Performance Analysis of a MIMO-OFDM Communication System with SFBC using MSK and GMSK Modulation

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Marzuka Ahmed Jumana

DEDICATION

To my dearest father, mother and younger sister

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First and above all, I praise God, the Almighty, the Creator and the Guardian and to whom I owe my very existence. I am grateful to Almighty Allah for providing me the opportunity and granting me the competence, fortitude and courage to proceed with my research. His constant grace and compassion was with me throughout my life and even more during this whole time of my research period.

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ABSTRACT

The use of multiple antennas at both the transmitting and receiving end of communication channel initiates the multiple input multiple output technology (MIMO). Orthogonal Frequency Division Multiplexing, OFDM with MIMO reduces the effect of fading on transmission rate without increasing the power and bandwidth. The bit error rate (BER) performance of time and frequency selective channels can be increased by combining MIMO with space time block coded OFDM (STBC-OFDM) and space frequency block coded OFDM (SFBC-OFDM) respectively. When Alamouti space time code is used in frequency domain over OFDM subcarriers, space frequency block code is obtained.

Mostly performance of SFBC-MIMO-OFDM scheme is evaluated in Rayleigh fading environment with MPSK (M-ary Phase Shift Keying) and MQAM (M-ary Quadrature Amplitude Modulation) modulation schemes. The Minimum Shift Keying (MSK) and Gaussian Minimum Shift Keying (GMSK) modulation schemes have better energy efficiency than the MPSK and MQAM modulation schemes while using with SFBC-MIMO-OFDM schemes.

This thesis contributes in deriving and evaluating the BER expressions using SFBC-MIMO-OFDM with MSK and GMSK modulation in both Rayleigh and Nakagami-m fading environment. Also the BER expressions for MQAM and MPSK in Nakagami-m environment are evaluated. Then the receiver sensitivity is also evaluated for all the schemes. Finally the BER performances and also the receiver sensitivities are compared among all the SFBC-OFDM schemes presented in the research.

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List of Symbols

Symbol	Meaning	Section
M_T	Number of transmit antenna	1.3
M_R	Number of receive antennas	1.3
η	Gaussian noise	1.3
s_1 and s_2	Modulated symbols used for Alamouti space-time encoding	1.8
S	Encoding matrix	1.8
NF_{Rx}	Noise Figure	1.10
N_S	Number of subbands	2.1
R_C	Code rate	2.1
$h_{j,i}(t)$	Fading process impulse response	2.1
$\alpha_{m,j,i}(t)$	Tap weight	2.1
$\tau_m(t)$	Time delay	2.1
S_i	Transmitted signal	2.1
r_j	Received signal	2.1
W_j	Additive white Gaussian noise parameter	2.1
$H_{j,i}$	Diagonal matrix with elements of DFT	2.1
E_S	Symbol energy	2.2
N_0	Variance of the real/imaginary part of the AWGN	2.2
erfc(x)	Complementary error function of x	2.2
γ_S	Signal to noise ratio	2.2
$\rho(\gamma)$	Probability density function of γ	2.2
α	Gaussian constant	2.2
m	Nakagami-m fading parameter	2.2
β	Bits/symbol	2.2

List of Abbreviation

Symbol	Meaning	Section
OFDM	Orthogonal Frequency Division Multiplexing	1.3
DFT	Discrete Fourier Transform	1.3
FFT	Fast Fourier Transform	1.3
MRC	Maximal Ratio Combining	1.5
EGC	Equal Gain Combining	1.5
SC	Selection Combining	1.5
STBC	Space Time Block Code	1.8
SFBC	Space Frequency Block Code	1.9
MSK	Minimum Shift Keying	1.7
GMSK	Gaussian Minimum Shift Keying	1.7
MPSK	M-ary Phase Shift Keying	1.7
MQAM	M-ary Quadrature Amplitude Modulation	1.7

CHAPTER 1

INTRODUCTION

1.1 Communication System

With the advancement of knowledge and technology, the means and modes of communication system is changing day by day. Communication system has now become more complicated and comprehensible. The increased demand of speed of transmission and accuracy of reception has now become a challenge for modern communication system. Latest communication system has already gained the speed of light and capability of exchanging a huge number of data with highest level of precision. Scientists and Engineers are always trying to enhance the speed, rate and accuracy of data passing through a communication network to fulfill the demand of today's competitive communicative world.

Communication networks are now consistent and reasonable. They are supporting their users to utilize their resources accurately to achieve their desired level of communication.

A standard electronic communication system has the following parts: (a) a source to generate the information that is to be communicated (b) an input transducer to convert the information into electrical signal to transmit through the channel (c) a transmitter to increase the quality of the signal (d) a channel to carry the signal to the receiver end (e) a receiver which collects the transmitted signal (f) an output transducer to convert the received signal to its user friendly original nature [1].

The signal is distorted by the channel by different noise sources. Channel acts as a filter and changes the wave shape of the transmitted signal. The distortion of signal increases with the length of the channel, internal noises like thermal motion of electrons and random emissions, and external noise and interference sources like thundering, presence of other signals nearby etc. Suitable steps must be taken to maintain the quality of the transmitted signal. Signal strength can be maintained by using amplification in some cases.

1.1.1 Analog and digital communication systems

Analog communication system has analog signals to transfer information from transmitter to receiver. The analog signals are continuous valued signals. They have continuous time, amplitude and phase properties. These signals also have infinite range of values. The process of transmitting messages through analog communication system is easier in practice as all the real signals around us are analog by nature. The hardware implementation of analog communication system is complicated and also requires lower bandwidth to transmit signals. Although the analog communication process requires low cost overall but they require high power. Above all the main drawback of analog communication is the easy accessibility of noise and interference to the system. Due to noise and interference of the channels and surrounding mediums, analog signals easily become erroneous and the original message can't be obtained from the receiver even sometimes after filtration. On this ground the inception of digital communication is commenced [2], [3].

Digital communication system works with discrete signals. These signals have discrete value in both time or frequency and amplitude or phase domain. The signals have finite values and the values are multiple of 2. They don't range to infinity. These signals can be received more perfect in nature than the analog signals. As their ranges minimum has only two values, even after addition of noise and interference they can be decoded properly. But to do so, the implementation of digital communication system requires high cost, miniature components, large bandwidth and long time. But after one successful design and implementation of a system, it can be used for several times with minor changes in design as required. Data can be stored and used later in these types of system. As the equipments are small in size, they require less power also. Most of the digital communication systems are software based, so they can be improved accordingly without rebuilding the full hardware system. Digital communication systems are now ruling the world. They are useful for long distance transmission whether analog communication lose their power of transmitted signals after a short distance. Analog signals can't provide enough confidentiality in communication then their digital counterparts. For today's high speed communication society, digital communication is the best means considering over all speed, accuracy, secrecy, cost and durability etc of transmitted and receiving communication signals [2], [3].

1.1.2 Simplex, half duplex and full duplex communication systems

According to the transmitting techniques, communication system is of three types: simplex, half duplex and full duplex communication. Through simplex communication, only transmitting of messages can occur to one direction. The receiver only will receive the transmitted signal. There is no chance of a receiver to send information to the transmitter in this type of communication system like radio. The listeners only hear the signals transmitted by the radio station but they can't send any signal.

The half duplex communication system gives the flexibility to both the users to transmit and receive signals or messages. But this can happen by only one side at a time. That means one user will send message and the other user will receive the message, then this user can send message and other will receive it. But two users simultaneously can't transmit messages in this type of communication system. Wireless receivers use this type of communication.

Finally, the full duplex communication system offers both the users to communicate simultaneously with each other. Both of them can send and receive signals at the same time. Telephone communication is like full duplex communication, both of the users have flexibility of transmitting and receiving at the same time. It makes communication faster than half duplex communication system.

1.1.3 Wired and wireless communication systems

Wired communication system uses physical medium for transmission and reception of data among users. These physical mediums include copper wire, Ethernet cable or fiber optic cables. Wired communication system requires a router or a hub and network switches to maintain connection among the devices or users of a specific wired network.

On the other hand wireless communication uses electromagnetic or radio frequency wave and infrared signals for transferring data. Now a days due to the advantages of mobility and accessibility the uses of wireless communication technology have been increased. Most of the devices like cellular phones, laptops, sensors and satellite receivers are using wireless technology. Advancement of wireless communication is playing an important role for today's high speed style of living [4].

Installation process of wired networks don't require much time or cost but sometimes designing of a wired network with more number of network devices are cumbersome. They require less time to be installed but difficult to moderate or redesign after primary installation. The connected devices of a single wired network can feel the presence of others where as they can't sense the availability of any other nearby network. Connection can be provided to the nearby devices among which the physical connection is passing through. Wired network has less mobility but they are reliable, high speedy and secured network connections. Wired networks require switches and hubs, they have low interference which causes less possibility of signal loss and fading. Overall wired networks have better performance and quality of service.

Wireless networks have the advantages of mobility, but they require high cost, much time and comparatively easy installation process then wired networks due to the lack of physical connectivity. The wireless networks are visible to one another and also connected devices can feel each other as well. Wireless networks can connect more number of devices than compared to a wired one due to the medium of operation. But wireless communication still is not much secured and reliable than the wired networks. They are easily accessed by the environmental interferences and cause loss of signal and as a result QoS is less in wireless networks [5]. Telephone system, video conferencing etc are the example of wired network where as cellular technology, satellite communication etc are examples of wireless networks.

1.2 Broadband Wireless Communication

Broadband communication means high speed data communication with instantaneous bandwidth. Recently, broadband technology is the technology with download speed of at least 25 Mbits/s and upload speed of at least 3 Mbits/s. Broadband communications is the most promising technology as compared to the dial up connection for high speed and always on nature. It can transfer data, audio and video at a tremendously high speed with better connectivity. Faster upload and download facility makes broadband technology much appropriate for today's communicative world.

Wireless communication was evolved in 1970s. From that time the wireless technology, especially the mobile technology is emerging at a high speed. Broadband wireless technology is facilitating the increasing demand of speedy and secured communication through cellular mediums of present world. Broadband wireless communication can be fixed like wi-fi and Wi-max and Bluetooth and it can also be mobile like present 3G, 4G technology and upcoming 5G technology [4], [5].

Wi-fi means wireless fidelity. It is the first broadband wireless technology. It is IEEE 802.11 standard. The major advantage of it is the easy operation and mobility of user upto 300 feet from the base station. It is also cost effective. But as the mobility ability for the users of wi-fi is only 300 feet which can be affirmed as its weakness. As wi-fi uses unlicensed radio signals interference from nearby devices and networks affect the wi-fi users.

Wi-max is the IEEE 802.16 standard. It has greater bandwidth and long coverage area than wi-fi network. Wi-max has bandwidth of upto 11 GHz and has network coverage range of nearly 30 miles. It has the advantages of higher data rate, long range of operation capacity and higher throughput. It does not require the users to be present near the base station. But Wi-max equipments must be used efficiently in case of power. Proper timing and power settings are required for wi-max application.

Mobile wireless technology includes the 3G and 4G techniques. 3G communication has frequency spectrum between 400MHz to 3GHz and it is originated in 1980 by International Telecommunication Union. 3G technology uses WCDMA and CDMA 2000 standard. 3G was evolved to incorporate all the standards of wireless technology but it was not completely possible because of differences in frequency ranges of different countries. 3G is used in wireless internet access, wireless telephony, video calls, mobile internet access and mobile television. 3G has played the vital role in the increased uses of mobile or cellular phones world wide. But the global standardization of wireless technology still is not possible with 3G which creates the necessity of 4G and further technologies.

4G technology has emerged due to the increasing demand and growth of users. The main purpose of introducing 4G technology was creating a common platform for all the existing technologies. Through 4G technology speed will increase in case of data transformation and cost will be reasonable for the users. It will provide better quality of service in case of flexibility and user friendly multi functional uses. 4G technology is taken as Long Term Evolution (LTE) technology and adds the feature of 3G with wi-max, wi-fi, video chat, HDTV contents etc. 4G is now the widely accepted network structure for access purpose [4], [5].

1.3 MIMO-OFDM Technology

With the advancement and increasing demand of broadband wireless networks, the use and requirement of multiple input multiple output technique with orthogonal frequency division multiplexing scheme is increasing. MIMO-OFDM gives a better solution of high data rate with high quality of service to the users of a wireless network like WLAN, WMANS and WWWW etc [6].

MIMO wireless technology offers high spectral efficiency with increased spatial multiplexing gain and improved link reliability through antenna diversity gain. Due to scarcity of spectrum and propagation constraints of channel path MIMO technology with the higher spectral efficiency is being used now-a-days. Formally multiple antennas were being used in one side of the propagation of channel to decrease or terminate the effect of interferences of wireless network. Array and diversity gain are observed by coherent combining. But the increased spectral efficiency and link reliability through spatial multiplexing gain and antenna diversity gain are the main features of using multiple antennas at both sides of the channels.

When number of data stream is transmitted over multiple antennas then it is called spatial multiplexing. It creates a capacity gain in relation to transmit antennas which is known as spatial multiplexing gain. This gain is realized by transmitting independent data signals from individual channels [7].

Antenna diversity means using of number of antennas in fixed position. Theses antennas will provide uncorrelated signals of same power level. At the receiver side all these signals are collected to create an enhanced signal. Antenna diversity gain improves the link reliability of a MIMO-OFDM system.

Spatial multiplexing increases capacity gain due to multiple antennas without expenditure of the spectrum. The receivers realize the differences in the spatial signatures coming out of the MIMO channels with the capacity gain.

Fading is now decreased by using diversity in antennas to improve link reliability. To obtain proper diversity gain multiple data signals are transmitted through different dimensions of time, frequency and space and they are combined correctly in the receiver. Spatial diversity gain uses the space rather than time or frequency to improve performance. MIMO can cancel or decrease the interference of propagation channels which increases the capability of wireless network [8].

OFDM is a modulation technique based on frequency division multiplexing. It utilizes IFFT or DFFT and works through number of parallel narrow band sub carriers instead of a single wideband carrier. OFDM mitigates the problems of single carrier modulation via the equalization of time domain over the frequency domain equalization of the single carrier signals. Frequency selective Fading channels are turned into flat fading channels through OFDM technique. OFDM introduces guard interval called cyclic prefix (CP) which is a copy of the last part of the OFDM symbol that is transmitted. The CP is long enough to withstand the delay spread of the spectrum. Due to the use of CP the linear convolution process of the channel converts into cyclic convolution which allows the use of IFFT and FFT at the transmitter and receiver of an OFDM channel respectively. Thus the frequency selective fading channels are converted into flat fading channels [9].

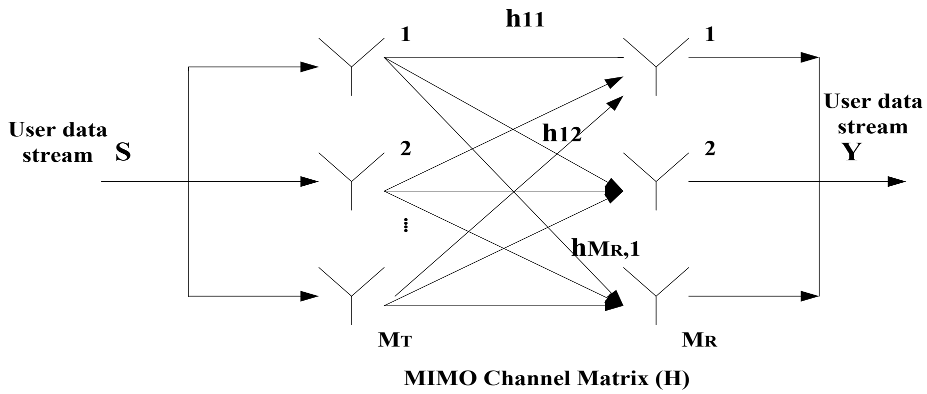

Fig. 1.1: MIMO antenna system [9]

Let us taken a transmitted data stream of $s= [s_1\ s_2\ s_3 \ldots\ldots\ldots s_M]^T$, received data stream of $y= [y_1\ y_2\ y_3 \ldots\ldots\ldots y_M]^T$ and additive white Gaussian noise of $\eta= [\eta_1\ \eta_2\ \eta_3 \ldots\ldots\ldots \eta_M]^T$. A MIMO system with M_T transmit antenna and M_R receive antenna is considered in the figure. A matrix format of $H = M_R \times M_T$ is obtained while calculating the impulse response between the i-th ($i = 1, 2, 3, \ldots\ldots M_T$) receive antenna and j-th ($j = 1, 2, 3, \ldots\ldots M_R$) transmit antenna of the MIMO system.

$$H = \begin{bmatrix} h_{1,1}\ h_{1,2} \ldots\ h_{1,M_T} \\ h_{2,1}\ h_{2,2} \ldots h_{2,M_T} \\ \cdots\ \cdots\ \vdots \cdots \\ h_{M_R,1}\ h_{M_R,2} \ldots\ h_{M_R,M_T} \end{bmatrix} \tag{1.1}$$

Here, $h_{i,j}$ is the representation of complex Gaussian random variable of the fading gain between the i-th transmit and j-th receive antenna.

If a transmitted signal is S_j from the j-th transmit antenna and a receive signal at the i-th receive antenna is y_i then we have got

$$y_i(t) = \sum_{j=1}^{M_T} h_{i,j} S_j(t), \tag{1.2}$$

The received signal vector $y(t)$ can be obtained as, $y(t) = H(t)s(t)$ (1.3)

For a 2×2 MIMO system the first receive antenna will receive,

$$y_1 = h_{1,1}s_1 + h_{1,2}s_2 + \eta_1 \tag{1.4}$$

The second receive antenna will receive,

$$y_2 = h_{2,1}s_1 + h_{2,2}s_2 + \eta_2 \tag{1.5}$$

OFDM technique has sub carriers to increase spectral efficiency. Theses carriers are orthogonal to each other. As a result although they are overlapping in nature, they don't interfere with one another. An OFDM system has the following steps to follow:

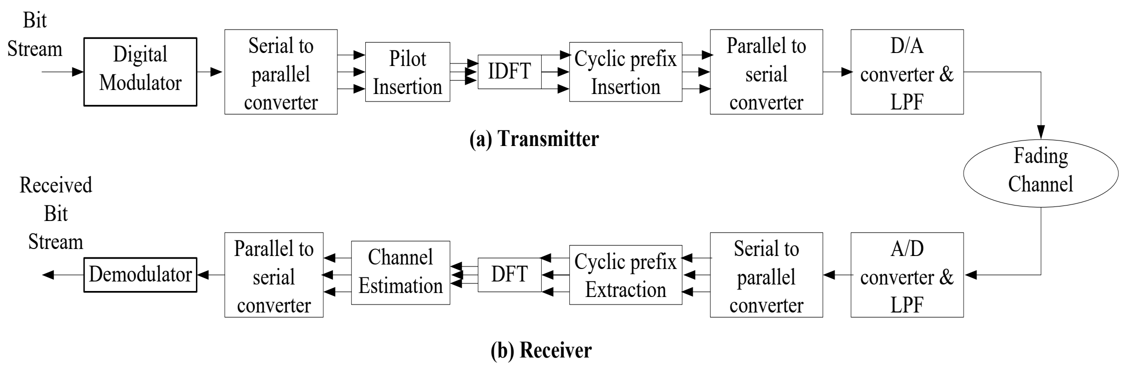

Fig. 1.2: OFDM system model [9]

For a N-tone OFDM system, first the incoming bits are modulated according to the modulation scheme. Next by a serial to parallel converter, the modulated bits are converted to parallel bit stream from serial one. Each parallel block has length of N. In a parallel data stream system, the total channel bandwidth is divided into number of narrow sub channel of flat response. A parallel block uses only a small part of the entire bandwidth. After that, DFFT is performed on each block of symbols including the pilot symbols for synchronization and the OFDM signal is obtained. Now, cyclic prefix of accurate bit size will be added with the OFDM signal to reduce ISI. Next, the signal is converted to serial and from digital to analog. Again, the signal is converted to parallel and the CP is removed. After DFT the data symbols will be detected and after demodulation original OFDM signal will be found.

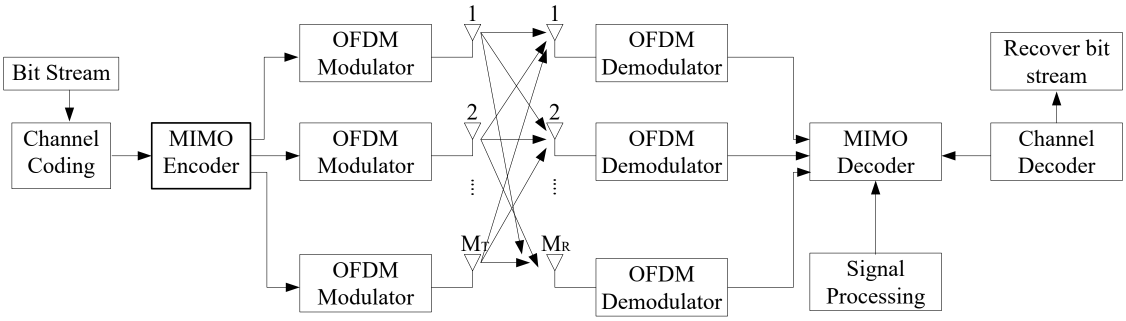

Fig. 1.3: MIMO-OFDM system model [9]

If $\{s_k\}_{k=0}^{N-1}$ is a complex signal, then OFDM modulated signal will be,

$$s(t)= \sum_{k=0}^{N-1} a_k^{j2\pi k\Delta ft} = \sum_{k=0}^{N-1} s_k \upsilon_k(t), \text{ for } 0 \leq t \leq T_s \tag{1.6}$$

The passband OFDM signal can be obtained as,

$$s(t)= \mathrm{Re}\left\{\sum_{k=0}^{N-1} a_k^{j2\pi(fc+k\Delta f)t}\right\}, \text{for } 0 \leq t \leq T_s \tag{1.7}$$

Here, for $k = 0, 1, 2, \ldots\ldots N-1$, a_k = complex valued modulated symbols, N = Number of subcarriers, f_c = carrier frequency, T_s = sampling period and Δf = sub carrier/channel spacing of OFDM symbol. To fulfill the orthogonality condition the symbol duration must be $T_s\, \Delta f = 1$.

MIMO-OFDM technology is used for wideband purpose. Multi user MIMO-OFDM system benefits from space and frequency domain freedom with multiuser diversity.

MIMO-OFDM meets the challenges of high data rate and high performance by eliminating the multipath fading and ISI.

1.4 Fading and Fading Channel Modeling

In wireless communication transmitted signals face different types of noise, delays, distortions, attenuations and phase shifts. As a result signals loss their transmitted power while propagating through channel from transmitter to receiver. Thus the signals are faded. To mitigate the fading problem, diversity technique is applied in wireless communication. Multiple copies of the same data are transmitted through multiple paths to reduce the loss and receiver receives the best option or combination of the signals from the copies. Destructive interferences through the channel causes the fading phenomenon of the signals.

