

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A function $f(x)$ is defined as follows: (12)

$$f(x) = \begin{cases} x^2 + 1 & \text{when } 0 \leq x < 1/2 \\ 0 & \text{when } x = 1/2 \\ x + 3 & \text{when } 1/2 < x \leq 1 \end{cases}$$

Discuss the continuity and differentiability of $f(x)$ at $x = 0$ and $x = \frac{1}{2}$. Also sketch the curve of $f(x)$.

- (b) If $x = \tan u$ then find $\{(1+x^2)u_{n+2} + 2(n+1)xu_{n+1} + n(n+1)u_n\}$ and hence find $(u_n)_0$. (13)

- (c) Evaluate: $\lim_{x \rightarrow \infty} (x + e^x)^{2/x}$ (10)

2. (a) Expand $(\sin^{-1} x)^2$ in a series of ascending powers of x . (14)

- (b) If the Mean Value theorem is $f(b) - f(a) = (b - a)f'(x_1)$, find x_1 , where $f(x) = x^2 - 3x - 1$, $a = 11/7$ and $b = 13/7$. (11)

- (c) Verify Euler's Theorem for the function $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$. (10)

3. (a) If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle θ with the axis of x , find the value of $(y \cos \theta - x \sin \theta)$. (18)

- (b) Find the largest rectangle that can be inscribed within the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (17)

4. (a) Evaluate: $\int_0^{\pi/2} \frac{\sin^2 x \, dx}{1 + \sin x \cos x}$ (12)

- (b) Evaluate: $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ (12)

- (c) Find the value of the following series: (11)

$$\lim_{n \rightarrow \infty} \left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \dots + \frac{n}{n^2+n^2} \right]$$

MATH 145/CSE

SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Show that $\int_0^{\frac{\pi}{2}} \sin^m \theta \cos^n \theta d\theta = \frac{1}{2} \beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right); m > -1, n > -1.$ (12)

(b) Prove that $\Gamma(1/2) = \sqrt{\pi}$. Hence evaluate $\int_0^{\infty} \sqrt{y} e^{-y^3} dy.$ (13)

(c) Evaluate $\int_0^{\sqrt{2}} \int_0^{2-x^2} \int_0^x xyz dz dy dx.$ (10)

6. (a) Find the exact arc length of the curve $24xy = y^4 + 48$ from $y = 2$ to $y = 4.$ (12)

(b) Find the area of the region enclosed by the rose curve $r = \cos 2\theta.$ (10)

(c) Find the volume of the solid formed by the revolution about the x -axis of a loop of the curve $(x - 4a)y^2 = ax(x - 3a).$ (13)

7. (a) Transform the equation $5x^2 - 2xy + 5y^2 + 2x - 10y - 7 = 0$ by suitable translation and rotation of axes. Also, identify the conic. (13)

(b) Show that the equation $2x^2 - 7xy + 3y^2 + x + 7y = 6$ represents a pair of straight lines. Also, find the point of intersection and the angle made by them. (10)

(c) Find the radical axis and the limiting points of the system of circles coaxial with the circles $x^2 + y^2 - 6x - 6y + 4 = 0$ and $x^2 + y^2 - 2x - 4y + 3 = 0.$ (12)

8. (a) Find the locus of the middle points of the normal chords of the parabola $y^2 = 4ax.$ (12)

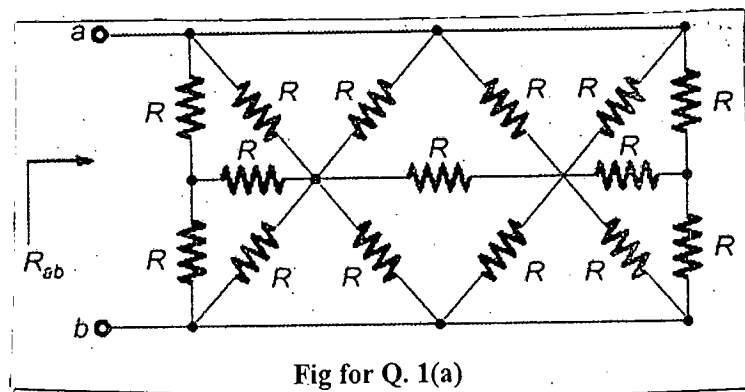
(b) Upon which condition the straight line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$? (8)

(c) Consider the hyperbola $6x^2 - 7xy - 3y^2 - 3x - 8y - 6 = 0.$ Find: (i) the asymptotes of the hyperbola, (ii) the centre of the hyperbola and (iii) the equation of the conjugate hyperbola. (15)

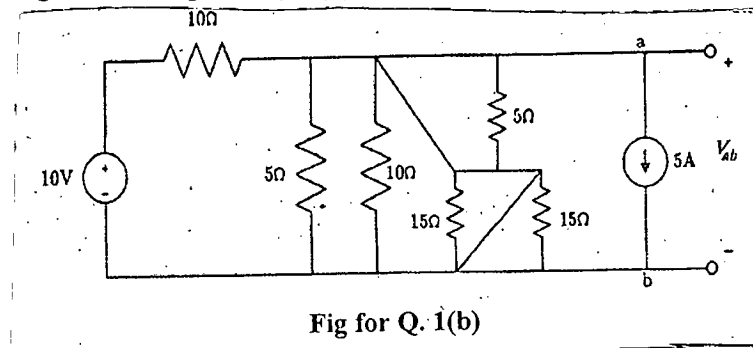
SECTION - A

There are **FOUR** questions in this section. Answer any **THREE**.

