is sample edge.

(10)

(c) Briefly explain how the start bit and data bits are sampled for UART in ATmega32. Your explanation must mention the key difference of sampling process between the normal and double speed modes.

(10)
```c
#include <avr/io.h>
#include <util/delay.h>

unsigned char UART_receive(void) {
    while (((UCSRA & (1<<RXC))) );
    return UDR;
}

int main(void) {
    DDRB = 0xFF;
    UCSRA = 0b00000010;
    UCSRB = 0b00001000;
    UCSRC = 0b00001100;
    UBRRL = 0x04;
    UBRRH = 0x16;

    while(1) {
        unsigned char c = UART_receive();
        PORTB = c;
        _delay_ms(1000);
    }
}
```

**Figure 3 (For Question 7(a))**

<table>
<thead>
<tr>
<th>Register Name</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>GICR</td>
<td>INT1</td>
</tr>
<tr>
<td>GIFR</td>
<td>INTF1</td>
</tr>
<tr>
<td>MCUCR</td>
<td>SE</td>
</tr>
<tr>
<td>MCUCSR</td>
<td>JTD</td>
</tr>
<tr>
<td>TCCR1A</td>
<td>COMA1A</td>
</tr>
<tr>
<td>TCCR1B</td>
<td>ICNC1</td>
</tr>
<tr>
<td>TIMSK</td>
<td>OCIE2</td>
</tr>
<tr>
<td>TIFR</td>
<td>OCF2</td>
</tr>
<tr>
<td>UCSRA</td>
<td>RXC</td>
</tr>
<tr>
<td>UCSRB</td>
<td>RXCIE</td>
</tr>
<tr>
<td>UCSRC</td>
<td>URSSEL</td>
</tr>
<tr>
<td>SPCR</td>
<td>SPIE</td>
</tr>
<tr>
<td>SPSR</td>
<td>SPIF</td>
</tr>
<tr>
<td>ADMUX</td>
<td>REFS1</td>
</tr>
<tr>
<td>ADCCSRA</td>
<td>ADEN</td>
</tr>
</tbody>
</table>

Table 1 List of registers
SECTION - A

There are FOUR questions in this section. Answer any THREE.

1. (a) What are the main problems in a monolithic system? Explain with a necessary diagram. (7)
   (b) How does the file subsystem interact with hardware in Unix? Explain with respect to Unix System Kernel structure. (10)
   (c) Some buffers are marked as “delayed write”. What does this mean and how are those handled while allocating a new buffer by Unix Kernel? (10)
   (d) What are the information kept in buffer headers? (8)

2. (a) Explain with necessary diagrams all possible scenarios while allocating a buffer (using getblk algorithm) when the buffer cache of free buffers is non-empty. (12)
   (b) What are the two types of inodes: disk inode and incore inode? What additional information are needed for incore inodes? Why are they absent in disk inode? Explain. (8)
   (c) What is the significance of inode link count while freeing an inode (in algorithm input)? What happens when inode reference count is zero but link count is non-zero? (7)
   (d) Write down the steps to convert filename into inode. (8)

3. (a) Consider that in a Unix system, each data block size is 4k bytes and 32 bits are required for data block addressing. Assuming usual table of contents structure of Unix System V, find out the maximum file size of such system. (10)
   (b) If 32-bytes are necessary to store information of a directly or a file, then how many files or subdirectories are possible inside a directly of a system explained in Question # 3(a). (5)
   (c) What is the basic difference of data structures for the following two: (i) Superblock free inode list (ii) Superblock free disk block list? Explain with necessary diagrams. Give the reason of having such different data structure. (15)
   (d) “Remember disk block number is used to search next available disk blocks” – Justify the statement. (5)

4. (a) What happens when a create file system call is invoked? Write down the algorithm and show the file data structures. (10)

Contd ........... P/2
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Contd... Q. No. 4

(b) Some region’s size can be altered while some other’s size cannot be altered (using growreg algorithm). Explain the reason and give examples.

(c) Is it possible to have an interrupt within the service routine of another interrupt? Explain with respect to the context of a process.

(d) What happens when the Kernel wakes up a sleeping process? Explain with necessary data structure.

SECTION-B
There are NINE questions in this section. Answer any SEVEN questions.

