THE ANALYTICAL SOLUTION OF THE NONLINEAR SHALLOW WATER EQUATIONS BY USING LAPLACE VARIATIONAL ITERATION METHOD

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CERTIFICATE OF APPROVAL

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AUTHOR'S DECLARATION

I hereby announce that the work which is being presented in this thesis entitled

"THE ANALYTICAL SOLUTION OF THE NONLINEAR SHALLOW WATER EQUATIONS BY USING LAPLACE VARIATIONAL ITERATION METHOD"

Submitted in partial fulfillment of the requirements for the decoration of the degree of Master of Science, Department of Mathematics, BUET, Dhaka, is an authentic record of my own work.

The work is also original except where indicated by and attached with special reference and figures in the context and no part of it has been submitted for any attempt to get other degrees or diplomas.

All views expressed in the dissertation are those of the author and in no way or by no means represent those of Bangladesh University of Engineering and Technology, Dhaka. This dissertation has not been submitted to any other University for examination either in home or abroad.

Shewly

(MOST SHEWLY AKTAR)

Date: 05 August, 2018

Dedicated To

My Parents & Teachers

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Author

ABSTRACT

In recent time, numerous attention from researchers has been engrossed by the various features of the well-known He's variational iteration method (VIM) and the reconstruction or modified variational iteration method. These methods are more powerful and effective to solve many problems. They provide a sequence which converges to the solution of the problem without discretization of the variables. The key advantage of the current method is that it can expand the convergence area of iterative approximate solutions.

The goal of this thesis is to present an advanced Laplace variational analytical outline, constructed by the Variational Iteration method and Laplace Transform, to solve certain modules of linear and nonlinear differential equations (DEs). Moreover, the method is successfully extended to develop the mathematical model applying the unsteady nonlinear 1D shallow water equations (SWEs) for computing the river depth and velocity flow. An initial solution are carried out to investigate the applicability of the model. Comparison of the result obtained using this method with existing numerical method reveals that the present method is more effective and appropriate for solving the nonlinear shallow water equations.

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NOMENCLATURE

τ_b	Bottom stress
A	Cross-sectional flow area
C_d	Water drag coefficient = 0.025
dv	Small length of a slice
dx	Small length of a slice
$d\tau$	Small time
$f(x)$	Algebraic function
F_p	Pressure force
F_r	Froude number
g	Acceleration due to gravity = 9.81 ms^{-2}
g(t)	Analytical function
h	Depth of river
h_0	Initial depth of river
h_n	n^{th} number iterated depth
L	Linear operator
m	mass of the particle
N	Nonlinear operator
n	Roughness of Chezy and Manning Coefficient = 0.025
P	Wetted perimeter
$P(s)$	Polynomial with the degree of the highest order derivative
$p(z)$	Pressure at level z
R_h	Hydraulic radius = 2.5 m
S_0	Friction slope = 0.0002
t	Time
U	Mean Velocity
u	Velocity
u', w'	Turbulent fluctuations
u_0	Initial velocity flow
$u_0(t)$	Initial iteration
u_n	n^{th} number iterated velocity
$u_n(t)$	Final iteration
W	Width
$w(z)$	Channel width at level z
X, Y	Dimensionless Cartesian coordinates
x_0	Initial value
x_{n+1}	Approximate solution
$z(x)$	Topography
R	Coefficient of Determination

Greek symbols

λ	Lagrange multiplier
τ	Reynolds stress
θ	Slope angle
ρ	Water density
δ	Classical variational operator

Subscripts

b	Bottom
n	Number of iteration

LIST OF FIGURES

CHAPTER 1
INTRODUCTION

1.1 Background of the Study

The largest class of linear and nonlinear differential equations, both ordinary over and above partial, can be solved by the Variational Iteration method (VIM) (He, 1998, 1999, 2000, 2006, 2007, 2010; He et al., 2007; Abdou and Soliman, 2005). This iteration method provides an effective procedure for the analytical solution of a wide and general class of dynamical systems representing real physical problems (Abbasbandy and Shivanian, 2009; Wazwaz, 2008; Das, 2009; Odibat and Momani, 2006; He et al., 2006; Yusufoglu, 2007; Sweilam and Khader, 2007). Recently, the implementations of VIM is applied to solve the singular fourth-order parabolic partial differential equations (PDEs) (Noor et al., 2009) and for the nonlinear equations arising for Heat Transfer (Geng, 2012). In addition, the modifications of VIM for solving differential equations (DEs) have been well established by notable researchers (Tatari and Dehghan, 2009; Soltani and Shirzadi, 2010; Ghorbani and Saberi, 2009). These methods efficiently work for initial value or boundary value problems and for linear or nonlinear, ordinary or partial differential equations and even for stochastic systems. Also, we have the advantage of a single global method for solving ordinary or partial differential equations as well as many types of other equations.

Our work is inspired by the analytical solution of the distributive thrust forces due to storm surges in coastal area (Marin, 2016) and by SWASHES: a compilation of shallow water analytic solutions for hydraulic and environmental studies (Delestre et al., 2016). In the present study, the analytical approximate solution of a nonlinear 1D shallow water equations (SWEs) i. e., governing equations has been deduced with the help of powerful Laplace Variational Iteration Method (LVIM). In what follows, we highlight the main steps of LVIM (Imani et al., 2012), where more details can be found in (Alawad et al., 2013; Eman and Tarig, 2014; Khuri and Sayfy, 2012; Abassy et al., 2007) among many others. The explicit solutions of the equations by using different initial values for different cases have been derived, which accelerate the rapid convergence of the series solution. The present method performs extremely well in terms of efficiency and simplicity. Analytical results of the problem are presented graphically using MAT LAB. An existing numerical result is compared to show the pertinent features of this method and a good agreement of the results is observed.

In this work, we consider the governing equations namely the one-dimensional nonlinear shallow water equations for the river where fluid (water) in a river of unit width was contemplated. The vertical velocity of the water was supposed to be insignificant and the horizontal velocity $u(x, t)$ as unevenly constant throughout the river cross section. This can be said to be true for small waves having a wavelength greater than the depth. The depth of fluid given by $h(x, t)$ and horizontal velocity $u(x, t)$ are variables for which we will seek analytical solutions. The hydrodynamics of coastal lagoons, shallow lakes, reservoirs, wide rivers and estuaries may be simulated using the nonlinear SWEs (Arshad et al., 2016; Aydin and Kanoglu, 2017). Natural flow geometries are invariably irregular and may contain significant local changes in bathymetry and bed roughness (Afzalimehr and Maddahi, 2017). This thesis describes an alternative approach whereby the unsteady shallow water equations for the river are solved on the adaptive analytical scheme.

1.2 Rationale of the Study

Nonlinear shallow water equations of river are considered in one spatial dimension. The first aim of this thesis is to find an initial condition which are physically suitable; that is, they let the waves move freely out of the domain and do not reflect them at the boundary in a nonphysical way. An initial condition with bed topography is proposed for solving shallow water equations. The second aim in this thesis is to analytically implement the initial condition in an analytical effective way. This is achieved by a suitable extension of LVIM by joining Variational Iteration Method (VIM) and the Laplace Transform (LT) of second order for the horizontal river flow in space and time evaluation.

1.3 Objective of the Study

The specific objectives of the present research work are twofold:

1. To develop an analytical solution of the nonlinear shallow water equations.
2. To derive the general formation of water depth and velocity flow of rivers, flooding, dam breaks, tsunami, etc.

The outcome of the study is the general formation of the solution of the shallow water equations by using LVIM. The results obtained from the general formation of the study in different cases may be used in rivers, flooding, dam brakes, tsunami etc.

1.4 Outline and Motivation

In this thesis, we present the analytical solutions of the nonlinear 1D shallow-water equations for the river, which to our knowledge has not been shown before. LVIM combines a Laplace Transform and several additional changes of VIM to apply the equations in nonlinear form. We obtain analytical approximate solutions of the equations by this reconstruction method. The model for shallow water over a river flow approaching is a nontrivial Cushman-Roisin's model (Cushman-Roisin, 2014). The motivation for adding a river flow to find velocity and depth is due to the fact that background of river flow changes the behavior of velocity and depth in nature.

In Chapter 1, we begin with some background and the general approach, objective and rationale of the study. The remainder of the thesis is organized as follows:

Chapter 2 reviews recent scientific works connected to the development of the model related to the shallow water. We explain about analytical solution.

Chapter 3 describes the methodology of the research work. We present about open channel flow specially river flow, depth of flow and natural flow variability. We introduce the equations of conservation of mass and the derivation of conservation of momentum. Moreover, Chapter 3 stands on the basic ideas of VIM and Limitations of VIM. We introduce new identification of Lagrange multiplier's by using the Laplace Transform. We simply provide the mathematical framework of the LVIM.

Chapter 4 introduces model development of the shallow water for river flow. We construct iteration formulations to solve Shallow water equations by using LVIM. The investigation of finding the analytical solution of the governing equations for river is the main focus.

Chapter 5 provides results and discussion with model applications of the SWEs. In order to illustrate the methodology, an initial condition is investigated and applied as model verification and validation.

Chapter 6 summarizes and concludes the study. We give some remarks and future works based on this study.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

In the applied sciences, most of the natural events can be modeled with the nonlinear equations because of the nature of the phenomena in the universe. Even though it is easy to find the solution of some problems by means of computers, it is still difficult to solve nonlinear problems either numerically or analytically in physics, engineering, chemistry and biology fields such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical kinematics, chemical physics and geochemistry. Besides, solving these nonlinear problems is a great purpose which can make researchers encounter difficulties in finding exact solutions and in some cases have been quite untouched. In recent years, advances were reported many numerical and analytical methods in literature to solve nonlinear problems of partial differential equations. Some of these well-known methods include Finite Difference Method (Kobelkov and Drutsa, 2013), Finite Volume Method (Benkhaldouna and Seaid, 2010), Homotopy Perturbation Method (Momani and Odibat, 2007; Wang, 2008; Kadem and Baleanu, 2011), Homotopy Analysis Method (Rashidi et al., 2008), Homogeneous Balance Method (Abdelsalam, 2017), Adomian Decomposition Method (Adomian, 1983, 1988; Ray and Bera, 2005; Duan et al., 2012), G'/G Method (Bekir and Aksoy, 2012), Differential Transportation Method (Hamed et al. 2016), and Variational Iteration Method (Mungkasi and Wiryanto, 2016; Fazhan andYingzhen, 2008; Saeid and Ahmad, 2008). Moreover, real problems can be modelled mathematically. A mathematical model needs to be solved in order to find solutions to the real problems. Exact solutions to mathematical models are generally hard to find. Therefore, approximations are another way to find solutions to the real problems. A well-known mathematical model for water flows is the Saint-Venant equations also known as the shallow water equations (SWEs). They are important tools to simulate a variety of problems related to coastal engineering, environment, ecology, etc. Research on methods of solution of the shallow-water equations has received considerable attention in the past two decades. Shallow-Water equations have been proposed by Adh'emar Barr'e de Saint-Venant to model flows in a channel (Barr´e de Saint-Venant, 1871). Nowadays, they are widely used to model flows in various contexts, such as rivers (Churuksaeva and Starchenko, 2015; Goutal and Maurel, 2002), flooding (Neal et al., 2015), dam breaks

(Zhang and Lin, 2016), tsunami (Al-Khaled and Al-Safeen, 2014). These equations consist of a nonlinear system of partial differential equations (PDEs), more precisely conservation laws describing the evolution of the depth and mean velocity of the fluid.