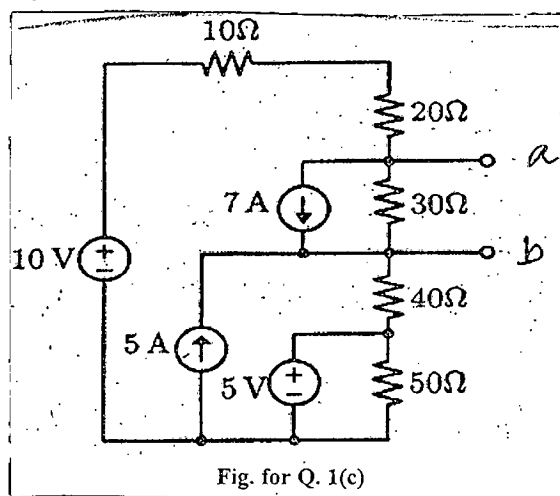
1. (a) Find the equivalent resistance R_{ab} in Fig. for Q. 1(a). (10)



- (b) Find the voltage V_{ab} in Fig. for Q. 1(b) (7)



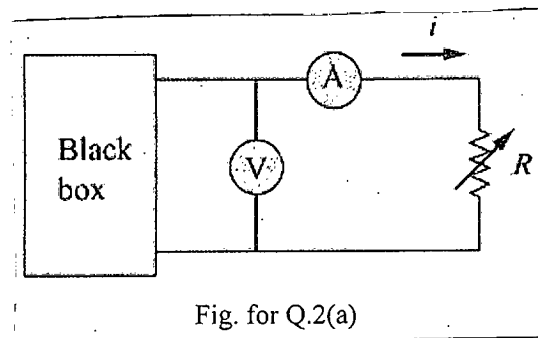
- (c) Using superposition theorem, find the Thevenin equivalent between terminals a-b of the circuit shown in Fig. for Q. 1(c). (18)



2. (a) A black box with a circuit in it is connected to a variable resistor. An ideal ammeter (with zero resistance) and an ideal voltmeter (with infinite resistance) are used to measure current and voltage as shown in Fig. for Q. 2(a). The results are shown in the table below. (10)

EEE 163/CSE

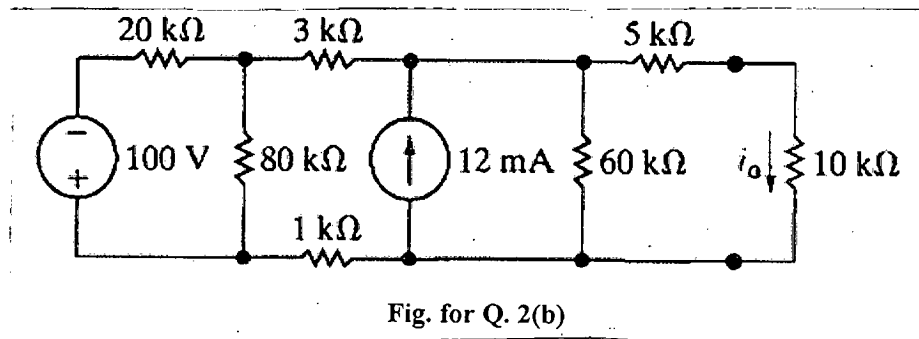
Contd... Q. No. 2(a)



$R(\Omega)$	$V(V)$	$i(A)$
2	3	1.5
8	8	1.0
14	10.5	0.75

Determine the maximum power available from the black box.

(b) Find the current in the 10kohm resistor (i_o) making a succession of appropriate source transformations in Fig. for Q. 2(b). (12)

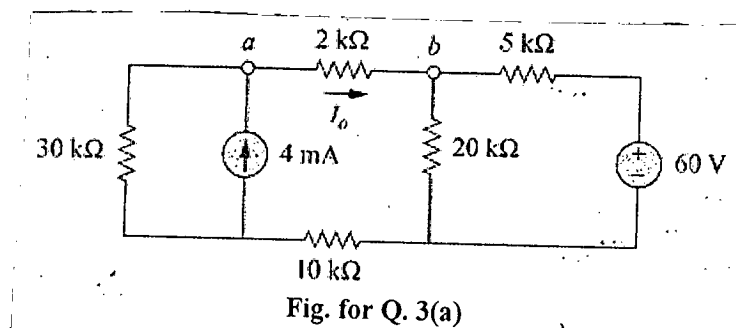


(c) Design a cascaded Op-Amp circuit that produces the output $v_{out} = 5\sin(\omega t + 100) + 8\sin(\omega t) + 2$ volts, given that you have the following signals available: $v_1 = \sin(\omega t + 10)$, $v_2 = \cos(\omega t)$, and a range of 0-100V DC source available.

Clearly specify the output at each stage of the Op-Amp circuit, the values of components and the minimum voltage source needed to power the opams, as well as label the function of each stage. However, you do not need to solve the designed circuit. (13)

3. (a) Consider the circuit in Fig. for Q. 3(a). An ammeter with internal resistance, R_i is the inserted between a and b to measure I_o . Determine the reading of the ammeter if:

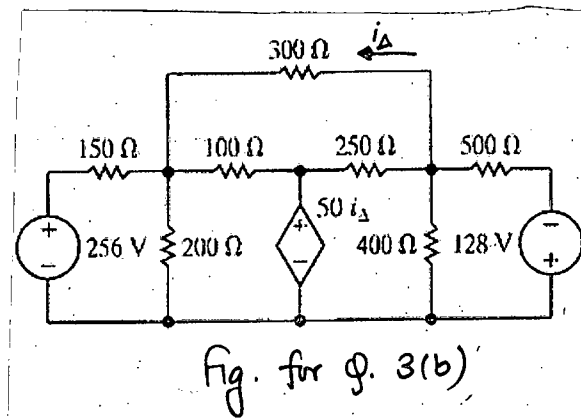
(i) $R_i = 0$ ohm (ii) $R_i = 500$ ohm. (17)



EEE 163/CSE

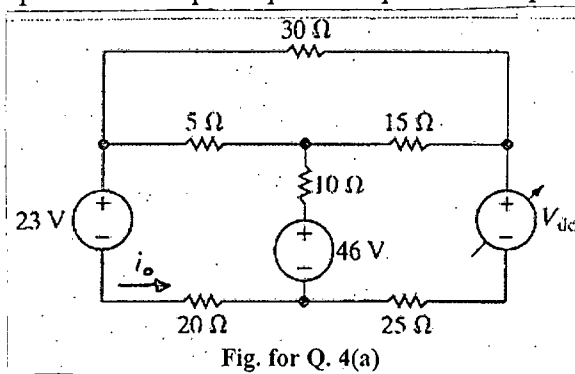
Contd... Q. No. 3

(b) Find the power dissipated in the 300 ohm resistor in the circuit shown in Fig. for Q. 3(b) using nodal analysis. (18)

