# Cost Optimization of Fully Continuous Prestressed Concrete I-Girder of Bridge 

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# Cost Optimization of Fully Continuous Prestressed Concrete I-Girder of Bridge 

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A thesis submitted to the Department of Civil Engineering of Bangladesh University of Engineering and Technology, Dhaka, in partial fulfilment of the requirements for the degree of

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## Declaration

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Munshi Galib Muktadir

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#### Abstract

Optimum design of a two-span continuous post-tensioned prestressed concrete I-girder of a bridge super-structure is presented in the thesis. The objective is to minimize the total cost of the girders of the bridge considering the cost of materials, fabrication and installation. The design variables considered for the cost minimization of the girders of the bridge, are girder spacing, various cross sectional dimensions of the girder, number of st rands $p$ er $t$ endon, number of $t$ endons, $t$ endon configuration, $s$ lab thickness a nd ordinary reinforcement for de ck slab a nd gi rder. E xplicit constraints on $t$ he de sign variables are considered on the basis of geometric requirements, practical dimension for construction a nd c ode re strictions. Im plicit c onstraints $f$ or $d$ esign a re c onsidered according to AASHTO LRFD 2007.


The pr esent opt imization pr oblem is characterized by having m ixed c ontinuous, discrete and integer design variables and having multiple local minima. Hence a global optimization a lgorithm called EVOP, is a dopted which is capable of locating directly with high probability the global minimum without any requirement for information on gradient or sub-gradient. A computer program is developed to formulate optimization problem which consists of mathematical expression required for the design and analysis of $t$ he br idge $s$ ystem, three functions: an obj ective $f$ unction, an implicit constraint function and an explicit constraint function and input control pa rameters required by the optimization algorithm. To determine the design moment and shear for the two-span continuous girder a $t$ va rious pos itions of $t$ he $s$ pan, the $c$ omputer pr ogram $w$ as incorporated with computer application of stiffness method to solve the indeterminate girder. No generalized equation for influence 1 ine of indeterminate $g$ irders $w$ as used, rather coordinates of the non-linear influence line were determined using basic stiffness method co ncept an $d$ w ere $u$ sed $t$ od etermine design 1 ive $1 \mathrm{oad} m$ oment a nd s hear. Finally, to solve the problem, the program is linked to the optimization algorithm.

As constant design parameters have influence on the optimum design, the optimization approach is $p$ erformed $f$ or $v$ arious su ch $p$ arameters $r$ esulting in considerable cost savings. Parametric st udies are performed for various girder spans ( $40 \mathrm{~m}, 60 \mathrm{~m}$ and 80 m ), girder concrete strengths ( 40 MPa and 50 MPa ) and three different unit costs of the $m$ aterials including fabrication and installation. From the parametric study, it is found $t$ hat, optimum $g$ irder $d$ epth increases $w$ ith increase in co st of s teels. On a $n$ average, girder depth increases $22 \%$ with increase in cost of steel for 40 MPa concrete. On the other hand, for 50 MPa concrete, the average increase in girder depth comes out to be $19 \%$. Optimum number of $s$ trand is higher in hi gher span $g$ irder. N umber of strand decreases $17 \%$ with increase in cost of steel for 40 MPa concrete. In case of 50 MPa co ncrete, the av erage d ecrease in $n$ umber of st rand is $16 \%$. Girder s pacing is found to be higher ins maller s pan than 1 arger span girder and optimum deck slab thickness comes out to be higher in shorter span as the girder s pacing is hi gher in shorter span.

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## List of Abbreviation

| AASHTO | American Association of State Highway and Transportation <br> Officials |
| :--- | :--- |
| BRTC | Bureau of Research Testing \& Consultation |
| EVOP | Evolutionary Operation |
| PC | Prestressed Concrete |
| PCI | Precast/ Prestressed Concrete Institute |
| RHD | Roads and Highway Department, Bangladesh. |

## Notation

| $A_{\text {net }}$ | $=$ Net area of the precast girder; |
| :---: | :---: |
| $A_{t f}$ | $=$ Transformed area of precast girder; |
| $A_{s}$ | = Area of prestressing steel; |
| $b$ | = Average flange width; |
| $d$ | = Effective depth for flexure; |
| $d_{s}$ | = Effective depth for shear; |
| $d_{\text {req }}, d_{\text {prov }}, d_{\text {min }}$ | $=$ Required, provided and minimum effective depth of deck slab respectively; |
| $e_{i}, e$ | $=$ Eccentricity of tendons at initial stage and at final stage of precast section respectively; |
| EFW | $=$ Effective flange width; |
| $f_{y}^{*}$ | $=$ Yield stress of pre-stressing steel $=0.9 \mathrm{f}_{\text {su }}$; |
| $f_{r}$ | $=$ Modulus of rupture of girder concrete $=0.625 \sqrt{f^{\prime}}{ }_{c}(\mathrm{MPa})$; |
| $f_{p u}$ | $=$ Ultimate strength of prestressing steel; |
| $f_{y}$ | $=$ Yield strength of ordinary steel; |
| $f_{c}$ | $=$ Compressive strength of girder concrete; |
| $f_{\text {cleck }}$ | = Compressive strength of deck slab concrete; |
| $f_{c i}$ | $=$ Compressive strength of girder concrete at initial stage; |
| $F_{1 i}, F_{2 i}, F_{3 i}, F_{4 i}$ | $=$ Prestressing force after instantaneous losses at various sections; |
| $I_{\text {net }}$ | $=$ Moment of inertia of net girder section; |
| $I_{t}$, | $=$ Transformed moment of inertia of precast section; |
| K | = Wobble coefficient; |
| $S_{t}, S_{t c}$ | $=$ Section Modulus of top fiber of transformed precast \& composite section respectively; |
| $S_{\text {deck }}$ | = Section Modulus of deck slab; |
| $Y_{1}, Y_{2}, Y_{3}, Y_{\text {end }}$ | $=$ Centroidal distance of tendons from bottom at section1, section 2 , section 3 and end section respectively; |
| $\mu$ | = Friction coefficient; |
| $\delta$ | = Anchorage Slip; |

## CHAPTER 1

### 1.1 General

In structural design process, the main objective of the designer is to design a structure that will perform its desired performance interms of st rength ( so that saf ety is assured) and serviceability (so that the desired use of the structure is assured). These main two objectives of the designer should be achieved at as low a cost as possible (so that eco nomy is assu red). But in the tra ditional process of structural d esign, the designer focuses mainly on attaining the first two objectives i.e. on attaining required strength and ser viceability and often sacrifices the economy issue. It is not that the designer willingly a voids to $r$ each the de sign which $w$ ill result in the lowest cost, rather he avoids this attempt because it is a very slow, difficult, tedious and therefore costly pr ocess. The reason of not making a a ttempt to reach most cost ef fective design is tried to be focused on in the following writings. The discussion is done from the perspective of bridge design.

### 1.2 Difficulties in Attaining the Most Cost Optimum Design

Suppose it is desired to construct a structure (bridge) across a river (Fig. 1.1, Fig. 1.2, and Fig. 1.3). The width of the bridge (no. of lanes) is fixed. There are many types of options (i.e. type of bridges) to be chosen for this purpose. F or the present case, a girder type br idge ha s be en c hosen. A gi rder b ridge t ransfers load through girder/beam, deck/slab, pier and footing system. For the above case, as the width of the riv er is s mall, nointermediate p ier a nd footing is c onsidered in between two abutments at two banks. The target is now to design the girders and the slab/deck.


Fig. 1.1 River cross section


Fig. 1.2 Girders / Beams


Fig. 1.3 Deck on girders


Fig. 1.4 Increasing span length


Fig. 1.5 Increasing no. of supports

Now, it is seen from Fig 1.4 and Fig. 1.5 that there c an be many combinations of girder and deck type to satisfy the required de sign strength and serviceability. It is evident that if the spacing between the girders is increased, the thickness of the deck has to be increased which will increase the cost of material. Again, if spacing between the girders is reduced, the thickness of the deck could be reduced which will reduce the cost of material. But reducing girder spacing means increased number of girders which will again increase the cost simultaneously. Now, determining which option is the best one in terms of cost-efficiency is a process of trial and error.

For de aling with onl y $t$ wo de sign va riables i.e. thickness of de ck a nd nu mber of girders makes the above problem easy to solve. But if it is considered that increasing number of girder actually $r$ educes 1 oad on $e$ ach gi rder, $t$ hen itise vident that increasing number of girders will not add to the cost in a linear-proportionality as the cross section of each girder is decreasing simultaneously.

Now, a larger bridge is considered in the following section and some other possible design variables are being introduced.

Fig. 1.6 shows the end part of a large bridge showing only two supports of the bridge. One of the supports is the abutment at the bank and the other is the pier and footing in the river.


Fig. 1.6 End portion of a deck girder bridge system

It can be said that, in the way girder spacing, number of girder, cross section of girder, and deck thickness were related to each other in the previous problem, in the same way, int his pr oblem, pi erto pi er s pacing (span length), num ber of pi er, pier dimension, pi er de pth ( according to r iver bo ttom profile), gi rder cross section a re inter-related.

Again, there is footing below every pier to transfer the load safely to underneath soil. There are various types of footing and each type has design variables of its own. The design of footing depends on 1 oad coming to it and the soil below the footing across the river bed.

So, here ar e t he al ready st ated inter-related d esign v ariables of f eam/deck-girder bridge system:

* Deck/slab thickness
* Beam/Girder dimension(cross-section)
* Beam/Girder Spacing (i.e. no. of girder)
* $\quad$ Pier to pier Spacing (i.e. span length)
* Foundation depth
* Foundation type/dimension

There can be thousands of combination of these design variables which will perform the desired strength and serviceability criteria pretty satisfactorily. But which option or combination is the most efficient in terms of cost is the point of interest.

Traditional manual iterative ap proaches $b$ ased on ex perience / heuristics oft he designers w ere very p oor t oh andle s uch c omplex pr oblems. A gain, t raditional computer aided optimum design (using optimization technique) that have been used so far were not efficient enough to deal with this very highly complex problem and could not find the global minimum point for cost.

### 1.3 Background and Present State of Problem

Prestressed Concrete (PC) I-girder Bridge systems are widely used bridge system for short to medium span ( 20 m to 60 m ) highway bridges due to its moderate self weight, structural efficiency, ease of fabrication, fast construction, low initial cost, long life expectancy, 1 ow maintenance an d si mple d eck r emoval \& r eplacement etc. (PCI, 2003). In order to compete with steel bridge systems, the design of PC I-girder Bridge must lead to the most economical use of the materials (PCI, 1999). Large numbers of design variables are involved in the design process of the present bridge system. All the v ariables ar e r elated $\mathrm{t} o$ each o ther l eading t on umerous al ternative f easible designs. In conventional de sign, br idge e ngineers follow aniterative pr ocedure to design the prestressed I-girder bridge structure. Most design offices do not advocate realistic estimate of material costs, just only satisfy all the specifications set forth by design codes. So there is no attempt to reach the best design that will yield minimum cost, weight or volume.

A gl obal opt imization algorithm n amed E VOP ( Evolutionary O peration) ( Ghani, 1989) was used in determining the most cost-efficient design of $30 \mathrm{~m}, 40 \mathrm{~m}$ and 50 m long post-tensioned pre-stressed concrete I-girder bridge system (Rana, 2010). EVOP is capable of lo cating d irectly w ith hi gh pr obability the gl obal m inimum. T o formulate the optimization problem a computer program was developed in $\mathrm{C}++$. The optimization approach was applied on a real life project (Teesta Bridge, Bangladesh) and a $30 \%$ cost efficient design was found.

In $t$ he $t$ hesis $t$ itled "Cost O ptimization of P ost-Tensioned P re-stressed co ncrete $\mathrm{I}-$ Girder B ridge System" (Rana, 2010), the author optimized a simply supported posttensioned pre-stressed co ncrete I-girder of br idge. Next step to ev en m ore co steffectiveness constitutes considering the girders to be continuous. That is, taking the number of span into consideration as a design variable and also taking into account the difference in design of piers and footings in case of corresponding number of spans.

### 1.4 Objectives of the Present Study

Ahsan et al. (2012) has recently demonstrated successful use of a global optimization algorithm - EVOP developed by Ghani (1995) in c ost o ptimum de sign of s imply supported post-tensioned I-girder of bridges. Simple span systems, however, can lead to 1 eakage through $t$ he de ck a nd deterioration of be am-ends, be arings a nd $t$ he substructure. When beams are made continuous, structural efficiency and long-term performance can be significantly improved (PCI, 2003). R ana et al. (2013) a dopted EVOP to o ptimize two-span post-tensioned br idge which was made continuous for superimposed de ad loads and live loads using deck reinforcement. The focus of the present $r$ esearch is $u$ sing EVOP effectively in ha ndling o ptimization problems of prestressed bridge structures made continuous by post-tensioning which by resisting the deck weight can significantly improve the structural performance of longer span bridges.

Hence, objective of the present study is cost minimization of a two-span continuous post-tensioned p restressed co ncrete I -girder o f br idge b y a dopting opt imization approach to obtain the optimum value of the following design variables:
(i) Cross-sectional dimensions of the components of the bridge superstructure (girder \& deck slab) and
(ii) Prestressing tendon size (number of strands per tendon), number of tendons, tendon arrangement \& layout, ordinary reinforcement in deck slab and girder.

### 1.5 Scope and Methodology of the Study

In the present study a cost optimum design approach of a two-span continuous posttensioned PC I-girder of bridge is presented considering the cost of materials, labor, fabrication and installation. The bridge system consists of precast girders with cast-in situ reinforced concrete deck. A large number of design variables and constraints are considered for c ost op timization of t he br idge s ystem. A gl obal opt imization algorithm na med E VOP (Evolutionary O peration) (Ghani, 1995) is u sed which is capable of 1 ocating directly $w$ ith hi gh pr obability the gl obal $m$ inimum. $T$ he optimization $m$ ethod s olves the optimization pr oblem a nd gi ves the opt imum solutions.

In their study, Ahsan et a l. (2012) a nd R ana et a l. (2013) de veloped a c omputer program written in C++ language to formulate the optimization problem which has the following components:
(i) Mathematical e xpression required for the de sign a nd a nalysis of the b ridge system,
(ii) An obj ective $f$ unction (it $d$ escribes $t$ he cost function of $t$ he $b$ ridge $t o$ be minimized)
(iii) Implicit c onstraints o r design c onstraints (it d escribes t he d esign or performance requirements of the bridge system)
(iv) Explicit constrains (it describes the upper and lower limit of design variables or parameters) and
(v) Input control parameters for the optimization method.

In the present study, the subroutines developed by Ahsan et al. (2012) and Rana et al. (2013) (written in C ++ l anguage) is upda ted for gi rder made c ontinuous by pos t tensioning. Considering the girders to be continuous makes the a nalysis part of the problem complicated. The design moment and shear force at different sections for a specific span length and number of spans cannot be found now using simpler formula, rather it can be found using different methods for solving indeterminate structure. In this study, stiffness method is used to determine the design bending moment and shear force at any section of a continuous bridge having any number of spans with equal
span length according to AASHTO specification. To make the basic stiffness method applicable to computer aided analysis, 'computer application of stiffness method' was used with required sequence of logic. To find out the live load and impact shear force and moment, it was necessary to construct influence lines for different sections. No general e quation of influence line has be en us ed; $r$ ather $t$ he coordinate va lues of different points of the influence line are determined using the basic stiffness concept. It was necessary to consider both primary and secondary moment due to prestress for the pos t -tensioned c ontinuous b eam. T wo de sign c odes, na mely American Association of S tate Highway a nd T ransportation O fficials (AASHTO)-2007 for highway bridges and Precast/ Prestressed Concrete Institute (PCI)- PCI bridge design manual were followed in de sign part. In the last step, the updated $\mathrm{C}++$ subroutines was linked with the EVOP algorithm written in FORTRAN language to figure out the most cost optimum solution for the bridge I-girders under consideration and to study relations a mong va rious de sign va riables a nd their ef fects on o verall co st of the Igirders. R epeating the s ame process, opt imum girder configurations of c ontinuous post-tensioned bridge I-girders for different spans will be determined.

### 1.6 Organization of the Thesis

Apart from this chapter, the remainder of the thesis has been divided into six chapters.

Chapter 2 presents literature review co ncerning $p$ ast research on the field of cost optimization of simply s upported PC bridge structures an dc ontinuous PC br idge structures.

Chapter 3 presents the various design criteria that should be satisfied for the design of continuous PC I-Girder bridge structures.

Chapter 4 presents the information a bout the va rious features of an optimization method a nd br ief de scription a bout $t$ he pr ocedure of global opt imization m ethod, EVOP, which is adopted in this study.

Chapter 5 presents the formulation of optimization problem of the bridge and linking process of optimization problem to the optimization method to obt ain the opt imum solution.

Chapter 6 presents the optimized results and discussions of the bridge system.

Finally, Chapter 7 presents major conclusions and recommendation for future scopes of study.

## Chapter 2

## Literature Review

### 2.1 Introduction

In the 1 ast three de cades, much work has be en done in structural opt imization, in addition to considerable de velopments in $m$ athematical optimization. Despite this, there ha s a lways be en a ga $p$ be tween $t$ he pr ogress of opt imization $t$ heory a nd its application to the practice of bridge engineering. In 1994, Cohn and Dinovitzer (1994) estimated that thep ublished record o ns tructural o ptimization s ince 1960 c an conservatively be placed at some 150 books a nd 2500 pa pers, the v ast majority of which deal w ith $t$ heoretical a spects o fo ptimization. $D$ ocumentation $i n t$ heir comprehensive catalogue of published examples shows that very little work has been done in the area of optimizing concrete highway bridges.

### 2.2 Past Research on Optimization of Prestressed Concrete Beams

The obj ective of most of the pa pers publ ished on opt imization of PC structures is minimization of cost of the structures. For the optimization of PC structure the general cost function for prestressed concrete structures considered in the past research can be expressed in the following form:

$$
\begin{equation*}
C_{m}=C_{c b}+C_{s b}+C_{p b}+C_{f b}+C_{s b v}+C_{f i b} \tag{2.1}
\end{equation*}
$$

where, $\mathrm{C}_{\mathrm{m}}$ is the total material cost, $\mathrm{C}_{\mathrm{cb}}$ is the cost of concrete in the girder, $\mathrm{C}_{\mathrm{sb}}$ is the cost of re inforcing steel, $\mathrm{C}_{\mathrm{pb}}$ is cost of p restressing s teel, $\mathrm{C}_{\mathrm{fb}}$ is the c ost of t he formwork and $\mathrm{C}_{\text {sbv }}$ is the cost of shear steel;

Goble and Lapay (1971) minimized the cost of post-tensioned prestressed concrete Tsection beams ba sed on $t$ he ACI code (ACI, 1963) by us ing the gradient projection method (Arora, 1989). The cost function included the first four terms of Eq. (2.1). They stated that the optimum design seemed to be unaffected by $t$ he changes in the cost co efficients. H owever, su bsequent researchers to be discussed later rebut t his conclusion.

Naaman (1976) compared minimum cost designs with minimum weight designs for simply supported prestressed rectangular beams and one-way slabs based on the ACI code. The cost function included the first, third, and fourth terms of Eq. (2.1) and was optimized by a direct search technique. He concluded that the minimum weight and minimum cost solutions give approximately similar results only when the ratio of cost of concrete per cubic yard to the cost of prestressing steel per pound is more than 60 . Otherwise, the minimum cost a pproach yi elds a more e conomical solution, a nd for ratios much smaller than 60 the cost optimization approach yields substantially more economical solutions. He also poi nted out that $f$ or $m$ ost $p$ rojects in the US the aforementioned ratio is less than 60 .

Cohn and MacRae (1984a) considered the minimum cost design of simply supported RC and partially or fully pre-tensioned and post-tensioned concrete beams of fixed cross-sectional $g$ eometry su bjected $t$ o ser viceability and $u$ ltimate lim it s tate constraints i ncluding constraints on flexural strength, deflection, duc tility, f atigue, cracking, a nd minimum $r$ einforcement, ba sed on the A CI c ode or $t$ he Canadian building code using the feasible conjugate-direction method (Kirsch, 1993). The beam can be of a ny c ross-sectional shape subjected to di stributed and concentrated loads. Their c ost function is s imilar to Eq. (2.1). For t he e xamples co nsidered they concluded that for post-tensioned members $p$ artial $p$ restressing ap pears to $b$ e $m$ ore economical than complete prestressing for a prestressing-to-reinforcing steel cost ratio greater than 4 . F or pr etensioned b eams, on t he ot her ha nd, c omplete pre-stressing seems to be the best solution. For partially prestressed concrete they also concluded that for a p restressing-to-reinforcing steel cost ratio in the range of $0.5 \mathrm{to} 6, \mathrm{t}$ he optimal s olutions vary a little. C ohn a nd M acRae (1984b) performed parametric studies on 240 simply s upported, reinforced, partially, or completely pre- and posttensioned prestressed concrete beams with different dimensions, depth-to-span ratios, and live load intensities. They concluded that, in general, RC beams are the most costeffective at high depth-to-span ratios and low live load intensities. On the other hand, completely prestressed beams are the most cost-effective at low depth-to-span ratios and high live load intensities. For intermediate values, partial prestressing is the most cost-effective option.

Saouma and Murad (1984) presented the minimum cost design of simply supported, uniformly 1 oaded, pa rtially pr estressed, I -shaped be ams w ith une qual flanges subjected to the c onstraints of the 1977 A CI code. The optimization p roblem was formulated in terms of nine design variables: six geometrical variables plus areas of tensile, co mpressive, an $\mathrm{d} p$ restressing st eel. The c onstrained opt imization pr oblem was transformed to an unconstrained optimization problem using the interior penalty function method (Kirsch, 1993) and was solved by the quasi-Newton method. They found the optimum solutions for several beams with spans ranging from 6 m to 42 m , assuming both cracked and un-cracked sections, and reported cost reductions in the range of $5 \%$ to $52 \%$. They also concluded that allowing cracking to occur does not reduce the cost by any significant measure.

Linear pr ogramming methods w ere us ed by $\operatorname{Kirsch}(1985,1993$, and 1997 ) to optimize in determinate p restressed c oncrete b eams w ith p rismatic c ross sec tions through a "bounding procedure". To simplify the problem, a two-level formulation was u sed to r educe the problem si ze an del iminate $p$ otential $n$ umerical difficulties encountered be cause of the fundamentally di fferent na ture of $t$ he de sign va riables. The c oncrete di mensions w ere op timized in one 1 evel, and t he tendon va riables (prestressing force and layout co ordinates) w ere determined in an other level. As a first step, a lower bound on $t$ he concrete volume was established without evaluating the tendon variables. The corresponding minimum prestressing force was an uppe r bound. S imilarly, a 1 ower bound on t he pr estressing f orce w as de termined by assuming the maximum concrete dimensions. Based on the two bounding solutions, a lower boun $d$ on $t$ he o bjective $f$ unction $w$ as evaluated. $T$ he be st of the boundi $n g$ solutions was first checked for optimality. If necessary, the search for the optimum was then co ntinued in the reduced space of the co ncrete $v$ ariables $u$ sing a $f$ easible directions $t$ echnique. $F$ or a ny a ssumed c oncrete di mensions, a r educed l inear programming problem was solved. The process was repeated until the optimum was reached.

Lounis a nd C ohn (1993a) presented a p restressed I-beam cost optimization method for individual bridge components using continuous design variables. They first found the maximum feasible girder spacing for each of the available precast girder shapes
and then minimized the prestressed and nonprestressed reinforcement in the I-beams and deck.

Lounis and C ohn (1993b) pr esented a mu lti-objective optimization formulation for minimizing the cost and maximizing the initial camber of post-tensioned floor slabs with s erviceability a nd ultimate lim it s tate c onstraints of t he A CI code. The c ost objective $f$ unction was chosen as $t$ he primary objective a nd the c amber obj ective function is transformed into a constraint with specified lower and upper bounds. The resulting single optimization problem was then solved by $t$ he projected Lagrangian method. The cost function for the slab included only the first and third terms of Eq. (2.1).

Khaleel an d I tani (1993) p resented the mi nimum cost de sign of s imply s upported partially $p$ restressed co ncrete $u$ nsymmetrical I-shaped gi rders as per AC I Building code. T he obj ective f unction was similar to Eq. (2.1). The s equential qua dratic programming method was used to solve the nonlinear optimization problem assuming both cracked and uncracked sections. They concluded that an increase in the concrete strength $d$ oes $n$ ot reduce $t$ he o ptimum co st significantly, a nd hi gher strength in prestressing st eel reduces the o ptimum co st to a cer tain extent. They claimed that some a mount ofreinforcing st eel facilitates $t$ he $d$ evelopment of c racking int he concrete, which reduces the cost of materials and improves ductility.

### 2.3 Past Research on Cost Optimization of Simply Supported Prestressed <br> Concrete Bridge Structures

Torres etal. (1966) presented the minimum cost design of prestressed co ncrete highway bridges subjected to AASHTO loading by using a piecewise LP method. The independent de sign va riables were the num ber a nd de pth of gi rders, prestressing force, and tendon eccentricity. They further defined dependent design variables as the spacing of girders, tendon cross sectional area, initial prestress, and the slab thickness and reinforcement. They claim their cost function includes the costs of transportation, erection, a nd bearings in addition to the material costs of concrete and steel, but do not give any detail. They presented results for bridges with spans ranging from 20 ft to $110 \mathrm{ft}(6.1 \mathrm{~m}$ to 33.5 m$)$ and with widths of $25 \mathrm{ft}(7.6 \mathrm{~m})$ and $50 \mathrm{ft}(15.2 \mathrm{~m})$.

Using i nteger pr ogramming, J ones (1985) formulated the minimum cost design of precast, pr estressed c oncrete s imply s upported box gi rders us ed in a m ulti-beam highway br idge a nd s ubjected tot he A ASHTO (1977) 1 oading assuming $t$ hat $t$ he cross-sectional geometry and the grid work of strands are given and fixed. The design variables are the concrete strength, and the number, location and draping of strands (moving the strands up at the end of the beam). The constraints used were release and service 1 oad st resses, ultimate $m$ oment cap acity, cr acking moment c apacity, an d release camber. The cost function included only the first and third terms of Eq. (2.1).

Yuetal. (1986) presented the minimum cost design of a p restressed concrete box bridge girder used in a ba lanced cantilever bridge (consisting of two end cantilever and overhang spans and one middle simple span) based on the British code and using general g eometric p rogramming (Beightler a nd Phillips, 1976). T he c ost f unction included the material co sts of concrete, p restressing steel, a nd the metal formwork. They included the labor cost of the metal formwork, roughly as 1.5 times the cost of the material for the formwork. The design variables were the prestressing forces, the eccentricities, and the girder depths for all spans.

Cohn and Lounis (1994) applied the above three-level cost optimization approach to multi-objective o ptimization of p artially an d fully p restressed concrete hi ghway bridges with span lengths of 10 m to 15 m and widths of 8 m to 16 m . Their objective functions included the minimum superstructure cost, minimum weight of prestressing steel, $m$ inimum vol ume f fc oncrete, maximum $g$ irder s pacing, m inimum superstructure de pth, maximum s pan-to-depth ratio, maximum feasible sp an length, and minimum superstructure camber. F or a four-lane 20m length single-span bridge, they c oncluded that t he voi ded s lab a nd t he pr ecast I -girder s ystems were more economical than the solid slab and one- and two-cell box $g$ irders. Lounis and Cohn (1995a) also concluded that voided slab decks are more economical than box girders for short spans (less that 20 m ) and wide decks (greater than 12 m ), a and single-cell box girders were more economical for medium spans (more than 20 m ) and narrow decks (less than 12 m ). The single-cell box $g$ irder, however, resulted in the deepest superstructure, which might be a drawback when there was restriction on the depth of the deck. Multi-criteria cost optimization of bridge structures was further discussed by Lounis and Cohn (1995b, 1996).

Fereig (1985) linearized the problem of prestressed concrete de sign op timization to determine $t$ he ad equacy of a $g$ iven co ncrete section an $d t$ he minimum $n$ ecessary prestressing force. He developed p reliminary design ch arts $t$ hat co uld beu sed to determine the required prestressing force for a given pretensioned, simply supported Canadian Precast-Prestressed Concrete Institute (CPCI) br idge girder, for any given span length and girder spacing.

Fereig (1996) presented the minimum cost preliminary de sign of single span bridge structures consisting of cast-in-place R C deck and girders based on the AAS HTO code (AASHTO, 1992 ). The a uthor 1 inearized the pr oblem by a pproximating $t$ he nonlinear constraints by straight lines and solves the resulting linear problem by the Simplex method. The author concluded that 'it is always more eco nomical to space the girder at the maximum p ractical spacing'. Fereig (1999) c ompared the required prestressing forces obtained in his latter study with those that would be obtained using concrete with cylinder strength of 69 MPa . It was found that using the higher concrete strength allowed a reduction in the prestressing force from 4 to $12 \%$ depending on the girder spacing and span for the example considered.

Ahsan et al. (2012 )has recently demonstrated successful use of a global optimization algorithm - EVOP developed by G hani in cost optimum design of simply supported post-tensioned I-girder of bridges. Rana et al. (2013) adopted EVOP to optimize twospan post-tensioned bridge which was made continuous for superimposed dead loads and live loads using deck reinforcement.

### 2.4 Past Research on Cost Optimization of Continuous Prestressed Concrete Bridge Structures

Kirsch (1972) presented the minimum cost design of continuous two-span prestressed concrete beams subjected to constraints on the stresses, pre- stressing force, and the vertical coordinates of the tendon by lin earizing the nonlinear optimization problem approximately a nd $s$ olving the reduced 1 inear pr oblem by $t$ he 1 inear pr ogramming (LP) method. His cost function included only the first and third terms of Eq. (2.1). Kirsch (1973) extended this work to prestressed concrete slabs.

Cohn and Lounis (1993a) p resented the minimum cost design of partially and fully prestressed co ncrete continuous be ams a nd on e-way s labs. T he optimization was based o n the 1 imit st ate d esign and an op timization method na med projected Lagrangian algorithm. They simultaneously satisfied both collapse and serviceability limit state criteria based on the ACI code. The material nonlinearity was idealized by an el astoplastic co nstitutive r elationship. A co nstant prestressing force a nd prestressing losses were assumed. Their cost function included the first three terms of Eq. (2.1). They reported that the total cost decreases with the increase in the allowable tensile stress.

Lounis and Cohn (1993a) presented the minimum cost design of short and medium span highway bridges consisting of RC slabs on pr ecast, post-tensioned, prestressed concrete I-girders satisfying the serviceability and ultimate limit state constraints of the Ontario Highway Bridge Design Code (OHBDC, 1983). They used a three-level optimization approach. In the first level they dealt with the optimization of the bridge components including di mensions of $t$ he $g$ irder c ross-sections, s lab t hickness, amounts of $r$ einforcing a nd pr estressing steel, a nd $t$ endon e ccentricities by $t$ he projected Lagrangian method. In the second level, they considered the optimization of the longitudinal layout such as the number of s pans, restraint type a nd span length ratios and transverse layout such as the number of girders and slab overhang length. In the third level, they considered various structural systems such as solid or voided slabs on precast I- or box girders. They used a sieve-search technique (Kirsch, 1993) for $t$ he $s$ econd a nd $t$ hird $l$ evels of opt imization. $T$ heir $c$ ost $f$ unction included the material costs of concrete, reinforcement, and connections at piers. They also included the costs of fabrication, transportation, a nd e rection of girders a ssuming a constant value per length of the girder. They concluded by optimizing a complete set of bridge system re sulting in a more e conomical structure than o ptimizing the in dividual components of $t$ he br idge. B ased on their op timization studies they recommended simply supported girders for prestressed concrete bridges up to 27 m ( 89 ft ) long, twospan continuous girders for span lengths from $28 \mathrm{~m}(92 \mathrm{ft})$ to $44 \mathrm{~m}(144 \mathrm{ft})$, three-span continuous girders for span lengths of $55 \mathrm{~m}(180 \mathrm{ft})$ to $100 \mathrm{~m}(328 \mathrm{ft})$, and two- span or three-span continuous girders for an intermediate range of $44 \mathrm{~m}(144 \mathrm{ft})$ to $55 \mathrm{~m}(180$ ft).

Han et al. (1995) discussed the minimum cost design of partially prestressed concrete rectangular, and T -shape beams based on the Australian code using the discretized continuum-type optimality criteria (DCOC) method. The cost function included the first four terms of Eq. (2.1). They concluded that for a simply supported beam, a Tshape is m ore eco nomical than a rectangular s ection. Han et al . (1996) us ed the DCOC method to minimize the cost of continuous, partially prestressed and singly reinforced T-beams with constant cross-sections within each span. A three-span and a four-span continuous beam example were also presented.

Rana et al. (2013) adopted EVOP to optimize two-span post-tensioned bridge which was made c ontinuous for s uperimposed de ad 1 oads a nd l ive loads using de ck reinforcement.

### 2.5 Concluding Remarks

The great majority of papers on c ost optimization of prestressed concrete st ructures include the material costs of concrete, steel, and formwork. Some researchers ignore the cost of the formwork. However, this cost is significant and should not be ignored. Other costs such as the cost of labor, fabrication and installation are often ignored. Most of $t$ he studies on $p$ restressed co ncrete $b$ ridge st ructures, e xcept the work of Ahsan et al. (2012) and Rana et al. (2013), either minimized the cost of individual components only or us ed standard AASHTO sections instead of considering crosssectional dimensions as d esign variables or co nsidered the co st of m aterials o nly. Most the studies considered prestressing strands to be located in a fixed position to obtain eccentricity which is not practical and the lump sum value (a fixed percentage of initial presress) of p restress losses. Only in their works, Ahsan et al. (2012) and Rana etal. (2013) considered variable location of prestressing strands and a ctual value of prestress loss. Most of the studies deal with optimization of pre-tensioned Igirder bridge systems. None of these studies, except the work of Ahsan et al. (2012) and Rana et al. (2013), deals with total cost optimization of the post-tensioned I-girder bridge systems considering all cross-sectional dimensions, prestressing tendons layout as design variables and al so cost of materials including fabrication and installation. Rana et al. (2013) adopted EVOP to optimize two-span post-tensioned bridge which was made c ontinuous for s uperimposed de ad l oads a nd live lo ads using de ck
reinforcement. The $f$ ocus oft he p resent research is using EVOP in ha ndling optimization pr oblems of pr estressed bridge structures made c ontinuous by pos ttensioning which by resisting the deck weight can significantly improve the structural performance of longer span bridges.

## Chapter 3

## Continuous Prestressed Concrete Bridge Design

### 3.1 Introduction

Prestressing can be defined as the application of pre-determined force or moment to a structural member in such a manner that the combined internal stresses in the member resulting from this force or moment and any anticipated condition of external loading will bec onfined $w$ ithin s pecific 1 imits. Thus $p$ restressing $r$ efers to the $p$ ermanent internal stress in a structure to improve performance by reducing the effect of external forces. T he co mpression p erformance o fco ncrete i s st rong b ut i ts tension performance is weak. The main idea of prestressing co ncrete is to counteract the tension stresses that are i nduced by ex ternal forces. For instance, prestressing w ire placed eccentrically, the force in tendon produces an axial compression and hogging moment in the beam. While under service loads the same beam will develop sagging moments. Thus, it is possible to have the entire section in compression when service loads are imposed on the beam. This is the main advantage of prestressed concrete. It is well known that reinforced concrete cracks in tension. But there is no cracking in fully prestressed concrete since the entire section is in compression. Thus, it can be said that prestress provides a means for efficient usage of the concrete cross-section in resisting the external loads.

### 3.2 Reinforced Concrete versus Prestressed Concrete

Both reinforced concrete (RC) and prestressed concrete (PC) consist of two materials, concrete an d st eel. B ut high st rength co ncrete an d st eel ar e u sed in prestressed concrete. Although they employ the same material, their structural behavior is quite different. In reinforced concrete structures, steel is an integral part and resists force of tension which concrete cannot resist. The tension force develops in the steel when the concrete begin to crack and the st rains of concrete are transferred to steel through bond. The stress in steel varies with the bending moment. The stress in steel should be limited in o rder to p revent ex cessive crack of co ncrete. In fact the st eel ac ts as a
tension flange of a beam. In prestressed concrete, on the other hand the steel is used primarily for inducing a prestress in concrete. If this prestress could be induced by other means, there is little need of steel. The stress in steel does not depend on $t$ he strain in concrete. There is no ne ed tolimit the stress in steel in order to control cracking of concrete. The st eel $d$ oes $n$ ot actas at ension $f$ lange of abeam. The behaviors of RC and PC flexural members are illustrated using Figure 3.1 and Figure 3.2 respectively.


Figure 3.1 Behaviors of reinforced concrete members (PCI 2003)


Figure 3.2 Behaviors of prestressed concrete members (PCI 2003)

Figure 3.1 shows the conditions in a reinforced concrete member that has mild steel reinforcement a nd no prestressing. Under ser vice 1 oad conditions, c oncrete on the tension s ide of $t$ he ne utral a xis is a ssumed to be cacked. O nly c oncrete on t he
compression side is effective in resisting loads. In comparison, a prestressed concrete member is normally designed to remain uncracked under service loads (Figure 3.2). Since the full cross-section is effective, the prestressed member is much stiffer than a conventionally reinforced $c$ oncrete $m$ ember $r$ esulting in $r$ educed $d$ eflection. No unsightly cracks a re expected to be seen. Reinforcement is better protected a gainst corrosion. Fatigue of $s$ trand due to repeated truck loading is ge nerally not a de sign issue when the concrete surrounding the strands is not allowed to crack. At ultimate load conditions, conventionally reinforced concrete and prestressed co ncrete behave similarly. However, due to the lower strength of mild bars, a larger steel quantity is needed to a chieve $t$ he same st rength as a $p$ restressed member. This increases the member material costs for a conventionally reinforced member.

### 3.3 Advantages and Disadvantages of Prestressed Concrete

The most important feature of prestressed concrete is that it is free of cracks under working loads a ndite nables the entire concrete section to t ake partinresisting moments. Due to no -crack condition in the member, c orrosion of s teel is a voided when $t$ he s tructure is e xposed $t$ o weather condition. The be havior of prestressed concrete is more p redictable than o rdinary reinforced co ncrete in sev eral asp ects. Once co ncrete cracks, the behavior of reinforced co ncrete becomes quite co mplex. Since there is no cracking in prestressed concrete, its behavior can be explained on a more rational basis. In prestressed concrete structures, sections are much smaller than that of the corresponding $r$ einforced $c$ oncrete $s$ tructure. This is due to the fact that dead load moments a re counterbalanced by $t$ he prestressing moment resulting from prestressing forces and shear resisting capacity of such section is also increased under prestressing. The reduced self-weight of the structure contribute to further reduction of material for r oundation e lements. O ther f eature ofprestres co ncrete is its increased quality tor esist i mpact, $h$ igh fatigue resistance an $d i n c r e a s e d ~ l i v e ~ l o a d ~$ carrying ca pacity. P restressed co ncrete i s most useful inc onstructing liquid containing structures and nuclear plant where no leakage is acceptable and also used in long span bridges and roof systems due to its reduced dead load. On the other hand, prestressed concrete also exhibit certain disadvantages. Some of the disadvantages of prestressed concrete construction are:
(i) It requires high strength concrete that may not be easy to produce
(ii) It uses high strength steel, which might not be locally available
(iii) It requires end anchorage, end plates, complicated formwork
(iv) Labor cost may be greater, as it requires trained labor and
(v) It calls for better quality control

Generally, prestressed concrete construction is economical, as for example a decrease in me mber sect ions $r$ esults in d ecreased d esign 1 oads to obt ain an e conomical substructure.

### 3.4 Prestressing Systems

The prestress in co ncrete st ructure is induced by ei ther of f he t wo p rocesses. Pre tensioning and post tensioning. Pre-tensioning is accomplished by stressing wires, or strands called endon toa pr e-determined a mount by stretching $t$ hem be tween anchoring p osts before pl acing t he c oncrete. T he c oncrete is t hen pl aced a nd t he tendons become bonded to the concrete throughout their length. After the concrete has hardened, the tendon will be released from the anchoring posts. The tendon will tend to re gain th eir o riginal 1 ength by $s$ hortening a nd int his pr ocess $t$ hey $t$ ransfer a compressive stress to the concrete through bond. The tendons are usually stressed by hydraulic j acks. T he o ther al ternative is post- tensioning. In pos t -tensioning, t he tendons a re st ressed af ter the co ncrete is ca st and $h$ ardened to cer tain st rength to withstand the prestressing force. The tendon are stressed and anchored at the end of the concrete sect ion. Here, the tendons are e ither co ated with g rease or bituminous material or encased with flexible metal hose be fore placing in forms to prevent the tendons from bonding to the concrete during placing and curing of $\mathrm{concrete}$. latter case, t he m etal hose is referred to as a sheath orduct and remains inthe structure. A fter the tendons are stressed, the void between tendon and the sheath is filled $w$ ith grout. T hus $t$ he $t$ endons a re bond ed $w$ ith $c$ oncrete a nd corrosion is prevented. Bonded s ystems a re more c ommonly us ed in br idges, bot hint he superstructure (the roadway) an d in cable-stayed bridges, the cable-stays. There are post-tensioning a pplications in a lmost a 1 f acets of c onstruction. P ost-tensioning allows $b$ ridges $t o$ be built $t o$ ve ry de manding ge ometry $r$ equirements, including complex cu rves, v ariable su per-elevation a nd significant gr ade c hanges. In m any
cases, post-tensioning allows construction that would otherwise be impossible due to either site constraints or architectural requirements. The main difference between the two pre stressing system is:-
(i) Pre tensioning is mostly used for small member, whereas post- tensioning is used for larger spans.
(ii) Post- tensioned tendon can be placed in the structure with little difficulties in smooth curved profile. Pre-tensioned tendon can be used for curved profile but needs extensive plant facilities.
(iii) Pre-tensioning $s$ ystem ha $s t$ he di sadvantage $t$ hat $t$ he abutment us ed $\mathrm{i} n$ anchoring the tendon has to be very strong and c annot be reused unt il the concrete in the member has sufficiently hardened and removed from bed.
(iv) Loss of pr estress in pr e-tensioning is more pr onounced $t$ han $t$ hat of pos $t$ tensioning.

### 3.5 Anchorage Zone and Anchorage System

In post-tensioned girders, the prestressing force is transferred to girders in their end portions known as anchorage zones. The anchorage zone is geometrically defined as the vol ume of $c$ oncrete through which $t$ he $c$ oncentrated pr estressing force at $t$ he anchorage device spreads transversely to a linear stress distribution across the entire cross section. The prestresssing force is transferred directly on the ends of the girder through bearing plate and anchors. As a result the ends are subjected to high bursting stresses. So it becomes necessary to increase the area of the girder's cross section in the end portion in order to accommodate the raised tendons, their anchorages, and the support bearing. This is accomplished by gradually increasing the web width to that of the flange; the resulting enlarged section is called end block. Design of the anchorage zone m ay be done by i ndependently ve rified manufacturer's recommendations for minimum cover, spacing and edge distances for a particular anchorage device.

An anchorage system consists of a cast iron guide incorporated in the structures which distributes $t$ he $t$ endon $f$ orce $i$ nto $t$ he $c$ oncrete e nd block. Ont he gui de $s$ its $t$ he anchorage block, into which the strands are anchored by means of three-piece jaws, each locked into a tapered hole. The anchorage guide is provided with an accurate and
robust $m$ ethod of fixing a nd $t$ he $t$ endon, a s it is pr ovided $w$ ith $s$ ubstantial $s$ hutter fixing holes and, at its opposite end, a firm screw type fixing for the sheath in addition it incorporates a large front access grout injection point which, by its careful transition design, is blockage free. All anchorage systems are designed to the same principles, varying only in size and numbers of strands. Freyssinet C range anchorage system for 15.2 mm di ameter strands is shown in Figure 3.3(a), 3.3(b) a nd 3.3(c) and metal sheath is shown in Figure 3.4.


Figure 3.3(a) 13C15 Post-tensioning anchorage system (C range, Freyssinet Inc.)


Figure 3.3(b) Range of anchorages (C range, Freyssinet Inc.)


| Size | A | B | C | D | $H$ | $\varnothing 1^{*}$ | $\varnothing 2^{* *}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 C 15 | 150 | 110 | 120 | 85 | 50 | 40 | 45 |
| 4 C 15 | 150 | 120 | 125 | 95 | 50 | 45 | 50 |
| 7 C 15 | 180 | 150 | 186 | 110 | 55 | 60 | 65 |
| 9 C 15 | 225 | 185 | 260 | 150 | 55 | 65 | 70 |
| 12 C 15 | 240 | 200 | 165 | 150 | 65 | 80 | 85 |
| 13 C 15 | 250 | 210 | 246 | 160 | 70 | 80 | 85 |
| 19 C 15 | 300 | 250 | 256 | 185 | 80 | 95 | 100 |
| 22 C 15 | 330 | 275 | 430 | 220 | 90 | 105 | 110 |
| 25 C 15 | 360 | 300 | 400 | 230 | 95 | 110 | 115 |

All dimensions are in mm

Figure 3.3(c) Freyssinet C range anchorage system (C range, Freyssinet Inc.)

##  <br> Yow

## Figure 3.4 Metal sheaths for providing duct in the girder

### 3.6 Prestress Losses

Loss of prestress is defined as the difference between the initial stresses in the strands (jacking stress), and the effective prestress in the member (at a t ime when concrete stresses a retobecal culated). There ar e three types of 1 osses in post-tensioned prestressed concrete, friction losses, short term (instantaneous) losses and long term (time dependent) losses. The short term losses are anchorage slip loss and the loss due to e lastic s hortening of co ncrete. The 1 ong term 1 osses a re loss due to creep of concrete, loss due to shrinkage of concrete a nd loss due to $s$ teel $r$ elaxation. These losses n eed to be accounted for be fore c hecking the ad equacy of a g irder s ection under the residual prestress. A number of methods a re a vailable to predict loss of prestress. T hey f all in to th ree m ain c ategories, listed in o rder of in creasing complexity:
(i) Lump sum estimate methods
(ii) Rational approximate methods and
(iii) Detailed time-dependent analyses

### 3.6.1 Frictional losses, $L_{W C}$

When a tendon is jacked from one or both ends the stress along the tendon decreases away from the jack due to the effects of friction. Frictional loss oc curs only in posttensioned member. The friction between tendons and the surrounding material is not small enough to be ignored. This loss may be considered partly to be due to length effect (wobble effect) and party to curvature effect. In straight elements, it occurs due to wobble effect and in curved ones; it occurs due to curvature and wobble effects.

If angle of curve is $\theta$ and $F_{1}$ is force on pulling end of the curve, then force $F_{2}$ on the other end of the curve is given as
$F_{2}=F_{1} e^{-\mu \theta}$

Similarly, the relation between $F_{1}$ and $F_{2}$ due to length effect (wobble effect) is given as
$F_{2}=F_{1} e^{-K x}$
The combined effect is
$F_{2}=F_{1} e^{-(\mu \theta+K x)}$
So the loss in the force is,
$F_{2}-F_{1}=F_{1}\left(1-e^{-(\mu \theta+K x)}\right)$
Where,
$x=$ length of a pr estressing $t$ endon f rom t he j acking end t oa ny poi nt unde r consideration;
$\mu=$ coefficient of friction;
$K=$ wobble friction coefficient per unit length of tendon;
$\theta=$ sum of the a bsolute va lues of a ngular change of post-tensioning tendon from jacking end to the point under investigation;
$F_{l}=$ Jacking force;

The value of $\mu$ and K for different type of cables can be read from Codes. The total loss for all tendons ( number of tendons $=\mathrm{N}_{\mathrm{T}}$ ) may be expressed by t he following equation:

$$
\begin{equation*}
L_{W C}=\sum_{i=1}^{i=N_{T}} F_{i}\left(1-e^{-\left(\mu \theta_{i}+K L\right)}\right) \tag{3.5}
\end{equation*}
$$

### 3.6.2 Instantaneous losses

### 3.6.2.1 Anchorage loss, $L_{A N C}$

Anchor set loss of prestress occurs in the vicinity of the jacking end of post-tensioned members as $t$ he post-tensioning force is transferred from the jack to the a nchorage block. During this process, the wedges move inward as they seat and grip the strand. This results in a loss of elongation and therefore force in the tendon. The value of the strand shortening generally referred to as anchor set, $\Delta \mathrm{L}$, varies from about 3 mm to 9 mm with an average value of 6 mm . It de pends on the anchorage $h$ ardware a nd jacking equipment. The a nchor set 1 oss is hi ghest a $t$ the anchorage. It di minishes
gradually due to friction effects as the distance from the anchorage increases. Anchor set loss can be calculated by Eq. (3.6).
$L_{A N C}=\frac{A_{s} E_{s}}{L} \Delta$
where
$A_{s}=$ Area of prestressing steel
$E_{s}=$ Modulus of elasticity of prestressing steel
$\Delta=$ Anchorage slip

### 3.6.2.2 Elastic shortening loss, $\mathrm{L}_{\mathrm{ES}}$

As the prestressed is transferred to the co ncrete the member itself shortens and the prestressing steel shorten with it. Therefore, there is a loss of prestress in the steel. In post tensioning, the tendons are not us ually stretched simultaneously. Moreover, the first tendon that is stretched is shortening by subsequent stretching of all other tendon. Only the last tendon is not shortening by any subsequent stretching. An average value of strain change c an be computed and equally a pplied to all tendons. The prestress loss due to elastic shortening in post-tensioned members is taken as the concrete stress at the centroid of the prestressing steel at transfer, $\mathrm{f}_{\text {cgp }}$, multiplied by the ratio of the modulus of elasticities of the prestressing steel and the concrete at transfer.
$L_{E S}=K_{e s} \frac{E_{s}}{E_{c i}} f_{c g p 1} A_{s}+K_{e s} \frac{E_{S}}{E_{c i}} f_{c g p 2} A_{s}=$
$K_{e s} \frac{E_{s}}{E_{c i}}\left(\frac{F_{o}}{A_{g}}+\frac{F_{o} e 1^{2}}{I}-\frac{M_{G 1} e 1}{I}\right) A_{s}+K_{e s} \frac{E_{s}}{E_{c i}}\left(\frac{F_{o}}{A_{g}}+\frac{F_{o} e 2^{2}}{I}-\frac{M_{G 2} e 2}{I}\right) A_{s}$
where,
$\mathrm{f}_{\text {cgp1 }}=$ sum of concrete stresses at the center of gravity of prestressing tendons due to the prestressing force at transfer and the self weight of the member at the sections of maximum positive moment.
$\mathrm{f}_{\text {cgp2 }}=$ sum of concrete stresses at the center of gravity of prestressing tendons due to the prestressing force at transfer and the self weight of the member at the sections of maximum negative moment.
$\mathrm{M}_{\mathrm{G} 1}=$ Maximum positive moment due to self weight of the girder.
$\mathrm{M}_{\mathrm{G} 2}=$ Maximum negative moment due to self weight of the girder.
$e_{1}=$ eccentricity at position of maximum positive moment.
$e_{2}=$ eccentricity at position of maximum negative moment.
$\mathrm{E}_{\mathrm{ci}}=$ modulus of elasticity of the concrete at transfer

Applying $t$ his equation requires e stimating $t$ he force in the st rands af ter $t$ ransfer. Initial Estimate of $F_{o}$ is unknown as yet loss for elastic shortening is undetermined. Let
$F_{o}=\mathrm{F}-\mathrm{L}_{\mathrm{WC}}-\mathrm{L}_{\mathrm{ANC}}$
With $F_{o}$ from E q. (3.8), $L_{E S}$ is cal culated from Eq. (3.7) and $F_{o}$ is continuously updated using Eq. (3.9) until the updated value becomes equal to previous value.
$F_{o}=\mathrm{F}-\mathrm{L}_{\mathrm{ES}}-\mathrm{L}_{\mathrm{WC}}-\mathrm{L}_{\mathrm{ANC}}$

### 3.6.3 Time-dependent losses

### 3.6.3.1 Loss due to shrinkage of concrete, $L_{S H}$

Shrinkage in concrete is a contraction due to drying and chemical ch anges. It is dependent on time and moisture condition but not on stress. The amount of shrinkage varies widely, depending on the individual conditions. For the propose of design, an average value of shrinkage strain would be about 0.0002 to 0.0006 for usual concrete mixtures employed in prestressed construction. Shrinkage of concrete is influenced by many factors but in this w ork the most important factors vol umes to surface ratio, relative $h$ umidity a nd tim ef rome ndofc uring to a pplication of prestress a re considered in calculation of shrinkage losses. The equation for estimating loss due to shrinkage of concrete, $\mathrm{L}_{\mathrm{SH}}$, is roughly based on an ultimate concrete shrinkage strain of approximately -0.00042 and a modulus of elasticity of approximately 193 GPa for prestressing strands. According to AASHTO 2007 the expression for prestress loss due to shrinkage is a function of the a verage annual ambient re lative humidity, $\mathrm{R}_{\mathrm{H}}$, and is given as for post-tensioned members.
$\mathrm{L}_{\mathrm{SH}}=0.8\left(117.3-1.03 \mathrm{R}_{\mathrm{H}}\right)(\mathrm{MPa})$
Where, $\mathrm{R}_{\mathrm{H}}=$ the a verage annual ambient relative humidity (\%). The average annual ambient relative humidity may be obtained from local weather statistics.

### 3.6.3.2 Loss due to creep of concrete, $L_{C R}$

Creep is the property of concrete by which it continues to de form with time unde $r$ sustained 1 oad at $u$ nit stresses $w$ ithin the acc epted e lastic ra nge. $T$ his in elastic deformation increase at decreasing rate during the time of loading and its magnitude may be several times larger than that of the short term elastic deformation. The strain due to creep varies with the magnitude of stress. It is a $t$ ime dependent phenomena. Creep of concrete result in loss in steel stress. The expression for prestress losses due to creep is,
$\mathrm{L}_{\mathrm{CR}}=0.083 \mathrm{f}_{\mathrm{cgp}}-0.048 \mathrm{f}_{\mathrm{cds}}(\mathrm{MPa})$
where,
$f_{c g p}=$ concrete stress at the center of gravity of the prestressing steel at transfer
$f_{c d s}=\mathrm{ch}$ ange in co ncrete st ress a t cen ter of gravity of prestressing st eel dueto permanent loads, except the load acting at the time the prestressing force is applied. Values of $f_{\text {cds }}$ should be calculated at the same section or at sections for which $f_{\text {cgp }}$ is calculated. The value of $\mathrm{f}_{\text {cds }}$ includes the effect of the weight of the diaphragm, slab and haunch, parapets, future wearing surface, utilities and any other permanent loads, other than the loads existing at transfer at the section under consideration, applied to the bridge.
$f_{c g p}=\left(\frac{F_{o}}{A_{t f}}+\frac{F_{o} e 1^{2}}{I}-\frac{M_{G 1} e 1}{I}\right)+\left(\frac{F_{o}}{A_{t f}}+\frac{F_{o} e 2^{2}}{I}-\frac{M_{G 2} e 2}{I}\right)$
$f_{c d s}=\frac{\left(M_{P 1}-M_{G 1}\right) e 1}{I}+\frac{M_{C 1} e_{c 1}}{I_{c}}+\frac{\left(M_{P 2}-M_{G 2}\right) e 2}{I}+\frac{M_{C 2} e_{c 2}}{I_{c}}$
where, $M_{G I}=$ Moment due to $g$ irder self weight at position of maximum pos itive moment; $M_{G 2}=$ Moment due to girder self weight at position of maximum ne gative moment; $e_{1}=$ eccentricity at position of maximum positive moment; $e_{2}=$ eccentricity at position of maximum negative moment; $M_{P I}=$ Non-composite dead load moment or M oment due t o girder self w eight, c ross girder a nd deck s lab at position of maximum positive moment; $M_{P 2}=$ Non-composite dead load moment or Moment due to girder self w eight, c ross g irder and de ck slab at pos ition of maximum ne gative moment; $M_{C l}=$ Composite dead load moment due to future wearing coarse and curb self weight at position of maximum positive moment; $M_{C 2}=$ Composite dead load moment due to future wearing coarse a nd curb self weight at position of maximum
negative moment; $I_{c}=$ moment of inertia of composite section; $e_{c l}=$ Eccentricity of tendons in composite s ection at p osition of maximum positive m oment; $e_{c 2}=$ Eccentricity oft endons in co mposite sec tion at pos ition of maximum ne gative moment;

### 3.6.3.3 Loss due to relaxation of steel, $L_{S R}$

Relaxation is assumed to find the loss of stress in steel under nearly constant strain at constant temperature. It is similar to creep of concrete. Loss due to relaxation varies widely f or di fferent s teels a nd its m agnitude m ay be s upplied by t he s teel manufactures based on test data. This loss is generally of the order of $2 \%$ to $8 \%$ of the initial steel stress. Losses due to relaxation should be based on a pproved test data. If test d ata is not a vailable, the loss may be assumed to be $2.5 \%$ of i nitial stress. AASHTO 2007 provides E q. 3.14 to e stimate re laxation a fter t ransfer f or posttensioned members with stress-relieved or low relaxation strands. The expression for prestress losses due to steel relaxation is,

$$
\begin{equation*}
\mathrm{L}_{\mathrm{SR}}=34.48-0.00069 \mathrm{~L}_{\mathrm{ES}}-0.000345\left(\mathrm{~L}_{\mathrm{SH}}+\mathrm{L}_{\mathrm{CR}}\right)(\mathrm{MPa}) \tag{3.14}
\end{equation*}
$$

### 3.7 Designs for Flexure

The design of a p restressed concrete member in flexure normally involve selection and proportioning of a concrete section, determination of the amount of prestressing force and eccentricity for given section. The design is done based on strength (load factor de sign) a nd on behavior at service condition (allowable stress design) at all load $s$ tages th at $m$ ay becritical d uring the life of the structure from the time the prestressing force is applied. In the next section the allowable stress design (Elastic design) and the load factor design (ultimate design) of a section for flexure is briefly described.

