

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Let z_1 and z_2 be two complex numbers such that $\bar{z}_1 + i\bar{z}_2 = 0$ and $\arg(z_1 z_2) = \pi$.
Then find $\arg(z_1)$. (12)
- (b) If $f(z) = x^2 + iy^2$, then show that $f'(z)$ exist at $z = x + ix$. (10)
- (c) Write down Cauchy Riemann equation in polar form. Test the differentiability of the function $f(z) = \sqrt{r}e^{i\frac{\theta}{2}}$, ($r > 0, \alpha < \theta < \alpha + 2\pi$) in the indicated domain and hence show that, $f'(z) = \frac{1}{2f(z)}$. (13 $\frac{2}{3}$)
- (d) Show that the transformation, $w = \frac{iz + 2}{4z + 1}$ transform the real axis in the z -plane into circle in the w -plane. Find the centre and radius of this. (11)
2. (a) Show that $u = \frac{1}{2} \ln(x^2 + y^2)$ is harmonic. Find an analytic function $f(z)$ in which $u(x, y)$ is the real part. Also express $f(z)$ in terms of z . (10)
- (b) Find the principle value of $(1 - i)^{4i}$. (10)
- (c) Solve the equation $\cos z = 2$ by equating the real and imaginary parts in the equation. (10)
- (d) Evaluate $\int_C z^2 dz$ where C is the part of the unit circle going anticlockwise from the point $z = 1$ to the point $z = i$. (16 $\frac{2}{3}$)
3. (a) Use Cauchy integral formula to evaluate $\oint_C \frac{z}{(9 - z^2)(z + i)} dz$, where C is the circle $|z| = 2$. (15)
- (b) Express $f(z) = \frac{z^2}{(z - 1)(z - 2)}$ in a Laurent series valid in the region $1 < |z| < 2$. (15 $\frac{2}{3}$)
- (c) Evaluate the integral $\oint_C \frac{\sin 3z}{(z - \frac{\pi}{4})^4} dz$ by Cauchy residue theorem, where $C = \{(x, y) : |x| \leq 2, |y| \leq 2\}$, positively oriented. (16)

MATH 263

4. Evaluate the following integral using the method of contour integration: (23+23 $\frac{2}{3}$)

(i)
$$\int_0^{2\pi} \frac{\cos 2\theta}{5-3\cos \theta} d\theta.$$

(ii)
$$\int_0^{\infty} \frac{\ln(1+x^2)}{(1+x^2)} dx.$$

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Apply Lagrange method to solve $x(y^2+z)p - y(x^2+z)q = z(x^2 - y^2)$. (15)

- (b) Find the integral surface of the first order linear partial differential equation (15)

$$(x-y)p + (y-x-z)q = z$$

which contains the circle $z=1, x^2+y^2=1$.

- (c) Use Charpit's method to find a complete integral of $(p^2+q^2)y=qz$. Hence find a singular integral (if it exists). (16 $\frac{2}{3}$)

6. Solve the following partial differential equations:

(i) $(D_x^2 - 6D_xD_y + 9D_y^2)z = 24x^2 + 72xy$, (15)

(ii) $(D_x^2 + D_xD_y - 6D_y^2)z = x^2 \sin(x+y)$, (15)

(iii) $(x^2D_x^2 - 2xyD_xD_y - 3y^2D_y^2 + xD_x - 3yD_y)z = 13x^2y \cos(\ln x^2)$. (16 $\frac{2}{3}$)

7. (a) Expand $f(x)$ in a half-range Fourier sine and cosine series, where (18 $\frac{2}{3}$)

$$f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 2-x, & 1 \leq x \leq 2. \end{cases}$$

Also, sketch the graph of the function.

- (b) Find the Fourier sine integral formula of the function $f(x) = e^{-x} \cos x$ for $x \geq 0$. (16)

- (c) Find the Fourier sine and cosine transform of $f(x) = e^{-x} \cos x$. (12)

8. (a) Solve Laplace's equation $\nabla^2 u(x, y, z) = 0$ in spherical polar coordinates (r, θ, ϕ) when u is independent of ϕ . (26 $\frac{2}{3}$)

(b) An infinitely long plane and a uniform plate is bounded by two parallel edges and an end at right angles to these. The breadth is π , the end is maintained at temperature v_0 at all points and the edge at temperature zero. Find the steady-state temperature within the plate. (20)

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols indicate their usual meaning. Assume reasonable values for any missing data.

1. (a) A rigid horizontal bar of negligible mass is connected to two rods as shown in Fig. 1(a). If the system is initially stress-free, calculate the temperature change that will cause a tensile stress of 90 MPa in the brass rod. Assume that both the rods are subjected to the same change in temperature. (15)
- (b) A circular bar is bent into the shape of a half-ring and supported in a vertical plane as shown in Fig. 1(b). Determine the horizontal movement of points *C* and *B*. Solve the problem by Castigliano's theorem. (20)
2. (a) A shaft composed of segments *AC*, *CD*, and *DB* is fastened to rigid supports and loaded as shown in Fig. 2(a). For bronze, $G = 35$ GPa; for aluminum, $G = 28$ GPa, and for steel, $G = 83$ GPa. Determine the maximum torsional shearing stress developed in segment *CD* and angle of twist of point *D* with respect to point *C*. (15)
- (b) A rigid bar of negligible weight transfers a load *P* to a combination of three helical springs arranged in parallel as shown in Fig. 2(b). Three springs are made up of the same material and the spring wires are of the same diameter. They are of same free length before loading. The numbers of coils in those three springs are 10, 12, and 15, respectively. The mean coil diameters are in the ratio of 1:1.2:1.4, respectively. Find the distance *x* as shown in the figure such that the rigid bar remains horizontal after the application of load *P*. Solve the problem in terms of dimension *a*. (20)
3. (a) A sphere is 50 mm mean diameter with a wall 0.5 mm thick. It has an inside pressure 0.5 MPa greater than the outside pressure. Calculate the change in diameter and change in volume. Take $E = 212$ GPa, and $\nu = 0.25$. (15)
- (b) A thick-walled cylinder is built up by shrinking a tube of 25.4 mm thickness on a hollow cylinder having an outside diameter of 150 mm and an inside diameter of 100 mm, thereby causing a contact pressure of 27.6 MPa. What is the greatest internal pressure that can be applied to the assembly without exceeding a tangential stress of 96.5 MPa at the inner surface? (20)

ME 243

4. A 60° strain rosette attached to the aluminum skin of an airplane fuselage measures the following strains: $\epsilon_a = 10^{-4}$, $\epsilon_b = -2 \times 10^{-4}$, $\epsilon_c = 4 \times 10^{-4}$. If $E = 70$ GPa and Poisson's ratio $\nu = 1/3$, (i) compute the principal stresses and the maximum shearing stress and their directions, (ii) the normal and shearing stresses on planes whose normals are at 45° and 135° with the x -axis.

