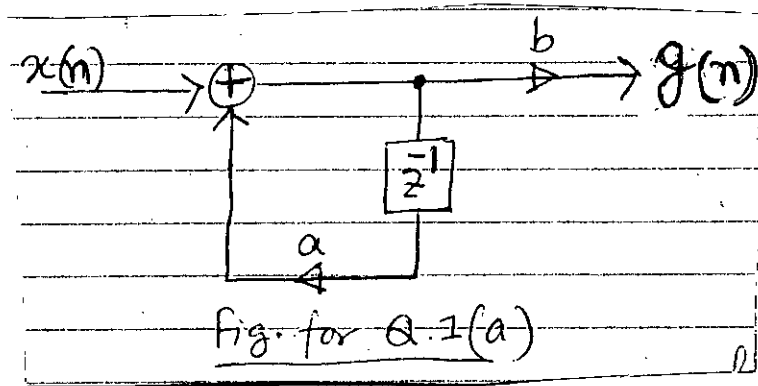


SECTION - A

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols and abbreviations have their usual meanings

1. (a) Consider the discrete-time system shown in Fig. for Q. 1(a).



(i) Determine b in terms of a such that

$$\sum_{n=-\infty}^{\infty} h(n) = 1$$

(ii) Compute the zero-state step response S(n) of the system and choose b so that S(∞) = 1. (18)

(b) An audio signal S(t) generated by a loudspeaker is reflected at two different walls with reflection coefficients r₁ and r₂. The signal x(t) recorded by a microphone close to the loudspeaker, after sampling, is (17)

$$x(n) = S(n) + r_1 S(n - K_1) + r_2 S(n - K_2) \text{ where } K_1 \text{ and } K_2 \text{ are the delays of the two echoes.}$$

(i) Determine the auto correlation r_{xx}(l) of the signal x(n).

(ii) Can we obtain r₁, r₂, K₁, and K₂ by observing r_{xx}(l)?

(iii) What happens if r₂ = 0?

2. (a) The causal system (17)

$$H(z) = \frac{1}{1 + \sum_{K=1}^N a_K z^{-K}}$$

is unstable. We modify this system by changing its impulse response

$$h(n) \text{ to } h'(n) = b^n h(n) u(n).$$

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Contd. to Q. 2

- (i) Show that by properly choosing b we can obtain a new stable system.
- (ii) What is the difference equation describing the new system?

(b) A causal linear time invariant discrete-time system has system function (18)

$$H(z) = \frac{(1 - 0.5z^{-1})(1 + 4z^{-2})}{1 - 0.64z^{-2}}$$

- (i) Express H(z) using a minimum-phase system H₁(z) and an all-pass system H_{ap}(z) such that H(z) = H₁(z) H_{ap}(z).
- (ii) Express H(z) employing a minimum-phase system H₂(z) and a generalization linear phase FIR system H_{lin}(z) such that H(z) = H₂(z) H_{lin}(z)

3. (a) A causal LTI system has a system function (19)

$$H(z) = \frac{1 + z^{-1}}{1 - \frac{1}{2}z^{-1}}$$

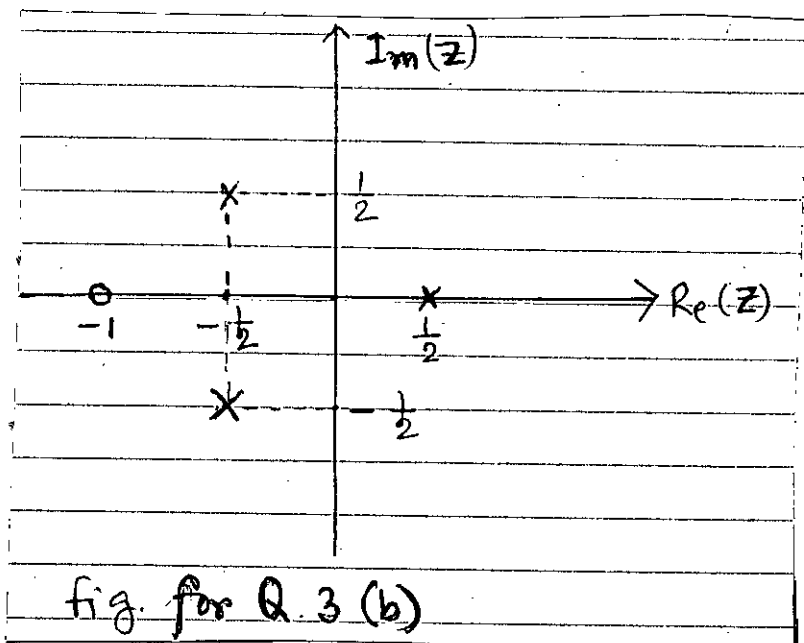
Find the z-transform of the input, x(n), that will produce the output

$$y(n) = \frac{1}{3} \left(\frac{1}{4}\right)^n u(n) - \frac{4}{3} (2)^n u(-n-1)$$

Also, find the input signal x(n).

(b) Let x(n) be a causal sequence with z-transform X(z) whose pole-zero plot is shown in Fig. for Q. 3(b). Sketch the pole-zero plots and the ROC of the following sequences: (16)

- (i) x₁(n) = x(-n + 2)
- (ii) x₂(n) = e^{jπ/3 n} x(n)



Contd. P/3

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4. (a) A major problem in the recording of ECGs is the appearance of unwanted 60-Hz interference in the output. Assume that the bandwidth of the signal of interest is 1 KHz, that is,

$$X_a(F) = 0, \quad |F| > 1000 \text{ Hz}$$

The analog signal is converted into a discrete-time signal with an ideal A/D converter operating using a sampling frequency F_s . The resulting signal $x(n) = x_a(nT)$ is then processed with a discrete-time system that is described by the difference equation

$$y(n) = x(n) + ax(n-1) + bx(n-2)$$

The filtered signal, $y(n)$, is then converted back into an analog signal using an ideal D/A converter. Design a system for removing the 60-Hz interference by specifying values for F_s , a , and b so that a 60-Hz signal of the form

$$V_a(t) = A \sin(120 \pi t)$$

will not appear in the output of the D/A converter.

- (b) Find the zero-input response of the causal system characterized by the following difference equation

$$y(n) = -y(n-1) + 6y(n-2) + x(n) + 3x(n-1)$$

Given that $y(-1) = 1$ and $y(-2) = 0.5$.

Is the system stable? Justify your answer.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols bear their usual meanings

5. (a) Given that $x(n) = \sin\left(\frac{n\pi}{2}\right)$, $n = 0, 1, 2, 3$

$$y(n) = 2^n, \quad n = 0, 1, 2, 3$$

Calculate the followings:

- (i) $X(k)$ using four-point DFT of $x(n)$
- (ii) $Y(k)$ using four-point DFT of $y(n)$
- (iii) $z(n) = x(n) \otimes y(n)$
- (iv) $z(n)$ using IDFT

- (b) Describe overlap-add method of filtering of long data sequence using DFT. What are the implications of finite data record in frequency analysis using DFT?

6. (a) Let $X_m(\omega)$ ($m = 1, 2, 3$) be the DTFT of the discrete-time signal $x_m(n)$. If $x_3(n) = x_1(n)x_2(n)$, then prove that

$$X_3(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\lambda)X_2(\omega - \lambda) d\lambda$$

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Contd. to Q. 6

(b) Consider $x_a(t)$ be an analog signal with bandwidth $B = 3$ kHz. It is required to find an $N = 2^m$ point DFT of the signal with a resolution less than or equal to 50 Hz. Determine (12)

(i) the minimum sampling rate and

(ii) the minimum number of samples required.

(c) Obtain the direct form II realization of the system given by (13)

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$$

7. (a) What are the two main problems of implementing an ideal FIR low-pass filter? Mention how to overcome the problems. (8)

(b) What is the problem of using a rectangular window? Explain why the use of Hamming window can reduce the problem. (7)

(c) Use the Kaiser window method to design a real-valued type II FIR filter with generalized linear phase that meets the following specifications:

$$\begin{aligned} 0.9 < |H(e^{j\omega})| < 1.1 & \quad 0 \leq |\omega| \leq 0.2\pi \\ -0.06 < |H(e^{j\omega})| < 0.06 & \quad 0.3\pi \leq |\omega| \leq 0.475\pi \\ 1.9 < |H(e^{j\omega})| < 2.1 & \quad 0.525\pi \leq |\omega| \leq \pi \end{aligned}$$

Find the Kaiser window function and the impulse response of the filter. (20)

8. (a) What is meant by the term "impulse invariance"? What is the major limitation of the Impulse-invariance method? Suggest a simple way to overcome the limitation. (10)

(b) A discrete-time low-pass filter is to be designed by applying the impulse invariance method to a continuous time Butterworth filter having the magnitude squared function of (25)

$$|H_c(j\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

The specifications for the discrete-time filter are:

$$\begin{aligned} 0.89125 \leq |H(e^{j\omega})| \leq 1, & \quad 0 \leq |\omega| \leq 0.2\pi \\ |H(e^{j\omega})| \leq 0.17783, & \quad 0.3\pi \leq |\omega| \leq \pi \end{aligned}$$

Design the continuous-time Butterworth filter to meet the passband and stopband specifications as determined by the desired discrete-time filter.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols and abbreviations have their usual meanings. Semi-log papers are to be supplied.