Fading has some parameters which define the various types of fading channels. They are as follows [10]:

Multipath spread: The maximum delay between paths of significant power in the channel is known as multipath spread (T_m).

Coherence Bandwidth: The range of frequencies that is allowed to pass through the propagating channel is known as coherence bandwidth (Δf_c).

Doppler Spread: The maximum of Doppler shift is known as Doppler spread (B_d).

Coherence Time: The time duration over which a propagating wave is considered predictable or coherent is known as coherent time (Δt_c).

1.4.1 Classification of fading: According to the parameters of fading channels and the transmitted signal, channels can be divided into the following categories:

1.4.1.1 Frequency selective and frequency non-selective fading

If a transmitted signal has smaller bandwidth than the coherence bandwidth, all the frequency components of that signal will undergo same degree of fading. This kind of fading is known as frequency non-selective fading. For this kind of fading, the symbol duration is larger than the coherence time. On the other hand, when a transmitted signal has larger bandwidth than the coherence bandwidth, the frequency components of that signal face different degrees of fading. This is then known as frequency selective fading. Here, the symbol duration will be less than the coherence time. For frequency non-selective fading, delays between different paths are relatively small with respect to the symbol duration and the situation is opposite for that of frequency selective fading. Frequency non-selective fading will receive only one copy of the signal which will be the superposition of all the copies within the coherence time in gain and phase. On the other hand, due to different degrees of fading there will be multiple copies of received signal in frequency selective fading [10], [11], [12].

1.4.1.2 Slow and fast fading

Slow fading is the long term fading effect. When receiver moves away from the transmitter and then faces reduction in the signal strength with the change in the mean value of the received signal, then it is the slow fading effect. When signaling interval is much smaller than the coherence time it is then called slow fading. Again, when signaling interval is much larger than the coherence time it will be then known as fast fading. Multipath propagation is related to fast fading as it is the short term component. Mobile terminal and bandwidth of transmitted signal influence fast fading [11], [12].

1.5 Diversity

Repetition of the transmitted information provides multiple copies of the same information to the receiver. Receiver takes the decision to receive the best option from the number of copies transmitted through number of transmitters. This is the technique of improving the performance of a channel influenced by fading. It is known as diversity. The decision of diversity taken by the receiver remains unknown to the transmitter [13].

1.5.1 Types of diversity techniques

Microscopic diversity techniques are applied to mitigate small scale fading and macroscopic diversity techniques are applied to prevent large scale fading. The main diversity techniques are given as follows [10-13]:

1.5.1.1 Space or spatial diversity

Space or spatial diversity is the most common and oldest diversity technique. Multiple antennas are used here in transmitting or receiving or at both ends of the channel. The antennas are physically separated and the receivers will receive different copies of signals suffering from independent fading. Space diversity provides higher data rates as well as large network coverage.

1.5.1.2 Frequency diversity

When the same information is transmitted through different frequency carriers separated by at least coherence bandwidth between them, is called frequency diversity. Here, also different copies of signals suffer from independent fading. The separation represents the frequency severance of uncorrelated signals.

1.5.1.3 Time diversity

If the same information bit is transmitted repetitively at short intervals of time, the time diversity is achieved. The time interval will be greater than the coherence time. As a result, different copies of the transmitted information will suffer from individual independent fading. The time interval has reciprocal relation with the rate of fading.

1.5.1.4 Polarization diversity

Multiple versions of the same signal with different polarization, that is with different electric and magnetic fields are transmitted to the receiver in polarization diversity technique. Orthogonal polarization with very small antennas can be obtained by this kind of diversity.

1.5.1.5 Angle diversity

Number of directional antennas with independent response to wave propagation causes angle diversity. When two omni-directional parasitic antennas change their pattern to receive signals at different angles then it is known as angle diversity technique.

1.5.2 Diversity combining techniques

Due to diversity multiple copies of the transmitted signal with independent fading are received by the receiver. One signal can be fade whether other may be with strong quality. By applying appropriate combining technique, the SNR can be improved at the receiver. The combining techniques are as follows [14]:

1.5.2.1 Selection Combining

In this combining process the best received signal is selected among the different signals. Switch logic is used to select the best SNR of the signal. If there are M independent Rayleigh fading channels with M number of diversity branches. The branch with highest SNR will be used by the receiver. Let us assume, same average SNR for each branch like,

$$\Gamma = \frac{E_b}{N_0}\overline{\alpha^2} \tag{1.8}$$

If $\overline{\alpha^2} = 1$ is considered, then the instantaneous SNR for each branch will be γ_i which can be defined as, $\gamma_i = \frac{\text{Instantaneous signal power per branch}}{\text{Mean noise power per branch}}$

If Rayleigh fading channel is considered, then its pdf is as follows,

$$\rho(\gamma_i) = \frac{1}{\Gamma} e^{-\frac{\gamma_i}{\Gamma}}; \gamma_i \geq 0 \tag{1.9}$$

The probability of all independent M branches receiving signals less than the threshold γ is,

$$\Pr[\gamma_1, \ldots\ldots\ldots, \gamma_M \leq \gamma] = \left(1-e^{-\frac{\gamma}{\Gamma}}\right)^M = P_M(\gamma) \tag{1.10}$$

The probability of SNR>γ for one or more branches is given by,

$$\Pr[\gamma_i \geq \gamma] = 1 - P_M(\gamma) = 1 - \left(1-e^{-\frac{\gamma}{\Gamma}}\right)^M \tag{1.11}$$

The pdf of γ is as follows,

$$\rho_M(\gamma) = \frac{d}{d\gamma} P_M(\gamma) = \frac{M}{\Gamma}\left(1-e^{-\frac{\gamma}{\Gamma}}\right)^{M-1} e^{-\frac{\gamma}{\Gamma}} \tag{1.12}$$

The mean SNR AND THE average SNR obtained by selection diversity are as follows:

$$\bar{\gamma} = \int_0^\infty \gamma \rho_M(\gamma) d\gamma = \Gamma \int_0^\infty Mx\left(1-e^{-x}\right)^{M-1} e^{-x}\, dx \tag{1.13}$$

Here, $x=\frac{\gamma}{\Gamma}$ and $\frac{\bar{\gamma}}{\Gamma}=\sum_{k=1}^{M}\frac{1}{k}$

1.5.2.2 Maximal Ratio Combining

MRC is the most complex combining technique as it combines all the branches. Scaling and cophasing of all the branches are required in MRC. Here, all the signals of M branches are weighted according to their individual SNR and then summed. The output SNR of MRC is the summation of all the individual SNRs. If none of the SNR of individual branches is acceptable, at that point MRC can provide an acceptable SNR. Best statistical fading reduction is obtained by this method.

The average SNR for maximal ratio combining is given as:

$$\overline{\gamma_M}=\sum_{i=1}^{M}\bar{\gamma}_i=\sum_{i=1}^{M}\Gamma=M\Gamma \tag{1.14}$$

1.5.2.3 Generalized Selection Combining

In MRC, the obtained SNR is the sum of the SNR's of all the individual branches. The selection combining process selects the largest SNR of the branches. In GSC, *m* number of large signals are chosen from *L* total diversity branches and then finally combined coherently. The average combined SNR for GSC is as follows:

$$\Gamma_{GSC}=\frac{1}{a}\left[m+\frac{m}{m+1}+\frac{m}{m+2}+\ldots\ldots\ldots+\frac{m}{L-1}+\frac{m}{L}\right] \tag{1.15}$$

$$\Gamma_{CSC}\leq\Gamma_{GSC}\leq\Gamma_{MRC} \tag{1.16}$$

The average SNR of GSC is upper bounded by the average SNR of MRC and lower bounded by SNR of selection combining.

1.6 Fading channel distributions

The commonly used distributions functions for modeling and designing of wireless communication systems are given below [15], [16]:

1.6.1 Rayleigh distribution

When the components of equivalent baseband signal, *h(t)* are independent, the probability density function of the amplitude $r = |h| = \alpha$ assumes Rayleigh pdf as:

$$\rho(r)=\frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right) \text{ where, } E\{r^2\}=2\sigma^2 \text{ and } r\geq 0 \tag{1.17}$$

Rayleigh is the most used signal model for wireless communication. In Rayleigh distribution the power is exponentially distributed whereas the phase is uniformly distributed and also remains independent from amplitude.

1.6.2 Nakagami distribution

For Nakagami distribution $h=re^{j\upsilon}$ where the angle υ is uniformly distributed over the range of [-π, π]. The variables r and υ are mutually independent of each other. The Nakagami pdf is as follows:

$$\rho(r)=\frac{2}{\Gamma(k)}\left(\frac{k}{2\sigma^2}\right)^k r^{2k-1}\exp\left(-\frac{kr^2}{2\sigma^2}\right) \text{ and } r\geq 0 \tag{1.18}$$

Where, $2\sigma^2=E\{r^2\}$, $\Gamma(\cdot)$ is the gamma function and $k\geq\frac{1}{2}$ fading figure that is the degrees of freedom related to the number of added Gaussian random variables. Here, receive power is Gamma distributed. When $k = 1$ Rayleigh=Nakagami.

1.6.3 Ricean distribution

For complex Gaussian channel $h= \alpha e^{j\upsilon}+ve^{j\upsilon}$ where α follows the Rayleigh distribution. The angles υ and θ are assumed to be mutually independent and uniformly distributed on [-π, π]. The ricean pdf is as follows:

$$\rho(r)=\frac{r}{\sigma^2}\exp\left(-\frac{r^2+v^2}{2\sigma^2}\right) I_0\left(\frac{rv}{\sigma^2}\right) \text{ and } r\geq 0 \tag{1.19}$$

Here, I_0 is the modified Bessel function.

1.6.4 Weibull distribution

Weibull distribution is another generalized form of Rayleigh distribution. The envelope of this distribution is Rayleigh distributed. The pdf of this distribution is as follows:

$$\rho(r)=\frac{kr^{k-1}}{2\sigma^2}\exp\left(-\frac{r^k}{2\sigma^2}\right) \tag{1.20}$$

1.7 Modulation Schemes

The digital signals are generated with low data rates. To increase the data rate, reduce the effect of interfering signals, baseband signals are modulated onto a radio frequency carrier for transmission from the transmitter to receiver. In this research paper four modulation schemes are used like: 16PSK, 16QAM, MSK and GMSK. They are briefly described below [17], [18-20].

1.7.1 M-PSK

Digital bit streams are converted to analog signal $a(t)cos(\omega t+\theta)$ when they are transmitted. This signal has amplitude $a(t)$, frequency $\omega/2\pi$ and phase θ. When $a(t)$ and ω remain unchanged and the phase θ is changed, the M-ary Phase Shift Keying is obtained. If the baseband signal is a binary 0, the transmitted signal is, $Acos(\omega t+\pi) = -Acos(\omega t)$ and for the binary 1, the transmitted signal is, $Acos(\omega t)$. When there are more than two phases, M number of phases M-ary PSK is obtained. Here, $M=2^b$. The SNR of PSK system with M phase is, $SNR= log_2 M(E_b/N_0)$ for $M \geq 2$. In M-PSK modulation, the amplitude of the transmitted signal remains constant, creating a circular constellation.

1.7.2 M-QAM

When $a(t)$ and θ are changed but ω remain fixed of the transmitted signal, then the modulation scheme is known as Quadrature Amplitude Modulation, QAM. The transmitted signal of this modulation is, $s(t)=I(t)\cos(2\pi f_0 t)-Q(t)\sin(2\pi f_0 t)$. When there is higher order of QAM, that is known as MQAM. Higher order QAM is known as rectangular modulation and lower order QAM is called non-rectangular modulation.

1.7.3 MSK

Minimum Shift Keying is related to dual nature if frequency shift keying and phase shift keying modulations. It is derived from orthogonal QPSK by replacing the rectangular pulse in amplitude with a half-cycle sinusoidal pulse. The MSK signal is defined as:

$$s(t)=a_I(t)\left|\cos\left(\frac{\pi(t-2nT_b)}{2T_b}\right)\right|\cos 2\pi f_c t+a_Q(t)\left|\sin\left(\frac{\pi(t-2nT_b)}{2T_b}\right)\right|\sin 2\pi f_c t \tag{1.21}$$

Here, $n = 0, 1, 2, 3\ldots$and T_b is the bit interval. The signal has constant envelope for modulated through MSK modulation. MSK makes the phase change linear and limited to $\pm\pi/2$ over a bit interval of T_b.

1.7.4 GMSK

The performance of MSK can be improved by using Gaussian pulse shape. The modulation obtained by this way is known as GMSK. The bandwidth of the Gaussian filter is quantified in the time-bandwidth product, BT_b. The improvement in bandwidth efficiency is obtained with the degradation of power efficiency. BER of GMSK is degraded to inter-symbol interference by the promulgation Gaussian filter.

1.8 Space Time Block Coding (STBC)

Wireless communication systems are now designed to integrate high data rates with high quality of services. To increase the demand of high network capacity and performance, multiple antennas are used both in transmitter and receiver end. Multiple input multiple output scheme (MIMO) can be exploited by using both space and frequency diversity. When space diversity is used to counter fading that is called Space Time Block Coding (STBC). Alamouti coding is the origin of space time block coding, STBC [21]. It is defined by using two transmit antennas using an orthogonal space time coding which is presented in Fig. 1.4:

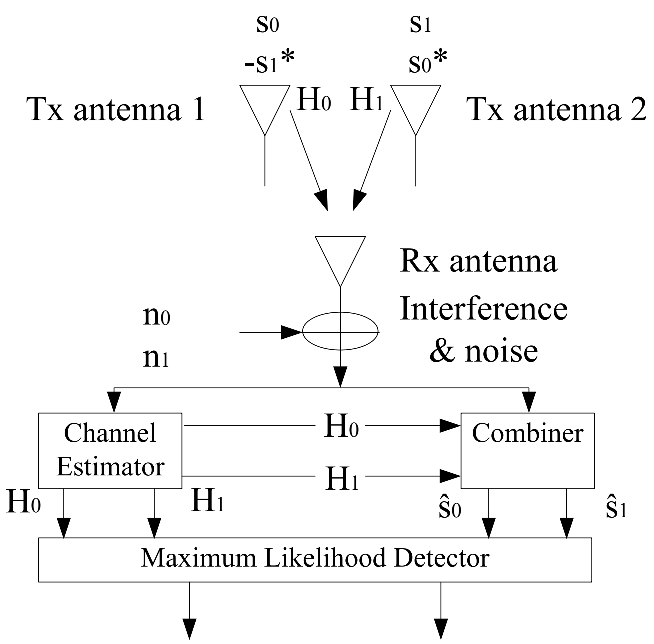

Fig. 1.4: Space time Alamouti coding [22]

The first antenna transmit s_0 and $-s_1*$ respectively and the second antenna will transmit s_1 and s_0*. If the channel remains invariant over the two consecutive transmissions, the received signals will be,

$$r_0=r(t)=s_0.H_0+s_1.H_1+w_0 \tag{1.22}$$

$$r_1=r(t+T)=-s_1^*.H_0+s_0^*.H_1+w_1 \tag{1.23}$$

The output r of the receiver is obtained at times t and $t+T$, w_0 and w_1 are complex random variables representing receiver noise. The signal can be recovered easily with a linear operation like:

$$\tilde{s}_0 = H_0^* r_0 + H_1 r_1^* \tag{1.24}$$

$$\tilde{s}_1 = H_1^* r_0 - H_0 r_1^* \tag{1.25}$$

Combining STBC with MIMO-OFDM has become a popular technique of mitigating the effect of fading and transforming frequency selective channels in flat fading channels. If there is a transmission sequence $\{s_1, s_2, \ldots\ldots\ldots, s_N\}$. Here, the symbols are grouped into two. According to Alamouti STBC-MIMO-OFDM, the signals transmitted at times t and $t+T$ from antennas 1 and 2 are found as follows:

Antenna 1: $S_0 = \left[s_1 s_3 \ldots\ldots s_{2N-1}, -s_2^* -s_4^* \ldots\ldots -s_{2N}^* \right]$

Antenna 2: $S_1 = \left[s_2 s_4 \ldots\ldots s_{2N}, s_1^* s_3^* \ldots\ldots s_{2N-1}^* \right]$

Here, S_0 and S_1 are the outputs for pathways 1 and 2 respectively of the space time encoder, $s_1, s_2, s_3, s_4, \ldots., s_{2N}$ are the signal symbols, $2N$ is representing the total symbol number and N is equal to the number of subcarriers. The received signal can be expressed as,

$$R = H_0.S_0 + H_1.S_1 + W \tag{1.26}$$

Where, $H_0 = [H_0(n,1) \ldots H_0(n,N), H_0(n+1,1) \ldots H_0(n+1,N)]^t$

$H_1 = [H_1(n,1) \ldots H_1(n,N), H_1(n+1,1) \ldots H_1(n+1,N)]^t$

$R = [R(n,1) \ldots R(n,N), R(n+1,1) \ldots R(n+1,N)]^t$

The combined signals $\tilde{s}_i$ and $\widetilde{s_{i+1}}$ are obtained as:

$$\tilde{s}_i(n,k) = H_0^*(n,k)R(n,k) + H_1(n,k)R^*(n+1,k) \tag{1.27}$$

$$\widetilde{s_{i+1}}(n,k) = H_1^*(n,k)R(n,k) - H_0(n,k)R^*(n+1,k) \tag{1.28}$$

Where, i is the symbol number and k refers to k^{th} subcarrier [22].

1.9 Space Frequency Block Coding (SFBC)

In SFBC-OFDM system the channel frequency response of adjacent subcarriers remains almost constant in the system. The use of OFDM offers the opportunity to code in frequency domain in the form of SFBC-OFDM system [23], [24-25].

For the case of channel variation over two consecutive OFDM symbols, SFBC is used rather than STBC. In SFBC, a single data sequence is applied to a standard Alamouti STBC encoder where the data sequence is, $a = [a_0, a_1, \ldots\ldots, a_{N-1}]$, then the output will be as follows:

$$[a_k \quad a_{k+1}] \xrightarrow{\text{STBC}} \begin{bmatrix} a_k & -a_{k+1}^* \\ a_{k+1} & a_k^* \end{bmatrix}, \text{k=0, 2, …, N-2} \tag{1.29}$$

The each row in the above equations, *N* symbol sequences are formed.

$$b^1 = \left[a_0, -a_1^*, a_2, -a_3^*, \ldots\ldots a_{N-2}, -a_{N-1}^*\right] \tag{1.30}$$

$$b^2 = \left[a_1, a_0^*, a_3, a_2^*, \ldots\ldots, a_{N-1}, a_{N-2}^*\right] \tag{1.31}$$

The sequences b^1 and b^2 are applied to an OFDM modulator and then are transmitted through antenna 1 and antenna 2 respectively. The received signal after FFT process at receiver v can be found as,

$$r^v = H^{1,v} b^1 + H^{2,v} b^2 + \eta^v \tag{1.32}$$

Here, $H^{s,v}$ is the channel frequency response. The FFT output at any two consecutive subcarriers *k* and *k+1* can be expressed as,

$$r_k^v = H_k^{1,v} a_k + H_k^{2,v} a_{k+1} + \eta_k^v \tag{1.33}$$

$$r_{k+1}^v = -H_{k+1}^{1,v} a_{k+1}^* + H_{k+1}^{2,v} a_k^* + \eta_{k+1}^v \tag{1.34}$$

Finally the output of FFT can be expressed as,

$$\begin{bmatrix} r_k^1 \\ r_k^2 \\ r_{k+!}^{*1} \\ r_{k+1}^{*2} \end{bmatrix} = \acute{H}_k \begin{bmatrix} a_k \\ a_{k+1} \end{bmatrix} + \begin{bmatrix} \eta_k^1 \\ \eta_k^2 \\ \eta_{k+!}^{*1} \\ \eta_{k+1}^{*2} \end{bmatrix} \tag{1.35}$$

The information symbols for SFBC can be extracted by using the STBC decoder.

1.10 Receiver Sensitivity

The minimum signal power, at which the receiver can provide adequate SNR at the receiver output and detect a signal at the same time, that power is called the receiver sensitivity [27].

The noise figure of a receiver is,

$$NF_{R_X}=SNR_i-SNR_o \tag{1.36}$$

Where, SNR_i = SNR at the input receiver and SNR_o = SNR at the output receiver. SNR at the input of a receiver is determined by input signal power and noise floor.

$$SNR_i(dB)=P_i(dBm)-NFloor(dBm) \tag{1.37}$$

Finally, the receiver sensitivity with the temperature 290K, Boltzmann constant k, and receiver bandwidth B_{R_X}, the receiver sensitivity will be as follows:

$$\text{Receiver Sensitivity} = kT_0+10\log\left(B_{R_X}\right)+SNR+NF_{R_X} \tag{1.38}$$

1.11 Convolutional Coding

Convolutional coding is different from the block coding as it offers a way to error correction in digital communication system. It consists of two parameters:(a) code rate and (b) the constraint length.

The proportion of data stream which is useful, known as code rate. That is, if the code rate is *k/n* this means, if the coder generates total *n* bits then *k* bits will be useful. Code rate can be 1/2, 2/3, 3/4, 5/6, 7/8 etc.

The constraint length is denoted by *K* which represents the length of the convolutional encoder. It means the number of k-bit stages which will be present to feed the combinational logic that produces the output symbols. Convolutional coding is based on construction techniques.

1.12 Literature Review

Numbers of research works have been carried out on the BER performances of SFBC with different fading environments. Some of them are briefly discussed below:

M. Torabi, S. Aissa and M. R. Soleymani [23] have presented closed form expressions for BER of SFBC-OFDM system. The performances were evaluated for MQAM and MPSK modulation under Rayleigh fading environment. The paper showed that the performances obtained by closed form expressions were nearly very close to the performances found by exact formulae.

M. A. Youssefi, N. Bounouader, Z. Guennoun and J. E. Abbadi [29] proposed an adaptive switching technique which enables switching between STBC and SFBC with MIMO-OFDM according to the performances over time and frequency selective fading channels. The research showed that, for severely frequency selective channels, SFBC has poor performance than STBC with MIMO-OFDM system.

Z. Mohammadian, M. Shahabinejad and S. Talebi [30] have introduced a full diversity space time frequency block code which can achieve maximum coding advantage. It is shown in the paper that, the proposed block code had better performance other than the previously proposed models for same order of receiver complexity.

N. N. M. Win and Z. M. Naing [31] have shown the performance of SFBC-MIMO-OFDM system under Rayleigh fading environment. The work demonstrated the Performance increment of SFBC with efficient data transmission over slow and fast fading environments.

S. R. Sabuj and M. S. Islam [32] have developed an analytical model with SFBC and data conjugate over Nakagami-m fading channel to reduce the SINR ratio with the incorporation of ICI and average probability of error. For larger values of the parameters, average probability of error has been reduced in Nakagami-m fading environment.

