L-2/T-2/CE
Date: 01/04/2019

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2  B. Sc. Engineering Examinations 2017-2018

Sub: **CE 205** (Numerical Methods)

Full Marks : 140  Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are FOUR questions in this section. Answer any THREE.

1. (a) For solving the system of linear equations what are the main Methods. Explain them with their subgroups. (5)

(b) Write short notes on

   (i) Cramer’s Rule

   (ii) Method of Chio

(c)

\[
\begin{pmatrix}
-3 & -2 & -1 & 0 & 1 & 2 & 3 \\
1 & 12 & 12' & 12'' & 12'' & 12'' & 12'' \\
2 & 2E & E \\
2E & E & I = 1500 in^4 \\
6.5 \times 10^{-4} & -1 & -1.37 & -1.37 & -1.37 & -1.37 & -1.37 \\
7.5 \times 10^{-4} & -0.5 & -0.5 & -0.5 & -0.5 & -0.5 & -0.5 \\
5 \times 10^{-4} & 1 & 1 & 1 & 1 & 1 & 1 \\
-5 \times 10^{-4} & 2 & 2 & 2 & 2 & 2 & 2 \\
\end{pmatrix}
\]

Figure 1 shows a simply supported beam and its deflected shape. Find slope, moment and shear forces at points \(-1, -2, +1\) and \(+2\). (12½)

2. (a) Derive the required expressions of Gauss-elimination method. (10½)

(b) Find the inverse of the matrix for the following system of linear equations. Also estimate the values of \(a, b\) and \(c\). (13)

\[
\begin{align*}
3a - 6b + 7c &= 3 \\
9a - 5c &= 3 \\
5a - 8b + 6c &= -4
\end{align*}
\]

3. (a) Derive the general expression for area calculation in Simpson’s rule. (10½)

(b) Estimate the deflection at point ‘A’ of the following beam. Relevant data are provided. (13)

\[
\begin{pmatrix}
\end{pmatrix}
\]

Contd ............. P/2
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4. (a) Define Gauss quadrature. Also derive associated points and weighting coefficients for
n = 3.

(b) For the following data find

(i) General polynomial equation (ii) Function value for x = 0.24

<table>
<thead>
<tr>
<th>x</th>
<th>0.1</th>
<th>0.18</th>
<th>0.30</th>
<th>0.38</th>
<th>0.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>1.40</td>
<td>1.54</td>
<td>1.75</td>
<td>1.98</td>
<td>2.32</td>
</tr>
</tbody>
</table>

SECTION – B

There are FOUR questions in this Section. Answer any THREE.

5. (a) Write down the system of linear equation (in matrix form) of the boundary value
problem: \( \frac{d^2y}{dx^2} = -4y + 4x \) \( y(0) = 0, \frac{dy}{dx} \bigg|_{x=\pi/2} = 0 \) \( (13 \frac{\text{Y}}{\text{S}}) \)
Use a step size of \( \pi/20 \). No need to solve the problem.

(b) Use Newton-Raphson method to determine the real root of \( f(x) = -0.9x^2 + 1.7x + 2.5 \)
using an initial guess of \( x_0 = 5 \). Perform computation until the approximate error goes below
0.1%.

6. (a) A mass balance for a pollution in a well-mixed lake can be written as:

\[ v \frac{dc}{dt} = W - Qc - kV \sqrt{c} \]

Given the parameter values \( V = 10^6 \text{ m}^3, Q = 10^5 \text{ m}^3/\text{year}, W = 10^6 \text{ gm/year}, k = 0.25 \). The
root can be located with fixed-point iteration as

\[ c = \left( \frac{W - Qc}{kV} \right) \text{ or } c = \frac{W - kV \sqrt{c}}{Q} \]

Only one will converge with initial guesses \( 2 < c < 6 \). Determine which one and state the
reason why. Solve the problem using fixed point iteration with an initial guess of \( c = 4 \text{ g/m}^3 \). 
Perform iteration until the approximate error goes below 1%.