### 3.7.1 Allowable stress design (ASD)

This design method ensures stress in concrete not to exceed the allowable stress value both at transfer and under service loads. The member is normally designed to remain un-cracked under service loads. Consequently, it is assumed that the member can be
analyzed a s ho mogeneous a nd e lastic unde r s ervice 1 oads that a ll a ssumptions of simple bending theory can be applied.

Stresses in concrete due to prestress and load:
Stresses in concrete due to prestress are usually computed by an elastic theory. For a prestressing force $F$ applied to a concrete section with an eccentricity $e$, the prestress is resolved in to two components a c oncentric force $F$ through the centroid a nd a moment $F \times e$. U sing el astic formula the st ress at top extreme fiber or bot tom extreme fiber due to axial compression, $F$ and moment, $M=F e$ is given by
$f=-\frac{F}{A} \pm \frac{F \times e}{S}$
where, $\mathrm{S}=$ section modulus for top or bottom fiber.
For post-tensioned member be fore being bonded, for the values of $A$ and $S$, the net concrete section has to be used. After the steel is bonded, these should be based on the transformed sect ion. The st resses produced $b y$ an $y$ ex ternal $l$ oad as $w$ ell as $o$ wn weight of the member is given by elastic theory as:
$f=\frac{M}{S}$
The resulting stresses due to prestress and loads are as follows:
$f=-\frac{F}{A} \pm \frac{F \times e}{S} \pm \frac{M}{S}$

### 3.7.2 Ultimate strength design

This method of design ensures that the section must have sufficient strength (resisting moment) under factored loads. The n ominal st rength of t he m ember is cal culated, based on $t$ he know ledge of member a nd material behavior. The nominal strength is modified by a strength reduction factor $\Phi$, less $t$ han $u$ nity, to obt ain the design strength. The required strength is an overload stage which is found by applying load factors $\gamma$, greater than unity, to the loads actually expected.

Stress in the prestressed steel at flexural failure:
For bonde d members with pr estressing onl y , t he average st ress in p restressing reinforcement at ultimate load, $\mathrm{f}_{\mathrm{su}}$, is:
$f_{s u}=f_{p u}\left(1-\frac{\gamma^{*}}{\beta_{1}} \rho \frac{f_{s}}{f_{c}^{\prime}}\right)$
where,
$\gamma^{*}=$ factor for type of pre stressing steel $=0.28$ (for low relaxation steel)
$\beta_{1}=$ ratio of depth of equivalent compression zone to depth of prestressing steel
$\mathrm{f}^{\prime}{ }_{\mathrm{c}}=$ compressive strength of concrete
for $\mathrm{f}_{\mathrm{c}}{ }^{\prime} \leq 27.6 \mathrm{MPa}, \beta_{1}=0.85$;
for $27.6 \mathrm{MPa} \leq \mathrm{f}_{\mathrm{c}}{ }^{\prime} \leq 55.2 \mathrm{MPa}, \beta_{1} \square=0.85-0.0725\left(\mathrm{f}_{\mathrm{c}}{ }^{\prime}[\mathrm{MPa}]-27.6\right.$ );
for $\mathrm{f}_{\mathrm{c}}{ }^{\prime} \geq 55.2 \mathrm{MPa}, \beta_{1}=0.65$
$\rho=\frac{A_{s}}{b d}$
$b=$ effective flange width
$d=$ effective depth for flexure

Design flexural strength:
With the stress in the prestressed tensile steel when the member fails in flexure, the design flexural strength is calculated as follows:

The depth of equivalent rectangular stress block, a ssuming a rectangular section, is computed by:
$a=\frac{A_{s} f_{s u}}{0.85 f_{c}^{\prime} b}$
For sections with prestressing strand only and the depth of the equivalent rectangular stress block less than the flange thickness ( $\mathrm{t}_{\mathrm{f}}$ ), the design flexural strength should be taken as:
$\Phi M_{n}=\Phi\left\{A_{s} f_{s u} d\left(1-0.6 \frac{\rho f_{s u}}{f_{c}^{\prime \prime}}\right)\right\}$

For sections with prestressing strand only and the depth of the equivalent rectangular stress block greater than the flange thickness, the design flexural strength should be taken as:

$$
\begin{equation*}
\Phi M_{n}=\Phi\left\{A_{s w} f_{s u} d\left(1-0.6 \frac{A_{s w} f_{s u}}{b_{w} d f_{c}^{\prime}}\right)\right\}+0.85 f_{c}\left(b-b_{w}\right) t_{f}\left(d-\frac{t_{f}}{2}\right) \tag{3.21}
\end{equation*}
$$

where,
$A_{s w}=A_{s}-A_{s f}$
$A_{s f}=0.85 f_{c}\left(b-b_{w}\right) \frac{t_{f}}{f_{s u}}$
$\mathrm{b}_{\mathrm{w}}=$ Web width

For negative moment region, the design flexural strength is calculated as follows: The depth of equivalent rectangular stress block is computed by:
$a=\frac{A_{s} f_{s u}}{0.85 f_{c}^{\prime} b w}$
The design flexural strength should be taken as:
$\Phi M_{n}=\Phi\left\{A_{s} f_{s u} d\left(1-0.6 \frac{\rho f_{s u}}{f_{c}^{\prime \prime}}\right)\right\}$

### 3.8 Ductility Limit

### 3.8.1 Maximum prestressing steel

According to the AASHTO Standard Specifications (AASHTO 2007), the prestressed concrete members must be designed so that the steel yields when the ultimate capacity is reached. Therefore, the maximum prestressing steel constraints for the composite section are given below:
$\omega \leq \omega_{u}$
where, $\omega=\mathrm{R}$ einforcement index; $\omega_{u}=\mathrm{U}$ pper bound t o R einforcement index $=$ $0.36 \beta_{1}$. This co nstraint en sures the st eel will $y$ ield as $t$ he ultimate cap acity is approached. AASHTO $g$ ives the $f$ ollowing $f$ ormula $f$ or c alculating r einforcement index ( $\omega$ )
$\omega=\frac{\rho f_{s u}}{f_{c}^{\prime}}$

### 3.8.2 Minimum prestressing steel

AASHTO limit the minimum value of pre stressing steel to be used in pre stressing concrete section. The pre stressing steel in a section should be a dequate to de velop ultimate moment at critical section at least 1.2 times the cracking moment $M_{c r}{ }^{*}$.
$\Phi M_{n} \geq 1.2 M_{c r}{ }^{*}$
At position of maximum positive moment,
$M_{c r}{ }^{*}=\left(f_{r}+f_{p e}\right) S_{b c}-M_{P 1}\left(\frac{S_{b c}}{s_{b}}-1\right)$
$f_{p e}=\frac{F_{e}}{A_{t f}}+\frac{F_{e} e_{c 1}}{S_{b}}$
where, $f_{r}=$ modulus of rupture; $f_{p e}=$ compressive stress in concrete due to effective prestress forces only (after al lowance for all 1 p restress losses) at ex treme fiber of section where tensile st ress is cau sed by ex ternally ap plied load; $S_{b}, S_{b c}=$ Section Modulus of bottom fiber of transformed precast \& composite section respectively; $e_{c l}$ $=$ eccentricity of composite section at position of maximum positive moment; $M_{P I}=$ Non-composite dead load moment or Moment due to girder self weight, cross girder and deck slab at position of maximum positive moment;

At position of maximum negative moment,
$M_{c r}{ }^{*}=\left(f_{r}+f_{p e}\right) S_{t c}-M_{P 2}\left(\frac{S_{t c}}{s_{t}}-1\right)$
$f_{p e}=\frac{F_{e}}{A_{t f}}+\frac{F_{e} e_{c 2}}{S_{t}}$
where, $f_{r}=$ modulus of rupture; $f_{p e}=$ compressive stress in concrete due to effective prestress forces only (after al lowance for all 1 p restress losses) at ex treme fiber of section w here tensile st ress is cau sed by ex ternally ap plied 1 oad; $S_{t}, S_{t c}=$ Section Modulus of top fiber of transformed precast \& composite section respectively; $e_{c 2}=$ eccentricity of composite section at position of maximum negative moment; $M_{P 2}=$ Non-composite dead load moment or Moment due to girder self weight, cross girder and deck slab at position of maximum negative moment;

### 3.9 Design for Shear

The design and analysis of precast, prestressed concrete bridge members for vertical shear is presented in this section. Unlike flexural design, for which conditions at both service and factored load are evaluated, shear design is on ly e valuated for factored loads (s trength limit state). The shear strength of a $p$ restressed co ncrete member is
taken as the sum of the shear strength contributions by concrete a nd by $t$ he web reinforcement. According $t$ o AASHTO s pecification for de sign purposes $t$ he relationship is written as,
$V_{\mathrm{u}} \leq \varphi\left(V_{\mathrm{c}}+V_{\mathrm{s}}\right)$
where,
$V_{u}=$ factored shear force at the section considered
$V_{c}=$ the concrete contribution taken as lesser of flexural shear, $V_{c i}$ and web shear, $V_{c w}$ $V_{s}=$ shear carried by the steel.

The concrete contribution $V_{c}$, is taken as the shear required to produce shear cracking. Two types of shear cracking have been flexural shear and web shear as illustrated in


Figure 3.5.

Figure 3.5 Types of cracking in prestressed concrete beams (PCI 2003)

Flexural shear c rack do minate the be havior of the portion of the girder where high flexural st resses co incide with si gnificant sh ear st resses. W eb sh ear crack forms in regions of high shear and small flexural stress such as near the support of the simply supported beam. The shears that produce these two types of cracking are $V_{c i}$ and $V_{c w}$. Therefore $\mathrm{V}_{\mathrm{c}}$ is taken as lesser of the $\mathrm{V}_{\mathrm{ci}}$ and $\mathrm{V}_{\mathrm{cw}}$. Procedures for computing these two shear capacity are presented below:

### 3.9.1 Flexure-shear, $V_{c i}$

A flexure-shear crack is initiated by a flexure crack forming at a distance $\mathrm{d} / 2$ from the section be ing considered. A st he sh ear increases, t he f lexure cr ack inclines an d becomes a shear cr ack with ah orizontal projection eq ual tot he d istance d. $V_{c i}$, nominal $s$ hear $s$ trength provided by $c$ oncrete $w$ hen di agonal c racking r esults from combined shear and moment is calculated as follows:
$V_{c i}=0.05 \sqrt{f_{c}} W_{t} d+V_{d}+\frac{V_{i} M_{c r}}{M_{\max }} \leq 0.141 \sqrt{f_{c}} W_{t} d$
$M_{c r}=S_{b c}\left(0.5 \sqrt{f_{c}}+f_{p e}-f_{d}\right)$
$M_{\max }=1.3\left(M_{D}+1.67 M_{L L}\right)-M_{D}$
$V_{i}=1.3\left(V_{D}+1.67 V_{L L}\right)-V_{D}$
$f_{d}=\frac{M_{D}}{S_{b c}}$
where,
$V_{D}=$ shear force at the section of investigation due to the unfactored dead load
$M_{\max }=$ maximum factored moment at the section due to externally applied loads
$V_{i}=$ factored shear force at the section that occurs simultaneously with $M_{\max }$
$M_{c r}=$ moment due to external 1 oad r equired tocr ack the co ncrete at t he cr itical section.

The term $f_{p e}$ and $f_{d}$ are the stresses at the ex treme tension fiber due to the effective prestress forces o nly, after al 11 oses, an d d ue t o the t otal unf actored de ad 1 oad , respectively.
The AASHTO specifications state that $V_{c i}$ need not be taken less than $0.141 \sqrt{f_{c}} W_{t} d$ $(\mathrm{kN})$ and that d need not be taken less than 0.8 h , where h is the height of the section.

### 3.9.2 Web-shear, $V_{c w}$

$\mathrm{V}_{\mathrm{cw}}$, no minal s hear strength provided by c oncrete w hen diagonal c racking results from excessive principal tensile stress in web is calculated as follows:
$V_{c w}=\left(0.283 \sqrt{f_{c}}+0.3 f_{p c}\right) W_{t} d+V_{p}$
$V_{p}=\sum_{i=1}^{i=N_{T}} F_{i} \sin \theta_{i}$
where,
$V_{p}=$ the vertical component of the prestress force.
$f_{p c}=$ stress that includes the effect of the prestress force after losses and the stresses due to any loads applied to the member as a non-composite section.

### 3.10 Design for Horizontal Interface Shear

Cast-in-place concrete decks designed to act compositely with precast concrete beams must be able to resist the horizontal shearing forces at the interface between the two elements. Design is carried out at various locations along the span, similar to vertical shear design. The Standard Specifications does not identify the location of the critical section. F or convenience, it may be assu med to be the same location as the critical section for vertical shear. Other sections, generally at tenth-point intervals along the span, are also designed for composite-action shear. This may be necessary to ensure that a dequate reinforcement is p rovided for h orizontal sh ear because reinforcement for $v$ ertical shear, $w$ hich is extended into $t$ he de ck a nd used for ho rizontal shear reinforcement, may va ry a long the length of the $m$ ember. Composite sect ions ar e designed for hor izontal shear at the interface between the precast beam anddeck using the equation:
$V_{u} \leq \varphi V_{n h}$
Where,
$V_{u}=$ factored sh ear force act ing on the interface; $\varphi=$ strength reduction factor for shear; $V_{n h}=$ nominal shear capacity of the interface

The nominal shear capacity is obtained from one of the following conditions given as:
a) When the contact surface is intentionally roughened but minimum vertical ties are not provided:
$V_{n h}=80 b_{v} d$
b) When minimum tie s a re provided but the c ontact s urface is notintentionally roughened:
$V_{n h}=80 b_{v} d$
c) When the contact surface is intentionally roughened to minimum amplitude of $1 / 4$ in and minimum vertical ties are provided:
$V_{n h}=350 b_{v} d$
d) When required area of ties, $\mathrm{A}_{\mathrm{v}} \mathrm{h}$, exceeds the minimum area:
$V_{n h}=330 b_{\imath} d+0.40 A_{v} h f_{y} d / s$
For the above equations,
$\mathrm{b}_{\mathrm{v}}=$ width of cross-section at the c ontact surface being investigated for horizontal shear
$d=$ distance from extreme compression fiber to centroid of the prestressing force. As for vertical shear design, d need not be taken less than 0.80 h .
$\mathrm{s}=$ maximum spacing not to exceed 4 t imes the least web width of support element, nor 24 in .

### 3.11 Design for Lateral Stability

Prestressed concrete members are generally stiff enough to prevent lateral buckling. However, during handling and transportation, support conditions may result in lateral displacements of the beam, thus producing lateral bending about the weak axis. For hanging beams, the tendency to roll is governed primarily by the properties of the beam. The equilibrium conditions for a ha nging beam are shown in Figure 3.6 and Figure 3.7. When a beam hangs from lifting points, it may roll about an axis through the lifting points. The safety and stability of long beams subject to roll are dependent upon:


Figure 3.6 Perspective of a beam free to roll and deflect laterally (PCI 2003)


Figure 3.7 Equilibrium of beam in tilted position (PCI 2003)
Where,
$e_{i}=$ the initial lateral eccentricity of the center of gravity with respect to the roll axis
$y_{r}=$ the height of the roll axis above the center of gravity of the beam
$\mathrm{z}_{\mathrm{o}}=$ the theoretical lateral deflection of the center of gravity of the beam, computed with the full weight applied as a lateral load, measured to the center of gravity of the deflected arc of the beam
$\theta_{\max }=$ tilt angle at which cracking begins, based on tension at the top corner equal to the modulus of rupture.

For a beam with overall length, 1 , and equal overhangs of length, a, at each end:

$$
\begin{equation*}
\overline{\mathrm{z}}_{\mathrm{o}}=\frac{\mathrm{w}}{12 \mathrm{EI}_{\mathrm{g}} \ell}\left[0.1\left(\ell_{1}\right)^{5}-\mathrm{a}^{2}\left(\ell_{1}\right)^{3}+3 \mathrm{a}^{4}\left(\ell_{1}\right)+1.2\left(\mathrm{a}^{5}\right)\right] \tag{3.40}
\end{equation*}
$$

Where $1_{1}=1-2 \mathrm{a}$
$I_{g}=$ moment of inertia of beam about weak axis
For a beam with no overhangs, $\left(a=0, l_{1}=1\right)$, and:

$$
\begin{equation*}
\overline{\mathrm{z}}_{\mathrm{o}}=\frac{\mathrm{w}(\ell)^{4}}{120 \mathrm{EI}_{\mathrm{g}}} \tag{3.41}
\end{equation*}
$$

It is to note that, for a two span continuous post-tensioned girder (with each span having a length $=1$ ), 1 of aforementioned equations should be replaced with 21 .

The factor of safety against cracking, $F S_{c}$, is given by:
$F S_{c}=\frac{1}{\frac{z_{o}}{y_{r}+}+\frac{\theta_{i}}{\theta_{\max }}} \geq 1.5$
where $\theta_{i}=$ the initial roll angle of a rigid beam.

It is recommended that $e_{i}$ be based, as a minimum, on $1 / 4 \mathrm{in}$. plus one-half the PCI tolerance for sweep. The PCI sweep tolerance is $1 / 8 \mathrm{in}$. per 10 ft of member length. When cracking oc curs, the lateral stiffness decreases and $Z_{o}$ increases. Thus, failure may occur shortly after cracking as the tilt angle increases rapidly due to the loss of stiffness. Consequently, the factor of $s$ afety a gainst failure, $\mathrm{FS}_{\mathrm{f}}$, is c onservatively taken equal to F $\mathrm{S}_{\mathrm{c}}$. The n ecessary f actor of saf ety can not b e d etermined f rom scientific laws; it must be determined from experience. It is suggested to use a factor of safety of 1.0 against cracking, $\mathrm{FS}_{\mathrm{c}}$, and 1.5 against failure, $\mathrm{FS}_{\mathrm{f}}$.

### 3.12 Control of Deflection

Flexural $m$ embers of $b$ ridge $s$ uperstructures $s$ hall be de signed $t o$ ha ve a dequate stiffness to $\lim$ it $d$ eflections or a ny de formations $t$ hat may a dversely a ffect $t$ he strength or serviceability of the structure at service load plus impact. When making deflection computations the following c riterion a ccording to A ASHTO S tandard Specification is recommended.

Members having simple or continuous spans preferably should be designed so that the deflection due to service live load plus impact shall not exceed $1 / 800$ of the span.

### 3.13 Composite Construction

Composite construction involves construction in which a precast member (usually a girder) acts in combination with cast-in-place concrete (usually a slab), that is poured at a 1 ater t ime a nd bon ded t ot he member, w ith s tirrups if ne cessary, t o de velop composite action. AASHTO (AASHTO 2007) defines a co mposite flexural member as one that "consists of precast and/or cast-in-place concrete elements constructed in separate placements but so interconnected that all elements respond to superimposed load as a unit".

### 3.14 Loads

### 3.14.1 General

There c an be va rious types of load c oming to a bridge structure. T hey a re 1 isted below:

1. Dead weight
2. Live load (vehicle load and pedestrian load)
3. Dynamic effect of live load
4. Wind load (directly on bridge, from vehicle, dynamic effect)
5. Earthquake load (static or dynamic)
6. Longitudinal forces (stopping vehicles)
7. Centrifugal forces (curved deck)
8. Thermal forces
9. Earth pressure
10. Buoyancy
11. Shrinkage stress
12. Rib shortening
13. Erection stresses
14. Ice loading
15. River current pressure

Among all these types of forces, in the present thesis, only the first three i.e. Dead Load, Live Load and Impact Load have been considered.

### 3.14.2 Dead Load

The dead loads on $t$ he bridge superstructure consist of self weight of the individual components ( girder weight, de ck s lab weight), w earing s urface on s lab, s idewalks, curbs, railings and diaphragm etc.

While finding out dead load, load coming on the girder from its ow $n$ weight, $t$ he weight of the deck and wearing surface that fall inside the girder's 'tributary a rea' should be considered (Figure 3.8).

In case of an exterior girder, extra load from the curb and railing has to be considered.


Figure 3.8 Tributary area of interior girder

### 3.14.3 Live Load

Vehicular live loading on the roadways of bridges or incidental structures, designated HL-93, shall consist of a combination of: i) Design truck or de sign tandem, a nd ii) Design lane load.

According to AASHTO LRFD HL 93 loading, each design lane should occupy either by the design truck or design tandem and lane load, which will be effective 3000 mm transversely within a design lane. (AASHTO, 2007 3.6.1.2.1)

### 3.14.3.1 Truck Load



Figure 3.9 Truck load coming to bridge (AASHTO, 2007)

### 3.14.3.2 Tandem Load

The design tandem shall consist of a pair of $110,000 \mathrm{~N}$ axles spaced 1200 mm apart. The t ransverse sp acing of w heels shall b e t aken as 1800 mm . (AASHTO, 20 07, 3.6.1.2.3)

### 3.14.3.3 Lane Load

The design lane load shall consist of a load of $9.3 \mathrm{~N} / \mathrm{mm}$ uniformly distributed in the longitudinal di rection. Transversely, $t$ he de sign 1 ane 1 oad $s$ hall be a ssumed to be uniformly distributed over a 3000 mm width. (AASHTO, 2007, 3.6.1.2.4)

### 3.14.3.4 Impact Load

Impact effect of live load is considered by increasing the live load effect by a certain factor.

Impact factor, $\mathrm{I}=50 /(\mathrm{L}+125)<0.3$; Where, $\mathrm{L}=$ Loaded span in ft .

According to A ASHTO 2007 e xplanation of $t$ he 1 oaded 1 ength, $t$ he 1 oaded 1 ength should be as follows:
(a) For roadway floors: the design span length.
(b) For t ransverse m embers, s uch a s f loor be ams: the s pan 1 ength of m ember center to center of support.
(c) For computing truck load moments: the span length, or for cantilever arms the length from the moment center to the farthermost axle.
(d) For shear due to truck loads: the length of the loaded portion of span from the point under consideration to the far reaction; except, for cantilever arms, use a $30 \%$ impact factor.
(e) For c ontinuous s pan: the 1 ength of s pan unde rc onsideration f or p ositive moment, and the average of two adjacent loaded spans for negative moment.

### 3.14.3.5 Distribution Factor for Moment

As per AASHTO 2007, while calculating design moment, it should be multiplied by certain D istribution F actor. V alue of Distribution F actor de pends on s everal parameters 1 ike number of 1 ane, s pan 1 ength of girder, de ck s lab thickness, 1 ateral spacing of girders etc. For in terior and exterior girder, AASHTO proposes different Distribution Factors.

## For Interior Beam/ Girder

According to A ASHTO 2007, Distribution F actors for moment f or interior beam/ girder are as follows:

Table 3.1 Distribution Factor for Moment (Interior Girder) [AASHTO, 2007]

| One Design Lane Loaded: | $1100 \leq S \leq 4900$ |
| :---: | :---: |
| $0.06+\left(\frac{S}{4300}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_{g}}{L t_{s}^{3}}\right)^{0.1}$ | $6000 \leq L \leq 73000$ |
| $N_{b} \geq 4$ |  |
| Two or More Design Lanes Loaded: | $4 \times 10^{9} \leq K_{g} \leq 3 \times 10^{12}$ |
| $0.075+\left(\frac{S}{2900}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_{g}}{L t_{s}^{3}}\right)^{0.1}$ |  |
| use lesser of the values obtained from the <br> equation above with $N_{b}=3$ or the lever rule | $N_{b}=3$ |

Here,
$\mathrm{S}=$ Spacing of Main Girder
$\mathrm{t}_{\mathrm{s}}=$ Thickness of Slab
L = Span Length
$\mathrm{N}_{\mathrm{b}}=$ Number of Beam/ Girder
$K_{g}=n\left(I+A_{g}{ }^{2}\right)$
$\mathrm{n}=\mathrm{E}_{\mathrm{B}} / \mathrm{E}_{\mathrm{D}}$
$\mathrm{E}_{\mathrm{B}}=$ Modulus of elasticity of beam/ girder material (MPa)
$\mathrm{E}_{\mathrm{D}}=$ Modulus of elasticity of deck/ slab material (MPa)
$I=$ Moment of inertia of beam/girder $\left(\mathrm{mm}^{4}\right)$
$\mathrm{e}_{\mathrm{g}}=$ distance between center of gravity of beam/ girder and deck/ slab (mm)

## For Exterior Beam/ Girder

According to A ASHTO 2007, D istribution F actors for moment for ex terior beam/ girder are as follows:

Table 3.2 Distribution Factor for Moment (Exterior Girder) [AASHTO, 2007]

| One Design <br> Lane Loaded | Two or More Design <br> Lanes Loaded | Range of <br> Applicability |
| :---: | :---: | :---: |
| Lever Rule | $g=e g_{\text {interior }}$ <br> $e=0.77+\frac{d_{e}}{2800}$ | $-300 \leq d_{e} \leq 1700$ |
|  | use lesser of the values <br> obtained from the <br> equation above with $N_{b}=$ <br> 3 or the lever rule | $N_{b}=3$ |

Here,
$\mathrm{g}=$ distribution factor for exterior beam/ girder
$g_{\text {interior }}=$ distribution factor for interior beam/ girder
$e=$ eccentricity of a lane from centre of gravity of the pattern of girders (mm)
$\mathrm{d}_{\mathrm{e}}=$ distance from the exterior web of exterior beam to the interior edge of curb or traffic barrier (mm).

### 3.14.3.6 Distribution Factor for Shear

As per A ASHTO 2007, while calculating de sign shear, it s hould be multiplied by certain D istribution F actor. V alue of Distribution F actor de pends on s everal parameters 1 ike num ber of 1 ane, s pan 1 ength of girder, de ck s lab thickness, 1 ateral spacing of girders etc. For interior and exterior girder, AASHTO proposes different Distribution Factors.

## For Interior Beam/ Girder

According to AASHTO 2007, D istribution Factors for shear for interior beam/ girder are as follows:

Table 3.3 Distribution Factor for Shear (Interior Girder) [AASHTO, 2007]

| One Design Lane Loaded | Two or More Design Lanes Loaded | Range of Applicability |
| :---: | :---: | :---: |
| $0.36+\frac{S}{7600}$ | $0.2+\frac{S}{3600}-\left(\frac{S}{10700}\right)^{2.0}$ | $\begin{aligned} & \hline 1100 \leq S \leq 4900 \\ & 6000 \leq L \leq 73000 \\ & 110 \leq t_{s} \leq 300 \\ & N_{b} \geq 4 \end{aligned}$ |
| Lever Rule | Lever Rule | $N_{b}=3$ |

Here,
$\mathrm{S}=$ Spacing of Main Girder
$\mathrm{t}_{\mathrm{s}}=$ Thickness of Slab
$\mathrm{L}=$ Span Length
$\mathrm{N}_{\mathrm{b}}=$ Number of Beam/ Girder

## For Exterior Beam/ Girder

According to AASHTO 2007, Distribution Factors for shear for exterior beam/ girder are as follows:

Table 3.4 Distribution Factor for Shear (Exterior Girder) [AASHTO, 2007]

| One Design <br> Lane Loaded | Two or More Design <br> Lanes Loaded | Range of <br> Applicability |
| :---: | :---: | :---: |
| Lever Rule | $g=e g_{\text {interior }}$ <br> $e=0.6+\frac{d_{e}}{3000}$ | $-300 \leq d_{e} \leq 1700$ |
|  | Lever Rule | $N_{b}=3$ |
|  |  |  |
|  |  |  |

Here,
$g=$ distribution factor for exterior beam/ girder
$g_{\text {interior }}=$ distribution factor for interior beam/ girder
$e=$ eccentricity of a lane from centre of gravity of the pattern of girders (mm)
$\mathrm{d}_{\mathrm{e}}=$ distance from the exterior web of exterior beam to the interior edge of curb or traffic barrier (mm).

## Chapter 4

## Optimization Method

### 4.1 Introduction

Optimization is the act of obtaining the best result under given circumstances. In the design, construction and $m$ aintenance of a ny e ngineering system, engineers ha ve to take many technological and managerial decisions at several stages. The ultimate aim of all such decision is to either minimize the effort required or maximize the desire benefit. Since the effort required or the benefit desired in any practical situation can be expressed as a function of a ce rtain design variables, optimization can be defined as the process of finding the conditions that give the minimum or maximum value of a function.

### 4.2 Classification of Optimization Problem

Generally o ptimization problems can be classified based on the nature of equation involved into $t$ wo categories. This is ba sed ont he e xpression fort he obj ective function and the constraints.
(i) Linear optimization problems: - where the expression for objective function and the expression for all constraints are linear function of design variable.
(ii) Non-linear optimization p roblem: - where $t$ he e xpression $f$ or ob jective function or the expressions for some or all of constraint are non linear function of the design variables.

### 4.3 Classification of Optimization Method

The available method of optimization may conveniently be divided into two distinctly different categories as follows:-
(i) Analytical me thod:-Which us ually e mploy the $m$ athematical th eory of calculus ( continuous $d$ ifferentiability, a vailability ofg radient $v$ ectors a nd existence of second derivatives), variation method etc.
(ii) Numerical method: - which a re u sually e mploying a branch in the field of numerical mathematics called programming method. The recent developments in th is branch are closely re lated to the rapid growth in computing capacity affected by $t$ he de velopment of c omputers. In nu merical methods, a ne ar optimal design is automatically generated in iterative manner. An initial guess is used as starting points for a sy stematic search for better design. The search is te rminated w hen c ertain c riteria are s atisfied; in dicating th at the c urrent design is sufficiently close to the optimum.

Many mathematical programming methods ha ve be en developed for solving linear and nonl inear opt imization pr oblems dur ing $t$ he last $t$ hree $d$ ecades. However, no single method has been found to be entirely efficient and robust for all different kinds of engineering optimization problems. Some methods, such as $t$ he penalty function method, t he a ugmented L agrangian method, and t he c onjugate gr adient m ethod, search for a local opt imum by moving in a direction related to the local gradient. Other methods apply the first and second order necessary conditions to seek a local minimum by solving a set of nonl inear equations. F or the optimum de sign of large structures, t hese methods be come i nefficient due t o a large a mount of gr adient calculations and finite element analyses. These methods usually seek a solution in the neighborhood of the starting point similar to local hill climbing. If there is more than one local optimum in the problem, the result will depend on the choice of the starting point, and the global optimum cannot be guaranteed. Furthermore, when the objective function and constraints have multiple or sharp peaks, the gradient search becomes difficult and unstable.

So a truly versatile optimization algorithm for realistic p roblems should possess, at the very least, the following capabilities (Ghani, 1989).
(i) Ability to deal with nonlinear objective and constraining functions directly without the requirement of gradients or sub-gradients.
(ii) Objective and constraining functions allowed possessing finite number of discontinuities.
(iii) Restart facility to truly check the previously obtained minimum and high probability of directly locating the global minimum.
(iv) Ability to minimize objective functions with a mix of continuous, discrete and integer variables as arguments.
(v) Scaling of objective and constraining functions unnecessary.
(vi) The optimization problem allowed possessing simultaneously some or all features from above.

### 4.4 Global Optimization Algorithm

Global opt imization a lgorithms are ba sed on nu merical or pr ogramming methods. These are an optimization algorithm that employs measures that prevent convergence to local optima and increase the probability of finding a global optimum. Figure 4.1 shows global a nd 1 ocal opt ima of a $t$ wo-dimensional $f$ unction, $f\left(X_{1}, X_{2}\right)$. Global optimization, so far, has been a rather difficult and illusive problem. It is still in its infancy, a nd consequently there is little in the literature compared to that for local optimization. Methods researched to date for global optimization are mainly for the unconstrained problem.


Figure 4.1Global and local optima of a two-dimensional function (Weise, 2008)

### 4.5 Statement of an Optimization Problem

An optimization or a mathematical programming problem can be stated as follows:Find $X=\left\{x_{1}, x_{2} \ldots x_{n}\right\}$ which minimize or maximize, $F(X)$

Subject to constraints
$g_{i}(X) \leq 0$ and $h_{j}(X)=0(\mathrm{j}=1,2, \ldots \ldots . \mathrm{m})$
where, $X$ is a n -dimensional ve ctor c alled t he de sign ve ctor, $F(X)$ is ca lled t he objective f unction. $\quad g_{i}(X)$ a nd $h_{i}(X)$ a re, respectively, t he e quality and i nequality constraints. The constrained stated in Eq. (4.1) is called a constrained optimization problem.

Some optimization problems do not involve any constraints and can be stated as:-
Find $X=\left\{x_{1}, x_{2} \ldots x_{n}\right\}$ which minimize or maximize, $F(X)$
Such problems are called unconstrained optimization problems.

Design vector: - any e ngineering system or component is described by set of quantities some of which are viewed as variables during the design process. In general certain quantities are u sually fixed at the outset and these are cal led pre assi gned parameter or c onstant design pa rameters. A 1 l he ot her quantities a re t reated as variables in the design process and are called design or decision variable, $X_{i}, i=1$, $2 \ldots \mathrm{n}$. The design variables are collectively represented as a design vector.

In s tructural de sign, f rom phys ical point of v iew, t he de sign va riables $\boldsymbol{X}$ that a re varied by o ptimization procedure $m$ ay $r$ epresent $t$ he $f$ ollowing pr operties of $t$ he structure
(i) The mechanical and physical properties of material
(ii) Topology of the structure i.e. the pattern of connection of members or the number of element in a structure.
(iii) The configuration or geometric layout of the structure
(iv) The cross-sectional dimensions or the member sizes

The types of design v ariables m aybe either continuous, integer, discrete or a combination of these types. Integer or discrete design variable is number of elements
in the structure, for e xample. From mathematical point of v iew, it is important to distinguish between continuous and discrete variables.

Design C onstraints: - In $m$ any pr actical problems, $t$ he design $v$ ariable $c$ annot be chosen arbitrarily; rather they have to satisfy certain specified functional, behavioral and other requirement. The restrictions that must be satisfied in order to produce an acceptable design are collectively called constraints. If thedesign m eets the entire requirement placed on it, it is called a feasible design. From the physical point of view we may identify two kind of constraint:
(i) Constraints imposed on the design variables and which restrict their range. For reasons other than behavior considerations will be called explicit constraint or side constraints. These constraints, which are explicit in from, may derive from va rious c onsiderations s uch a s functionality, fabrication, or aesthetics thus, a side constraints is a specified limitation (upper or lower bound) on a design variable, or a relationship which fixes the relative value of a group of design va riable, e xample of s uch c onstrain in s tructural de sign i nclude minimum thickness of plate, maximum height of a shell structure, minimum slope of a roof structure.
(ii) Constraints th at are derived from be havior r equirements w ill be c alled behavior constraints or implicit constraints in structural design. For example, limitations on maximum s tresses, de flections, flexural strength, or bu ckling strength are implicit constraints.

Explicit a nd im plicit c onstraints a re o ften g iven b y formulas according to design codes or sp ecifications. H owever im plicit c onstraints a re g enerally im plicit in a ny case the constraint $m$ ust be a c omputable function of $t$ he de sign va riable. Form a mathematical p oint of v iew, b oth explicit and implicit constraints m ay us ually be expressed as a set of inequalities, $g_{i}(X) \leq 0 ;(j=1,2 \ldots . m)$. Where $m$ is the number of inequality constraints and $\boldsymbol{X}$ is the vector of design variables. In a structural design problem, one has also to consider equality constraints of the general form, $h_{j}(X)=0$; $(j=1 \ldots \mathrm{p})$. Where $p$ is the number of equalities.

Objective $f$ unction: - The conventional de sign pr ocedures a im a $t \mathrm{f}$ inding a n acceptable or adequate design, which merely sat isfies the $f$ unctional an $d o$ ther requirements of $t$ he $p$ roblem. In ge neral $t$ here will be $m$ ore $t$ han o ne accep table designs a nd the pur pose of opt imization istochoose the be st out ofthe many acceptable design va riable. Thus a c riterion hastobechosen for omparing the different a lternate acceptable de sign a nd selecting the be st one. T he criteria w ith respect to which the design is optimized when expressed as a function of the design variable is called objective function. The choice of the objective function is governed by the $n$ ature of the problem. For instant, in aircraft and a erospace structure de sign problem, $t$ he obj ective $f$ unction is us ually $b$ e $w$ eight oft he structure, in civil engineering structure designs, the objective is u sually taken as the minimization of cost.

### 4.6 Optimization Algorithm (EVOP)

A global optimization a lgorithm EVOP (Evolutionary O peration) for constrained parameter optimization has been presented. Few current methods cope with real world problems involving discontinuous objective and constraining functions where there is ac ombination of continuous, di screte and integer set of arguments a nd global minimum is sought. For noisy data, solutions are possible with genetic algorithms but costly parallel processing would be needed to locate the global minimum. Solutions remain elusive with genetic algorithms for problems with hard real-time constraints. The robust algorithm EVOP surmounts these difficulties with a much faster and more accurate solution.

## Virtues of EVOP

Searching the Internet will yield a number of algorithms with the name EVOP. But this EVOP is unique in its speed and accuracy and flexibility. It appears this EVOP is the ' silver bullet' t hat ha s s ucceeded in s laying t he dr agon of di mensionality in multiple minima bound objective function. Nothing comparable is available to date. It has the c apability to locate directly with high probability the global minimum. It is also c apable to de al with pos sible finite num ber of di scontinuities in the nonl inear objective a nd c onstraining f unctions. I th as t he a bility $\mathrm{t} o \mathrm{~m}$ inimize di rectly an objective function without requiring information on gr adient or sub-gradient. It can
also de al with objective functions having a mix of integer, discrete and continuous variables a s a rguments. There is nor equirement $f$ or $s$ caling of o bjective a nd constraining functions. It has the capability for optimization even when there are more than one of the above difficulties simultaneously present. It has facility for automatic restarts to check whether the previously obtained minimum is the global minimum. It can optimize physical systems in real-time or accelerated time; e.g. optimal adaptive control of phys ical s ystems. S ince obj ective f unction is ne ver evaluated in the infeasible region, as a consequence the safety of the plant or system is not in jeopardy at a ny time be cause of opt imization. Gradient or sub-gradient is notr equired thus ensuring that noise in measurement will not be accen tuated to ad versely af fect the optimization process. It has inherent ability to cope with realistic hard time constraint requirement imposed by real-time systems.

An updated version of EVOP is available that is capable for minimization of objective function ha ving combination of integer, discrete a nd continuous a rguments (design variables). The method treats all arguments as continuous but for discrete and integer variables, picks values from thin strips cen tered on specified values. The procedure EVOP has successfully minimized a large number of internationally recognized test problems ( Ghani, 1 995). The p roblems w ere ca tegorized as u nconstrained, constrained, multiple minima and mixed variable problems.

The algorithm can minimize an objective function
$F(X)=F\left(x_{1}, x_{2} \ldots x_{n}\right)$
Where, $F(x)$ is a function of n independent variables $\left(x_{1}, x_{2} \ldots x_{n}\right)$. The n independent variables $x_{i}{ }^{\text {s }} \mathrm{s}(i=1,2 \ldots . n)$ are subject to explicit constraints
$l_{i} \leq x_{i} \leq u_{i}$
Where, $l_{i}{ }^{\text {s }} \mathrm{s}$ a nd $u_{i}{ }^{\text {s }} \mathrm{s}$ a re lo wer a nd u pper lim its o n th e v ariables. T hey a re e ither constants or f unctions of n i ndependent va riables (movable bounda ries). These n independent variables $x_{i}$ 's are also subject to $m$ numbers of implicit constraints
$L_{j} \leq f_{j}\left(x_{1}, x_{2} \ldots x_{n}\right) \leq U_{j}$
Where, $j=1,2 \ldots . m . L_{j}$ 's and $U_{j}$ 's a re lower and upper limits on the m implicit constraints. They are either constants or functions of n independent variables.

### 4.6.1 The Procedure

The method is subdivided into six fundamental processes (Figure 4.2) which are fully described in the reference (Ghani, 1989). They are,
(i) Generation of a 'complex',
(ii) Selection of a 'complex' vertex for penalization,
(iii) Testing for collapse of a 'complex',
(iv) Dealing with a collapsed 'complex',
(v) Movement of a 'complex' and
(vi) Convergence tests.


Figure 4.2 General outline of EVOP Algorithm (Rana, 2010)

A 'complex' is a 'living' object spanning an $n$-dimensional space defined by $k \geq(n+1)$ vertices inside the feasible region. It has the intelligence to move towards a minimum located on the boundary or inside the allowed space. It can rapidly change its shape and s ize f or ne gotiating di fficult terrain. F igure 4.3 s hows a ' complex' with four vertices in a two dimensional parameter space. The 'complex' vertices are identified
by lower case 1 etters ' $a$ ', ' $b$ ', 'c' and 'd' in an ascending or der of function values, i.e. $\mathrm{f}(\mathrm{a})<\mathrm{f}(\mathrm{b})<\mathrm{f}(\mathrm{c})<\mathrm{f}(\mathrm{d})$. S traight l ine parallel to th e c o-ordinate axes ar e ex plicit constraints with fixed upper a nd 1 ower 1 imits. The curved lin es represent implicit constraints set to either upper or lower limits. The hatched area is the two dimensional feasible search spaces.


Figure 4.3 A "complex" with four vertices inside a two dimensional feasible search space (Ghani, 1989)

### 4.6.1.1 Generation of a 'complex'

Referring to Figure 4.4, for any feasible parameter space a random point is generated in such a w ay that all the ex plicit constraints are au tomatically s atisfied. T he co ordinates of this random point are given by
$x_{i} \triangleq l_{i}+r_{i}\left(u_{i}-l_{i}\right)(i=1,2 \ldots n)$
where $r_{i}$ is a pseudo-random deviate of rectangular distribution over the interval $(0,1)$.

The implicit constraints are satisfied by continually moving the newly generated point halfway towards the feasible centroid of all feasible vertices al ready g enerated. The new point $\mathbf{x}$ is obtained from the old feasible point $\mathbf{x}^{\prime}$ and the feasible centroid $\mathbf{C}$ as follows,
$\boldsymbol{x}=\frac{1}{2}\left(\boldsymbol{C}+\boldsymbol{x}^{\prime}\right)$


Figure 4.4 Generation of initial "complex" (Ghani, 1989)

Once $x$ has satisfied all constraints it is added to the list of feasible complex vertices. This p rocess is re peated till a 11 k v ertices th at s atisfy a 11 explicit a nd a 1 im plicit constraints have been generated beginning from a single feasible starting point.

In simpler language, a starting point is required that satisfies all explicit and implicit constraint sets. A second point is randomly generated within the bound de fined by the e xplicit c onstraints. If th is second p oint a lso h appens to s atisfy, a ll implicit constraints, e verything is goi ng fine. C entroid of $t$ he $t$ wo $f$ easible points is determined. If itsatisfies all constraints, then things a re really going fine and the 'complex' is upda ted with this second point. I f, how ever, the randomly ge nerated point fails to satisfy implicit constraints it is continually moved half way towards the feasible starting point till all constraints are satisfied. Feasibility of the centroid of the two points is next checked. If the centroid satisfies all constraints, then we have an acceptable 'complex', and we proceed to generate the third point for the 'complex'. If, how ever, the centroid fails to s atisfy a ny of the constraints this second point is randomly once again generated in the space defined by the explicit constraints.

Referring to the Figure 4.4, the first random point is ' d ', and the centroid is the feasible starting point ' $a$ '. The point ' $d$ ' is moved halfway towards ' $a$ ' in or der to satisfy the violated im plicit constraint. Next the centroid of the feasible vertices ' a ' a nd 'd' is determined which itself is feasible. Another random point ' c ' is next generated and is moved to 'c' to satisfy the violated implicit constraint. Repeating the procedure all the
(k-1) feasible vertices of the in itial 'complex' are obtained. The in itial 'complex' for the two dimensional example is the object 'abcd'. It can be seen that the same feasible point 'a' can be used again for generating a new initial 'complex' which would be of a completely different shape and size. The method allows repeated re-use of the same feasible starting point for checking whether the global minimum has been located.

For a convex feasible parameter space the above method would, without fail, succeed in generating a 'complex' with k vertices. If the parameter space is non-convex and the centroid happens to lie in the infeasible area, there is every chance that a ' complex' cannot be generated. Figure 4.5 shows such a possibility. Three vertices 'a', 'b' and 'c' in the feasible parameter space have already been generated. In order to generate the fourth feasible ve rtex a trial point ' $\mathrm{T}_{1}$ ' sat isfying the explicit constraints is created. However, ' $\mathrm{T}_{1}$ ' is infeasible as it violates an implicit constraint. In order to make ' $\mathrm{T}_{1}{ }^{\prime}$ feasible it is continually moved halfway towards the centroid ' $\mathbf{C}$ '. Since the centroid itself is infeasible no amount of such moves would make ' $\mathrm{T}_{1}$ ' feasible, and a 'complex' with four vertices can never be generated. In such case if a $n$ ew feasible 'complex' vertex re sults in the n ew c entroid to lie in the in feasible area, that n ew v ertex is discarded and another is generated until a feasible centroid is obtained.

In minimization involving combination of continuous, integer and discrete variables rounding of $f$ of a ppropriate co-ordinates of the trial point is conducted in the us er written explicit constraint function. The co-ordinates of the centroid should never be rounded off.


Figure 4.5 A 'complex' with four vertices 'abcd' cannot be generated
(Ghani, 1989)

### 4.6.1.2 Selection of a 'complex' vertex for penalization

In the present minimization process the worst vertex 'ng' of a 'complex' is that with the highest f unction va lue w hich is penalized b y ove r -reflecting on the c entroid. Referring to Figure 4.6, the penalized point ' $d$ ' is re flected over the centroid ' $\mathbf{C}$ ' to create a trial point ' $\mathrm{T}_{1}$ ' which violates an implicit constraint. Since the centroid itself is in the infeasible region, repeated movement of point ' $\mathrm{T}_{1}$ ' halfway towards the centroid would result in collapse of the point on the centroid. The new complex has now three vertices 'a', 'b' and 'c'. One more such collapse would result in complete collapse of the 'complex' because an object with two vertices cannot span a two dimensional space. In ge neral a s pace of n di mension c an onl y be s panned by obj ects de fined by k vertices where $\mathrm{k} \geq(\mathrm{n}+1)$.


Figure 4.6 The possibility of collapse of a trial point onto the centroid.
(Ghani, 1989)
For selection of a 'complex' vertex for penalization the procedure as shown in Figure 4.7 is followed until a preset number of calls to the three functions (Implicit, Explicit and Objective) are collectively exceeded.

### 4.6.1.3 Testing for collapse of a 'complex'

A 'complex' is said to have collapsed in a subspace if the $\mathrm{i}^{\text {th }}$ coordinate of the centroid is identical to $t$ he sameof all' $\mathrm{k}^{\prime} \mathrm{v}$ ertices oft he complex'. This is a su fficiency condition and detects collapse of a ' complex' when it lies parallel along a co ordinate axis. Once a 'complex' has collapsed to a subspace it can never again be able to span the original space. The word "identical" implies here "identical within the resolution of $\Phi_{\mathrm{cpx}}$ which is a parameter for detection of 'complex' collapse. Numbers x and y are identical w ithin the resolution of $\Phi_{\mathrm{cpx}}$ if x a $\mathrm{nd}\left\{\mathrm{x}+\Phi_{\mathrm{cpx}}(\mathrm{x}-\mathrm{y})\right\}$ h ave t he sam e numerical values. For $\Phi_{\mathrm{cpx}}$ set to $10^{-2}$, if x and y differs by not more than the last two significant digits they will be considered identical.


Figure 4.7 Selection of a 'complex' vertex for penalization (Rana, 2010)

Figure 4.8 shows a 'complex' with vertices 'a', 'b', 'c' and 'd', which has collapsed to a one dimensional search space. The $\mathrm{X}_{2}$ coordinates of all vertices and the centroid are identical within the resolution of $\Phi_{\mathrm{cpx}}$. As can be seen the 'complex' vertices can now move onl y along the $X_{1}$ coordinate direction. $S$ uch collapses a long o ther angular directions have not been accounted. Such a failure would rapidly lead to the type of collapse discussed above, albeit additional computations will be performed.


Figure 4.8 Collapse of a 'complex' to a one dimensional subspace. (Ghani, 1989)

### 4.6.1.4 Dealing with a collapsed 'complex'

On d etecting co llapse of a' complex' so me act ions ar et aken su ch $t$ hat an ew 'complex' is ge nerated within the full feasible s pace de fined by the explicit a nd implicit constraints or a 'complex' spanning smaller feasible space. The process for movement of a 'complex' as explained below is continued.

### 4.6.1.5 Movement of a 'complex'

The pr ocess be gins by over-reflecting the worst ve rtex 'ng' of a ' complex' on $t$ he feasible centroid ' $\mathbf{C}$ ' of the remaining vertices to generate a new trial point $\mathbf{x}_{\mathbf{r}}$,
$x_{r}=(1+\alpha) C-\alpha x_{g}$
where $\alpha$ is reflection coefficient.

A check is then made to determine whether the trial point violates any constraints. If an explicit constraint is violated, the trial point is moved just inside the boundary by a small amount $\Delta$ called the explicit constraint retention coefficient. If a ny implicit constraint is violated the trial point is repeatedly moved halfway towards the centroid until the constraint is satisfied.

Figure 4.9 shows $t$ he penalized point ' d ' on ove r-reflection $v$ iolates a $n$ e xplicit constraint. The trial point ' $\mathrm{T}_{1}$ ' is moved to ' $\mathrm{T}_{2}$ ' just inside the constraint boundary by a factor, $\Delta$.


Figure 4.9 The reflected point violating an explicit constraint. (Ghani, 1989)

Figure 4.10 shows the case when an implicit constraint is violated. The infeasible trial point ' $T_{1}$ ' is moved halfway towards the feasible centroid ' $\mathbf{C}$ ' to ' $\mathrm{T}_{2}$ ' which satisfies all constraints.

The function value at the feasible trial point is next evaluated. The reflection step is considered successful if the function value at this new trial point is lower than that at vertex ' $n h$ ', and the vertex ' $n g$ ' is replaced by the trial point. If, however, the function value at the trial point is greater than that at vertex 'nh' of the current 'complex', the trial poi nt would be $t$ he $w$ orst $v$ ertex in $t$ he ne $w$ ' complex' $c$ onfiguration. $T$ he reflection step is, therefore, considered unsuccessful and contraction step applied.


Figure 4.10 The reflected trial point violating an implicit constraint (Ghani, 1989)
Depending on situation anyone of the three stages (Stages 1-3) of the contraction step can be called. If the function value at the feasible trial point after over-reflection is less than that at vertex 'ng' but equal to or greater than that at vertex 'nh', Stage 1 of contraction step is ap plied. This is essen tially an $u$ nder-reflection, and he coordinates of the new trial point $\mathbf{x}$ is given by
$x=(1+\beta) C-\beta x_{g}$
where $\beta$ is contraction coefficient.

Figure 4.11 shows the worst vertex 'd' of the current 'complex' 'abcd' over-reflected on the f easible cen troid ' $\mathbf{C}$ '. The tria 1 p oint ' $\mathrm{T}_{1}$ ' so obt ained is moved j ust i nside a n explicit constraint resulting in trial point ' $\mathrm{T}_{2}$ ' which still violates an implicit constraint.
' $\mathrm{T}_{2}$ ' is moved halfway towards the centroid ' $\mathbf{C}$ ' along the line joining the two resulting in a completely feasible trial point ' $\mathrm{T}_{3}$ '. Function value at ' $\mathrm{T}_{3}$ ' is calculated, and found to be intermediate be tween the second hi ghest function value at vertex ' c ', a nd the highest function value at vertex ' $d$ '. If ' $d$ ' is replaced by the feasible trial point ' $T_{3}$ ' to form an ew co mplex ' $\operatorname{abcT}_{3}$ ', t hen ' $\mathrm{T}_{3}$ ' w ould be the w orst v ertex in the new configuration. The reflection step is, therefore, considered unsuccessful and point ' $\mathrm{T}_{3}$ ' is rejected, and Stage 1 of contraction step is applied. The worst vertex 'd' is underreflected on $n$ he f easible cen troid ' $\mathbf{C}^{\prime}$ to ' $\mathrm{T}_{4}$ '. Since the trial p oint ' $\mathrm{T}_{4}$ ' v iolates an implicit constraint it is moved halfway towards the centroid to ' $\mathrm{T}_{5}$ '. The trial point ' $\mathrm{T}_{5}$ ' is feasible, and replaces the worst vertex ' d ' to form a new complex 'abc $\mathrm{T}_{5}$ '.


Figure 4.11 Unsuccessful over-reflection and Stage of contraction step applied. (Ghani, 1989)

Stage 2 of contraction step is applied if at the end of over-reflection the function value at the feasible trial point is equal to or greater than that at the worst vertex ' $n g^{\prime}$ ' of the current complex. The co-ordinates of the new trial point are given by:

$$
\begin{equation*}
x=\beta x_{g}+(1-\beta) C \tag{4.9}
\end{equation*}
$$

Figure 4.12 shows that the point ' $d$ ' of a current 'complex' 'abcd' is reflected over the centroid ' $\mathbf{C}$ ' of the remaining po ints ' $a$ ', ' $b$ ' a nd ' $c$ ' to obtain a trial point ' $T_{1}$ ' which violates both explicit and implicit constraints. The explicit constraint is satisfied by moving the point ' $\mathrm{T}_{1}$ ' to ' $\mathrm{T}_{2}$ ' just inside the boundary of the explicit constraint. The implicit constraint is satisfied by moving the point ' $\mathrm{T}_{2}$ ' to ' $\mathrm{T}_{3}$ ' halfway towards the feasible centroid ' $\mathbf{C}$ '. The function value at the feasible trial point ' $T_{3}$ ' is found to be
greater $t$ han $t$ he hi ghest function va lue of $t$ he $c$ urrent 'complex' at $v$ ertex ' $d$ '. The reflection step is, therefore, considered unsuccessful, and point ' $\mathrm{T}_{3}$ ' is rejected. Stage 2 of contraction step is applied penalizing the worst vertex ' $\mathrm{d}^{\prime}$ to ' $\mathrm{T}_{4}$ '. The trial point ' $\mathrm{T}_{4}$ ' violates an implicit constraint. It is made feasible by moving halfway to wards the feasible centroid ' $\mathbf{C}$ ' to ' $\mathrm{T}_{5}$ '. ' $\mathrm{T}_{5}$ ' replaces the worst vertex ' d ' to form a new 'complex' 'abcT5'.


Figure 4.12 Unsuccessful over-reflection and Stage 2 of contraction step applied (Ghani, 1989)
Stage 3 of contraction step is called only after Stages 1 and/or 2 have been previously applied consecutively for more than ' 2 k ' times. A small 'complex' is generated using vertex 'ns' as the starting point. If on over-reflection the trial point has not violated any constraints, has a function value lower than the lowest function value at vertex 'ns' of the current 'complex', and the previous move was not a contraction step, this overreflection is considered over-successful. Expansion step is then applied to generate a new trial point further a way from the feasible centroid a long the same straight line used for over-reflection. The co-ordinates of this accelerated trial point is given by

$$
\begin{equation*}
x=\gamma x_{r}+(1-\gamma) C \tag{4.10}
\end{equation*}
$$

Where, $\gamma$ is expansion coefficient.

Feasibility of this accelerated trial point is next checked. If any constraint is violated, the acceleration step is considered unsuccessful, and a new 'complex' is formed with the ove r-reflected feasible trial point $\boldsymbol{x}_{r}$ as the upda ted ve rtex r eplacing $t$ he w orst vertex ' $n g$ ' of $t$ he $c$ urrent ' complex'. O therwise $t$ he $f$ unction va lue a $t$ the $f$ easible
accelerated trial point is evaluated. If it is lower than that at x the acceleration step is considered successful. The accelerated point then replace the worst vertex ' $n g$ ' to form the $n$ ew ' complex'. E lse, ift he function value at the a ccelerated $p$ oint eq uals or exceeds that at the ove r-reflected point $\boldsymbol{x}_{\boldsymbol{r}}$ the acceleration st ep is a lso co nsidered unsuccessful. The accelerated point is rejected in favor of the point $\boldsymbol{x}_{\boldsymbol{r}}$ to form the new 'complex'.

Figure 4.13 shows a successful acc eleration step. The over-reflected trial point ${ }^{\prime} \mathrm{T}_{1}{ }^{\prime}$ does not vi olate a ny c onstraint, a nd has a f unction va lue 1 ower t han t he 1 owest function value at vertex 'a' of the current 'complex' 'abcd'. Since contraction step was not applied previously acceleration step is called to obtain the trial point ' $\mathrm{T}_{2}$ '. ' $\mathrm{T}_{2}$ ' does not violate any constraint and yet has a function value lower than that at ' $\mathrm{T}_{1}$ '. Trial point ' $\mathrm{T}_{2}$ ' replaces the worst vertex ' d ' of the c urrent 'complex' to form the updated 'complex' 'abcT ${ }_{2}$ '.


