L-2/T-2/IPE **Date : 01/04/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2017-2018

Sub : **IPE 205** (Manufacturing Process-I)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What are some of the materials used in making casting patterns? What features should be considered when selecting a pattern material? Explain briefly. **(10)**

   (b) Show by means of neat sketch the section of a typical finished mold, prior to casting. Name the various elements and gating system and explain their functions. **(10)**

   (c) With the help of suitable diagrams describe the following: **(15)**

      (i) Split pattern and Sweep pattern

      (ii) Centrifugal casting

2. (a) List the advantages and limitations of metal as the pattern material. Discuss the dry sand mold and skin dry mold used in the casting process. **(10)**

   (b) With the help of sketches describe the different types of casting defect in sand casting. How might defective castings be repaired to permit successful use in their intended application? **(10)**

   (c) Describe briefly the investment casting with necessary sketch. What are the advantages and limitations of investment casting in comparison with sand mold casting? **(15)**

3. (a) Sketch the various types of weld joints and weld used in making a joint. Differentiate among autogenous welding, homogeneous welding and heterogeneous welding. **(10)**

   (b) Explain the similarities and differences of consumable and non-consumable electrodes. Why is shielded metal arc welding a commonly used process? Why it is also called stick welding? **(10)**

   (c) With the help of neat sketches, describe briefly the principles of operation of Submersed arc welding. For what types of applications might Submersed arc welding be attractive? **(15)**

4. (a) Explain the similarities and differences between Electron-beam welding and Laser-beam welding. Give some typical applications for each. **(10)**

   (b) Enumerate common defects encountered with welding products and suggest methods to counter these defects. Identify the factors that affect weldability. **(10)**

   (c) What are the desirable properties of a metal that would provide good weldability for resistance welding? With the help of suitable diagram describe Percussion welding. **(15)**

Contd P/2

IPE 205

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Explain different forging operations based on the nature of work material flow which is constrained by dies. **(15)**

   (b) Describe any 5 types of forming processes in sheet metal working with necessary sketches. **(20)**

6. (a) Distinguish different forging presses regarding their mechanism. **(12)**

   (b) What do you mean by extrusion? Describe various extrusion processes with appropriate sketches. **(15)**

   (c) With the help of diagram, discuss the following operations: **(8)**

   (i) Upsetting

   (ii) Metal hobbing

7. (a) Using schematic diagram briefly describe different types of dies used for shearing of sheet metal. **(12)**

   (b) Discuss the manufacturing process of an glass bottle with appropriate sketches. **(15)**

   (c) Describe the general properties of ceramics and glasses. **(8)**

8. (a) What are the steps involved in making powder metallurgy parts? Describe briefly. **(12)**

   (b) Discuss the design considerations for powder metallurgy? Give examples for poor and good design of powder metallurgy parts. **(15)**

   (c) How the density can influences the mechanical and physical properties of powder metallurgy parts. **(8)**

L-2/T-2/IPE **Date : 07/04/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2017-2018

Sub : **ME 243** (Mechanics of Solids)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Before the 400-kN load is applied, the rigid platform rests on two steel bars, each of cross-sectional area 1400 mm^2, as shown in the Fig. for Q. No. 1(a). The cross-sectional area of the aluminium bar is 2800 mm^2. Compute the stress in the aluminium bar after the 400-kN load is applied. Use E = 200 GPa for steel and E = 70 GPa for aluminum. Neglect the weight of the platform. **(15)**

   (b) (i) For the axial loading shown in the Fig. for Q. 1(b), determine the change in height and the change in volume of the brass cylinder.

   (ii) Solve the problem in part (i), assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa. **(20)**

2. (a) Four pulleys are attached to the 50-mm-diameter aluminium shaft. If torques are applied to the pulleys as shown in Fig. for Q. No. 2(a), determine the angle of rotation of pulley D relative to pulley A. Use G = 28 GPa for aluminium. **(17)**

   (b) A flanged bolt coupling consists of six $\frac{1}{2}$ in. steel bolts evenly spaced around a bolt circle 12 in. in diameter, and four $\frac{3}{4}$ in. aluminium bolts on a concentric bolt circle 8 in. in diameter. What torque can be applied without exceeding 9000 psi in the steel or 6000 psi in the aluminium? Assume $G_{st} = 12 \times 10^6$ psi and $G_{al} = 4 \times 10^6$ psi. **(18)**

3. (a) An aluminium column of length L and rectangular cross-section has a fixed end at B and supports an axial load at A as shown in Fig. for Q. No. 3(a). Two smooth and rounded fixed plates restrain end A from moving in one of the vertical planes of symmetry but allow it to move in the other plane. **(18)**

   (i) Determine the ratio a/b of the two sides of the corss-section corresponding to the most efficient design against buckling and

   (ii) Design the most efficient cross-section for the column, knowing that, L = 20in., $E = 10.1 \times 10^6$ psi, $P = 5$ kips and factor of safety $FS = 2.5$ are required.

   (b) A pinned-end column of length L = 2.1 m is constructed of steel pipe (E = 210 GPa) having insider diameter $d_1 = 60$ mm and outside diameter $d_2 = 68$ mm as shown in Fig. for Q. No. 3(b). A compressive load P = 10 kN acts with eccentricity $e = 30$ mm. **(17)**

   (i) What is the maximum compressive stress in the column?

   (ii) If the allowable stress in the steel is 50 MPa, what is the maximum permissible length L_{max} of the column?

Contd P/2

4. An element in plane stress is subjected to stresses as shown in Fig. for Q. No. 4. Using Mohr's Circle, determine **(35)**

   (i) the stresses acting on an element oriented at an angle $\theta = 60°$ from the x-axis.

   (ii) the principal stresses and

   (iii) the maximum shear stresses and associated normal stresses.

   Show all results on sketches of properly oriented elements.

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Draw the shear force and bending moment diagrams for the loaded beam as shown in Figure 5(a). Specify values at all changes of loading positions. Also specify the value of maximum bending moment and the location where it occurs. **(17)**

   (b) A box overhanging beam carries a uniformly distributed load of 5 kN/m and a concentrated load of 10 kN, as shown in Figure 5(b). If the allowable stress in either tension or compression is 120 MPa, determine the thickness "t". **(18)**

6. (a) A cantilever beam of length L carries a uniformly distributed load of "**w**" over its entire length and a concentrated load of **P** at its free end, as shown in Figure 6(a). Using double integration method, find the equation of the elastic curve of the beam and an expression of maximum deflection. **(17)**

   (b) A cantilever beam of length 4 m is subjected to a triangular distribution of load with maximum value of 20 kN and an end moment M_o = 5 kNm, as shown in Figure 6(b). Using area moment method, determine the deflection at the free end. Consider flexural rigidity EI = 50×10^6 Nm^2. **(18)**

7. (a) A timber beam is reinforced at the bottom only by a steel plate, as shown in Figure 7(a). Determine the moment that can be resisted by the beam if the allowable stress in timber is 8 MPa and that in steel is 130 MPa. Consider n = 15. **(17)**

   (b) A crane hook of trapezoidal cross-section supports a load P, as shown in Figure 7(b). If the allowable stress of the hook material either in tension or compression is 140 MPa, determine the maximum load P that can be supported, Solve the problem using curve-beam theory. **(18)**

8. (a) Derive the expression for tangential and longitudinal stresses of a thin-walled closed ends cylindrical pressure vessel subjected to internal pressure and hence show that the magnitude of tangential stress is twice the value of longitudinal stress. **(17)**