5. (a) Show the states of a process and the possible transitions between the states in a diagram. Give an example for each possible transition.

(b) Suppose you are going to write a program that will compute the sum of all possible 64 bit integers. Discuss how a multithreaded version of the program will affect the time required in comparison to that by single threaded program.

(c) What do you mean by busy waiting? What are the problems of busy waiting?

6. (a) What do you mean by CPU bound and I/O bound processes? Why I/O bound processes are given higher priority in priority scheduling algorithms?

(b) Discuss the convey effect in First Come First Serve scheduling algorithm.

(c) Consider the following code of producer process for a producer consumer system.

```c
semaphore empty = 10;
semaphore full = 0;
mutex m;
void producer(void) {
    int item;
    while(TRUE) {
        item = produce_item();
        lock(&m);
        down(&empty);
        insert_item(item);
        up(&full);
        unlock(&m);
    }
}
```

Will the above code work correctly? If not, explain the problems and write down the correct code for producer. If you think this code to be correct, write down the corresponding code for consumer.
7. (a) Consider the following 4 processes:

<table>
<thead>
<tr>
<th>Process</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>30</td>
</tr>
<tr>
<td>P2</td>
<td>10</td>
</tr>
<tr>
<td>P3</td>
<td>19</td>
</tr>
<tr>
<td>P4</td>
<td>56</td>
</tr>
</tbody>
</table>

Assume all processes arrived at the same time. Draw Gantt charts and then calculate average turnaround times for the processes using the following scheduling algorithms:

(i) Shortest Job First
(ii) Round Robin with quantum 8.

(b) Discuss the limitations of Shortest Remaining Time Next scheduling algorithm.

8. Write short notes on the following terms:

(i) Priority Scheduling
(ii) Guaranteed Scheduling
(iii) Lottery Scheduling.

9. (a) What do you mean by preempt able and non-preempt able resources? Briefly discuss the four conditions for deadlock.

(b) Suppose a system is detected to be in deadlock. Discuss some ways to recover from it.

10. (a) What are the problems of exposing physical memory to processes?

(b) Suppose you are going to use bitmaps to keep track of memory usage. Discuss the issues you need to consider while determining the size of each allocation unit.

(c) The Not Frequently Used (NFU) page replacement algorithm keeps a software counter associated with each page and increments it if the page had been referenced after previous clock interrupt. When a page fault occurs, the page with the lowest counter is chosen for replacement. Discuss a case where this algorithm will not perform optimally. Can you suggest a modification to the NFU algorithm to overcome this limitation?

11. (a) Describe how deadlock can be detected by monitoring CPU utilization.

(b) Suppose there are four processes P1, P2, P3, P4 and four resources R1, R2, R3, R4 in a system. The current allocation matrix and request matrix are as followed:
The availability resource vector is $[3 \ 2 \ 2 \ 1]$. Determine whether the processes will be in deadlock or not.

12. (a) Suppose you have four processes with length 48, 64, 120, and 28 bytes respectively. The memory available to your system is 100 bytes. Discuss a way by which you can run all the four processes parallelly in your system.

(b) What are the disadvantages if the page size is too large or too small?

13. (a) What do you mean by pre-paging and demand paging?

(b) What is TLB? Briefly describe how the TLB functions.

(c) Briefly discuss how page sharing between processes can be handled by a memory management unit.
1. (a) A routing protocol works with five consecutive steps as follows: i) Discover neighbor routers, ii) Measure cost to each neighbor, iii) Construct a packet incorporating costs to all neighbors, iv) Send the packet to all other routers, and v) Compute shortest path to all routers. The trickiest part among these steps is the fourth one, i.e., sending the packet (consisting costs to neighbors) to all other routers. To facilitate this part, a notion of Sequence Number is used. The sequence number helps in identifying fresh packets, and thus, helps to decide on whether or not to send a newly-arrived packet consisting costs to neighbors.