(b) Solve the following equation for \( y(3) \) using Euler’s method with a step size of 0.5 and
with initial conditions \( y(0) = 1 \) and \( y'(0) = 0 \):

\[ 2 \frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^2 + 6x = 0 \]

7. (a) Two forms of Michaelis-Mention equations are typically used to study the chemical
reaction in living cells in terms of reactions velocities (v) and substrate concentrations ([S]):

\[ v = \frac{v_m[S]}{k_s + [S]} \text{ and } v = \frac{v_m[S]^2}{k_s + [S]^2} \]

Contd .......... P3
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Contd... Q. No. 7(a)

Fit both the models to the following data to determine the maximum initial reaction velocity ($v_m$) and half-saturation constant ($k_s$):

<table>
<thead>
<tr>
<th>[S]</th>
<th>1.3</th>
<th>1.8</th>
<th>3</th>
<th>4.5</th>
<th>6</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>0.07</td>
<td>0.13</td>
<td>0.22</td>
<td>0.275</td>
<td>0.335</td>
<td>0.35</td>
<td>0.36</td>
</tr>
</tbody>
</table>

(b) Determine which one of the above models is a better representation (fit) of the data compared to the other.

(c) What is the difference between round-off error and truncation error in numerical methods? Explain the relationship between total numerical error and step-size. Why do these types of errors are not a concern in analytical methods?

8. (a) Use the Taylor series to predict $f(2)$ for the function $f(x) = \ln(x)$ using a base point at $x = 1$. Employ the zero-, first-, second-, third- and fourth-order versions for the Taylor series and compute the true % error for each case.

(b) Solve the equation $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^3 T}{\partial y^3} = 2x^2y^2$ over the square domain $0 \leq x \leq 3$ and $0 \leq y \leq 3$.

Given the boundary conditions as shown in figure 3, below, find the temperatures $T_1$, $T_2$, $T_3$, $T_4$.

(c) Explain graphically how

(i) Regula falsi method can perform worse than bi-section method

(ii) Newton-Raphson method fails to converge at a root

(iii) Solution of PDE using explicit method can become unstable.

(6)
1. (a) What is center of pressure? Derive the equation for the location of center of pressure of a plane area submerged fully in a static liquid. \( (2+10=12) \)

(b) Prove that for an inclined plane surface submerged in a static fluid, center of pressure is always below center of gravity. \( (6) \)

(c) A vertical, triangular gate with water on one side is shown in Figure 1. Determine the total resulting force acting on the gate and the location of the center of pressure. \( (8) \)

(d) Curved wall ABC in Figure 2 is a quarter circle 9 ft wide perpendicular to the paper. Compute the horizontal and vertical hydrostatic forces on the wall and the line of action of the resultant force. \( (9) \)

2. (a) What is viscosity of fluid? Derive the equation of viscosity of fluid. In this respect differences between Newtonian and non-Newtonian fluid with proper figure. \( (2+4+6=12) \)
WRE 211

Contd...Q.No.2

(b) Explain Reynolds experiment to distinguish between laminar and turbulent flow with a sketch. Also define critical Reynolds number. \( \text{(5+2=7)} \)

(c) A 6 inch shaft rides in a 6.01 inch sleeve 8 inch long, the clearance space assumed to be uniform being filled with lubricating oil \( (\mu=0.0018 \text{ lb.s/ft}^2) \). Calculate the rate at which heat is generated when the shaft turns at 96 rpm. \( \text{(8)} \)

(d) In Figure 3, atmospheric pressure = 101.3 kN/m\(^2\) abs, the gage reading is 35 kN/m\(^2\); the vapor pressure of the alcohol = 12 kN/m\(^2\) abs. Compute x and y. \( \text{(8)} \)

3. (a) Write short notes on

   (i) Compressibility of fluid
   (ii) Absolute and gage pressure
   (iii) Inverted differential manometer
   (iv) Specific gravity and Specific weight
   (v) Buoyancy

   (b) Derive the general equation which gives the variation of pressure in a static fluid on a plane area. \( \text{(8)} \)

   (c) A hydrometer weighs 0.00485 lb and has a stem in the upper end which is cylindrical and 0.110 inch in diameter. How much deeper will it float in oil of specific gravity 0.780 than in alcohol of specific gravity 0.821? \( \text{(7)} \)

4. (a) Derive Darcy-Weisbach equation for pipe friction. \( \text{(13)} \)

   (b) Write down the minor losses in pipe flow. \( \text{(7)} \)

   (c) For the given pipe network as shown in Figure 4, determine the five pipe flows, given the head loss from A to D is 91 ft and all pipes have friction factor, \( f=0.017 \). \( \text{(10)} \)
(d) Calculate the discharge for the pipe of Figure 5, the fluid is water at standard temperature and pressure. Use value of friction factor, $f = 0.019$.