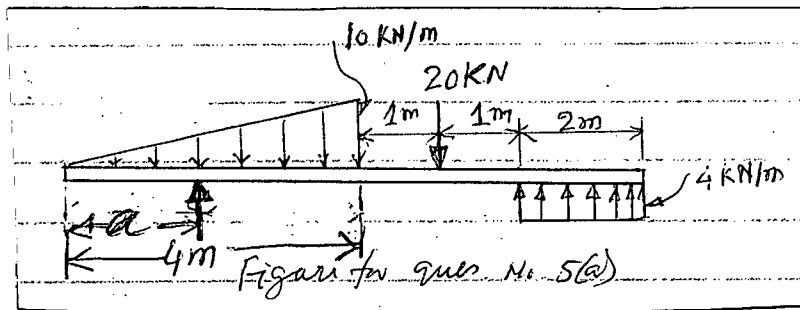
(35)

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

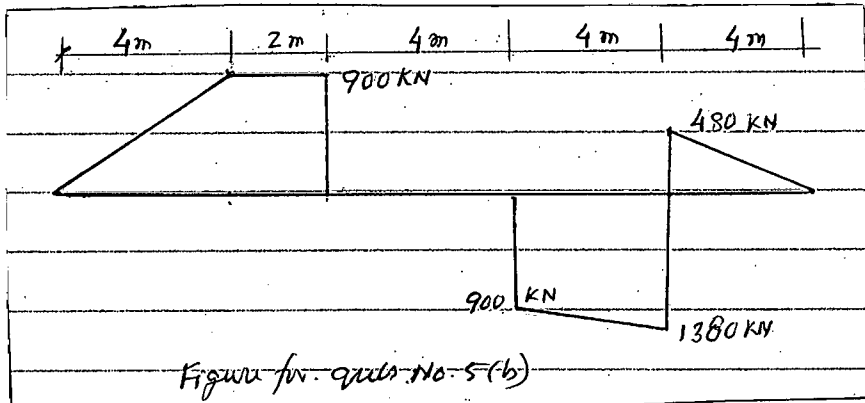
5. (a) Write shear and moment equations for the beam shown in Figure for Q. No. 5(a). Also draw shear and moment diagrams, specifying values at all change of loading positions and at all points of zero shear. Neglect the mass of the beam.

(20)



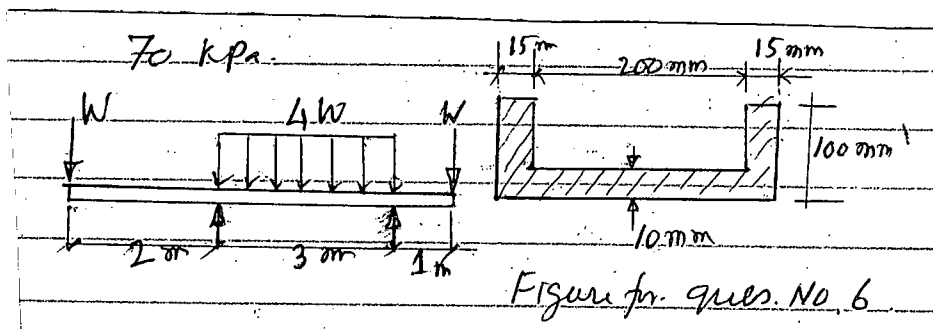
- (b) Draw moment and load diagrams corresponding to the shear diagram shown in Figure for Q. No. 5(b). Specify values at all change of load positions.

(15)



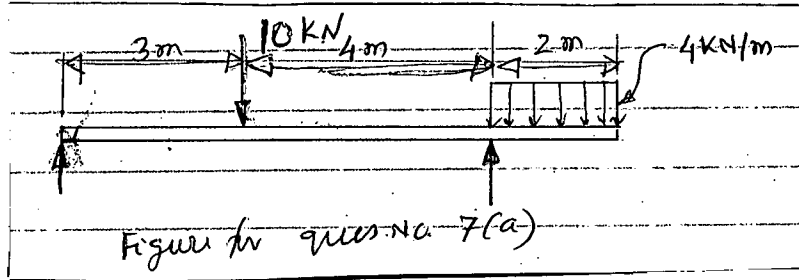
6. A channel section carries two concentrated loads W and a total distributed load of $4W$ as shown in Figure for Q. No. 6. Determine maximum value of W that will not exceed allowable stresses in tension 50 kPa, in compression 100 kPa or in shear of 70 kPa.

(35)

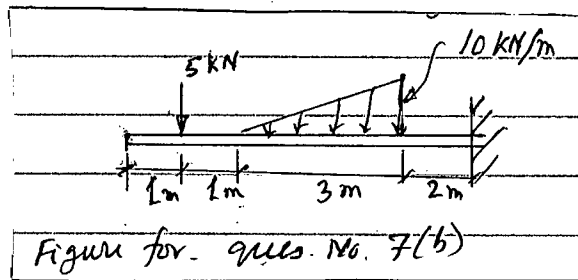


ME 243

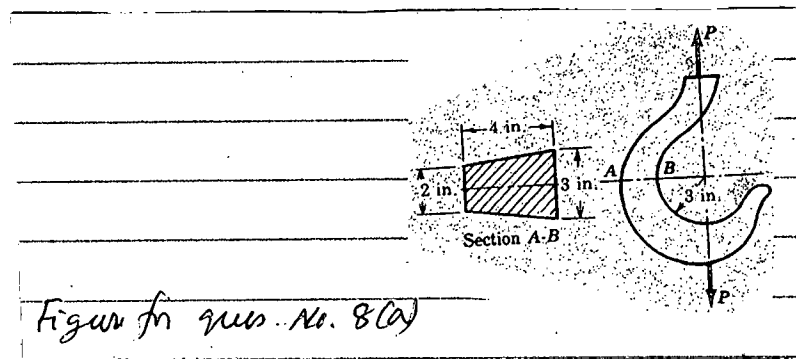
7. (a) For the beam loaded as shown in Figure for Q. No. 7(a), determine the maximum deflection between the supports. (20)



- (b) Find the maximum value of $EI\delta$ for the beam shown in Figure for Q. No. 7(b) using area moment method. (15)



8. (a) A crane hook has a cross-section that is approximated by the trapezoidal section shown in Figure for Q. No. 8(a). What is the maximum load P that will not exceed a stress of 10 ksi? (18)



- (b) Two C310 x45 channels are latticed together so they have equal moments of inertia about the principal axes. Determine the minimum length of a column having this section, assuming pinned ends. $E = 200$ GPa and a proportional limit 240 MPa. What safe load will the column carry for a length of 12 m with a factor of safety of 2.5? (See Table for Q. No. 8(b)). (17)

