MULTIDISCIPLINARY DESIGN OPTIMIZATION OF INJECTION MOLDING SYSTEMS UNDER UNCERTAINTY

Pramiti Sarker
Student No: 1015082009P
Session: October, 2015

DEPARTMENT OF INDUSTRIAL AND PRODUCTION ENGINEERING
BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY
DHAKA-1000, BANGLADESH

MULTIDISCIPLINARY DESIGN OPTIMIZATION OF INJECTION MOLDING SYSTEMS UNDER UNCERTAINTY

By
Pramiti Sarker

A thesis submitted to the Department of Industrial & Production Engineering, Bangladesh University of Engineering & Technology, in partial fulfillment of the requirements for the degree of Master of Science in Industrial & Production Engineering

DEPARTMENT OF INDUSTRIAL AND PRODUCTION ENGINEERING
BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY
DHAKA-1000, BANGLADESH

December, 2017

CERTIFICATE OF APPROVAL

The thesis titled **"Multidisciplinary Design Optimization of Injection Molding Systems under Uncertainty"** submitted by Pramiti Sarker, Student no: 1015082009 has been accepted as satisfactory in partial fulfillment of the requirements for the degree of Master of Science in Industrial & Production Engineering on December 9, 2017.

BOARD OF EXAMINERS

Dr. AKM Kais Bin Zaman
Professor
Department of IPE, BUET, Dhaka.

Chairman
(Supervisor)

Head
Department of IPE, BUET, Dhaka.

Member
(Ex-Officio)

Dr. Nikhil Ranjan Dhar
Professor
Department of IPE, BUET, Dhaka.

Member

Dr. A. K. M. Masud
Professor
Department IPE, BUET, Dhaka.

Member

Dr. A.B.M. Toufique Hasan
Professor
Department ME, BUET, Dhaka.

Member
(External)

CANDIDATE’S DECLARATION

It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

Pramiti Sarker

To the Almighty

To my family

ACKNOWLEDGEMENT

All credits go to the Almighty, for his boundless grace in successful completion of this thesis.

At the very beginning the author expresses her sincere gratitude and profound indebtedness to her thesis supervisor Dr. AKM Kais Bin Zaman, Professor & Head, Department of Industrial & Production Engineering, BUET, Dhaka-1000, under whose continuous supervision this thesis was carried out. His affectionate guidance, valuable suggestions and inspirations throughout this work made this study possible.

The author also expresses her gratitude to Kapot Kallol Tarafder, Reliability Engineer, Koch Fertilizers Inc. Fort Dodge, IA, USA for his constant co-operation and assistance in developing the algorithm and computer coding and revising the work.

The author is also indebted to Shib Shankar Dey, Lecturer Department of IPE, Md. Gulam Kibria, Lecturer Department of IPE, BUET and Rashid Anzoom, Lecturer, Department of IPE, BUTEX for their valuable instructions in performing this thesis work.

Finally, the author would like to thank all of her colleagues and friends for their co-operation and motivation to complete the thesis timely. The author would also like to extend her thanks to her parents whose continuous inspiration, sacrifice and support encouraged her to complete the thesis successfully.

ABSTRACT

Injection molding machine is a versatile machine in the field of manufacturing. It is mainly used in mass production to replicate plastic parts. Now-a-days, a quick shipment of finished products to the customer is necessary. Thus, cycle time needs to be minimized. However, besides quick delivery of products, maintaining quality of the finished plastic products is also necessary. Pressure drop is a vital factor to maintain the quality of the finished products. If pressure drop is large in magnitude then it can hamper the quality of the products. Loss of pressure hinders plastics to travel through the nozzle into the mold and plastic may be dried up in before reaching the mold cavity. Thus, to shorten lead time and to obtain good quality products both cycle time and pressure drop need to be minimized. However, these two objectives are conflicting in nature, which means minimizing cycle time, maximizes pressure drop and vice versa. Optimization of injection molding system is also multidisciplinary in nature. Multi-disciplinary injection molding system is a complex engineering system consisting of four distinctive physically different sub-systems among which feed-forward and feed-back coupling variables are also present. The role of uncertainty management is increasingly being recognized in the design of complex systems that require multidisciplinary analyses. Inclusion of uncertainty in the design variables and the system parameters further adds another level of complexity in the design of injection molding systems. The overall objective of this thesis is to find optimum values of design variables of this injection molding system using multidisciplinary design optimization MDO methodology and considering both feed-forward and feed-back couplings as well as uncertainty in both design variables and system parameters. Specifically, this thesis accomplishes this objective through development of formulations and algorithms for design optimization of injection molding system under uncertainty, from the perspective of system robustness.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Background

(MDO) can be defined as a methodology for the design of systems in which strong interaction between disciplines motivates designers to simultaneously manipulate variables in several disciplines [1]. If a system is optimized including interactions among its different disciplines, the obtained result is better than the result obtained if the system was optimized individually under each discipline. MDO uses optimization techniques to solve design problems involving multiple disciplines in such a way that the desired output is either minimized or maximized. Engineers always work in team to develop a robust design. Each team wants to optimize its output; however, sometimes there is interaction among different disciplines which should not be ignored. Ignoring interactions among disciplines may optimize disciplinary level output but fails to optimize system level output. MDO thus considers the interactions among disciplinary level outputs. Till now multidisciplinary design optimization has been applied successfully to automobile industry, aero plane design, naval architecture, electronics etc. Increasing complexity of such engineering systems has sparked the need of applying multidisciplinary optimization methods. In MDO, computational complexity might be an issue as there would be a huge number of design and coupling variables due to the interactions among different disciplines along with nonlinear constraints. Despite this, the usage of MDO is increasing rapidly in modern engineering practices.

The MDO methodology has been frequently used to solve design problems in aerospace and automobile engineering. However, it has hardly been used in the sector of manufacturing engineering. A single manufacturing process is completed with the contribution of different disciplines even within one machine. A machine system may look simple, but the complex interactions among different disciplines make a manufacturing process complete. If the final output of the system is minimized or maximized without considering internal dependency of variables (i.e., coupling), then the final output obtained will not constitute actual result.

The manufacturing sector is a vast one. In this thesis, injection molding system is chosen as an illustration of the application of MDO in manufacturing, it clearly consists of different disciplines. Injection molding is a high precision and sophisticated tool used for the production of plastic parts. This machine manufactures plastic parts by replicating shape of the mold. With the rapid pace of modernization, time needed to deliver final product to the customer has been reduced. Concept of mass production and inventory policy has also been changed. Increasing customer satisfaction is today's prime market need. Now-a-days, products are being produced on customer demand and are expected to be delivered as soon as possible. Reducing cycle time can increase customer satisfaction in this regard a lot. However, in addition to customer satisfaction, manufacturer's satisfaction should be taken into account as well. While delivering the products to customers at a reduced cycle time, if the cost of production increases significantly, the manufacturers will not be satisfied. However, if the energy consumption or pressure drop can be reduced, then the production cost will also be reduced, delighting the customer. Therefore, satisfying both the customer and the manufacturer is the main challenge and this thesis attempts to pursue this through simultaneously minimizing cycle time and pressure drop. However, these two objectives can be conflicting to some extent.

Finding the optimum design by minimizing these two objectives is the contribution of this thesis.

The design of injection molding machine-system is a highly interactive manual process that involves substantial knowledge of several disciplines [6]. Injection molding is a machine where a plastic part is replicated to the shape of the mold. However, to convert the granular plastic to the desired shape, different processes take place before replicating the mold. They are- i) Plasticizing-Heating and melting the plastic in the plasticator. ii) Injection-Injecting controlled volume of shot of molten plastic into die under pressure. iii) Packing-To maintain the injected polymer in mold even after filling the die. iv) Cooling-To solidify the molten metal in solid state so that desired shape of part can be obtained in sufficiently rigid state. v) Mold Release Part-Opening the mold, ejecting the part and closing the mold.

Together, all these processes produce the desired part. The summation of their process times also makes up the cycle time. As mentioned earlier, one of the objectives of this research is to determine the design variables of injection molding machine in such a way that cycle time and pressure drop are minimized.

However, if we closely observe the certain processes of injection molding machine, then we can figure out distinctive physical systems like melting system, injection system, cooling system, and ejection system. These different systems contain some interactions among themselves, i.e., output variable of injection system is an input to cooling system. However, this interaction will not always happen in feed forward manner, rather in feed-back way too, i.e., output of cooling system can be an input to injection system simultaneously. These types of variables are known as coupling variables. If the effect of mutual interaction of these variables is not considered, system level optimization cannot be obtained. Optimization of a system that contains multiple disciplines and considers the coupling variables is thus called multidisciplinary design optimization.

These design variables as well as some system parameters (e.g., flowrate. Temperature of the mold, ejection temperature) may not be deterministic rather probabilistic too. That is why inclusion of uncertainty in the design of injection molding process is also a major contribution of this thesis.

1.2 Objectives with Specific Aims

The specific objectives of this research are-

i) To model an injection molding system as a fully coupled multidisciplinary system considering four sub-systems such as- injection, heat transfer, ejection systems and structural.

ii) To develop a deterministic MDO formulation with the objective being minimization of both pressure drop and cycle time subject to disciplinary constraints as well as the system level constraints

iii) To develop an MDO formulation under uncertainty with the objective being minimization of both pressure drop and cycle time subject to probabilistic disciplinary and system level constraints

The research will help manufacturers minimize cycle time, pressure drop and energy consumption of injection molding process which will eventually reduce cost due to inclusion of uncertainty and both feed forward and feed-back couplings, leading it to a more realistic multidisciplinary system.

1.3 Outline of Methodology

The proposed research methodology is outlined below:

i. The injection molding system has been divided into four sub-systems based on the underlying physics such as- melting system, heat transfer system, cooling and ejection system.
ii. The input variables, system parameters and uncertainty associated with each input variable and system parameters have been identified using existing literature.
iii. A deterministic MDO formulation has been developed that considers multiple objectives, i.e., minimization of cycle time, and pressure drop subject to disciplinary constraints of individual sub-systems and inter-disciplinary couplings such as coupling between feeding system and heat transfer system.
iv. An MDO formulation has been developed based on the deterministic formulation developed in Step (iii) that takes into account the uncertainty quantified in Step (ii).
v. Classical MDO methods e.g., MDF (Multiple Disciplinary Feasible) has been used to solve the MDO formulation developed in Step (iii).
vi. The stochastic MDO formulation developed in Step (iv) has been solved MDO methods under uncertainty named multidisciplinary robust design optimization.
vii. Results obtained from previous two formulations have been compared with each other

1.4 Scope and Limitation

Multidisciplinary optimization is highly used in modern engineering. With advancement of technology and research, design complexity of different systems is increasing day by day. Co-ordination of different systems is necessary. For ease of analysis, large system is broken into multiple disciplines. Traditionally different disciplines have developed their own local design tool to optimize their discipline-specific variables. However, this cannot produce optimal system design[2]. One approach is to partition original design problem in smaller and easier to solve sub-problems and then co-ordinate them towards consistent and optimal system solution. The focus of the MDO methods is to facilitate this interdisciplinary communication and enable the designers to exploit the synergy that exists among constituent disciplines [3]. However, computational complexity in this approach is quite intensive. Higher order mathematical calculation is necessary which costs a lot and consumes a lot of time.

In this research, Multidisciplinary Design Optimization is discussed with all different varieties after discussing about the works previously done by numerous researchers. All three approaches, i.e. Multidisciplinary Design Feasibility approach, All At Once and Individual Disciplinary Feasibility approach are explained. Then uncertainty based and deterministic design optimization processes are described with a generalized presentation of Injection Molding system. All the subsystems of Injection Molding System are described along with Design Variables, Process Parameters and Coupling Variables. Feed-forward and Feed-back coupling is explained then. When all the basics are explained, numerical illustration is presented which includes mathematical illustration of both deterministic and robust-based (uncertainty consideration) design optimization system. Matlab is used as the solving tool and after obtaining converging satisfactory results, graphical and numerical presentation of the results are then illustrated. Depicted results are explained to such extent that can represent that the vision of this research is fulfilled. Finally discussing about the conclusion and future improvements to this study, this research paper draws a successful bottom-line. Now previous works on this topic will be presented below.

Chapter 2

Literature Review

Decomposition method can make any complex design problem simple and easy to manipulate. One of the decomposition approaches is multidisciplinary design optimization. MDO is mainly based on multidisciplinary analysis (MDA) which shares input and output data that interact with each other. Several types of MDA analysis include AAO (all-at once), IDF (individual disciplinary feasible), and MDF (multidisciplinary feasible) methods. Among these, AAO and IDF use single level optimizer and MDF use multiple level optimizers.

