

SECTION – A

There are **FIVE** questions in this section. Answer any **FOUR**.

Assume reasonable value for any missing data.

1. A cantilever beam 2 metre in length supports a point load at the free end inclined to the vertical axis as shown in Fig. 1. Determine the flexural stresses at the fixed end of the beam and hence locate the neutral axis at that section. (26 ¼)

2. (a) Given the state of stress in Fig. 2, use Mohr's Circle of stress to determine (i) the principal stresses and the corresponding planes on which they act, (ii) maximum in-plane shearing stress along with associated normal stress. Show the result for both cases on properly oriented elements. (16)
 (b) Rework the problem using stress transformation formulae. (10 ¼)

3. An overhanging beam is loaded as shown in Fig. 3. Find the elastic curve equation for portions of the beam AD and DB. Determine the maximum deflection of the beam between supports AB and compare it with the deflection at midspan i.e 2 m from the supports. Given: $w_0 = 2 \text{ kN/m}$, $EI = 2055 \text{ kN-m}^2$. (26 ¼)

4. (a) A $150 \times 100 \times 10 \text{ mm}$ angle is welded to a gusset plate with its longer leg attached to the gusset as shown in Fig. 4. The weld length L_3 covers the entire end of the angle. Determine the lengths of side fillet welds required at the heel and toe of the angle for a non-eccentric connection. Fillet side = 8 mm, $\tau_{\text{allow}} = 145 \text{ MPa}$. (16)
 (b) Recalculate the lengths of these two welds (L_1 & L_2) assuming that the end weld is not possible due to congestion at the joint. (10 ¼)

5. (a) A fixed beam is loaded with concentrated load as shown in Fig. 5. Using area-moment method, determine the reactions and draw shear force and bending moment diagrams. (14)
 (b) For the beam shown in Fig. 5 derive the elastic curve and rotation equations for left hand section i.e AC portion of the beam. Hence, find the maximum deflection and its location in this segment. (12 ¼)

CE 213**SECTION - B**

There are **FIVE** questions in this section. Answer any **FOUR**.

6. (a) Derive and state general cable theorem. (10 ¼)
 (b) Draw shear force and bending moment diagrams for the stiffening girders of the suspension bridge shown in Fig. 6. (16)

7. Describe the necessity of theories of failure. Name at least six yielding theories of failure. Describe maximum shear stress theory of failure. (26 ¼)

8. (a) Using AISC ASD column formulas, select the lightest W shape for a 20 ft long, pin-ended column to carry a concentric concentrated load of 230 kips. Given $\sigma_{yp} = 36$ ksi.

For long columns when $(L_e/r) > C_c$ is

(16 ¼)

$$\sigma_{allow} = \frac{12\pi^2 E}{23(L_e/r)^2}$$

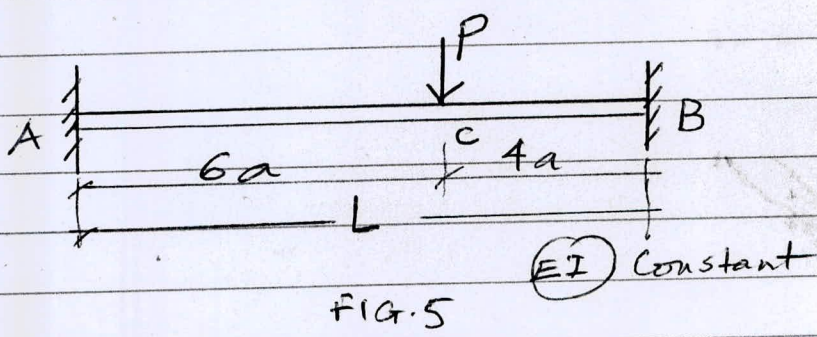
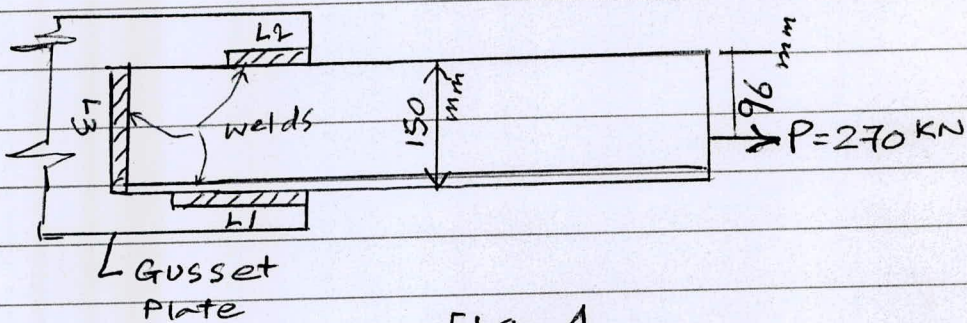
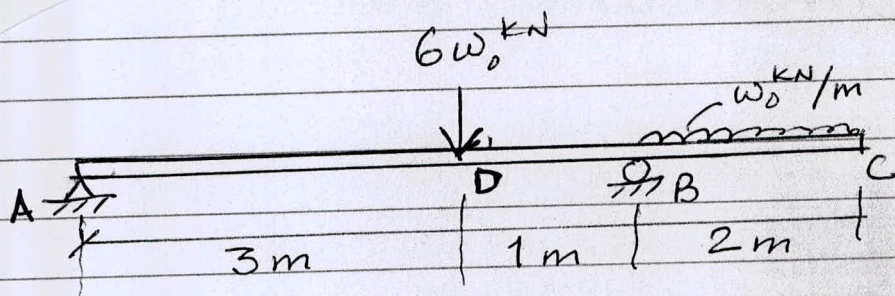
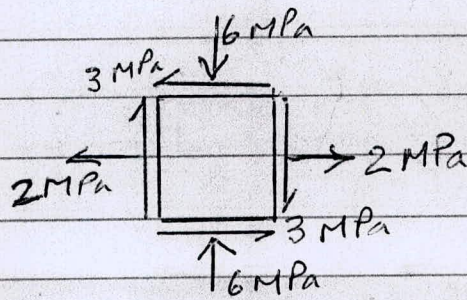
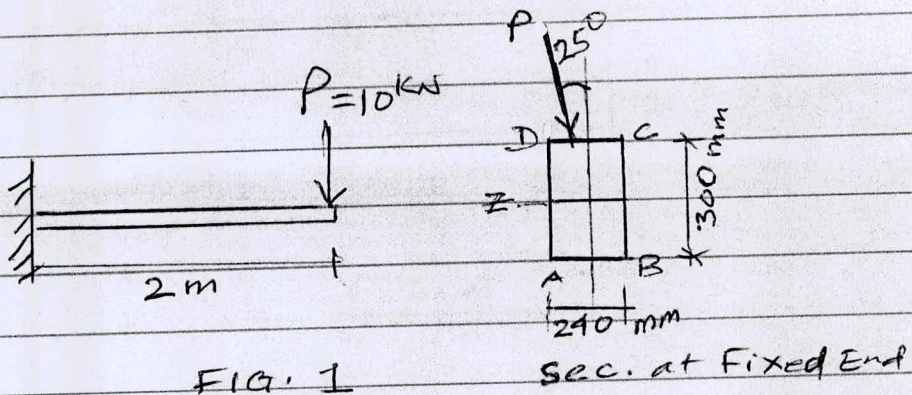
For an L_e/r ratio less than C_c AISC specifies a parabolic formula:

$$\sigma_{allow} = \frac{[1 - (L_e/r)^2 / 2C_c^2] \sigma_{yp}}{F.S.}$$

where F.S., the factor of safety, is defined as

$$F.S. = \frac{5}{3} + \frac{3(L_e/r)}{8C_c} - \frac{(L_e/r)^3}{8C_c^3}$$

- (b) Using AISC LRFD column formulas, determine the design compressive strength P_u for the steel column as designed in question 8(a). (10)
9. (a) Using energy method, derive equation for total strain energy absorbed due to axial load and calculate the deflection of point B of the structure in Fig. 7. Given that $E = 29000$ ksi. (13 ¼)
 (b) Using energy method, derive equation for total strain energy absorbed due to bending and calculate the deflection of point A of the structure in Fig. 8. Given that size of the beam is $6" \times 8"$ and $E = 29000$ ksi. (13)
10. (a) A load of 36 kips acts on the repeating section of the triple-row riveted butt joint in Fig. 9. The length of section is 8 in., diameter of rivet hole is $15/16$ in., thickness of main plate is $1/2$ in., and of each cover plate is $3/8$ in. Determine the shearing, bearing, and tensile stresses developed in the joint. (16 ¼)
 (b) A plate is lapped over and welded to a gusset plate as shown in Fig. 10. Determine the size of fillet welds to be specified using allowable shear stress 21 ksi through the throats of the welds. (10)