F. Fazel and H. Jafarkhani [33] have proposed a novel class of space frequency and space time frequency block codes over quasi orthogonal designs under Rayleigh fading environment. Complexity of maximum likelihood technique is reduced through the proposed model.

M. Torabi, A. Jemmali and J. Conan [34] have demonstrated performance analysis of SFBC-OFDM and frequency switched transmit diversity OFDM over MIMO fading channels. The papers showed Monte-Carlo simulation results. Monte-carlo results matched closely with the derived mathematical expressions.

K. Woradit, S. Siwamogsatham, and L. Wuttisittikulkij [35] have proposed a high-rate full-diversity SFC for both trellis-based and block-based cases. The obtained simulation results showed that the proposed rate-one SFBC provided larger coding gain than the conventional SFBC with higher rate, i.e., rate-two.

1.13 Motivations, Objectives and Possible Outcomes of the Thesis

This thesis work has the following motivations:

a. Previously, BER performances are obtained with MQAM and MPSK modulation schemes by SFBC-MIMO-OFDM system.

b. Mostly, the performances are obtained with Rayleigh fading and some of the researches are carried out in Nakagami-m fading environment.

c. Performances regarding receiver sensitivity were not performed in many researches.

d. Analytical expressions of MSK and GMSK-SFBC-OFDM were not shown in number of papers considering both the Rayleigh and Nakagami-m fading.

e. Comparisons of the results of MQAM, MPSK, MSK and GMSK are not demonstrated in previous works.

The objectives of this particular research work are:

a. To carry out analysis of a MIMO-OFDM wireless communication system with SFBC to find out the closed form expressions of average BER for MSK and GMSK modulations schemes considering frequency selective Rayleigh fading channels and expressions of average BER for M-ary QAM, M-ary PSK, MSK and GMSK modulations considering frequency selective Nakagami-m fading channels.

b. To evaluate the BER performance results and receiver sensitivities for both Rayleigh and Nakagami-m distribution with the above four modulation schemes for various MIMO-OFDM-SFBC configurations.

c. To find and compare the improvement in BER performances as well as receiver sensitivities of MIMO-SFBC-OFDM system with the MSK and GMSK modulations than the M-ary QAM and M-ary PSK modulation schemes.

The possible outcomes of the research work are as follows:

a. Analytical expressions for MSK and GMSK-SFBC-OFDM with both Rayleigh and Nakagami-m fading environment are to be found.

b. Comparisons among the BER performances with Rayleigh fading for MQAM, MPSK, MSK and GMSK modulations are to be performed.
c. Comparisons among the BER performances with Nakagami-m fading for MQAM, MPSK, MSK and GMSK modulations are also to be performed.
d. Receiver sensitivity comparison among all the modulation schemes are to be performed with both the fading environment.

1.14 Organization of the Thesis

The thesis paper is organized chapter wise as follows:

Chapter 1 includes the brief descriptions of wireless communications, fadings, fading models, modulation schemes, combining techniques, space time block coding and finally space frequency block coding. This chapter also includes the literature review and the motivations and possible outcomes of the thesis with the organization of the whole research paper.

Chapter 2 will show the proposed system model with the analytical evaluation of the average BER for MQAM, MPSK, MSK and GMSK-SFBC-OFDM under Rayleigh and Nakagami-m fading environment. Receiver sensitivity will also be evaluated in this chapter.

Chapter 3 will demonstrate the results and discussions of the simulated results for the analytical expressions of the average BER obtained in chapter 2. Comparisons among the results of BER performances will be shown by tables and figures. Also receiver sensitivities of different modulation schemes will be compared and shown in this chapter.

Finally chapter 4 will include the conclusion and proposed future works of the research work. Last of all there will be list of references used for this research work.

CHAPTER 2
SYSTEM MODEL AND ANALYTICAL EVALUATION

2.1 System Model

The SFBC-OFDM system used for the analytical evaluation of the BER performances of various modulation schemes is shown in Fig. 2.1. Number of transmitters is referred as M_T**,** number of receivers is referred as M_R**.** N is the number of OFDM subcarriers, $N_S = N/q$ where N_S is the number of subbands with q as the symbol period of SFBC system. All subbands are modulated by one of the four modulation schemes like MQAM, MPSK, R_C MSK and GMSK as they provide the input to the SFBC encoder as a vector $S = \{s[0], s[1], \ldots\ldots\ldots, s[N_t\text{-}1]\}$. Here, N_t is the multiplication of N with code rate. To utilize the space frequency block coding, input blocks of every transmitter must be of length N. M_T blocks are obtained by SFBC, each of length N and with N/q subblocks as $S_i = (s_i[0]\ s_i[1]\ldots\ldots s_i[N/q\text{-}1])^T$ for $i = 1, 2, 3\ldots M_T$. The modulators generates $X_1, X_2, \ldots, X_{M_T}$ which are to be transmitted by the 1st, 2nd… and M_T th transmitters. To avoid inter signal interference, the guard time interval is made longer than the delay spread of the multipath channels [23].

For the slow varying fading environment the impulse response of the link between the i-th transmitter and j-th receiver can be written as, $h_{j,i}(t) = \sum_{m=0}^{L\text{-}1} \alpha_{m,j,i}(t)\delta\left(t\text{-}\tau_m(t)\right)$ where, $\alpha_{m,j,i}(t)$ is the tap weight with $\tau_m(t)$ is the time delay of the m-path. Here, L is the total number of resolvable paths. When the cyclic prefix is removed and FFT is done at the receiver side, the demodulated signal can be found as:

$$r_j = \sum_{i=1}^{M_T} H_{j,i} S_i + W_j, \quad 1 \le j \le M_R \tag{2.1}$$

Here, $r_j = \left(r_j[0],\ r_j[1], \ldots\ldots, r_j[N\text{-}1]\right)^T$, is the received signal.

$S_i = \left(s_i[0],\ s_i[1], \ldots\ldots, s_i\left[\frac{N}{q}\text{-}1\right]\right)^T$ is the transmitted signal.

$W_j = \left(W_j[0],\ W_j[1], \ldots\ldots, W_j[N\text{-}1]\right)^T$ *is the AWGN and*

$H_{j,i} = diag\{H_{j,i}[k]\}_{k=0}^{N\text{-}1}$ *is a* matrix with size NxN.

When, the channel information is known, by any suitable detection technique the decoding of the SFBC modulated signal can be done.

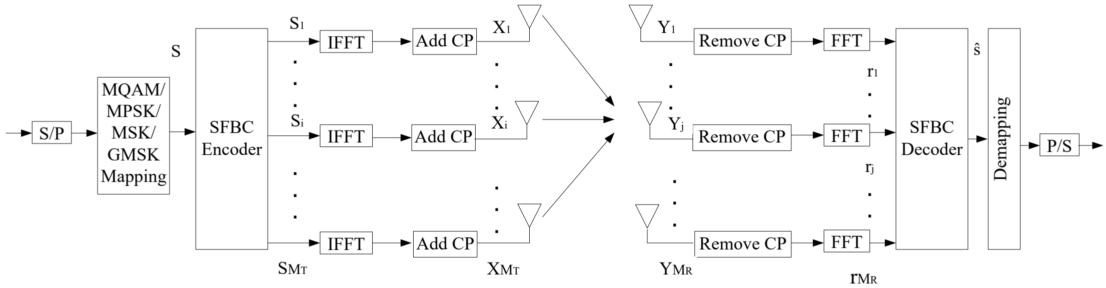

Fig. 2.1: SFBC-OFDM block diagram

2.2 Analytical Evaluation

If $S = \left(s[0],\ldots s[N\text{-}1]\right)^T$ is the input signal, $r = \left(r[0],\ldots r[N\text{-}1]\right)^T$ is the received signal, $W = \left(W[0],\ldots W[N\text{-}1]\right)^T$ is the AWGN and $H = diag\{H[k]\}_{k=0}^{N\text{-}1}$ is the matrix with the elements of DFT corresponding to the impulse response, then after removing cyclic prefix and FFT performance, the received signal will be,

$$r[k]=H[k]s[k]+W[k],\ \ k=0,\ 1,\ \ldots\ldots,N\text{-}1 \tag{2.2}$$

The BER expression for any OFDM system can be found by,

$$\mathrm{BER}=\frac{1}{N}\sum_{k=0}^{N\text{-}1}\mathrm{BER}[k] \tag{2.3}$$

Where, *BER[k]* is the notation of instantaneous BER of the k-th subchannel in the OFDM block.

2.2.1 BER performance of MQAM-OFDM in Rayleigh Fading Environment

The instantaneous BER of the k-th subchannel for MQAM ($M = 2^{\beta}$, β is the bits/symbol) with gray bit mapping can be found as [23],

$$\mathrm{BER}_{\mathrm{MQAM}}[k] = \frac{2\left(1-\frac{1}{\sqrt{2^{\beta}}}\right)}{\beta}\mathrm{erfc}\left(\sqrt{\frac{1.5\gamma_s|H[k]|^2}{2^{\beta}\text{-}1}}\right) \tag{2.4}$$

Here, $\gamma_s = \frac{E_s}{N_0}$ where, E_s is the symbol energy at the transmitter and $\frac{N_0}{2}$ is the variance of the real or imaginary part of the AWGN.

An approximate expression for $BER_{MQAM}[k]$ can be written as [23],

$$\mathrm{BER}_{\mathrm{MQAM}}[k] = 0.2\exp\left(\frac{-1.6\gamma_s|H[k]|^2}{2^{\beta}\text{-}1}\right) \tag{2.5}$$

Now the BER expression for the OFDM system can be stated as,

$$\mathrm{BER_{MQAM}} = \frac{2\left(1-\frac{1}{\sqrt{2^{\beta}}}\right)}{N\beta}\sum_{k=0}^{N-1}\mathrm{erfc}\left(\sqrt{\frac{1.5\gamma_s|H[k]|^2}{2^{\beta}-1}}\right) \tag{2.6}$$

This expression can be approximated as,

$$\mathrm{BER_{MQAM}} = \frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-1.6\gamma_s|H[k]|^2}{2^{\beta}-1}\right) \tag{2.7}$$

Now, the average BER will be found by the following equation,

$$\overline{\mathrm{BER_{MQAM}}} = \int_0^{\infty}\mathrm{BER_{MQAM}}\,\rho(\gamma)d\gamma \tag{2.8}$$

Here, $\rho(\gamma)$ is the probability density function (PDF) of $\gamma=\gamma_s|H[k]|^2$. As it is the Rayleigh distributed response, $|H[k]|^2$ is chi-square distributed with two degrees of freedom. So, γ is also chi-square distributed and $\rho(\gamma) = \frac{1}{\bar{\gamma}}exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \gamma \geq 0$ and also, $\bar{\gamma} = \gamma_s E\{|H[k]|^2\}$. Now the average BER for MQAM-OFDM can be found as [23],

$$\overline{\mathrm{BER_{MQAM}}} = \int_0^{\infty}\mathrm{BER_{MQAM}}\,\rho(\gamma)d\gamma$$

$$= \int_0^{\infty}\frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-1.6\gamma_s|H[k]|^2}{2^{\beta}-1}\right)\rho(\gamma)d\gamma$$

$$= \int_0^{\infty}\frac{0.2}{N}\times N\times\exp\left(\frac{-1.6\gamma}{2^{\beta}-1}\right)\frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)d\gamma$$

$$= \int_0^{\infty}0.2\exp\left(\frac{-1.6\gamma}{2^{\beta}-1}\right)\frac{1}{\gamma_s}\exp\left(-\frac{\gamma}{\gamma_s}\right)d\gamma$$

$$= \frac{0.2}{\gamma_s}\int_0^{\infty}\exp\left(\frac{-1.6\gamma}{2^{\beta}-1}-\frac{\gamma}{\gamma_s}\right)d\gamma$$

$$= \frac{0.2}{\gamma_s}\int_0^{\infty}\exp\left(\frac{-1.6\gamma\gamma_s-\gamma(2^{\beta}-1)}{\gamma_s(2^{\beta}-1)}\right)d\gamma$$

$$= \frac{0.2}{\gamma_s}\int_0^{\infty}\exp\left(\frac{-\gamma\left(1.6\gamma_s+(2^{\beta}-1)\right)}{\gamma_s(2^{\beta}-1)}\right)d\gamma$$

$$= \frac{0.2}{\gamma_s} \times \frac{-\gamma_s\left(2^\beta\text{-}1\right)}{1.6\gamma_s + \left(2^\beta\text{-}1\right)}\left[\exp\left(\frac{-\gamma\left(1.6\gamma_s + \left(2^\beta\text{-}1\right)\right)}{\gamma_s\left(2^\beta\text{-}1\right)}\right)\right]_0^\infty$$

$$= \frac{-0.2\left(2^\beta\text{-}1\right)}{1.6\gamma_s + \left(2^\beta\text{-}1\right)}[\exp(-\infty) - \exp(0)]$$

$$= \frac{-0.2\left(2^\beta\text{-}1\right)}{1.6\gamma_s + \left(2^\beta\text{-}1\right)}[0 - 1]$$

$$= \frac{0.2\left(2^\beta\text{-}1\right)}{1.6\gamma_s + \left(2^\beta\text{-}1\right)}$$

$$= 0.2\left[\frac{1.6\gamma_s + \left(2^\beta\text{-}1\right)}{2^\beta\text{-}1}\right]^{-1}$$

$$= 0.2\left[1 + \frac{1.6\gamma_s}{2^\beta\text{-}1}\right]^{-1}$$

So, $\overline{\mathrm{BER}_{\mathrm{MQAM}}} = 0.2\left[1 + \frac{1.6\gamma_s}{2^\beta\text{-}1}\right]^{-1}$ (2.9)

2.2.2 BER performance of MQAM-SFBC-OFDM in Rayleigh Fading Environment

When MQAM-SFBC-OFDM system is provided with M_T transmitters and M_R receivers then it can be written as [23],

$$\tilde{s}[k] = \frac{1}{R_C}\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2 s[k] + \eta[k] \tag{2.10}$$

Here, $H_{j,i}[k]$ is the k-th subchannel with i-th transmitter and j-th receiver, R_c is the code rate with $\eta[k]$ as the noise component.

Now the normalized SNR can be written as [23],

$$\gamma = \frac{1}{M_T R_C}\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2 \gamma_s \tag{2.11}$$

Now the BER of MQAM-SFBC-OFDM for frequency selective fading channels can be obtained as [23],

$$\mathrm{BER}_{\mathrm{MQAM}} = \frac{2}{N\beta}\left(1 - \frac{1}{\sqrt{2^\beta}}\right)\sum_{k=0}^{N-1}\mathrm{erfc}\left(\sqrt{\frac{1.5\gamma_s\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{R_C M_T\left(2^\beta\text{-}1\right)}}\right) \tag{2.12}$$

The above expression can be approximated as [23],

$$\mathrm{BER}_{\mathrm{MQAM}} = \frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-1.6\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}|H_{j,i}[k]|^2}{R_C M_T(2^{\beta}-1)}\right) \tag{2.13}$$

The average BER can be found by the following equation,

$$\overline{\mathrm{BER}_{\mathrm{MQAM}}} = \int_0^{\infty}\cdots\int_0^{\infty}\mathrm{BER}_{\mathrm{MQAM}}\ \rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T} \tag{2.14}$$

Here, $\gamma_{j,i}=\gamma_s\left|H_{j,i}[k]\right|^2$ and $\rho\left(\gamma_{j,i}\right) = \frac{1}{\overline{\gamma_{j,i}}}exp\left(-\frac{\gamma_{j,i}}{\overline{\gamma_{j,i}}}\right)$, $\gamma_{j,i}\geq 0$ where, $i = 1,\dots,M_T$ and $j = 1, \dots, M_R$.

Now the average BER will be found as follows [23]:

$$\overline{\mathrm{BER}_{\mathrm{MQAM}}} = \int_0^{\infty}\cdots\int_0^{\infty}\mathrm{BER}_{\mathrm{MQAM}}\ \rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty}\cdots\int_0^{\infty}\frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-1.6\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}|H_{j,i}[k]|^2}{R_C M_T(2^{\beta}-1)}\right)\rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty}\cdots\int_0^{\infty}\frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-1.6\gamma_{1,1}}{R_C M_T\left(2^{\beta}-1\right)}\right)\frac{1}{\overline{\gamma_{1,1}}}\exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty}\cdots\int_0^{\infty}\frac{0.2}{N}\times N\exp\left(\frac{-1.6\gamma_{1,1}}{R_C M_T\left(2^{\beta}-1\right)}\right)\frac{1}{\overline{\gamma_{1,1}}}\exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty}\cdots\int_0^{\infty}0.2\exp\left(\frac{-1.6\gamma_{1,1}}{R_C M_T\left(2^{\beta}-1\right)}\right)\frac{1}{\overline{\gamma_{1,1}}}\exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty}\cdots\int_0^{\infty}\frac{0.2}{\overline{\gamma_{1,1}}}\exp\left(\frac{-1.6\gamma_{1,1}}{R_C M_T\left(2^{\beta}-1\right)}-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty}\cdots\int_0^{\infty}\frac{0.2}{\overline{\gamma_{1,1}}}\exp\left(\frac{-1.6\gamma_{1,1}\overline{\gamma_{1,1}}-\gamma_{1,1}R_C M_T\left(2^{\beta}-1\right)}{\overline{\gamma_{1,1}}R_C M_T\left(2^{\beta}-1\right)}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty}\cdots\int_0^{\infty}\frac{0.2}{\overline{\gamma_{1,1}}}\exp\left(\frac{-\gamma_{1,1}(1.6\overline{\gamma_{1,1}}+R_C M_T\left(2^{\beta}-1\right))}{\overline{\gamma_{1,1}}R_C M_T\left(2^{\beta}-1\right)}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty}\cdots\frac{0.2}{\overline{\gamma_{1,1}}}\times\frac{-\left(2^{\beta}-1\right)R_C M_T\left(\overline{\gamma_{1,1}}\right)}{\left(1.6\overline{\gamma_{1,1}}+R_C M_T\left(2^{\beta}-1\right)\right)}\left[\exp\left(\frac{-\gamma_{1,1}(1.6\overline{\gamma_{1,1}}+R_C M_T\left(2^{\beta}-1\right))}{\overline{\gamma_{1,1}}R_C M_T\left(2^{\beta}-1\right)}\right)\right]_0^{\infty}\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^{\infty} \cdots 0.2 \times \frac{-(2^{\beta}-1)R_C M_T}{\left(1.6\overline{\gamma_{1,1}}+R_C M_T(2^{\beta}-1)\right)} \left[\exp(-\infty) - \exp(0)\right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^{\infty} \cdots 0.2 \times \frac{-(2^{\beta}-1)R_C M_T}{\left(1.6\overline{\gamma_{1,1}}+R_C M_T(2^{\beta}-1)\right)} (0-1) \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^{\infty} \cdots 0.2 \times \frac{(2^{\beta}-1)R_C M_T}{\left(1.6\overline{\gamma_{1,1}}+R_C M_T(2^{\beta}-1)\right)} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^{\infty} \cdots 0.2 \times \left[\frac{1.6\overline{\gamma_{1,1}}+R_C M_T(2^{\beta}-1)}{(2^{\beta}-1)R_C M_T}\right]^{-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$\overline{BER_{MQAM}} = 0.2\left(1+\frac{1.6\gamma_s}{(2^{\beta}-1)R_C M_T}\right)^{-M_R M_T} \tag{2.15}$$

2.2.3 BER performance of MPSK-OFDM in Rayleigh Fading Environment

The instantaneous BER of the k-th subchannel for MPSK can be found as,

$$BER_{MPSK}[k] = \frac{1}{\beta} \operatorname{erfc}\left(\sqrt{\gamma_s |H[k]|^2} \sin\left(\frac{\pi}{2^{\beta}}\right)\right) \tag{2.16}$$

An approximate expression for $BER_{MPSK}[k]$ can be written as [23],

$$BER_{MPSK}[k] = 0.2\exp\left(\frac{-7\gamma_s |H[k]|^2}{2^{1.9\beta}+1}\right) \tag{2.17}$$

Now the BER expression for the MPSK-OFDM system can be stated as,

$$BER_{MPSK} = \frac{1}{N\beta} \sum_{k=0}^{N-1} \operatorname{erfc}\left(\sqrt{\gamma_s |H[k]|^2} \sin\left(\frac{\pi}{2^{\beta}}\right)\right) \tag{2.18}$$

This expression can be approximated as [23],

$$BER_{MPSK} = \frac{0.2}{N} \sum_{k=0}^{N-1} \exp\left(\frac{-7\gamma_s |H[k]|^2}{2^{1.9\beta}+1}\right) \tag{2.19}$$

Now, the average BER will be found by the following equation [23],

$$\overline{BER_{MPSK}} = \int_0^{\infty} BER_{MPSK}\, \rho(\gamma) d\gamma \tag{2.20}$$

$$= \int_0^{\infty} \frac{0.2}{N} \sum_{k=0}^{N-1} \exp\left(\frac{-7\gamma_s |H[k]|^2}{2^{1.9\beta}+1}\right) \rho(\gamma) d\gamma$$

$$= \int_0^{\infty} \frac{0.2}{N} \times N \times \exp\left(\frac{-7\gamma}{2^{1.9\beta}+1}\right) \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) d\gamma, \ [\because \gamma = \gamma_s |H[k]|^2]$$

$$= \int_0^{\infty} 0.2 \exp\left(\frac{-7\gamma}{2^{1.9\beta}+1}\right) \frac{1}{\gamma_s} \exp\left(-\frac{\gamma}{\gamma_s}\right) d\gamma$$

$$= \frac{0.2}{\gamma_s} \int_0^{\infty} \exp\left(\frac{-7\gamma}{2^{1.9\beta}+1} - \frac{\gamma}{\gamma_s}\right) d\gamma$$

$$= \frac{0.2}{\gamma_s} \int_0^{\infty} \exp\left(\frac{-7\gamma\gamma_s - \gamma\left(2^{1.9\beta}+1\right)}{\gamma_s(2^{1.9\beta}+1)}\right) d\gamma$$

$$= \frac{0.2}{\gamma_s} \int_0^{\infty} \exp\left(\frac{-\gamma\left(7\gamma_s + \left(2^{1.9\beta}+1\right)\right)}{\gamma_s(2^{1.9\beta}+1)}\right) d\gamma$$

$$= \frac{0.2}{\gamma_s} \times \frac{-\gamma_s\left(2^{1.9\beta}+1\right)}{7\gamma_s + \left(2^{1.9\beta}+1\right)} \left[\exp\left(\frac{-\gamma\left(7\gamma_s + \left(2^{1.9\beta}+1\right)\right)}{\gamma_s\left(2^{1.9\beta}+1\right)}\right)\right]_0^{\infty}$$

$$= \frac{-0.2\left(2^{1.9\beta}+1\right)}{7\gamma_s + \left(2^{1.9\beta}+1\right)} [\exp(-\infty) - \exp(0)]$$

$$= \frac{-0.2\left(2^{1.9\beta}+1\right)}{7\gamma_s + \left(2^{1.9\beta}+1\right)} [0 - 1]$$

$$= \frac{0.2\left(2^{1.9\beta}+1\right)}{7\gamma_s + \left(2^{1.9\beta}+1\right)}$$

$$= 0.2 \left[\frac{7\gamma_s + \left(2^{1.9\beta}+1\right)}{2^{1.9\beta}+1}\right]^{-1}$$

$$= 0.2 \left[1 + \frac{7\gamma_s}{2^{1.9\beta}+1}\right]^{-1}$$

$$\text{So, } \overline{BER_{MPSK}} = 0.2 \left[1 + \frac{7\gamma_s}{2^{1.9\beta}+1}\right]^{-1} \tag{2.21}$$