1. (a) A mass-spring model of a biomedical system is shown in Fig. 1. The parameters are defined as follows: (15)

$f(t)$ is the input force,

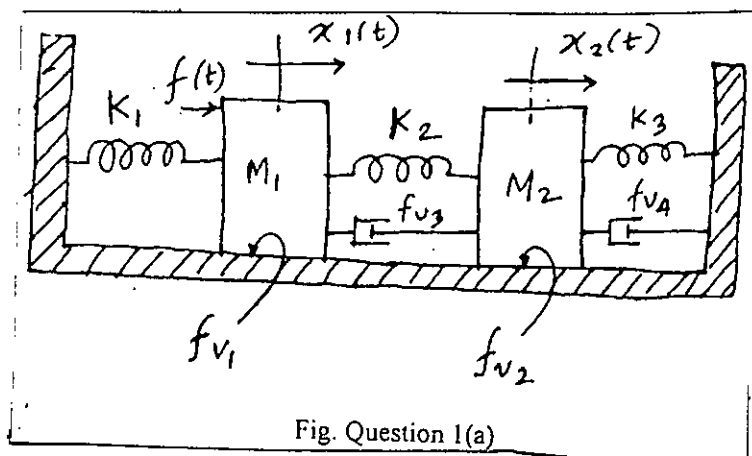
$x_1(t)$ and $x_2(t)$ are displacement,

M_1 and M_2 are masses,

f_{v1} , f_{v2} , f_{v3} , f_{v4} are coefficients of viscous friction and

K_1 , K_2 and K_3 are spring constants.

Find the transfer function, $X_2(s)/F(s)$ for the system.



- (b) Draw a series circuit analog for the system provided in 1(a). Find the state-space representation of the obtained electrical circuit considering the parameter $v_2(t)$ (velocity of the mass M_2) as the output of the state-space model. Finally, write the state-space model in a matrix form. (5+15)

2. (a) Write the general form of a second-order system using natural frequency (ω_n) and damping ratio (ζ). Define and explain these parameters. For an underdamped second-order system excited by a unit step function, derive the expressions for (i) Peak time, (ii) Percent overshoot, and (iii) Settling time, in terms of its natural frequency and damping ratio. The step-response of a general second-order system is given by (15)

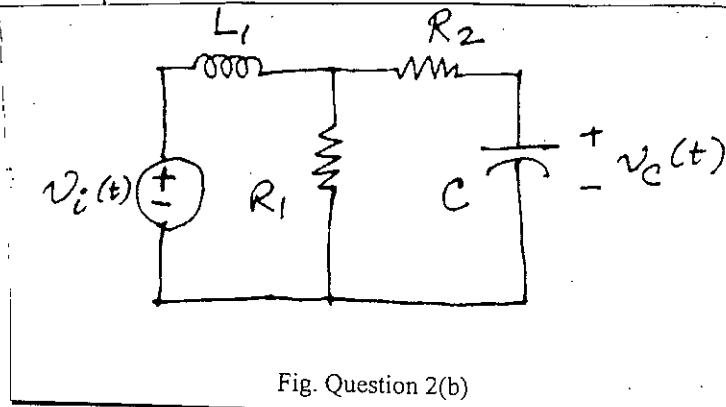
$$c(t) = 1 - e^{-\zeta\omega_n t} \left(\cos \omega_n \sqrt{1-\zeta^2} t + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_n \sqrt{1-\zeta^2} t \right)$$

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Contd ... Q. No. 2

(b) An equivalent circuit of a physiological system is shown in the figure below. The input and output of the system are $v_i(t)$ and $v_c(t)$, respectively. For this system, find the values of R_2 and C to yield an overshoot of 8% with a settling time of 1 ms. In the model, assume $R_1 = 1 \text{ M}\Omega$ and $L_1 = 1 \text{ H}$.

(10)



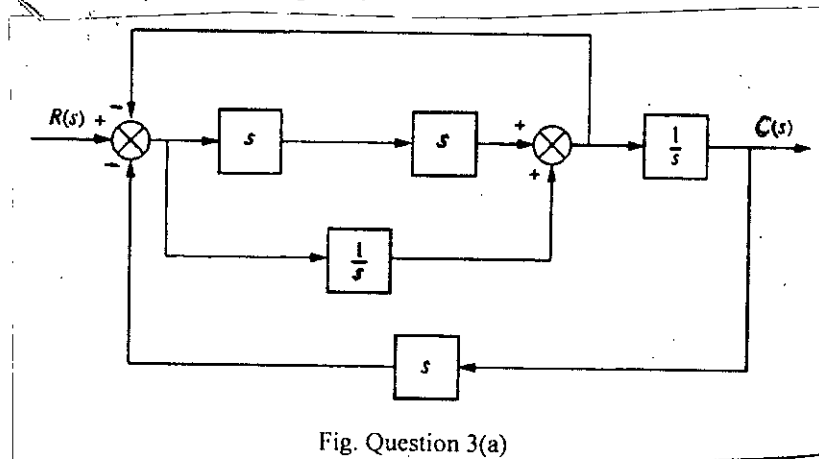
(c) The transfer function of a system $G(s)$ is provided below. Find a state-space representation of the system and draw its equivalent block diagram.

(10)

$$G(s) = \frac{24}{(s^3 + 9s^2 + 26s + 24)}$$

3. (a) A control system is shown below. Use block diagram reduction methods to obtain the transfer function of the system. Compare your answer with that obtained by Mason's rule.

(15)



(b) For a unity-gain negative feedback system, the open loop transfer function $G(s)$ is provided below. Determine the ranges/values of K that will result in a stable, unstable, and marginally stable system. Use the Routh-Hurwitz table.

(10)

$$G(s) = \frac{K(s+6)}{s(s+1)(s+2)}$$

(c) A system is represented below in the state-space form. Find the transfer function of the system $T(s) = Y(s)/U(s)$ using the matrix method.

(10)

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = [1 \ 0 \ 0] \mathbf{x}$$

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4. (a) For the system $G(s)$ provided below, sketch the Bode asymptotic magnitude and asymptotic phase plots. (10)

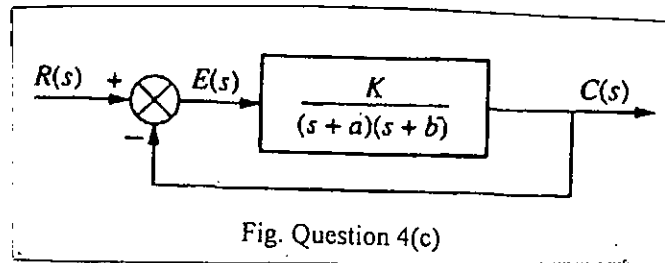
$$G(s) = \frac{(s + 5)}{(s + 2)(s + 4)}$$

- (b) A unity-gain negative feedback system has an open loop transfer function: (15)

$$G(s) = \frac{K(s + \alpha)}{(s + \beta)^2}$$

Design the system to meet the following specifications: steady-state error for a unit step input = 0.1; damping ratio = 0.5; natural frequency = $\sqrt{10}$.

- (c) A unity-gain negative feedback system has an open-loop transfer function of $\frac{K}{(s + a)(s + b)}$ as shown below. Determine the system type and the values of the static error constants. Find the sensitivity of the steady-state error to changes in parameter K and parameter a (assuming an appropriate input). (10)



SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

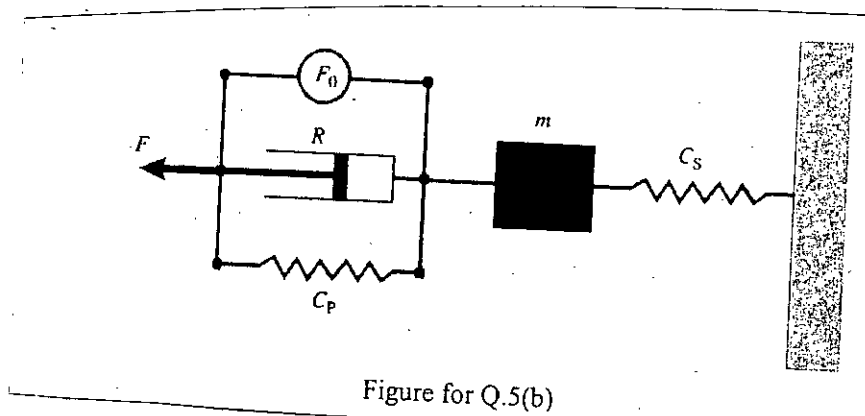
5. (a) Based on the description of the following physiological reflex system, construct a block diagram to represent the major control mechanisms involved. Clearly identify the physiological correlates of the controller, the plant, and the feedback element, as well as the controlling, controlled, and feedback variables. Also, identify the type of feedback and explain how it is achieved. (12+3)

"The control system that regulates water balance is intimately coupled with the control of sodium excretion. When sodium is reabsorbed by the distal tubules of the kidneys, water will also be reabsorbed if the permeability of the tubular epithelium is lowered. This is achieved in the following way. When there is a drop in plasma volume, mean systemic pressure decreases, leading to a change in stimulation of the left atrial pressure receptors. The latter send signals to a group of neurons in the hypothalamus, increasing its production of vasopressin or antidiuretic hormone (ADH). As a result, the ADH concentration in blood plasma increases, which leads to an increase in water permeability of the kidney distal tubules and collecting ducts."