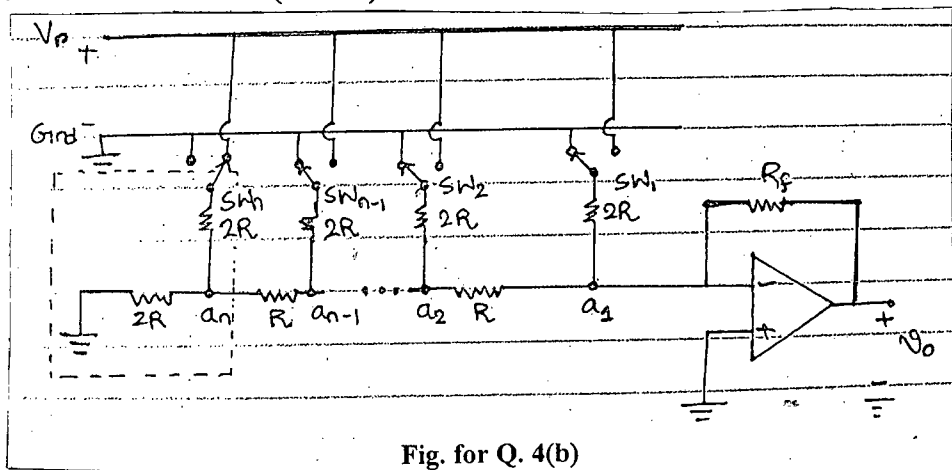


4. (a) The variable dc voltage source in the circuit in Fig. for Q. 4(a) is adjusted so that i_o is zero. (17)

- (i) Find the value of V_{dc} .
- (ii) Show that the power developed equals the power dissipated.



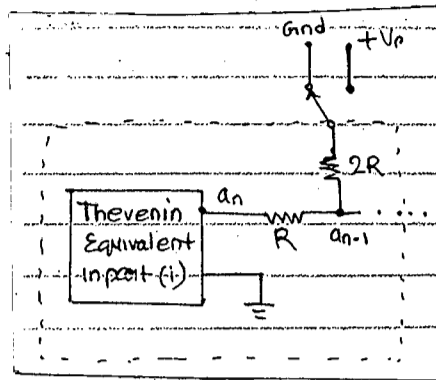
(b) Consider the following n-bit R-2R Ladder DAC circuit in Fig. for Q. 4(b), where the switch is either closed ($b_k = 1$, connected to V_r volts) or open ($b_k = 0$, connected to ground voltage). Suppose that, only switch SW_n is connected to V_r volts, and all other switches are connected to GND (0 volts). (18)



- (i) For the dashed box shown in the circuit, find the Thevenin equivalent of this box at node a_n with respect to ground.
- (ii) Suppose the dashed box is replaced with its Thevenin equivalent in part (i). Now suppose, the dashed box is extended to include node a_{n-1} , as shown in the figure below. Find the Thevenin equivalent of the new box.

EEE 163/CSE

Contd... Q. No. 4(b)



(iii) Using your results in part (i) and part (ii), deduce the voltage at node a_1 and hence, find the output of the Opamp, V_0 .

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Assume that a current $i(t) = I_m \sin(\omega t)_A$ flows through a series RLC circuit. Derive the expressions for applied voltage (V), total impedance (Z), instantaneous power (p), real power (P), reactive power (Q) and apparent power (S). Also draw the wave shape for power. (15)

(b) Given that $V_{in} = 10\cos(\omega t)$ V, the current flowing through the inductor is $-j2$ A and the current flowing through the RC branch is $3\angle 45^\circ$ A in the circuit of Fig. for Q.5(b). The product of values of L and C is 10^{-9} . (20)

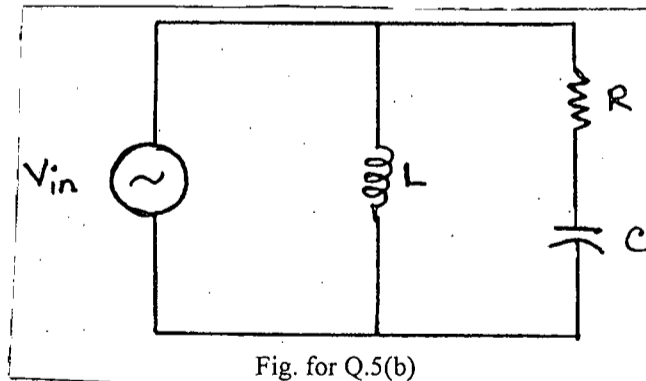


Fig. for Q.5(b)

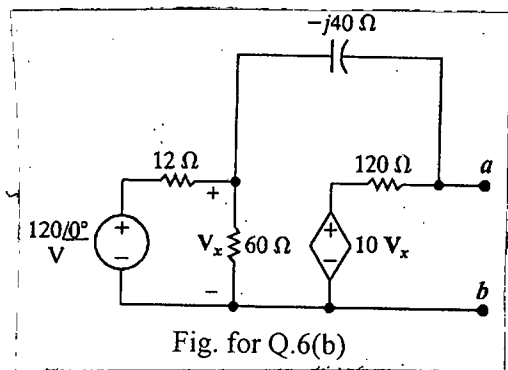
- (i) Find the operating frequency (ω) of the voltage source at this condition.
- (ii) The frequency (ω) at which the power factor of the load will be unity.

6. (a) For a Thevenin equivalent circuit, show that the maximum average power transfer occurs when the load impedance (Z_L) is equal to the complex conjugate of the Thevenin impedance (Z_{TH}). (10)