Figure 4.13 Successful acceleration steps (Ghani, 1989)

### 4.6.1.6 Convergence tests

While executing the process of movement of a 'complex', test for convergence are made periodically after certain preset number of calls to the objective function. There are two levels of convergence tests. The first convergence test would succeed only if a predefined number of consecutive function values are identical within the resolution of co nvergence p arameter $\Phi$, which s hould be finer than $\Phi_{\mathrm{cpx}}$. T he second convergence test is attempted only if the first convergence test succeeds. This second test for co nvergence verifies whether function values at all vertices of the cu rrent 'complex' are also identical within the resolution of $\Phi$.

## Chapter 5

## Problem Formulation

### 5.1 Introduction

Appropriate o ptimization pr oblem has $t o$ be $f$ ormulated $f$ or $t$ he pr esent br idge structure (a two-span continuous post-tensioned girder of bridge) to be solved by an optimization $m$ ethod (EVOP). Then, o ptimization method solves the problem and gives $t$ he optimum $s$ olutions. In $t$ his $c$ hapter $t$ he $v$ arious $c$ omponents of $t$ he optimization problem of the present study are described. The various components are mathematical expression required for the design and analysis of the bridge system, an objective f unction, implicitc onstraints, explicitc onstraints and inputc ontrol parameters for $t$ he opt imization $m$ ethod.For $d$ esign a nd a nalysis of the tw o-span continuous gi rder, e ight s ections/ positions a long $t$ he $s$ pan of $t$ he girder w ere considered. The s tructure being a ni ndeterminate one, Stiffness Met hodwas incorporated in the mathematical expression required for the design and analysis of it. Prestress losses, allowable and ultimate strength design criteria, ductility limits, lateral stability and deflection criteria were applied/ considered by taking into account the continuity effect of the structure. The program is finally linked to the opt imization method to obtain the optimumsolutionsof cost optimum design.

### 5.2 Objective Function

In this study, the objective is the cost minimization of the present bridge systems by taking in to a ccount the cost of a ll materials, fabrication, a nd in stallation. The to tal cost of a bridge system is formulated as:
$C_{T}=C_{G C}+C_{D C}+C_{P S}+C_{O S}$
where, $C_{G C}, C_{D C}, C_{P S}$ and $C_{O S}$ are the cost of materials, fabrication and installation of Girder Concrete, Deck slab Concrete, Prestressing Steel and Ordinary Steel for deck reinforcement an $d g$ irder's sh ear reinforcement $r$ espectively. $C$ osts ofindividual components arecalculated as:

$$
\begin{equation*}
C_{G C}=\left(U P_{G C} V_{G C}+U P_{G F} S A_{G}\right) N_{G} \tag{5.2}
\end{equation*}
$$

$$
\begin{align*}
& C_{D C}=\left(U P_{D C} V_{D C}+U P_{D F}\left(S-T F_{w}\right)\right) N_{G}  \tag{5.3}\\
& C_{P S}=\left(U P_{P S} W_{P S}+2 U P_{A N C} N_{T}+U P_{S H} N_{T} L\right) N_{G}  \tag{5.4}\\
& C_{N S}=U P_{O S}\left(W_{O S D}+W_{O S G}\right) N_{G} \tag{5.5}
\end{align*}
$$

where, $U P_{G C}, U P_{D C}, U P_{P S}$ and $U P_{O S}$ are the $u$ nit prices including $m$ aterials, 1 abor, fabrication and installation of (i) the precast g irder co ncrete, (ii) deck concrete, (iii) prestressing steel and (iv) ordinary steel respectively; $U P_{G F}, U P_{D F}, U P_{A N C}, U P_{S H}$ are the unit prices of girder formwork, deck formwork, a nchorage set a nd metal sheath for duct re spectively; $V_{G C}, V_{D C}, W_{P S}, W_{O S D}$ and $W_{O S G}$ are the volume of the precast girder concrete and deck slab concrete, weight of prestressing steel and ordinary steel in deck slab and in girder respectively; $L$ is the girder span; $N_{G}$ is number of girders; $S$ is girder spacing.

### 5.3 Design Variables and Constant Design Parameters

For a particular girder span and bridge width, a large number of parameters control the design of the bridge such as girder spacing, cross sectional dimensions of girder, deck $s$ lab $t$ hickness, $n$ umber of $s$ trands pertendon, nu mber of $t$ endons, de ck slab reinforcement, configuration of tendons, anchorage system, pre-stress losses, concrete strength e tc. The de sign va riables a nd va riable type considered int he study a re tabulated in Table 5.1. A typical cross-section of the two-span continuous PC I-girder at 0.4 L distance from end support is illustrated in Figure 5.1 to highlight several of the design variables. Besides, typical cross-sections i) above end support (at position of zero moment), ii) at positive moment region and iii) just above the interior support (at position of maximum negative moment) are illustrated in Figure 5.2 as well.


Figure 5.1 Section with design variables at positive moment region (Rana, 2010).


Figure 5.2 Typical cross-section at various positions along the span

The constant design $p$ arameters under consideration are various $m$ aterial properties, superimposed dead loads, AASHTO live load, strand size, post-tensioning anchorage system a nd u nitc osts ofm aterials including $f$ abricationa nd installatione tc. Optimization is based on the analysis of an interior girder arranged as shown in Figure 5.3. The girder and the deck are assumed to act as a composite section during service condition. Prestress is considered to be applied in two st ages, a p ercentage of total prestress at initial stage to carry only the girder self weight \& stress produced during lifting and transportation and full prestress during casting of deck slab. In the present study the tendons arrangement is not assumed as fixed rather it is considered as design variable as it has significant effects on prestress losses and flexural stress at various sections along the girder. Tendons layout along the span is assumed as parabolic. The vertical a nd hor izontal arrangement of $t$ endons de pends on va rious cross sectional dimensions of girder such as depth, bottom flange, and web. Typical arrangements of tendons at various sections are shown in Figure 5.2. The arrangement of tendons also depends on duct size and spacing, a nchorage s pacing a nd a nchorage e dge di stance. These parameters depend on a design variable, namely, number of strand per tendon and on a constant pa rameter, na mely, c oncrete strength, and a re de termined us ing values listed in the Table 5.2.


Figure 5.3 Girders arrangement in the bridge

Table 5.1 Design variables with variable type

| Design variables | Variabletype |
| :--- | :---: |
| Girder spacing $(\mathrm{S})(\mathrm{m})$ | Discrete |
| Girder depth $\left(\mathrm{G}_{\mathrm{d}}\right)(\mathrm{mm})$ | Discrete |
| Top flange width $\left(\mathrm{TF}_{\mathrm{w}}\right)(\mathrm{mm})$ | Discrete |
| Top flange thickness $\left(\mathrm{TF}_{\mathrm{t}}\right)(\mathrm{mm})$ | Discrete |
| Top flange transition thickness $\left(\mathrm{TFT}_{\mathrm{t}}\right)(\mathrm{mm})$ | Discrete |
| Bottom flange width $\left(\mathrm{BF}_{\mathrm{w}}\right)(\mathrm{mm})$ | Discrete |
| Bottom flange thickness $\left(\mathrm{BF}_{\mathrm{t}}\right)(\mathrm{mm})$ | Continuous |
| Web width $\left(\mathrm{W}_{\mathrm{w}}\right)(\mathrm{mm})$ | Discrete |
| Number of strands per tendon $\left(\mathrm{N}_{\mathrm{s}}\right)$ | Integer |
| Number of tendons per girder $\left(\mathrm{N}_{\mathrm{T}}\right)$ | Integer |
| Lowermost tendon position at the end from bottom $\left(\mathrm{y}_{1}\right)(\mathrm{mm})$ | Continuous |
| Initial stage prestress $(\%$ of full prestress $)(\eta)$ | Continuous |
| Slab thickness $(\mathrm{t})(\mathrm{mm})$ | Discrete |
| Slab main reinforcement ratio $(\rho)$ | Continuous |

Table 5.2 Minimum dimensions for $\mathbf{C}$ range anchorage system

| No. of strands per <br> tendon | $\mathbf{1 - 3}$ | $\mathbf{4}$ | $\mathbf{5 - 7}$ | $\mathbf{8 - 9}$ | $\mathbf{1 0 - 1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 4 - 1 9}$ | $\mathbf{2 1 - 2 2}$ | $\mathbf{2 3 - 2 7}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Duct diameter | 45 | 50 | 65 | 70 | 85 | 85 | 100 | 110 | 115 |
| Duct clear spacing $\left(\mathrm{D}_{\mathrm{S}}\right)$ | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 50 | 50 |
| $\mathrm{~A}_{\mathrm{D}}$ | 110 | 120 | 150 | 185 | 200 | 210 | 250 | 275 | 300 |
| $\mathrm{~A}_{\mathrm{M}}$ | 128 | 150 | 188 | 210 | 248 | 255 | 300 | 323 | 345 |

$\overline{A S}=\frac{F}{f_{c} \times B F_{w}} ; f_{c}=$ concrete strength at stressing time; $F=$ Prestressing force; $A_{\mathrm{D}}=$ Anchorage dimension; $\mathrm{A}_{\mathrm{M}}=$ Anchorage minimum vertical edge distance

### 5.4 Explicit Constraints

These are specified 1 imitation (upper or lower limit) on design variables which are derived $f$ romg eometric $r$ equirements ( superstructure depth, cl earances e tc.), minimum practical dimension for construction, code restriction etc. The constraint is defined as
$X_{L} \leq X \leq X_{U}$
Where, $X=$ Design variable, $X_{L}=$ Lower limit of the design variable, $X_{U}=$ Upper limit of the design variable.

Explicit constraints for girder spacing: Lower and upper limit of girder spacing is considered such that number of girder in the bridge can vary from 1 to 10 .

Explicit constraints for top flange: The lower limit of top flange width is assumed as 300 mm from lateral stability and be aring considerations and upper limit equal to girder spacing. The lower limit of top flange thickness is considered as 75 mm to resist damage during handling and proper placement of transverse reinforcement and upper limit is assumed as 300 mm . The lower limit of top flange transition thickness is considered as 50 m m to facilitate pl acement a nd consolidation of concrete a nd upper limit is assumed as 300 mm . The haunch thickness and width is assumed as 50 mm .

Explicit constraints for web: The lower limit of web width is equal to diameter of duct plus web rebars and clear cover (Figure 5.4) and upper limit is assumed as 300 mm .


Figure 5.4 Width of web (Rana, 2010)

Explicit constraints for bottom flange: The lower limit of bottom flange width is assumed as 300 mm to accommodate anchorage setup and upper limit equal to girder spacing. The lower limit of thickness is equal to clear cover and duct diameter to fit at
least one $r$ ow of $t$ endons. The up per 1 imit is a ssumed as 600 mm . The width to thickness ratio of bottom flange transition area is assumed as 2 to 1 frompractical construction point of view.

Explicit constraints for girder depth: The lower limit of girder depth is considered as 1000 m m and upper limit 3500 mm which is common range of girder de pth to minimize the cost of substructure and approach roads and from aesthetics and limited clear space criterion.

Explicit constraints for num ber of strand pertendon: Within the available anchorage system one tendon may consist of several seven-wire strands like 1 to 55 . Here $t$ he effect of num ber of $s$ trands in a tendon is studied. F or this study it is considered that each tendon may consist of 1 to 27 strands.

Explicit constraints for num ber of tendon: The amount of pre-stressing force required for cost optimum design are directly associated with the number of tendons required in the girder. For this study it is considered that the number of tendon may vary from 1 to 20.

Explicit constraints for lowermost tendon position: To vary the profile of tendon along the girder span the lower most tendon position from bottom at the end section is considered as a design variable and the other tendon positions are determined from anchorage s pacing. The lower limit of vertical position of the tendon is considered equal to an chorage minimum vertical edge distance an $d$ upper limit is assu med as 1000 mm .

Explicit constraints for deck slab: The lo wer lim it ofdeck s lab thickness is considered as 175 mm to control deflection and ex cessive crack and upper limit as 300 m m . The lower and uppe r limits of de ck s lab reinforcement a re considered according to A ASHTO standard s pecification. The e xplicit c onstraints for all the above design variables are shown in Table 5.3.

Table 5.3 Design variables with Explicit Constraints

| Design variables | Explicit Constraint |
| :--- | :---: |
| Girder spacing $(\mathrm{S})(\mathrm{m})$ | $\mathrm{B}_{\mathrm{W}} / 10 \leq \mathrm{S} \leq \mathrm{B}_{\mathrm{W}}$ |
| Girder depth $\left(\mathrm{G}_{\mathrm{d}}\right)(\mathrm{mm})$ | $1000 \leq \mathrm{G}_{\mathrm{d}} \leq 3500$ |
| Top flange width $\left(\mathrm{TF}_{\mathrm{w}}\right)(\mathrm{mm})$ | $300 \leq \mathrm{TF}_{\mathrm{w}} \leq \mathrm{S}$ |
| Top flange thickness $\left(\mathrm{TF}_{\mathrm{t}}\right)(\mathrm{mm})$ | $75 \leq \mathrm{TF}_{\mathrm{t}} \leq 300$ |
| Top flange transition thickness $\left(\mathrm{TFT}_{\mathrm{t}}\right)(\mathrm{mm})$ | $50 \leq \mathrm{TFT}_{\mathrm{t}} \leq 300$ |
| Bottom flange width $\left(\mathrm{BF}_{\mathrm{w}}\right)(\mathrm{mm})$ | $300 \leq \mathrm{BF}_{\mathrm{w}} \leq \mathrm{S}$ |
| Bottom flange thickness $\left(\mathrm{BF}_{\mathrm{t}}\right)(\mathrm{mm})$ | $\mathrm{a} \leq \mathrm{BF}_{\mathrm{t}} \leq 600$ |
| Web width $\left(\mathrm{W}_{\mathrm{w}}\right)(\mathrm{mm})$ | $\mathrm{b} \leq \mathrm{W}_{\mathrm{w}} \leq 300$ |
| Number of strands per tendon $\left(\mathrm{N}_{\mathrm{s}}\right)$ | $1 \leq \mathrm{N}_{\mathrm{s}} \leq 27$ |
| Number of tendons per girder $\left(\mathrm{N}_{\mathrm{T}}\right)$ | $1 \leq \mathrm{N}_{\mathrm{T}} \leq 20$ |
| Lowermost tendon position at the end from bottom $\left(\mathrm{y}_{1}\right)(\mathrm{mm})$ | $\mathrm{A}_{\mathrm{M}} \leq \mathrm{y}_{\mathrm{l}} \leq 1000$ |
| Initial stage prestress (\% of full prestress) $(\eta)$ | $1 \% \leq \eta \leq 100 \%$ |
| Slab thickness (t) (mm) | $175 \leq \mathrm{t} \leq 300$ |
| Slab main reinforcement ratio $(\rho)$ | $\rho_{\min } \leq \rho \leq \rho_{\max }$ |

$\mathrm{a}=$ clear cover + duct diameter; $\mathrm{b}=$ clear cover + web rebar's diameter + duct diameter; $\mathrm{A}_{\mathrm{M}}=$ Anchorage minimum vertical edge distance

### 5.5 Implicit Constraints

These constraints represent the performance requirements or response of the bridge system. A to tal 46 im plicit constraints are considered a ccording to the A ASHTO Standard Specifications (AASHTO 2007). These constraints are categorized into eight groups:
(i) Flexural working stress constraints
(ii) Flexural ultimate strength constraints
(iii) Shear constraints (ultimate strength)
(iv) Ductility constraints
(v) Deflection constraints
(vi) Lateral stability constraint
(vii) Tendons eccentricity constraint and
(viii) Deck slab design constraint

These constraints are formulated as below:

### 5.5.1 Flexural working stress constraints:

These constraints limit the working stresses in concrete and are given by:

$$
\begin{align*}
& f^{L} \leq f_{j} \leq f^{U}  \tag{5.7}\\
& f_{j}=-\frac{F_{j}}{\mathrm{~A}} \pm \frac{F_{j} e_{j}}{\mathrm{~s}_{\mathrm{j}}} \pm \frac{M_{j}}{\mathrm{~s}_{\mathrm{j}}} \tag{5.8}
\end{align*}
$$

Where, $f^{L}=$ allowable compressive stress (lower limit), $f^{U}=$ allowable tensile stress (upper limit) and $f_{j}$ is the actual working stress in concrete; $F_{j}, e_{j}, S_{j}, M_{j}=$ prestressing force, tendons eccentricity section modulus a nd moment at $\mathrm{j}^{\text {th }}$ section respectively . These constraints are considered at eight critical sections along the span of the girder as s hown in Figure 5.5 and f or various 1 oading s tages (initial s tage a nd service conditions). The eight sections are (describing from left support):

Section 4: Section where anchorages are placed
Section 2: End of anchorage and transition zone (Considered at a distance of 1.5 times girder depth).

Section 3: Section where prestress is at its maximum value.
Section 1: Section at 0.4L distance from end support.
Section 7: Midpoint of section 1 and 6.
Section 6: Section at 0.1 L di stance from interior s upport (where parabolic t endon changes its curvature)

Section 8: Midpoint of section 6 and 5.
Section 5: Section at interior support.

## Allowable stresses for prestressed concrete (AASHTO 2007)

## Compression stress:

1. The stress limit due to the sum of the effective prestress, permanent loads, and transient 1 oads a nd dur ing s hipping a nd ha ndling is $t$ aken a s $\mathbf{0 . 6} \boldsymbol{f}_{\boldsymbol{c}}^{\prime}$ and $0.55 f_{c i}^{\prime}$ at transfer.
2. The stress limit in prestressed concrete at the service limit state after losses for fully pr estressed components in br idges ot her t han s egmentally c onstructed
due to the sum of ef fective prestress an $d$ permanent loads shall be taken as $0.45 f_{c}^{\prime}$
3. The stress limit in prestressed concrete at the service limit state after losses for fully pr estressed c omponents in br idges ot her t han s egmentally c onstructed due to live load plus one-half the sum of the effective prestress and permanent loads shall be taken as $\boldsymbol{0 . 4 0 \boldsymbol { f } _ { \boldsymbol { c } } ^ { \prime }}$

## Tension stress:

The stress limit in prestressed concrete at the service limit state after losses for fully prestressed components in bridges other than segmentally constructed, which include bonded prestressing tendons and are subjected to not worse than moderate corrosion conditions s hall be taken a st he f ollowing: $0.25 \sqrt{\boldsymbol{f}_{\boldsymbol{c i}}^{\prime}}(\mathrm{MPa})$ (initial) and $0.5 \sqrt{\boldsymbol{f}_{\boldsymbol{c}}^{\prime}}(\mathrm{MPa})$ (Final).


Section 1: Section at 0.4 L distance from end support; Section 2: End of anchorage and transition zone (Considered at a di stance of 1.5 t imes girder depth); Section 3: Section where prestress is at its maximum value; Section 4: Section where anchorages are placed; Section 5: Section at interior support; Section 6: Section at 0.1L distance from interior support (where parabolic tendon changes its curvature); Section 7: Midpoint of section 1 and 6 ; Section 8: Midpoint of section 6 and 5.

Figure 5.5 Tendons profile along the girder

The initial loading stage includes the girder self weight and prestressing force after instantaneous losses (friction loss, anchorage loss and elastic shortening loss). In this stage net cross sectional properties of precast girder are used excluding duct. At initial stage a portion of total prestress is applied only to carry girder self weight. At service the first lo ading stage includes initial loading stage in addition slab a nd di aphragm weight. In this stage transformed cross sectional properties of precast girder are used and full prestress is applied. The second loading stage includes first loading stage in addition loads due to wearing course a nd median strip superimposed on composite section an d prestress force after total losses is considered. The third loading st age includes live load and impact load superimposed on composite section in addition to second 1 oading $s$ tage. The $f$ ourth loading stage i ncludes ha lf of de ad 1 oad and prestress force plus full live load. Loading stages are summarized in Table 5.4.

Table 5.4 Loading stages and implicit constraints

| Load stage | Resisting section | Section properties | Load Combination | Implicit constraint |
| :---: | :---: | :---: | :---: | :---: |
| Initial stage | Precast section | $\begin{gathered} A_{\text {net }}, e_{i}, \\ S_{\text {net }} \\ \hline \end{gathered}$ | $\eta F+G$ | Eq. (5.9) |
| 1 | Precast section | $A_{t}$, $, e, S$ | $F_{i}+G+S B+D P$ | Eq. (5.10) |
| 2 | Precast section $+$ Composite section | $\begin{gathered} A_{t ;}, e, S \\ + \\ S_{C} \end{gathered}$ | $\begin{gathered} \hline F_{e}+G+S B+D P \\ + \\ S D \end{gathered}$ | Eq. (5.11) |
| 3 | Precast section $+$ <br> Composite section | $\begin{gathered} A_{t f}, e, S \\ + \\ S_{C} \end{gathered}$ | $\begin{gathered} F_{e}+G+S B+D P \\ + \\ S D+L+I \end{gathered}$ | Eq. (5.12) |
| 4 | Precast section $+$ <br> Composite section | $\begin{gathered} A_{t ;}, e, S \\ + \\ S_{C} \end{gathered}$ | $0.5\left(F_{e}+D L\right)$ + $L+I$ | Eq. (5.13) |

$\overline{\overline{\mathrm{G}}=\text { Girder self weight; } \mathrm{SB}=\text { slab weight; } \mathrm{DP}=\text { diaphragm weight; } \mathrm{SD}=\text { superimposed dead }}$ load for wearing coarse and curb weight; $\mathrm{DL}=$ total dead load; $\mathrm{L}=$ live load; $\mathrm{I}=$ impact load. $\mathrm{F}=$ Jacking force; $\mathrm{F}_{\mathrm{i}}, \mathrm{F}_{\mathrm{e}}=$ Prestressing force after initial losses and total losses respectively;

$$
\begin{equation*}
-0.55 f_{c i}^{\prime} \leq f_{i} \leq 0.25 \sqrt{f_{c i}^{\prime}} \tag{5.9}
\end{equation*}
$$

$$
\begin{align*}
& -0.60 f_{c}^{\prime} \leq f \leq 0.5 \sqrt{f_{c}^{\prime}}  \tag{5.10}\\
& -0.40 f_{c}^{\prime} \leq f \leq 0.5 \sqrt{f_{c}^{\prime}}  \tag{5.11}\\
& -0.60 f_{c}^{\prime} \leq f \leq 0.5 \sqrt{f_{c}^{\prime}}  \tag{5.12}\\
& -0.40 f_{c}^{\prime} \leq f \leq 0.5 \sqrt{f_{c}^{\prime}} \tag{5.13}
\end{align*}
$$

Prestress losses are estimated according to AASHTO 2007 instead of using lump sum value for greater accuracy because prestress losses are also implicit functions of some of $d$ esign $v$ ariables. The i nstantaneous 1 osses depend on jacking e quipment and anchorage $h$ ardware us ed a nd the design variables (number of tendons, num ber of strands per tendon, layout of tendon in the girder, prestressing of tendon and girder cross sectional properties). The long term losses are loss due to creep of concrete, loss due to sh rinkage of co ncrete and loss due to steel relaxation an da re a lso implicit functions of so me ofdesign variables. In post-tensioned girder, prestressing forces varies a long the length of the girder due to friction losses a nd a nchorage losses as shown in $F$ igure 5.6. The $p$ restressing $f$ orces af ter $i$ nstantaneous 1 osses at sev en critical sections and at the end are determined as follows:
$F_{1 i}=F-\sum_{i=1}^{i=N_{T}} F_{i}\left(1-e^{-\left(\mu \theta_{i}+0.4 K L\right)}\right)-L_{E S}$
$F_{3 i}=F-0.5 L_{A N C}-L_{E S}$
$F_{2 i}=F_{3 i}-0.5 L_{A N C}\left(\frac{x_{2}-x_{3}}{x_{2}}\right)-L_{E S}$
$F_{4 i}=F-L_{A N C}-L_{E S}$
$L_{E S}=K_{e s} \frac{E_{s}}{E_{c i}} f_{c g p} A_{s}$
$L_{A N C}=\frac{4\left(F-F_{1 i}\right)}{L} \sqrt{\frac{L A_{S} E_{S}}{2\left(F-F_{1 i}\right)} \delta}$
$F_{5 i}=F_{8 i}-\sum_{i=1}^{i=N_{T}} F_{i}\left(1-e^{-\left(\mu \theta_{i}+0.05 K L\right)}\right)-L_{E S}$
$F_{6 i}=F_{7 i}-\sum_{i=1}^{i=N_{T}} F_{i}\left(1-e^{-\left(\mu \theta_{i}+0.25 K L\right)}\right)-L_{E S}$
$F_{7 i}=F_{1 i}-\sum_{i=1}^{i=N_{T}} F_{i}\left(1-e^{-\left(\mu \theta_{i}+0.25 K L\right)}\right)-L_{E S}$
$F_{8 i}=F_{6 i}-\sum_{i=1}^{i=N_{T}} F_{i}\left(1-e^{-\left(\mu \theta_{i}+0.05 K L\right)}\right)-L_{E S}$

The prestress forces after all losses at seven sections are $F_{1 e}, F_{2 e,}, F_{3 e}, F_{5 e}, F_{6 e}, F_{7 e}$ and $F_{8 e}$ respectively. F or p ost-tensioned m embers a ccording t o A ASHTO a llowable prestress immediately after seating at anchorage $0.7 f_{s u}$, at the end of the seating loss zone $0.83 f_{y}^{*}$ and st ress at ser vice 1 oad af ter 1 osses $0.80 f_{y}^{*}$. In $t$ he pr esent study tensioning to $0.9 f_{y}^{*}$ (jacking s tress) f or short p eriod of time p rior to s eating is considered to offset anchorage andfriction losses and implicit constraints are applied such th at the stresses in the te ndon re main within the allowable limit. The implicit constraints are as follows:

$$
\begin{align*}
& 0.0 \leq F_{4 i} \leq 0.7 f_{s u} A_{s}  \tag{5.20}\\
& 0.0 \leq F_{3 i} \leq 0.83 f_{y}^{*} A_{s}  \tag{5.21}\\
& 0.0 \leq F_{3 e} \leq 0.80 f_{y}^{*} A_{s} \tag{5.22}
\end{align*}
$$



Figure 5.6 Variation of prestressing force along the length of girder

The working stresses at various loading stages are determined as follows:

## Initial stage

At positive moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{\eta F_{i}}{A_{\text {net }}}+\frac{\eta F_{i} e_{i}}{S_{\text {tnet }}}-\frac{M_{G}}{S_{\text {tnet }}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{\eta F_{i}}{A_{\text {net }}}-\frac{\eta F_{i} e_{i}}{S_{\text {bnet }}}+\frac{M_{G}}{S_{\text {bnet }}}
$$

At negative moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{\eta F_{i}}{A_{\text {net }}}-\frac{\eta F_{i} e_{i}}{S_{\text {tnet }}}+\frac{M_{G}}{S_{\text {tnet }}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{\eta F_{i}}{A_{\text {net }}}+\frac{\eta F_{i} e_{i}}{S_{\text {bnet }}}-\frac{M_{G}}{S_{\text {bnet }}}
$$

## First loading stage

At positive moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{F_{i}}{A_{t f}}+\frac{F_{i} e}{S_{t}}-\frac{M_{P}}{S_{t}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{F_{i}}{A_{t f}}-\frac{F_{i} e}{S_{b}}+\frac{M_{P}}{S_{b}}
$$

At negative moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{F_{i}}{A_{t f}}-\frac{F_{i} e}{S_{t}}+\frac{M_{P}}{S_{t}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{F_{i}}{A_{t f}}+\frac{F_{i} e}{S_{b}}-\frac{M_{P}}{S_{b}}
$$

## Second loading stage

At positive moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{F_{e}}{A_{t f}}+\frac{F_{e} e}{S_{t}}-\frac{M_{P}}{S_{t}}-\frac{M_{C}}{S_{t c}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{F_{e}}{A_{t f}}-\frac{F_{e} e}{S_{b}}+\frac{M_{P}}{S_{b}}+\frac{M_{C}}{S_{b c}}
$$

At negative moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{F_{e}}{A_{t f}}-\frac{F_{e} e}{S_{t}}+\frac{M_{P}}{S_{t}}+\frac{M_{C}}{S_{t c}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{F_{e}}{A_{t f}}+\frac{F_{e} e}{S_{b}}-\frac{M_{P}}{S_{b}}-\frac{M_{C}}{S_{b c}}
$$

## Third loading stage

At positive moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{F_{e}}{A_{t f}}+\frac{F_{e} e}{S_{t}}-\frac{M_{P}}{S_{t}}-\frac{M_{C}}{S_{t c}}-\frac{M_{L}}{S_{t c}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{F_{e}}{A_{t f}}-\frac{F_{e} e}{S_{b}}+\frac{M_{P}}{S_{b}}+\frac{M_{C}}{S_{b c}}+\frac{M_{L}}{S_{b c}}
$$

At negative moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{F_{e}}{A_{t f}}-\frac{F_{e} e}{S_{t}}+\frac{M_{P}}{S_{t}}+\frac{M_{C}}{S_{t c}}+\frac{M_{L}}{S_{t c}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{F_{e}}{A_{t f}}+\frac{F_{e} e}{S_{b}}-\frac{M_{P}}{S_{b}}-\frac{M_{C}}{S_{b c}}-\frac{M_{L}}{S_{b c}}
$$

## Fourth loading stage

At positive moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{1}{2} \frac{F_{e}}{A_{t f}}+\frac{1}{2} \frac{F_{e} e}{S_{t}}-\frac{\left(M_{L}+\frac{M_{D}}{2}\right)}{S_{t c}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{1}{2} \frac{F_{e}}{A_{t f}}-\frac{1}{2} \frac{F_{e} e}{S_{b}}+\frac{\left(M_{L}+\frac{M_{D}}{2}\right)}{S_{b c}}
$$

At negative moment sections:
Stress at top fiber,

$$
f_{t}=-\frac{1}{2} \frac{F_{e}}{A_{t f}}-\frac{1}{2} \frac{F_{e} e}{S_{t}}+\frac{\left(M_{L}+\frac{M_{D}}{2}\right)}{S_{t c}}
$$

Stress at bottom fiber,

$$
f_{b}=-\frac{1}{2} \frac{F_{e}}{A_{t f}}+\frac{1}{2} \frac{F_{e} e}{S_{b}}-\frac{\left(M_{L}+\frac{M_{D}}{2}\right)}{S_{b c}}
$$

### 5.5.2 Ultimate flexural strength constraints

The u ltimate f lexural s trength c onstraints f or the p recast sect ion a nd co mposite section are considered as:
$0.0 \leq M_{p u} \leq \varphi M_{p n}$
$0.0 \leq M_{c u} \leq \varphi M_{c n}$
where, $M_{p u}$ and $M_{c u}$ are factored be nding moments; $\varphi M_{p n}$ and $\varphi M_{c n}$ are flexural strength of the precast and composite section respectively.

To cal culate $t$ he $f$ lexural st rength of $t$ he co mposite section at pos ition of pos itive moments, following four cases are considered and detail calculations are tabulated in Table 5.5.

Case 1: Compression block remains within the deck slab.
Case 2: Compression block remains within the Top flange.
Case 3: Compression block remains within the Top flange transition area.
Case 4: Compression block falls in web (Flanged section calculation is used assuming T shape stress block).

Table 5.5 Flexural strength calculations
Case Equations

$$
b=E F W ; \rho=\frac{A_{s}}{b d}
$$

Case 1: $\quad f_{s u}=f_{p u}\left(1-\frac{\gamma^{*}}{\beta} \rho \frac{f_{p u}}{f^{\prime}{ }_{\text {cdeck }}}\right) ; z=\frac{A_{s} f_{s u}}{0.85 f_{\text {cdeck }} b}$

$$
\boldsymbol{M}_{\boldsymbol{n}}=A_{s} f_{s u}\left(d-\frac{Z}{2}\right)
$$

$$
b=\left(\frac{f_{\text {cdeck }}}{f_{c}^{\prime}} E F W \times t+\frac{T F_{w} \times T F_{t}}{t+T F_{t}}\right) ; \rho=\frac{A_{s}}{b d}
$$

Case 2:

$$
\begin{aligned}
& f_{s u}=f_{p u}\left(1-\frac{\gamma^{*}}{\beta} \rho \frac{f_{p u}}{f_{c}^{\prime}}\right) ; z=\frac{A_{s} f_{s u}}{0.85 f_{c} b} \\
& \boldsymbol{M}_{\boldsymbol{n}}=A_{s} f_{s u}\left(d-\frac{z}{2}\right)
\end{aligned}
$$

$$
b=\left(\frac{f_{\text {cdeck }}}{f_{c}^{\prime}} E F W \times t+\frac{T F_{w} \times T F_{t}+\frac{2\left(T F_{w}-T F S_{w}\right)}{2} \times T F S_{t}}{t+T F_{t}+T F S_{t}}\right)
$$

Case 3: $\quad \rho=\frac{A_{s}}{b d}$

$$
f_{s u}=f_{p u}\left(1-\frac{\gamma^{*}}{\beta} \rho \frac{f_{p u}}{f_{c}^{\prime}}\right) ; z=\frac{A_{s} f_{s u}}{0.85 f_{c} b}
$$

$$
\boldsymbol{M}_{\boldsymbol{n}}=A_{s} f_{s u}\left(d-\frac{z}{2}\right)
$$

$$
A_{s w}=A_{s}-\frac{0.85 * f_{c} *\left(b-W_{w}\right) \times\left(t+T F_{t}+T F S_{t}\right)}{f_{s u}}
$$

Case 4:

$$
\rho=\frac{A_{s w}}{W_{w} d}
$$

$$
\begin{aligned}
& \boldsymbol{M}_{\boldsymbol{n}}=0.85 f_{c}\left(b-W_{w}\right)\left(t+T F_{t}+T F S_{t}\right)\left(d-\frac{t+T F_{t}+T F S_{t}}{2}\right) \\
& +A_{s w}+f_{s u} d\left(1-0.6 \frac{\rho f_{s u}}{f_{c}^{\prime}}\right)
\end{aligned}
$$

To cal culate $t$ he f lexural s trength of theg irder section at pos ition of ne gative moments, corresponding equations that are used are as follows:
$b=b_{w} ; \rho=\frac{A_{s}}{b_{w} d}$
$f_{s u}=f_{p u}\left(1-\frac{\gamma^{*}}{\beta} \rho \frac{f_{p u}}{f^{\prime}{ }_{c}}\right) ; z=\frac{A_{s} f_{s u}}{0.85 f^{\prime}{ }_{c} b_{w}}$

$$
\boldsymbol{M}_{\boldsymbol{n}}=A_{s} f_{s u}\left(d-\frac{z}{2}\right)
$$

### 5.5.3 Ductility (maximum and minimum prestressing steel) constraints

The maximum prestressing steel constraint for the composite section is given below:
$0 \leq \omega \leq \omega_{u}$
Where, Reinforcement index, $\omega=\frac{\rho f_{s u}}{f_{c}^{\prime}}$ and $\omega_{u}=$ Upper limit to reinforcement index $=$ $0.36 \beta_{1}$
The constraints which limit the minimum value of reinforcement are,
$1.2 M_{c r}^{*} \leq \varphi M_{n}$
Where, for composite girder,
At position of maximum positive moment,
$M_{c r}{ }^{*}=\left(f_{r}+f_{p e}\right) S_{b c}-M_{P 1}\left(\frac{S_{b c}}{S_{b}}-1\right)$
$f_{p e}=\frac{F_{e}}{A_{t f}}+\frac{F_{e} e_{c 1}}{S_{b}}$
where, $f_{r}=$ modulus of rupture; $f_{p e}=$ compressive stress in concrete due to effective prestress forces only (after al lowance for all 1 p restress losses) at ex treme fiber of section where tensile st ress is cau sed by ex ternally ap plied load; $S_{b}, S_{b c}=$ Section Modulus of bottom fiber of transformed precast \& composite section respectively; $e_{c l}$ $=$ eccentricity of composite section at position of maximum positive moment; $M_{P I}=$ Non-composite dead load moment or Moment due to girder self weight, cross girder and deck slab at position of maximum positive moment;

At position of maximum negative moment,
$M_{c r}{ }^{*}=\left(f_{r}+f_{p e}\right) S_{t c}-M_{P 2}\left(\frac{S_{t c}}{S_{t}}-1\right)$
$f_{p e}=\frac{F_{e}}{A_{t f}}+\frac{F_{e} e_{c 2}}{S_{t}}$
where, $f_{r}=$ modulus of rupture; $f_{p e}=$ compressive stress in concrete due to effective prestress forces only (after al lowance for all 1 p restress losses) at ex treme fiber of
section w here tensile st ress is cau sed by ex ternally ap plied 1 oad; $S_{t}, S_{t c}=$ Section Modulus of top fiber of transformed precast \& composite section respectively; $e_{c 2}=$ eccentricity of composite section at position of maximum negative moment; $M_{P 2}=$ Non-composite dead load moment or Moment due to girder self weight, cross girder and deck slab at position of maximum negative moment;

For deck slab,

$$
\begin{equation*}
M_{c r s l a b}^{*}=f_{r} S_{d e c k} \tag{5.29}
\end{equation*}
$$

### 5.5.4 Ultimate shear strength and horizontal interface shear constraints

The $u$ ltimate sh ear $s$ trength is co nsidered at $t$ wo sect ions, sect ion at $t$ he en $d$ of transition zone and section where the prestress is maximum and the related implicit constraint is defined as,

$$
\begin{equation*}
\varphi V_{s}=\left(V_{u}-\varphi V_{c}\right) \leq 0.666 \sqrt{f_{c}^{\prime}} W_{w} d_{s} \tag{5.30}
\end{equation*}
$$

where, $V_{u}=$ factored shear at a section, $V_{c}=$ the concrete contribution taken as lesser of flexural shear, $V_{c i}$ and web shear, $V_{c w}, V_{s}=$ shear carried by the steel in kN . These two shear capacity are determined according to AASHTO specification.

Composite sect ions ar ed esigned for h orizontal sh ear at $t$ he interface between $t$ he precast beam and deck and the related constraint is:

$$
\begin{equation*}
V_{u} \leq \varphi V_{n h} \tag{5.31}
\end{equation*}
$$

Where, $V_{n h}=$ nominal horizontal shear strength.

### 5.5.5 Deflection constraint

Deflection at mid span due to initial prestress (For parabolic tendon profile) is computed as:

$$
\begin{equation*}
\Delta_{P T}=\frac{13}{136} \sum_{i=1}^{i=N_{T}} \eta F_{1 i} h_{i} \frac{L^{2}}{E_{c i} I_{n e t}}+\frac{1}{8} \sum_{i=1}^{i=N_{T}} \eta F_{1 i} e_{i} \frac{L^{2}}{E_{c i} I_{n e t}} \tag{5.32}
\end{equation*}
$$

Where, $h_{i}, e_{i}$ are the sag and eccentricity of the $\mathrm{i}^{\text {th }}$ tendon respectively.

Deflection due to dead load:
$\Delta_{D L}=\frac{13}{136} M_{G} \frac{L^{2}}{E I}$

Initial camber $=\Delta_{P T}-\Delta_{D L}$

Deflection due to live load (AISC Mkt 1986):
$\Delta_{L L}=\frac{324}{E I_{c}} P_{T}\left(L^{3}-555 L+4780\right)$
The live load deflection constraint is as follows:
$\Delta_{L L} \leq L / 800$

### 5.5.6 End section tendon eccentricity constraint

Eccentricity of tendons at the end section becomes a co nstraint because eccentricity has to remain within the kern distances of the section to avoid extreme fiber tension both a $t$ in itial stage a nd at final stage. $T$ he following $c$ onstraint lim its $t$ he $t$ endon eccentricity at end section so that the eccentricity remains within the kern distances,
$\frac{G_{d}}{6}+0.25 \sqrt{f_{c i}} \frac{A_{4} G_{d}}{6 F_{4 i}} \leq e_{4} \leq \frac{G_{d}}{6}+0.5 \sqrt{\mathrm{f}_{\mathrm{c}}} \frac{A_{4} G_{d}}{6 F_{4 e}}$

### 5.5.7 Lateral stability constraint

The following constraint according to PCI (PCI 2003) limits the safety and stability during lifting of long girder subject to roll about weak axis,
$F S_{c}=\frac{1}{\frac{z_{o}}{y_{r}+}+\frac{\theta_{i}}{\theta_{\max }}} \geq 1.5$

Where, $\mathrm{FS}_{\mathrm{c}}=$ factor of s afety a gainst c racking of top flange when the girder ha ngs from lifting loop.

### 5.5.8 Deck slab constraints

The co nstraint co nsidered for deck sl ab thickness acco rding todesign cr iteria of ODOT (ODOT 2000) is,
$t \geq \frac{S_{d}+17}{3}$
The constraint which limit the required effective depth for deck slab is,
$d_{\text {min }} \leq d_{\text {req }} \leq d_{\text {prov }}$
Where, $S_{d}=$ effective slab span in feet $=S-T F_{w} / 2 ; \mathrm{t}=$ slab thickness in inch.

### 5.6 Stiffness method

### 5.6.1 General

One of the $b$ asic ad vantages of st iffness method is that whatever be the st ructural idealization the main steps of stiffness method are always same and as stated below:

1. Identify t he unknow n displacement f or e ach joint. T hat is, de termine t he degree of kinematic indeterminacy.
2. Make the s tructure $k$ inematically determinate by re straining a ll degrees of freedom.
3. Apply 1 oads an dc alculate j oint f orces corresponding t oe ach D egree of Freedom (D.O.F.). That is finally obtain the member force vector [ Pm ].
4. Apply unknow $n$ di splacements on eata t ime a nd c alculate j oint forces corresponding to each D.O.F. That is, calculate the stiffness terms and finally obtain the stiffness matrix [k].
5. Write e quilibrium e quations c orresponding to each D .O.F. i.e. the S tiffness Equations. Solve for unknown displacements.
6. Superimpose the effects of loads and displacements to obtain stress resultants and reactions.

### 5.6.2 Computer Application of Stiffness Method

Although the step by step approach is convenient as a common method to different structures, $t$ he $m$ ethod is $n$ ot $q$ uite $y$ et a ppropriate $f$ or $w$ riting $p$ rograms to $s$ olve problems using stiffness method. For computer application the same stiffness method is used following a slightly different sequence.

First stiffness matrix of each member is derived. Then they are assembled to form a global stiffness matrix of the whole structure. Then global force vectors are derived. At 1 ast bou ndary conditions are i mposed. Stiffness equations a re then solved to determine the unknow ns. The process of solving indeterminate structure following this approach will be demonstrated by a beam problem (Figure 5.7).


Figure 5.7 Two-span continuous indeterminate beam with UDL
First a single beam member without any boundary condition is considered:


Figure 5.8 Single beam member without any boundary condition
The beam member has got four de grees of freedom (local), namely, one translation $\left(u_{1}\right)$ and one rotation $\left(u_{2}\right)$ at node 1 and one translation $\left(u_{3}\right)$ and one rotation $\left(u_{4}\right)$ at node 2 (Figure 5.8). The stiffness matrix ( $4 \times 4$ ) of the member will be:

$$
\left[\begin{array}{cccc}
\frac{12 E I}{L^{3}} & \frac{6 E I}{L^{2}} & \frac{-12 E I}{L^{3}} & \frac{6 E I}{L^{2}} \\
\frac{6 E I}{L^{2}} & \frac{4 E I}{L} & \frac{-6 E I}{L^{2}} & \frac{2 E I}{L} \\
\frac{-12 E I}{L^{3}} & \frac{-6 E I}{L^{2}} & \frac{12 E I}{L^{3}} & \frac{-6 E I}{L^{2}} \\
\frac{6 E I}{L^{2}} & \frac{2 E I}{L} & \frac{-6 E I}{L^{2}} & \frac{4 E I}{L}
\end{array}\right]
$$

Now, for the time being, if loading and boundary conditions are ignored, the present structure is $b$ asically a $n$ assem blage of $t$ wo beam members $c$ onnected at a no de (Figure 5.9).


Figure 5.9 Assemblage of two beam members without boundary conditions

In total the structure has 3 node $s$ and 6 (global) corresponding de grees of freedom. The members have following attributes assigned to them:
Membe
1
2
Length
I
E
1
1
2
L
I
E
2
2
3
L
I
E

Using the last three attributes member stiffness matrix for each member can easily be calculated. For the first member its local first (i) and second (j) node, respectively, corresponding to the global node 1 a nd 2 . Thus the four rows and columns of the member stiffness matrix of member 1 will fill up the first four rows and columns of global stiffness matrix.

However formember 2 , its first (i) a nd second (j) l ocal node, respectively, corresponding to the global node 2 a nd 3 . Thus the four rows and columns of the member stiffness matrix will fill up $3^{\text {rd }}(2 \mathrm{i}-1), 4^{\text {th }}(2 \mathrm{i}), 5^{\text {th }}(2 \mathrm{j}-1)$ and $6^{\text {th }}(2 \mathrm{j})$ rows and columns of the global stiffness matrix. Thus the $6 \times 6$ global stiffness matrix will be:

$$
\left[\begin{array}{cccccc}
\frac{12 E I}{L^{3}} & \frac{6 E I}{L^{2}} & \frac{-12 E I}{L^{3}} & \frac{6 E I}{L^{2}} & 0 & 0 \\
\frac{6 E I}{L^{2}} & \frac{4 E I}{L} & \frac{-6 E I}{L^{2}} & \frac{2 E I}{L} & 0 & 0 \\
\frac{-12 E I}{L^{3}} & \frac{-6 E I}{L^{2}} & \frac{12 E I}{L^{3}}+\frac{12 E I}{L^{3}} & \frac{-6 E I}{L^{2}}+\frac{6 E I}{L^{2}} & \frac{-12 E I}{L^{3}} & \frac{6 E I}{L^{2}} \\
\frac{6 E I}{L^{2}} & \frac{2 E I}{L} & \frac{-6 E I}{L^{2}}+\frac{6 E I}{L^{2}} & \frac{4 E I}{L}+\frac{4 E I}{L} & \frac{-6 E I}{L^{2}} & \frac{2 E I}{L} \\
0 & 0 & \frac{-12 E I}{L^{3}} & \frac{-6 E I}{L^{2}} & \frac{12 E I}{L^{3}} & \frac{-6 E I}{L^{2}} \\
0 & 0 & \frac{6 E I}{L^{2}} & \frac{2 E I}{L} & \frac{-6 E I}{L^{2}} & \frac{4 E I}{L}
\end{array}\right]
$$

The process of de riving gl obal stiffness $m$ atrix from member st iffness matrices is called assembling. The global stiffness matrix for beam problem becomes banded.

Here it is easily noticeable that, now the same structure can have different loading and support conditions without any need for recalculating the stiffness matrix.

Now the loading will be considered. Member loads are considered as assignments to individual members an dj oint loads ar e considered as assignments toindividual degrees of freedom (Figure 5.10).


Figure 5.10 Assigning loads to degrees of freedom

The local member force matrix ( 4 x 1 ) for a single beam member will be:

$$
\left\{\operatorname{Pm}^{1}\right\}=\left[\begin{array}{c}
\frac{w L}{2} \\
\frac{w L^{2}}{16} \\
\frac{w L}{2} \\
-\frac{w L^{2}}{16}
\end{array}\right]
$$

After assem blage of two $s$ uch local member $f$ orce $m$ atrices of $t$ wo si ngle $b$ eam members, the global member force matrix ( $6 \times 1$ ) will be found:

$$
\{\operatorname{Pm}\}=\left[\begin{array}{c}
\frac{w L}{2} \\
\frac{w L^{2}}{16} \\
w L \\
0 \\
\frac{w L}{2} \\
-\frac{w L^{2}}{16}
\end{array}\right]
$$

Since there are no loads on joints, the joint force matrix $\{\mathrm{Pj}\}$ will be a null-vector:

$$
\{\mathrm{Pj}\}=\left[\begin{array}{l}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{array}\right]
$$

Considering degrees of freedom matrix $\{\mathrm{u}\}$ to be:

$$
\{\mathrm{u}\}=\left[\begin{array}{l}
u_{1} \\
u_{2} \\
u_{3} \\
u_{4} \\
u_{5} \\
u_{6}
\end{array}\right]
$$

The stiffness equations then becomes,

$$
\begin{aligned}
& \{\operatorname{Pm}\}+[\mathrm{K}]^{*}\{\mathrm{u}\}=\{\mathrm{Pj}\} \\
& {\left[\begin{array}{c}
\frac{w L}{2} \\
\frac{w L^{2}}{16} \\
w L \\
0 \\
\frac{w L}{2} \\
-\frac{w L^{2}}{16}
\end{array}\right]+\left[\begin{array}{cccccc}
\frac{12 E I}{L^{3}} & \frac{6 E I}{L^{2}} & \frac{-12 E I}{L^{3}} & \frac{6 E I}{L^{2}} & 0 & 0 \\
\frac{6 E I}{L^{2}} & \frac{4 E I}{L} & \frac{-6 E I}{L^{2}} & \frac{2 E I}{L} & 0 & 0 \\
\frac{-12 E I}{L^{3}} & \frac{-6 E I}{L^{2}} & \frac{24 E I}{L^{3}} & 0 & \frac{-12 E I}{L^{3}} & \frac{6 E I}{L^{2}} \\
\frac{6 E I}{L^{2}} & \frac{2 E I}{L} & 0 & \frac{8 E I}{L} & \frac{-6 E I}{L^{2}} & \frac{2 E I}{L} \\
0 & 0 & \frac{-12 E I}{L^{3}} & \frac{-6 E I}{L^{2}} & \frac{12 E I}{L^{3}} & \frac{-6 E I}{L^{2}} \\
0 & 0 & \frac{6 E I}{L^{2}} & \frac{2 E I}{L} & \frac{-6 E I}{L^{2}} & \frac{4 E I}{L}
\end{array}\right]\left[\begin{array}{l}
u_{1} \\
u_{2} \\
u_{3} \\
u_{4} \\
u_{5} \\
u_{6}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{array}\right]}
\end{aligned}
$$

It is noticeable that the diagonal elements of the stiffness matrix are as usual positive and $t$ he $m$ atrix is symmetric. It is to remind that, support conditions have not y et considered in the stiffness equations. Thus, if these equations are solved, no uni que solution can be found. That is, all the six stiffness equations above are not linearly independent.

Now, the support conditions will be considered. There are two ways of doing that. One method is easier but requires more memory space and computation time and the other method is more difficult but takes less memory and computation time. In this thesis, the easier procedure is used for simplicity.

In or der to impose boundary condition it is required to know, nu mber of restrained nodes.

No. of restrained nodes, $\mathrm{NRN}=3$.

In order to make $\mathrm{u}_{\mathrm{i}}=0$,
(i) Make all the e lements of the $i$ th row and $i$ th column of the a ssembled structure stiffness matrix $=0$;
(ii) Make diagonal members of the stiffness matrix, $\mathrm{K}_{\mathrm{ii}}=1$;
(iii) Make corresponding element of member force matrix, $\mathrm{P}_{\mathrm{i}}=0$;

In this way, boundary conditions can be imposed and Modified Stiffness Matrix and Modified Stiffness Equation can be obtained.

So, finally the stiffness equations for the free degrees of freedom will be:

$$
\left[\begin{array}{c}
\frac{w L^{2}}{16} \\
0 \\
-\frac{w L^{2}}{16}
\end{array}\right]+\left[\begin{array}{ccc}
\frac{4 E I}{L} & \frac{2 E I}{L} & 0 \\
\frac{2 E I}{L} & \frac{8 E I}{L} & \frac{2 E I}{L} \\
0 & \frac{2 E I}{L} & \frac{4 E I}{L}
\end{array}\right]\left[\begin{array}{l}
u_{2} \\
u_{4} \\
u_{6}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

### 5.7 Constructing Influence Line for Indeterminate Structure

To find out de sign moment a nd shear, the statically indeterminate structure had to solve for hundreds of times. For solving the indeterminate structure, stiffness method was used. To make the basic stiffness method applicable to computer aided analysis, 'computer application of stiffness method' was used with required sequence of logic.

To find out the live load and impact sh ear force and moment, it w as necessary to construct influence lines for different sections. No general equation of influence line has been used; rather the coordinate values of different points of the influence line are determined $u$ sing $t$ he $b$ asic stiffness co ncept. The $g$ eneral si mple co ncept $u$ sed to construct the influence lines is described below:

The total length of the bridge is divided into a series of 0.25 meter long segments. At every 0.25 meter, a node/ coordinate have been considered. For constructing influence line, a 1-kip load was placed at each of these nodes and for that 1-kip load, the whole structure is solved using stiffness method. After solving the whole structure, the shear force and bending moment values at every 0.25 meter apart nodes has been stored in the columns of $t$ wo matrices of su fficient size. While st oring $t$ he sh ear force an $d$ moment values, it was assured that, the nodal values were placed column wise. It is evident from $t$ he $c$ oncept / de finition of influence 1 ine $t$ hat $t$ he $n$th $r$ ow of $t$ hose matrices will $g$ ive the coordinate values of the in fluence line for sh ear force and moment for that particular section (i.e. $\mathrm{n}^{\text {th }}$ section) (Fig 5.11).


Fig. 5.11 Constructing Influence Line Matrix
For example, the coordinates of the influence line of shear force for the 60 meter long two-span continuous girder came out to be:


Fig. 5.12 Influence Line for Shear at section 1 (at 0.4L distance from left support) of first span for two-span continuous girder with 60 meter span length

For the 60 meter long two-span continuous girder, influence line for shear at section 1 which is at 0.4 L (i.e. 24 ft ) distance from left s upport has be en p lotted (Fig 5.12) using the values of shear force at every 0.25 m nodes along the span. In accordance with the concept with which the influence line matrix for shear has been developed, it is evident that, the $96^{\text {th }}$ row (24X4) of the influence line matrix will have the 480 nos. $(120 \mathrm{X} 4=480)$ of nodal values (Fig 5.13) of the influence line.

|  | 1 | 2 | 3 | ... | 96 | 97 | 98 | 99 | ... | 239 | 240 | 241 | 242 |  | 479 | 480 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| : |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| : |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 95 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 96 | $\begin{aligned} & -0.02 \\ & \mathrm{kN}-\mathrm{m} \end{aligned}$ | -0.04 | -0.06 | ... | $\begin{gathered} -0.48 \\ 0.52 \\ \hline \end{gathered}$ | 0.50 | 0.47 | 0.44 | ... | 0.03 | 0.00 | -0.01 | -0.02 | ... | -0.01 | 0.00 |
| 97 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 98 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 478 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 479 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 480 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Fig. 5.13 Coordinates of Influence Line for Shear at section 1 (at 0.4L distance from left support) of first span for two-span continuous girder with 60 meter span length

Similarly, the coordinates of the influence line of bending moment for the 60 m eter long two-span continuous girder came out to be:


Fig. 5.14 Influence Line for Moment at section 1 (at 0.4L distance from left support) of first span for two-span continuous girder with 60 meter span length

While determining the design value of live load shear / moment, the wheel loads had to be placed on the influence lines and the pick value had to be found out. It is to be noted that, a HL-93 truck has the spacing of 4.3 meter between the front and middle wheel, and a spacing of 4.3 meter between the middle and rear wheel. Both these two
numbers, i.e. 4.3 m and 4.3 m are almost dividable by 0.25 w ith a fraction of 0.05 meter. So, it will not be too much erroneous if the wheel loads are placed only on the nodes de scribed be fore. F or a ny particular position of the truck, the shear/ moment can be e asily found by multiplying the $w$ heel loads $w$ ith corresponding va lues of influence line respective coordinates.

### 5.8 Linking Optimization Problem with EVOP and Solve

In thep resent o ptimization p roblem a la rge number of de sign v ariables a nd constraints are asso ciated. $T$ he $d$ esign $v$ ariables ar e cl assified as co mbination of continuous, discrete and integer variables. Expressions for the objective function and the c onstraints a re non 1 inear functions of $t$ hese de sign $v$ ariables. S o the o ptimal design pr oblem be comes hi ghly $n$ onlinear a nd non -convex ha ving multiple 1 ocal minima which requires an optimization method to de rive the global optimum. As a result the global optimization algorithm named EVOP (Ghani 1989) is used.

The algorithm EVOP requires three user written functions the objective function, the explicit constraint function and implicit constraint function, some user input control parameters and a st arting point inside the feasible sp ace (Figure 5.15). Given the coordinates of a feasible pointin an N -dimensional s pace t he obj ective f unction calculates the functional va lue. Explicit constraint function evaluates the upper and the lower limits of the explicit constraints. Implicit constraint function evaluates the implicit c onstraints v alues a nd their u pper a nd lo wer 1 imits. T he in put c ontrol parameters with their default values and ranges are, $\alpha=1.2$ (1.0 to 2.0); $\beta=0.5$ ( 0 to 1.0); $\Delta=10^{-12} ; \gamma=2.0$ (greater than 1.0 to upwards), $\Phi=10^{-14}\left(10^{-16}\right.$ to $\left.10^{-8}\right)\left(\Phi=10^{-12}\right.$ will yield higher accuracy for convergence compared to $\Phi=10^{-14}$ ) and $\Phi_{\mathrm{cpx}}=10^{-9}$ $\left(10^{-16}\right.$ to $\left.10^{-8}\right)$.