2.2.3 BER performance of MPSK-SFBC-OFDM in Rayleigh Fading Environment

The BER of MPSK-SFBC-OFDM for frequency selective fading channels can be obtained as,

$$BER_{MPSK} = \frac{1}{N\beta} \sum_{k=0}^{N-1} \operatorname{erfc}\left(\sqrt{\frac{\gamma_s \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{R_C M_T}} \ \sin\left(\frac{\pi}{2^{\beta}}\right)\right) \tag{2.22}$$

The above expression can be approximated as [23],

$$\mathrm{BER_{MPSK}} = \frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-7\gamma_s\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{R_C M_T\left(2^{1.9\beta}+1\right)}\right), \text{[Also here, } \gamma_{j,i}=\gamma_s\left|H_{j,i}[k]\right|^2\text{]} \tag{2.23}$$

The average BER can be found by the following equation,

$$\overline{\mathrm{BER_{MPSK}}} = \int_0^\infty \cdots \int_0^\infty \mathrm{BER_{MPSK}}\ \rho\left(\gamma_{1,1}\right)\cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T} \tag{2.24}$$

So, the average BER is found as follows [23],

$$\overline{\mathrm{BER_{MPSK}}} = \int_0^\infty \cdots \int_0^\infty \mathrm{BER_{MPSK}}\ \rho\left(\gamma_{1,1}\right)\cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-7\gamma_s\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{R_C M_T\left(2^{1.9\beta}+1\right)}\right)\rho\left(\gamma_{1,1}\right)\cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-7\gamma_{1,1}}{R_C M_T\left(2^{1.9\beta}+1\right)}\right)\frac{1}{\overline{\gamma_{1,1}}}\exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2}{N}\times N\exp\left(\frac{-7\gamma_{1,1}}{R_C M_T\left(2^{1.9\beta}+1\right)}\right)\frac{1}{\overline{\gamma_{1,1}}}\exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty 0.2\exp\left(\frac{-7\gamma_{1,1}}{R_C M_T\left(2^{1.9\beta}+1\right)}\right)\frac{1}{\overline{\gamma_{1,1}}}\exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2}{\overline{\gamma_{1,1}}}\exp\left(\frac{-7\gamma_{1,1}}{R_C M_T\left(2^{1.9\beta}+1\right)}-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2}{\overline{\gamma_{1,1}}}\exp\left(\frac{-7\gamma_{1,1}\overline{\gamma_{1,1}}-\gamma_{1,1}R_C M_T\left(2^{1.9\beta}+1\right)}{\overline{\gamma_{1,1}}R_C M_T\left(2^{1.9\beta}+1\right)}\right)\cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2}{\overline{\gamma_{1,1}}}\exp\left(\frac{-\gamma_{1,1}\left(7\overline{\gamma_{1,1}}+R_C M_T\left(2^{1.9\beta}+1\right)\right)}{\overline{\gamma_{1,1}}R_C M_T\left(2^{1.9\beta}+1\right)}\right)\cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2}{\overline{\gamma_{1,1}}}\times\frac{-\left(2^{1.9\beta}+1\right)R_C M_T\left(\overline{\gamma_{1,1}}\right)}{\left(7\overline{\gamma_{1,1}}+R_C M_T\left(2^{1.9\beta}+1\right)\right)}\left[\exp\left(\frac{-\gamma_{1,1}\left(7\overline{\gamma_{1,1}}+R_C M_T\left(2^{1.9\beta}+1\right)\right)}{\overline{\gamma_{1,1}}R_C M_T\left(2^{1.9\beta}+1\right)}\right)\right]_0^\infty$$

$$\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots 0.2\times\frac{-\left(2^{1.9\beta}+1\right)R_C M_T}{\left(7\overline{\gamma_{1,1}}+R_C M_T\left(2^{1.9\beta}+1\right)\right)}\left[\exp(-\infty)-\exp(0)\right]\cdots \rho\left(\gamma_{M_R,M_T}\right)\cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots 0.2 \times \frac{-\left(2^{1.9\beta}+1\right)R_C M_T}{\left(7\overline{\gamma_{1,1}}+R_C M_T\left(2^{1.9\beta}+1\right)\right)} \text{ (0-1)} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots 0.2 \times \frac{\left(2^{1.9\beta}+1\right)R_C M_T}{\left(7\overline{\gamma_{1,1}}+R_C M_T\left(2^{1.9\beta}+1\right)\right)} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots 0.2 \times \left[\frac{7\overline{\gamma_{1,1}}+R_C M_T\left(2^{1.9\beta}+1\right)}{\left(2^{1.9\beta}+1\right)R_C M_T}\right]^{-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= 0.2\left(1+\frac{7\gamma_s}{\left(2^{1.9\beta}+1\right)R_C M_T}\right)^{-M_R M_T}$$

So, $$\overline{BER_{MPSK}} = 0.2\left(1+\frac{7\gamma_s}{\left(2^{1.9\beta}+1\right)R_C M_T}\right)^{-M_R M_T} \tag{2.25}$$

2.2.5 BER performance of MSK-OFDM in Rayleigh Fading Environment

The instantaneous BER of the k-th subchannel for MSK can be found as,

$$BER_{MSK}[k] = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\gamma_s |H[k]|^2}\right) \tag{2.26}$$

An approximate expression for $BER_{MSK}[k]$ can be written as,

$$BER_{MSK}[k] = \frac{1}{2}\exp\left(-\gamma_s |H[k]|^2\right) \tag{2.27}$$

Now the BER expression for the OFDM system can be stated as,

$$BER_{MSK} = \frac{1}{2N}\sum_{k=0}^{N-1}\operatorname{erfc}\left(\sqrt{\gamma_s |H[k]|^2}\right) \tag{2.28}$$

This expression can be approximated as,

$$BER_{MSK} = \frac{1}{2N}\sum_{k=0}^{N-1}\exp\left(-\gamma_s |H[k]|^2\right) \tag{2.29}$$

Now, the average BER will be found by the following equation,

$$\overline{BER_{MSK}} = \int_0^\infty BER_{MSK}\, \rho(\gamma) d\gamma \tag{2.30}$$

Now the average BER will be found as follows,

$$\overline{BER_{MSK}} = \int_0^\infty BER_{MSK}\, \rho(\gamma) d\gamma$$

$$= \int_0^{\infty} \frac{1}{2N} \sum_{k=0}^{N-1} \exp\left(-\gamma_s |H[k]|^2\right) \ \rho(\gamma) d\gamma$$

$$= \int_0^{\infty} \frac{1}{2N} \times N \times \exp(-\gamma_s |H[k]|^2) \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) d\gamma$$

$$= \int_0^{\infty} \frac{1}{2} \exp(-\gamma_s |H[k]|^2) \frac{1}{\gamma_s} \exp\left(-\frac{\gamma}{\gamma_s}\right) d\gamma$$

$$= \frac{1}{2\gamma_s} \int_0^{\infty} \exp\left(-\gamma - \frac{\gamma}{\gamma_s}\right) d\gamma$$

$$= \frac{1}{2\gamma_s} \int_0^{\infty} \exp\left(\frac{-\gamma\gamma_s - \gamma}{\gamma_s}\right) d\gamma$$

$$= \frac{1}{2\gamma_s} \int_0^{\infty} \exp\left(\frac{-\gamma\left(1+\gamma_s\right)}{\gamma_s}\right) d\gamma$$

$$= \frac{1}{2\gamma_s} \times \frac{-\gamma_s}{1+\gamma_s} \left[\exp\left(\frac{-\gamma\left(1+\gamma_s\right)}{\gamma_s}\right)\right]_0^{\infty}$$

$$= \frac{1}{2\gamma_s} \times \frac{-\gamma_s}{1+\gamma_s} [\exp(-\infty) - \exp(0)]$$

$$= \frac{1}{2\gamma_s} \times \frac{-\gamma_s}{1+\gamma_s} [0-1]$$

$$= \frac{1}{2\gamma_s} \times \frac{\gamma_s}{1+\gamma_s}$$

$$= \frac{1}{2} \left(1+\gamma_s\right)^{-1}$$

So, $\overline{BER_{MSK}} = \frac{1}{2} \left(1+\gamma_s\right)^{-1}$ [28] (2.31)

2.2.6 BER performance of MSK-SFBC-OFDM in Rayleigh Fading Environment

The BER of MSK-SFBC-OFDM for frequency selective fading channels can be obtained as,

$$BER_{MSK} = \frac{1}{2N} \sum_{k=0}^{N-1} \operatorname{erfc}\left(\sqrt{\frac{\gamma_s \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{R_C M_T}}\right) \tag{2.32}$$

The above expression can be approximated as,

$$BER_{MSK} = \frac{1}{2N}\sum_{k=0}^{N-1}\exp\left(\frac{-\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{R_C M_T}\right), \text{[Also here, } \gamma_{j,i}=\gamma_s\left|H_{j,i}[k]\right|^2\text{]} \tag{2.33}$$

The average BER can be found by the following equation,

$$\overline{BER_{MSK}} = \int_0^\infty \cdots \int_0^\infty BER_{MSK}\ \rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T} \tag{2.34}$$

So the average BER for MSK-SFBC-OFDM is found as follows,

$$\overline{BER_{MSK}} = \int_0^\infty \cdots \int_0^\infty BER_{MSK}\ \rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2N}\sum_{k=0}^{N-1}\exp\left(\frac{-\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{R_C M_T}\right)\rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2N}\sum_{k=0}^{N-1}\exp\left(\frac{-\gamma_{1,1}}{R_C M_T}\right)\frac{1}{\overline{\gamma_{1,1}}}\exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2N}\times N\exp\left(\frac{-\gamma_{1,1}}{R_C M_T}\right)\frac{1}{\overline{\gamma_{1,1}}}\exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2}\exp\left(\frac{-\gamma_{1,1}}{R_C M_T}\right)\frac{1}{\overline{\gamma_{1,1}}}\exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2\overline{\gamma_{1,1}}}\exp\left(\frac{-\gamma_{1,1}}{R_C M_T}-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2\overline{\gamma_{1,1}}}\exp\left(\frac{-\gamma_{1,1}\overline{\gamma_{1,1}}-\gamma_{1,1}R_C M_T}{\overline{\gamma_{1,1}}R_C M_T}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2\overline{\gamma_{1,1}}}\exp\left(\frac{-\gamma_{1,1}(\overline{\gamma_{1,1}}+R_C M_T)}{\overline{\gamma_{1,1}}R_C M_T}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{1}{2\overline{\gamma_{1,1}}}\times\frac{-R_C M_T\left(\overline{\gamma_{1,1}}\right)}{\left(\overline{\gamma_{1,1}}+R_C M_T\right)}\left[\exp\left(\frac{-\gamma_{1,1}(\overline{\gamma_{1,1}}+R_C M_T)}{\overline{\gamma_{1,1}}R_C M_T}\right)\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots \frac{1}{2}\times\frac{-R_C M_T}{\left(\overline{\gamma_{1,1}}+R_C M_T\right)}\left[\exp(-\infty)-\exp(0)\right]\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots \frac{1}{2}\times\frac{-R_C M_T}{\left(\overline{\gamma_{1,1}}+R_C M_T\right)}(0-1)\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^{\infty} \cdots \frac{1}{2} \times \frac{R_C M_T}{\left(\overline{\gamma_{1,1}}+R_C M_T\right)} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^{\infty} \cdots \frac{1}{2} \times \left[\frac{\overline{\gamma_{1,1}}+R_C M_T}{R_C M_T}\right]^{-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

So, $\overline{BER_{MSK}} = \frac{1}{2}\left(1+\frac{\gamma_s}{R_C M_T}\right)^{-M_R M_T}$ [28] (2.35)

2.2.7 BER performance of GMSK-OFDM in Rayleigh Fading Environment

The instantaneous BER of the k-th subchannel for GMSK can be found as,

$$BER_{GMSK}[k] = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\alpha\gamma_s|H[k]|^2}{2}}\right) \tag{2.36}$$

An approximate expression for $BER_{GMSK}[k]$ can be written as,

$$BER_{GMSK}[k] = \frac{1}{2}\exp\left(\frac{-\alpha\gamma_s|H[k]|^2}{2}\right) \tag{2.37}$$

Now the BER expression for the OFDM system can be stated as,

$$BER_{GMSK} = \frac{1}{2N}\sum_{k=0}^{N-1}\operatorname{erfc}\left(\sqrt{\frac{\alpha\gamma_s|H[k]|^2}{2}}\right) \tag{2.38}$$

This expression can be approximated as,

$$BER_{GMSK} = \frac{1}{2N}\sum_{k=0}^{N-1}\exp\left(\frac{-\alpha\gamma_s|H[k]|^2}{2}\right) \tag{2.39}$$

Now, the average BER will be found by the following equation,

$$\overline{BER_{GMSK}} = \int_0^{\infty} BER_{GMSK}\, \rho(\gamma)d\gamma \tag{2.40}$$

So the average BER for GMSK-OFDM can be found as follows,

$$\overline{BER_{GMSK}} = \int_0^{\infty} BER_{GMSK}\, \rho(\gamma)d\gamma$$

$$= \int_0^{\infty} \frac{1}{2N}\sum_{k=0}^{N-1} \exp\left(\frac{-\alpha\gamma_s|H[k]|^2}{2}\right) \rho(\gamma)d\gamma$$

$$= \int_0^{\infty} \frac{1}{2N} \times N \times \exp\left(\frac{-\alpha\gamma}{2}\right)\frac{1}{\overline{\gamma}}\exp\left(-\frac{\gamma}{\overline{\gamma}}\right) d\gamma$$

$$= \int_0^\infty \frac{1}{2}\exp(\frac{-\alpha\gamma}{2})\frac{1}{\gamma_s}\exp\left(-\frac{\gamma}{\gamma_s}\right)d\gamma$$

$$= \frac{1}{2\gamma_s}\int_0^\infty \exp\left(\frac{-\alpha\gamma}{2} - \frac{\gamma}{\gamma_s}\right)d\gamma$$

$$= \frac{1}{2\gamma_s}\int_0^\infty \exp\left(\frac{-\alpha\gamma\gamma_s - \gamma\times 2}{2\gamma_s}\right)d\gamma$$

$$= \frac{1}{2\gamma_s}\int_0^\infty \exp\left(\frac{-\gamma(\alpha\gamma_s + 2)}{2\gamma_s}\right)d\gamma$$

$$= \frac{1}{2\gamma_s}\times\frac{-2\gamma_s}{\alpha\gamma_s + 2}\left[\exp\left(\frac{-\gamma(\alpha\gamma_s+2)}{2\gamma_s}\right)\right]_0^\infty$$

$$= \frac{1}{2}\times\frac{-2}{\alpha\gamma_s + 2}[\exp(-\infty) - \exp(0)]$$

$$= \frac{1}{2}\ \frac{-2}{\alpha\gamma_s + 2}[0-1]$$

$$= \frac{2}{2(\alpha\gamma_s + 2)}$$

$$= \frac{1}{2}\left[\frac{\alpha\gamma_s+2}{2}\right]^{-1}$$

$$= \frac{1}{2}\left[1+\frac{\alpha\gamma_s}{2}\right]^{-1}$$

So, $\overline{BER_{GMSK}} = \frac{1}{2}\left[1+\frac{\alpha\gamma_s}{2}\right]^{-1}$ [28] (2.41)

2.2.8 BER performance of GMSK-SFBC-OFDM in Rayleigh Fading Environment

The BER of GMSK-SFBC-OFDM for frequency selective fading channels can be obtained as,

$$BER_{GMSK} = \frac{1}{2N}\sum_{k=0}^{N-1}\text{erfc}\left(\sqrt{\frac{\alpha\gamma_s\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}|H_{j,i}[k]|^2}{2R_C M_T}}\right) \quad (2.42)$$

The above expression can be approximated as,

$$BER_{GMSK} = \frac{1}{2N}\sum_{k=0}^{N-1}\exp\left(\frac{-\alpha\gamma_s\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}|H_{j,i}[k]|^2}{2R_C M_T}\right), \text{[Also here, } \gamma_{j,i}=\gamma_s|H_{j,i}[k]|^2\text{]} \quad (2.43)$$

The average BER can be found by the following equation,

$$\overline{\mathrm{BER_{GMSK}}} = \int_0^\infty \cdots \int_0^\infty \mathrm{BER_{GMSK}}\ \rho\left(\gamma_{1,1}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T} \tag{2.44}$$

So, the average BER for GMSK-SFBC-OFDM is found as follows,

$$\overline{\mathrm{BER_{GMSK}}} = \int_0^\infty \cdots \int_0^\infty \mathrm{BER_{GMSK}}\ \rho\left(\gamma_{1,1}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2N} \sum_{k=0}^{N-1} \exp\left(\frac{-\alpha\gamma_s \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{2R_C M_T}\right) \rho\left(\gamma_{1,1}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2N} \sum_{k=0}^{N-1} \exp\left(\frac{-\alpha\gamma_{1,1}}{2R_C M_T}\right) \frac{1}{\overline{\gamma_{1,1}}} \exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2N} \times N \exp\left(\frac{-\alpha\gamma_{1,1}}{2R_C M_T}\right) \frac{1}{\overline{\gamma_{1,1}}} \exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2} \exp\left(\frac{-\alpha\gamma_{1,1}}{2R_C M_T}\right) \frac{1}{\overline{\gamma_{1,1}}} \exp\left(-\frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2\overline{\gamma_{1,1}}} \exp\left(\frac{-\alpha\gamma_{1,1}}{2R_C M_T} - \frac{\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2\overline{\gamma_{1,1}}} \exp\left(\frac{-\alpha\gamma_{1,1}\overline{\gamma_{1,1}} - 2\gamma_{1,1} R_C M_T}{2\overline{\gamma_{1,1}} R_C M_T}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2\overline{\gamma_{1,1}}} \exp\left(\frac{-\gamma_{1,1}(\alpha\overline{\gamma_{1,1}} + 2R_C M_T)}{2\overline{\gamma_{1,1}} R_C M_T}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{1}{2\overline{\gamma_{1,1}}} \times \frac{-2R_C M_T\left(\overline{\gamma_{1,1}}\right)}{\left(\alpha\overline{\gamma_{1,1}} + 2R_C M_T\right)} \left[\exp\left(\frac{-\gamma_{1,1}(\alpha\overline{\gamma_{1,1}} + 2R_C M_T)}{2\overline{\gamma_{1,1}} R_C M_T}\right)\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots \frac{1}{2} \times \frac{-2R_C M_T}{\left(\alpha\overline{\gamma_{1,1}} + 2R_C M_T\right)} \left[\exp(-\infty) - \exp(0)\right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots \frac{1}{2} \times \frac{-2R_C M_T}{\left(\alpha\overline{\gamma_{1,1}} + 2R_C M_T\right)} (0-1) \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots \frac{1}{2} \times \frac{2R_C M_T}{\left(\alpha\overline{\gamma_{1,1}} + 2R_C M_T\right)} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

$$= \int_0^\infty \cdots \frac{1}{2} \times \left[\frac{\alpha\overline{\gamma_{1,1}}+2R_CM_T}{2R_CM_T}\right]^{-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\left(\gamma_{M_R,M_T}\right)$$

So, $\overline{BER_{GMSK}} = \frac{1}{2}\left(1+\frac{\alpha\gamma_s}{2R_CM_T}\right)^{-M_RM_T}$ [28] (2.45)

2.2.9 BER performance of MQAM-OFDM in Nakagami-m Fading Environment

The PDF for Nakagami-m fading environment is [27],

$$\rho(\gamma) = \frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma m} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right) \quad (2.46)$$

The approximate BER expression for OFDM system can be written as,

$$BER_{MQAM} = \frac{0.2}{N}\sum_{k=0}^{N-1} \exp\left(\frac{-1.6\gamma_s|H[k]|^2}{2^\beta-1}\right) \quad (2.47)$$

Considering the previous conditions, the average BER with Nakagami-m fading can be found as,

$$\overline{BER_{MQAM}} = \int_0^\infty BER_{MQAM}\, \rho(\gamma)d\gamma$$

$$= \int_0^\infty \frac{0.2}{N}\sum_{k=0}^{N-1} \exp\left(\frac{-1.6\gamma_s|H[k]|^2}{2^\beta-1}\right) \frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma m} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right) d\gamma$$

$$= \int_0^\infty \frac{0.2}{N} \times N \exp\left(\frac{-1.6\gamma}{2^\beta-1}\right) \frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma m} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right) d\gamma$$

$$= \int_0^\infty 0.2 \exp\left(\frac{-1.6\gamma}{2^\beta-1} - \frac{m\gamma}{\gamma_s}\right) \frac{m^m\gamma^{m-1}}{\gamma_s^m\Gamma m} d\gamma$$

$$= \int_0^\infty \frac{0.2\, m^m}{\gamma_s^m\Gamma m} \exp\left(\frac{-1.6\gamma\gamma_s - m\gamma(2^\beta-1)}{\gamma_s(2^\beta-1)}\right) \gamma^{m-1} d\gamma$$

$$= \int_0^\infty \frac{0.2\, m^m}{\gamma_s^m\Gamma m} \exp\left(\frac{-\gamma(1.6\gamma_s + m(2^\beta-1))}{\gamma_s(2^\beta-1)}\right) \gamma^{m-1} d\gamma$$

Let, $f = \frac{1.6\gamma_s + m(2^\beta-1)}{\gamma_s(2^\beta-1)}$

$$= \frac{0.2\, m^m}{\gamma_s^m\Gamma m} \int_0^\infty \exp(-\gamma f)\gamma^{m-1} d\gamma \quad (2.48)$$