Allison et. al. [3] applied MDO in anchor design problem. This problem had an MDO flavor due to feedback coupling. Sobieszczanski-Sobieski et. al. [3] surveyed different publications in the field of aerospace, where MDO is of particular interest. They addressed the interdisciplinary coupling inherent in MDO as major cause of computational and organization changes. Cramer et.al. [4] found an alternative way to formulate MDO problem. Though they stated its application in the field of aero-elastic problem, they mainly focused on alternative way of problem formulation. They also showed the formulation difference of three methods namely i.e. MDF, AAO and IDF. Ilan et. al. [5] tried to apply MDO method to aircraft design problem in a simplified way. They tried solving the same problem in a new manner to reduce the complexity of MDO problem. Very few work can be found with the application of MDO in the field of manufacturing. There exist only a few methods in the literature for the applications of MDO methods in manufacturing, (e.g. Ferreira et al. [6] and Ferreira et. al. [7]) that proposed optimization methods for injection molding system recognizing its multidisciplinary nature. However, all these methods considered injection molding system as a lightly coupled system taking into account only the feed-forward coupling among the disciplinary analyses. In addition, the existing methods solved a single objective optimization problem with the objective of minimizing cycle time and also did not take into account the uncertainty about different parameters. The design and analysis of injection molding systems have been extensively studied in the literature (e.g., [6-9]). Nannapaneni et. al. [7] proposed a framework using Bayesian network to predict the energy consumption of injection molding systems under uncertainty. However, they did not optimize any performance parameter of injection molding machine or consider injection molding system as a multidisciplinary system. Now-a-days, cycle time is an important performance parameter of injection molding machine, as the minimum the cycle time leads to minimum energy consumption thus reducing cost. However, minimizing pressure drop is also a crucial performance parameter, because if pressure drop is large enough in magnitude, then enough material will not travel into the mold and would rather dry up in the runner [8]. Plastic travels through nozzle, sprue, runner, gate and finally part. Each stage eats up some pressure and leads to pressure drop. Eric et. al. described [9] two methods to determine pressure drop so that pressure losses through an injection mold can be better understood. However, they did not minimize pressure drop, which is necessary as high pressure minimizes shrinkage.

Uncertainty prevails in design and observed data. Uncertainty is often ignored in deterministic design problem. Previous work mentioned above hardly considered multidisciplinary optimization problem as a probabilistic. But even in the major application of MDO methods like automobile or aero-space industry, uncertainty prevails. Taguchi [10] first introduced uncertainty in the field of engineering. He delivered a way of choosing design variables in such a way that performance parameter become less sensitive to change in design variables. Wei et.al. [10] Identified methods developed by Taguchi were based on statistical modeling and were unable to solve nonlinear problems with nonlinear constraints. However, there exist a lot of methods in the literature to solve MDO problems under uncertainty which include collaborative optimization, multidisciplinary reliability based design optimization and multidisciplinary robust design optimization. Zaman et. al. [10] proposed a decoupled approach for robustness based design optimization using both interval and sparse data. Park. et. al. [11] described three methods such as, Taguchi approach, robust optimization, and axiom method to include uncertainty in engineering design and their relative advantage and disadvantages. In this thesis, robust design optimization is used. The essential elements of robust design optimization are 1) ensuring objective robustness, 2) ensuring feasibility robustness, 3) estimating mean and measure of variation (e.g., variance) of the performance function, and 4) multi-objective optimization.

There are different methods available in literature to estimate mean and variance of performance function including Taylor series expansion [1]. The following two equations (2.1) and (2.2) describes mean and variance of a function after using Taylor series expansion:

$$\mu_f \cong f(\mu_x, \mu_f) \tag{2.1}$$

$$\sigma_f^2 = \sum_{i=1}^{n} \left(\frac{df}{dx_i} \right)^2 \sigma_{x_i}^2 \tag{2.2}$$

Du et. al. [12] formulated a framework that includes inverse reliability strategy with percentile performance for assessing both objective robustness and feasibility robustness. In this research, Taylor series expansion has been used to estimate the mean and variance of cycle time - main performance function of multidisciplinary design optimization of injection molding system.

Design of a multi-disciplinary system under uncertainty means optimization of mean and minimization of variance. If variance is reduced then feasibility region is also reduced. If performance function is minimized, under these newly formulated constraints, then it is called a robust design [13]. In order to achieve improvement in product quality and reliability of manufacturing processes in industrial engineering, idea of robust design optimization was introduced by Taguchi for the first time. There are different methods available to optimize mean and minimize variance in design problem. As minimization of mean and variance is done simultaneously, then it also turns into a multi-objective optimization technique. A huge number of methods are available in literature to solve a multi-objective optimization. A very common method is weighted sum method [11]. Besides this weighted sum approach, some other methods are also available like ε-constraint method [14], and genetic algorithm method to solve the multi-objective optimization problem. Among these available methods, weighted sum approach is taken into consideration in this research due to its simplicity. Often units of the multiple objectives are not the same. That is why normalization is a must in weighted sum approach. Normalization can be achieved in any of the three ways [15]:

i) Normalize by dividing the objective function by its value at x_0

ii) Normalize by diving objective function by its value at minimum point and

iii) Normalize by the minimum of objective functions and nadir and utopia points. Nadir point the highest value of a point and utopia point is a point which optimizes almost all objective functions

Second method with normalizing the objective function by its value at minimum optimization condition has been applied in the proposed robustness-based multidisciplinary design optimization problem. Each method has its own advantages and disadvantages. Although solution distribution of weighted sum method is not uniform and inefficient to obtain non-convex region of Pareto front, considering implementation cost, it has been used in this research.

Therefore, application of MDO methodology considering the effect of both feed-back and feed-forward coupling along with uncertainty is hardly seen. Though individual work has been observed, the current research is an attempt to apply industrial engineering methodology in solving manufacturing engineering problems i.e., injection molding systems. Injection molding system is considered as a multi-disciplinary system with feed forward and feed-back couplings. Details are described in the following chapter.

Chapter 3

Multidisciplinary Design Optimization

Multi-disciplinary design optimization simply means the optimization of a complex system containing at least two or more disciplines, which use different optimization methodologies to get the optimum value of the performance parameters. Output of MDO gives such values of design variables that optimize the overall system level output. Benefits of this system are vast yet limited to only a few fields, like aero-space, automobile etc. Multidisciplinary design optimization highly depends on multi-disciplinary analysis.

Design Optimization or in short DO, is the process where we find the best design parameters for satisfying the system requirements. To do that, we use different sets of tools like Design of Experiment (DoE), statistics and optimization techniques. The target is to evaluate trade-offs and select the best design. It might be easier to achieve for simple and small systems. However, in case of large systems, which involve different disciplines, complexity arises. The traditional approach suggests dividing the system into different disciplines. Now, different systems have different design variables and different optimization tools as well are available to solve them. However, this approach fails to achieve the optimal design because it optimizes the system design from the perspective of a certain discipline, not the entire system. Besides, communication among the disciplines gets increasingly difficult, which hinders the local disciplines from having information regarding the effect of local design change on the whole system. That is where multidisciplinary optimization (MDO) comes into play. It facilitates that interdisciplinary communication which enables the designers to exploit the existing synergy between constituent disciplines. MDO is believed to have tremendous effect on the design optimization system by reducing cycle time and producing products at lower cost. The following figure shows a two-discipline system for the sake of illustration [2]:

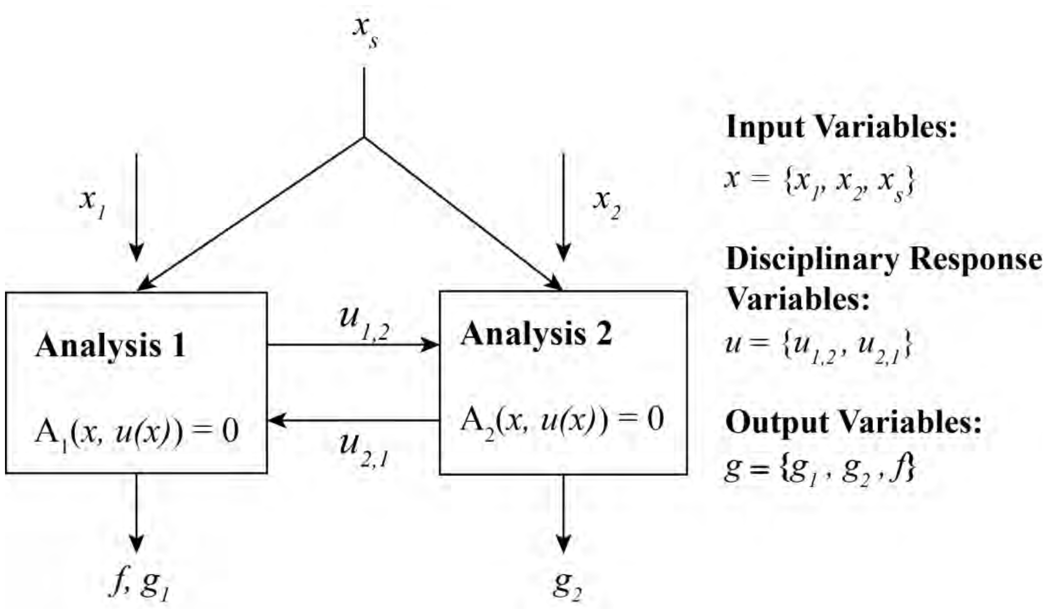

Fig 3-1: A two-discipline system

In Figure 3.1 multidisciplinary design optimization, of two disciplines are present. x_1 and x_2 are input variables to individual systems and x_s is the variable which is shared by both disciplines. However, the catalyst that converts a system with multiple disciplines into multidisciplinary system is coupling variables which are $u_{1,2}$ *and* $u_{2,1}$ here. Values of $u_{1,2}$ *and* $u_{2,1}$ are mutually dependent on each other and one of these variables are input to a subsystem whereas another is the output of the same subsystem.

The mathematical formulation of an MDO problem can be written as follows in equation (3.1):

$$\begin{aligned} &\min c(x) \\ &g(x,u(x),v(x)) > 0 \\ &h_1(x,u,v) = 0 \\ &h_2(x,u,v) = 0 \end{aligned} \tag{3.1}$$

Here, $u(x)$ and $v(x)$ are intermediate variables (also known as behavior or state variable) and h_1 and h_2 are implicit functions. Objective function, $c(x)$ is cycle time and pressure drop in our research and x is design variables of injection molding system.

Now, solving even a deterministic MDO problems can be difficult- especially if effective handling of the multidisciplinary system of equations is taken into account. That is why multi-disciplinary analysis (MDA) is considered very important in MDO method. It capitalizes on the individual disciplinary analysis codes that interact with each other through shared input and output data. A feasible multidisciplinary analysis provides a solution that simultaneously satisfies all individual disciplinary analyses. The problem however is the computational expense of it. Even at conceptual level of design process, it is quite expensive. If we add uncertainty, the required computational effort increases furthermore. There are three different basic MDO methods based on how the system analysis is handled [4]. These are:

1. Multi-Disciplinary Feasibility (MDF) Method
2. All At Once (AAO) Method
3. Individual Disciplinary Feasibility (IDF) Method

Among these three we are going to use the MDF method. A brief description of this method is given below:

3.1 MDF Approach

The most common way to perform MDA analysis is MDF or Multidisciplinary Feasibility Analysis. In MDF approach, the optimizer tries to control only the design variables, not the coupling variables [2]. Its formulation is quite basic. Though easy to use, a complicated system analysis has to be carried out in each step. The coupled relationship is solved in that system analysis. Full multidisciplinary problem feasibility is maintained at each optimization iteration through repeated iteration between individual disciplinary analyses until convergence. Solving for coupling variables at each iteration makes the system quite expensive. In complete multidisciplinary feasibility approach, iteration feasibility is maintained at each and every step. In MDF methodology, values of design variables $\boldsymbol{X_D}$ are sent to the coupling variables that are used to find the value of objective function satisfying constraints [2].

An MDF-based design optimization formulation is given below as in equation (3.2):

$$\begin{aligned} & \min f\,(U_D, X_D) \\ & \text{with respect to } X_D, \\ & s.t.\, C_D(X_D, U_D) > 0 \\ & where, U_D(A(X_D, G(X_D, U(X_D)) = 0 \end{aligned} \tag{3.2}$$

Where, f is the objective function, which depends on the value of coupling variables U_D and design variables X_D. C_D is the constraint. Value of C_D and U_D is also dependent on other coupling variables and design variables.

Figure (3-2) shows how a MDF methodology works. In MDF methodology, coupling variables are not optimized rather solved outside the optimization. After each solving value of coupling variables are sent to the optimization algorithm, then current design value is obtained which is again sent to multi-disciplinary system. Thus MDF method is optimized.