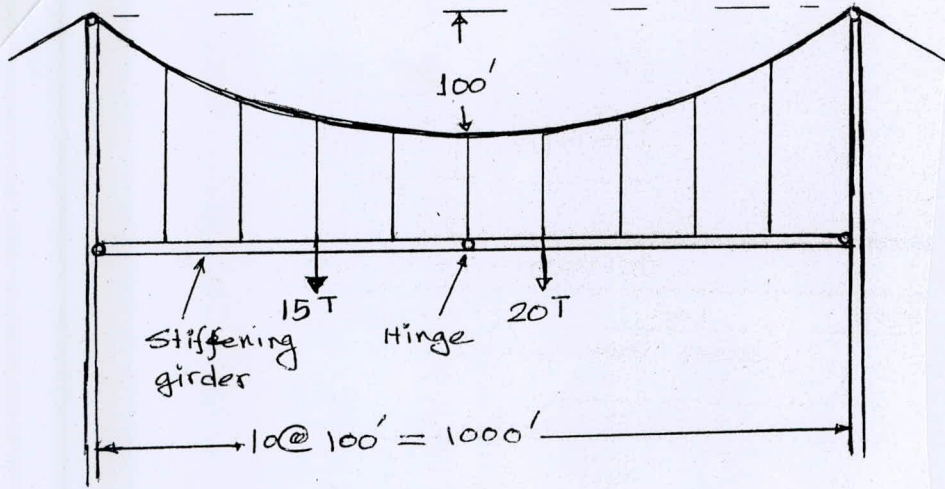


Fig-6

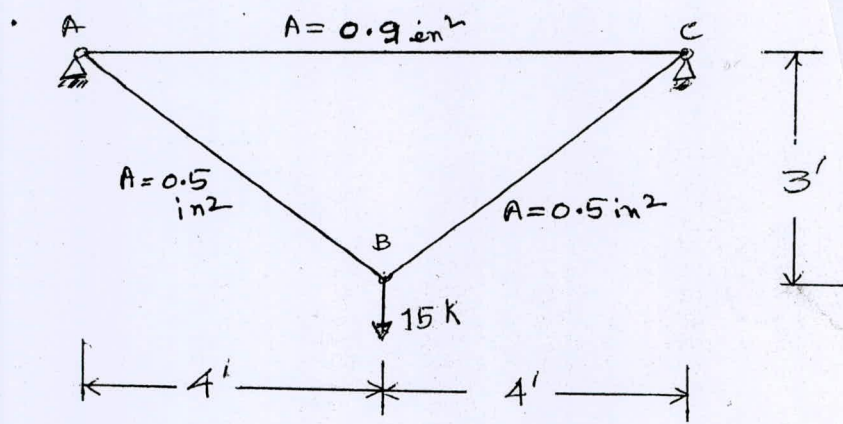


Fig-7

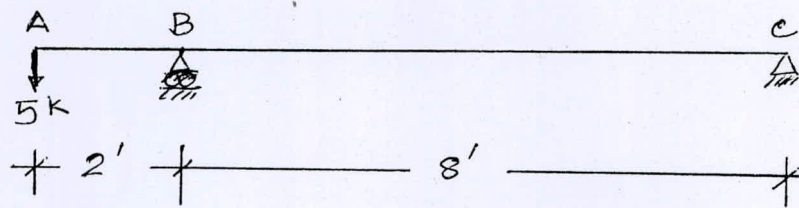


Fig-8

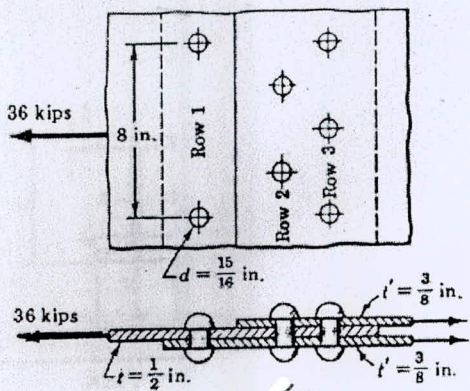


Fig-9

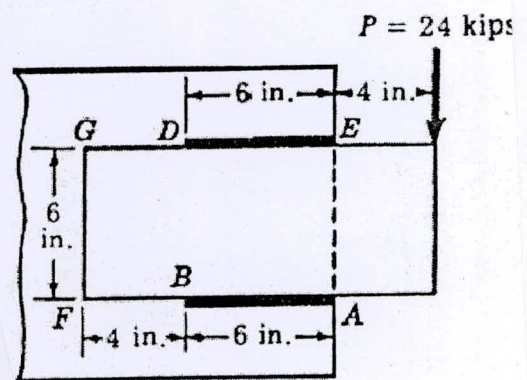
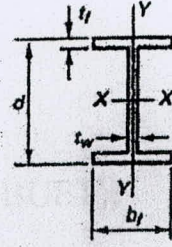


Fig-10

Table-1



**TABLE 4A. AMERICAN STANDARD STEEL W SHAPES DIMENSIONS AND PROPERTIES
U.S. CUSTOMARY UNITS (ABRIDGED LIST)**

Designation ^a	Area A	Depth d	Web Thickness t _w	Flange		Axis X-X		Axis Y-Y	
				Width b _f	Thickness t _f	I _x	r _x	I _y	r _y
in × lb/ft	in ²	in	in	in	in	in ⁴	in	in ⁴	in
W36 × 245	72.1	36.08	0.800	16.510	1.350	16100	15.0	1010	3.75
230	67.6	35.90	0.760	16.470	1.260	15000	14.9	940	3.73
150	44.2	35.85	0.625	11.975	0.940	9040	14.3	270	2.47
135	39.7	35.55	0.600	11.950	0.790	7800	14.0	225	2.38
W33 × 201	59.1	33.68	0.715	15.745	1.150	11500	14.0	749	3.56
130	38.3	33.09	0.580	11.510	0.855	6710	13.2	218	2.39
118	34.7	32.86	0.550	11.480	0.740	5900	13.0	187	2.32
W30 × 191	56.1	30.68	0.710	15.040	1.185	9170	12.8	673	3.46
173	50.8	30.44	0.655	14.985	1.065	8200	12.7	598	3.43
W27 × 161	47.4	27.59	0.660	14.020	1.080	6280	11.5	497	3.24
146	42.9	27.38	0.605	13.965	0.975	5630	11.4	443	3.21
94	27.7	26.92	0.490	9.990	0.745	3270	10.9	124	2.12
84	24.8	26.71	0.460	9.960	0.640	2850	10.7	106	2.07
W18 × 60	17.6	18.24	0.415	7.555	0.695	984	7.47	50.1	1.69
50	14.7	17.99	0.355	7.495	0.570	800	7.38	40.1	1.65
46	13.5	18.06	0.360	6.060	0.605	712	7.25	22.5	1.29
35	10.3	17.70	0.300	6.000	0.425	510	7.04	15.3	1.22
W16 × 26	7.68	15.69	0.250	5.500	0.345	301	6.26	9.59	1.12
W14 × 193	56.8	15.48	0.890	15.710	1.440	2400	6.50	931	4.05
159	46.7	14.98	0.745	15.565	1.190	1900	6.38	748	4.00
99	29.1	14.16	0.485	14.565	0.780	1110	6.17	402	3.71
90	26.5	14.02	0.440	14.520	0.710	999	6.14	362	3.70
W12 × 72	21.1	12.25	0.430	12.040	0.670	597	5.31	195	3.04
65	19.1	12.12	0.390	12.000	0.605	533	5.28	174	3.02
50	14.7	12.19	0.370	8.080	0.640	394	5.18	56.3	1.96
45	13.2	12.06	0.335	8.045	0.575	350	5.15	50.0	1.94
40	11.8	11.94	0.295	8.005	0.515	310	5.13	44.1	1.93
W10 × 112	32.9	11.36	0.755	10.415	1.250	716	4.66	236	2.68
60	17.6	10.22	0.420	10.080	0.680	341	4.39	116	2.57
49	14.4	9.98	0.340	10.000	0.560	272	4.35	93.4	2.54
45	13.3	10.10	0.350	8.020	0.620	248	4.33	53.4	2.01
39	11.5	9.92	0.315	7.985	0.530	209	4.27	45.0	1.98
33	9.71	9.73	0.290	7.960	0.435	170	4.19	36.6	1.94
W8 × 67	19.7	9.00	0.570	8.280	0.935	272	3.72	88.6	2.12
58	17.1	8.75	0.510	8.220	0.810	228	3.65	75.1	2.10
40	11.7	8.25	0.360	8.070	0.560	146	3.53	49.1	2.04
31	9.13	8.00	0.285	7.995	0.435	110	3.47	37.1	2.02
28	8.25	8.06	0.285	6.535	0.465	98.0	3.45	21.7	1.62
24	7.08	7.93	0.245	6.495	0.400	82.8	3.42	18.3	1.61
21	6.16	8.28	0.250	5.270	0.400	75.3	3.49	9.77	1.26
18	5.26	8.14	0.230	5.250	0.330	61.9	3.43	7.97	1.23