5. (a) What is laminar flow and turbulent flow? A Newtonian fluid with a dynamic viscosity of $0.38 \text{ Ns/m}^2$ and specific gravity of 0.91 flows through a 25 mm diameter pipe with a velocity of 2.6 m/s, find whether the flow is laminar or turbulent?

(b) Define streamline and path line. A 2D flow can be described by $u = -\left(\frac{y}{b^2}\right)$ and $u = -\left(\frac{x}{a^2}\right)$. Find the equation of streamline which passes through (a, 0).

(c) A circular pipe 10 cm in diameter has a 2 m length which is porous. In this porous section the velocity of exit is known to be constant. If the velocities at inlet and outlet of the pipe are 2 m/s and 1.2 m/s respectively. Estimate the discharge emitted out through the walls of the porous pipe and the average velocity of this emitted discharge. See figure 6.
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6. (a) Derive the Bernoulli's equation. 

(b) A tank has two outlets (i) a rounded entry orifice A of diameter $D = 2$ m and (ii) a pipe B with rounded entry diameter $D = 2$ m and of length $L = 5$ m as shown in figure 7. For a head of water $H = 4$ m in the tank, determine the discharge from the outlets A and B, also determine velocity of outlet B at level 1 and 2 as shown in the figure 7.

(c) What is energy correction factor? Why correction factor is greater than unity in both laminar and turbulent flow?

(d) The velocity profile for turbulent flow can be approximated by a parabola $u = U_{\text{max}} [1 - 0.4(r/r_0)^2]$, where $U_{\text{max}}$ is the maximum velocity at the axis.

7. (a) Water entering a pump through an 8 inch diameter pipe at 4 psi has a flow rate of 3.5 cfs. It leaves the pump through a 4 inch diameter pipe at 15 psi. Assume that the suction and discharge side of the pump are at the same elevation, find the horse power delivered to the water by the pump.

(b) What is hydraulic grade line and energy grade line? Draw hydraulic grade line and energy grade from the following information. A pipeline with a pump leads to a nozzle as shown in the figure 8. The pump develops a head of 80 ft. Assume that the head loss in the 6 inch diameter pipe may be expressed by $h_L = 5V_1^2 / 2g$ while the head loss in the 4 inch diameter pipe is $h_L = 8V_2^2 / 2g$. The jet has a diameter of 3 inch.
WRE 211
Contd...Q. No. 7

(c) What is cavitation? Gasoline which has vapor pressure of $5.5 \times 10^4$ Pa (abs) and density $\rho = 680 \text{ kg/m}^3$ flows through a constriction in a pipe where the diameter is reduced from 20 cm to 10 cm. The pressure in the 20 cm pipe just upstream of the constriction is 50 kPa. If the atmospheric pressure is 75 mercury, calculate the maximum discharge that can be passed through the constriction without cavitation occurring.

8. (a) Determine the magnitude and direction of the force exerted by the liquid ($\gamma = 62.4 \text{ lb/ft}^3$) on the double nozzle shown in the figure 9. The 6 inch nozzle has velocity of 35 fps and the 4 inch one has velocity of 40 fps. Assume frictionless flow on horizontal plane.

(b) Find the pull on the bolts in the figure 10. The diameter at the entrance $d_1 = 5 \text{ cm}$, in the middle $d_2 = 10 \text{ cm}$ and at the exit $d_3 = 2.5 \text{ cm}$; the manometer reading $Y = 180 \text{ cm}$ where the liquid in the manometer has specific gravity of 0.80. Neglect the weight of water and assume ideal fluid.

(c) Assume ideal fluid is flowing in a horizontal plane in figure 11. Calculate the magnitude and direction of the resultant force on a single blade moving with a velocity of 5 m/s. The entering jet has 150 mm diameter and velocity of 12 m/s which is divided by the splitter so that one third of the water is diverted towards A of total water.
SECTION – A

There are FOUR questions in this section. Answer any THREE questions.