Now, do you think the usage of only sequence number perfectly solves the problem of deciding whether to send a newly-arrived packet consisting costs to neighbors or not? If yes, elaborate how the problem gets solved. If not, then elaborate what are the remaining issues to be solved and describe how they can be solved? (23)

(b) Draw a high-level networking architecture of Internet and show where Path Vector Protocol (related to BGP) is generally used. Besides, justify or refute the following statement with necessary elaboration – “Path Vector Protocol extends Link State Routing Protocol and deals with all routers on the way to find a shortest path”. (23⅔)

2. (a) Two typical approaches for broadcast routing could be to send distinct packets to each router or to flood the same packet over a network. Do you think these approaches possess any limitation? If, not then elaborate all the advantages of these approaches. If, yes then what are the limitations and what could be the possible way(s) to alleviate the limitations? You need to elaborate your answer with necessary figures(s) and description. (23)

(b) A network consists of four sub-networks namely A, B, C, and D. These are connected as follows: A-B-C-D. Here, the packet handling capabilities of the sub-networks are as follows: A and D can handle packets with a maximum size of 2KB, B can handle packets with a maximum size of 1KB, and C can handle packets with a maximum size of 512 B. Now, you are given a job to transmit packets of 2KB from A to D. What will be the problem you are going to face to perform the transmission? Is there any way to solve the problem? If yes, then describe the ultimate consequence. (23⅔)
3. (a) "In TCP, transmissions of three messages are enough to establish a connection except the case of Call Collision." – Justify or refute with necessary figure(s) and elaboration.

(b) "In UDP, a slow sender may pose a threat of substantial overhead, which can be minimized by Nagle’s solution. Similarly, in UDP, Clark’s solution solves the problem exposed by a slow receiver." – Justify or refute with necessary figure(s) and elaboration.

4. (a) A network protocol designer is given the task of controlling network congestion with the primary target of ensuring stability. To do so, he enables the method of additive increase of congestion window in case of finding favorable situation (packets are being successfully transmitted) and additive decrease of congestion window in case of finding adverse situation (packets are being dropped). Do you think this choice will ensure stability over multiple transmitters? If so, then describe how it ensures stability over multiple transmitters. If not, then describe why not and elaborate a better alternative.

(b) Distinguish between TCP Tahoe and TCP Reno with necessary figure(s) and elaboration.

SECTION-B

There are FOUR questions in this section. Answer any THREE questions.

5. (a) When bit stuffing is used as framing method, is it possible for the loss, insertion or modification of single bit to cause an error not detected by the checksum? If not, why not? If so, how can it be ensured that the single bit error is detected? Does the checksum length play a role here?

(b) What are the odds that can arise when two hosts simultaneously send an initial packet in Protocol 4 (Stop and Wait Protocol)? Explain with appropriate figures.

(c) Both Go Back N and Selective Repeat Protocol supports the concept of multiple outstanding frames. If a 3 bit sequence number is used what is the range of distinct sequence number that can be used for frames outstanding at any instant for both the mentioned protocols? Also explain the reasons behind choosing such range for both Go Back N and Selective Repeat Protocol.

(d) A block of bits with n rows and k columns uses horizontal and vertical parity bits for error detection. Suppose that exactly 4 bits are inverted due to transmission errors. Will the mentioned scheme be able to detect this error? Will the scheme be sufficient to correct it?
6. (a) Consider the delay of pure ALOHA versus slotted ALOHA at high load. Which one of the mentioned protocols is preferable in this case? Explain your answer.

(b) A LAN uses binary countdown method as Multiple Access Protocol. At a certain instant, the ten stations have the virtual station numbers 8, 2, 4, 5, 1, 7, 3, 6, 9 and 0. The next three stations to send are 4, 3 and 9, in the mentioned order. What are the new virtual station numbers after all three have finished their transmissions?

(c) The wireless LANs use MACA instead of CSMA/CD protocol. What are the odds of using CSMA/CD protocol for wireless LANs? Under what conditions, if any, would it be possible to use CSMA/CD instead?

(d) What is hidden and exposed station problem? Describe the process how these are problems are solved in 802.11 MAC sub layer protocol.

(e) To increase reliability, some sites use two or more bridges in parallel between pairs of LANs. This arrangement, however, introduces some problems because it creates loops in the topology. Explain the problems with an appropriate figure. How these problems can be overcome?

7. (a) Show DNS operations for a site, say, www.buet.ac.bd with diagram when the local DNS server does not have this URL in its cache.

