

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2017-2018

Sub : **ME 243** (Mechanics of Solids)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

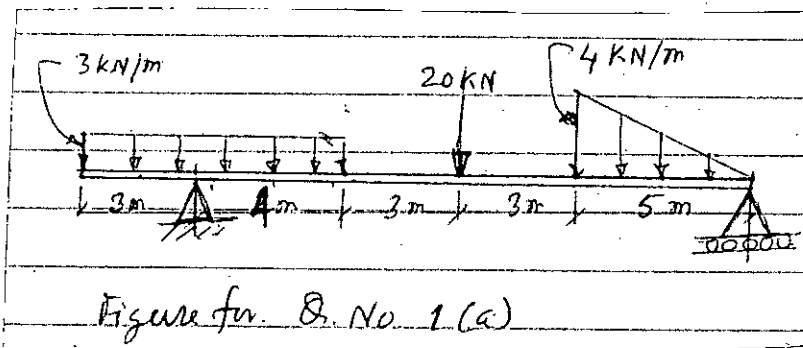
USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION - A

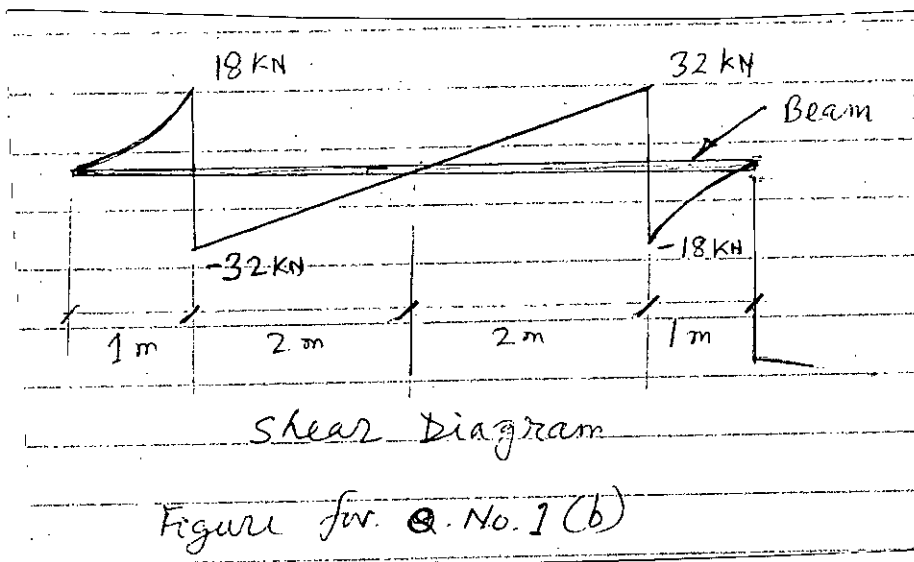
There are **FOUR** questions in this section. Answer any **THREE**.

Symbols indicate their usual meaning. Assume reasonable values for any missing data, if there is any.

1. (a) Write shear and moment equations the loaded beam as shown in Figure for Q. No. 1(a). Also draw shear force and bending moment diagrams. Specifying values at all change of loading position and at all points of zero shear. Neglect the mass of the beam. (20)



- (b) Draw moment and load diagrams corresponding to the shear force diagram as shown in Figure for Q. No. 1(b). Specify values at all change of loads positions. (15)



2. Determine the maximum tensile and compression stresses developed in the beam as shown in Figure for Q. No. 2. Also compute the shearing stress developed at horizontal layer 20 mm apart from top for a section 10 mm from the left end. (35)

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Contd. Q. No. 2

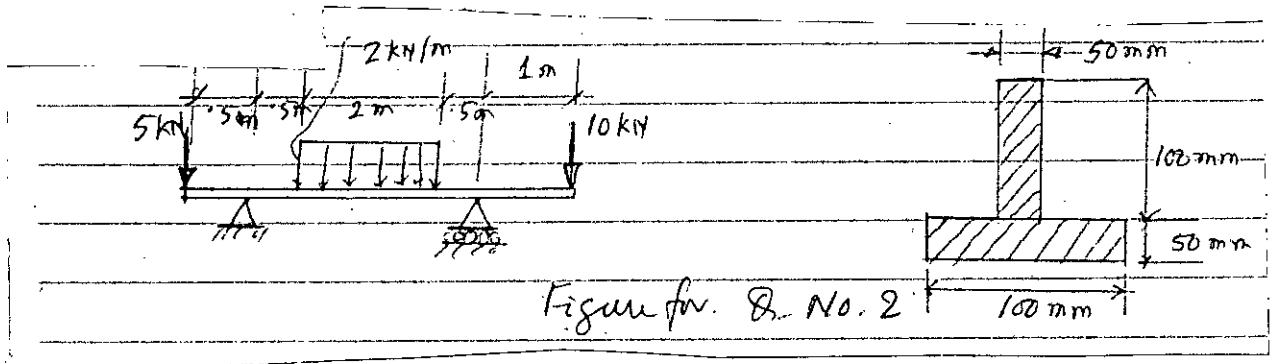


Figure for Q. No. 2

3. (a) For the beam loaded as shown in Figure for Q. No. 3(a). Determine the maximum deflection ($EI\delta$) between the supports, using double Integration Method. (20)

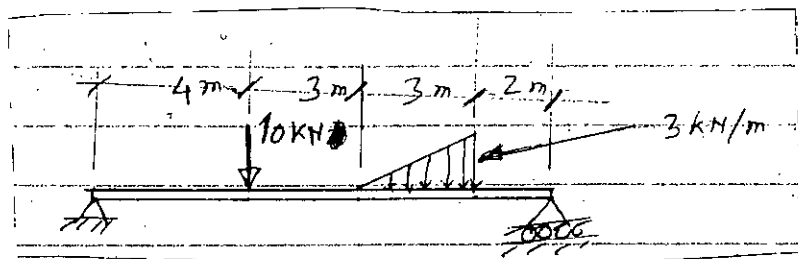


Fig. for Q. No. 3(a)

- (b) Find the maximum value of $EI\delta$ for the beam shown in Figure for Q. No. 3(b) using area moment method. (15)

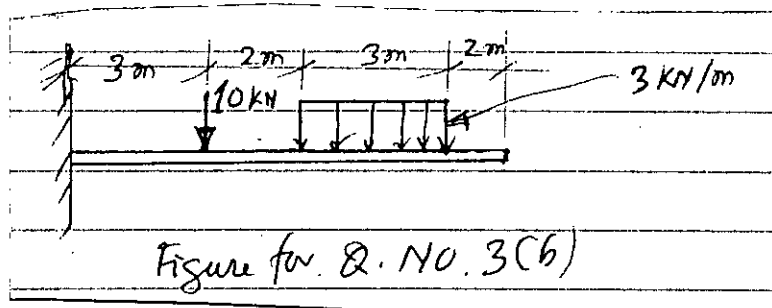


Figure for Q. No. 3(b)