1. (a) What do you understand by short-run and long-run in the theory of production. Describe the relationship between total physical product (TPP), average physical product (APP) and marginal physical product (MPP). Use diagrams. (10)
   (b) Explain the law of diminishing marginal returns in production. Illustrate the optimum combination of factors using (i) isoquant/isocost approach and (ii) marginal product approach. (13½)

2. (a) Describe the basic features of the main four-types of market structures. Illustrate the following situations under perfect competition: (10)
   (i) Super normal/excess profit
   (ii) Loss minimizing level of production
   (iii) Shutdown point
(b) Explain the conditions for profit maximization of a firm. Given the total revenue (TR) and total cost (TC) functions of a firm
   \[ TR = 1400Q - 7.5Q^2 \]
   \[ TC = Q^3 - 6Q^2 + 140Q + 750 \]  
   [Q refers to quantity of output] 
   Set up the profit function and calculate the maximum profit and level of output. (13½)

3. (a) What are the major macroeconomic policy objectives that governments typically pursue? Explain in your own words. (10)
   (b) What is meant by aggregate demand (AD) in economics? Illustrate the circular flow income model and describe the relationship between withdrawals and injections. (13½)

4. Write short notes on any THREE of the following: (23½)
   (a) Economies and diseconomies of scale of production
   (b) Monopolistic competition
   (c) Any two methods of measuring national income
   (d) Long run average cost (LRAC) curve of a firm.

Contd ........... P/2
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SECTION - B

There are FOUR questions in this section. Answer any THREE.

5. Suppose we are analyzing the market for hot chocolate. Graphically illustrate the impact each of the following would have on demand or supply. Also show how equilibrium price and equilibrium quantity would change.
   (a) Winter starts, and the weather turns sharply colder. (4)
   (b) Consumer income falls because of a recession, and hot chocolate is considered a normal good. (4)
   (c) Producers expect the price of hot chocolate to increase next month. (4)
   (d) Currently, the price of hot chocolate is $0.50 per cup above equilibrium. (4)
   (e) A better method of harvesting cocoa beans is introduced. (4)
   (f) The ministry of health announces that hot chocolate cures acne. (3/5)

6. Consider the following pairs of goods. For which of the two goods would you expect the demand to be more price elastic? Why?
   (a) water or diamonds (5)
   (b) insulin or nasal decongestant spray (5)
   (c) food in general or breakfast bread (5)
   (d) gas over the course of a week or gas over the course of a year (5)
   (e) personal computers or Dell personal computers (3/5)

7. (a) Explain three characteristics or assumptions of rationality with examples. (8)
   (b) Show that in equilibrium slope of indifference curve is equal to the slope of budget line. (7)
   (c) Using indifference curve explain why demand curve slopes downward. (8/5)

8. Suppose you are constructing a road (say project X) with the cost of BDT 1000 million. This project will yield BDT 200 million return every year for 6 years. Another project (say project Z) costs same but the returns are different. Project Z will yield BDT 150 million each year for 10 years.
   (a) Using payback period method, find the better project. Why is payback period method termed as 'dirty method'? (7)
   (b) What are the NPVs of both projects, and which one is better project? (use 5% discounting rate). (8)
   (c) What is IRR? Find the IRR of the project X. [Hint: if can not measure exact value, give a range or likely value] (8/5)
1. (a) If $X_1, X_2, \ldots$ are independent Poisson random variables each having mean $\lambda$, determine the maximum likelihood estimator of $\lambda$ (Derive expression). 

(b) Piston rings are mass-produced. The target internal diameter is 45 mm but records show that the diameters are normally distributed with mean 45 mm and standard deviation 0.05 mm. An acceptable diameter is one within the range 44.95 mm to 45.05 mm. What proportion of the output is unacceptable?

(c) Civil Engineers believe that $W$, the amount of weight (in units of 1000 pounds) that a certain span of a bridge can withstand without structural damage resulting, is normally distributed with mean 400 and standard deviation 40. Suppose that the weight (again in units of 1000 pounds) of a car is a random variable with mean 3 and standard deviation 0.3. How many cars would have to be on the bridge span for the probability of structural damage to exceed 0.1?

2. (a) Daily probability that a major earthquake occurs $P[E] = 10^{-5}$. Probability that premonitory event $A$ or $B$ occurs given that major earthquake occurs is $P[A|E] = P[B|E] = 0.1$. Probability that premonitory event $A$ or $B$ occurs given that major earthquake does not occur is $P[A|E'] = P[B|E'] = 0.001$.

(i) Determine the probability of a major earthquake, given that premonitory event $A$ is observed.

(ii) Also, determine the probability of a major earthquake, given that both premonitory events $A$ and $B$ are observed at the same time.