The Shallow Water Equations are derived from the depth-averaged Navier-Stokes equations (Azerad and Guillen, 1999; Randall, 2006; Bresch and Noble, 2011). The SWEs do not have a general exact solution that can be written in an explicit (closed) form. Therefore, a method to approximate the exact solution is needed. Among the numerical schemes: Well-Balanced Scheme (Audusse et al., 2004), Central Scheme (Balbas and Karni, 2009), Upwind Scheme (Bermudez et. al., 1998) can't properly handle shocks and contact discontinuities. Furthermore, on the whole, these schemes are well-known to be quite imprecise for near steady states, by means of the structure of their numerical truncation errors is in general not compatible with exact physical steady state conditions. Also the well-balanced version is less affected—despite the order of resolution—than the standard scheme where the derivative of the bottom topography presents strong variations. Again, these schemes can't balance, when the bottom surface is not flat and when the shallow water equations have a source term containing the gradient of the depth from a fixed reference level. Another difficulty arises from the non-homogeneity of the physical flux when we deal with the two-dimensional problem by using unstructured meshes. The finite-volume method (Benkhaldoun and Seaid, 2010) has the advantage of being able to conserve the basic quantities such as mass and momentum but no boundary conditions are needed for the numerical fluxes at some stage. And this arises a great problem because in many practical situations the geometrical domains have irregular boundary. However, the inclusion of source terms, e.g., those terms relevant to bed topography and bed shear stress, is often necessary to permit the modeling of realistic problems. For example, modeling tidal flows in estuary and coastal water regions usually requires consideration of the bed topography (Zhou et al., 2002; Shiue et al., 2011; Delestre et al., 2016). The SWEs can be shown to fail to predict a hydraulic jump accurately if the bed shear stress terms are neglected.

Therefore, computing the numerical solutions of the SWEs is not trivial due to nonlinearity, the presence of the convective term in some form of shallow water model and the coupling of the equations through the source term. In many applications, the convective terms are distinctly more important than the source terms; particularly when certain non-dimensional parameters reach high values (e.g. the Froude number), these convective terms are a source of computational difficulties and oscillations (San and Kara, 2011; Santillan et al., 2016).

Although, at all times it is likely to apply numerical solutions to resolve a number of nonlinear equations, the precision and computational manual labor demanded for these methods greatly respond to their numerical competency and stability. Additionally, the advantage of analytical approximation is that they allow extensive comprehension of the nature and quality of nonlinear equations. The analytic solutions are described in a unified formalism to make a consistent set of test cases. HPM (Rashidi et. al., 2008; Ganji et al., 2010; Al-Khaled and Al-Safeen, 2014), ADM (Al-Khaled and Allan, 2004; Dispini and Mungkasi, 2016) methods can be used to solve the non-linear shallow water equations with precise approximation, but these approximations are suitable only for a small range, because boundary conditions are satisfied via these methods. And therefore, unsatisfied conditions play no roles in the final results. This shows that most of these methods encounter the in-built deficiencies and involve huge computational work. Furthermore, we find that the ADM is relevant for small time and it is not relevant for large time in dealing with unsteady flow problems. Thus, the study of methods for solving the full shallow water equations continues to receive attention. One of the available methods in the literatures is the VIM, proposed by Ji-Huan He (He, 1999, 2000, 2006, 2007; Abdou et al., 2005; He et al., 2007). This technique provides fast convergent consecutive approximations of the precise solution, if such a solution exists; or else some approximations can be used for numerical purposes. Many researchers worked on the convergence of VIM. In order to decrease the computational work that exists in VIM and get better results after a few iterations specially for problems that the linear part is derivative with respect to x and t and can’t be solved by VIM, then a new reconstruction of VIM is proposed. This reconstruction is a combination of Laplace Transform and VIM and is called LVIM or RVIM (Abassy et al., 2007; Khuri and Sayfy, 2012; Imani et al., 2012; Alawad et al., 2013; Eman and Tarig, 2014). This modification reduces the computations and accelerates the convergence. This method is very powerful and efficient technique for solving different kinds of linear and nonlinear system of PDEs. The method presents a useful way to develop an analytic treatment for these systems. The results reveal that this method is very effective and highly promising in comparison with numerical method namely Finite Difference Method (FDM).

In this thesis, we will focus on researching the application of the LVIM to solve nonlinear problems of shallow water flows. Consequently, this effort provides a significant analytic solution to the SWEs that is presently dispersed through the literature of various scientific disciplines. In particular, we shall investigate how relevant the LVIM is to solve nonlinear problems of the shallow water equations.

In current work, we will propose a new analytical scheme that incorporate the technique from the method of characteristics into the reconstruction of VIM. Our main goal is to present an effective method that are simple, easy to implement, and accurately solves the shallow water equations without relying on a Riemann solver (Roe, 1981; Wu and Cheung, 2008). This goal is reached by solving the shallow water equations by applying LVIM with observing an initial condition.

2.2 What is an Analytical Solution?

The exact solution to a differential equation (DE) that can be expressed in terms of polynomial, logarithmic or exponential and trigonometric functions is an analytical solution e.g., Bessel or Error functions. These library functions are so-called “closed-form” solutions. Nowadays, computers have become very commonplace and the definition of ’analytical solution’ has shifted. Solutions to DEs can now mechanically generate from symbolic management correspondences such as Maple, MAT LAB, Mathematica and are normally too erudite to be evaluated without a computer. Once the solution can no longer be evaluated by hand, the distinction between numerical and analytical starts to blur. If we develop algorithms or modeling engineering systems, analytical solutions often offer important advantages:

- ❖ **Pellucidity:** By means of analytical solutions provide math expressions, they propose a pure outlook how variables and interactions between variables affect the result.
- ❖ **Efficiency:** Algorithms and models articulated with analytical explanations are often extra efficient than equivalent numeric executions.

For instance, if we calculate the solution of an ordinary differential equation for different values of its parametric inputs, it is every so often faster, further accurate, and further appropriate to evaluate an analytical solution than to perform numerical integration.

Craig and Read (2010) posited that a renaissance of analytical and hybrid analytical numerical solutions may be forthcoming. They presented multiple recent advances in analytical modelling approaches, some necessary ingredients for future success, and some reasons they showed analytical and semi-analytical approaches succeed. They proposed the following definition in advance:

Analytical Solution: Any solution to a differential equation that can be evaluated to any desired degree of accuracy at a given point in space and time, without modifying the structure of the solution.

Most of the PDEs can be solved approximately by an analytical method such as analytical approximate method (Shawagfeh, 2002), Adomian's decomposition method (Adomian, 1983, 1988) and Liaos's Homotopy Analysis method (Liao, 1997). But these methods could not always satisfy all its boundary conditions that lead error near boundaries. A successful approximation of solution for PDEs is established with no boundary problems by Variational Iteration method (VIM) (He, 1997, 1998, 1999).

Moramarco et al. (1999) presented an analytical solution to the linearized Saint-Venant equation with lateral inflow uniformly distributed along the channel. He showed a run time comparison between the numerical and the analytical solutions that the latter is more efficient computationally and hence is ideal as an element of a distributed rainfall-runoff model.

In this study, we solve the governing equations having no boundary problems by the reconstruction of VIM i. e., by LVIM. Here the governing equations used are the Saint-Venant equations or 1D SWEs which are characteristically non-linear PDEs.

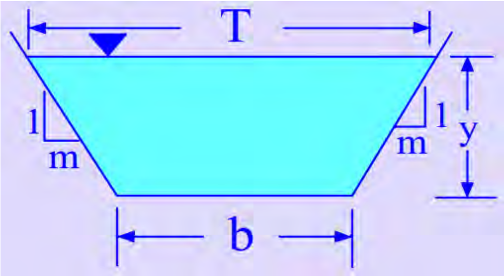
CHAPTER 3
METHODOLOGY

3.1 Basic of the Study

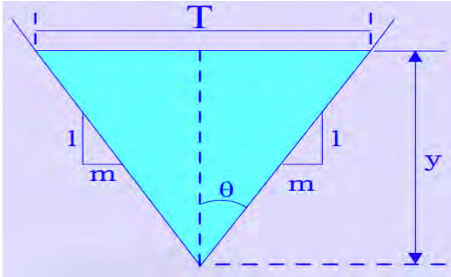
A major computational topic for geophysical fluid dynamics is described here as a contextual of this study. Over a region of interest, limited area models are often used to obtain high resolution. The challenge of applying such models arises from an open channel flow with lateral initial condition. An open-channel flow analysis is more complex because the flow area, wetted perimeter, and hydraulic radius are not necessarily constant as they are in a uniform pipe section under full-flow conditions (see Fig 3.1). Because of this estimating difference, additional characteristics become important when dealing with an open-channel flow (Bautista et al., 2003).

An open channel is a physical structure which conduits water with a free surface at the atmospheric pressure. An open channel may be categorized as either artificial or natural channel corresponding to its formation and origin (Chow, 1959). Exist naturally in the earth, natural channels consist of all water sources of varying sizes from streams, miniature hillside rivulets, small and large rivers to tidal estuaries. Streams with subsurface carrying water with a free surface are also considered as natural open channels. The cross sections of natural channels are uneven and discontinuous and hydraulic properties also vary from section to section. A completed and detailed study of the function of flow in natural channels requires knowledge of other fields, such as hydrology, geomorphology, sediment transportation, geochemistry, physics and solid state physics. Mainly, river mechanics called fluvial hydraulics deal with the details with these appearances (Papadimitrakis and Orphanos, 2009; Pittaluga et al., 2014).

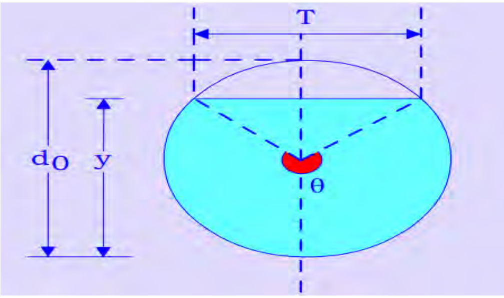
Thandaveswara (2009) drawn the figures (see Fig. 3.1) in his lecture notes and derived the equations and solutions of these channel cross-sections.