4. (a) Select the lightest W shape section (using the attached table) that with support an axial load of 100 kN on a length of 10 m. Use AISC column specification with yield strength = 250 MPa and $E = 210$ GPa. One end of the column is fixed and other end hinged. (20)
- (b) A W 250 × 67 section (use the attached table) is to be used as a column with a length of 10 m, both ends fixed condition. The column supports an axial load of 130 kN and an eccentric load of 100 kN acting on the y axis (as shown in the table). Determine the maximum eccentricity for the 100 kN load using AISC specification with $\sigma_{yp} = 250$ MPa and $E = 200$ GPa. (15)

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) A rectangular piece of wood, 50 mm by 100 mm in cross section, is used as a compression block as shown in Fig. 5(a). Determine the maximum axial force P that can be safety applied to the block if the compressive stress in the wood is limited to 20 MN/m² and

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Contd. Q. No. 5(a)

- the shearing stress parallel to the grain is limited to 5 MN/m^2 . The grain makes an angle of 20° with the horizontal as shown. (5)
- (b) Calculate the increase in stress for each segment of the compound bar shown in Fig. 5(b) if the temperature increases by 100°F . Assume that the supports are unyielding and that the bar is suitably braced against buckling. (10)
- (c) As shown in Fig. 5(c), a rigid bar with negligible mass is pinned at O and attached to two vertical rods. Assuming that the rods were initially stress free, what maximum load P can be applied without exceeding stresses of 150 MPa in the steel rod and 70 MPa in the bronze rod? (20)
6. (a) A water supply pipe is 2 m in diameter and built of longitudinal wood staves held in place by hoops of 24-mm round steel rod. What spacing of hoops is required when the pressure is 350 kPa ? The allowable stress in the steel rod is 140 MPa . (15)
- (b) Two thick-walled cylinders of the same dimensions, outer diameter being twice the inner diameter, are subjected to fluid pressure. One cylinder is subjected to internal fluid pressure and the other is subjected to external fluid pressure. Determine the ratio of pressures if the magnitude of the maximum tangential stress of each cylinder is the same. (20)
7. (a) A solid steel shaft is loaded as shown in Fig. 7(a). Using $G = 83 \text{ GPa}$, determine the required diameter of the shaft if the shearing stress is limited to 60 MPa and the angle of rotation at the free end is not to exceed 4° . (15)
- (b) A rigid bar, hinged at one end, is supported by two identical springs as shown in Fig. 7(b). Each spring consists of 20 turns of 10-mm wire having a mean diameter of 150 mm . Compute the maximum shearing stress in the spring considering the effect of curvature of spring wire. (20)
8. (a) A round bar of length 2 m and diameter 100 cm is subjected to an axial load of 100 kN . If the elongation of the bar due to the application of the load is 10 cm , find the true stress and true strain developed in the bar. (5)
- (b) (i) For the state of stress of an element shown in Fig. 8(b), determine the principal stresses and the maximum in-plane shearing stress. Show all results on complete sketches of differential element. (10)
- (ii) Also find the stress components on planes whose normal are at $+30^\circ$ and $+120^\circ$ with the x axis. Show your answers on a complete sketch of a differential element. (20)
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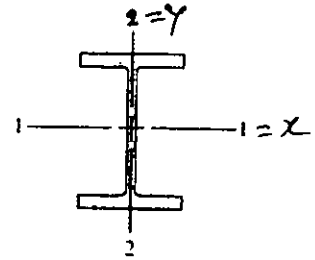


TABLE F-1(b) PROPERTIES OF WIDE-FLANGE SECTIONS (W SHAPES) - SI UNITS
(ABRIDGED LIST)

Designation	Mass per meter	Area	Depth	Web thickness	Flange		Axis 1-1			Axis 2-2		
					Width	Thickness	I	S	r	I	S	r
	kg	mm ²	mm	mm	mm	mm	$\times 10^6$ mm ⁴	$\times 10^3$ mm ³	mm	$\times 10^6$ mm ⁴	$\times 10^3$ mm ³	mm
W 760 × 314	314	40100	785	19.7	384	33.5	4290	10900	328	315	1640	88.6
W 760 × 196	196	25100	770	15.6	267	25.4	2400	6230	310	81.6	610	57.2
W 610 × 241	241	30800	635	17.9	330	31.0	2150	6780	264	184	1120	77.5
W 610 × 140	140	17900	617	13.1	230	22.2	1120	3640	251	45.4	393	50.3
W 460 × 177	177	22600	483	16.6	287	26.9	912	3790	201	105	736	68.3
W 460 × 106	106	13400	470	12.6	194	20.6	487	2080	191	25.1	259	43.2
W 410 × 149	149	19000	432	14.9	264	25.0	620	2870	180	77.4	585	63.8
W 410 × 114	114	14600	419	11.6	262	19.3	462	2200	178	57.4	441	62.7
W 410 × 85	85.0	10800	417	10.9	181	18.2	316	1510	171	17.9	198	40.6
W 410 × 46.1	46.1	5890	404	6.99	140	11.2	156	773	163	5.16	73.6	29.7
W 360 × 179	179	22800	368	15.0	373	23.9	574	3110	158	206	1110	95.0
W 360 × 122	122	15500	363	13.0	257	21.7	367	2020	154	61.6	480	63.0
W 360 × 79	79.0	10100	353	9.40	205	16.8	225	1270	150	24.0	234	48.8
W 360 × 39	39.0	4960	353	6.48	128	10.7	102	578	144	3.71	58.2	27.4
W 310 × 129	129	16500	318	13.1	307	20.6	308	1930	137	100	651	78.0
W 310 × 74	74.0	9420	310	9.40	205	16.3	163	1050	132	23.4	228	49.8
W 310 × 52	52.0	6650	318	7.62	167	13.2	119	747	133	10.2	122	39.1
W 310 × 21	21.0	2680	302	5.08	101	5.72	36.9	244	117	0.982	19.5	19.1
W 250 × 89	89.0	11400	259	10.7	257	17.3	142	1090	112	48.3	377	65.3
W 250 × 67	67.0	8580	257	8.89	204	15.7	103	805	110	22.2	218	51.1
W 250 × 44.8	44.8	5700	267	7.62	148	13.0	70.8	531	111	6.95	94.2	34.8
W 250 × 17.9	17.9	2280	251	4.83	101	5.33	22.4	179	99.1	0.907	18.0	19.9
W 200 × 52	52.0	6650	206	7.87	204	12.6	52.9	511	89.2	17.7	174	51.6
W 200 × 41.7	41.7	5320	205	7.24	166	11.8	40.8	398	87.6	9.03	109	41.1
W 200 × 31.3	31.3	3970	210	6.35	134	10.2	31.3	298	88.6	4.07	60.8	32.0
W 200 × 22.5	22.5	2860	206	6.22	102	8.00	20.0	193	83.6	1.42	27.9	22.3

Note: Axes 1-1 and 2-2 are principal centroidal axes.