(b) DNS uses UDP instead of TCP. If a DNS packet is lost, there is no automatic recovery. Does this cause a problem? If so, how is it solved?

(c) From an ISP's point of view, POP3 and IMAP differ in an important way. POP3 users generally empty their mailboxes every day. IMAP users keep their mail on the server indefinitely. Imagine that you were called to advise an ISP on which protocol it should support. What considerations would you bring up?

(d) Briefly describe how e-mail works for ISP customers using both SMTP and POP3 protocol. Use the intervention of DNS server whenever you find it necessary.

8. (a) The CEO of Cisco Inc. gets an idea of developing a mutating routing algorithm (an algorithm that can mutate depending on the network automatically) with collaboration of Google. The CEO asks his legal department to look into it, and they in turn ask engineering department for help. As a result, the chief engineer calls his counter-part at Google to discuss the technical aspects of the project. The engineers then report back to their respective legal departments, which then confer by telephone to arrange the legal aspects. Finally, the two CEOs discuss the financial side of the deal. In this an example of a multi-layer protocol model? Write the advantages of using multi-layer protocol in terms of networking concept.

(b) Write a short note on HTTP protocol of World Wide Web.

(c) What is the principle difference between connection-oriented and connection-less communication?

(d) What is a Virtual LAN? Briefly describe the working procedure of a Virtual LAN.

(e) What is frequency hopping spread spectrum?
SECTION – A
There are Four questions in this section. Answer any THREE.

1. (a) Briefly explain the predator-prey model. What is the significance of the critical point in the phase-plane representation for the Lotka-Volterra model? (11)
(b) What is the Lorenz system of ordinary differential equations? Briefly explain how it can have chaotic solutions for certain parameter values and initial representation of xy projection and xz projection. What is a strange attractor? (14)
(c) What is a slope field? Explain how it can help to visualize the graph of the general solution of a differential equation for the falling parachutist problem. (10)

2. (a) Given the differential equation:
\[ y'' - xy' - y = 0 \]
with the initial condition \( y(0) = 1 \) and \( y'(0) = 0 \), use Taylor’s series method to find out the value of \( y(0.1) \). The solution should include at least three non-zero terms. (10)
(b) How does Runge-Kutta (RK) methods achieve the accuracy of a Taylor series approach without requiring the calculation of higher derivatives? Show how you can obtain different second-order methods for different values of parameter \( a \). (10)
(c) Use the fourth-order RK method to solve the following system of equations over the range from \( x = 0 \) to \( x = 1 \) using a step-size of 0.5 and with initial conditions \( y(0) = 2 \) and \( z(0) = 4 \). (15)

\[
\begin{align*}
\frac{dy}{dx} &= -2y + 4e^{-x} \\
\frac{dz}{dx} &= -\frac{yz^2}{3}
\end{align*}
\]

3. (a) Distinguish between (i) accuracy and precision (ii) truncation error and round-off error of a numerical method. Briefly explain how estimation by Richardson extrapolation achieves improved accuracy over the multiple-application. Simpson’s rules method of numerical integration. (15)
(b) Numerically integrate the function \( f(t) = t \cos t \) using the trapezoidal rule and the Simpson’s \( 3/8 \) rule from the lower limit of 0 to the upper limit \( 2\pi \). (10)
(c) Given the differential equation \( \frac{dx}{dy} = \frac{x^2}{y^2 + 1} \) with the initial condition \( y(0) = 0 \), use Picard’s method to obtain \( y \) for \( x = 0.25 \) and \( x = 0.5 \) (10)

Contd .......... P/2
4. (a) What is the complex form of the Fourier series and how can it be derived? Also show how the complex Fourier coefficients $c_k$ and $c_k^*$ can be expressed in terms of $a_k$ and $b_k$, where symbols carry their usual meaning. 

(b) For the given periodic data in the following table, it is required to find the least-squares fit by a linear combination of sinusoidal functions.

\[
f(t) = A_0 + A_1 \cos \omega t + B_1 \sin \omega t + \ldots + A_n \cos n \omega t + B_n \sin n \omega t
\]

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(t)$</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Calculate $A_0, A_1, B_1, A_2$ and $B_2$.

(c) What is the distinction between Fourier integral and Fourier transform? Write the expressions for discrete Fourier transform and inverse transform. Explain why they are especially suitable for engineering applications.