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Contd ... Q. No. 5

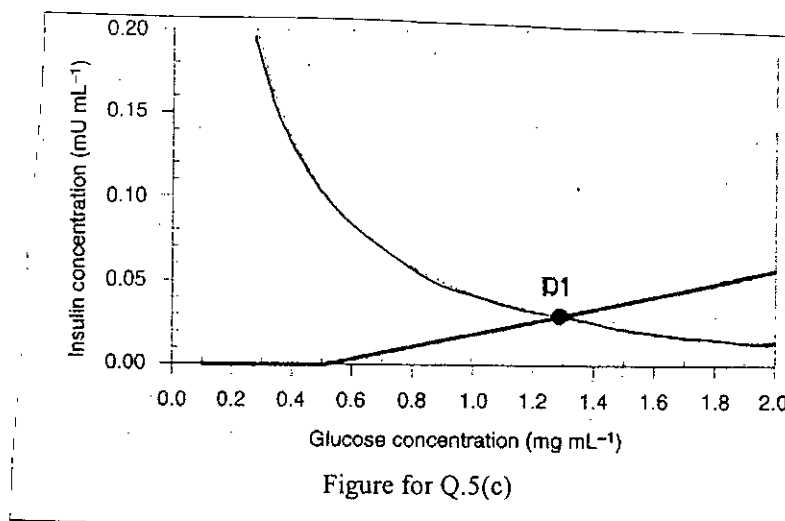
(b) A mechanical model of linear muscle mechanics from is shown in the Figure for Q. 5(b) below. Here, the elastic element C_p is placed in parallel to the viscous damping element R and the contractile element, and the entire parallel combination is placed in series with the elastic element C_s and the lumped representation of the muscle mass m . Draw the equivalent electrical model. (10)



(c) In the Figure for Q. 5(c) below, the thin curve represents the glucose response to insulin, while the bold line represents the insulin response to glucose. In this case, the subject represented by these curves is a type-1 diabetic patient (one whose pancreas is unable to produce sufficient insulin), who has high glucose and low insulin levels. Sketch the figure in your answer script (exact sketch is not required).

(i) Draw necessary lines on the figure to show the new steady-state operating point for glucose-insulin, if this patient were to wear an insulin pump that continuously infuses insulin at a constant rate into his body (independent of blood glucose level). Label the new steady-state operating point D2. Explain how you arrived at your answer. (3+2)

(ii) Next, assume that this patient does not receive insulin therapy. Instead, he is prescribed a drug that increases his insulin resistance, thus reducing the ability of insulin to stimulate glucose uptake by the body tissues. Draw necessary lines on the figure to show the new steady-state operating point for glucose-insulin. Label the new steady-state operating point D3. Explain how you arrived at your answer. (3+2)

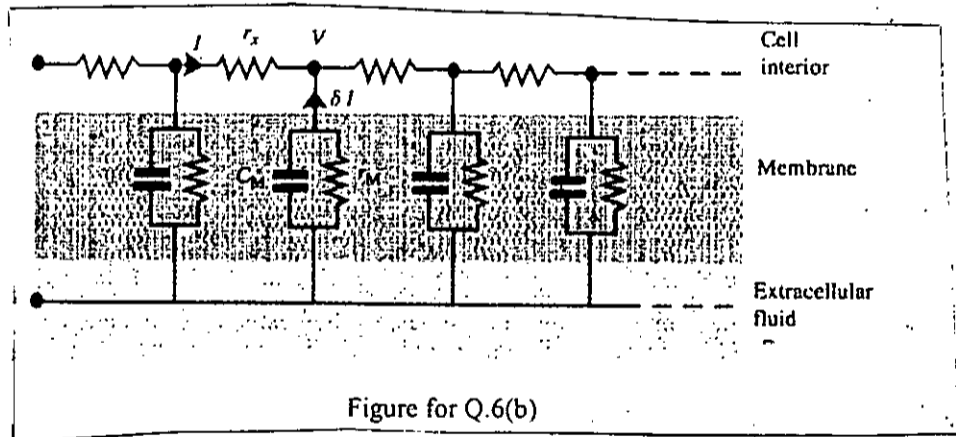


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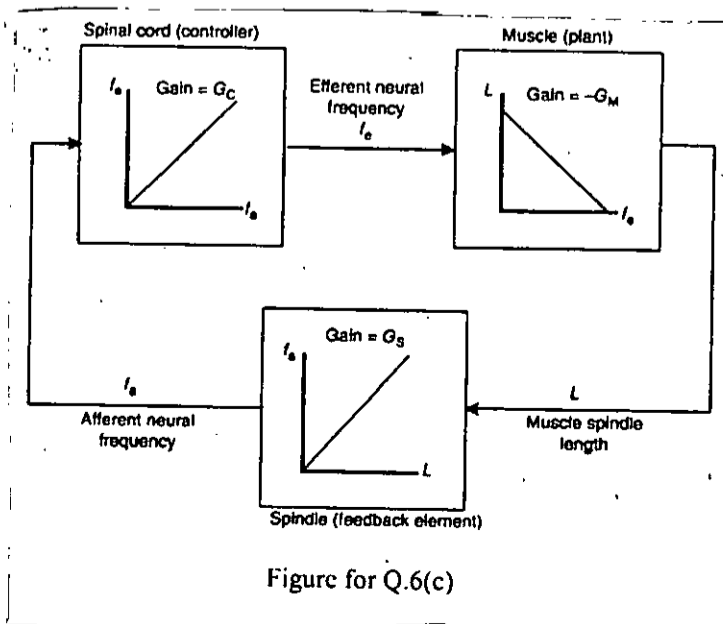
6. (a) Based on the description of the following physiological reflex system, construct a block diagram to represent the major control mechanisms involved. Clearly identify the physiological correlates of the controller, the plant, and the feedback element, as well as the controlling, controlled, and feedback variables. Also, describe how negative (or positive) feedback is achieved: (12+3)

"The Bainbridge reflex is a cardiac reflex that aids in the matching of cardiac output (the flow rate at which blood is pumped out of the heart) to venous return (the flow rate at which blood returns to the heart). Suppose there is a transient increase in the amount of venous blood returning to the right atrium. This increases blood pressure in the right atrium, stimulating the atrial stretch receptors. As a result, neural traffic in the vagal afferents to the medulla is increased. This, in turn, leads to an increase in efferent activity in the cardiac sympathetic nerves as well as a parallel decrease in efferent parasympathetic activity. Consequently, both heart rate and cardiac contractility are increased, raising cardiac output. In this way, the reflex acts like a servomechanism, adjusting cardiac output to track venous return."

- (b) A nerve fiber is modeled as a network containing serially connected multiple subunits, each with circuit elements r_x , r_M , and c_M as shown in Figure for Q. 6(b) below. r_x represents the axial resistance of 1 cm of nerve tissue per cm^2 of cross-sectional area, and is given in $\Omega\text{-cm}$. r_M and c_M represent the resistance and capacitance of 1 cm^2 of nerve membrane surface area, respectively. Assume that the extracellular medium bathing the nerve fiber represents the electrical ground in this model. Draw the equivalent mechanical model.



- (c) The block diagram shown below in Figure for Q. 6(c) displays the steady-state characteristics of the muscle stretch reflex model components. Given these characteristics and the fact that there is no explicit reference input in this case, explain graphically how the steady-state operating point of the system is reached. (10)



Contd P/6

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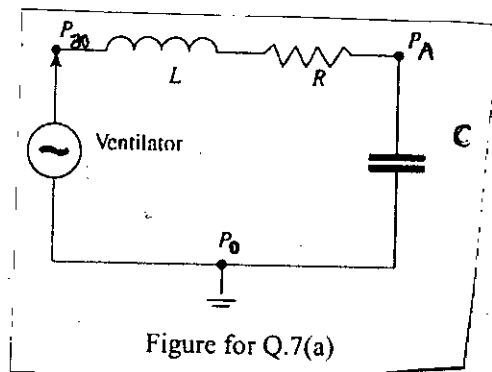
7. (a) Consider a simplified version of the linearized lung mechanics model shown in the Figure for Q. 7(a). R represents a combination of resistance to airflow in the airways, lung tissue resistance, and chest wall resistance. C represents the combined compliance of lung tissue, chest wall, and airways and L represents the fluid inertance. P_A , P_{ao} , and P_o are the alveolar pressure, pressure at airway opening, and ambient pressure, respectively. The transfer function of the model is given by (15)

$$\frac{P_A(s)}{P_{ao}(s)} = \frac{1}{LCs^2 + RC + \beta}$$

where $\beta = 1$ for the open loop case, and $\beta = 1 + k$ for the closed loop case.