   (b) For a thick-walled cylindrical pressure vessel subjected to an internal pressure of 10 MPa, determine the wall thickness, if the internal diameter of the cylinder is 500 mm and the tangential stress is not to exceed 110 MPa. What maximum internal pressure may be applied, if this cylinder for the same limiting tangential stress is to be a thin-walled cylinder? **(18)**

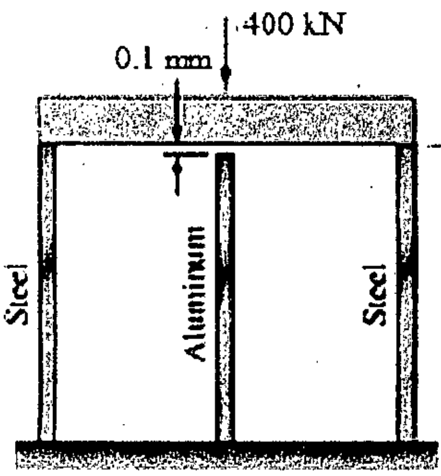

Fig. for Que. No. 1(a)

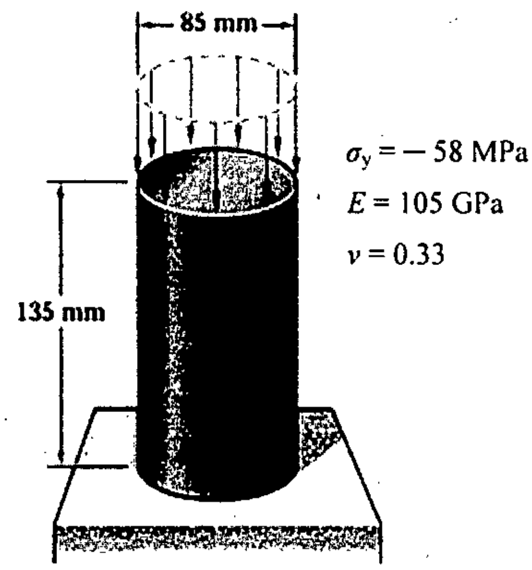

Fig. for Que. No. 1(b)

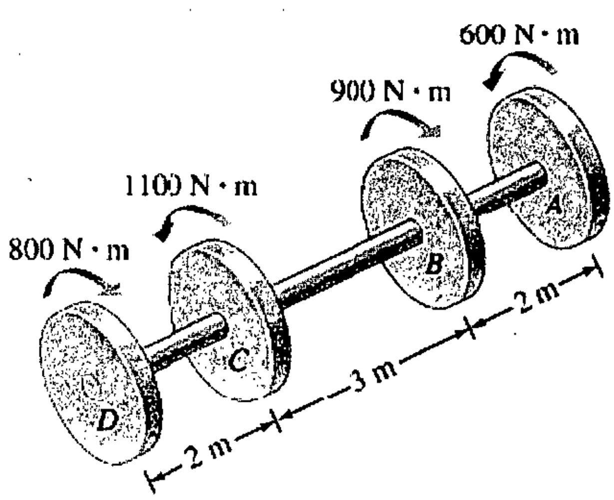

Fig. for Que. No. 2(a)

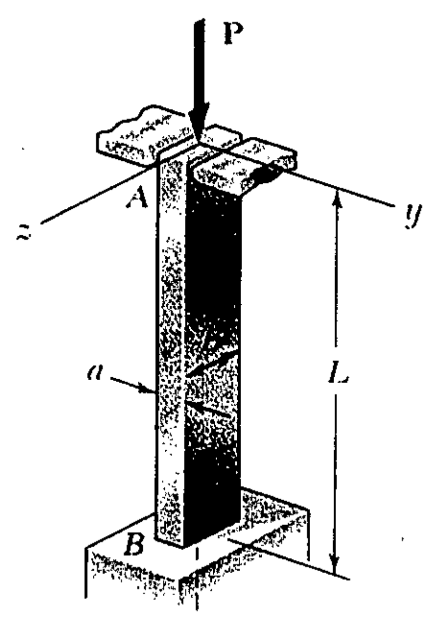

Fig. for Que. No. 3(a)

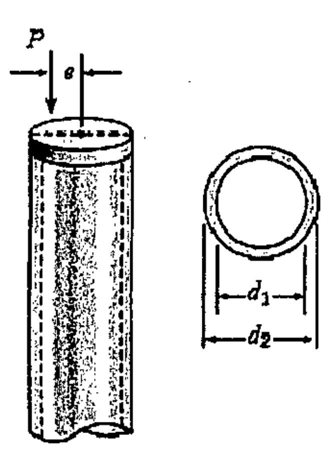

Fig. for Que. No. 3(b)

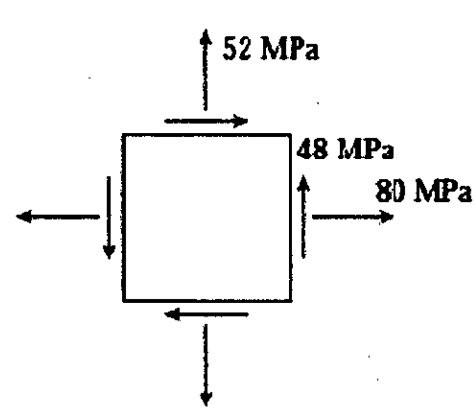

Fig. for Que. No. 4

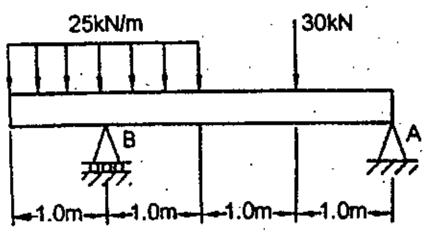

Figure 5(a)

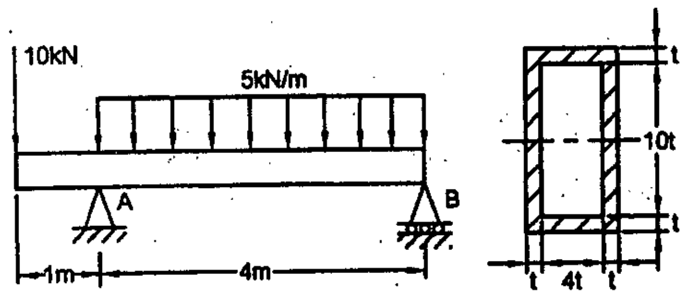

Figure 5(b)

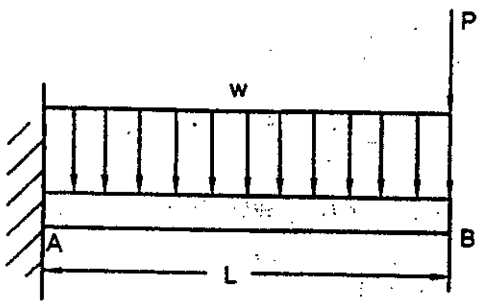

Figure 6(a)

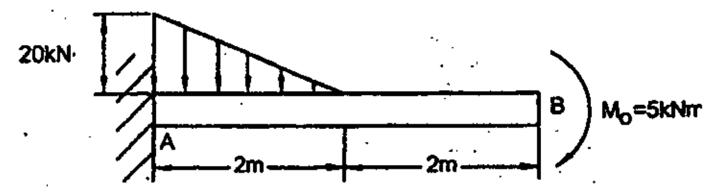

Figure 6(b)