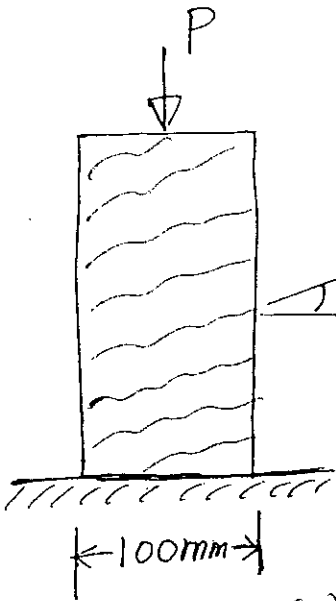
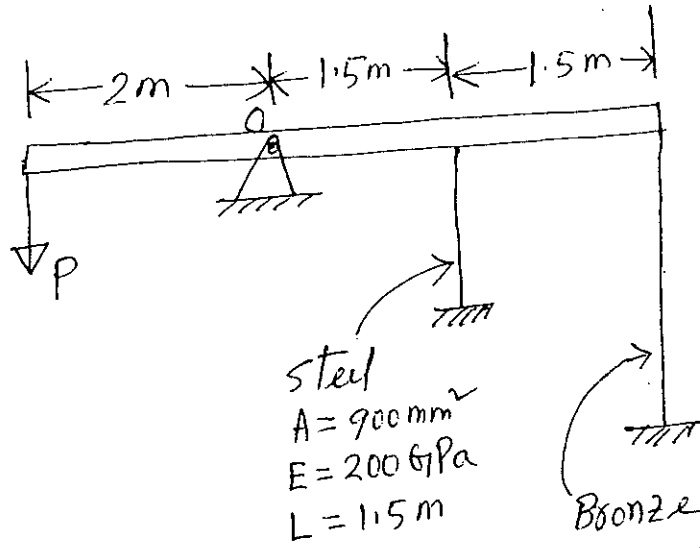


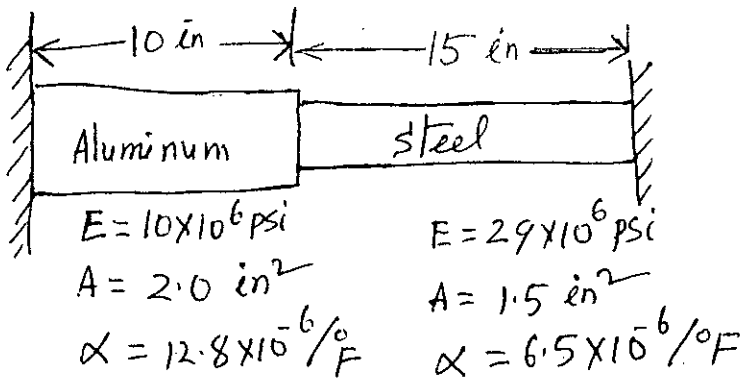
Fig. 5(a)



Steel
 $A = 900 \text{ mm}^2$
 $E = 200 \text{ GPa}$
 $L = 1.5 \text{ m}$

Bronze
 $A = 300 \text{ mm}^2$
 $E = 83 \text{ GPa}$
 $L = 2 \text{ m}$

Fig. 5(c)



$E = 10 \times 10^6 \text{ psi}$

$A = 2.0 \text{ in}^2$

$\alpha = 12.8 \times 10^{-6} / ^\circ\text{F}$

$E = 29 \times 10^6 \text{ psi}$

$A = 1.5 \text{ in}^2$

$\alpha = 6.5 \times 10^{-6} / ^\circ\text{F}$

Fig. 5(b)

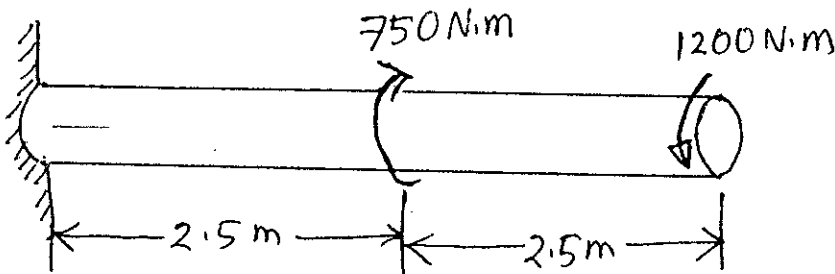


Fig. 7(a)

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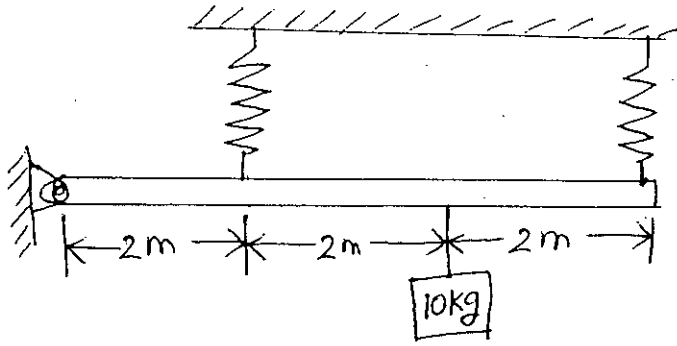


Fig. 7 (b)

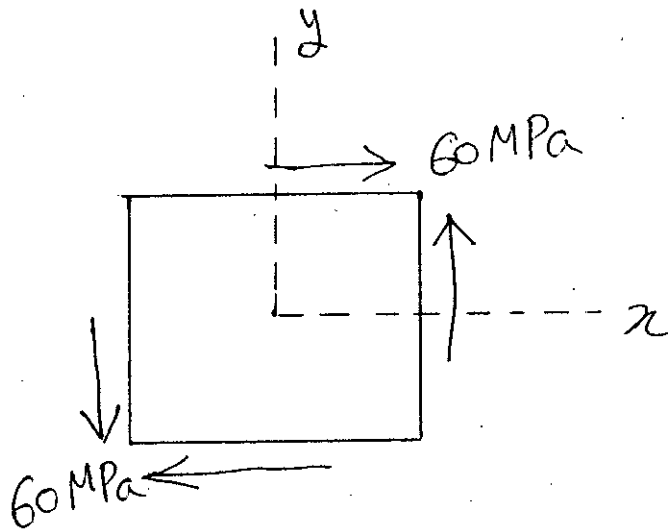


Fig. 8 (b)

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) The table below gives the values of $f(x)$ for $1 \leq x \leq 9$. Fit a Fourth-order Newton's interpolating polynomial and estimate the value of $f(4)$. (13)

x	1	3	5	6	9
$f(x)$	9	18	1	-6	5

- (b) The following data defines the sea-level concentration of dissolved oxygen for fresh water as a function of temperature: (15)

$T^\circ(C)$	8	16	24	32
Dissolved O_2 (mg/L)	14	11	9	8

In order to fit the above data with quadratic splines, describe the procedure to generate the equations needed. Then find out the equations and show them in matrix form.