Figure 5.15 Steps for Optimization Problem Formulations and Linking with Optimization Algorithm (EVOP) (Rana, 2010)

The other parameters relevant to the usage of the program EVOP are as follows.

IJK --- For first entry, this variable should always be set to 1 . It will subsequently be changed by 'EVOP'.

K --- Number of 'complex' vertices. If ' $n$ ' is the dimension of the parameter space, for $\mathrm{n}<=5, \mathrm{k}=2 \mathrm{n}$; and for $\mathrm{n}>5, \mathrm{k}>=(\mathrm{n}+1)$.

KNT --- Number of co nsecutive times the objective function is called af ter which tests are conducted for convergence. (Typically 25).

LIMIT --- Maximum number of times the three functions: the objective function, the explicit constraint function a nd the implicit c onstraint function can be co llectively called.

NRSTRT --- Number of automatic re start of EVOP t o c heck t hat t he pr eviously obtained va lue is the g lobal m inimum. If NRS TRT $=5$, t he E VOP p rogram will execute 5 times. For first time ex ecution a starting point of the complex inside the feasible space has to be given. For further restart the complex is generated taking the coordinates of the previous minimum (values obtained from previous execution of EVOP) as the starting point of the complex.

IER --- Error flag.
$=1 \mathrm{i}$ ndicates us er provided starting point is violating upper 1 imit of a $n$ explicit constraint.
$=2 \mathrm{i}$ ndicates us er pr ovided s tarting point is violating lower lim it of a n explicit constraint.
$=3$ i ndicates us er pr ovided starting point is violating u per lim it of an implicit constraint.
$=4$ i ndicates user provided starting point is violating the lower limit of an implicit constraint.
$=5$ indicates randomly generated $(\mathrm{k}-1)$ tests points not obtainable in the 'LIMIT' to which the three functions can be collectively called.
$=6 \mathrm{i}$ ndicates minimum of the obj ective function not obtainable w ithin the de sired accuracy of convergence. The results are those obtained after exceeding 'LIMIT'.
$=7$ i ndicates final 'complex' has not reduced its size to satisfy convergence test 2 . Results are those obtained after exceeding 'LIMIT'.
$=8 \mathrm{i}$ ndicates m inimum of t he objective f unction has b een 1 ocated t ot he desired degree of accuracy to satisfy both convergence tests.

XMAX(N) --- Array of di mension ' $N$ ' c ontaining the upper limits of $t$ he explicit constraints. They are calculated and supplied by the explicit constraint function for a given trial point provided by 'EVOP'.

XMIN(N) --- Array of di mension ' $N$ ' c ontaining t he 1 ower limits of the e xplicit constraints. They are calculated and supplied by the explicit constraint function for a given trial point provided by 'EVOP'.
$\mathrm{XT}(\mathrm{N})$--- Array of dimension ' N ' c ontaining the coordinates of the trial point. On first entry ' $\mathrm{XT}(\mathrm{N})$ ' contains the feasible trial point, and at the end of minimization it returns with the coordinates of the minimum located.

XX(NIC) --- Array of dimension 'NIC' containing the implicit constraint function values. They a re calculated and su pplied by the implicit constraint function, for a given trial point ' $\mathrm{XT}(\mathrm{N})$ ' provided by 'EVOP'.

XXMAX(NIC) --- Array of di mension 'NIC' containing $t$ he uppe r limit oft he implicit c onstraints. T hey a re calculated a nd supplied by the im plicit c onstraint function, for a given trial point ' $\mathrm{XT}(\mathrm{N})$ ' provided by 'EVOP'.

XXMIN(NIC) --- Array of dimension 'NIC' containing the lower limit of the implicit constraints. They are calculated and supplied by the implicit constraint function, for a given trial point ' $\mathrm{XT}(\mathrm{N})$ ' provided by 'EVOP'.

## Values for ' $\alpha$ ', ' $\beta$ ', ' $\gamma$ ', ' $\boldsymbol{\Phi}$ ' and ' $\Phi_{\text {cpx }}$ '

(i) Initially 'NRSTRT' has to be set to a high integer value, say 10 or 20 .
(ii) Initially for low convergence accuracy, the value of $\Phi=10^{-14}$ has to be set and the value of $\Phi_{\mathrm{cpx}}=10^{-9}$ has to be set.
(iii) ' $\boldsymbol{\alpha}$ ', ' $\boldsymbol{\beta}$ ', ' $\boldsymbol{\gamma}$ ' have t o be set to t heir de fault va lues of $1.2,0.5$ and 2.0 respectively, and the program has to be run.
(iv)Keeping $\boldsymbol{\beta}$ and ' $\boldsymbol{\gamma}$ ' fixed, $\boldsymbol{\alpha}$ has to be varied from a value greater than 1.0 to a value less than 2.0 for convergence 'IER $=8$ ', with lowest number of function evaluation 'NFUNC', and lowest function value ' F '.
(v) ' $\Phi$ ' has to be increased upto $10^{-10}$ for double precision in steps for tighter convergence, and ' $\Phi$ ' has to be set the highest value that would still yield 'IER $=8^{\prime}$. Note: $\boldsymbol{\alpha}$ is the most sensitive parameter.
(vi)Keeping $\boldsymbol{\alpha}$ and $\gamma$ fixed $\beta$ has to be varied above 0.0 to less than 1.0 f or the criterion set out in (iv) above.
(vii) Keeping $\alpha$ and $\beta$ fixed $\gamma$ has to be varied from 2.0 up wards for the criterion set out in (iv) above.
(viii) Repeat from step (iv) only if lo wer values of ' $N F U N C$ ' and ' F ' are required.
(ix) $\Phi_{\text {cpx }}$ has to be changed from a value two decades higher to two decades lower compared to $\Phi$ and observe the effects on 'NFUNC' and ' $F$ '. $\Phi_{\text {cpx }}$ has to be chosen for least ' NFUNC ' and ' F '.
(x) Using opt imum 'XT(N)' and c orresponding 'XMAX(N)', 'X MIN(N)', 'XXMAX(NIC)', ‘XXMIN(NIC)', from ( viii) a bove, the program has to be run with s ame v alues fo r ' $\boldsymbol{\alpha}$ ', ' $\boldsymbol{\beta}$ ', ' $\boldsymbol{\gamma}$ ', ' $\boldsymbol{\Phi}$ ' a nd ' $\boldsymbol{\Phi}_{\text {cpx }}$ '. Whether a b etter minimum is obtained has to be checked.

A computer program coded in $\mathrm{C}++$ (Appendix A ) is used to input control parameters and to define three functions: an objective function, an explicit constraint function and an implicit constraint function. First the values of the control parameters are assigned with th eir $d$ efault $v$ alues a nd $o$ ther in put $p$ arameters a re set to specific numerical values. $T$ hese $o$ ther in put $p$ arameters $f$ or thep resent o ptimization problem a re: number of complex vertices, $K=15$; maximum number of times the three functions can be collectively called, limit $=100000$; dimension of the design variable space, N = 14; number of implicit constraint, NIC $=73$ and number of EVOP restart, NRSTRT $=10$.

Determination ofa feasible s tarting p oint (the va lues of de sign va riables corresponding to the feasible point satisfy all the explicit and implicit constraints) is simple. B efore c alling s ubroutine E VOP a r andom poi nt s atisfying a llexplicit constraints is $g$ enerated an $d t$ ested $f$ or sat isfying al 1 i mplicit constraints. If these constraints are also satisfied then control is passed to the function EVOP. Otherwise the process is repeated till a feasible starting point is found. Next the function EVOP
is called. Next suitable values of the control parameters are obtained by varying the parameters within the range sequentially a nd setting $\Phi$ to highest value that would still yield convergence and number of function evaluation becomes lowest with least function va lue. T he pr ogram is r erun us ing optimum d esign va riables obt ained previously as starting point $w$ ith same va lues of $c$ ontrol pa rameters a nd $c$ hecked whether a better minimum is obtained.

## Chapter 6

## Results andDiscussions

### 6.1 General

The cost optimum design method has been performed for $\mathbf{4 0} \mathbf{m}, \mathbf{6 0} \mathbf{m}, \mathbf{8 0} \mathbf{m}$ girder span and for 3 L ane or 4 Lane bridges. AASHTO HL-93 live load is considered for each case a nd su perimposed d ead loads are ac cording to AASHTO a lso. O ptimum design is d ependent ont he design constant parameters i.e. unit co st of materials, labor, f abrication and installation, c oncrete st rength, st rand size, an chorage sy stem etc. As different design constant parameters will result in different optimum design, the cost optimum design method has been performed for two types of girder concrete strengths (28 days) $\mathbf{4 0} \mathbf{M P a}$ and $\mathbf{5 0} \mathbf{M P a}$ and for three different unit costs of the materials as shown in Table 6.1 so that the variation in design with respect to change in the parameters can be observed. Concrete strength at initial stage is taken as $75 \%$ of 28 da ys strength. Deck slab concrete strength is considered as $\mathbf{2 5} \mathbf{~ M P a}$. Ultimate strength of prestressing st eel and y ield strength of o rdinary st eel a re considered as 1861 MPa and 410 MPa respectively. Freyssinet C-Range anchorage system is used for posttensioning tendons consisting of 15.2 mm diameter 7-wire strands. Unit cost of $g$ irder co ncrete an deck slab concrete is considered fixed and the u nit costs of steels and anchorage systems are varied such that in Cost2, these costs are two times those in Cost1 and in Cost3, these costs are three times those in Cost1.

Table 6.1 Relative cost parameters used for cost minimum design (As per RHD schedule of rates 2015)

| Item | Unit | Cost1 | Cost2 | Cost3 |
| :--- | :---: | :---: | :---: | :---: |
|  |  |  | (C1) | (C2) |
| (C3) |  |  |  |  |

## 6．2 Parametric Studies

## 6．2．1 Optimum design for $\mathbf{4 0} \mathbf{m}$ double span continuous girder

Table 6．2 Optimum values of design variables for 3 Lane 40 m double span continuous girder and Concrete strength $=\mathbf{5 0} \mathbf{~ M P a}$

| Cost | $\begin{gathered} \hline \hline \mathrm{S} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} \hline \mathrm{Gd} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \hline \mathrm{TF}_{\mathrm{w}} \\ & (\mathrm{~mm}) \end{aligned}$ | $(\mathrm{mm})$ | $\begin{aligned} & \hline \hline \mathrm{TFS}_{\mathrm{t}} \\ & (\mathrm{~mm}) \end{aligned}$ | (mm) | $(\mathrm{mm})$ | $\begin{gathered} \hline \hline \mathrm{W}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ | $\mathrm{N}_{\mathrm{S}}$ | $\mathrm{N}_{\mathrm{T}}$ | $\begin{gathered} \mathrm{t} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \rho \\ & \% \end{aligned}$ | $\begin{gathered} \mathrm{y}_{1} \\ (\mathrm{~mm}) \end{gathered}$ | $\eta$ $\%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C1 | 4.0 | 1700 | 450 | 75 | 50 | 325 | 260 | 150 | 9 | 4 | 255 | 0.68 | 430 | 27 |
| C2 | 4.0 | 2200 | 750 | 75 | 50 | 325 | 235 | 150 | 9 | 4 | 265 | 0.55 | 720 | 30 |
| C3 | 4.0 | 2250 | 1250 | 75 | 50 | 300 | 195 | 150 | 8 | 3 | 285 | 0.48 | 680 | 50 |

Table 6．3 Optimum values of design variables for 3 Lane 40 m double span continuous girder and Concrete strength $=\mathbf{4 0} \mathrm{MPa}$

| Cost | $\begin{gathered} \hline \hline \mathrm{S} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} \hline \hline \mathrm{Gd} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \hline \hline \mathrm{TF}_{\mathrm{w}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \hline \hline \mathrm{TF}_{\mathrm{t}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \hline \hline \mathrm{TFS}_{\mathrm{t}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{gathered} \hline \hline \mathrm{BF}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \hline \hline \mathrm{BF}_{\mathrm{t}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \hline \hline \mathrm{W}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ | $\mathrm{N}_{\mathrm{S}}$ | $\mathrm{N}_{\mathrm{T}}$ | $\begin{gathered} \hline \hline \mathrm{t} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \rho \\ & \% \end{aligned}$ | $\begin{gathered} \mathrm{y}_{1} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \hline \eta \\ & \% \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C1 | 4.0 | 1850 | 725 | 75 | 50 | 310 | 265 | 160 | 9 | 4 | 245 | 0.67 | 425 | 27 |
| C2 | 4.0 | 2250 | 950 | 75 | 50 | 325 | 150 | 160 | 8 | 4 | 245 | 0.52 | 795 | 37 |
| C3 | 4.0 | 2625 | 1375 | 75 | 50 | 305 | 130 | 145 | 7 | 3 | 290 | 0.45 | 810 | 29 |

Table 6．4 Cg of tendons from bottom fiber of girder in the optimum design

|  | Girder concrete strength $=50 \mathrm{MPa}$ |  |  |  |  |  |  |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underset{\sim}{x}$ | $\cdots$ | $\rangle^{\frac{0}{3}}$ | $\cdots$ | $\grave{\sim}$ | خ | $\cdots$ | 4 | $\bar{\chi}$ | $\underset{\sim}{\sim}$ | $\cdots$ | ${ }^{\text {号 }}$ | $\cdots$ | $\grave{\sim}$ | $\lambda$ | $\stackrel{\sim}{\sim}$ | 4 |
| C1 | － | \％ | İ | $\xrightarrow[\sim]{n}$ | $\stackrel{\sim}{\sim}$ | － |  | $\stackrel{\circ}{1}$ | － | ミ | તิర | $\stackrel{\circ}{9}$ | ก | $\stackrel{\infty}{\text { ¢ }}$ | 资 | ¢ |  |
| C2 | $\cong \stackrel{\otimes}{\square}$ | $\stackrel{\text { ® }}{\sim}$ | त | \％ | $\stackrel{\sim}{\infty}$ | $\stackrel{\text { ® }}{\infty}$ | $\stackrel{\circ}{\circ}$ | $\stackrel{8}{8}$ | Э | $\stackrel{\square}{\square}$ | ® | $\stackrel{n}{\circ}$ | $\stackrel{\text {－}}{\sim}$ | － | $\underset{\infty}{ \pm}$ | $\stackrel{\circ}{\text { ® }}$ | $\stackrel{\text { ® }}{ }$ |
| C3 | $\cong$ | $\bigcirc$ | \％ |  | 过 | ๕ | $\stackrel{\stackrel{-}{\infty}}{\sim}$ | $\stackrel{8}{1}$ | $\pm$ | $\bigcirc$ | ก | $\stackrel{\text { ® }}{\square}$ | $\stackrel{\text { ç }}{\text { c }}$ | $\stackrel{\text { ® }}{\text { ® }}$ | § | N／ | $\stackrel{\otimes}{\sim}$ |

Optimum d esign for 40 m double spancontinuous girder (Girder co ncrete strength $=\mathbf{5 0} \mathbf{~ M P a}$ )


Figure 6.1(a) Optimum design for double span cont. girder at section 1 for Cost1


Figure 6.1(b) Optimum design for double span cont. girder at section 5 for Cost1


Figure 6.1(c) Optimum design for double span cont. girder at section 4 for Cost1


Figure 6.1(d) Optimum design for double span continuous girder for Cost1


Figure 6.2(a) Optimum design for double span cont. girder at section 1 for Cost2


Figure 6.2(b) Optimum design for double span cont. girder at section 5 for Cost2


Figure 6.2(c) Optimum design for double span cont. girder at section 4 for Cost2


Figure 6.2(d) Optimum design for double span continuous girder for Cost2


Figure 6.3(a) Optimum design for double span cont. girder at section 1 for Cost3


Figure 6.3(b) Optimum design for double span cont. girder at section 5 for Cost3


Figure 6.3(c) Optimum design for double span cont. girder at section 4 for Cost3


Figure 6.3(d) Optimum design for double span continuous girder for Cost3

Table 6.5 Cost of individual materials for 3 Lane 40 m double span cont girder

|  | Girder concrete strength $=50 \mathrm{MPa}$ |  |  |  |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Cost } \\ \text { (BDT) } \end{gathered}$ | $\mathrm{C}_{\mathrm{GC}}$ * | $\mathrm{C}_{\text {DC }}{ }^{*}$ | $\mathrm{C}_{\text {PS }}{ }^{*}$ | Cos* | TC* | $\mathrm{C}_{\mathrm{GC}}$ * | $\mathrm{C}_{\text {DC }}$ * | $\mathrm{CPS}^{*}$ | Cos* | TC* |
| C1 | 1880 | 1790 | 1250 | 1250 | 4,540 | 1590 | 1690 | 1215 | 1265 | 4,970 |
| C2 | 2290 | 1860 | 1975 | 2255 | 7,980 | 1930 | 1720 | 1750 | 2350 | 7,420 |
| C3 | 2350 | 1720 | 2770 | 3380 | 9,750 | 2350 | 1930 | 2320 | 2910 | 9,010 |

* Cost in (BDT) per square meter of deck slab; TC = Total cost;

Table 6.6 Computational effort and control parameters used


* Number of evaluations; OF = Objective function; EC = Explicit constraint; IC = Implicit constraint; $\mathrm{T}=$ Time (sec)

From the parametric study of the cost optimum design for $\mathbf{4 0} \mathbf{m}$ double span continuous girder of the present bridge system it is observed that:
(i) Optimum $g$ irder s pacing f or 3 L ane B ridge is $\mathbf{4 . 0} \mathbf{~ m}$ for bot hc oncrete strengths ( 50 MPa and 40 M Pa ) and for all the cost cases, Cost1, Cost2 and Cost3 (Table 6.2 a nd $T$ able 6.3). It indicates that it is m ore eco nomical to space the girder at the maximum practical spacing.
(ii) Optimum girder depth for Cost1, Cost2, Cost3 are $1700 \mathrm{~mm}, 2200 \mathrm{~mm}$ and 2250 mm respectively for 50 MPa concrete strength and $1850 \mathrm{~mm}, 2250 \mathrm{~mm}$, 2625 mm respectively for 40 MPa concrete strength (Table 6.2 and Table 6.3).

So o ptimum girder depth i ncreases with increase in co sts of st eels in bo th cases which indicates $t$ hat $r$ elative co st difference of materials influence optimum design of bridge. The optimum depth is smaller in higher concrete strength. Optimum designs of the 40 m double span continuous girder bridge for concrete strength of 50 MPa are shown in Figure 6.1, 6.2 and 6.3.
(iii) Due to composite construction, deck slab thickness is adequate to satisfy the compression area required for flexural strength of the girder and so top flange width is controlled by the effective span of deck slab to satisfy serviceability criteria of the deck and lateral stability effects of the girder. Top flange width increases in Cost2 and in Cost3. Optimum top flange widths are $450 \mathrm{~mm}, 750$ mm and 1250 mm in Cost1, C ost2, C ost3 respectively in case of co ncrete strength of 50 MPa and $725 \mathrm{~mm}, 950 \mathrm{~mm}$ and 1375 mm respectively in case of concrete st rength of 40 M Pa . Optimum to p f lange $w$ idth in creases with relative increase in costs of steels. Optimum top flange width decreases with increases in concrete strength. Top flange thickness and top flange transition thickness remain to their lower limit.
(iv) Optimum bottom flange width is about 300 mm to 325 mm for both concrete strengths which is close to the lower limit. It indicates that it is not necessary to have large width to accommodate all the tendons in the lowermost position to have greater eccentricity. Thus bottom flange transition area is minimized to keep the concrete area smaller. Optimum bottom flange thickness decreases with increase in relative costs of steels as $n$ umber of tendon decreases with increase in relative costs of steels.
(v) Optimum web width in all the three cases is about 145 mm to 160 mm a nd number of strands per tendon is 8 to 9 for concrete strength of 50 MPa and 7 to 9 for concrete strength of 40 MPa . It indicates that the C-Range anchorage system which accommodates 7 to 9 tendons is the optimum value (Table 6.2, Table 6.3). Number of tendons i.e. prestressing steel required decreases with increase in cost of steels.
(vi) Deck slab thickness increases a little which indicates that even the steel cost is high, deck thickness does not increase comparatively because larger thickness induces larger dead load. So optimum value of deck slab thickness is 255 mm to 285 mm for concrete s trength of 50 M Pa a nd 245 mm to 290 mm for concrete strength of 40 MPa . Reinforcement ratio in the deck decreases with increase in steel costs for cost minimization of the bridge.
(vii) Percentage of steel to be prestressed at initial stage increases with the increase in steel cost which indicates that as steel cost increase the girder weight also increases which require more prestress at initial stage. In this study tendons arrangement al ong $t$ he girder is co nsidered as $v$ ariables and $t$ he $v$ ertical position of tendons at various sections are shown in the Table 6.4.
(viii) Optimum costs ofbridge f or d ifferent r elative c osts of f aterials a nd f or different concrete strengths are tabulated in Table 6.5. Total cost of steels is higher in higher concrete strength.
(ix) The most a ctive constraints gov erning the op timum de sign are co mpressive stress at top fiber of $g$ irder for $p$ ermanent dead lo ad at service $c$ ondition, tensile stress at bottom fiber due to all loads, prestress force at the end of seating loss zone, deck thickness and factor of safety against lateral stability, deflection at s ervice c ondition due to full lo ad in most of the th ree cases. When steel cost is higher, flexural strength of co mposite $g$ irder becomes an active constraint as amount of prestressing steel decreases.
(x) Computational e fforts us ed by E VOP a nd control pa rameters u sed ar e tabulated in Table 6.6 which shows that the optimization problem with a large number of $m$ ixed $t$ ype de sign va riables and implicit constraints converges with a s mall n umber of f unction e valuations. Intel C OREi5 processor h as been used in this study and computational time required for optimization by EVOP is about only 7-8 seconds.

### 6.2.2 Optimum design for $\mathbf{6 0} \mathbf{m}$ double span continuous girder

The opt imum de signs for 60 m do uble span continuous $g$ irder for va rious relative costs and concrete strengths are tabulated in Table 6.7 and Table 6.8. The optimized costs are tabulated in Table 6.10.

Table 6.7 Optimum values of design variables for $\mathbf{3}$ Lane $\mathbf{6 0} \mathbf{m}$ double span continuous girder and Concrete strength $=\mathbf{5 0} \mathbf{~ M P a}$

| Cost | $\begin{gathered} \hline \mathrm{S} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} \hline \hline \mathrm{G}_{\mathrm{d}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \hline \hline \mathrm{TF}_{\mathrm{w}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{gathered} \hline \mathrm{TF}_{\mathrm{t}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \hline \hline \mathrm{TFS}_{\mathrm{t}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \hline \mathrm{BF}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \hline \hline \mathrm{BF}_{\mathrm{t}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \hline \hline \mathrm{W}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ |  |  | $\begin{gathered} \mathrm{t} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \rho \\ & \% \end{aligned}$ | $\begin{gathered} \mathrm{y}_{1} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \eta \\ & \% \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C1 | 3.0 | 2430 | 1075 | 75 | 50 | 365 | 225 | 150 | 9 | 5 | 240 | 0.65 | 750 | 55 |
| C2 | 3.0 | 2670 | 1150 | 75 | 50 | 350 | 240 | 150 | 9 | 4 | 230 | 0.59 | 880 | 68 |
| C3 | 3.0 | 3030 | 1075 | 75 | 50 | 335 | 180 | 150 | 8 | 4 | 240 | 0.55 | 760 | 73 |

Table 6.8 Optimum values of design variables for $\mathbf{3}$ Lane $\mathbf{6 0} \mathbf{m}$ double span continuous girder and Concrete strength $=\mathbf{4 0} \mathbf{~ M P a}$

| Cost | $\begin{gathered} \mathrm{S} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{aligned} & \hline \mathrm{Gd} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \hline \mathrm{TF}_{\mathrm{w}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \hline \mathrm{TF}_{\mathrm{t}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \hline \hline \mathrm{TFS}_{\mathrm{t}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \hline \mathrm{BF}_{\mathrm{w}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{gathered} \mathrm{BF}_{\mathrm{t}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \hline \mathrm{W}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ | $\mathrm{N}_{\text {S }}$ | N | $\begin{gathered} \mathrm{t} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \rho \\ \% \end{gathered}$ | $\begin{gathered} \hline \mathrm{y}_{1} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \eta \\ & \% \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C1 | 3.0 | 2690 | 1225 | 75 | 50 | 360 | 240 | 150 | 9 | 7 | 235 | 0.67 | 920 | 51 |
| C2 | 3.0 | 3120 | 1350 | 75 | 50 | 370 | 270 | 150 | 9 | 5 | 240 | 0.58 | 820 | 59 |
| C3 | 3.0 | 3450 | 1125 | 75 | 50 | 320 | 175 | 150 | 9 | 4 | 270 | 0.53 | 875 | 72 |

Table 6.9 Cg of tendons from bottom fiber of girder in the optimum design

|  | Girder concrete strength $=50 \mathrm{MPa}$ |  |  |  |  |  |  |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\stackrel{\sim}{2}$ | $\cdots$ | $\stackrel{\square}{\square}$ | $\cdots$ | $\stackrel{\sim}{2}$ | $\stackrel{\infty}{\sim}$ |  |  | $\cdots$ | $>$ |  | $\cdots$ | $\stackrel{\sim}{*}$ | $\stackrel{\sim}{2}$ | $\stackrel{\infty}{\sim}$ | \％ |
| C1 | － | $\underset{\sim}{\sim}$ | הి | $\stackrel{\infty}{\infty}$ | $\underset{\sim}{\underset{\sim}{N}} \underset{\sim}{\underset{\sim}{2}}$ | త్రి | $\stackrel{\circ}{\mathrm{N}}$ | N |  | N | $\stackrel{\infty}{n}$ | $\stackrel{\sim}{0}$ | त | $\stackrel{\rightharpoonup}{i}$ | $\stackrel{\circ}{\square}$ | $\stackrel{\text { g }}{\text { d }}$ |  |
| C2 |  | $\mathfrak{F}$ | $\stackrel{\otimes}{\circ}$ | $\stackrel{n}{n}$ | $\stackrel{\circ}{\stackrel{\rightharpoonup}{\sim}} \stackrel{\infty}{\stackrel{1}{\lambda}}$ | $\stackrel{\infty}{0}$ | $\underset{\sim}{\mathrm{N}}$ | ते |  | $\stackrel{\infty}{\%}$ | $\stackrel{+}{6}$ | $\stackrel{\sim}{n}$ | ત્ત入入 | $\stackrel{\otimes}{\square}$ | $\stackrel{\text { ® }}{\text { ® }}$ | 人े |  |
| C3 | $\underset{=}{\circ}$ | $\stackrel{\sim}{\infty}$ | ఠి | $\stackrel{\infty}{\rightrightarrows}$ | $\underset{\sim}{\text { ti }}$ | $\stackrel{\infty}{\square}$ | $\stackrel{\rightharpoonup}{\underset{\sim}{c}}$ | $\frac{m}{n}$ |  | ＋ | \％ | I | べへ্入ী | $\underset{\sim}{\text { J }}$ | $\stackrel{\stackrel{\infty}{\infty}}{=}$ | $\underset{\sim}{\tilde{N}}$ | \％ |

[^0]Table 6．10 Cost of individual materials for $\mathbf{3}$ Lane $\mathbf{6 0} \mathbf{m}$ double span cont girder

| $\begin{gathered} \text { Cost } \\ \text { (BDT) } \end{gathered}$ | Girder concrete strength $=50 \mathrm{MPa}$ |  |  |  |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{Cag}^{*}$ | $\mathrm{C}_{\mathrm{DC}}{ }^{*}$ | $\mathrm{C}_{\mathrm{PS}}$＊ | Cos＊ | TC＊ | $\mathrm{CGG}^{*}$ | $\mathrm{C}_{\mathrm{DC}}{ }^{*}$ | $\mathrm{C}_{\text {PS }}$＊ | Cos＊ | TC＊ |
| C1 | 3550 | 1390 | 2280 | 1160 | 7，850 | 2920 | 1390 | 2070 | 1160 | 6，920 |
| C2 | 3860 | 1370 | 3650 | 2350 | 10，700 | 3290 | 1300 | 3300 | 2530 | 9，760 |
| C3 | 4220 | 1370 | 4960 | 3430 | 13，910 | 3470 | 1440 | 4320 | 3650 | 11，460 |

[^1]Optimum d esign for 60 m double s panc ontinuous girder ( Girder co ncrete strength $=\mathbf{4 0} \mathbf{~ M P a}$ )


Figure 6.2(a) Optimum design for double span continuous girder for Cost1


Figure 6.2(b) Optimum design for double span continuous girder for Cost2


Figure 6.2(c) Optimum design for double span continuous girder for Cost3

Table 6.11 Computational effort and control parameters used

| Girder concrete strength $=50 \mathrm{MPa}$ |  |  |  |  |  |  |  |  |  |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | OF* | EC* | IC* | $\begin{gathered} \mathrm{T} \\ (\mathrm{~s}) \\ \hline \end{gathered}$ | $\alpha$ | $\beta$ | $\gamma$ | Ф |  | $\Phi_{\text {cpx }}$ |  | EC | C* |  | T (s) | $\alpha$ | $\beta$ | $\gamma$ | $\Phi$ | $\Phi_{\text {cpx }}$ |
| C1 | 153 | 863 | 569 | 7 | 1.2 | 0.5 | 2 |  |  | $10^{-16}$ |  | 45 | 58 | 276 | 7 | 1.3 | 0.5 | 2 | $10^{-13}$ | $10^{-16}$ |
| C2 | 184 | 652 | 652 | 8 | 1.5 | 0.5 | 2 | $10^{-13}$ |  | $10^{-16}$ |  | 56 | 68 | 547 | 7 | 1.6 | 0.5 | 2 | $10^{-13}$ | $10^{-16}$ |
| C3 | 164 | 763 | 459 | 8 | 1.7 | 0.5 | 2 | $10^{-13}$ |  | $10^{-16}$ | 127 | 53 | 39 | 379 | 9 | 1.2 | 0.5 | 2 | $10^{-13}$ | $10^{-16}$ |

* Number of evaluations; OF = Objective function; EC = Explicit constraint; IC = Implicit constraint; $\mathrm{T}=$ Time (sec)

Table 6.12 Values of design variables for $\mathbf{4}$ Lane $\mathbf{6 0} \mathbf{m}$ double span continuous girder and Concrete strength $=\mathbf{5 0} \mathbf{~ M P a}$

|  | $\begin{gathered} \hline \hline \mathrm{S} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} \hline \hline \mathrm{G}_{\mathrm{d}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \hline \hline \mathrm{TF}_{\mathrm{w}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \hline \hline \mathrm{TF}_{\mathrm{t}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \hline \hline \mathrm{TFS}_{\mathrm{t}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \hline \mathrm{BF}_{\mathrm{w}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{gathered} \hline \mathrm{BF}_{\mathrm{t}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \hline \hline \mathrm{W}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ | $\mathrm{N}_{\mathrm{S}}$ | $\mathrm{N}_{\mathrm{T}}$ | $\begin{gathered} \hline \hline \mathrm{t} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \hline \rho \\ & \% \end{aligned}$ | $\begin{gathered} \hline \hline \mathrm{y}_{1} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \eta \\ & \% \end{aligned}$ | TC* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C1 | 3.2 | 2530 | 950 | 75 | 50 | 355 | 240 | 150 | 9 | 6 | 210 | 0.72 | 837 | 41 | 7990 |
| C2 | 3.2 | 2790 | 1150 | 75 | 50 | 365 | 190 | 150 | 9 | 5 | 210 | 0.67 | 714 | 45 | 10950 |
| C3 | 3.2 | 2880 | 1150 | 75 | 50 | 375 | 220 | 150 | 9 | 5 | 220 | 0.58 | 950 | 46 | 13780 |

From the parametric study of the cost optimum design for $\mathbf{6 0} \mathbf{m}$ girder of the present bridge system it is observed that:
(i) Optimum girder spacing for 3 L ane Bridge is 3 m for both concrete strengths and for 4 Lane Bridge is 3.2 m for all the cost cases, Cost1, Cost 2 and Cost 3 (Table 6.7, Table 6.8 and Table 6.12).
(ii) Optimum girder depth for Cost1 is 2430 mm (3 lanes) and 2530 mm (4 lanes) for 50 M pac oncrete s trength a nd 2690 mm ( 3 lanes) for 40 Mpa co ncrete strength. Optimum g irder depth increases with increase in costs of steels in both cases.
(iii) Optimum top flange width increases in C ost2 compared to C ost1 while it remains ne arly unc hanged $f$ or Cost3. In C ost3 r equired d eck t hickness is greater for optimization which $n$ eed $n$ ot $s$ maller s pan. O ptimum to $p$ flange width is in between 1075 mm to 1150 mm in case of concrete strength of 50 Mpa a nd 1125 mm to 1350 mm in case of co ncrete st rength of 40 Mpa . It indicates that the wider top flange reduces the formwork cost of the deck slab and increase safety factor ag ainst lateral st ability. Top flange thickness a nd top flange transition thickness remain to their lower limit.
(iv) Optimum bottom flange width is about 370 m m for both concrete strengths which is close to the lower limit which in dicates that it is not necessary to have large width to accommodate all the tendons in the lowermost position to have greater eccentricity. Thus bottom flange transition area is minimized to keep the concrete area smaller. Bottom flange thickness increases a little with increase in cost of steel in Cost2 but decrease in Cost3.
(v) Optimum web width in all the three cases is 150 mm and number of strands per tendon is 8 or 9 for both concrete strengths. It indicates that the C-Range anchorage s ystem which a ccommodates 9 tendons is the opt imum value (Table 6.7, Table 6.8). Number of tendons required decreases with increase in cost ofs teels. Number of $t$ endons required is lower in hi gher c oncrete strength.
(vi) Deck slab thickness increases a little which indicates that even the steel cost is high, deck thickness does not increase comparatively because larger thickness induces larger dead load. So optimum value of deck slab thickness is 215 mm to 230 mm irrespective of the relative cost differences. Reinforcement ratio in the deck decreases with increase in steel costs for cost minimization of the bridge.
(vii) The vertical positions of tendons at various sections are shown in the Table 6.9 .
(viii) Optimum costs of b ridge f or d ifferent r elative c osts of m aterials a nd f or different concrete strengths are tabulated in Table 6.10. Total cost of steels is higher in higher concrete strength.
(ix) The most a ctive constraints gov erning the op timum de sign are co mpressive stress at top $f$ iber of $g$ irder for $p$ ermanent $d$ ead lo ad at service $c$ ondition, tensile stress at bottom fiber due to all loads, prestressing force at the end of seating loss zone, deck thickness and factor of safety against lateral stability, deflection at service condition due to full load in most of the three cost cases. When steel cost is higher, flexural st rength of composite girder be comes an active constraint as amount of prestressing steel decreases.
( x ) It is interesting to n ote that co sts p er square m eter of deck ar $\mathrm{e} v$ ery c lose irrespective of number of lanes.
(xi) Computational ef forts $u$ sed by E VOP an d control parameters $u$ sed ar e tabulated in Table 6.11.

### 6.2.3 Optimum design for $\mathbf{8 0} \mathbf{m}$ double span continuous girder

The opt imum de signs for 80 m double spancontinuous girder for v arious re lative costs an dc oncrete st rengths ar e tabulated in T able 6.13 a nd T able 6.14 . T he optimized costs are tabulated in Table 6.15.

Table 6.13 Optimum values of design variables for $\mathbf{3}$ Lane $\mathbf{8 0} \mathbf{m}$ double span continuous girder and Concrete strength $=50 \mathrm{MPa}$

| Cost | $\begin{gathered} \mathrm{S} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} \mathrm{G}_{\mathrm{d}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \mathrm{TF}_{\mathrm{w}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{gathered} \mathrm{TF}_{\mathrm{t}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \mathrm{TFS}_{\mathrm{t}} \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{gathered} \mathrm{BF}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{BF}_{\mathrm{t}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{W}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ |  |  | $\begin{gathered} \mathrm{t} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{aligned} & \rho \\ & \% \end{aligned}$ | $\begin{gathered} \mathrm{y}_{1} \\ (\mathrm{~mm}) \end{gathered}$ | $\eta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C1 | 4.0 | 3450 | 725 | 75 | 50 | 300 | 255 | 145 | 9 | 8 | 235 | 0.61 | 832 | 42 |
| C2 | 3.0 | 3675 | 800 | 75 | 50 | 300 | 220 | 145 | 9 | 7 | 210 | 0.55 | 843 | 59 |
| C3 | 3.0 | 3750 | 700 | 75 | 50 | 300 | 210 | 135 | 7 | 7 | 220 | 0.51 | 762 | 52 |

Optimum d esign for 80 m double spancontinuous girder (Girder co ncrete strength $=50 \mathrm{MPa}$ )


Figure 6.3(a) Optimum design for double span continuous girder for Cost1


Figure 6.3(b) Optimum design for double span continuous girder for Cost2


Figure 6.3(c) Optimum design for double span continuous girder for Cost3

Table 6.14 Optimum values of design variables for $\mathbf{3}$ Lane $\mathbf{8 0} \mathbf{m}$ double span continuous girder and Concrete strength $=40 \mathrm{MPa}$

| Cost | $\begin{gathered} \hline \hline \mathrm{S} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} \hline \mathrm{G}_{\mathrm{d}} \\ (\mathrm{~mm}) \end{gathered}$ | $(\mathrm{mm})$ | $\begin{gathered} \hline \mathrm{TF}_{\mathrm{t}} \\ (\mathrm{~mm}) \end{gathered}$ | (mm) | $(\mathrm{mm})$ | $(\mathrm{mm})$ | $\begin{gathered} \hline \mathrm{W}_{\mathrm{w}} \\ (\mathrm{~mm}) \end{gathered}$ |  |  | $(\mathrm{mm})$ | $\begin{gathered} \rho \\ \% \end{gathered}$ | $\begin{gathered} \mathrm{y}_{1} \\ (\mathrm{~mm}) \end{gathered}$ | $\eta$ $\%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C1 | 3.0 | 3650 | 1050 | 75 | 50 | 300 | 200 | 145 | 8 | 8 | 230 | 0.61 | 921 | 45 |
| C2 | 3.0 | 3850 | 950 | 75 | 50 | 300 | 270 | 145 | 9 | 8 | 235 | 0.57 | 932 | 42 |
| C3 | 3.0 | 3900 | 1150 | 75 | 50 | 325 | 155 | 145 | 9 | 7 | 260 | 0.42 | 719 | 41 |

Table 6.15 Cost of individual materials for $\mathbf{3}$ Lane $\mathbf{8 0} \mathbf{m}$ double span cont girder

|  | Girder concrete strength $=50 \mathrm{MPa}$ |  |  |  |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Cost } \\ \text { (BDT) } \end{gathered}$ | $\mathrm{C}_{\mathrm{GC}}$ * | $\mathrm{C}_{\text {DC }}$ * | $\mathrm{C}_{\text {PS }}{ }^{*}$ | Cos* | TC* | $\mathrm{C}_{\mathrm{GC}}$ * | $\mathrm{C}_{\mathrm{DC}}$ * | $\mathrm{C}_{\text {PS }}{ }^{*}$ | Cos* | TC* |
| C1 | 2690 | 1740 | 1460 | 1230 | 6,490 | 2210 | 1690 | 1330 | 1210 | 4,990 |
| C2 | 3780 | 1560 | 2320 | 2300 | 9,750 | 2360 | 1800 | 2400 | 2430 | 8,390 |
| C3 | 3750 | 1150 | 3600 | 2580 | 10,980 | 2420 | 1800 | 3620 | 3410 | 10,690 |

* Cost in (BDT) per square meter of deck slab; TC = Total cost;

Table 6.16 shows total optimum number of prestressing strands $\left(\mathrm{N}_{\mathrm{S}} \times \mathrm{N}_{T}\right)$ required for various girder span and concrete strength. Number of strands increases with the increase in span of girder.

Table 6.17 shows optimum girder spacing for various girder span a nd concrete strength. G irder spacing is higher in smaller s pan than 1 arger sp an. G irder sp acing depends on $t$ he maximum 1 imit of gi rder de pth. If maximum de pth of girder $c$ an exceed the practical limit, the girder spacing will increase for more optimized design of bridge.

Table 6.16 Total optimum number of prestressing strands ( $\mathbf{N}_{S} \times \mathbf{N}_{T}$ )

| Cost | Girder concrete strength $=50 \mathrm{MPa}$ |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 40 m | 60 m | 80 m | 40 m | 60 m | 80 m |
| C 1 | 36 | 45 | 72 | 36 | 63 | 64 |
| C 2 | 36 | 36 | 63 | 32 | 45 | 72 |
| C 3 | 24 | 32 | 49 | 21 | 36 | 63 |

Table 6.17 Optimum girder spacing (meter)

| Cost | Girder concrete strength $=50 \mathrm{MPa}$ |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{4 0} \mathbf{~ m}$ | $\mathbf{6 0} \mathbf{~ m}$ | $\mathbf{8 0} \mathbf{~ m}$ | $\mathbf{4 0} \mathbf{~ m}$ | $\mathbf{6 0} \mathbf{~ m}$ | $\mathbf{8 0} \mathbf{~ m}$ |
| C 1 | 4.0 | 3.0 | 4.0 | 4.0 | 3.0 | 3.0 |
| C 2 | 4.0 | 3.0 | 3.0 | 4.0 | 3.0 | 3.0 |
| C 3 | 4.0 | 3.0 | 3.0 | 4.0 | 3.0 | 3.0 |

Table 6.18 shows optimum deck slab thickness for various girder span and concrete strength. Optimum deck slab thickness is higher in shorter span as the girder spacing is higher in shorter span. The higher girder spacing, the higher effective span of deck slab which requires thicker depth of deck slab. Table 6.19 shows Optimum deck slab main reinforcement. It can be observed that girder concrete strength has no effect on optimum deck slab main reinforcement.

Table 6.18 Optimum deck slab thickness (mm)

| Cost | Girder concrete strength $=50 \mathrm{MPa}$ |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{4 0} \mathbf{~ m}$ | $\mathbf{6 0 ~ m}$ | $\mathbf{8 0} \mathbf{~ m}$ | $\mathbf{4 0} \mathbf{~ m}$ | $\mathbf{6 0} \mathbf{~ m}$ | $\mathbf{8 0} \mathbf{~ m}$ |
| C 1 | 255 | 240 | 235 | 245 | 235 | 230 |
| C 2 | 265 | 230 | 210 | 245 | 240 | 235 |
| C 3 | 285 | 240 | 220 | 290 | 270 | 260 |

Table 6.19 Optimum deck slab main reinforcement (\%)

| Cost | Girder concrete strength $=50 \mathrm{MPa}$ |  | Girder concrete strength $=40 \mathrm{MPa}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{4 0} \mathbf{~ m}$ | $\mathbf{6 0} \mathbf{~ m}$ | $\mathbf{8 0} \mathbf{~ m}$ | $\mathbf{4 0} \mathbf{~ m}$ | $\mathbf{6 0} \mathbf{~ m}$ | $\mathbf{8 0} \mathbf{~ m}$ |
| C 1 | 0.68 | 0.65 | 0.61 | 0.67 | 0.67 | 0.61 |
| C 2 | 0.55 | 0.59 | 0.55 | 0.52 | 0.58 | 0.57 |
| C 3 | 0.48 | 0.55 | 0.51 | 0.45 | 0.53 | 0.42 |

## Chapter 7

## Conclusions and Summary of Suggestions

### 7.1 Conclusions

The present research work commenced with an aim to achieve the cost minimization of a double span continuous post-tensioned PC I-girder bridge superstructure system by a dopting a $n$ opt imization a pproach $t$ o obt ain $t$ he opt imum design a nd a lso to perform various parametric studies for the constant design parameters of the bridge system to o bserve the effects of su ch parameters on the optimum design. A global optimization algorithm named E VOP (Evolutionary O peration) is us ed which is capable of locating directly with high probability the global minimum. A program is developed for the optimization which may be beneficial to designers and contractors interested in cost minimum design of the present bridge system. Influence of constant design $p$ arameters su ch as $u$ nit co sts of $m$ aterials an $d c$ oncrete $s$ trength o $n$ the optimum design is presented.

Under the scope of the present study, following conclusions can be made:
(i) Optimum girder spacing is higher in smaller span than larger span bridges.
(ii) Optimum girder depth increases with increase in cost of steels. On an average, girder depth increases $22 \%$ with every $100 \%$ increase in cost of steel for 40 MPa concrete. On the other hand, for 50 MPa concrete, the average increase in girder depth came out to be $19 \%$.
(iii) Optimum top flange width is controlled by deck slab span and lateral stability effect. Top flange thickness and top flange transition thickness remain to their lower limit.
(iv) Optimum bottom flange width remains ne arly to the lower lim it. Optimum bottom flange thickness decreases with increase in relative cost of steels.
(v) Optimum web width remains nearly constant irrespective of girder span and concrete strength.
(vi) Optimum number of strand is higher in higher span girder. Number of strand decreases $17 \%$ with every $100 \%$ increase in cost of steel for 40 MPa concrete. On the o ther hand, for 50 MP a concrete, the average decrease in number of strand came out to be $16 \%$.
(vii) Optimum number of strand per tendon is 8 or 9 for both concrete strengths for 80 m girder and 7 t o 9 f or 40 m and 60 m girder spans studied. Number of strands required is higher in higher concrete strength.
(viii) Optimum deck slab thickness is higher in shorter span as the girder spacing is higher in shorter span. The higher girder spacing, the higher effective span of deck slab which requires thicker depth of deck slab.
(ix) The present constrained optimization problem of 14 nu mber design variables having a combination of continuous, integer, discrete types and 73 numbers of implicit constraints is easily solved by EVOP with a relatively small number of function evaluations by simply adjusting the EVOP control parameters.

### 7.2 Summary of Suggestions

It is recommended that the study can be extended further in the following fields:
(i) Application of optimization approach on the I-Girder bridges system or other types of bridge system considering both superstructure and substructure.
(ii) Application of high strength concrete in the optimization of I- Girder bridge system. High st rength co ncrete ( HSC) $h$ as sev eral ad vantages o ver conventional strength concrete. These include increased compressive strength, modulus of e lasticity, tensile strength. In a ddition, high strength concrete is nearly a lways en hanced by these other benefits, a s maller creep co efficient, less shrinkage strain, lower permeability and improved durability.
(iii) Application of optimization approach on various types of prestressed concrete structures under flexure considering the various classes (Class U, Class T and Class C) of flexure.