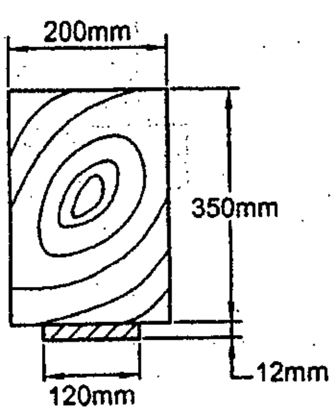

Figure 7(a)

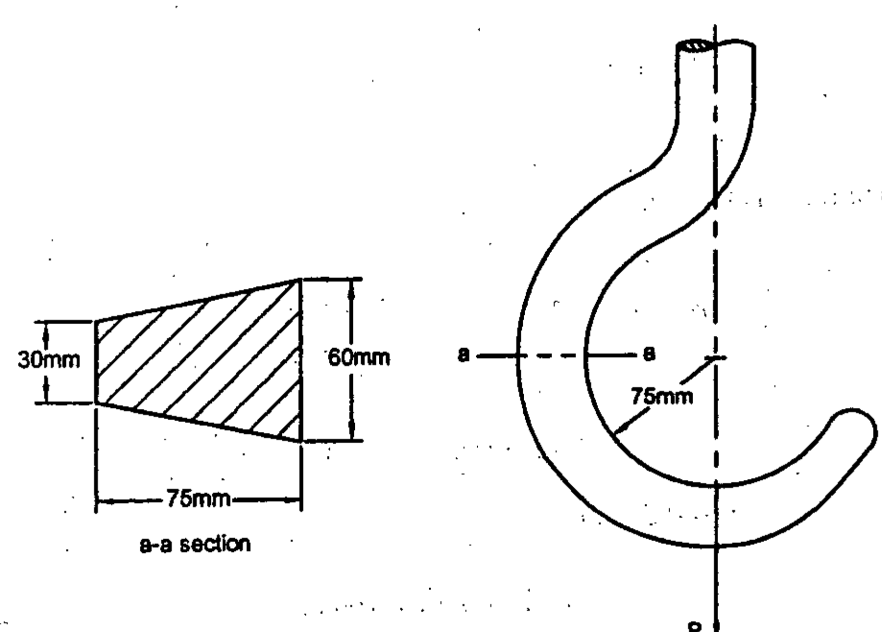

Figure - 7(b)

L-2/T-2/IPE **Date: 11/04/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2017-2018

Sub: **IPE 209** (Engineering Economy)

Full Marks: 140 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

Tables related to Interest, Annuity, Depreciable Property Classification and GDS recovery rate are provided

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SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

Draw cash flow diagrams if applicable.

1. (a) What do you understand by *Minimum Attractive Rate of Return (MARR)*? How does top management of an organization set value of *MARR*? Briefly explain. **($3\frac{1}{3}$)**

(b) A project your firm is considering for implementation has these estimated costs and revenues: an investment cost of \$50,000; maintenance costs that start at \$5,000 at the end of year (EOY) 1 and increase by \$1,000 for each of the next 4 years, and then remain constant for the following 5 years; savings of \$20,000 per year (EOY 1–10); and finally a resale value of \$35,00 at the EOY 10. If the project has a 10-year life and the firm's MARR is 10% per year, what is the present worth of the project? Is it a sound investment opportunity? **(10)**

(c) Stan Moneymaker has the opportunity to purchase a certain U.S. Treasury bond that matures in eight years and has a face value of \$10,000. This means that Stan will receive \$10,000 cash when the bond's maturity date is reached. The bond stipulates a fixed nominal interest rate of 8% per year, but interest payments are made to the bondholder every three months; therefore, each payment amounts to 2% of the face value. Stan would like to earn 10% nominal interest (compounded quarterly) per year on his investment, because interest rates in the economy have risen since the bond was issued. How much should Stan be willing to pay for the bond? **(10)**

2. (a) What is Capitalized Worth (CW)? Discuss in short. **($3\frac{1}{3}$)**

(b) A new bridge across the Cumberland River is being planned near a busy high-way intersection in the commercial part of a mid-western town. The construction (first) cost of the bridge is \$1,900,000 and annual upkeep is estimated to be \$25,000. In addition to annual upkeep, major maintenance work is anticipated every eight years at a cost of \$350,000 per occurrence. The town government's MARR is 8% per year. (i) For this problem, what analysis period (N) is, practically speaking, defined as forever? (ii) If the bridge has an expected life of 50 years, what is the capitalized worth (iii) (CW) of the bridge over a 100-year study period? **(10)**

Contd P/2

IPE 209

Contd... Q. No. 2

(c) The world's largest carpet maker has just completed a feasibility study of what to do with the 16,000 tons of overruns, rejects, and remnants it produces every year. The company's CEO launched the feasibility study by asking, why pay someone to dig coal out of the ground and then pay someone to put our waste into a landfill? Why not just burn our own waste? The company is proposing to build a $10 million power plant to burn its waste as fuel, thereby saving $2.8 million a year in coal purchases. Company engineers have determined that the waste burning plant will be environmentally sound, and after its four-year study period the plant can be sold to a local electricity utility for $5 million. (i) What is the IRR of the proposed power plant? (ii) If the firm's MARR is 15% per year, should this project be undertaken? **(10)**

3. (a) Four mutually exclusive alternatives are being evaluated, and their costs and revenues are itemized below. If the MARR is 15% per year and the analysis period is 12 years, use the PW method to determine which alternatives are economically acceptable and which one should be selected. If the total capital investment budget available is $200,000, which alternative should be selected? Which rule applies? Why? **(11 $\frac{1}{3}$)**

	Mutually Exclusive Alternative			
	I	II	III	IV
Capital investment	$100,000	$152,000	$184,000	$220,000
Annual revenues less expenses	15,200	31,900	35,900	41,500
Market value (end of useful life)	10,000	0	15,000	20,000
Useful life (years)	12	12	12	12

(b) You have been requested to offer a recommendation of one of the mutually exclusive industrial sanitation control systems shown in the table below. If the MARR is 15% per year, which system would you select? Use the IRR method. **(6)**

	Gravity-fed	Vacuum-led
Capital investment	$24,500	$37,900
Annual receipts less expenses	8,000	8,000
Life (in years)	5	10
IRR	18.9%	16.5%

(c) A firm purchased and placed in service a new piece of semiconductor manufacturing equipment. The cost basis for the equipment is $100,000. Determine (i) the depreciation charge permissible in the fourth year, (ii) the BV at the end of the fourth year, (iii) the cumulative depreciation through the third year, (iv) the BV at the end of the fifth year if the equipment is disposed of at that time. **(6)**

IPE 209

4. (a) Acme purchased a pump for Tk. 25,000,000 and expended Tk. 2,000,000 for shipping and installation. The addition of this pump wil result in an increase in revenue of Tk. 8,000,000 with associated increased expenses of Tk. 1,000,000 each year. The pump has a GDS recovery period of five years, and Acme's effective tax rate is 40%. Calculate BTCFs and ATCFs for this project for different years of service of the asset. **(10)**

(b) List some difficulties associated with evaluating public sector projects. The city of Columbia is considering extending the runways of its municipal airport so that commercial jets can use the facility. The land necessary for the runway extension is currently a farmland that can be purchased for $350,000. Construction costs for the runway extension are projected to be $600,000, and the additional annual maintenance costs for the extension are estimated to be $22,500. If the runways are extended, a small terminal will be constructed at a cost of $250,000. The annual operating and maintenance costs for the terminal are estimated at $75,000. Finally, the projected increase in flight will required the addition of two air traffic controller at an annual cost of $100,000. Annual benefits of the runway extension have been estimated as follows: **(11)**

$325,000 Rental receipts from airlines leasing space at the facility

$65,000 Airport tax charged to passengers

$50,000 Convenience benefit for residents of Columbia

$50,000 Additional tourism dollars for Columbia

Apply the B-C ratio method with a study period of 20 years and a MARR of 10% per year to determine whether the runways at Columbia Municipal Airport should be extended.