- (c) Derive the Lagrange form from Newton's First order Interpolating polynomial. (7)

2. (a) According to the IEEE double precision format, mention how computer stored infinite binary number in a finite number of bits. Also write down the mantissa of floating point number that represent 9.4. (7)

- (b) Taylor Series can be written as: (7)

$$f(x_{i+1}) = f(x_i) + hf'(x_i) + \frac{h^2}{2!} f''(x_i) + \dots + \frac{h^n}{n!} f^n(x_i) + R_n$$

where, $h = x_{i+1} - x_i$ and the remainder term is

$$R_n = \frac{h^{n+1}}{(n+1)!} f^{n+1}(\xi)$$

Now, graphically represent the physical significance of ξ in the remainder term.

- (c) Evaluate and interpret the condition numbers for (7)

$$f(x) = \frac{\sin x}{1 + \cos x}$$

at $x = 1.001\pi$.

- (d) Find the inverse of the following matrix in order to solve a system of linear equations on the form of $[A]\{x\} = \{b\}$ (14)

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 0 \end{bmatrix}$$

Apply Gauss-Elimination Method.

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3. (a) The Trajectory of a ball thrown by a fieldsman is defined by the (x, y) coordinates and can be modeled as:

(16)

$$y = (\tan \theta_0)x - \left[\frac{gx^2}{2v_0^2 \cos^2 \theta_0} \right] + 1.8$$

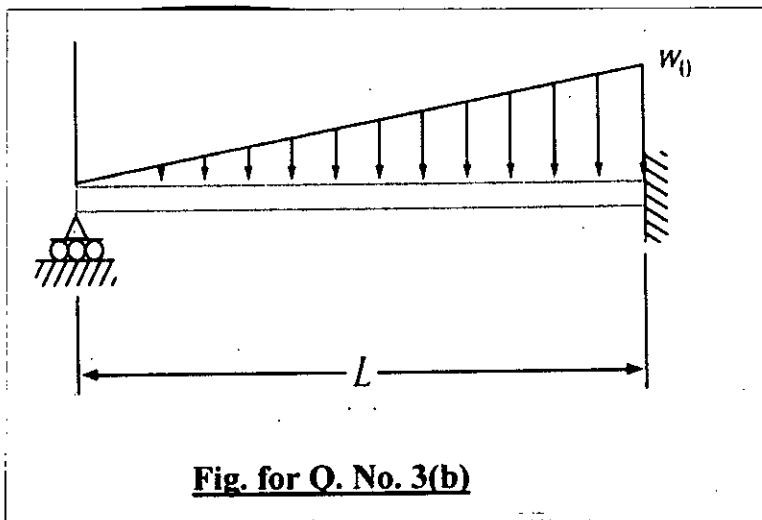
Find the appropriate initial angle θ_0 if $v_0 = 20$ m/s and the horizontal distance covered by the ball is 40 m. The throw leaves the fielder's hand at an elevation of 1.8 m and the catcher receives it at 1 m. Use Newton-Raphson method. Guess the initial value and show that it satisfies the convergence criteria. Show only 3 iterations.

- (b) A uniform beam subject to a linearly increasing distributed load is shown in the Fig. for Q. No. 3(b). The equation for the resulting elastic curve is:

(12)

$$y = \frac{w_0}{120EIL} (-x^5 + 2L^2x^3 - L^4x)$$

Use bisection method to determine the point of maximum deflection that is the point where slope of the elastic curve is zero. Use the following parameter values: $L = 600$ cm, $E = 50,000$ kN/cm², $I = 30,000$ cm⁴, and $w_0 = 2.5$ kN/cm. Show only three iterations. Can you predict how many iterations required to find the solution up-to five correct decimal places?



- (c) Lee and Duffy (1976) relate the friction factor for flow of a suspension of fibrous particles to the Reynolds number by this empirical equation:

(7)

$$\frac{1}{\sqrt{f}} = \left(\frac{1}{k} \right) \ln(\text{Re} \sqrt{f}) + \left(14 - \frac{5.6}{k} \right)$$

In their relation, f is the friction factor, Re is the Reynolds number, and k is a constant determined by the concentration of the suspension. For a suspension with 0.08% concentration, $k = 0.28$. What is the value of f if $Re = 3750$? Use Secant method with initial guesses of 0.005 and 0.006. Show only one iteration.

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4. (a) Why is pivoting necessary in elimination method of solution? (7)

(b) Solve the following system of linear equations by Jacobi's method: (14)

$$\begin{bmatrix} 6 & -2 & 1 \\ 1 & 2 & -5 \\ -2 & 7 & 2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 11 \\ -1 \\ 5 \end{Bmatrix}$$

Assume any suitable initial guess. Consider $\|E_{absolute}\|_{\infty} \leq 0.1$

(c) Determine the roots of the following non-linear system of equations using Newton-Raphson method: (14)

$$xyz - x^2 + y^2 = 1.34$$

$$xy - z^2 = 0.09$$

$$e^x - e^y + z = 0.41$$

Assume any suitable initial guesses and perform one iteration.

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols used have their usual meaning.

5. (a) Describe the mathematical details of Faddeev-Leverrier method to obtain the characteristic equation of an eigen-value problem. (10+8)

Discuss the potential use of Faddeev-Leverrier method with reference to the following issues:

- (i) determination of the smallest eigen-value and the corresponding eigen vectors
- (ii) solution of a set of nonhomogeneous linear algebraic equations.

(b) With the help of Faddeev-Leverrier method, determine the smallest eigen-value and the corresponding eigen-vectors for the following homogeneous system using an iterative scheme. (17)

$$\begin{aligned} 3.556X_1 - 1.778X_2 &= \lambda X_1 \\ -1.778X_1 + 3.556X_2 - 1.778X_3 &= \lambda X_2 \\ -1.778X_2 + 3.556X_3 &= \lambda X_3 \end{aligned}$$

Show the results of atleast three iterations. Retain three decimal digits in eigen value and eigen vectors at each step. Comment on the convergence of your solution.

6. (a) What do you understand by Local and Global Truncation Error? (17)

Show that the local and global truncation errors of the second-order Runge-Kutta method are of $O(h^3)$ and $O(h^2)$, respectively.