(b) A plane is missing and it is presumed that it was equally likely to have gone down in any of 3 possible regions. Let $\beta_i$ denote the probability that the plane will be found upon a search of the $i$th region when the plane is, in fact, in that region, $i = 1, 2, 3$. (The constants $\beta_i$ are called overlook probabilities because they represent the probability of overlooking the plane; they are generally attributable to the geographical and environmental conditions of the regions). What is the conditional probability that the plane is in the $i$th region, given that a search of region 1 is unsuccessful, $i = 1, 2, 3$?
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Contd ... Q. No. 2

(c) California Department of Transportation did a study a number of years ago that showed that the proportion of cars tested which failed to meet the state pollution standard was 0.37. The document would like to be able to say that the cars have improved since then. In a sample of \( n = 100 \) cars more recently, the proportion not meeting the standards was 0.28. Are the cars better at meeting the standards than they used to be? Clearly state the null and alternative hypotheses. Perform the hypothesis test at level \( \alpha = 0.01 \), by computing the test statistic and explain the meaning of your conclusion in words.

3. (a) Solve the following boundary value problem:

\[ u'' - 9u = 50e^{-2x}, \quad (0 < x < \infty) \]

Given, \( u'(0) = v_0, u(\infty) = \text{bounded} \)

(b) The following function (Fig. 1) is given as the forcing function of an ordinary differential equation. How will you express the function to solve the equation?

![Fig. 1](image1)

4. (a) Derive Fourier series for the following function (Fig. 2).

![Fig. 2](image2)

(b) Using Fourier transform derive the expression of deformation of an infinite beam subjected to uniformly distributed load \( w \) at a stretch from \(-a\) to \(a\) (Fig. 3).

![Fig. 3](image3)
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SECTION – B

There are SEVEN questions in this Section. Answer any FIVE.

Symbols have their usual meaning

5. The tensile strength of a glued joint is related to the glue thickness. A sample of six values gave the following results:

<table>
<thead>
<tr>
<th>Glue thickness (inches)</th>
<th>0.12</th>
<th>0.12</th>
<th>0.13</th>
<th>0.13</th>
<th>0.14</th>
<th>0.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Strength (lbs)</td>
<td>49.8</td>
<td>46.1</td>
<td>46.5</td>
<td>45.8</td>
<td>44.3</td>
<td>45.9</td>
</tr>
</tbody>
</table>

(i) Calculate the sample correction coefficient \( r \) for these data
(ii) Use the fitted least square regression line to predict the tensile strength of a joint for a glue thickness of 0.14 inches.

6. (a) A manufacturer of car batteries claims that his batteries will last, on average, 3 years with a variance of 1 year. If 5 of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5 and 4.2 years, construct a 95% confidence interval for \( \sigma^2 \) and decide if the manufacturer’s claim that \( \sigma^2 = 1 \) is valid. Assume the population of battery lives to be approximately normally distributed.
(b) What do you understand by the terms ‘order’, ‘explicit form’, ‘implicit form’ of an Ordinary Different Equation?

7. A cylindrical water tank of 2 m diameter has a hole of 2 cm diameter at the bottom. The initial height of the water when the hole is opened is 2.56 m, when will the tank be empty?

It's given that, under the influence of gravity the out flowing water has velocity.

\[ v(t) = 0.600 \sqrt{2gh(t)} \]

Where, \( h(t) \) is the height of the water.

8. Check for exactness of the equation and solve. \((x^2 + y^2) \, dx - 2xy \, dy = 0.\)

9. (a) Apply power series method and solve the following differential equation: \( y'' + y = 0.\)
(b) Determine the radius of convergence of the following power series, and find the interval of convergence.

\[ \sum_{m=0}^{\infty} \frac{x^{2m+j}}{(2m+1)!} \]

10. (a) What are the operations that are permissible on a power series to solve a differential equation.
(b) Write down (i) Legendre Equation (ii) Legendre Polynomial. Use general notations.
(c) Bessel’s equation is a special form of Frobenius equation. Explain with an example.