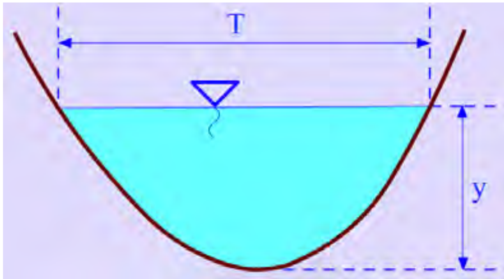
Trapezoidal channel section



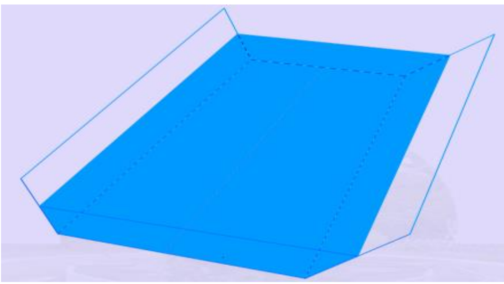
Triangular Channel section



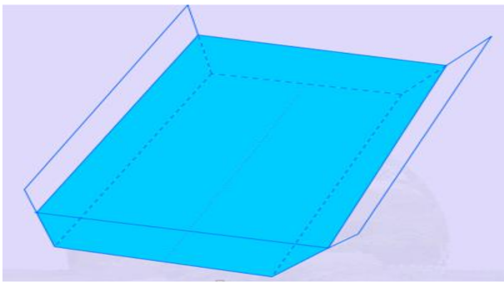
Circular Channel section



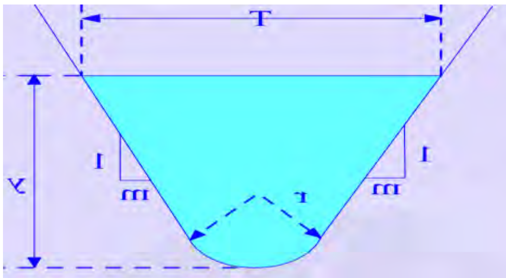
Parabola Channel section



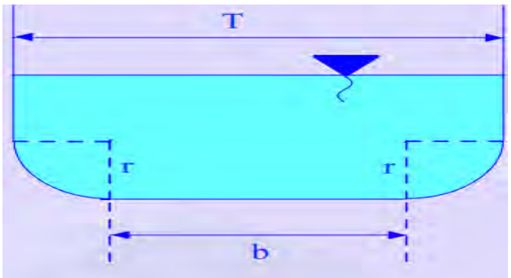
Prismatic Channel



Non-prismatic Channel



Round bottomed triangle



Round cornered rectangle (y > r)

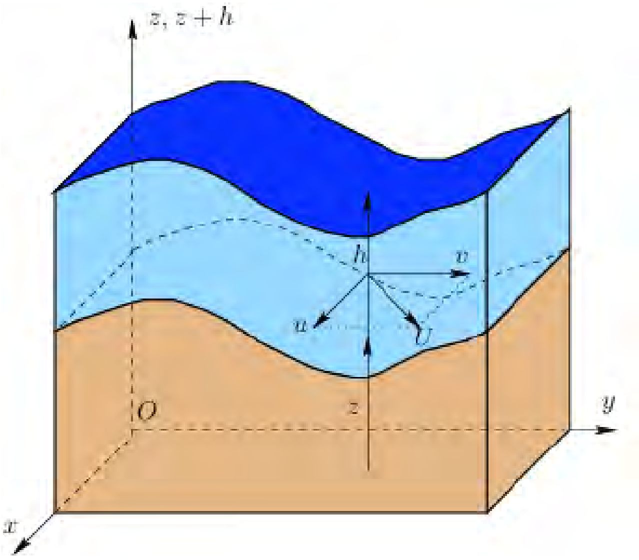
(Source: http://nptel.ac.in/courses/105106114/pdfs)

Fig. 3.1: Geometric elements of different channel sections

Rivers and river mechanisms are one of the most crucial geomorphic structures regulating on the earth system. Rivers are one kind of open channels that conduit of water over a free surface. Contrary to the aqueducts, canals, ditches, and other structures designed and created by humans, the rivers are the product of natural geological techniques and, consequently, are quite irregular. It has the competence to scour its beds, import sediments and deposit these sediments, forever altering its own channels. Rivers are deeper, wider and more tranquil. Moreover, rivers are, in first approximation, nearly one-dimensional flows driven by gravity down a slope and resisted by friction. Notwithstanding, this may have the feature of simple form from a physical aspect, nonlinearities in the dynamic engender complex nature.

We study the Saint-Venant equations for shallow water flows, with non-flat bottom. They are a hyperbolic construction of conservation laws that refer to approximate numerous geophysical flows, such as rivers, coastal areas, and oceans what time accomplished with a Coriolis term, or granular flow what time accomplished with roughness. Numerical approximate solutions to these equations may be generated to appropriately handle surprises and deal discontinuities. But, as a whole, these schemes are quite inappropriate for near steady states, as the structure of their numerical truncation errors is generally not compatible with exact physical steady state conditions. This difficulty can be overcome by means of the so-called analytical scheme. We describe a universal approach, created by a reconstruction that allows us to derive an analytical scheme to solve the nonlinear shallow water equations. When the initial solver satisfies some classical stability properties, it produces a simple and fast analytical scheme that preserves the non-negativity of the water velocity and depth.

3.1.1 Depth of flow



(Source: https://hal.archives-ouvertes.fr/hal-00628246/file/SW_analytic-complements.pdf)
Fig. 3.2: Three dimensional shallow water flow

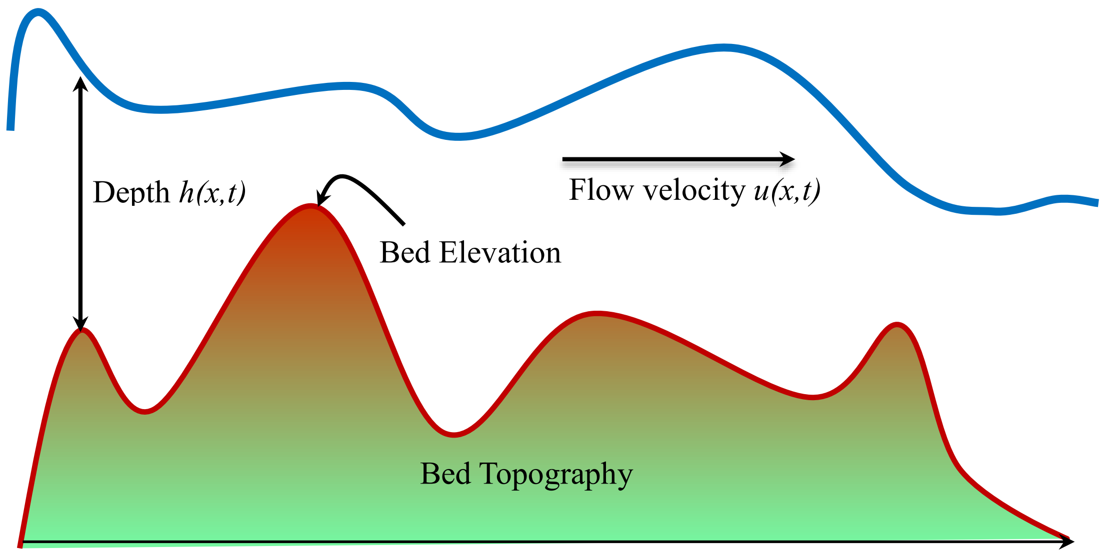


Fig. 3.3: One dimensional shallow water flow over the bed function of a river

With a free surface exposed to the atmosphere, the water depth in a river gradually varies with respect to space and time. River flow is really three-dimensional because the velocity depends respectively on downstream distance, depth and transverse position (see Fig. 3.2). This happens as the friction against the bottom and banks causes the velocity to decrease from a maximum at the surface near the middle of the current flow to zero along the bottom and sides (Vila et al., 2017). Moreover, centrifugal effects in river bends generate secondary circulations that render the velocity a full three-dimensional vector.

For the reason that, the flow varies in the downstream direction, we will disregard cross-flow velocity components along with cross-flow variations of the downstream component, by taking into account the speed u by means of water velocity averaged across the current flow and a function of only the downstream distance x and time t. Again, since the flow in a river almost never reverses, the fact that we take x directed downstream implies that u is a positive quantity. This implies a second flow variable, namely the water depth, which we denote h and consider as a function of x and t. Similar to the velocity u, h is also positive everywhere. The presence of these two dependent variables, $u(x, t)$ and $h(x, t)$, calls for two governing equations (see Fig. 3.3). Naturally, these represent the statements of mass conservation and momentum budget.

3.1.2 Natural river flow variability

Hickin (1995) described by considering briefly the range and variety of potential controls on natural river flow, it is scarcely unexpected that the overwhelming characteristic of natural river flow is its variability. Moreover, flows in natural channels vary over a wide range of time scales:

- Hourly fluctuations can occur in small channels as a response to rainstorms;
- Diurnal fluctuations occurring glacier-fed channels as temperature and melt-rate vary over the day;
- Monthly variations relate to seasonal controls such as the nature of the snowmelt freshet;
- Seasonal aspects of climate can cause differences in storminess.

Additionally, some rivers have high flows that yield two diverse populations: the snowmelt-made seasonal freshet that occurs in the spring each year and quite more erratic rainfall-made storm floods that commonly transpire in the fall.

Rivers are characterized by five primary riverine components and the complex interactions among those components. As noted in Annear et al. (2004) these include the river's hydrology, biology, geomorphology, water quality and connectivity. When the hydrology is changed, each of the other components is influenced to varying degrees. To address the full spectrum of those changes associated with flow modification, studies are typically needed to address each of these five components.

3.2 Derivation of Governing Equation for River

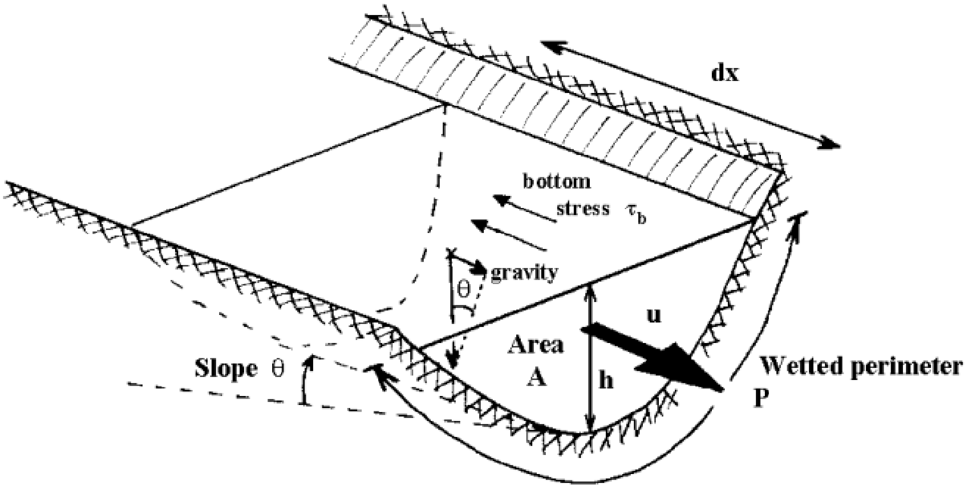
Shallow water equations describing unsteady open channel flows are typically used to model river flows also known as the governing equations.