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Cont. Q. 6

(b) The ordinary differential equation (18)

$$x = \frac{y^2}{10} + \frac{dy}{dx}$$

has the following solutions:

$$y(0) = 1.0, y(0.1) = 0.99507$$

Find the next two steps of solution for the above ODE using the following methods and show them on a single table.

- (i) Euler method
- (ii) Heun's method
- (iii) Ralston's method

7. (a) With suitable examples, briefly explain the following concepts: (8+10)

- (i) Multiple linear regression
- (ii) Weighted linear regression

Derive the general weighted least-squares matrix formulation for a set of n data points $(X_i, Y_i), i = 1, 2, 3, \dots, n$ by a cubic polynomial of the form.

$$Y = C_1 + C_2 X + C_3 X^2 + C_4 X^3$$

(b) Consider the following three data points (17)

$$(0, 0), (1, 1) \text{ and } (2, 1)$$

- (i) Find the best linear least-squares fit for the data
- (ii) Repeat (i) giving the data point (2,1) double weight
- (iii) Determine which of the straight lines obtained best represents the data.

8. (a) Consider the following ODE (15)

$$10 \frac{d^2 Y}{dX^2} + 4 \frac{dY}{dX} + 6Y = 2X^2$$

With reference to Taylor Series expansions, derive the equivalent central-difference approximation to the ODE for the following two different levels of truncation error

- (i) order of error of $\Delta X^2 (O(h^2))$
- (ii) order of error of $\Delta X^4 (O(h^4))$

(b) Derive the Simpson's $\frac{3}{8}$ -rule of integration for the numerical evaluation of the following integral (10)

$$I = \int_a^b f(x) dx$$

(c) $I = \int_{0.5}^{1.0} \cos \sqrt{x} dx$ (10)

The above integral is to be evaluated by Trapezoidal rule of integration. How many intervals should be taken to assure that the error in the answer is no worse than 0.00001?

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2017-2018

Sub : **ME 249** (Engineering Mechanics -II)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

Symbols carry usual significance and meanings. Assume reasonably any missing data.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) The two blocks as shown in Fig. for Q. 1(a) are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block A and the incline, determine (i) the acceleration of each block, (ii) the tension in the cable. (17)
 (b) As shown in Fig. for Q. 1(b) a 54-kg pilot flies a jet trainer in a half vertical loop of 1200-m radius so that the speed of the trainer decreases at a constant rate. Knowing that the pilot's apparent weights at Points A and C are 1680 N and 350 N, respectively, determine the force exerted on her by the seat of the trainer when the trainer is at Point B. (18)
2. (a) A 500-g collar can slide without friction on the curved rod BC in a horizontal plane as shown in Fig. for Q. 2(a). Knowing that the undeformed length of the spring is 80 mm and that $k = 400$ kN/m, determine the velocity that the collar should be given at A to reach B with zero velocity. (17)
 (b) Rod BDE is partially guided by a roller at D which moves in a vertical track as shown in Fig. for Q. 2(b). Knowing that at the instant shown the angular velocity of crank AB is 5 rad/s clockwise and that $\beta = 25^\circ$, using the method of instantaneous center of rotation, determine (i) the angular velocity of the rod, (ii) the velocity of Point D. (18)
3. (a) For direct central impact of two particles A and B, show that the relative velocity of two particles after impact can be obtained by multiplying their relative velocity before impact by the coefficient of restitution, i. e., $v'_s - v'_A = e(v_A - v_s)$ (17)
 where the symbols refer the usual meaning.
 (b) A 1-kg block B is moving with a velocity v_0 of magnitude $v_0 = 2$ m/s as it hits the 0.5-kg sphere A, which is at rest and hanging from a cord attached at O as shown in Fig. for Q. 3(b). Knowing that $\mu_k = 0.6$ between the block and the horizontal surface and $e = 0.8$ between the block and the sphere, determine after impact the maximum height h reached by the sphere. (18)
4. In the toggle mechanism as shown in Fig. for Q. 4 D is constrained to move on a horizontal path. The dimensions of various links are: AB = 200 mm; BC = 300 mm; OC = 150 mm; and BD = 450 mm. The crank OC is rotating in a counter clockwise direction at a speed of 180 rpm, increasing at the rate of 50 rad/s². Find, for the given configuration, velocity and acceleration of D. Solve graphically. (35)

ME 249**SECTION – B**

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) The elevator shown (Fig. for Q. 5(a)) starts from rest moves downward with a constant acceleration. The counterweight W moves through 10 m in 5 sec. Find- (i) Elevator's acceleration, (ii) Cable's acceleration at point C, (iii) Elevator's velocity after 5 sec. **(17½)**

(b) As shown in Fig. for Q. 5(b), airplane A is flying horizontally in a straight line, and its speed increases at the rate of 8 m/s^2 . Airplane B is flying at the same altitude as airplane A and follows a circular path. Given, at the positions shown, speed of B decreases at the rate of 3 m/s^2 . Find the acceleration of A relative to B . **(17½)**

6. (a) The helicopter (Fig. for Q. 6(a)) produces a maximum downward air speed of 25 m/s in a 10-m diameter slip stream. Give, weight of the helicopter and its crew is 20 kN and air density is 1.20 kg/m^3 . Calculate the maximum load that the helicopter can carry while hovering in mid air. **(15)**

(b) As shown in Fig. for Q. 6(b), mass m (3 kg) travels towards right at 10 m/s. It suddenly bursts into two parts A (2 kg) and B (1 kg). Given at an instant: **(20)**

	Position (m)	Velocity (m/s)
Part A	$2\mathbf{i} + \mathbf{j}$	$3\mathbf{i}$
Part B	$1\mathbf{i} + r_{By}\mathbf{j}$	$v_{Bx}\mathbf{i} + v_{By}\mathbf{j}$

Calculate: v_{Bx} , v_{By} and r_{By} . Assume, motion in xy plane only.

7. (a) Length of the uniform rod AB is 1.2 m (Fig. for Q. 7(a)) and its mass is 25 kg. The rod is released from rest from the position shown. Ends of AB remains in contact with the smooth surfaces. (i) Identify the type of motion of the rod. (ii) Find the end reactions. Take $\mu = 0$. **(20)**

(b) The composite disc (Fig. for Q. 7(b)) rolls without sliding has following data: mass = 5 kg, $\bar{k} = 0.4 \text{ m}$, $a = 0.3 \text{ m}$, $b = 0.6 \text{ m}$, \bar{a} (mass centre's acceleration) = 10 m/s^2 towards right. Find (i) The force P (ii) the minimum value of μ_s compatible with the problem. **(15)**

8. (a) A uniform and slender rod AC has a mass of 4 kg and is pivoted at B . A spring of $k = 400 \text{ N/m}$ and of unstretched length 150 mm is connected at C and D points. Knowing that the rod is released from rest in the position shown, determine its angular velocity after it has rotated through 90° . See Fig. for Q. 8(a) for other information. **(17)**