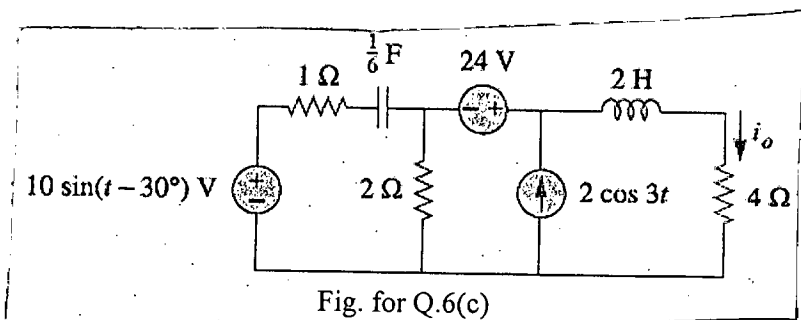
EEE 163/CSE

Contd... Q. No. 6

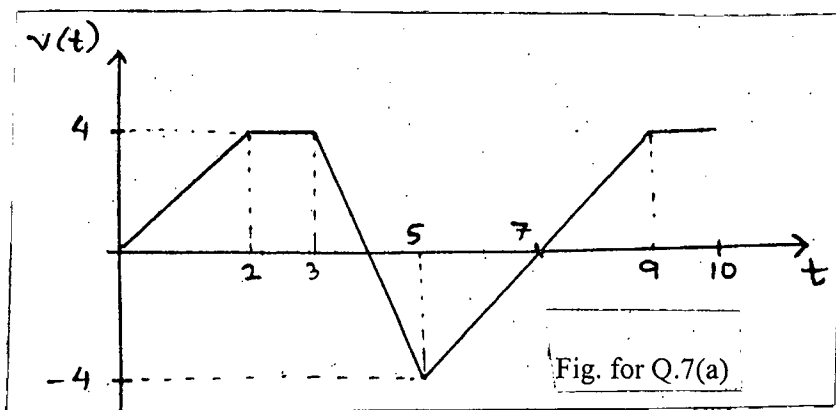
(b) Find the Norton equivalent circuit with respect to terminals a-b for the circuit shown Fig. for Q. 6(b). Find the resistance for which maximum average power transfer occurs. (13)



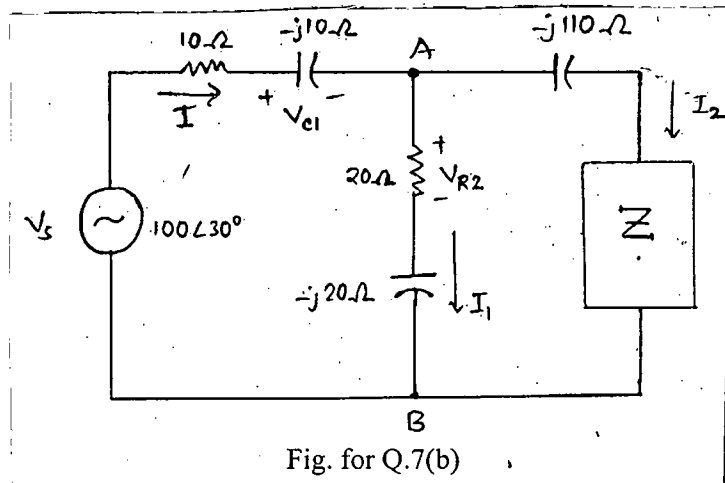
(c) Determine i_o for the circuit in Fig. for Q. 6(c). (12)



7. (a) If the voltage $v(t)$ as shown in Fig. for Q. 7(a) is applied across an impedance of $3 + 4j$, find the complex power of the load impedance. (15)

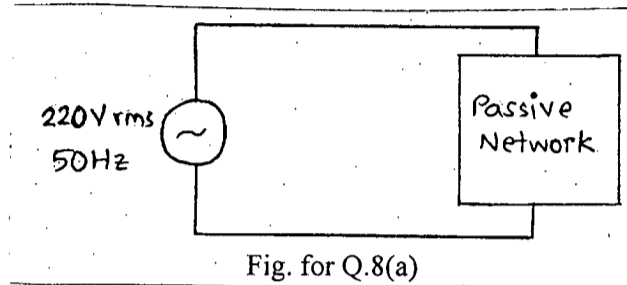


(b) Draw the phasor diagram indicating V_{C1} , V_{R2} , I , I_1 , I_2 and V_S for the circuit shown in Fig. for Q. 7(b). Here Z is an R-L block with impedance $20 + j130$. Take V_{AB} as reference. Also find the phase difference between V_{AB} and source voltage V_S . (20)

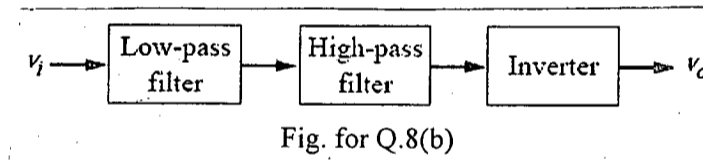


EEE 163/CSE

8. (a) The unknown network shown in Fig. for Q. 8(a) operates at 0.8 lagging power factor. When a capacitor C is added in parallel to this network, the capacitor draws a current of $j73.3$ A and improves the power factor to 0.95 lagging. Find: (15)
- (i) The value of the capacitance C.
 - (ii) The nature of the passive network and values of the circuit parameters.



- (b) Design an active bandpass filter by cascading successive filters as shown in Fig. for Q. 8 (b) and determine its overall transfer function $H(\omega)$. Given that the desired lower and upper cut-off frequencies of this filter are 49 *krad/s* and 51 *krad/s*. Find: (20)
- (i) The center frequency ω_0
 - (ii) Bandwidth
 - (iii) Quality factor of this filter.



SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) In the Figure for Question 1(a), a continuous cable of total length 4 m is wrapped around the small pulleys at A, B, C, and D. If each spring is stretched 300 mm from their equilibrium positions, determine the mass of each block. Neglect the weight of the pulleys and cords. The springs are un-stretched when $d = 2$ m. (20)

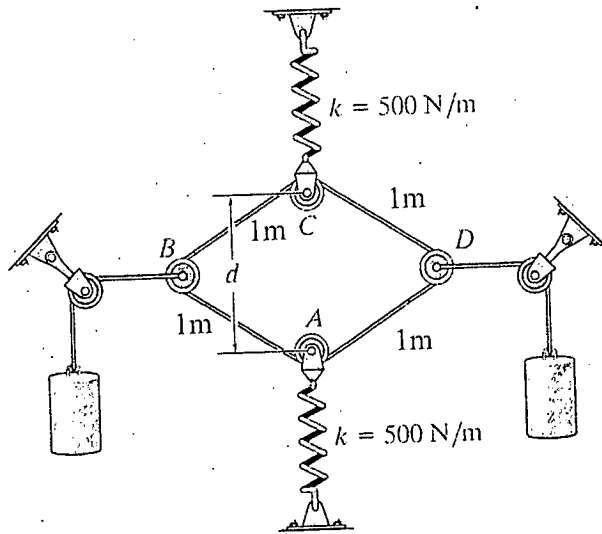


Figure for Question 1(a)

