Integrated Economic Design of Quality Control and Maintenance Management Using CUSUM Chart with VSIFT Sampling Policy

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CERTIFICATE OF APPROVAL

The thesis titled "**INTEGRATED ECONOMIC DESIGN OF QUALITY CONTROL AND MAINTENANCE MANAGEMENT USING CUSUM CHART WITH VSIFT SAMPLING POLICY**" submitted by Rajesh Saha, Roll no: 1014082135F has been accepted as satisfactory in partial fulfillment of the requirements for the degree of Master of Engineering in Advanced Engineering Management on October 16, 2017.

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It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma

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Rajesh Saha

To the Almighty

To my family

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All credits go to God, the most benevolent and the Almighty, for his boundless grace in successful completion of this thesis.

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ABSTRACT

Due to the close interrelation between statistical process control and maintenance management policy and the necessity of these two key tools in running a smooth production system, this paper presents an integrated economic model for joint optimization of quality control parameters and preventive maintenance policy with cumulative sum (CUSUM) control chart. In this model CUSUM chart is used to monitor both process mean and variance and joint average run length (ARL) is determined by combining mean and variance through absorbing Markov chain approach. Here the CUSUM chart is designed using variable sampling interval fixed time (VSIFT) sampling policy. In order to determine the in control and out of control chart for both mean and variance, Taguchi quadratic loss function and modified linear loss function are used respectively in this model. In this proposed model two types of maintenance policy i.e. imperfect preventive maintenance and minimal corrective maintenance have been considered. The proposed model determines the optimum values of eight test parameters (the sample size (n), the fixed sampling interval (h), the number of subintervals between two consecutive sampling times (η), the control limit coefficient for CUSUM mean chart (k), the warning limit coefficient for CUSUM mean chart (w), the time interval of preventive maintenance (t_{pm}), the control limit coefficient for CUSUM variance chart (k_1) and the warning limit coefficient for CUSUM variance chart (w_1)) so that expected total cost per unit time is minimized. A numerical example is presented to demonstrate the effectiveness of the test model in cost minimization. Nelder-Mead downhill simplex method and Genetic algorithm approaches are applied to search for the optimal values of the eight test parameters for the economic statistical design of VSIFT CUSUM charts. A sensitivity analysis has also been performed to observe the effect of different process parameters on total cost.

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NOMENCLATURE

$(ARLj2)_{E}$	joint average run length during an out-of-control period owing to external reasons
$(ARLj2)_{m/c}$	joint average run length during an out-of-control period owing to machine failure
ARLj1	joint average run length during in-control period
k	control limit coefficient for CUSUM-m chart
w	warning limit coefficient for CUSUM-m chart
k_1	control limit coefficient for CUSUM-S^2 chart
w_1	warning limit coefficient for CUSUM-S^2 chart
C_{lp}	cost of lost production (Tk/job)
C_{Frej}	cost of rejection while the process moves out-of-control
$C_{resetting}$	cost of resetting
T_{evel}	evaluation period
$E[C_{CM}]$ FM1	expected cost of corrective maintenance (CM) owing to failure mode1
$E[C_{PM}]$	expected cost of preventive maintenance (PM)
$E[T_{Cycle}]$	expected cycle length
$E[T_1]$	expected in-control period
$E[N_{CM}]$	expected number of corrective maintenance
$E[f_{alarm}]$	expected number of false alarms during the process
T_{false}	expected search time for false alarm
t_0	expected time spent searching for a false alarm
t_1	expected time to determine occurrence of assignable cause
$E[T_{restore}]$	expected time to restore the process which may be moved out-of-control owing to machine degradation or external causes
$E[TCQ]_{process-failure}$	expected total cost of quality owing to process failure
$[ETCPUT]_{M*Q}$	expected total cost per unit time of integrated maintenance and quality policy
λ	process failure rate
λ_1	failure owing to external causes
λ_2	failure owing to machine degradation
C_{FCPCM}	fixed cost per CM (Tk/component)
C_{FCPPM}	fixed cost per PM (Tk/preventing component)
LC	maintenance personnel cost (Rs/h of preventing machine)
τ	mean elapse time from the last sample before the assignable cause to the occurrence of assignable cause when the maintenance and quality policies are integrated
$MTTR_{CM}$	mean time required for corrective repair (h)
$MTTR_{PM}$	mean time required for preventive repair (h)
N_f	number of failures
t_{PM}	preventive maintenance interval
β_E	probability of nonconforming items produced owing to external cause
P_{FM1}	probability of occurrence of failure owing to failure mode 1
P_{FM2}	probability of occurrence of failure owing to failure mode 2
PR	production rate (job/h)
n	sample size
$E[(C_{repair})_{FM2}]$	the expected cost of corrective maintenance owing to failure mode 2

$T_{resetting}$	time to perform the resetting of the process which moves out-of-control owing to external reason
T_s	time to sample and chart one item
α	type I error probability
β	type II error probability h
R	proportion of non-conforming units when the process is in-control state
$(R_\delta)_{m/c}$	probability of nonconforming items produced owing to machine failure
$(R_\delta)_E$	probability of nonconforming items produced owing to external reasons
$(R'_\delta)_{m/c}$	probability of nonconforming items produced owing to machine failure
$(R'_\delta)_E$	probability of nonconforming items produced owing to external reasons
ɳ	no. of sampling subinterval in between two consecutive fixed sampling interval
$\varphi(.)$	standard normal probability density function
$\phi(.)$	standard normal cumulative distribution function

ABBREVIATIONS

GA	Genetic algorithm
CUSUM	Cumulative Sum
CUSUM-m	Cumulative sum- mean
CUSUM-S^2	Cumulative Sum- variance
FM	Failure Mode
PM	Preventive Maintenance
CM	Corrective Maintenance
MTTR	Mean Time to Repair
VSIFT	Variable sampling interval at Fixed Times
VSR	Variable sampling rate
ETCPUT	Expected Total Cost Per Unit Time
EWMA	Exponentially weighted moving average
CTQ	Critical to Quality
TCQ	Total Cost of Quality
ATS	Average Time to Signal
SPC	Statistical process control
ARL	Average run length
USL	Upper Specification Limit
LSL	Lower Specification Limit
UCL	Upper Control Limit
FCPPM	Fixed Cost per Preventive Maintenance
FCPCM	Fixed Cost per Corrective Maintenance

CHAPTER I

INTRODUCTION

In today's competitive business world, quality plays the role of prime catalyst to satisfy the customers. To achieve desired quality, flawless products and services with least variances are necessary. Statistical process control (SPC) can be a key tool in restricting products and processes to deviate from the desired level. Along with this, the performance of a production system highly depends on the breakdown-free operations of equipment and processes. Because the reduction in the performance of machine or equipment causes deterioration in product quality. Thus a proper maintenance policy becomes handy in reducing breakdowns and process variations to ameliorate quality level. It is evident from this context, quality and maintenance are interrelated to each other. This relationship between quality and maintenance has lead the researchers to develop integrated economic models using the concept of SPC and maintenance to reduce total cost of quality and maintenance.

A control chart is one of the seven basic tools of total quality management (TQM). It is used in monitoring the variations in the characteristics of products or services to maintain a process in a state of statistical control. Conventionally, control charts are designed considering statistical criteria only. Its performance may be unsatisfactory from the economic point of view. But economic control charts are designed considering both economic and statistical criteria and this design is used to determine and evaluate some design parameters. These parameters have large significance because the value of these parameters are determined in a way so that all costs associated with control chart, such as cost of sampling, downtime costs, cost of searching assignable cause due to out of control signal and resetting it, cost of false alarm, cost of accepting nonconforming items etc. are optimized. Besides assignable causes production can be hampered by machine breakdowns which incur corrective maintenance cost. Before complete breakdown machine may degrade from its desirable working conditions. This menial functionality of machines may lead to higher operating cost and rejection. Therefore increase repair and replacement cost. A suitable preventive maintenance (PM) policy can reduce these breakdown rates as well as improve machine function ability. So economic control chart and proper maintenance policy both are necessary to maintain quality and to minimize total costs. Moreover, both PM and quality control add costs in terms of sampling, inspection, down time, repair/replacement, etc. so it's better to integrate these two shop floor policies rather than operating separately.

1.1 Rationale of the Study

Although the economic design of control chart and proper maintenance policy both are important in terms of quality as well as different costs and these two areas have already been proven to be highly correlated [1,2], the usual practice of industries indicates that quality control and maintenance plan are optimized independently. However academic community has shown interests in studying the interrelation of process control and maintenance management and many researchers has developed several integrated models of process quality and maintenance policy. Though many of these integrated models focus on the integration of process quality and preventive maintenance action, most of them ignored the possibility of a machine/equipment failure in terms of machine breakdown or inferior functionality of the equipment which results in deterioration of product quality from the desired level and call for maintenance action. Rather they focused on only on determining a warning limit or fixed time interval beyond which planned maintenance will be carried out.

In most cases of integrating process control and maintenance management, X-bar control chart has been used to monitor the process mean [3]. Although some recent studies have taken Exponentially Weighted Moving Average (EWMA) chart into consideration [4, 5, 6] but, Cumulative Sum (CUSUM) chart, which is very effective to detect a small shift, has been considered by only a few [7]. Since it is cumulative, even minor drifting in the process mean or variance may lead to steadily increasing or decreasing cumulative deviation values. Therefore, this chart is especially useful in detecting slow shifts away from the target value due to machine wear, calibration problems, and so on. It is also useful for process industries, such as chemical products, liquid pharmaceuticals, cold drinks etc., where sample size n=1 is justified.

In a comparative study [8] it is found that in detecting shifts in order of 1 standard deviation or less EWMA chart shows better performance. Whereas CUSUM shows a prompt and accurate response to detecting shifts in between standard deviation 1 and

2. Since in most of the practical cases, allowable slack is almost 1 standard deviation, so CUSUM can be more effective than EWMA chart.

In case of designing these integrated economic models, different sampling interval and sample size policy are getting more attention of the researchers nowadays. Although variable sampling interval (VSI) and variable sample size have (VSS) already been applied in these integrated models in case of sampling interval policy, but most promising Variable Sampling Interval at Fixed Times (VSIFT) [8] action has not yet been explored by many researchers. Reynolds [21] has shown that using VSIFT sampling policy is more advantageous and effective than VSI or VSS policies in designing different control charts specially EWMA and CUSUM. But VSIFT policy has never been used in designing CUSUM chart, in case of an integrated economic model of quality control and maintenance management using.

In most of the control chart models, in control and out of control costs are calculated based on traditional American goal post view which doesn't consider losses due to deviation from the target value, unlike Taguchi Loss Function. Moreover, every model considers only Process means to monitor the process and to determine Average Run Length (ARL) while considering both mean and variance can be more effective. Though a few study [10, 23] has been conducted on EWMA chart by combining mean and variances, in case of CUSUM chart, it has not yet been done.

Considering the significance of integration of process quality with maintenance management using CUSUM chart and importance of joint ARL combining both mean and variance, the effectiveness of VSIFT sampling policy and Taguchi Loss function to calculate the in control and out of control cost an integrated general economic model is developed to optimize the design parameter and preventive maintenance schedule.

1.2 Objectives of the study

The objective of this thesis is to determine the joint ARL combining both mean and variance and to develop an integrated model that can be used to minimize the expected total cost of process failures, sampling and inspection and maintenance action by jointly optimizing maintenance and CUSUM quality control chart parameters. The specific objectives of this research are

- To compute ARL combining both mean and variance in CUSUM chart.
- To develop an integrated economic model of process quality control and maintenance actions in case of CUSUM control chart parameters, using combined ARL with VSIFT sampling policy incorporating Taguchi Loss Function and modified linear loss function.

- To optimize the integrated model with a suitable nonlinear optimization technique to determine the value of design parameters of the model.
- To design and develop a numerical hypothetical example for clear understanding of the model

1.3 Outline of Methodology

A mathematical cost model has been developed to be used to minimizing the expected total cost of process failures, inspection, sampling and corrective and Preventive maintenance action by jointly optimizing maintenance and quality control chart parameter considering both mean and variance. This mathematical model is used to determine the values of control chart design parameters and preventive maintenance schedule (sample size, fixed sampling interval, number of subintervals between two consecutive sampling times, control limit coefficient for mean, warning limit coefficient for mean, preventive maintenance interval, control limit coefficient for variance and warning limit coefficient for variance) The proposed research methodology is outlined below:

- Machine and process failure is categorized such that whenever a complete machine failure occurs, a corrective maintenance action is used to restore the machine. Besides process is monitored through CUSUM mean (CUSUM-m) and CUSUM variance (CUSUM-S^2) control charts. Whenever a shift is being detected in any of these control charts due to machine degradation a corrective action is carried out, if the shift occurs due to external reasons, resetting of the process is done. Preventive maintenance is also carried out at some fixed interval which is also a decision variable along with control chart parameters.

- Combined ARL for mean and variance in CUSUM control chart is computed using absorbing Markov Chain approach.

- The equation to determine the cost of process failure has been developed by adopting Lorenzen-Vance general cost model.

- The equation of both in control and out of control cost for CUSUM-m and CUSUM-S^2 chart is determined using the concept of Taguchi Loss function and modified Kapoor and Wang's [9] linear loss function respectively.

- The cost functions for carrying out both corrective and preventive maintenance have been developed.

- Finally, a total cost function or objective function is formulated for determining the cost of CUSUM chart with VSIFT policy integrating both preventive and corrective maintenance action as well as Taguchi Loss function.

- The objective function is optimized using Genetic Algorithm and Nelder Mead Downhill Simplex Algorithm.

- Then a sensibility analysis is done to analyze the robustness of the model.

CHAPTER II

LITERATURE REVIEW

As a usual practice, control chart has been used for many years in industries to monitor the variances in process and product.Because, an appropriately designed quality control chart may help in identifying any abnormal behavior of the process, thereby helping to initiate a restoration action. Similarly, different kinds of maintenance actions are used to maintain the performance of machine and equipment. Because, an appropriate Preventive Maintenance (PM) policy not only reduces the probability of machine failure but also improves the performance of the machine in terms of lower production costs and higher product quality.So, Quality control and maintenance management are key tools in daily industrial practice. However, researchers have shown that a relationship exists between equipment maintenance and process quality [1], and joint consideration of these two policies can be more effective and lucrative.

Although these integrated models are gaining popularity nowadays among researchers but it all started when at first Tagaras [10] developed an integrated cost model for the joint optimization of process control and maintenance. Following him, Rahim [11] jointly determined the optimal design parameters of an X- bar control chart and preventive maintenance time for a production system with an increasing failure rate. They generalized the model for the economic design of X- bar control charts of Duncan [12], starting from the papers of Lorenzen and Vance [13] and Banerjee and Rahim [14]. The classical model of Duncan [12] and its several extensions including the unified model of Lorenzen and Vance [13] assumed exponentially distributed in-control periods and provided uniform sampling schemes. Banerjee and Rahim [14], however, assumed a Weibull-distributed in-control period having an increasing failure rate and used variable sampling intervals. This article was an extension of the work of Banerjee and Rahim [14], where a general distribution of in-control periods having an increasing failure rate was assumed and the possibility of age-dependent repair before failure was considered. A general distribution of the in-control period was considered and the salvage value of the equipment was introduced. The model allowed the possibility of age-dependent replacement before failure. A replacement before failure was meaningful only when such a replacement yields economic benefits. Intuitively, the residual life beyond a certain age for systems involving increasing hazard rate shock models would be rather short. Consequently, more frequent sampling could be necessary after the system attains a certain age. This, in turn, might increase the operational cost as a result of frequent sampling. Therefore, they argued that it is conceivable that terminating a production cycle at some time beyond this age might yield additional economies. A truncated production cycle was defined to be a production cycle which terminates after the detection of a failure or at a certain pre-specified age, whichever comes first. The question of replacement before failure did not arise for the Markovian shock model because of its memoryless property. Following an approach similar to that of Banerjee and Rahim [14], the focus of their study was to propose a manner by which the frequency of sampling was to be regulated, while taking into account the underlying probability distribution of the in-control duration. The criterion for choosing the sampling plan was that the expected cost per hour of operation should be minimized when the lengths of the sampling intervals were chosen in such a way as to maintain a constant integrated hazard over each sampling interval. Several different truncated and nontruncated probability models were chosen. It was proposed that economic benefits could be achieved by adopting a non-uniform inspection scheme and by truncating a production cycle when it attains a certain age. Numerical examples were presented to support this proposition. Finally, the effect of model specification in the choice of failure mechanism was investigated.

Rahim [15] presented a model for jointly determining an economic production quantity, inspection schedule and control chart design of an imperfect production process. The process was subjected to the occurrence of a non- Markovian shock having an increasing failure rate. The product quality of the process was monitored under the surveillance of an X-bar control chart. The objective was to determine the optimal control chart design parameters and production quantity so as to minimize the expected total cost (the quality control cost and inventory control cost) per unit time. Both uniform and non-uniform inspection schemes were considered. For non-uniform inspection schemes, the lengths of the inspection intervals were regulated to maintain a constant integrated hazard rate over each inspection interval. Examples of weibull shock models having increasing failure rate were provided. It was shown that the non-uniform and decreasing process inspection intervals scheme resulted in a lower cost than the uniform inspection scheme.

Ben-Daya and Duffuaa [16] discussed the relationship and interaction between maintenance, production and quality. Models relating these three important components of any manufacturing system had been briefly reviewed. Two approaches had been proposed to link between maintenance and quality to incorporate that relationship into a model so that they could be jointly optimized. One approach was based on the idea that maintenance affects the failure pattern of the equipment and it should be modeled using the concept of imperfect maintenance. The second approach was based on Taguchi's approach to quality.

Ben-Daya [17] developed an integrated model for the joint optimization of the economic production quantity, the economic design of X- bar control chart and the optimal maintenance level. This model considered a deteriorating process where the in-control period follows a general probability distribution with increasing hazard rate. The proposed model consists of the four different costs: the production setup cost; the inventory holding cost; the quality control cost; and the preventive maintenance cost. In the proposed model, the Preventive Maintenance (PM) actions were supposed to change the failure pattern of the equipment. It was assumed that, after PM, the age of the system would be reduced proportionally to the PM level. This reduction in the age of the equipment would affect the time to shift distribution and consequently control chart design. It would also affect the production run length and consequently the economic production quantity (EPQ). Thus the modeling of the effect of maintenance on the time shift distribution provided the underlying link and allowed the integration of EPQ, the economic design of control chart, and the optimization of the preventive maintenance effort. Compared to the case with no PM, the extra cost of maintenance results in lower quality control cost which would lead to lower overall expected cost. These issues were illustrated using an example of a Weibull shock model with an increasing hazard rate.

Lindeman et al. [18] demonstrated the value of integrating Statistical Process Control and maintenance by jointly optimizing their policies to minimize the total costs associated with quality, maintenance, and inspection. While maintenance is often scheduled periodically, this analysis encourages ''adaptive'' maintenance where the maintenance schedule accelerates when the process becomes unstable. The completion of maintenance was supposed to return the process to its original operating condition and resulted in a process renewal. The analysis demonstrated considerable economic benefit in coordinating process-control and maintenance policies and indicated conditions for which the benefits were most pronounced. In addition, this research provided expanded insight into the implications of Process Failure Mechanism when determining process-control and maintenance policies. The model derived an optimal policy to minimize the cost per unit time. Finally, a sensitivity analysis was conducted to develop insights into the economic and process variables that influence the integration efforts.

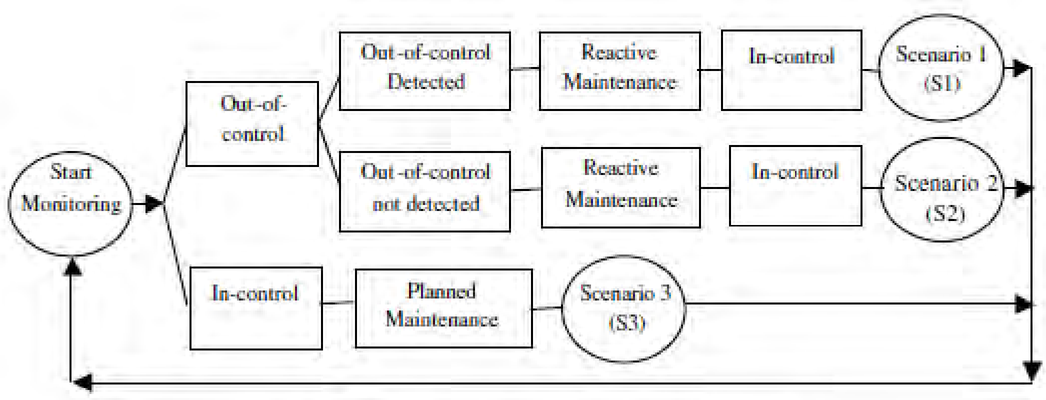


Fig. 2.1 Three scenarios proposed by Lindeman et al. [18]

Three scenarios were considered in their paper as showed in Fig.2.1. They assumed that the process begins in-control and inspections occur after h hours of production to determine whether the process has shifted from an in-control to an out-of-control state. Sometime between the jth and (j+1)th sampling interval an assignable cause occurs and the process shifts to an out-of-control state. The process continue to operate; however, the control chart does not detect an out-of-control condition until the (j +i)th sample. A time lag is associated with collecting the data and plotting the results on the chart. The control chart then signals an out-of-control condition and a search for an assignable cause takes place to validate the signal. A valid control-chart signal then results in Reactive Maintenance that restores the equipment to a ‘‘good-as new’’ condition (a renewal) this is scenario 1. In Scenario 2 in Fig.2.1, the process shifts to an out-of-control state, but the control chart does not signal an out-of-control condition before the Planned Maintenance. As in Scenario 1, the process begins in an in-control state and sometime between the jth and (j +1)th sampling interval, the process shifts to an out-of-control state. However, the process continues to operate because the control chart does not detect an out-of-control condition. At the (k +1)th sampling interval, maintenance begins, and the out-of-control state is identified. They considered that to be Reactive Maintenance because the out-of-control condition occurred before the scheduled maintenance and additional time and expense would be incurred to identify and resolve the equipment problem. Completing the maintenance causes the process to renew and return to the in-control state. In Scenario 3 in Fig.2.1, the process still remains in an in-control state at the time of the Planned Maintenance. The Planned Maintenance would take place at the (k + 1)th sampling interval to preempt a process failure. Typically, the activities associated with planned maintenance are less costly than those associated with Reactive Maintenance because preparation activities can be conducted off-line before the Planned Maintenance. Finally, after maintenance completion, the process renews itself. The approach assumed that the process monitoring and maintenance schedule followed a rolling schedule.

Zhou and Zhu et al. [19] developed an integrated model of control chart and maintenance management with reference to the integrated model proposed by Linderman et al. [18]. In their model, control chart was used to monitor the equipment and to provide signals that indicate equipment deterioration, while Planned Maintenance was scheduled at regular intervals to pre-empt equipment failure. Based on Alexander's cost model, they investigated the economic behavior of the integrated model, and gave an optimal design for determining the four policy variables (the sample size (n), the sampling interval (h), scheduled sampling times (k), control limit, (L) that minimize hourly cost.