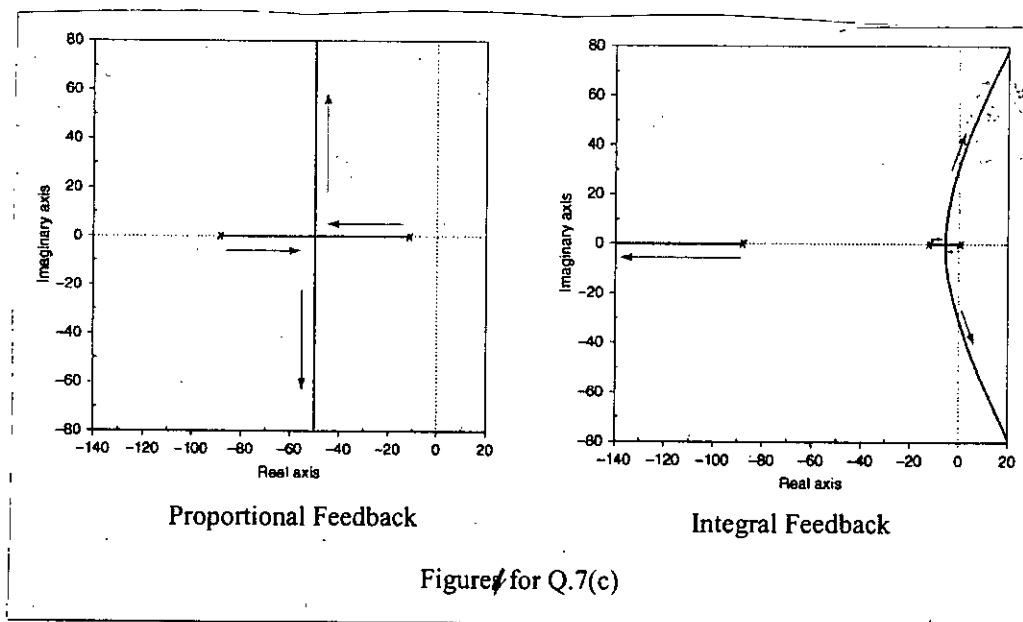
Sketch and explain the response of a first-order lung mechanics model to a unit impulse for both the open-loop and closed loop cases. Assume $R = 1 \text{ cm H}_2\text{O s L}^{-1}$, $C = 0.1 \text{ L cm H}_2\text{O}^{-1}$, and $\lambda = 2$.

Explain when a first-order lung mechanics model may be a valid approximation.



- (b) Sketch and explain the frequency response of the system in (a) for both the open-loop and close-loop cases. (7+3)

- (c) For the same system in (a), but assuming higher order dynamics, the root locus plots for a proportional feedback and an integral feedback are shown in the Figure for Q. 7(c). Explain, quantitatively from the plots, the impact of the two types of feedback on the stability of the system. (10)



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8. (a) The Figures for Q. 8(a) below shows the Nichols charts for the linearized lung mechanics model in open-loop and closed-loop modes.

(i) Using the value of $\omega = 30$ rad/s from both the plots, explain, quantitatively, the difference between the frequency responses of the open-loop and closed-loop modes. (5)

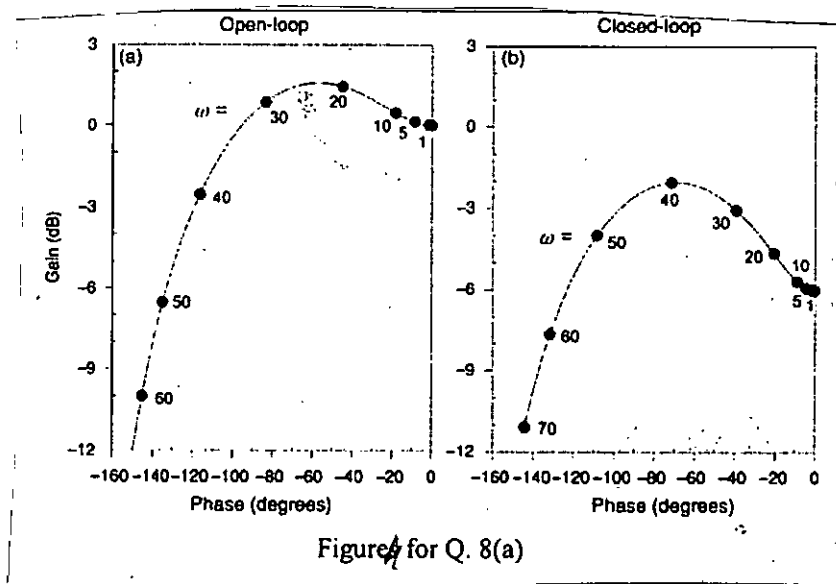
(ii) Draw charts (approximately) in your answer sheet and add necessary plots to show the impact of (with explanations) (3+3+4)

- an increase in the lung airway resistance due to pulmonary disease on the open-loop mode.
- doubling the value of proportionate feedback on the closed-loop mode.

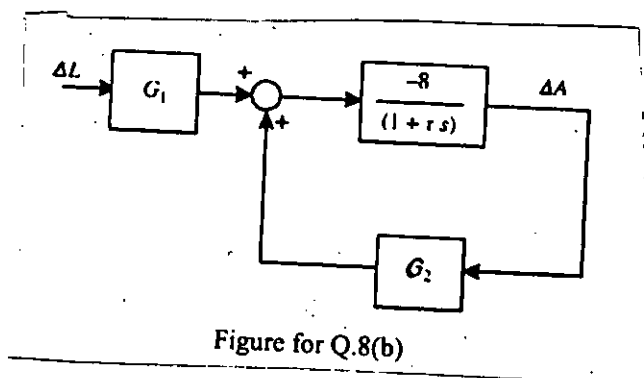
You may use the transfer function of the linearized lung mechanics model given by

$$\frac{P_A(s)}{P_{ao}(s)} = \frac{1}{LCs^2 + RC + \beta}$$

to assist your explanation.



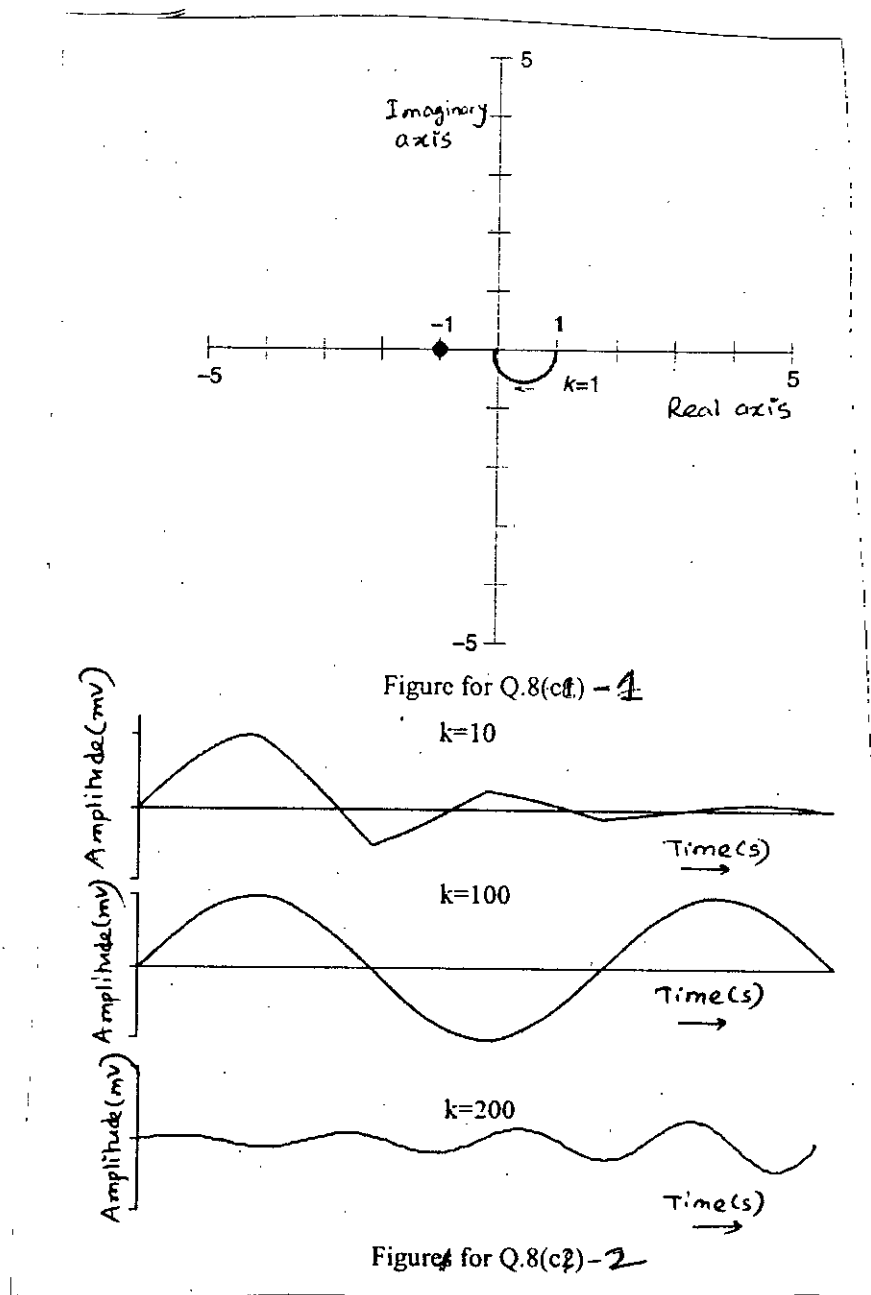
(b) The Figure for Q. 8(b) below displays the block diagram of a simplified, linear closed-loop model used to characterize the pupillary light reflex. The input is ΔL (change in light intensity) and the output is ΔA (change in pupil area), and the gains G_1 and G_2 are non-negative. Derive expressions for the open and closed-loop transfer functions of this system. τ represents delay in the neural pathways. (10)