11. Find the basis of solution by Frobenius Method for following Ordinary Differential Equation

\( y'' + (x - 1)y = 0. \)
Table of Fourier Transform

<table>
<thead>
<tr>
<th>f(x)</th>
<th>( \hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x} , dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{1}{a^2 + x^2} ) ((a &gt; 0))</td>
<td>( \frac{\pi}{a} \sin \omega a )</td>
</tr>
<tr>
<td>2. ( H(x)e^{-\omega x} ) ((\text{Re} \omega &gt; 0))</td>
<td>( \frac{1}{a + i\omega} )</td>
</tr>
<tr>
<td>3. ( H(-x)e^{\omega x} ) ((\text{Re} \omega &gt; 0))</td>
<td>( \frac{1}{a - i\omega} )</td>
</tr>
<tr>
<td>4. ( e^{-\omega x} ) ((\omega &gt; 0))</td>
<td>( \frac{1}{\sqrt{\omega^2 + a^2}} )</td>
</tr>
<tr>
<td>5. ( e^{-\omega x^2} )</td>
<td>( e^{-\omega^2 a^2} )</td>
</tr>
<tr>
<td>6. ( \frac{1}{2\sqrt{\pi x}} e^{-x^2/(4\omega^2)} ) ((\omega &gt; 0))</td>
<td>( e^{-\omega^2 a^2} )</td>
</tr>
<tr>
<td>7. ( \frac{1}{\sqrt{\pi}} )</td>
<td>( \sqrt{\frac{x}{\omega}} )</td>
</tr>
<tr>
<td>8. ( e^{-\omega x^2/4} \sin \left( \frac{\omega}{\sqrt{\omega}} x + \frac{\pi}{2} \right) ) ((\omega &gt; 0))</td>
<td>( \frac{2\sin \omega a}{\omega} )</td>
</tr>
<tr>
<td>9. ( H(x + a) - H(x - a) )</td>
<td>( e^{-\omega a} )</td>
</tr>
<tr>
<td>10. ( \delta(x - a) )</td>
<td>( e^{-\omega a} )</td>
</tr>
<tr>
<td>11. ( f(ax + b) ) ((\omega &gt; 0))</td>
<td>( \frac{1}{a} e^{i\omega b/a} f \left( \frac{\omega}{a} \right) )</td>
</tr>
<tr>
<td>12. ( \frac{1}{a} e^{-i\omega x} f \left( \frac{\omega}{a} \right) ) ((\omega &gt; 0, b \text{ real}))</td>
<td>( f(\omega a + b) )</td>
</tr>
<tr>
<td>13. ( f(\omega) \cos cx ) ((\omega &gt; 0, c \text{ real}))</td>
<td>( \frac{1}{2a} \left[ f \left( \frac{\omega + c}{a} \right) + f \left( \frac{\omega - c}{a} \right) \right] )</td>
</tr>
<tr>
<td>14. ( f(\omega) \sin cx ) ((\omega &gt; 0, c \text{ real}))</td>
<td>( \frac{1}{2a} \left[ f \left( \frac{\omega + c}{a} \right) - f \left( \frac{\omega - c}{a} \right) \right] )</td>
</tr>
<tr>
<td>15. ( f(x + c) + f(x - c) ) ((c \text{ real}))</td>
<td>( 2f(\omega) \text{ cos } wc )</td>
</tr>
<tr>
<td>16. ( f(x + c) - f(x - c) ) ((c \text{ real}))</td>
<td>( 2f(\omega) \sin wc )</td>
</tr>
<tr>
<td>17. ( \pi^n f(x) ) ((n = 1, 2, \ldots))</td>
<td>( \pi^n \frac{\partial^n}{\partial \omega^n} \hat{f}(\omega) )</td>
</tr>
</tbody>
</table>

Linearity of transform and inverse:

18. \( \alpha f(x) + \beta g(x) \) | \( \alpha \hat{f}(\omega) + \beta \hat{g}(\omega) \) |

Transform of derivative:

19. \( f^{(n)}(x) \) | \( (i\omega)^n \hat{f}(\omega) \) |

Transform of integral:

20. \( f(x) = \int_{-\infty}^{x} g(t) \, dt \) | \( \hat{f}(\omega) = \frac{1}{i\omega} \hat{g}(\omega) \) |

where \( f(x) \to 0 \) as \( x \to \infty \)

Fourier convolution theorem:

21. \( (f * g)(x) = \int_{-\infty}^{\infty} f(x - \xi)g(\xi) \, d\xi \) | \( \hat{f}(\omega)\hat{g}(\omega) \)

contd... P5
# Tables of Fourier Cosine and Sine Transforms

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( \hat{f}(x) = \int_0^\infty f(x) \cos zx , dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^{-\alpha x} ) ((\alpha &gt; 0))</td>
<td>( \frac{\alpha}{\alpha^2 + s^2} )</td>
</tr>
<tr>
<td>( x^n e^{-\alpha x} ) ((\alpha &gt; 0))</td>
<td>( \frac{n!(\alpha + i\omega)^{n+1}}{(\alpha^2 + s^2)^{n+1}} ) ((\text{Re} , s &gt; \alpha))</td>
</tr>
<tr>
<td>( \frac{1}{\sqrt{2\pi}} e^{-\alpha x} ) ((\alpha &gt; 0))</td>
<td>( \frac{\alpha}{2\alpha^2 + s^2} )</td>
</tr>
</tbody>
</table>