## Appendix-A

## Computer Program

## (Written in C++ Language)

```
//***********************************************************************************
//__01_01__01__01_Header Files Declaration Zone___ 01__01__01__01__
/ / * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ~
#include <iostream>
#include <fstream>
#include <stdio.h>
#include <cmath>
#include <string>
#include <time.h>
#include<math.h>
#define SWAP(a,b){temp=(a);(a)=(b);(b)=temp;}
#include <stddef.h>
#include <stdlib.h>
#define NR_END 1
#define FREE_ARG char*
using namespace std;
extern "C"
{
    void __stdcall EVOP(double*,double*,double*,double*,double*,double*,double*,
```



```
                double*,double*,double*,double*,double*,double*,double*,double*,double*);
            void __stdcall DINTG2(int*,double*,double*,double*,double*);
            void __stdcall DISCR2(double*,int*,}\mp@subsup{\mathrm{ int*}}{}{*},\mathrm{ double*,double*,double*,double*);
            void __stdcall EXPCON(int*,int*,int*,int*,double*,double*,double*);
            void __stdcall FUNC(double*,int*,int*,int*,double*);
            double __stdcall RNDOFF(double*);
            void __stdcall IMPCON(int*,int*,double*,double*,double*,double*);
}
//**********************************************************************************
//__02__02_02__02__Variable Declaration Zone___ 02_02__02_02__02_
//***********************************************************************************
int No_of_Span=2;
int No_of_Node=No_of_Span+1;
int y=1;
int xx=1;
double xw;
```

```
//Area and other
double Ag,Gd,Gdc,Anet,Atf,Atfc,EFW;
//I
double I,Inet,Ic,Itf;
//Y
double Y1,Y2,Y3,Y_end,Y_int_sup,Y_inf,Y7,Y8; //cg of strands
//double Y1i,Y2i,Y3i,Y_end_i,Y_int_sup_i,Y_inf_i,Y7i,Y8i; //cg of strands
double
Yb,Y1bnet,Y1tnet,Y2bnet,Y2tnet,Y3bnet,Y3tnet,Y_int_sup_bnet,Y_int_sup_tnet,Y_inf_bnet,Y_inf_tnet,Y
7bnet,Y7tnet,Y8bnet,Y8tnet,Y1b,Y2b,Y3b,Y_int_sup_b,Y_inf_b,Y7b,Y8b,Y1t,Y2t,Y3t,Y_int_sup_t,Y_inf_t,Y
7t,Y8t,Y1bc,Y1tc,Y2bc,Y2tc,Y3bc,Y3tc,Y_int_sup_bc,Y_int_sup_tc,Y_inf_bc,Y_inf_tc,Y7bc,Y7tc,Y8bc,Y8tc;
    //cg of section
//e
double
e1,e2,e3,e_end,e_int_sup,e_inf,e7,e8,e1i,e2i,e3i,e_end_i,e_int_sup_i,e_inf_i,e7i,e8i,ec1,ec2,ec3,ec_en
d,ec_int_sup,ec_inf,ec7,ec8; //eccentricity
//s
double
S1tnet,S1bnet,S1t,S1b,S1tc,S1bc,S2tnet,S2bnet,S2t,S2b,S2tc,S2bc,S3tnet,S3bnet,S3t,S3b,S3tc,S3bc,S_en
d_tnet,S_end_bnet,S_end_t,S_end_b,S_end_tc,S_end_bc;
double
S_int_sup_tnet,S_int_sup_bnet,S_int_sup_t,S_int_sup_b,S_int_sup_tc,S_int_sup_bc,S_inf_tnet,S_inf_b
net,S_inf_t,S_inf_b,S_inf_tc,S_inf_bc,S7tnet,S7bnet,S7t,S7b,S7tc,S7bc,S8tnet,S8bnet,S8t,S8b,S8tc,S8bc;
double
Cable_Loc_mid[31],Cable_Loc_end[31],Cable_Loc_int_sup[31],Cable_Loc_inf[31],Cable_Loc_8[31],Cable
_Loc_7[31],alpha[31],alpha_xw[31],alpha3[31],alpha7[31],alpha8[31],Layer_dist_bottom_mid[31],Layer
_dist_bottom_inf[31];
double
TFRd,TFRw,TFFHd,TFFHtw,TFFHw,TFFHbw,TFSHd=75,TFSHtw,TFSHw=75,TFSHbw,W,Wt,BFHd,BFHw,BFR
d, BFRw,ts,GS,Nstrand,Ncable,cable_1st_position_end;
double Wd,Rh,Rw,FPw,FPt,Kcr,T,UPcondeck,UPnonprest,UPcon,UPst;
double
Duct_dia,Ancg_C2C,Ancg_Edge_dist,Ancg_C2C_Lay1,Ancg_Edge_dist_Lay1,Ancg_Edge_dist_vertical,Duc
t_clear_spacing,fricncoeff,Nstrandt,Mu,Wri,Anchor_dim;
double deflectiont,deflectione,deflectionf,deflection;
double
MG1,MG2,MG3,MG4,MG5,MG6,MG7,MG8,MCG1,MCG2,MCG3,MCG4,MCG5,MCG6,MCG7,MCG8,MS1,
MS2,MS3,MS4,MS5,MS6,MS7,MS8,MWC1,MWC2,MWC3,MWC4,MWC5,MWC6,MWC7,MWC8;
double
MMS1,MMS2,MMS3,MMS4,MMS5,MMS6,MMS7,MMS8,MFP,MC1,MC2,MC3,MC4,MC5,MC6,MC7,MC8
,DF,MLL1,MLL2,MLL3,MLL4,MLL5,MLL6,MLL7,MLL8;
```

```
double
IMF,MT1,MT2,MT3,MT4,MT5,MT6,MT7,MT8,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MD1,MD2,MD3,
MD4,MD5,MD6,MD7,MD8,MFP_xw,MFP3; //??
double Fend,F3i,F1i,F11,As,F2i,F21,F31,x,Fend2;
double f4ti,F4i,e4i,S4tnet,f5ti,F5i,e5i,S5tnet,f6ti,F6i,e6i,S6tnet,f7ti,F7i,f8ti,F8i;
double
fti,fbi,ftc,fbc,ftt,fbt,fttt,fbtt,f3ti,f3bi,f3tc,f3bc,f3tt,f3bt,f3ttt,f3btt,fti_xw,fbi_xw,ftc_xw,fbc_xw,ftt_xw,fbt
_xw,fttt_xw,fbtt_xw;
double f4bi,f5bi,f6bi,f7bi,f8bi,f4tc,f5tc,f6tc,f7tc,f8tc,F41,F51,F61,F71,F81,f4bc,f5bc,f6bc,f7bc,f8bc;
double
f4tt,f5tt,f6tt,f7tt,f8tt,f4bt,f5bt,f6bt,f7bt,f8bt,f4ttt,f5ttt,f6ttt,f7ttt,f8ttt,f4btt,f5btt,f6btt,f7btt,f8btt;
double
VDL2,VLL2,IMF2,Vc,Mcr2,fpe,Vu,dshear,Vs,Vnh,ds,R,Asnp,Asnpd,rho,d_min,Muslab,IMFS,MSS,MSWC,M
SDL,MSLL,dreq;
int LayerNo_mid,LayerNo_inf,Cable_Layer_mid[31],Cable_Layer_inf[31];
double NoGirder;
double DX[8],pt1,pt2,pt3,Ig,DECKT[26],DX1[69],DX2[69],DX4[101],DX11[10], DX12[11], DX13[36];
int const nv = 14;
int const icn = 73;
double Cable_Loc_3[31],Cable_Loc_xw[31];
int cost = 1;
double Anchcost,sheathcost,UPgf,UPdf;
double Cpcon,Cpst,Cdconc,Cnpst,SA;
// Bridge Data:
// Length of girder,
double L=40000; //mm
// Width of bridge =
double BW=12000; //mm
// Cross girder, wearing coarse and Median strip constant
int NCG = 9;
double CGt = 250; //mm
double WCt = 50; }//\textrm{mm
double MSh = 600; //mm
double MSw = 450; //mm
// Material constants:
double Gammawc = 25; //!KN/m3
// Unit weight of concrete,
double Gammacon = 24; //!KN/m3
// Unitweight of steel,
double Gammast=7850e-9; //Kg/mm3
double Astrand=140.0; //mm2
// Astrand = 98.7;
// Wobble coefficient,
```

```
double Kwc =0.000005;//!per mm
// Anchorage Slip
double Delta=6; //!mm
// Modulus of elasticiy of steel
double Es= 197000; // !MPa //AASHTO LRFD (2007) 5.4.4.2
// Ultimate strength of prestressing steel,
double fpu= 1860; //ASTM A416 M
// Concrete:
// Compressive strength of concrete, MPa
double fc=40;
// Compressive strength of concrete for deck slab, MPa
double fcdeck=25;
// Concrete compressive strength at transfer,
double fci =0.75*fc;
// Coefficient of elastic shortening,
double Kes =0.5;
// Design Data:
// Specification AASHTO 2007
// Live Load HL-93
// Load from frontal wheel,
double P1=35; //!KN
// Load from Rear wheel,
double P2=145; //!KN
// Modulus of elasticiy of concrete
double Ec=33.0*pow(150,1.5)*sqrt(fc*145.0)/145.0; // !MPa
// Modulus of elasticiy of concrete at initial stage
double Eci = 33.0*pow(150,1.5)*sqrt(fci*145.0)/145.0;
double Ecdeck = 33.0*pow(150,1.5)*sqrt(fcdeck*145.0)/145.0;
double mratio = Ecdeck/Ec;
int whichSection_intermsof_ILmatrixrow=0;
double max_wheelLoad_shearPositive=0;
double max_wheelLoad_shearNegative=0;
double max_wheelLoad_momentPositive=0;
double max_wheelLoad_momentNegative=0;
double max_laneLoad_shearPositive=0;
double max_laneLoad_shearNegative=0;
double max_laneLoad_momentPositive=0;
double max_laneLoad_momentNegative=0;
double laneLoad_pick=0;
double loadedLength_max_wheelLoad_shearPositive=0;
double loadedLength_max_wheelLoad_shearNegative=0;
double loadedLength_max_wheelLoad_momentPositive=0;
double loadedLength_max_wheelLoad_momentNegative=0;
```

```
double loadedLength_max_laneLoad_shearPositive=0;
double loadedLength_max_laneLoad_shearNegative=0;
double loadedLength_max_laneLoad_momentPositive=0;
double loadedLength_max_laneLoad_momentNegative=0;
double ImpactFactor_max_wheelLoad_shearPositive=0;
double ImpactFactor_max_wheelLoad_shearNegative=0;
double ImpactFactor_max_wheelLoad_momentPositive=0;
double ImpactFactor_max_wheelLoad_momentNegative=0;
double ImpactFactor_max_laneLoad_shearPositive=0;
double ImpactFactor_max_laneLoad_shearNegative=0;
double ImpactFactor_max_laneLoad_momentPositive=0;
double ImpactFactor_max_laneLoad_momentNegative=0;
double w_dyn=0 ;
double UDL_SW_girder=0;
double UDL_SW_CG=0;
double UDL_SW_slab=0;
double UDL_SW_WC=0;
double UDL_SW_MS=0;
double UDL_SW_LL=0;
//************************************************************************************
//__03_03__03__03___Matrix/Array Declaration Zone
03__03__03__03_
//***********************************************************************************
double local_stiffness_matrix[4][4];
double global_stiffness_matrix[32][32];
double **global_stiffness_matrix_pointer;
double global_member_force_matrix[32][1];
double **global_member_force_matrix_pointer;
double final_DOF_matrix[16][1];
double total_DL_end_shearforce_bendingmoment_matrix[15][4];
double for_ILvalue_LL_end_shearforce_bendingmoment_matrix[15][4];
double total_DL_sectionwise_shearforce_matrix[1][3500];
double total_DL_sectionwise_bendingmoment_matrix[1][3500];
double for_ILvalue_LL_sectionwise_shearforce_matrix[1][3500];
double for_ILvalue_LL_sectionwise_bendingmoment_matrix[1][3500];
double IL_matrix_shear[3500][3500];
double IL_matrix_moment[3500][3500];
double matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[1][3500];
double matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[1][3500];
double matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[1][3500];
double matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[1][3500];
//***********************************************************************************
//__04__04__04_04__04__Matrix Initialization Zone___ 04__04__04__04__04_
//***********************************************************************************
void matrix_initialization_function()
{
// cout<<"matrix_initialization function"<<"\t";
```

```
for(int i=0;i<32;i++)
{
    for(int j=0;j<32;j++)
    {
        global_stiffness_matrix[i][j]=0.0; //global_stiffness_matrix[32][32];
    }
}
for(int k=0;k<32;k++)
{
    global_member_force_matrix[k][0]=0.0; //global_member_force_matrix[32][1];
    //DOF_matrix[32][1];
    //Pj_matrix[32][1];
}
for(int l=0;|<16;l++)
{
    final_DOF_matrix[I][0]=0.0; //final_DOF_matrix[16][1];
}
for(int m=0;m<15;m++)
{
    for(int n=0;n<4;n++)
    {
        total_DL_end_shearforce_bendingmoment_matrix[m][n]=0.0;
                            //total_DL_end_shearforce_bendingmoment_matrix[15][4];
                for_ILvalue_LL_end_shearforce_bendingmoment_matrix[m][n]=0.0;
                            //for_ILvalue_LL_end_shearforce_bendingmoment_matrix[15][4];
    }
}
for(int o=0;o<3500;o++)
{
    total_DL_sectionwise_shearforce_matrix[0][0]=0.0;
                            //total_DL_sectionwise_shearforce_matrix[1][3500];
        total_DL_sectionwise_bendingmoment_matrix[0][0]=0.0;
                            //total_DL_sectionwise_bendingmoment_matrix[1][3500];
        for_ILvalue_LL_sectionwise_shearforce_matrix[0][0]=0.0;
            //for_ILvalue_LL_sectionwise_shearforce_matrix[1][3500];
        for_ILvalue_LL_sectionwise_bendingmoment_matrix[0][0]=0.0;
            //for_ILvalue_LL_sectionwise_bendingmoment_matrix[1][3500];
}
for(int p=0;p<3500;p++)
```

```
    {
        for(int q=0;q<3500;q++)
        {
            IL_matrix_shear[p][q]=0.0; //IL_matrix_shear[3500][3500];
        IL_matrix_moment[p][q]=0.0; //IL_matrix_moment[3500][3500];
        }
    }
    for(int r=0;r<3500;r++)
    {
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][r]=0.0;
            //double matrix_forSorting_maXof_wheelLoadorlaneLoad_shear[1][3500];
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][r]=0.0;
        //double matrix_forSorting_maXof_wheelLoadorlaneLoad_moment[1][3500];
            matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][r]=0.0;
            //double matrix_forSorting_maXof_wheelLoadorlaneLoad_shear[1][3500];
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][r]=0.0;
        //double matrix_forSorting_maXof_wheelLoadorlaneLoad_moment[1][3500];
    }
}
//************************************************************************************
//__05__05__05__05____Function Declaration Zone___05__05__05__05__05__
//**********************************************************************************
void local_stiffness_matrix_function(); //calculates [k]_local matrix
void global_stiffness_matrix_function(); //calculates [k]_global matrix
void global_member_force_matrix_function(double w_dyn); //calculates [Pm]_golbal matrix.
void impose_boundarycondition_function(); /*imposes boundary condition by making kii=1;
Pi=0; ith row and column values=0 to make
ui=0*/
void modified_global_stiffness_matrix_function(); //considers boundary condition void modified_global_member_force_matrix_function(); //considers boundary condition void DOF_matrix_solution_function(double **a,int n,double **b,int m);
void final_DOF_matrix_function(); /*call within DOF_matrix_solution_function(); ..not from main()*/
void total_DL_end_shearforce_bendingmoment_function(double w_dyn);
void for_ILvalue_LL_end_shearforce_bendingmoment_function(int increment_a,int increment_c);
void total_DL_sectionwise_shearforce_function(double w_dyn);
void total_DL_sectionwise_bendingmoment_function(double w_dyn);
void influence_line_function();
void global_member_force_matrix_function_forlL(int increment_a,int increment_c);
    //calculates [Pm]_golbal matrix for influence line
void for_ILvalue_LL_sectionwise_shearforce_function(int increment_a,int increment_c);
void for_ILvalue_LL_sectionwise_bendingmoment_function(int increment_a,int increment_c);
void matrix_initialization_function(); //initializes all matrices with zero value
void test_function(); //common output function to test different funtions
```

```
void nrerror(char error_text[]);
int *ivector(long nl, long nh);
void free_ivector(int *v, long nl, long nh);
void dynamic_allocation_01();
void dynamic_allocation_02();
void factored_DLplusLL_shearCombination_atSpecificSection_function
(int whichSection_intermsof_ILmatrixrow);
void factored_DLplusLL_momentCombination_atSpecificSection_function
(int whichSection_intermsof_ILmatrixrow);
double total_LL_atSpecific_section_positive_shearforce_function
(int whichSection_intermsof_ILmatrixrow);
double total_LL_atSpecific_section_negative_shearforce_function
(int whichSection_intermsof_ILmatrixrow);
double total_LL_atSpecific_section_positive_bendingmoment_function
(int whichSection_intermsof_ILmatrixrow);
double total_LL_atSpecific_section_negative_bendingmoment_function
(int whichSection_intermsof_ILmatrixrow);
double maxm(double value_a, double value_b);
double minm(double value_c, double value_d);
double absolue_value_function(double value_r)
    if (value_r>=0)
    return value_r;
        else
        return -(value_r);
}
void Anchorage_system() // Duct and Anchorage System
{
// cout<<"anchorage_system"<<"\t";
double Fcable;
Nstrandt = RNDOFF(&Nstrand);
Fcable = 0.7*fpu*Astrand*Nstrand/1000;
if(Nstrandt <= 3)
{
    Duct_dia = 45;
    Duct_clear_spacing = 38;
    Ancg_Edge_dist_vertical = 127.5;//Ancg_Edge_dist_vertical = 1.5*boo
    fricncoeff = 0.25;
    Anchor_dim = 110;
}
else if(Nstrandt <= 4 )
{
    Duct_dia = 50;
    Duct_clear_spacing = 38;
    Ancg_Edge_dist_vertical = 150;
    fricncoeff = 0.25;
    Anchor_dim = 120;
    }
```

```
else if(Nstrandt <= 7 )
{
    Duct_dia = 65;
    Duct_clear_spacing = 38;
    Ancg_Edge_dist_vertical = 187.5;
    fricncoeff = 0.25;
    Anchor_dim = 150;
}
else if(Nstrandt <= 9 )
{
    Duct_dia = 70;
    Duct_clear_spacing = 38;
    Ancg_Edge_dist_vertical = 210;
    fricncoeff = 0.20;
    Anchor_dim = 185;
}
else if(Nstrandt <= 12 )
{
    Duct_dia = 85;
    Duct_clear_spacing = 38;
    Ancg_Edge_dist_vertical = 247.5;
    fricncoeff = 0.20;
    Anchor_dim = 200;
}
else if(Nstrandt <= 13 )
{
    Duct_dia = 85;
    Duct_clear_spacing = 38;
    Ancg_Edge_dist_vertical = 255;
    fricncoeff = 0.20;
    Anchor_dim = 210;
}
else if(Nstrandt <= 19)
{
    Duct_dia = 100;
    Duct_clear_spacing = 38;
    Ancg_Edge_dist_vertical = 300;
    fricncoeff = 0.20;
    Anchor_dim = 250;
}
else if(Nstrandt <= 22)
{
    Duct_dia = 110;
    Duct_clear_spacing = 50;
    Ancg_Edge_dist_vertical = 322.5;
    fricncoeff = 0.20;
    Anchor_dim = 275;
}
```

```
    else
    {
        Duct_dia = 115;
        Duct_clear_spacing = 50;
        Ancg_Edge_dist_vertical = 345;
        fricncoeff = 0.20;
        Anchor_dim = 300;
    }
    Ancg_Edge_dist_Lay1 = Ancg_Edge_dist_vertical/1.50;
    Ancg_C2C_Lay1 = Fcable*1000.0/fci/Ancg_Edge_dist_Lay1;
    Ancg_Edge_dist = BFRw/2;
    Ancg_C2C = Fcable*1000.0/fci/Ancg_Edge_dist;
    Wt = Duct_dia + 80;
}
double minm(double a, double b)
{
    double temp;
    if(a<b)
        temp = a;
    else
        temp = b;
    return temp;
}
double maxm(double a, double b)
{
    double temp;
    if(a>b)
        temp = a;
    else
        temp = b;
    return temp;
}
void Sectional_Properties()
{
// cout<<"sectional_properties"<<"\t";
// Non Composite Section Properties
    Ag =
(TFRd*TFRw)+((TFFHtw+TFFHbw)/2*TFFHd)+((TFSHtw+TFSHbw)/2*TFSHd)+Wd*Wt+((Wt+BFRw)/2*BFH
d)+(BFRd*BFRw);
    Anet = Ag-Ncable*3.1416/4*(Duct_dia)*(Duct_dia);
    Atf = Ag + (Es/Ec-1)*As;
    Yb = ((TFRd*TFRw)*(Gd-TFRd/2))+((TFFHw*TFFHd)*(Gd-TFRd-TFFHd/3));
    Yb = Yb +((TFFHtw-2*TFFHw)*TFFHd*(Gd-TFRd-TFFHd/2));
    Yb = Yb +((TFSHd*TFSHw)*(Gd-TFRd-TFFHd-TFSHd/3))+((Wt*TFSHd)*(Gd-TFRd-TFFHd-
TFSHd/2));
    Yb = Yb +((Wt*Wd)*(BFRd+BFHd+Wd/2))+((BFHw*BFHd)*(BFRd+BFHd/3));
    Yb = Yb +((BFHd*Wt)*(BFRd+BFHd/2))+((BFRd*BFRw)*BFRd/2);
```

```
    Y1bnet = Yb - Ncable*3.1416/4*pow((Duct_dia),2)*Y1; // section 1 and 2
    Y2bnet = Yb - Ncable*3.1416/4*pow((Duct_dia),2)*Y2;
    Y1b = Yb + (Es/Ec-1)*As*Y1;
        Y2b = Yb + (Es/Ec-1)*As*Y2;
    Y1b = Y1b/Atf;
        Y2b = Y2b/Atf;
        Y1t = Gd-Y1b;
        Y2t = Gd-Y2b;
    Y1bnet = Y1bnet/Anet;
    Y2bnet = Y2bnet/Anet;
    Y1tnet = Gd-Y1bnet;
    Y2tnet = Gd-Y2bnet;
//----------------------------------------------------------------------
    Y3b = Yb + (Es/Ec-1)*As*Y3;
        Y3b = Y3b/Atf;
        Y3t = Gd-Y3b;
        Y3bnet = Y3bnet/Anet;
        Y3tnet = Gd-Y3bnet;
//-------------------------------------------------------------------------------
    Y_int_sup_bnet = Yb - Ncable*3.1416/4*pow((Duct_dia),2)*Y_int_sup; //section int support
    Y_int_sup_b = Yb + (Es/Ec-1)*As*Y_int_sup;
    Y_int_sup_b = Y_int_sup_b/Atf;
    Y_int_sup_t = Gd-Y_int_sup_b;
    Y_int_sup_bnet = Y_int_sup_bnet/Anet;
    Y_int_sup_tnet = Gd-Y_int_sup_bnet;
//-----------------------------------------------------------------------------
    Y_inf_bnet = Yb - Ncable*3.1416/4*pow((Duct_dia),2)*Y_inf; // section Inflection point
    Y_inf_b = Yb + (Es/Ec-1)*As*Y_inf;
    Y_inf_b = Y_inf_b/Atf;
    Y_inf_t = Gd-Y_inf_b;
    Y_inf_bnet = Y_inf_bnet/Anet;
    Y_inf_tnet = Gd-Y_inf_bnet;
```



```
    Y7bnet = Yb - Ncable*3.1416/4*pow((Duct_dia),2)*Y7; // section 7
    Y7b = Yb + (Es/Ec-1)*As*Y7;
    Y7b = Y7b/Atf;
    Y7t = Gd-Y7b;
    Y7bnet = Y7bnet/Anet;
    Y7tnet = Gd-Y7bnet;
//-------------------------------------------------------------------------------
    Y8bnet = Yb - Ncable*3.1416/4*pow((Duct_dia),2)*Y8; // section 8
    Y8b = Yb + (Es/Ec-1)*As*Y8;
```

```
        Y8b = Y8b/Atf;
        Y8t = Gd-Y8b;
        Y8bnet = Y8bnet/Anet;
        Y8tnet = Gd-Y8bnet;
// Ecentricity,
//------------------------
    e1i = Y1bnet-Y1; //!eccentricity at section 1
//----------------------------------------------------------------------------
    e2 = Y2b - Y2; //!eccentricity at section 2 (i.e section xw)
    e2i = Y2bnet - Y2;
//--------------------------------------------------------------------------------------
    e3 = Y3b - Y3; //!eccentricity at section 3 (i.e. section x)
    e3i = Y3bnet - Y3;
//----------------------------------------------------------------------------
    e_int_sup = Y_int_sup_b - Y_int_sup; //!eccentricity at section int_sup
    e_int_sup_i = Y_int_sup_bnet - Y_int_sup;
//--------------------------------------------------------------------------------
    e_inf = Y_inf_b - Y_inf; //!eccentricity at section inflection
    e_inf_i = Y_inf_bnet - Y_inf;
//------------------------------------------------------------------
    e7i = Y7bnet - Y7;
//--------------------------------------------------------------------------------
    e8 = Y8b - Y8; //!eccentricity at section 8
    e8i = Y8bnet - Y8;
    I = pow(TFRd,3)*TFRw/12+(TFRd*TFRw)*pow((Y1t-TFRd/2),2);
    I = I + TFFHw*pow(TFFHd,3)/36*2+(TFFHw*TFFHd)*pow((Y1t-TFRd-TFFHd/3),2);
    I = I + (pow(TFFHd,3)*TFFHbw/12)+(TFFHbw*TFFHd)*pow((Y1t-TFRd-TFFHd/2),2);
    I = I + (TFSHw*pow(TFSHd,3)/36)*2+(TFSHw*TFSHd)*pow((Y1t-TFRd-TFFHd-TFSHd/3),2);
    I = I + (Wt* pow(TFSHd,3)/12)+(Wt*TFSHd)*pow((Y1t-TFRd-TFFHd-TFSHd/2),2);
    I = I + (Wt*pow(Wd,3)/12)+(Wt*Wd)*pow((Y1b-BFRd-BFHd-Wd/2),2);
    I = I + (BFHw*pow(BFHd,3)/36)*2+(BFHw*BFHd)*pow((Y1b-BFRd-BFHd/3),2);
    I = I + Wt*pow(BFHd,3)/12+(Wt*BFHd)*pow((Y1b-BFRd-BFHd/2),2);
    I = I + (BFRw*pow(BFRd,3)/12)+(BFRd*BFRw)*pow((Y1b-BFRd/2),2);
    Inet = I - (3.1416*pow(Duct_dia,4)/32*Ncable + 3.1416*pow(Duct_dia,2)/4*Ncable*e1*e1);
    Itf = I + (Es/Ec-1)*As*e1*e1;
//-------------------------------------------------------------------------------------------------------
    S1tnet = Inet/Y1tnet;
                                //section 1 and 2
    S2tnet = Inet/Y2tnet;
    S1bnet = Inet/Y1bnet;
    S2bnet = Inet/Y2bnet;
    S1t = Itf/Y1t;
    S2t = Itf/Y2t;
    S1b = Itf/Y1b;
```

```
    S2b = Itf/Y2b;
//-------------------------------------------------------------------------
    S3bnet = Inet/Y3bnet;
    S3t = Itf/Y3t;
    S3b = Itf/Y3b;
//---------------------------------------------------------------------------------------
    S_int_sup_tnet = Inet/Y_int_sup_tnet; //section int_sup
    S_int_sup_bnet = Inet/Y_int_sup_bnet;
    S_int_sup_t = Itf/Y_int_sup_t;
    S_int_sup_b = ltf/Y_int_sup_b;
//-----------------------------------------------------------------------------------------
    S_inf_bnet = Inet/Y_inf_bnet;
    S_inf_t = ltf/Y_inf_t;
    S_inf_b = Itf/Y_inf_b;
//----------------------------------------------------------------------------------------
    S7tnet = Inet/Y7tnet; //section 7
    S7bnet = Inet/Y7bnet;
        S7t = Itf/Y7t;
        S7b = Itf/Y7b;
//---------------------------------------------------------------------------------------
    S8tnet = Inet/Y8tnet; //section 8
    S8bnet = Inet/Y8bnet;
        S8t = Itf/Y8t;
        S8b = Itf/Y8b;
//---------------------------------------------------------------------------------
}
void Comp_Sectional_Properties()
{
// cout<<"comp_sectional_properties"<<"\t";
// Composite Section Properties
    double EWW;
        EWW = minm(TFRw,12*(TFRd+TFFHd)+Wt+2*TFSHd);
        EFW = 12*ts+EWW;
        EFW = minm(L/4,EFW);
        EFW = minm(EFW,GS);
        Gdc = Gd + ts;
        Atfc = Atf + mratio*EFW*ts;
//---------------------------------------------------------------------------
        Y1bc = (Y1b*Atf+(mratio*EFW*ts)*(Gdc-ts/2))/Atfc;
        Y2bc = (Y2b*Atf+(mratio*EFW*ts)*(Gdc-ts/2))/Atfc;
```

```
    ec1= Y1bc-Y1; //section 1 and 2
    ec2 = Y2bc - Y2;
    Y1tc = Gdc-Y1bc;
    Y2tc = Gdc-Y2bc;
//---------------------------------------------------------------------------
    ec3= Y3bc-Y3; //section 3
    Y3tc = Gdc-Y3bc;
//-----------------------------------------------------------------------------------
    Y_int_sup_bc = (Y_int_sup_b*Atf+(mratio*EFW*ts)*(Gdc-ts/2))/Atfc;
    ec_int_sup= Y_int_sup_bc-Y_int_sup; //section int_sup
    Y_int_sup_tc = Gdc-Y_int_sup_bc;
//-------------------------------------------------------------------------
    Y_inf_bc = (Y_inf_b*Atf+(mratio*EFW*ts)*(Gdc-ts/2))/Atfc;
    ec_inf= Y_inf_bc-Y_inf; //section inf
    Y_inf_tc = Gdc-Y_inf_bc;
//---------------------------------------------------------------------------
    Y7bc = (Y7b*Atf+(mratio*EFW*ts)*(Gdc-ts/2))/Atfc;
    ec7= Y7bc-Y7; //section 7
    Y7tc = Gdc-Y7bc;
    //
    Y7bc = (Y7b*Atf+(mratio*EFW*ts)*(Gdc-ts/2))/Atfc;
    ec7= Y7bc-Y7; //section 8
    Y7tc = Gdc-Y7bc;
//------------------------------------------------------------------------------
    Ic = (mratio*EFW* pow(ts,3)/12)+((mratio*EFW*ts)*pow((Y1tc-ts/2),2));
    Ic = Ic +pow(TFRd,3)*TFRw/12+(TFRd*TFRw)*pow((Y1tc-ts-TFRd/2),2);
    Ic = Ic +((TFFHw* pow(TFFHd,3)/36)*2+((TFFHw*TFFHd)*pow((Y1tc-ts-TFRd-TFFHd/3),2)));
    Ic = Ic +((pow(TFFHd,3)*TFFHbw/12)+(TFFHbw*TFFHd)*pow((Y1tc-ts-TFRd-TFFHd/2),2));
    Ic = Ic +((TFSHw*pow(TFSHd,3)/36)*2+(TFSHw*TFSHd)*pow((Y1tc-ts-TFRd-TFFHd-TFSHd/3),2));
    Ic = Ic +((Wt*pow(TFSHd,3)/12)+(Wt*TFSHd)*pow((Y1tc-ts-TFRd-TFFHd-TFSHd/2),2));
    Ic = Ic +((Wt*pow(Wd,3)/12)+(Wt*Wd)*pow((Y1bc-BFRd-BFHd-Wd/2),2));
    Ic = Ic +((BFHw* pow(BFHd,3)/36)*2+(BFHw*BFHd)*pow((Y1bc-BFRd-BFHd/3),2));
    Ic = Ic +((Wt*pow(BFHd,3)/12+(Wt*BFHd)*pow((Y1bc-BFRd-BFHd/2),2)));
    Ic = Ic +((BFRw*pow(BFRd,3)/12)+(BFRd*BFRw)*pow((Y1bc-BFRd/2),2));
    Ic = Ic +(Es/Ec-1)*As*ec1*ec1;
//--------------------------------------------------------------------------------
    S1tc = Ic/(Y1tc - ts); //section 1 and 2
    S2tc = Ic/(Y2tc - ts);
    S1bc = Ic/Y1bc;
    S2bc = Ic/Y2bc;
//----------------------------------------------------------------------------------
```

```
        S3tc = Ic/(Y3tc - ts); //section 3
        S3bc = Ic/Y3bc;
//-------------------------------------------------------------------------------
    S_int_sup_bc = Ic/Y_int_sup_bc;
//----------------------------------------------------------------------------
    S_inf_bc = Ic/Y_inf_bc;
//---------------------------------------------------------------------------------------
        S7tc = Ic/(Y7tc - ts); //section 7
        S7bc = Ic/Y7bc;
//------------------------------------------------------------------------------------------
// S8tc = Ic/(Y8tc - ts); //section 8
// S8bc = Ic/Y8bc;
//------------------------------------------------------------------------------------
}
//***********************************************************************************
//__06__06__06__06_____Stiffness Equation Development Zone
                                    Stiffness Equation Development Zone___06__06
                                    06__06__06
//
void local_stiffness_matrix_function()
{
// cout<<"local_stiffness_matrix_function"<<"\t";
    local_stiffness_matrix[0][0]=12*Ec*Itf/(L*L*L);
    local_stiffness_matrix[0][1]=6*Ec*Itf/(L*L);
    local_stiffness_matrix[0][2]=-12*Ec*Itf/(L*L*L);
    local_stiffness_matrix[0][3]=6*Ec*Itf/(L*L);
    local_stiffness_matrix[1][0]=6*Ec*Itf/(L*L);
    local_stiffness_matrix[1][1]=4*Ec*Itf/(L);
    local_stiffness_matrix[1][2]=-6*Ec*Itf/(L*L);
    local_stiffness_matrix[1][3]=2*Ec*Itf/(L);
    local_stiffness_matrix[2][0]=-12*Ec*Itf/(L*L*L);
    local_stiffness_matrix[2][1]=-6*Ec*Itf/(L*L);
    local_stiffness_matrix[2][2]=12*Ec*Itf/(L*L*L);
    local_stiffness_matrix[2][3]=-6*Ec*Itf/(L*L);
    local_stiffness_matrix[3][0]=6*Ec*Itf/(L*L);
    local_stiffness_matrix[3][1]=2*Ec*Itf/L;
    local_stiffness_matrix[3][2]=-6*Ec*Itf/(L*L);
    local_stiffness_matrix[3][3]=4*Ec*Itf/L;
}
//*************************************************************************************
void global_stiffness_matrix_function()
{
// cout<<"global_stiffness_matrix_function"<<"\t";
    int increment=0;
    for(int i=0;i<No_of_Span;i++)
    {
                for(int j=0;j<4;j++)
                for(int k=0;k<4;k++)
```

```
                                    global_stiffness_matrix[j+increment][k+increment]=
                global_stiffness_matrix[j+increment][k+increment]+
        local_stiffness_matrix[j][k];
            }
        increment=increment+2;
    }
}
//***********************************************************************************
void global_member_force_matrix_function(double w_dyn)
{
// cout<<"global_member_force_matrix_function"<<"\t";
double local_member_force_matrix[4][1]={{w_dyn*L/2},{w_dyn*L*L/12},
            {w_dyn*L/2},{-w_dyn*L*L/12}};
        int increment=0;
        for(int i=0;i<No_of_Span;i++)
        {
        for(int j=0;j<4;j++)
        {
            global_member_force_matrix[j+increment][0]=
            global_member_force_matrix[j+increment][0]+
            local_member_force_matrix[j][0];
            }
            increment=increment+2;
        }
        for(int k=0;k<2*No_of_Node;k++)
        {
            global_member_force_matrix[k][0]=
            (-global_member_force_matrix[k][0]);
        }
}
//************************************************************************************
//__07__07__07___Boundary Condition Application Zone____0___07__07__07
//************************************************************************************
void impose_boundarycondition_function()
{
// cout<<"impose_boundary_condition_function"<<"\t";
    modified_global_stiffness_matrix_function();
    modified_global_member_force_matrix_function();
}
//*********************************************************************************
void modified_global_stiffness_matrix_function()
{
// cout<<"modified_global_stiffness_matrix_function"<<"\t";
    int increment=0;
    for(int i=0;i<No_of_Node;i++)
    {
        for(int j=0;j<2*No_of_Node;j++)
```

```
        {
            global_stiffness_matrix[0+increment][j]=0.0;
        }
        for(int k=0;k<2*No_of_Node;k++)
        {
        global_stiffness_matrix[k][0+increment]=0.0;
        }
        global_stiffness_matrix[increment][increment]=1.0;
        increment=increment+2;
    }
}
//*****************************************************************************************
void dynamic_allocation_01()
{
// cout<<"dynamic_allocation_01"<<"\t";
        int i,j,M,N;
        N = 2*No_of_Node;
        M = 2*No_of_Node;
        global_stiffness_matrix_pointer= new double* [N];
        for(i=0; i<N; i++)
        {
            global_stiffness_matrix_pointer[i] = new double[M];
            for(j=0; j<M; j++)
            {
            global_stiffness_matrix_pointer[i][j] = global_stiffness_matrix[i][j];
        }
    }
}
//*****************************************************************************************
void modified_global_member_force_matrix_function()
{
// cout<<"modified_global_member_force_matrix_function"<<"\t";
    int increment=0;
    for(int i=0;i<No_of_Node;i++)
    {
        global_member_force_matrix[increment][0]=0.0;
        increment=increment+2;
    }
}
//****************************************************************************************
void dynamic_allocation_02()
{
// cout<<"dynamic_allocation_02"<<"\t";
    int i,j,M,N;
N = 2*No_of_Node;
M = 1;
```

```
global_member_force_matrix_pointer= new double* [N];
for(i=0;i<N; i++)
{
    global_member_force_matrix_pointer[i] = new double[M];
    for(j=0; j<M; j++)
    {
        global_member_force_matrix_pointer[i][j] = global_member_force_matrix[i][j];
    }
}
}
//***************************************************************************************
//__08__08__08__08__08__Stiffness Equation Solution Zone____ 08__08__08_
//****************************************************************************************
//Modified stiffness equation (boundary condition apllied)/stiffness equation solution zone
void nrerror(char error_text[])
/* Numerical Recipes standard error handler */
{
    fprintf(stderr,"Numerical Recipes run-time error...\n");
    fprintf(stderr,"%s\n",error_text);
    fprintf(stderr,"...now exiting to system...\n");
    exit(1);
}
//************************************************************************************
int *ivector(long nl, long nh)
/* allocate an int vector with subscript range v[nl..nh] */
{
int *v;
v=(int *)malloc((size_t) ((nh-nl+1+NR_END)*sizeof(int)));
if (!v) nrerror("allocation failure in ivector()");
return v-nl+NR_END;
}
//***********************************************************************************
void free_ivector(int *v, long nl, long nh)
/* free an int vector allocated with ivector() */
{
free((FREE_ARG) (v+nl-NR_END));
}
//*************************************************************************************
void DOF_matrix_solution_function(double **a,int n,double **b,int m)
{
// cout<<"DOF_matrix_solution_function"<<"\t";
    int *indxc,*indxr,*ipiv;
    int i,icol,irow,j,k,l,Il;
    double big,dum,pivinv,temp;
    indxc=ivector(1,n); /* the integer arrays ipiv,indxr,and indxc are used for
                                    bookkeeping on the pivoting*/
    indxr=ivector(1,n);
ipiv=ivector(1,n);
```

```
for(j=0;j<n;j++) ipiv[j]=0;
for(i=0;i<n;i++) //this is the main loop over the column to be reduced
{
    big=0.0;
for(j=0;j<n;j++) //this is the outer loop of the search for a pivot element
if(ipiv[j]!=1)
for(k=0;k<n;k++)
{
            if(ipiv[k]==0)
            {
                                if(fabs(a[j][k])>=big)
                                {
                                    big=fabs(a[j][k]);
                                    irow=j;
                                    icol=k;
                                    }
            }
    }
    ++(ipiv[icol]);
    if(irow!=icol)
    {
                for(l=0;l<n;l++) SWAP(a[irow][I],a[icol][I])
                    for(l=0;l<n;l++) SWAP(a[irow][I],a[icol][I])
    }
/*we are now ready to divide the pivot row by the pivot element,located at irow and icol*/
    indxr[i]=irow;
    indxc[i]=icol;
    if(a[icol][icol]==0.0) nrerror("gaussj:Singular Matrix");
    pivinv=1.0/a[icol][icol];
    a[icol][icol]=1.0;
    for(l=0;l<n;l++) a[icol][l]*=pivinv;
    for(l=0;l<m;l++) b[icol][l]*=pivinv;
    for(II=0;|l<n;Il++)
        if(II!=icol)
    {
                dum=a[II][icol];
                a[III[icol]=0.0;
                for(l=0;l<n;l++) a[II][I]-=a[icol][I]*dum;
                for(l=0;l<m;l++) b[II][I]-=b[icol][I]*dum;
            }
}
for(l=n;l>=1;l--)
{
```

```
        if(indxr[l]!=indxc[l])
        for(k=0;k<n;k++)
            SWAP(a[k][indxr[l]],a[k][indxc[l]]);
    }
    free_ivector(ipiv,1,n);
    free_ivector(indxr,1,n);
    free_ivector(indxc,1,n);
    for(int p=0;p<2*No_of_Node;p++)
    {
        final_DOF_matrix[p][0]=global_member_force_matrix_pointer[p][0];
    }
}
//***********************************************************************************
//__09__09__09__09_____End Shear and Moment Calculation Zone_____________ 
//**********************************************************************************
/*For each member- end shear and moment calculation zone (using solved values of DOFs)
(matrix form)*/
void total_DL_end_shearforce_bendingmoment_function(double w_dyn)
{
// cout<<"total_DL_end_shearforce_bendingmoment_function"<<"\t";
int increment=0;
for(int i=0;i<No_of_Span;i++)
{
    total_DL_end_shearforce_bendingmoment_matrix[i][0]=
    +w_dyn*L/2.0
    +final_DOF_matrix[i+increment+1][0]*6.0*Ec*Itf/(L*L)
    +final_DOF_matrix[i+increment+3][0]*6.0*Ec*Itf/(L*L); //left end shear
    total_DL_end_shearforce_bendingmoment_matrix[i][1]=
    -w_dyn*L*L/12.0
    -final_DOF_matrix[i+increment+1][0]*4.0*Ec*Itf/L
    -final_DOF_matrix[i+increment+3][0]*2.0*Ec*Itf/L; //left end moment
    total_DL_end_shearforce_bendingmoment_matrix[i][2]=
    -w_dyn*L/2.0
    +final_DOF_matrix[i+increment+1][0]*6.0*Ec*Itf/(L*L)
    +final_DOF_matrix[i+increment+3][0]*6.0*Ec*Itf/(L*L); //right end shear
    total_DL_end_shearforce_bendingmoment_matrix[i][3]=
    -w_dyn*L*L/12.0
    +final_DOF_matrix[i+increment+1][0]*2.0*Ec*Itf/L
    +final_DOF_matrix[i+increment+3][0]*4.0*Ec*Itf/L; //right end moment
    increment=increment+1;
}
```

```
}
//************************************************************************************
void for_ILvalue_LL_end_shearforce_bendingmoment_function(int increment_a,int increment_c)
{
// cout<<"for_ILvalue_LL_end_shearforce_bendingmoment_function"<<"\t";
        int constant=0;
        int increment=0;
        for(int i=0;i<No_of_Span;i++)
        {
            for_ILvalue_LL_end_shearforce_bendingmoment_matrix[i][0]=
            +final_DOF_matrix[i+increment+1][0]*6.0*Ec*Itf/(L*L)
            +final_DOF_matrix[i+increment+3][0]*6.0*Ec*Itf/(L*L); //left end shear
            for_ILvalue_LL_end_shearforce_bendingmoment_matrix[i][1]=
            -final_DOF_matrix[i+increment+1][0]*4.0*Ec*Itf/L
            -final_DOF_matrix[i+increment+3][0]*2.0*Ec*Itf/L; //left end moment
    for_ILvalue_LL_end_shearforce_bendingmoment_matrix[i][2]=
            +final_DOF_matrix[i+increment+1][0]**.0*Ec*Itf/(L*L)
        +final_DOF_matrix[i+increment+3][0]*6.0*Ec*Itf/(L*L); //right end shear
            for_ILvalue_LL_end_shearforce_bendingmoment_matrix[i][3]=
            +final_DOF_matrix[i+increment+1][0]*2.0*Ec*Itf/L
            +final_DOF_matrix[i+increment+3][0]*4.0*Ec*Itf/L; //right end moment
            increment=increment+1;
    }
    constant=increment_c/2;
    for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][0]=
    +(increment_a*0.25*y)*(50.0*y-increment_a*0.25*y)
    *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y*50.0*y)
    -(increment_a*0.25*y)*(increment_a*0.25*y)
    *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y*50.0*y)
    +(50-increment_a*0.25)/50.0
    +final_DOF_matrix[increment_c+1][0]*6.0*Ec*Itf/(L*L)
    +final_DOF_matrix[increment_c+3][0]*6.0*Ec*Itf/(L*L); //left end shear
    for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][1]=
    -(increment_a*0.25*y)*(50.0*y-increment_a*0.25*y)
    *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y)
    -final_DOF_matrix[increment_c+1][0]*4.0*Ec*Itf/L
    -final_DOF_matrix[increment_c+3][0]*2.0*Ec*Itf/L; //left end moment
for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][2]=
    -(increment_a*0.25*y)*(increment_a*0.25*y)
    *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y*50.0*y)
    +(increment_a*0.25*y)*(50.0*y-increment_a*0.25*y)
```

```
            *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y*50.0*y)
            -(increment_a*0.25)/50.0
        +final_DOF_matrix[increment_c+1][0]*6.0*Ec*Itf/(L*L)
    +final_DOF_matrix[increment_c+3][0]*6.0*Ec*Itf/(L*L); //right end shear
    for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][3]=
    -(increment_a*0.25*y)*(increment_a*0.25*y)
    *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y)
    +final_DOF_matrix[increment_c+1][0]*2.0*Ec*Itf/L
    +final_DOF_matrix[increment_c+3][0]*4.0*Ec*Itf/L; //right end moment
}
/*above formula are written for following sign convention:
---> left end/section: upward shear positive; clockwise moment positive
---> right end/section: downward shear positive; anti-clockwise moment positive*/
//***********************************************************************************
//__10__10__10__10____Sectionwise DL Shear And Moment Calculation Zone____10__10__10__
//**********************************************************************************
// for each member sectionwise shear and moment calculation zone (matrix form)
/*for finding shear at every 0.25 metre interval*/
    void total_DL_sectionwise_shearforce_function(double w_dyn)
{
// cout<<"total_DL_sectionwise_shearforce_function"<<"\t";
        int increment=0;
        for(int i=0;i<No_of_Span;i++)
        {
        for(int j=0;j<201;j++)
        {
            total_DL_sectionwise_shearforce_matrix[0][j+increment]=
            +total_DL_end_shearforce_bendingmoment_matrix[i][0]
            -w_dyn*j*(0.25)*(y); //in fps
        }
        increment=increment+201;
    }
}
//************************************************************************************
/*for finding moment at every 0.25 metre*/
void total_DL_sectionwise_bendingmoment_function(double w_dyn)
{
// cout<<"total_DL_sectionwise_bendingmoment_function"<<"\t";
    int increment=0;
    for(int i=0;i<No_of_Span;i++)
    {
        for(int j=0;j<201;j++)
        {
            total_DL_sectionwise_bendingmoment_matrix[0][j+increment]=
            +total_DL_end_shearforce_bendingmoment_matrix[i][1]
```

```
            +total_DL_end_shearforce_bendingmoment_matrix[i][0]*j*(0.25)*(y)
                -w_dyn*j*(0.25)*(y)*j*(0.25)*(y)/2.0;
            }
            increment=increment+201;
    }
}
//****************************************************************************************
//__11__11__11__11_Influence Line Development Zone__11__11__11__11
//*
void influence_line_function()
{
// cout<<"influence_line_function"<<"\t";
int constant=0;
int increment_a=1;
int increment_b=1;
int increment_c=0;
for(int k=0;k<No_of_Span;k++)
{
    for(int l=0;|<199;|++) /*for influence line values at every 0.25 metre interval.*/
    {
            dynamic_allocation_01();
                    global_member_force_matrix_function_forIL(increment_a,increment_c);
                    modified_global_member_force_matrix_function();
                    dynamic_allocation_02();
            DOF_matrix_solution_function(global_stiffness_matrix_pointer,2*No_of_Node,
                                    global_member_force_matrix_pointer,x);
for_ILvalue_LL_end_shearforce_bendingmoment_function(increment_a,increment_c);
                        for_ILvalue_LL_sectionwise_shearforce_function(increment_a,increment_c);
for_ILvalue_LL_sectionwise_bendingmoment_function(increment_a,increment_c);
    for(int n=0;n<No_of_Span*201;n++)
        {
            IL_matrix_shear[n][0+increment_b]=
            for_ILvalue_LL_sectionwise_shearforce_matrix[0][n];
            IL_matrix_moment[n][0+increment_b]=
            for_ILvalue_LL_sectionwise_bendingmoment_matrix[0][n];
        }
        increment_a=increment_a+1;
        increment_b=increment_b+1;
    }
    increment_a=1;
    increment_b=increment_b+1;
    increment_c=increment_c+2;
}
```

```
}
//***********************************************************************************
void global_member_force_matrix_function_forlL(int increment_a,int increment_c)
{
// cout<<"global_member_force_matrix_function_forIL"<<"\t";
    for(int m=0;m<2*No_of_Node;m++)
    {
                global_member_force_matrix[m][0]=0.0;
    }
    global_member_force_matrix[0+increment_c][0]=
        +(increment_a*0.25*y)*(50.0*y-increment_a*0.25*y)
        *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y*50.0*y)
        -(increment_a*0.25*y)*(increment_a*0.25*y)
        *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y*50.0*y)
        +(50-increment_a*0.25)/50.0;
            global_member_force_matrix[1+increment_c][0]=
            (increment_a*0.25*y)*(50.0*y-increment_a*0.25*y)
            *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y);
    global_member_force_matrix[2+increment_c][0]=
        +(increment_a*0.25*y)*(increment_a*0.25*y)
        *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y*50.0*y)
        -(increment_a*0.25*y)*(50.0*y-increment_a*0.25*y)
        *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y*50.0*y)
        +(increment_a*0.25)/50.0;
    global_member_force_matrix[3+increment_c][0]=
    -(increment_a*0.25*y)*(increment_a*0.25*y)
    *(50.0*y-increment_a*0.25*y)/(50.0*y*50.0*y);
    for(int k=0;k<2*No_of_Node;k++)
    {
        global_member_force_matrix[k][0]=
        (-global_member_force_matrix[k][0]);
    }
}
//***********************************************************************************
void for_ILvalue_LL_sectionwise_shearforce_function(int increment_a,int increment_c)
{
// cout<<"for_ILvalue_LL_sectionwise_shearforce_function"<<"\t";
        int constant=0;
        int increment=0;
                                /*for finding shear at every 0.25 metre interval*/
    for(int i=0;i<No_of_Span;i++)
    {
            for(int j=0;j<201;j++)
```

```
        {
            for_ILvalue_LL_sectionwise_shearforce_matrix[0][j+increment]=
            +for_ILvalue_LL_end_shearforce_bendingmoment_matrix[i][0];
    }
    increment=increment+201;
    }
    constant=increment_c/2;
    for(int k=0;k<=increment_a;k++)
    {
        for_ILvalue_LL_sectionwise_shearforce_matrix[0][k+201*constant]=
        +for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][0];
    }
    for(int l=1;|<=200-increment_a;|++)
    {
        for_ILvalue_LL_sectionwise_shearforce_matrix[0][I+201*constant+increment_a]=
        +for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][0]-1;
    }
}
//***************************************************************************************
void for_ILvalue_LL_sectionwise_bendingmoment_function(int increment_a,int increment_c)
{
// cout<<"for_ILvalue_LL_sectionwise_bendingmoment_function"<<"\t";
int constant=0;
int increment=0; /*for finding moment at every 0.25 metre interval*/
for(int i=0;i<No_of_Span;i++)
{
        for(int j=0;j<201;j++)
        {
            for_ILvalue_LL_sectionwise_bendingmoment_matrix[0][j+increment]=
            +for_ILvalue_LL_end_shearforce_bendingmoment_matrix[i][1]
            +for_ILvalue_LL_end_shearforce_bendingmoment_matrix[i][0]*j*(0.25)*(y);
        }
        increment=increment+201;
    }
    constant=increment_c/2;
    for(int k=0;k<=increment_a;k++)
    {
        for_ILvalue_LL_sectionwise_bendingmoment_matrix[0][k+201*constant]=
        +for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][1]
        +for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][0]***(0.25)*(y);
        }
        for(int l=1;|<=200-increment_a;l++)
    {
        for_ILvalue_LL_sectionwise_bendingmoment_matrix[0][I+201*constant+increment_a]=
        +for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][1]
        +for_ILvalue_LL_end_shearforce_bendingmoment_matrix[constant][0]
        *(l+increment_a)*(0.25)*(y)
```

```
        -1*I*(0.25)*(y);
    }
}
//***********************************************************************************
//__12__12__12___GGirder DL & LL (Moment & Shear) Combination Zone______12__12__12_
//*********************************************************************************
void factored_DLplusLL_shearCombination_atSpecificSection_function
(int whichSection_intermsof_ILmatrixrow)
{
//cout<<"factored_DLplusLL_shearCombination_atSpecificSection_function"<<"\t";
    double factored_DLplusLL_shear_01=0;
    double factored_DLplusLL_shear_02=0;
    double factored_DLplusLL_shear_absolute=0;
        factored_DLplusLL_shear_01=1.3*total_DL_sectionwise_shearforce_matrix
                            [0][whichSection_intermsof_ILmatrixrow]+2.17
*total_LL_atSpecific_section_positive_shearforce_function
        (whichSection_intermsof_ILmatrixrow);
        factored_DLplusLL_shear_01=absolue_value_function(factored_DLplusLL_shear_01);
        factored_DLplusLL_shear_02=1.3*total_DL_sectionwise_shearforce_matrix
                            [0][whichSection_intermsof_ILmatrixrow]+2.17
*total_LL_atSpecific_section_negative_shearforce_function
                        (whichSection_intermsof_ILmatrixrow);
        factored_DLplusLL_shear_02=absolue_value_function(factored_DLplusLL_shear_02);
        factored_DLplusLL_shear_absolute=maxm(factored_DLplusLL_shear_01,
        factored_DLplusLL_shear_02);
}
//****************************************************************************************
void factored_DLplusLL_momentCombination_atSpecificSection_function
(int whichSection_intermsof_ILmatrixrow)
{
//cout<<"factored_DLplusLL_momentCombination_atSpecificSection_function"<<"\t";
    double factored_DLplusLL_moment_01=0;
    double factored_DLplusLL_moment_02=0;
    double factored_DLplusLL_moment_positive=0;
    double factored_DLplusLL_moment_negative=0;
    factored_DLplusLL_moment_01=1.3*total_DL_sectionwise_bendingmoment_matrix
    [0][whichSection_intermsof_ILmatrixrow]+2.17
```

```
*total_LL_atSpecific_section_positive_bendingmoment_function
                        (whichSection_intermsof_ILmatrixrow);
    factored_DLplusLL_moment_02=1.3*total_DL_sectionwise_bendingmoment_matrix
                                    [0][whichSection_intermsof_ILmatrixrow]+2.17
*total_LL_atSpecific_section_negative_bendingmoment_function
                        (whichSection_intermsof_ILmatrixrow);
    if(factored_DLplusLL_moment_01>0 && factored_DLplusLL_moment_02>0)
    {
        factored_DLplusLL_moment_positive=
        maxm(factored_DLplusLL_moment_01,factored_DLplusLL_moment_02);
    }
    else if(factored_DLplusLL_moment_01<0 && factored_DLplusLL_moment_02<0)
    {
        factored_DLplusLL_moment_negative=
        minm(factored_DLplusLL_moment_01,factored_DLplusLL_moment_02);
    }
    else if(factored_DLplusLL_moment_01>0 && factored_DLplusLL_moment_02<0)
    {
        factored_DLplusLL_moment_positive=factored_DLplusLL_moment_01;
        factored_DLplusLL_moment_negative=factored_DLplusLL_moment_02;
    }
    else
    {
        factored_DLplusLL_moment_positive=factored_DLplusLL_moment_02;
        factored_DLplusLL_moment_negative=factored_DLplusLL_moment_01;
    }
}
//************************************************************************************
double total_LL_atSpecific_section_positive_shearforce_function
(int whichSection_intermsof_ILmatrixrow)
{
// cout<<"total_LL_atSpecific_section_positive_shearforce_function"<<"\t";
//maxm positive 'wheel load' shear calculation
//----------------------------------------
//for vehicle going from left to right
    for(int i=0;i<=whichSection_intermsof_ILmatrixrow-1;i++)
    {
            matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][i]=
        IL_matrix_shear[whichSection_intermsof_ILmatrixrow][i];
    }
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01
```

```
[0][whichSection_intermsof_ILmatrixrow]=
IL_matrix_shear
[whichSection_intermsof_ILmatrixrow][whichSection_intermsof_ILmatrixrow]-1;
for(int i2=whichSection_intermsof_ILmatrixrow;i2<200*No_of_Span+1;i2++)
{
            matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][i2+1]=
    IL_matrix_shear[whichSection_intermsof_ILmatrixrow][i2];
}
for(int j=0;j<17;j++)
{
        if(wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][j]>0
            &&
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][j]>
        max_wheelLoad_shearPositive)
        {
        max_wheelLoad_shearPositive=
                wheel_load_frontAxle*
                matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][j];
    }
}
for(int k=17;k<34;k++)
{
    if(wheel_load_frontAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k]+
    wheel_load_rearAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k-17]>0
            &&
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k-17]>
        max_wheelLoad_shearPositive)
        {
            max_wheelLoad_shearPositive=
            wheel_load_frontAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k-17];
    }
}
for(int l=34;|<=200*No_of_Span+2;|++)
{
```

```
        if(wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-17]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-34]>0
            &&
    wheel_load_frontAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l]+
    wheel_load_rearAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-17]+
    wheel_load_rearAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-34]>
    max_wheelLoad_shearPositive)
    {
        max_wheelLoad_shearPositive=
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-17]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-34];
    }
}
```

//for vehicle going from right to left

```
for(int ii=0;ii<200*No_of_Span+2;ii++)
{
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][ii]=
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][200*No_of_Span+1-ii];
}
for(int jj=0;jj<17;jj++)
{
    if(wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][j]]>0
            &&
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][j]>
        max_wheelLoad_shearPositive)
    {
            max_wheelLoad_shearPositive=
            wheel_load_frontAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][j]];
    }
}
```

```
for(int kk=17;kk<34;kk++)
{
    if(wheel_load_frontAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk]+
    wheel_load_rearAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk-17]>0
                &&
    wheel_load_frontAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk]+
    wheel_load_rearAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk-17]>
    max_wheelLoad_shearPositive)
    {
        max_wheelLoad_shearPositive=
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk-17];
    }
}
for(int Il=34;|l<200*No_of_Span+2;|l++)
{
    if(wheel_load_frontAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II]+
    wheel_load_rearAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][Il-17]+
    wheel_load_rearAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II-34]>0
            &&
    wheel_load_frontAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II]+
    wheel_load_rearAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][Il-17]+
    wheel_load_rearAxle*
    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II-34]>
    max_wheelLoad_shearPositive)
    {
        max_wheelLoad_shearPositive=
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheeILoadorlaneLoad_shear_02[0][Il-17]+
        wheel_load_rearAxle*
        matrix forSorting_maXof wheeILoadorlaneLoad shear 02[0][Il-34];
    }
}
```


## //Impact Load Factor//loaded length

```
//-
```

    loadedLength_max_wheeILoad_shearPositive=(whichSection_intermsof_ILmatrixrow\%200)*y;
        if (loadedLength_max_wheelLoad_shearPositive \(<100^{*} y\) )
        \{
            loadedLength_max_wheelLoad_shearPositive=
            200 *y-loadedLength_max_wheelLoad_shearPositive;
        \}
        ImpactFactor_max_wheelLoad_shearPositive=
        50/(loadedLength_max_wheelLoad_shearPositive+125);
        if(ImpactFactor_max_wheelLoad_shearPositive>0.3)
        \{
        ImpactFactor_max_wheelLoad_shearPositive=0.3;
    \}
    max_wheelLoad_shearPositive=max_wheelLoad_shearPositive
            *(1+ImpactFactor_max_wheelLoad_shearPositive);
    
//maxm positive 'lane load' shear calculation
for(int m=0;m<200*No_of_Span+2;m++)
\{
if(matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][m]>=0
\&\&
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][m+1]>=0)
\{
max_laneLoad_shearPositive=
max_laneLoad_shearPositive+
lane_load_UDL_forShear*0.25* ${ }^{*}$ *
(matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][m]+
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][m+1])/2;//checksequence
loadedLength_max_laneLoad_shearPositive=
loadedLength_max_laneLoad_shearPositive+1;
\}
\}
//finding laneLoad_pick
for(int $\mathrm{n}=0 ; \mathrm{n}<200^{*}$ No_of_Span+2;n++)
\{
if(matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][n]>=0
\&\&
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][n+1]>=0)

```
        {
            laneLoad_pick=
            maxm(matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][n],
                        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][n+1]);
                    }
    }
    laneLoad_pick=laneLoad_pick*lane_load_Concentrated_forShear;
    max_laneLoad_shearPositive=
    max_laneLoad_shearPositive+laneLoad_pick;
//Impact Load Factor//loaded length
loadedLength_max_laneLoad_shearPositive=loadedLength_max_laneLoad_shearPositive*0.25*y;
    ImpactFactor_max_laneLoad_shearPositive=
    50/(loadedLength_max_laneLoad_shearPositive+125);
    if(ImpactFactor_max_laneLoad_shearPositive>0.3)
    {
        ImpactFactor_max_laneLoad_shearPositive=0.3;
    }
    max_laneLoad_shearPositive=max_laneLoad_shearPositive
            *(1+ImpactFactor_max_laneLoad_shearPositive);
    return maxm(max_wheelLoad_shearPositive,max_laneLoad_shearPositive);
}
//****************************************************************************************
double total_LL_atSpecific_section_negative_shearforce_function
(int whichSection_intermsof_ILmatrixrow)
{
// cout<<"total_LL_atSpecific_section_negative_shearforce_function"<<"\t";
//maxm negative 'wheel load' shear calculation
//--------------------------------------------
//for vehicle going from left to right
```

```
for(int i=0;i<=whichSection_intermsof_ILmatrixrow-1;i++)
```

for(int i=0;i<=whichSection_intermsof_ILmatrixrow-1;i++)
{
{
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][i]=
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][i]=
IL_matrix_shear[whichSection_intermsof_ILmatrixrow][i];
IL_matrix_shear[whichSection_intermsof_ILmatrixrow][i];
}
}
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01
[0][whichSection_intermsof_ILmatrixrow]=

```
```

IL_matrix_shear
[whichSection_intermsof_ILmatrixrow][whichSection_intermsof_ILmatrixrow]-1;
for(int i2=whichSection_intermsof_ILmatrixrow;i2<200*No_of_Span+1;i2++)
{
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][i2+1]=
IL_matrix_shear[whichSection_intermsof_ILmatrixrow][i2];
}
for(int j=0;j<17;j++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][j]<0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][j]<
max_wheelLoad_shearNegative)
{
max_wheelLoad_shearNegative=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][j];
}
}
for(int k=17;k<34;k++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k-17]<0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k-17]<
max_wheelLoad_shearNegative)
{
max_wheelLoad_shearNegative=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][k-17];
}
}
for(int l=34;l<=200*No_of_Span+2;I++)
{
if(wheel_load_frontAxle*

```
```

    matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][I]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-17]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-34]<0
            &&
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][I]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-17]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-34]<
        max_wheelLoad_shearNegative)
    {
        max_wheelLoad_shearNegative=
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][I]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][l-17]+
        wheel_load_rearAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][I-34];
    }
    }

```
//for vehicle going from right to left
```

for(int ii=0;ii<200*No_of_Span+2;ii++)
{
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][ii]=
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][200*No_of_Span+1-ii];
}
for(int jj=0;jj<17;jj++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][j]]<0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][j]<
max_wheelLoad_shearNegative)
{
max_wheelLoad_shearNegative=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][j]];
}
}
for(int kk=17;kk<34;kk++)

```
```

{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk-17]<0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk-17]<
max_wheelLoad_shearNegative)
{
max_wheelLoad_shearNegative=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][kk-17];
}
}
for(int Il=34;|l<200*No_of_Span+2;|l|+)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][Il-17]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II-34]<0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][ll-17]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II-34]<
max_wheelLoad_shearNegative)
{
max_wheelLoad_shearNegative=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][II]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheeILoadorlaneLoad_shear_02[0][Il-17]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_02[0][Il-34];
}
}

```
```

//----------------------------------
loadedLength_max_wheelLoad_shearNegative=(whichSection_intermsof_ILmatrixrow%200)*y;
if(loadedLength_max_wheelLoad_shearNegative<100*y)
{
loadedLength_max_wheelLoad_shearNegative=
200*y-loadedLength_max_wheelLoad_shearNegative;
}
ImpactFactor_max_wheelLoad_shearNegative=
50/(loadedLength_max_wheelLoad_shearNegative+125);
if(ImpactFactor_max_wheelLoad_shearNegative>0.3)
{
ImpactFactor_max_wheelLoad_shearNegative=0.3;
}
max_wheelLoad_shearNegative=max_wheelLoad_shearNegative
*(1+ImpactFactor_max_wheelLoad_shearNegative);
//***********************************************************************************
//maxm Negative 'lane load' shear calculation
for(int m=0;m<200*No_of_Span+2;m++)
{
if(matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][m]<=0
\&\&
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][m+1]<=0)
{
max_laneLoad_shearNegative=
max_laneLoad_shearNegative+
lane_load_UDL_forShear*0.25*y*
(matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][m]+
matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][m+1])/2;//checksequence
loadedLength_max_laneLoad_shearNegative=
loadedLength_max_laneLoad_shearNegative+1;
}
}
//finding laneLoad_pick
for(int n=0;n<200*No_of_Span+2;n++)
{
if(matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][n]<=0
\&\&

```
```

            matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][n+1]<=0)
        {
            laneLoad_pick=
            minm(matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][n],
                matrix_forSorting_maXof_wheelLoadorlaneLoad_shear_01[0][n+1]);
    }
    }
    laneLoad_pick=laneLoad_pick*lane_load_Concentrated_forShear;
    max_laneLoad_shearNegative=
    max_laneLoad_shearNegative+laneLoad_pick;
    //Impact Load Factor//loaded length
loadedLength_max_laneLoad_shearNegative=loadedLength_max_laneLoad_shearNegative*0.25*y;
ImpactFactor_max_laneLoad_shearNegative=
50/(loadedLength_max_laneLoad_shearNegative+125);
if(ImpactFactor_max_laneLoad_shearNegative>0.3)
{
ImpactFactor_max_laneLoad_shearNegative=0.3;
}
max_laneLoad_shearNegative=max_laneLoad_shearNegative
*(1+ImpactFactor_max_laneLoad_shearNegative);
return minm(max_wheelLoad_shearNegative,max_laneLoad_shearNegative);
}
//****************************************************************************************
double total_LL_atSpecific_section_positive_bendingmoment_function
(int whichSection_intermsof_ILmatrixrow)
{
// cout<<"total_LL_atSpecific_section_positive_bendingmoment_function"<<"\t";
//maxm positive 'wheel load' moment calculation
//-----------------------------------------------
//for vehicle going from left to right
for(int i=0;i<200*No_of_Span+1;i++)
{
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][i]=
IL_matrix_moment[whichSection_intermsof_ILmatrixrow][i];
}
for(int j=0;j<17;j++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][j]>0

```
```