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Contd ... Q. No. 8

(c) The Nyquist plot below shown in the Figure for Q. 8(c)-1. for the linear lung mechanics model with proportional feedback. k represents the feedback gain. Response of the linear lung mechanics model to a sinusoidal input for values of $k = 10, 100,$ and $200,$ are shown in the Figure for Q. 8(c)-2. Draw (approximately) the Nyquist plot in your answer script and add approximate plots on the Nyquist plot for values of $k = 10, 100,$ and $200.$ Give necessary explanations for the additional plots. (2+2+2+4)



BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2017-2018

Sub : **EEE 377** (Random Signals and Processes)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

Symbols and abbreviations have their usual meanings

- (a) Consider the random processes $X(t) = A \cos(\omega_0 t + \theta)$ where $\theta \sim u(0, 2\pi)$, and A and ω_0 are arbitrary constants. Find if $X(t)$ is Mean-ergodic and correlation-ergodic.

(b) Let $X(t) = At + B$ where A and B are independent random variables and uniformly distributed over the interval $[-1, 1]$. Is it possible to compute the PSD of $X(t)$?
- (a) The output of a filter is $Y(t)$ where $Y(t) = X(t) + X(t - t_0) + X(t - 2t_0)$, $X(t)$ and t_0 are the input and arbitrary constant, respectively. Determine (i) $R_{yy}(\tau)$ as function of $R_{xx}(\tau)$ and (ii) $E\{Y^2(t)\}$.

(b) With suitable examples, explain strict-range stationarity of a random process. Differentiate between ergodicity and strict-range stationarity of the random process.
- (a) The auto correlation of a WSS process $X(t)$ is given by

$$R_{xx}(\tau) = 36 + \frac{4}{1 + \tau^2}$$

Now $Y(t) = Q + X(t)$ where Q is a deterministic quantity. Find (i) $E(Y)$, (ii) auto correlation and auto covariance of Y .

(b) A random process $X_c(t)$ is sampled with a sampling period of 10s to produce $X(n)$ which is then given as input to an LTI system where impulse response is defined as $h(n) = e^{-an} u(n)$.

Find the PSD of the output of the system given that

$$R_{X_c X_c}(\tau) = e^{-6|\tau|}$$

- (a) The output of a biomedical measurement system is the random Variable X where $X \sim u(1, 2)$. Now S is the sum of 40 experimental values of X . Find $\Pr[55 < S \leq 65]$. Repeat your calculations for $X \sim u(1, 4)$. Comment on the results.

(b) The point pdf of X and Y is given by

$$f_{XY}(x, y) = e^{-(x/y)} e^{-y}; 0 \leq x < \alpha$$

$$0 \leq y < \alpha$$

$$= 0; \text{ otherwise}$$

Find $E[X|Y = y]$ and $E[E(Y|X)]$.

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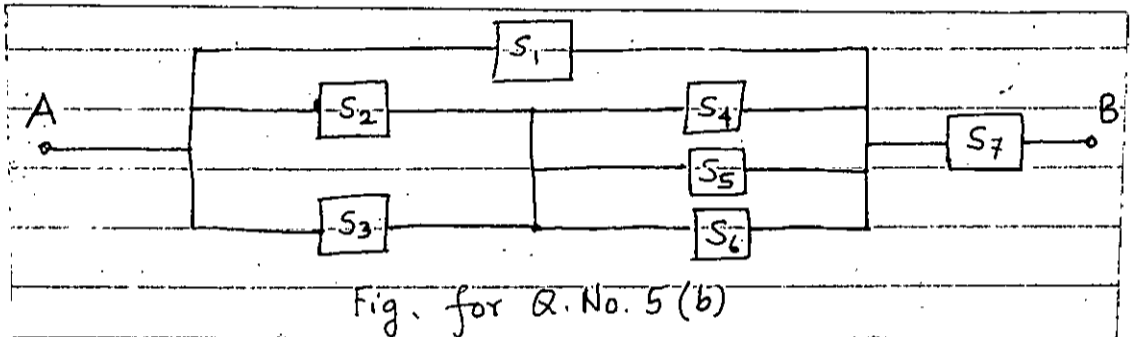
SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) A certain manufacturer produces biomedical instruments at two factories labeled A and B. Ten percent of the instruments produced at factory A are found to be defective, while 5% of the instruments produced at factory B are defective. If factory A produces 1,00,000 instruments per year and factory B produces 50,000 instruments per year, compute the following: (12)

- (i) Probability of purchasing a defective instrument from the manufacturer.
 (ii) If an instrument purchased from the manufacturer is defective, what is the probability it came from factory A?

- (b) Consider the network shown in Fig. for Q. No. 5(b). It is given that the switches S_1 to S_7 have the probabilities of independently failing within the next two years: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06 and 0.07. If the switches fail independently, find the reliability of the composite system. (13)



- (c) The probability of bleeding in case of gastrointestinal (GI) disease is 0.25. The prediction is correct 80% of the time when it is bleeding and 50% of the time for other GI diseases. What is the probability that the prediction for a particular case selected at random is correct? (10)

6. (a) The discrete random variable K has the following probability mass function (PMF): (12)

$$P_K(k) = \begin{cases} b, & k=0 \\ 2b, & k=1 \\ 3b, & k=2 \\ 0, & \text{otherwise} \end{cases}$$

- (i) What is the value of b?
 (ii) Determine the values of $P[K \leq 2]$ and $P[0 < K < 2]$.
 (iii) Determine the cumulative distribution function (CDF) of K.

- (b) A random variable X has the following probability density function (PDF), where $a > 0$. (13)

$$f_X(x) = \begin{cases} 0, & x < 1 \\ a(x-1), & 1 \leq x \leq 2 \\ a(3-x), & 2 \leq x \leq 3 \\ 0, & x > 3 \end{cases}$$

- (i) Find the value of a and sketch $f_X(x)$.
 (ii) What is the CDF of X? (iii) What is $P[1 \leq X \leq 2]$?
 (c) Let N be a random variable with the following CDF: (10)

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Contd ... Q. No. 6(c)

$$F_N(n) = \begin{cases} 0, & n < 1 \\ 0.2, & 1 \leq n < 2 \\ 0.5, & 2 \leq n < 3 \\ 0.8, & 3 \leq n < 4 \\ 1, & n \geq 4 \end{cases}$$

(i) What is the PMF of N (ii) Find the expected value and variance of N.

7. (a) Two random variables X and Y have the joint PDF given by (12)

$$f_{XY}(x, y) = \begin{cases} be^{-(2x+3y)}, & x \geq 0, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Find the value of the constant b that makes $f_{XY}(x, y)$ a true joint PDF
- (ii) Determine the marginal PDFs of X and Y
- (iii) Find $P[X < 1, Y < 0.5]$

(b) The joint PDF of two variables X and Y is given by (13)

$$f_{XY}(x, y) = \frac{6}{7} \left(x^2 + \frac{2y}{2} \right), \quad 0 < x < 1, \quad 0 < y < 2$$

(i) Find the CDF of X, (ii) Find $P[X > Y]$, (iii) $P\left[Y > \frac{1}{2} \mid X < \frac{1}{2}\right] = ?$

(c) A random variable X has the PDF $f_X(x) = 2e^{-2x}, x \geq 0$. (10)

Obtain an upper bound for $P[|X - E[X]| \geq 1]$

8. (a) X and Y are two independent random variables with PDFs (12)

$$f_X(x) = 4e^{-4x}, \quad x \geq 0$$

$$f_Y(y) = 2e^{-2y}, \quad y \geq 0$$

Let the random variable $U = X+Y$. Find the PDF of U and $P[U > 0.2]$

(b) Find $f_{UV}(u, v)$ in terms of $f_{XY}(x, y)$ if (13)

$$U = X^2 + Y^2$$

$$V = X^2 - Y^2$$

Here $f_{XY}(x, y)$ is the joint PDF of two random variables X and Y.