3.2.1 Conservation of mass

In any control volume consisting of the fluid (water) under consideration, the net change of mass in the control volume due to inflow and outflow is equal to the net rate of change of mass in the control volume. This leads to the classical continuity equation balancing the inflow, out flow and the storage change in the control volume. Therefore, the equation of continuity is

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0 \tag{3.1}$$

3.2.2 Derivation of conservation of momentum



(Source: http://www.dartmouth.edu/~cushman/books/EFM/chap15.pdf)
Fig. 3.4: A slice of length dx along a river for the formulation of mass conservation and momentum budget.

Cushman Roisin (2014) derived the conservation of momentum (see Fig. 3.4). We know that in the control volume, the rate of change of momentum is equal to the net forces acting on the control volume. Then, the time rate of change of momentum inside our slice is the momentum flux entering upstream, minus the momentum flux downstream, plus the sum of accelerating forces (acting in the direction of the flow), and minus the sum of the decelerating forces (acting against the flow).

$$\frac{d}{dt}[momentum\ inside\ the\ slice]$$
$$\begin{aligned} &= momentum\ lux\ entering\ at\ x \\ &- Momentum\ flux\ exiting\ at\ (x+dx) \\ &+ pressure\ force\ in\ the\ rear - Pressure\ force\ ahead \\ &+ Downslope\ gravitational\ force \\ &- Frictional\ force\ along\ the\ bottom \end{aligned} \tag{3.2}$$

The momentum is the mass times the velocity, i.e., $(\rho Au)u = \rho Au^2$. The pressure force F_p action at each end of the slice is obtained from the integration of the depth dependent pressure over the cross section.

$$Pressure\ force = F_p = \iint p dA = \int_0^h p(z)w(z)\,dz \tag{3.3}$$

In which $p(z)$ and $w(z)$ are, respectively, the pressure and channel width at level z, with z varying from zero at the bottom-most point to h at the surface. Under the assumption of a hydrostatic balance, the pressure increases linearly with depth according to

$$p(z) = \rho g(h-z) \tag{3.4}$$

Discounting the atmospheric pressure which acts all around and has no effect on the flow. The pressure force is thus equal to:

$$F_p = \int_0^h pg(h-z)w(z)\,dz \tag{3.5}$$

And is a function of how filled the channel is. In other words, it is a function of depth h. Taking the h derivative (which will be needed later), we have:

$$\frac{dF_p}{dh} = [pg(z-h)w(z)]_{z=h} + \int_0^h pgw(z)\,dz = pg\int_0^h w(z)\,dz = pgA \tag{3.6}$$

The gravitational force is the weight of the water slice projected along the x-direction, which is $mg\ (=\rho \upsilon)$ times the sin of the slope angle θ,

$$Gravitational\ force = [(\rho dv)g]sin\theta = \rho AgS_0dx \tag{3.7}$$

The frictional force is the bottom stress τ_b multiplied by the wetted surface area:

$$Frictional\ force = \tau_b pdx \tag{3.8}$$

The bottom stress is proportional to the square of the velocity. Invoking a drag coefficient C_d, we write:

$$Bottom\ stress = \tau_b = C_d \rho u^2 \tag{3.9}$$

Which resembles a Reynolds stress *($\tau = -\rho\, u'w'$)*, with the turbulent fluctuations u' and w' each proportional to the average velocity u. The frictional force exerted on the slice of water is then:

$$Frictional\ force = \tau_b p dx = C_d \rho u^2 p dx \tag{3.10}$$

We now gather the momentum budget as

$$\begin{gathered}\left[\frac{\rho A u dx|_{at\ (t+dt)} - \rho A u dx|_{at\ t}}{dt}\right] = \rho A u^2|_{at\ x} - \rho A u^2|_{at\ x+dx} + F_p|_{at\ x} \\ -F_p|_{at\ x+dx} + \rho g A S_0 dx - C_d \rho u^2 p dx\end{gathered} \tag{3.11}$$

In differential form,

$$\frac{\partial}{\partial t}(\rho A u) + \frac{\partial}{\partial x}(\rho A u^2) = -\frac{\partial F_p}{\partial x} + \rho g A S_0 - C_d \rho p u^2 \tag{3.12}$$

The gradient of the pressure force becomes

$$\frac{\partial F_p}{\partial x} = \frac{dF_p}{dh} \cdot \frac{\partial h}{\partial x} = \rho g A \frac{\partial h}{\partial x} \tag{3.13}$$

Which simplifies into

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} + g S_0 - C_d \frac{u^2}{R_h} \tag{3.14}$$

In this equation the ratio of the cross-sectional area A over the wetted perimeter P, which has the dimension of a length, is defined as

$$R_h = \frac{A}{P} \tag{3.15}$$

This is called hydraulic radius. Because we consider wide water body, which is much wider than they are deep, the wetted perimeter is generally not much more than the width *($P \approx W$)*, and so the hydraulic radius is approximately

$$R_h = \frac{A}{W} \tag{3.16}$$

So the momentum equation reduces to

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x} + gS_0 - C_d\frac{u^2}{R_h} \tag{3.17}$$

Equations (3.1) and (3.17) are called the Saint-Venant equation i.e., 1D shallow water equation for open channel flow.

In present study, Eqs. (3.1) and (3.17) will be solved analytically to compute the depth and velocity of a river.

3.3. Introduction of Laplace Variational Iteration Method

A novel reconstruction of the variational iteration method (VIM) is presented and new identity of Lagrange multiplier's is developed by means of the Laplace Transform.

3.3.1 Basics Ideas of He's Variational Iteration Method (VIM)

The variational iteration method, which was first proposed by He (He, 1998, 2000; He and Wu, 2006) has been verified by many authors to be a powerful and influential mathematical tool for various kinds of linear and nonlinear complications (Odibat and Momani, 2006, 2007; Mei and Zhang, 2007; Tari and Ganji, 2007; Sweilam and Khader, 2007; Yusufoglu, 2007;Wazwaz, 2008). Contrasting to the traditional numerical methods, VIM needs no discretization, linearization, transformation or perturbation.

Inokuti et al. (1978) proposed a general Lagrange multiplier method to solve non-linear problems, which was first proposed to solve problems in quantum mechanics (Inokuti et al., 1978 and the references cited therein). The main feature of the method is as follows: the solution of a mathematical problem with linearization assumption is used as initial approximate on or trial-function, then a more highly precise approximation at some special point can be obtained. We consider the following general non-linear system

$$Lu(t) + Nu(t) = g(t) \tag{3.18}$$

where, L is a linear operator, N is a non-linear operator and *g(t)* is a known analytical function.

Ji-Huan He (1998) modified the above method into an iteration method (Finlayson et al., 1972; He, 1997, 1998, 1999) in the following way:

$$u_{n+1} = u_n + \int_0^t \lambda[Lu_n(\xi) + N\overline{u_n}(\xi) - g(\xi)]d\xi \tag{3.19}$$

where, λ is a general Lagrange multiplier (Inokuti et al., 1978), which can be identified optimally via the variational theory (Finlayson et al., 1972; Inokuti et al., 1978; Nayfeh et al., 1985), the subscript n introduces the nth approximation, and u_n is reflected as a limited variation (Finlayson et al., 1972), i.e. $\delta\overline{u_n} = 0$.

The successive approximations $u_n(x); n \geq 1$, of the solution $u(x)$ will be readily found upon using the obtained Lagrange multiplier via any selective function $u_0(x)$. Consequently, the exact solution may be obtained by using

$$u(x) = \lim_{n\to\infty} u_n(x) \tag{3.20}$$

Abassy et al. (2007) proposed an amendment of the variational iteration method and used it to provide an approximate power series solutions for some well-known nonlinear problems. Their proposed modification of the VIM facilitates and minimizes the computational work and effectively improves the speed of convergence. Noor et al. (2009) applied a modified VIM for solving singular fourth order parabolic partial differential equations. The proposed modification is made by introducing He's polynomials in the correction functional. Ghorbani and Saberi (2009) modified the VIM by constructing an initial trial function without unknown parameters.

3.3.2. Limitations of VIM

Wu and Baleanu (2013) showed that in some applications when series solution is searched for, variational iteration method has some draw backs which reduce the efficiency of the method due to repeated calculations and calculations of massive unneeded terms.

Moreover, we have studied few problems with the variational iteration method, which does not require small parameter in an equation as the perturbation techniques do.

Generally, in applications of VIM to initial value problems of differential equations, one usually follows the following three steps:

I. Establishing the correction functional;

II. Identifying the Lagrange multipliers;

III. Determining the initial iteration.

The step (II) is very crucial. The applications of VIM method to fractional differential equations FDEs generally used the Lagrange multipliers in ordinary differential equations

(ODEs) which led to poor convergences. This point of view needs some explanations will elucidate the target of the suggested improvement, among them:

- When the Riemann-Liouville (RL) integral appears in the constructed correctional functional, the integration by parts is challenging to apply
- To avoid this problem, the RL integral is replaced by an integer one which allows the integration by parts. This is a very strong simplification but it affects the next steps of the application of the method
- Therefore, the Lagrange multiplier is determined by a simplification not reasonably explained in the literature, so far.

To overcome these drawbacks, Wu and Baleanu (2013) conceived a method how the Lagrange has to be defined from Laplace transform. The technique can be readily and universally extended to solve both differential equations and FDEs with initial value conditions.