American standard wide-flange shapes are designated by the letter W followed by the nominal depth in inches with the weight in pounds per linear foot given last.

SECTION - AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Explain the following terminologies (9)
- (i) Back Substitution
 - (ii) Pivotal Condensation
 - (iii) Upper Triangular Matrix

- (b) Solve the following using Romberg's Quadrature. (14 $\frac{1}{3}$)

$$I = \int_1^3 \frac{1}{2x+3} dx$$

2. (a) Explain Gauss-Jordan method analytically. (10 $\frac{1}{3}$)
- (b) (13)

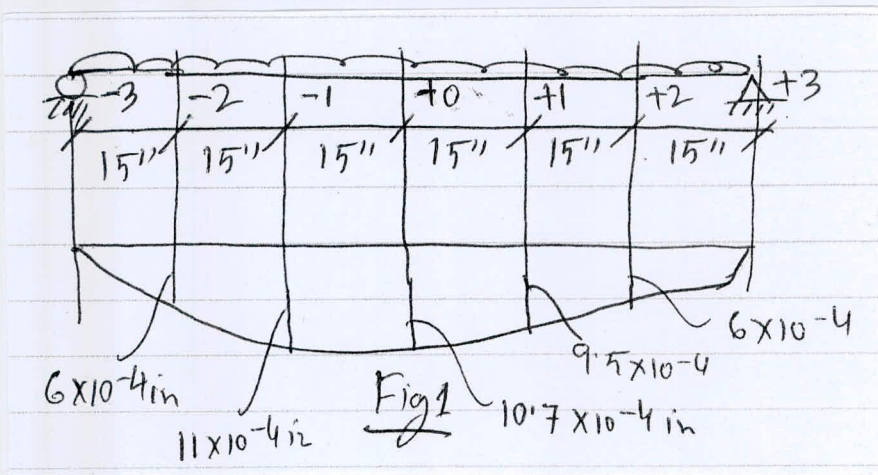


Figure 1 shows a simply-supported beam and its deflected shape. Given: $E = 30 \times 10^6$ psi, $I = 1000 \text{ in}^4$.

Find Moments and shear forces at points -1 , -2 , $+1$ and $+2$.

3. (a) Explain the method of Chio analytically for evaluating a determinant. (8 $\frac{1}{3}$)
- (b) Table 1 provides x (degree) and corresponding, $\text{Sin } x$ values. Find $\text{Sin } 24^\circ$ using: (15)
- (i) Gregory-Newton Interpolation (Do not use difference Table)
 - (ii) Lagrangian Interpolation.

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Table 1

x(degree)	Sinx
0	0.00000
10	0.17365
20	0.34202
30	0.50000
40	0.64279

4. (a) Explain the Difference Table analytically. (8/3)

(b) Approximate the following definite integrals by using Gauss quadrature with n = 3. (15)

(i) $\int_{-1}^1 x^2 \sin x \, dx$

(ii) $\int_0^3 x^2 \sin x \, dx$

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) The following expression describes the fall velocity (v) of a bungee jumper as a function of time (t): (15)

$$v(t) = \sqrt{\frac{gm}{c_d}} \tanh \left[\left(\sqrt{\frac{gc_d}{m}} \right) t \right]$$

where, m = mass of bungee jumper, c_d = drag coefficient, g = 9.81 m/s²

- (i) Use bisection method to determine the drag coefficient needed so that an 80-kg bungee jumper has a velocity of 36 m/s after 4 sec free fall. Start with initial guesses 0.1 and 0.2 and iterate until the approximate relative error falls below 2%
- (ii) How many bisection iterations would it require to determine the drag coefficient to an absolute error of 0.001 with your initial guess range?

(b) What do you mean by total numerical error? Provide a graphical depiction of the trade-off between roundoff and truncation error. What is the difference between Dirichlet and Neumann boundary conditions? (8/3)

6. (a) Three disease carrying organisms decay exponentially in seawater according to the following equation: (14)

$$p(t) = Ae^{-1.5t} + Be^{-0.3t} + Ce^{-0.05t}$$

Use multiple linear regression to estimate the initial concentration of each organism (A, B, C) given the following measurements:

t	0.5	1	2	3	4	5	6	7	9
p(t)	6	4.4	3.2	2.7	2	1.9	1.7	1.4	1.1

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(b) Use Newton-Raphson method to find the root of $f(x) = 7e^{-x} \sin(x)$ with an initial guess of $x = 0.3$. Perform 4 iterations and determine the approximate error in each iteration.

(9 1/3)

7. (a) The parabolic equation: $y = 0.17x^2 + 0.48x + 2.96$ has been derived after performing least-square regression on the following dataset:

(10)

x	1	3	5	7	9	11
y	4	5.5	13	14	18	31

Determine the R^2 value of the fit.

(b) The velocity (v) and distance travelled (x) by a bungee jumper can be expressed by the following system of differential equation:

(13 1/3)

$$\frac{dx}{dt} = 0$$

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

where, m = mass of bungee jumper = 68.1 kg, c_d = drag coefficient = 0.25, $g = 9.81$ m/s²

- (i) Assuming that at $t = 0$, $x = v = 0$, solve for the velocity and position upto $t = 10$ s. Use Euler's method with a step-size of 2s.
- (ii) Compute the true percent relative error for velocity for each time step if the analytical solution for velocity is the same as that provided for question 5(a).

8. (a) The temperature distribution of a long, thin rod of length 10 cm is described by the equation:

(10)

$$k \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

where, $k = 0.835$ cm²/sec. Write down the simultaneous equations (in matrix form) to solve the temperature distribution of the rod at $t = 0.1$ sec (implicit method) given the following:

$$\Delta x = 2 \text{ cm}, \Delta t = 0.1 \text{ sec},$$

initial condition: the temperature of the rod is 10°C at $t = 0$

boundary condition: $T(0) = 50^\circ\text{C}$ and $\left. \frac{dT}{dx} \right|_{x=10} = 0$ at all times.

(b) Solve the same problem for $t = 0.1$ sec using 'explicit' method.