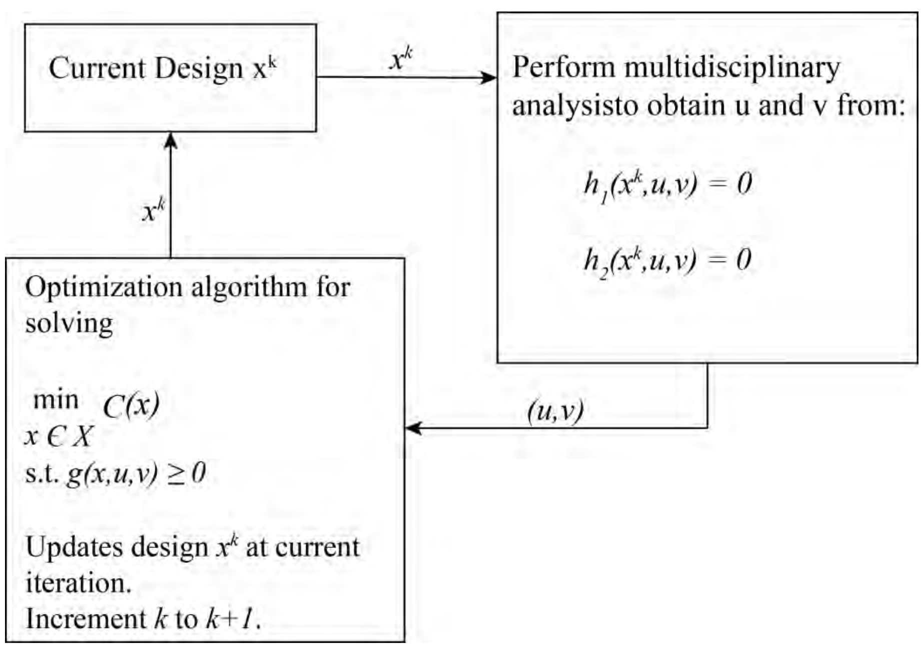

Fig 3-2: Diagram of MDF Method

3.2 AAO Approach

In terms of problem formulation, the AAO approach is at the opposite end of MDF. It performs neither the system analysis nor the individual analysis for each discipline. Here, both the design and coupling variables are controlled by the optimizer and the system compatibility requirements are appeared as constraints in the optimization.

3.3 IDF Approach

The IDF approach falls between MDF and AAO. Its formulation has been developed to eliminate the system analysis, and each discipline is independently solved. Here, individual discipline feasibility is maintained at each iteration and multidisciplinary feasibility is satisfied at the optimization convergence. While eliminating the system analysis, IDF uses the complimentary variables and the compatibility conditions of coupling variables [2].

Among these three, MDF method was used to analyze multidisciplinary design optimization in this study. MDF method could be used only in deterministic problem where variables are deterministic in nature and no uncertainty is taken into consideration while optimizing the system. However, uncertainty prevails in engineering design problem regardless the nature of the problem. Consideration of uncertainty while designing makes the design system robust. Often safety factor is used to make the system safe, but it does not make the system robust. Robust system is insensitive to changes in the design variables. As engineers always want to have firm output from their systems, uncertainty is not desirable in engineering design. Robust design methods make the system less sensitive to changes in design variables.

3.4 Sources of Uncertainty

There are two types of uncertainty: aleatory uncertainty and epistemic uncertainty[16]. Aleatory uncertainty arises from natural variability whose reason is unknown. Epistemic uncertainty arises from lack of knowledge. When proper knowledge or data is not known, then uncertainty due to lack of knowledge is known as epistemic uncertainty. Epistemic uncertainty can be reduced if data is properly collected. Distinction between these two uncertainties is necessary because risk assessment decisions in engineering design sectors may be tough. Thus, if we run the same experiments, due to aleatory uncertainty, value of the output will be changed every time. On the other hand, epistemic uncertainty is the systematic uncertainty where lack of proper systematic knowledge to collect data is the cause of uncertainty. If data is collected in proper manner then epistemic uncertainty can be greatly reduced. This research deals with aleatory uncertainty only.

Aleatory uncertainty arises from natural variability like human error, machine error, environmental change which is out of control. For example: a worker may have to remove an injection part at 29°C, but due to fatigue he may waits a bit and remove part from injection molding at a lower temperature. Uncertainty due to this type of reason is called aleatory uncertainty.

3.5 Robustness-based Design Optimization

Robustness-based design optimization or in short RDO is used to optimize a design problem under uncertainty. Uncertainty prevails in a problem when input variables as well as system outputs are uncertain.

RDO can be implemented by optimizing mean and minimizing variance. For multi-disciplinary design optimization under uncertainty, often variance of a function should be known. This variance of a function could be either objective function or the constraints. Using first-order Taylor series expansion, mean and variance of a function could be written as follows in equation (3.3) and (3.4):

$$\mu_f \approx g(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}) \tag{3.3}$$

$$\sigma_f^2 = \sqrt{\sigma_{x_1}^2 \left(\frac{df}{dx_1}\right)^2 + \sigma_{x_2}^2 \left(\frac{df}{dx_2}\right)^2 + \ldots\ldots + \sigma_{xi}^2 \left(\frac{df}{dx_i}\right)^2} \tag{3.4}$$

Besides Taylor series expansion, sampling based methods and point estimate methods are also available in literature to estimate the mean and variance of performance cost function ([11], [12], [16]). Among these, Taylor series expansion is quite popular, although at high value of variance of the variables it may produce error.

Feasibility robustness can be achieved by minimizing boundary of the constraints. On the other hand, minimizing variance under these constraints provides objective robustness. Zaman et. al. [13] proposed a RDO formulation as follows in equation (3.5):

$$
\begin{aligned}
&\min_{\mathbf{d}} \; f(\mu,\sigma) = (w * \mu_f + (1-w) * \sigma_f) \\
&s.t. \qquad \mathbf{LB} + k\sigma(g_i(\mathbf{d},\mathbf{z})) \le E(g_i(\mathbf{d},\mathbf{z})) \le \mathbf{UB} - k\sigma(g_i(\mathbf{d},\mathbf{z})) \;\; \textit{for all } i \\
&\qquad\qquad lb_i + k\sigma(x_i) \le d_i \le ub_i - k\sigma(x_i) \qquad \textit{for } i = 1,2,\ldots,nrdv \\
&\qquad\qquad lb_i \le d_i \le ub_i \qquad \textit{for } i = 1,2,\ldots,nddv
\end{aligned}
\tag{3.5}
$$

where, in equation (3.5) μ_f and σ_f are the mean and standard deviation of the objective functions. *LB* and *UB* are the lower and upper bounds of the constraints. Here $\omega \leq 1$, *nrdv* and *nddv* are the number of random design variables and deterministic design variables. In our problem, value of *k* was taken as 1 and random design variables were design variables of the injection molding design problem and deterministic variables were process parameters. As we are minimizing both mean and variance, it could be defined as multi-objective optimization problem.

The following chapter proposes a RDO methodology for injection molding system.

Chapter 4

Representation of Injection Molding System as a Multidisciplinary System

Injection molding is a manufacturing process for producing parts in a very large quantity, sometimes in thousands or even in millions to reduce cost and increase the rate of production. In this process, molten metal or plastic is inserted into a mold under pressure which through heat transformation, changes phase to become solid and turns into the form of the mold. The ability to scale production in a large quantity is the most lucrative advantages of injection molding process to manufacturers. Once initial cost gets paid, the price of parts becomes very low and gets even lower with the production of more parts.

To facilitate injection molding process, the parts that need to be injection molded, need rigorous engineering considerations. Material of the part, material of the mold, features and shapes of the part, properties of the machine, pressure in the chamber, melting temperature, injection temperature, ejection temperature, heat transfer rate- all these terms need to be considered very carefully to prevent economic loss and production time.

The history of injection molding goes back to 1872 when American Inventor John Wesley Hyatt along with his brother Isaiah Hyatt patented the first ever injection molding machine. It was a very simple machine with way fewer capabilities than the machines used now-a-days industrially. Arthur Eichengrun and Theodore Becker invented first soluble form of cellulose acetate in 1903. Arthur Eichengrun made his first molding press in 1919 and then in 1939 he patented the injection molding of plasticized cellulose acetate. World War II created a huge demand in low cost mass productions which triggered injection molding as one the most useful methods for manufacturing. American inventor James Watson Hendry invented screw injection machine in 1946, which offered more precision and faster production with less material waste which in later years got tremendously modified for even higher efficiency. In 1970, Hendry developed first gas assisted injection molding machine which permitted the production of complex, hollow articles that cools fast.

In USA and Europe, injection molding machine is widely used to replicate parts. It bloomed in its production due to the capability of creating mass products of same size and shape in shorter time period. Not only does this machine create part with almost full accuracy also saves time of producers. Manufactured parts can be used in almost all sectors of daily life. Thus injection molding machine is a versatile machine too.

Injection molding machine is a machine which can replicate any part and produce them in plastic form. It can produce parts in different cycles. Each cycle can produce one or more parts. However, in our research, per cycle production of one part was considered. Thus the less is the cycle time the more production can be obtained. However, uniform filling and cooling of the plastic part may be hampered if cycle time is reduced extensively. In injection molding machine, granular plastic is melted at first. Then this melted liquid flows through a channel, sprue and runner. Lastly, it is injected into a hole to mold cavity. Mold has two parts - cavity and core halves. Top half of the mold is called cavity half and bottom half is known as core or moveable half.

Size of the mold cavity depends on dimension of the manufactured part. Thus, length and diameter of runner, sprue and gates are important design variables for injection molding. The mold has two halves. One half contains injection system and the other half contains cavity and ejection system. In core, liquid plastic is filled for some moments and then it shrinks to solid until ejected from the core. After completing the production of every part, the mold has to be opened. Therefore, this back and forth opening movement of the mold is controlled by a structural system. The structural system is also responsible for giving proper clamping force to the mold while drying the part.

Thus, injection molding has 4 important sub-systems [6]. They are:

1. Feeding System
2. Heat Transfer System
3. Ejection System
4. Structural System

4.1 Feeding system

Feeding system is the system where molten liquid is fed into the cavity. Function of it is to transfer the molten liquid from the channel to the cavity. Feeding system consists of runner system, sprue system and gating system. A properly well designed runner system helps producers to obtain optimum number of cavities, delivering melt to cavities, balancing feeling up of multiple cavities, minimizing waste by pouring almost all liquids to the cavity insert, reducing energy consumption, pressure drop and cycle time. There are two types of runner systems in injection molding system. First one is hot runner system and second one is cold runner system. Hot runner is comparatively expensive but it is really suitable for smaller pressure drop and minimized cycle time. On the other hand cold runner is cheaper but it gives slower cycle time and huge pressure drop. As our aim is to design an injection molding system with minimum cycle time and pressure drop, our system was considered as hot runner system. Design of feeding system is highly dependent on geometry and size of the finished part and its gating,[6].

4.2 Heat Transfer System

After mold cavity is filled up with molten plastic, it should be dried to solid state so that it could be ejected. To draw away the heat from the cavity cooling channels are present around this system. When filling starts, partial filling also starts. During this period mainly conduction, convection and viscous flow helps to reduce the temperature of the liquid plastic. However, while holding the liquid plastic mainly conduction predominates. Filling time is negligible compared to holding time, that is why only heat transfer by conduction is considered in our research [17].

4.3 Ejection System

Ejection system ejects the solid part after cooling the part. Its main function is to release the part from the mold cavity. To ease the release of the part without any hamper to the part, proper ejection temperature, clamping force, stroke and distance of the release should be proper. The quality of ejected part is highly dependent on ejection system. It also determines the production efficiency because when every cycle finished, product needs to be taken out of mold. This process is known as de-molding.

4.4 Structural System

Another important subsystem of injection molding system is structural system. This system gives proper clamping stoke to the mold so the liquid molten plastic does not slip through the cavity wall. Again structural system ensures clamp opening force which must be enough to open the mold and take out the part in completely perfect condition. Thus, injection molding system can be considered as a multi-disciplinary system consisting of four sub-systems mentioned above. In this multi-disciplinary system different variables enter each system and give output (Figure 4.1). Even output of a system can be input to other systems. When this happens in forward manner, it is called feed-forward coupling. When output of a system becomes the input of a previous system then it is known as feed-back coupling, like injection pressure and shot volume (ρ_{inj}, v_s). The multi-disciplinary system is explained below in short with the help of a figure 4.1.