(c) Briefly explain the importance of sensitivity analysis in case of engineering economy. Explain with an example. **(2⅓)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) While studying for the Engineering Economy final exam, you and two friends find yourselves craving a fresh pizza. You can't spare the time to pick up the pizza and must have it delivered, "Pick-UP-Sticks" offers a 1.25-inch-thick (including toppings), 20-inch square pizza with your choice of two toppings for $15 plus 5% VAT and a $1.50 delivery charge (no Vat on delivery charge). "Pizza King" offers the round, deep-dish pizza, which is 20 inches in diameter. It is 1.75 inches thick, includes two toppings, and costs $17.25 plus 5% VAT and free delivery. **(18)**

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Contd... Q. No. 5(a)

(i) What is the problem in this situation? Please state it in an explicit and precise manner.

(ii) Systematically apply the seven principles of engineering economy to the problem you have defined in part (i).

(iii) Assuming that your common unit of measure is dollars (i.e., cost), what is the better value for getting a pizza based on the criterion of minimizing cost per unit of volume?

(iv) What other criteria might be used to select which pizza to purchase?

(b) Differentiate between necessities and luxuries. Explain with examples. **(5⅓)**

6. (a) Briefly explain the acquisition phase of life cycle cost and its relative cost. **(10)**

(b) Write down the differences between cash cost and book cost briefly with examples. **(5)**

(c) A large wood products company is negotiating a contract to sell plywood overseas. The fixed cost that can be allocated to the production of plywood is \$800,000 per month. The variable cost per thousand board feet is \$155.50. The price charged will be determined by $p = \$600 - (0.5)D$ per 1,000 board feet. **(8⅓)**

(i) For this situation, determine the optimal monthly sales volume for this product and calculate the profit (or loss) at the optimal volume.

(ii) What is the domain of profitable demand during a month?

7. (a) It is estimated that you will pay about \$80,000 into the Social Security system (FICA) over your 40-year work span. For simplicity, assume this is an annuity of \$2,000 per year, starting with your 26th birthday and continuing through your 65th birthday. **(11)**

(i) What is the future equivalent worth of your Social Security savings when you retire at age 65 if the government's interest rate is 9% per year?

(ii) What annual withdrawal can you make if you expect to live 20 years in retirement? Let i = 9% per year.

(b) Suppose that a \$200 lump-sum amount is invested for 15 years at a nominal interest rate of 15% compounded quarterly. How much is it worth at the end of the 10th year? **(6⅓)**

(c) Describe the differences between **(6)**

(i) Simple interest and compound interest

(ii) Nominal interest and effective interest rate

(iii) Consumer goods and producer goods

Contd P/5

IPE 209

8. (a) What value of T makes these two cash flow diagrams (see Figure 8(a)) economically equipment at 9% annual interest? **(10)**

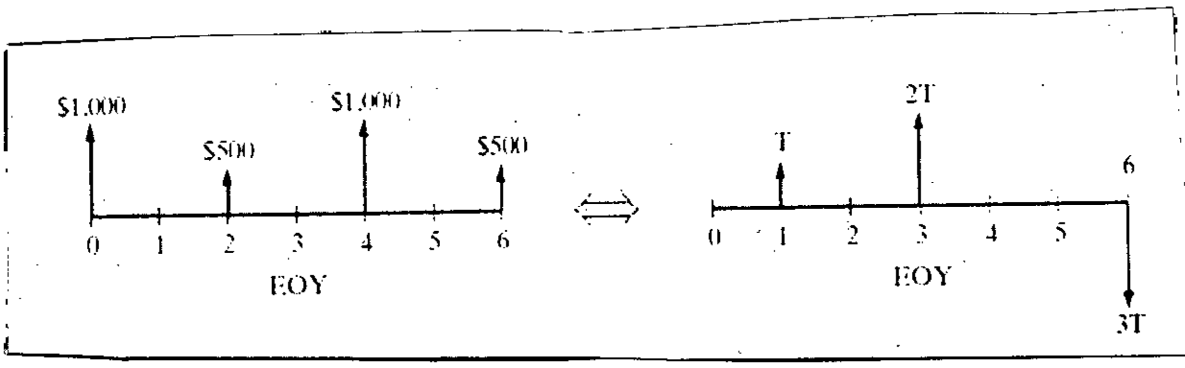

Figure: 8(a)

(b) Determine the present equivalent and annual equivalent (uniform) value of the cash flow pattern shown below when i = 15% per year. **(7⅓)**

End of Year	Amount ($)
0	–1600
1	+700
2	+800
3	+400
4	+200
5	+100
6	+200
7	+300

(c) Write short notes on **(6)**

(i) Direct and indirect cost
(ii) Incremental cost
(iii) Opportunity cost

L-2/T-2/IPE **Date : 27/03/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2017-2018

Sub : **ME 265** (Thermodynamics and Heat Transfer)

Full Marks : 280 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE.**

1. (a) Briefly explain the primary energy supplies in Bangladesh. **(15)**

(b) Write about per capita electricity consumption in major SAARC countries. Also mention their corresponding carbon emission scenario. **(10)**

(c) Write a note on 'Applications of Heat Transfer' – Also mention about empirical laws of each of the heat transfer modes. **(11⅔)**

(d) Give examples of heat generation in solids. How is it taken care of in one dimensional heat conduction analysis? **(10)**

2. (a) Consider a long resistance wire of radius 2 mm and thermal conductivity k = 15 W/m.K in which heat is generated uniformly as a result of resistance heating at a constant rate of 50 W/cm^3. The wire is embedded in a 5 mm-thick layer of ceramic (k = 1.2 W/m.K). If the outer surface of the ceramic layer is measured to be 45°C, determine the temperatures at the center of the wire and the interface of the wire and the ceramic layer under steady state conditions. **(22)**

(b) A 3-mm-diameter an 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is 0.15 W/m.K. Electrical measurements indicate that a current of 10A passes through the wire and there is a voltage drop of 10 V along the wire. If the insulated wire is exposed to a medium at 30°C with a heat transfer coefficient of 15 $W/m^2.K$, determine the temperature at the interface of the wire and the plastic cover under steady operation Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature. **(16)**

(c) Write what you know about convection heat transfer coefficient. **(8⅔)**

3. (a) With physical significance, briefly explain three important non-dimensional parameters used in forced convection. Also write a general expression for their relationship. **(11⅔)**

(b) Water is to be heated from 15°C to 65°C as it flows through a 3 cm-internal diameter 5-m long tube. The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface-temperature of the pipe at the exit. **(15)**

Contd P/2

ME 265

Contd ... Q. No. 3(b)

Properties of water at 40°C:

$\rho = 992.1$ kg/m^3; $c_p = 4179$ J/kg°C; $p_r = 4.32$ k = 0.631 W/m°C; $\nu = 0.658 \times 10^{-6}$ m^2/s.