(8)

(c) What is the convergence criteria in fixed point iteration? Give examples (with diagrams) of convergence and divergence in fixed point iteration.

(5 1/3)

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Explain the condition of the consumer equilibrium for a single commodity under the cardinal approach to utility analysis and mathematically derive the condition. (8)
 (b) How would you draw the demand curve of the consumer based on the axiom of diminishing marginal utility? (8)
 (c) Briefly discuss the main challenges that every economy primarily struggles to overcome. (7 1/3)

2. (a) Define market demand. How would you draw a market demand curve of a commodity? Explain how the interactions between market demand and market supply curves determine equilibrium price and output of a commodity. (13 1/3)
 (b) The demand and supply functions of a commodity (say, X) are given respectively (10)

$$Q_D = 1550 - 75P_x \text{ and}$$

$$Q_s = 680 + 25P_x$$
 where, P, is the price of the commodity. Find the equilibrium price and quantity of the commodity X. If the Government provides 21% subsidy on unit price, what would be the new equilibrium price and quantity? What is the proportion of the subsidy that the consumers would actually enjoy.

3. (a) Define substitution effect and income effect of a price change. Graphically show the amount of substitution effect and income effect of a price rise for a normal commodity. (13 1/3)
 (b) Illustrate the concept of marginal rate of substitution (MRS) when two commodities are consumed. Describe the relationship between MRS and marginal utility with the numerical examples. (10)

4. Write short notes on any THREE of the following: (23 1/3)
 (a) 'Change in supply' and 'change in quantity supplied'
 (b) Importance of cross elasticity of demand
 (c) Factors that govern the price elasticity of demand in general
 (d) Optimal consumption point of a consumer.

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SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) A manufacturer has a fixed cost of \$50,000 and a variable cost of \$2.60 per unit made and sold. Selling price is \$3 per unit. **(10)**
- (i) Find the revenue, cost and profit functions using q for the number of units.
 - (ii) Compute profit if 250000 units are made and sold.
 - (iii) Find the break-even quantity.
 - (iv) Construct the break-even chart. Label the cost and revenue lines, the fixed cost line, and the break-even point.

- (b) Complete the following table and sketch the graph explaining the relations among the various short run cost curves. **(13 1/3)**

Quantity of output	Total fixed cost	Total variable cost	Total cost	Average fixed cost	Average variable cost	Average Total cost	Marginal cost
1	100	50					
2	100	60					
3	100	65					
4	100	75					
5	100	95					
6	100	140					

6. (a) What are the assumptions of a perfectly competitive market? Explain in brief. **(10)**
- (b) Graphically explain the long run equilibrium of a firm under perfect competition. **(13 1/3)**

7. (a) What do you understand by monopoly? When does a firm emerge as a monopolist? **(10)**
- (b) Explain the short run equilibrium of a firm under monopoly. **(13 1/3)**

8. (a) Given that, $C = 100 + 0.80 Y_d$ **(10)**
- $I = 150, G = 200, X = 70, M = 150, TR = 200, T = 0.15Y$
- (i) Calculate the equilibrium level of income and multiplier in this model.
 - (ii) If tax rate is increased to 20%, what will be the new equilibrium level of income and multiplier?
 - (iii) What will happen to the equilibrium level of income if investment is increased to 500?

- (b) What are the problems of measuring national income in a developing country like Bangladesh? **(13 1/3)**

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Derive the expression for friction loss in pipes for Laminar flow. (10)
 (b) Determine the diameter of steel pipe ($e = 0.045$ mm) to carry 30 l/s of water if the permissible head loss per meter of the pipe length is 0.05 m. Use Moody diagram. Take $\nu = 1 \times 10^{-6}$ m²/s. (10)
 (c) A pipe 60 m long and 15 cm in diameter is connected to a water tank at one end and flows freely into the atmosphere at other end. The height of the water level in the tank is 2.8 m above the center of the pipe. The pipe is horizontal with $f = 0.04$. Determine the discharge through the pipe. (10)
 (d) Why water rises in capillary glass tube and mercury depress below the glass tube. (5)

2. (a) Derive Newton's equation of viscosity. Explain why the viscosity of all liquids decrease and that of all gases increase with temperature. (10)
 (b) A cubical block weighing 200 gms and having a 20 cm edge is allowed to slide down on an inclined plane surface making an angle of 20° with the horizontal on which there is a thin film having a viscosity of 2.2×10^{-3} N-s/m². What terminal velocity will be attained if the film thickness is 0.025 mm. (10)
 (c) For the system shown in Figure 1, pipe dimensions are as follows (10)

$L_1 = 300$ m	$L_2 = 350$ m	$L_3 = 320$ m
$D_1 = 0.6$ m	$D_2 = 0.4$ m	$D_3 = 0.8$ m
$f_1 = 0.021$	$f_2 = 0.018$	$f_3 = 0.019$

- Find Q considering minor losses.
- (d) Differentiate between (5)
 - (i) Adhesion and cohesion
 - (ii) Ideal and real fluid.

3. (a) Prove that for a constant rate of discharge and a constant value of f , the frictional head loss in a pipe varies inversely as the fifth power of the diameter. (5)

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(b) Explain the relative advantages and disadvantages of piezometer and double column manometers. (5)

(c) For the pipes connected in parallel as shown in Figure 2, the pipes dimensions and friction factors are as follows: (10)

$L_1 = 900 \text{ m}$	$D_1 = 0.3 \text{ m}$	$f_1 = 0.021$
$L_2 = 600 \text{ m}$	$D_2 = 0.2 \text{ m}$	$f_2 = 0.018$
$L_3 = 1200 \text{ m}$	$D_3 = 0.4 \text{ m}$	$f_3 = 0.019$

For a total discharge of $0.6 \text{ m}^3/\text{s}$, determine the flow through each pipe and head loss from A to B.

(d) Initial distribution of flows through a pipe network is shown in Figure 3. Taking $n = 2$ for all pipes, obtain flows in each pipe. Discharge is in l/s . Apply correction twice. (15)

4. (a) The head loss in 60 m of 15 cm diameter pipe is known to be 8 m when oil ($s = 0.9$) of viscosity $0.04 \text{ N}\cdot\text{s}/\text{m}^2$ flows at $0.06 \text{ m}^2/\text{s}$. Determine the centerline velocity, the shear stress at the wall of the pipe, and the velocity at 5 cm from the centerline and the friction factor. (10)

(b) Find the total pressure force acting on the gate per meter length, which is quadrant of a circle of radius 2 m. At what angle will it be acting to the horizontal? Prove that the resultant force passes through the hinge C. See figure 4. (10)

(c) Write short notes on (15)

- (i) Viscous sublayer
- (ii) Solid and fluid
- (iii) Uniform and Nonuniform flow
- (iv) Compressible and Incompressible fluid
- (v) Center of pressure and center of gravity.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

Assume any reasonable value if necessary.

5. (a) Sketch typical velocity profiles for ideal and real fluid flowing in a circular diameter pipe. (4)

(b) Distinguish path line, stream line and streak line with necessary sketches. (6)

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(c) Velocity along the centerline of a 1 m long uniformly diverging conical pipe is expressed as

(13)

$$V_x = \frac{Q}{0.00785(1+x)^2}$$

Where Q , is discharge through the pipe in m^3/s and x is distance along the pipe length in meter. Determine local, convective and total acceleration at the mid-section of the pipe if rate of flow is $0.12 m^3/s$ and it remains constant.

(d) Three pipes steadily deliver water to a large exit pipe as shown in figure (5). The velocity $V_2 = 5m/s$, and the exit flow rate is $Q_4 = 120 m^3/hour$. Find the velocity V_1, V_3 and V_4 . It is known that, by increasing Q_3 by 20% would increase Q_4 by 10%.