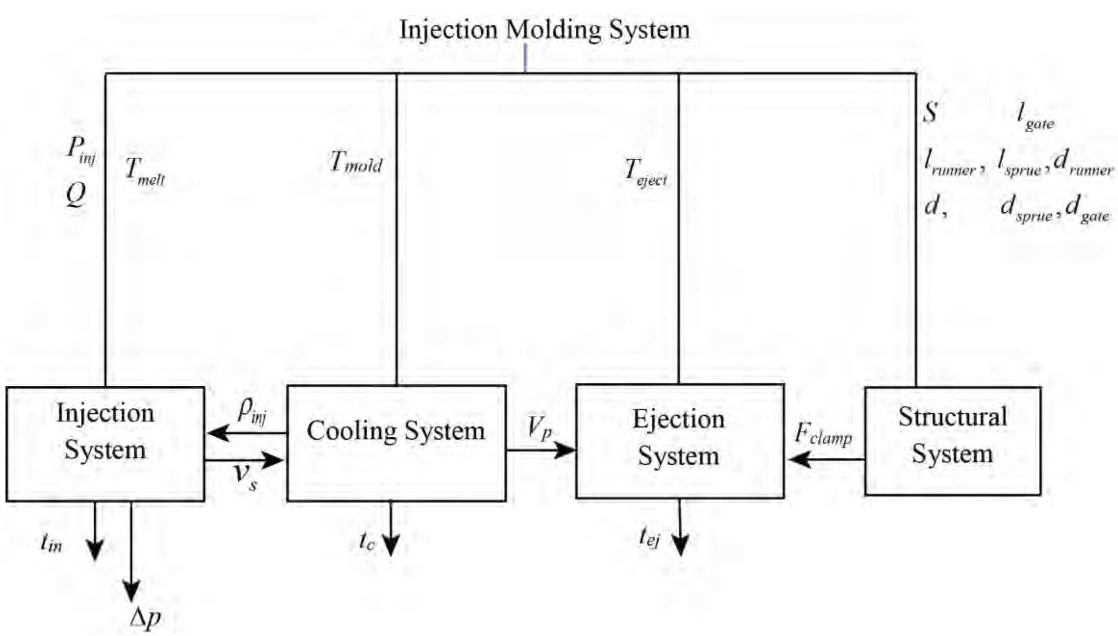

Fig 4-1: Multidisciplinary Injection Molding System

where,

$v_s = Volume\ of\ shot$
$P_{inj} = Power\ of\ injection$
$h_{\max} = Wall\ thickness\ of\ the\ mold$
$T_{melt} = Melting\ temperature\ of\ plasticing\ temperature\ of\ the\ plastic$
$T_{eject} = Ejection\ temperature\ when\ molten\ plastic\ solidifies$
$t_{eject} = Time\ to\ ejecct\ the\ finished\ part$
$F_{clamp} = Clamping\ force\ to\ close\ and\ open\ the\ mold$
$V_p = Volume\ of\ the\ finished\ part$
$\Delta p = pressue\ drop$
$t_d = Dry\ cycle\ time$
$p_{inj} = Pressure\ of\ injection$
S=Length of the stroke
$t_{inj} = Time\ to\ inject\ the\ liquid\ polymer$
$t_c = Time\ to\ cool\ the\ polymer$

Quality and production efficiency of injection molding system are highly dependent on design of injection molding which determines the value of the above variables. Previous studies show that if the runner length and diameter are smaller, cycle time is reduced but results in a huge pressure drop. Huge pressure drop can result in defective and sticky product. Liquid plastic may dry in runner due to pressure drop and never reach the cavity. However, if runner diameter is larger, it results in less pressure drop but higher cycle time. Duration of cycle time determines production efficiency and pressure drop amount determines quality of the product. Therefore, design of injection molding system determines production efficiency and product quality. Pressure drop and cycle time minimization is good for injection molding machine. However, these two objectives are conflicting in nature. If value of cycle time is minimized, value of pressure drop increases and vice versa. Thus, to obtain optimum value of cycle time and pressure drop injection molding system should be optimized taking into account both objectives in consideration. Values of design variables should be chosen in such a way that both cycle time and pressure drop are minimized.

Cycle time consists of five stages [18] i.e.

i. **Plasticizing**: plasticizing means heating and melting of the polymer

ii. **Injection:** injecting refers to the insertion of liquid polymer into the mold cavity

iii. **Packing**: filling the cavity even after cavity is filled up with liquid polymer to stop back flow

iv. **Cooling** : cooling the polymer to solid state

v. **Ejection:** ejecting solid part from the mold cavity

Among these five stages injection and plasticizing happen almost simultaneously, that is why plasticizing is ignored during cycle time calculation. Only injection time is taken into account. Packing time also is a part of cooling time so this is ignored too. Summation of injection time, cooling time and ejection time is considered as cycle time of injection molding system (Equation 4.1) [19].

Cycle time can be written as follows

Cycle Time= injection time + cooling time + ejection time=

$$= \frac{2 \times V_s \times p_{inj}}{P_{inj}} + \frac{h_{max}^2}{\pi^2 \alpha} ln(4) \times \left(\frac{(T_{melt} - T_{mold})}{(T_{ejection} - T_{mold})} \right) + [1 + 1.17 t_d \sqrt{\frac{2d+5}{S}}] \qquad (4.1)$$

During minimization among these three types of time, time to cool the polymer should not be decreased to below 3sec (equation 4.2)[19]. In other words, cooling time must be more than 3 secs.

$$\frac{h_{\max}^2}{\pi^2 \alpha} \ln(4) \times \left(\frac{(T_{melt} - T_{mold})}{(T_{ejection} - T_{mold})} \right) > 3 \qquad (4.2)$$

In injection molding machine, parameters like temperature of mold, melting temperature and ejection temperature completely depend on material to be replicated through injection molding machine. Therefore, these variables were considered as process parameters and other variables can be determined during design of the process. That is why they are considered as design variables. Thermal diffusivity, α is taken as constant, based on particular material type. Coupling variables, design variables and process parameters of injection molding are explained below.

4.5 Coupling Variables

4.5.1 Volume of shot

It is the amount of molten plastic that goes to the cavity. While deciding volume of shot shrinkage allowance of the working plastic material should be taken into account. Mold cavity is the negative of the plastic part to be molded. In an injection molding there can be one or more numbers of mold cavities. Volume of shot also depends on number of volume of shot [20]. Volume of shot is an important variable in injection molding design. Shot volume is an output of injection system and an input to cooling system. Its value is dependent on injection pressure (equation 4.3) [21].

$$v_s = \frac{P_{inj} \times t_{inj}}{2 \times \rho_{inj}} \tag{4.3}$$

4.5.2 Injection Pressure

Amount of pressure required to inject plastic in the cavity mold is called the injection pressure. This pressure is required when the plunger in the screw rotates to insert molten plastic in the cavity. While inserting molten plastic a significant amount of pressure is lost. That is why injection pressure must be greater than empirical pressure required for a material. Often researchers argue that, it must be more than 1.25 times the injection pressure (equation 4.4) [20].

$$p_{inj}\max \leq 1.25\, p_{inj} \tag{4.4}$$

where, p_{injmax} is maximum pressure and p_{inj} is required injection pressure. Once pressure is decided then the task is to select cavity number. After optimization, required injection pressure can be obtained, which is a very important parameter because volume of shot or injection volume in a projected area depends on this parameter. Therefore value of injection pressure is dependent on shot volume. Injection pressure is an input to injection system which depends on an input variable of cooling system as in equation (4.5).

$$\rho_{inj} = \frac{Q \times F_{clamp} \times t_{inj} \times v_p}{2 \times v_s^2 \times A_{part}} \tag{4.5}$$

Now these two parameters, injection pressure and shot volume are coupling variables in the considered multi-disciplinary system. Coupling variables are those variables, value of one variable depends on the value of another but these are either inputs or outputs of different systems. Hence output of one system can be the input of another system and vice versa. As these two feed-back coupling variables belong to two different systems, optimizing the whole system without considering mutual interaction between these two systems can result in erroneous optimization result. Increment in value of volume of shot decreases injection pressure and vice versa. However, to minimize objective functions both of these variables should be minimized. Major contribution of this research is to incorporate these two coupling variables during optimization. Individual optimization of each system thus cannot provide actual minimized value. MDF method, popular for solving deterministic optimization problem, including the effect of coupling variable has been used to solve the multidisciplinary optimization problem of injection molding machine.

4.6 Design Variables

Design variables are those variables, values of which decide the performance function before machine is designed. Thus, careful choosing of the values of design variables can affect output of performance function.

4.6.1 Clamping Force

Clamping force in injection molding machine is the amount of force which is exerted by clamping unit to the mold of injection molding so that it can withstand the separating force of the mold. Separating force is the force which occurs when molten liquid enters the mold under huge injection pressure and tries to separate the mold. That is why clamping force is necessary to keep the mold halves in proper place. Clamping force is an important design variable as it determines amount of defects in the part, i.e., amount of flash in the part. Clamping force depends on different parameters, especially on the projected area of the part and inside cavity pressure or injection pressure of the mold.

Clamping force must need to be greater than the force generating from this cavity pressure. This force is called separating force. Thus design of injection molding machine structural system should incorporate the fact that melting pressure acting in the projected area of the mold should not surpass the maximum clamp force (equation 4.6) [5].

$$-p_{inj} + \frac{F_{clamp}}{A_{proj}} < 0 \tag{4.6}$$

4.6.2 Length and Diameter of the sprue, runner and gate

Sprue, runner and gate are the important design variables of injection molding system. They control cycle time and pressure drop, the two most important performance functions of injection molding system. Sprue is a passage through which molten liquid passes and reaches the mold. The extra material dries up in the sprue and it is cut off when part is removed. Length and diameter of sprue are fundamental design variables, because when diameter and length of the sprue are too large cycle time increases, on the other hand when it is too small pressure drop increases. Too small diameter lets the liquid plastic to become solidified when it touches the wall of the sprue. The less the diameter and length are, the more the pressure drop is. Runner is small passage through which molten plastic flows from sprue to mold. Gate is nothing but a small opening which allows melted plastic to run through the passage into the mold. Length and diameter of the gate are also important factors because these determine the amount of liquid to pass to the mold. Single gate is always preferred as it puts less gate mark on the finished part. Lengths of these three design variables determine required injection pressure which can be determined from Hagen-Pouseulle's law [6] as follows (equation 4.7) :

$$\begin{aligned}
&-\rho_{inj} + \frac{32 \times (l_{sprue} + l_{runner} + l_{gate} + l_{part}) \times \varphi \times v_F \times \eta_{eff}}{0.004} < 0 \\
&where,\ l_{sprue} = length\ of\ the\ sprue, l_{runner} = length\ of\ the\ runner \\
&\qquad l_{gate} = length\ of\ the\ gate, l_{part} = length\ of\ the\ part \\
&\qquad \varphi = ratio\ between\ width\ and\ thickness, v_F = front\ flow\ velocity \\
&\qquad \eta_{eff} = apparent\ effecctive\ viscosity
\end{aligned} \tag{4.7}$$

There are also some others constraints related to these design variables for example, sprue must have enough capacity to fill up the downstream runners [6]. This can be expressed through the following equation (4.8):

$$d_{sprue} + d_{runner}\sqrt{n_{downstream}} < 0 \tag{4.8}$$

Figure (4-2) shows the relative positions of the runner, sprue and gate. From the figure, it can be seen that how their relative positions control the flow of liquid plastic.