(c) Random variable X has the following PDF (10)

$$f_X(x) = \begin{cases} \frac{1}{3}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

If we define $Y = 2X + 3$, what is the PDF of Y?

$$= 4 =$$

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Table of Φ Function

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000

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Table of Fourier Transform

$x(\tau)$	$X(\omega)$
$e^{-a \tau }, a > 0$	$\frac{2a}{a^2 + \omega^2}$
$e^{-a\tau}, a > 0, \tau \geq 0$	$\frac{1}{a + j\omega}$
$e^{b\tau}, b > 0, \tau < 0$	$\frac{1}{b - j\omega}$
$\tau e^{-a\tau}, a > 0, \tau \geq 0$	$\frac{1}{(a + j\omega)^2}$
1	$2\pi\delta(\omega)$
$\delta(\tau)$	1
$e^{j\omega_0\tau}$	$2\pi\delta(\omega - \omega_0)$
$\begin{cases} 1 & -T/2 < \tau < T/2 \\ 0 & \text{otherwise} \end{cases}$	$T \frac{\sin(\omega T/2)}{(\omega T/2)}$
$\begin{cases} 1 - \tau /T & \tau < T \\ 0 & \text{otherwise} \end{cases}$	$T \left[\frac{\sin(\omega T/2)}{(\omega T/2)} \right]^2$
$\cos(\omega_0\tau)$	$\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$
$\sin(\omega_0\tau)$	$-j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$e^{-a \tau } \cos(\omega_0\tau)$	$\frac{a}{a^2 + (\omega - \omega_0)^2} + \frac{a}{a^2 + (\omega + \omega_0)^2}$

Fig

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define Isopycnic Centrifugation. Differentiate between two major methods of Isopycnic Centrifugation. (2+8=10)
- (b) List the elements that are most essential to animal life and health. Why are they important? (7)
- (c) Formulate 2 (two) methods along with suitable diagrams for fractionation of cells. (8)
- (d) Write definitions for the following items. (5×2=10)
 - (i) Dalton (ii) Conformation (iii) Active site (iv) Coenzyme (v) Holoenzyme

2. (a) 'The enzymatic catalysis of reactions is essential to living systems.'- Justify the statement. (8)
- (b) Choose a parameter to compare the catalytic efficiencies of different enzymes or the turnover of different substrates by the same enzyme. Explain the reasons behind choosing this parameter. (12)
- (c) When 10µg of an enzyme of M_r 50,000 is added to a solution containing its substrate at a concentration one hundred times the K_m , it catalyzes the conversion of 75µmol of substrate into product 3 min. What is the enzyme's turnover number? (5)
- (d) Methanol (wood alcohol) is highly toxic because it is converted to formaldehyde in a reaction catalyzed by the enzyme alcohol dehydrogenase. Part of the medical treatment for methanol poisoning is to administer ethanol (ethyl alcohol) in amounts large enough to cause intoxication under normal circumstances. Explain this phenomenon along with suitable diagrams and equations. (10)

3. (a) Distinguish between Epimers and Anomers. (6)
- (b) Describe a colorimetric method to determine glucose concentration levels in human tear samples. (12)
- (c) Graphically represent the formation of Pyranose and Furanose forms from D Glucose and D Ribose. (10)
- (d) Show the Catabolic pathway for Proteins, Carbohydrates and Fats in a single diagram. (7)

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4. (a) Predict the lipids that are localized in the thylakoid membranes of chloroplasts. Write short note on it. (6)
- (b) Describe the transport processes for nonpolar and polar compounds through membranes along with diagrams. (12+6=18)
- Calculate the energy cost for pumping Na^+ from cytosol where its concentration is about 10^{-6} M, to the extracellular fluid where its concentration is about 1 mM. Assume a temperature of 37°C and a standard transmembrane potential of 100 mV (inside positive) for the plasma membrane.
- (c) List the major storage lipids and structural lipids. Point out the importance of sterols in animal body. (6+5=11)

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Show mathematically why the pH of pure water is 7 at 25°C . (8)
- (b) Explain with the help of a sketch of a titration curve, how you might use phosphoric acid to buffer a biological reaction in the lab? (Phosphoric acid is tribasic, with pK_a 's of 2.14, 6.86, and 12.4) (14)
- (c) Ammonium sulfate, $(\text{NH}_4)_2\text{SO}_4$ is a water-soluble salt. Explain the thermodynamic feasibility of dissolving this molecule in water. Draw the relevant structures that form when ammonium sulfate dissolves in water. (13)
6. (a) Describe in detail one characterization technique for Proteins and one for Nucleic acids. Include appropriate diagrams if needed. (12)
- (b) Give a few examples of commonly found secondary structures in RNA strands? Explain why they are formed. (8)
- (c) Given the following DNA template sequence write down the corresponding DNA coding strand and the corresponding mRNA sequence. Finally write down the sequence of amino acids that are coded by the sequence given using the chart given. Choose the correct reading frame based on the start codon. (NOTE: the start codon is present within the first 6 bases thus you should ignore all bases coming before that) (15)
- DNA TEMPLATE sequence:
- 5' GGATGCCGAAACGCGAGAAGCATGCGAGTAATCAG 3'

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7. (a) Taking help of a neat schematic diagram, mention how the structure is stabilized. Describe the salient features of the α -helix structure, mark (and state) the length of the pitch of the helix in your diagram. **(12)**
- (b) What are some of the factors that cause instability in the α -helix? **(8)**
- (c) With the aid of appropriate diagrams explain the rules of DNA replication. **(15)**
8. (a) Draw the *fully protonated* structure and the titration curve for the pentapeptide CHAIR. Mark on the titration curve the net charge on this pentapeptide at various pHs. Show the corresponding schematic diagrams at these pHs. What is the pI and the molecular weight of this pentapeptide? **(20)**
- (b) **(15)**
- (i) Calculate the pH of a buffer system that is 0.25 M benzoic acid and 0.75 M ammonium benzoate if the $pK_a = 4.2$
- (ii) Calculate the number of grams of benzoate and benzoic acid required to make a liter of 0.5M buffer solution at the pH calculated in (i)
- (MW benzoic acid, $C_6H_5COOH = 122g\ mol^{-1}$);
- (MW ammonium benzoate, $C_6H_5COONH_4 = 139g\ mol^{-1}$);
-

Amino Acid Properties

Amino acid name	Molecular weight amino acid	Molecular weight residue	pK ₁	pK ₂	pK _R
Alanine	89.10	71.08	2.34	9.69	
Arginine	174.20	156.18	2.17	9.04	12.48
Asparagine	132.12	114.10	2.02	8.80	
Aspartic Acid	133.11	115.09	1.88	9.60	3.65
Cysteine	121.16	103.14	1.96	10.28	8.18
Glutamic Acid	147.13	129.11	2.19	9.67	4.25
Glutamine	146.15	128.13	2.17	9.13	
Glycine	75.07	57.05	2.34	9.60	
Histidine	155.16	137.14	1.82	9.17	6.00
Hydroxyproline	131.13	113.11	1.82	9.65	
Isoleucine	131.18	113.16	2.36	9.60	
Leucine	131.18	113.16	2.36	9.60	
Lysine	146.19	128.17	2.18	8.95	10.53
Methionine	149.21	131.19	2.28	9.21	
Phenylalanine	165.19	147.17	1.83	9.13	
Proline	115.13	97.11	1.99	10.60	
Serine	105.09	87.07	2.21	9.15	
Threonine	119.12	101.10	2.09	9.10	
Tryptophan	204.23	186.21	2.83	9.39	
Tyrosine	181.19	163.17	2.20	9.11	10.07
Valine	117.15	99.13	2.32	9.62	

mRNA Codon Chart

	U	C	A	G	
U	Phe	Ser	Tyr	Cys	U
	Phe	Ser	Tyr	Cys	C
	Leu	Ser	stop	stop	A
	Leu	Ser	stop	Trp	G
C	Leu	Pro	His	Arg	U
	Leu	Pro	His	Arg	C
	Leu	Pro	Gln	Arg	A
	Leu	Pro	Gln	Arg	G
A	Ile	Thr	Asn	Ser	U
	Ile	Thr	Asn	Ser	C
	Ile	Thr	Lys	Arg	A
	Met	Thr	Lys	Arg	G
G	Val	Ala	Asp	Gly	U
	Val	Ala	Asp	Gly	C
	Val	Ala	Glu	Gly	A
	Val	Ala	Glu	Gly	G