                            &&
            wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][j]>
        max_wheelLoad_momentPositive)
        {
        max_wheelLoad_momentPositive=
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][j];
    }
    }
//cout<<max_wheelLoad_momentPositive<<endl;
for(int k=17;k<34;k++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k-17]>0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k-17]>
max_wheelLoad_momentPositive)
{
max_wheelLoad_momentPositive=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k-17];
}
}
//cout<<max_wheelLoad_momentPositive<<endl;
//cout<<max_wheelLoad_momentPositive<<endl;
//for vehicle going from right to left

```
```

for(int ii=0;ii<200*No_of_Span+1;ii++)

```
for(int ii=0;ii<200*No_of_Span+1;ii++)
{
{
    matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][ii]=
    matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][ii]=
    matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][200*No_of_Span-ii];
    matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][200*No_of_Span-ii];
}
}
for(int jj=0;jj<17;jj++)
for(int jj=0;jj<17;jj++)
{
{
    if(wheel_load_frontAxle*
    if(wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][j]]>0
```

        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][j]]>0
    ```
```

                &&
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][jj]>
        max_wheelLoad_momentPositive)
        {
        max_wheelLoad_momentPositive=
        wheel_load_frontAxle*
        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][jj];
    }
    }
for(int kk=17;kk<34;kk++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk-17]>0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk-17]>
max_wheelLoad_momentPositive)
{
max_wheelLoad_momentPositive=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk-17];
}
}
for(int |l=34;|<=200*No_of_Span+1;|++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II-17]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheeILoadorlaneLoad_moment_02[0][II-34]>0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II-17]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II-34]>
max_wheelLoad_momentPositive)

```
```

        {
            max_wheelLoad_momentPositive=
            wheel_load_frontAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II]+
            wheel load rearAxle*
            matrix_forSorting_maXof_wheeILoadorlaneLoad_moment_02[0][Il-17]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][Il-34];
            }
    }
    //cout<<max_wheelLoad_momentPositive<<endl;
    //Impact Load Factor//loaded length
//--------------------------------
loadedLength_max_wheelLoad_momentPositive=50*y;
ImpactFactor_max_wheelLoad_momentPositive=
50/(loadedLength_max_wheelLoad_momentPositive+125);
if(ImpactFactor_max_wheelLoad_momentPositive>0.3)
{
ImpactFactor_max_wheelLoad_momentPositive=0.3;
}
max_wheelLoad_momentPositive=max_wheelLoad_momentPositive
*(1+ImpactFactor_max_wheelLoad_momentPositive);
//************************************************************************************
//maxm positive 'lane load' moment calculation
for(int m=0;m<200*No_of_Span;m++)
{
if(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m]>=0
\&\&
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m+1]>=0)
{
max_laneLoad_momentPositive=
max_laneLoad_momentPositive+
lane_load_UDL_forMoment*0.25*y*
(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m]+
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m+1])/2;//checksequenc
loadedLength_max_laneLoad_momentPositive=
loadedLength_max_laneLoad_momentPositive+1;
}
}
//finding laneLoad_pick

```
```

    for(int n=0;n<200*No_of_Span;n++)
    {
        if(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n]>=0
            &&
        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n+1]>=0)
    {
            laneLoad_pick=
            maxm(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n],
        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n+1]);
    }
    }
//cout<<laneLoad_pick<<endl;
laneLoad_pick=laneLoad_pick*lane_load_Concentrated_forMoment;
max_laneLoad_momentPositive=
max_laneLoad_momentPositive+laneLoad_pick;
//Impact Load Factor//loaded length
loadedLength_max_laneLoad_momentPositive=
loadedLength_max_laneLoad_momentPositive*0.25*y;
// cout<<loadedLength_max_laneLoad_momentPositive<<endl;
ImpactFactor_max_laneLoad_momentPositive=
50/(loadedLength_max_laneLoad_momentPositive+125);
if(ImpactFactor_max_laneLoad_momentPositive>0.3)
{
ImpactFactor_max_laneLoad_momentPositive=0.3;
}
max_laneLoad_momentPositive=max_laneLoad_momentPositive
*(1+ImpactFactor_max_laneLoad_momentPositive);
return maxm(max_wheelLoad_momentPositive,max_laneLoad_momentPositive);
}
//*************************************************************
(int whichSection_intermsof_ILmatrixrow)
{
// cout<<"total_LL_atSpecific_section_negative_bendingmoment_function"<<"\t";
//maxm negative 'wheel load' moment calculation
//--------------------------------------------

```
```

//for vehicle going from left to right
for(int i=0;i<200*No_of_Span+1;i++)
{
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][i]=
IL_matrix_moment[whichSection_intermsof_ILmatrixrow][i];
}
for(int j=0;j<17;j++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][j]<0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][j]<
max_wheelLoad_momentNegative)
{
max_wheelLoad_momentNegative=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][j];
}
}
for(int k=17;k<34;k++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k-17]<0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k-17]<
max_wheelLoad_momentNegative)
{
max_wheelLoad_momentNegative=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k]+
wheel_load_rearAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][k-17];
}
}
for(int l=34;|<=200*No_of_Span+1;I++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][I]+
wheel_load_rearAxle*

```
```

            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][l-17]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][l-34]<0
                    &&
            wheel_load_frontAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][l]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][l-17]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][l-34]<
            max_wheelLoad_momentNegative)
            {
            max_wheelLoad_momentNegative=
            wheel_load_frontAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][l]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][l-17]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][l-34];
    }
    }

```
//for vehicle going from right to left
```

for(int ii=0;ii<200*No_of_Span+1;ii++)
{
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][ii]=
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][200*No_of_Span-ii];
}
for(int jj=0;jj<17;jj++)
{
if(wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][j]]<0
\&\&
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][j]<
max_wheelLoad_shearNegative)
{
max_wheelLoad_momentNegative=
wheel_load_frontAxle*
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][j]];
}
}
for(int kk=17;kk<34;kk++)
{
if(wheel_load_frontAxle*

```
```

            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk-17]<0
                &&
            wheel_load_frontAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk-17]<
            max_wheelLoad_momentNegative)
                {
            max_wheelLoad_shearNegative=
            wheel_load_frontAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][kk-17];
    }
    }
    for(int |=34;|<=200*No_of_Span+1;|++)
    {
            if(wheel_load_frontAxle*
                        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II-17]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II-34]<0
                &&
            wheel_load_frontAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II-17]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II-34]<
            max_wheelLoad_momentNegative)
            {
            max_wheelLoad_momentNegative=
            wheel_load_frontAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II-17]+
            wheel_load_rearAxle*
            matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_02[0][II-34];
            }
    }
    //Impact Load Factor//loaded length
//-----------------------------------
loadedLength_max_wheelLoad_momentNegative=50*y;

```
```

    ImpactFactor_max_wheelLoad_momentNegative=
    50/(loadedLength_max_wheelLoad_momentNegative+125);
    if(ImpactFactor_max_wheelLoad_momentNegative>0.3)
    {
ImpactFactor_max_wheelLoad_momentNegative=0.3;
}
max_wheelLoad_shearNegative=max_wheelLoad_momentNegative
*(1+ImpactFactor_max_wheelLoad_momentNegative);
//***********************************************************************************
//maxm Negative 'lane load' moment calculation

```
```

for(int m=0;m<201*No_of_Span;m++)

```
for(int m=0;m<201*No_of_Span;m++)
{
{
        if(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m]<=0
        if(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m]<=0
            &&
            &&
        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m+1]<=0)
        matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m+1]<=0)
    {
    {
            max_laneLoad_momentNegative=
            max_laneLoad_momentNegative=
            max_laneLoad_momentNegative+
            max_laneLoad_momentNegative+
            lane_load_UDL_forMoment*0.25*y*
            lane_load_UDL_forMoment*0.25*y*
            (matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m]+
            (matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m]+
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][m+1])/2;//checsequence
            loadedLength_max_laneLoad_momentNegative=
            loadedLength_max_laneLoad_momentNegative+1;
    }
}
//finding laneLoad_pick
```

```
for(int n=0;n<201*No_of_Span;n++)
```

for(int n=0;n<201*No_of_Span;n++)
{
{
if(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n]<=0
if(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n]<=0
\&\&
\&\&
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n+1]<=0)
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n+1]<=0)
{
{
laneLoad_pick=
laneLoad_pick=
minm(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n],
minm(matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n],
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n+1]);
matrix_forSorting_maXof_wheelLoadorlaneLoad_moment_01[0][n+1]);
}
}
}
}
laneLoad_pick=laneLoad_pick*lane_load_Concentrated_forMoment;
laneLoad_pick=laneLoad_pick*lane_load_Concentrated_forMoment;
max_laneLoad_momentNegative=

```
max_laneLoad_momentNegative=
```

```
    max_laneLoad_momentNegative+laneLoad_pick;
//Impact Load Factor//loaded length
    loadedLength_max_laneLoad_momentNegative=
    loadedLength_max_laneLoad_momentNegative*0.25*y;
    ImpactFactor_max_laneLoad_momentNegative=
    50/(loadedLength_max_laneLoad_momentNegative+125);
    if(ImpactFactor_max_laneLoad_momentNegative>0.3)
    {
        ImpactFactor_max_laneLoad_momentNegative=0.3;
    }
    max_laneLoad_momentNegative=max_laneLoad_momentNegative
            *(1+ImpactFactor_max_laneLoad_momentNegative);
    return minm(max_wheelLoad_momentNegative,max_laneLoad_momentNegative);
}
//***********************************************************************************
//13__13__13__13__13__Cable layout Function Zone____13__13__13__13__13_
//***********************************************************************************
void Cable_layout(double cable_1st_position_end)
{
// cout<<"cable_layout"<<"\t";
    double Layer_dist_bottom_end[31],N_total,Ncablet;
    int Cable_Layer_end[31],LayerNo_end,k1;
// Cable Layout
    Anchorage_system();
// No of cable per Layer determination at mid section
    Cable_Layer_mid[1] = (int)(BFRw-76-Duct_dia)/(Duct_clear_spacing+Duct_dia)+1;
    Layer_dist_bottom_mid[1] = 38 + Duct_dia/2;
    LayerNo_mid = 1;
    N_total = 0;
    Ncablet = RNDOFF(&Ncable);
    for(; ;)
    {
        N_total = N_total+Cable_Layer_mid[LayerNo_mid];
        if(N_total<Ncablet)
        {
            LayerNo_mid = LayerNo_mid+1;
                            Layer_dist_bottom_mid[LayerNo_mid] = Layer_dist_bottom_mid[LayerNo_mid-
1] + Duct_clear_spacing+Duct_dia;
    if(Layer_dist_bottom_mid[LayerNo_mid]<= BFRd)
```

Cable_Layer_mid[LayerNo_mid] = Cable_Layer_mid[LayerNo_mid-1]; else if(Layer_dist_bottom_mid[LayerNo_mid]<= (BFRd+BFHd))

Cable_Layer_mid[LayerNo_mid] = int((BFRw-
(Layer_dist_bottom_mid[LayerNo_mid]-BFRd)/BFHd*BFHw*2-76-
Duct_dia)/(Duct_clear_spacing+Duct_dia) + 1);
else
Cable_Layer_mid[LayerNo_mid] = 1;
\}
else
\{
Cable_Layer_mid[LayerNo_mid] = int(Ncablet-(N_total -
Cable_Layer_mid[LayerNo_mid]));
break;
\}
\}
// $\quad \mathrm{Cg}$ of cables at mid section from bottom, Y1 = 0;
for(int $i=1 ; i<=$ LayerNo_mid;i++)
\{
Y1 = Y1 + Cable_Layer_mid[i]*Layer_dist_bottom_mid[i]; if(i==LayerNo_mid) Y1 = Y1/Ncablet;
\}

//No of cable per Layer determination at end section
Cable_Layer_end[1] = (int)(BFRw-2*Ancg_Edge_dist_Lay1)/(Anchor_dim+30)+1;
Layer_dist_bottom_end[1] = cable_1st_position_end;

```
    LayerNo_end = 1;
```

    N_total = 0;
    for( ; ; )
    \{
        N_total = N_total+Cable_Layer_end[LayerNo_end];
        if(N_total<Ncablet)
        \{
            LayerNo_end = LayerNo_end+1;
            if(Cable_Layer_end[1]>1)
                            Layer_dist_bottom_end[LayerNo_end] =
    Layer_dist_bottom_end[LayerNo_end-1]+ Ancg_C2C_Lay1;
else
Layer_dist_bottom_end[LayerNo_end] =
Layer_dist_bottom_end[LayerNo_end-1]+ Ancg_C2C;
Cable_Loc_xw[LayerNo_end] =
Layer_dist_bottom_mid[1]+4*(Layer_dist_bottom_end[LayerNo_end]-
Layer_dist_bottom_mid[1])*pow((0.4L-xw),2)/pow(0.8L,2);
if(Cable_Loc_xw[LayerNo_end]<= (BFRd + BFHd))
\{
Cable_Layer_end[LayerNo_end] = Cable_Layer_end[LayerNo_end-1];
$\}$

```
        else
        {
            Cable_Layer_end[LayerNo_end] = 1;
            Layer_dist_bottom_end[LayerNo_end] =
Layer_dist_bottom_end[LayerNo_end-1]+ Ancg_C2C;
                }
    }
        else
    {
                            Cable_Layer_end[LayerNo_end] = int(Ncablet-(N_total -
Cable_Layer_end[LayerNo_end]));
                break;
            }
    }
// Cg of cables at end section from bottom,
    Y_end = 0;
    for(i=1;i<=LayerNo_end;i++)
    {
        Y_end = Y_end + Cable_Layer_end[i] * Layer_dist_bottom_end[i];
        if(i == LayerNo_end) Y_end = Y_end/Ncablet;
    }
//*****************************************************************************************
//No of cable per Layer determination at int_sup section
    double Layer_dist_bottom_int_sup[31];
    int Cable_Layer_int_sup[31],LayerNo_int_sup;
    Cable_Layer_int_sup[1] = 1;
    Layer_dist_bottom_int_sup[1] = Gd-2*(Duct_clear_spacing+Duct_dia)-(Ncablet-
1)*(Duct_clear_spacing+Duct_dia)-Duct_dia/2;
    LayerNo_int_sup = 1;
    N_total = 0;
    for(; ; )
    {
        N_total = N_total+Cable_Layer_int_sup[LayerNo_int_sup];
        if(N_total<Ncablet)
        {
            LayerNo_int_sup = LayerNo_int_sup+1;
            Cable_Layer_int_sup[LayerNo_int_sup]=1;
    Layer_dist_bottom_int_sup[LayerNo_int_sup]=Layer_dist_bottom_int_sup[LayerNo_int_sup-
1]+Duct_clear_spacing+Duct_dia;
    }
    else
    {
        break;
}
```

```
    }
// Cg of cables at int sup section from bottom,
    Y_int_sup = 0;
    for(i = 1;i<=LayerNo_int_sup;i++)
    {
        Y_int_sup = Y_int_sup + Cable_Layer_int_sup[i] * Layer_dist_bottom_int_sup[i];
        if(i == LayerNo_int_sup) Y_int_sup = Y_int_sup/Ncablet;
    }
//*
//No of cable per Layer determination at inf section
double Layer_dist_bottom_inf[31];
    int Cable_Layer_inf[31],LayerNo_inf;
    Cable_Layer_inf[1] = 1;
    Layer_dist_bottom_inf[1] = Gd-4*(Duct_clear_spacing+Duct_dia)-(Ncablet-
1)*(Duct_clear_spacing+Duct_dia)-Duct_dia/2;
    LayerNo_inf = 1;
    N_total = 0;
    for(; ; )
    {
        N_total = N_total+Cable_Layer_inf[LayerNo_inf];
        if(N_total<Ncablet)
        {
            LayerNo_inf = LayerNo_inf+1;
            Cable_Layer_inf[LayerNo_inf]=1;
            Layer_dist_bottom_inf[LayerNo_inf]=Layer_dist_bottom_inf[LayerNo_inf-
1]+Duct_clear_spacing+Duct_dia;
            }
            else
            {
            break;
            }
    }
// Cg of cables at int sup section from bottom,
    Y_inf = 0;
    for(i=1;i<=LayerNo_inf;i++)
    {
        Y_inf = Y_inf + Cable_Layer_inf[i] * Layer_dist_bottom_inf[i];
        if(i == LayerNo_inf) Y_inf = Y_inf/Ncablet;
    }
//**
// Location of individual cable
    k1 = 0;
    for(i = 1;i<=LayerNo_mid;i++)
    {
        for(int j = 1;j<=Cable_Layer_mid[i];j++)
```

```
    {
        k1 = k1 + 1;
        Cable_Loc_mid[k1] = Layer_dist_bottom_mid[i];
    }
}
k1 = 0;
for(i = 1;i<=LayerNo_end;i++)
{
        for(int j = 1;j<=Cable_Layer_end[i];j++)
        {
            k1 = k1 + 1;
            Cable_Loc_end[k1] = Layer_dist_bottom_end[i];
    }
}
k1 = 0;
for(i = 1;i<=LayerNo_int_sup;i++)
{
        for(int j = 1;j<=Cable_Layer_int_sup[i];j++)
        {
            k1 = k1 + 1;
            Cable_Loc_int_sup[k1] = Layer_dist_bottom_int_sup[i];
        }
}
k1 = 0;
for(i = 1;i<=LayerNo_inf;i++)
{
        for(int j = 1;j<=Cable_Layer_inf[i];j++)
        {
            k1 = k1 + 1;
            Cable_Loc_inf[k1] = Layer_dist_bottom_inf[i];
        }
}
k1 = 0;
for(i=1;i<=LayerNo_mid;i++)
{
    for(int j = 1;j<=Cable_Layer_mid[i];j++)
    {
        k1 = k1 + 1;
                            Cable_Loc_xw[k1] = Layer_dist_bottom_mid[i]+4*(Cable_Loc_end[k1]-
Cable_Loc_mid[k1])*pow((0.4L-xw),2)/pow(0.8L,2);
            alpha_xw[k1] = 4*(Cable_Loc_xw[k1]-Cable_Loc_mid[k1])/(0.8L);
        }
        }
```

```
    Y2 = 0;
    for( i = 1;i<=k1;i++)
    {
        Y2 = Y2 + Cable_Loc_xw[i];
        if(i==k1)
        {
            Y2 = Y2/Ncablet;
    }
    }
}
//*************************************************************************************
//14__14__14__14__14____ Flexural Strength Determination Zone
                                    14 14 14
//*****************************************************************************************
double Flexural_Strength(double As,double &Wri)
{
// cout<<"Flexural_Strength"<<"\t";
    double gamma,beff,deff,Pp,fsu,Mu,Asw,z;
// Flexural Strength
    gamma = 0.28;
    deff = Gdc-Y1;
    Pp = As/(EFW*deff);
    fsu = fpu*(1-gamma/0.85*Pp*fpu/fcdeck);
    Wri = Pp*fsu/fcdeck;
    z = As*fsu/(0.85*fcdeck*EFW);
    if(z <=ts)
        Mu = 0.95*As*fsu*(deff-z/2)/1000;
        else if(As*fsu>0.85*fcdeck*EFW*ts)
        {
            beff = (fcdeck/fc*EFW*ts + TFRw*TFRd)/(ts + TFRd);
    Pp = As/(beff*deff);
    fsu = fpu*(1-gamma/0.75*Pp*fpu/fc);
    Wri = Pp*fsu/fc;
    z = As*fsu/(0.85*fc*beff);
    if(z<=(TFRd+ts))
                Mu = 0.95*As*fsu*(deff-z/2)/1000;
    else
    {
    beff = (fcdeck/fc*EFW*ts + TFRw*TFRd+(TFFHtw+TFFHbw)/2*TFFHd)/(ts +
TFRd+TFFHd);
```

```
    Pp = As/(beff*deff);
```

    Pp = As/(beff*deff);
    fsu = fpu*(1-gamma/0.75*Pp*fpu/fc);
    fsu = fpu*(1-gamma/0.75*Pp*fpu/fc);
    Wri = Pp*fsu/fc;
    Wri = Pp*fsu/fc;
    z = (As*fsu)/(0.85*fc*beff);
    z = (As*fsu)/(0.85*fc*beff);
    if(z<=(ts + TFRd+TFFHd))
    if(z<=(ts + TFRd+TFFHd))
        Mu = 0.95*As*fsu*(deff-z/2)/1000;
    ```
        Mu = 0.95*As*fsu*(deff-z/2)/1000;
```

```
        else
        {
            Asw = As - 0.85*fc*(beff-Wt)*(ts + TFRd+TFFHd)/fsu;
            Pp = Asw/(Wt*deff);
            Wri = Pp*fsu/fc;
            Mu=0.95*(0.85*fc*(beff-Wt)*(ts + TFRd+TFFHd)*(deff-(ts +
TFRd+TFFHd)/2)
                                    +Asw*fsu*deff*(1-0.6*(Asw*fsu/Wt/deff/fc)))/1000;
            }
        }
    }
    return Mu;
}
double Flexural_Strength_precastgirder(double &Wri2)
{
// cout<<"Flexural_Strength_precastgirder"<<"\t";
    double gamma,beff,deff,Pp,fsu,Mu2,Asw,z;
// Flexural Strength
    gamma = 0.28;
    deff = Gd-Y1;
    Pp = pt1*As/(TFRw**deff);
    fsu = fpu*(1-gamma/0.75*Pp*fpu/fc);
    Wri2 = Pp*fsu/fc;
    z = pt1*As*fsu/(0.85*fc*TFRw);
    if(z <= TFRd)
    Mu2 = 0.95*pt1*As*fsu*(deff-z/2)/1000;
    else if(pt1*As*fsu>0.85*fc*TFRw*TFRd)
    {
    beff = (TFRw*TFRd+(TFFHtw+TFFHbw)/2*TFFHd)/(TFRd+TFFHd);
    Pp = pt1*As/(beff*deff);
    fsu=fpu*(1-gamma/0.75*Pp*fpu/fc);
    Wri2 = Pp*fsu/fc;
    z = pt1*As*fsu/(0.85*fc*beff);
    if(z<=(TFRd+TFFHd))
                Mu2 = 0.95*pt1*As*fsu*(deff-z/2)/1000;
        else
        {
                Asw = pt1*As - 0.85*fc*(beff-Wt)*(TFRd+TFFHd)/fsu;
                Pp = Asw/(Wt*deff);
                Wri2 = Pp*fsu/fc;
                Mu2 = 0.95*(0.85*fc*(beff-Wt)*(TFRd+TFFHd)*(deff-(TFRd+TFFHd)/2)
                            + Asw*fsu*deff*(1-0.6*(Asw*fsu/Wt/deff/fc)))/1000;
            }
        }
        return Mu2;
}
```

```
//************************************************************************************
//15__15__15__15__15__Deflection Calculation Zone____15__15__15__15__15_
//***********************************************************************************
void Deflection(double Ncablet,double MD1,double MG1,double MLL1)
{
// cout<<"Deflection"<<"\t";
    // Deflection
    deflectiont = 0.0;
    deflectione = 0.0;
    deflectionf = 0.0;
    for(int i = 1; i<= Ncablet;i++)
    {
            deflectiont = deflectiont+(13.0/136.0*F1i*pt1/Ncablet*(Cable_Loc_end[i] -
Cable_Loc_mid[i])-F1i*pt1/Ncablet*(Cable_Loc_end[i] - Gd/2)/8)*L*L/Eci/I*1000;
    }
    for(i = 1;i<= Ncablet;i++)
    {
            deflectione = deflectione +(13.0/136.0*F1i/Ncablet*(Cable_Loc_end[i] -
Cable_Loc_mid[i])-F1i/Ncablet*(Cable_Loc_end[i] - Gd/2)/8)*L*L/Ec/Itf*1000;
    }
    deflectionf = deflectione*2.2;
    deflectiont = 13.0/136.0*MG1*1000*L*L/Ec/Itf - deflectiont;
    deflectione = 13.0/136.0*(MG1*1.85+(MP1-MG1))*L*L/Ec/Itf*1000 - deflectione;
    deflectionf = 13.0/136.0*((2.4*MG1/Itf+3.0*((MP1-MG1)/Itf+MC1/Ic)+MLL1/Ic))*1000*L*L/Ec -
deflectionf;
// deflection = 13.0/136.0*MLL1*1000*L*L/Ec/lc;
    deflection = 324.0*pow(25.4,4)/(Ec*0.145*Ic)*24*DF*(1+IMF)*(pow(L/1000*3.28,3)-
555*L/1000*3.28+4780)*25.4/NoGirder;
// deflection=
22.5*pow(L/1000*3.28,3)*pow(25.4,4)/(Ec*0.145*Ic)*(1.6*9+0.32)*3*DF*(1+IMF)*25.4/NoGirder;
}
//***********************************************************************************
//16__16__16__16__16__Prestress Loss Calculation Zone____16__16__16__16__
//************************************************************************************
void Prestress_Loss(double MG1,double MP1,double MD1)
{
// cout<<"Prestress_Loss"<<"\t";
    double Lt,LWC,LAN,LES,Lo,LCR,LSR,LSH,Fof,Fmid;
    int Ncablet;
// Loss Calculation
    As = Astrand*Nstrand*Ncable;
// Jacking force,
```

```
Fend=0.9*0.9*fpu*As/1000; //!yield strength = 0.9 * ultimate KN
Ncablet = RNDOFF(&Ncable); // !jacking force = 0.9* yield strength
// Wobble and curvature loss,
        LWC = 0;
        for(int i = 1;i<=Ncablet;i++)
        {
        alpha[i] = (0.4L)*8*(Cable_Loc_end[i]-Cable_Loc_mid[i])/(L*L);
        LWC = LWC + Fend/Ncablet * (1-exp(-fricncoeff*alpha[i]-0.4*Kwc*L));
    }
```

    Fmid = Fend - LWC;
    // Anchorage Loss,
$x=\operatorname{sqrt}($ Delta*Es*L/2/((Fend-Fmid)*1000/As));
LAN $=2 *($ Fend-Fmid)*x/(L/2);
F3i = Fend - LAN/2;
Fend = Fend - LAN;
F2i $=$ F3i- $(x-x w)^{*}$ LAN/ $2 / x$;
// Elastic shortening loss,
F1i = Fmid;
for( ; ;)
\{
LES $=$ Kes*Es/Ec*(F1i/Atf+F1i*e1*e1/Itf-MG1*e1/Itf)*As;
Fof = Fmid - LES;
if (fabs((F1i-Fof)/F1i) <= 0.0001)
break;
else
F1i = Fof;
\}
for(int $\mathrm{i}=1$; $\mathrm{i}<=$ Ncablet; $\mathrm{i}++$ )
\{
alpha[i] $=(0.25 \mathrm{~L}){ }^{*} 8^{*}($ Cable_Loc_inf[i]-Cable_Loc_mid[i])/(L*L);
LWC = LWC + Fend/Ncablet * (1-exp(-fricncoeff*alpha[i]-0.25*Kwc*L));
\}
F7i = F1i - LWC;
for(int $\mathrm{i}=1 ; \mathrm{i}<=$ Ncablet; $\mathrm{i}++$ )
\{
alpha[i] $=(0.25 \mathrm{~L}) * 8^{*}($ Cable_Loc_inf[i]-Cable_Loc_mid[i])/(L*L);
LWC $=$ LWC + Fend/Ncablet * (1-exp(-fricncoeff*alpha[i]-0.25*Kwc*L));
\}
F6i = F7i - LWC;

```
    for(int i = 1;i<=Ncablet;i++)
    {
        alpha[i] = (0.05L)*8*(Cable_Loc_inf[i]-Cable_Loc_mid[i])/(L*L);
    LWC = LWC + Fend/Ncablet * (1-exp(-fricncoeff*alpha[i]-0.05*Kwc*L));
}
    F8i = F6i - LWC;
    for(int i=1;i<=Ncablet;i++)
{
    alpha[i] = (0.05L)*8*(Cable_Loc_inf[i]-Cable_Loc_mid[i])/(L*L);
    LWC = LWC + Fend/Ncablet * (1-exp(-fricncoeff*alpha[i]-0.05*Kwc*L));
}
F5i = F8i - LWC;
F2i = F2i - LES;
F3i = F3i - LES;
Fend = Fend - LES;
F5i = F5i - LES;
F6i = F6i - LES;
F7i = F7i - LES;
F8i = F8i - LES;
// Losses of prestress at transfer,
Lo = LWC+LES+LAN;
// Time dependent Loss
// Loss due to creep of concrete
    LCR = (12*(F1i/Atf+F1i*e1*e1/Itf-MG1*e1/Itf)-7*((MP1-MG1)*e1/Itf+MC1*ec1/Ic))*1000*145;
//!psi
    LSH = 0.8*(17000-150*77.916); //!psi RH = 77.916
    LSR = 5000-0.10*LES*1000/As*145-0.05*(LSH+ LCR);// !psi
    if(LSR<0) LSR = 2185; //!2.0%Loss of initial prestress considering
// Time dependent Loss of prestress,
// Lt=(LCR+LSH+LSR)/145.0*As/1000;
    Lt= (LCR+LSH+LSR)/145.0*As/1000*(1-pt1/20.0);
// Effective force,
F11= F1i-Lt;
F21 = F2i - Lt;
F31 = F3i - Lt;
Fend2 = Fend - Lt;
F51 = F5i - Lt;
F61 = F6i - Lt;
F71 = F7i - Lt;
```

```
        F81 = F8i - Lt;
}
/* void Moment3()
{
        MG3 = w_stage_03/2.0*(L*x-x*x)*1.0e-9;
        MCG3=(GS*Gd-BFRd*(GS-BFRw)-Ag)*Gammacon*CGt*(NCG/L)/2*(L*x-x*x)*1.0e-9;
        MS3 = (ts+12.5)*GS*Gammacon/2*(L*x-x*x)*1.0e-9;
        MP3=MG3+MCG3+MS3;
        MWC3 =WCt*Gammawc*GS/2*(L*x-x*x)*1.0e-9;
        MMS3=MSh*MSw*Gammacon/2*(L*x-x*x)/(NoGirder)*1.0e-9;
        MC3 = MWC3 + MMS3;
        MD3=MP3+MC3;
        MLL3 = maxm((4*P2*((L-x)/L + (L-x-4.27*1000)/L + (L-x-
8.54*1000)/L/4))*DF*x,(0.5*L*9.34/1000+80.064)*x*(L-x)/L*DF/2)*(1+IMF);
        MT3 = MLL3+MD3;
} */
void cablelayout3()
{
// cout<<"cablelayout3"<<"\t";
    int k1, Ncablet;
    k1 = 0;
    for(int i = 1;i<=LayerNo_mid;i++)
    {
        for(int j = 1;j<=Cable_Layer_mid[i];j++)
        {
            k1 = k1 + 1;
            Cable_Loc_3[k1] = Layer_dist_bottom_mid[i]+4*(Cable_Loc_end[k1]-
Cable_Loc_mid[k1])*pow((0.4L-x),2)/pow(0.8L,2);
            alpha3[k1] = 4*(Cable_Loc_3[k1]-Cable_Loc_mid[k1])/(0.8L);
        }
    }
    Ncablet = RNDOFF(&Ncable);
    Y3 = 0;
    for( i = 1;i<=k1;i++)
    {
        Y3 = Y3 + Cable_Loc_3[i];
        if(i==k1)
        {
            Y3 = Y3/Ncablet;
        }
    }
    Y3bnet = Yb - Ncable*3.1416/4*pow((Duct_dia),2)*Y3;
    Y3b = Yb + (Es/Ec-1) * As* Y3;
    Y3b = Y3b/Atf;
    Y3t = Gd-Y3b;
```

```
    Y3bnet = Y3bnet/Anet;
    Y3tnet = Gd-Y3bnet;
    Y3bc = (Y3b*Atf+(mratio*EFW*ts)*(Gdc-ts/2))/Atfc;
    Y3tc = Gdc-Y3bc;
    e3i = Y3bnet - Y3;
    e3 = Y3b - Y3;
    ec3 = Y3bc - Y3;
    S3tnet = Inet/Y3tnet;
    S3bnet = Inet/Y3bnet;
    S3t = Itf/Y3t;
    S3b = Itf/Y3b;
    S3tc = Ic/(Y3tc - ts);
    S3bc = Ic/Y3bc;
}
void cablelayout7()
{
// cout<<"cablelayout7"<<"\t";
    int k1, Ncablet;
    k1 = 0;
    for(int i = 1;i<=LayerNo_mid;i++)
    {
        for(int j = 1;j<=Cable_Layer_mid[i];j++)
        {
            k1 = k1 + 1;
            Cable_Loc_7[k1] = Layer_dist_bottom_mid[i]+4*(Cable_Loc_inf[k1]-
Cable_Loc_mid[k1])*pow((L/2-L/4),2)/pow(L,2);
            alpha7[k1] = 4*(Cable_Loc_7[k1]-Cable_Loc_mid[k1])/L;
        }
    }
    Ncablet = RNDOFF(&Ncable);
    Y7 = 0;
    for(i = 1;i<=k1;i++)
    {
        Y7 = Y7 + Cable_Loc_7[i];
        if(i==k1)
        {
            Y7 = Y7/Ncablet;
        }
    }
}
void cablelayout8()
{
// cout<<"cablelayout8"<<"\t";
    int k1, Ncablet;
    k1 = 0;
```

```
    for(int i = 1;i<=LayerNo_inf;i++)
    {
        for(int j = 1;j<=Cable_Layer_inf[i];j++)
        {
            k1 = k1 + 1;
            Cable_Loc_8[k1] = Layer_dist_bottom_inf[i]+4*(Cable_Loc_int_sup[k1]-
Cable_Loc_inf[k1])*pow((L/10-L/20),2)/pow(L/5,2);
            alpha8[k1] = 4*(Cable_Loc_8[k1]-Cable_Loc_inf[k1])/(0.2L);
    }
    }
    Ncablet = RNDOFF(&Ncable);
    Y8 = 0;
    for(i = 1;i<=k1;i++)
    {
        Y8 = Y8 + Cable_Loc_8[i];
        if(i==k1)
        {
            Y8 = Y8/Ncablet;
        }
    }
}
//***********************************************************************************
//17_17__17_17_17_ Moment Calculation Zone___ 17__17_17__17_
//***************************************************************************************
void Moment()
{
// cout<<"Moment"<<"\t";
// cout<<"Moment SWg"<<"\t";
// Girder selfweight moment @ different postitions :
    UDL_SW_girder= Ag*Gammacon/1000000; //N/mm
    matrix_initialization_function();
    local_stiffness_matrix_function();
    global_stiffness_matrix_function();
    global_member_force_matrix_function(UDL_SW_girder);
    impose_boundarycondition_function();
    dynamic_allocation_01();
    dynamic_allocation_02();
    DOF_matrix_solution_function(global_stiffness_matrix_pointer,2*No_of_Node,
                                    global_member_force_matrix_pointer,x);
    total_DL_end_shearforce_bendingmoment_function(UDL_SW_girder);
    total_DL_sectionwise_shearforce_function(UDL_SW_girder);
    total_DL_sectionwise_bendingmoment_function(UDL_SW_girder);
```

MG1 = total_DL_sectionwise_bendingmoment_matrix[0][80]; //@ section 1 MG2 = total_DL_sectionwise_bendingmoment_matrix[0][30]; //@ section 2
MG3 = total_DL_sectionwise_bendingmoment_matrix[0][50]; //@ section 3
MG4 = total_DL_sectionwise_bendingmoment_matrix[0][0]; //@ section 4
MG5 = total_DL_sectionwise_bendingmoment_matrix[0][200]; //@ section 5
MG6 = total_DL_sectionwise_bendingmoment_matrix[0][180]; //@ section 6
MG7 = total_DL_sectionwise_bendingmoment_matrix[0][130]; //@ section 7
MG8 = total_DL_sectionwise_bendingmoment_matrix[0][190]; //@ section 8
//--
// cout<<"Moment SWcg"<<"\t";
// Cross Girder Moment @ different postitions :

UDL_SW_CG= (GS*Gd-BFRd*(GS-BFRw)-Ag)*Gammacon*CGt*(NCG/L)/1000000;
matrix_initialization_function();
local_stiffness_matrix_function();
global_stiffness_matrix_function();
global_member_force_matrix_function(UDL_SW_CG);
modified_global_stiffness_matrix_function();
dynamic_allocation_01();
modified_global_member_force_matrix_function();
dynamic_allocation_02();
DOF_matrix_solution_function(global_stiffness_matrix_pointer,2*No_of_Node, global_member_force_matrix_pointer,x);
total_DL_end_shearforce_bendingmoment_function(UDL_SW_CG);
total_DL_sectionwise_shearforce_function(UDL_SW_CG);
total_DL_sectionwise_bendingmoment_function(UDL_SW_CG);

MCG1 = total_DL_sectionwise_bendingmoment_matrix[0][80]; //@ section 1
MCG2 = total_DL_sectionwise_bendingmoment_matrix[0][30]; //@ section 2
MCG3 = total_DL_sectionwise_bendingmoment_matrix[0][50]; //@ section 3
MCG4 = total_DL_sectionwise_bendingmoment_matrix[0][0]; //@ section 4
MCG5 = total_DL_sectionwise_bendingmoment_matrix[0][200]; //@ section 5
MCG6 = total_DL_sectionwise_bendingmoment_matrix[0][180]; //@ section 6
MCG7 = total_DL_sectionwise_bendingmoment_matrix[0][130]; //@ section 7
MCG8 = total_DL_sectionwise_bendingmoment_matrix[0][190]; //@ section 8

//cout<<"Moment SWs"<<"\t";
// Slab Moment @ different positions:

UDL_SW_slab=(ts+12.5)*GS*Gammacon/1000000;
matrix_initialization_function();
local_stiffness_matrix_function();
global_stiffness_matrix_function();
global_member_force_matrix_function(UDL_SW_slab);
modified_global_stiffness_matrix_function();

```
            dynamic_allocation_01();
            modified_global_member_force_matrix_function();
    dynamic_allocation_02();
    DOF_matrix_solution_function(global_stiffness_matrix_pointer,2*No_of_Node,
                                    global_member_force_matrix_pointer,x);
        total_DL_end_shearforce_bendingmoment_function(UDL_SW_slab);
        total_DL_sectionwise_shearforce_function(UDL_SW_slab);
        total_DL_sectionwise_bendingmoment_function(UDL_SW_slab);
        MS1 = total_DL_sectionwise_bendingmoment_matrix[0][80]; //@ section 1
        MS2 = total_DL_sectionwise_bendingmoment_matrix[0][30]; //@ section 2
        MS3 = total_DL_sectionwise_bendingmoment_matrix[0][50]; //@ section 3
        MS4 = total_DL_sectionwise_bendingmoment_matrix[0][0]; //@ section 4
        MS5 = total_DL_sectionwise_bendingmoment_matrix[0][200]; //@ section 5
        MS6 = total_DL_sectionwise_bendingmoment_matrix[0][180]; //@ section 6
        MS7 = total_DL_sectionwise_bendingmoment_matrix[0][130]; //@ section 7
        MS8 = total_DL_sectionwise_bendingmoment_matrix[0][190]; //@ section 8
//----------------------------------------------------------------------------
// Moment due to self weight, cross girder, deck slab @ different positions
MP1=MG1+MCG1+MS1; //@ section 1
MP2=MG2+MCG2+MS2; //@ section 2
MP3=MG1+MCG1+MS1; //@ section 3
MP4=MG2+MCG2+MS2; //@ section 4
MP5=MG1+MCG1+MS1; //@ section 5
MP6=MG2+MCG2+MS2; //@ section 6
MP7=MG1+MCG1+MS1; //@ section 7
MP8=MG2+MCG2+MS2; //@ section 8
//-
//cout<<"Moment SWwc"<<"\t";
// Wearing course moment @ different positions
    UDL_SW_WC=WCt*Gammawc*GS/1000000;
    matrix_initialization_function();
    local_stiffness_matrix_function();
    global_stiffness_matrix_function();
    global_member_force_matrix_function(UDL_SW_WC);
    modified_global_stiffness_matrix_function();
    dynamic_allocation_01();
    modified_global_member_force_matrix_function();
    dynamic_allocation_02();
    DOF_matrix_solution_function(global_stiffness_matrix_pointer,2*No_of_Node,
                                    global_member_force_matrix_pointer,x);
    total_DL_end_shearforce_bendingmoment_function(UDL_SW_WC);
    total_DL_sectionwise_shearforce_function(UDL_SW_WC);
    total_DL_sectionwise_bendingmoment_function(UDL_SW_WC);
```

MWC1 = total_DL_sectionwise_bendingmoment_matrix[0][80]; //@ section 1 MWC2 = total_DL_sectionwise_bendingmoment_matrix[0][30]; //@ section 2 MWC3 = total_DL_sectionwise_bendingmoment_matrix[0][50]; //@ section 3 MWC4 = total_DL_sectionwise_bendingmoment_matrix[0][0]; //@ section 4
MWC5 = total_DL_sectionwise_bendingmoment_matrix[0][200]; //@ section 5
MWC6 = total_DL_sectionwise_bendingmoment_matrix[0][180]; //@ section 6
MWC7 = total_DL_sectionwise_bendingmoment_matrix[0][130]; //@ section 7
MWC8 = total_DL_sectionwise_bendingmoment_matrix[0][190]; //@ section 8

UDL_SW_MS=MSh*MSw*Gammacon/(NoGirder)/1000000;
matrix_initialization_function();
local_stiffness_matrix_function();
global_stiffness_matrix_function();
global_member_force_matrix_function(UDL_SW_MS);
modified_global_stiffness_matrix_function();
dynamic_allocation_01();
modified_global_member_force_matrix_function();
dynamic_allocation_02();
DOF_matrix_solution_function(global_stiffness_matrix_pointer,2*No_of_Node, global_member_force_matrix_pointer,x);
total_DL_end_shearforce_bendingmoment_function(UDL_SW_MS);
total_DL_sectionwise_shearforce_function(UDL_SW_MS);
total_DL_sectionwise_bendingmoment_function(UDL_SW_MS);

MMS1 = total_DL_sectionwise_bendingmoment_matrix[0][80]; //@ section 1
MMS2 = total_DL_sectionwise_bendingmoment_matrix[0][30]; //@ section 2
MMS3 = total_DL_sectionwise_bendingmoment_matrix[0][50]; //@ section 3
MMS4 = total_DL_sectionwise_bendingmoment_matrix[0][0]; //@ section 4
MMS5 = total_DL_sectionwise_bendingmoment_matrix[0][200]; //@ section 5
MMS6 = total_DL_sectionwise_bendingmoment_matrix[0][180]; //@ section 6
MMS7 = total_DL_sectionwise_bendingmoment_matrix[0][130]; //@ section 7
MMS8 = total_DL_sectionwise_bendingmoment_matrix[0][190]; //@ section 8