For m = 2, from equ^n (2.48) we get,

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \int_0^\infty \exp(-\gamma f)\gamma^{2-1} d\gamma$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \int_0^\infty \gamma \exp(-\gamma f) d\gamma$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[\gamma \int \exp(-\gamma f) d\gamma - \int \left(\frac{d}{d\gamma}\gamma \int \exp(-\gamma f) d\gamma\right) d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[-\frac{\gamma \exp(-\gamma f)}{f} - \int -\frac{\exp(-\gamma f)}{f} d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[-\frac{\gamma \exp(-\gamma f)}{f} + \frac{1}{f}\left(-\frac{\exp(-\gamma f)}{f}\right)\right]_0^\infty$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[-\frac{\gamma \exp(-\gamma f)}{f} - \frac{1}{f^2}\exp(-\gamma f)\right]_0^\infty$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[\left\{-\frac{\infty \exp(-\infty f)}{f} - \frac{\exp(-\infty f)}{f^2}\right\} - \frac{0.\exp(-0.f)}{f} - \frac{\exp(-0.f)}{f^2}\right]$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[0 - \left(-\frac{1}{f^2}\right)\right]$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \times \frac{1}{f^2}$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \times \left(\frac{\gamma_s(2^\beta - 1)}{1.6\gamma_s + 2(2^\beta - 1)}\right)^2$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \times \left(\frac{1.6\gamma_s + 2(2^\beta - 1)}{\gamma_s(2^\beta - 1)}\right)^{-2} \tag{2.49}$$

For m = 3, from equ^n (2.48) we get,

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \int_0^\infty \exp(-\gamma f)\gamma^{3-1} d\gamma$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[\gamma^2 \int \exp(-\gamma f) d\gamma - \int \left(\frac{d}{d\gamma}\gamma^2 \int \exp(-\gamma f) d\gamma\right) d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[-\frac{\gamma^2 \exp(-\gamma f)}{f} - \int 2\gamma \times \left(-\frac{1}{f}\right) \exp(-\gamma f) d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3}\left[-\frac{\gamma^2 \exp(-\gamma f)}{f} + \frac{1}{f}\int 2\gamma \exp(-\gamma f) d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3}\left[-\frac{\gamma^2 \exp(-\gamma f)}{f} + \frac{2}{f}\left\{\gamma \int \exp(-\gamma f) d\gamma - \int (\frac{d}{d\gamma}\gamma \int \exp(-\gamma f) d\gamma) d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3}\left[-\frac{\gamma^2 \exp(-\gamma f)}{f} + \frac{2}{f}\left\{-\frac{\gamma \exp(-\gamma f)}{f} - \int -\frac{\exp(-\gamma f)}{f} d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3}\left[-\frac{\gamma^2 \exp(-\gamma f)}{f} + \frac{2}{f}\left\{-\frac{\gamma \exp(-\gamma f)}{f} - \frac{1}{f^2}\exp(-\gamma f)\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3}\left[-\frac{\gamma^2 \exp(-\gamma f)}{f} - \frac{2\gamma \exp(-\gamma f)}{f^2} - \frac{2}{f^3}\exp(-\gamma f)\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3}\left[0 - \left(-\frac{2}{f^3}\right)\right]$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \times \frac{2}{f^3}$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \times 2 \times \left(\frac{1.6\gamma_s + 3(2^\beta - 1)}{\gamma_s(2^\beta - 1)}\right)^{-3} \tag{2.50}$$

For m = 4, from equ^n (2.48) we get,

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\int_0^\infty \exp(-\gamma f)\gamma^{4-1} d\gamma$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[\gamma^3 \int \exp(-\gamma f) d\gamma - \int (\frac{d}{d\gamma}\gamma^3 \int \exp(-\gamma f) d\gamma) d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma f)}{f} + \frac{3}{f}\int \gamma^2 \exp(-\gamma f) d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma f)}{f} + \frac{3}{f}\left\{\gamma^2 \int \exp(-\gamma f) d\gamma - \int (\frac{d}{d\gamma}\gamma^2 \int \exp(-\gamma f) d\gamma) d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma f)}{f} + \frac{3}{f}\left\{-\frac{\gamma^2 \exp(-\gamma f)}{f} + \frac{2}{f}\int \gamma \exp(-\gamma f) d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma f)}{f} - \frac{3\gamma^2 \exp(-\gamma f)}{f^2} + \frac{6}{f^2}\left\{\gamma \int \exp(-\gamma f) d\gamma - \int (\frac{d}{d\gamma}\gamma \int \exp(-\gamma f) d\gamma) d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma f)}{f} - \frac{3\gamma^2 \exp(-\gamma f)}{f^2} + \frac{6}{f^2}\left\{-\frac{\gamma \exp(-\gamma f)}{f} + \frac{1}{f}\int \exp(-\gamma f) d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma f)}{f} - \frac{3\gamma^2 \exp(-\gamma f)}{f^2} - \frac{6\gamma \exp(-\gamma f)}{f^3} - \frac{6\exp(-\gamma f)}{f^4}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[0-\left(-\frac{6}{f^4}\right)\right]$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4} \times \frac{6}{f^4}$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4} \times 6 \times \left[\frac{1.6\gamma_s + 4(2^\beta - 1)}{\gamma_s (2^\beta - 1)}\right]^{-4} \tag{2.51}$$

Now, by observing the equ^n (2.49), (2.50) and (2.51) we get, the generalized form of average BER_{MQAM} is,

$$\overline{BER_{MQAM}} = 0.2\ m^m \left[m + \frac{1.6\gamma_s}{2^\beta - 1}\right]^{-m} \tag{2.52}$$

2.2.10 BER performance of MQAM-SFBC-OFDM in Nakagami-m fading environment

The BER of MQAM-SFBC-OFDM for frequency selective fading channels can be approximated as,

$$BER_{MQAM} = \frac{0.2}{N} \sum_{k=0}^{N-1} \exp\left(\frac{-1.6\gamma_s \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{R_C M_T (2^\beta - 1)}\right) \tag{2.53}$$

Again considering all the previous conditions, the average BER for MQAM-SFBC-OFDM in Nakagami-m fading environment is,

$$\overline{BER_{MQAM}} = \int_0^\infty \cdots \int_0^\infty BER_{MQAM}\ \rho(\gamma_{1,1}) \cdots \rho(\gamma_{M_R,M_T})\, d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2}{N} \sum_{k=0}^{N-1} \exp\left(\frac{-1.6\gamma_s \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{R_C M_T (2^\beta - 1)}\right) \rho(\gamma_{1,1}) \cdots \rho(\gamma_{M_R,M_T})\, d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty 0.2 \exp\left(\frac{-1.6\gamma_{1,1}}{R_C M_T (2^\beta - 1)}\right) \frac{m^m \gamma_{1,1}^{m-1}}{\overline{\gamma_{1,1}}^m \Gamma m} \exp\left(-\frac{m\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right) \cdots \rho(\gamma_{M_R,M_T})\, d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2 m^m}{\overline{\gamma_{1,1}}^m \Gamma m} \exp\left(\frac{-1.6\gamma_{1,1}}{R_C M_T (2^\beta - 1)} - \frac{m\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right) \gamma_{1,1}^{m-1} \cdots \rho(\gamma_{M_R,M_T})\, d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2m^m}{\overline{\gamma_{1,1}}^m \Gamma m} \exp\left(\frac{-\gamma_{1,1}(1.6\overline{\gamma_{1,1}}+m(R_C M_T(2^\beta-1)))}{\overline{\gamma_{1,1}} R_C M_T(2^\beta-1)}\right)\gamma_{1,1}^{m-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

Let, $\frac{1.6\overline{\gamma_{1,1}}+m(R_C M_T(2^\beta-1))}{\overline{\gamma_{1,1}} R_C M_T(2^\beta-1)} = f$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2m^m}{\overline{\gamma_{1,1}}^m \Gamma m} \exp\left(-\gamma_{1,1} f\right) \gamma_{1,1}^{m-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T} \tag{2.54}$$

Now, for m = 2 from equ^n (2.54) we get,

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \int_0^\infty \gamma_{1,1} \exp\left(-\gamma_{1,1} f\right) d\gamma_{1,1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \left[\gamma_{1,1} \int \exp\left(-\gamma_{1,1} f\right) d\gamma_{1,1} - \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1} \int \exp\left(-\gamma_{1,1} f\right) d\gamma_{1,1}\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \times \left[-\frac{\gamma_{1,1} \exp\left(-\gamma_{1,1} f\right)}{f} + \frac{1}{f} \int \exp\left(-\gamma_{1,1} f\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \times \left[-\frac{\gamma_{1,1} \exp\left(-\gamma_{1,1} f\right)}{f} - \frac{\exp\left(-\gamma_{1,1} f\right)}{f^2}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \times \left[0 - \left(-\frac{1}{f^2}\right)\right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \times \frac{1}{f^2} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T} \tag{2.55}$$

Now, for m = 3 from equ^n (2.54) we get,

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3 \Gamma 3} \int_0^\infty \gamma_{1,1}^{3-1} \exp\left(-\gamma_{1,1} f\right) d\gamma_{1,1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3 \Gamma 3} \times \left[\gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} f\right) d\gamma_{1,1} - \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} f\right) d\gamma_{1,1}\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)}{f}+\frac{2}{f}\int\gamma_{1,1}\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[\begin{array}{c}-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{2}{f}\left\{\begin{array}{c}\gamma_{1,1}\int\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}-\\ \int\left(\frac{d}{d\gamma_{1,1}}\gamma_{1,1}\int\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}\right)d\gamma_{1,1}\end{array}\right\}\end{array}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[\begin{array}{c}-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{2}{f}\left\{-\frac{\gamma_{1,1}\exp\left(-\gamma_{1,1}f\right)}{f}+\frac{1}{f}\int\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}\right\}\end{array}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[\begin{array}{c}-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{2}{f}\left\{-\frac{\gamma_{1,1}\exp\left(-\gamma_{1,1}f\right)}{f}-\frac{1}{f^2}\exp\left(-\gamma_{1,1}f\right)\right\}\end{array}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)}{f}-\frac{2\gamma_{1,1}\exp\left(-\gamma_{1,1}f\right)}{f^2}-\frac{2\exp\left(-\gamma_{1,1}f\right)}{f^3}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[0-\left(-\frac{2}{f^3}\right)\right]\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\frac{2}{f^3}\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T} \tag{2.56}$$

Again, for m = 4 from equ^n (2.54) we get,

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4\Gamma 4}\int_0^\infty\gamma_{1,1}^{4-1}\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4\Gamma 4}\times\left[\gamma_{1,1}^3\int\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}-\int\left(\frac{d}{d\gamma_{1,1}}\gamma_{1,1}^3\int\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}\right)d\gamma_{1,1}\right]_0^\infty$$
$$\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4\Gamma 4}\times\left[-\frac{\gamma_{1,1}^3\exp\left(-\gamma_{1,1}f\right)}{f}+\frac{3}{f}\int \gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}\right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4\Gamma 4}\times\left[\begin{array}{c}-\frac{\gamma_{1,1}^3\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{3}{f}\left\{\begin{array}{c}\gamma_{1,1}^2\int\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}-\\ \int\left(\frac{d}{d\gamma_{1,1}}\gamma_{1,1}^2\int\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}\right)d\gamma_{1,1}\end{array}\right\}\end{array}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4\Gamma 4}\times\left[\begin{array}{c}-\frac{\gamma_{1,1}^3\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{3}{f}\left\{\begin{array}{c}-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{2}{f}\int\gamma_{1,1}\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}\end{array}\right\}\end{array}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4\Gamma 4}\times\left[\begin{array}{c}-\frac{\gamma_{1,1}^3\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{3}{f}\left\{\begin{array}{c}-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{2}{f}\left(\gamma_{1,1}\int\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}-\int\left(\frac{d}{d\gamma_{1,1}}\gamma_{1,1}\int\exp\left(-\gamma_{1,1}f\right)d\gamma_{1,1}\right)d\gamma_{1,1}\right)\end{array}\right\}\end{array}\right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4\Gamma 4}\times\left[\begin{array}{c}-\frac{\gamma_{1,1}^3\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{3}{f}\left\{\begin{array}{c}-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)}{f}+\\ \frac{2}{f}\left(-\frac{\gamma_{1,1}\exp\left(-\gamma_{1,1}f\right)}{f}-\frac{\exp\left(-\gamma_{1,1}f\right)}{f^2}\right)\end{array}\right\}\end{array}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4\Gamma 4}\times\left[-\frac{\gamma_{1,1}^3\exp\left(-\gamma_{1,1}f\right)}{f}-\frac{3\gamma_{1,1}^2\exp\left(-\gamma_{1,1}f\right)}{f^2}-\frac{6\gamma_{1,1}\exp\left(-\gamma_{1,1}f\right)}{f^3}-\frac{6\exp\left(-\gamma_{1,1}f\right)}{f^4}\right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times4^4}{\gamma_{1,1}{}^4\Gamma 4}\times\left[0+\frac{6}{f^4}\right]\cdots \rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times4^4}{\gamma_{1,1}{}^4\Gamma 4}\times\frac{6}{f^4}\cdots \rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T} \quad (2.57)$$

Now, by observing the equ^n (2.55), (2.56) and (2.57) we get, the generalized form of average BER_{MQAM} with SFBC and Nakagami-m fading is,

$$\overline{BER_{MQAM}} = \frac{0.2\times m^m}{\gamma_s^m\Gamma m}\times\Gamma m\times\left[\frac{1.6\gamma_s + m(R_C M_T(2^\beta-1))}{\gamma_s R_C M_T(2^\beta-1)}\right]^{-mM_RM_T}$$

$$= 0.2\times m^m\times\left[\frac{1.6\gamma_s + m(R_C M_T(2^\beta-1))}{R_C M_T(2^\beta-1)}\right]^{-mM_RM_T}$$

$$= 0.2\times m^m\times\left[m+\frac{1.6\gamma_s}{R_C M_T(2^\beta-1)}\right]^{-mM_RM_T}$$

So, $$\overline{BER_{MQAM}} = 0.2\times m^m\times\left[m+\frac{1.6\gamma_s}{R_C M_T(2^\beta-1)}\right]^{-mM_RM_T} \quad (2.58)$$

2.2.11 BER performance of MPSK-OFDM in Nakagami-m fading environment

The approximate BER expression for MPSK-OFDM system can be written as,

$$BER_{MPSK} = \frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-7\gamma_s|H[k]|^2}{2^{1.9\beta}+1}\right) \quad (2.59)$$

With all the previous conditions, the average BER for MPSK-OFDM is obtained as,

$$\overline{BER_{MPSK}} = \int_0^\infty BER_{MPSK}\,\rho(\gamma)d\gamma$$

$$= \int_0^\infty \frac{0.2}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-7\gamma_s|H[k]|^2}{2^{1.9\beta}+1}\right)\frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma m}\exp\left(-\frac{m\gamma}{\bar{\gamma}}\right)d\gamma$$

$$= \int_0^\infty \frac{0.2}{N}\times N\exp\left(\frac{-7\gamma}{2^{1.9\beta}+1}\right)\frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma m}\exp\left(-\frac{m\gamma}{\bar{\gamma}}\right)d\gamma$$

$$= \int_0^\infty 0.2\exp\left(\frac{-7\gamma}{2^{1.9\beta}+1}-\frac{m\gamma}{\gamma_s}\right)\frac{m^m\gamma^{m-1}}{\gamma_s^m\Gamma m}d\gamma$$

$$= \int_0^\infty \frac{0.2\,m^m}{\gamma_s^m\Gamma m}\exp\left(\frac{-7\gamma\gamma_s - m\gamma(2^{1.9\beta}+1)}{(2^{1.9\beta}+1)}\right)\gamma^{m-1}d\gamma$$

$$= \int_0^\infty \frac{0.2\ m^m}{\gamma_s^m \Gamma m} \exp\left(\frac{-\gamma(7\gamma_s + m(2^{1.9\beta}+1))}{\gamma_s(2^{1.9\beta}+1)}\right)\gamma^{m-1} d\gamma$$

Let, $g = \frac{7\gamma_s + m(2^{1.9\beta}+1)}{\gamma_s(2^{1.9\beta}+1)}$

$$= \frac{0.2\ m^m}{\gamma_s^m \Gamma m} \int_0^\infty \exp(-\gamma g)\gamma^{m-1} d\gamma \qquad (2.60)$$

For m = 2, from equ^n (2.60) we get,

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \int_0^\infty \exp(-\gamma g)\gamma^{2-1} d\gamma$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \int_0^\infty \gamma \exp(-\gamma g) d\gamma$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[\gamma \int \exp(-\gamma g)d\gamma - \int (\frac{d}{d\gamma}\gamma \int \exp(-\gamma g)d\gamma)d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[-\frac{\gamma \exp(-\gamma g)}{g} - \int -\frac{\exp(-\gamma g)}{g} d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[-\frac{\gamma \exp(-\gamma g)}{g} + \frac{1}{g}\left(-\frac{\exp(-\gamma g)}{g}\right)\right]_0^\infty$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[-\frac{\gamma \exp(-\gamma g)}{g} - \frac{1}{g^2}\exp(-\gamma g)\right]_0^\infty$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[\left\{-\frac{\infty \exp(-\infty f)}{g} - \frac{\exp(-\infty g)}{g^2}\right\} - \frac{0.\exp(-0.g)}{g} - \frac{\exp(-0.g)}{g^2}\right]$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \left[0 - \left(-\frac{1}{g^2}\right)\right]$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \times \frac{1}{g^2}$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \times \left(\frac{\gamma_s(2^{1.9\beta}+1)}{7\gamma_s + 2(2^{1.9\beta}+1)}\right)^2$$

$$= \frac{0.2 \times 2^2}{\gamma_s^2 \Gamma 2} \times \left(\frac{7\gamma_s + 2(2^{1.9\beta}+1)}{\gamma_s(2^{1.9\beta}+1)}\right)^{-2} \qquad (2.61)$$

For m = 3, from equ^n (2.60) we get,

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \int_0^\infty \exp(-\gamma g)\gamma^{3-1} d\gamma$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[\gamma^2 \int \exp(-\gamma g)d\gamma - \int (\frac{d}{d\gamma}\gamma^2 \int \exp(-\gamma g)d\gamma)d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[-\frac{\gamma^2 \exp(-\gamma g)}{g} - \int 2\gamma \times \left(-\frac{1}{g}\right) \exp(-\gamma g)d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[-\frac{\gamma^2 \exp(-\gamma g)}{g} + \frac{1}{f}\int 2\gamma \exp(-\gamma g)d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[-\frac{\gamma^2 \exp(-\gamma g)}{g} + \frac{2}{f}\left\{\gamma \int \exp(-\gamma g)d\gamma - \int (\frac{d}{d\gamma}\gamma \int \exp(-\gamma g)d\gamma)d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[-\frac{\gamma^2 \exp(-\gamma g)}{g} + \frac{2}{g}\left\{-\frac{\gamma \exp(-\gamma g)}{g} - \int -\frac{\exp(-\gamma g)}{g} d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[-\frac{\gamma^2 \exp(-\gamma g)}{g} + \frac{2}{g}\left\{-\frac{\gamma \exp(-\gamma g)}{g} - \frac{1}{g^2}\exp(-\gamma g)\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[-\frac{\gamma^2 \exp(-\gamma g)}{g} - \frac{2\gamma \exp(-\gamma g)}{g^2} - \frac{2}{g^3}\exp(-\gamma g)\right]_0^\infty$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \left[0 - \left(-\frac{2}{g^3}\right)\right]$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \times \frac{2}{g^3}$$

$$= \frac{0.2 \times 3^3}{\gamma_s^3 \Gamma 3} \times 2 \times \left(\frac{7\gamma_s + 3(2^{1.9\beta}+1)}{\gamma_s(2^{1.9\beta}+1)}\right)^{-3} \tag{2.62}$$

For m = 4, from equ^n (2.60) we get,

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4} \int_0^\infty \exp(-\gamma g)\gamma^{4-1} d\gamma$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4} \left[\gamma^3 \int \exp(-\gamma g)d\gamma - \int (\frac{d}{d\gamma}\gamma^3 \int \exp(-\gamma g)d\gamma)d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4} \left[-\frac{\gamma^3 \exp(-\gamma g)}{g} + \frac{3}{g}\int \gamma^2 \exp(-\gamma g)d\gamma\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma g)}{g} + \frac{3}{g}\left\{\gamma^2 \int \exp(-\gamma g) d\gamma - \int \left(\frac{d}{d\gamma}\gamma^2 \int \exp(-\gamma g) d\gamma\right) d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma g)}{g} + \frac{3}{g}\left\{-\frac{\gamma^2 \exp(-\gamma g)}{g} + \frac{2}{g}\int \gamma \exp(-\gamma g) d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma g)}{g} - \frac{3\gamma^2 \exp(-\gamma g)}{g^2} + \frac{6}{g^2}\left\{\gamma \int \exp(-\gamma g) d\gamma - \int \left(\frac{d}{d\gamma}\gamma \int \exp(-\gamma g) d\gamma\right) d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma g)}{g} - \frac{3\gamma^2 \exp(-\gamma g)}{g^2} + \frac{6}{g^2}\left\{-\frac{\gamma \exp(-\gamma g)}{g} + \frac{1}{g}\int \exp(-\gamma g) d\gamma\right\}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[-\frac{\gamma^3 \exp(-\gamma g)}{g} - \frac{3\gamma^2 \exp(-\gamma g)}{g^2} - \frac{6\gamma \exp(-\gamma g)}{g^3} - \frac{6 \exp(-\gamma g)}{g^4}\right]_0^\infty$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4}\left[0 - \left(-\frac{6}{g^4}\right)\right]$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4} \times \frac{6}{g^4}$$

$$= \frac{0.2 \times 4^4}{\gamma_s^4 \Gamma 4} \times 6 \times \left[\frac{7\gamma_s + 4(2^{1.9\beta}+1)}{\gamma_s(2^{1.9\beta}+1)}\right]^{-4} \tag{2.63}$$

Now, by observing the equ^n (2.61), (2.62) and (2.63) we get, the generalized form of average BER_{MPSK} is,

$$\overline{BER_{MPSK}} = 0.2\, m^m \left[m + \frac{7\gamma_s}{2^{1.9\beta}+1}\right]^{-m} \tag{2.64}$$

2.2.12 BER performance of MPSK-SFBC-OFDM in Nakagami-m fading environment

The BER of MPSK-SFBC-OFDM for frequency selective fading channels can be approximated as,

$$BER_{MPSK} = \frac{0.2}{N}\sum_{k=0}^{N-1} \exp\left(\frac{-7\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}|H_{j,i}[k]|^2}{R_C M_T (2^{1.9\beta}+1)}\right) \tag{2.65}$$