(c) What is natural convection? What are the factors that influence natural convection? **(10)**

(d) Schematically show the natural convection flows in hot and cold plates oriented horizontally and vertically. **(10)**

4. (a) A six meter long section of an 8-cm-diameter horizontal water pipe passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection and radiation. Assume the emissivity of the pipe surface to be 0.8. Comment on the results. **(20)**

Properties of air at 45°C:

k = 0.02699 W/m°C; $p_r = 0.7241$; $\nu = 1.749 \times 10^{-5}$ m^2/s

(b) What are the factors on which thermal radiation depend? Write about view factor algebra. **(10)**

(c) Distinguish between refrigeration and air conditioning. **(6⅔)**

(d) What is a heat exchanger? Mention its types. Explain fouling in it. **(10)**

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

Abbreviations have their standard meaning.

5. (a) Compare the characteristics of reciprocating CI and SI engines. **(14)**

(b) What do you understand by "Firing Order" of a SI engine? With a schematic diagram explain how the high voltage spark is created and distributed up to the spark plugs. **(16⅔)**

(c) Why do we need a flywheel in a reciprocating IC engine? Briefly explain. **(8)**

(d) What do you understand by the following engine specification– $r_v = 10$, $B_{sfc} = 230$ g/kW-h, SAE 20W50, 16-valve. **(8)**

6. (a) State the 2nd law of Thermodynamics. Briefly explain how it is applicable for a Diesel engine and a domestic refrigerator. **(12)**

(b) Briefly explain the advantages of a MPFI system compared to a carburetor in a reciprocating SI engine. **(10)**

(c) Why do we need as cooling system in an engine? With a schematic diagram identify the components of the water cooling system of a typical car engine. **(14⅔)**

(d) State the functions that need to be fulfilled by the fuel system of a Diesel engine. **(10)**

7. (a) Briefly explain the advantages and limitations of using Gas Turbines. Where are they used? **(12)**

(b) With a schematic diagram briefly explain how a Jet Engine is different from a Gas Turbine. **(14)**

Contd P/3

ME 265

Contd … Q. No. 7

(c) What do you understand by "Bypass-ratio"? Which type of Jet engine is most commonly used in modern passenger aircrafts? **(8⅔)**

(d) What do you understand by an "Isentropic" process? Give an example. Deduce an expression of Non-flow energy equation for an isentropic process. **(12)**

8. (a) A boiler capacity is specified as "5 Ton", what does it mean? State the essentials features of a good boiler. **(13⅔)**

(b) What do you understand by "Super heater" and "Economiser" of a boiler. Briefly explain how they influence boiler performance. **(10)**

(c) Distinguish between "Impulse" and "Reaction" turbines. **(8)**

(d) What do you understand by "Compounding"? Briefly explain the different methods of compounding practiced in impulse turbines. **(15)**

TABLE A–15

Properties of air at 1 atm pressure

Temp. T, °C	Density ρ, kg/m³	Specific Heat c_p, J/kg·K	Thermal Conductivity k, W/m·K	Thermal Diffusivity α, m²/s	Dynamic Viscosity μ, kg/m·s	Kinematic Viscosity ν, m²/s	Prandtl Number Pr
−150	2.866	983	0.01171	4.158×10^{-6}	8.636×10^{-6}	3.013×10^{-6}	0.7246
−100	2.038	966	0.01582	8.036×10^{-6}	1.189×10^{-5}	5.837×10^{-6}	0.7263
−50	1.582	999	0.01979	1.252×10^{-5}	1.474×10^{-5}	9.319×10^{-6}	0.7440
−40	1.514	1002	0.02057	1.356×10^{-5}	1.527×10^{-5}	1.008×10^{-5}	0.7436
−30	1.451	1004	0.02134	1.465×10^{-5}	1.579×10^{-5}	1.087×10^{-5}	0.7425
−20	1.394	1005	0.02211	1.578×10^{-5}	1.630×10^{-5}	1.169×10^{-5}	0.7408
−10	1.341	1006	0.02288	1.696×10^{-5}	1.680×10^{-5}	1.252×10^{-5}	0.7387
0	1.292	1006	0.02364	1.818×10^{-5}	1.729×10^{-5}	1.338×10^{-5}	0.7362
5	1.269	1006	0.02401	1.880×10^{-5}	1.754×10^{-5}	1.382×10^{-5}	0.7350
10	1.246	1006	0.02439	1.944×10^{-5}	1.778×10^{-5}	1.426×10^{-5}	0.7336
15	1.225	1007	0.02476	2.009×10^{-5}	1.802×10^{-5}	1.470×10^{-5}	0.7323
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111

- **For Natural convection in a horizontal pipe**

$$\mathrm{Nu} = \left\{0.6 + \frac{0.387\,\mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$\mathrm{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\mathrm{Pr}$$

=4=

TABLE A-9

Properties of saturated water

Temp. T, °C	Saturation Pressure P_{sat}, kPa	Density ρ, kg/m³ Liquid	Density Vapor	Enthalpy of Vaporization h_{fg}, kJ/kg	Specific Heat c_p, J/kg·K Liquid	Specific Heat Vapor	Thermal Conductivity k, W/m·K Liquid	Thermal Conductivity Vapor	Dynamic Viscosity μ, kg/m·s Liquid	Dynamic Viscosity Vapor	Prandtl Number Pr Liquid	Prandtl Number Vapor	Volume Expansion Coefficient β, 1/K Liquid
0.01	0.6113	999.8	0.0048	2501	4217	1854	0.561	0.0171	1.792×10^{-3}	0.922×10^{-5}	13.5	1.00	-0.068×10^{-3}
5	0.8721	999.9	0.0068	2490	4205	1857	0.571	0.0173	1.519×10^{-3}	0.934×10^{-5}	11.2	1.00	0.015×10^{-3}
10	1.2276	999.7	0.0094	2478	4194	1862	0.580	0.0176	1.307×10^{-3}	0.946×10^{-5}	9.45	1.00	0.733×10^{-3}
15	1.7051	999.1	0.0128	2466	4185	1863	0.589	0.0179	1.138×10^{-3}	0.959×10^{-5}	8.09	1.00	0.138×10^{-3}
20	2.339	998.0	0.0173	2454	4182	1867	0.598	0.0182	1.002×10^{-3}	0.973×10^{-5}	7.01	1.00	0.195×10^{-3}
25	3.169	997.0	0.0231	2442	4180	1870	0.607	0.0186	0.891×10^{-3}	0.987×10^{-5}	6.14	1.00	0.247×10^{-3}
30	4.246	996.0	0.0304	2431	4178	1875	0.615	0.0189	0.798×10^{-3}	1.001×10^{-5}	5.42	1.00	0.294×10^{-3}
35	5.628	994.0	0.0397	2419	4178	1880	0.623	0.0192	0.720×10^{-3}	1.016×10^{-5}	4.83	1.00	0.337×10^{-3}
40	7.384	992.1	0.0512	2407	4179	1885	0.631	0.0196	0.653×10^{-3}	1.031×10^{-5}	4.32	1.00	0.377×10^{-3}
45	9.593	990.1	0.0655	2395	4180	1892	0.637	0.0200	0.596×10^{-3}	1.046×10^{-5}	3.91	1.00	0.415×10^{-3}
50	12.35	988.1	0.0831	2383	4181	1900	0.644	0.0204	0.547×10^{-3}	1.062×10^{-5}	3.55	1.00	0.451×10^{-3}
55	15.76	985.2	0.1045	2371	4183	1908	0.649	0.0208	0.504×10^{-3}	1.077×10^{-5}	3.25	1.00	0.484×10^{-3}
60	19.94	983.3	0.1304	2359	4185	1916	0.654	0.0212	0.467×10^{-3}	1.093×10^{-5}	2.99	1.00	0.517×10^{-3}
65	25.03	980.4	0.1614	2346	4187	1926	0.659	0.0216	0.433×10^{-3}	1.110×10^{-5}	2.75	1.00	0.548×10^{-3}
70	31.19	977.5	0.1983	2334	4190	1936	0.663	0.0221	0.404×10^{-3}	1.126×10^{-5}	2.55	1.00	0.576×10^{-3}
75	38.58	974.7	0.2421	2321	4193	1948	0.667	0.0225	0.378×10^{-3}	1.142×10^{-5}	2.38	1.00	0.607×10^{-3}
80	47.39	971.8	0.2935	2309	4197	1962	0.670	0.0230	0.355×10^{-3}	1.159×10^{-5}	2.22	1.00	0.653×10^{-3}
85	57.83	968.1	0.3536	2296	4201	1977	0.673	0.0235	0.333×10^{-3}	1.176×10^{-5}	2.08	1.00	0.670×10^{-3}
90	70.14	965.3	0.4235	2283	4206	1993	0.675	0.0240	0.315×10^{-3}	1.193×10^{-5}	1.96	1.00	0.702×10^{-3}
95	84.55	961.5	0.5045	2270	4212	2010	0.677	0.0246	0.297×10^{-3}	1.210×10^{-5}	1.85	1.00	0.716×10^{-3}
100	101.33	957.9	0.5978	2257	4217	2029	0.679	0.0251	0.282×10^{-3}	1.227×10^{-5}	1.75	1.00	0.750×10^{-3}