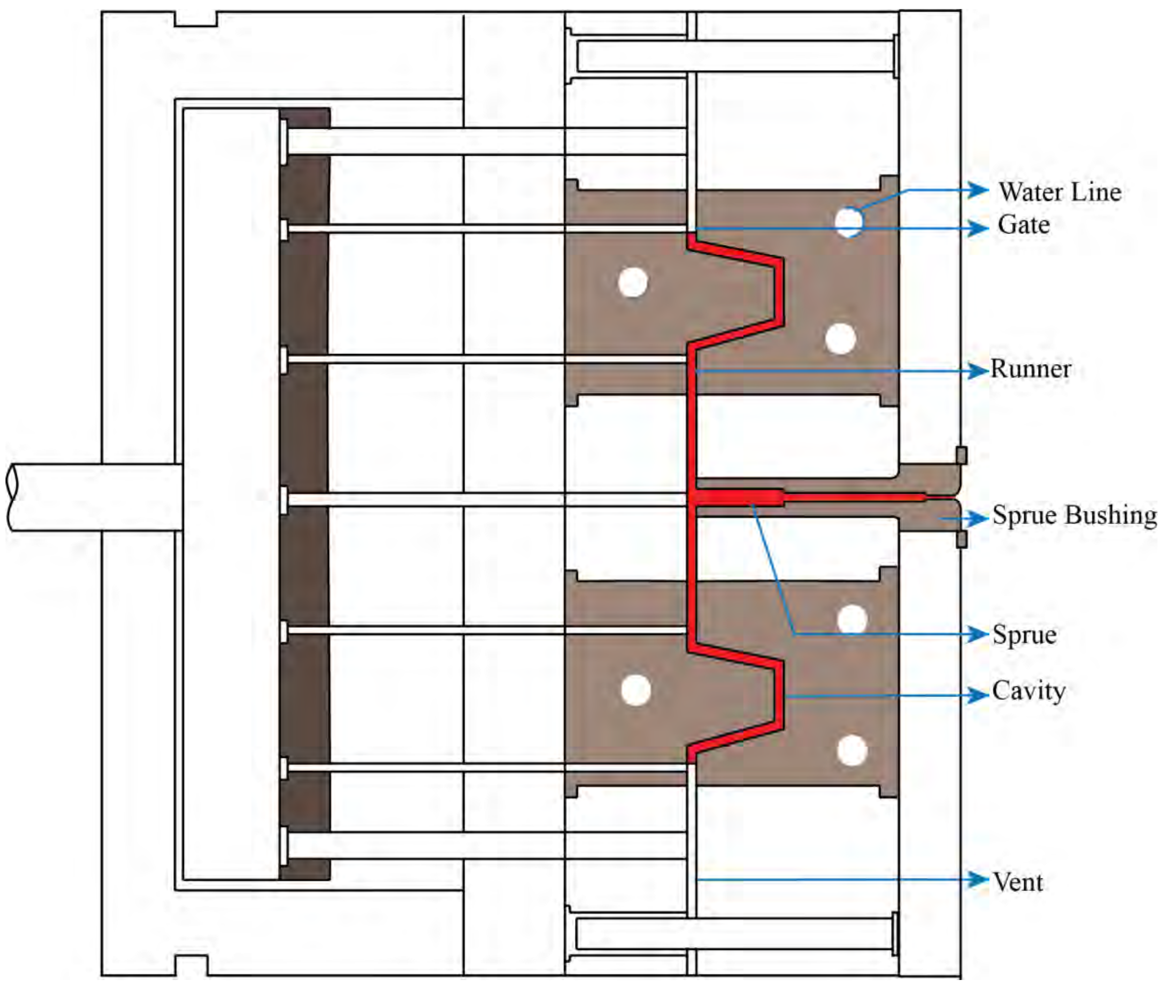

Fig 4-2: Schematic Diagram Injection Molding system showing primary design Variables

4.7 Process Parameters

Process parameters are those variables that depend on the process and material selection but they do affect the performance function like cycle time and pressure drop. These variables are not defined as design variables rather process parameters. As throughout our research we consider only one type of material (polypropylene), these process parameters were considered constant during design optimization.

4.7.1 Viscosity, Shear rate and Mold flow Index

Injection molding machine replicates polymers into different shapes of mold. The more shear rate of a polymer is increased the more easily it can be pushed in to the mold. Fluid form of polymer in injection molding system is non-Newtonian flow. Non-Newtonian flow does not obey Newton's law of viscosity. Non-Newtonian flow power law index becomes between 0 to 1. If power law is less than 1, then it means effective viscosity would decrease with increasing shear rate. Thus, viscosity is a property of any material which provides resistance to every material to flow. Viscosity can be written as the ratio of shear stress to shear rate [22].

$$\eta = \frac{SHEAR\ STRESS}{SHEAR\ RATE} = \frac{\tau}{\gamma} \tag{4.9}$$

where, γ =shear rate

τ =shear stress

Every material will flow if enough time is given. Viscosity decreases with the increment of shear rate, γ . Thus increasing value of shear rate is good for injection molding system. However shear stress should not be increased in such a way that, shear rate for flow through gates does not surpass the maximum allowable shear. This rule can be specified according to the power law as follows [6]:

$$-d_{gate} + 2 \times [\frac{(3+1/n) \times Q}{\pi \times \gamma_{max}}]^{1/3} < 0 \tag{4.10}$$

4.7.2 Injection Temperature and Mold Temperature

Injection temperature generally varies between 180-260°C and mold temperature 20-30°C. During design optimization process parameter is assumed fixed and design parameters are optimized. Injection temperature and mold temperature are kept constant during deterministic design optimization.

4.7.3 Length and Projected Area of Finished Part

Length and projected area of part determines few process parameters such as amount of time to inject, shot volume, time to eject, force required to clamp the mold halves etc. Multi-disciplinary design optimization has been applied to minimize cycle time and pressure drop for a particular product length and projected area of the finished part was taken as constant and mean value of their range was used in robust design optimization problem.

4.7.4 Flow Rate

Flow rate is also considered as a process parameter as it is completely determined during process. However, process parameter, flow rate depends on injection pressure and injection power. While considering deterministic multidisciplinary optimization average flow rate was considered as a constant value.

$$Q = \frac{P_{inj}}{\rho_{inj}} \tag{4.11}$$

4.7.5 Dry Cycle Time

Other than melting the polymer, injecting the liquid polymer into the cavity of the mold, drying the polymer to finished part there are also some other activities in injection molding machine. These activities include opening and closing the mold, actuating ejection, touching nozzle to the mold etc. This time is called dry cycle time, t_d. Dry cycle time is also a process parameter which is in the same manner taken as constant in deterministic design and its mean value is used in robust based design optimization.

Chapter 5

Numerical Illustration

In this research, MDF (Multidisciplinary Feasible) method which is a method for system analysis available in multi-disciplinary optimization is used to optimize injection molding system during design. A theory of industrial engineering has been applied in sector of manufacturing engineering, to extend the application of industrial engineering out of its border.

In this example, the whole, injection molding system is divided into four subsystems and interaction between these systems has also been identified. Presence of feed-forward and feed-back coupling was identified. Thus, optimizing the sub-systems individually will not give actual minimized value. The presence of coupling variables minimizing objective function with respect to each system may lead output of another system to maximization.

Multidisciplinary design optimization was performed considering the system as a deterministic problem. However, design variables can be uncertain too. Uncertainty is the natural quality of any system. Optimizing any system including uncertainty makes the system robust. Thus, in this research multidisciplinary optimization has been performed incorporating and without incorporating uncertainty. Multiple disciplinary feasible and Robust-Design Optimization were used respectively in these cases. The mathematical illustrations for both methodologies are explained below.

5.1 Mathematical Illustration of Deterministic Design Problem

This research has focused to minimize cycle time and pressure drop. Minimum cycle time ensures shorter time period to send products to the customer. Pressure drop minimization ensures good quality of products. Thus, objective function of multi-disciplinary design optimization along with constraints are described below (equation 5.1-5.9) [6,20],

Minimize,

i)Cycle Time,

$$f_1 = \frac{2 \times V_s \times p_{inj}}{P_{inj}} + \frac{h_{\max}^2}{\pi^2 \alpha} \ln(4) \times \left(\frac{(T_{melt} - T_{mold})}{(T_{ejection} - T_{mold})} \right) + [1 + 1.17 t_d \sqrt{\frac{2d+5}{S}}] \qquad (5.1)$$

ii)Pressure Drop,

$$f_2 = 4k[\frac{l_{sprue}}{d_{sprue}}(\frac{(3+\frac{1}{n})\times Q}{\pi\times(\frac{d_{sprue}}{2})^3})^n + \frac{l_{runner}}{d_{runner}}(\frac{(3+\frac{1}{n})\times Q}{\pi\times(\frac{d_{runner}}{2})^3})^n + \frac{l_{gate}}{d_{gate}}(\frac{(3+\frac{1}{n})\times Q}{\pi\times(\frac{d_{sprue}}{2})^3})^n] \quad (5.2)$$

Subject to,

i) $$\frac{h_{max}^2}{\pi^2\alpha} ln(4)\times\left(\frac{(T_{melt} - T_{mold})}{(T_{ejection} - T_{mold})}\right) > 3 \quad (5.3)$$

ii) $$-p_{inj} + \frac{F_{clamp}}{A_{proj}} < 0 \quad (5.4)$$

iii) $$-\rho_{inj} + \frac{32\times(l_{sprue} + l_{runner} + l_{gate} + l_{part})\times\varphi\times v_F \times\eta_{eff}}{0.004} < 0 \quad (5.5)$$

iv) $$-d_{sprue} + d_{runner}\sqrt{n_{downstream}} < 0 \quad (5.6)$$

v) $$-d_{gate} + 2\times[\frac{(3+1/n)\times Q}{\pi\times\gamma_{max}}]^{1/3} < 0 \quad (5.7)$$

Coupling variables:

i) $$v_s = \frac{P_{inj}\times t_{inj}}{2\times\rho_{inj}} \quad (5.8)$$

ii) $$\rho_{inj} = \frac{Q\times F_{clamp}\times t_{inj}\times v_p}{2\times v_s^2\times A_{part}} \quad (5.9)$$

Where, Flow rate, $Q = \frac{P_{inj}}{\rho_{inj}}$,[19].

Values of some above mentioned input variables were assumed to be constant, which depends on polymer property, (polypropylene)[22].

Table 5-1: Properties of deterministic process parameters

Properties	Value
Thermal diffusivity, α	$0.096\times10^{-6}\,m^2 s$
Viscosity evaluated at a shear rate of one reciprocal second, k	$400s^{-1}$
Ration between width and thickness, φ	1.5
Apparent effective viscosity, η_{aeff}	$25s^{-1}$
Number of downstreams, $n_{downstream}$	2
Power law Index	0.4

Thus, the optimization formulation given in (5.10)-(5.16) becomes:

Minimize,

$$f_1 = \frac{2\times V_s \times p_{inj}}{P_{inj}} + \frac{h_{\max}^2}{\pi^2\alpha}\ln(4)\times\left(\frac{(T_{melt}-T_{mold})}{(T_{ejection}-T_{mold})}\right)+[1+1.17t_d\sqrt{\frac{2d+5}{S}}] \quad (5.10)$$

$$f_2 = 4k[\frac{l_{sprue}}{d_{sprue}}(\frac{8\times5.5\times P_{inj}}{\pi\times(d_{sprue})^3\times\rho_{inj}})^{0.4}+\frac{l_{runner}}{d_{runner}}(\frac{8\times5.5\times P_{inj}}{\pi\times(d_{runner})^3\times\rho_{inj}})^{0.4}+\frac{l_{gate}}{d_{gate}}(\frac{8\times5.5\times P_{inj}}{\pi\times(d_{gate})^3\times\rho_{inj}})^{0.4}] \quad (5.11)$$

Subject to,

$$\text{i) } g_1 = 3 - \frac{h_{max}^2}{\pi^2\alpha}ln(4)\times\left(\frac{(T_{melt}-T_{mold})}{(T_{ejection}-T_{mold})}\right) \quad (5.12)$$

$$\text{ii) } g_2 = -p_{inj} + \frac{F_{clamp}}{A_{proj}} \quad (5.13)$$

iii) $g_3 = -\rho_{inj} + \frac{32 \times (l_{sprue} + l_{runner} + l_{gate} + l_{part}) \times 1.5 \times v_F \times \eta_{eff}}{0.004}$ (5.14)

iv) $g_4 = -d_{sprue} + d_{runner}\sqrt{2}$ (5.15)

v) $g_5 = -d_{gate} + 2 \times [\frac{5.5 \times P_{inj}}{\pi \times 1000 \times \rho_{inj}}]^{1/3}$ (5.16)

Among these, P_{inj}, d, S, l_{sprue}, l_{runner}, l_{gate}, d_{sprue}, d_{runner}, d_{gate}, F_{clamp}, t_{inj} are considered as design variables and the rest, T_{melt}, T_{mold}, T_{ejecct}, t_d, Q, V_p, A_{part}, l_{part}, v_F are considered as non-design variables. Design variables have lower and upper bounds. Though non-design variables have interval values, during design optimization a constant value within the range was considered. Following interval data of the design and non-design variables have been tabulated (Table 5-2).

Table 5-2: Ranges of Values of Process Parameters

Process Parameters (Non-design variables)	Range of Value
Mold temperature, T_{mold}	29-85°C
Melting Temperature, T_{melt}	218-271°C
Ejecting temperature, $T_{ejection}$	130-190°C
Dry cycle time, t_d	2-7sec
Flow rate of material, Q	0.2-0.3m^3/s
Projected area of the part, A_{part}	0.1-0.3m^2
Part length, l_{part}	0.0005-0.03m
Front flow velocity of the melted liquid, v_F	0.2-0.6ms^{-1}

Table 5-3: Ranges of Values of Design Variables

Design Variables	Range of Value
Injection Power, P_{inj}	1000-5000W
Depth of mold, d	0.004-0.2m
Maximum clamp stroke, S	0.004-0.2m
Length of sprue l_{sprue}	0.01-0.025m
Length of runner, l_{runner}	0.002-0.02m
Length of gate, l_{gate}	0.002-0.02m
Diameter of sprue, d_{sprue}	0.002-0.02m
Diameter of runner, d_{runner}	0.0005-0.01m
Diameter of gate, d_{gate}	0.02-0.2m
Clamp stroke, F_{clamp}	1000-5000N
Injection Time, t_{inj}	2-50sec

The multi-disciplinary optimization problem was solved using MATLAB solvers called 'fmincon' and 'fsolve'[23]. 'Fmincon' can solve only a single objective at a time. That is why two objective functions were summed up to one objective using weighted sum approach. Two objectives have different units. Thus, normalization method was necessary. Thus our optimization problem becomes (equation 5.17):

$$\begin{aligned} \min, \ & \frac{f_1(x,p,u)}{f_1^m(x,p,u)} + \frac{f_2(x,p,u)}{f_2^m(x,p,u)} \\ & \text{subject to, } g_i(x,p,u) \le 0 \\ & u_{m,n}(x,p) = 0 \\ & i = 1,2....5 \\ & m,n = 1,2. \end{aligned} \tag{5.17}$$

where, $f_1^m(x,p,u)$ and $f_2^m(x,p,u)$ are the minimum value of the objective functions. As, discussed previously MDF method does not consider optimize coupling variables as optimization variables, rather just solve their functions to deliver value of coupling variables at each optimization iteration. When the values of coupling variables are sent to the main function it is optimized using solver ‘fmincon’ and minimum values of pressure drop and cycle time are obtained [23]. Values of design variables are also obtained when pressure drop and cycle time are minimized. While optimizing design variables, process parameters are kept constant.