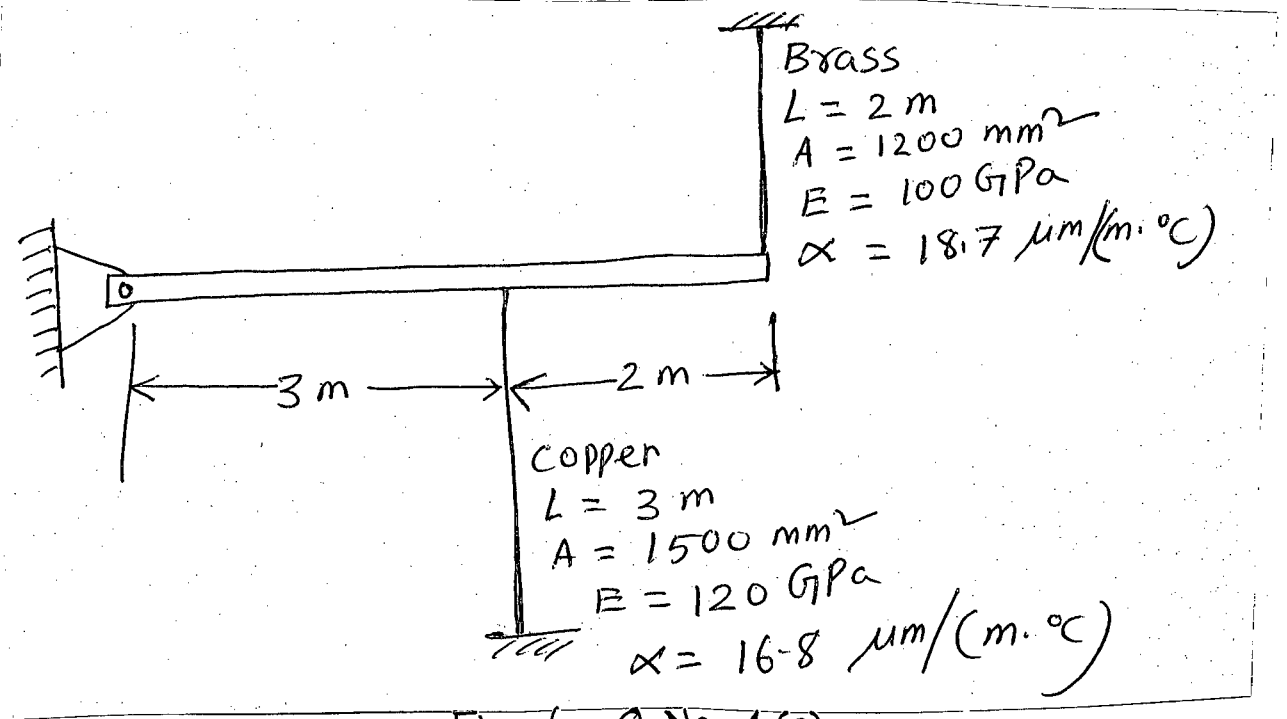


Fig. for Q.No. 1(a)

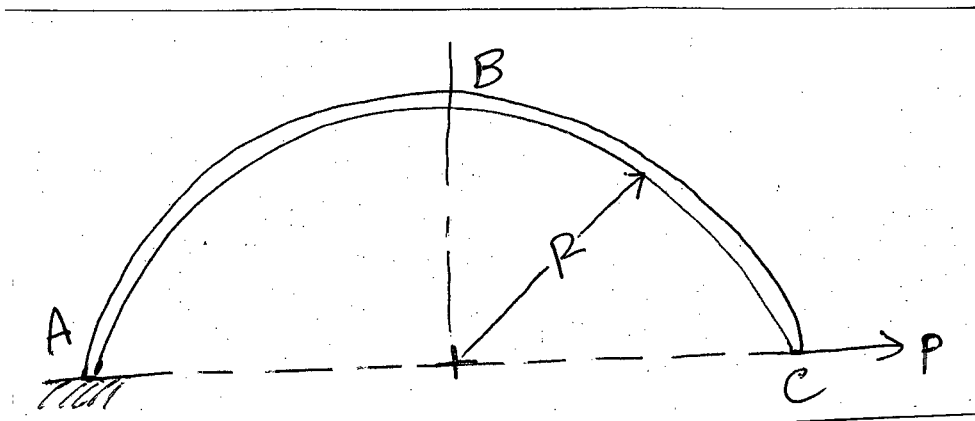


Fig. for Q.No. 1(b)

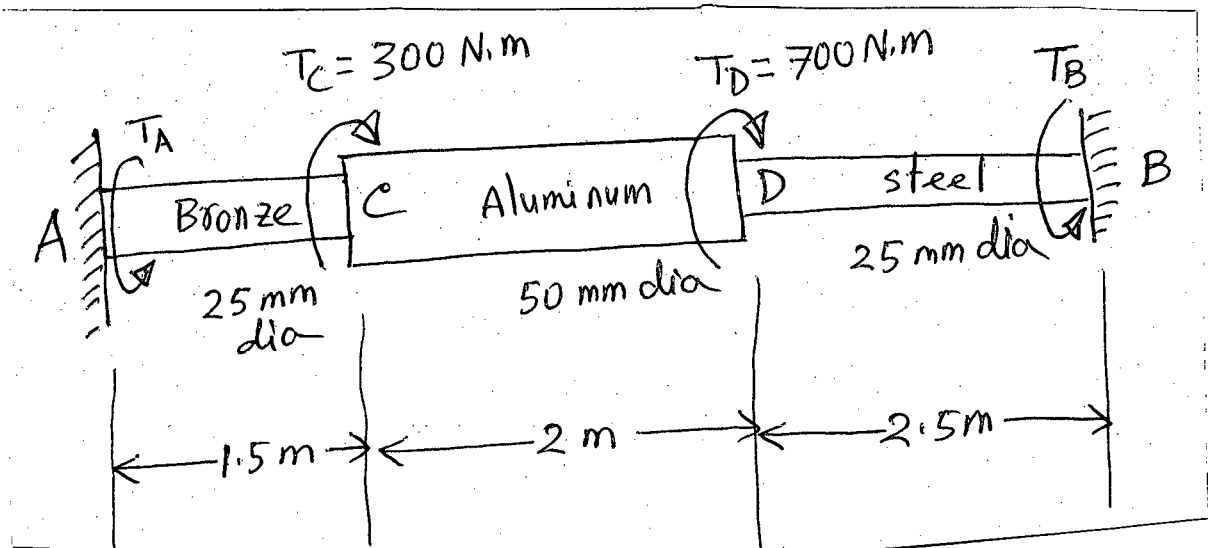


Fig. for Q.No. 2(a)

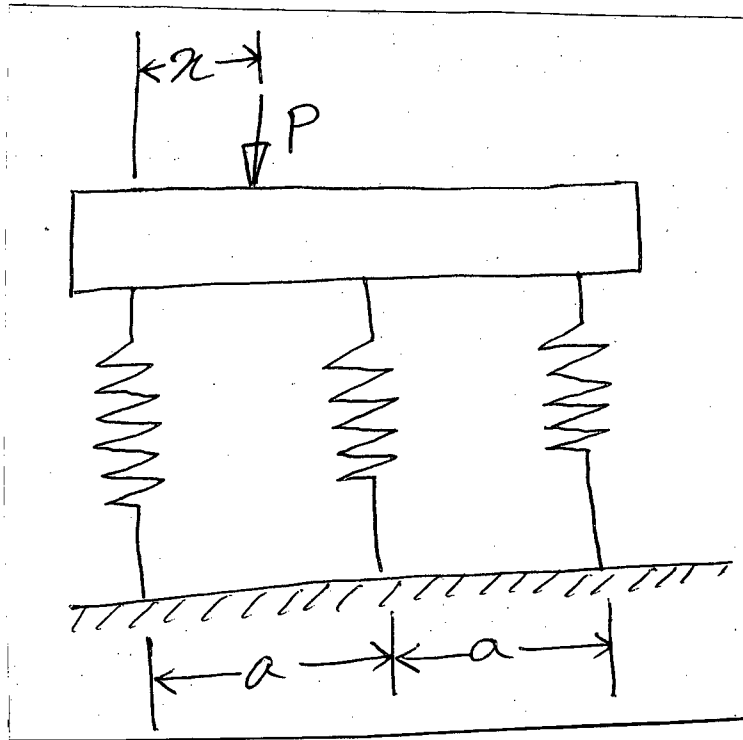


Fig. for Q. No. 2(b)

Table for Q. No. 8(b)