// Composite dead load moment
MC1 = MWC1 + MMS1;
MC2 = MWC2 + MMS2;
MC3 = MWC3 + MMS3;
MC4 = MWC4 + MMS4;
MC5 = MWC5 + MMS5;
MC6 = MWC6 + MMS6;
MC7 = MWC7 + MMS7;
MC8 = MWC8 + MMS8;

```
// Total dead load Moment
    MD1=MP1+MC1;
        MD2=MP2+MC2;
        MD3=MP3+MC3;
        MD4=MP4+MC4;
        MD5=MP5+MC5;
        MD6=MP6+MC6;
        MD7=MP7+MC7;
        MD8=MP8+MC8;
//--------------------------------------------------------------------------------------
// Live load moment @ different positions
    influence_line_function();
            MLL1 = maxm(total_LL_atSpecific_section_positive_bendingmoment_function(80),
            total_LL_atSpecific_section_negative_bendingmoment_function(80)); //@ section 1
        MLL2 = maxm(total_LL_atSpecific_section_positive_bendingmoment_function(30),
            total_LL_atSpecific_section_negative_bendingmoment_function(30)); //@ section 2
            MLL3 = maxm(total_LL_atSpecific_section_positive_bendingmoment_function(50),
            total_LL_atSpecific_section_negative_bendingmoment_function(50)); //@ section 3
            MLL4 = maxm(total_LL_atSpecific_section_positive_bendingmoment_function(0),
            total_LL_atSpecific_section_negative_bendingmoment_function(0)); //@ section 4
            MLL5 = maxm(total_LL_atSpecific_section_positive_bendingmoment_function(200),
            total_LL_atSpecific_section_negative_bendingmoment_function(200)); //@ section 5
            MLL6 = maxm(total_LL_atSpecific_section_positive_bendingmoment_function(180),
            total_LL_atSpecific_section_negative_bendingmoment_function(180)); //@ section 6
            MLL7 = maxm(total_LL_atSpecific_section_positive_bendingmoment_function(130),
            total_LL_atSpecific_section_negative_bendingmoment_function(130)); //@ section 7
            MLL8 = maxm(total_LL_atSpecific_section_positive_bendingmoment_function(190),
            total_LL_atSpecific_section_negative_bendingmoment_function(190)); //@ section 8
//-------------------------------------------------------------------------------------
// Total Moment.
        MT1 = MLL1+MD1;
        MT2 = MLL2+MD2;
        MT3 = MLL3+MD3;
        MT4 = MLL4+MD4;
        MT5 = MLL5+MD5;
        MT6 = MLL6+MD6;
        MT7 = MLL7+MD7;
        MT8 = MLL8+MD8;
}
void momentslab()
{
// cout<<"momentslab"<<"\t";
    IMFS= minm(50/((GS-TFRw/2)/1000*3.28+125),0.3);
    MSS = (ts+12.5)*Gammacon*pow((GS-TFRw/2),2)/10*1.0e-6;
    MSWC =WCt*Gammawc*pow((GS-TFRw/2),2)/10*1.0e-6;
    MSDL = MSS + MSWC;
    if(NoGirder >= 2.98)
```

```
MSLL = ((GS-TFRw/2)/1000*3.28+2)/32.0*16.0*4451*0.8;
    else
        MSLL = ((GS-TFRw/2)/1000*3.28+2)/32.0*16.0*4451;
        Muslab = 1.3*(MSDL + 1.67*MSLL*(1+IMFS));
        dreq = sqrt(Muslab/(0.9*410*rho*(1-0.59*rho*fcdeck/410)));
        d_min = sqrt(Muslab/(0.9*410*0.0195*(1-0.59*0.0195*fcdeck/410)));
        ds = ts - 57;
// R = Muslab/(0.9*ds*ds); //ref pci chap 8.2.3
// Asnp = 0.85*fcdeck/410*(1-sqrt(1-(2/0.85/fcdeck)*R))*ds;
}
//************************************************************************************
//18__18__18__18__18__Shear Calculation Zone____18_18__18__18__
//**************************************************************************************
void Shear()
{
// cout<<"shear"<<"\t";
    double Vi,fd,Mcr,Mmax,Vci,Vp,Vcw,d_xw,fpe_xw,fpc,Ncablet,V2s;
    double V3i,fd3,Mcr3,Mmax3,V3ci,V3cw,d3,fpe3,fpc3,V3s,IMF3,VDL3,VLL3,V3c,V3u;
    IMF2 = minm( 50/((L-xw)/1000*3.28+125),0.3);
    VDL2 = 4*MD1/L - 8*MD1/(L*L)*xw;
    VLL2 = maxm((4*P2*((L-xw)/L + (L-xw-4.27*1000)/L + (L-xw-8.54*1000)/L/4))*DF,
        (0.5*9.34*(L-xw)/1000+115.65)*DF/2)*(1+IMF2);
// Evaluation of Vci
    Vi=1.3*(VDL2+1.67*VLL2)-VDL2;
    d_xw = Gdc - Y2;
    e2 = Y2b - Y2;
    ec2 = Y2bc-Y2;
    Vnh = 350*(TFRw*d_xw)/(25.4*25.4)/1000*4.45;
    if(d_xw< 0.8*Gdc)
        d_xw = 0.8*Gdc; // !As per AASHTO 2007
        fpe_xw = (F21/Atf+F21*e2/S2b)*1000;
        fd = (MP2/S2b+MC2/S2bc)*1000;
        Mcr =S2bc*(0.5* sqrt(fc) + fpe_xw - fd)/1000;
        Mmax = 1.3*(MD2+1.67*MLL2)-MD2;
        Vci = 0.05*sqrt(fc)*Wt*d_xw/1000 + VDL2 + Vi*Mcr/Mmax;
        if(Vci < 0.141*sqrt(fc)*Wt*d_xw/1000)
            Vci = 0.141*sqrt(fc)*Wt*d_xw/1000;
//***********************************************************************************
// Evaluation of Vcw
    Ncablet = RNDOFF(&Ncable);
    Vp=0;
    for(int i=1;i<=Ncablet;i++)
    {
        Vp=Vp + F21/Ncablet*sin(alpha_xw[i]);
    }
```

```
    fpc = F21/Atf - F21*e2*(Y2bc-Y2b)/Itf + MP2*(Y2bc-Y2b)/Itf;
    Vcw = (0.283*sqrt(fc)/1000 + 0.3*fpc)*Wt*d_xw + Vp;
    dshear = d_xw;
    Vc = minm(Vci,Vcw);
    Vu =1.3*(VDL2+1.67*VLL2); //!KN
    V2s = maxm((Vu/0.9 - Vc),0.1); //!phi = 0.9 for shear unit = KN
//***************************************************************************//
    IMF3 = minm( 50/((L-x)/1000*3.28+125),0.3);
    VDL3 = 4*MD3/L - 8*MD3/(L*L)*x;
    VLL3 = maxm((4*P2*((L-x)/L + (L-x-4.27*1000)/L + (L-x-8.54*1000)/L/4))*DF,
            (0.5*9.34*(L-x)/1000+115.65)*DF/2)*(1+IMF2);
// Evaluation of Vci
    V3i =1.3*(VDL3+1.67*VLL3)-VDL3;
    d3 = Gdc - Y3;
    e3 = Y3b - Y3;
    if(d3< 0.8*Gdc)
            d3 = 0.8*Gdc; // !As per AASHTO 2007
        fpe3 = (F31/Atf+F31*e3/S3b)*1000;
        fd3 = (MP3/S3b+MC3/S3bc)*1000;
        Mcr3 =S3bc*(0.5* sqrt(fc) + fpe3 - fd3)/1000;
        Mmax3 = 1.3*(MD3+1.67*MLL3)-MD3;
        V3ci = 0.05*sqrt(fc)*Wt*d3/1000 + VDL3 + V3i*Mcr3/Mmax3;
        if(V3ci < 0.141*sqrt(fc)*Wt*d3/1000)
            V3ci = 0.141*sqrt(fc)*Wt*d3/1000;
//***********************************************************************************
// Evaluation of Vcw
    Ncablet = RNDOFF(&Ncable);
    Vp = 0;
    for( i = 1;i<=Ncablet;i++)
    {
        Vp = Vp + F31/Ncablet*sin(alpha3[i]);
        }
        fpc3 = F31/Atf - F31*e3*(Y3bc-Y3b)/Itf + MP3*(Y3bc-Y3b)/Itf;
        V3cw = (0.283*sqrt(fc)/1000 + 0.3*fpc3)*Wt*d3 + Vp;
        V3c = minm(V3ci,V3cw);
        V3u =1.3*(VDL3+1.67*VLL3); //!KN
        V3s = maxm((V3u/0.9 - V3c),0.1); //!phi = 0.9 for shear unit = KN
        Vs = maxm(V2s,V3s);
        if(Vs == V2s)
            dshear = d_xw;
        else
            dshear = d3;
}
//**************************************************************************************
//19__19__19__19___19
EXPCON function Zone
                                    19 19 19 19
//****************************************************************************************
void __stdcall EXPCON(int *IFLG,int *ISKP,int *KKT,int *KOUNT,double XMAX[nv],double
XMIN[nv],double XT[nv])
```

```
{
// cout<<"EXPCON"<<"\t";
        double STRIP;
        int nx = 8,ny = 26,nt=69,nb=69,nd=101,na=10,ne=11,nf=36;
        Nstrand = XT[5];
        *KOUNT = *KOUNT+1;
        *KKT = *КKT+1;
        Anchorage_system();
        XMIN[0]=1499.99;
// XMIN[0]=2399.99;
// XMIN[0]=2999.99;
        XMIN[1]=300;
        XMIN[2]=300;
        XMIN[3]=Duct_dia + Duct_clear_spacing;
        XMIN[4]=1000.01;
        XMIN[5]=2.99;
        XMIN[6]=0.99;
    XMIN[7]=Ancg_Edge_dist_vertical;
        XMIN[8]=0.001;
        XMIN[9]=174.99;
        XMIN[10]=0.0015;
        XMIN[11]=75;
        XMIN[12]=50;
        XMIN[13]=Duct_dia + 80;
        XMAX[0]= 12000.01;
// XMAX[0]= 2400.01;
// XMAX[0]= 3000.01;
        XMAX[1]= 2000.01;
        XMAX[2]= 1150.01;
        XMAX[3]= 600.01;
        XMAX[4]= 3500.01;
        XMAX[5]= 19.001;
        XMAX[6]= 15.001;
        XMAX[7]= 1000.01;
        XMAX[8]= 0.999;
        XMAX[9]= 300.01;
        XMAX[10]=0.32*fcdeck/410.0;
        XMAX[11]= 300;
        XMAX[12]= 300;
        XMAX[13]= 300;
        if(*IFLG == 0)
        {
        STRIP=1e-4;
// DX[0] = 2400;
```

```
//
    DX[0] = 3000;
    DX[0]= 1500.0;
    DX[1]= 1715.0;
    DX[2]= 2000.0;
    DX[3]= 2400.0;
    DX[4]= 3000.0;
    DX[5]= 4000.0;
    DX[6]= 6000.0;
    DX[7]= 12000.0;
/* DX[0]= 2000.0;
    DX[1]= 2300.0;
    DX[2]= 2650.0;
    DX[3]= 3200.0;
    DX[4]= 4000.0;
    DX[5]= 5330.0;
    DX[6]= 8000.0;
    DX[7]= 16000.0;*/
    DISCR2(DX,ISKP,&nx,&STRIP,&XT[0],&XMAX[0],&XMIN[0]);
    DISCR2(DX1,ISKP,&nt,&STRIP,&XT[1],&XMAX[1],&XMIN[1]);
    DISCR2(DX2,ISKP,&nb,&STRIP,&XT[2],&XMAX[2],&XMIN[2]);
    DISCR2(DX4,ISKP,&nd,&STRIP,&XT[4],&XMAX[4],&XMIN[4]);
    DINTG2(ISKP,&STRIP,&XT[5],&XMAX[5],&XMIN[5]);
            DINTG2(ISKP,&STRIP,&XT[6],&XMAX[6],&XMIN[6]);
            DISCR2(DECKT,ISKP,&ny,&STRIP,&XT[9],&XMAX[9],&XMIN[9]);
            DISCR2(DX11,ISKP,&na,&STRIP,&XT[11],&XMAX[11],&XMIN[11]);
            DISCR2(DX12,ISKP,&ne,&STRIP,&XT[12],&XMAX[12],&XMIN[12]);
            DISCR2(DX13,ISKP,&nf,&STRIP,&XT[13],&XMAX[13],&XMIN[13]);
        }
}
//***********************************************************************************
//20__20__20__20__20__stdcall FUNC function Zone____ 20__20__20__20_
//************************************************************************************
void __stdcall FUNC(double *F,int *KOUNT,int *KUT,int *N,double XT[nv])
{
// cout<<"FUNC"<<"\t";
double
Av,smax,s,shearbar_length,shearbar_no,Ncablet,Wtst,Volcon,Wtnonprestd,Wtnonprestg;
GS = XT[0];
TFRw = XT[1];
\(B F R w=X T[2] ;\)
\(\mathrm{BFRd}=\mathrm{XT}[3]\);
Gd = XT[4];
Nstrand = XT[5];
Ncable \(=\) XT[6];
```

```
    cable_1st_position_end = XT[7];
    pt1 = XT[8];
    ts = XT[9];
    rho = XT[10];
    TFRd = XT[11];
    TFFHd = XT[12];
    Wt = XT[13];
//*****************************************************************************************
    TFSHbw = Wt;
    TFSHtw = TFSHbw + 2*TFSHw;
    TFFHbw = TFSHtw;
    TFFHtw = TFRw;
    TFFHw = (TFFHtw - TFFHbw)/2;
    BFHw = (BFRw-Wt)/2;
    BFHd = BFHw/2;
    NoGirder = BW/GS;
    SA =
(TFRd+sqrt(pow(TFFHw,2)+pow(TFFHd,2))+TFSHd*1.414+Wd+sqrt(pow(BFHw,2)+pow(BFHd,2))+BFRd)*
L;
    Ncablet = RNDOFF(&Ncable);
    Anchorage_system();
    Wd = Gd - (TFRd+TFFHd+TFSHd+BFHd+BFRd);
    xw =1.5*Gd;
    Cable_layout(cable_1st_position_end);
    Sectional_Properties();
    Comp_Sectional_Properties();
    Ag =
(TFRd*TFRw)+((TFFHtw+TFFHbw)/2*TFFHd)+((TFSHtw+TFSHbw)/2*TFSHd)+Wd*Wt+((Wt+BFRw)/2*BFH
d)+(BFRd*BFRw);
    Anet = Ag-Ncable*3.1416/4*(Duct_dia)*(Duct_dia);
    Volcon =Anet*(L-1.5*Gd)+(BFRw*Gd-Ncable*3.1416/4*(Duct_dia)*(Duct_dia))*Gd +
((Anet+BFRw*Gd)/2.0-Ncable*3.1416/4*(Duct_dia)*(Duct_dia))*Gd/2.0;
    As = Astrand*Nstrand*Ncable;
    Moment();
    Prestress_Loss(MG1,MP1,MD1);
// Moment3();
    cablelayout3();
    cablelayout7();
    cablelayout8();
    Shear();
    Av = maxm(Vs*1000.0/(410.0*dshear),50*Wt/410*0.00683);
    if(Vs <= 0.333 * sqrt(fc)*Wt*dshear/1000)
    smax = minm(0.75*(Gd + ts + 12.5),610);
    else
        smax = minm(0.75* (Gd + ts + 12.5)/2.0,305);
    s = minm(226.0/Av,smax);
```

```
/* AASHTO 8.20-12.7 mm @ 18" c/c temperature reinforcement at top = As = 0.265 mm2/mm*/
// shearbar_length = 113.0*(2.0*(Gd+ts)+2*(150+4.123*BFHd+(BFRd-40))+(BFRw-80)-60 +
2*(200+TFFHw+TFRd+ts-40+120-30));
    shearbar_length = 113.0* (2.0* (Gd+ts)+2* (150+4.123*BFHd+(BFRd-
40))+2*(200+TFFHw+TFRd+ts-40+120-30));
    if ( }s==610
            shearbar_no = 2*((L/2-Gd)/3/s+(L/2-Gd)/3/s+(L/2-Gd)/3/s);
    else if(s <=510)
            shearbar_no = 2*((L/2-Gd)/3/s+(L/2-Gd)/3/(s+50)+(L/2-Gd)/3/(s+100));
    else
            shearbar_no = 2*((L/2-Gd)/3/s+(L/2-Gd)/3/(s+50)+(L/2-Gd)/3/(s+50));
    ds = ts - 57;
    Asnp = rho*ds;
    Asnpd = minm(220/sqrt((GS-TFRw/2)/1000*3.28),67)/100*Asnp;
    Wtnonprestd = (2*Asnp*GS*L + Asnpd*L*GS+ 0.265*L*GS)*Gammast;
    Wtnonprestg = shearbar_length*shearbar_no*Gammast;
    Wtst = Astrand*Nstrand*Ncable*L*Gammast;
    Cpcon = Volcon*UPcon+UPgf*2*SA;
    Cdconc = GS*ts*L*UPcondeck+UPdf*(GS-TFRw)*L;
    Cpst = Wtst*UPst+Anchcost*2*Ncable+sheathcost*(Duct_dia/50.0)*Ncable*(L/1000);
    Cnpst = Wtnonprestd*UPnonprest + Wtnonprestg*UPnonprest;
// Cnpst = Wtnonprestd*UPnonprest;
    *KOUNT=*KOUNT+1;
    *KUT=*KUT+1;
    *F=(Cpcon+Cpst+Cdconc+Cnpst)*NoGirder;
}
//***********************************************************************************
//21__21__21__21__21__IMPCON function Zone
21 21 21 21
//**********************************************************************************
void __stdcall IMPCON(int *KOUNT,int *M,double XT[nv],double XX[icn],double XXMAX[icn],double
XXMIN[icn])
{
// cout<<"IMPCON"<<"\t";
double Mfactored,Ncablet;
*KOUNT = *KOUNT + 1;
*M = *M + 1;
GS = XT[0];
TFRw = XT[1];
BFRw = XT[2];
BFRd = XT[3];
Gd = XT[4];
Nstrand = XT[5];
Ncable = XT[6];
cable_1st_position_end = XT[7];
```

```
    pt1 = XT[8];
    ts = XT[9];
    rho = XT[10];
    TFRd = XT[11];
    TFFHd = XT[12];
    Wt = XT[13]
//****************************************************************************************
    TFSHbw = Wt;
    TFSHtw = TFSHbw + 2*TFSHw;
    TFFHbw = TFSHtw;
    TFFHtw = TFRw;
    TFFHw = (TFFHtw - TFFHbw)/2;
    BFHw = (BFRw-Wt)/2;
    BFHd = BFHw/2;
    Wd = Gd - (TFRd+TFFHd+TFSHd+BFHd+BFRd);
    xw =1.5*Gd;
    Ncablet = RNDOFF(&Ncable);
    As = Astrand*Nstrand*Ncable;
    Cable_layout(cable_1st_position_end);
    Sectional_Properties();
    Comp_Sectional_Properties();
    NoGirder = BW/GS;
//****************************************************************************************
    Moment();
// Factored Moment
    Mfactored = 1.3*(MD1+1.67*MLL1);
//***************************************************************************************
    Prestress_Loss(MG1,MP1,MD1);
// Moment3();
    cablelayout3();
    cablelayout7();
    cablelayout8();
//***********************************************************************************
// Flexural stress at transfer
// Initial stress at top,
    fti=(-F1i*pt1/Anet+F1i*pt1*e1i/S1tnet-MG1/S1tnet)*1000;
    fti_xw=(-F2i*pt1/Anet+F2i*pt1*e2i/S2tnet-MG2/S2tnet)*1000;
    f3ti=(-F3i*pt1/Anet+F3i*pt1*e3i/S3tnet-MG3/S3tnet)*1000;
// f4ti=(-Fend*pt1/Anet+Fend*pt1*e4i/S4tnet-MG4/S4tnet)*1000;
    f5ti=(-F5i*pt1/Anet+F5i*pt1*e_int_sup_i/S_int_sup_tnet-MG5/S_int_sup_tnet)*1000;
    f6ti=(-F6i*pt1/Anet+F6i*pt1*e_inf_i/S_inf_tnet-MG6/S_inf_tnet)*1000;
    f7ti=(-F7i*pt1/Anet+F7i*pt1*e7i/S7tnet-MG7/S7tnet)*1000;
// f8ti=(-F8i*pt1/Anet+F8i*pt1*e8i/S8tnet-MG8/S8tnet)*1000;
```

```
XX[0]= fti;
XXMAX[0]= 0.25* sqrt(fci);
XXMIN[0]= -0.55*fci;
XX[1]= fti_xw;
XXMAX[1]= 0.25* sqrt(fci);
XXMIN[1]= -0.55*fci;
XX[2]= f3ti;
XXMAX[2]= 0.25* sqrt(fci);
XXMIN[2]= -0.55*fci;
XX[3]= f5ti;
XXMAX[3]= 0.25* sqrt(fci);
XXMIN[3]= -0.55*fci;
XX[4]= f6ti;
XXMAX[4]= 0.25* sqrt(fci);
XXMIN[4]= -0.55*fci;
XX[5]= f7ti;
XXMAX[5]= 0.25* sqrt(fci);
XXMIN[5]= -0.55*fci;
// Initial stress at bottom,
fbi =-(F1i*pt1/Anet+F1i*pt1*e1i/S1bnet-MG1/S1bnet)*1000;
fbi_xw =-(F2i*pt1/Anet+F2i*pt1*e2i/S2bnet-MG2/S2bnet)*1000;
f3bi =-(F3i*pt1/Anet+F3i*pt1*e3i/S3bnet-MG3/S3bnet)*1000;
// f4ti=-(Fend*pt1/Anet+Fend*pt1*e4i/S4tnet-MG4/S4tnet)*1000;
f5bi=-(F5i*pt1/Anet+F5i*pt1*e_int_sup_i/S_int_sup_bnet-MG5/S_int_sup_bnet)*1000;
f6bi=-(F6i*pt1/Anet+F6i*pt1*e_inf_i/S_inf_bnet-MG6/S_inf_bnet)*1000;
f7bi=-(F7i*pt1/Anet+F7i*pt1*e7i/S7bnet-MG7/S7bnet)*1000;
f8bi=-(F8i*pt1/Anet+F8i*pt1*e8i/S8bnet-MG8/S8bnet)*1000;
XX[6]= fbi;
XXMAX[6]= 0.25* sqrt(fci);
XXMIN[6]= -0.55*fci;
XX[7]= fbi_xw;
XXMAX[7]= 0.25* sqrt(fci);
XXMIN[7]= -0.55*fci;
XX[8]= f3bi;
XXMAX[8]= 0.25* sqrt(fci);
XXMIN[8]= -0.55*fci;
XX[9]= f5bi;
```

```
XXMAX[9]= 0.25* sqrt(fci);
XXMIN[9]= -0.55*fci;
XX[10]= f6bi;
XXMAX[10]= 0.25* sqrt(fci);
XXMIN[10]= -0.55*fci;
XX[11]= f7bi;
XXMAX[11]= 0.25* sqrt(fci);
XXMIN[11]= -0.55*fci;
// Flexural stress at (Service II)Moment due to self weight, cross girder, deck slab,Wearing
course,Median strip
// Stress at top fiber of girder,
    ftc= (-F11/Atf+F11*e1/S1t-MP1/S1t-MC1/S1tc)*1000;
    ftc_xw= (-F21/Atf+F21*e2/S2t- MP2/S2t-MC2/S2tc)*1000;
    f3tc= (-F31/Atf+F31*e3/S3t- MP3/S3t-MC3/S3tc)*1000;
// f4tc= (-F41/Atf+F41*e4/S4t- MP4/S4t-MC4/S4tc)*1000;
    f5tc= (-F51/Atf+F51*e_int_sup/S_int_sup_t- MP5/S_int_sup_t-MC5/S_int_sup_tc)*1000;
    f6tc= (-F61/Atf+F61*e_inf/S_inf_t- MP6/S_inf_t-MC6/S_inf_tc)*1000;
    f7tc= (-F71/Atf+F71*e7/S7t- MP7/S7t-MC7/S7tc)*1000;
// f8tc= (-F81/Atf+F81*e8/S8t-MP8/S8t-MC8/S8tc)*1000;
XX[12]= ftc;
XXMAX[12]= 0.5* sqrt(fc);
XXMIN[12]= -0.40*fc;
XX[13]= ftc_xw;
XXMAX[13]= 0.5* sqrt(fc);
XXMIN[13]= -0.40*fc;
XX[14]= f3tc;
XXMAX[14]= 0.5* sqrt(fc);
XXMIN[14]= -0.40*fc;
XX[15]= f5tc;
XXMAX[15]= 0.5* sqrt(fc);
XXMIN[15]= -0.40*fc;
XX[16]= f6tc;
XXMAX[16]= 0.5* sqrt(fc);
XXMIN[16]= -0.40*fc;
XX[17]= f7tc;
XXMAX[17]= 0.5* sqrt(fc);
XXMIN[17]= -0.40*fc;
```

```
// Stress at bottom fiber,
    fbc = -(F11/Atf+F11*e1/S1b-MP1/S1b-MC1/S1bc)*1000;
    fbc_xw = -(F21/Atf+F21*e2/S2b-MP2/S2b-MC2/S2bc)*1000;
    f3bc = -(F31/Atf+F31*e3/S3b-MP3/S3b-MC3/S3bc)*1000;
// f4bc = -(F41/Atf+F41*e4/S4b-MP4/S4b-MC4/S4bc)*1000;
    f5bc = -(F51/Atf+F51*e_int_sup/S_int_sup_b-MP5/S_int_sup_b-MC5/S_int_sup_bc)*1000;
    f6bc = -(F61/Atf+F61*e_inf/S_inf_b-MP6/S_inf_b-MC6/S_inf_bc)*1000;
    f7bc = -(F71/Atf+F71*e7/S7b-MP7/S7b-MC7/S7bc)*1000;
// f8bc = -(F81/Atf+F81*e8/S8b-MP8/S8b-MC8/S8bc)*1000;
XX[18]= fbc;
XXMAX[18]= 0.5* sqrt(fc);
XXMIN[18]=-0.40*fc;
XX[19]= fbc_xw;
XXMAX[19]= 0.5* sqrt(fc);
XXMIN[19]=-0.40*fc;
XX[20]= f3bc;
XXMAX[20]= 0.5* sqrt(fc);
XXMIN[20]=-0.40*fc;
XX[21]= f5bc;
XXMAX[21]= 0.5* sqrt(fc);
XXMIN[21]=-0.40*fc;
XX[22]= f6bc;
XXMAX[22]= 0.5* sqrt(fc);
XXMIN[22]=-0.40*fc;
XX[23]= f7bc;
XXMAX[23]= 0.5* sqrt(fc);
XXMIN[23]=-0.40*fc;
// Flexural stress at (Service III)Moment due to all dead Load + Live load
// Stress at top fiber of girder,
ftt= (-F11/Atf+F11*e1/S1t-MP1/S1t-(MC1+MLL1)/S1tc)*1000;
ftt_xw= (-F21/Atf+F21*e2/S2t-MP2/S2t-(MC2+MLL2)/S2tc)*1000;
f3tt= (-F31/Atf+F31*e3/S3t-MP3/S3t-(MC3+MLL3)/S3tc)*1000;
// f4tt= (-F41/Atf+F41*e4/S4t-MP4/S4t-(MC4+MLL4)/S4tc)*1000;
f5tt= (-F51/Atf+F51*e_int_sup/S_int_sup_t-MP5/S_int_sup_t-(MC5+MLL5)/S_int_sup_tc)*1000;
f6tt= (-F61/Atf+F61*e_inf/S_inf_t-MP6/S_inf_t-(MC6+MLL6)/S_inf_tc)*1000;
f7tt= (-F71/Atf+F71*e7/S7t-MP7/S7t-(MC7+MLL7)/S7tc)*1000;
f8tt= (-F81/Atf+F81*e8/S8t-MP8/S8t-(MC8+MLL8)/S8tc)*1000;
XX[24]= ftt;
XXMAX[24]= 0.5* sqrt(fc);
XXMIN[24]= -0.6*fc;
```

```
        XX[25]= ftt_xw;
        XXMAX[25]= 0.5* sqrt(fc);
        XXMIN[25]= -0.6*fc;
        XX[26]= f3tt;
        XXMAX[26]= 0.5* sqrt(fc);
        XXMIN[26]= -0.6*fc;
        XX[27]= f5tt;
        XXMAX[27]= 0.5* sqrt(fc);
        XXMIN[27]= -0.6*fc;
        XX[28]= f6tt;
        XXMAX[28]= 0.5* sqrt(fc);
        XXMIN[28]= -0.6*fc;
        XX[29]= f7tt;
        XXMAX[29]= 0.5* sqrt(fc);
        XXMIN[29]= -0.6*fc;
// Stress at bottom fiber,
        fbt = -(F11/Atf+F11*e1/S1b-MP1/S1b-(MC1+MLL1)/S1bc)*1000;
        fbt_xw = -(F21/Atf+F21*e2/S2b-MP2/S2b-(MC2+MLL2)/S2bc)*1000;
        f3bt = -(F31/Atf+F31*e3/S3b-MP3/S3b-(MC3+MLL3)/S3bc)*1000;
// f4bt = -(F41/Atf+F41*e4/S4b-MP4/S4b-(MC4+MLL4)/S4bc)*1000;
    f5bt = -(F51/Atf+F51*e_int_sup/S_int_sup_b-MP5/S_int_sup_b-
(MC5+MLL5)/S_int_sup_bc)*1000;
            f6bt = -(F61/Atf+F61*e_inf/S_inf_b-MP6/S_inf_b-(MC6+MLL6)/S_inf_bc)*1000;
            f7bt = -(F71/Atf+F71*e7/S7b-MP7/S7b-(MC7+MLL7)/S7bc)*1000;
// f8bt = -(F81/Atf+F81*e8/S8b-MP8/S8b-(MC8+MLL8)/S8bc)*1000;
XX[30]= fbt;
XXMAX[30]= 0.5* sqrt(fc);
XXMIN[30]= -0.6*fc;
XX[31]= fbt_xw;
XXMAX[31]= 0.5* sqrt(fc);
XXMIN[31]= -0.6*fc;
XX[32]= f3bt;
XXMAX[32]= 0.5* sqrt(fc);
XXMIN[32]= -0.6*fc;
XX[33]= f5bt;
XXMAX[33]= 0.5* sqrt(fc);
XXMIN[33]= -0.6*fc;
```

```
XX[34]= f6bt;
XXMAX[34]= 0.5* sqrt(fc);
XXMIN[34]= -0.6*fc;
XX[35]= f7bt;
XXMAX[35]= 0.5* sqrt(fc);
XXMIN[35]= -0.6*fc;
// Flexural stress at (Service IIII)Moment due to 1/2( dead Load + PS) + Live load
// Stress at top fiber of girder,
fttt= (-(F11/2)/Atf+(F11/2)*e1/S1t-(MLL1+MD1/2)/S1tc)*1000;
fttt_xw= (-(F21/2)/Atf+(F21/2)*e2/S2t-(MLL2+MD2/2)/S2tc)*1000;
f3ttt= (-(F31/2)/Atf+(F31/2)*e3/S3t-(MLL3+MD3/2)/S3tc)*1000;
// f4ttt= (-(F41/2)/Atf+(F41/2)*e4/S4t-(MLL4+MD4/2)/S4tc)*1000;
f5ttt= (-(F51/2)/Atf+(F51/2)*e_int_sup/S_int_sup_t-(MLL5+MD5/2)/S_int_sup_tc)*1000;
f6ttt= (-(F61/2)/Atf+(F61/2)*e_inf/S_inf_t-(MLL6+MD6/2)/S_inf_tc)*1000;
f7ttt= (-(F71/2)/Atf+(F71/2)*e7/S7t-(MLL7+MD7/2)/S7tc)*1000;
// f8ttt= (-(F81/2)/Atf+(F81/2)*e8/S8t-(MLL8+MD8/2)/S8tc)*1000;
XX[36]= fttt;
XXMAX[36]= 0.5*sqrt(fc);
XXMIN[36]= -0.40*fc;
XX[37]= fttt_xw;
XXMAX[37]= 0.5* sqrt(fc);
XXMIN[37]= -0.40*fc;
XX[38]= f3ttt;
XXMAX[38]= 0.5* sqrt(fc);
XXMIN[38]= -0.40*fc;
XX[39]= f5ttt;
XXMAX[39]= 0.5* sqrt(fc);
XXMIN[39]= -0.40*fc;
XX[40]= f6ttt;
XXMAX[40]= 0.5* sqrt(fc);
XXMIN[40]= -0.40*fc;
XX[41]= f7ttt;
XXMAX[41]= 0.5* sqrt(fc);
XXMIN[41]= -0.40*fc;
// Stress at bottom fiber,
fbtt = -((F11/2)/Atf+(F11/2)*e1/S1b-(MLL1+MD1/2)/S1bc)*1000;
fbtt_xw = -((F21/2)/Atf+(F21/2)*e2/S2b-(MLL2+MD2/2)/S2bc)*1000;
f3btt = -((F31/2)/Atf+(F31/2)*e3/S3b-(MLL3+MD3/2)/S3bc)*1000;
```

```
// f4btt = -((F41/2)/Atf+(F41/2)*e4/S4b-(MLL4+MD4/2)/S4bc)*1000;
    f5btt = -((F51/2)/Atf+(F51/2)*e_int_sup/S_int_sup_b-(MLL5+MD5/2)/S_int_sup_bc)*1000;
    f6btt = -((F61/2)/Atf+(F61/2)*e_inf/S_inf_b-(MLL6+MD6/2)/S_inf_bc)*1000;
    f7btt = -((F71/2)/Atf+(F71/2)*e7/S7b-(MLL7+MD7/2)/S7bc)*1000;
// f8btt = -((F81/2)/Atf+(F81/2)*e8/S8b-(MLL8+MD8/2)/S8bc)*1000;
XX[42]= fbtt;
XXMAX[42]= 1* sqrt(fc);
XXMIN[42]= -0.4*fc;
XX[43]= fbtt_xw;
XXMAX[43]= 1* sqrt(fc);
XXMIN[43]= -0.4*fc;
XX[44]= f3btt;
XXMAX[44]= 1* sqrt(fc);
XXMIN[44]= -0.4*fc;
XX[45]= f5btt;
XXMAX[45]= 1* sqrt(fc);
XXMIN[45]= -0.4*fc;
XX[46]= f6btt;
XXMAX[46]= 1* sqrt(fc);
XXMIN[46]= -0.4*fc;
XX[47]= f7btt;
XXMAX[47]= 1* sqrt(fc);
XXMIN[47]= -0.4*fc;
Mu = Flexural_Strength(As,Wri);
XX[48]= Mfactored;
XXMAX[48]= Mu;
XXMIN[48]= 0.0;
Shear();
XX[49]= Vs;
XXMAX[49]= 0.666 * sqrt(fc)*Wt*dshear/1000;
XXMIN[49]= 0.0;
// Ductility Limit
fpe = (F11/Atf+F11*e1/S1b)*1000;
Mcr2 = S1bc*(0.625*sqrt(fc)+fpe)/1000-MP1*(S1bc/S1b-1);
XX[50]= Mcr2;
XXMAX[50]= Mu/1.2;
XXMIN[50]= 0.0;
```

```
XX[51]= Wri;
XXMAX[51]= 0.36*0.75;
XXMIN[51]= 0.0;
XX[52]= Y_end;
XXMAX[52]= Gd/2+Gd/6+0.5* sqrt(fc)* (BFRw*Gd)*Gd/6/(Fend2*1000);
XXMIN[52]= Gd/2-Gd/6-0.25* sqrt(fci)* (BFRw*Gd)*Gd/6/(Fend*1000);
Deflection(Ncablet,MD1,MG1,MLL1);
XX[53]= fabs(deflectiont);
XXMAX[53]= L/360;
XXMIN[53]= 0.0;
XX[54]= fabs(deflectione);
XXMAX[54]= L/360;
XXMIN[54]= 0.0;
XX[55]= fabs(deflectionf);
XXMAX[55]= L/100;
XXMIN[55]= 0.0;
XX[56]= fabs(deflection);
XXMAX[56]= L/800;
XXMIN[56]= 0.0;
XX[57]= Fend;
XXMAX[57]= 0.70*fpu*As/1000;
XXMIN[57]= 0.0;
XX[58]= F3i;
XXMAX[58]= 0.747*fpu*As/1000;
XXMIN[58]= 0.0;
XX[59]= F31;
XXMAX[59]= 0.72*fpu*As/1000;
XXMIN[59]= 0.0;
XX[60]= Vnh;
XXMAX[60]= 100000;
XXMIN[60]= Vu/0.9;
XX[61]= ts;
XXMAX[61]= 300.0;
XXMIN[61]= ((GS-TFRw/2)/1000*3.28+17)/3*25.4;
momentslab();
XX[62]= dreq;
```

```
    XXMAX[62]= ds;
    XXMIN[62]= d_min;
    double Wri2;
    double Mupregirder = Flexural_Strength_precastgirder(Wri2);
    XX[63]= 1.3*MG1;
    XXMAX[63]= Mupregirder;
    XXMIN[63]= 0.0;
    double Mcrslab = 0.625*sqrt(fcdeck)*ts*ts/6;
    XX[64]= Mcrslab;
    XXMAX[64]= Muslab/1.2;
    XXMIN[64]= 0.0;
    XX[65]= Wri2;
    XXMAX[65]= 0.36*0.75;
    XXMIN[65]= 0.0;
    double a1 = 0.1*L;
    double L1 = L-2*a1;
    double ei = (1.025*(pow(L1/L,2)-0.333)+0.25)*25.4;
    double yr = Y1t-deflectiont;
    Ig = pow(TFRw,3)*TFRd/12;
    Ig = Ig + TFFHd*pow(TFFHw,3)/36*2+(TFFHw*TFFHd)*pow((TFFHbw/2+TFFHw/3),2);
    Ig = Ig + (pow(TFFHbw,3)*TFFHd/12);
    Ig = Ig + (TFSHd*pow(TFSHw,3)/36)*2+(TFSHw*TFSHd)*pow((Wt/2+TFSHw/3),2);
    lg = Ig + (TFSHd*pow(Wt,3)/12);
    lg = Ig + (Wd*pow(Wt,3)/12);
    lg = Ig + (BFHd*pow(BFHw,3)/36)*2+(BFHw*BFHd)*pow((Wt/2+BFHw/3),2);
    Ig = Ig + BFHd*pow(Wt,3)/12;
    Ig = Ig + (BFRd*pow(BFRw,3)/12);
    double zo = UDL_SW_slab/(12*Eci*Ig*L)*(0.1*pow(L1,5)-
pow(L1,3)*a1*a1+3*pow(a1,4)*L1+1.2*pow(a1,5))*1e-6;
// double oi = maxm(ei/yr,0.0001);
    double oi = ei/yr;
    //double res1=(0.625*sqrt(fci)+(-fti));
    //res1=fabs(res1);
    double Mlat = fabs((0.625*sqrt(fci)+(-fti)))*Ig/(TFRw/2);
    double Mg1 = (UDL_SW_slab*L1*L1/8.0)*1e-6;
    double omax = maxm(Mlat/Mg1,0.00001);
// double ab = maxm((zo/yr),0.0001);
    double ab = zo/yr;
// double cd = maxm((oi/omax),0.0001);
```

```
            double cd = oi/omax;
            double Fsc = 1/(ab+cd);
            XX[66]= Fsc;
            XXMAX[66]= 100.0;
            XXMIN[66]= 1.5;
            double ft,ft_xw,f3t,fb,fb_xw,f3b;
            ft= (-F1i/Atf+F1i*e1/S1t-MP1/S1t)*1000;
            ft_xw= (-F2i/Atf+F2i*e2/S2t-MP2/S2t)*1000;
                f3t = (-F3i/Atf+F3i*e3/S3t-MP3/S3t)*1000;
                    XX[67]= ft;
                    XXMAX[67]= 0.5* sqrt(fc);
XXMIN[67]= -0.60*fc;
XX[68]= ft_xw;
XXMAX[68]= 0.5* sqrt(fc);
XXMIN[68]= -0.60*fc;
XX[69]= f3t;
XXMAX[69]= 0.5* sqrt(fc);
XXMIN[69]= -0.60*fc;
// Stress at bottom fiber,
fb = -(F1i/Atf+F1i*e1/S1b-MP1/S1b)*1000;
fb_xw = -(F2i/Atf+F2i*e2/S2b-MP2/S2b)*1000;
f3b = -(F3i/Atf+F3i*e3/S3b-MP3/S3b)*1000;
XX[70]= fb;
XXMAX[70]= 0.5* sqrt(fc);
XXMIN[70]= -0.60*fc;
XX[71]= fb_xw;
XXMAX[71]= 0.5* sqrt(fc);
XXMIN[71]= -0.60*fc;
XX[72]= fb;
XXMAX[72]= 0.5* sqrt(fc);
XXMIN[72]= -0.60*fc;
}
//****************************************************************************************
//22__22__22__22_22_MMIN function Zone___ 22__22__22__22
//***********************************************************************************
void main()
{
// cout<<"main"<<"\t";
```

```
    time_t start, stop;
    time(&start);
    double
C[nv],FF[nv+1],H[nv*(nv+1)],OLDCC[nv],XDN[nv],XG[nv],XMAX[nv],XMIN[nv],XUP[nv],XX[icn],XXMAX[icn
],XXMIN[icn],XT[nv];
    double ALPHA,BETA,DEL,GAMA,PHI,PHICPX;
    int ICON,IJK,IMV,IPRINT,K,KNT,LIMIT,N,NRSTRT,NIC;
    // initial Value:
    GS = 1600;
    TFRw = 400;
    BFRw = 400;
    BFRd = 250;
    Gd = 1500; //3lane 50mpa
    Nstrand = 5.0;
    Ncable = 4.0;
    cable_1st_position_end = 350.0;
    pt1 = 0.44;
    ts = 200.00;
    rho = 0.001773;
    TFRd = 160;
    TFFHd = 55;
    Wt =250;
    XT[0]= GS;
    XT[1]= TFRw;
    XT[2]= BFRw;
    XT[3]= BFRd;
    XT[4]= Gd;
    XT[5]= Nstrand;
    XT[6]= Ncable;
    XT[7]= cable_1st_position_end;
    XT[8]= pt1;
    XT[9]= ts;
    XT[10] = rho;
    XT[11] = TFRd;
    XT[12] = TFFHd;
    XT[13] = Wt;
    matrix_initialization_function();
//***********************************************************************************
    for(int i = 0;i<26;i++)
    {
        DECKT[i] = 175+5*i;
    }
```

```
for(i = 0;i<69;i++)
{
    DX1[i] = 300+25*i;
    DX2[i] = 300+25*i;
}
    for( i = 0;i<101;i++)
{
    DX4[i] = 1000+25*i;
}
    for(i=0;i<10;i++)
{
    DX11[i] = 75+25*i;
}
    for(i = 0;i<11;i++)
{
    DX12[i] = 50+25*i;
}
    for(i=0;i<36;i++)
{
    DX13[i] = 125+5*i;
}
cout<<"cost = ?"<<endl;
cin>>cost;
if(cost == 1)
{
    UPcon=12500e-9; //!per mm3
    UPcondeck=6000e-9; // !per mm3
    UPst=90;// !per Kg
    UPnonprest = 45;//UPst/4.0;
    Anchcost = 4500;
    sheathcost = 90;
    UPgf = 400e-6;
    UPdf = 415e-6;
}
else if(cost == 2)
{
    UPcon=12500e-9; //!per mm3
    UPcondeck=6000e-9; // !per mm3
    UPst=180;// !per Kg
    UPnonprest = 90;//UPst/4.0;
    Anchcost = 9000;
    sheathcost = 180;
    UPgf = 400e-6;
    UPdf = 415e-6;
}
```

```
        else
        {
            UPcon=12500e-9; //!per mm3
            UPcondeck=6000e-9; // !per mm3
                UPst=270;// !per Kg
                UPnonprest = 135;//UPst/4.0;
                Anchcost = 13500;
                sheathcost = 270;
                UPgf = 400e-6;
                UPdf = 415e-6;
    }
//***********************************************************************************
// CONTROL PARAMETERS FOR "EVOP"
    ALPHA = 1.2;
    BETA=0.5;
    GAMA=2.0;
    DEL=1e-12;
    PHI=1e-13;
    PHICPX=1e-8;
    ICON=5;
    LIMIT=100000;
    KNT=25;
    N=nv;
    NIC=icn;
    if(nv<=5)
    {
        K=2*nv;
    }
    else
    {
        K=nv+1;
    }
    IPRINT=2;
    NRSTRT=10;
    IMV=0;
    IJK=1;
line1:
EVOP(&ALPHA,&BETA,C,&DEL,FF,&GAMA,H,&ICON,&IJK,&IMV,&IPRINT,&K,&KNT,&LIMIT,&N,&NRSTRT,
                &NIC,OLDCC,&PHI,&PHICPX,XDN,XG,XMAX,XMIN,XT,XUP,XX,XXMAX,XXMIN);
    if (IJK < 9) goto line1;
    time(&stop);
    cout<<difftime(stop, start)<<endl;
    cout<<"Cpcon"<<Cpcon/(GS*L*70)*1e6<<endl;
    cout<<"Cdconc"<<Cdconc/(GS*L*70)*1e6<<endl;
    cout<<"Cpst"<<Cpst/(GS*L*70)*1e6<<endl;
    cout<<"Cnpst"<<Cnpst/(GS*L*70)*1e6<<endl;
    cout<<UPgf*2*SA/(GS*L*70)*1e6<<endl;
```

```
    cout<<UPdf*(GS-TFRw)*L/(GS*L*70)*1e6<<endl;
    cout<<(Cpcon+Cpst+Cdconc+Cnpst)*NoGirder/(BW*L*70)*1e6<<endl;
    ofstream fout("output.txt");
    fout<<"Cpcon"<<Cpcon/(GS*L*70)*1e6<<endl;
    fout<<"Cdconc"<<Cdconc/(GS*L*70)*1e6<<endl;
    fout<<"Cpst"<<Cpst/(GS*L*70)*1e6<<endl;
    fout<<"Cnpst"<<Cnpst/(GS*L*70)*1e6<<endl;
    fout<<UPgf*2*SA/(GS*L*70)*1e6<<endl;
    fout<<UPdf*(GS-TFRw)*L/(GS*L*70)*1e6<<endl;
    fout<<(Cpcon+Cpst+Cdconc+Cnpst)*NoGirder/(BW*L*70)*1e6<<endl;
    fout<<Y1<<endl;
    fout<<Y2<<endl;
    fout<<Y3<<endl;
    fout<<Y_end<<endl;
    fout<<Ancg_C2C<<endl;
test_function();
    fout.close();
}
```


## Appendix.B

## Output Summary of Optimization Results of the $\mathbf{4 0} \mathbf{m}$ double span continuous girder from EVOP Program

## INPUT PARAMETERS FOR OPTIMISATION SUBROUTINE EVOP

REFLECTION COEFFICIENT
CONTRACTION COEFFICIENT EXPANSION COEFFICIENT
$\mathrm{ALPHA}=.13000000 \mathrm{E}+01$
$\mathrm{BETA}=.50000000 \mathrm{E}+00$
$\mathrm{GAMA}=.20000000 \mathrm{E}+01$

EXPLICIT CONSTRAINT RETENTION COEFFICIENT

ACCURACY PARAMETER FOR CONVERGENCE

$$
\begin{aligned}
& \mathrm{DEL}=.10000000 \mathrm{E}-11 \\
& \mathrm{PHI}=.10000000 \mathrm{E}-12
\end{aligned}
$$

PARAMETER FOR DETERMINING COLLAPSE OF A COMPLEX IN A
SUBSPACE
PHICPX $=.10000000 \mathrm{E}-15$
GLOBAL LIMIT ON THE NUMBER OF CALLS TO FUNCTION SUBROUTINE

$$
\text { LIMIT }=100000
$$

NUMBER OF COMPLEX RESTARTS
NUMBER OF CALLS TO FUNCTION SUBROUTINE AFTER WHICH
CONVERGENCE TESTS ARE MADE
$\mathrm{KNT}=25$
NUMBER OF CONSECUTIVE CONVERGENCE TEST_1
$\mathrm{ICON}=5$
NUMBER OF VARIABLES $=$ NUMBER OF EXPLICIT CONSTRAINTS $N=14$
NUMBER OF IMPLICIT CONSTRAINTS
$\mathrm{NIC}=73$
NUMBER OF COMPLEX VERTICES
$K=15$

COORDINATES OF THE STARTING POINT

| Serial No. | Design variables | Design variables |  |
| :---: | :---: | :--- | :--- |
| 1 | $\mathrm{~S}=3500 ;$ | $\mathrm{XT}(1)=.35000000 \mathrm{E}+04$ |  |
| 2 | $\mathrm{TF}_{\mathrm{w}}=650 ;$ | $\mathrm{XT}(2)=.65000000 \mathrm{E}+03$ |  |
| 3 | $\mathrm{BF}_{\mathrm{w}}=450 ;$ | $\mathrm{XT}(3)=.45000000 \mathrm{E}+03$ |  |
| 4 | $\mathrm{BF}_{\mathrm{t}}=325 ;$ | $\mathrm{XT}(4)=.32500000 \mathrm{E}+03$ |  |
| 5 | $\mathrm{G}_{\mathrm{d}}=2500 ;$ | $\mathrm{XT}(5)=.25000000 \mathrm{E}+04$ |  |
| 6 | $\mathrm{~N}_{\mathrm{S}}=9.0 ;$ | $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ |  |


| 7 | $\mathrm{~N}_{\mathrm{T}}=7.0 ;$ | $\mathrm{XT}(7)=$ | $.70000000 \mathrm{E}+01$ |
| :---: | :---: | :--- | :--- |
| 8 | $\mathrm{y}_{1}=645 ;$ | $\mathrm{XT}(8)=$ | $.64500000 \mathrm{E}+03$ |
| 9 | $\eta=0.55 ;$ | $\mathrm{XT}(9)=$ | $.55000000 \mathrm{E}+00$ |
| 10 | $\mathrm{t}=250 ;$ | $\mathrm{XT}(10)=$ | $.25000000 \mathrm{E}+03$ |
| 11 | $\rho=0.005373 ;$ | $\mathrm{XT}(11)=$ | $.53730000 \mathrm{E}-02$ |
| 12 | $\mathrm{TF}_{\mathrm{t}}=125 ;$ | $\mathrm{XT}(12)=$ | $.12500000 \mathrm{E}+03$ |
| 13 | $\mathrm{TFT}_{\mathrm{t}}=75 ;$ | $\mathrm{XT}(13)=$ | $.75000000 \mathrm{E}+02$ |
| 14 | $\mathrm{~W}_{\mathrm{w}}=190 ;$ | $\mathrm{XT}(14)=$ | $.19000000 \mathrm{E}+03$ |

FUNCTION VALUE AT THE STARTING POINT $\mathbf{F F}(\mathbf{1})=\mathbf{~} \mathbf{5 4 0 8 7 8 3 9 E + 0 7}$

| UPPER BOU CONSTRAINTS | OF EXPLICIT <br> T THE STARTING INT | LOWER BOUND OF EXPLICIT CONSTRAINTS AT THE STARTING POINT |  |
| :---: | :---: | :---: | :---: |
| XMAX( 1 ) = | . $12000010 \mathrm{E}+05$ | XMIN(1) $=$ | . $14999900 \mathrm{E}+04$ |
| $\operatorname{XMAX}(2)=$ | .20000100E+04 | $\operatorname{XMIN}(2)=$ | . $30000000 \mathrm{E}+03$ |
| XMAX (3) = | . $11500100 \mathrm{E}+04$ | $\operatorname{XMIN}(3)=$ | . $30000000 \mathrm{E}+03$ |
| $\operatorname{XMAX}(4)=$ | . $60001000 \mathrm{E}+03$ | $\operatorname{XMIN}(4)=$ | . $10800000 \mathrm{E}+03$ |
| $\operatorname{XMAX}(5)=$ | . $35000100 \mathrm{E}+04$ | $\operatorname{XMIN}(5)=$ | . $99999000 \mathrm{E}+03$ |
| $\operatorname{XMAX}(6)=$ | . $19001000 \mathrm{E}+02$ | $\operatorname{XMIN}(6)=$ | . $29900000 \mathrm{E}+01$ |
| XMAX (7) = | . $15001000 \mathrm{E}+02$ | XMIN( 7 ) $=$ | . $99000000 \mathrm{E}+00$ |
| $\operatorname{XMAX}(8)=$ | . $10000100 \mathrm{E}+04$ | $\operatorname{XMIN}(8)=$ | . $21000000 \mathrm{E}+03$ |
| $\operatorname{XMAX}(9)=$ | . $99900000 \mathrm{E}+00$ | $\operatorname{XMIN}(9)=$ | . $10000000 \mathrm{E}-02$ |
| $\operatorname{XMAX}(10)=$ | . $30001000 \mathrm{E}+03$ | $\mathrm{XMIN}(10)=$ | . $17499000 \mathrm{E}+03$ |
| XMAX (11) $=$ | . $19512195 \mathrm{E}-01$ | XMIN(11) $=$ | . $15000000 \mathrm{E}-02$ |
| $\operatorname{XMAX}(12)=$ | . $30001000 \mathrm{E}+03$ | XMIN(12) $=$ | $.74990000 \mathrm{E}+02$ |
| $\operatorname{XMAX}(13)=$ | . $30001000 \mathrm{E}+03$ | $\mathrm{XMIN}(13)=$ | $.49990000 \mathrm{E}+02$ |
| XMAX (14) $=$ | . $30001000 \mathrm{E}+03$ | $X \mathrm{MIN}(14)=$ | . $14999000 \mathrm{E}+03$ |

## IMPLICIT CONSTRAINTS

| Implicit <br> Constraints | Description |
| :---: | :---: |
| $\begin{aligned} & \mathrm{XX}(1)=\mathrm{f} 1 \mathrm{ti} \\ & \mathrm{XX}(2)=\mathrm{f} 2 \mathrm{ti} \\ & \mathrm{XX}(3)=\mathrm{f} 3 \mathrm{ti} \\ & \mathrm{XX}(4)=\mathrm{f} 5 \mathrm{ti} \\ & \mathrm{XX}(5)=\mathrm{f} 6 \mathrm{ti} \\ & \mathrm{XX}(6)=\mathrm{f} 7 \mathrm{ti} \end{aligned}$ | f1ti, f2ti, f3ti, f5ti, f6ti, f7ti are top fiber flexural stresses at section1, section2, section3, section5, section6, section7 respectively at initial stage $f_{t i}=-\frac{\eta F_{i}}{A_{\text {net }}} \pm \frac{\eta F_{i} e_{i}}{S_{\text {tnet }}} \mp \frac{M_{G}}{S_{\text {tnet }}}$ |
| $\begin{aligned} & \mathrm{XX}(7)=\mathrm{f} 1 \mathrm{bi} \\ & \mathrm{XX}(8)=\mathrm{f} 2 \mathrm{bi} \\ & \mathrm{XX}(9)=\mathrm{f} 3 \mathrm{bi} \\ & \mathrm{XX}(10)=\mathrm{f} 5 \mathrm{bi} \\ & \mathrm{XX}(11)=\mathrm{f} 6 \mathrm{bi} \\ & \mathrm{XX}(12)=\mathrm{f} 7 \mathrm{bi} \end{aligned}$ | f1bi, f2bi, f3bi, f5bi, f6bi, f7bi are bottom fiber flexural stresses at section1, section2, section3, section5, section6, section7 respectively at initial stage $f_{b i}=-\frac{\eta F_{i}}{A_{n e t}} \mp \frac{\eta F_{i} e_{i}}{S_{\text {bnet }}} \pm \frac{M_{G}}{S_{\text {bnet }}}$ |
| $\begin{aligned} & \mathrm{XX}(13)=\mathrm{fltc} \\ & \mathrm{XX}(14)=\mathrm{f} 2 \mathrm{tc} \\ & \mathrm{XX}(15)=\mathrm{f} 3 \mathrm{tc} \\ & \mathrm{XX}(16)=\mathrm{f} 5 \mathrm{tc} \\ & \mathrm{XX}(17)=\mathrm{f} 6 \mathrm{tc} \\ & \mathrm{XX}(18)=\mathrm{f} 7 \mathrm{tc} \end{aligned}$ | $\mathrm{fltc}, \mathrm{f} 2 \mathrm{tc}, \mathrm{f} 3 \mathrm{tc}, \mathrm{f} 5 \mathrm{tc}, \mathrm{f} 6 \mathrm{tc}, \mathrm{f} 7 \mathrm{tc}$ are top fiber flexural stresses at section1, section2, section3, section5, section6, section7 respectively at second loading stage $f_{t}=-\frac{F_{e}}{A_{t f}} \pm \frac{F_{e} e}{S_{t}} \mp \frac{M_{P}}{S_{t}} \mp \frac{M_{C}}{S_{t c}}$ |
| $\begin{aligned} & \mathrm{XX}(19)=\mathrm{f} 1 \mathrm{bc} \\ & \mathrm{XX}(20)=\mathrm{f} 2 \mathrm{bc} \\ & \mathrm{XX}(21)=\mathrm{f} 3 \mathrm{bc} \\ & \mathrm{XX}(22)=\mathrm{f} 5 \mathrm{bc} \\ & \mathrm{XX}(23)=\mathrm{f} 6 \mathrm{bc} \\ & \mathrm{XX}(24)=\mathrm{f} 7 \mathrm{bc} \end{aligned}$ | $\mathrm{flbc}, \mathrm{f} 2 \mathrm{bc}, \mathrm{f} 3 \mathrm{bc}, \mathrm{f} 5 \mathrm{bc}, \mathrm{f} 6 \mathrm{bc}, \mathrm{f} 7 \mathrm{bc}$ are bottom fiber flexural stresses at section1, section2, section3, section5, section6, section7 respectively at second loading stage $f_{b}=-\frac{F_{e}}{A_{t f}} \mp \frac{F_{e} e}{S_{b}} \pm \frac{M_{P}}{S_{b}} \pm \frac{M_{C}}{S_{b c}}$ |
| $\begin{aligned} & \mathrm{XX}(25)=\mathrm{f} 1 \mathrm{tt} \\ & \mathrm{XX}(26)=\mathrm{f} 2 \mathrm{tt} \\ & \mathrm{XX}(27)=\mathrm{f} 3 \mathrm{tt} \\ & \mathrm{XX}(28)=\mathrm{f} 5 \mathrm{tt} \\ & \mathrm{XX}(29)=\mathrm{f} 6 \mathrm{tt} \\ & \mathrm{XX}(30)=\mathrm{f} 7 \mathrm{tt} \end{aligned}$ | $\mathrm{fltt}, \mathrm{f} 2 \mathrm{tt}, \mathrm{f} 3 \mathrm{tt}, \mathrm{f} 5 \mathrm{tt}, \mathrm{f} 6 \mathrm{tt}, \mathrm{f} 7 \mathrm{tt}$ are top fiber flexural stresses at section1, section2, section3, section5, section6, section7 respectively at third loading stage $f_{t}=-\frac{F_{e}}{A_{t f}} \pm \frac{F_{e} e}{S_{t}} \mp \frac{M_{P}}{S_{t}} \mp \frac{M_{C}}{S_{t c}} \mp \frac{M_{L}}{S_{t c}}$ |


| $\begin{aligned} & \mathrm{XX}(31)=\mathrm{f} 1 \mathrm{bt} \\ & \mathrm{XX}(32)=\mathrm{f} 2 \mathrm{bt} \\ & \mathrm{XX}(33)=\mathrm{f} 3 \mathrm{bt} \\ & \mathrm{XX}(34)=\mathrm{f} 5 \mathrm{bt} \\ & \mathrm{XX}(35)=\mathrm{f} 6 \mathrm{bt} \\ & \mathrm{XX}(36)=\mathrm{f} 7 \mathrm{bt} \end{aligned}$ | f1bt, f2bt, f3bt, f5bt, f6bt, f7bt are bottom fiber flexural stresses at section1, section2, section3, section5, section6, section7 respectively at third loading stage $f_{b}=-\frac{F_{e}}{A_{t f}} \mp \frac{F_{e} e}{S_{b}} \pm \frac{M_{P}}{S_{b}} \pm \frac{M_{C}}{S_{b c}} \pm \frac{M_{L}}{S_{b c}}$ |
| :---: | :---: |
| $\begin{aligned} & \mathrm{XX}(37)=\mathrm{f} 1 \mathrm{ttt} \\ & \mathrm{XX}(38)=\mathrm{f} 2 \mathrm{ttt} \\ & \mathrm{XX}(39)=\mathrm{f} 3 \mathrm{ttt} \\ & \mathrm{XX}(40)=\mathrm{f} 5 \mathrm{tt} \\ & \mathrm{XX}(41)=\mathrm{f} 6 \mathrm{ttt} \\ & \mathrm{XX}(42)=\mathrm{f} 7 \mathrm{ttt} \end{aligned}$ | f1ttt, f2ttt, f3ttt, f5ttt, f6ttt, f7ttt are top fiber flexural stresses at section1, section2, section3, section5, section6, section7 respectively at fourth loading stage $f_{t}=-\frac{1}{2} \frac{F_{e}}{A_{t f}} \pm \frac{1}{2} \frac{F_{e} e}{S_{t}} \mp \frac{\left(M_{L}+\frac{M_{D}}{2}\right)}{S_{t c}}$ |
| $\begin{aligned} & \mathrm{XX}(43)=\mathrm{flbtt} \\ & \mathrm{XX}(44)=\mathrm{f} 2 \mathrm{btt} \\ & \mathrm{XX}(45)=\mathrm{f} 3 \mathrm{btt} \\ & \mathrm{XX}(46)=\mathrm{f} 5 \mathrm{btt} \\ & \mathrm{XX}(47)=\mathrm{f} 6 \mathrm{btt} \\ & \mathrm{XX}(48)=\mathrm{f} 7 \mathrm{btt} \end{aligned}$ | $\mathrm{flbtt}, \mathrm{f} 2 \mathrm{btt}, \mathrm{f} 3 \mathrm{btt}, \mathrm{f5btt}, \mathrm{f} 6 \mathrm{btt}, \mathrm{f} 7 \mathrm{btt}$ are bottom fiber flexural stresses at section1, section2, section3, section5, section6, section7 respectively at fourth loading stage $f_{b}=-\frac{1}{2} \frac{F_{e}}{A_{t f}} \mp \frac{1}{2} \frac{F_{e} e}{S_{b}} \pm \frac{\left(M_{L}+\frac{M_{D}}{2}\right)}{S_{b c}}$ |
| $\mathrm{XX}(49)=M_{c u}$ | Calculations are done according to Table 5.5 |
| $\begin{aligned} & \mathrm{XX}(50)=\mathrm{V}_{\mathrm{S}} \\ & \mathrm{XX}(51)=M_{c r}^{*} \\ & \mathrm{XX}(52)=w_{c} \\ & \mathrm{XX}(53)=\mathrm{Y}_{\mathrm{end}} \\ & \mathrm{XX}(54)=\Delta_{L L} \\ & \mathrm{XX}(55)=F_{4 i} \\ & \mathrm{XX}(56)=F_{2 i} \\ & \mathrm{XX}(57)=F_{2 e} \\ & \mathrm{XX}(58)=\mathrm{V}_{\mathrm{nh}} \\ & \mathrm{XX}(59)=\mathrm{t} \\ & \mathrm{XX}(60)=\mathrm{d}_{\mathrm{req}} \\ & \mathrm{XX}(61)=M_{p u} \\ & \mathrm{XX}(62)=M_{c r s l a b}^{*} \end{aligned}$ | Eq. (3.28) and Eq. (3.34); Detail calculations are done in Appendix-A <br> Eq. (5.27) and Eq. (5.28) <br> Eq. (3.23); Reinforcement index of composite girder <br> Centroidal distance of tendons at end section <br> Eq. (5.35) <br> Eq. (5.12) <br> Eq. (5.10) <br> $F_{2 i}$ - Time dependent losses $350 *\left(\mathrm{TF}_{\mathrm{w}} * \mathrm{~d}_{\mathrm{S}}\right) /(25.4 * 25.4) / 1000 * 4.45$ <br> Deck slab thickness <br> Detail calculations are done in Appendix-A <br> Calculations are done according to Table 5.5 <br> Eq. (5.29) |

\(\left.$$
\begin{array}{|l|l|}\hline \mathrm{XX}(66)=w_{p} \\
\mathrm{XX}(67)=\mathrm{F}_{\text {sc }}\end{array}
$$ \quad \begin{array}{l}Eq. (3.23); Reinforcement index of precast girder <br>
Eq. (3.36) to Eq. (3.38); Detail calculations are done in <br>

Appendix-A\end{array}\right]\)| $\mathrm{XX}(68)=\mathrm{flt}$ |
| :--- | :--- |
| $\mathrm{XX}(69)=\mathrm{f} 2 \mathrm{t}$ |
| $\mathrm{XX}(70)=\mathrm{f3t}$ |$\quad$| f1t, f2t, f3t are top fiber flexural stresses at section1, section2, |
| :--- |
| section3 respectively at first loading stage |
| $f_{t}=-\frac{F_{i}}{A_{t f}} \pm \frac{F_{i} e}{S_{t}} \mp \frac{M_{P}}{S_{t}}$ |




|  |  |
| :---: | :---: |


| $\operatorname{XXMAX}(68)=0.5 \sqrt{f_{c}^{\prime}}$ |  |
| :--- | :--- |
| $\operatorname{XXMAX}(69)=0.5 \sqrt{f_{c}^{\prime}}$ |  |
| $\operatorname{XXMAX}(70)=0.5 \sqrt{f_{c}^{\prime}}$ |  |
| $\operatorname{XXMAX}(71)=0.5 \sqrt{f_{c}^{\prime}}$ |  |
| $\operatorname{XXMAX}(72)=0.5 \sqrt{f_{c}^{\prime}}$ |  |
| $\operatorname{XXMAX}(73)=0.5 \sqrt{f_{c}^{\prime}}$ |  |


| UPPER BOUND OF IMPLICIT CONSTRAINTS AT THE STARTING POINT |  | LOWER BOUND OF IMPLICIT CONSTRAINTS AT THE STARTING POINT |  |
| :---: | :---: | :---: | :---: |
| XXMAX(1) = | . $13693064 \mathrm{E}+01$ | XXMIN( 1 ) = | $-.16500000 \mathrm{E}+02$ |
| XXMAX(2) $=$ | . $13693064 \mathrm{E}+01$ | $\operatorname{XXMIN}(2)=$ | $-.16500000 \mathrm{E}+02$ |
| XXMAX(3) $=$ | . $13693064 \mathrm{E}+01$ | XXMIN(3) $=$ | $-.16500000 \mathrm{E}+02$ |
| XXMAX(4) $=$ | . $13693064 \mathrm{E}+01$ | XXMIN(4) $=$ | $-.16500000 \mathrm{E}+02$ |
| $\operatorname{XXMAX}(5)=$ | . $13693064 \mathrm{E}+01$ | $\operatorname{XXMIN}(5)=$ | $-.16500000 \mathrm{E}+02$ |
| $\operatorname{XXMAX}(6)=$ | . $13693064 \mathrm{E}+01$ | XXMIN(6) $=$ | $-.16500000 \mathrm{E}+02$ |
| XXMAX(7) $=$ | . $13693064 \mathrm{E}+01$ | XXMIN(7) $=$ | $-.16500000 \mathrm{E}+02$ |
| $\operatorname{XXMAX}(8)=$ | . $13693064 \mathrm{E}+01$ | XXMIN( 8) $=$ | $-.16500000 \mathrm{E}+02$ |
| $\operatorname{XXMAX}(9)=$ | . $13693064 \mathrm{E}+01$ | XXMIN(9) $=$ | $-.16500000 \mathrm{E}+02$ |
| XXMAX (10) = | . $13693064 \mathrm{E}+01$ | XXMIN(10) $=$ | $-.16500000 \mathrm{E}+02$ |
| XXMAX (11) = | . $13693064 \mathrm{E}+01$ | XXMIN(11) $=$ | $-.16500000 \mathrm{E}+02$ |
| XXMAX $(12)=$ | . $13693064 \mathrm{E}+01$ | XXMIN(12) $=$ | $-.16500000 \mathrm{E}+02$ |
| XXMAX(13) = | .31622777E+01 | XXMIN(13) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(14)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(14) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX (15) = | . $31622777 \mathrm{E}+01$ | XXMIN(15) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(16)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(16) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(17)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(17) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX (18) = | . $31622777 \mathrm{E}+01$ | XXMIN(18) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(19) = | . $31622777 \mathrm{E}+01$ | XXMIN(19) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(20)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(20) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(21)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(21) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(22)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(22) $=$ | $-.16000000 \mathrm{E}+02$ |


| XXMAX(23) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(23) $=$ | $-.16000000 \mathrm{E}+02$ |
| :---: | :---: | :---: | :---: |
| XXMAX(24) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(24) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(25)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(25) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX $(26)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(26) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(27) = | . $31622777 \mathrm{E}+01$ | XXMIN(27) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX $(28)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(28) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(29) = | . $31622777 \mathrm{E}+01$ | XXMIN(29) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX $(30)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(30) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(31) = | . $31622777 \mathrm{E}+01$ | XXMIN(31) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX $(32)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(32) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(33) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(33) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX (34) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(34) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(35) = | . $31622777 \mathrm{E}+01$ | XXMIN(35) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(36) = | . $31622777 \mathrm{E}+01$ | XXMIN(36) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(37) = | . $31622777 \mathrm{E}+01$ | XXMIN(37) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(38)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(38) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(39) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(39) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(40) = | . $31622777 \mathrm{E}+01$ | XXMIN(40) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(41)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(41) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX $(42)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(42) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(43) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(43) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(44) = | . $31622777 \mathrm{E}+01$ | XXMIN(44) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(45) = | . $31622777 \mathrm{E}+01$ | XXMIN(45) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(46) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(46) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(47) = | . $31622777 \mathrm{E}+01$ | XXMIN(47) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(48) = | . $31622777 \mathrm{E}+01$ | XXMIN(48) $=$ | $-.16000000 \mathrm{E}+02$ |
| XXMAX(49) = | . $36280958 \mathrm{E}+08$ | XXMIN(49) $=$ | . $00000000 \mathrm{E}+00$ |
| XXMAX(50) = | . $17478754 \mathrm{E}+04$ | XXMIN(50) $=$ | . $00000000 \mathrm{E}+00$ |
| XXMAX $(51)=$ | . $30234132 \mathrm{E}+08$ | XXMIN(51) $=$ | . $00000000 \mathrm{E}+00$ |
| XXMAX(52) $=$ | . $27000000 \mathrm{E}+00$ | XXMIN(52) $=$ | . $00000000 \mathrm{E}+00$ |
| XXMAX(53) $=$ | .18561776E+04 | XXMIN(53) $=$ | $.77051056 \mathrm{E}+03$ |
| XXMAX(54) = | . $61000000 \mathrm{E}+02$ | XXMIN(54) $=$ | $.00000000 \mathrm{E}+00$ |
| XXMAX $(55)=$ | . $11489814 \mathrm{E}+05$ | XXMIN(55) $=$ | . $00000000 \mathrm{E}+00$ |


| XXMAX (56) = | . $12261273 \mathrm{E}+05$ | XXMIN(56) $=$ | . $00000000 \mathrm{E}+00$ |
| :---: | :---: | :---: | :---: |
| XXMAX(57) $=$ | .11818094E+05 | XXMIN(57) $=$ | . $00000000 \mathrm{E}+00$ |
| XXMAX(58) $=$ | . $10000000 \mathrm{E}+06$ | XXMIN(58) $=$ | . $17714722 \mathrm{E}+04$ |
| XXMAX(59) = | . $30000000 \mathrm{E}+03$ | XXMIN(59) $=$ | .19704473E+03 |
| XXMAX $(60)=$ | . $17300000 \mathrm{E}+03$ | XXMIN(60) $=$ | . $79049716 \mathrm{E}+02$ |
| XXMAX $(61)=$ | . $14889011 \mathrm{E}+08$ | XXMIN(61) $=$ | $.00000000 \mathrm{E}+00$ |
| XXMAX(62) $=$ | . $37443427 \mathrm{E}+05$ | XXMIN(62) $=$ | . $000000000 \mathrm{E}+00$ |
| XXMAX(66) $=$ | . $27000000 \mathrm{E}+00$ | XXMIN(66) $=$ | . $00000000 \mathrm{E}+00$ |
| XXMAX(67) $=$ | . $10000000 \mathrm{E}+03$ | XXMIN(67) $=$ | . $15000000 \mathrm{E}+01$ |
| XXMAX(68) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(68) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(69) = | . $31622777 \mathrm{E}+01$ | XXMIN(69) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX $(70)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(70) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX $(71)=$ | . $31622777 \mathrm{E}+01$ | XXMIN(71) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(72) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(72) $=$ | $-.24000000 \mathrm{E}+02$ |
| XXMAX(73) $=$ | . $31622777 \mathrm{E}+01$ | XXMIN(73) $=$ | $-.24000000 \mathrm{E}+02$ |