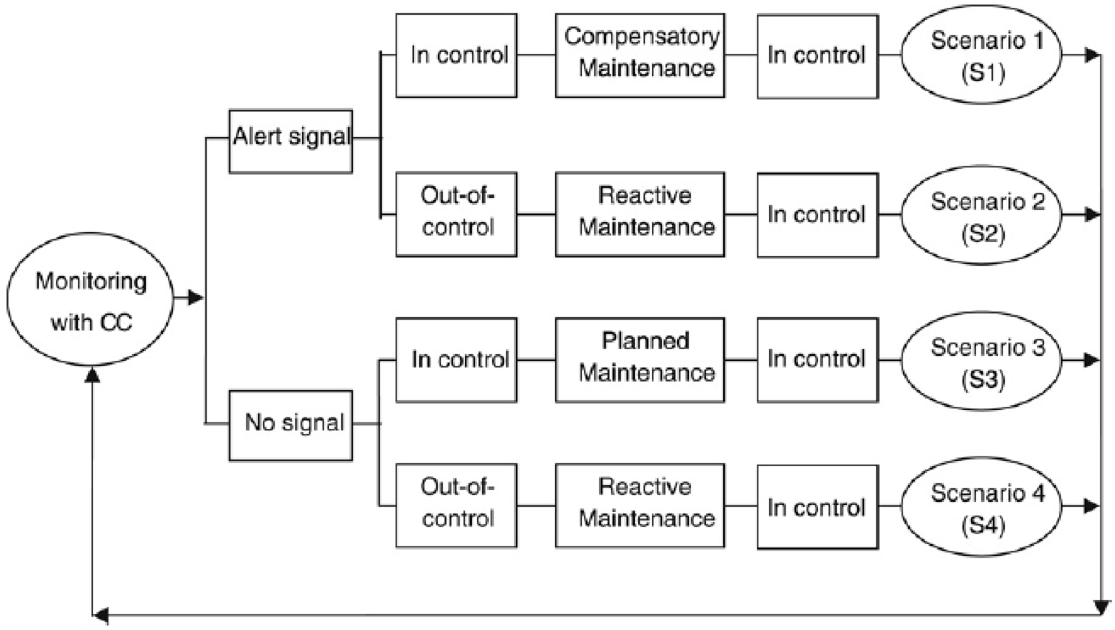


Fig. 2.2 Four scenarios proposed by Zhou and Zhu et al. [19]

The integrated model espoused the framework as shown in Fig. 2.2. The process begins with an in-control state with a Process Failure Mechanism that follows a Weibull distribution. They assumed that process inspections with control chart occur after h hours of production to determine whether the process has shifted from an in-control to an out-of-control state. The quality characteristic was measured and plotted on a control chart to assess the state of the process. If the control chart does not signal an out-of-control condition after k inspection intervals, then scheduled or Planned Maintenance occurs at the (k + 1) th sampling interval. However, if the control chart signals an out-of-control condition at any of the j inspections, a search for an assignable cause takes place to validate the signal. A valid control-chart signal then results in Reactive Maintenance and false signal results in Compensatory Maintenance. They assumed that the completion of Planned, Reactive and Compensatory maintenance restores the equipment to a “good-as-new” condition (a renewal). As shown in Fig. 2.2., the integrated model might result in four different scenarios. In S1, the process begins with an “in-control” state and inspections occur after h hours of monitoring as to whether the process has shifted from an “in-control” to an “out-of-control” state. And there is an alert signal in the control chart before the scheduled time when maintenance should be performed. But if the signal is false, that is to say, the process is still “in-control”. Since searching and determining false signal take time and incur cost, Compensatory Maintenance would be performed. Similar to S1, there is also a signal in S2. While the signal is valid and the process shifts to an “out-of-control” state, it results in Reactive Maintenance. In S3 and S4, no signal occurs in the control chart before the scheduled time. Then at the (k + 1) th sampling interval, appropriate maintenance should be arranged. In S3, the process is always “in-control”, and Planned Maintenance was suggested to be performed. When the process shifts to an “out-of-control” state in S4, Reactive Maintenance would take place because the “out-of-control” condition occurred before the scheduled time and additional time and expense would be incurred to identify and solve the equipment problem.

The interesting fact is almost all of the above researches proposed integration of SPC and maintenance and developed their model using X-bar control chart which has the disadvantage of not detecting small changes in mean efficiently. So nowadays researchers are paying attention to other types of control charts like EWMA and CUSUM charts for the purpose of developing an integrated model to detect even small changes quickly.

Average run length (ARL) is the measure of expected number of constructive samples taken before the sample statistics falls outside the control limits. ARL has become the traditional measure to determine the control chart’s performance. So in designing economic control chart, proper computation of ARL has large significance. Since determining ARL is a difficult job many researchers tried to find out a simple and credible ways to find out ARL precisely.

Lucas and Saccucci [20] developed a model to determine the average run length (ARL) for exponentially weighted moving average (EWMA) and combined Shewhart-EWMA control schemes. The model was developed using markov chain approach to calculate zero-state and steady-state ARLs.

Knoth [21] discussed the importance of ARL in detecting changes in the process mean using EWMA control chart. They also discussed different methods of determining accurate ARL. According to the paper, the most accurate method—solving a Fredholm integral equation with the Nystr¨om method—fails due to an improper kernel in the case of chi-squared distributions. Here, they exploited the collocation method and the product Nystrom method. These methods were compared to Markov chain based approaches. Collocation allowed fast and very precise computation of the ARL. With the exception of two-sided charts based on individual observations (leads to $k = 1$) the collocation results were very accurate. Even in problematic cases collocation provided suitable results.

Morais and Pacheo [22] considered the problem of the joint monitoring of process mean (μ) and the process standard deviation (σ)- when the quality characteristic follows a normal distribution -, using a combined Exponentially Weighted Moving Average (CEWMA) scheme. Three performance measures of this joint control scheme were investigated under shifts in the process mean or inflation of the process standard deviation, and under the adoption of head starts: the average run length, the run length percentage points and the probability of a misleading signal. The Markovian approach and the independence between the horizontal and vertical transitions of the approximating two-dimensional Markov chain played an important role in providing tractable expressions for these three performance measures. A numerical comparison between these three performance measures and the corresponding ones of the matched combined *Shewhart* (CShewhart) scheme was also provided in the paper, which concluded that the substitution of this combined scheme by the CEWMA scheme can improve the joint monitoring of the process mean and standard deviation.

Since ensuring optimal quality in the process and product is the prime concern to satisfy customers, so proper identification of quality loss is necessary. Most of the cases in determining quality loss American traditional goal post view were used which doesn’t consider the loss due to deviation from the target value. It only considers loss when sample values goes beyond the control limits of respective control charts. Realizing the importance of determining both in control and out of control losses of quality, several researchers tried to incorporate different loss functions to determine proper quality loss.

Al-Ghazi et al. [23] incorporated Taguchi’s quadratic loss function in the economic design of the control chart. This was done by redefining the in-control and out-of-control costs using Taguchi’s loss function in the general model for the economic design of *x- bar*-control charts developed by Banerjee and Rahim. Both cases of increasing hazard rate (Weibull failure rate) and constant hazard rate (exponential distribution) was presented and followed by numerical examples. Finally, sensitivity analysis was conducted to study the effect of important parameters on the cost. The resulting model combined the advantages of the economic design and Taguchi's philosophy.

Serel [24] studied the economic design of EWMA-based mean and dispersion charts where a linear, quadratic, or exponential loss function was used for computing the costs arising from poor quality. The chart parameters (sample size, sampling interval, control limits and smoothing constant) minimizing the overall cost of the control scheme are determined via computational methods. Using numerical examples, he compared the performances of the EWMA charts with Shewhart X-bar and S charts, and investigated the sensitivity of the chart parameters to changes in process parameters and loss functions. His computational study suggested that using a different type of quality loss function (linear versus quadratic) leads to a significant change in sampling interval while affecting the sample size and control limits very little. It was also observed that the overall costs are insensitive to the choice of Shewhart or EWMA charts. Numerical results implied that rather than sample size or control limits, the users need to adjust the sampling interval in response to changes in the cost of poor quality.

With the development of economic control charts, different types of sampling policies got attention of the researchers to design economic control charts more effectively by saving costs.

Reynolds [25] discussed various sampling policy such as fixed sampling interval (FSI), variable sampling interval (VSI) and variable sample size (VSS). This paper also considered a type of VSI control chart in which samples were always taken at specified equally spaced fixed time points, but additional samples were allowed between these fixed times when indicated by the data from the process. The location of the fixed times would typically be determined by administrative considerations such as testing schedules or by the desirability of sampling according to natural periods in the process. Markov process methods were given for analyzing the performance of these VSI with fixed times (VSIFT) charts. The VSIFT feature had been considered for the X bar-chart, the EWMA chart and the CUSUM chart and general expressions for the ATS had been developed. It was shown that VSIFT charts will detect most process shifts substantially faster than FSI charts. It also showed that VSIFT charts are just as effective in detecting shifts as standard VSI charts that are not constrained to sample at the specified fixed times. In comparison VSS charts, the VSIFT charts were shown to be better at detecting large shifts but not as good at detecting small shifts, in case of X bar Chart. But, the VSS EWMA and CUSUM charts did not perform as well as the corresponding VSIFT EWMA and CUSUM charts except at very small shifts.Finally, this paper recommended for the use of the design of VSIFT X bar-charts, VSIFT EWMA charts and VSIFT CUSUM charts, over other sampling policies.

Park et al. [26] compared the Variable Sampling Rate (VSR) approach with Fixed Sampling Rate (FSR) approach in case of a EWMA chart. A VSR EWMA chart is a EWMA chart with the VSR sampling scheme. The properties of the VSR EWMA chart were obtained by using a Markov chain approach. The model contained cost parameters which allow the specification of the costs associated with sampling, false alarms and operating off target as well as search and repair. Control charts that vary the sample size are called Variable Sample Size (VSS) charts. A large sample size is used when there is some indication of a problem and a small sample size is used when there is no indication of a problem. Variable Sampling Rate (VSR) charts allow both the sample size and the sampling interval to vary depending on the previous value of the control statistic. The idea of the VSR chart was to combine the VSI and VSS features. The economic design model of Park and Reynolds had been applied to evaluate the expected cost per hour associated with the operation of VSR and FSR EWMA charts in the single and double occurrence models. The optimal parameters of the VSR and FSR EWMA charts were obtained in this economic model for some given sets of values of the process and cost parameters. For the parameter combinations considered here, it was shown that the optimal control limit of the VSR chart was considerably higher than that of the FSR chart. This results in a much lower false alarm rate for the VSR chart. The percent reductions in cost presented in the tables showed that applying the VSR scheme in place of the FSR scheme in the EWMA chart could result in substantial cost savings. A double occurrence model was studied for a possible situation in which, given that a special cause has occurred, a second special cause may arrive before a signal is given. It was shown that such a model change has little effect on determining the optimal chart parameters.

Chou et al. [27] developed the economic design of the exponentially weighted moving average (EWMA) chart using variable sampling intervals with sampling at fixed times (VSIFT) control scheme to determine the values of the six test parameters of the chart (i.e., the sample size, the fixed sampling interval, the number of subintervals between two consecutive sampling times, the warning limit coefficient, the control limit coefficient, and exponential weight constant) such that the expected total cost per hour is minimized. The sampling scheme of a VSIFT chart was to use the sampling interval (denoted as h) between fixed time points as long as the sample point is close to the target so that there is no indication of process change. However, if the sample point is far away from the target, but still within the control limits, so that there is some indication of process shift, then additional samples were allowed between the two fixed sampling time points. The control charts using variable sampling intervals with sampling at fixed times (VSIFT) policy had been shown to give substantially faster detection of most process shifts than the conventional control charts. The cost function was established based on the cost model in Lorenzen and Vance [13] with the VSIFT control scheme. The genetic algorithms (GA) were applied to search for the optimal values of the six test parameters of the VSIFT EWMA chart, and then an example and its solution were provided. Finally, a sensitivity analysis was carried out to investigate the effect of model parameters on the solution of the economic design.

Haq et al [4] proposed new EWMA control charts for monitoring the process mean and the process dispersion. These EWMA control charts were based on the best linear unbiased estimators obtained under ordered double ranked set sampling (ODRSS) and ordered imperfect double ranked set sampling (OIDRSS) schemes, named EWMA-ODRSS and EWMA-OIDRSS charts, respectively.They used Monte Carlo simulations to estimate the average run length, median run length, and standard deviation of run length of the proposed EWMA charts.The performances of the proposed EWMA chart was compared with the existing EWMA charts when detecting shifts in the process mean and in the process variability. It turned out that the EWMA-ODRSS mean chart performed uniformly better than the classical EWMA, fast initial response-based EWMA, Shewhart-EWMA, and hybrid EWMA mean charts. The EWMA-ODRSS mean chart also outperformed the Shewhart-EWMA mean charts based on ranked set sampling (RSS) and median RSS schemes and the EWMA mean chart based on ordered RSS scheme. Moreover, the graphical comparisons of the EWMA dispersion charts reveal that the proposed EWMA-ODRSS and EWMA-OIDRSS charts are more sensitive than their counterparts. They also provided illuminating examples to illustrate the implementation of the proposed EWMA mean and dispersion charts.

Pandey et al. [2] introduced a new integrated approach for joint optimization of maintenance (preventive maintenance interval) and process control policy (control chart parameters) using "Taguchi Loss Function" and a way to categorize the machine and process failure. Two failure modes were considered (showed in fig.2 4.), failure mode 1 leads to the immediate breakdown of the machine. This machine failure was assumed to follow a two-parameter weibull distribution. Failure mode 2 could be caused either by external reasons or malfunction of the machine to some extent which

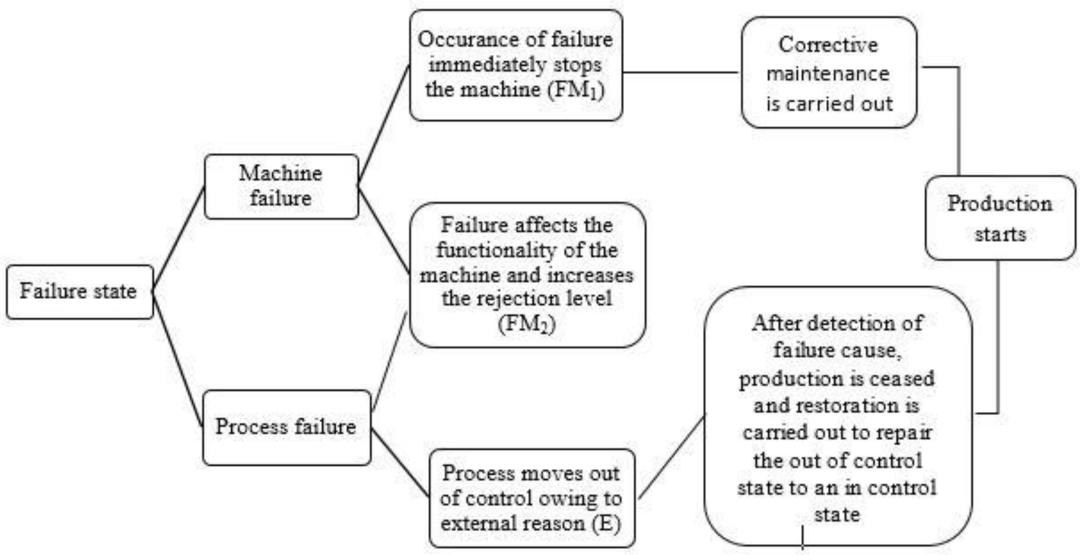


Fig. 2.3 Failure modes and actions proposed by Pandey et al. [2]

immediately affects the process mean. To monitor these types of shift an X-bar control chart was suggested to use. Whenever a complete failure occurs, corrective maintenance action, which restores the machine, would be implemented immediately. In parallel, the process would be monitored through a control chart in order to identify the actual operating state. Whenever a quality shift would be detected owing to machine degradation (partial failure), a corrective maintenance action was suggested to be performed to restore an in-control state through a repair action. Whenever quality shift would be detected owing to external reasons, a resetting of the process was suggested to be performed to restore the process to its in-control state. Thus, this type of maintenance action was expected to result in twin benefits, i.e. eliminating the quality cost related to out-of-control operation owing to external reasons and machine degradation and also improving the machine's reliability by protecting it against failures. Two types of maintenance policies were considered: minimal corrective maintenance that maintains the state of the equipment without affecting the age and imperfect preventive maintenance that upgrades the equipment in between 'as good as new' and 'as bad as old' condition. The proposed model enabled the determination of the optimal value of each of the four decision variables, i.e. sample size (n), sample frequency (h), control limit coefficient (k), and preventive maintenance interval (t_{PM}) that minimizes the expected total cost of the integration per unit time. A numerical example was presented to demonstrate the effect of the cost parameters on the joint economic design of preventive maintenance and process quality control policy. The sensitivity analysis was alsoconducted to study the effect of various model parameters on the behaviour of the system. This may help the manufacturer to identify the parameters which are more sensitive from those which are not.

Sultana et al. [6] developed an economic statistical design of the exponentially weighted moving average (EWMA) chart using variable sampling intervals with sampling at fixed times (VSIFT) control scheme considering preventive maintenance and Taguchi Loss function to determine the values of the seven test parameters of the chart (i.e., the sample size, the fixed sampling interval, the number of subintervals between two consecutive sampling times, the warning limit coefficient, the control limit coefficient, and exponential weight constant and the time interval of preventive maintenance) such that the expected total cost per hour is minimized.In this model taguchi loss function was incorporated to determine in control and out of control cost.this paper alo gave emphasis on preventive maintenance to maintain proper functionality of equipments and to restrict the process from moving out of control state due to machone degradation or deteriorartion.That's why this paper influenced on integarted optimization of preventive maintenance interval and EWMA control chart parameters.They used Genetic algorithm and nealder mead downhill simplex algorithm to optimize the value of seven test parameters.

Serel and Moskowitz [5] used control charts with exponentially weighted moving average (EWMA) statistics (mean and variance) to jointly monitor the mean and variance of a process. A EWMA cost minimization model was presented to design the joint control scheme based on pure economic or both economic and statistical performance criteria. The pure economic model was extended to the economic-statistical design by adding constraints associated with in-control and out-of-control average run lengths. The quality related production costs were calculated using Taguchi's quadratic loss function. The optimal values of smoothing constants, sampling interval, sample size, and control chart limits were determined by using a numerical search method. The average run length of the control scheme was computed by using the Markov chain approach. Computational study indicated that optimal sample sizes decrease as the magnitudes of shifts in mean and/or variance increase, and higher values of quality loss coefficient lead to shorter sampling intervals. The sensitivity analysis results regarding the effects of various inputs on the chart parameters provided useful guidelines for designing an EWMA-based process control scheme when there exists an assignable cause generating concurrent changes in process mean and variance.

Shrivastava et al [7] developed an integrated model for joint optimization of preventive maintenance and quality control policy with cumulative sum (CUSUM) control chart parameters. Two types of maintenance policies were considered, namely minimal corrective maintenance and imperfect preventive maintenance. The proposed model enables the determination of the optimal value of each of the five decision variables, i.e. sample size (n), sample interval (h), the reference value (K), the decision interval (h1) and preventive maintenance interval(tPM), that minimize the expected total system cost per unit time. A numerical example was presented to demonstrate the effect of the cost parameters on the integrated design of preventivemaintenance and quality control policy. It also compared the system performance employing the proposed integrated approach with that obtained by considering maintenance and CUSUM chart independently. Substantial economic benefit of 19 % was observedin the joint optimization.

Zaman et al [28] proposed a new control chart named as mixed CUSUM-EWMA (called MCE) control chart for the efficient monitoring of process dispersion. The proposed MCE chart has been compared with other existing control charts and some of their modifications. Average run length, extra quadratic loss, relative average run length, and performance comparison index were the measures that were used to judge the performance of charts. For practical considerations, an illustrative example with real data was also provided.

CHAPTER III

COMPUTATIONAL OPTIMIZATION PROCEDURE

3.1 Nelder Mead Downhill Simplex Algorithm

The Nelder-Mead algorithm was originally published in 1965 is one of the best known algorithms for multidimensional unconstrained optimization without derivatives. The algorithm is stated using the term simplex (a generalized triangle in N dimensions) and finds the minimum of a function of N variables. It is effective and computationally compact. Since, this method does not require any derivative information; therefore, it is quite suitable for problems with non-smooth functions. It is widely used to solve parameter estimation and similar statistical problems, where the function values are uncertain or subject to noise and that is a particular cause of choosing this particular algorithm for my analysis.

The Nelder-Mead algorithm is designed to solve the classical unconstrained optimization problem of minimizing a given nonlinear function f: Rn→R . The method

- uses only function values at some points in R^n , and
- Does not try to form an approximate gradient at any of these points.

The Nelder-Mead method is simplex-based. A simplex K in R^n is defined as the convex hull of n+1 vertices x ….., $x_n \in R^n$. For example, a simplex in $R^2$is a triangle, and a simplex in R^3 is a tetrahedron. A simplex-based direct search method begins with a set of n+1 points $x_0,\ldots,x_n \in R^n$ that are considered as the vertices of a working simplex K , and the corresponding set of function values at the vertices $f_j{:=}f(x_j)$, for j=0,…,n . The initial working simplex S has to be non degenerate, i.e., the points $x_0\ldots, x_n$ must not lie in the same hyper plane.

The method then performs a sequence of transformations of the working simplex K, aimed at decreasing the function values at its vertices. At each step, the transformation is determined by computing one or more test points, together with their function values, and by comparison of these function values with those at the vertices.

This process is terminated when the working simplex K becomes sufficiently small in some sense, or when the function values f_j are close enough in some sense (provided f is continuous).

The Nelder-Mead algorithm typically requires only *one* or *two* function evaluations at each step, while many other direct search methods use *n* or even more function evaluations.

3.1.1 Initial simplex

Initial simplex K is usually constructed by generating $n+1$ vertices $x_0,\ldots,x_n$ around a given input point $x_{in} \in R^n$. In practice, the most frequent choice is $x_0 = x_{in}$ to allow proper restarts of the algorithm. The remaining n vertices are then generated to obtain one of two standard shapes of K.

- K is right angled at x_0, based on coordinated axes, or

  $x_j = xo + h_j e_j$,   j=1…….n

  Where h_j is a step size in the direction of unit vector e_j in R^n .
- K is a regular simplex, where all edges have the same specified length.

3.1.2 General Simplex transformation algorithm

One iteration of the Nelder-Mead method consists of the following three steps.

1. **Ordering**: Determine the indices of the worst (w), good (G), best (B) vertex, respectively, in the current working simplex K,

   $f_W = \max_j f_{j,}$ $f_G = \max_{j \neq W} f_{j,}$ $f_B = \min_{j \neq W} f_j$

2. **Centroid**: Calculate the *centroid* c of the *best* side—this is the one opposite the *worst* vertex x_w

   c =1n∑j≠hxj.

3. **Transformation**: Compute the new working simplex from the current one. First, try to replace only the *worst* vertex x_w with a better point by using reflection, expansion or contraction with respect to the best side. All test points lie on the line defined by x_w and *c*, and at most two of them are computed in one iteration. If this succeeds, the accepted point becomes the new vertex of the

working simplex. If this fails, shrink the simplex towards the *best* vertex x_b. In this case, n new vertices are computed.

Simplex transformations in the Nelder-Mead method are controlled by four parameters α for reflection, β for contraction, γ for expansion and δ for shrinkage. They should satisfy the conditions, $\alpha>0$, $0<\beta<1$, $\gamma>1$, $\gamma>\alpha$, $0<\delta<1$.The standard values used in most implementations are $\alpha = 1$, $\beta = ½$, $\gamma = 2$, $\delta = ½$.

3.1.3 Simplex transformation algorithm for two variables

For two variables, a simplex is a triangle, and the method is a pattern search that compares function values at the three vertices of a triangle. The worst vertex, where f(x,y) is largest, is rejected and replaced with a new vertex. A new triangle is formed and the search is continued. The process generates a sequence of triangles, for which the function values at the vertices get smaller and smaller. Eventually, the size of the triangles is reduced and the coordinates of the minimum point are found.