(12)

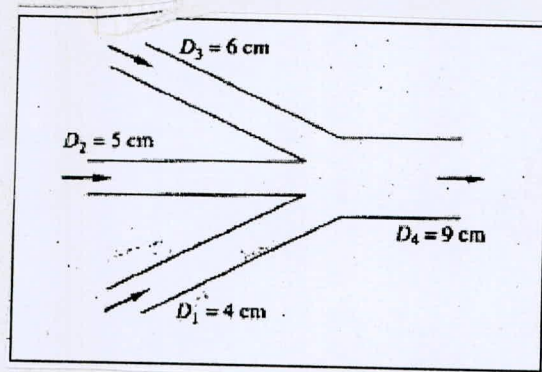


Figure (5)

6. (a) What is hydraulic gradient? Derive the equation for steady motion along a streamline for real fluid.

(10)

(b) Figure (6) shows a pump employed for lifting water from a sump. If it is required to pump 60 liters/sec of water through a 0.1 m diameter pipe from the sump to a point 10 m above, determine the power required. Also determine pressure intensities at L and M. Assume an overall efficiency of pump is 70%.

(15)

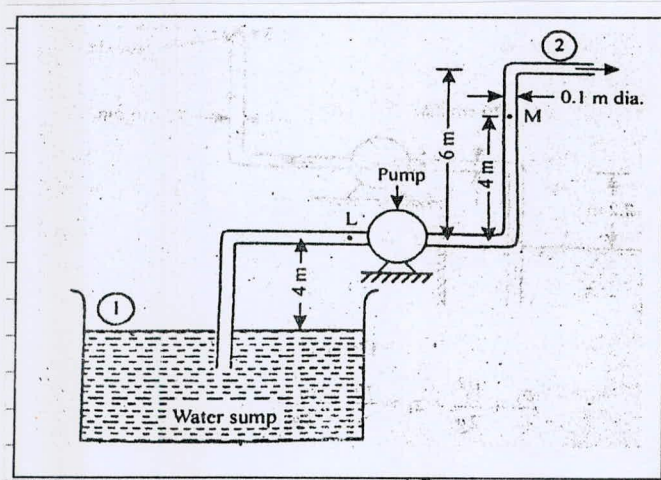


Figure (6)

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Contd... Q. No. 6

(c) Define uniform flow in a pipe. Water flows over a spillway in Figure (7). The velocity is uniform at a section 1 and 2 where the pressure is approximately hydrostatic. Neglecting losses determine discharge and velocity at the upstream (v_1) and downstream (v_2) of the spillway. Assume unit width. (10)

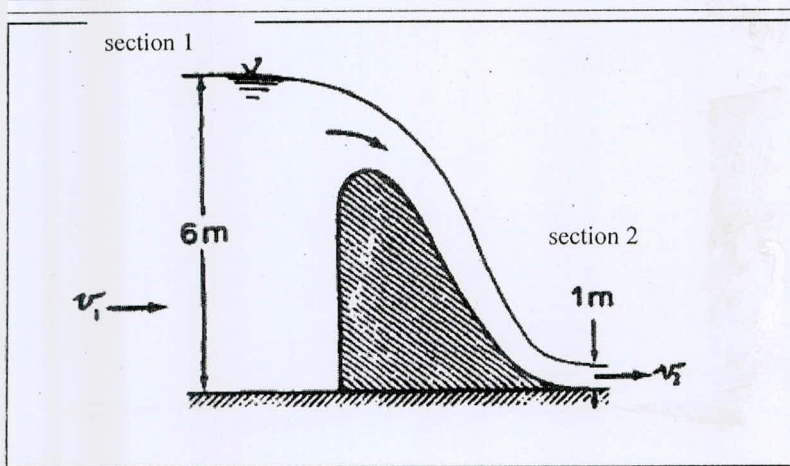


Figure (7)

7. (a) What is cavitation? How cavitation produce drop in efficiency of machine or propeller contact with water and what could be done to avoid this problem. (8)

(b) For low speed laminar flow in circular pipe as shown in the Figure (8), the velocity distribution takes the form $u = (\gamma/\mu)(r_0^2 - r^2)$, where μ is the fluid viscosity.

Determine (13)

- (i) Maximum velocity in terms of γ , μ and r_0 and
- (ii) The mass flow rate in kg/s if and 40 mm diameter of the pipe is carrying fluid a of density 998 kg/m^3 whose centerline maximum velocity is 2.07 m/s.

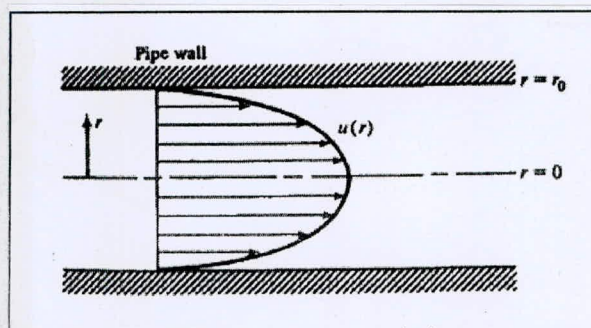


Figure (8)

WRE 211/CE

Contd... Q. No. 7

(c) Determine the value of normal force (F) when a jet of water of 4 cm diameter at a velocity of 2 m/s strikes a stationary flat plate as shown in Figure (9). (7)

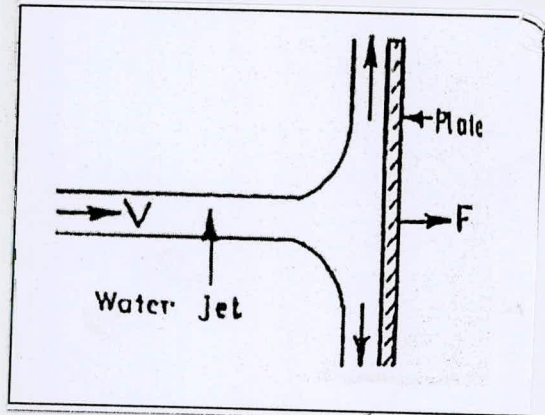


Figure (9)

(d) Derive the equation of resultant force exerted by fluid on a reducing pipe bend. (7)

8. (a) A jet of water having a velocity of 10 m/s and flow rate of 60 litres/s is deflected through a right angled chute as shown in Figure (10). Determine the magnitude and direction of resultant reaction force in chute. Neglect the friction of the chute. (12)

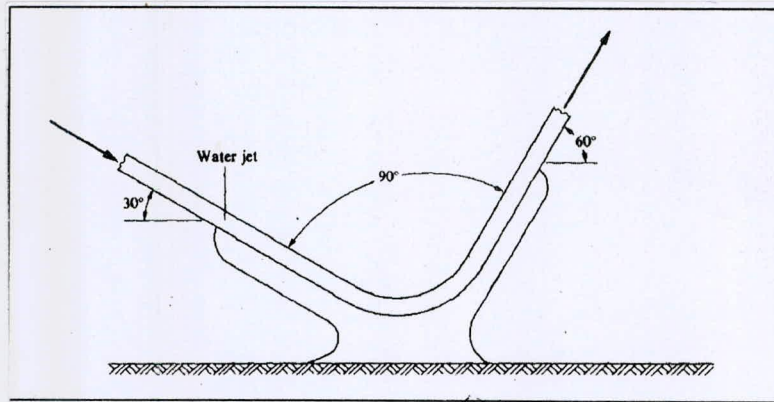


Figure (10)

(b) Derive equation for momentum correction factor and state the significance of it for real fluid. (6)

(c) A centrifugal pump impeller has $r_1 = 0.3$ m, $r_2 = 1$ m, $\beta_1 = 120^\circ$, $\beta_2 = 135^\circ$ and the thickness of 0.1 m parallel to the axis of rotation. If it delivers water at $2 \text{ m}^3/\text{s}$ with no tangential velocity component at the entrance (i.e radial inflow pump) what is the rotational speed? For this, condition calculate- (i) torque; (ii) power of the machine and (iii) energy given to each unit weight of water. (17)