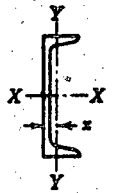


TABLE B-4 PROPERTIES OF CHANNEL SECTIONS: SI UNITS

Designation	Theoretical mass (kg/m)	Area (mm ²)	Depth (mm)	Flange			Web thickness (mm)	Axis X-X			Axis Y-Y		
				Width (mm)	Thickness (mm)			I (10 ⁶ mm ⁴)	$S = \frac{I}{c}$ (10 ³ mm ³)	$r = \sqrt{I/A}$ (mm)	I (10 ⁶ mm ⁴)	$S = \frac{I}{c}$ (10 ³ mm ³)	$r = \sqrt{I/A}$ (mm)
C380 x74	74.4	9 480	381	94	16.5	18.2	168	881	133	4.60	62.4	22.0	20.3
x60	59.4	7 570	381	89	16.5	13.2	145	760	138	3.84	55.5	22.5	19.7
x50	50.5	6 430	381	86	16.5	10.2	131	687	143	3.39	51.4	23.0	20.0
C310 x45	44.7	5 690	305	80	12.7	13.0	67.3	442	109	2.12	33.6	19.3	17.0
x37	37.1	4 720	305	77	12.7	9.8	59.9	393	113	1.85	30.9	19.8	17.1
x31	30.8	3 920	305	74	12.7	7.2	53.5	351	117	1.59	28.2	20.1	17.5
C250 x45	44.5	5 670	254	76	11.1	17.1	42.8	337	86.9	1.60	26.8	16.8	16.3
x37	37.3	4 750	254	73	11.1	13.4	37.9	299	89.3	1.40	24.3	17.2	15.7
x30	29.6	3 780	254	69	11.1	9.6	32.7	257	93.0	1.16	21.5	17.5	15.3
x23	22.6	2 880	254	65	11.1	6.1	27.8	219	98.2	0.922	18.8	17.9	15.9
C230 x30	29.8	3 800	229	67	10.5	11.4	25.5	222	81.9	1.01	19.3	16.3	14.8
x22	22.3	2 840	229	63	10.5	7.2	21.3	186	86.6	0.806	16.8	16.8	14.9
x20	19.8	2 530	229	61	10.5	5.9	19.8	173	88.5	0.716	15.6	16.8	15.1
C200 x28	27.9	3 560	203	64	9.9	12.4	18.2	180	71.5	0.825	16.6	15.2	14.4
x21	20.4	2 600	203	59	9.9	7.7	14.9	147	75.7	0.627	13.9	15.5	14.0
x17	17.0	2 170	203	57	9.9	5.6	13.5	133	78.9	0.544	12.8	15.8	14.5
C180 x22	21.9	2 700	178	58	9.3	10.6	11.3	127	63.8	0.568	12.8	14.3	13.3
x18	18.7	2 310	178	55	9.3	8.0	10.0	113	65.8	0.476	11.4	14.4	13.3
x15	14.5	1 850	178	53	9.3	5.3	8.86	99.6	69.2	0.405	10.3	14.8	13.8
C150 x19	19.2	2 450	152	54	8.7	11.1	7.12	93.7	53.9	0.425	10.3	13.2	12.9
x16	15.5	1 980	152	51	8.7	8.0	6.22	81.9	56.0	0.351	9.13	13.3	12.6
x12	12.1	1 540	152	48	8.7	5.1	5.36	70.6	59.0	0.279	7.93	13.5	12.8
C130 x17	17.2	2 190	127	52	8.1	12.0	4.36	68.7	44.6	0.346	8.85	12.6	12.9
x13	13.3	1 700	127	47	8.1	8.3	3.66	57.6	46.4	0.252	7.20	12.2	11.9
x10	9.9	1 260	127	44	8.1	4.8	3.09	48.6	49.5	0.195	6.14	12.4	12.2
C100 x11	10.8	1 370	102	43	7.5	8.2	1.91	37.4	37.3	0.174	5.52	11.3	11.5
x9	9.4	1 190	102	42	7.5	6.3	1.77	34.6	38.6	0.158	5.18	11.5	11.6
x8	8.0	1 020	102	40	7.5	4.7	1.61	31.6	39.7	0.132	4.65	11.4	11.6
C75 x9	8.8	1 120	76	40	6.9	9.0	0.85	22.3	27.5	0.123	4.31	10.5	11.4
x7	7.3	933	76	37	6.9	6.6	0.75	19.7	28.3	0.096	3.67	10.1	10.8
x6	6.0	763	76	35	6.9	4.3	0.67	17.6	29.6	0.077	3.21	10.1	10.9

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A cylindrical specimen of steel ($E = 210$ GPa) having an original diameter of 12.8 mm is tensile tested to fracture and found to have an engineering yield strength of 300 MPa and an engineering fracture strength of 460 MPa. If its cross-sectional diameter at fracture is 10.7 mm, determine: (4+4+4=12)

- (i) The ductility of the steel
- (ii) The true stress at fracture
- (iii) The true strain at yield point

- (b) A cylindrical rod 120 mm long and having a diameter of 15.0 mm is to be deformed using a tensile load of 35,000 N. It must not experience either plastic deformation or a diameter reduction of more than 1.2×10^{-2} mm. Of the materials listed below, which are possible candidates? Justify your choice(s). (12)

Material	Modulus of Elasticity (GPa)	Yield Strength (MPa)	Poisson's Ratio
Aluminum alloy	70	250	0.33
Titanium alloy	105	850	0.36
Steel alloy	205	550	0.27

- (c) As an NDT specialist you are asked to detect both surface and sub-surface defects in a wooden block whose one side is penetrated in soil. You can't uproot the block. Now, select a NDT technique to counter this problem with appropriate justification. (11)
2. (a) What do you understand by 'Critical resolved shear stress (CRSS)'? Deduce an expression for CRSS for single crystal. (2+10=12)
- (b) Consider a single crystal of BCC iron having a diameter of 0.5 m oriented such that a tensile stress is applied along a [010] direction. Compute the resolved shear stress along a (110) plane and in a $[\bar{1}11]$ direction when a tensile load of 52 kN is applied. (11)
- (c) How can ductility be restored after work hardening? Explain with appropriate diagram. (12)

MME 291/ME

3. (a) A structural component in the form of a wide plate is to be fabricated from a steel alloy that has a plane strain fracture toughness of $98.9 \text{ MPa}\sqrt{\text{m}}$ and a yield strength of 860 MPa. The flaw size resolution limit of the flaw detection apparatus is 3.0 mm. If the design stress is one half of the yield strength and the value of Y is 0.9, determine whether or not a critical flaw for this plate is subject to detection. (12)

(b) Three identical fatigue specimens (denoted A, B, and C) are fabricated from a nonferrous alloy. Each is subjected to one of the maximum-minimum stress cycles listed below; the frequency is the same for all three tests. (15)

Specimen	σ_{max} (MPa)	σ_{min} (MPa)
A	+450	-150
B	+300	-300
C	+500	-200

(i) Rank the fatigue lifetimes of these three specimens from the longest to the shortest.

(ii) Now justify this ranking using a schematic S-N plot.

(c) 'There always exist a considerable degree of scatter in fatigue data, which may lead to significant design uncertainties.' — What can be done to solve this problem? (8)

4. (a) 'For normalizing, hypereutectoid steel is heated above the upper critical whereas for full annealing, it is only heated above the lower critical' — explain the assertion. (10)

(b) Explain the changes that will occur to a thick steel sample when it is quenched in water from austenitic region. (15)

(c) In carburizing process, after carburization heat treatment is necessary to get desired hardness whereas in nitriding process, no such post treatment is needed — why? (10)

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Explain how phosphorus and sulphur are removed in Electric Arc furnace to improve the quality of the steel. Is it possible in L.D converter? Why? (15)

(b) What purposes do the Stoves serve in iron making in blast furnace? (10)

(c) Explain briefly the term 'anodizing'. (10)

MME 291/ME

6. (a) A component of red alpha brass is operated in an ammonia rich environment. The component cracks after few days. What is this Phenomena called? What are its reasons and how can it be prevented? (15)
- (b) Which type of cast iron is obtained by heat treating white cast iron? Explain the mechanism of formation of its microstructure. (10)
- (c) How does sensitization affect stainless steel? Explain with figure. (10)
7. (a) In a hypoeutectoid steel, both eutectoid and proeutectoid ferrite exist. Explain the difference between them. What will be the carbon concentration in each? (12)
- (b) How does 'coring' occur? What problem occurs when cored microstructure is formed and how can it be eliminated? (13)
- (c) Explain various types of point defects with schematic figure. (10)
8. (a) Briefly explain why, upon solidification, an alloy of eutectic composition forms a microstructure consisting of alternating layers of two solid phases. (15)
- (b) "Gray cast iron has low strength and almost zero ductility in tension but has better properties in compression" — explain the assertion with figure. (12)
- (c) How are the coarse needles in Al-Si alloys are modified to obtain a ductile material? What is the purpose of basic lining in L.D converter? (8)
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BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2016-2017

Sub: **ME 261** (Numerical Analysis)

Full Marks: 210

Time: 3 Hours

The figures in the margin indicate full marks.