**Linearity of transform and inverse:**
- \( a f(x) + b g(x) \) \( \rightarrow \) \( a \hat{f}(x) + b \hat{g}(x) \)

**Transform of derivative:**
- \( f'(x) \) \( \rightarrow \) \( \sqrt{2\pi} \hat{f}(0) \)
- \( g''(x) \) \( \rightarrow \) \( \sqrt{2\pi} \hat{g}(0) \)

**Convolution theorem:**
- \( \frac{1}{\sqrt{2\pi}} \int_0^\infty f(x-t)g(x) \, dx = \hat{f}(x) \hat{g}(x) \)

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( \hat{f}(x) = \int_0^\infty f(x) \sin zx , dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^{-\alpha x} ) ((\alpha &gt; 0))</td>
<td>( \frac{\alpha}{\alpha^2 + s^2} )</td>
</tr>
<tr>
<td>( x^n e^{-\alpha x} ) ((\alpha &gt; 0))</td>
<td>( \frac{n!(\alpha + i\omega)^{n+1}}{(\alpha^2 + s^2)^{n+1}} ) ((\text{Im} , s &gt; \alpha))</td>
</tr>
<tr>
<td>( \frac{1}{\sqrt{2\pi}} e^{-\alpha x} ) ((\alpha &gt; 0))</td>
<td>( \frac{\alpha}{2\alpha^2 + s^2} )</td>
</tr>
</tbody>
</table>

**Linearity of transforms and inverse:**
- \( a f(x) + b g(x) \) \( \rightarrow \) \( a \hat{f}(x) + b \hat{g}(x) \)

**Transform of derivative:**
- \( f'(x) \) \( \rightarrow \) \( -\sqrt{2\pi} \hat{f}(0) \)
- \( g''(x) \) \( \rightarrow \) \( -\sqrt{2\pi} \hat{g}(0) \)

**Convolution theorem:**
- \( \frac{1}{\sqrt{2\pi}} \int_0^\infty f(x-t)g(x) \, dx = \hat{f}(x) \hat{g}(x) \)
SECTION A
There are FIVE questions in this section. Answer any FOUR.

1. (a) Using energy method, prove that due to torsion the end rotation of a uniform shaft
\[ \phi = \frac{TL}{GJ}, \]
where all the symbols carry their usual meaning.

(b) Using energy method, calculate the deflection of point C of the beam shown in
Figure 1. Given that, size of the beam is 4" x 6" and \( E = 29000 \text{ ksi} \).

2. (a) Calculate the force carrying capacity of the riveted butt joint shown in Figure 2.
The main plates are 13 mm thick by 250 mm wide; the two cover plates are each 7 mm
thick. The rivets are 22 mm in diameter in 25 mm standard holes. The allowable
stresses are 140 MPa in tension, 100 MPa in shear and 340 MPa in bearing.

(b) A bracket is to be attached to a body of a machine by means of three welds as
shown in the Figure 3. If the applied force is 70 kN, what size fillet weld is required.
The tensile strength of the weld material is 450 MPa. All dimensions are in mm.
3. (a) A steel column of W10 x 45 section is 25 ft long and fixed supported at both ends. The column is laterally supported at mid height to resist the buckling around minor axis. Determine the Euler's critical buckling load of the column. The yield stress of the steel is 36 ksi and young's modulus is 29000 ksi. Properties of W-sections are given in Table-1.

(b) Use AISC/LRFD column formulas, determine the design strength $P_u$ for the steel column stated in question 3(a).

$$\lambda_c = \frac{L_e}{r_n} \sqrt{\frac{\sigma_{yp}}{E}}$$

$$\sigma_{cr} = \left[ \frac{0.877}{\lambda_c^2} \right] \sigma_{yp} \quad \text{For} \quad \lambda_c > 1.5$$

$$\sigma_{cr} = \left[ 0.658 \lambda_c^2 \right] \sigma_{yp} \quad \text{For} \quad \lambda_c \leq 1.5$$

4. (a) Using direct integration method, determine the deflection and rotation of the cantilever beam of Figure 4 at the free end. Consider, $EI$ of the beam as constant.

(b) Using moment area method, determine the maximum deflection between two supports of the beam shown in Figure 5. Consider, $EI$ of the beam as constant.