$= 6 =$

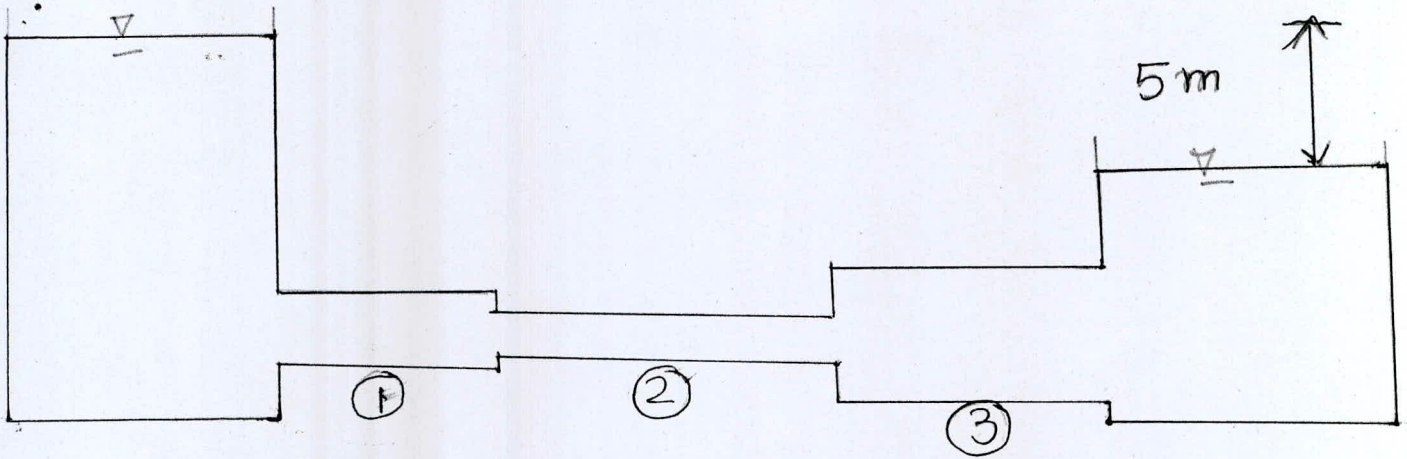


Figure 1 for Question 2(c)

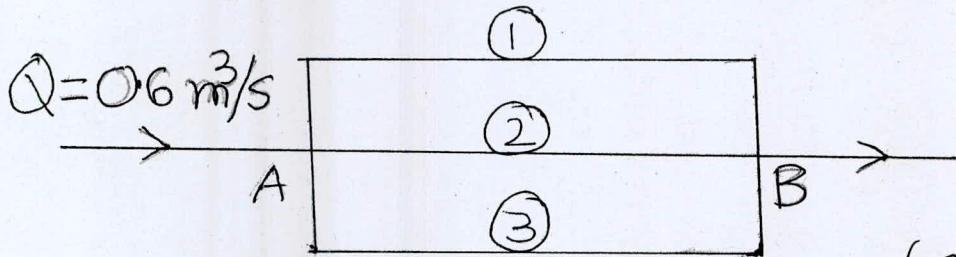


Figure 2 for Question 3(c)

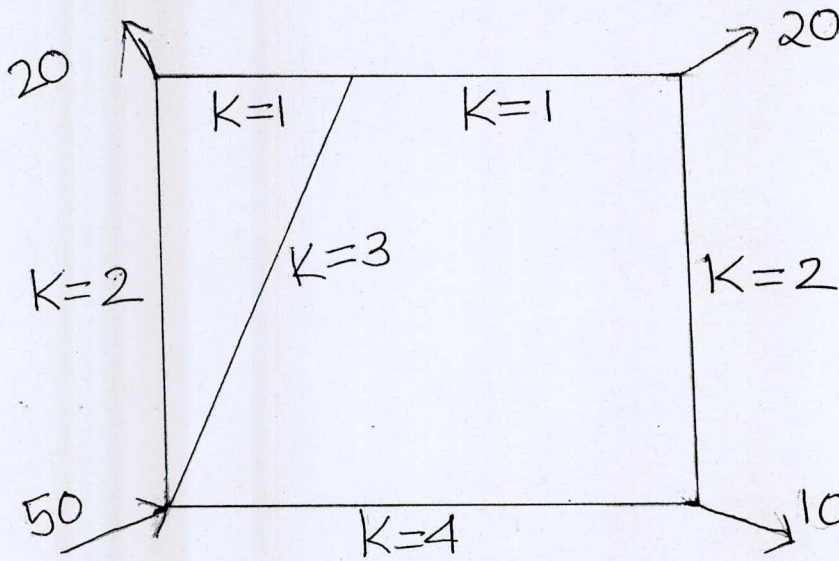


Figure 3 for Question 3(c)

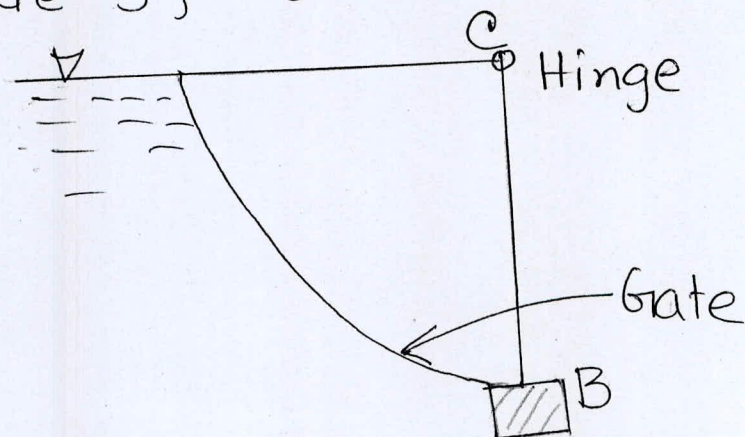


Figure 4 for Question 4(b)