Considering all the previous conditions, the average BER for MPSK-SFBC-OFDM in Nakagami-m fading environment is,

$$\overline{BER_{MPSK}} = \int_0^\infty \cdots \int_0^\infty BER_{MPSK}\ \rho\left(\gamma_{1,1}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2}{N}\sum_{k=0}^{N-1} \exp\left(\frac{-7\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{R_C M_T\left(2^{1.9\beta}+1\right)}\right)\rho\left(\gamma_{1,1}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty 0.2 \exp\left(\frac{-7\gamma_{1,1}}{R_C M_T\left(2^{1.9\beta}+1\right)}\right)\frac{m^m \gamma_{1,1}^{m-1}}{\overline{\gamma_{1,1}}^m \Gamma m}\exp\left(-\frac{m\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right) \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2m^m}{\overline{\gamma_{1,1}}^m \Gamma m} \exp\left(\frac{-7\gamma_{1,1}}{R_C M_T\left(2^{1.9\beta}+1\right)} - \frac{m\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\gamma_{1,1}^{m-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2m^m}{\overline{\gamma_{1,1}}^m \Gamma m} \exp\left(\frac{-\gamma_{1,1}(7\overline{\gamma_{1,1}}+m(R_C M_T\left(2^{1.9\beta}+1\right)))}{\overline{\gamma_{1,1}} R_C M_T\left(2^{1.9\beta}+1\right)}\right)\gamma_{1,1}^{m-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

Let, $\frac{7\overline{\gamma_{1,1}}+m(R_C M_T\left(2^{1.9\beta}+1\right))}{\overline{\gamma_{1,1}} R_C M_T\left(2^{1.9\beta}+1\right)} = g$

$$= \int_0^\infty \cdots \int_0^\infty \frac{0.2m^m}{\overline{\gamma_{1,1}}^m \Gamma m} \exp\left(-\gamma_{1,1} g\right)\gamma_{1,1}^{m-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T} \quad (2.66)$$

Now, for m = 2 from equ^n (2.66) we get,

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2}\int_0^\infty \gamma_{1,1} \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2}\left[\gamma_{1,1}\int \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1} - \int\left(\frac{d}{d\gamma_{1,1}}\gamma_{1,1}\int \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1}\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \times \left[-\frac{\gamma_{1,1}\exp\left(-\gamma_{1,1} g\right)}{g} + \frac{1}{g}\int \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \times \left[-\frac{\gamma_{1,1}\exp\left(-\gamma_{1,1} fg\right)}{g} - \frac{\exp\left(-\gamma_{1,1} g\right)}{g^2}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \times \left[0-\left(-\frac{1}{g^2}\right)\right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 2^2}{\overline{\gamma_{1,1}}^2 \Gamma 2} \times \frac{1}{g^2} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T} \quad (2.67)$$

Now, for m = 3 from equ^n (2.66) we get,

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\int_0^\infty \gamma_{1,1}^{3-1}\exp\left(-\gamma_{1,1}g\right)d\gamma_{1,1}\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[\gamma_{1,1}^2\int \exp\left(-\gamma_{1,1}g\right)d\gamma_{1,1}-\int\left(\frac{d}{d\gamma_{1,1}}\gamma_{1,1}^2\int \exp\left(-\gamma_{1,1}g\right)d\gamma_{1,1}\right)d\gamma_{1,1}\right]_0^\infty$$

$$\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}g\right)}{g}+\frac{2}{g}\int \gamma_{1,1}\exp\left(-\gamma_{1,1}g\right)d\gamma_{1,1}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[\begin{matrix}-\dfrac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}g\right)}{f}+\\ \dfrac{2}{g}\left\{\begin{matrix}\gamma_{1,1}\int \exp\left(-\gamma_{1,1}g\right)d\gamma_{1,1}-\\ \int\left(\dfrac{d}{d\gamma_{1,1}}\gamma_{1,1}\int \exp\left(-\gamma_{1,1}g\right)d\gamma_{1,1}\right)d\gamma_{1,1}\end{matrix}\right\}\end{matrix}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[\begin{matrix}-\dfrac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}g\right)}{g}+\\ \dfrac{2}{g}\left\{-\dfrac{\gamma_{1,1}\exp\left(-\gamma_{1,1}g\right)}{g}+\dfrac{1}{g}\int \exp\left(-\gamma_{1,1}g\right)d\gamma_{1,1}\right\}\end{matrix}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[\begin{matrix}-\dfrac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}g\right)}{g}+\\ \dfrac{2}{g}\left\{-\dfrac{\gamma_{1,1}\exp\left(-\gamma_{1,1}g\right)}{g}-\dfrac{1}{g^2}\exp\left(-\gamma_{1,1}g\right)\right\}\end{matrix}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}g\right)}{g}-\frac{2\gamma_{1,1}\exp\left(-\gamma_{1,1}g\right)}{g^2}-\frac{2\exp\left(-\gamma_{1,1}g\right)}{g^2}\right]_0^\infty\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\left[0-\left(-\frac{2}{g^3}\right)\right]\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 3^3}{\overline{\gamma_{1,1}}^3\Gamma 3}\times\frac{2}{g^3}\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T} \qquad (2.68)$$

Again, for m = 4 from equ^{n} (2.66) we get,

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \int_0^\infty \gamma_{1,1}^{4-1} \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[\gamma_{1,1}^3 \int \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1} - \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1}^3 \int \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1}\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[-\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} g\right)}{g} + \frac{3}{g} \int \gamma_{1,1}^2 \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[\begin{array}{c} -\dfrac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} g\right)}{g} + \\ \dfrac{3}{g}\left\{\begin{array}{c} \gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1} - \\ \int \left(\dfrac{d}{d\gamma_{1,1}} \gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1}\right) d\gamma_{1,1} \end{array}\right\} \end{array}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[\begin{array}{c} -\dfrac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} g\right)}{g} + \\ \dfrac{3}{g}\left\{\begin{array}{c} -\dfrac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} g\right)}{g} + \\ \dfrac{2}{g} \int \gamma_{1,1} \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1} \end{array}\right\} \end{array}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[\begin{array}{c} -\dfrac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} g\right)}{g} + \\ \dfrac{3}{g}\left\{\begin{array}{c} -\dfrac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} g\right)}{g} + \\ \dfrac{2}{g}\left(\gamma_{1,1} \int \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1} - \int \left(\dfrac{d}{d\gamma_{1,1}} \gamma_{1,1} \int \exp\left(-\gamma_{1,1} g\right) d\gamma_{1,1}\right) d\gamma_{1,1}\right) \end{array}\right\} \end{array}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[\begin{array}{c} -\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} g\right)}{g} + \\ \frac{3}{g} \left\{ \begin{array}{c} -\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} g\right)}{g} + \\ \frac{2}{g}\left(-\frac{\gamma_{1,1} \exp\left(-\gamma_{1,1} g\right)}{g} - \frac{\exp\left(-\gamma_{1,1} g\right)}{g^2} \right) \end{array} \right\} \end{array} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[-\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} g\right)}{g} - \frac{3\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} g\right)}{g^2} - \frac{6\gamma_{1,1} \exp\left(-\gamma_{1,1} g\right)}{g^3} - \frac{6\exp\left(-\gamma_{1,1} g\right)}{g^4} \right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[0 + \frac{6}{g^4} \right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{0.2\times 4^4}{\overline{\gamma_{1,1}}^4 \Gamma 4} \times \frac{6}{g^4} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T} \qquad (2.69)$$

Now, by observing the equ^n (2.67), (2.68) and (2.69) we get, the generalized form of average BER_{MPSK} with SFBC and Nakagami-m fading is,

$$\overline{BER_{MPSK}} = \frac{0.2\times m^m}{\gamma_s^m \Gamma m} \times \Gamma m \times \left[\frac{7\gamma_s + m(R_C M_T(2^{1.9\beta}+1))}{\gamma_s R_C M_T (2^{1.9\beta}+1)} \right]^{-mM_R M_T}$$

$$= 0.2\times m^m \times \left[\frac{7\gamma_s + m(R_C M_T(2^{1.9\beta}+1))}{R_C M_T (2^{1.9\beta}+1)} \right]^{-mM_R M_T}$$

$$= 0.2\times m^m \times \left[m + \frac{7\gamma_s}{R_C M_T (2^{1.9\beta}+1)} \right]^{-mM_R M_T}$$

So, $\overline{BER_{MPSK}} = 0.2\times m^m \times \left[m + \frac{7\gamma_s}{R_C M_T (2^{1.9\beta}+1)} \right]^{-mM_R M_T}$ (2.70)

2.2.13 BER performance of MSK-OFDM in Nakagami-m Fading Environment

The approximate BER expression for MSK-OFDM system can be written as,

$$BER_{MSK} = \frac{1}{2N} \sum_{k=0}^{N-1} \exp\left(-\gamma_s |H[k]|^2\right) \qquad (2.71)$$

So, the average BER with Nakagami-m fading (with all previous conditions) can be found as,

$$\overline{BER_{MSK}} = \int_0^{\infty} BER_{MSK}\ \rho(\gamma)d\gamma$$

$$= \int_0^{\infty} \frac{1}{2N}\sum_{k=0}^{N-1} \exp\left(-\gamma_s|H[k]|^2\right)\frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma m}\ \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right)d\gamma$$

$$= \int_0^{\infty} \frac{1}{2N} \times N\exp(-\gamma)\frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma m}\ \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right)d\gamma$$

$$= \int_0^{\infty} \frac{1}{2}\exp\left(-\gamma-\frac{m\gamma}{\gamma_s}\right)\frac{m^m\gamma^{m-1}}{\gamma_s^m\Gamma m}d\gamma$$

$$= \int_0^{\infty} \frac{m^m}{2\gamma_s^m\Gamma m}\exp\left(\frac{-\gamma\gamma_s-m\gamma}{\gamma_s}\right)\gamma^{m-1}d\gamma$$

$$= \int_0^{\infty} \frac{m^m}{2\gamma_s^m\Gamma m}\exp\left(\frac{-\gamma(\gamma_s+m)}{\gamma_s}\right)\gamma^{m-1}d\gamma$$

Let, $r = \frac{\gamma_s+m}{\gamma_s}$

$$= \frac{m^m}{2\gamma_s^m\Gamma m}\int_0^{\infty}\exp\left(-\gamma r\right)\gamma^{m-1}d\gamma \tag{2.72}$$

For m = 2, from equ^n (2.72) we get,

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\int_0^{\infty}\exp(-\gamma r)\gamma^{2-1}d\gamma$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\int_0^{\infty}\gamma\exp(-\gamma r)d\gamma$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[\gamma\int\exp(-\gamma r)d\gamma-\int\left(\frac{d}{d\gamma}\gamma\int\exp(-\gamma r)d\gamma\right)d\gamma\right]_0^{\infty}$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[-\frac{\gamma\exp(-\gamma r)}{r}-\int-\frac{\exp(-\gamma r)}{r}d\gamma\right]_0^{\infty}$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[-\frac{\gamma\exp(-\gamma r)}{r}+\frac{1}{r}\left(-\frac{\exp(-\gamma r)}{r}\right)\right]_0^{\infty}$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[-\frac{\gamma\exp(-\gamma r)}{r}-\frac{1}{r^2}\exp(-\gamma r)\right]_0^\infty$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[\left\{-\frac{\infty\exp(-\infty r)}{r}-\frac{\exp(-\infty r)}{r^2}\right\}-\frac{0.\exp(-0.r)}{r}-\frac{\exp(-0.r)}{r^2}\right]$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[0-\left(-\frac{1}{r^2}\right)\right]$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\times\frac{1}{r^2}$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\times\left(\frac{\gamma_s}{\gamma_s+2}\right)^2$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\times\left(\frac{\gamma_s+2}{\gamma_s}\right)^{-2} \tag{2.73}$$

For m = 3, from equ^n (2.72) we get,

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\int_0^\infty \exp(-\gamma r)\gamma^{3-1}d\gamma$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[\gamma^2\int\exp(-\gamma r)d\gamma-\int\left(\frac{d}{d\gamma}\gamma^2\int\exp(-\gamma r)d\gamma\right)d\gamma\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma r)}{r}-\int 2\gamma\times\left(-\frac{1}{r}\right)\exp(-\gamma r)d\gamma\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma r)}{r}+\frac{1}{r}\int 2\gamma\exp(-\gamma r)d\gamma\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma r)}{r}+\frac{2}{r}\left\{\gamma\int\exp(-\gamma r)d\gamma-\int\left(\frac{d}{d\gamma}\gamma\int\exp(-\gamma r)d\gamma\right)d\gamma\right\}\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma r)}{r}+\frac{2}{r}\left\{-\frac{\gamma\exp(-\gamma r)}{r}-\int-\frac{\exp(-\gamma r)}{r}d\gamma\right\}\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma r)}{r}+\frac{2}{r}\left\{-\frac{\gamma\exp(-\gamma r)}{r}-\frac{1}{r^2}\exp(-\gamma r)\right\}\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma r)}{r}-\frac{2\gamma\exp(-\gamma r)}{r^2}-\frac{2}{r^3}\exp(-\gamma r)\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[0-\left(-\frac{2}{r^3}\right)\right]$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3} \times \frac{2}{r^3}$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3} \times 2 \times \left(\frac{\gamma_s+3}{\gamma_s}\right)^{-3} \tag{2.74}$$

For m = 4, from equ^n (2.72) we get,

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\int_0^\infty \exp(-\gamma r)\gamma^{4-1}d\gamma$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[\gamma^3 \int \exp(-\gamma r)d\gamma - \int \left(\frac{d}{d\gamma}\gamma^3 \int \exp(-\gamma r)d\gamma\right)d\gamma\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma r)}{r} + \frac{3}{r}\int \gamma^2\exp(-\gamma r)d\gamma\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma r)}{r} + \frac{3}{r}\left\{\gamma^2 \int \exp(-\gamma r)d\gamma - \int \left(\frac{d}{d\gamma}\gamma^2 \int \exp(-\gamma r)d\gamma\right)d\gamma\right\}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma r)}{r} + \frac{3}{r}\left\{-\frac{\gamma^2\exp(-\gamma r)}{r} + \frac{2}{r}\int \gamma\exp(-\gamma r)d\gamma\right\}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma r)}{r} - \frac{3\gamma^2\exp(-\gamma r)}{r^2} + \frac{6}{r^2}\left\{\gamma \int \exp(-\gamma r)d\gamma - \int \left(\frac{d}{d\gamma}\gamma \int \exp(-\gamma r)d\gamma\right)d\gamma\right\}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma r)}{r} - \frac{3\gamma^2\exp(-\gamma r)}{r^2} + \frac{6}{r^2}\left\{-\frac{\gamma\exp(-\gamma f)}{r} + \frac{1}{r}\int \exp(-\gamma f)d\gamma\right\}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma r)}{r} - \frac{3\gamma^2\exp(-\gamma r)}{r^2} - \frac{6\gamma\exp(-\gamma r)}{r^3} - \frac{6\exp(-\gamma r)}{r^4}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[0-\left(-\frac{6}{r^4}\right)\right]$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4} \times \frac{6}{r^4}$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4} \times 6 \times \left[\frac{\gamma_s+4}{\gamma_s}\right]^{-4} \tag{2.75}$$

Now, by observing the equ^n (2.73), (2.74) and (2.75) we get, the generalized form of average BER_{MSK} is,

$$\overline{BER_{MSK}} = \frac{m^m}{2}\left(m+\gamma_s\right)^{-m} \tag{2.76}$$

2.2.14 BER performance of MSK-SFBC-OFDM in Nakagami-m fading environment

The BER of MSK-SFBC-OFDM for frequency selective fading channels can be approximated as,

$$\mathrm{BER_{MSK}} = \frac{1}{2N}\sum_{k=0}^{N-1} \exp\left(\frac{-\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{R_C M_T}\right) \tag{2.77}$$

So, the average BER for MSK-SFBC-OFDM in Nakagami-m fading environment is,

$$\overline{\mathrm{BER_{MSK}}} = \int_0^\infty \cdots \int_0^\infty \mathrm{BER_{MSK}}\ \rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2N}\sum_{k=0}^{N-1} \exp\left(\frac{-\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{R_C M_T}\right)\rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{2}\exp\left(\frac{-\gamma_{1,1}}{R_C M_T}\right)\frac{m^m \gamma_{1,1}^{m-1}}{\overline{\gamma_{1,1}}^m \Gamma m}\exp\left(-\frac{m\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{m^m}{2\overline{\gamma_{1,1}}^m \Gamma m}\ \exp\left(\frac{-\gamma_{1,1}}{R_C M_T}-\frac{m\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\gamma_{1,1}^{m-1}\ \cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{m^m}{2\overline{\gamma_{1,1}}^m \Gamma m}\ \exp\left(\frac{-\gamma_{1,1}(\overline{\gamma_{1,1}}+mR_C M_T)}{\overline{\gamma_{1,1}}R_C M_T}\right)\gamma_{1,1}^{m-1}\ \cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

Let, $\frac{\overline{\gamma_{1,1}}+m(R_C M_T)}{\overline{\gamma_{1,1}}R_C M_T}=r$

$$= \int_0^\infty \cdots \int_0^\infty \frac{m^m}{2\overline{\gamma_{1,1}}^m \Gamma m}\ \exp\left(-\gamma_{1,1}r\right)\gamma_{1,1}^{m-1}\ \cdots\rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1}\cdots d\gamma_{M_R,M_T} \tag{2.78}$$

Now, for m = 2 from equ^n (2.78) we get,

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2 \Gamma 2}\int_0^\infty \gamma_{1,1}\exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1}\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2 \Gamma 2}\left[\gamma_{1,1}\int \exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1} - \int\left(\frac{d}{d\gamma_{1,1}}\gamma_{1,1}\int \exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1}\right) d\gamma_{1,1}\right]_0^\infty$$

$$\cdots\rho\left(\gamma_{M_R,M_T}\right)\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2\Gamma 2} \times \left[-\frac{\gamma_{1,1}\exp\left(-\gamma_{1,1}r\right)}{r} + \frac{1}{r}\int \exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2\Gamma 2} \times \left[-\frac{\gamma_{1,1}\exp\left(-\gamma_{1,1}r\right)}{r} - \frac{\exp\left(-\gamma_{1,1}r\right)}{r^2} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2\Gamma 2} \times \left[0 - \left(-\frac{1}{r^2} \right) \right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2\Gamma 2} \times \frac{1}{r^2} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T} \tag{2.79}$$

Now, for m = 3 from equ^n (2.78) we get,

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \int_0^\infty \gamma_{1,1}^{3-1}\exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[\gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1} - \int \left(\frac{d}{d\gamma_{1,1}}\gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1} \right) d\gamma_{1,1} \right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[-\frac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}r\right)}{r} + \frac{2}{r}\int \gamma_{1,1}\exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[\begin{array}{c} -\dfrac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}r\right)}{r} + \\ \dfrac{2}{r}\left\{ \begin{array}{c} \gamma_{1,1}\int \exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1} - \\ \int \left(\dfrac{d}{d\gamma_{1,1}}\gamma_{1,1}\int \exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1} \right) d\gamma_{1,1} \end{array} \right\} \end{array} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[\begin{array}{c} -\dfrac{\gamma_{1,1}^2\exp\left(-\gamma_{1,1}r\right)}{f} + \\ \dfrac{2}{r}\left\{ -\dfrac{\gamma_{1,1}\exp\left(-\gamma_{1,1}r\right)}{r} + \dfrac{1}{r}\int \exp\left(-\gamma_{1,1}r\right) d\gamma_{1,1} \right\} \end{array} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3 \Gamma 3} \times \left[\begin{matrix} -\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} r\right)}{f} + \\ \frac{2}{r}\left\{ -\frac{\gamma_{1,1} \exp\left(-\gamma_{1,1} r\right)}{r} - \frac{1}{r^2} \exp\left(-\gamma_{1,1} r\right) \right\} \end{matrix} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3 \Gamma 3} \times \left[-\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} r\right)}{r} - \frac{2\gamma_{1,1} \exp\left(-\gamma_{1,1} r\right)}{r^2} - \frac{2\exp\left(-\gamma_{1,1} r\right)}{r^3} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3 \Gamma 3} \times \left[0 - \left(-\frac{2}{r^3} \right) \right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3 \Gamma 3} \times \frac{2}{r^3} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T} \tag{2.80}$$

Again, for m = 4 from equ^n (2.78) we get,

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4 \Gamma 4} \int_0^\infty \gamma_{1,1}^{4-1} \exp\left(-\gamma_{1,1} r\right) d\gamma_{1,1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[\gamma_{1,1}^3 \int \exp\left(-\gamma_{1,1} r\right) d\gamma_{1,1} - \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1}^3 \int \exp\left(-\gamma_{1,1} r\right) d\gamma_{1,1} \right) d\gamma_{1,1} \right]_0^\infty$$
$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[-\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} r\right)}{r} + \frac{3}{r} \int \gamma_{1,1}^2 \exp\left(-\gamma_{1,1} r\right) d\gamma_{1,1} \right]_0^\infty$$
$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4 \Gamma 4} \times \left[\begin{matrix} -\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} r\right)}{r} + \\ \frac{3}{r} \left\{ \begin{matrix} \gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} r\right) d\gamma_{1,1} - \\ \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} r\right) d\gamma_{1,1} \right) d\gamma_{1,1} \end{matrix} \right\} \end{matrix} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[\frac{3}{r}\left\{ \begin{array}{c} -\dfrac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} r\right)}{r} + \\ -\dfrac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} r\right)}{r} + \\ \dfrac{2}{r}\int \gamma_{1,1} \exp\left(-\gamma_{1,1} r\right) d\gamma_{1,1} \end{array} \right\} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[\frac{3}{r}\left\{ \begin{array}{c} -\dfrac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} r\right)}{r} + \\ -\dfrac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} r\right)}{r} + \\ \dfrac{2}{r}\left(\gamma_{1,1} \int \exp\left(-\gamma_{1,1} r\right) d\gamma_{1,1} - \int \left(\dfrac{d}{d\gamma_{1,1}}\gamma_{1,1} \int \exp\left(-\gamma_{1,1} r\right) d\gamma_{1,1}\right) d\gamma_{1,1}\right) \end{array} \right\} \right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[\frac{3}{r}\left\{ \begin{array}{c} -\dfrac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} r\right)}{r} + \\ -\dfrac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} r\right)}{r} + \\ \dfrac{2}{r}\left(-\dfrac{\gamma_{1,1} \exp\left(-\gamma_{1,1} r\right)}{r} - \dfrac{\exp\left(-\gamma_{1,1} r\right)}{r^2}\right) \end{array} \right\} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[-\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} r\right)}{r} - \frac{3\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} r\right)}{r^2} - \frac{6\gamma_{1,1} \exp\left(-\gamma_{1,1} r\right)}{r^3} - \frac{6\exp\left(-\gamma_{1,1} r\right)}{r^4} \right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[0 + \frac{6}{r^4}\right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \frac{6}{r^4} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T} \qquad (2.81)$$