3.3.3 New Identification of the Lagrange Multipliers

Let us go back to the original idea of the Lagrange multipliers in the case of an algebraic equation. Primarily, an iteration formula for finding the solution of the algebraic equation $f(x) = 0$ can be constructed as

$$x_{n+1} = x_n + \lambda f(x_n) \tag{3.21}$$

The optimality condition for the extreme $\frac{\delta x_{n+1}}{\delta x_n} = 0$ provides to

$$\lambda = -\frac{1}{f'(x_n)} \tag{3.22}$$

where, δ is the classical variational operator. From (3.21) and (3.22), for a given initial value x_0, we can find the approximate solution x_{n+1} by the iterative scheme for (3.22)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, f'(x_0) \neq 0, n = 0,1,2,\cdots \tag{3.23}$$

This algorithm is well known as the Newton-Raphson method and has quadratic convergence. Now, we extend this idea to finding the unknown Lagrange multiplier. The main step is to first take the Laplace transform to Eq. (3.18). Then the linear part is transformed into an algebraic equation as follows:

$$\mathcal{L}\big[L[u]\big] + \mathcal{L}\big[N[u]\big] - \mathcal{L}[g(t)] = 0 \tag{3.24}$$

The iteration formula of (3.24) can be used to suggest the main iterative scheme involving the Lagrange multiplier as

$$U_{n+1}(s) = U_n(s) + \lambda(s)[\mathcal{L}(L[u_n] + N[u_n] - g(t))] \tag{3.25}$$

In view of $\mathcal{L}$ ($L[u_n]+N[u_n]$) as restricted terms, one can derive a Lagrange multiplier as

$$\lambda(s) = -\frac{1}{s^m} \tag{3.26}$$

With Eq. (3.26) and the inverse-Laplace Transform $\mathcal{L}^{-1}$, the iteration formula (3.25) can be explicitly given as

$$\begin{aligned} U_{n+1}(t) &= U_n(t) - \mathcal{L}^{-1}[\frac{1}{s^m}[\mathcal{L}(L[u_n] + N[u_n] - g(t))] \\ &= \mathcal{L}^{-1}\left(-\frac{1}{s^m}\mathcal{L}(L[u_n] + N[u_n] - g(t))\right) \end{aligned} \tag{3.27}$$

where, the initial iteration $u_0(t)$ can be determined and explained in the classical VIM.

Here, this modified VIM handovers the problem into the PDE in the Laplace s-domain and take away the differentiation with respect to time. This idea has been used in other analytical methods such as the Laplace ADM (Tsai and Chen, 2010; Zeng and Qin, 2012) and the Laplace HPM (Javidi and Raji, 2012) respectively.

3.3.4 Laplace Variational Iteration Method

With the help of Imani et al. (2012), we present here the elementary knowledge of RVIM. We consider the independent variables t as the principle variable and x as the secondary variable and u(x, t) as a function of x and t. When the Laplace Transform is applied to t as a variable, the definition of Laplace Transform is

$$\mathcal{L}\{u(x,t)\} = \int_0^\infty e^{-st}\, u(x,t)dt$$

Also

$$\mathcal{L}\left\{\frac{\partial u(x,t)}{\partial t}\right\} = \int_0^\infty (e^{-st}\,\frac{\partial u(x,t)}{\partial t})dt$$

$$= sU(x,s) - u(x,0)$$

$$where, U(x,s) = \mathcal{L}\{u(x,t)\}$$

Now the convolution of $u(x, t)$ and $v(x, t)$ written as $u(x, t)*v(x, t)$ is defined as the integral of the product of the two functions after one is reversed and shifted.

Convolution Theorem:

If *U(x, s)*, *V(x, s)* are the Laplace Transform of *u(x, t)* and *v(x, t)*, when the Laplace Transform is applied to *t* as a variable, then

$$u(x,t) * v(x,t) = \int_0^t u(x, t-\tau) \cdot v(x,t) d\tau$$

$$i.e., \qquad \mathcal{L}^{-1}\{U(x,s) * V(x,s)\} = \int_0^t u(x, t-\tau) \cdot v(x,t) d\tau \tag{3.28}$$

To illustrate the basic concept of the RVIM, we now consider the following general differential equation

$$L\big(u(x,t)\big) + N\big(u(x,t)\big) = f(x,t) \tag{3.29}$$

where, *L* is a linear operator, *N* is a nonlinear operator and *f(x, t)* is the forcing term.

To facilitate our discussion, we introduce the new linear or nonlinear function

$$H\big(u(x,t)\big) = f(x,t) - N(u(x,t)) \tag{3.30}$$

$$L\big(u(x,t)\big) = H\big(u(x,t)\big) \tag{3.31}$$

Take Laplace Transform on both sides, we get

$$\mathcal{L}\{L\big(u(x,t)\big)\} = \mathcal{L}\{H\big(u(x,t)\big)\} \tag{3.32}$$

Now we introduce artificial initial conditions to zero for the main problem, then the left hand side of (3.32) becomes

$$\mathcal{L}\{L\big(u(x,t)\big)\} = U(x,s) \cdot P(s) \tag{3.33}$$

where, *P(s)* is a polynomial with the degree of the highest order derivative of the selected linear operator.

$$\therefore U(x,s) = \frac{\mathcal{L}\{H\big(u(x,t)\big)\}}{P(s)} \tag{3.34}$$

Suppose, $\frac{1}{P(s)} = D(s)$ & $\mathcal{L}\big\{H\big(u(x,t)\big)\big\} = M(x,s)$, $\therefore\ U(x,s) = D(s) \cdot M(x,s)$. Appling the convolution theorem, we have

$$U(x,s) = D(s) \cdot M(x,s) = \mathcal{L}\big\{d(t) * H\big(u(x,t)\big)\big\}$$

$$i.e., \mathcal{L}\{u(x,t)\} = \mathcal{L}\big\{d(t) * H\big(u(x,t)\big)\big\} = \mathcal{L}[\int_0^t d(t-\tau) \cdot H(u,x,\tau) d\tau] \tag{3.35}$$

Taking the inverse Laplace Transform on both sides of (3.35),

$$u(x,t) = \int_0^t d(t-\tau) \cdot H(u,x,\tau)d\tau \tag{3.36}$$

Thus, the reconstructed method of variational iteration formula can be written as

$$u_{n+1}(x,t) = u_0(x,t) + \int_0^t d(t-\tau) \cdot H(u_n,x,\tau)d\tau \tag{3.37}$$

where, $u_0(x, t)$ is initial solution with or without unknown parameters. In absence of unknown parameters, $u_0(x, t)$ should satisfy initial or boundary conditions.

CHAPTER 4
MODEL DEVELOPMENT

4.1 Introduction

In this chapter, we introduce the shallow water equations as a model for the flow of a fluid and their basic mathematical properties explained. Several extensions required for the source of the depth of flow are described and added to the model. Shallow water equations describing unsteady open channel flows are typically used to model river flows.

4.2 The Shallow Water Model for River Flow

In Fluid Dynamics, the Saint-Venant Equations (Raisinghania, 2003) were formulated in the 19th century by two mathematicians, Adhemar Jean Claude Barre de Saint Venant and Bousinnesque (Raisinghania, 2003). Saint-Venant equations are derived from Navier-Stokes Equations (Vreukdenhil, 1994) for shallow water flow conditions (Kubatko, 2015). The Navier-Stokes Equations are a general model which can be used to model water flows (Besson et al., 2007). A general flood wave for 1-D situation (Besson et al., 2007; Bulatov, 2013) can be described by the Saint-Venant equations.

The following assumptions are used in our model:

- Flow is one-dimensional
- Flow is unsteady and non-uniform
- Hydrostatic pressure prevails and vertical accelerations are negligible
- Streamline curvature is small
- Bottom slope of the channel is small
- Manning’s and Chezy’s equation are used to describe resistance effects
- The fluid is incompressible
- The gravity force is the only one taken into account. So the influence of the Coriolis force is neglected

These assumptions do not affect the basic of the governing equations.

In this study, the shallow water equations, also known as the Saint-Venant equations, for one-dimensional plane flow (Napiorkowski and Dooge, 1988; Moramarco et al., 1999 and Yen, 2004) are used to compute the depth and velocity flow of the river.

4.3 Solutions of Shallow Water Equations

We rewrite the shallow water equations (3.1) and (3.17) as follows

$$\frac{\partial h}{\partial t}+\frac{\partial (hu)}{\partial x}=0 \tag{4.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial h}{\partial x}-gs_0+c_d\frac{u^2}{R_h}=0 \tag{4.2}$$

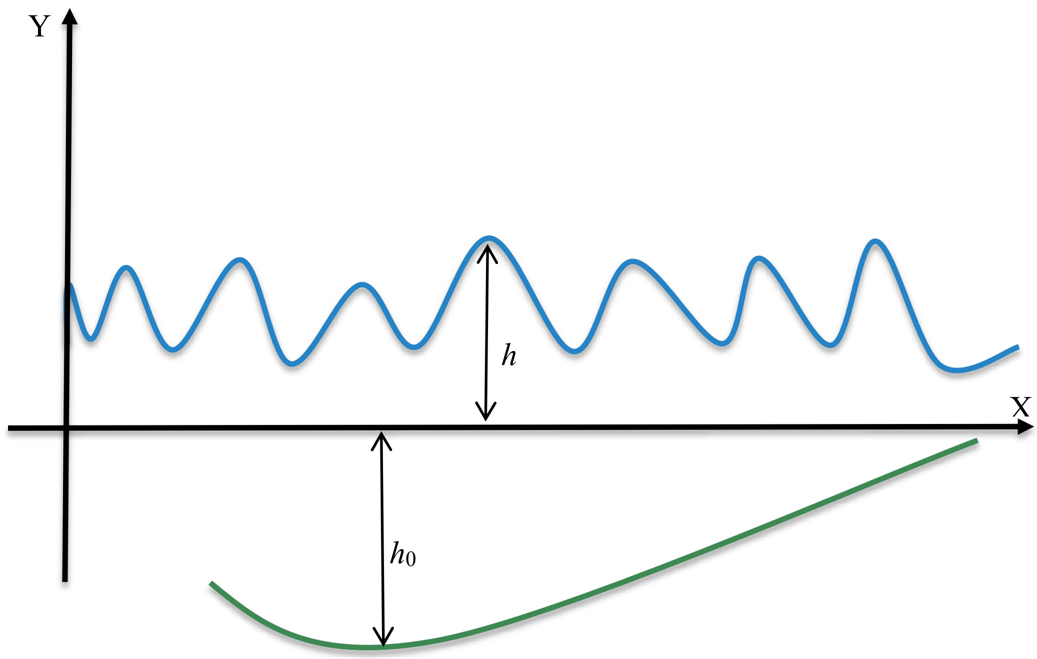


Fig. 4.1: The geometry of shallow water

Klarbring (2015) and Roberts (1994) showed that functions $h = h(x,t)$ and $u = u(x,t)$ represent the total height above the bottom of the channel and the fluid velocity respectively and $h_0 = h(x,0)$ is the depth of the same point but from a fixed reference level (see Fig. 4.1), where, $h_0 = h(x,0) = 0$, h_0 gives the water depth (bottom to reference level) and $u_0 = u(x,0) = 0$.