Symbols used have their usual meaning and interpretation.

Assume reasonable value for any missing data.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Using False-Position method of root finding, determine a root of the following equation up to four decimal places within the interval $[1, 3]$: $\sin(x) - x + 2 = 0$.

Comment on the advantage of False-Position method over Bisection method.

(15)

- (b) Given that the velocity of an object thrown out from a plane is governed by the following equation:

(20)

$$v(t) = \frac{gm}{C} \left(1 - e^{-\frac{Ct}{m}} \right).$$

Find the value of the drag coefficient, C using Newton-Raphson method such that the mass of the object $m = 5$ kg, can attain a prescribed velocity, $v = 10$ m/s at a set period of time, $t = 9$ s. Start with the initial guess of $C_0 = 3.0$ and continue the iteration for an accurate root up to five significant digits.

2. (a) Find the inverse of the following matrix in order to solve a system of linear equations in the form of $[A]\{x\}=\{b\}$ using Gauss-Elimination method:

(15)

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 0 \end{bmatrix}$$

- (b) Determine the roots of the following non-linear system of equations using Gauss-Seidel and Newton-Raphson methods:

(20)

$$x^2 + y^2 = 4$$

$$e^x + y = 1$$

Use initial guesses as $[x^{(0)}, y^{(0)}] = [1, -1.7]$. Perform two sets of iterations and comment on the results.

Contd P/2

ME 261

3. (a) Show that the local truncation error associated with the Simpson's 1/3 rule of integration is $-\frac{h^5}{90} f^{(4)}(\xi)$. How can one obtain the exact result using this rule? (15)
- (b) Determine the maximum step sizes and the corresponding minimum number of panels required to integrate the function $x^3(\sin x + \cos x)$ between $x = 0$ and $x = 2$ to obtain an answer within 0.1% of the true answer, 3.657126, using: (20)
- (i) Trapezoidal method,
 - (ii) Simpson's 1/3 rule of integration,
 - (iii) Simpson's 3/8 rule of integration.

4. (a) What is Richardson's extrapolation scheme? Derive Richardson's extrapolation formula for Trapezoidal method of integration. How can this scheme be applied to numerical differentiation? (10)
- (b) Derive the finite difference formula for the first derivative with an error of $O(h^3)$ using forward difference approximation. (10)
- (c) When a fluid flows over a surface, the shear stress τ (N/m²) at the surface is given by the following expression: (15)

$$\tau = \mu \left. \frac{du}{dy} \right|_{\text{surface}}$$

where viscosity, $\mu = 0.00024$ Pa.s, u is the velocity parallel to the surface (m/s), and y is the distance normal to the surface (m). Measurements of the velocity of an air stream flowing above the surface are given in the following table:

y (m)	0.0	0.01	0.02	0.03	0.04
u (m/s)	0.00	45.56	70.16	90.38	100.00

Calculate the shear stress at the surface ($y = 0$) based on first-, second-, and third-order accurate approximations.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What is meant by 'machine dependent error' in numerical computation? Explain the rule followed by the computing machine to introduce the error in the results. (10)
- (b) Define 'relative error'. (10)

Show that the percentage error of an approximate number, u can be expressed by

$$\epsilon_p \leq \frac{100 \times \Delta u}{|u|}$$

where, Δu is the maximum limit on the magnitude of absolute error.

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ME 261

Contd... Q. No. 5

(c) The following function is to be evaluated for various values of x , which is subjected to an experimental error of one unit.

(15)

$$z = \frac{k}{x^2} \exp(-\beta x)$$

The constants are $\beta = 3.0 \times 10^{-3}$ and $k = 1.3 \times 10^7$, each accurate to the number of significant digits indicated. Find the absolute and the relative errors in z for $x = 700$.

6. (a) The solution of a physical system involving three unknowns leads to the following set of linear algebraic equations

(17)

$$\left. \begin{aligned} q_1 + 2q_2 + q_3 &= 6 \\ 2q_1 + q_2 + 2q_3 &= 6 \\ q_1 + q_2 + 3q_3 &= 2 \end{aligned} \right\}$$

Demonstrate the application of Faddeev-Leverrier method to obtain the unknowns.

(b) For a particular case of the system, the resulting equations assume the following homogeneous form:

(18)

$$\begin{bmatrix} 10-\alpha & 4 & -1 \\ 4 & 2-\alpha & 3 \\ -1 & 3 & 1-\alpha \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \{0\}$$

where α is an additional system unknown.

- (i) Find the largest value of α and the corresponding $\{q\}$ using an iterative scheme. Perform at least three iterations.
- (ii) Draw a flow-chart to describe an iterative algorithm to find the smallest value of α and the corresponding $\{q\}$ simultaneously.

7. (a)

(17)

Time (t)	t_1	t_2	t_3	t_4	t_n
Variable (y)	y_1	y_2	y_3	y_4	y_n

For the purpose of fitting the above data set, deduce the least-squares matrix formulation to find the best values of the constants used in the following trigonometric function

$$y = A_0 + A_1 \cos(\omega t) + B_1 \sin(\omega t)$$

where, ω is assumed to be constant throughout the analysis.

ME 261

Contd... Q. No. 7

(b) Determine the constants α and β by the method of least-squares such that the following function fits the given data set shown below. (18)

$$y = \alpha \exp(\beta x)$$

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

8. (a) Give the graphical and mathematical details of Ralston's method to numerically solve ODEs. (17)

List the relative advantages and disadvantages of the method over Heun's method.

(b) Suppose an electrical circuit contains a source of emf, E , an inductance L , a resistor R and a switch, S . The resistor can be expressed as a function of current flow, i as follows: (18)

$$R = a + bi^2 \quad (i)$$

Applying Kirchoff's voltage law to the circuit loop, the following DEQ is obtained:

$$E - L \frac{di}{dt} - Ri = 0 \quad (ii)$$

Switch S is closed at time $t = 0$, and the current flow is desired as a function of time for $t > 0$.

Generate the first two steps of solution of $i(t)$ with a time interval of 0.002 sec using the Classical form of the fourth-order R-k method.