Useful Formula

Solid Geometry	Governing Equation For 1D-SS with heat generation	Thermal Resistance, R
Plane Wall	$\frac{d^2T}{dx^2} + \frac{\dot{e}_{gen}}{k} = 0$	$R = \frac{L}{kA}$
Cylinder (pipe, rod)	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{gen}}{k} = 0$	$R = \frac{ln(^{r_2}/_{r_1})}{2\pi Lk}$
Sphere	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{e}_{gen}}{k} = 0$	$R = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$

- **Net Radiation exchange between opaque, diffuse-gray surfaces:**

$$\dot{Q}_{ij} = \frac{\sigma(T_i^4 - T_j^4)}{\frac{1-\varepsilon_i}{\varepsilon_i A_i} + \frac{1}{A_i F_{ij}} + \frac{1-\varepsilon_j}{\varepsilon_j A_j}}$$

- **For fully developed turbulent flow in a smooth tube:**

Dettus-Boelter Equation:

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^n \quad \begin{Bmatrix} \mathrm{Re} > 10,000 \\ 0.7 \le \mathrm{Pr} \le 160 \end{Bmatrix} \quad \begin{vmatrix} n = 0.4\ heating \\ n = 0.3\ cooling \end{vmatrix}$$

L-2/T-2/IPE **Date: 16/03/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2017-2018

Sub: **IPE 207** (Probability and Statistics)

Full Marks: 280 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Explain ordinal-level data and interval-level data. **(12)**

(b) Explain the advantages of cumulative frequency distribution. **(8)**

(c) Suppose that we roll a pair of fair dice, so each of the 36 possible outcomes is equally likely. Let A denote the event that the first dice lands on 3, let B be the event that the sum of the dice is 8, and let C be the event that the sum of the dice is 7. **(16)**

(i) Are A and B independent?

(ii) Are A and C independent?

(d) A blood test is 99 percent effective in detecting a certain disease when the disease is present. However, the test also yields a false-positive result for 2 percent of the healthy patients tested. Suppose 0.5 percent of the population has the disease. Find the conditional probability that a randomly tested individual actually has the disease given that his or her test result is positive. **(10⅔)**

2. (a) Explain the characteristics of hypergeometric distribution. **(10)**

(b) How many three-digit numbers can be formed from the digits 0, 1, 2, 3, 4, 5, and 6, if each digit can be used only once? **(14)**

(i) How many of these are odd numbers?

(ii) How many are greater then 330?

(c) A pair of fair dice is tossed. Find the probability of getting (i) a total of 8 (ii) at most a total of 5. **(12)**

(d) Two cards are drawn in succession from a deck without replacement. What is the probability that both cards are greater than 2 and less than 8? **(10⅔)**

3. (a) Prove that both the mean and variance of the binomial distribution are $\mu = np$, and $\sigma^2 = npq$. **(12)**

(b) The probability that an automobile being filled with gasoline will also need an oil change is 0.25, the probability that it needs a new oil filter is 0.40, and the probability that both the oil and filter need changing is 0.14. **(16)**

(i) If the oil had to be changed, what is the probability that a new oil filter is needed?

(ii) If a new oil filter is needed, what is the probability that the oil has to be changed?

Contd P/2

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Contd... Q. No. 5

(b) A random variable X has the geometric distribution $g(x;p) = pq^{x-1}$ for x = 1, 2, 3, ... Show that the moment-generating function of X is **(16)**

$$M_x(t) = \frac{pe^t}{1-qe^t}, \quad t < \ln q,$$

and then use $M_x(t)$ to find the mean and variance of the geometric distribution.

(c) The Edison Electric Institute has published figures on the number of kilowatt hours used annually by various home appliances. It is claimed that a vacuum cleaner uses an average of 46 kilowatt hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners use an average of 42 kilowatt hours per year with a standard deviation of 11.9 kilowatt hours, does this suggest at the 0.05 level of significance that vacuum cleaners use, on average, less than 46 kilowatt hours annually? Assume the population of kilowatt hours to be normal. **(15)**

6. (a) Engineers at a large automobile manufacturing company are trying to decide whether to purchase brand A or brand B tires for the company's new models. To help them arrive at a decision, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are as follows: **(15⅔)**

Brand A: $\bar{x}_1 = 37{,}900$ kilometers,

$s_1 = 5100$ kilometers.

Brand B: $\bar{x}_1 = 39{,}800$ kilometers,

$s_2 = 5900$ kilometers.

Test the hypothesis that there is no difference in the average wear of the two brands of tires at the 0.05 level of significance. Assume the populations to be approximately normally distributed with equal variances. Use a P-value.