Applying Eq. (3.31), on Eqs. (4.1) and (4.2) equations, we have

$$L(h,t)=\frac{\partial h}{\partial t}=-\frac{\partial}{\partial x}(hu) \tag{4.3}$$

$$L(u,t) = \frac{\partial u}{\partial t} = -\left(u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} - gs_0 + c_d\frac{u^2}{R_h}\right) \tag{4.4}$$

Appling Laplace Transform with respect to independent variable t to both sides of Eqs. (4.3) & (4.4) and setting all the defined artificial initial conditions to zero, we get

$$\mathcal{L}\left\{\frac{\partial h}{\partial t}\right\} = \mathcal{L}\{H(h,x,t)\}\ i.e., s\mathcal{L}\{h(x,t)\} - h(x,0) = \mathcal{L}\{H(h,x,t)\}\ or, sH(x,s) = \mathcal{L}\{H(h,x,t)\} \tag{4.5}$$

$$\mathcal{L}\left\{\frac{\partial u}{\partial t}\right\} = \mathcal{L}\{H(u,x,t)\}\ i.e., s\mathcal{L}\{u(x,t)\} - u(x,0) = \mathcal{L}\{H(u,x,t)\}\ or, sU(x,s) = \mathcal{L}\{H(u,x,t)\} \tag{4.6}$$

$$\therefore H(x,s) = \frac{\mathcal{L}\{H(h,x,t)\}}{s}, U(x,s) = \frac{\mathcal{L}\{H(u,x,t)\}}{s} \tag{4.7}$$

Using the inverse Laplace Transform and Convolution theorem, we have

$$\mathcal{L}^{-1}\{H(x,s)\} = \mathcal{L}^{-1}\{\mathcal{L}\left\{H(h,x,t) * \frac{1}{s}\right\} = H(h,x,t) * 1$$

$$\mathcal{L}^{-1}\{U(x,s)\} = \mathcal{L}^{-1}\{\mathcal{L}\left\{H(u,x,t) * \frac{1}{s}\right\} = H(u,x,t) * 1$$

$$i.e., h(x,t) = \int_0^t H(h,x,\tau)d\tau$$

$$u(x,t) = \int_0^t H(u,x,\tau)d\tau$$

Therefore, in exchange with applying recursive algorithm, following relations are achieved.

$$h_{n+1}(x,t) = h_0(x,t) - \int_0^t \frac{\partial}{\partial x}(h_n(x,\tau)\cdot u_n(x,\tau))d\tau \tag{4.8}$$

$$u_{n+1}(x,t) = u_0(x,t) - \int_0^t (u_n(x,t)\cdot\frac{\partial u_n(x,t)}{\partial x} + g\frac{\partial h_n(x,\tau)}{\partial x} - gs_0 + c_d\frac{u_n^2(x,\tau)}{R_h})d\tau \tag{4.9}$$

CHAPTER 5
RESULTS AND DISCUSSION

5.1 Introduction

We consider the initial condition with bed topography to develop the approximate results obtained from LVIM.

5.2 Initial Value with Bed Topography

Al-Khaled and Allan (2004) provided numerical solutions of the shallow water equations which verified by considering the initial velocity and the initial height like Eqs. (5.1) and (5.3) respectively with bed topography Eqs. (5.2) of the water to be a known point from a fixed reference level of the water. Mungkasi and Wiryanto (2016) took the initial conditions (5.15) and (5.17) with slight modification of Al-Khaled and Allan to solve governing equations. Later Dispini and Mungkasi (2016) presented their investigation on the relevance of the ADM for the shallow water equations by considering the initial conditions (5.1) and (5.3) for the depth and velocity.

To calculate the related terms of the Laplace Variational series (4.8) and (4.9) for *h(x, t)* and *u(x, t)*, we substitute the initial conditions (5.15) and (5.17) and obtain some of the terms as follows:

$$h_0(x,t) = h(x,0) = \frac{1}{10} + \frac{1}{4}\operatorname{sech}(x) + z(x) \tag{5.1}$$

where,

$$z(x) = \frac{e^{-x^2}}{1+e^{-x^2}} \tag{5.2}$$

$$u_0(x,t) = u(x,0) = 0 \tag{5.3}$$

$$h_1(x,t) = \frac{1}{10} + \frac{1}{4}\operatorname{sech}(x) + \frac{e^{-x^2}}{1+e^{-x^2}} \tag{5.4}$$

$$u_1(x,t) = gt\left\{\frac{1}{4}\tanh(\mathrm{x})\operatorname{sech}(x) + \frac{2xe^{-x^2}}{1+e^{-x^2}} - \frac{2x(e^{-x^2})^2}{(1+e^{-x^2})^2}\right\} - \mathrm{g}S_0\mathrm{t} \tag{5.5}$$

$$
\begin{aligned}
h_2(x,t) = \frac{1}{10} + \frac{1}{4}\operatorname{sech}(x) + \frac{e^{-x^2}}{1+e^{-x^2}} \\
- g\frac{t^2}{2}\left\{\frac{1}{10} + \frac{1}{4}\operatorname{sech}(x) + \frac{e^{-x^2}}{1+e^{-x^2}}\right\}\left\{-\frac{1}{4}\operatorname{sech}(x)\tanh(x)^2\right. \\
+ \frac{1}{4}\operatorname{sech}(x)\left(1-\tanh(x)^2\right) - \frac{4x^2e^{-x^2}}{1+e^{-x^2}} + \frac{2e^{-x^2}}{1+e^{-x^2}} \\
\left. + \frac{12x^2(e^{-x^2})^2}{(1+e^{-x^2})^2} - \frac{8x^2(e^{-x^2})^3}{(1+e^{-x^2})^3} - \frac{2(e^{-x^2})^2}{(1+e^{-x^2})^2}\right\} \\
+ g\frac{t^2}{2}\left\{\frac{1}{4}\tanh(\mathrm{x})\operatorname{sech}(x) + \frac{2xe^{-x^2}}{1+e^{-x^2}} - \frac{2x(e^{-x^2})^2}{(1+e^{-x^2})^2}\right. \\
\left. - S_0\right\}\left\{-\frac{1}{4}\tanh(\mathrm{x})\operatorname{sech}(x) - \frac{2xe^{-x^2}}{1+e^{-x^2}} + \frac{2x(e^{-x^2})^2}{(1+e^{-x^2})^2}\right\}
\end{aligned}
\tag{5.6}
$$

$$
\begin{aligned}
u_2(x,t) = -\frac{t^3g^2}{3}\left\{\frac{1}{4}\tanh(\mathrm{x})\operatorname{sech}(x) + \frac{2xe^{-x^2}}{1+e^{-x^2}} - \frac{2x(e^{-x^2})^2}{(1+e^{-x^2})^2}\right. \\
\left. - S_0\right\}\left\{\frac{1}{4}\operatorname{sech}(x)\left(1-\tanh(x)^2\right) - \frac{1}{4}\operatorname{sech}(x)\tanh(x)^2\right. \\
+ \frac{2e^{-x^2}}{1+e^{-x^2}} - \frac{4x^2e^{-x^2}}{1+e^{-x^2}} + \frac{12x^2(e^{-x^2})^2}{(1+e^{-x^2})^2} - \frac{2(e^{-x^2})^2}{(1+e^{-x^2})^2} \\
\left. - \frac{8x^2(e^{-x^2})^3}{(1+e^{-x^2})^3}\right\} \\
+ gt\left\{\frac{1}{4}\tanh(\mathrm{x})\operatorname{sech}(x) + \frac{2xe^{-x^2}}{1+e^{-x^2}} - \frac{2x(e^{-x^2})^2}{(1+e^{-x^2})^2} + S_0\right\} \\
- \frac{C_dt^3g^2}{3R_h}\left\{\frac{1}{4}\tanh(\mathrm{x})\operatorname{sech}(x) + \frac{2xe^{-x^2}}{1+e^{-x^2}} - \frac{2x(e^{-x^2})^2}{(1+e^{-x^2})^2} - S_0\right\}^2
\end{aligned}
\tag{5.7}
$$

The corresponding figures of the solutions (5.1), (5.4), (5.5), (5.6) and (5.7) are drawn in Figs. 5.1-5.5.