IMPLICIT CONSTRAINTS AT THE STARTING POINT

| $\mathrm{XX}(1)=$ | $-.51534951 \mathrm{E}+01$ | $\mathrm{XX}(38)=$ | $.91364319 \mathrm{E}+04$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{XX}(2)=$ | $-.78275140 \mathrm{E}+01$ | $\mathrm{XX}(39)=$ | $.38575891 \mathrm{E}+04$ |
| $\mathrm{XX}(3)=$ | $-.58464857 \mathrm{E}+01$ | $\mathrm{XX}(40)=$ | $.23000000 \mathrm{E}+03$ |
| $\mathrm{XX}(4)=$ | $-.90183803 \mathrm{E}+01$ | $\mathrm{XX}(41)=$ | $.15055625 \mathrm{E}+03$ |
| $\mathrm{XX}(5)=$ | $-.62977662 \mathrm{E}+01$ | $\mathrm{XX}(42)=$ | $.66089805 \mathrm{E}+07$ |
| $\mathrm{XX}(6)=$ | $-.94254173 \mathrm{E}+01$ | $\mathrm{XX}(43)=$ | $.27552083 \mathrm{E}+05$ |
| $\mathrm{XX}(7)=$ | $-.12798985 \mathrm{E}+02$ | $\mathrm{XX}(44)=$ | $.13905571 \mathrm{E}+00$ |
| $\mathrm{XX}(8)=$ | $-.13424276 \mathrm{E}+02$ | $\mathrm{XX}(45)=$ | $.18554627 \mathrm{E}+01$ |
| $\mathrm{XX}(9)=$ | $-.12657727 \mathrm{E}+02$ | $\mathrm{XX}(46)=$ | $-.93922106 \mathrm{E}+01$ |
| $\mathrm{XX}(10)=$ | $-.78140814 \mathrm{E}+01$ | $\mathrm{XX}(47)=$ | $-.15481798 \mathrm{E}+02$ |
| $\mathrm{XX}(11)=$ | $-.85757223 \mathrm{E}+01$ | $\mathrm{XX}(48)=$ | $-.11272259 \mathrm{E}+02$ |
| $\mathrm{XX}(12)=$ | $-.10593743 \mathrm{E}+02$ | $\mathrm{XX}(49)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(13)=$ | $-.14913684 \mathrm{E}+02$ | $\mathrm{XX}(50)=$ | $-.13616007 \mathrm{E}+02$ |
| $\mathrm{XX}(14)=$ | $-.13994308 \mathrm{E}+02$ | $\mathrm{XX}(51)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(15)=$ | $-.14200353 \mathrm{E}+02$ | $\mathrm{XX}(52)=$ | $.38575891 \mathrm{E}+04$ |
| $\mathrm{XX}(16)=$ | $-.29470536 \mathrm{E}+01$ | $\mathrm{XX}(53)=$ | $.23000000 \mathrm{E}+03$ |


| $\mathrm{XX}(17)=$ | $-.71646409 \mathrm{E}+01$ | $\mathrm{XX}(54)=$ | $.15055625 \mathrm{E}+03$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{XX}(18)=$ | $-.69501662 \mathrm{E}+01$ | $\mathrm{XX}(55)=$ | $.66089805 \mathrm{E}+07$ |
| $\mathrm{XX}(19)=$ | $-.16537192 \mathrm{E}+01$ | $\mathrm{XX}(56)=$ | $.27552083 \mathrm{E}+05$ |
| $\mathrm{XX}(20)=$ | $-.54253256 \mathrm{E}+01$ | $\mathrm{XX}(57)=$ | $.13905571 \mathrm{E}+00$ |
| $\mathrm{XX}(21)=$ | $-.28598239 \mathrm{E}+01$ | $\mathrm{XX}(58)=$ | $.18554627 \mathrm{E}+01$ |
| $\mathrm{XX}(22)=$ | $-.15041360 \mathrm{E}+01$ | $\mathrm{XX}(59)=$ | $-.93922106 \mathrm{E}+01$ |
| $\mathrm{XX}(23)=$ | $-.36658537 \mathrm{E}+01$ | $\mathrm{XX}(60)=$ | $-.15481798 \mathrm{E}+02$ |
| $\mathrm{XX}(24)=$ | $-.85757223 \mathrm{E}+01$ | $\mathrm{XX}(61)=$ | $-.11272259 \mathrm{E}+02$ |
| $\mathrm{XX}(25)=$ | $-.10593743 \mathrm{E}+02$ | $\mathrm{XX}(62)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(26)=$ | $-.14913684 \mathrm{E}+02$ | $\mathrm{XX}(63)=$ | $-.13616007 \mathrm{E}+02$ |
| $\mathrm{XX}(27)=$ | $-.13994308 \mathrm{E}+02$ | $\mathrm{XX}(64)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(28)=$ | $-.14200353 \mathrm{E}+02$ | $\mathrm{XX}(65)=$ | $.13905571 \mathrm{E}+00$ |
| $\mathrm{XX}(29)=$ | $-.29470536 \mathrm{E}+01$ | $\mathrm{XX}(66)=$ | $.18554627 \mathrm{E}+01$ |
| $\mathrm{XX}(30)=$ | $-.71646409 \mathrm{E}+01$ | $\mathrm{XX}(67)=$ | $-.93922106 \mathrm{E}+01$ |
| $\mathrm{XX}(31)=$ | $-.69501662 \mathrm{E}+01$ | $\mathrm{XX}(68)=$ | $-.15481798 \mathrm{E}+02$ |
| $\mathrm{XX}(32)=$ | $-.16537192 \mathrm{E}+01$ | $\mathrm{XX}(69)=$ | $-.11272259 \mathrm{E}+02$ |
| $\mathrm{XX}(33)=$ | $-.54253256 \mathrm{E}+01$ | $\mathrm{XX}(70)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(34)=$ | $-.28598239 \mathrm{E}+01$ | $\mathrm{XX}(71)=$ | $-.13616007 \mathrm{E}+02$ |
| $\mathrm{XX}(35)=$ | $-.15041360 \mathrm{E}+01$ | $\mathrm{XX}(72)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(36)=$ | $-.36658537 \mathrm{E}+01$ | $\mathrm{XX}(73)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(37)=$ | $-.36658537 \mathrm{E}+01$ |  |  |

## INITIAL COMPLEX CONFIGURATION

All the vertices of the initial complex shown below are feasible solution or design of the bridge. It indicates that a lot of design of the bridge can be done with the different costs of the bridge. The design of the bridge which yields the minimum cost is the optimum design.

| VERTEX NUMBER 1 FUNCTION VALUE $=.53064839 \mathrm{E}+07$ COORDINATES $\mathrm{XT}(1)=.3563000 \mathrm{E}+04$ $\mathrm{XT}(2)=.5680000 \mathrm{E}+03$ $\mathrm{XT}(3)=.26800000 \mathrm{E}+03$ $\mathrm{XT}(4)=.34700000 \mathrm{E}+03$ $\mathrm{XT}(5)=.19000000 \mathrm{E}+04$ $\mathrm{XT}(6)=.80000000 \mathrm{E}+01$ $\mathrm{XT}(7)=.50000000 \mathrm{E}+01$ $\mathrm{XT}(8)=.56400000 \mathrm{E}+03$ $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ $\mathrm{XT}(10)=.35500000 \mathrm{E}+03$ $\mathrm{XT}(11)=.58849000 \mathrm{E}-02$ $\mathrm{XT}(12)=.67000000 \mathrm{E}+02$ $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ XT | VERTEX NUMBER 9 FUNCTION VALUE $=.57796738 \mathrm{E}+07$ COORDINATES $\mathrm{XT}(1)=.3583000 \mathrm{E}+04$ $\mathrm{XT}(2)=.5730000 \mathrm{E}+03$ $\mathrm{XT}(3)=.34100000 \mathrm{E}+03$ $\mathrm{XT}(4)=.33400000 \mathrm{E}+03$ $\mathrm{XT}(5)=.19400000 \mathrm{E}+04$ $\mathrm{XT}(6)=.80000000 \mathrm{E}+01$ $\mathrm{XT}(7)=.50000000 \mathrm{E}+01$ $\mathrm{XT}(8)=.56400000 \mathrm{E}+03$ $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ $\mathrm{XT}(10)=.35500000 \mathrm{E}+03$ $\mathrm{XT}(11)=.58849000 \mathrm{E}-02$ $\mathrm{XT}(12)=.67000000 \mathrm{E}+02$ $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ VERTEX |
| :---: | :---: |


| $\begin{aligned} & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |
| :---: | :---: |
| VERTEX NUMBER 3 | VERTEX NUMBER 11 |
| FUNCTION VALUE $=.54864721 \mathrm{E}+07$ | FUNCTION VALUE $=.52410747 \mathrm{E}+07$ |
| COORDINATES | COORDINATES |
| $\mathrm{XT}(1)=.3583000 \mathrm{E}+04$ | $\mathrm{XT}(1)=.36900000 \mathrm{E}+04$ |
| $\mathrm{XT}(2)=.5730000 \mathrm{E}+03$ | $\mathrm{XT}(2)=.3200000 \mathrm{E}+03$ |
| $\mathrm{XT}(3)=.34100000 \mathrm{E}+03$ | $\mathrm{XT}(3)=.42900000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.33400000 \mathrm{E}+03$ | $\mathrm{XT}(4)=.32000000 \mathrm{E}+03$ |
| $\mathrm{XT}(5)=.19400000 \mathrm{E}+04$ | $\mathrm{XT}(5)=.19000000 \mathrm{E}+04$ |
| $\mathrm{XT}(6)=.80000000 \mathrm{E}+01$ | $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ |
| $\mathrm{XT}(7)=.50000000 \mathrm{E}+01$ | $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ |
| $\mathrm{XT}(8)=.56400000 \mathrm{E}+03$ | $\mathrm{XT}(8)=.39000000 \mathrm{E}+03$ |
| $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ | $\mathrm{XT}(9)=.34170000 \mathrm{E}+00$ |
| $\mathrm{XT}(10)=.35500000 \mathrm{E}+03$ | $\mathrm{XT}(10)=.32500000 \mathrm{E}+03$ |
| $\mathrm{XT}(11)=.58849000 \mathrm{E}-02$ | $\mathrm{XT}(11)=.54989000 \mathrm{E}-02$ |
| $\mathrm{XT}(12)=.67000000 \mathrm{E}+02$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |
| VERTEX NUMBER 4 |  |
| FUNCTION VALUE $=.56230794 \mathrm{E}+07$ | VERTEX NUMBER 12 |
| COORDINATES | FUNCTION VALUE $=.58728819 \mathrm{E}+07$ |
| $\mathrm{XT}(1)=.36900000 \mathrm{E}+04$ | COORDINATES |
| $\mathrm{XT}(2)=.3200000 \mathrm{E}+03$ | $\mathrm{XT}(1)=.3583000 \mathrm{E}+04$ |
| $\mathrm{XT}(3)=.42900000 \mathrm{E}+03$ | $\mathrm{XT}(2)=.5730000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.32000000 \mathrm{E}+03$ | $\mathrm{XT}(3)=.34100000 \mathrm{E}+03$ |
| $\mathrm{XT}(5)=.19000000 \mathrm{E}+04$ | $\mathrm{XT}(4)=.33400000 \mathrm{E}+03$ |
| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ | $\mathrm{XT}(5)=.19400000 \mathrm{E}+04$ |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(6)=.80000000 \mathrm{E}+01$ |
| $\mathrm{XT}(8)=.39000000 \mathrm{E}+03$ | $\mathrm{XT}(7)=.50000000 \mathrm{E}+01$ |
| $\mathrm{XT}(9)=.34170000 \mathrm{E}+00$ | $\mathrm{XT}(8)=.56400000 \mathrm{E}+03$ |
| $\mathrm{XT}(10)=.32500000 \mathrm{E}+03$ | $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ |



| $\begin{aligned} & \mathrm{XT}(9)=.27170000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.35500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.58849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.67000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ <br> VERTEX NUMBER 7 <br> FUNCTION VALUE $=.50098596 \mathrm{E}+07$ COORDINATES $\begin{aligned} & \mathrm{XT}(1)=.3563000 \mathrm{E}+04 \\ & \mathrm{XT}(2)=.5680000 \mathrm{E}+03 \\ & \mathrm{XT}(3)=.26800000 \mathrm{E}+03 \\ & \mathrm{XT}(4)=.34700000 \mathrm{E}+03 \\ & \mathrm{XT}(5)=.19000000 \mathrm{E}+04 \\ & \mathrm{XT}(6)=.80000000 \mathrm{E}+01 \\ & \mathrm{XT}(7)=.50000000 \mathrm{E}+01 \\ & \mathrm{XT}(8)=.56400000 \mathrm{E}+03 \\ & \mathrm{XT}(9)=.27170000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.35500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.58849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.67000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ <br> VERTEX NUMBER 8 <br> FUNCTION VALUE $=.55388064 \mathrm{E}+07$ COORDINATES $\begin{gathered} \mathrm{XT}(1)=.36900000 \mathrm{E}+04 \\ \mathrm{XT}(2)=.3200000 \mathrm{E}+03 \\ \mathrm{XT}(3)=.42900000 \mathrm{E}+03 \\ \mathrm{XT}(4)=.32000000 \mathrm{E}+03 \\ \mathrm{XT}(5)=.19000000 \mathrm{E}+04 \\ \mathrm{XT}(6)=.90000000 \mathrm{E}+01 \\ \mathrm{XT}(7)=.40000000 \mathrm{E}+01 \end{gathered}$ | $\begin{aligned} & \mathrm{XT}(6)=.80000000 \mathrm{E}+01 \\ & \mathrm{XT}(7)=.50000000 \mathrm{E}+01 \\ & \mathrm{XT}(8)=.56400000 \mathrm{E}+03 \\ & \mathrm{XT}(9)=.27170000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.35500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.58849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.67000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ <br> VERTEX NUMBER 15 <br> FUNCTION VALUE $=.56236387 \mathrm{E}+07$ <br> COORDINATES $\begin{aligned} & \mathrm{XT}(1)=.3583000 \mathrm{E}+04 \\ & \mathrm{XT}(2)=.5730000 \mathrm{E}+03 \\ & \mathrm{XT}(3)=.34100000 \mathrm{E}+03 \\ & \mathrm{XT}(4)=.33400000 \mathrm{E}+03 \\ & \mathrm{XT}(5)=.19400000 \mathrm{E}+04 \\ & \mathrm{XT}(6)=.80000000 \mathrm{E}+01 \\ & \mathrm{XT}(7)=.50000000 \mathrm{E}+01 \\ & \mathrm{XT}(8)=.56400000 \mathrm{E}+03 \\ & \mathrm{XT}(9)=.27170000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.35500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.58849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.67000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ |
| :---: | :---: |

```
\(\mathrm{XT}(8)=.39000000 \mathrm{E}+03\)
\(\mathrm{XT}(9)=.34170000 \mathrm{E}+00\)
\(\mathrm{XT}(10)=.32500000 \mathrm{E}+03\)
\(\mathrm{XT}(11)=.54989000 \mathrm{E}-02\)
\(\mathrm{XT}(12)=.75000000 \mathrm{E}+02\)
\(\mathrm{XT}(13)=.50000000 \mathrm{E}+02\)
\(\mathrm{XT}(14)=.15000000 \mathrm{E}+03\)
```


## OUTPUT SUMMARY FROM SUBROUTINE EVOP

MINIMUM OF THE OBJECTIVE FUNCTION HAS BEEN LOCATED TO THE DESIRED DEGREE OF ACCURACY FOR CONVERGENCE. IER $=8$ TOTAL NUMBER OF OBJECTIVE FUNCTION EVALUATION.

NFUNC = 1267

NUMBER OF TIMES THE SUBROUTINE FUNCTION IS CALLED DURING THE PRESENT CONVERGENCE TESTS. KUT = 6

NUMBER OF TIMES THE EXPLICIT CONSTRAINTS WERE EVALUATED $\mathrm{KKT}=4346$

NUMBER OF TIMES THE IMPLICIT CONSTRAINTS WERE EVALUATED $\mathrm{M}=2142$

COORDINATES OF THE MINIMUM

| $\mathrm{XT}(1)=.40000000 \mathrm{E}+04$ | $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |
| :---: | :--- |
| $\mathrm{XT}(2)=.4500000 \mathrm{E}+03$ | $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ |
| $\mathrm{XT}(3)=.32500000 \mathrm{E}+03$ | $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.26000000 \mathrm{E}+03$ | $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |
| $\mathrm{XT}(5)=.17000000 \mathrm{E}+04$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |

IMPLICIT CONSTRAINT VALUES AT THE MINIMUM

| $\mathrm{XX}(1)=$ | -. $51534951 \mathrm{E}+01$ | $\mathrm{XX}(38)=$ | . $91364319 \mathrm{E}+04$ |
| :---: | :---: | :---: | :---: |
| $X X(2)=$ | $-.78275140 \mathrm{E}+01$ | $\mathrm{XX}(39)=$ | . $38575891 \mathrm{E}+04$ |
| $\mathrm{XX}(3)=$ | -. $58464857 \mathrm{E}+01$ | $\mathrm{XX}(40)=$ | . $23000000 \mathrm{E}+03$ |
| $X X(4)=$ | $-.90183803 \mathrm{E}+01$ | $X X(41)=$ | . $15055625 \mathrm{E}+03$ |
| $X X(5)=$ | $-.62977662 \mathrm{E}+01$ | $\mathrm{XX}(42)=$ | . $66089805 \mathrm{E}+07$ |
| $\mathrm{XX}(6)=$ | $-.94254173 \mathrm{E}+01$ | $\mathrm{XX}(43)=$ | . $27552083 \mathrm{E}+05$ |
| $X X(7)=$ | $-.12798985 \mathrm{E}+02$ | $X X(44)=$ | . $13905571 \mathrm{E}+00$ |
| $X X(8)=$ | $-.13424276 \mathrm{E}+02$ | $X X(45)=$ | . $18554627 \mathrm{E}+01$ |
| $\mathrm{XX}(9)=$ | $-.12657727 \mathrm{E}+02$ | $\mathrm{XX}(46)=$ | $-.93922106 \mathrm{E}+01$ |
| $X X(10)=$ | $-.78140814 \mathrm{E}+01$ | $X X(47)=$ | $-.15481798 \mathrm{E}+02$ |
| $X X(11)=$ | $-.85757223 \mathrm{E}+01$ | $\mathrm{XX}(48)=$ | $-.11272259 \mathrm{E}+02$ |
| $\mathrm{XX}(12)=$ | $-.10593743 \mathrm{E}+02$ | $\mathrm{XX}(49)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(13)=$ | $-.14913684 \mathrm{E}+02$ | $\mathrm{XX}(50)=$ | $-.13616007 \mathrm{E}+02$ |
| $X X(14)=$ | $-.13994308 \mathrm{E}+02$ | $X X(51)=$ | $-.18917406 \mathrm{E}+02$ |
| $X X(15)=$ | $-.14200353 \mathrm{E}+02$ | $\mathrm{XX}(52)=$ | . $38575891 \mathrm{E}+04$ |
| $\mathrm{XX}(16)=$ | $-.29470536 \mathrm{E}+01$ | $\mathrm{XX}(53)=$ | $.23000000 \mathrm{E}+03$ |
| $\mathrm{XX}(17)=$ | $-.71646409 \mathrm{E}+01$ | $X X(54)=$ | . $15055625 \mathrm{E}+03$ |
| $X X(18)=$ | $-.69501662 \mathrm{E}+01$ | $\mathrm{XX}(55)=$ | . $66089805 \mathrm{E}+07$ |
| $X X(19)=$ | $-.16537192 \mathrm{E}+01$ | $\mathrm{XX}(56)=$ | . $27552083 \mathrm{E}+05$ |
| $X X(20)=$ | $-.54253256 \mathrm{E}+01$ | $\mathrm{XX}(57)=$ | . $13905571 \mathrm{E}+00$ |
| $X X(21)=$ | $-.28598239 \mathrm{E}+01$ | $\mathrm{XX}(58)=$ | . $18554627 \mathrm{E}+01$ |
| $\mathrm{XX}(22)=$ | $-.15041360 \mathrm{E}+01$ | $\mathrm{XX}(59)=$ | $-.93922106 \mathrm{E}+01$ |
| $X X(23)=$ | $-.36658537 \mathrm{E}+01$ | $\mathrm{XX}(60)=$ | $-.15481798 \mathrm{E}+02$ |
| $X X(24)=$ | $-.85757223 \mathrm{E}+01$ | $\mathrm{XX}(61)=$ | $-.11272259 \mathrm{E}+02$ |
| $\mathrm{XX}(25)=$ | $-.10593743 \mathrm{E}+02$ | $\mathrm{XX}(62)=$ | $-.18917406 \mathrm{E}+02$ |
| $X X(26)=$ | $-.14913684 \mathrm{E}+02$ | $\mathrm{XX}(63)=$ | $-.13616007 \mathrm{E}+02$ |
| $X X(27)=$ | $-.13994308 \mathrm{E}+02$ | $\mathrm{XX}(64)=$ | $-.18917406 \mathrm{E}+02$ |
| $X X(28)=$ | $-.14200353 \mathrm{E}+02$ | $X X(65)=$ | . $13905571 \mathrm{E}+00$ |
| $X X(29)=$ | $-.29470536 \mathrm{E}+01$ | $\mathrm{XX}(66)=$ | . $18554627 \mathrm{E}+01$ |
| $X X(30)=$ | $-.71646409 \mathrm{E}+01$ | $\mathrm{XX}(67)=$ | $-.93922106 \mathrm{E}+01$ |
| $X X(31)=$ | $-.69501662 \mathrm{E}+01$ | $X X(68)=$ | $-.15481798 \mathrm{E}+02$ |
| $X X(32)=$ | $-.16537192 \mathrm{E}+01$ | $\mathrm{XX}(69)=$ | $-.11272259 \mathrm{E}+02$ |


| $\mathrm{XX}(33)=$ | $-.54253256 \mathrm{E}+01$ | $\mathrm{XX}(70)=$ | $-.18917406 \mathrm{E}+02$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{XX}(34)=$ | $-.28598239 \mathrm{E}+01$ | $\mathrm{XX}(71)=$ | $-.13616007 \mathrm{E}+02$ |
| $\mathrm{XX}(35)=$ | $-.15041360 \mathrm{E}+01$ | $\mathrm{XX}(72)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(36)=$ | $-.36658537 \mathrm{E}+01$ | $\mathrm{XX}(73)=$ | $-.18917406 \mathrm{E}+02$ |
| $\mathrm{XX}(37)=$ | $-.36658537 \mathrm{E}+01$ |  |  |

## FINAL COMPLEX CONFIGURATION

The co ordinates of $t$ he $v$ ertices of the final co mplex af ter co nvergence are sh own below.

| VERTEX NUMBER 1 | VERTEX NUMBER 9 |
| :---: | :---: |
| FUNCTION VALUE $=.42824304 \mathrm{E}+07$ | FUNCTION VALUE $=.42826755 \mathrm{E}+07$ |
| COORDINATES | COORDINATES |
| $\mathrm{XT}(1)=.389000000 \mathrm{E}+04$ | $\mathrm{XT}(1)=.387500000 \mathrm{E}+04$ |
| $\mathrm{XT}(2)=.4450000 \mathrm{E}+03$ | $\mathrm{XT}(2)=.4320000 \mathrm{E}+03$ |
| $\mathrm{XT}(3)=.32000000 \mathrm{E}+03$ | $\mathrm{XT}(3)=.32500000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.25500000 \mathrm{E}+03$ | $\mathrm{XT}(4)=.25000000 \mathrm{E}+03$ |
| $\mathrm{XT}(5)=.18000000 \mathrm{E}+04$ | $\mathrm{XT}(5)=.19000000 \mathrm{E}+04$ |
| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ | $\mathrm{XT}(6)=.80000000 \mathrm{E}+01$ |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ |
| $\mathrm{XT}(8)=.42500000 \mathrm{E}+03$ | $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |
| $\mathrm{XT}(9)=.26370000 \mathrm{E}+00$ | $\mathrm{XT}(9)=.26370000 \mathrm{E}+00$ |
| $\mathrm{XT}(10)=.26500000 \mathrm{E}+03$ | $\mathrm{XT}(10)=.26500000 \mathrm{E}+03$ |
| $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ | $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |
| $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |
| VERTEX NUMBER 2 | VERTEX NUMBER 10 |
| FUNCTION VALUE $=.42834434 \mathrm{E}+07$ | FUNCTION VALUE $=.42812646 \mathrm{E}+07$ |
| COORDINATES | COORDINATES |
| $\mathrm{XT}(1)=.39900000 \mathrm{E}+04$ | $\mathrm{XT}(1)=.389000000 \mathrm{E}+04$ |
| $\mathrm{XT}(2)=.4520000 \mathrm{E}+03$ | $\mathrm{XT}(2)=.4450000 \mathrm{E}+03$ |


| $\mathrm{XT}(3)=.32000000 \mathrm{E}+03$ | XT(3) = |
| :---: | :---: |
| $\mathrm{XT}(4)=.25500000 \mathrm{E}+03$ | $\mathrm{XT}(4)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(5)=.16500000 \mathrm{E}+04$ | $\mathrm{XT}(5)=.18000000 \mathrm{E}+04$ |
| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ | $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ |
| $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ | $\mathrm{XT}(8)=.42500000 \mathrm{E}+03$ |
| $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ | $\mathrm{XT}(9)=.26370000 \mathrm{E}+00$ |
| $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ | $\mathrm{XT}(10)=.26500000 \mathrm{E}+03$ |
| $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ | $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |
| $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |
|  | VERTEX NUMBER 11 |
| VERTEX NUMBER 3 | FUNCTION VALUE $=.42828413 \mathrm{E}+07$ |
| FUNCTION VALUE $=.42809856 \mathrm{E}+07$ | COORDINATES |
| COORDINATES | $\mathrm{XT}(1)=.39900000 \mathrm{E}+04$ |
| $\mathrm{XT}(1)=.387500000 \mathrm{E}+04$ | $\mathrm{XT}(2)=.4520000 \mathrm{E}+03$ |
| $\mathrm{XT}(2)=.4320000 \mathrm{E}+03$ | $\mathrm{XT}(3)=.32000000 \mathrm{E}+03$ |
| $\mathrm{XT}(3)=.32500000 \mathrm{E}+03$ | $\mathrm{XT}(4)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.25000000 \mathrm{E}+03$ | $\mathrm{XT}(5)=.16500000 \mathrm{E}+04$ |
| $\mathrm{XT}(5)=.19000000 \mathrm{E}+04$ | $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ |
| $\mathrm{XT}(6)=.80000000 \mathrm{E}+01$ | $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |
| $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ | $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ |
| $\mathrm{XT}(9)=.26370000 \mathrm{E}+00$ | $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(10)=.26500000 \mathrm{E}+03$ | $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |
| $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |
| $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ | VERTEX NUMBER 12 |
| VERTEX NUMBER 4 | FUNCTION VALUE $=.42819999 \mathrm{E}+07$ |
| FUNCTION VALUE $=.42824698 \mathrm{E}+07$ | COORDINATES |
| COORDINATES | $\mathrm{XT}(1)=.387500000 \mathrm{E}+04$ |



## COORDINATES

$\mathrm{XT}(1)=.387500000 \mathrm{E}+04$
$\mathrm{XT}(2)=.4320000 \mathrm{E}+03$
$\mathrm{XT}(3)=.32500000 \mathrm{E}+03$
$\mathrm{XT}(4)=.25000000 \mathrm{E}+03$
$\mathrm{XT}(5)=.19000000 \mathrm{E}+04$
$\mathrm{XT}(6)=.80000000 \mathrm{E}+01$
$\mathrm{XT}(7)=.40000000 \mathrm{E}+01$
$\mathrm{XT}(8)=.43000000 \mathrm{E}+03$
$\mathrm{XT}(9)=.26370000 \mathrm{E}+00$
$\mathrm{XT}(10)=.26500000 \mathrm{E}+03$
$\mathrm{XT}(11)=.62849000 \mathrm{E}-02$
$\mathrm{XT}(12)=.75000000 \mathrm{E}+02$
$\mathrm{XT}(13)=.50000000 \mathrm{E}+02$
$\mathrm{XT}(14)=.15000000 \mathrm{E}+03$

## VERTEX NUMBER 7

FUNCTION VALUE $=.42815025 \mathrm{E}+07$
COORDINATES
$\mathrm{XT}(1)=.389000000 \mathrm{E}+04$
$\mathrm{XT}(2)=.4450000 \mathrm{E}+03$
$\mathrm{XT}(3)=.32000000 \mathrm{E}+03$
$\mathrm{XT}(4)=.25500000 \mathrm{E}+03$
$\mathrm{XT}(5)=.18000000 \mathrm{E}+04$
$\mathrm{XT}(6)=.90000000 \mathrm{E}+01$
$\mathrm{XT}(7)=.40000000 \mathrm{E}+01$
$\mathrm{XT}(8)=.42500000 \mathrm{E}+03$
$\mathrm{XT}(9)=.26370000 \mathrm{E}+00$
$\mathrm{XT}(10)=.26500000 \mathrm{E}+03$
$\mathrm{XT}(11)=.62849000 \mathrm{E}-02$
$\mathrm{XT}(12)=.75000000 \mathrm{E}+02$
$\mathrm{XT}(13)=.50000000 \mathrm{E}+02$
$\mathrm{XT}(14)=.15000000 \mathrm{E}+03$

FUNCTION VALUE $=.42837932 \mathrm{E}+07$
COORDINATES
$\mathrm{XT}(1)=.389000000 \mathrm{E}+04$
$\mathrm{XT}(2)=.4450000 \mathrm{E}+03$
$\mathrm{XT}(3)=.32000000 \mathrm{E}+03$
$\mathrm{XT}(4)=.25500000 \mathrm{E}+03$
$\mathrm{XT}(5)=.18000000 \mathrm{E}+04$
$\mathrm{XT}(6)=.90000000 \mathrm{E}+01$
$\mathrm{XT}(7)=.40000000 \mathrm{E}+01$
$\mathrm{XT}(8)=.42500000 \mathrm{E}+03$
$\mathrm{XT}(9)=.26370000 \mathrm{E}+00$
$\mathrm{XT}(10)=.26500000 \mathrm{E}+03$
$\mathrm{XT}(11)=.62849000 \mathrm{E}-02$
$\mathrm{XT}(12)=.75000000 \mathrm{E}+02$
$\mathrm{XT}(13)=.50000000 \mathrm{E}+02$
$\mathrm{XT}(14)=.15000000 \mathrm{E}+03$
VERTEX NUMBER 15
FUNCTION VALUE $=.42828669 \mathrm{E}+07$
COORDINATES
$\mathrm{XT}(1)=.387500000 \mathrm{E}+04$
$\mathrm{XT}(2)=.4320000 \mathrm{E}+03$
$\mathrm{XT}(3)=.32500000 \mathrm{E}+03$
$\mathrm{XT}(4)=.25000000 \mathrm{E}+03$
$\mathrm{XT}(5)=.19000000 \mathrm{E}+04$
$\mathrm{XT}(6)=.80000000 \mathrm{E}+01$
$\mathrm{XT}(7)=.40000000 \mathrm{E}+01$
$\mathrm{XT}(8)=.43000000 \mathrm{E}+03$
$\mathrm{XT}(9)=.26370000 \mathrm{E}+00$
$\mathrm{XT}(10)=.26500000 \mathrm{E}+03$
$\mathrm{XT}(11)=.62849000 \mathrm{E}-02$
$\mathrm{XT}(12)=.75000000 \mathrm{E}+02$
$\mathrm{XT}(13)=.50000000 \mathrm{E}+02$
$\mathrm{XT}(14)=.15000000 \mathrm{E}+03$

| VERTEX NUMBER 8 |  |
| :---: | :---: |
| FUNCTION VALUE $=.42824856 \mathrm{E}+07$ |  |
| COORDINATES |  |
| $\mathrm{XT}(1)=.39900000 \mathrm{E}+04$ |  |
| $\mathrm{XT}(2)=.4520000 \mathrm{E}+03$ |  |
| $\mathrm{XT}(3)=.32000000 \mathrm{E}+03$ |  |
| $\mathrm{XT}(4)=.25500000 \mathrm{E}+03$ |  |
| $\mathrm{XT}(5)=.16500000 \mathrm{E}+04$ |  |
| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ |  |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ |  |
| $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |  |
| $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ |  |
| $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ |  |
| $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |  |
| $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |  |
| $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |  |
| $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |  |
|  |  |

## FIRST RESTART OF "EVOP" TO CHECK THE MINIMUM

Automatic $r$ estart of E VOP $t$ akes $p$ lace $t o$ check $w$ hether $t$ he $p$ reviously obtained minimum is the global minimum. The initial complex as shown be low is generated taking $t$ he coordinates of $t$ he pr evious $m$ inimum (values obt ained from pr evious execution of EVOP) as the starting point of the complex.

INITIAL COMPLEX CONFIGURATION

| VERTEX NUMBER 1 | VERTEX NUMBER 9 |
| :---: | :---: |
| FUNCTION VALUE $=.42805835 \mathrm{E}+07$ | FUNCTION VALUE $=.43559536 \mathrm{E}+07$ |
| COORDINATES | COORDINATES |
| $\mathrm{XT}(1)=.39900000 \mathrm{E}+04$ | $\mathrm{XT}(1)=.36900000 \mathrm{E}+04$ |
| $\mathrm{XT}(2)=.4520000 \mathrm{E}+03$ | $\mathrm{XT}(2)=.3200000 \mathrm{E}+03$ |
| $\mathrm{XT}(3)=.32000000 \mathrm{E}+03$ | $\mathrm{XT}(3)=.42900000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.25500000 \mathrm{E}+03$ | $\mathrm{XT}(4)=.32000000 \mathrm{E}+03$ |


|  |  |
| :---: | :---: |
| $\mathrm{XT}(5)=.16500000 \mathrm{E}+04$ | $\mathrm{XT}(5)=.19000000 \mathrm{E}+04$ |
| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ | $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ |
| $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ | $\mathrm{XT}(8)=.39000000 \mathrm{E}+03$ |
| $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ | $\mathrm{XT}(9)=.34170000 \mathrm{E}+00$ |
| $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ | $\mathrm{XT}(10)=.32500000 \mathrm{E}+03$ |
| $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ | $\mathrm{XT}(11)=.54989000 \mathrm{E}-02$ |
| $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |
| VERTEX NUMBER 2 |  |
| FUNCTION VALUE $=.42278859 \mathrm{E}+07$ | VERTEX NUMBER 10 |
| COORDINATES | FUNCTION VALUE $=.44066776 \mathrm{E}+07$ |
| $\mathrm{XT}(1)=.36900000 \mathrm{E}+04$ | COORDINATES |
| $\mathrm{XT}(2)=.3200000 \mathrm{E}+03$ | $\mathrm{XT}(1)=.39900000 \mathrm{E}+04$ |
| $\mathrm{XT}(3)=.42900000 \mathrm{E}+03$ | $\mathrm{XT}(2)=.4520000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.32000000 \mathrm{E}+03$ | $\mathrm{XT}(3)=.32000000 \mathrm{E}+03$ |
| $\mathrm{XT}(5)=.19000000 \mathrm{E}+04$ | $\mathrm{XT}(4)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ | $\mathrm{XT}(5)=.16500000 \mathrm{E}+04$ |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ |
| $\mathrm{XT}(8)=.39000000 \mathrm{E}+03$ | $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ |
| $\mathrm{XT}(9)=.34170000 \mathrm{E}+00$ | $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |
| $\mathrm{XT}(10)=.32500000 \mathrm{E}+03$ | $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ |
| $\mathrm{XT}(11)=.54989000 \mathrm{E}-02$ | $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ | $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |
| $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| VERTEX NUMBER 3 | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |
| FUNCTION VALUE $=.44438093 \mathrm{E}+07$ | VERTEX NUMBER 11 |
| COORDINATES | FUNCTION VALUE $=.47584549 \mathrm{E}+07$ |
| $\mathrm{XT}(1)=.389000000 \mathrm{E}+04$ | COORDINATES |
| $\mathrm{XT}(2)=.4450000 \mathrm{E}+03$ | $\mathrm{XT}(1)=.389000000 \mathrm{E}+04$ |
| $\mathrm{XT}(3)=.32000000 \mathrm{E}+03$ | $\mathrm{XT}(2)=.4450000 \mathrm{E}+03$ |

$\mathrm{XT}(4)=.25500000 \mathrm{E}+03$
$\mathrm{XT}(5)=.18000000 \mathrm{E}+04$
$\mathrm{XT}(6)=.90000000 \mathrm{E}+01$
$\mathrm{XT}(7)=.40000000 \mathrm{E}+01$
$\mathrm{XT}(8)=.42500000 \mathrm{E}+03$
$\mathrm{XT}(9)=.26370000 \mathrm{E}+00$
$\mathrm{XT}(10)=.26500000 \mathrm{E}+03$
$\mathrm{XT}(11)=.62849000 \mathrm{E}-02$
$\mathrm{XT}(12)=.75000000 \mathrm{E}+02$
$\mathrm{XT}(13)=.50000000 \mathrm{E}+02$
$\mathrm{XT}(14)=.15000000 \mathrm{E}+03$

## VERTEX NUMBER 4

FUNCTION VALUE $=.45273709 \mathrm{E}+07$ COORDINATES
$\mathrm{XT}(1)=.389000000 \mathrm{E}+04$
$\mathrm{XT}(2)=.4450000 \mathrm{E}+03$
$\mathrm{XT}(3)=.32000000 \mathrm{E}+03$
$\mathrm{XT}(4)=.25500000 \mathrm{E}+03$
$\mathrm{XT}(5)=.18000000 \mathrm{E}+04$
$\mathrm{XT}(6)=.90000000 \mathrm{E}+01$
$\mathrm{XT}(7)=.40000000 \mathrm{E}+01$
$\mathrm{XT}(8)=.42500000 \mathrm{E}+03$
$\mathrm{XT}(9)=.26370000 \mathrm{E}+00$
$\mathrm{XT}(10)=.26500000 \mathrm{E}+03$
$\mathrm{XT}(11)=.62849000 \mathrm{E}-02$
$\mathrm{XT}(12)=.75000000 \mathrm{E}+02$
$\mathrm{XT}(13)=.50000000 \mathrm{E}+02$
$\mathrm{XT}(14)=.15000000 \mathrm{E}+03$
VERTEX NUMBER 5
FUNCTION VALUE $=.52749319 \mathrm{E}+07$ COORDINATES
$\mathrm{XT}(1)=.389000000 \mathrm{E}+04$

```
XT(3) = .32000000E +03
XT(4) = .25500000E +03
XT(5) = .18000000E +04
XT(6) = .90000000E +01
XT(7) = .40000000E+01
XT( 8) = .42500000E +03
XT(9) = .26370000E+00
XT(10) = .26500000E +03
XT(11)=.62849000E-02
XT(12) = .75000000E+02
XT(13) = .50000000E+02
XT(14) = .15000000E+03
```


## VERTEX NUMBER 12

FUNCTION VALUE $=.43274612 \mathrm{E}+07$
COORDINATES
$\mathrm{XT}(1)=.39900000 \mathrm{E}+04$
$\mathrm{XT}(2)=.4520000 \mathrm{E}+03$
$\mathrm{XT}(3)=.32000000 \mathrm{E}+03$
$\mathrm{XT}(4)=.25500000 \mathrm{E}+03$
$\mathrm{XT}(5)=.16500000 \mathrm{E}+04$
$\mathrm{XT}(6)=.90000000 \mathrm{E}+01$
$\mathrm{XT}(7)=.40000000 \mathrm{E}+01$
$\mathrm{XT}(8)=.43000000 \mathrm{E}+03$
$\mathrm{XT}(9)=.27170000 \mathrm{E}+00$
$\mathrm{XT}(10)=.25500000 \mathrm{E}+03$
$\mathrm{XT}(11)=.62849000 \mathrm{E}-02$
$\mathrm{XT}(12)=.75000000 \mathrm{E}+02$
$\mathrm{XT}(13)=.50000000 \mathrm{E}+02$
$\mathrm{XT}(14)=.15000000 \mathrm{E}+03$

VERTEX NUMBER 13

| $\begin{aligned} & \mathrm{XT}(2)=.4450000 \mathrm{E}+03 \\ & \mathrm{XT}(3)=.32000000 \mathrm{E}+03 \\ & \mathrm{XT}(4)=.25500000 \mathrm{E}+03 \\ & \mathrm{XT}(5)=.18000000 \mathrm{E}+04 \\ & \mathrm{XT}(6)=.90000000 \mathrm{E}+01 \\ & \mathrm{XT}(7)=.40000000 \mathrm{E}+01 \\ & \mathrm{XT}(8)=.42500000 \mathrm{E}+03 \\ & \mathrm{XT}(9)=.26370000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.26500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.62849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.75000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ <br> VERTEX NUMBER 6 <br> FUNCTION VALUE $=.46799539 \mathrm{E}+07$ COORDINATES $\begin{gathered} \mathrm{XT}(1)=.389000000 \mathrm{E}+04 \\ \mathrm{XT}(2)=.4450000 \mathrm{E}+03 \\ \mathrm{XT}(3)=.32000000 \mathrm{E}+03 \\ \mathrm{XT}(4)=.25500000 \mathrm{E}+03 \\ \mathrm{XT}(5)=.18000000 \mathrm{E}+04 \\ \mathrm{XT}(6)=.90000000 \mathrm{E}+01 \\ \mathrm{XT}(7)=.40000000 \mathrm{E}+01 \\ \mathrm{XT}(8)=.42500000 \mathrm{E}+03 \\ \mathrm{XT}(9)=.26370000 \mathrm{E}+00 \\ \mathrm{XT}(10)=.26500000 \mathrm{E}+03 \\ \mathrm{XT}(11)=.62849000 \mathrm{E}-02 \\ \mathrm{XT}(12)=.75000000 \mathrm{E}+02 \\ \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{gathered}$ <br> VERTEX NUMBER 7 | FUNCTION VALUE $=.57948675 \mathrm{E}+07$ <br> COORDINATES $\begin{aligned} & \mathrm{XT}(1)=.389000000 \mathrm{E}+04 \\ & \mathrm{XT}(2)=.4450000 \mathrm{E}+03 \\ & \mathrm{XT}(3)=.32000000 \mathrm{E}+03 \\ & \mathrm{XT}(4)=.25500000 \mathrm{E}+03 \\ & \mathrm{XT}(5)=.18000000 \mathrm{E}+04 \\ & \mathrm{XT}(6)=.90000000 \mathrm{E}+01 \\ & \mathrm{XT}(7)=.40000000 \mathrm{E}+01 \\ & \mathrm{XT}(8)=.42500000 \mathrm{E}+03 \\ & \mathrm{XT}(9)=.26370000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.26500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.62849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.75000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ <br> VERTEX NUMBER 14 <br> FUNCTION VALUE $=.47675648 \mathrm{E}+07$ <br> COORDINATES $\begin{aligned} & \mathrm{XT}(1)=.39900000 \mathrm{E}+04 \\ & \mathrm{XT}(2)=.4520000 \mathrm{E}+03 \\ & \mathrm{XT}(3)=.32000000 \mathrm{E}+03 \\ & \mathrm{XT}(4)=.25500000 \mathrm{E}+03 \\ & \mathrm{XT}(5)=.16500000 \mathrm{E}+04 \\ & \mathrm{XT}(6)=.90000000 \mathrm{E}+01 \\ & \mathrm{XT}(7)=.40000000 \mathrm{E}+01 \\ & \mathrm{XT}(8)=.43000000 \mathrm{E}+03 \\ & \mathrm{XT}(9)=.27170000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.25500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.62849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.75000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ |
| :---: | :---: |


| FUNCTION VALUE $=.42092963 \mathrm{E}+07$ <br> COORDINATES $\begin{aligned} & \mathrm{XT}(1)=.40000000 \mathrm{E}+04 \\ & \mathrm{XT}(2)=.4500000 \mathrm{E}+03 \\ & \mathrm{XT}(3)=.32500000 \mathrm{E}+03 \\ & \mathrm{XT}(4)=.26000000 \mathrm{E}+03 \\ & \mathrm{XT}(5)=.17000000 \mathrm{E}+04 \\ & \mathrm{XT}(6)=.90000000 \mathrm{E}+01 \\ & \mathrm{XT}(7)=.40000000 \mathrm{E}+01 \\ & \mathrm{XT}(8)=.43000000 \mathrm{E}+03 \\ & \mathrm{XT}(9)=.27170000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.25500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.62849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.75000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ <br> VERTEX NUMBER 8 <br> FUNCTION VALUE $=.49540199 \mathrm{E}+07$ COORDINATES $\begin{aligned} & \mathrm{XT}(1)=.40000000 \mathrm{E}+04 \\ & \mathrm{XT}(2)=.4500000 \mathrm{E}+03 \\ & \mathrm{XT}(3)=.32500000 \mathrm{E}+03 \\ & \mathrm{XT}(4)=.26000000 \mathrm{E}+03 \\ & \mathrm{XT}(5)=.17000000 \mathrm{E}+04 \\ & \mathrm{XT}(6)=.90000000 \mathrm{E}+01 \\ & \mathrm{XT}(7)=.40000000 \mathrm{E}+01 \\ & \mathrm{XT}(8)=.43000000 \mathrm{E}+03 \\ & \mathrm{XT}(9)=.27170000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.25500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.62849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.75000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ | VERTEX NUMBER 15 <br> FUNCTION VALUE $=.45442755 \mathrm{E}+07$ <br> COORDINATES $\begin{aligned} & \mathrm{XT}(1)=.40000000 \mathrm{E}+04 \\ & \mathrm{XT}(2)=.4500000 \mathrm{E}+03 \\ & \mathrm{XT}(3)=.32500000 \mathrm{E}+03 \\ & \mathrm{XT}(4)=.26000000 \mathrm{E}+03 \\ & \mathrm{XT}(5)=.17000000 \mathrm{E}+04 \\ & \mathrm{XT}(6)=.90000000 \mathrm{E}+01 \\ & \mathrm{XT}(7)=.40000000 \mathrm{E}+01 \\ & \mathrm{XT}(8)=.43000000 \mathrm{E}+03 \\ & \mathrm{XT}(9)=.27170000 \mathrm{E}+00 \\ & \mathrm{XT}(10)=.25500000 \mathrm{E}+03 \\ & \mathrm{XT}(11)=.62849000 \mathrm{E}-02 \\ & \mathrm{XT}(12)=.75000000 \mathrm{E}+02 \\ & \mathrm{XT}(13)=.50000000 \mathrm{E}+02 \\ & \mathrm{XT}(14)=.15000000 \mathrm{E}+03 \end{aligned}$ |
| :---: | :---: |

## OUTPUT SUMMARY FROM SUBROUTINE EVOP

MINIMUM OF THE OBJECTIVE FUNCTION HAS BEEN LOCATED TO THE
DESIRED DEGREE OF ACCURACY FOR CONVERGENCE. IER $=8$ TOTAL NUMBER OF OBJECTIVE FUNCTION EVALUATION

NFUNC = 371

NUMBER OF TIMES THE SUBROUTINE FUNCTION IS CALLED DURING THE PRESENT CONVERGENCE TESTS. KUT = 6

NUMBER OF TIMES THE EXPLICIT CONSTRAINTS WERE EVALUATED
$\mathrm{KKT}=3514$
NUMBER OF TIMES THE IMPLICIT CONSTRAINTS WERE EVALUATED
$\mathrm{M}=2186$

## COORDINATES OF THE MINIMUM

| $\mathrm{XT}(1)=.40000000 \mathrm{E}+04$ | $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |
| :---: | :--- |
| $\mathrm{XT}(2)=.4500000 \mathrm{E}+03$ | $\mathrm{XT}(9)=.29170000 \mathrm{E}+00$ |
| $\mathrm{XT}(3)=.32000000 \mathrm{E}+03$ | $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.26000000 \mathrm{E}+03$ | $\mathrm{XT}(11)=.62549000 \mathrm{E}-02$ |
| $\mathrm{XT}(5)=.17500000 \mathrm{E}+04$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(6)=.9000000 \mathrm{E}+01$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| $\mathrm{XT}(7)=.4000000 \mathrm{E}+01$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |

OBJECTIVE FUNCTION VALUE AT THE MINIMUM F = .41296593E+07

## SECOND RESTART OF "EVOP" TO CHECK THE MINIMUM

OUTPUT SUMMARY FROM SUBROUTINE EVOP
MINIMUM OF THE OBJECTIVE FUNCTION HAS BEEN LOCATED TO THE DESIRED DEGREE OF ACCURACY FOR CONVERGENCE. IER $=8$ TOTAL NUMBER OF OBJECTIVE FUNCTION EVALUATION.

NFUNC $=$ 342

NUMBER OF TIMES THE SUBROUTINE FUNCTION IS CALLED DURING THE PRESENT CONVERGENCE TESTS. KUT =6

NUMBER OF TIMES THE EXPLICIT CONSTRAINTS WERE EVALUATED $\mathrm{KKT}=4592$

NUMBER OF TIMES THE IMPLICIT CONSTRAINTS WERE EVALUATED $\mathrm{M}=2810$

COORDINATES OF THE MINIMUM

| $\mathrm{XT}(1)=.40000000 \mathrm{E}+04$ | $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |
| :---: | :--- |
| $\mathrm{XT}(2)=.4500000 \mathrm{E}+03$ | $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ |
| $\mathrm{XT}(3)=.32500000 \mathrm{E}+03$ | $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.26000000 \mathrm{E}+03$ | $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |
| $\mathrm{XT}(5)=.17000000 \mathrm{E}+04$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |

OBJECTIVE FUNCTION VALUE AT THE MINIMUM $\mathbf{F}=\boldsymbol{. 4 1 1 3 7 6 0 7 E + 0 7}$

## THIRD RESTART OF "EVOP" TO CHECK THE MINIMUM

OUTPUT SUMMARY FROM SUBROUTINE EVOP
MINIMUM OF THE OBJECTIVE FUNCTION HAS BEEN LOCATED TO THE DESIRED DEGREE OF ACCURACY FOR CONVERGENCE. IER $=8$ TOTAL NUMBER OF OBJECTIVE FUNCTION EVALUATION.
NFUNC $=122$
NUMBER OF TIMES THE SUBROUTINE FUNCTION IS CALLED DURING THE PRESENT CONVERGENCE TESTS. KUT $=6$

NUMBER OF TIMES THE EXPLICIT CONSTRAINTS WERE EVALUATED
KKT $=2122$
NUMBER OF TIMES THE IMPLICIT CONSTRAINTS WERE EVALUATED
$M=1209$
COORDINATES OF THE MINIMUM

| $\mathrm{XT}(1)=.40000000 \mathrm{E}+04$ | $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |
| :---: | :--- |
| $\mathrm{XT}(2)=.4500000 \mathrm{E}+03$ | $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ |
| $\mathrm{XT}(3)=.32500000 \mathrm{E}+03$ | $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.26000000 \mathrm{E}+03$ | $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |
| $\mathrm{XT}(5)=.17000000 \mathrm{E}+04$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |


| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| :--- | :--- |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |

OBJECTIVE FUNCTION VALUE AT THE MINIMUM $\mathbf{F}=\boldsymbol{. 4 1 8 8 3 0 9 5 E + 0 7}$

## FOURTH RESTART OF "EVOP" TO CHECK THE MINIMUM

OUTPUT SUMMARY FROM SUBROUTINE EVOP
MINIMUM OF THE OBJECTIVE FUNCTION HAS BEEN LOCATED TO THE DESIRED DEGREE OF ACCURACY FOR CONVERGENCE. $\operatorname{IER}=8$ TOTAL NUMBER OF OBJECTIVE FUNCTION EVALUATION.

NFUNC = 32
NUMBER OF TIMES THE SUBROUTINE FUNCTION IS CALLED DURING THE PRESENT CONVERGENCE TESTS. KUT = 7

NUMBER OF TIMES THE EXPLICIT CONSTRAINTS WERE EVALUATED $\mathrm{KKT}=256$

NUMBER OF TIMES THE IMPLICIT CONSTRAINTS WERE EVALUATED $\mathrm{M}=171$

COORDINATES OF THE MINIMUM

| $\mathrm{XT}(1)=.40000000 \mathrm{E}+04$ | $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |
| :---: | :--- |
| $\mathrm{XT}(2)=.4500000 \mathrm{E}+03$ | $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ |
| $\mathrm{XT}(3)=.32500000 \mathrm{E}+03$ | $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ |
| $\mathrm{XT}(4)=.26000000 \mathrm{E}+03$ | $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |
| $\mathrm{XT}(5)=.17000000 \mathrm{E}+04$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |

OBJECTIVE FUNCTION VALUE AT THE MINIMUM $\quad \mathbf{F}=\mathbf{. 4 1 4 1 3 6 4 7 E + 0 7}$

Further restart of EVOP gives the same coordinates of the minimum as obtained in the fourth restart. So these coordinates of the minimum obt ained in this restart a re the optimum s olutions a nd obj ective f unction value a t hem inimumis, $\mathbf{F}=$ $.46413647 \mathrm{E}+\mathbf{0 7}$. The $p$ rogram is $r$ erun us ing $t$ hese opt imum de sign variables a $s$ starting point with same values of control parameters and the minimum remains same.

So the global minimum is $\mathbf{F}=\mathbf{. 4 1 1 3 7 6 0 7 E}+\mathbf{0 7}$ and optimum value of the design variables are as follows:

| Serial No. | Design variables | Design variables |
| :---: | :---: | :---: |
| 1 | $\mathrm{~S}=4000 ;$ | $\mathrm{XT}(1)=.40000000 \mathrm{E}+04$ |
| 2 | $\mathrm{TF}_{\mathrm{w}}=450 ;$ | $\mathrm{XT}(2)=.4500000 \mathrm{E}+03$ |
| 3 | $\mathrm{BF}_{\mathrm{w}}=325 ;$ | $\mathrm{XT}(3)=.32500000 \mathrm{E}+03$ |
| 4 | $\mathrm{BF}_{\mathrm{t}}=260 ;$ | $\mathrm{XT}(4)=.26000000 \mathrm{E}+03$ |
| 5 | $\mathrm{G}_{\mathrm{d}}=1700 ;$ | $\mathrm{XT}(5)=.17000000 \mathrm{E}+04$ |
| 6 | $\mathrm{~N}_{\mathrm{S}}=9.0 ;$ | $\mathrm{XT}(6)=.90000000 \mathrm{E}+01$ |
| 7 | $\mathrm{~N}_{\mathrm{T}}=4.0 ;$ | $\mathrm{XT}(7)=.40000000 \mathrm{E}+01$ |
| 8 | $\mathrm{y}_{1}=430 ;$ | $\mathrm{XT}(8)=.43000000 \mathrm{E}+03$ |
| 9 | $\eta=0.27 ;$ | $\mathrm{XT}(9)=.27170000 \mathrm{E}+00$ |
| 10 | $\mathrm{t}=255 ;$ | $\mathrm{XT}(10)=.25500000 \mathrm{E}+03$ |
| 11 | $\rho=0.006284 ;$ | $\mathrm{XT}(11)=.62849000 \mathrm{E}-02$ |
| 12 | $\mathrm{TF}_{\mathrm{t}}=75 ;$ | $\mathrm{XT}(12)=.75000000 \mathrm{E}+02$ |
| 13 | $\mathrm{TFT}_{\mathrm{t}}=50 ;$ | $\mathrm{XT}(13)=.50000000 \mathrm{E}+02$ |
| 14 | $\mathrm{~W}_{\mathrm{w}}=150 ;$ | $\mathrm{XT}(14)=.15000000 \mathrm{E}+03$ |


[^0]:    AS $=$ Anchorage spacing

[^1]:    ＊Cost in（BDT）per square meter of deck slab；TC＝Total cost；