Assume the following parameters

$$\begin{cases} E = 200 \text{ volts} \\ L = 3 \text{ Henries} \\ a = 100 \Omega \\ b = 50 \Omega / \text{amp}^2 \end{cases}$$

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) At the instant shown in Fig. for Q. 1(a), slider block B is moving with a constant acceleration, and its speed is 150 mm/s. Knowing that after slider block A has moved 240 mm to the right its velocity is 60 mm/s, determine (i) the accelerations of A and B , (ii) the acceleration of portion D of the cable. (17)
- (b) At a given instant in an airplane race as shown in Fig. for Q. 1(b), airplane A is flying horizontally in a straight line, and its speed is being increased at the rate of 8 m/s^2 . Airplane B is flying at the same altitude as airplane A and, as it rounds a pylon, is following a circular path of 300-m radius. Knowing that at the given instant the speed of B is being decreased at the rate of 3 m/s^2 , determine, for the positions shown, (i) the velocity of B relative to A , (ii) the acceleration of B relative to A . (18)
2. (a) Two spheres as shown in Fig. for Q. 2(a), each of mass m , can slide freely on a frictionless, horizontal surface. Sphere A is moving at a speed $v_0 = 5 \text{ m/s}$ when it strikes sphere B which is at rest and the impact causes sphere B to break into two pieces, each of mass $m/2$. Knowing that 0.7 s after the collision one piece reaches point C and 0.9 s after the collision the other piece reaches point D , determine (i) the velocity of sphere A after the collision, (ii) the angle θ after the collision. (17)
- (b) A high-speed jet as shown in Fig. for Q. 2(b), of air issues from nozzle A with a velocity of v_A and mass flow rate of 0.36 kg/s. The air impinges on a vane causing it to rotate to the position shown. The vane has a mass of 6 kg. Knowing that the magnitude of the air velocity is equal at A and B , determine (i) the magnitude of the velocity at A , (ii) the components of the reactions at O . (18)
3. (a) A 20-kg cabinet is mounted on casters that allow it to move freely ($\mu = 0$) on the floor as shown in Fig. for Q. 3(a). If a 100-N force is applied as shown, determine (i) the acceleration of the cabinet, (ii) the range of values of h for which the cabinet will not tip. (17)
- (b) The ends of the 10-kg uniform rod AB are attached to collars of negligible mass that slide without friction along fixed rods as shown in Fig. for Q. 3(b). If the rod is released from rest when $\theta = 25^\circ$, determine immediately after release (i) the angular acceleration of the rod, (ii) the reaction at A , (iii) the reaction at B . (18)
4. (a) A slender rod of length l and weight W is pivoted at one end as shown in Fig. for Q. 4(a). It is released from rest in a horizontal position and swings freely. Determine the angular velocity of the rod as it passes through a vertical position and determine the corresponding reaction at the pivot. (17)

ME 249**Contd... Q. No. 4**

(b) Member ABC has a mass of 2.4 kg and is attached to a pin support at B as shown in Fig. for Q. 4(b). An 800-g sphere D strikes the end of member ABC with a vertical velocity v_1 of 3 m/s. Knowing that $L = 750$ mm and that the coefficient of restitution between the sphere and member ABC is 0.5, determine immediately after the impact (i) the angular velocity of member ABC , (ii) the velocity of the sphere. (18)

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) The coefficients of friction between the blocks A and C (see Figure for Question 5(a)) and the horizontal surfaces are $\mu_s = 0.24$ and $\mu_k = 0.20$. The masses of the blocks are $m_A = 5$ kg, $m_B = 10$ kg and, $m_C = 10$ kg. Determine (i) the tension in the cord, (ii) the acceleration of each block. (18)

(b) The horizontal rod OA of Figure for Question 5(b) rotates about a vertical shaft according to the relation $\dot{\theta} = 10t$ where $\dot{\theta}$ and t are expressed in rad/s and seconds, respectively. A 250 g collar B is held by a cord with a breaking strength of 18 N. Neglecting friction, determine, immediately after the cord breaks, (i) the relative acceleration of the collar with respect to the rod, (ii) the magnitude of the horizontal force exerted on the collar by the rod. (17)

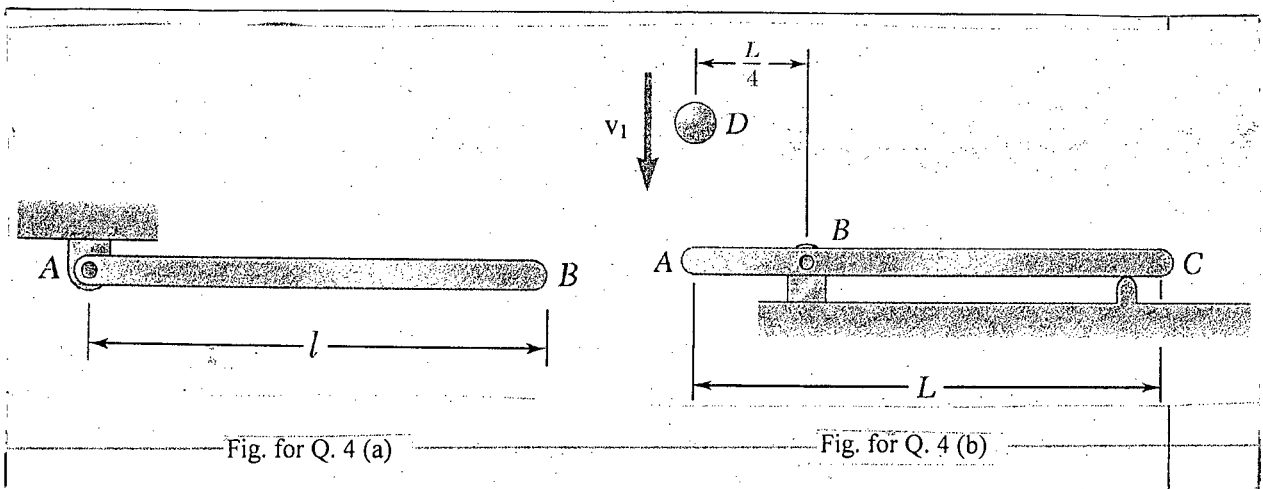
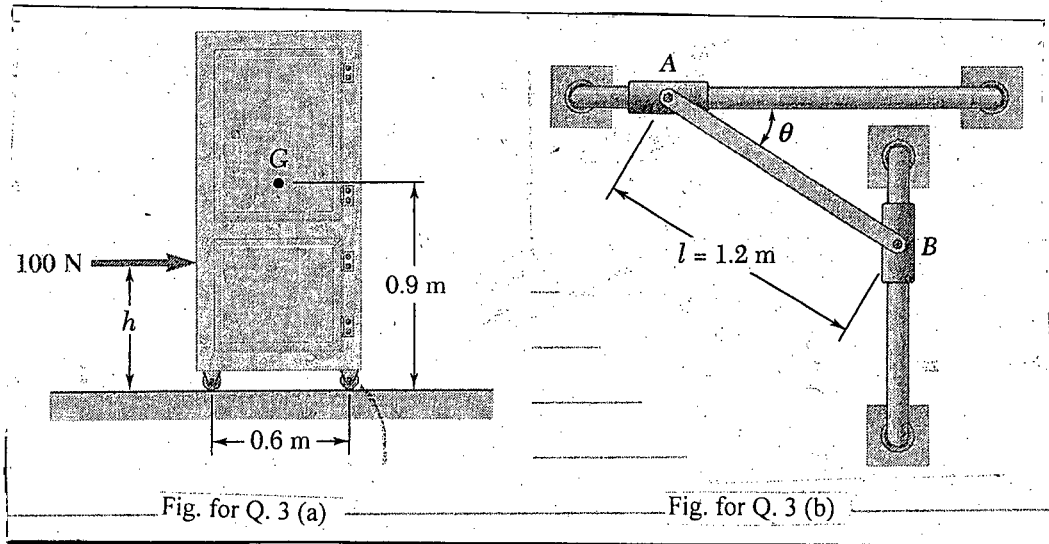
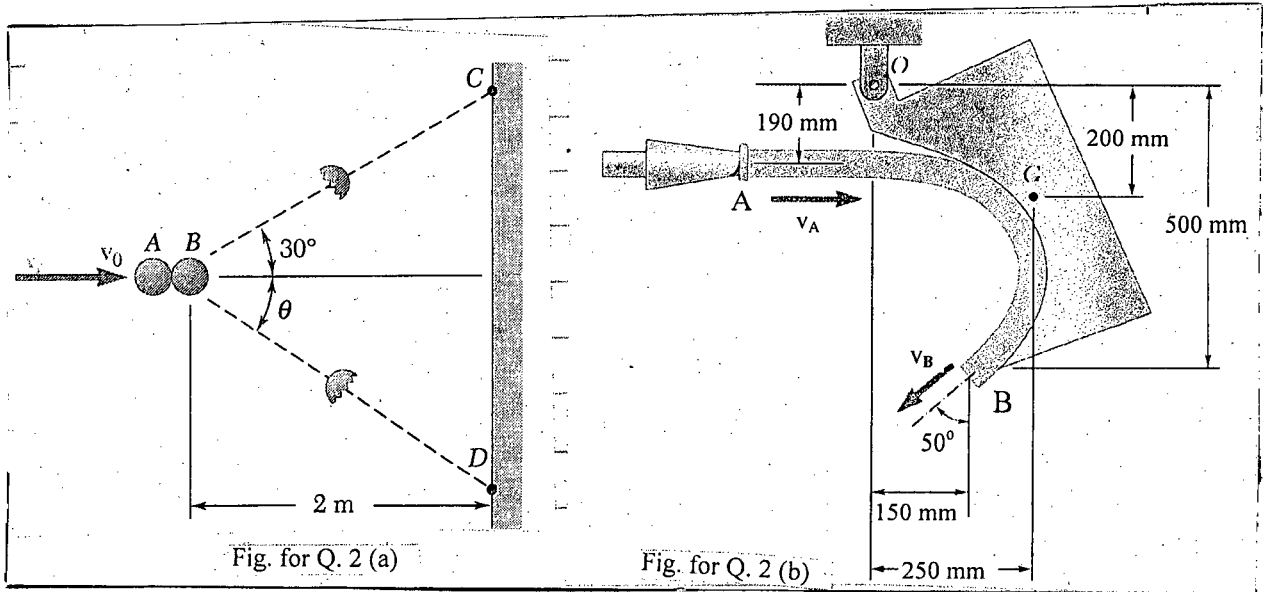
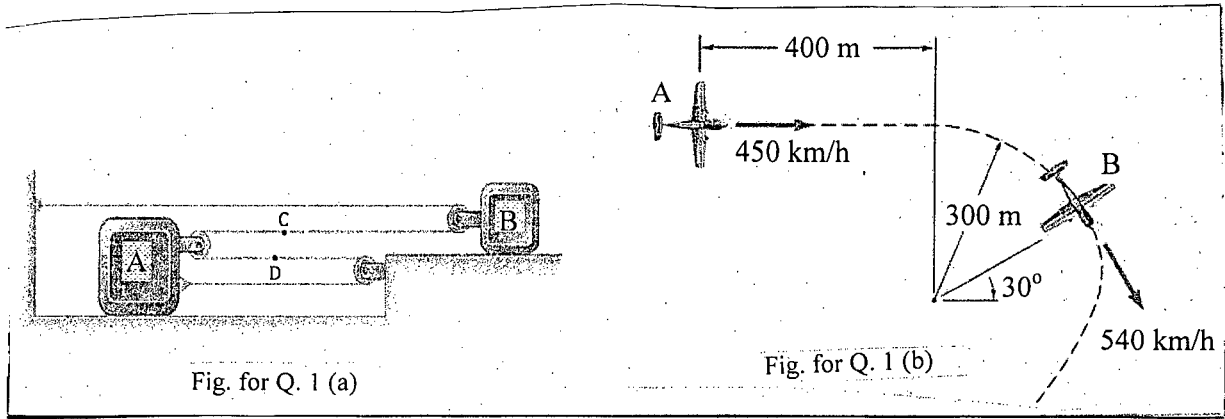
6. (a) A spring is used to stop a 50-kg package which is moving down a 20° incline (see Figure for Question 6(a)). The spring has a constant $k = 30$ kN/m and is held by cables so that it is initially compressed 50 mm. Knowing that the velocity of the package is 2 m/s when it is 8 m from the spring and the kinetic friction coefficient between the package and the incline is 0.2, determine the maximum additional deformation of the spring in bringing the package to rest. (18)