In figure 4, MDF method has been applied to multidisciplinary injection molding system where values of coupling variables are solved outside the optimizer and supplied to main optimizer

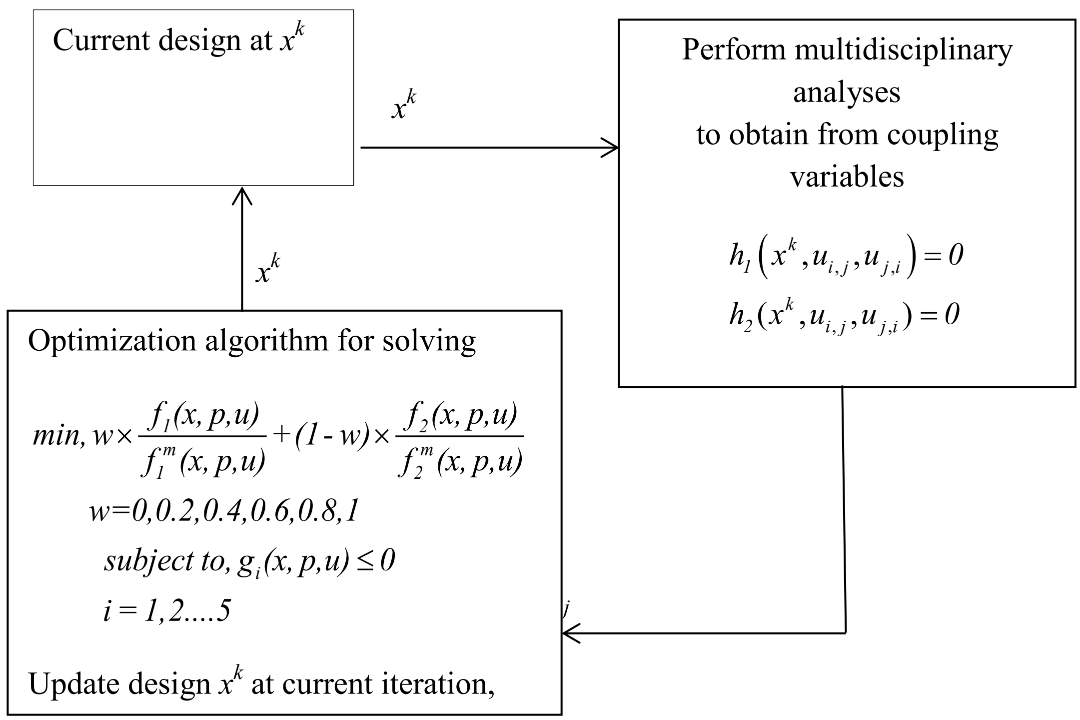

Fig 5-1: MDF methodology applied in Injection Molding Machine

Weighted-sum method is applied; different weights are assigned to the objective functions. Value of w can be between 0 and 1. Value of w was varied with an increment of 0.2 i.e. w=0, 0.2, 0.4, 0.6, 0.8, 1. For each of the values of w, objective function was minimized. That means each objective function was given different weights and different values of each objective function were obtained. It is observed that when cycle time was minimized, pressure drop function gave maximized value. When pressure drop was minimized, cycle time gave maximized value.

5.2 Robust –Design Optimization

MDF method was applied in multi-disciplinary optimization considering both design and non-design variables deterministic. However, design and non-design variables will always not be deterministic rather probabilistic. Probabilistic variables can lead to uncertainty. In this case, robust design optimization for multi-disciplinary system can yield more realistic results.

5.3 Uncertainty-based design

Methodology to obtain uncertainty based design optimization includes:

i) **Obtaining Objective Robustness:** Objective robustness can be obtained by both minimizing the mean and variance. Thus, in robust design optimization multi-objective optimization becomes minimization of mean and variance.

ii) **Obtaining Feasibility Robustness**: Feasibility robustness can be obtained when function converges within feasible limit even when feasible bound of the constraints is reduced. In our multi-disciplinary problem, boundary was reduced my one standard deviation.

iii) **Estimating mean and variance of performance function**: Means and variances of objective (5.18, 5.19) and constrain functions (5.20-5.26) are obtained using first-order Taylor series expansion[10].

As data are available as only interval data, averages of upper and lower bound were considered as the mean of non-design variables and 10% values of mean were considered as standard deviation for both design and non-design variables. In robust-based design optimization, we have taken mean and variance of cycle time as objective function and pressure drop was added as another constraint. Minimized and maximized value of pressure drop during deterministic optimization of multi-disciplinary system of injection molding machine have been used as lower and upper bound of pressure drop. Thus, two new constraints were added to the robust multi-disciplinary optimization of injection molding system. Therefore, the robust multi-disciplinary optimization formulation now becomes (equation 5.18-5.26):

i)

$$cycle\ time\ ,\ f_1 = \frac{2 \times V_s \times p_{inj}}{P_{inj}} + \frac{h_{max}^2}{\pi^2 \alpha} ln(4) \times \left(\frac{(T_{melt} - T_{mold})}{(T_{ejection} - T_{mold})} \right) + [1 + 1.17 t_d \sqrt{\frac{2d+5}{S}}] \quad (5.18)$$

ii)

$$variance\ of\ cycle\ time = [\frac{2 \times v_s \times \rho_{inj}}{P_{inj}}]^2 \times var\_P_{inj} + [\frac{0.002^2}{9.457 \times 10^{-5}} \times ln(4) \times \frac{1}{T_{ej} - T_m}]^2 \times var\_T_{inj}$$
$$+ [\frac{0.002^2}{9.457 \times 10^{-5}} \times ln(4) \times \frac{T_{inj} - T_m}{(T_m - T_{ej})^2}] var\_T_{ej} + [\frac{0.002^2}{9.457 \times 10^{-5}} \times ln(4) \times \frac{T_{inj} - T_m}{(T_{ej} - T_m)^2}]^2 \times var\_T_m \quad (5.19)$$
$$+ [1.17 \times \sqrt{\frac{2 \times d + 5}{S}}]^2 \times var\_t_d + [1.17 \times t_d \times \frac{1}{2} \times (\frac{2 \times d + 5}{S})^{\frac{-1}{2}} \times \frac{2}{S}]^2 \times var\_d$$
$$+ [1.17 \times t_d \times \frac{1}{2} \times (\frac{2 \times d + 5}{S})^{\frac{-1}{2}} \times (\frac{2 \times d + 5}{S^2})]^2 \times var\_S$$

Subject to,

i)

$$\frac{v_s \times A_{part} \times \rho_{inj}}{v_p} - F_{clamp} + k \times \sqrt{\sigma_{A_{part}}^2 \times (\frac{v_s \times \rho_{inj}}{v_p})^2 + \sigma_{v_p}^2 \times (\frac{v_s \times A_{part} \times \rho_{inj}}{v_p^2}) + \sigma_{F_{clamp}}^2} < 0 \quad (5.20)$$

ii)

$$3 - \frac{0.002^2}{\pi^2 \alpha} \times \ln(4) \times \frac{T_{melt} - T_{mold}}{T_{eject} - T_{mold}} + k \times \sqrt{\begin{array}{l} \sigma_{T_{melt}}^2 \times (\frac{0.002^2}{\pi^2 \alpha} \times \ln(4) \times \frac{1}{(T_{eject} - T_{mold})})^2 + \\ \sigma_{T_{eject}}^2 \times (\frac{0.002^2}{\pi^2 \alpha} \times \ln(4) \times \frac{T_{melt} - T_{mold}}{(T_{eject} - T_{mold})})^2 + \\ \sigma_{T_{mold}}^2 \times (\frac{0.002^2}{\pi^2 \alpha} \times \ln(4) \times \frac{T_{melt} - T_{mold}}{(T_{eject} - T_{mold})})^2 \end{array}} < 0 \quad (5.21)$$

iii)

$$-\rho_{inj}+32\times(l_{sprue}+l_{runner}+l_{gate}+l_{part})\times 1.5\times v_F\times\frac{25}{0.004}6666+k\sqrt{\begin{array}{l}(\sigma^2_{l_{sprue}}\times 32\times 1.5\times\frac{v_F\times 25}{0.004})^2+(\sigma^2_{l_{runner}}\times 32\times 1.5\times\frac{v_F\times 25}{0.004})^2\\+(\sigma^2_{l_{gate}}\times 32\times 1.5\times\frac{v_F\times 25}{0.004})^2+(\sigma^2_{l_{part}}\times 32\times 1.5\times\frac{v_F\times 25}{0.004})^2\\+(\sigma^2_{v_F}\times 3\times 10^5\times(l_{sprue}+l_{runner}+l_{gate}+l_{part})^2\end{array}}<0 \tag{5.22}$$

iv)

$$-d_{gate}+2\times(\frac{5.5\times P_{inj}}{3141.6\times\rho_{inj}})^{\frac{1}{3}}+\sqrt{\begin{array}{l}(\sigma^2_{d_{gate}}+\sigma^2_{P_{inj}}\times(\frac{2}{3}\times(\frac{5.5}{3141.6\times\rho_{inj}})^{\frac{1}{3}}\\\times P_{inj}^{-\frac{2}{3}})^2\end{array}}<0 \tag{5.23}$$

v) $$-d_{sprue}+d_{gate}\sqrt{d_{downstream}}+k\sqrt{\sigma^2_{d_{sprue}}+2\times\sigma^2_{gate}}<0 \tag{5.24}$$

iv)

$$\begin{aligned}&-2.98\times 10^6+4K[\frac{l_{sprue}}{d_{sprue}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{sprue}^3})^{0.4}+\frac{l_{runner}}{d_{runner}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{runner}^3})^{0.4}+\\&\frac{l_{gate}}{d_{gate}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{gate}^3})^{0.4}]+k\times sqrt[\text{var\_}l_{sprue}[\frac{4K}{d_{sprue}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{sprue}^3})^{0.4}]^2+\\&\text{var\_}d_{sprue}[\frac{4K\times l_{sprue}\times 2.2}{d_{sprue}^{3.2}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}})^{0.4}]^2+\text{var\_}l_{runner}[\frac{4K}{d_{sprue}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{sprue}^3})^{0.4}]^2+\\&\text{var\_}d_{sprue}[\frac{4K\times l_{sprue}\times 2.2}{d_{sprue}^{3.2}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}})^{0.4}]^2\ \text{var\_}l_{gate}[\frac{4K}{d_{gate}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{gate}^3})^{0.4}]^2+\\&\text{var\_}d_{gate}[\frac{4K\times l_{gate}\times 2.2}{d_{gate}^{3.2}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}})^{0.4}]^2]<0\end{aligned} \tag{5.25}$$

v)

$$1.91\times 10^{5} - 4K[\frac{l_{sprue}}{d_{sprue}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{sprue}^{3}})^{0.4} + \frac{l_{runner}}{d_{runner}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{runner}^{3}})^{0.4} + \frac{l_{gate}}{d_{gate}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{gate}^{3}})^{0.4}] + k\times sqrt[\text{var\_}l_{sprue}[\frac{4K}{d_{sprue}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{sprue}^{3}})^{0.4}]^{2} + \text{var\_}d_{sprue}[\frac{4K\times l_{sprue}\times 2.2}{d_{sprue}^{3.2}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}})^{0.4}]^{2} + \text{var\_}l_{runner}[\frac{4K}{d_{sprue}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{sprue}^{3}})^{0.4}]^{2} + \text{var\_}d_{sprue}[\frac{4K\times l_{sprue}\times 2.2}{d_{sprue}^{3.2}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}})^{0.4}]^{2}\ \text{var\_}l_{gate}[\frac{4K}{d_{gate}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}\times d_{gate}^{3}})^{0.4}]^{2} + \text{var\_}d_{gate}[\frac{4K\times l_{gate}\times 2.2}{d_{gate}^{3.2}}(\frac{44\times P_{inj}}{\pi\times\rho_{inj}})^{0.4}]^{2}] < 0 \qquad (5.26)$$

If RDO methodology can be presented in a short form then it could be summarized as below (equation 5.27):

$$\begin{aligned}
&min,\ \frac{f_1(x,\mu_p,\mu_u)}{f_1^m(x,\mu_p,\mu_u)} + \frac{f_2(x,\mu_p,\mu_u)}{f_2^m(x,\mu_p,\mu_u)} \\
&\quad subject\ to,\ LB + k\sigma(g_i(x,\mu_p,u)) \le E(g_i(x,p,u)) \le UB - k\sigma(g_i(x,\mu_p)) \\
&\quad for\ all\ i \qquad (5.27) \\
&\quad lb + k\sigma(x) \le x \le ub - k\sigma(x) \\
&\quad u_{m,n}(x,p) = 0 \\
&\quad i = 1,2....5;\ m,n = 1,2.
\end{aligned}$$

Among these, $x = P_{inj}$, d, S, l_{sprue}, l_{runner}, l_{gate}, d_{sprue}, d_{runner}, d_{gate}, F_{clamp}, t_{inj} are considered as design variables and the rest, T_{melt}, T_{mold}, T_{ejecct}, t_d, Q, V_p, A_{part}, l_{part}, v_F are considered as non-design variables. IN RDO problem mean value of non-design variables were chosen. ***LB*** and ***UB*** are lower and upper bound of the constraints and ***lb*** and ***ub*** are lower and upper bound of design variables. $\boldsymbol{\sigma}$'s are the standard deviations. Value of *k* determines how far feasible region should be reduced. *k* can take any value. In this research value of *k* was taken as 1[20].