(b) The 10-kg rod AB is attached by pins to two 5-kg uniform discs as shown in Fig. for Q. 8(b). The assembly rolls without sliding on a horizontal surface. Given for the position shown $\theta = 20^\circ$ and $\bar{V}_c = 1 \text{ m/s}$ to the left. Calculate the angular speed of disc D when $\theta = 90^\circ$. Given, $AC = BD = 0.15 \text{ m}$, radius of each disc = 0.2 m. **(18)**

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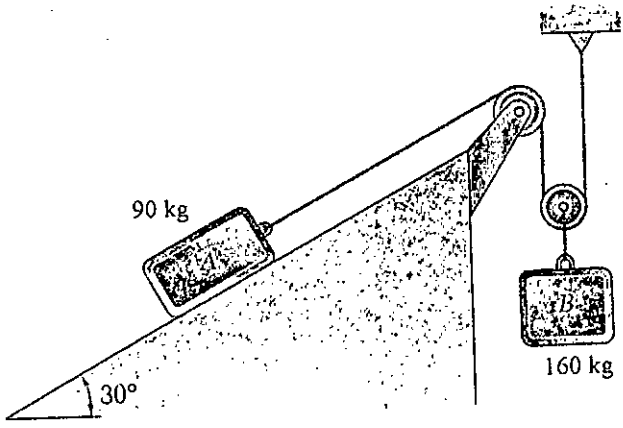


Fig. for Q.1(a)

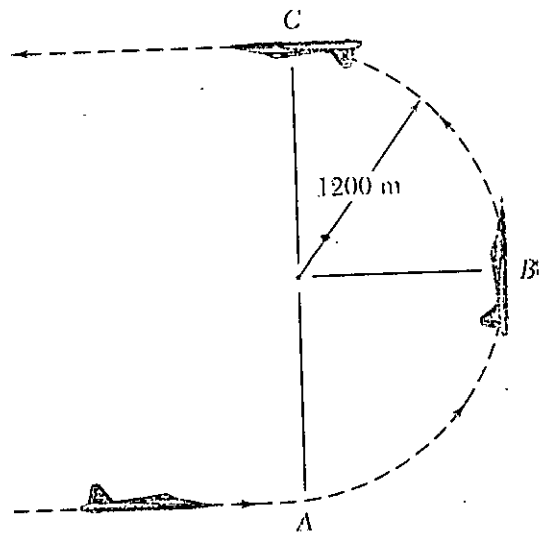


Fig. for Q.1(b)

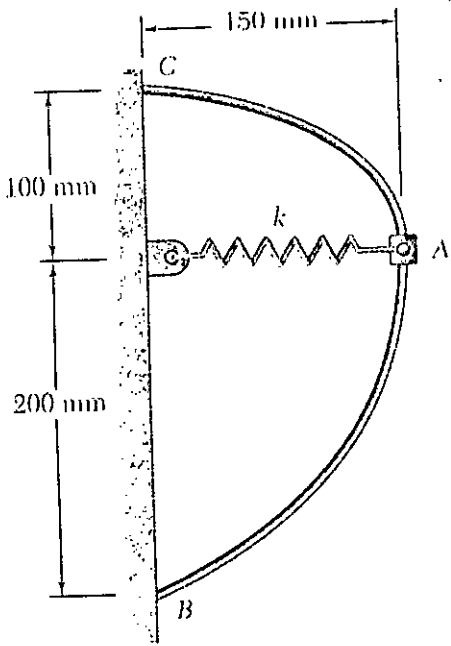


Fig. for Q.2(a)

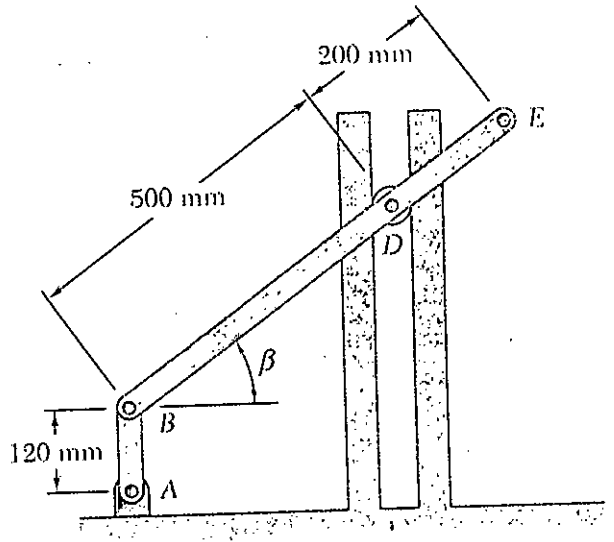


Fig. for Q.2(b)

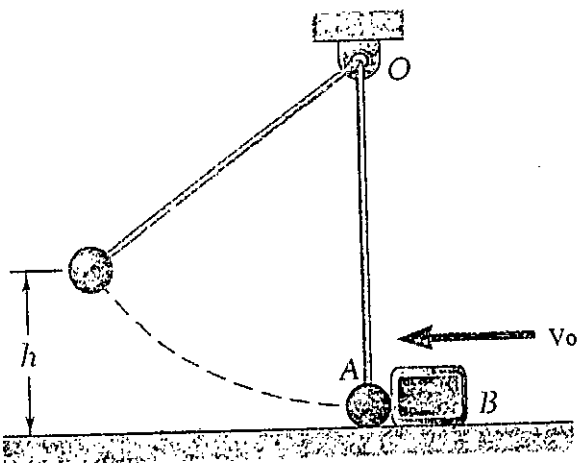


Fig. for Q.3(b)

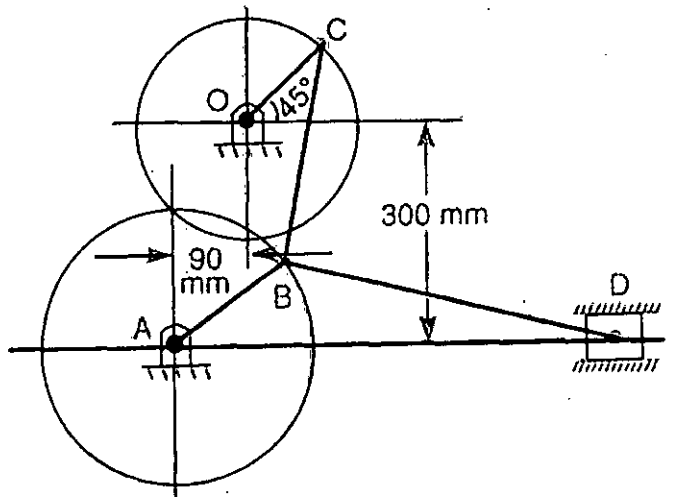


Fig. for Q.4

FIG. for Q. 5 (a)