Now, by observing the equ^n (2.79), (2.80) and (2.81) we get, the generalized form of average BER_{MSK} with SFBC and Nakagami-m fading is,

$$\overline{BER_{MSK}} = \frac{m^m}{2\gamma_s^m \Gamma m} \times \Gamma m \times \left[\frac{\gamma_s + m(R_C M_T)}{\gamma_s R_C M_T}\right]^{-mM_R M_T}$$

$$= \frac{1}{2}\times m^m \times \left[\frac{\gamma_s + m(R_C M_T)}{R_C M_T}\right]^{-mM_R M_T}$$

$$= \frac{1}{2}\times m^m \times \left[m + \frac{\gamma_s}{R_C M_T}\right]^{-mM_R M_T}$$

So, $\overline{BER_{MSK}} = \frac{1}{2}\times m^m \times \left[m + \frac{\gamma_s}{R_C M_T}\right]^{-mM_R M_T}$ (2.82)

2.2.15 BER performance of GMSK-OFDM in Nakagami-m Fading Environment

The approximate BER expression for GMSK-OFDM system can be written as,

$BER_{GMSK} = \frac{1}{2N}\sum_{k=0}^{N-1} \exp\left(\frac{-\alpha\gamma_s |H[k]|^2}{2}\right)$ (2.83)

So, with all the previous conditions, the average BER with Nakagami-m fading can be found as,

$$\overline{BER_{GMSK}} = \int_0^\infty BER_{GMSK}\, \rho(\gamma) d\gamma$$

$$= \int_0^\infty \frac{1}{2N}\sum_{k=0}^{N-1} \exp\left(\frac{-\alpha\gamma_s |H[k]|^2}{2}\right) \frac{m^m \gamma^{m-1}}{\overline{\gamma}^m \Gamma m} \exp\left(-\frac{m\gamma}{\overline{\gamma}}\right) d\gamma$$

$$= \int_0^\infty \frac{1}{2N} \times N \exp\left(\frac{-\alpha\gamma}{2}\right) \frac{m^m \gamma^{m-1}}{\overline{\gamma}^m \Gamma m} \exp\left(-\frac{m\gamma}{\overline{\gamma}}\right) d\gamma$$

$$= \int_0^\infty \frac{1}{2} \exp\left(\frac{-\alpha\gamma}{2} - \frac{m\gamma}{\gamma_s}\right) \frac{m^m \gamma^{m-1}}{\gamma_s^m \Gamma m} d\gamma$$

$$= \int_0^\infty \frac{m^m}{2\gamma_s^m \Gamma m} \exp\left(\frac{-\alpha\gamma\gamma_s - 2m\gamma}{2\gamma_s}\right) \gamma^{m-1} d\gamma$$

$$= \int_0^\infty \frac{m^m}{2\gamma_s^m \Gamma m} \exp\left(\frac{-\gamma(\alpha\gamma_s + 2m)}{2\gamma_s}\right) \gamma^{m-1} d\gamma$$

Let, $v = \frac{\alpha\gamma_s + 2m}{2\gamma_s}$

$= \frac{m^m}{2\gamma_s^m \Gamma m}\int_0^\infty \exp(-\gamma v)\gamma^{m-1} d\gamma$ (2.84)

For m = 2, from equ^n (2.84) we get,

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\int_0^\infty \exp(-\gamma v)\gamma^{2-1}d\gamma$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\int_0^\infty \gamma\exp(-\gamma v)d\gamma$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[\gamma\int \exp(-\gamma v)d\gamma - \int (\frac{d}{d\gamma}\gamma\int \exp(-\gamma f)d\gamma)d\gamma\right]_0^\infty$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[-\frac{\gamma\exp(-\gamma v)}{v} - \int -\frac{\exp(-\gamma v)}{v}d\gamma\right]_0^\infty$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[-\frac{\gamma\exp(-\gamma v)}{v} + \frac{1}{v}\left(-\frac{\exp(-\gamma v)}{v}\right)\right]_0^\infty$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[-\frac{\gamma\exp(-\gamma v)}{v} - \frac{1}{v^2}\exp(-\gamma v)\right]_0^\infty$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[\left\{-\frac{\infty\exp(-\infty v)}{v} - \frac{\exp(-\infty v)}{v^2}\right\} - \frac{0.\exp(-0.v)}{v} - \frac{\exp(-0.v)}{v^2}\right]$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\left[0-\left(-\frac{1}{v^2}\right)\right]$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\times\frac{1}{v^2}$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\times\left(\frac{2\gamma_s}{\alpha\gamma_s+2\times 2}\right)^2$$

$$= \frac{2^2}{2\gamma_s^2\Gamma 2}\times\left(\frac{\alpha\gamma_s+2\times 2}{2\gamma_s}\right)^{-2} \tag{2.85}$$

For m = 3, from equ^n (2.84) we get,

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\int_0^\infty \exp(-\gamma v)\gamma^{3-1}d\gamma$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[\gamma^2\int \exp(-\gamma v)d\gamma - \int (\frac{d}{d\gamma}\gamma^2\int \exp(-\gamma v)d\gamma)d\gamma\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma v)}{v} - \int 2\gamma\times\left(-\frac{1}{v}\right)\exp(-\gamma v)d\gamma\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma v)}{v}+\frac{1}{v}\int 2\gamma\exp(-\gamma v)d\gamma\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma v)}{v}+\frac{2}{v}\left\{\gamma\int\exp(-\gamma v)d\gamma - \int\left(\frac{d}{d\gamma}\gamma\int\exp(-\gamma v)d\gamma\right)d\gamma\right\}\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma v)}{v}+\frac{2}{v}\left\{-\frac{\gamma\exp(-\gamma v)}{v}-\int -\frac{\exp(-\gamma v)}{v}d\gamma\right\}\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma v)}{v}+\frac{2}{v}\left\{-\frac{\gamma\exp(-\gamma v)}{v}-\frac{1}{v^2}\exp(-\gamma v)\right\}\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[-\frac{\gamma^2\exp(-\gamma v)}{v} - \frac{2\gamma\exp(-\gamma v)}{v^2}-\frac{2}{v^3}\exp(-\gamma v)\right]_0^\infty$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3}\left[0-\left(-\frac{2}{v^3}\right)\right]$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3} \times \frac{2}{v^3}$$

$$= \frac{3^3}{2\gamma_s^3\Gamma 3} \times 2 \times \frac{3^3}{2\gamma_s^3\Gamma 3} \tag{2.86}$$

For m = 4, from equ^n (2.84) we get,

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\int_0^\infty \exp(-\gamma v)\gamma^{4-1}d\gamma$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[\gamma^3\int\exp(-\gamma v)d\gamma-\int\left(\frac{d}{d\gamma}\gamma^3\int\exp(-\gamma v)d\gamma\right)d\gamma\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma v)}{v}+\frac{3}{v}\int\gamma^2\exp(-\gamma v)d\gamma\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma v)}{v}+\frac{3}{v}\left\{\gamma^2\int\exp(-\gamma v)d\gamma-\int\left(\frac{d}{d\gamma}\gamma^2\int\exp(-\gamma v)d\gamma\right)d\gamma\right\}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma v)}{v}+\frac{3}{v}\left\{-\frac{\gamma^2\exp(-\gamma v)}{v}+\frac{2}{v}\int\gamma\exp(-\gamma v)d\gamma\right\}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma v)}{v}-\frac{3\gamma^2\exp(-\gamma v)}{v^2}+\frac{6}{v^2}\left\{\gamma\int\exp(-\gamma v)d\gamma - \int\left(\frac{d}{d\gamma}\gamma\int\exp(-\gamma v)d\gamma\right)d\gamma\right\}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma v)}{v}-\frac{3\gamma^2\exp(-\gamma v)}{v^2}+\frac{6}{v^2}\left\{-\frac{\gamma\exp(-\gamma v)}{v}+\frac{1}{v}\int\exp(-\gamma v)d\gamma\right\}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[-\frac{\gamma^3\exp(-\gamma v)}{v}-\frac{3\gamma^2\exp(-\gamma v)}{v^2}-\frac{6\gamma\exp(-\gamma v)}{v^3}-\frac{6\exp(-\gamma v)}{v^4}\right]_0^\infty$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\left[0-\left(-\frac{6}{v^4}\right)\right]$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\times\frac{6}{v^4}$$

$$= \frac{4^4}{2\gamma_s^4\Gamma 4}\times 6\times\left[\frac{\alpha\gamma_s+4\times 2}{2\gamma_s}\right]^{-4} \tag{2.87}$$

Now, by observing the equ^n (2.85), (2.86) and (2.87) we get, the generalized form of average BER_{GMSK} is,

$$\overline{BER_{GMSK}} = \frac{1}{2}\, m^m\left[m+\frac{\alpha\gamma_s}{2}\right]^{-m} \tag{2.88}$$

2.2.16 BER performance of GMSK-SFBC-OFDM in Nakagami-m fading environment

The BER of GMSK-SFBC-OFDM for frequency selective fading channels can be approximated as,

$$BER_{GMSK} = \frac{1}{2N}\sum_{k=0}^{N-1}\exp\left(\frac{-\alpha\gamma_s\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{2R_C M_T}\right) \tag{2.89}$$

So, the average BER for GMSK-SFBC-OFDM in Nakagami-m fading environment is,

$$\overline{BER_{GMSK}} = \int_0^\infty\cdots\int_0^\infty BER_{GMSK}\ \rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty\cdots\int_0^\infty\frac{1}{2}\sum_{k=0}^{N-1}\exp\left(\frac{-\alpha\gamma_s\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{2R_C M_T}\right)\rho\left(\gamma_{1,1}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty\cdots\int_0^\infty\frac{1}{2}\exp\left(\frac{-\alpha\gamma_{1,1}}{2R_C M_T}\right)\frac{m^m\gamma_{1,1}^{m-1}}{\overline{\gamma_{1,1}}^m\Gamma m}\exp\left(-\frac{m\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty\cdots\int_0^\infty\frac{m^m}{2\overline{\gamma_{1,1}}^m\Gamma m}\exp\left(\frac{-\alpha\gamma_{1,1}}{2R_C M_T}-\frac{m\gamma_{1,1}}{\overline{\gamma_{1,1}}}\right)\gamma_{1,1}^{m-1}\cdots\rho\left(\gamma_{M_R,M_T}\right)d\gamma_{1,1}\cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{m^m}{2\overline{\gamma_{1,1}}^m \Gamma m} \exp\left(\frac{-\gamma_{1,1}(\alpha\overline{\gamma_{1,1}}+2mR_C M_T)}{2\overline{\gamma_{1,1}} R_C M_T}\right) \gamma_{1,1}^{m-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T}$$

Let, $\frac{\alpha\overline{\gamma_{1,1}}+2mR_C M_T}{2\overline{\gamma_{1,1}} R_C M_T}=v$

$$= \int_0^\infty \cdots \int_0^\infty \frac{m^m}{2\overline{\gamma_{1,1}}^m \Gamma m} \exp\left(-\gamma_{1,1} v\right) \gamma_{1,1}^{m-1} \cdots \rho\left(\gamma_{M_R,M_T}\right) d\gamma_{1,1} \cdots d\gamma_{M_R,M_T} \quad (2.90)$$

Now, for m = 2 from equ^n (2.90) we get,

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2 \Gamma 2} \int_0^\infty \gamma_{1,1} \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2 \Gamma 2} \left[\gamma_{1,1} \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} - \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1} \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1}\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2 \Gamma 2} \times \left[-\frac{\gamma_{1,1}\exp\left(-\gamma_{1,1} v\right)}{v} + \frac{1}{v} \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2 \Gamma 2} \times \left[-\frac{\gamma_{1,1}\exp\left(-\gamma_{1,1} v\right)}{v} - \frac{\exp\left(-\gamma_{1,1} v\right)}{v^2}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2 \Gamma 2} \times \left[0-\left(-\frac{1}{v^2}\right)\right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{2^2}{2\overline{\gamma_{1,1}}^2 \Gamma 2} \times \frac{1}{v^2} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T} \quad (2.91)$$

Now, for m = 3 from equ^n (2.90) we get,

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3 \Gamma 3} \int_0^\infty \gamma_{1,1}^{3-1} \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3 \Gamma 3} \times \left[\gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} - \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1}\right) d\gamma_{1,1}\right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[-\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right)}{v} + \frac{2}{v}\int \gamma_{1,1} \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[\begin{array}{c} -\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right)}{v} + \\ \frac{2}{v}\left\{ \begin{array}{c} \gamma_{1,1} \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} - \\ \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1} \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \right) d\gamma_{1,1} \end{array} \right\} \end{array} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[\begin{array}{c} -\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right)}{f} + \\ \frac{2}{v}\left\{ -\frac{\gamma_{1,1} \exp\left(-\gamma_{1,1} v\right)}{v} + \frac{1}{v}\int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \right\} \end{array} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[\begin{array}{c} -\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right)}{f} + \\ \frac{2}{v}\left\{ -\frac{\gamma_{1,1} \exp\left(-\gamma_{1,1} v\right)}{v} - \frac{1}{v^2} \exp\left(-\gamma_{1,1} v\right) \right\} \end{array} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[-\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right)}{v} - \frac{2\gamma_{1,1} \exp\left(-\gamma_{1,1} v\right)}{v^2} - \frac{2\exp\left(-\gamma_{1,1} v\right)}{v^3} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \left[0 - \left(-\frac{2}{v^3} \right) \right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{3^3}{2\overline{\gamma_{1,1}}^3\Gamma 3} \times \frac{2}{v^3} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T} \tag{2.92}$$

Again, for m = 4 from equ^{n} (2.90) we get,

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \int_0^\infty \gamma_{1,1}^{4-1} \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[\gamma_{1,1}^3 \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} - \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1}^3 \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \right) d\gamma_{1,1} \right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[-\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} v\right)}{v} + \frac{3}{v} \int \gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[\begin{array}{c} -\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} v\right)}{v} + \\ \frac{3}{v} \left\{ \begin{array}{c} \gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} - \\ \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1}^2 \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \right) d\gamma_{1,1} \end{array} \right\} \end{array} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[\begin{array}{c} -\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} v\right)}{v} + \\ \frac{3}{v} \left\{ \begin{array}{c} -\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right)}{v} + \\ \frac{2}{v} \int \gamma_{1,1} \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \end{array} \right\} \end{array} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[\begin{array}{c} -\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} v\right)}{v} + \\ \frac{3}{v} \left\{ \begin{array}{c} -\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right)}{v} + \\ \frac{2}{v} \left(\gamma_{1,1} \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} - \int \left(\frac{d}{d\gamma_{1,1}} \gamma_{1,1} \int \exp\left(-\gamma_{1,1} v\right) d\gamma_{1,1} \right) d\gamma_{1,1} \right) \end{array} \right\} \end{array} \right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[\begin{array}{c} -\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} v\right)}{v} + \\ \frac{3}{v} \left\{ \begin{array}{c} -\frac{\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right)}{v} + \\ \frac{2}{v} \left(-\frac{\gamma_{1,1} \exp\left(-\gamma_{1,1} v\right)}{v} - \frac{\exp\left(-\gamma_{1,1} v\right)}{v^2} \right) \end{array} \right\} \end{array} \right]_0^\infty \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^\infty \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[-\frac{\gamma_{1,1}^3 \exp\left(-\gamma_{1,1} v\right)}{v} - \frac{3\gamma_{1,1}^2 \exp\left(-\gamma_{1,1} v\right)}{v^2} - \frac{6\gamma_{1,1} \exp\left(-\gamma_{1,1} v\right)}{v^3} - \frac{6\exp\left(-\gamma_{1,1} v\right)}{v^4} \right]_0^\infty$$

$$\cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty} \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \left[0+\frac{6}{v^4}\right] \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T}$$

$$= \int_0^{\infty} \cdots \frac{4^4}{2\overline{\gamma_{1,1}}^4\Gamma 4} \times \frac{6}{v^4} \cdots \rho\left(\gamma_{M_R,M_T}\right) \cdots d\gamma_{M_R,M_T} \tag{2.93}$$

Now, by observing the equ^n (2.91), (2.92) and (2.93) we get, the generalized form of average BER_{GMSK} with SFBC and Nakagami-m fading is,

$$\overline{BER_{GMSK}} = \frac{m^m}{2\gamma_s^m \Gamma m} \times \Gamma m \times \left[\frac{\alpha\gamma_s + 2mR_C M_T}{2\gamma_s R_C M_T}\right]^{-mM_R M_T}$$

$$= \frac{1}{2} m^m \times \left[\frac{\alpha\gamma_s + 2mR_C M_T}{2\gamma_s R_C M_T}\right]^{-mM_R M_T}$$

$$= \frac{1}{2} m^m \times \left[m + \frac{\alpha\gamma_s}{2R_C M_T}\right]^{-mM_R M_T}$$

$$\text{So, } \overline{BER_{GMSK}} = \frac{1}{2} m^m \times \left[m + \frac{\alpha\gamma_s}{2R_C M_T}\right]^{-mM_R M_T} \tag{2.94}$$

2.2.17 Analytical evaluation of receiver sensitivity

The receiver sensitivity is calculated by the following equation:

$$\text{Rx Sensitivity} = kT_0 + 10\log\left(B_{R_X}\right) + SNR + NF_{R_X} + \text{Implementation Margin} \tag{2.95}$$

Here, k is the Boltzmann constant, T_0 is the reference room temperature, B_{R_X} is the bandwidth of the receiver and finally NF_{R_X} is the noise figure of the receiver.

Considering, $k = 1.38\times10^{-23}$ J/K, $T_0 = 290K$, $B_{R_X} = 20$ MHz we get the following equation to find out the receiver sensitivity with different modulation schemes.

$$\text{Rx Sensitivity} = -174dBm + 73 + SNR + NF_{R_X} + \text{Implementation Margin} \tag{2.96}$$

CHAPTER 3

RESULTS AND DISCUSSIONS

3.1 Simulated Results and Comparisons

The BER performances of MIMO-OFDM with uncoded and SFBC coded MSK, GMSK, MQAM and MPSK modulation schemes are evaluated in this research work. Two fading environments are taken into consideration – Rayleigh and Nakagami-m fading. The considered channels are frequency selective fading channels. The OFDM system has N = 512 subcarriers. 16QAM and 16PSK modulation schemes are used as MQAM and MPSK modulation. 1/2 and 3/4 are used as code rate for the coded performances. The Gaussian constant for GMSK modulation used here is, α = 0.5 and 0.9. The value of used for Nakagami-m fading is m = 0.5, 2, 3 and 4. The considered parameters of the obtained results are given in the following table, Table 3.1.

Table 3.1: Key Parameters for Evaluation of BER Performances MIMO-OFDM with SFBC

Ser No	Parameters	Abbreviation and Value
1.	Modulation Schemes	16QAM, 16PSK, MSK, GMSK
2.	Fading Environment	Rayleigh, Nakagami-m
3.	Number of Subcarriers	N = 512
4.	Number of Transmitters	M_T = 1, 2, 3, 4
5.	Number of Receivers	M_R = 1, 2, 3, 4
6.	Code Rate	R_C = 1/2, 3/4
7.	Average BER	BER = 10^{-6}, 10^{-9}
8.	Gaussian Constant	α = 0.5, 0.9
9.	Nakagami-m Constant	m = 0.5, 2, 3, 4

The BER performances of uncoded MIMO-OFDM systems with 16QAM, 16PSK, MSK and GMSK modulation schemes are shown in the following figures with two different fading environments. The results are compared with each other.

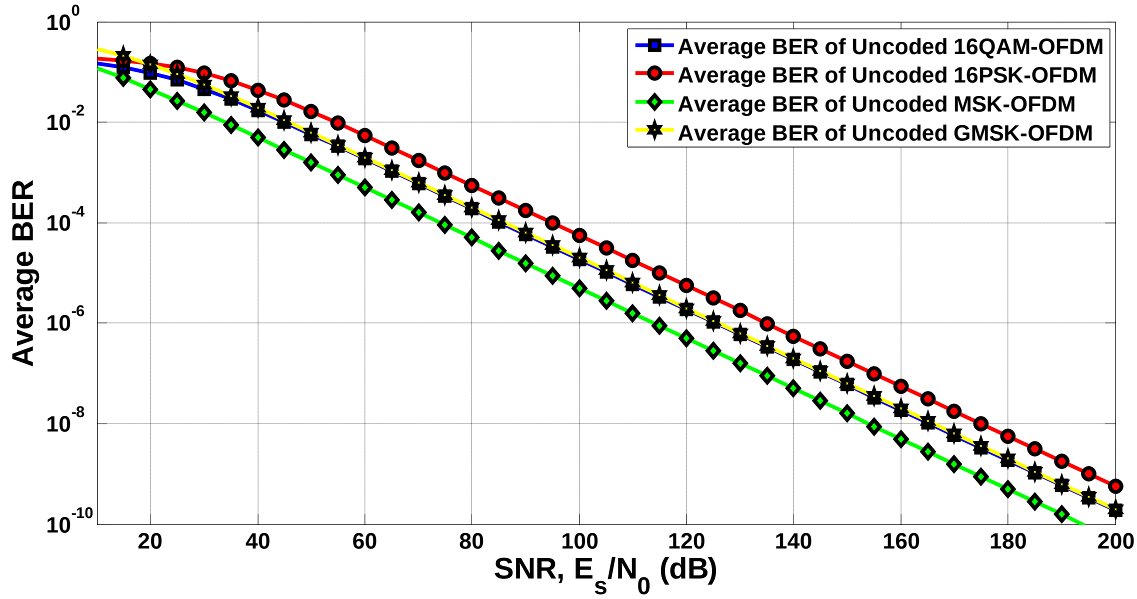

Fig. 3.1: BER performances of uncoded 16QAM, 16PSK, MSK and GMSK with $\alpha = 0.5$ in Rayleigh fading environment

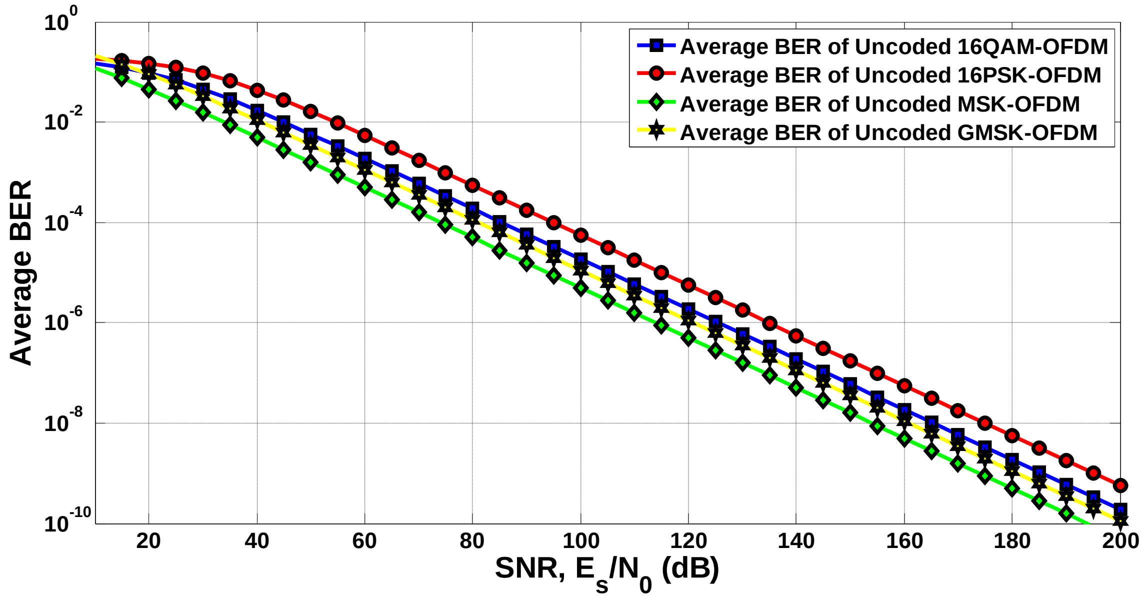

Fig. 3.2: BER performances of uncoded 16QAM, 16PSK, MSK and GMSK with $\alpha = 0.9$ in Rayleigh fading environment

From the above 02 (two) figures it is observed that, when the modulation schemes are used without coding, the average BER of 10^{-6} and 10^{-9} are nearly obtained for the same SNR values. Although the Gaussian constants are different but it does not have any significant affect on the BER performance. It is also observed from the above figures that the best result is obtained from the uncoded MSK OFDM system. 16QAM and GMSK nearly have same results.