5. Draw shear force and bending moment diagram of the indeterminate beam shown in Figure 6. Also calculate the deflection and rotation of the beam at the mid span. The cross section of the beam is 250 x 400 mm and $E = 25$ GPa.
CE 213

SECTION – B

There are FIVE questions in this section. Answer any FOUR. Assume any reasonable value for any missing data.

6. A 12-foot long cantilever beam is constructed from an S 24 × 80 steel section (A = 23.5 in², beam depth = 24 in, flange width = 7 in, $I_{zz} = 2100$ in⁴, $S_{zz} = 175$ in³, $I_{yy} = 42.2$ in⁴, $S_{yy} = 12.1$ in³) shown in Figure 7. A load $P = 10$ kip acts in the vertical direction at the end of the beam.

(a) Determine the maximum bending stresses in the beam if the beam is inclined at a small angle $\alpha = 1^\circ$ to the load $P$.

(b) Calculate the neutral axis $n-n$ inclination angle $\beta$ from the $z$ axis.

(c) Compute % reduction in maximum bending stress if $\alpha = 0^\circ$ in comparison to $\alpha = 1^\circ$.

![Figure 7](image)

7. (a) The horizontal force of $P = 80$ kN acts at the free end of the fixed supported plate as shown in Figure 8. The plate has a thickness of 10 mm and $P$ acts along the centerline of this thickness such that $d = 50$ mm. Plot the distribution of normal stress acting along section a-a.

(b) Determine the normal stress and shear stress acting on the inclined plane AB as shown in Figure 9. Solve the problem using the stress transformation equations. Show the result on the sectioned element.

![Figure 8](image)  
![Figure 9](image)
8. Given the state of stress shown in Figure 10, use Mohr's Circle of stress to determine (i) the principal stresses and the corresponding plane on which they act, (ii) maximum in-plane shearing stress along with associated normal stress, (iii) the stresses acting on an element inclined at an angle \( \theta = 45^\circ \). Show the result for all cases on properly oriented elements. (8+8+10 \( \frac{3}{4} \))

![Figure 10](image)

9. (a) Derive and state general cable theorem. (10 \( \frac{3}{4} \))

(b) Determine the reactions at the supports (A & C) produced by the 120 kips load at midspan as shown in Figure 11 (i) using the equations of static equilibrium and (ii) using the general cable theorem. Neglect the self weight of the cable ABC. (8+8)

![Figure 11](image)

10. (a) Describe the necessity of theories of failure. Write down six theories of failure. (10 \( \frac{1}{4} \))

(b) For plane stress condition \( \sigma_3 = 0 \), derive and show graphically the yield criteria based on,

(i) Maximum principal stress theory
(ii) Maximum principal strain theory

(8+8)
### Table 4A. American Standard Steel W Shapes: Dimensions and Properties

<table>
<thead>
<tr>
<th>Designation</th>
<th>Area A (in²)</th>
<th>Depth d (in)</th>
<th>Flange Width b (in)</th>
<th>Flange Thickness t (in)</th>
<th>Axis X-X</th>
<th>Axis Y-Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>W8 × 8</td>
<td>0.600</td>
<td>14.510</td>
<td>1.350</td>
<td>16100</td>
<td>15.0</td>
<td>10100</td>
</tr>
<tr>
<td>W10 × 10</td>
<td>0.760</td>
<td>16.470</td>
<td>1.260</td>
<td>15000</td>
<td>14.9</td>
<td>960</td>
</tr>
<tr>
<td>W12 × 12</td>
<td>0.825</td>
<td>18.430</td>
<td>1.180</td>
<td>15000</td>
<td>14.7</td>
<td>10900</td>
</tr>
<tr>
<td>W14 × 14</td>
<td>0.900</td>
<td>20.390</td>
<td>1.100</td>
<td>15000</td>
<td>14.5</td>
<td>10900</td>
</tr>
<tr>
<td>W16 × 16</td>
<td>0.975</td>
<td>22.350</td>
<td>1.020</td>
<td>15000</td>
<td>14.3</td>
<td>10900</td>
</tr>
<tr>
<td>W18 × 18</td>
<td>1.050</td>
<td>24.310</td>
<td>0.940</td>
<td>15000</td>
<td>14.1</td>
<td>10900</td>
</tr>
</tbody>
</table>

**American standard wide-flange shapes are designated by the letter W followed by the nominal depth in inches with the weight in pounds per linear foot given last.**