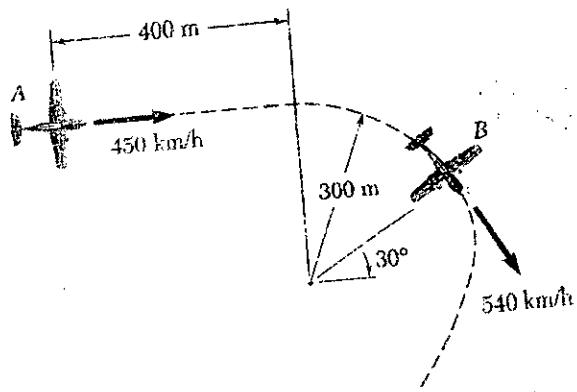
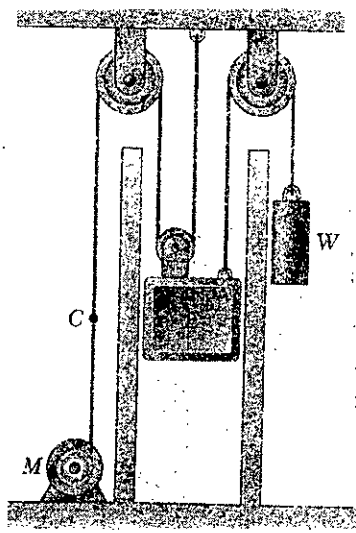


FIG. for Q. 5 (b)

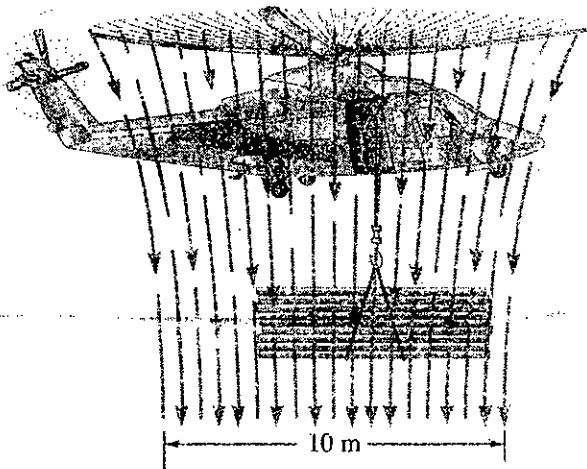


FIG. for Q. 6 (a)

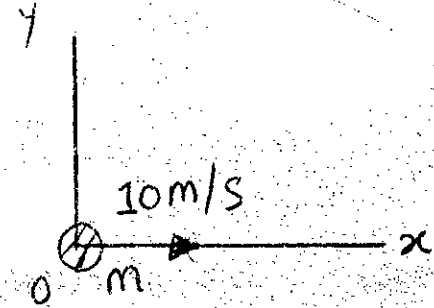
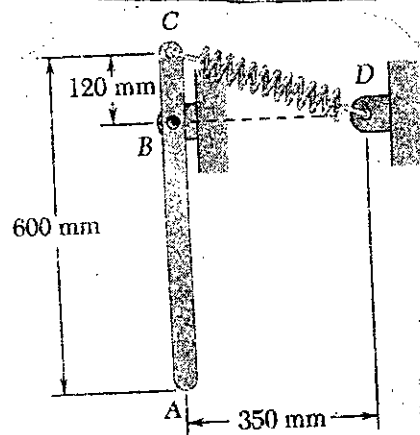
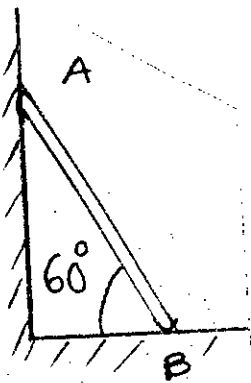
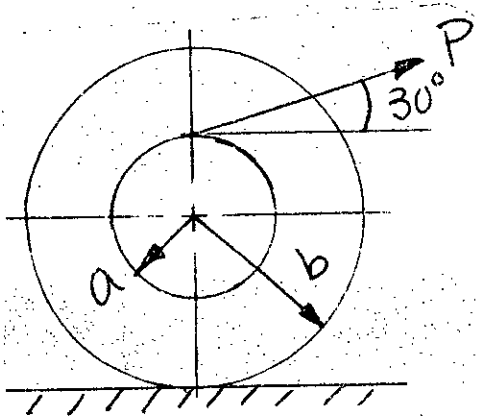


FIG. for Q. 6 (b)

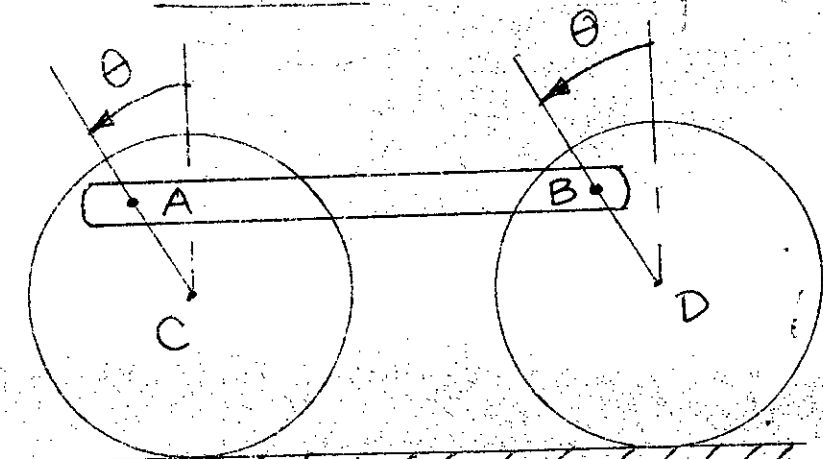
Q. 7 (a)



Q. 8 (a)



FIGS. for Q. 7 (b)



FIGS. for Q. 8 (b)

Sub: **MATH 263** (Complex Variables, Harmonic Analysis and
Partial Differential Equations)

Full Marks: 280

Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

Symbols have their usual meaning.