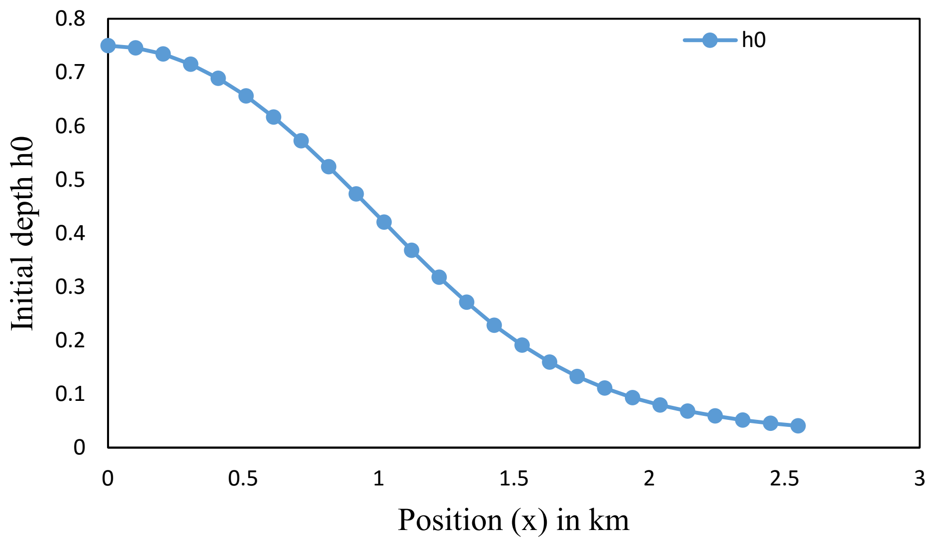


Fig. 5.1: Initial depth h0 with bed topography

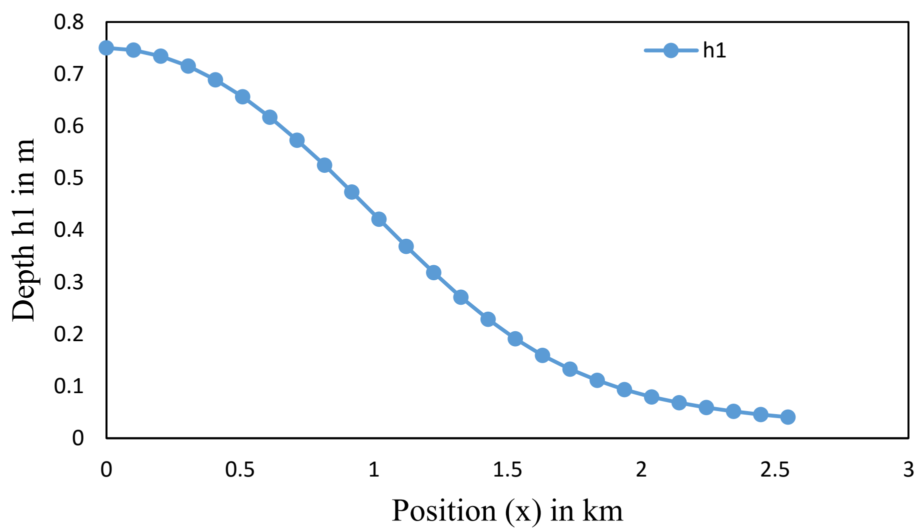


Fig. 5.2: First approximation of depth h1 with bed topography

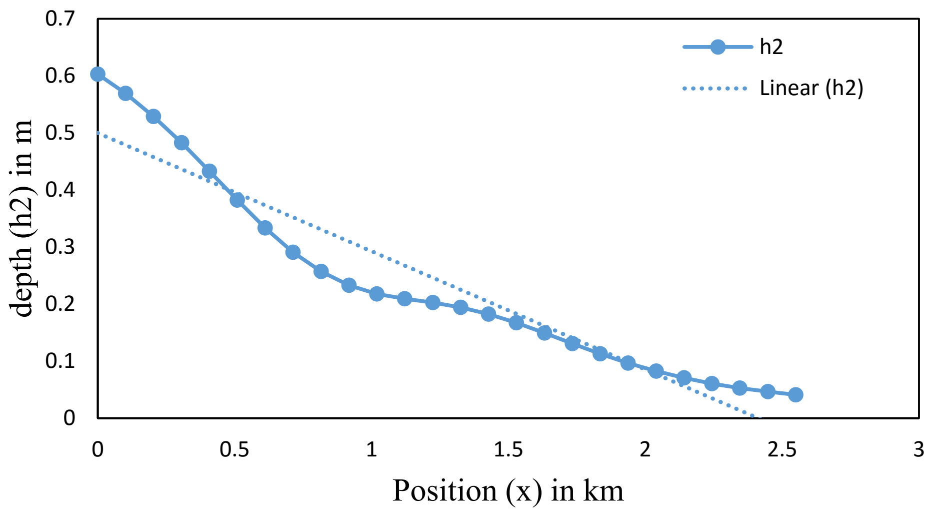


Fig. 5.3: Second approximation of depth h2 with bed topography

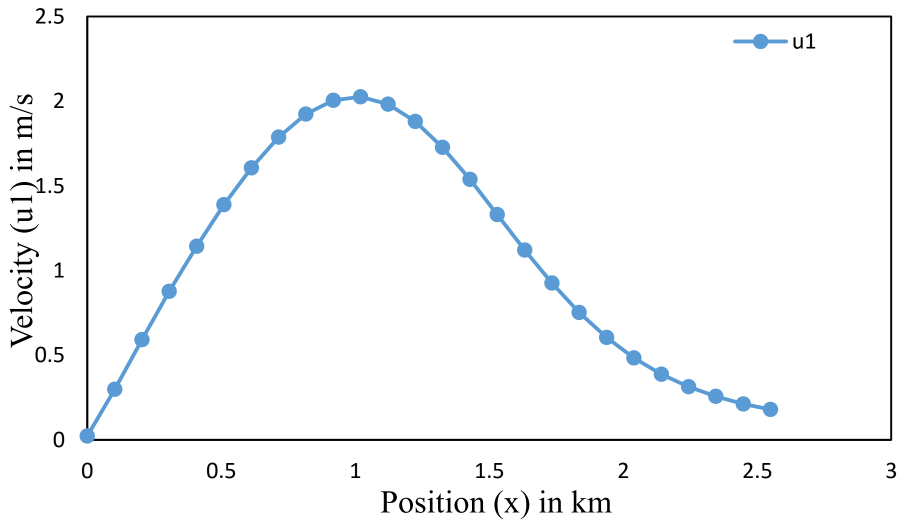


Fig. 5.4: First approximation of velocity u1 with bed topography

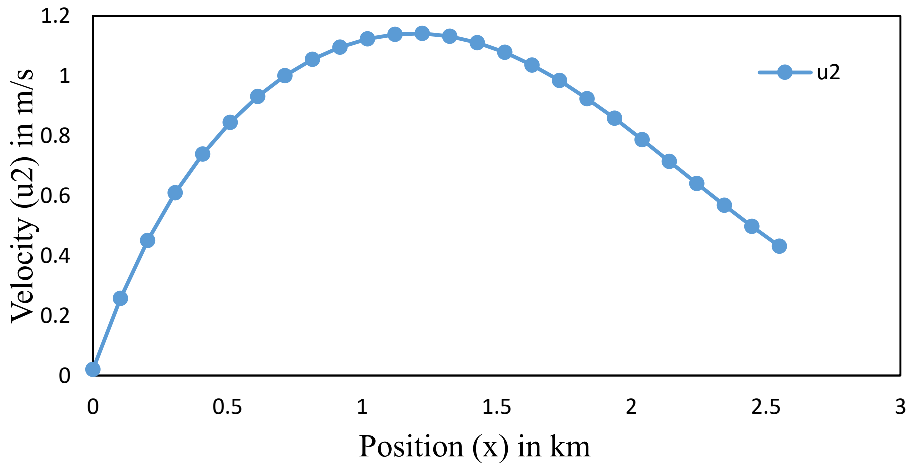


Fig. 5.5: Second approximation of velocity u2 with bed topography

The result shown in Figs. 5.1 and 5.2 represents the initial and first profile of depth of the water. Fig. 5.8 represents the velocity flow of the water, which begins with zero and, at any later instant of time, it changes with positive direction. Fig. 5.3 shows second profile for the depth for different values of space and time. Fig. 5.5 shows again second approximation for the velocity flow. Moreover, many terms can be calculated with no trouble in order to achieve a high level of accuracy of the LVIM method with help of Mat Lab.

In this work, we successfully obtain a reliable approximations up to second order, which accelerates the rate of the convergence of the solution. The main difference now is that there is a no variation in the selection of the components h_0 and h_1. Again the variation of h_1 and h_2 shows that the effect is large in that the modification demonstrates reliability and effectiveness of the newly developed methodology.

5.3 Numerical Solution using Finite Difference Method

The available numerical method for solving the shallow water equations is known as Finite difference method (FDM). The standard definition of finite difference method in elementary calculus is the following:

$$\frac{\partial u}{\partial x} = \frac{u(x+\Delta x) - u(x)}{\Delta x} \ and \ \frac{\partial u}{\partial t} = \frac{u(t+\Delta t) - u(t)}{\Delta t} \tag{5.8}$$

Then the solution of the SWEs (5.1) and (5.2) is solved by applying this finite difference method is given as:

$$u_i = \frac{u_{i-1} + u_{i+1}}{2} - \frac{1}{2\frac{dx}{dt}}(F_{i+1} - F_{i-1}) + \frac{s_{i+1} + s_{i-1}}{2} dt; i = 1,2,3,\cdots \tag{5.9}$$

To verify the analytical solution of shallow water equations with finite difference method, the depth and velocity profiles Eqs. (5.6) and (5.7) is compared with this solution Eqs. (5.9) obtained by FDM.

5.4 Description of a Hypothetical Channel

For our simulations, a hypothetical channel of 5.1km long, 100m wide is considered for the verification of the model which is shown in the Fig. (5.6) (Marin, 2016). As no measured data is available, the model is verified indirectly by comparing the water depth and velocity computed from the analytical solutions Eqs. (5.6) and (5.7) the SWEs by LVIM with the water depth and velocity computed from a FDM using Eqs. (5.9).

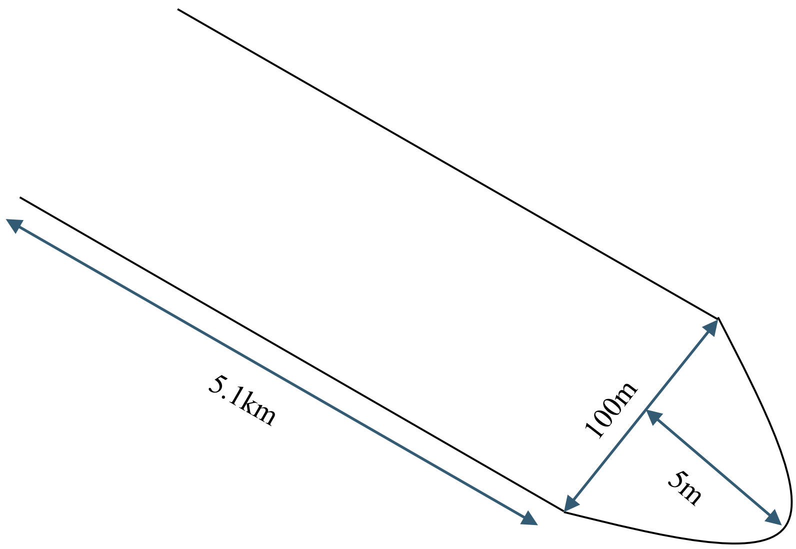


Fig. 5.6: A Hypothetical Channel

5.5 Discussion and Comparison with Results

The second approximations for depth and velocity has been solved analytically and numerically (by Eq. (5.9)) under the hypothetical channel mentioned in Fig. 5.6. The results are validated and the best approximations are presented here. Fig. 6.7 demonstrates the correlation between numerical and analytical results for depth found by FDM and LVIM respectively, and Fig. 5.8 shows their corresponding regression analysis with the coefficient of determination, $R^2 = 0.9741$. Similarly, Fig. 5.9 illustrates the correlation between the velocity profiles found by FDM and LVIM. In Fig 5.10 the result is found by observing the regression analysis with the coefficient of determination, $R^2 = 0.98921$.