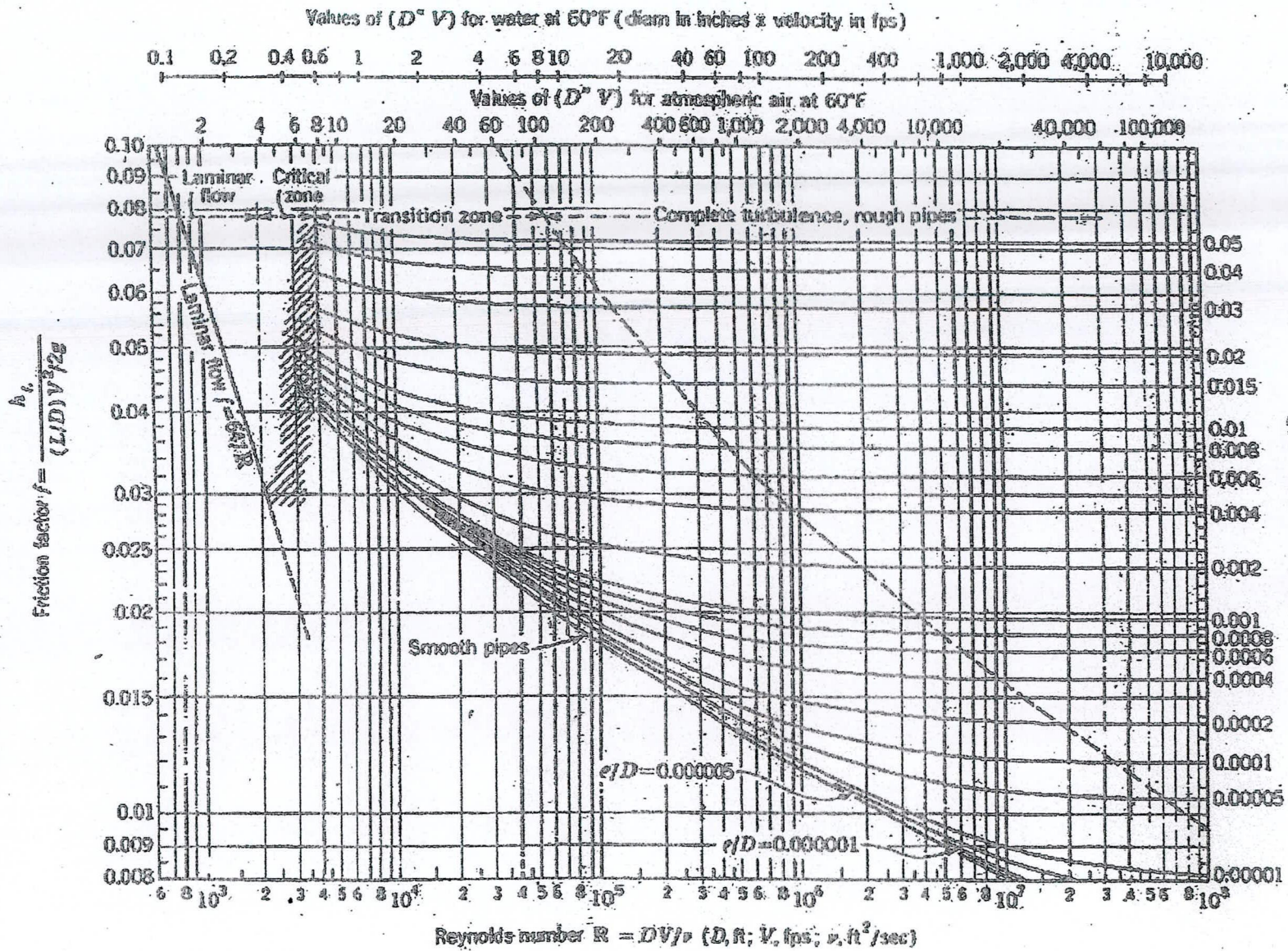


Figure: Moody Diagram for friction factor.

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2016-2017

Sub : **CE 207** (Applied Mathematics of Engineers)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

Assume reasonable value of missing data only if necessary.

Symbols have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What do you mean by control charts of a manufacturing process? Show that there is a chance of 0.0027 that a subgroup average may fall outside the control limits. (10)
- (b) Derive expressions for A and B using sample data for linear regression equation $Y = A + BX$ (9)
- (c) Write short notes on (Answer any four): (4×4=16)
- (i) Inferential Statistics
 - (ii) Hypothesis Testing
 - (iii) Total Probability Theorem
 - (iv) Conditional Probability and Statistical Independence
 - (v) Standard Normal Distribution
2. (a) There are 10 boxes containing green balls, 20 boxes containing red balls and 20 boxes containing yellow balls. The probability of green balls, red balls and yellow balls being defective is 0.0, 0.1 and 0.2 respectively. A box is chosen at random and two balls are tested and found to be satisfactory (i.e., no defect). What is the probability that the box contains (i) green balls (ii) red balls and (iii) yellow balls? (13)

Apply Bayes Theorem:
$$P(C_j|S) = \frac{P(C_j) \times P(S|C_j)}{\sum P(C_j) \times P(S|C_j)}$$

Event S: Two balls tested are satisfactory

Event C_j : Box chosen contains balls of color C_j

= 2 =

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Contd ... Q. No. 2

(b) The continuous random variable X has probability density function given by

$$f(x) = k(2-x)(x-5) \text{ for } 2 \leq x \leq 5$$
$$= 0 \quad \text{otherwise}$$

Determine: (i) k

(ii) Mean of X

(iii) Variance of X

Note $\text{var}(X) = E(X^2) - \{E(X)\}^2$

(c) The probability, that a disk is defective is 0.05. Find the probability that a pack of 25 disks will contain two or more defective disks.

(i) Apply Poisson's distribution,

(ii) Apply Binomial distribution

3. (a) Bolts are manufactured for a specified length of 10 cm. It is known from past experience that the variance of the length of such bolts is 0.05 cm^2 . A random sample of 12 bolts are collected from a large number of production bolts. Their lengths are found to be (in cm):

10.68	10.13	10.62	10.51	10.36	10.52
10.29	10.77	10.45	10.39	10.12	10.25

(i) Check the hypothesis H_0 that the bolt length is 10 cm at 5% level of significance (i.e., 95% confidence interval)

(ii) What happen if 0.1% level of significance is considered?

(b) Suppose that the variance of the length of steel bolts of Prob. 3(a) is unknown. Using the same data, perform t-test to check the hypothesis, H_0 that bolt length is 10 cm for 95% confidence interval.

(c) The time required X (in days) to fix an air conditioner has the following density function:

$$f(x) = 0.4 - 0.075(x-1) \text{ for } 1 \leq x \leq 5$$
$$= 0 \quad \text{otherwise}$$

Determine the expected cost (in Taka) for repairing the air-conditioner, where

$$Y = 1000 + 500X + 40X^2.$$

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4. (a) Identify if each of the following equations is Ordinary or Partial Differential Equation, Linear or Non-Linear, Homogenous or Non- Homogenous Equation: (18)

(i) $y' + p(x)y = q(x)$

(ii) $(-x^2)y'' - 2xy' + \lambda y = 0$

(iii) $x^2y'' + xy' + x^2y = 0$

(iv) $mx'' + kx = f(t)$

(v) $x'' + x + x^3 = 0$

(vi) $\frac{\partial^2 u}{\partial x^2} - 5\frac{\partial u}{\partial y} + u = xy^2$

- (b) A dynamical system with constant mass m and constant stiffness k is perpetually driven by arbitrary force $p(t)$. The governing differential equation is $m\frac{d^2u}{dt^2} + ku = p(t)$. (17)

Solve the equation for the arbitrary load.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) A steel bar is subjected to tension at a slow rate until failure occurs. Write: (20)

(i) a description how you observe the test.

(ii) List the variables and identify the dependent and independent variables for the associated governing laws.

- (b) What will be the changes in your observation and formulation if the same bar is subjected to an impact load? (15)

6. (a) Differentiate between Implicit Solution and Explicit solution. (5)

(b) Explain the term, “Approximation” in terms of (i) Experimental observation

(ii) Analytical solution. (iii) Numerical solution. (15)

(c) Explain the difference between an “arithmetical solution” and “engineering solution” in terms of an example. (5)

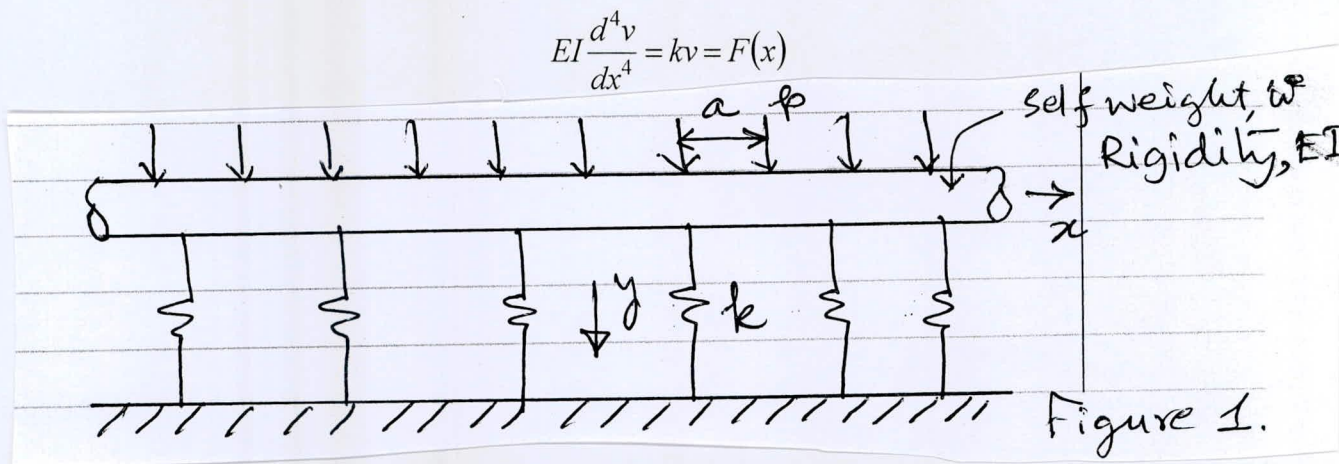
(d) Explain the following terms with example: (i) Initial value problem (ii) Orthogonality property in Legendre Polynomials. (10)