Table 5-4: Means and variances of non-design variables

Non design Variable	Mean	Variance
Mold temperature, T_{mold}	$57^{\circ}C$	$5.7^{\circ}C$
Melting Temperature, T_{melt}	$244.5^{\circ}C$	$24.45^{\circ}C$
Ejecting temperature, $T_{ejection}$	$160^{\circ}C$	$16^{\circ}C$
Dry cycle time, t_d	4.5sec	0.45 sec
Flow rate of material, Q	$0.25m^3$	$0.025m^3$
Projected area of the part, A_{part}	$0.2m^2$	$0.02m^2$
Part length, l_{part}	0.01525m	0.001525m
Front flow velocity of the melted liquid, v_F	$0.4ms^{-1}$	$0.04ms^{-1}$
Volume of the finished part	$0.25m^3$	$0.025m^3$

Table 5-5: Variances of design variables

Design Variables	*Lower Bound, LB*	*Upper Bound, UB*	*Standard Deviation,* σ_X
Injection Power, P_{inj}	1000W	5000W	300W
Depth of mold, d	0.004m	0.2m	0.0102m
Maximum clamp stroke, S	0.004m	0.2m	0.0102m
Length of sprue l_{sprue}	0.01m	0.025m	0.00175m
Length of runner, l_{runner}	0.002m	0.02m	0.0011m
Length of gate, l_{gate}	0.002m	0.02m	0.0011m
Diameter of sprue, d_{sprue}	0.002m	0.02m	0.0011m
Diameter of runner, d_{runner}	0.0005m	0.01m	0.000525m

Diameter of gate, d_{gate}	0.02m	0.2m	0.011m
Clamp stroke, F_{clamp}	1000m	5000m	300m
Injection Time, t_{inj}	2sec	50sec	2.6sec

The robust-based design optimization problem was also solved using MATLAB solvers called 'fmincon' and 'fsolve'. 'fmincon' can solve only a single objective at a time [23]. That's why two objective functions were summed up to one objective using weighted sum approach. Two objectives have different units. Thus, normalization method was necessary. As in this robust design optimization, variables were optimized with more reduced bound of constraint and variables, output becomes less sensitive to change of the input values of design variable. Thus robust-design is obtained. Different optimized values were obtained for different values of 'w' from 0 to 1.

Chapter 6

Results and Discussions

6.1 Conclusion

Multidisciplinary design optimization was performed in injection molding system to obtain the values of design variables in such a way that the performance functions are optimized. Performance functions of injection molding design system are cycle time to produce a part, energy consumption and pressure drop.

Multi-disciplinary design optimization was performed on injection molding system both in deterministic and probabilistic condition. Deterministic system considers variables of injection molding system without effect of uncertainty. On the other hand, probabilistic approach considers uncertainty of each design and non-design variable. Multidisciplinary Feasible Approach and Robust-design optimization have been applied to optimize deterministic and probabilistic optimization. Results obtained from this optimization have been described below.

6.2 Results Obtained from Deterministic formulation

Multi-disciplinary optimization using MDF method was solved by weighted sum method. Value of weight, w was used from 0 to 1 i.e. w=0, 0.2, 0.4, 0.6, 0.8, 1. Values obtained for the design variables are tabulated below:

Table 6-1: Results obtained for different values of *w*

w=0

Injection Power, P_{inj}	3281.446W
Depth of mold, d	0.102m
Maximum clamp stroke, S	0.102m
Length of sprue l_{sprue}	0.01m
Length of runner, l_{runner}	0.02m
Length of gate, l_{gate}	0.002m
Diameter of sprue, d_{sprue}	0.014142m

Diameter of runner, d_{runner}	0.0005m
Diameter of gate, d_{gate}	0.2m
Clamp stroke, F_{clamp}	2821.302N
Injection Time, t_{inj}	2.702147s

w=0.2

Injection Power, P_{inj}	4770.078W
Depth of mold, d	0.004m
Maximum clamp stroke, S	0.2m
Length of sprue l_{sprue}	0.01m
Length of runner, l_{runner}	0.02m
Length of gate, l_{gate}	0.002m
Diameter of sprue, d_{sprue}	0.014142m
Diameter of runner, d_{runner}	0.0005m
Diameter of gate, d_{gate}	0.2m
Clamp stroke, F_{clamp}	4682.471N
Injection Time, t_{inj}	3.085136s

w= 0.4

Injection Power, P_{inj}	3257.284W
Depth of mold, d	0.02307m
Maximum clamp stroke, S	0.198334m
Length of sprue l_{sprue}	0.01m
Length of runner, l_{runner}	0.02m
Length of gate, l_{gate}	0.002m
Diameter of sprue, d_{sprue}	0.014142m
Diameter of runner, d_{runner}	0.0005m

Diameter of gate, d_{gate}	0.199928m
Clamp stroke, F_{clamp}	2952.926N
Injection Time, t_{inj}	2.84919s

w=0.6

Injection Power, P_{inj}	3250.75W
Depth of mold, d	0.004m
Maximum clamp stroke, S	0.2m
Length of sprue l_{sprue}	0.01m
Length of runner, l_{runner}	0.02m
Length of gate, l_{gate}	0.002m
Diameter of sprue, d_{sprue}	0.014142m
Diameter of runner, d_{runner}	0.0005m
Diameter of gate, d_{gate}	0.2m
Clamp stroke, F_{clamp}	2068.659N
Injection Time, t_{inj}	2s

w=0.8

Injection Power, P_{inj}	3299.112W
Depth of mold, d	0.004m
Maximum clamp stroke, S	0.2m
Length of sprue l_{sprue}	0.01m
Length of runner, l_{runner}	0.02m
Length of gate, l_{gate}	0.002m
Diameter of sprue, d_{sprue}	0.014142m
Diameter of runner, d_{runner}	0.0005m
Diameter of gate, d_{gate}	0.2m

Clamp stroke, F_{clamp}	2913.254N
Injection Time, t_{inj}	2.775274s

w=1

Injection Power, P_{inj}	3265.364W
Depth of mold, d	0.004001m
Maximum clamp stroke, S	0.2m
Length of sprue l_{sprue}	0.017011m
Length of runner, l_{runner}	0.015326m
Length of gate, l_{gate}	0.010307m
Diameter of sprue, d_{sprue}	0.005729m
Diameter of runner, d_{runner}	0.00502m
Diameter of gate, d_{gate}	0.181172m
Clamp stroke, F_{clamp}	2985.304N
Injection Time, t_{inj}	2.000002s

Therefore, for the values of design variables for which both cycle time and pressure drops are minimized to some extent are the optimum values of design variables. Different values of cycle time and pressure drop are tabulated below:

Table 6-2: Cycle Time and Pressure Drop for different values of *w*

w	*Cycle Time, (sec), t*	*Pressure Drop, Δp*
0	850.41	1.9061×10^5
0.2	30.187891	1.9061×10^5
0.4	30.095111	1.9061×10^5
0.6	29.102755	1.9061×10^5
0.8	29.87830	1.9601×10^5
1	29.102763	2.9585×10^6

The following figure summarizes these results.

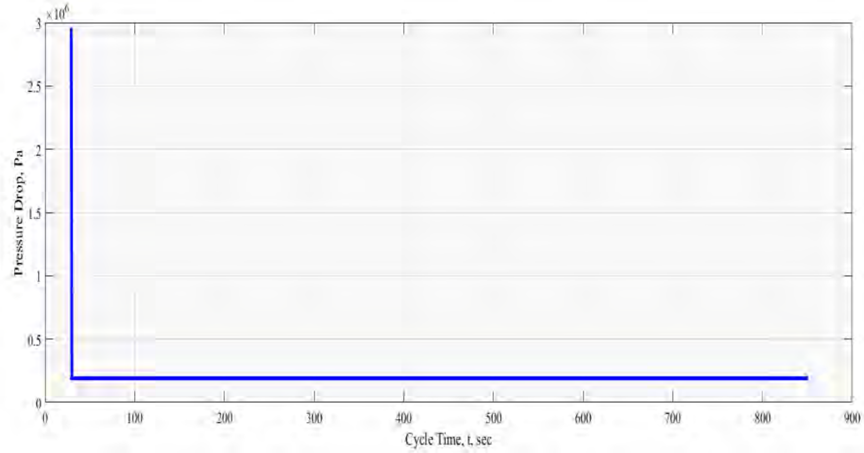

Fig 6-1: Cycle time Vs Pressure Drop for Different Weights of Objective Function

Graph in Figure 6.1 shows when w=0 only pressure drop was minimized, cycle time was not minimized. Thus, a lower value of pressure drop (1.95×10^5 Pa) comes along with a higher value of cycle time (850 sec). However, when, w=1, which means only cycle time was minimized. This time, cycle time was minimized (29.102763 sec), but pressure drop was maximized (2.9585×10^6 Pa). Therefore, to get an optimized multi-disciplinary system, values of design variables should be chosen for w = 0.2 to w = 0.8 values, where, values of both cycle time and pressure drop are minimized to some extent.

6.3 Results Obtained from Robust Design Optimization

Multi-disciplinary optimization of injection molding system under uncertainty was solved by robust design optimization methodology. This includes optimization of mean and minimization of variance, which is also a multi-objective optimization problem. Thus, weighted sum method was used. Value of weight, w was used from 0 to 1 i.e. w=0, 0.2, 0.4, 0.6, 0.8, 1. Values obtained for the design variables are tabulated below.