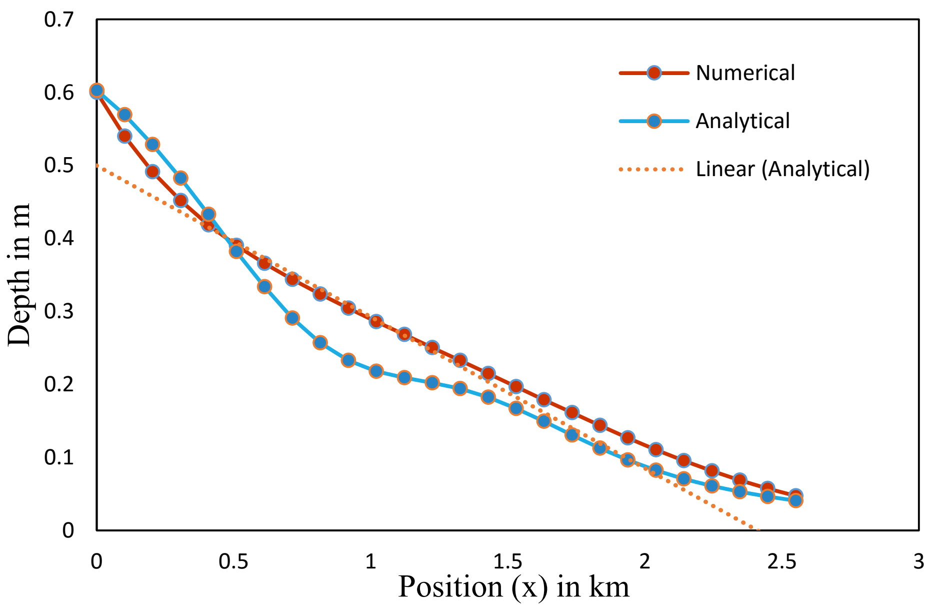


Fig. 5.7: Comparison of numerical and analytical solution of depth

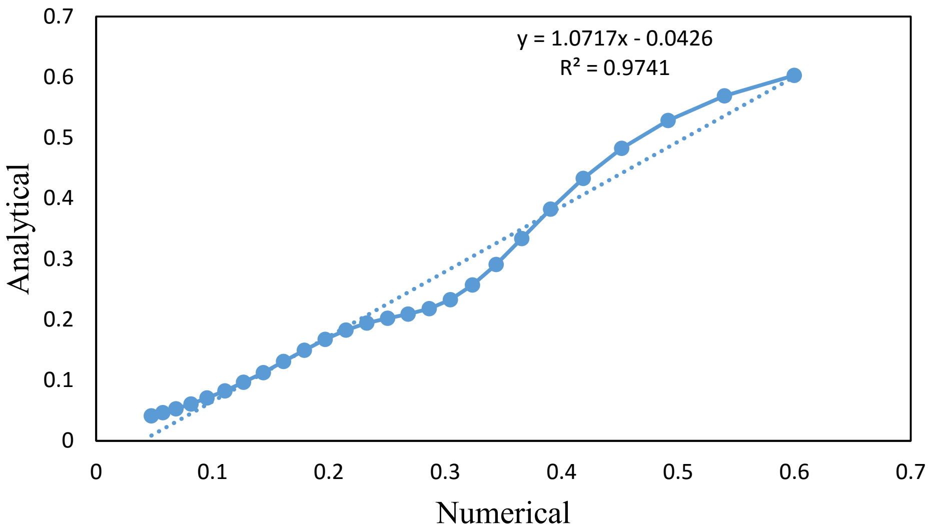


Fig. 5.8: Regression analysis between numerical and analytical solution of depth

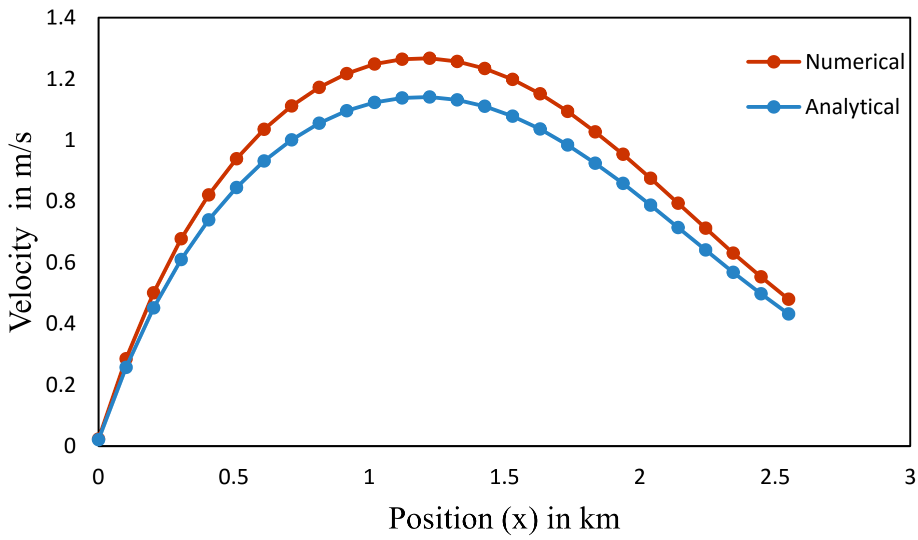


Fig. 5.9: Comparison between numerical and analytical solution of velocity

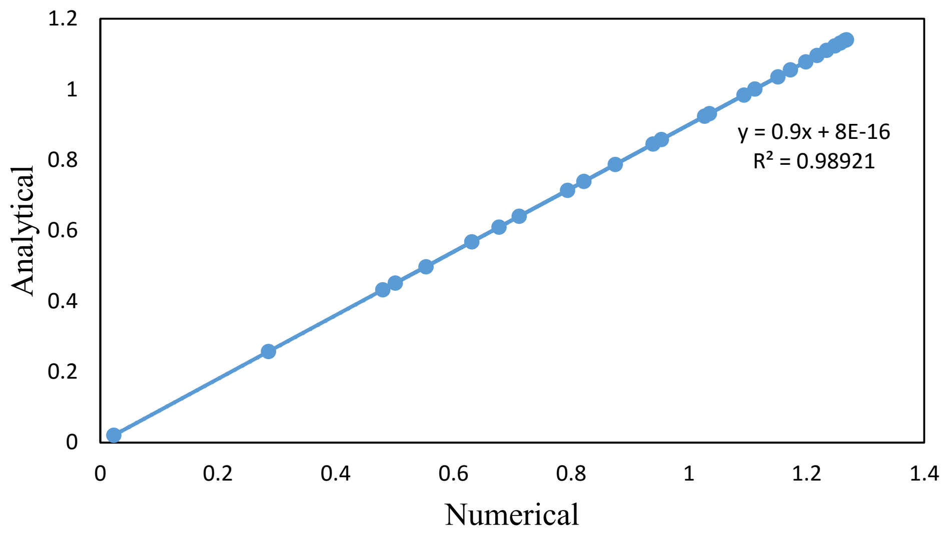


Fig. 5.10: Regression analysis between numerical and analytical solution of velocity

A similar behavior was predicted from the theory of LVIM with respect to FDM and verified from the experiments of SWEs by FDM (Marin, 2016). Therefore, based on these results, we conclude that Eqs. (4.8) and (4.9) are inherently more close approximate solutions of nonlinear SWEs. Eqs. 5.1-5.8 provide one dimensional view of space time evolution of depth and velocity profiles obtained from Eqs. (4.8) and (4.9) respectively. From the images from Figs. 5.1-5.5 and Figs. 5.7-5.10, it is evident that the increase or decrease in depth and velocity profiles can be attributed to strong nonlinear effects in space and time. These effects arise due to nonlinear interactions among the corresponding components and the bed topography due to small channel as mentioned in Fig. 5.6. Numerical studies by FDM (Kobelkov and Drutsa, 2013) help to understand the factors affecting the depth decreasing and nonlinear interactions, which can be analyzed during experimental study.

CHAPTER 6
CONCLUSIONS

6.1 Conclusions

This work reviews the current status of research and development of variational iteration method (VIM); the most useful iteration formulations are listed in a convenient form for later reference and systematic use. Compared to previous convenient tutorial review of this method (He, 2010; Imani et al., 2012; Khuri and Sayfy, 2012; Alawad et al., 2013; Eman and Tarig, 2014; Wang and Atluri, 2016), this thesis aims at communicating necessary theoretical background knowledge required for an in-depth study of the variational iteration method and its application.

This work has constructed the approximate solutions of one dimensional nonlinear shallow water equations (SWEs) by using the Laplace variational iteration method (LVIM). The analytical solutions for depth and velocity flow of the SWEs are up to second order approximations which are obtained by a direct way without using linearization, perturbation or restrictive assumptions. The results for depth demonstrate that there is no variation between the initial and first approximation of the depth, but the variation between first and second approximation shows that the effect is very large and the second approximation converge to the numerical solutions. The corresponding regression analysis of depth and velocity with the coefficient of determination, $R^2 = 0.9741$ and $R^2 = 0.98921$ respectively indicate that the second order approximations for depth and velocity are very close to those obtaining from numerical results. The average error between numerical and analytical solution for depth is 0.0259 and for velocity is 0.01079. These errors are due to second order approximations of the solutions and these errors can be overcome by computing further approximations with similar procedure. The obtained analytical solutions for depth and velocity flow obtained by this methodology can be applied for further works in future. The method was used in so that the results provide more consistency of the functions of depth and velocity of a river and acquire approximate solutions for the depth and velocity of any kind of river.

The advantages of using LVIM are that the solution is easily calculated and that it can be computed at any time and space. Moreover, this method does not change the problem into a convenient one for the use of linear theory. The method delivers more realistic series solutions that generally converge very rapidly in real physical problems. Another benefit of the Laplace Variational iteration methodology is that it does not need any discretization to get analytical or numerical solutions. There are two important facts to make here. First, unlike the implicit and explicit numerical methods used by many authors, the solution here is given in a closed form and the proposed method is successfully implemented by using the initial condition only. Second, the LVIM (RVIM) avoids the cumbersome nature of the computational methods while still maintaining a high level of accuracy. The initial approximation was selected arbitrary not in the form of the exact solution with unknown constants.

Finally, we conclude that LVIM, a reconstruction of He's variational iteration method is successful to solve the SWEs for river, which accelerates the rate of convergence of the solutions for the shallow water model for river. However, there are many events related to shallow water model such as flooding, dam breaks, tsunami etc., which are needed to develop with respect to LVIM. And then the ideas have been provided here will be computationally efficient in applying the proposed technique to several models of shallow water that are important in research. In all cases of the applied fields, this method is needed to implement for solving shallow water related problems. But, it is mentioned that the arrangements of the equations are important in the use of this method.

6.2 Remarks

In this work we demonstrated that the LVIM, which is well suited to solve linear and nonlinear differential equations, is efficient and powerful in solving the one dimensional nonlinear shallow water equations. Some remarks are worth mentioning.

- The conceived reconstruction of the VIM is a universal approach to both ODEs and FDEs as well as PDEs. As a result, it becomes possible to design a 'universal' software package in future work like SWASHES.
- Now one can consider implementing other linearized techniques, i.e., the Adomian series and the Homotopy series to handle the nonlinear terms and improve the accuracy of the approximate solutions by using LVIM.
- The LVIM is straightforward, without restrictive assumptions, and the components of the series solution can be easily computed as far as we like using any mathematical symbolic package.

- ❖ In general the method produces rapidly convergent series, however, in the case of slow convergence, any accelerating technique can be used.

6.3 Limitations of the Work

Following limitations are observed to deal the results of the SWEs of river obtained from the thesis:

1. Because of the imposed initial condition, it is only develop based on initial condition and it is not appropriate to apply the model without verifying the initial conditions with respect to the river.
2. The results are computed up to second order approximation and further approximations are necessary to verify for the accuracy of the solutions with respect to numerical results for the SWEs.

6.4 Future Work

Future directions of research include many other non-linear problems. Possible future works relating to the LVIM are:

1. To derive the general formation of water depth and velocity flow of flooding, dam breaks, tsunami, etc. by using LVIM.
2. To study further about initial and boundary conditions which effect the depth and velocity of a river.
3. To derive the other approximations of the solutions and to verify for more accurate results with respect to numerical solution of SWEs.

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