Initial Triangle: Let f (x, y) be the function that is to be minimized. At the beginning, three vertices of a triangle are considered such that, $V_k=(x_k, y_k)$ where, k=1, 2, 3. For simplicity let's assume the points as $B=(x_1, y_1)$, $G=(x_2, y_2)$, and $W=(x_3, y_3)$. The function f (x, y) is then evaluated at each of the three points i.e. $z_k=f(x_k, y_k)$ for k=1, 2, 3. The subscripts are then reordered so that $z_1 \le z_2 \le z_3$.

Midpoint of the Good Side: The construction process uses the midpoint of the line segment joining B and G.

$$M = \frac{B+G}{2} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \tag{3.1}$$

Reflection using the Point R: The function decreases as the points move along the side of the triangle from W to B, and also along the side from W to G. Hence it is feasible that f (x, y) takes on smaller values at points that lie away from W on the opposite side of the line between B and G. So, a test point R is chosen, that is obtained by "reflecting" the triangle through the side BG.

$$R = M + \alpha(M - W) \tag{3.2}$$

If $f_B \le f_R < f_G$, then W is replaced by R and iteration terminated.

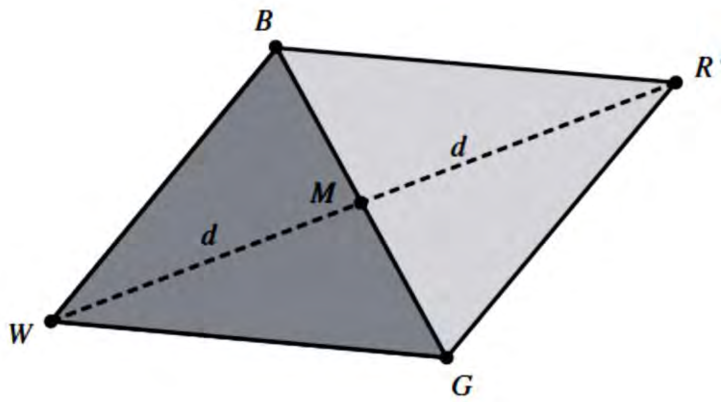


Fig. 3.1 Initial triangle BGW, midpoint (M) and reflection point (R) in Nelder Mead method

Expansion using the Point E: If the function value at R is smaller than the function value at B, then the point has moved in the correct direction toward the minimum. Perhaps the minimum is just a bit farther than the point R. So in the next step, the line segment through W, M and R is extended to the point E. This forms an expanded triangle BGE. The point E is found by moving an additional distance d along the line joining M and R.

$$E = R + \gamma(R - M) \tag{3.3}$$

If $f_E < f_R$, then E is accepted, otherwise R is accepted and iteration is terminated.

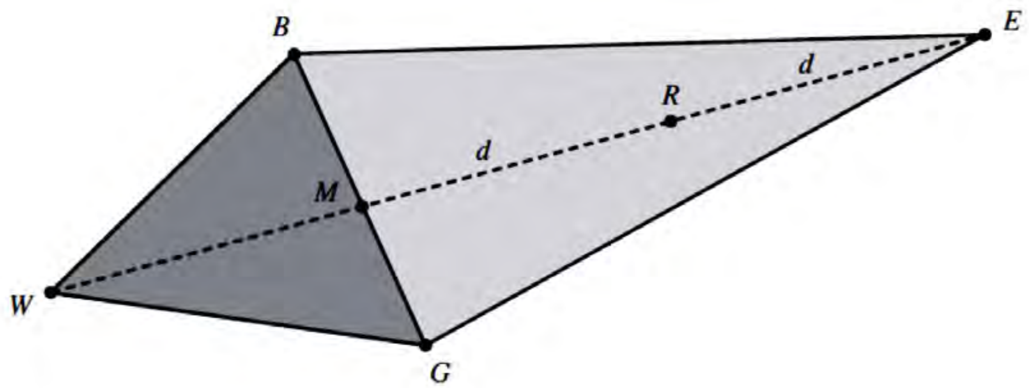


Fig. 3.2 Expanded triangle BGE, reflection point (R) and extended point (E) in Nelder Mead method

Contraction using the Point C: If the function values at R is greater or equal to G, another point must be tested. Perhaps the function is smaller at M, but W cannot be replaced with M because the points must form a simplex, in this case a triangle. So, the two points C_1 and C_2 of the line segments WM and MR is considered, respectively. If function value of R is greater than or equal of W inside contraction (C_1) should be done. If function value of R is less than of W outside contraction (C_2) should be done

$$C_1 = M + \beta\,(W - M) \tag{3.4}$$

If $f_{C1} < f_{W,}$ then C1 is accepted, otherwise shrink transformation is performed.

$$C2 = M + \beta\,(R - M) \tag{3.5}$$

If $f_{C2} \leq f_{R,}$ then C2 is accepted, otherwise shrink transformation is performed.

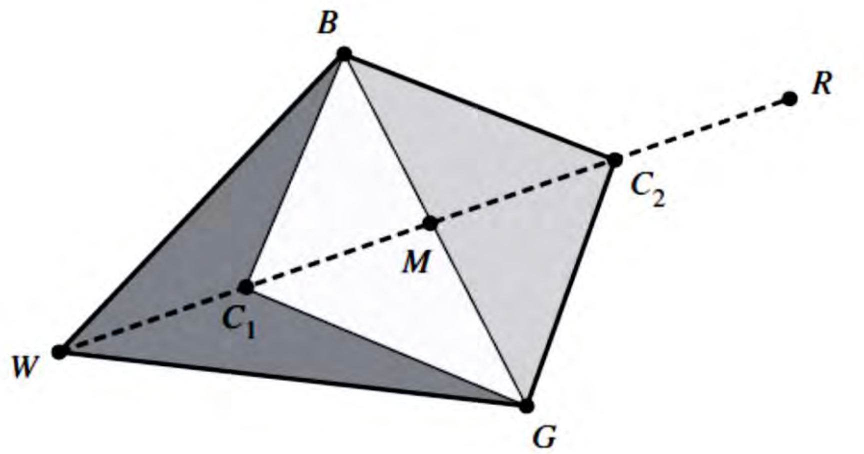


Fig. 3.3 Contraction point C_1 and C_2 in Nelder Mead method

Shrink Towards B: If the function value at C1 is not less than the value at W or function value at C2 is not less than the value at R, the points G and W must be shrunk towards B. The point G is replaced with M, and W is replaced with S, which is the midpoint of the line segment joining B with W (for δ = ½).So, in shrinking n new vertices are formed for n variable optimization.

General formula for shrinking is,

$X_j = B + \delta\,(x_j - B)$ and $f_j = f(x_j)$; (3.6)

for $j = 1,2,3\ldots\ldots n+1$; where, $j \neq l$; here l indicates lowest or best value, $x_l = B$.

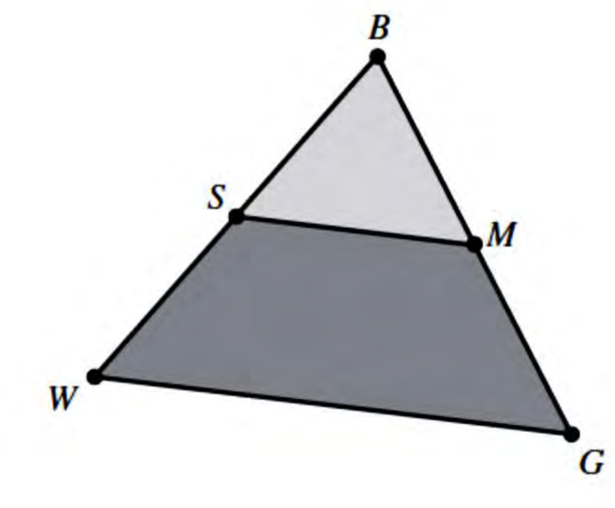


Fig. 3.4 Shrinking of simplex (triangle) towards B

3.1.4 Termination Tests

A practical implementation of the Nelder-Mead method must include a test that ensures termination in a finite amount of time. The termination test is often composed of three different parts i.e.

1. term_x ,
2. term_f and
3. fail.

'term_x' is the domain convergence or termination test. It becomes true when the working simplex S is sufficiently small in some sense (some or all vertices xj are close enough). 'term_f' is the function-value convergence test. It becomes true when (some or all) function values f_j are close enough in some sense. 'fail' is the no-convergence test. It becomes true if the number of iterations or function evaluations exceeds some prescribed maximum allowed value. The algorithm terminates as soon as at least one of these tests becomes true. The flow chart of the working procedure of the algorithm is as follows:

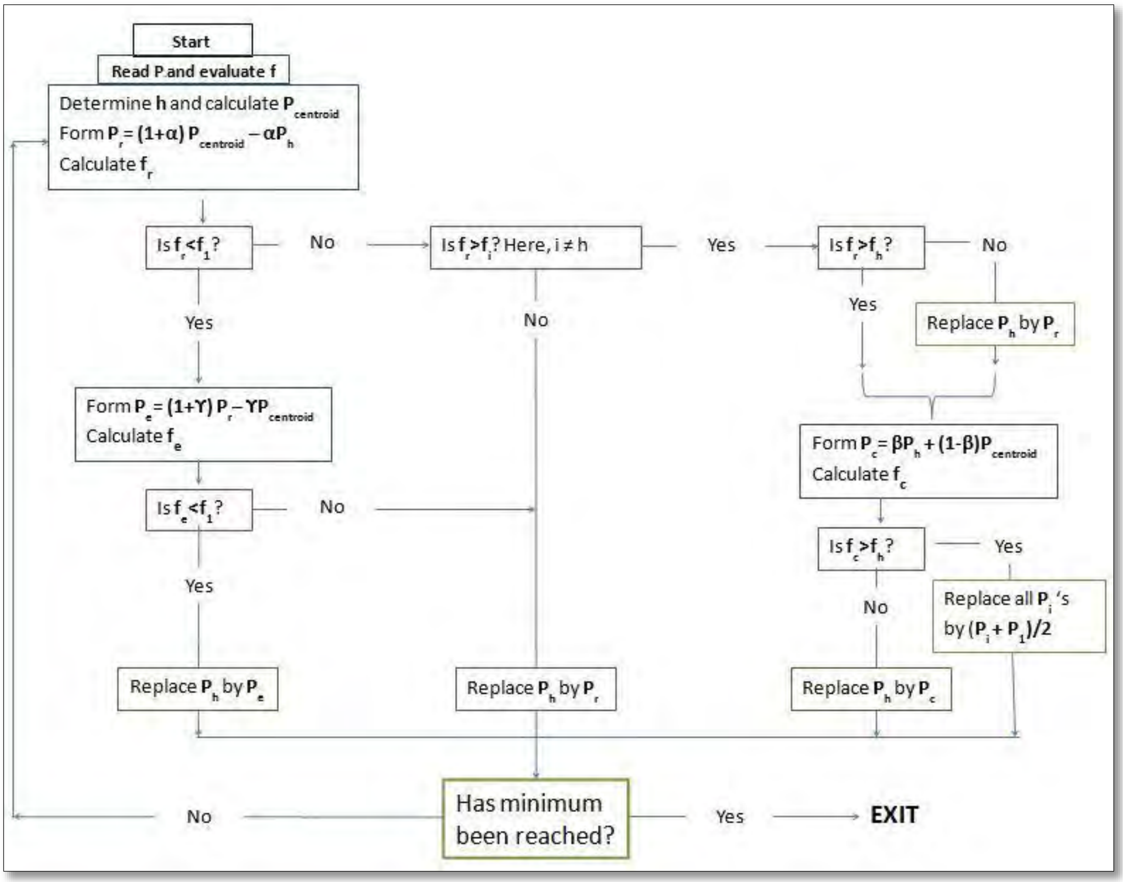


Fig. 3.5 Flow chart of Nelder Mead algorithm's working procedure

3.1.5 Convergence of Nelder Mead Method

Rigorous analysis of the Nelder-Mead method seems to be a very hard mathematical problem. Known convergence results for direct search methods in simplex terms rely on one or both of the following properties:

(a) The angles between adjacent edges of the working simplex are uniformly bounded away from 0 and π throughout the iterations, i.e., the simplex remains uniformly non-degenerate.

(b) Some form of "sufficient" descent condition for function values at the vertices is required at each iteration.

In general, the original Nelder-Mead method does not satisfy either of these properties. By design, the shape of the working simplex can almost *degenerate* while "adapting itself to the local landscape", and the method uses only *simple* decrease of function values at the vertices to transform the simplex. Hence, very little is known about the convergence properties of the method

3.1.6 Advantages and Disadvantages

In many practical problems, like parameter estimation and process control, the function values are uncertain or subject to noise. Therefore, a highly accurate solution is not necessary, and may be impossible to compute. All that is desired is an improvement in function value, rather than full optimization.

The Nelder-Mead method frequently gives significant improvements in first few iterations and quickly produces quite satisfactory results. Also, the method typically requires only one or two function evaluations per iteration, except in shrink transformations, which are extremely rare in practice. This is very important in applications where each function evaluation is very expensive or time-consuming. For such problems, the method is often faster than other methods, especially those that require at least n function evaluations per iteration. In many numerical tests, the Nelder-Mead method succeeds in obtaining a good reduction in the function value using a relatively small number of function evaluations.

Apart from being simple to understand and use, this is the main reason for its popularity in practice.

On the other hand, the lack of convergence theory is often reflected in practice as a numerical breakdown of the algorithm, even for smooth and well-behaved functions.

The method can take an enormous amount of iterations with negligible improvement in function value, despite being nowhere near to a minimum. This usually results in premature termination of iterations. A heuristic approach to deal with such cases is to restart the algorithm several times, with reasonably small number of allowed iterations per each run.

3.2 Genetic Algorithm

A genetic algorithm (GA) is a search algorithm or method developed by Holland (1975) for solving both constrained and unconstrained optimization problems based on a natural selection process that mimics biological evolution. In evolution, the problem that each species faces is to search for beneficial adaptations to the complicated and changing environment. In other words, each species has to change its chromosome combination to survive in the living world. In GA, a string represents a set of decisions (chromosome combination), that is a potential solution to a problem. Each string is evaluated on its performance with respect to the fitness function (objective function). The ones with better performance (fitness value) are more likely to survive than the ones with worse performance. Then the genetic information is exchanged between strings by crossover and perturbed by mutation. The result is a new generation with (usually) better survival abilities. This process is repeated until the strings in the new generation are identical, or certain termination conditions are met. A generic flow of GA is given in Fig.3.6 this algorithm is continued since the stopping criterion is reached.

The genetic algorithm uses three main types of rules at each step to create the next generation from the current population:

- Selection rules select the individuals, called parents, which contribute to the population at the next generation.
- Crossover rules combine two parents to form children for the next generation.
- Mutation rules apply random changes to individual parents to form children.

The generic cycle and operations of basic GAs can be explained as follows:

Step 1: The individuals resulting from these three operations for the next generation's population are obtained. This process is repeated until the system stops its evolution.

Step 2: Individuals contribute to the gene pool in proportion to their relative fitness; occasionally good individuals contribute more copies and poor individuals contribute fewer copies than before.

Step 3: The recombination happen depends on crossover operators. Next population consists of a new chromosome.

Step 4: The mutation happen depends on mutation operators which help to assure population variety.

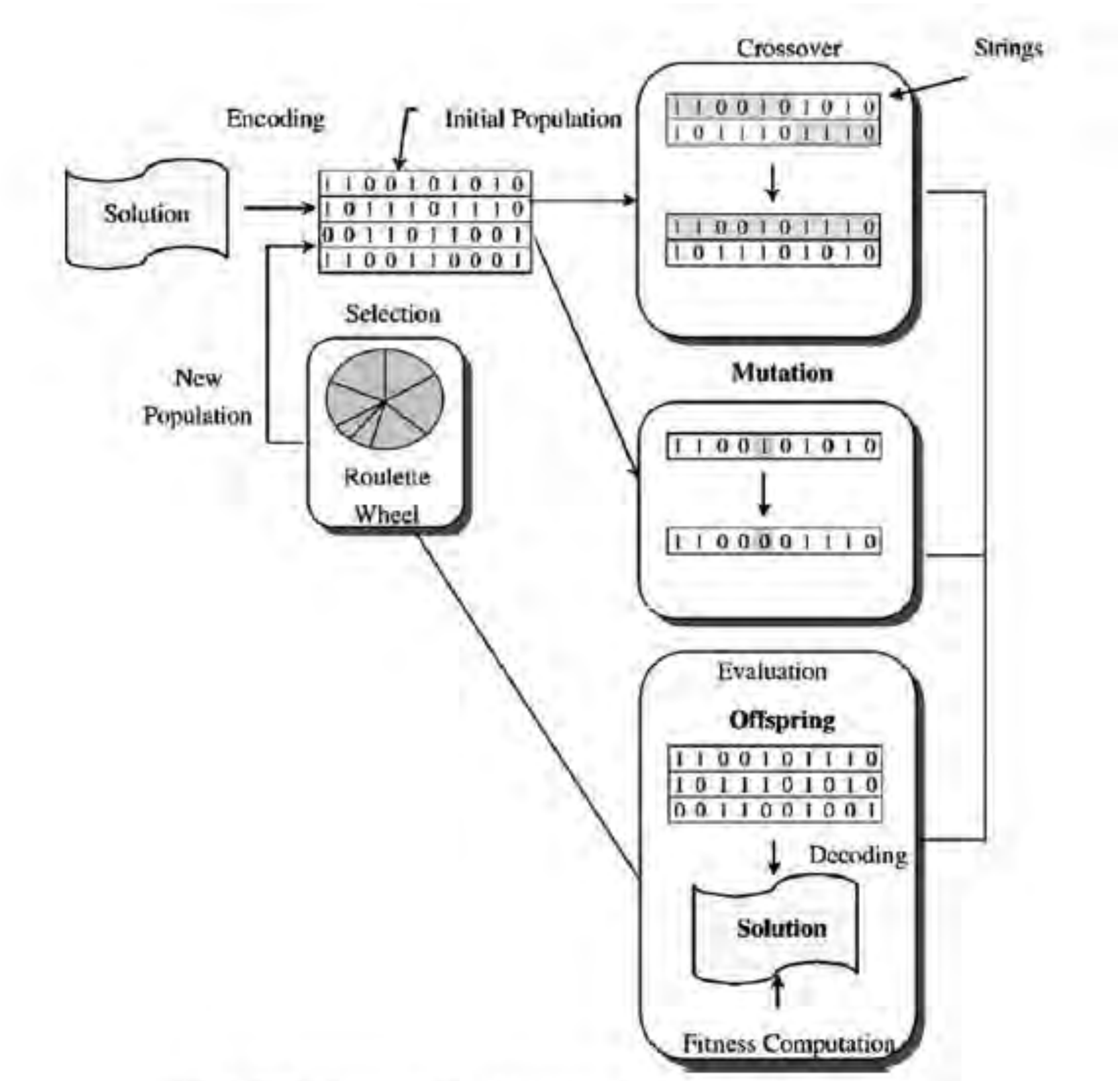


Fig. 3.6 Flow chart of Genetic Algorithm's working procedure

GAs is used in forming models to solve optimization problems. Readers can find more details of GAs in Gen and Cheng [29], Kaya [30].GAs are different from other search procedures in the following ways [31].

3.2.1 Outline of the Basic Genetic Algorithm

[Start] Generate random population of *n* chromosomes (suitable solutions for the problem)

[Fitness] Evaluate the fitness *f(x)* of each chromosome *x* in the population

[New population] Create a new population by repeating following steps until the new population is complete

[Selection] Select two parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to be selected)

[Crossover] With a crossover probability crossover the parents to form a new offspring (children). If no crossover was performed, offspring is an exact copy of parents.

[Mutation] With a mutation probability mutate new offspring at each locus (position in chromosome).

[Accepting] Place new offspring in a new population

[Replace] Use new generated population for a further run of algorithm

[Test] If the end condition is satisfied, stop, and return the best solution in current population

The GA, based on the concept of natural genetics, is directed toward a random optimization search technique. The GA solves problems using the approach inspired by the process of Darwinian evolution. In the GA, the solution of a problem is called a ‘‘chromosome’’. A chromosome is composed of genes (i.e., features or characters). Although there are several kinds of numerical optimization methods, such as neural network, gradient-based search, GA, etc., the GA has advantages in the following aspects:

1. The operation of GA uses the fitness function values and the stochastic way (not deterministic rule) to guide the search direction of finding the optimal solution. Therefore the GA can be applied for many kinds of optimization problems.

2. The GA can lead to a global optimum by mutation and crossover technique to avoid trapping in the local optimum.

3. The GA is able to search for many possible solutions (or chromosomes) at the same time. Hence, it can obtain the global optimal solution efficiently.

Based on these points, GA is considered as an appropriate technique for solving the problems of combinatorial optimization and has been successfully applied in many areas to solve optimization problems.

3.2.2 Parameters of GA

Crossover probability says how often will be crossover performed. If there is no crossover, offspring is exact copy of parents. If there is a crossover, offspring is made from parts of parents' chromosome. If crossover probability is 100%, then all offspring is made by crossover. If it is 0%, whole new generation is made from exact copies of chromosomes from old population (but this does not mean that the new generation is the same!).

Mutation probability says how often will be parts of chromosome mutated. If there is no mutation, offspring is taken after crossover (or copy) without any change. If mutation is performed, part of chromosome is changed. If mutation probability is 100%, whole chromosome is changed, fit is 0%, nothing is changed.

Population size says how many chromosomes are in population (in one generation). If there are too few chromosomes, GA has a few possibilities to perform crossover and only a small part of search space is explored. On the other hand, if there are too many chromosomes, GA slow down.

CHAPTER IV

MODEL DEVELOPMENT

4.1 Problem Identification

Since two major tools for control of production process i.e. Statistical Process Control (SPC) and Maintenance Management (MM) are closely related to each other and their goals overlap a great deal, integration of these two may be more effective than their separate parallel application. Realizing this fact several researchers have already developed a great deal of integrated models of these two. But the following omissions are observed in the literature in this context:

1. Most of the integrated general cost models integrate maintenance management policy and process quality control considering $\overline{X}$ control chart and EWMA control chart. But CUSUM chart can be very effective in detecting small shifts and it is also known for its prompt response. Since it is cumulative in nature, it represents even very small drift in steadily increasing or decreasing cumulative deviation values. . It is also effective for process industries, where sample size n=1 is justified.
2. Most of the models consider only Process mean to monitor the process and to determine Average Run Length (ARL), while considering both mean and variance chart in case of process monitoring can be more effective. In this context joint ARL (combining mean and variance) should be determined.
3. Most of the integrated models focus on process quality problems and completely ignore the possibility of an equipment failure in terms of machine breakdown or improper functioning of the equipment that result in poor product quality and call for maintenance action.
4. Most of the integrated models focus on the investigation of preventive replacement or perfect PM policy that restores the equipment to an ‘as-good-as-new’ state. Relatively very few papers have appeared that incorporate the aspect of imperfect PM. But in real life aspect imperfect PM has attained more acceptances.
5. Most of the integrated models consider the conventional goal-post approach in the economic design of control chart to determine the quality loss. But it doesn’t

consider the loss due to deviation from the target value; however, any deviation of quality characteristics from the target value is an indirect loss to the customers. Thus, incorporating the taguchi quadratic loss function approach in designing integrated economic control chart and maintenance policy should give better results.

6. In case of sampling policy, variable sampling interval has been considered to be more effective than fixed sampling interval and has become more realistic one. Variable Sampling Interval at Fixed Times (VSIFT) is one of the latest variable sampling technique which has been used in a very few study and recently it has already been accepted as the most promising technique of sampling policy. But only one integrated model has ever been developed with VSIFT approach and that integrated model was based on EWMA control chart. It has never been applied in integrated model based on CUSUM control chart.