(b) In a study to estimate the proportion of residents in a certain city and its suburbs who favor the construction of the nuclear power plant, it is found that 63 of 100 urban residents favor the construction while only 59 of 125 suburban residents are in favor. Is there a significant difference between the proportions of urban and suburban residents who favor construction of the nuclear plant at the 0.05 level of significance? Make use of a P-value. **(15)**

(c) What assumption do you make regarding the shape of the population distribution in goodness of fit test? Justify your answer. **(4)**

(d) The bank credit card department of Carolina Bank knows from experience that 5 percent of its card holders have had some high school, 15 percent have completed high school, 25 percent have had some college, and 55 percent have completed college. Of the 500 card holders whose cards have been called in for failure to pay their charges this month, 50 had some high school, 100 had completed high school, 190 had some college, and 160 had completed college. Can we conclude that the distribution of card holders who do not pay their charges in different from all others? Use the 0.01 significance level. **(12)**

Contd P/4

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Contd... Q. No. 3

(c) Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pound per square inch (psi). Let x denote the actual air pressure for the right tire and y denote the actual air pressure for the left tire. Suppose that x and y are random variables with the joint density **($18\frac{2}{3}$)**

$$f(x,y)=\begin{cases} k(x^2+y^2), & 30 \le x < 50, \\ & 30 \le y < 50 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Find k.

(ii) Find P ($30 \le x \le 40$ and $40 \le y < 50$).

(iii) Find the probability that both tires are under filled.

4. (a) State the 'Chebyshev's theorem'. **(6)**

(b) Explain the central limit theorem. **(8)**

(c) Explain Stratified Random Sampling and Systematic Random Sampling. **(16)**

(d) In November 1990 issue of chemical engineering progress, a study discussed the present purity of oxygen from a certain supplier. Assume that the mean was 99.61 with a standard deviation of 0.08. Assume that the distribution of percent purity was approximately normal. **($16\frac{2}{3}$)**

(i) What percentage of the purity values would you expect to be between 99.5 and 99.7?

(ii) What purity value would you expect to exceed exactly 5% of the population?

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

Normal distribution, t-distribution, chi-square distribution and F-distribution tables attached.

5. (a) The hospital period, in days, for patients following treatment for a certain type of kidney disorder is a random variable $Y = X + 4$, where X has the density function **($15\frac{2}{3}$)**

$$f(x)=\begin{cases} \frac{32}{(x+4)^3}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}.$$

(i) Find the probability density function of the random variable Y.

(ii) Using the density function of Y, find the probability that the hospital period for a patient following this treatment will exceed 8 days.

IPE 207

7. (a) There are important applications in which, due to know scientific constraints, the regression line **must go through the origin** (i.e., the intercept must be zero). In other words, the model should read

$$Y_i = \beta_1 x_i + \in_i, \quad i = 1, 2, ..., n,$$ **(16⅔)**

and only a simple parameter requires estimation. The model is often called the **regression through the origin model.**

(i) Show that the least squares estimator of the slop is

$$b_1 = \left(\sum_{i=1}^{n} x_i y_i\right) \Big/ \left(\sum_{i=1}^{n} x_i^2\right)$$

(ii) Show that

$$\sigma_{B_1}^2 = \sigma^2 \Big/ \left(\sum_{i=1}^{n} x_i^2\right)$$

(iii) Show that b_1 in part (*i*) is an unbiased estimator for β_1.

(b) Show, in the case of a least squares fit to the simple linear regression model **(12)**

$$Y_i = \beta_0 + \beta_1 x_i + \in_i, \quad i = 1, 2, ..., n,$$

that

$$\sum_{i=1}^{n}(y_i - \hat{y}_i) = \sum_{i=1}^{n} e_i = 0$$

(ii) Consider the situation of part (*i*) but suppose $n = 2$ (i.e., only two data points are available). Give an argument that the least squares regression line will result in $(y_1 - \hat{y}_1) = (y_2 - \hat{y}_2) = 0$. Also show that for this case $R^2 = 1.0$.

(iii) Does the condition in part (*i*) hold for a model with zero intercept? Show why or why not.

(c) A study was made on the amount of converted sugar in a certain process at various temperatures. The data were coded and recorded as follows: **(18)**

Temperature, x	Converted Sugar, y
1.0	8.1
1.1	7.8
1.2	8.5
1.3	9.8
1.4	9.5
1.5	8.9
1.6	8.6
1.7	10.2
1.8	9.3
1.9	9.2
2.0	10.5

(i) Estimate the linear regression line.

(ii) Estimate the mean amount of converted sugar produced when the coded temperature is 1.75.

(iii) Evaluate s^2.

(iv) Construct a 95% confidence interval for β_0.

(v) Construct a 95% confidence interval for β_1.

Contd P/5

8. (a) A study was performed on a type of bearing to find the relationship of amount of wear y to x_1 = oil viscosity and x_2 = load. The following data were obtained. **(30⅔)**

y	x_1	x_2	y	x_1	x_2
193	1.6	851	230	15.5	816
172	22.0	1058	91	43.0	1201
113	33.0	1357	125	40.0	1115

(i) Estimate the unknown parameters of the multiple linear regression equation

$$\mu_{Y|x_1,x_2} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 .$$

(ii) Estimate σ^2.

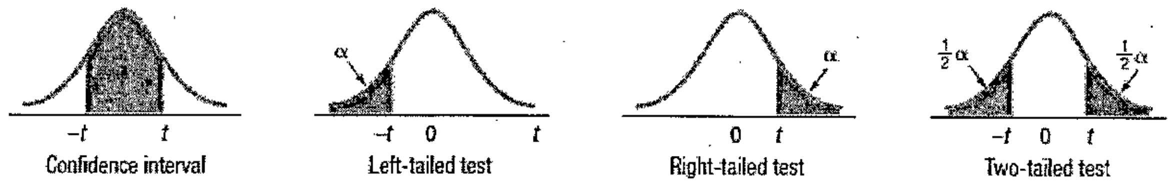
(iii) Compute predicted values, a 95% confidence interval for mean wear, and a 95% prediction interval for observed wear if $x_1 = 20$ and $x_2 = 1000$.

(iv) Test the following at the 0.05 level. $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$; $H_0 : \beta_2 = 0$ versus $H_1 : \beta_2 \neq 0$. Do you have any reason to believe that the model in part (*i*) should be changed? Why or why not?

(b) The data in the following table represent the number of hours of relief provided by five different brands of headache tablets administered to 25 subjects experiencing fevers of 38°C or more. Perform the analysis of variance and test the hypothesis at the 0.05 level of significance that the mean number of hours of relief provided by the tablets is the same for all five brands. Discuss the results. **(16)**