CE 207

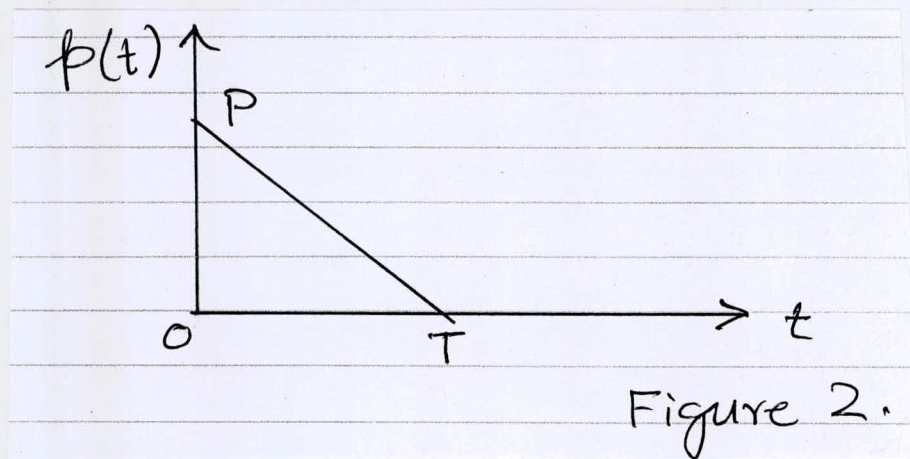
7. (a) Write down the first theorem of Frobenius Method and show that Bessel's Equation is a special case of the theorem. (10)
- (b) What is indicial equation? Derive it from Bessel's equation. (10)
- (c) Why the Bessel function of the first kind of order n converges for all x very rapidly. (5)
- (d) Apply power series method and solve the following differential equation. (10)

$$y' = 2xy$$

8. (a) An infinitely long beam weighing w per unit length with flexural rigidity EI is supported by an elastic medium of stiffness k per unit length is subjected to a very long train of concentrated loads of amplitude p as shown in Figure 1. Determine the deflection of the beam at $x = a/2$. Governing differential equation of the problem is given by the following equation: (20)



- (b) Determine Fourier Integral expression of the blast load shown in Figure 2. (15)



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

PERCENTAGE POINTS OF THE T DISTRIBUTION

Tail Probabilities									
One Tail	0.10	0.05	0.025	0.01	0.005	0.001	0.0005		
Two Tails	0.20	0.10	0.05	0.02	0.01	0.002	0.001		
D	1	3.078	6.314	12.71	31.82	63.66	318.3	637	1
E	2	1.886	2.920	4.303	6.965	9.925	22.330	31.6	2
G	3	1.638	2.353	3.182	4.541	5.841	10.210	12.92	3
R	4	1.533	2.132	2.776	3.747	4.604	7.173	8.610	4
E	5	1.476	2.015	2.571	3.365	4.032	5.893	6.869	5
E	6	1.440	1.943	2.447	3.143	3.707	5.208	5.959	6
S	7	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7
	8	1.397	1.860	2.306	2.896	3.355	4.501	5.041	8
O	9	1.383	1.833	2.262	2.821	3.250	4.297	4.781	9
F	10	1.372	1.812	2.228	2.764	3.169	4.144	4.587	10
	11	1.363	1.796	2.201	2.718	3.106	4.025	4.437	11
F	12	1.356	1.782	2.179	2.681	3.055	3.930	4.318	12
R	13	1.350	1.771	2.160	2.650	3.012	3.852	4.221	13
E	14	1.345	1.761	2.145	2.624	2.977	3.787	4.140	14
E	15	1.341	1.753	2.131	2.602	2.947	3.733	4.073	15
D	16	1.337	1.746	2.120	2.583	2.921	3.686	4.015	16
O	17	1.333	1.740	2.110	2.567	2.898	3.646	3.965	17
M	18	1.330	1.734	2.101	2.552	2.878	3.610	3.922	18
	19	1.328	1.729	2.093	2.539	2.861	3.579	3.883	19
	20	1.325	1.725	2.086	2.528	2.845	3.552	3.850	20
	21	1.323	1.721	2.080	2.518	2.831	3.527	3.819	21
	22	1.321	1.717	2.074	2.508	2.819	3.505	3.792	22
	23	1.319	1.714	2.069	2.500	2.807	3.485	3.768	23
	24	1.318	1.711	2.064	2.492	2.797	3.467	3.745	24
	25	1.316	1.708	2.060	2.485	2.787	3.450	3.725	25

Table of Fourier Transform

$f(x)$	$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x} dx$
1. $\frac{1}{x^2 + a^2} \quad (a > 0)$	$\frac{\pi}{a} e^{-a \omega }$
2. $H(x)e^{-ax} \quad (\text{Re } a > 0)$	$\frac{1}{a + i\omega}$
3. $H(-x)e^{ax} \quad (\text{Re } a > 0)$	$\frac{1}{a - i\omega}$
4. $e^{-a x } \quad (a > 0)$	$\frac{2a}{\omega^2 + a^2}$
5. e^{-x^2}	$\sqrt{\pi} e^{-\omega^2/4}$
6. $\frac{1}{2a\sqrt{\pi}} e^{-x^2/(2a)^2} \quad (a > 0)$	$e^{-a^2\omega^2}$
7. $\frac{1}{\sqrt{ x }}$	$\sqrt{\frac{2\pi}{ \omega }}$
8. $e^{-a x /\sqrt{2}} \sin\left(\frac{a}{\sqrt{2}} x + \frac{\pi}{4}\right) \quad (a > 0)$	$\frac{2a^3}{\omega^4 + a^4}$
9. $H(x+a) - H(x-a)$	$\frac{2 \sin \omega a}{\omega}$
10. $\delta(x-a)$	$e^{-i\omega a}$
11. $f(ax+b) \quad (a > 0)$	$\frac{1}{a} e^{i\omega b/a} \hat{f}\left(\frac{\omega}{a}\right)$
12. $\frac{1}{a} e^{-ibx/a} f\left(\frac{x}{a}\right) \quad (a > 0, b \text{ real})$	$\hat{f}(a\omega + b)$
13. $f(ax) \cos cx \quad (a > 0, c \text{ real})$	$\frac{1}{2a} \left[\hat{f}\left(\frac{\omega-c}{a}\right) + \hat{f}\left(\frac{\omega+c}{a}\right) \right]$
14. $f(ax) \sin cx \quad (a > 0, c \text{ real})$	$\frac{1}{2ai} \left[\hat{f}\left(\frac{\omega-c}{a}\right) - \hat{f}\left(\frac{\omega+c}{a}\right) \right]$
15. $f(x+c) + f(x-c) \quad (c \text{ real})$	$2\hat{f}(\omega) \cos \omega c$
16. $f(x+c) - f(x-c) \quad (c \text{ real})$	$2i\hat{f}(\omega) \sin \omega c$
17. $x^n f(x) \quad (n = 1, 2, \dots)$	$i^n \frac{d^n}{d\omega^n} \hat{f}(\omega)$
Linearity of transform and inverse:	
18. $\alpha f(x) + \beta g(x)$	$\alpha \hat{f}(\omega) + \beta \hat{g}(\omega)$
Transform of derivative:	
19. $f^{(n)}(x)$	$(i\omega)^n \hat{f}(\omega)$
Transform of integral:	
20. $f(x) = \int_{-\infty}^x g(\xi) d\xi,$ where $f(x) \rightarrow 0$ as $x \rightarrow \infty$	$\hat{f}(\omega) = \frac{1}{i\omega} \hat{g}(\omega)$
Fourier convolution theorem:	
21. $(f * g)(x) = \int_{-\infty}^{\infty} f(x-\xi)g(\xi) d\xi$	$\hat{f}(\omega)\hat{g}(\omega)$