(b) A 25-g steel-jacketed bullet is fired horizontally with a velocity of 600 m/s and ricochets off a steel plate along the path CD with a velocity of 400 m/s (see Figure for Question 6(b)). Knowing that the bullet leaves a 10-mm scratch on the plate and assuming that its average speed is 500 m/s while it is in contact with plate, determine the magnitude and direction of the average impulsive force exerted by the bullet on the plate. (17)

7. (a) A 600-g ball A is moving with a velocity of magnitude 6 m/s when it is hit as shown by a 1-kg ball B (see Figure for Question 7(a)) which has a velocity of magnitude 4 m/s. Knowing that the coefficient of restitution is 0.8 and assuming no friction, determine the velocity of each ball after impact. (20)

(b) In the Figure for Question 7(b), if the collar at C is moving downward to the left at a velocity of 8 m/s, determine the angular velocity of the link AB at the instant shown. Use the instantaneous center of rotation method for analysis. (15)

8. For the quick return mechanism in the Figure for Question 8, the driving crank OA rotates at 120 rpm in clockwise direction. In the Figure, $CD = 500$ mm, $QC = 150$ mm, and $OA = 200$ mm. For the instant shown, find out the magnitude and direction of the velocity and acceleration of the block D in graphical method. (35)



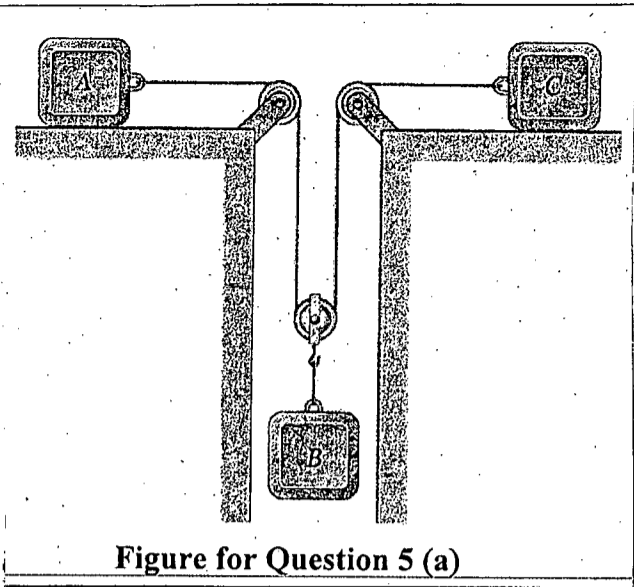


Figure for Question 5 (a)

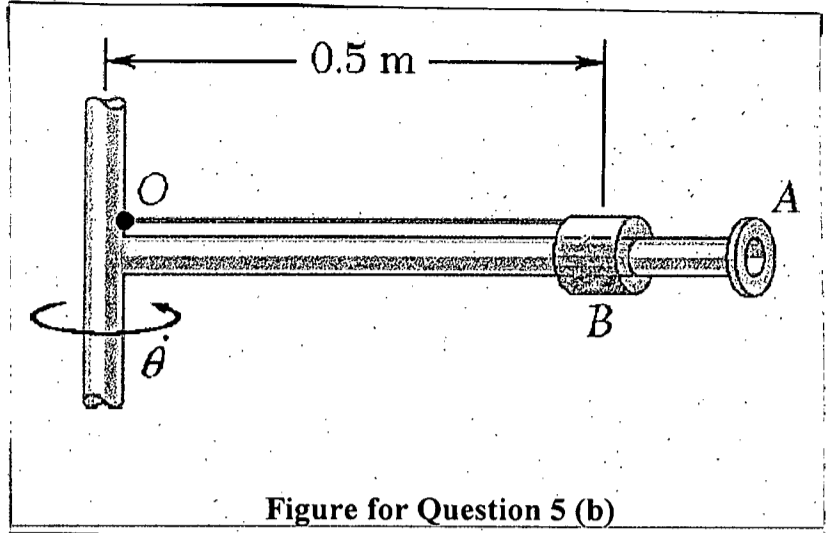


Figure for Question 5 (b)