Table 6-3: Result of Robust-based Design Optimization for different values of *w*

$w = 0$

Injection Power, P_{inj}	2202.717561W
Depth of mold, d	0.014203605m

Maximum clamp stroke, S	0.189799757m
Length of sprue l_{sprue}	0.016497975m
Length of runner, l_{runner}	0.016035624m
Length of gate, l_{gate}	0.007802651m
Diameter of sprue, d_{sprue}	0.007443704m
Diameter of runner, d_{runner}	0.004682203m
Diameter of gate, d_{gate}	0.181223606m
Clamp stroke, F_{clamp}	4699.996873N
Injection Time, t_{inj}	4.600034198s

$w = 0.2$

Injection Power, P_{inj}	2202.743W
Depth of mold, d	0.0142m
Maximum clamp stroke, S	0.1898m
Length of sprue l_{sprue}	0.016354m
Length of runner, l_{runner}	0.016006m
Length of gate, l_{gate}	0.007679m
Diameter of sprue, d_{sprue}	0.007384m
Diameter of runner, d_{runner}	0.004591m
Diameter of gate, d_{gate}	0.181237m
Clamp stroke, F_{clamp}	4699.999N
Injection Time, t_{inj}	4.6s

$w = 0.4$

Injection Power, P_{inj}	2202.492W
Depth of mold, d	0.014206m
Maximum clamp stroke, S	0.1898m

Length of sprue l_{sprue}	0.016346m
Length of runner, l_{runner}	0.015997m
Length of gate, l_{gate}	0.007702m
Diameter of sprue, d_{sprue}	0.007361m
Diameter of runner, d_{runner}	0.00459m
Diameter of gate, d_{gate}	0.181253m
Clamp stroke, F_{clamp}	4699.702N
Injection Time, t_{inj}	4.600051s

$w = 0.6$

Injection Power, P_{inj}	2202.54W
Depth of mold, d	0.014205m
Maximum clamp stroke, S	0.1898m
Length of sprue l_{sprue}	0.016354m
Length of runner, l_{runner}	0.016007m
Length of gate, l_{gate}	0.007679m
Diameter of sprue, d_{sprue}	0.007381m
Diameter of runner, d_{runner}	0.00459m
Diameter of gate, d_{gate}	0.181236m
Clamp stroke, F_{clamp}	4699.753N
Injection Time, t_{inj}	4.600034s

$w = 0.8$

Injection Power, P_{inj}	2202.486W
Depth of mold, d	0.0142m
Maximum clamp stroke, S	0.1898m
Length of sprue l_{sprue}	0.016401m

Length of runner, l_{runner}	0.015979m
Length of gate, l_{gate}	0.007627m
Diameter of sprue, d_{sprue}	0.007365m
Diameter of runner, d_{runner}	0.004573m
Diameter of gate, d_{gate}	0.181254m
Clamp stroke, F_{clamp}	4699.488N
Injection Time, t_{inj}	4.600003s

$w = 1$

Injection Power, P_{inj}	1971.714W
Depth of mold, d	0.014213m
Maximum clamp stroke, S	0.189799m
Length of sprue l_{sprue}	0.01555m
Length of runner, l_{runner}	0.015882m
Length of gate, l_{gate}	0.006856m
Diameter of sprue, d_{sprue}	0.007211m
Diameter of runner, d_{runner}	0.004096m
Diameter of gate, d_{gate}	0.182559m
Clamp stroke, F_{clamp}	4380.323N
Injection Time, t_{inj}	4.600069m

For different values of *w,* values of mean and variance of the objective function cycle time were obtained. These values have been tabulated below:

Table 6-4: Mean and Variance of Cycle Time for Robust Design for different values of *w*

w	$Cycle\ Time, t$	$variance\ of\ \ cycle\ time, \sigma^2_{cycletime}$
0	43.38169432	13.10243865
0.2	43.38162436	13.10239874
0.4	43.3817393	13.10255401
0.6	43.38170852	13.10252164
0.8	43.3816272	13.10250693
1	43.3818315	13.21677228

The following figure summarizes these results.

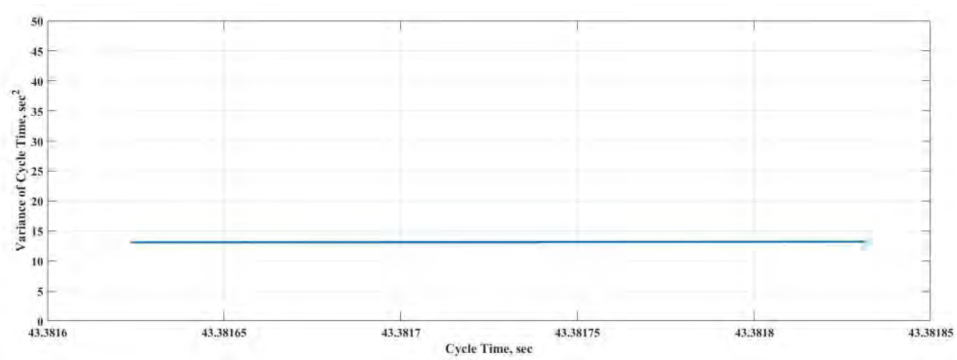

Fig 6-2: Mean of Cycle Time vs. Variance of Cycle Time

The above figure 6-2 is a graph of mean of cycle time versus variance of cycle time which demonstrates that, due to the inclusion of uncertainty; mean value of cycle time does not change to a great extent with the change of variance of cycle time. In other words, when robust design optimization is implemented, there is less effect on output with the changes in input variables.

Results obtained from multidisciplinary design optimization with the effect of coupling variables, along with uncertainty and without uncertainty have been compared here with results from multi-disciplinary optimization without the effect of coupling variables and uncertainty [6] for validation of the above methodology.

Table 6-5: Comparison between results obtained from previous studies and present study

Design Variables	Without effect of Coupling variables	With effect of coupling variables but not with uncertainty	With effect of coupling variables and uncertainty
Injection Power, ρ_{inj}	3567W	3250.75W	2202.54W
Length of sprue l_{sprue}	0.102m	0.01m	0.016354m
Length of runner, l_{runner}	0.084m	0.02m	0.016007m
Length of gate, l_{gate}	0.000502m	0.002m	0.007679m
Diameter of sprue, d_{sprue}	0.0085m	0.014142m	0.007385m
Diameter of runner, d_{runner}	0.009m	0.005m	0.00459m
Diameter of gate, d_{gate}	0.002m	0.2m	0.181236m
Cycle Time, t_{inj}	65.7sec	29.102 sec	43.3817sec

Chapter 7

Conclusion and Future Work

7.1 Conclusion

Optimization plays a pivotal role in shaping the current world and is expected to do so in future as well. The objective of optimization may vary depending on the perspective or discipline. However, a large system can consist of different disciplines. Therefore, optimizing from perspective of one discipline might not necessarily optimize the system from overall perspective. The reason is not taking the interaction between the disciplines into consideration. That is why Multi-disciplinary Design Optimization was considered in this thesis.

7.2 Recommendations

This research applied MDF (Multi-disciplinary Feasibility) optimization technique to optimize cycle time and pressure drop of an injection molding machine. This could be done from several perspectives, as the mechanism of injection molding machine consists of several disciplines. The previous work has described injection molding system as Multi-disciplinary system with four disciplines. However, though they claimed that, Multi-disciplinary Design Optimization (MDO) was applied, they skipped the main concept of coupling variables between disciplines. We, however, considered four disciplines, including the interaction between two of these disciplines. This leads this research to the actual application of Multi-disciplinary design Optimization what is called MDO. MDO can be applied to multi-disciplinary system in various ways, like MDF, IDF, AAO. Only MDF was applied in this research. MDF method controls only the design variables, not the coupling variables. AAO controls both design and coupling variables. IDF is medium of these two extreme types MDO methods. Though, MDF method has been applied in this research, future work can be extended by using IDF and AAO method. Collaborative Optimization (CO) can also be applied. CO used system level optimization instead of system level single optimization like MDF.

Another contribution of this thesis was to incorporate uncertainty in the model. In all previous works, the parameters were considered deterministic. However, in real world, nothing is certain. Therefore, the problem was solved considering the design variables as well as the parameters having uncertainty. Robust design Optimization was applied to reduce effect of uncertainty in the final output. If a system is sensitive to uncertain input parameters, then the output becomes uncertain too. Robust based design optimization includes optimization of mean and minimization of variance. However, this method only includes aleatory uncertainty which arises from natural variability. However, epistemic uncertainty has not been considered here, which occurs when enough data are not available. Incorporation of epistemic uncertainty can improve the results. While collecting data to reduce epistemic uncertainty, data of many variables can be obtained in multiple interval value other than single interval value. In cooperating this interval values of design variables during optimization will make the system more robust. During uncertainty inclusion, estimating mean and variance was done by using Taylor series expansion. With higher order variability this can provide erroneous result. This can be improved too in future works.

Finally, MDF has not been widely used in the manufacturing sector. However, for manufacturing sector, optimization is more crucial than ever now-a-days. The continuous depletion of resources and the increased demand and competition have forced the manufacturers to concentrate heavily on increasing productivity while sustaining reasonable profit. Also, manufacturing requires knowledge of several disciplines as well. That is why we believe MDF or any multi-disciplinary optimization techniques should be applied more frequently in this sector. We believe, our work will prove to be helpful for users of injection molding machine and help the world reach sustainability in terms of both economy and energy.

References

[1] Yi, S. I., Shin, J. K., and Park, G. J., "Comparison Of MDO Methods with Mathematical Examples," *Struct. Multidiscip. Optim.*, vol. 35, pp. 391–402, 2008.

[2] Zaman, K., *Modeling and management of Epistemic Uncertainty for Mulstidisciplinary System Analysis and Design*, Dissertation to the faculty of the Graduate School of Vanderbilt, (2010).

[3] Allison, J., Kokkolaras, M., Zawislak, M., and Papalambros, J. Y., "On the Use of Analytical Target Cascading and Collaborative Optimization for Complex System Design," *Structural and Multidisciplinary Optimization*, pp. 1–10, 2005.

[4] Cramer, E.J., Frank, J. E. D., Frank, P.D., Lewis, R.M., and Shubin, G.R., "Problem Formulation for Multidisciplinary Optimization," *SIAM J. Optim.*, vol. 4, p. 754-776, 1994.

[5] Kroo,I., Altus, S., Braun, R., Gage, P., and Sobieski, I.,"Multidisciplinary Optimization Methods for Aircraft Preliminary Design," *5th Symp. Multidiscip. Anal. Optim.*, vol. 4, p. 1994-4325, 1994.

[6] Ferreira, I., Weck, O., Saraiva, P., and Cabral, J., "Multidisciplinary Optimization of Injection Molding Systems," *Struct. Multidiscip. Optim.*, vol. 41, pp. 621–635, 2010.

[7] Ferreira, I., Cabral, J. A., Saraiva, P., and Oliveira, M. C., "A Multidisciplinary Framework to Support the Design of Injection Mold Tools," *Struct. Multidiscip. Optim.*, vol. 49, pp. 501–521, 2014.

[8] Kamal, M. R., and Kenig, S., "The Injection Molding of Thermoplastics Part I: Theoretical Model," *Polym. Eng. Sci.*, vol. 12, pp. 294–301, 1972.

[9] Bowersox , E. D., and Hoffman, D. A.,"Evaluation Of Methodoligies Utilized to Determine the Pressure Drop Throughout an Injection Mold", *American Injection Molding Institute Background & Theory*, pp. 1122–1129, 2016.

[10] Arora, J., *Introduction to Optimum Design, Third Edition*.(2011).

[11] Park, G. J., Lee, T. H., Lee, K. H., and Hwang, K. H., "Robust Design: An Overview," *AIAA J.*, vol. 44, pp. 181–191, 2006.

[12] Du, X., Sudjianto, A., and Chen, W., "An Integrated Framework for Optimization Under Uncertainty Using Inverse Reliability Strategy," *J. Mech. Des.*, vol. 126, pp. 562-570, 2004.

[13] Zaman, K., and Mahadevan, S., "Robustness-Based Design Optimization of Multidisciplinary System Under Epistemic Uncertainty," *AIAA J.*, vol. 51, pp. 1021–1031, 2013.

[14] Mavrotas, G., "Effective Implementation of the ε-constraint Method in Multi-Objective Mathematical Programming Problems," *Appl. Math. Comput.*, vol. 213, pp. 455–465, 2009.

[15] Halevy, I., Kava, Z, and Seeman, T. "Normalization and Other Topics in Multi - Objective Optimization," *Optimization*, vol. 2, pp. 89–101, 2006.

[16] Mullins, J., Ling, Y., Mahadevan, S., Sun, L., and Strachan, A., "Separation of Aleatory and Epistemic Uncertainty in Probabilistic Model Validation," *Reliab. Eng. Syst. Saf.*, vol. 147, pp. 49–59, 2016.

[17] Jansen, K. M. B., "Heat Transfer in Injection Moulding Systems with Insulation Layers And Heating Elements," *Int. J. Heat Mass Transf.*, vol. 38, no. 2, pp. 309–316, 1995.

[18] Rosato D. V., and Rosato, M. G, *Injection molding handbook*., (2000).

[19] Madan, J., Mani, M., and Lyons, K. W., "Characterizing Energy Consumption of the Injection Molding Process," *Syst. Micro Nano Technol. Sustain. Manuf.*, vol. 2 pp. V002T04A015, 2013.

[20] Weissman, A., Gupta, S. K., Ananthanarayanan, A., and Sriram R. D., "A Systematic Methodology for Accurate Design-Stage Estimation of Energy Consumption for Injection Molded Parts," *Proc. ASME Int. Des. Eng. Tech. Conf. Comput. Inf. Eng. Conf.*, pp. 1–13, 2010.

[21] Nannapaneni, S., and Mahadevan, S., "Uncertainty Quantification In Performance Evaluation of Manufacturing Processes," *in 2014 IEEE Int. Conf. Big Data (Big Data)*, pp. 996–1005, (2014).

[22] Vlachopoulos, J., and Strutt, D.,*The Role of Rheology in Polymer Extrusion, Department of Chemical Engineering, (2010).*

[23] MATLAB and Statistics Toolbox Release 2016, The MathWorks, Inc., Natick, Massachusetts, United States.