- (b) Figure for Question 1(b), shows a light bar AB that supports a 15-kg block at its midpoint C . Rollers at A and B rest against frictionless surfaces, and a horizontal cable AD is attached at A . Determine (a) the tension in cable AD , (b) the reactions at A and B . (15)

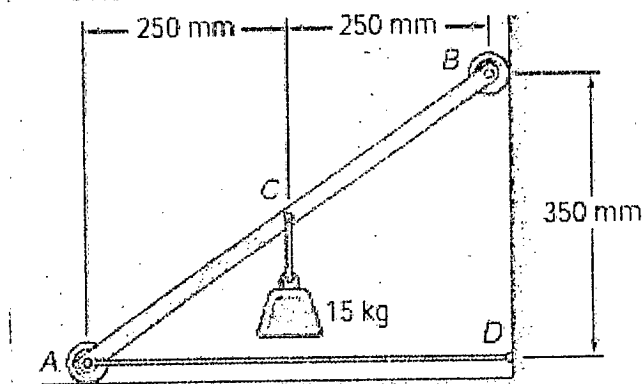


Figure for Question 1(b)

ME 165/CSE

2. (a) In the Figure for Question 2(a), the boom is supported by a ball-and-socket joint at *A* and a guy wire at *B*. If the two 5-kN loads lie in a plane which is parallel to the *x-y* plane, determine the *x*, *y*, *z* components of reactions at *A* and the tension in the cable at *B*. (17)

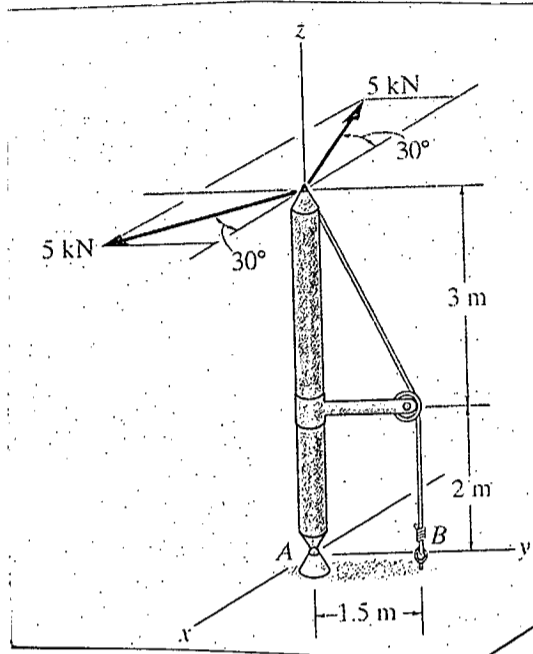


Fig. for Q. No. 2(a)

- (b) Determine the forces in members *CE*, *FH*, and *EF* of the truss in the Figure for Question 2(b). Mention whether the members are in tension or compression. (18)

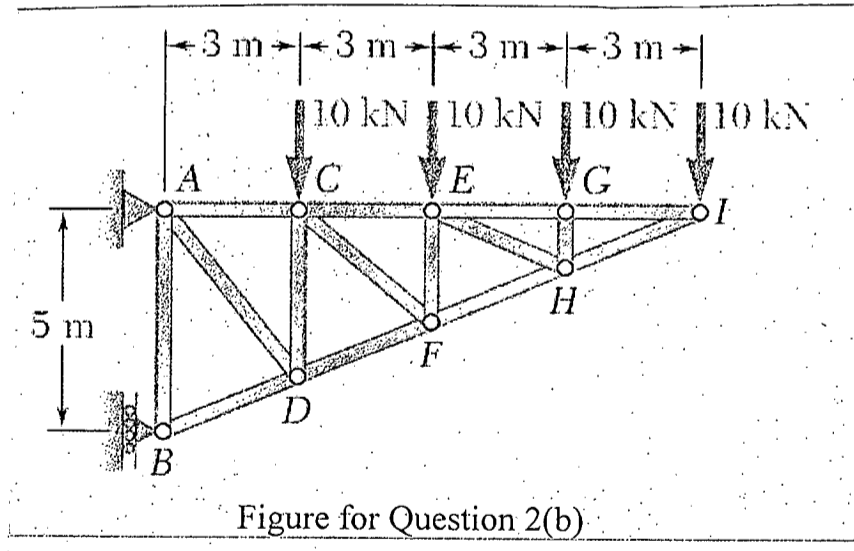
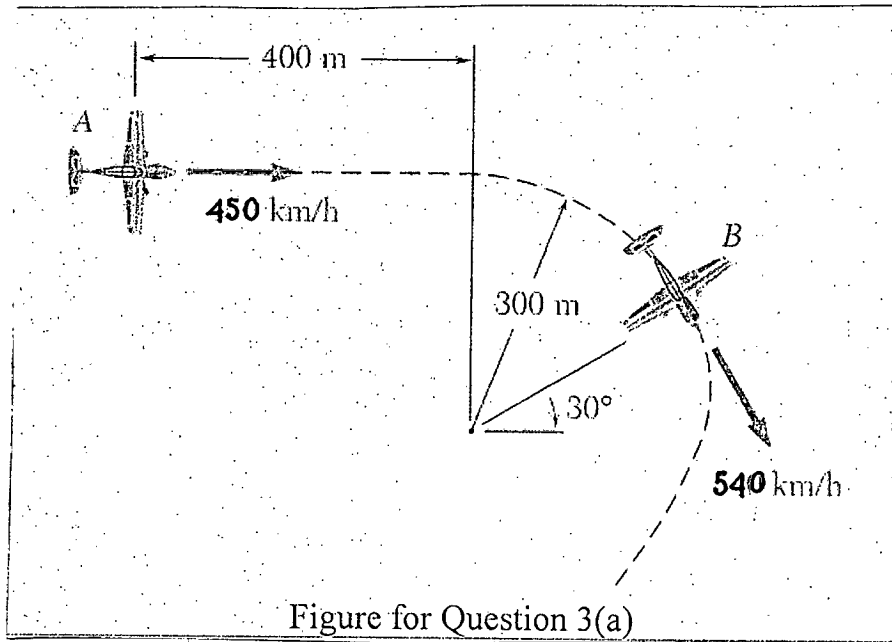


Figure for Question 2(b)