Tablet

A	*B*	*C*	*D*	*E*
5.2	9.1	3.2	2.4	7.1
4.7	7.1	5.8	3.4	6.6
8.1	8.2	2.2	4.1	9.3
6.2	6.0	3.1	1.0	4.2
3.0	9.1	7.2	4.0	7.6



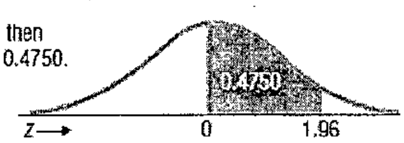
df	80%	90%	95%	98%	99%	99.9%
	Confidence Intervals, *c*					
	Level of Significance for One-Tailed Test, α					
	0.10	0.05	0.025	0.01	0.005	0.0005
	Level of Significance for Two-Tailed Test, α					
	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
31	1.309	1.696	2.040	2.453	2.744	3.633
32	1.309	1.694	2.037	2.449	2.738	3.622
33	1.308	1.692	2.035	2.445	2.733	3.611
34	1.307	1.691	2.032	2.441	2.728	3.601
35	1.306	1.690	2.030	2.438	2.724	3.591
36	1.306	1.688	2.028	2.434	2.719	3.582
37	1.305	1.687	2.026	2.431	2.715	3.574
38	1.304	1.686	2.024	2.429	2.712	3.566
39	1.304	1.685	2.023	2.426	2.708	3.558
40	1.303	1.684	2.021	2.423	2.704	3.551
41	1.303	1.683	2.020	2.421	2.701	3.544
42	1.302	1.682	2.018	2.418	2.698	3.538
43	1.302	1.681	2.017	2.416	2.695	3.532
44	1.301	1.680	2.015	2.414	2.692	3.526
45	1.301	1.679	2.014	2.412	2.690	3.520
46	1.300	1.679	2.013	2.410	2.687	3.515
47	1.300	1.678	2.012	2.408	2.685	3.510
48	1.299	1.677	2.011	2.407	2.682	3.505
49	1.299	1.677	2.010	2.405	2.680	3.500
50	1.299	1.676	2.009	2.403	2.678	3.496
51	1.298	1.675	2.008	2.402	2.676	3.492
52	1.298	1.675	2.007	2.400	2.674	3.488
53	1.298	1.674	2.006	2.399	2.672	3.484
54	1.297	1.674	2.005	2.397	2.670	3.480
55	1.297	1.673	2.004	2.396	2.668	3.476
56	1.297	1.673	2.003	2.395	2.667	3.473
57	1.297	1.672	2.002	2.394	2.665	3.470
58	1.296	1.672	2.002	2.392	2.663	3.466
59	1.296	1.671	2.001	2.391	2.662	3.463
60	1.296	1.671	2.000	2.390	2.660	3.460
61	1.296	1.670	2.000	2.389	2.659	3.457
62	1.295	1.670	1.999	2.388	2.657	3.454
63	1.295	1.669	1.998	2.387	2.656	3.452
64	1.295	1.669	1.998	2.386	2.655	3.449
65	1.295	1.669	1.997	2.385	2.654	3.447
66	1.295	1.668	1.997	2.384	2.652	3.444
67	1.294	1.668	1.996	2.383	2.651	3.442
68	1.294	1.668	1.995	2.382	2.650	3.439
69	1.294	1.667	1.995	2.382	2.649	3.437
70	1.294	1.667	1.994	2.381	2.648	3.435

B.2 Student's *t* Distribution (*concluded*)

df	80%	90%	95%	98%	99%	99.9%
	Confidence Intervals, *c*					
	Level of Significance for One-Tailed Test, α					
	0.10	0.05	0.025	0.01	0.005	0.0005
	Level of Significance for Two-Tailed Test, α					
	0.20	0.10	0.05	0.02	0.01	0.001
71	1.294	1.667	1.994	2.380	2.647	3.433
72	1.293	1.666	1.993	2.379	2.646	3.431
73	1.293	1.666	1.993	2.379	2.645	3.429
74	1.293	1.666	1.993	2.378	2.644	3.427
75	1.293	1.665	1.992	2.377	2.643	3.425
76	1.293	1.665	1.992	2.376	2.642	3.423
77	1.293	1.665	1.991	2.376	2.641	3.421
78	1.292	1.665	1.991	2.375	2.640	3.420
79	1.292	1.664	1.990	2.374	2.640	3.418
80	1.292	1.664	1.990	2.374	2.639	3.416
81	1.292	1.664	1.990	2.373	2.638	3.415
82	1.292	1.664	1.989	2.373	2.637	3.413
83	1.292	1.663	1.989	2.372	2.636	3.412
84	1.292	1.663	1.989	2.372	2.636	3.410
85	1.292	1.663	1.988	2.371	2.635	3.409
86	1.291	1.663	1.988	2.370	2.634	3.407
87	1.291	1.663	1.988	2.370	2.634	3.406
88	1.291	1.662	1.987	2.369	2.633	3.405
89	1.291	1.662	1.987	2.369	2.632	3.403
90	1.291	1.662	1.987	2.368	2.632	3.402
91	1.291	1.662	1.986	2.368	2.631	3.401
92	1.291	1.662	1.986	2.368	2.630	3.399
93	1.291	1.661	1.986	2.367	2.630	3.398
94	1.291	1.661	1.986	2.367	2.629	3.397
95	1.291	1.661	1.985	2.366	2.629	3.396
96	1.290	1.661	1.985	2.366	2.628	3.395
97	1.290	1.661	1.985	2.365	2.627	3.394
98	1.290	1.661	1.984	2.365	2.627	3.393
99	1.290	1.660	1.984	2.365	2.626	3.392
100	1.290	1.660	1.984	2.364	2.626	3.390
120	1.289	1.658	1.980	2.358	2.617	3.373
140	1.288	1.656	1.977	2.353	2.611	3.361
160	1.287	1.654	1.975	2.350	2.607	3.352
180	1.286	1.653	1.973	2.347	2.603	3.345
200	1.286	1.653	1.972	2.345	2.601	3.340
∞	1.282	1.645	1.960	2.326	2.576	3.291

B.1 Areas under the Normal Curve

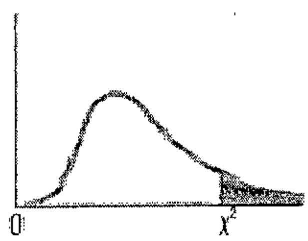
Example:
If $z = 1.96$, then
$P(0 \text{ to } z) = 0.4750$.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

B.3 Critical Values of Chi-Square

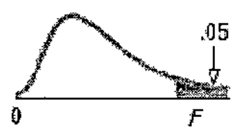
This table contains the values of χ^2 that correspond to a specific right-tail area and specific number of degrees of freedom.



Example: With 17 *df* and a .02 area in the upper tail, $\chi^2 = 30.995$

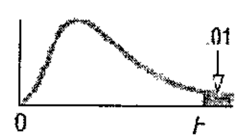
Degrees of Freedom, *df*	Right-Tail Area			
	0.10	**0.05**	**0.02**	**0.01**
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.345
4	7.779	9.488	11.668	13.277
5	9.236	11.070	13.388	15.086
6	10.645	12.592	15.033	16.812
7	12.017	14.067	16.622	18.475
8	13.362	15.507	18.168	20.090
9	14.684	16.919	19.679	21.666
10	15.987	18.307	21.161	23.209
11	17.275	19.675	22.618	24.725
12	18.549	21.026	24.054	26.217
13	19.812	22.362	25.472	27.688
14	21.064	23.685	26.873	29.141
15	22.307	24.996	28.259	30.578
16	23.542	26.296	29.633	32.000
17	24.769	27.587	30.995	33.409
18	25.989	28.869	32.346	34.805
19	27.204	30.144	33.687	36.191
20	28.412	31.410	35.020	37.566
21	29.615	32.671	36.343	38.932
22	30.813	33.924	37.659	40.289
23	32.007	35.172	38.968	41.638
24	33.196	36.415	40.270	42.980
25	34.382	37.652	41.566	44.314
26	35.563	38.885	42.856	45.642
27	36.741	40.113	44.140	46.963
28	37.916	41.337	45.419	48.278
29	39.087	42.557	46.693	49.588
30	40.256	43.773	47.962	50.892

B.4 Critical Values of the F Distribution at a 5 Percent Level of Significance



Degrees of Freedom for the Denominator	Degrees of Freedom for the Numerator: 1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39

B.4 Critical Values of the F Distribution at a 1 Percent Level of Significance (*concluded*)



Degrees of Freedom for the Denominator	Degrees of Freedom for the Numerator: 1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	4052	5000	5403	5625	5764	5859	5928	5981	6022	6056	6106	6157	6209	6235	6261	6287
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59