4.2 Model Formulation

It is always required to consider the design of a control chart from an economical point of view. Because the choice of control chart parameters affect the whole cost. Selection of these control chart parameters is usually called the "Design of the Control Chart". Economic models are generally formulated using a total cost function, which expresses the relationships between the control chart design parameters and associated costs of process control. The production, monitoring and adjustment process are a series of independent cycles over time. Each cycle begins with the production process in the in-control state and continues until process monitoring via the control chart results in an out-of-control signal. After adjustment it returned to the in-control state to begin another cycle. Then the expected cost per unit time is,

$$E(A) = \frac{E(c)}{E(T)} \tag{4.1}$$

Where C, T are dependent random variables.

The above equation is optimized to design economically optimal control chart. The sequence of production-monitoring-adjustment, with accumulation of costs over the cycle, can be represented by a particular type of stochastic process called Renewal

Reward Process. Where time cost is given by the ratio of the expected reward per cycle to the expected cycle length.

4.2.1 The VSIFT CUSUM chart

The sampling scheme of a VSIFT chart is to use the sampling interval (denoted as h) between fixed time points as long as the sample point is close to the target so that there is no indication of process change. However, if the sample point is far away from the target, but still within the control limits, so that there is some indication of process shift, then additional samples are allowed between the two fixed sampling time points. Suppose that the interval h between two fixed times is divided into η subintervals of length d, where d = h/η. For example, if h = 1 h and η = 5 (i.e., d = 12 min), then samples would always be taken every hour. But if the sample point indicates a problem, then the next sample would be taken 12 min later.

CUSUM chart for mean (CUSUM- m): The ith sample statistic of a CUSUM chart for mean is

$$C_i^+ = \max[C_{i-1}^+ + x_i - (\mu_0 + s), 0] \tag{4.2}$$

$$C_i^- = \max[C_{i-1}^+ + (\mu_0 - s) - x_i, 0] \tag{4.3}$$

Where C_i^+ and C_i^- are the one sided upper and lower CUSUM for ith sample statistic. μ_0 is the process target value, the sequentially recorded observations x_i can either be individually observed values from the process or sample averages obtained from rational subgroups and s is the allowable slack and usually it is denoted by s = $\frac{\delta * \sigma_x}{2}$; usually δ =1. The upper and lower control limits (denoted by UCL and LCL, respectively) for the CUSUM-m chart are

$$UCL = \mu_0 + k\sigma_x \tag{4.4}$$

$$LCL = \mu_0 - k\sigma_x \tag{4.5}$$

Here, k is the control limit coefficient of the CUSUM-m chart that determines the size of the critical region of the chart, and σ_x is the asymptotic standard deviation of the sample statistic for mean and is equal to

$$\sigma_x = \frac{\sigma}{\sqrt{n}} \tag{4.6}$$

Where σ is the standard deviation of the process characteristic and n is the sample size. The upper and lower warning limits (denoted by UWL and LWL, respectively) for the CUSUM -m chart are

$$\text{UWL} = \mu_0 + w\sigma_x \tag{4.7}$$

$$\text{LWL} = \mu_0 - w\sigma_x \tag{4.8}$$

Where w is the warning limit coefficient of the CUSUM chart for mean.

CUSUM chart for variance (CUSUM-S^2) : Castagolia et al [32] proposed a new type of CUSUM chart to monitor process variance. In this paper that CUSUM-S^2chart will be used to monitor the variance.

Let $x_{k,1}, \ldots, x_{k,n}$ be a sample of n independent normal (μ_0,σ) random variables, where μ_0 is the nominal process mean, σ is the nominal process standard-deviation and k is the subgroup number. The investigated chart is used to monitor the process dispersion, therefore the "out-of-control" condition for the process corresponds to the occurrence of a special cause, which leaves unchanged the process mean and shifts the standard deviation. Let S_k^2 be the sample variance of subgroup k, i.e.

$$S_k^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x_{kj} - \bar{x}_k)^2 \tag{4.9}$$

Where $\bar{x}_k$ is the sample mean of subgroup k.

$$T_k = a + b\ln(S_k^2 + c) \tag{4.10}$$

Here, $b = B(n)$

$c = C(n)\sigma^2$

$a = A(n) - 2B(n)\ln(\sigma)$

The value of A(n), B(n), C(n) can be determined from the Appendix A of Castagolia(2005a). The kth static to monitor the process variance is,

$$Z_k^- = \max\,[0, Z_{k-1}^- + (E(T_k) - L) - T_k] \tag{4.11}$$

$Z_k^+ = \max[0, Z_{k-1}^+ + T_k - (E(T_k) + L)]$ (4.12)

$Z_k = \max(Z_k^-, Z_k^+)$ (4.13)

Here, $E(T_k)$ is the expected mean value of T_k.. L is the allowable slack for CUSUM-S^2 chart. $L \geq 0$ is a constant. The upper and lower control limits (denoted by UCL and LCL, respectively) for the CUSUM-S^2 chart are

$UCL = k_1 \sigma_x$ (4.14)

$LCL = 0$ (4.15)

Here, k_1 is the control limit coefficient of the CUSUM-S^2 chart that determines the size of the critical region of the chart, and σ_x *is* the standard deviation of the sample statistic for variance and is equal to

$\sigma_x = \frac{\sigma}{\sqrt{n}}$ (4.16)

Where σ is the standard deviation of the process characteristic and n is the sample size. The upper warning limit (denoted by UWL) for the chart is

$UWL = w_1 \sigma_x$ (4.17)

Where w_1 is the warning limit coefficient of the CUSUM-S^2 chart.

If the last sample point falls in the safe region (i.e., $LWL \leq x_i \leq UWL$) for both the mean and variance chart, then take the next sample at the next fixed sampling time point. If the last sample point falls in the warning region for at least one chart i.e. CUSUM-m or CUSUM-S^2 (i.e., $UWL < x_i \leq UCL$ or $LCL \leq x_i < LWL$ or xi>UWL), then take the next sample using the sampling interval d. A search for the assignable cause is under taken when the sample point falls outside the control limits. Thus, when the VSIFT CUSUM chart is applied to maintain current control of a process, seven test parameters (i.e., n, h, η, k, w, k_1 and w_1) should be determined. The economic design of VSIFT CUSUM chart is to determine the optimal values of the seven test parameters such that the average total cost per hour associated with test procedure may be minimized.

4.2.2 Process assumptions for VSIFT CUSUM control chart

To simplify the mathematical manipulation and analysis, the following assumptions are made:

1. The process characteristic monitored by the VSIFT CUSUM chart follows a normal distribution with mean μ and standard deviation σ.
2. In the start of the process, the process is assumed to be in the safe state; that is, $\mu = \mu_0$
3. The process mean may be shifted to the out-of-control region; that is, $\mu = \mu_0 + \delta\,\sigma$.
4. The process standard deviation σ remains unchanged.
5. The time between occurrences of the assignable cause is exponential with a mean of λ occurrences per hour.
6. When the process goes out of control, it stays out of control until detected and corrected.
7. During each sampling interval, there exists at most one assignable cause which makes the process out of control. The assignable cause will not occur at sampling time.
8. The measurement error is assumed to be zero.

4.2.3 Problem Statement and Assumptions

In the present work, an integrated model of preventive maintenance and quality control policy with CUSUM chart is presented to determine the optimal values sample size (n), fixed sampling interval (h), sampling sub interval (η), control limit coefficient of CUSUM-m chart (k), warning limit coefficient of CUSUM-m chart (w), control limit coefficient of CUSUM-S^2 chart (k_1), warning limit coefficient of CUSUM-S^2 chart (w_1) and preventive maintenance interval (t_{pm}), to minimize the expected total cost per unit time of this integrated model. Here VSIFT sampling policy has been considered for both CUSUM-m and CUSUM-S^2 charts. Joint ARL for mean and variance has been computed using absorbing Markov chain approach. In control and out of control cost has been determined using Taguchi quadratic and linear loss function for CUSUM-m and CUSUM-S^2 chart, respectively.

Like Pandey et al. [2] here, a production system has been considered consisting of a single machine, produce products of the same type with a constant Production Rate (PR, items per hour) on a continuous basis (three shifts of seven hours each, six days a week). Further, considered a single component operating as a part of machine with time-to-failures following a two-parameter Weibull distribution. The failures of most components in a mechanical system like production machines can be modeled using a two-parameter Weibull distribution. However, the approach proposed in this thesis is generic and can be used with other distributions also.

Here, machine failures are divided into two failure modes:

(1) Failure mode 1 (FM_1): leads to immediate breakdown of the machine.

(2) Failure mode 2 (FM_2): leads to deterioration in process quality owing to shifting of the desired level.

Similar classifications were also used by Lad and Kulkarni [33]. They have defined the failure of a machine tool as any event that either brings the machine down or leads to the machine still running but producing higher rejections. This means that if a failure occurs, it is not necessary that it will be detected immediately, and the machine will be stopped, but it may also affect the quality of products being manufactured on the machine. It is therefore necessary to consider these types of failures, and the corresponding failure costs, which may be situation-specific, in maintenance-planning decisions. The failures belonging to the second type of failure mode can also be considered as partial failures and are defined by Black and Mejabi [34] as degradation in the machine performance without complete failure. Thus, the problem is to jointly optimize design parameters for the preventive maintenance and process quality policy with CUSUM chart

Let us consider the following assumptions:

1. A single part gets manufactured on the machine with a single critical to quality (CTQ) characteristic.
2. Corrective maintenance is minimal in nature, i.e. after corrective action, the equipment has the same age as it did at the time of failure.
3. Preventive maintenance is imperfect in nature.

4. In this model it is assumed that failure modes FM_1 and FM_2 are independent and result in failures with an appropriate time to failure distribution. For a given failure, let the probability that it is due to FM be P_{FM1} and P_{FM2} respectively. Since it is assumed that these are the only failure mode types, $P_{FM1} + P_{FM2}=1$. These probabilities can be obtained from the failure reports maintained by maintenance personnel from production lines. The failure reports mainly cover the following information: Component ID, Time to failure, Time of repair, Failure mode, Action taken, Failure cost.
5. The process is jointly monitored by a VSIFT CUSUM-m and CUSUM-S^2 control chart.
6. At the time of detection and restoration, the whole system is stopped. After restoration, the system returns to perfect condition.

4.2.4 Problem Description

If FM_1 occurs, it immediately stops the machine. Corrective actions are taken to repair the machine to its operating condition. Thus, the expected cost of corrective maintenance $E\,[C_{CM}]_{FM1}$ includes the cost of down time, and the cost of repair/restore action. FM_2 affects the functionality of the machine and in turn increases the rejection level. In other words, FM_2 affects the process rejection rate. It is assumed that whenever FM_2 is detected, the process is stopped immediately, and corrective actions are taken to repair the process back to the normal condition. Apart from failures owing to FM_2, the process may also deteriorate owing to external causes (E) such as environmental effects, operators' mistakes, use of wrong tool, etc. The process is reset to the in-control state if an external event 'E' is detected. The time-to-failure of the process is assumed to follow an exponential distribution (Ben-Daya [17]). The detection of FM_2 or an external cause is achieved by monitoring the process. In this thesis a VSIFT CUSUM control chart mechanism is considered for process monitoring. Thus, the expected total cost of process failure $E\,[TCQ]_{process\text{-}failure}$ owing to FM and external events considering the cost of down time, cost of rejections owing to process shifts, cost of repair/resetting, cost of sampling and inspection, cost of investigation of false/valid alarm, and cost of deviation from the target value of the CTQ. Apart from the above corrective actions, the machine can undergo preventive maintenance t_{PM} to minimize the unplanned downtime losses. In this thesis, imperfect preventive maintenance has been considered. This means that the PM upgrades the equipment to a state between the as-good-as-new and as-bad-as-old conditions. The frequency of failures can be significantly decreased through PM. Reduction in FM_2 reduces the quality costs related to the out-of-control operation. However, PM also consumes some resources and productive machine time that could otherwise be used for production. The expected cost of PM (E [C_{PM}]) comprises the cost of downtime and cost of performing preventive maintenance actions.

4.3 Mathematical Model

4.3.1 Joint ARL Computation for mean and variance

Average run length (ARL) is a measure of the expected number of consecutive samples taken before the sample statistic falls outside the control limits. Since ARL is a function of the prevailing process mean and standard deviation, its value depends on whether the process is in-control or out-of-control. When multiple charts are used jointly for monitoring the process, the investigation for an assignable cause is initiated when at least one of the charts shows an out-of-control signal. Hence, not the ARLs of the individual charts but the joint ARL of the overall control scheme is the relevant performance measure when multiple charts are used simultaneously. Joint ARL of mean and variance for the VSIFT CUSUM chart can be computed using absorbing Markov chain approach. According to Morais and Pacheco [22] to determine the ARL of combined scheme, the run length distribution of mean and variance charts need to be determined. Then using this results complementary cumulative distribution function can be determined.

Joint ARL, $ARL_j = [\sum_{s=0}^{\infty} \bar{F}_{RL_{\mu}^{i}}(s) * \bar{F}_{RL_{s^2}^{j}}(s)]_{\substack{i=1....u+1 \\ j=1....v+1}}$ (4.18)

$$\bar{F}_{RL_{\mu}^{i}}(s) = \begin{cases} 1, & S<1 \\ e_{\mu.i}^{T} * [Q_{\mu}]^{s} * 1_{\mu}, & s>1 \end{cases} \quad (4.19)$$

$$F_{RL_{s^2}^{j}}(s) = \begin{cases} 1, & s<1 \\ e_{s.^2\, j}^{T} * [Q_{s^2}]^{s} * 1_{s^2}, & s>1 \end{cases} \quad (4.20)$$

Where $\bar{F}_{RL_{\mu}^{i}}(s)$ and $F_{RL_{s^2}^{j}}(s)$ are the probability that the expected run length of CUSUM-m and CUSUM-S^2 is greater than or equal s, respectively. Here u+1 and v+1 are the number of in control state for CUSUM-m and CUSUM-S^2 chart respectively. Here $e_{\mu.i}^{T}$ and $e_{s.^2\,j}^{T}$ are the transpose of orthonormal basis of R^{u+1} and R^{v+1}, respectively. Q_{μ} and Q_{s^2} are [u x u] and [v x v] matrix, that represents initial transition probabilities for CUSUM-m and CUSUM-S^2 chart, respectively. 1_{μ} and 1_{s2} are the column vector of ones.

4.3.2 Development of cost function

The cost function, which is the objective function of the economic design of the VSIFT CUSUM chart integrating maintenance management, is developed based on the economic model given in Lorenzen and Vance [13].

Expected Cycle Length Determination

The cycle length is defined as the total time from which the process starts in control, shifts to an out-of-control condition, has the out-of-control condition detected, and results in the assignable cause being identified and the process being corrected. Four time intervals in an expected cycle length are respectively

1. The interval the process is in control, denoted by T_1,

2. The interval the process is out of control before the final sample of the detecting subgroup is taken, denoted by T_2,

3. The interval to sample, inspect, evaluate and plot the subgroup result, denoted by T_3, and

4. The interval to search for the assignable cause and correct the process, denoted by T_4.

When the expected cycle length is determined, the cost components can be converted to a ''per hour of operation'' basis.

The average time interval that the process is in control can be expressed by

$$E\,(T_{1}) = \frac{1}{\lambda} + (1-\gamma_1)*t_0 * \frac{s}{\mathrm{ARL}_j 1} \quad (4.21)$$

Where t_0 is expected time of searching for an assignable cause under a false alarm, s is the expected sampling frequency while in control and ARL_{j1} denotes Joint Average Run Length when process is in control.

$\gamma_1 = 1$; if production continues during searches

0; if production ceases during searches

λ is the process failure rate. Let the rate of failure due to external causes be λ_1 and that due to poor machine condition be λ_2. Since we assume that λ_1 and λ_2 are independent and do not occur simultaneously, the process failure rate (λ) is the sum of failure rates due to external causes and due to machine degradation. Thus, It can be written as

$\lambda = \lambda_1 + \lambda_2$

Process failure rate due to external causes is calculated as

$\lambda_1 =$ (1 / Mean time between process failures due to external causes)

Pandey et al [2] used a simulation approach to generate a regression model to establish a relationship between N_f and t_{PM}

$$N_f = 0.0437 * (t_{PM})^{0.8703} \tag{4.22}$$

The process failure rate owing to machine degradation can be calculated as

$$\lambda_2 = \frac{N_f * P_{FM_2}}{T_{eval}} \tag{4.23}$$

Where, N_f is the number of machine failures and T_{eval} is the evaluation period and t_{PM} is the preventive maintenance interval.

Calculation of the expected sampling frequency while in control

Here the sampling frequency is calculated based on the concept given by Chou et. al. [27].The first sampling occurs at time point h (i.e., this is the only one sampling during the first interval [0, h]). When the process is in control, except for the first interval, the possible values of the sampling frequency for any interval with fixed length h (i.e., the intervals [h, 2h], [2h, 3h], [3h, 4h] ...) are 1, 2,…. η. For any interval between two consecutive fixed sampling time points (except for the first interval), define A_i as the event that the process is in control at the ith sampling given that the (i -1)st sample point falls in the warning region, for i =1,2,..., η. Then the expected sampling frequency for any interval with fixed length h (excluding the first interval), denoted by v, is

$$v = \sum_{i=1}^{\eta} iP(A_i)$$

$$= (1-\rho) + 2\rho(1-\rho) + 3\rho^2(1-\rho) + \ldots\ldots + (\eta-1)\rho^{\eta-2}(1-\rho) + \eta\rho^{\eta-1}$$

$$= \frac{1-\rho^{\eta-1}}{1-\rho} + (2\eta-1)\rho^{\eta-1} \tag{4.24}$$

Let B_j be the event that the process is in control at the start of the jth interval with fixed length h, for j =2,3,..., ∞. Then, based on the model assumptions, we have

$$P(B_j) = 1 - \int_0^{(j-1)h} \lambda e^{-\lambda t}\, dt$$

$$= e^{-(j-1)\lambda h} \tag{4.25}$$

for j =2,3,..., ∞.

From Eqs. (4.8) and (4.9), the expected sampling frequency while in control is,

$$s = 1 + \left(\frac{1-\rho e^{\eta-1}}{1-\rho} + (2\eta-1)\rho^{\eta-1}\right) * \sum_{j=2}^{\infty} e^{-(j-1)\lambda h}$$

$$= 1 + \left(\frac{1-\rho e^{\eta-1}}{1-\rho} + (2\eta-1)\rho^{\eta-1}\right) * \left(\frac{e^{-\lambda h}}{1-e^{-\lambda h}}\right) \tag{4.26}$$

$$\eta = \frac{h}{d} \tag{4.27}$$

Where ρ is the conditional probability that the sample point is plotted in the warning region given that the process is in control and is equal to

$$\rho = \frac{2[\phi(k) - \phi(\omega)]}{\phi(k) - \phi(-k)} + \frac{[\phi(k1) - \phi(\omega1)]}{\phi(k1) - \phi(0)} - \left(\frac{2[\phi(k) - \phi(\omega)]}{\phi(k) - \phi(-k)} * \frac{[\phi(k1) - \phi(\omega1)]}{\phi(k1) - \phi(0)}\right) \tag{4.28}$$

Where $\phi(.)$ is the cumulative function of the standard normal random distribution,

α is the probability of false alarm and for EWMA chart it can be calculated by

$$\alpha = 1/ARL_{j1} \tag{4.29}$$

The average time interval that the process is out of control before the final sample of the detecting subgroup is taken may be written as

$$E(T_2)=ATS_2 - \xi \tag{4.30}$$

$$ATS_2 = (ATS_2)_{m/c} + (ATS_2)_{external} \tag{4.31}$$

Where ATS_2 is defined as the average time from the occurrence of an assignable cause to the time when the chart indicates an out-of-control signal.

$$ATS_2 = [\rho \sum_{i=0}^{\eta-2} \rho^i d + (1-\rho) \sum_{i=o}^{\eta-1} \rho^i (h - id)] * [(ARL_{j2})_{mc} * \frac{\lambda_2}{\lambda} + (ARL_{j2})_E * \frac{\lambda_1}{\lambda}] \tag{4.32}$$

Where $(ARL_{j2})_{mc}$ and $(ARL_{j2})_E$ are joint out of control ARL machine and external cause respectively. $= \rho \sum_{i=0}^{\eta-2} \rho^i d + (1-\rho) \sum_{i=o}^{\eta-1} \rho^i (h - id)$ is average sampling interval.

ξ is average time lag between the sampling time point, which is just prior to the occurrence of the assignable cause, and the time point that the assignable cause occurs and it can be shown that

$$\xi = \sum_{j=0}^{\eta-1} \left(\rho_{1j}\, \tau_{1j}\right) + \rho_2\, \tau_2 \tag{4.33}$$

Where p_{1j} is the ratio of the sampling interval h- jd to the average sampling interval and is equal to

$$p_{1j} = \frac{\rho^j (1-\rho)(h - jd)}{\rho \sum_{i=0}^{\eta-2} \rho^i d + (1-\rho) \sum_{i=o}^{\eta-1} \rho^i (h - id)} \tag{4.34}$$

p_2 is the ratio of the sampling interval d to the average sampling interval and is equal to

$$p_2 = \frac{\rho \sum_{i=0}^{\eta-2} \rho^i d}{\rho \sum_{i=0}^{\eta-2} \rho^i d + (1-\rho) \sum_{i=o}^{\eta-1} \rho^i (h - id)} \tag{4.35}$$

τ_{1j} is defined as, given that the assignable cause occurs between the sampling time points $ih + jd$ and $(i+1)h$, the expected in-control time interval during this period and, from the definition, it may be shown that

$$\tau_{10} = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda\,(1-e^{-\lambda d})} \tag{4.36}$$

$$\tau_{1j} = \frac{1}{\lambda} - \frac{(\eta-j)de^{-\lambda(\eta-j)h}}{1-e^{-\lambda(\eta-j)h}} \tag{4.37}$$

For j = 1, 2…..η-1

and τ_2 is defined as, given that the assignable cause occurs between the ith and $(i+1)$st sampling time points with sampling interval d, the expected in-control time interval during this period and is equal to

$$\tau_2 = \frac{1-(1+\lambda d)e^{-\lambda d}}{\lambda(1-e^{-\lambda d})} \tag{4.38}$$

The average time interval to sample, inspect, evaluate and plot the subgroup result is equal to

$$E(T_3)=n*Ts \tag{4.39}$$

where Ts is the average sampling, inspecting, evaluating and plotting time for each sample. The average time interval to search for the assignable cause and correct the process is

$$E\,(T_4) = t_1 + E\,(T_{restore})$$

$$=t_1+(T_{resetting}*\frac{\lambda 1}{\lambda}+MTTR_{cr}*\frac{\lambda 2}{\lambda}) \tag{4.40}$$

where t_1 is the average time to search for the assignable cause and $E(T_{restore})$is the expected time to repair or reset the process. Therefore, from Eqs. (4.20), (4.29), (4.37) and (4.38), the expected cycle length is

$$E\,(T_{cycle}) = E\,(T_1) + E\,(T_2) + E\,(T_3) + E\,(T_4) \tag{4.41}$$

Expected Cost Calculation

According to Pandy et al [2], to estimate the expected cost of corrective maintenance owing to FM_1 and preventive maintenance, the analyst must have the following information:

1. The amount of time that the equipment is expected to be down each time CM/PM is required. This can include the time to perform the maintenance as well as any logistical delays (i.e. waiting for labor and/or materials required).