3. (a) At a given instant in an airplane race (see Figure for Question 3(a)), airplane *A* is flying horizontally in a straight line, and its speed is being increased at the rate of 8 m/s^2 . Airplane *B* is flying at the same altitude as airplane *A* and, as it rounds a pylon, is following a circular path of 300-m radius. Knowing that at the given instant the speed of *B* is being decreased at the rate of 3 m/s^2 , determine, for the positions shown, (a) the velocity of *B* relative to *A*, (b) the acceleration of *B* relative to *A*. (17)

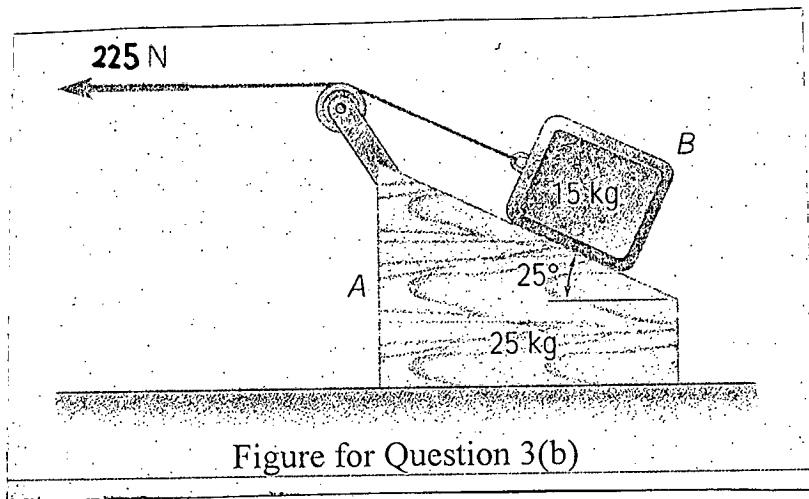
ME 165/CSE

Contd... Q. No. 3 (a)



(b) In Figure for Question 3(b), the 15-kg block B is supported by the 25-kg block A and is attached to a cord to which a 225-N horizontal force is applied as shown. Neglecting friction, determine (a) the acceleration of block A, (b) the acceleration of block B relative to A.

(18)



4. (a) Draw and label the valve timing diagram of a typical 4 stroke-cycle SI engine. With relation to this diagram, explain a) Valve overlap, and b) Spark advance. (10)
- (b) A typical car engine has bore \times stroke = 79 mm \times 77 mm and its speed is 3000 rpm. What is the capacity of the engine in cc? Calculate the mean piston speed for this engine. (6)
- (c) With neat sketch explain how cams control the timing of inlet and exhaust valve's movement. (7)
- (d) What is the key difference between knocking in CI engine and SI engine? (6)
- (e) Write very brief notes on VVTi technology and piston rings. (6)

ME 165/CSE

SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols have their usual meanings. Assume any data if necessary.

Two items (Psychrometric Chart and P-h diagram for refrigerant R-134a)

5. (a) Differentiate between the vapor compression and vapor absorption refrigeration system. (5)
- (b) With necessary sketch, discuss how the central air-conditioning system works. (8)
- (c) What are ODP and GWP? Write the chemical formula of the following refrigerants: R22, R600a, R114, R744, and R11. (5+5)
- (d) Tupli wants to make ice using their domestic refrigerator which uses vapor compression refrigeration cycle and R-134a as refrigerant. The refrigerant enters the compressor as saturated vapor at 1.5 bar and leaves the condenser as saturated liquid at 7 bar. She will be using 10 kg of water per hour at a temperature of 30°C for her ice production and she wants to make ice of temperature -5°C. Determine the power input needed to run her refrigerator. The specific heat of ice and water are 2.1 and 4.18 kJ/kgK, respectively, and the latent heat of fusion of ice is 334 kJ/kg. (12)
6. (a) Mention the key challenges in the energy sector of Bangladesh. What are your recommendations to tackle those challenges? (8)
- (b) Write short note on the following (any three): (15)
- (i) OTEC
 - (ii) Wind turbine
 - (iii) Geothermal energy
 - (iv) Pressurized water nuclear reactor
 - (v) Bio-diesel
- (c) An air-conditioning system is to take in outdoor air at 10°C and 30% relative humidity at a steady rate of 45 m³/min and to condition it to 25°C and 60% relative humidity. The outdoor air is first heated to 22°C in the heating section and then humidified by injecting the hot steam in the humidifying section. Assuming the entire process takes place at a pressure of 100 kPa, determine the heat supply needed for this heating purpose. (12)
7. (a) What are the specifications used to describe a robot? What are different joint drive systems used in robotics? (10)
- (b) What is a robot manipulator? How robot manipulators are classified? With necessary sketch discuss SCARA robot arm. (10)
- (c) A transformation in a cylindrical robot can be expressed as following representation:
- $$T_{cyl}(r, \alpha, 1) = \text{Trans}(0, 0, 1) \text{Rot}(z, \alpha) \text{Trans}(r, 0, 0).$$
- Then the arm of the robot is rotated at an angle of $-\alpha$ about the z axis to bring it back parallel to the reference frame. (10+5)

ME 165/CSE

Contd... Q. No. 7 (c)

- (i) Derive the necessary matrix to describe this motion.
- (ii) If we desire to place the origin of the hand frame of this cylindrical robot at $[3, 4, 7]^T$, calculate the joint variables of the robot.