1. (a) Find the principle argument $\text{Arg}z$ of $z = (\sqrt{3} - i)^6$. Hence show that $\text{Log}(-1+i)^2 \neq 2\text{Log}(-1+i)$ (15)
- (b) If $f(z) = x^3 + i(1-y)^3$, then show that $f'(z)$ exist at $z = i$. (10)
- (c) If the imaginary part of $\frac{(2Z+1)}{(iZ+1)}$ is -2 , then show that the locus of the points representing in the argand plane is the straight line. (10)
- (d) Prove that under the transformation, $w = \frac{(z-i)}{(iz-1)}$ the region $\text{Im}(z) \geq 0$ is mapped into the region $|w| \leq 1$. (11 2/3)
2. (a) Write down Cauchy Riemann equations in polar form. Test the differentiability of the function $f(z) = \frac{1}{r^4} \cos 4\theta + i \left(-\frac{1}{r^4} \sin 4\theta \right)$, $r > 0, 0 < \theta < 2\pi$ in the indicated domain and hence show that, $f'(z) = -\frac{4}{r^5 e^{i5\theta}}$. (15)
- (b) Show that $v(x, y) = \frac{x}{x^2 + y^2} + \cosh x \cos y$ is a harmonic function. Find an analytic function $f(z) = u(x, y) + iv(x, y)$ and express $f(z)$ in terms of z . (16 2/3)
- (c) Evaluate $\int z^2 dz$ along the curve $x = t, y = t^2$ joining the points (1,1) and (2,4). (15)
3. (a) Use Cauchy integral formula to show that (16 2/3)
- $$\oint_C \frac{e^{3z}}{z - \pi i} dz = \begin{cases} -2\pi i, & \text{if } C \text{ is the circle } |z - 1| = 4 \\ 0, & \text{if } C \text{ is the ellipse } |z - 2| + |z + 2| = 6 \end{cases}$$
- (b) Express $f(z) = \frac{z-1}{(z+2)(z+3)}$ in a Laurent series valid in the region $2 < |z| < 3$. (15)
- (c) Evaluate the integral $\oint_C \frac{2z^2 - z + 1}{(2z-1)(z+1)^2} dz$ by Cauchy residue theorem, (15)
- where, $C = \{C : r = 2 \cos \theta; 0 \leq \theta \leq 2\pi\}$.

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4. Evaluate the following integral using the method of contour integration: (23+23 $\frac{2}{3}$)

(a) $\int_0^{2\pi} \frac{\sin 2\theta}{5 - 3 \cos \theta} d\theta$

(b) $\int_0^{\infty} \frac{\cos x}{a^2 - x^2} dx$

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Use Lagrange's method to solve $(y - x)p + (y + x)q = \frac{x^2 + y^2}{z}$. (15)

- (b) Find the integral surface of the first order linear partial differential equation $4yzp + q + 2y = 0$ which passes through $y^2 + z^2 = 1$ and $x + z = 2$. (16 $\frac{2}{3}$)

- (c) Solve $2(z + xp + yq) = yp^2$ by charpit's method. (15)

6. Solve the following partial differential equations:

(a) $(2D_x^2 + D_x D_y - 15D_y^2)z = y^2 \cos(2x + y)$ (16)

(b) $(D_x^2 - D_x D_y - 2D_y^2 + 2D_x + 2D_y)z = e^{3x-y}$ (14)

(c) $(x^2 D_x^2 - xy D_x D_y - 2y^2 D_y^2 + x D_x - 2y D_y)z = xy \cos(\ln x)$. (16 $\frac{2}{3}$)

7. (a) Find the Fourier series expansion of $f(x)$, where (18 $\frac{2}{3}$)

$$f(x) = \begin{cases} -\pi, & -\pi \leq x < 0 \\ x, & 0 < x \leq \pi \end{cases}$$

Hence obtain the value of the infinite series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

- (b) Find the Fourier sine integral formula of xe^{-x} for $x \geq 0$. (15)

- (c) Find the Fourier cosine transform of (13)

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

8. (a) Write down Laplace's equation in polar coordinates and hence find the circular harmonics of degree 0 and of degree n. (30)

- (b) A very long circular cylinder is made of two halves; one half is at a constant temperature of u_0 and the other half is at a constant temperature u_1 . Find the temperature distribution inside the cylinder. (16 $\frac{2}{3}$)

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What are the purposes of annealing? In case of annealing of hypereutectic steels, why heating above upper critical is not done like hypoeutectic steels? - explain. (5+3=8)
 (b) Show the major microstructural changes during very slow cooling of 0.2% carbon steel, from austenite to room temperature. (12)
 (c) In case of normalizing of steels, what are the effects of faster cooling are on: (15)
 - (i) proeutectoid constituents.
 - (ii) austenite transformation temperature.
 - (iii) fineness of pearlite.

2. (a) In carburizing process, after carburization heat treatment is necessary to get desired hardness where as in nitriding process, no such post treatment is needed- why? (10)
 (b) Describe various reactions that occur in stack, bosh and hearth regions of a blast furnace. (25)

3. (a) "Iron making is a reductive process, while steelmaking is an oxidative process"- justify this statement. (7)
 (b) "The one major drawback to induction furnace usage in a foundry is the lack of refining capacity"- how can you solve this problem? (10)
 (c) What types of cast iron should be used for base of heavy machinery? If you use the same material in a component that has to carry tensile loads, will you face any problem? - explain with suitable figures. (18)

4. (a) Which type of cast iron is obtained by heat treating white cast iron? Explain the mechanism of formation of its microstructure. (13)
 (b) Suppose you have ordinary low carbon steel (%C<0.2) and the option of adding C, Ni, Cr as alloying elements. Your target is to make steel blades suitable for surgical instruments. State with clear reasoning(s) which alloying elements you will add and in what amount. (15)
 (c) What is shape memory alloy? Write down some important applications of shape memory alloy in biomedical. (3+4=7)

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SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Define yield stress. Why do upper and lower yield points appear in stress-strain diagram during tensile testing of plain carbon steels? (13)
- (b) "Hardness of a material can be properly measured using proper quantitative hardness testing method". Explain this statement for the case of low carbon steel and cast iron. (12)
- (c) Name different strategies for strengthening of metals and explain in detail the mechanism of strain hardening of metals. (10)
6. (a) Differentiate between Izod and Charpy impact tests. Explain the effect of crystal structure and grain size on the ductile-brittle transition temperature (DBTT) of metallic materials. (12)
- (b) Describe the effect of temperature and applied stress on the creep failure of metals. Name few metallic materials that are resilient to creep. (13)
- (c) Explain how we can get useful information on the fatigue property both for a ferrous metal and a non ferrous metal from the curve produced by a series of test results. (10)
7. Draw the iron and iron carbide thermal equilibrium diagram labeling all points, lines and phase fields. Then consider 2.0 kg of a 99.6 wt% Fe-0.4 wt% C alloy that is cooled to a temperature just below the eutectoid. During this cooling (35)
- (i) how many kilograms of proeutectoid ferrite form?
- (ii) how many kilograms of eutectoid ferrite form?
- (iii) how many kilograms of cementite form?
- Use your own iron-iron carbide diagram to answer the aforementioned questions.
8. (a) With reference to a hypothetical binary equilibrium diagram describe how coring occurs during solidification, under normal industrial condition of a solid solution alloy. Describe how coring is (i) prevented and (ii) removed. (17)
- (b) How can the mechanical properties of brass be tailored by increasing zinc content and other alloying elements? (10)
- (c) List the problem that aluminum alloys face during casting and mention some remedies of these problems. (8)
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