2. The cost of CM/PM including the downtime, labor, materials, and other costs.

3. The degree to which the equipment will be restored by CM/PM (e.g. 'as good as new,' 'as bad as old,' or 'Imperfect'). This is quantified in terms of a restoration factor. The restoration factor can be determined empirically or based on expert judgment as calculated in Reliasoft [35] and Lad and Kulkarni [33] respectively.

4. The probability that the equipment will fail owing to a particular failure mode.

The expected cost of minimal corrective maintenance owing to FM_1 is given as:

$$E\,[C_{CM}]_{FM1}=\{MTTR_{CM}[PR.C_{lP}+LC]+C_{FCPCM}\}*P_{FM1}*N_f \tag{4.42}$$

$MTTR_{CM}$ [PR. C_{lP} +LC] is the down time cost owing to corrective maintenance

The expected total cost of preventive maintenance action of component will be

$$E[C_{PM}]=\{MTTR_{PM}[PR.C_{lP}+LC]+C_{FCPPM}\}*\frac{T_{eval}}{t_{PM}} \tag{4.43}$$

Where $MTTR_{PM}$ [PR. C_{lP} +LC] is the down time cost owing to Preventive maintenance.

Let the cost per unit time for investigating a false alarm is $C_{false.}$ This includes the cost of searching and testing for the cause. Then the expected cost of false alarm

$$E[C_{false}] = C_{false}\,.\,\frac{S}{ARL_{j\,1}}\,.\,t_0 \tag{4.44}$$

Let 'a' be the fixed cost per sample of sampling and 'b' be the variable cost per unit sampled. Thus, the expected cost per cycle for sampling is the sum of the fixed cost per sample and variable cost per unit sampled, and is given as:

$$E\ [\text{Cost of sampling}] = \frac{(a+bn)[\frac{1}{\lambda} + \left\{(1-\gamma 1)*t0*\frac{s}{ARL_{j}1}\right\} + ATS_2 - \xi + \eta Ts + r_1 t_1 + r_2\, E(T_{restore})]}{\rho \sum_{i=0}^{\eta-2} \rho^i d + (1-\rho) \sum_{i=0}^{\eta-1} \rho^i (h - id)} \tag{4.45}$$

.

$\gamma_2 = 1$; if production continues during process correction

0; if production ceases during process correction

Let $C_{resetting}$ be the cost for finding and resetting the assignable cause owing to external reasons, downtime if process ceases functioning, and for finding and resetting the process. The expected value of $C_{resetting}$ can be calculated as:

$$E[C_{resetting}]=[C_{resetting}*T_{resetting}]*\frac{\lambda_1}{\lambda} \tag{4.46}$$

The expected cost of corrective maintenance action owing to failure mode FM_2 of the component and for finding and repairing the assignable cause owing to machine failure is given by:

$$E[(C_{repair})_{FM2}]=\{MTTR_{CM}[PR.\ C_{IP}+LC]+C_{FCPPM}\}*\frac{\lambda_2}{\lambda} \tag{4.47}$$

Consideration of Taguchi Loss

To calculate the cost of quality loss incurred owing to defectives produced while the process is in control and out of control a Taguchi loss function has been used. Consider a Critical to Quality (CTQ) with bilateral tolerances of equal value (Δ) for CUSUM-m chart and unilateral tolerance of value (Δ_1) for CUSUM-S^2 chart. The cost to society for manufacturing a product out of specification is A Tk/unit, and uniform rejection cost is incurred beyond the control limits.

CUSUM-m chart

[$L_{\text{in control}}$] determination: The quality loss per unit time incurred while process is in control state is given as

$$[L_{incontrol}]_{mean}=[PR*\frac{A}{\Delta^2}\int_{\mu-\frac{k\sigma}{\sqrt{n}}}^{\mu+\frac{k\sigma}{\sqrt{n}}}(x-\mu)^2 f(x)dx] + (PR*R*C_{frej}) \quad (4.48)$$

Where PR is production rate, x is a random variable denoting sample means of the quality characteristic and $f(x)$is its normal density function with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$ .Now any deviation from the target value will incur a loss. This was not the case under the classical SPC approach. The proportion of non-conforming units 'R' when the process is in-control state is given as $R = 1 - \{\phi(k) - \phi(-k)\}$.$C_{frej}$ represents cost of rejection per unit.

From algebraic manipulations given in Appendix-A,

$$[L_{in\ control}]_{mean} = PR*\frac{A}{\Delta^2}*\frac{\sigma^2}{n}[1-\frac{2k}{\sqrt{2\pi}}e^{\frac{-k^2}{2}}-\alpha] + (PR*R*C_{frej}) \quad (4.49)$$

Where $\alpha = 2\,\phi(-k)$

[L $_{out\ of\ control}$] determination

$$[L_{out\ of\ control}]_{mean} = PR*\frac{A}{\Delta^2}\int_{-\infty}^{\infty}(x'-\mu)^2 f(x')dx' - \int_{\mu-\frac{k\sigma}{\sqrt{n}}}^{\mu+\frac{k\sigma}{\sqrt{n}}}(x'-\mu)^2 f(x')dx'\}$$

Where $x'$is a random variable denoting sample means and $f(x')$ is its normal density function with mean $\mu + \delta\sigma$ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

From algebraic manipulation given in Appendix-A,

$$[L_{out\ of\ control}]_{mean} = PR*\frac{A}{\Delta^2}*\frac{\sigma^2}{n}[(1+\delta^2 n)*(1-\beta)+(\frac{k+\delta\sqrt{n}}{\sqrt{2\pi}})*e^{\frac{-(k-\delta\sqrt{n})^2}{2}}$$
$$+(\frac{k-\delta\sqrt{n}}{\sqrt{2\pi}})*e^{\frac{-(k+\delta\sqrt{n})^2}{2}}] + (PR*R_\delta*C_{frej}) \quad (4.50)$$

$\beta = \phi(k - \delta*\sqrt{n}) - \phi(-k-\delta*\sqrt{n})$

Where R_δ is proportion of non-conforming unit while the process moves out of control state.

The quality loss per unit time incurred while the process is operating in out-of-control state owing to machine failure is given as:

$[L_{\text{out of control}}]_{\text{mean m/c}} = PR * \frac{A}{\Delta^2} * \frac{\sigma^2}{n}[(1+\delta_{m/c}^2 n) * \{1-\phi(k-\delta_{m/c} * \sqrt{n}) + \phi(-k-\delta_{m/c} * \sqrt{n})\} + (\frac{k+\delta_{m/c}\sqrt{n}}{\sqrt{2\pi}}) * e^{\frac{-(k-\delta_{m/c}\sqrt{n})^2}{2}} + (\frac{k-\delta_{m/c}\sqrt{n}}{\sqrt{2\pi}}) * e^{\frac{-(k+\delta_{m/c}\sqrt{n})^2}{2}}] + (PR * R_{\delta\, m/c} * C_{frej})$ (4.51)

Where $R_{\delta\, m/c} = 1-\{\phi(k-\delta_{m/c}\, \sigma) - \phi\left(-k-\delta_{\, m/c}\sigma\right)\}$.

Similarly, the quality loss per unit time incurred while the process is operating in out-of- control state owing to external reasons is given as:

$[L_{\text{out of control}}]_{\text{mean E}} = PR * \frac{A}{\Delta^2} * \frac{\sigma^2}{n}[(1+\delta_E^2 n) * \{1-\phi(k-\delta_E * \sqrt{n}) + \phi(-k-\delta_E * \sqrt{n})\} + (\frac{k+\delta_E\sqrt{n}}{\sqrt{2\pi}}) * e^{\frac{-(k-\delta_E\sqrt{n})^2}{2}} + (\frac{k-\delta_E\sqrt{n}}{\sqrt{2\pi}}) * e^{\frac{-(k+\delta_E\sqrt{n})^2}{2}}] + (PR * R_{\delta\, E} * C_{frej})$ (4.52)

Where $R_{\delta\, m/c} = 1-\{\phi(k-\delta_E\sigma) - \phi(-k-\delta_E\, \sigma)\}$ (4.53)

CUSUM- S^2 chart

Since it's a smaller the better situation. I.e. it's better if variance is smaller in CUSUM-S^2 chart, the desirable value for variance is 0.in this case only upper control limit is considered to monitor the chart. Trietsch [36] stated that when the expected cost of exceeding the tolerance limits is not equal to the right and to the left of the target, Taguchi quadratic loss function is inappropriate in that situation. That's why here in control and out of control loss for CUSUM-S^2 chart is determined considering modified Kapoor and Wang'[37] model stated in Chen and Chou [9] , which is a linear loss function.

$[L_{\text{in control}}]$ determination: The quality loss per unit time incurred while process is in control state is given as

$$[L_{\text{in control}}]_{\text{variance}} = PR * \frac{A}{\Delta 1}\int_{-\infty}^{\frac{k1*\sigma}{\sqrt{n}}} y f(y) dy + (PR * R' * C_{frej}) \quad (4.54)$$

$$f(y) = \frac{1}{\phi(k1) * \frac{\sigma}{\sqrt{n}} * \sqrt{2\pi}} * e^{-\frac{(\frac{y-\mu}{\frac{\sigma}{\sqrt{n}}})^2}{2}} \quad (4.55)$$

Where PR is production rate, y is a random variable denoting sample variance of the quality characteristic and $f(y)$is the probability density function of truncated normal random variable with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$ .Now any deviation from the target value will incur a loss. In this work $\frac{\mu}{\sigma}$>5, probability that y<0 is tense to 0.

From algebraic manipulations given in Appendix-B,

$$[L_{incontrol}]_{variance} = PR * \frac{A}{\Delta 1} * \frac{1}{\phi(k1)} \{\mu * \phi\left(k1 \frac{\sigma}{\sqrt{n}} - \mu\right) - \frac{\sigma}{\sqrt{n}} * \varphi\left(k1 \frac{\sigma}{\sqrt{n}} - \mu\right) + (PR * R' * C_{frej}) \tag{4.56}$$

Where $\varphi(.)$ Signifies standard normal probability density function. R' denotes proportion defective item while the process is in control state.

$$R' = 1 - \phi(k1) \tag{4.57}$$

[L $_{\text{out of control}}$] determination

$$[L_{\text{out of control}}]_{variance} = PR * \frac{A}{\Delta 1} \{\int_{-\infty}^{\infty} y' f(y') dy' - \int_{-\infty}^{\frac{k\sigma}{\sqrt{n}}} y' f(y') dy'\} + (PR * R'_{\delta} * C_{frej})$$

Where $y'$is a random variable denoting sample variances and $f(y')$ is the probability density function of truncated normal random variable with mean $\mu + \delta 1\sigma$ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

$$f(y') = \frac{1}{\phi(k1) * \frac{\sigma}{\sqrt{n}} * \sqrt{2\pi}} * e^{-\frac{\left(\frac{y-\mu-\delta 1\sigma}{\frac{\sigma}{\sqrt{n}}}\right)^2}{2}} \tag{4.58}$$

From algebraic manipulation given in Appendix-B,

$$[L_{\text{out of control}}]_{variance} = PR * \frac{A}{\Delta 1} * \frac{1}{\phi(k1)} [\varphi\left(k1 - \frac{\mu\sqrt{n}}{\sigma} - \delta 1\sigma\right) + \{\{1 - \phi(k1 - \frac{\mu\sqrt{n}}{\sigma} - \delta 1\sigma)\}(\mu + \delta 1\sigma)\}] + (PR * R'_{\delta} * C_{frej}) \tag{4.59}$$

Where R'_{δ} is proportion of non-conforming unit while the process moves out of control state .

The quality loss per unit time incurred while the process is operating in out-of-control state owing to machine failure is given as:

$[L_{\text{out of control}}]_{\text{variance m/c}}$ = PR* $\frac{A}{\Delta 1} * \frac{1}{\phi(k1)}[\varphi\left(k1 - \frac{\mu\sqrt{n}}{\sigma} - \delta 1_{m/c}\sigma\right) + \{\{1 - \phi(k1 - \frac{\mu\sqrt{n}}{\sigma} - \delta 1_{m/c}\sigma)\}(\mu + \delta 1_{m/c}\sigma)\}]+ (PR * R'_{\delta\, m/c} * C_{frej})$ (4.60)

Where $R'_{\delta\, m/c} = 1 - \phi(k1 - \delta_{m/c}\,\sigma)$ (4.61)

Similarly, the quality loss per unit time incurred while the process is operating in out-of- control state owing to external reasons is given as:

$[L_{\text{out of control}}]_{\text{variance E}}$ = PR* $\frac{A}{\Delta 1} * \frac{1}{\phi(k1)}[\varphi\left(k1 - \frac{\mu\sqrt{n}}{\sigma} - \delta 1_{E}\sigma\right) + \{\{1 - \phi(k1 - \frac{\mu\sqrt{n}}{\sigma} - \delta 1_{E}\sigma)\}(\mu + \delta 1_{E}\sigma)\}]+ (PR * R'_{\delta\, E} * C_{frej})$ (4.62)

Where $R'_{\delta\, E} = 1 - \phi(k1 - \delta_{E}\,\sigma)$ (4.63)

Thus, the expected process quality loss incurred per cycle in the in-control state is:

$E[L_{\text{incontrol}}] = \{[L_{\text{in control}}]_{\text{mean}} + [L_{\text{in control}}]_{\text{variance}}\} * \frac{1}{\lambda}$ (4.64)

Thus, the expected quality loss incurred per cycle in the out-of-control state owing to machine failure is:

$E[(\text{cost of } L_{\text{out of control}})_{M/C}] = [[L_{\text{out of control}}]_{\text{mean m/c}} + [L_{\text{out of control}}]_{\text{variance m/c}}] * \{ATS_2 - \xi + n{*}Ts + r_1t_1 + r_2 * E(T_{\text{restore}})\} * \frac{\lambda_2}{\lambda}$ (4.65)

Thus, the expected quality loss cost incurred in out-of-control state owing to external reasons is:

$E[(\text{cost of } L_{\text{out of control}})_{E}] = [[L_{\text{out of control}}]_{\text{mean E}} + [L_{\text{out of control}}]_{\text{variance E}}] * \{ATS_2 - \xi + n{*}Ts + r_1t_1 + r_2 * E(T_{\text{restore}})\} * \frac{\lambda_1}{\lambda}$ (4.66)

Adding Equations (4.42), (4.43), (4.44), (4.45), (4.59), (4.60), and (4.61) gives the expected cost of process failure per cycle as:

$E[C_{process}] = E[C_{false}] + E$ [Cost of sampling] $+ E[C_{resetting}] + E[(C_{repair})_{FM2}] + E[L_{in\ control}] + E[(\text{cost of } L_{out\ of\ control})_{M/C}] + E[(\text{cost of } L_{out\ of\ control})_E]$ (4.67)

The process failure is assumed to be repetitive in nature, i.e. every time when the process moves out-of-control from the in-control state and is again restored, it will take the same expected time (having fixed expected cycle length).If there are M process failure cycles in a given evaluation period, the expected process quality control cost for the evaluation period will be:

$$E[TCQ]_{process\text{-}failure} = E[C_{process}] * M \quad (4.68)$$

where , $M = \frac{T_{eval}}{E[T_{cycle}]}$ (4.69)

Optimization Model

The problem is to determine the optimal values of the decision variables (n, h, η, k, w, t_{pm}, $k1$ *and* $w1$) that minimize the expected total cost per unit time of the system ETCPUT. Recall that the age of the equipment after a PM is reduced according to the restoration factor. The expected total cost per unit time of preventive maintenance and control chart policy ETCPUT is the ratio of the sum of the expected total cost of the process quality control ($E[TCQ]_{process\text{-}failure}$), expected total costs of the preventive maintenance $E[C_{PM}]$ and expected total cost of machine failure ($E[C_{CM}]_{FM1}$) to the evaluation time. Therefore, the expected total cost per unit time for the integrated model is given as:

$$\text{Minimize } [ETCPUT] = \frac{E[C_{CM}]FM1 + E[C_{PM}] + E[TCQ]process\text{-}failure}{Teval} \quad (4.70)$$

Where $[ETCPUT] = f(n, h, \eta, k, w, t_{pm}, k1 \text{ and } w1)$ and T_{eval} is the Planning period for which the analysis is done.

CHAPTER V

NUMERICAL EXAMPLE

Equation (4.70) indicates that optimizing the eight variables (n, h, η, k, w, $t_{pm,}$ $k1$ *and* $w1$) is not a simple process. In this section, we present a numerical example (According to Pandey et al. [2]) to illustrate the nature of the solution obtained from the economic design of the proposed integrated model.

To illustrate, consider a single component operating as a part of a machine. Machine failure is assumed to follow a two-parameter Weibull distribution. The machine considered here is expected to operate for three shifts of seven hours each for six days in a week. Time to execute a preventive maintenance action T_{PM} = 7 time units with a restoration factor RF_{PM} = 0.6 (it implies 60% restoration of life and sets the age of the block to 40% of the age of the block at the time of the maintenance action) and time to execute corrective maintenance action T_{CM}=12 time units with restoration factor $RF_{CM\,=}$ 0 (repair is minimal, i.e. the age of a repaired machine is the same as its age when it failed).The time to failure for the component was obtained through simulation used in Pandey et al. [2].

A hypothetical example is illustrated in this section to compute joint ARL and to analyze the proposed integrated model. It is assumed that the CUSUM control chart is used to monitor a CTQ characteristic. Assuming that the process, in its in-control state, is characterized by a process mean of μ_0 = 24 mm and a process standard deviation of σ = 0.01 and that the magnitude of the shift owing to external reasons is δ_E = 1 and owing to machine failure is $\delta_{m/c\,=}$ 0.5, which occurs at random and results in a shift of process mean from μ_0 to $\mu_{0\,+}\,\delta\,\sigma$.

The initial values of necessary parameters to obtain joint ARL is given below.

Initial transition probability matrix for CUSUM-m chart,

$$Q_\mu = \begin{bmatrix} .47 & .47 & .03 & 0 & 0 & 0 & 0 & 0 \\ .09 & .59 & .31 & .01 & 0 & 0 & 0 & 0 \\ 0 & .15 & .64 & .15 & .06 & 0 & 0 & 0 \\ 0 & .01 & .19 & .65 & .09 & .06 & 0 & 0 \\ 0 & 0 & .03 & .35 & .45 & .13 & .04 & 0 \\ 0 & 0 & 0 & .09 & .3 & .41 & .16 & .04 \\ 0 & 0 & 0 & 0 & .05 & .25 & .45 & .2 \\ 0 & 0 & 0 & 0 & .02 & .13 & .32 & .39 \end{bmatrix}$$

Initial transition probability matrix for CUSUM-S^2 chart,

$$Q_{S^2} = \begin{bmatrix} .48 & .45 & 0 & 0 & 0 & 0 & 0 & 0 \\ .28 & .45 & .19 & .05 & 0 & 0 & 0 & 0 \\ 0 & .25 & .61 & .12 & .02 & 0 & 0 & 0 \\ 0 & .01 & .3 & .45 & .21 & .03 & 0 & 0 \\ 0 & 0 & .02 & .28 & .41 & .28 & .01 & 0 \\ 0 & 0 & 0 & .05 & .21 & .49 & .24 & .01 \\ 0 & 0 & 0 & 0 & 0 & .45 & .47 & .08 \\ 0 & 0 & 0 & 0 & 0 & 0 & .43 & .57 \end{bmatrix}$$

The initial values of necessary parameters to develop the integrated model are given in Table 5.1.

Table 5.1 Initial values of necessary parameters for the hypothetical numerical example

Parameter	Value	Parameter	Value
δ_E	1	PR	10
$\delta_{m/c}$	0.5	T_{eval}	1000
$\delta 1_E$	0.004	C_{lp}	400
$\delta 1_{m/c}$	0.001	$T_{resetting}$	2
a	50	LC	500
b	5	Ts	0.4
C_{false}	1200	$MTTR_{CM}$	12
A	500	$MTTR_{PM}$	6
C_{Frej}	2750	Δ	0.03
C_{FCPPM}	1000	$\Delta 1$	0.005
C_{FCPCM}	10000	μ_0	24
C_{reset}	2000	σ	0.01
t_0	1	λ_1	0.05
t_1	1	P_{FM1}	0.4
γ_1	0	P_{FM2}	0.6
γ_2	0		

CHAPTER VI

RESULTS AND DISCUSSIONS

Since this thesis work is theoretical in nature, here a mathematical hypothetical problem has been considered to show better demonstration about the implementation of the proposed model. The proposed model is composed of some mathematical equations which are required to determine the optimal values of eight decision variables i.e. sample size (n), fixed sampling interval (h), number of subintervals between two consecutive sampling times (η), control limit coefficient for mean (k), warning limit coefficient for mean (w), preventive maintenance interval (t_{pm}), control limit coefficient for variance (k_1) and warning limit coefficient for variance (w_1). A computer programming code of the proposed model has been generated in Matlab 7.8.0 (R2009a) and two of the optimization methods have been used to determine the optimal values.

6.1 Result Discussion

In this work the Nelder Mead Downhill simplex algorithm is mainly used as numerical search method to find the optimum results for a given set of input parameters. The numerical results obtained from three of the Nelder Mead Simplex runs are summarized in Table 6.1. Although the Nelder Mead method does not guarantee convergence to the global optimal solution, but this method succeeds in obtaining a good reduction in the function value using a relatively small number of function evaluations. In this case, it is apparent that the cost values resulting from implementations with different initial points are close to each other. In the table 6.1 three of such implementations' results are summarized and it is observed that the results are fairly similar for both cost and decision variables. So it is evident from the results that as an optimization technique Nelder-Mead algorithm has exhibited good convergence.