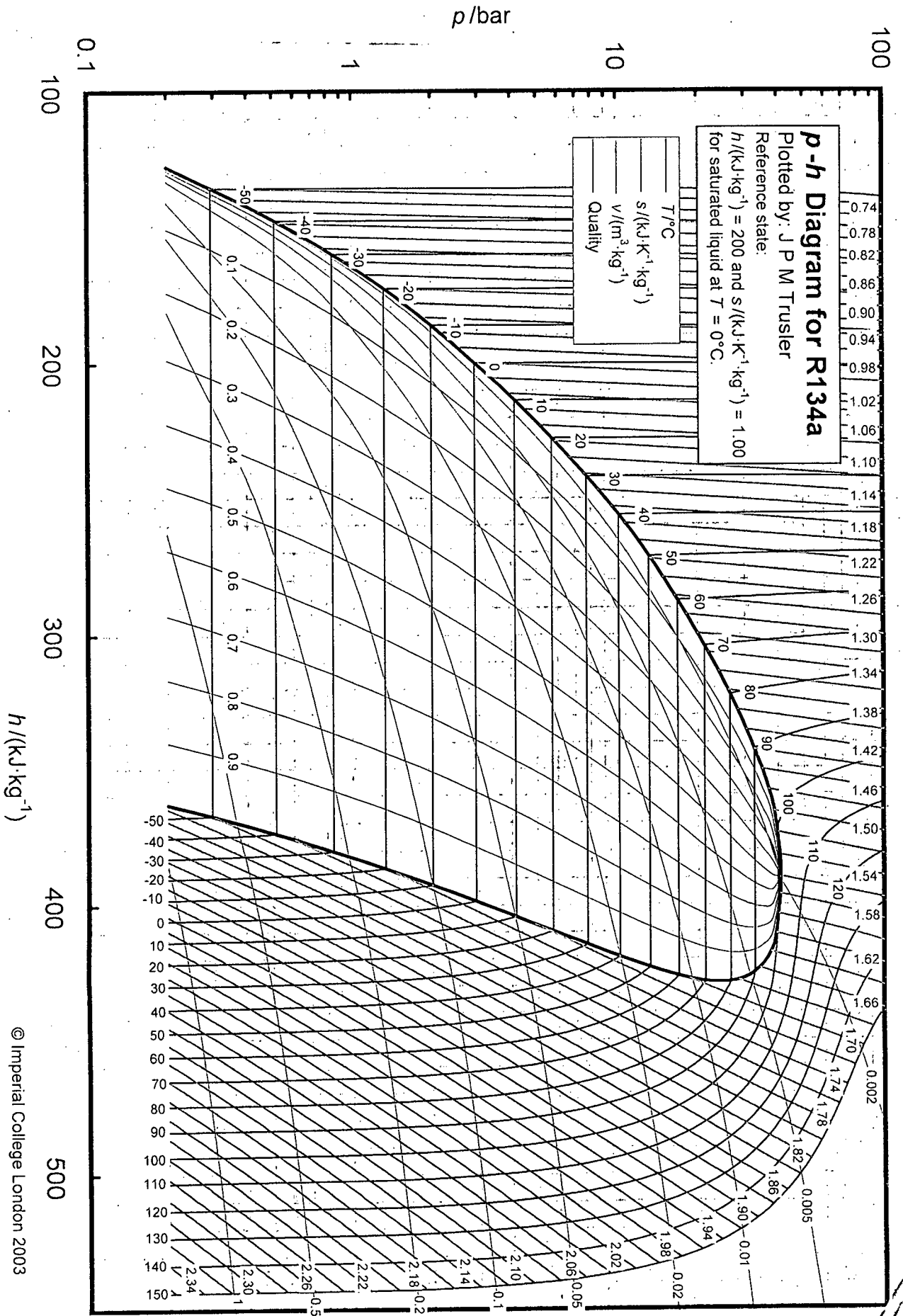
8. (a) What is robot vision? Discuss how a robot vision system works. (10)

(b) Derive the matrix representation of differential rotation about a general axis k . (10)

(c) Find the effect of a differential rotation of 0.1 rad about the y-axis followed by differential translation of $[0.1, 0, 0.2]$ on the given frame B. (10)

where, $B = \begin{bmatrix} 0 & 0 & 1 & 10 \\ 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $\Delta = \begin{bmatrix} 0 & -\delta z & \delta y & dx \\ \delta z & 0 & -\delta x & dy \\ -\delta y & \delta x & 0 & dz \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(d) What do you understand by "Degeneracy" and "Dexterity"? (5)



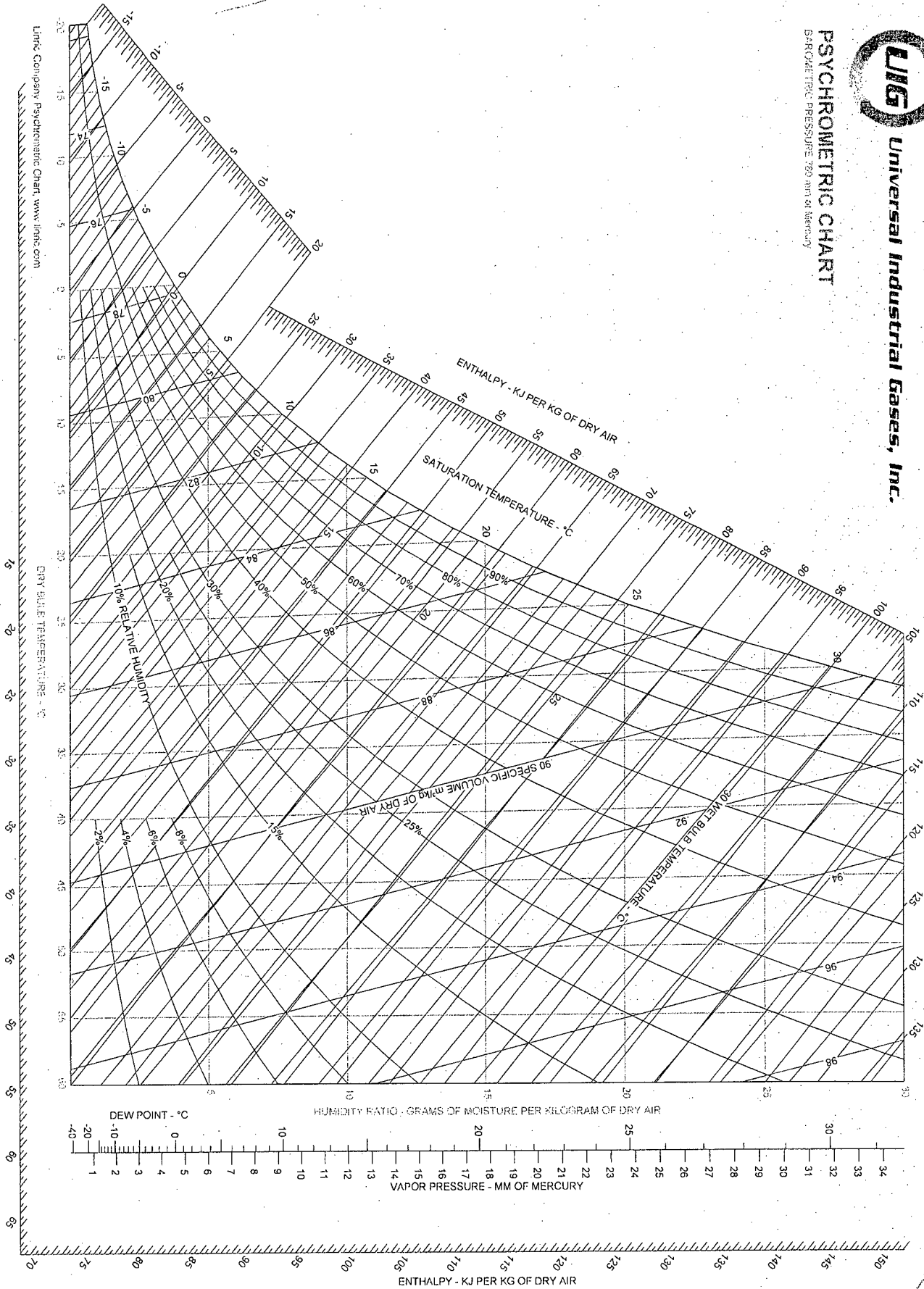
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Sub: **PHY 109** (Heat and Thermodynamics, Electricity and Magnetism, Waves and Oscillations and Mechanics)