5. (a) What are the differences between an open method and a bracketing method? Which method is suitable in what situation? Explain it with the help of appropriate examples.

(b) Use (i) fixed-point iteration and (ii) the Newton-Raphson method to determine a root of \( f(x) = x^3 + 1.8x + 2.5 \) using $x_0 = 5$. Perform the computation until $\epsilon_x$ is less than $\epsilon_x = 0.05\%$. Also perform an error check of your final answer.

(c) Explain the problems of the Newton-Raphson method and their remedial measures. Derive the formula used by the Secant method to find the roots of algebraic and transcendental equations.

(b) Determine the real root of \( x^3 = 80 \) using the false-position method within $\epsilon_x = 2.5\%$. Use initial guesses of 2.0 and 5.0.

(c) Given the system of equations

\[
\begin{align*}
-3x_1 + 7x_3 &= 2 \\
x_1 + 2x_2 - x_3 &= 3 \\
5x_1 - 2x_2 &= 2
\end{align*}
\]

(i) Use Gauss elimination with partial pivoting to solve for the $x$'s.

(ii) Substitute your results back into the original equations to check your solution.

7. (a) Explain the problems associated with the Graphical method, Cramer's rule and Gauss elimination method for finding the roots of a system of equations.
(b) Use Gauss-Jordan elimination to solve:

\[
\begin{align*}
2x_1 + x_2 - x_3 &= 1 \\
5x_1 + 2x_2 + 2x_3 &= -4 \\
3x_1 + x_2 + x_3 &= 5
\end{align*}
\]

Do not employ pivoting. Check your answer by substituting them into the original equations.

(c) Solve the following systems of equation by LU decomposition method.

\[
\begin{align*}
x_1 + 7x_2 - 4x_3 &= -51 \\
4x_1 - 4x_2 + 9x_3 &= 62 \\
12x_1 - x_2 + 3x_3 &= 8
\end{align*}
\]

8. (a) What is meant by least square regression? How does it differ from interpolation? Derive the equations of the two constants (say, \( k_0 \) and \( k_0 \)) used by the least square regression criterion.

(b) Given the data

\[
\begin{array}{ccccccc}
   x & 1 & 2 & 3 & 5 & 7 & 8 \\
 f(x) & 3 & 6 & 19 & 99 & 291 & 444 \\
\end{array}
\]

Calculate \( f(4) \) using Newton’s interpolating polynomials of order 1 through 4. Choose your base points to attain good accuracy. What do your results indicate regarding the order of the polynomial used to generate the data in the table?
SECTION – A
There are FOUR questions in this section. Answer any THREE.

1. (a) Establish all recurrence relations satisfied by \( J(n) \) of Josephus problem. (10)
   (b) Compute \( J_5(10,000) \) where every 5th man is deleted. (10)
   (c) Construct a binary free sharing how multi-peg tower of Hanoi is solved using presumed optimal solution where \( p = 7 \) and \( n = 489 \). (15)

2. (a) Deduce average case complexity of Quick sort algorithm using summation factor. (10)
   (b) Deduce \( \sum x^2 H_x \delta_x \) by summation by parts. (10)
   (c) Use combinational argument to establish the value of \( \sum_{k=0}^{m} \binom{k}{m} \). (15)

3. (a) Find the multiplicity of 72 in 200!. (10)
   (b) Deduce recurrence relations satisfied by the Stirling numbers of the first and second kind. (10)
   (c) A group of \( n \) fans of the winning football team throw their hats high into the air. The hats came back randomly, one hat to each of the \( n \) fans. How many ways are there for all \( n \) fans not to end up in having their hats? Solve it using generating function. (15)

4. (a) Construct a generating function for Fibonacci numbers and find their values. (10)
   (b) Given \( n \) cards and \( 1 \) table what is the largest possible overhang by stacking the cards up over the table’s edge? (10)
   (c) prove that \( x^n = \sum_{k=0}^{\lfloor n/k \rfloor} \binom{n}{k} x^k \), integers \( n \geq 0 \). (15)

SECTION-B
There are FOUR questions in this section. Answer any THREE questions.