Table 6.1 Optimization using Nelder-Mead downhill simplex method

No. of obs.	n	h	η	k	w	t_{pm}	K_1	W_1	Cost
1	8.0002	15.999	10	4.9992	1.5001	220	5	2	1119.303
2	8	16	9.9997	5	1.5	219	4	2	1119.083
3	8.0001	15.599	9.999	4.9997	1.5	219.35	4.97	2	1118.835

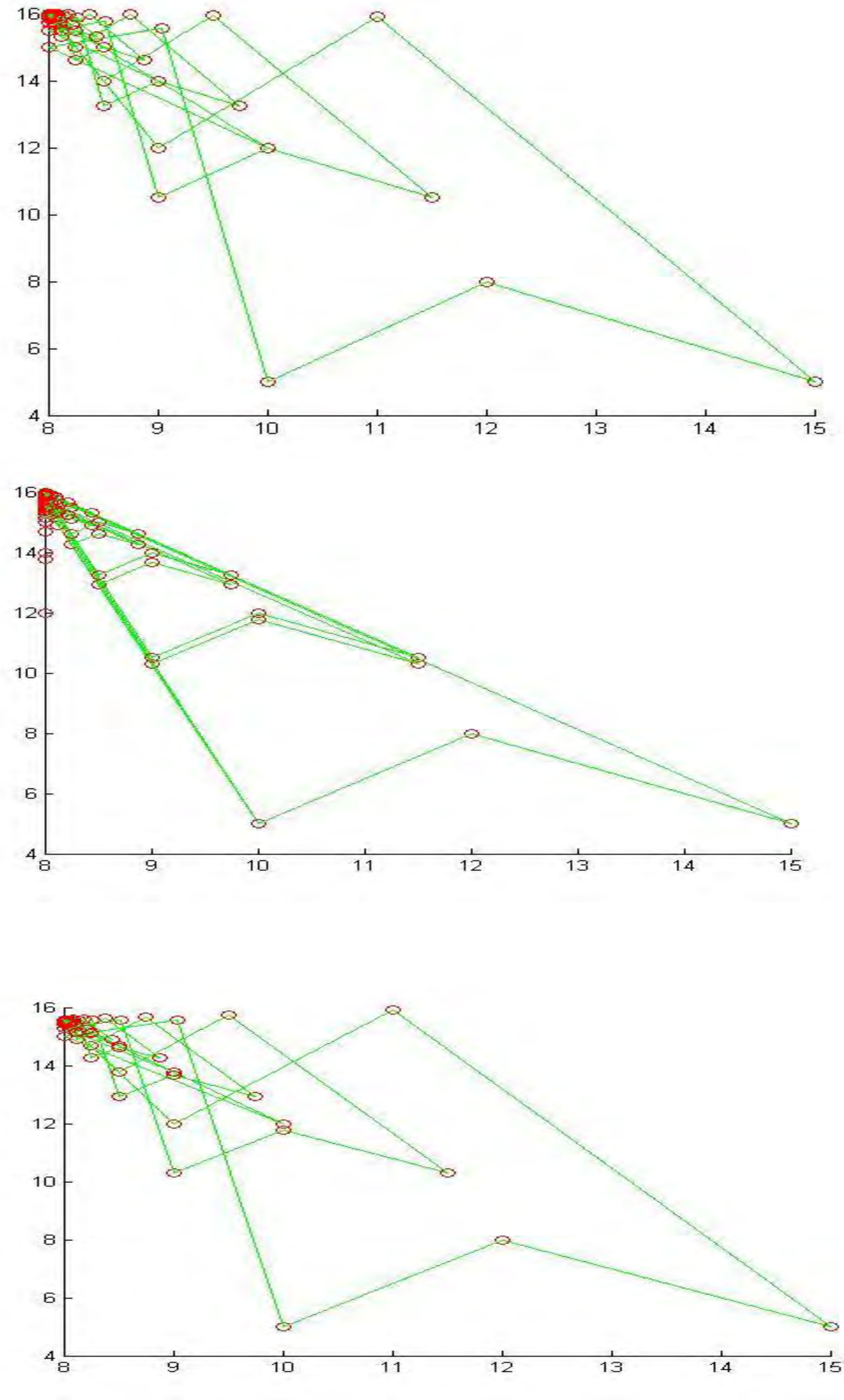


Fig. 6.1 Convergence path of points in the domain in Nelder-Mead method

The Convergence paths of points in the domain in Nelder-Mead method are shown in the Fig 6.1. From Fig 6.1 it can be seen all of the three figures look different although they are converging the same problem. It happened because the initial parameters were set different for each run and therefore the starting simplex of the Nelder Mead algorithms are different. And due to different initial values, the transition of the simplex from larger approximation to the optimal solution is different. But the destinations of the paths or the convergence points are same at the upper left corner of the graph. So it can be concluded that the solutions found using this method might reach to the global optimal value or close to that value.

Moreover, to validate the effectiveness of the result obtained in Nelder Mead approach, another technique "Genetic Algorithm (GA) approach" is also used to find the optimal values of decision variables that minimize the expected total cost of system per unit time ETCPUT. It has been observed that, the results are very close to the results of Nelder-Mead method. There are four results shown in table 6.2 obtained using GA. Stall generation (G) and mutation rate (m) have been changed to get better view of the result. The Convergence steps with number of generations in GA are shown in the Fig 6.2. Since both approaches exhibit quite similar results, so it can be concluded that we have got best economic solution of the problem.

Table 6.2 Optimization data using Genetic Algorithm

G	m	n	h	η	k	w	t_{pm}	K_1	W_1	Cost
1200	.02	8.002	15.994	9.991	5	1.5	220	4.999	2.0	1119.18
1000	.02	8.0001	15.999	9.998	4.99	1.5	219.997	4.999	2.0	1119.30
800	.02	8	16	9.984	4.98	1.5	219.997	5	2.0	1120.17
800	.015	8.0001	15.996	9.986	4.99	1.5	219.996	4.997	2.0	1119.38

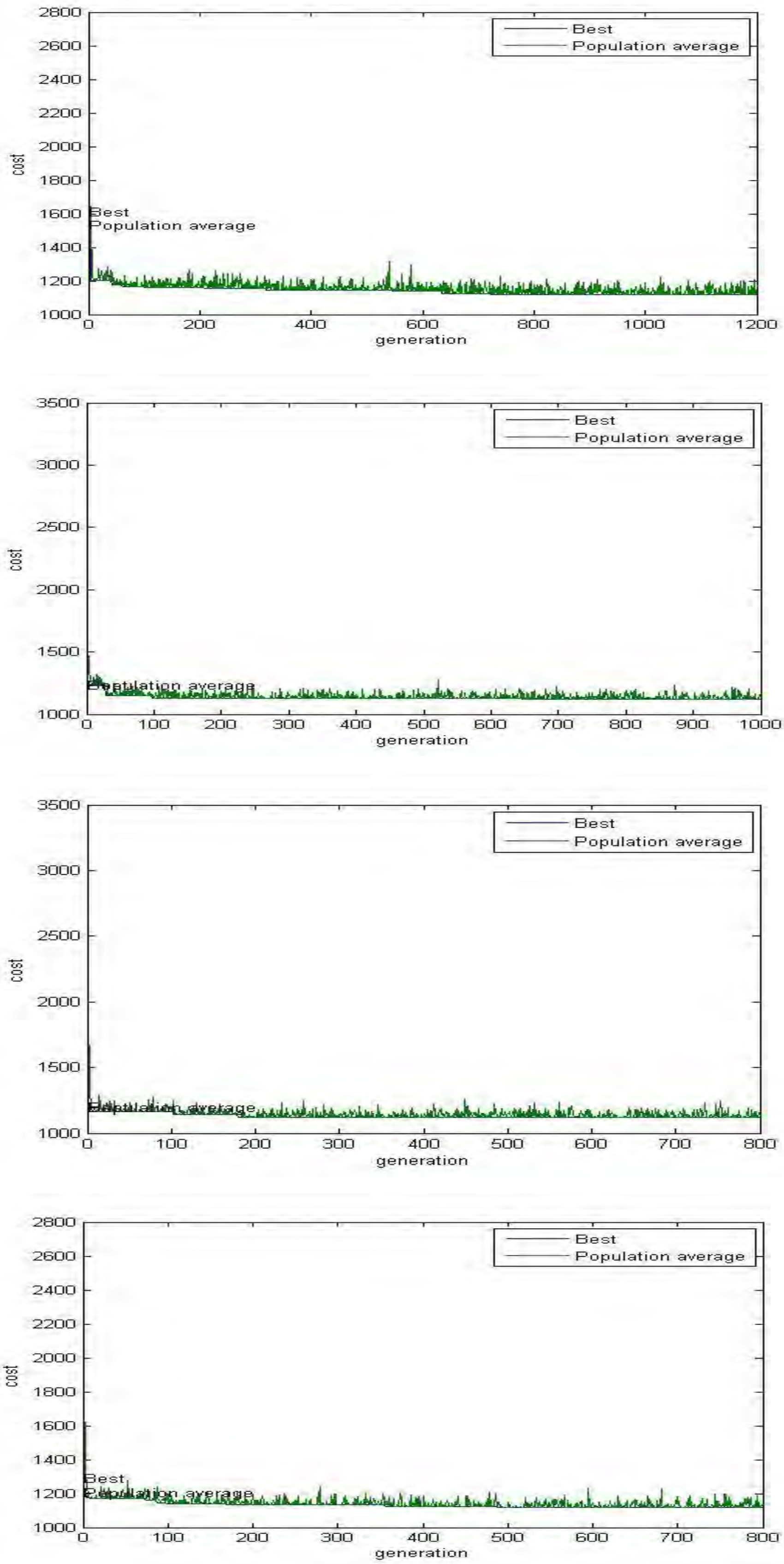


Fig. 6.2 Minimization of cost (Best cost and Population average) with number of generations in GA

6.2 Sensitivity Analysis

In order to study the effects of some of the model parameters, a sensitivity analysis is performed with the illustrative example shown in chapter 5. In Table.6.3, basic level which was used to solve the example in Section 6.1.Levels 1 and 2 represent the values of these parameters at -10 and +10% of the basic level respectively. The values of the objective function at this three different levels of the model parameters are also shown in Table.6.3.The column 'changes from level 1 to level 2' shows how sensitive the optimum value is with the change of the value of these parameters. From this table it can be seen that four parameters ($\delta_{E\text{ m}}$, $\delta_{E\text{ v}}$, λ_1 and A) affect more in changing the optimum cost value than any other parameters.

Table 6.3 Experimental data set and result of sensitivity analysis

parameters	Basic level	Level 1	Level 2	B.L Cost	L.1 Cost	L.2 Cost	Range
delta E m	1	0.9	1.1	1120	563.893	2512	1948.107
delta m/c m	0.5	0.45	0.55	1120	1121.99	1120.6	1.39
delta E V	0.004	0.0036	0.0044	1120	1118.3	1124.39	6.09
delta m/c V	0.001	0.0009	0.0011	1120	1118.7	1121.8	3.1
lemda 1	0.05	0.045	0.055	1120	1115.5	1128.7	13.2
b	5	4.5	5.5	1120	1119.7	1124.2	4.5
Ts	0.4	0.36	0.44	1120	1118.9	1120.8	1.9
A	500	450	550	1120	1039	1207.7	168.7
Cfrej(c7)	2500	2250	2750	1120	1119.6	1120.5	0.9
tr	2	1.8	2.2	1120	1119.8	1120.1	0.3
t0	1	0.9	1.1	1120	1120	1120	0
t1	1	0.9	1.1	1120	1120	1120	0
a	50	45	55	1120	1119.2	1124.1	4.9

To analyze the effect of these four parameters precisely, a DOE is performed by using ½ fraction factorial analysis. From table 6.4 and fig. 6.3 it can easily be observed that shift due to assignable cause in CUSUM-m chart ($\delta_{E\text{ m}}$) is the most significant parameter for this model other than the rest 3 competitive parameters. Because change in $\delta_{E\text{ m}}$ changes the objective function's value drastically. Though A and $\delta_{E\text{ m}}$ *A contribute to the change of optimum cost value but it is much less than the change causes by $\delta_{E\text{ m}}$. So, It can be concluded from this model, production process should be designed in a way to restrict $\delta_{E\text{ m}}$ as minimum as possible to minimize the overall cost.

Table 6.4 Results from ½ fraction factorial analysis

Parameters	Effect	F value	P value
$\delta_{E\,\mathrm{m}}$	1949.01	750815.78	0.00
$\delta_{E\,\mathrm{v}}$	1.01	0.20	0.665
λ_1	11.16	24.63	0.001
A	253.21	12672.50	0.00
$\delta_{E\,m} * \delta_{E\,v}$	0.98	0.19	0.675
$\delta_{E\,m} * \lambda_1$	8.33	13.71	0.006
$\delta_{E\,m} * A$	193.48	7398.74	0.00

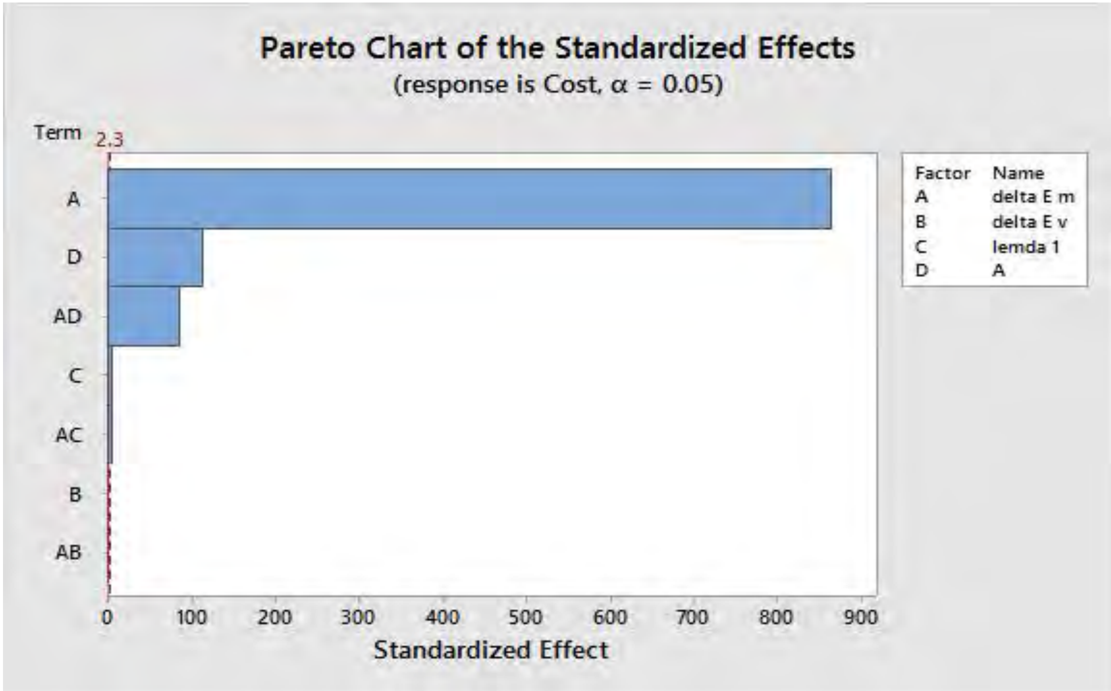


Fig. 6.3 Pareto Chart of the standardized effects in ½ fraction factorial analysis

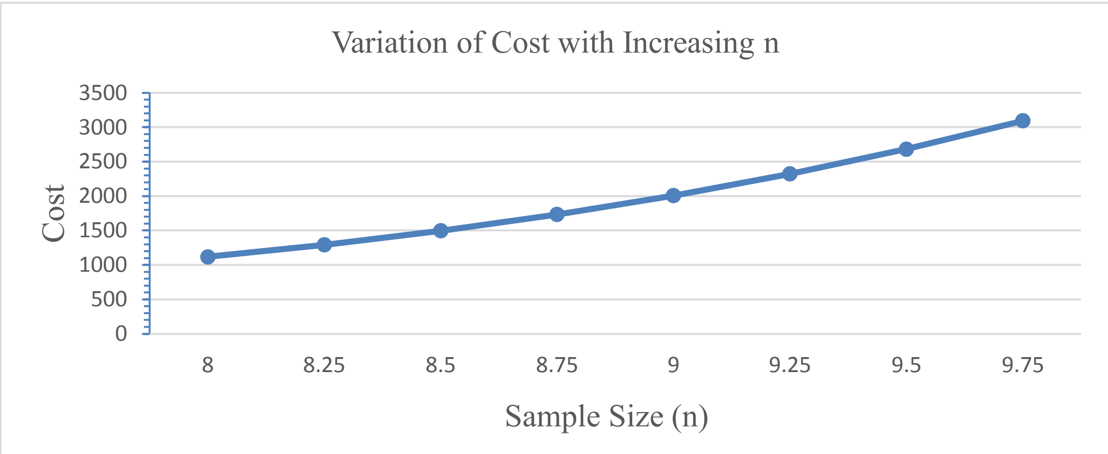
6.3 Effect of changes in variable parameter on cost

To observe the effect of the decision variables on the total cost, an analysis for all the decision variables was performed. In total 16 data points were used for each of the independent variable keeping all other variables constant. In this data set, 8 points were taken following increasing trend and the rest 8 points were taken following decreasing trend of the respective variable for better understanding about the relationship or effect of the variable change on the total cost.

Table 6.5 Variation of cost with the change of sample size and fixed sampling interval

Variation of cost with sample size				Variation of cost with fixed sampling interval			
n *increasing*	cost	*n* *decreasing*	cost	*h* *increasing*	cost	*h* *decreasing*	cost
8	1117.17	8	1117.17	16	1119.29	16	1119.29
8.25	1291.9	7.75	968.26	17	1119.26	15	1119.35
8.5	1495.95	7.5	842.05	18	1119.18	14	1119.42
8.75	1732.95	7.25	735.73	19	1119.12	13	1119.49
9	2006.82	7	646.7	20	1119.07	12	1119.55
9.25	2321.94	6.75	572.66	21	1119.04	11	1119.61
9.5	2682.85	6.5	511.51	22	1118.97	10	1119.66
9.75	3094.37	6.25	461.43	23	1118.93	9	1119.68

The variation of cost with the change of sample size and fixed sampling interval is shown in the Table 6.5 and the patterns of the relationships are shown in the Fig 6.4 and Fig 6.5 respectively. The green curve shows pattern for decreasing the value of the variable and the blue curve shows the pattern for increasing the value of the variable.



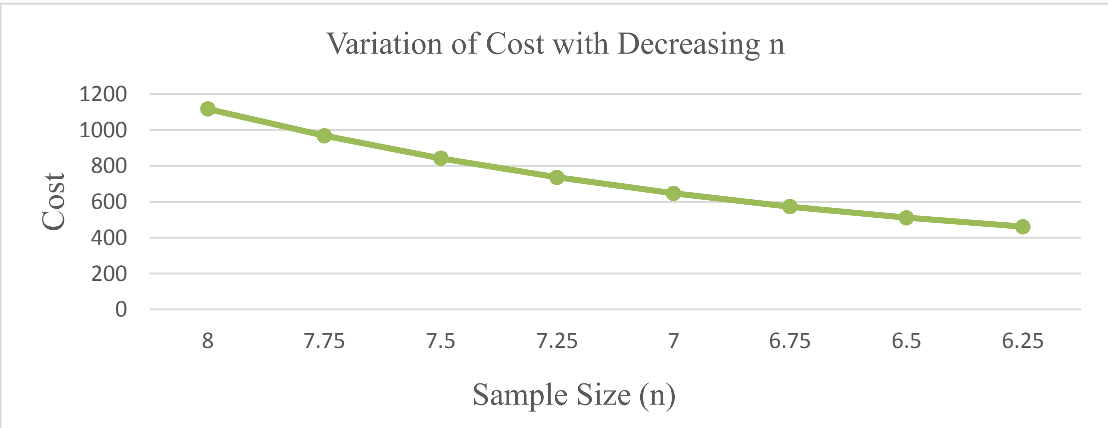
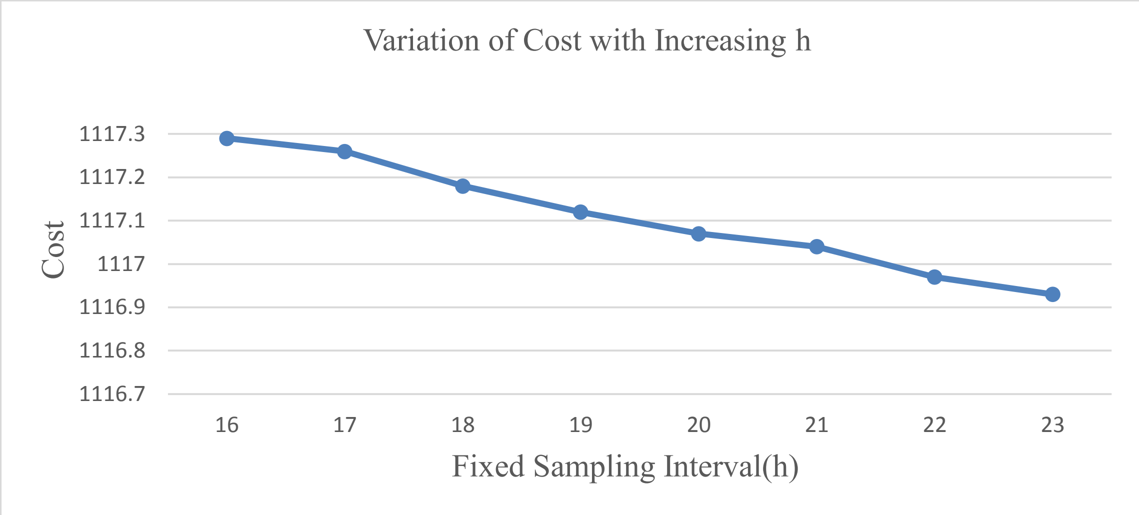


Fig. 6.4 Relationship between sample size variability and cost (Increasing and decreasing respectively)

It is evident from table 6.5 and figure 6.4 that with the increase of sample size cost also increase. It is quite justified as increase in sample size will definitely increase the sampling cost, again it will also influence the in control and out control cost and also make them increase.Due to the same reason with the decrease of sample size cost also decreases.



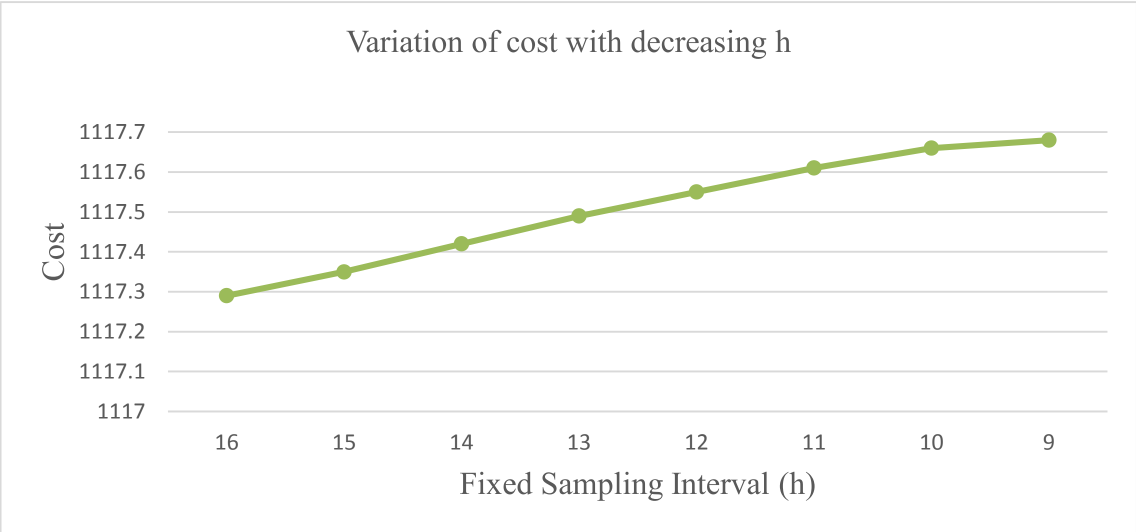


Fig. 6.5 Relationship between fixed sampling interval variability and cost (increasing and decreasing respectively)

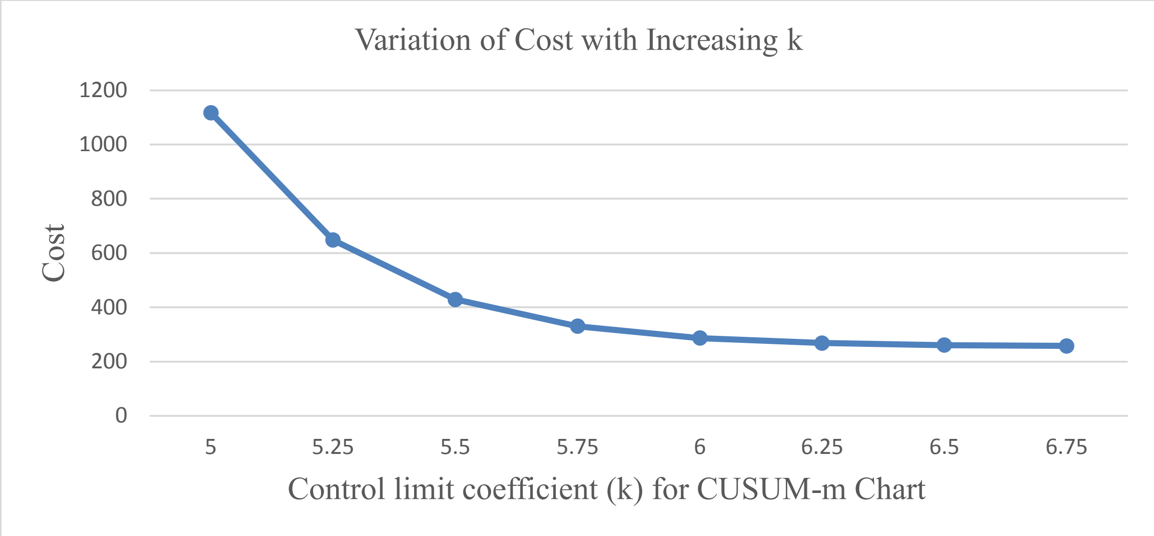
From the Fig.6.5 it can be found that with the increase of fixed sampling interval the cost is decreasing and vice versa. The reason is with higher sampling interval sampling frequency decreases which in turn decrease the sampling cost and so decrease the total cost. It is also evident from table 6.5 and fig.6.4 that, although with the change of fixed sampling interval cost changes but the rate of cost change is very low, which is negligible in terms of total cost.