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Biomedical systems tend to give rise to more complex transport processes than do inert systems. What are the characteristics and behaviors that make dealing with biomedical system challenging? (10)

(b) Show that the velocity distribution for a power law fluid with a value of 2 as the behavior index or power law index in the sliding plate viscometer is $v_x = \left(\sqrt{\frac{\tau_w}{K}} \right) y$;

where K is the flow consistency index. (8)

(c) What is Fahraeus effect? In a laboratory setup blood is flowing within a small hollow fiber module containing a total of 10,000 fibers. Each fiber has an internal diameter of 60 μm and a length of 20 cm. The total flow rate of the blood is 10 mL min^{-1} . Assume the discharge hematocrit of the blood is 0.40 and that the Newtonian viscosity of blood in large tubes is 3 cP. The plasma viscosity is 1.2 cP. Find (i) the value of tube hematocrit, H_T and (ii) the pressure drop in mmHg across the hollow fibers. The relevant equations are given below (17)

For in-vitro setup:

The ratio of tube hematocrit and discharge hematocrit can be given by

$$\frac{H_T}{H_D} = H_D + (1 - H_D)(1 + 1.7e^{-0.415d} - 0.6e^{-0.011d})$$

Apparent viscosity can be found from

$$\frac{\mu_{\text{apparent}}}{\mu_{\text{plasma}}} = 1 + (\eta_{0.45} - 1) \frac{(1 - H_D)^c - 1}{(1 - 0.45)^c - 1}$$

Where, $\eta_{0.45} = 220e^{-1.3d} + 3.2 - 2.44e^{-0.06d^{0.645}}$

and $C = (0.8 + e^{-0.075d}) \left(-1 + \frac{1}{1 + 10^{-11}d^{12}} \right) + \frac{1}{1 + 10^{-11}d^{12}}$

For in-vivo setup,

Apparent viscosity can be found from

$$\frac{\mu_{\text{apparent}}}{\mu_{\text{plasma}}} = \left[1 + (\eta_{0.45}^* - 1) \frac{(1 - H_D)^c - 1}{(1 - 0.45)^c - 1} \left(\frac{d}{d - 1.1} \right)^2 \right] \left(\frac{d}{d - 1.1} \right)^2$$

Where, $\eta_{0.45}^* = 6e^{-0.085d} + 3.2 - 2.44e^{-0.06d^{0.645}}$

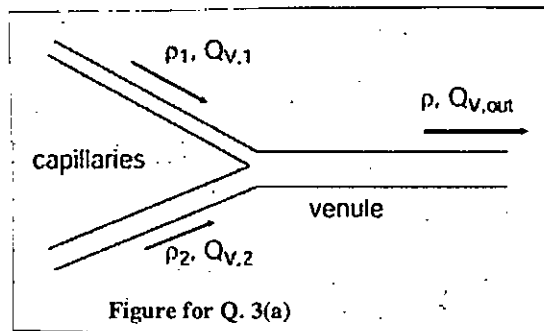
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2. (a) Describe four boundary conditions which are commonly used in the field of fluid mechanics and/or transport phenomena. (8)

(b) A catheter is initially filled with blood and subsequently flushed with saline. Depending on how the blood and saline interact in the catheter, what are the different approaches to solve this problem? (7)

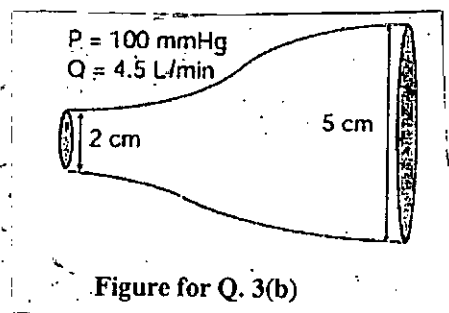
(c) 1 mL catheter is initially filled with blood having density ρ_b . At time $t = 0$, the catheter is flushed at a constant rate of 1 mL/s with saline having a density ρ_s . How long will it take to flush out 99% of the blood? What volume of saline is needed? Solve this problem considering the blood and saline can be well mixed. Mention the initial considerations, system definition and environmental interactions, governing equation to solve the problem. (20)

3. (a) Blood from two capillaries merge to form a small venule as shown in Figure 3(a). If the flow rate and density in each capillary are constant, find the flow rate and density of fluid emerging from the venule. State the initial considerations, system definition and governing equation. (11)



(b) An aortic aneurysm shown in Figure 3(b) is a ballooning of the aorta caused by a weakened vessel wall. Find the followings: (24)

- (i) Use a macroscopic mass balance to find the velocity at the outlet of an aortic aneurysm. Assume a steady horizontal flow of 4.5 L/min.
- (ii) Use a mechanical energy balance to estimate the pressure in the aneurysm. The inlet pressure is 100 mmHg. Does this tend to increase or decrease the diameter of the aneurysm? Explain.
- (iii) Use a momentum balance to compute the force by the blood on the vessel wall.



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4. (a) Bronchi that are located two generations down the respiratory tree from the trachea have a mean diameter of 9.4 mm and a length of 30.3 mm. The air flow in the lung during inspiration is 1 L/s (0.25 L/s per bronchus). The kinematic viscosity of the air is $1.57 \times 10^{-5} \text{ m}^2/\text{s}$. Estimate the pressure difference across these bronchi considering the entry effect and also not considering the entry effect. The relevant graphs are given in Figure 4(a). (18)

(b) Starting from a general statement of conservation of mass and using a three dimensional fluid shell in rectangular co-ordinates, show that for incompressible flow: (17)

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols and abbreviation have their usual meanings.

5. (a) Consider heat transfer from a slab (having a uniform cross-sectional area) that is heated at a constant rate on one side and is in contact with a moving fluid on the other side. Heat flows by conduction from the slab interior to the surface and then by convection away from the surface into the fluid. Draw the equivalent thermal resistance model of the system. If the heat transfer rate is constant and steady heat flows through the slab, show that in 1-D the surface temperature (T_s) can be derived as (12)

$$T_s = \frac{T_s + Bi(T_\infty)}{(1 + Bi)}$$

(b) Everolimus that treats restenosis after balloon angioplasty is infused on to the circulating system long enough for the concentration of the drug in the blood to be constant at 5 mmol. Consider steady-state one-dimensional diffusion of the drug through the arterial tissue. Measurements in coronary arteries have found that the drug-concentration in the vessel is 0.25 mmol in the outer part of the artery. If the inner radius of the artery is 2.5 mm and the outer radius of the vessel is 3.5 mm, determine the concentration at 3 mm from the center of the artery. Neglect chemical reaction. Mention all the assumptions. (18)

(c) What are the importance of dimensional analysis during the design of a medial device? (5)

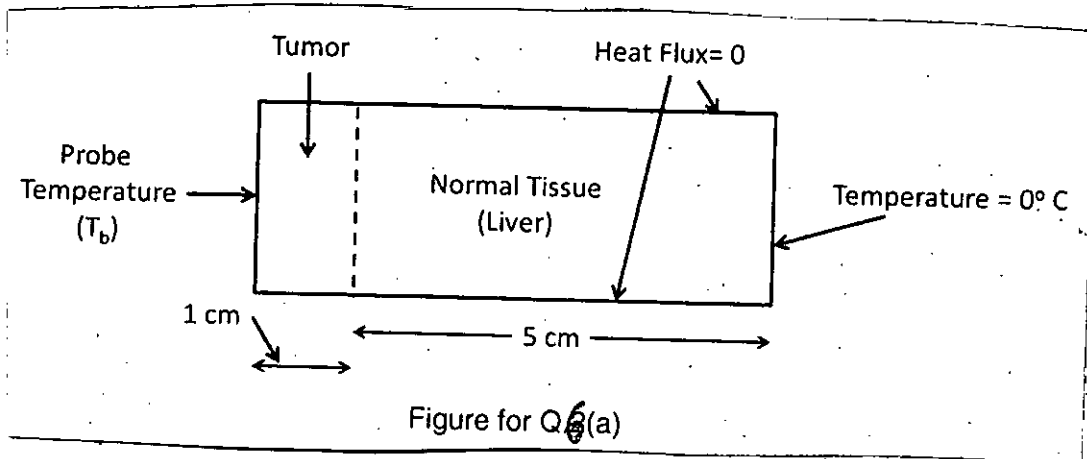
6. (a) Qualitatively show that the velocity distribution is independent of any position for one-dimensional flow through a porous medium. Mention all the assumptions. (15)

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Contd ... Q. No. 6

(b) Heated probes can be used to kill tumors in the liver as well as in other parts of the body. Such a procedure is preferable over open surgery because it is minimally invasive. However, thermal ablation must be planned accurately so that the minimum temperature for the tumor destruction is 50° C, and there is minimum damage to the healthy tissue. Assume that the entire heating probe is in contact with the tumor to be destroyed and the heating is only along the x-direction. However, for better illustration of the process a 2D domain is given below:

(20)



What should be the minimum temperature and flux at the heating probe surface so that all the tumor tissue will be destroyed at steady-state? Assume that the thermal properties of the normal and tumor tissues are same as given below:

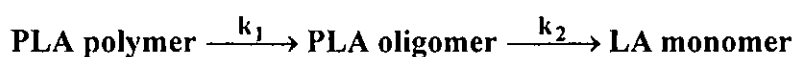
- Density 1060 kg/m³,
- Thermal conductivity 0.512 W/(m-K) and
- Specific heat 3600 J/(kg-K).