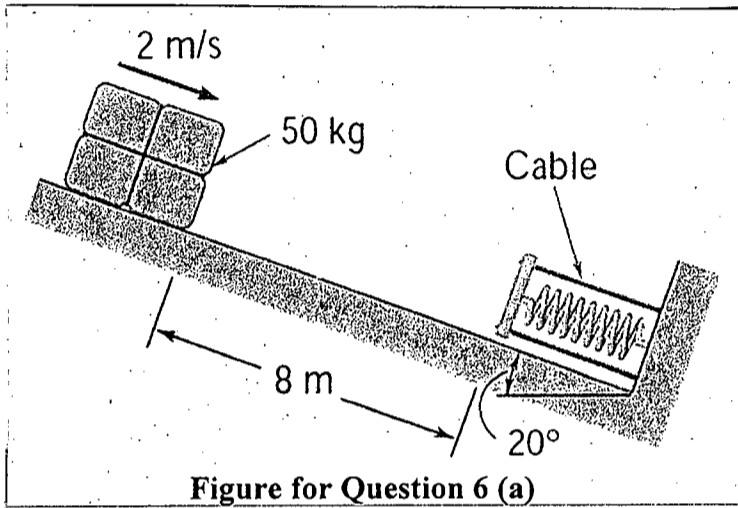


Figure for Question 6 (a)

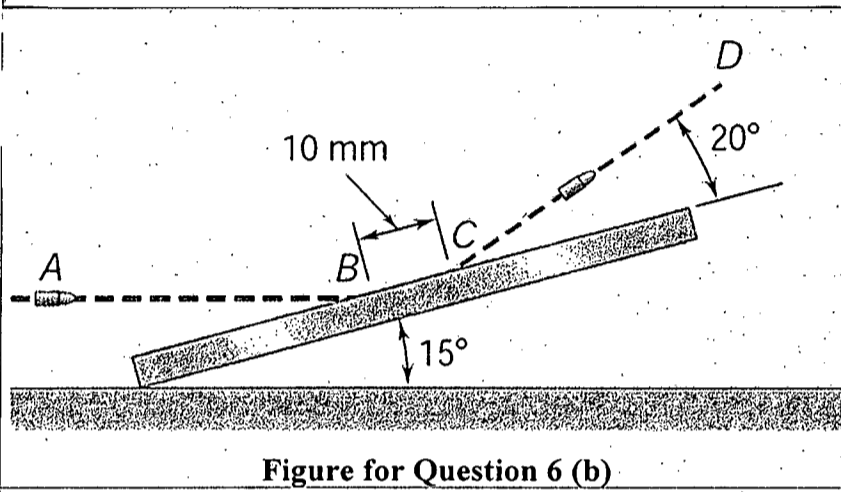


Figure for Question 6 (b)

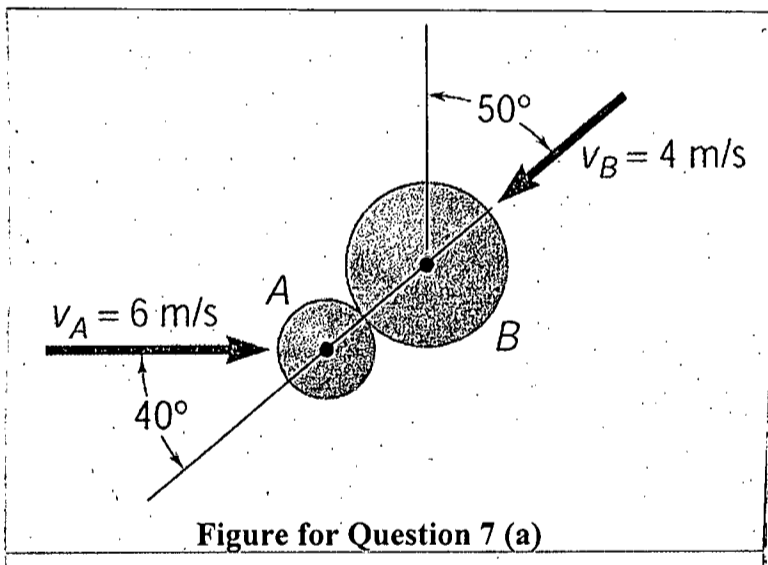


Figure for Question 7 (a)

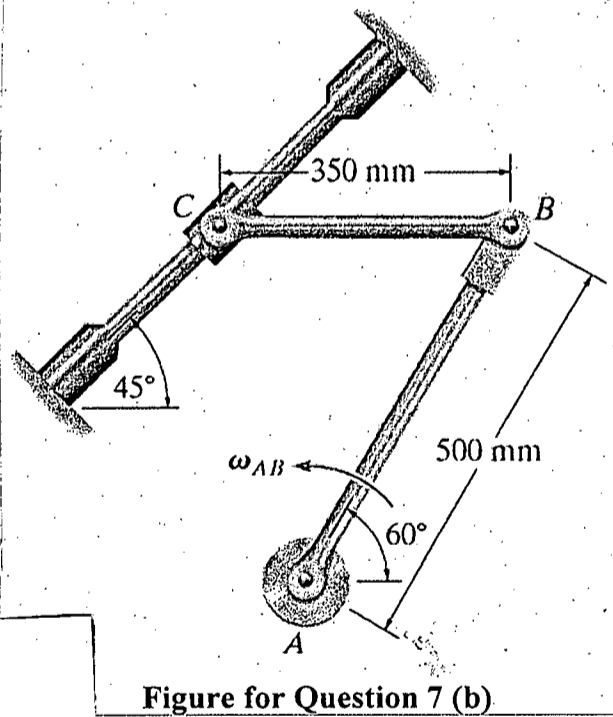


Figure for Question 7 (b)

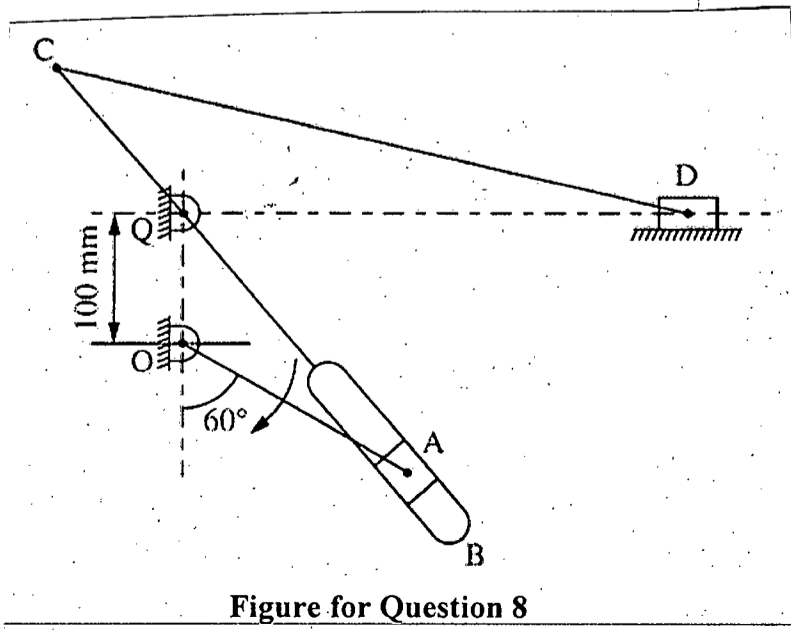


Figure for Question 8