Table 6.6 Variation of cost with change of control limit coefficient and warning limit coefficient of CUSUM-m chart

Variation of cost with control limit coefficient of CUSUM-m chart				Variation of cost with warning limit coefficient of CUSUM-m chart			
k increasing	cost	*k decreasing*	cost	*w increasing*	cost	*w decreasing*	cost
5	1117.3	5	1117.3	2.5	1119.57	2.5	1119.57
5.25	647.73	4.75	2069.6	2.75	1119.58	2.25	1119.55
5.5	428.63	4.5	3856.9	3	1119.59	2	1119.51
5.75	330.04	4.25	6896.35	3.25	1119.6	1.75	1119.4
6	286.8	4	11492.13	3.5	1119.600	1.5	1119.3
6.25	268.29	3.75	17564.8	3.75	1119.600	1.25	1119.08
6.5	260.61	3.5	24449.36	4	1119.601	1	1118.7
6.75	257.61	3.25	30985.1	4.25	1119.60	0.75	1117.99

The variation of cost with the change of control limit coefficient and warning limit coefficient for CUSUM-m chart is shown in the Table 6.6 and the patterns of the relationships are shown in the Fig 6.6 and Fig 6.7 respectively. The blue curve shows pattern for increasing the value of the variable and the green curve shows the pattern for decreasing the value of the variable.

It can be found from fig. 6.6 that with the increase of control limit coefficient (k) the cost is decreasing rapidly but at the value 6 and higher the rate of reduction of the cost become very low and almost converge above the value 6. On the other hand with the decrease of k, cost is increasing rapidly at first but little bit slower at the later part. The possible reason is with higher control limit coefficient value probability of rejection decreases. Since the value of standard deviation is fixed for this problem, with the increase of k the area between two control limit also increases which in turn increases the probability of accepting bad products. Thus rejection, repair and out of control cost decreases. On the other hand with the decrease of k area between control limits decreases which increases the probability of rejection and demands high precision production system with very little margin of error. Thus the rejection cost and out of control cost increases. But while k increases, due to the application of Taguchi loss function in control cost also increases. That’s why rate of increasing of cost is much lower than the decreasing of cost with respect to the decrease and increase of k.



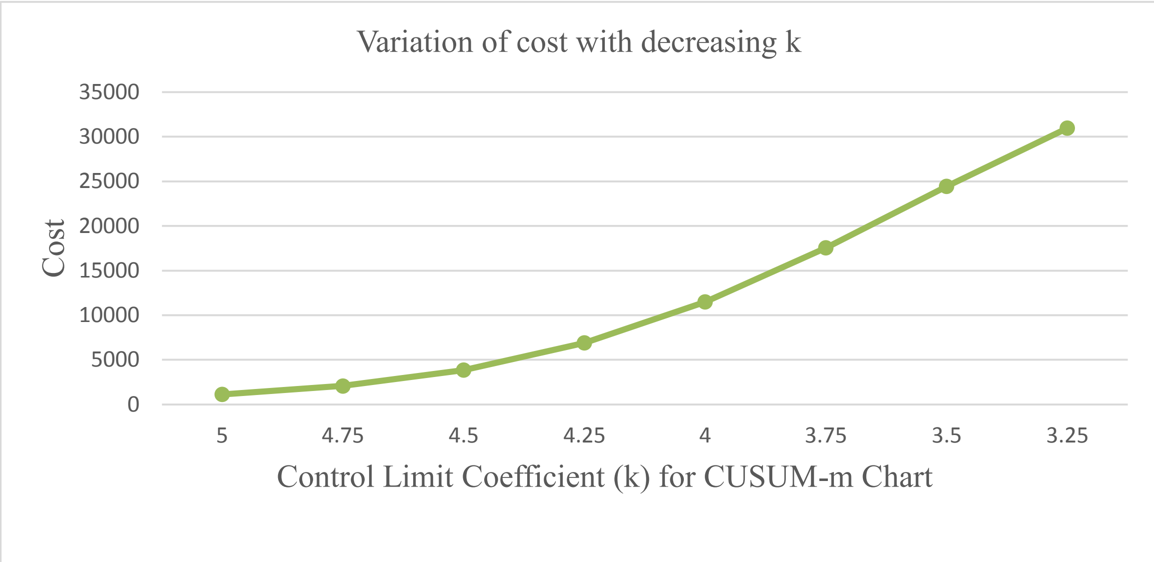
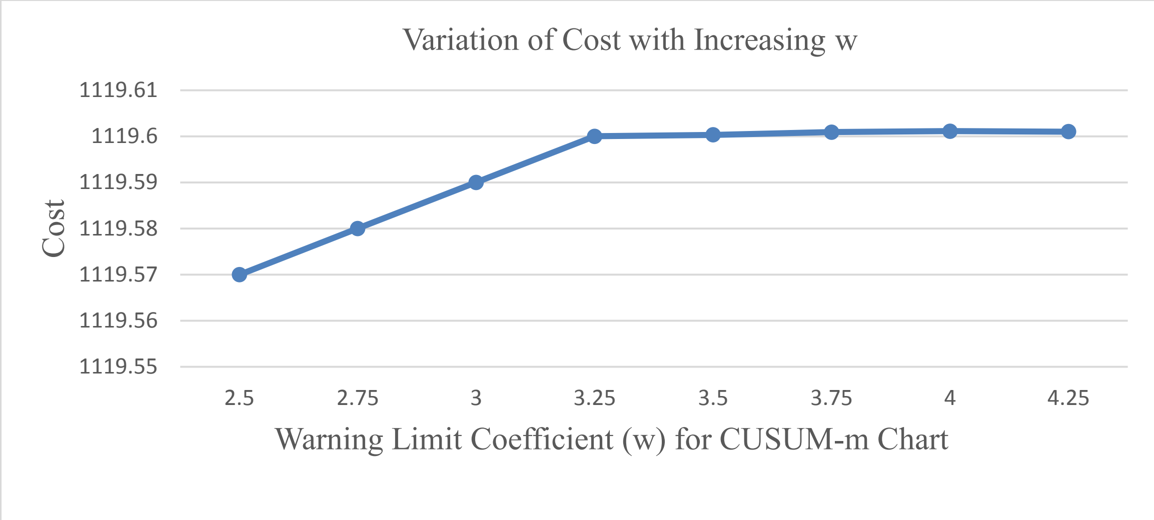


Fig. 6.6 Relationship between control limit coefficient of CUSUM-m chart variability and cost (increasing and decreasing respectively)

In case of warning limit coefficient, with the increase of the coefficient (w), cost is increasing at first but from a certain value 3.25 it become constant. With the decrease of the w cost remain almost constant at first but began to decrease below 1.75 but at a very slow rate. But it can be seen that in both cases (increasing and decreasing of w) the change of cost is very little because low warning limit results in higher no. of sampling and more sampling results in high sampling cost and higher warning limit increases in control cost in Taguchi loss function. Thus these two costs almost nullify each other.



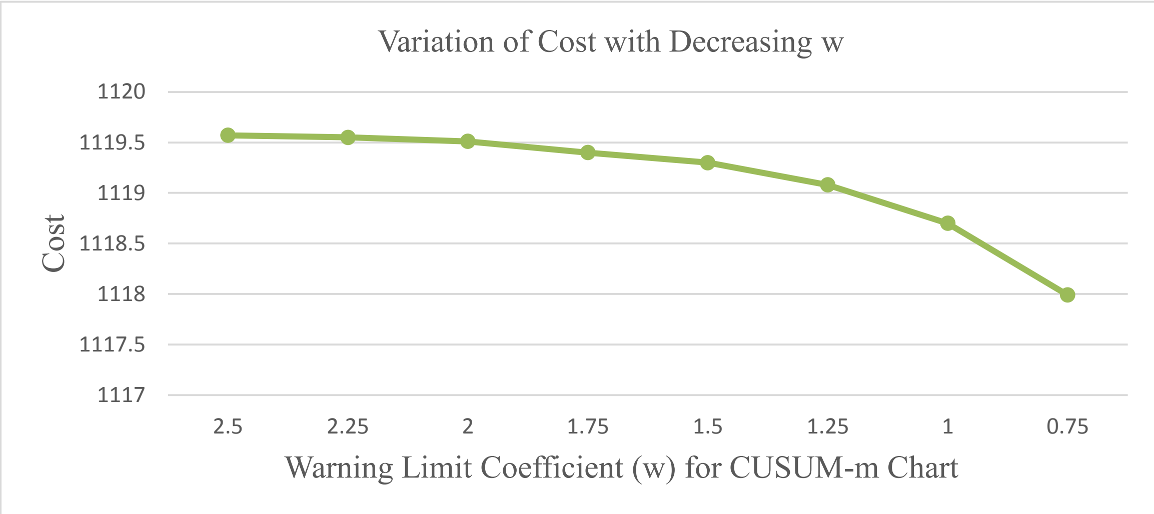
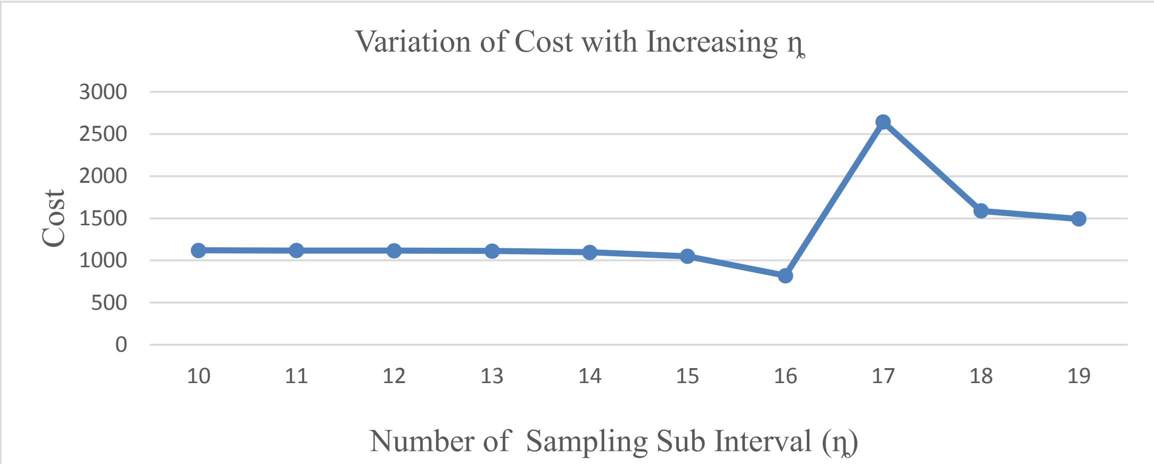


Fig. 6.7 Relationship between warning limit coefficient variability and cost (increasing and decreasing respectively)

The variation of cost with the change of no. of subinterval between two consecutive sampling time and preventive maintenance intervals is shown in the Table 6.7 and the patterns of the relationships are shown in the Fig 6.8 and Fig 6.9 respectively. The green curve shows pattern for decreasing the value of the variable and the blue curve shows the pattern for increasing the value of the variable.

Table 6.7 Variation of cost with the change of no of subinterval between two consecutive sampling times and preventive maintenance interval

Variation of cost with no of subinterval between two consecutive sampling times				Variation of cost with preventive maintenance interval			
η *increasing*	cost	η *decreasing*	cost	t_{pm} *increasing*	cost	t_{pm} *decreasing*	cost
10	1119.304	10	1119.304	220	1119.3	220	1119.3
11	1118.62	9	1119.55	222	1118.8	218	1119.83
12	1116.8	8	1119.64	224	1118.3	216	1120.38
13	1111.65	7	1119.68	226	1117.83	214	1120.95
14	1096.9	6	1119.7	228	1117.38	212	1121.55
15	1049.48	5	1119.7	230	1116.95	210	1122.17
16	821.07	4	11502.52	232	1116.53	208	1122.81
17	2642	3	35949.82	234	1116.13	206	1123.5
18	1589	2	38933				
19	1494.3	1	45202				



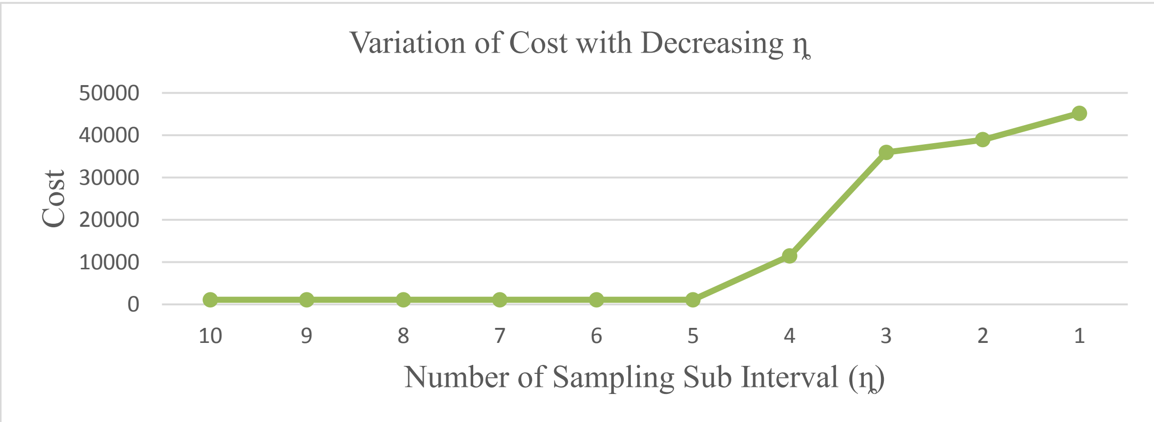
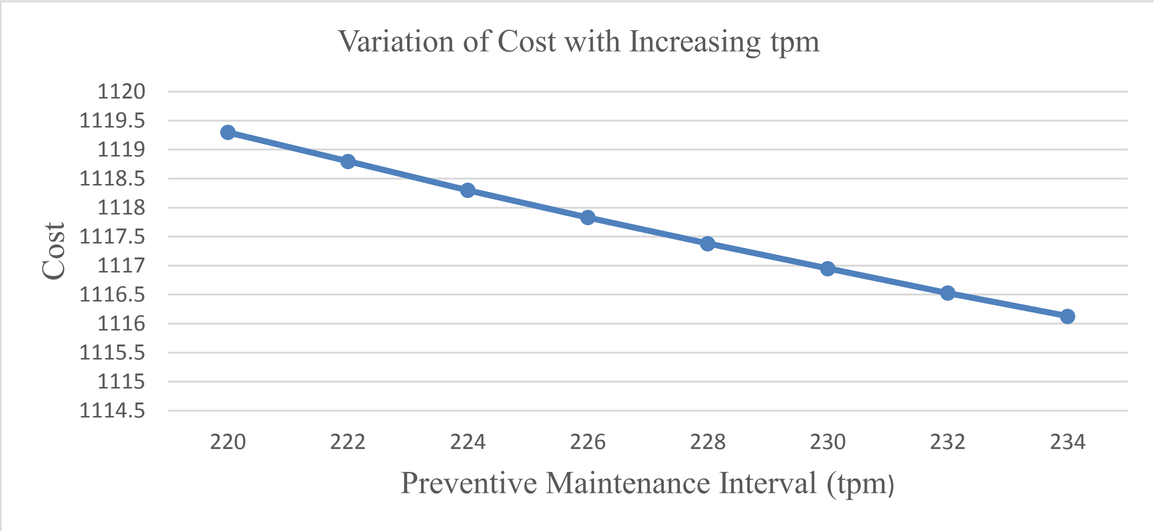


Fig. 6.8 Relationship between number of subinterval between two consecutive sampling time's variability and cost (increasing and decreasing respectively)

It can be found from Fig 6.8 that with the increase of no. of subinterval between two consecutive sampling times the cost is decreasing very slowly but at the value 17 cost increase drastically then for upper values cost started deteriorating slowly. With the decrease of no. of subinterval between two consecutive sampling times the cost remains constant for first few values then started to increase and increase rapidly.so basically for the value (5-16) cost remains almost unchanged and beyond this range cost increases and for values less than 5 it increases rapidly.it happened because when no. of subintervals become very high sampling cost increases and when the no of η is very low no. of out of control ARL increases which in turn increases the probability of repair and rejection and out of control costs.



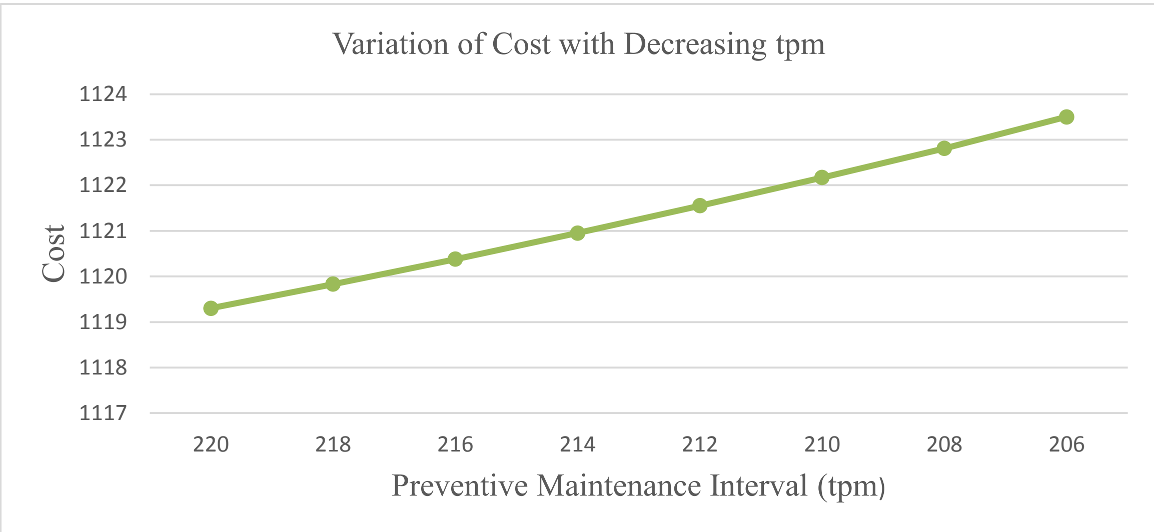


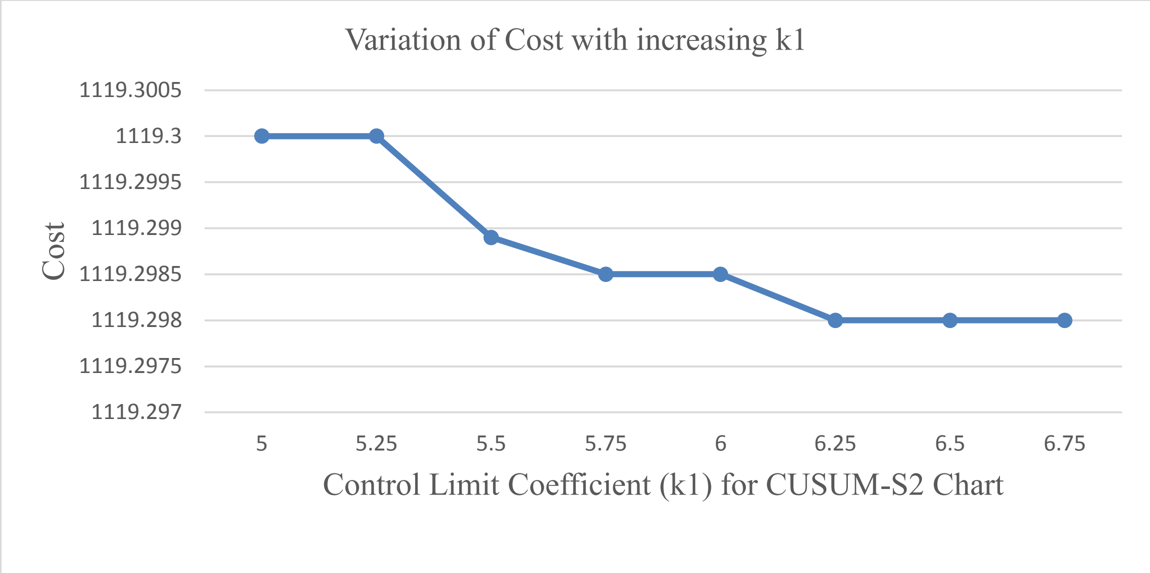
Fig. 6.9 Relationship between preventive maintenance interval variability and cost (increasing and decreasing respectively)

It can be observed from the Fig.6.9 that with the increase and decrease of $t_{pm,}$ cost decreases and increases respectively and the relationship is linear. It occurs, because with the increase of t_{pm} preventive maintenance cost decreases. But it is also found that change of cost is very low.so small change of t_{pm} from its optimum value doesn't affect the cost much

The variation of cost with the change of control limit coefficient and warning limit coefficient for CUSUM-S^2 chart is shown in the Table 6.8 and the patterns of the relationships are shown in the Fig 6.10 and Fig 6.11 respectively. The blue curve shows pattern for increasing the value of the variable and the green curve shows the pattern for decreasing the value of the variable.