5. (a) It is known that DVDs produced by a certain company will be defective with probability 0.01, independent of each other. The company sells the DVDs in packages of size 10 and offers a money-back guarantee that if at least 1 of the 10 DVDs in a package is defective, money will be returned. If someone buys 3 packages, what is the probability that he or she will return exactly 1 of them? (13)
(b) An airline knows that 5 percent of the people making reservations on a certain flight will not show up. Consequently, their policy is to sell 52 tickets for a flight that can hold only 50 passengers. What is the probability that there will be a seat available for every passenger who shows up?

(c) Suppose that two teams are playing a series of games, each of which is independently won by team A with probability \( p \) and by team B with probability \( 1-p \). The winner of the series is the first team to win \( i \) games. Find the expected number of games that are played when (a) \( i = 2 \) and (b) \( i = 3 \).

6. (a) At a party \( n \) men take off their hats. The hats are then mixed up and each man randomly selects one. We say that a match occurs if a man selects his own hat. What is the probability of no matches? What is the probability of exactly \( k \) matches?

(b) A computer receives requests for elements stored in its memory. Consider that \( n \) elements \( e_1, e_2, \ldots, e_n \) are initially arranged in some ordered list. At each unit of time a request is made for one of these elements – \( e_j \), being requested, independently of the past, with probability \( P_j \). After being requested, the element is then moved to the front of the list. That is, for instance, if the present ordering is \( e_1, e_2, e_3, e_4 \) and \( e_3 \) is requested, then the next ordering is \( e_3, e_1, e_2, e_4 \). Determine the expected position of the element requested after this process has been in operation for a long time.

(c) In an election, candidate A receives \( n \) votes, and candidate B receives \( m \) votes where \( n > m \). Assuming that all orderings are equally likely, show that the probability that A is always ahead in the count of votes is \( \frac{n-m}{n+m} \).

7. (a) A certain town never has two sunny days in a row. Each day is classified as being either sunny, cloudy (but dry), or rainy. If it is sunny one day, then it is equally likely to be either cloudy or rainy the next day. If it is rainy or cloudy one day, then there is one chance in two that it will be the same next day, and if it changes then it is equally likely to be either of the other two possibilities. In the long run, what proportions of days are sunny? What proportions are cloudy?

(b) Consider a gambler who at each play of the game has probability \( p \) of winning one unit and probability \( q = 1-p \) of losing one unit. Assume that successive plays of the game are independent, what is the probability that, starting with \( i \) units, the gambler’s fortune will reach \( N \) before reaching \( 0 \)?

(c) Define Markov chain. Derive the Chapman-Kolmogorov equations for computing n-step transition probabilities in a Markov chain.
(d) Consider a large population of individuals, each of whom possesses a particular pair of genes, of which each individual gene is classified as being of type $\alpha$ or type $\beta$. Assume that the proportions of individuals whose gene pairs are $\alpha\alpha$, $\beta\beta$, or $\alpha\beta$, respectively are $p_o q_o + q_o + p_o$, such that $p_o + q_o + r_o = 1$ respectively. When two individuals mate, each contributes one of his or her genes, chosen at random, to the resultant offspring. Assuming that the mating occurs at random, in which each individual is equally likely to mate with other individual, determine the proportions $p$, $q$, and $r$ of individuals in the next generation whose genes are $\alpha\alpha$, $\beta\beta$, or $\alpha\beta$ respectively.

8. (a) If $X$ and $Y$ are independent Poisson random variables with respective means $\lambda_1$ and $\lambda_2$, calculate the conditional expected value of $X$ given that $X + Y = n$.

(b) Define moments of a Random Variable. How can we obtain them from moment generating function? Derive the moment generating function for the binomial distribution with parameters $n$ and $p$. If $X$ and $Y$ are independent binomial random variables with parameters $(n, p)$ and $(m, p)$, respectively, then what is the distribution of $X + Y$?

(c) Two chips are drawn at random without replacement from a box that contains five chips numbered 1 through 5. If the sum of chips drawn is even, the random variable $X$ equals 5; if the sum of chips drawn is odd, $X = -3$.

(i) Find the moment-generating function (MGF) for $X$.

(ii) Use the MGF to find the first and second moments.

(iii) Find the expected value and variance of $X$. 

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