7. (a) Direction of blood and dialysate flows play an important role on the efficacy of a dialysis machine. Show that molar flow rate depends on the log mean concentration difference in terms of concentration entering and existing on the blood and dialysate sides of a dialysis system where the blood flow direction is opposite to that of dialysate flow.

(20)

(b) Poly lactic acid (PLA) is a very commonly used biomaterial to develop polymer-based medical device and controlled drug delivery. When water molecules penetrate into the PLA polymer it breaks the ester bonds by a random scission process and produce PLA oligomers and subsequently lactic acid (LA) monomers. This is a sequential reaction as shown below:

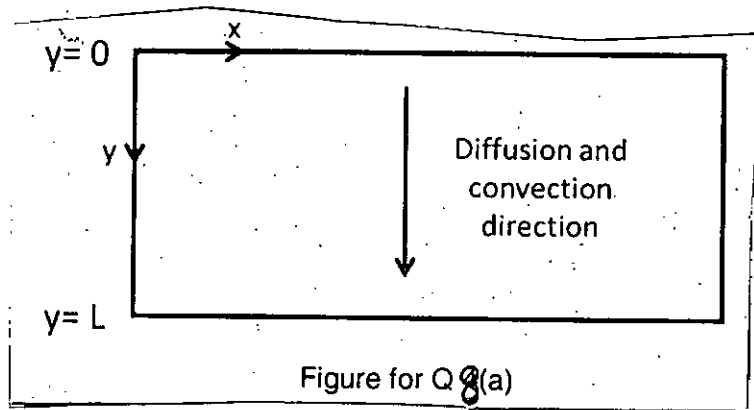
(15)



If only PLA polymer with concentration of C₀ is present at the initial stage, what is the concentration of PLA oligomer at any given time t. k₁ and k₂ are the reaction rate constants.

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8. (a) Consider steady-state one-dimensional diffusion and convection through a cellular layer thickness L as shown below. Assume negligible pressure drop across the cellular layer that results a constant fluid velocity. Determine the concentration expression at any point inside the cellular layer if the entrance and exit concentration are C_0 and C_L . (20)

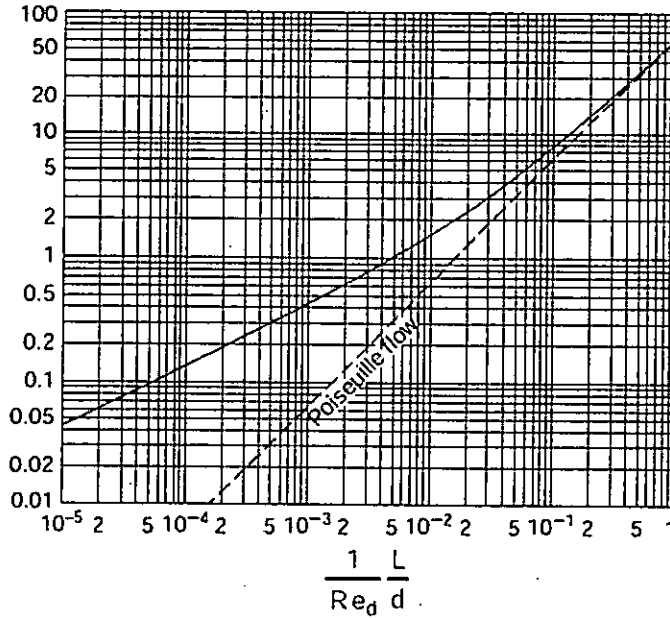


- (b) What is dilute solution approximation? Why is it important in modeling biological system? (10)
- (c) With appropriate assumptions show that when drug is administered using a bolus intravenous injection in our body, total amount of drug exposure depends on the initial drug concentration and drug clearance rate. (5)
-

Entrance to a tube, laminar flow:

= 6 =

$$\frac{\Delta P}{\frac{1}{2} \rho \langle v \rangle^2} = 4f \frac{L}{d}$$



Moody Chart

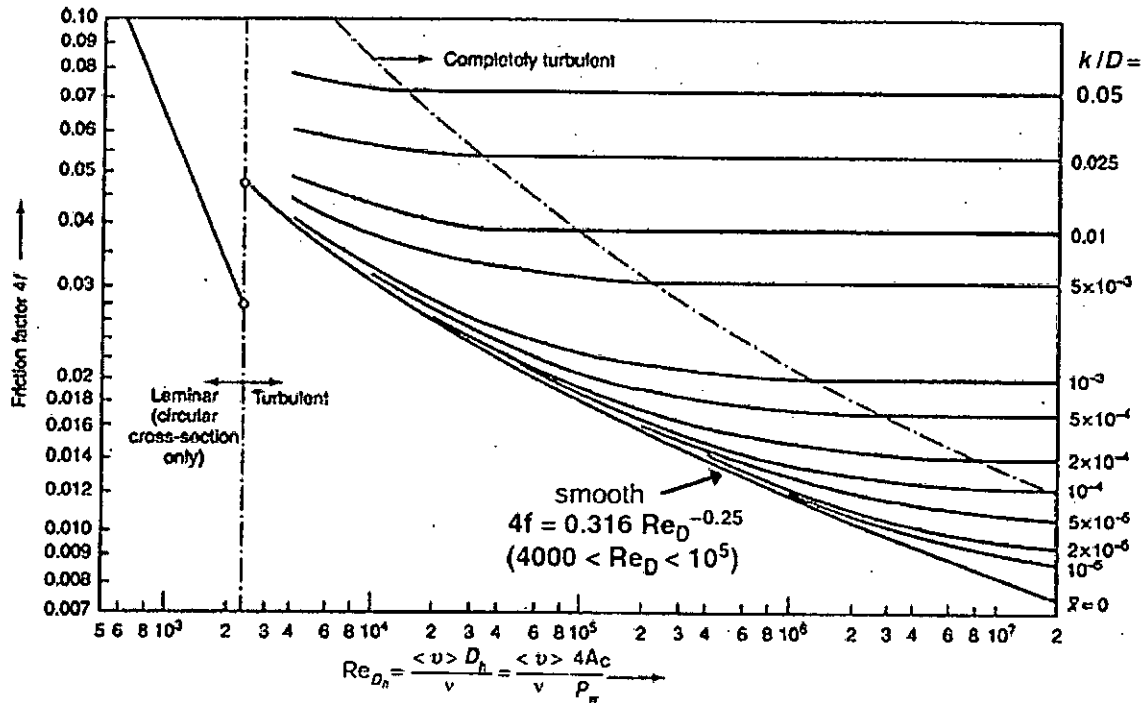


Figure for Q 4(a)

= 7 =

Table 1: Equations of continuity in various coordinate systems

Rectangular	$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$
Cylindrical	$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$
Spherical	$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(\rho \sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}(\rho v_\phi) = 0$

Table 2: Mass conservation relations for dilute solutions in various coordinate systems

Rectangular	$\frac{\partial C_i}{\partial t} + v_x \frac{\partial C_i}{\partial x} + v_y \frac{\partial C_i}{\partial y} + v_z \frac{\partial C_i}{\partial z} = D_{ij} \left(\frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2} + \frac{\partial^2 C_i}{\partial z^2} \right) + R_i$
Cylindrical	$\frac{\partial C_i}{\partial t} + v_r \frac{\partial C_i}{\partial r} + \frac{v_\theta}{r} \frac{\partial C_i}{\partial \theta} + v_z \frac{\partial C_i}{\partial z} = D_{ij} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C_i}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 C_i}{\partial \theta^2} + \frac{\partial^2 C_i}{\partial z^2} \right) + R_i$
Spherical	$\begin{aligned} \frac{\partial C_i}{\partial t} + v_r \frac{\partial C_i}{\partial r} + \frac{v_\theta}{r} \frac{\partial C_i}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial C_i}{\partial \phi} \\ = D_{ij} \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C_i}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial C_i}{\partial \theta} \right) \right. \\ \left. + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 C_i}{\partial \phi^2} \right) + R_i \end{aligned}$

Table 3: Governing equations for heat conduction in various coordinate systems

Rectangular	$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\dot{q}_{met}}{\rho c_p}$
Cylindrical	$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\dot{q}_{met}}{\rho c_p}$
Spherical	$\begin{aligned} \frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) \right. \\ \left. + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right) \\ + \frac{\dot{q}_{met}}{\rho c_p} \end{aligned}$