Table 6.8 Variation of cost with change of control limit coefficient and warning limit coefficient of CUSUM-S^2 chart

Variation of cost with control limit coefficient of CUSUM-S^2 chart				Variation of cost with warning limit coefficient of CUSUM-S^2 chart			
K_1 *increasing*	cost	K_1 *decreasing*	cost	W_1 *increasing*	cost	W_1 *decreasing*	cost
5	1117.3	5	1117.3	2.5	1119.57	2.5	1119.57
5.25	647.73	4.75	2069.6	2.75	1119.58	2.25	1119.55
5.5	428.63	4.5	3856.9	3	1119.59	2	1119.51
5.75	330.04	4.25	6896.35	3.25	1119.6	1.75	1119.4
6	286.8	4	11492.13	3.5	1119.600	1.5	1119.3
6.25	268.29	3.75	17564.8	3.75	1119.600	1.25	1119.08
6.5	260.61	3.5	24449.36	4	1119.601	1	1118.7
6.75	257.61	3.25	30985.16	4.25	1119.601	0.75	1117.99



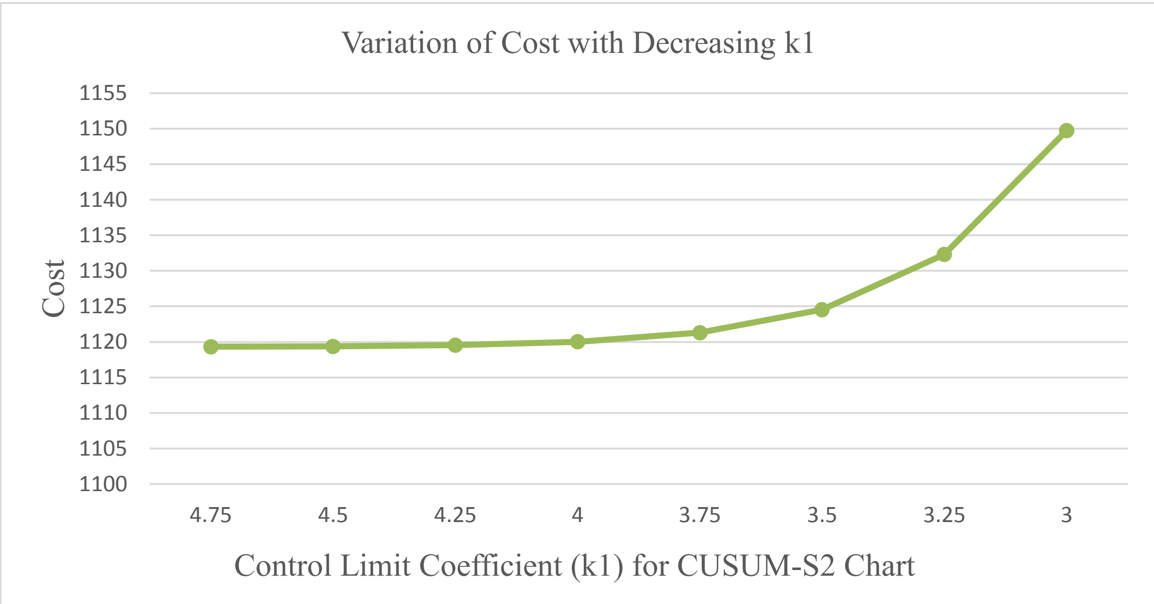
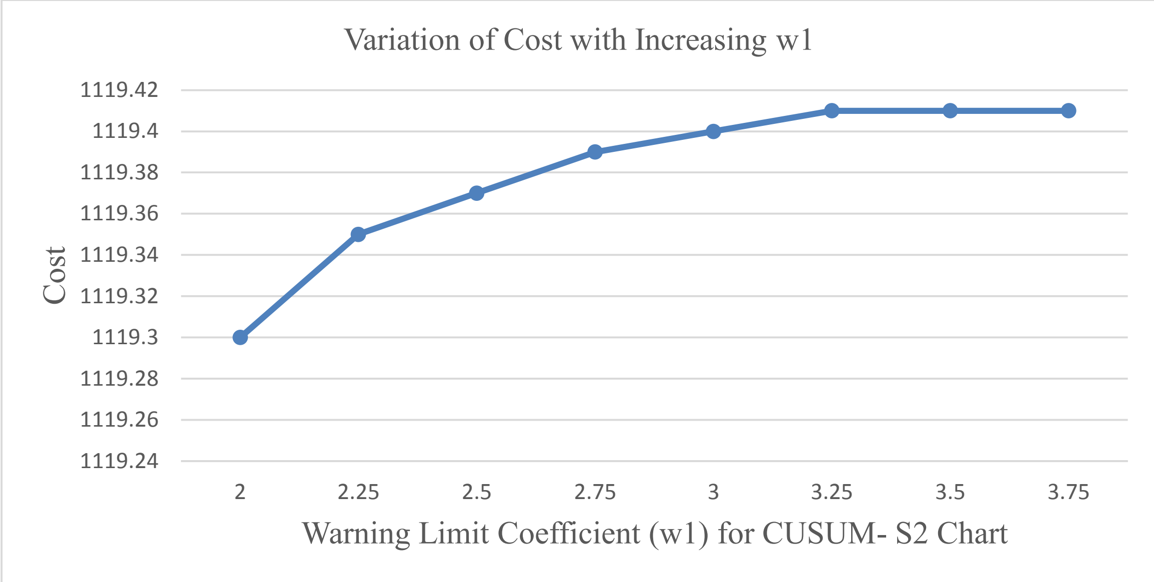


Fig. 6.10 Relationship between control limit coefficient of CUSUM-S^2 chart variability and cost (increasing and decreasing respectively)

It can be found from the table 6.8 that with the increase of k_1 cost almost remains unchanged and with the decrease of k1 cost remains constant for time being and started increasing from 3.75 and below. Due to very small value of k_1, the margin of error decreases at very low level for sample variances which in turn increase the probability of rejection and thus increase rejection cost.



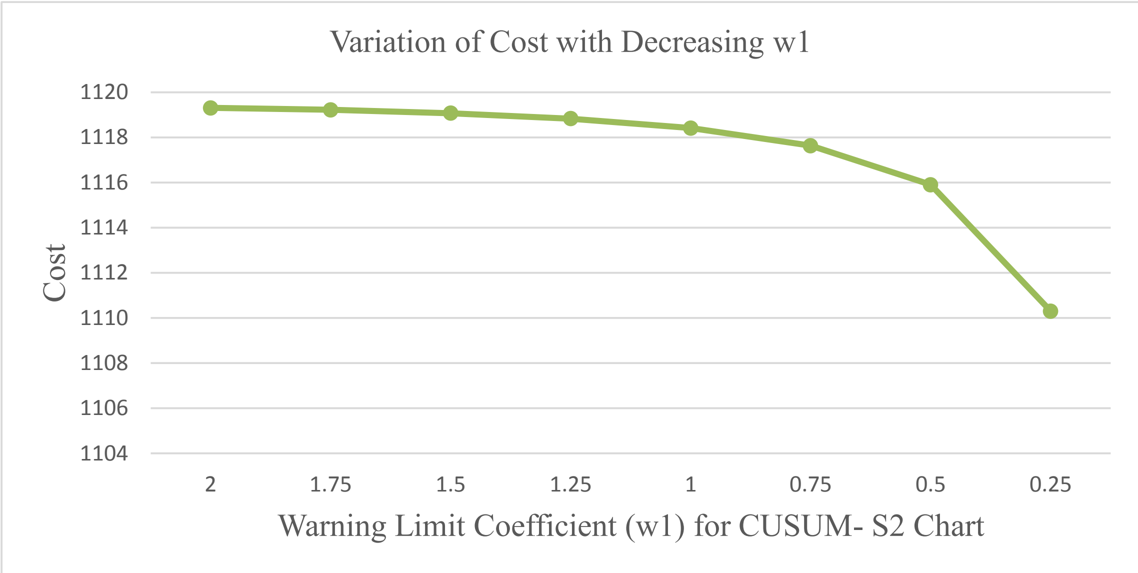


Fig. 6.11 Relationship between warning limit coefficient of CUSUM-S^2 chart variability and cost (increasing and decreasing respectively)

It is evident from table 6.8 that cost remains almost constant with the increase of w_1 and cost started to decrease very slowly with the decrease of w_1. With the increase of control limit coefficient, in control cost increases and with the decrease of warning limit coefficient, probability of sampling increase which lead to increase the sampling cost.so both costs almost nullify each other and total cost remains almost unchanged.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

This present thesis work stated that by combining statistical process control and maintenance management policy in a production system considerable economic benefit can be achieved rather than the separate use of these two key tools. For this purpose, an integrated economic model has been developed where CUSUM chart is used to monitor both process mean and variance. Because CUSUM chart is well known for detecting small shifts and it can detect shifts faster. Moreover, deviation from any one of the desired mean or variance deteriorates product's quality which incurs costs. So, Calculating Joint ARL for integrating both mean and variance is justified. Due to the use of VSIFT sampling policy in this model, substantially faster detection of process shifts has become possible and the probability of running the process at out of control condition has been decreased.Moreover, incorporation of Taguchi loss function and modified Kapoor and Wang's [9] linear loss function in the model have helped in minimizing in control and out of control costs for both mean and variance which perform far better than the conventional approach.

In this work two approaches, Nelder Mead downhill simplex algorithm and Genetic Algorithm, have been used to find out the optimum values of the decision variables (n,h,η,k,w,t_{pm}, k_1, w_1) which minimize the total cost. Both approaches have given almost similar results which justify the credibility of the model.So it can be concluded that the proposed model is a complete model which can give at most possible economic benefit.

7.2 Recommendations

In this paper CUSUM control chart has been used in developing the model and in many research EWMA has already been considered.So it can be effective to use mixed CUSUM and EWMA control chart to monitor both mean and variance.Although VSIFT is the most promising sampling policy, Variable Sampling Rate (VSR) can also be an effective alternative to VSIFT.Because VSR sampling policy allows both the sample size and the sampling interval to vary depending on the previous values of the control statistic.Moreover, multiple equipment manufacturing processes can be considered in the model.However further research can be conducted assuming Weibull or any other distribution in case of determining the time to failure or out of control conditions due to the assignable causes.

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APPENDICES

***Appendix A*:**

[$L_{\text{in control}}$] determination for CUSUM-m chart

$L_{\text{in control}} = PR * \frac{A}{\Delta^2} \int_{\mu-k\sigma/\sqrt{n}}^{\mu+k\sigma/\sqrt{n}} (x-\mu)^2 f(x)dx$

$= PR * \frac{A}{\Delta^2} \left(\frac{1}{\sqrt{2\pi}} * \frac{1}{\sigma/\sqrt{n}} \int_{\mu-k\sigma/\sqrt{n}}^{\mu+k\sigma/\sqrt{n}} (x-\mu)^2 e^{\frac{-(x-\mu)^2}{2*\sigma/\sqrt{n}}}\right) dx$

Let, $\frac{x-\mu}{\sigma/\sqrt{n}} = z$

Therefore,

$L_{\text{in control}} = PR * \frac{A}{\Delta^2} * \frac{1}{\sqrt{2\pi}} * \frac{1}{\sigma/\sqrt{n}} \int_{-k}^{+k} \left(z^2 * \frac{\sigma^2}{n} e^{\frac{-z^2}{2}}\right) \frac{\sigma}{\sqrt{n}} dz$

$= PR * \frac{A}{\Delta^2} * \frac{1}{\sqrt{2\pi}} * \frac{\sigma^2}{n} \int_{-k}^{+k} \left(z^2 e^{\frac{-z^2}{2}}\right) dz$

$= PR * \frac{A}{\Delta^2} * \frac{1}{\sqrt{2\pi}} * \frac{\sigma^2}{n} \{-2ke^{\frac{-k^2}{2}} + \int_{-k}^{+k} e^{\frac{-z^2}{2}} dz\}$

$= PR * \frac{A}{\Delta^2} * \frac{\sigma^2}{n} \{ \frac{1}{\sqrt{2\pi}} * (-2ke^{\frac{-k^2}{2}}) + (1 - 2\phi(-k)) \}$

$[L_{\text{in control}}] = PR * \frac{A}{\Delta^2} * \frac{\sigma^2}{n} \{ 1 - \frac{2ke^{\frac{-k^2}{2}}}{\sqrt{2\pi}} - 2\phi(-k) \}$

[$L_{\text{out of control}}$] determination for CUSUM-m chart

$L_{\text{out of control}} = PR * \frac{A}{\Delta^2} \{\int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx - \int_{\mu-k\sigma/\sqrt{n}}^{\mu+k\sigma/\sqrt{n}} (x-\mu)^2 f(x) dx\}$

$= PR * \frac{A}{\Delta^2} * \frac{1}{\sqrt{2\pi}} * \frac{1}{\frac{\sigma}{\sqrt{n}}} \{\int_{-\infty}^{\infty} (x-\mu)^2 e^{\frac{-(x-\mu-\delta\sigma)^2}{2*\frac{\sigma}{\sqrt{n}}}} dx - \int_{\mu-k\sigma/\sqrt{n}}^{\mu+k\sigma/\sqrt{n}} (x-\mu)^2 e^{\frac{-(x-\mu-\delta\sigma)^2}{2*\sigma/\sqrt{n}}} dx\}$

Let, $\frac{x-\mu-\delta\sigma}{\sigma/\sqrt{n}} = z$

Therefore,

$L_{\text{out of control}} = PR * \frac{A}{\Delta^2} * \frac{1}{\sqrt{2\pi}} * \frac{1}{\frac{\sigma}{\sqrt{n}}} \{\int_{-\infty}^{\infty} \left(\frac{z\sigma}{\sqrt{n}} + \delta\sigma\right)^2 e^{\frac{-z^2}{2}} * \frac{\sigma}{\sqrt{n}} dz - \int_{-k-\delta\sqrt{n}}^{k-\delta\sqrt{n}} \left(\frac{z\sigma}{\sqrt{n}} + \delta\sigma\right)^2 e^{\frac{-z^2}{2}} * \frac{\sigma}{\sqrt{n}} dz\}$

$$= \text{PR}* \frac{A}{\Delta^2} * \frac{1}{\sqrt{2\pi}} \{\int_{-\infty}^{\infty} \left(\frac{z^2\sigma^2}{n} + \frac{2\delta\sigma^2 z}{\sqrt{n}} + \delta^2\sigma^2\right) e^{\frac{-z^2}{2}} dz - \left(\frac{z^2\sigma^2}{n} + \frac{2\delta\sigma^2 z}{\sqrt{n}} + \delta^2\sigma^2\right) e^{\frac{-z^2}{2}} dz\}$$

$$= \text{PR} * \frac{A}{\Delta^2} * \frac{1}{\sqrt{2\pi}} \left[\frac{\sigma^2}{n} \int_{-\infty}^{\infty} z^2\, e^{\frac{-z^2}{2}} dz + \frac{2\delta\sigma^2}{\sqrt{n}} \int_{-\infty}^{\infty} z e^{\frac{-z^2}{2}} dz + \delta^2\sigma^2 \int_{-\infty}^{\infty} e^{\frac{-z^2}{2}} dz - \frac{\sigma^2}{n} \int_{-k-\delta\sqrt{n}}^{k-\delta\sqrt{n}} z^2\, e^{\frac{-z^2}{2}} dz - \frac{2\delta\sigma^2}{\sqrt{n}} \int_{-k-\delta\sqrt{n}}^{\infty k-\delta\sqrt{n}} z e^{\frac{-z^2}{2}} dz - \delta^2\sigma^2 \int_{-k-\delta\sqrt{n}}^{k-\delta\sqrt{n}} e^{\frac{-z^2}{2}} dz \right]$$

$$= \text{PR} * \frac{A}{\Delta^2} [\frac{\sigma^2}{n*\sqrt{2\pi}} * \sqrt{2\pi} + 0 + \delta^2\sigma^2 - \frac{\sigma^2}{n*\sqrt{2\pi}} \left\{ -(k+\delta\sqrt{n}) * e^{\frac{-(k-\delta\sqrt{n})^2}{2}} - (k-\delta\sqrt{n}) * e^{\frac{-(k+\delta\sqrt{n})^2}{2}} + \int_{-k-\delta\sqrt{n}}^{k-\delta\sqrt{n}} e^{\frac{-z^2}{2}} dz \right\} - 0 - \delta^2\sigma^2 \{\phi(k-\delta\sqrt{n}) - \phi(-k-\delta\sqrt{n})\}]$$

$$= PR * \frac{A}{\Delta^2} [\frac{\sigma^2}{n} + \delta^2\sigma^2 + \frac{\sigma^2}{n} * \frac{(k+\delta\sqrt{n})}{\sqrt{2\pi}} e^{\frac{-(k-\delta\sqrt{n})^2}{2}} + \frac{\sigma^2}{n} * \frac{(k-\delta\sqrt{n})}{\sqrt{2\pi}} e^{\frac{-(k+\delta\sqrt{n})^2}{2}} - \frac{\sigma^2}{n} \{\phi(k-\delta\sqrt{n}) - \phi(-k-\delta\sqrt{n})\} - \delta^2\sigma^2 \{\phi(k-\delta\sqrt{n}) - \phi(-k-\delta\sqrt{n})\}]$$

$$= \text{PR} * \frac{A}{\Delta^2} [(\frac{\sigma^2}{n} + \delta^2\sigma^2) + \frac{\sigma^2}{n} * \frac{(k+\delta\sqrt{n})}{\sqrt{2\pi}} e^{\frac{-(k-\delta\sqrt{n})^2}{2}} + \frac{\sigma^2}{n} * \frac{(k-\delta\sqrt{n})}{\sqrt{2\pi}} e^{\frac{-(k+\delta\sqrt{n})^2}{2}} - (\frac{\sigma^2}{n} + \delta^2\sigma^2) \{ \phi(k-\delta\sqrt{n}) - \phi(-k-\delta\sqrt{n})\}]$$

$$= \text{PR} * \frac{A}{\Delta^2} [\{(\frac{\sigma^2}{n} + \delta^2\sigma^2)(1 - \{ \phi(k-\delta\sqrt{n}) - \phi(-k-\delta\sqrt{n})\}) + \frac{\sigma^2}{n} * \frac{(k+\delta\sqrt{n})}{\sqrt{2\pi}} e^{\frac{-(k-\delta\sqrt{n})^2}{2}} + \frac{\sigma^2}{n} * \frac{(k-\delta\sqrt{n})}{\sqrt{2\pi}} e^{\frac{-(k+\delta\sqrt{n})^2}{2}}]$$

Appendix B:

[L in control] determination for CUSUM-S²chart

$L_{in\ control} = PR* \frac{A}{\Delta 1} \int_{-\infty}^{k\sigma/\sqrt{n}} y\, f(y) dy$

$= PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} (\frac{1}{\sqrt{2\pi}} * \frac{1}{\sigma/\sqrt{n}} \int_{-\infty}^{k\sigma/\sqrt{n}} y\, e^{\frac{-(y-\mu)^2}{2*\sigma/\sqrt{n}}})dy$

Let, $\frac{y-\mu}{\sigma/\sqrt{n}} = z$

Therefore,

$L_{in\ control} = PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} * \frac{1}{\left(\frac{\sigma}{\sqrt{n}}\right)\sqrt{2\pi}} [\int_{-\infty}^{\frac{k\sigma}{\sqrt{n}}-\mu} \{\frac{z\sigma}{\sqrt{n}} + \mu) * e^{\frac{-z^2}{2}}\} \frac{\sigma}{\sqrt{n}} dz]$

$= PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} * \frac{1}{\sqrt{2\pi}} [\int_{-\infty}^{k\sigma/\sqrt{n}-\mu} \{\frac{z\sigma}{\sqrt{n}} * e^{\frac{-z^2}{2}} + \mu * e^{\frac{-z^2}{2}}\} dz]$

$= PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} [\frac{\sigma}{\sqrt{n}\sqrt{2\pi}} \int_{-\infty}^{k\sigma/\sqrt{n}-\mu} z * e^{\frac{-z^2}{2}} dz + \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{k\sigma/\sqrt{n}-\mu} e^{\frac{-z^2}{2}} dz]$

$= PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} [\frac{\sigma}{\sqrt{n}} \{\frac{1}{\sqrt{2\pi}} \left(-e^{\frac{(k\sigma/\sqrt{n}-\mu)^2}{2}}\right)\} + \mu * \phi\left(\frac{k\sigma}{\sqrt{n}} - \mu\right)]$

$= PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} \{ \mu * \phi\left(\frac{k\sigma}{\sqrt{n}} - \mu\right) - \frac{\sigma}{\sqrt{n}} * \varphi\left(\frac{k\sigma}{\sqrt{n}} - \mu\right)\}$

[L out of control] determination for CUSUM-S² chart

$L_{out\ of\ control} = PR* \frac{A}{\Delta 1} \int_{k\sigma/\sqrt{n}}^{\infty} y\, f(y) dy$

$= PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} * \frac{1}{\frac{\sigma}{\sqrt{n}}\sqrt{2\pi}} \int_{k\sigma/\sqrt{n}}^{\infty} y\, e^{\frac{-(y-\mu-\delta\sigma)^2}{\sqrt{n}}} dy$

$= PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} * \frac{1}{\frac{\sigma}{\sqrt{n}}\sqrt{2\pi}} [\int_{-\infty}^{\infty} y\, e^{\frac{-(y-\mu-\delta\sigma)^2}{\sqrt{n}}} dy - \int_{-\infty}^{\frac{k\sigma}{\sqrt{n}}} y\, e^{\frac{-(y-\mu-\delta\sigma)^2}{\sqrt{n}}} dy]$

Let, $\frac{y-\mu-\delta\sigma}{\sigma/\sqrt{n}} = z$

Therefore,

$= PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} * \frac{1}{\frac{\sigma}{\sqrt{n}}\sqrt{2\pi}} [\int_{-\infty}^{\infty} \left(\frac{z\sigma}{\sqrt{n}} + \mu + \delta\sigma\right) e^{\frac{-z^2}{2}} \frac{\sigma}{\sqrt{n}} dz - \int_{-\infty}^{k-\frac{\mu\sqrt{n}}{\sigma}-\delta\sqrt{n}} y\, e^{\frac{-z^2}{2}} \frac{\sigma}{\sqrt{n}} dz]$

$= PR* \frac{A}{\Delta 1} * \frac{1}{\phi(k)} * \frac{1}{\sqrt{2\pi}} [\int_{-\infty}^{\infty} \frac{z\sigma}{\sqrt{n}} * e^{\frac{-z^2}{2}} dz + \int_{-\infty}^{\infty} \mu * e^{\frac{-z^2}{2}} dz + \int_{-\infty}^{\infty} \delta\sigma * e^{\frac{-z^2}{2}} dz -$
$\int_{-\infty}^{k-\frac{\mu\sqrt{n}}{\sigma}-\delta\sqrt{n}} \frac{z\sigma}{\sqrt{n}} * e^{\frac{-z^2}{2}} dz - \int_{-\infty}^{k-\frac{\mu\sqrt{n}}{\sigma}-\delta\sqrt{n}} \mu * e^{\frac{-z^2}{2}} dz - \int_{-\infty}^{k-\frac{\mu\sqrt{n}}{\sigma}-\delta\sqrt{n}} \delta\sigma * e^{\frac{-z^2}{2}} dz]$

$= \quad \text{PR}* \quad \frac{\text{A}}{\Delta 1} * \frac{1}{\phi(k)} \quad [\quad \frac{\sigma}{\sqrt{2\pi}\sqrt{n}} \int_{-\infty}^{\infty} z * e^{\frac{-z^2}{2}} dz + \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-z^2}{2}} dz + \frac{\delta\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-z^2}{2}} dz -$

$\frac{\sigma}{\sqrt{2\pi}\sqrt{n}} \int_{-\infty}^{k-\frac{\mu\sqrt{n}}{\sigma}-\delta\sqrt{n}} z * e^{\frac{-z^2}{2}} dz - \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{k-\frac{\mu\sqrt{n}}{\sigma}-\delta\sqrt{n}} e^{\frac{-z^2}{2}} dz - \frac{\delta\sigma}{\sqrt{2\pi}} \int_{-\infty}^{k-\frac{\mu\sqrt{n}}{\sigma}-\delta\sqrt{n}} e^{\frac{-z^2}{2}} dz]$

$= \quad \text{PR}* \quad \frac{\text{A}}{\Delta 1} * \frac{1}{\phi(k)} \quad [0+ \quad \mu + \delta\sigma - \frac{\sigma}{\sqrt{2\pi}\sqrt{n}} \left\{ e^{\frac{-\left(k-\frac{\mu\sqrt{n}}{\sigma}-\delta\sqrt{n}\right)^2}{2}} \right\} - \mu\phi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right) -$

$\delta\sigma\phi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right)]$

$= \quad \text{PR}* \quad \frac{\text{A}}{\Delta 1} * \frac{1}{\phi(k)} \quad [\quad \mu + \delta\sigma - \varphi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right) - \mu\phi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right) -$

$\delta\sigma\phi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right)]$

$= \quad \text{PR}* \quad \frac{\text{A}}{\Delta 1} * \frac{1}{\phi(k)} \quad [\quad \varphi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right) + \mu\left\{1 - \phi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right)\right\} + \delta\sigma\{1 -$

$\phi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right)\}]$

$= \text{PR}*\frac{\text{A}}{\Delta 1} * \frac{1}{\phi(k)}[\varphi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right) + \left\{1 - \phi\left(k - \frac{\mu\sqrt{n}}{\sigma} - \delta\sqrt{n}\right)\right\} * (\mu + \delta\sigma)]$