Full Marks: 280

Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What is meant by ‘mean free path’ of a molecule in a gas assembly? Derive Maxwell’s expression for the mean free path, (20)

$$\lambda = \frac{1}{\sqrt{2} \pi n d^2}$$

where the symbols have their usual meanings.

- (b) State and explain Maxwell-Boltzmann distribution of molecular speeds. Using Maxwell-Boltzmann distribution law of molecular speeds show that r.m.s. velocity of the molecules is given by (18 $\frac{2}{3}$)

$$v_{rms} = \sqrt{\frac{3KT}{m}}$$

- (c) Calculate the number of molecules per cm^3 of a gas if the mean free path of the molecules is 2.6×10^{-6} cm and the molecular diameter is equal to 2.2×10^{-8} cm. What will be collision frequency if the r. m. s. speed of the molecules is 1×10^5 cm? (8)

2. (a) Explain the concept of entropy? (4)

- (b) Show that the sum of the entropies of all systems taking part in a reversible process remains constant and increases in all irreversible processes. (18)

- (c) Define and explain four fundamental thermodynamic potentials U, F, H and G, where symbols have their usual meanings. What is the significance of these potentials in thermodynamics? (16 $\frac{2}{3}$)

- (d) 10 g of water at 60°C is mixed with 30 g of water at 20°C . Calculate the temperature of mixture. Will the entropy of the system increase or decrease? Calculate the net change in entropy. (8)

3. (a) What are Lissajous figures? By what factors it depends on? Mention a few uses of Lissajous figures. (10 $\frac{2}{3}$)

- (b) Derive a general expression for the combination of two simple harmonic motions acting at right angle to each other having frequency ratio 2:1. From the expression find out the equation of parabola. (26)

- (c) A block of mass m moving at speed v collides with a spring of restoring force $F = -k_1x - k_2x^3$ on a frictionless surface. Find the maximum compression of the spring. (10)

PHY 109/CSE

4. (a) What is particle velocity and wave velocity? Find out a relation between them. (10 $\frac{2}{3}$)
- (b) Show that phase velocity is the same as wave velocity. Derive the differential equation of a progressive wave. (26)
- (c) A plane progressive wave train of frequency 400 Hz has a phase velocity of 480 m/s. (10)
- (i) How far apart are two points 30° out of phase? (ii) What is the phase difference between two displacements of the given point at times 10⁻³s apart?

SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Explain the terms electric flux and solid angle. Derive Gauss's law of electrostatics for an isolated point charge, Q. (12)
- (b) Discuss the advantages of Gauss's law over Coulomb's law. Deduce Coulomb's law from Gauss's law. (12)
- (c) Using Gauss's law, calculate \vec{E} for an infinity long line of charge (e.g. a wire with negligible diameter) with a constant charge per unit length, λ . (12 $\frac{2}{3}$)
- (d) An α -particle, approaching the surface of a nucleus of gold, is a distance equal to one nuclear radius (6.9×10^{-15} m) away from that surface. What are the forces on the α -particle and its acceleration at that point? The mass of the α -particle, which may be treated here as a point, is 6.7×10^{-27} kg. (10)
6. (a) What are solenoid and toroid? Derive expression of \vec{B} for solenoid and toroid. (18)
- (b) Briefly explain the term self inductance and hence derive expressions for self inductances for toroids having circular and rectangular cross sections. (20 $\frac{2}{3}$)
- (c) A rectangular loop of N turns and of length, a , and width, b , is rotated at a frequency, ν , in a uniform magnetic field of induction \vec{B} . (i) Show that an induced emf given by $E = 2\pi\nu NabB \sin(2\pi\nu t) = E_0 \sin(2\pi\nu t)$ appears in the loop. (ii) Design a loop that will produce an emf with $E_0 = 220$ V when rotated at 50 rev/s in a field of magnetic induction of 5000 Gauss. (8)
7. (a) Briefly explain (i) Conservation of linear momentum, and (ii) Conservation of angular momentum. (8)

PHY 109/CSE

Contd... Q. No. 7

(b) (i) Define the terms: Torque acting on a particle and angular momentum of a particle. (ii) Derive the relations: $\tau = I\alpha$ and $L = I\omega$, where their symbols have their usual meaning.

(23 $\frac{2}{3}$)

(c) Consider the motion of a body of mass m (e.g. planet or satellite) about a massive body of mass M (e.g. sun or earth), where M to be at rest with the body m moving about it in a circular orbit of radius, r . Show that the total energy, $E = -\frac{GMm}{2r}$, where G is the universal gravitational constant. What is the significance of this negative energy? Justify your answer graphically.

(15)

8. (a) Derive the time independent Schrödinger equation for a particle moving in a potential. Write down the physical significance of wave function.

(15)

(b) Consider a particle moving freely in a one dimensional box of length L . The potential energy of the particle can be written as:

(21 $\frac{2}{3}$)

$$V(x) = 0, \quad 0 \leq x \leq L$$
$$= \infty, \quad x < 0, x > L$$

(i) Find out the energy, and (ii) the normalized wave function of the particle.

(c) Define Hamilton operator. Show that momentum operator commutes with the free particle Hamilton operator, $[\hat{H}, \hat{p}] = 0$.

(10)