The following 02 (two) figures show the performances of the uncoded MIMO-OFDM with the above modulation schemes under Nakagami-m fading environment.

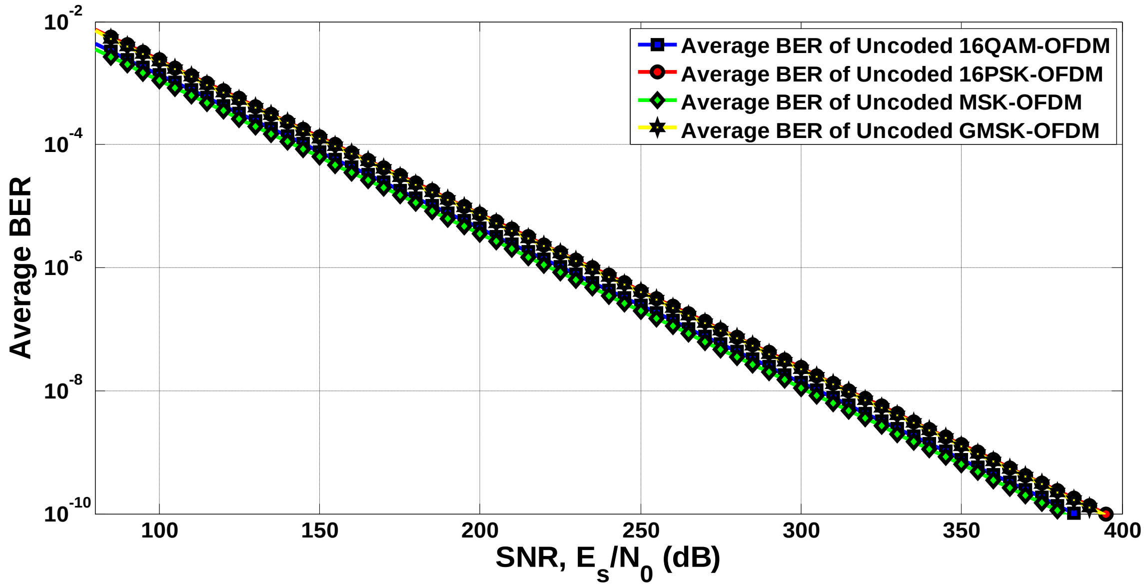

Fig. 3.3: BER performances of uncoded 16QAM, 16PSK, MSK and GMSK with $m = 0.5$ and $\alpha = 0.5$ in Nakagami-m fading environment

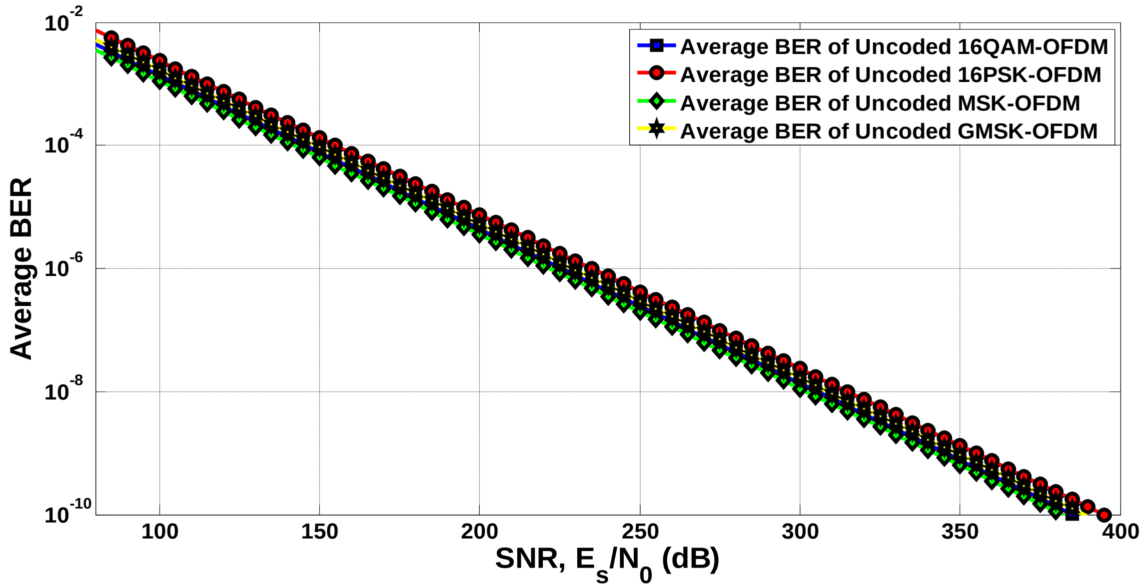

Fig. 3.4: BER performances of uncoded 16QAM, 16PSK, MSK and GMSK with $m = 0.5$ and $\alpha = 0.9$ in Nakagami-m fading environment

From the Fig. 3.3 and Fig. 3.4 it is seen that with the same value of Nakagami constant, m and with 02 different values of the GMSK constant, α, the results are nearly same. Here, the four modulation schemes show nearly same SNR for 10^{-6} and 10^{-9} average BER. Now again with the change of the value of m and the previous values of α, the following results are obtained.

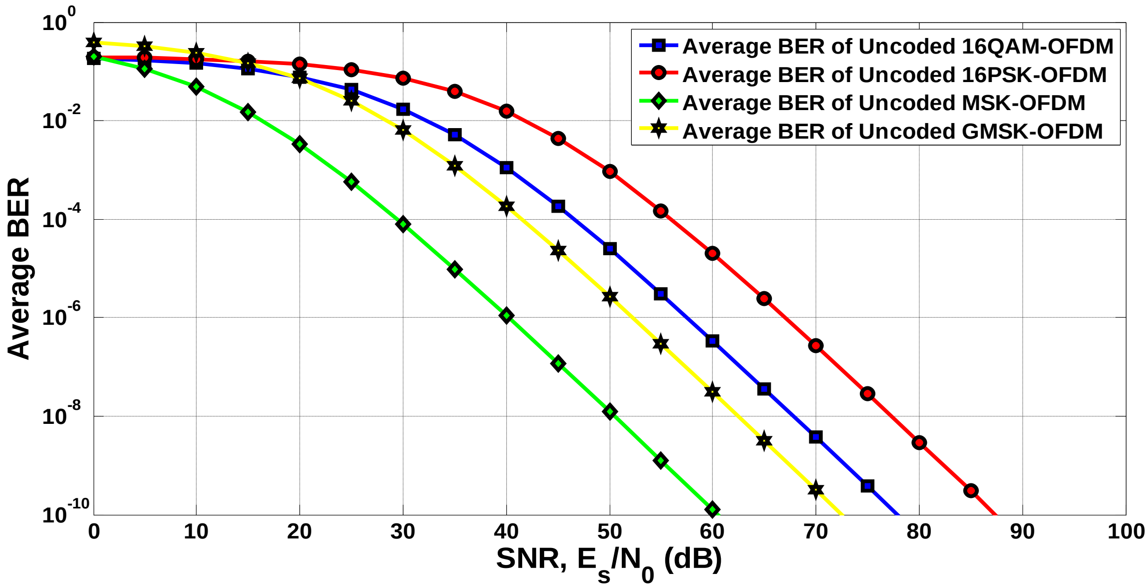

Fig. 3.5: BER performances of uncoded 16QAM, 16PSK, MSK and GMSK with $m = 4$ and $\alpha = 0.5$ in Nakagam-m fading environment

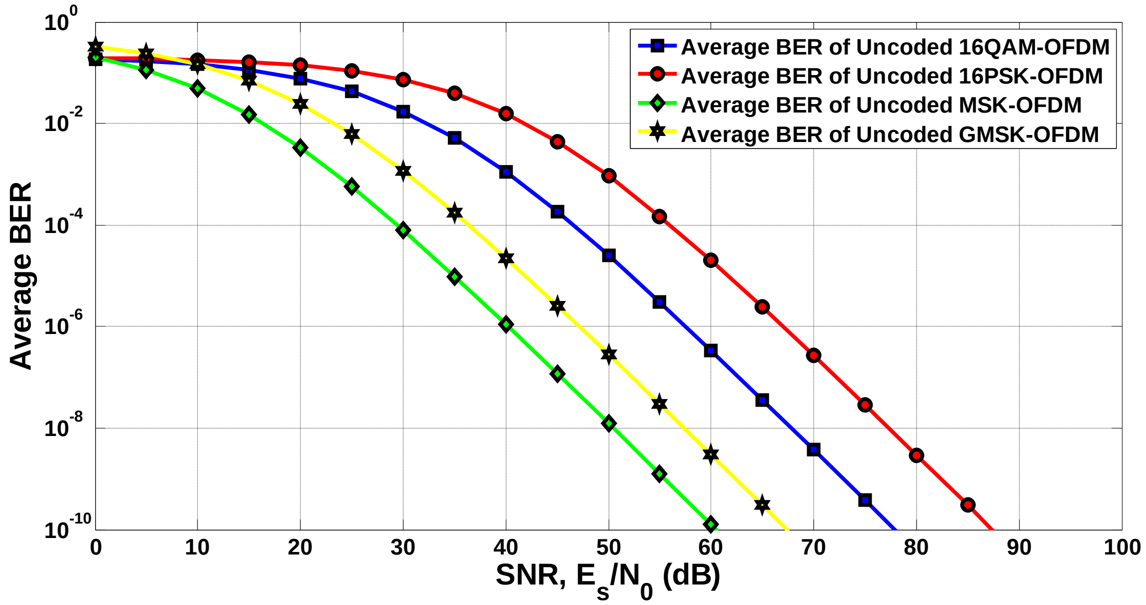

Fig. 3.6: BER performances of uncoded 16QAM, 16PSK, MSK and GMSK with $m = 4$ and $\alpha = 0.9$ in Nakagam-m fading environment

Fig. 3.5 and Fig. 3.6 show significant improvement in the result. With $m = 4$ and $\alpha = 0.5$ or 0.9, it is viewed from the figures that, the 10^{-6} and 10^{-9} average BER are obtained for low values of SNR in both cases. From the above 04 figures it is clear that, by increasing the value of m, we can obtain our expected BER in low SNR. The results obtained from the Nakagami-m faded environment with low m show less efficient result than that of the results achieved from Rayleigh faded environment. But with the increment of the value of m, results are improved.

Now the following table shows the values of SNR which are obtained for the 10^{-6} and 10^{-9} average BER respectively with code rate, R_C = 1/2 and 3/4 in 16QAM-SFBC-OFDM system. Number of transmitters used are, M_T = 2, 3 and 4 and the number of receivers are M_R = 1, 2, 3 and 4. The overall system is in Rayleigh fading environment.

Table 3.2: BER Performances 16QAM-SFBC-OFDM with M_T = 2, 3, 4 and M_R = 1, 2, 3, 4 with Code Rate, R_C = 1/2 and 3/4 in Rayleigh fading environment

Ser No	Code Rate, R_C	#Tx, M_T	$\overline{\text{BER}}$	SNR, $\gamma_S = E_S/N_0$(dB)			
				#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	2	10^{-6}	72.4303	45.5238	35.8923	30.5622
2	1/2	3	10^{-6}	58.1516	39.4141	32.1537	27.8979
3	1/2	4	10^{-6}	51.5444	36.5828	30.3967	26.6319
4	1/2	2	10^{-9}	102.4491	60.8712	46.7427	39.3566
5	1/2	3	10^{-9}	78.2866	50.2645	40.3018	34.8216
6	1/2	4	10^{-9}	66.8918	45.3772	37.3204	32.7032
7	3/4	2	10^{-6}	75.9521	49.0456	39.4141	34.0840
8	3/4	3	10^{-6}	61.6735	42.9359	35.6755	31.4198
9	3/4	4	10^{-6}	55.0662	40.1046	33.9158	30.1537
10	3/4	2	10^{-9}	105.9709	64.3931	50.2645	42.8784
11	3/4	3	10^{-9}	81.8084	53.7864	43.8236	38.3435
12	3/4	4	10^{-9}	70.4137	48.8990	40.8422	36.2250

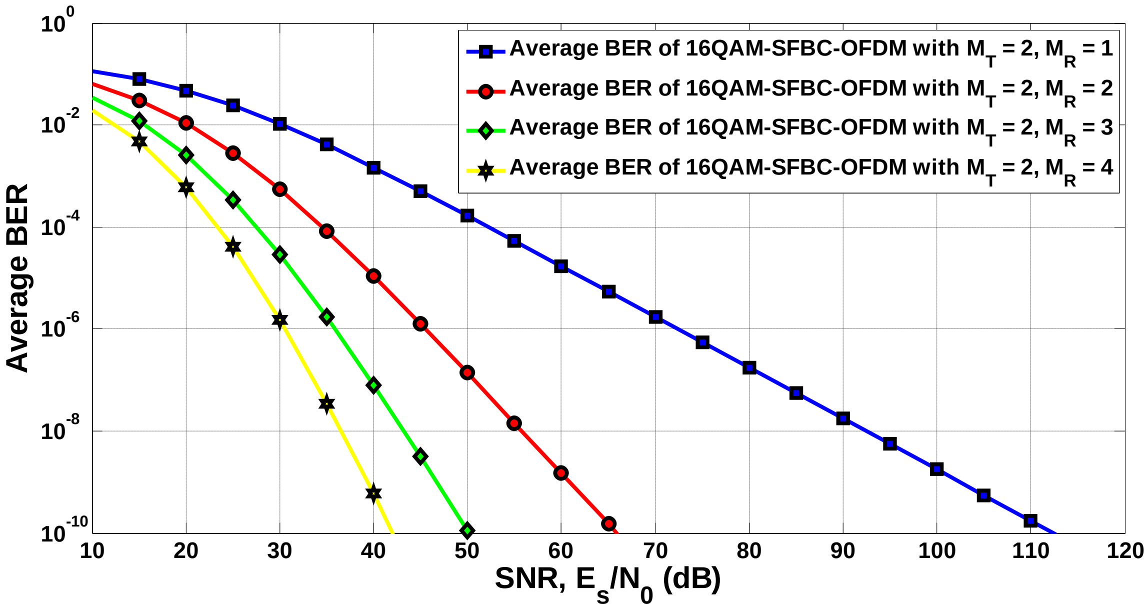

Fig. 3.7: BER performances of 16QAM-SFBC-OFDM with $M_T = 2$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 1/2$ in Rayleigh fading environment

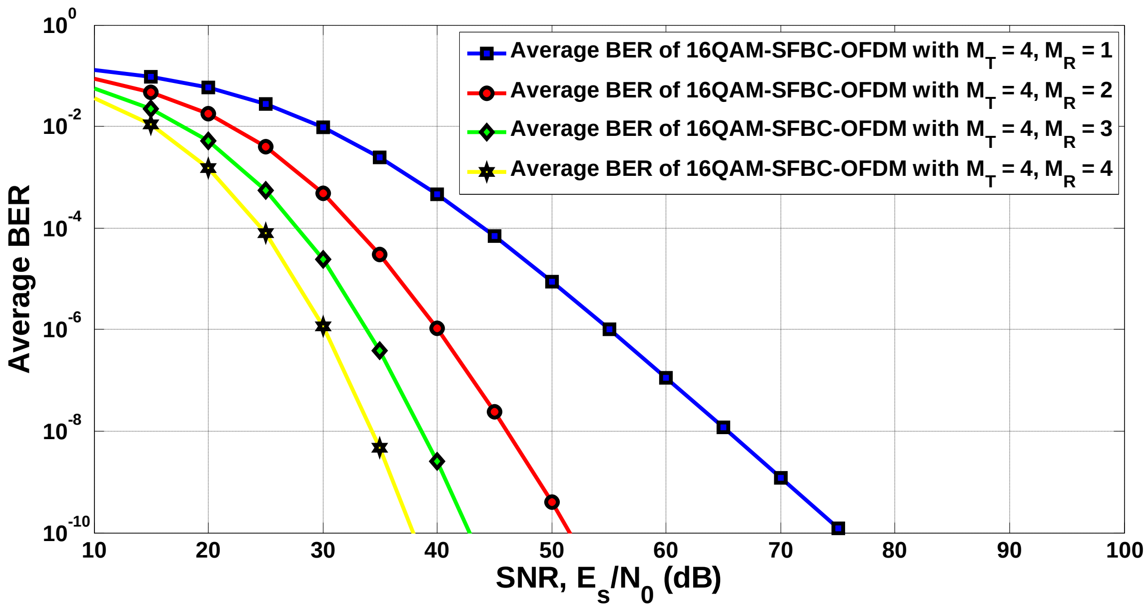

Fig. 3.8: BER performances of 16QAM-SFBC-OFDM with $M_T = 4$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 3/4$ in Rayleigh fading environment

Table 3.2 shows some remarkable changes in the results by using SFBC with 16QAM-OFDM system. We get, same average BER with lower value of SNR if the number of transmitters is increased having same code rate. It is also seen that, with the increment in the number of receivers, the SNR value also decreases for the same number of transmitters and code rate. If the average BER changes, than if we use higher rate, the SNR values will decrease. But if we require better performance with lower BER, the

SNR values will increase and will show the same criterion for number of transmitters, number of receivers and code rate. Also with the change in code rate, if the code rate increases, the SNR values for obtaining same BER, will increase otherwise will decrease. The Fig. 3.7 and Fig. 3.8 show the above observations for 16QAM-SFBC-OFDM system under Rayleigh fading environment.

Similar results are obtained by using 16PSK and MSK-SFBC-OFDM system. The results are shown in Table 3.3 and Table 3.4 respectively with Fig. 3.9 and Fig. 3.10 for 16PSK system and Fig. 3.11 and Fig. 3.12 for MSK system in Rayleigh environment.

Table 3.3: BER Performances 16PSK-SFBC-OFDM with $M_T = 2, 3, 4$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 1/2$ and $3/4$ in Rayleigh fading environment

Ser No	Code Rate, R_C	#Tx, M_T	$\overline{\text{BER}}$	SNR, $\gamma_S = E_S/N_0$(dB)			
				#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	2	10^{-6}	81.8988	54.9923	45.3608	40.0307
2	1/2	3	10^{-6}	67.6201	48.8826	41.6222	37.3665
3	1/2	4	10^{-6}	61.0129	46.0513	39.8652	36.1004
4	1/2	2	10^{-9}	111.9176	70.3397	56.2112	48.8251
5	1/2	3	10^{-9}	87.7551	59.7330	49.7703	44.2901
6	1/2	4	10^{-9}	76.3603	54.8457	46.7889	42.1717
7	3/4	2	10^{-6}	85.4206	58.5142	48.8826	43.5525
8	3/4	3	10^{-6}	71.1420	52.0044	45.1441	40.8883
9	3/4	4	10^{-6}	64.5348	49.5731	43.3871	39.6222
10	3/4	2	10^{-9}	115.4394	73.8616	59.7330	52.3469
11	3/4	3	10^{-9}	91.2769	63.2549	53.2921	47.8120
12	3/4	4	10^{-9}	79.8822	58.3675	50.3107	45.6935

From the Table 3.3 it is clearly seen that, the results follow the same nature shown by the 16QAM system. The key difference among the performances of the two modulation schemes is, 16PSK requires higher SNR to achieve the same average BER than the 16QAM system with all other parameters remaining same.

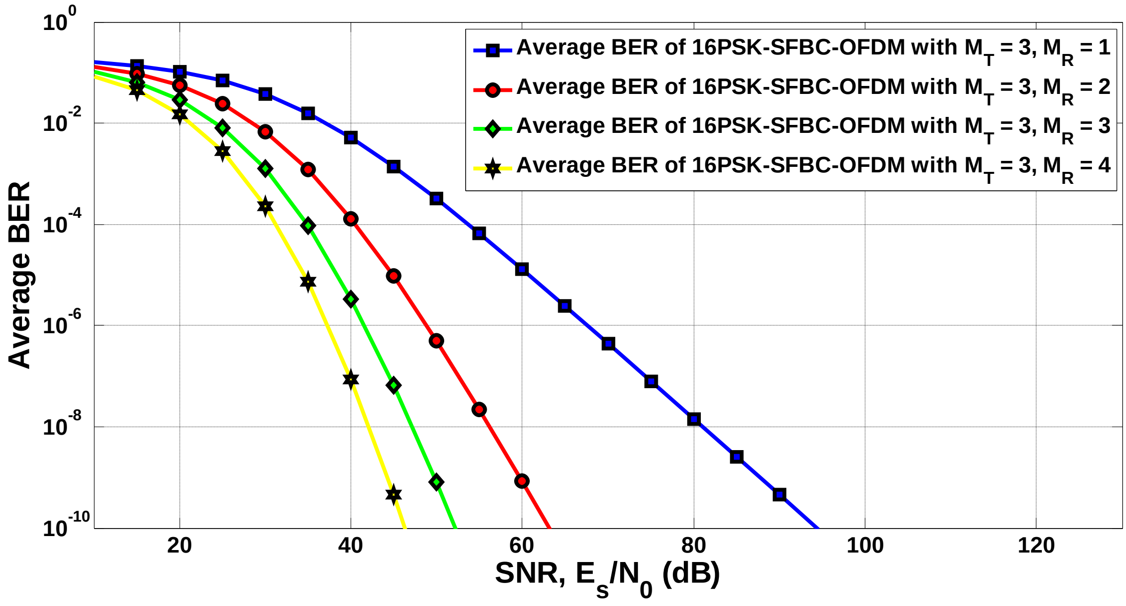

Fig. 3.9: BER performances of 16PSK-SFBC-OFDM with $M_T = 3$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 1/2$ in Rayleigh fading environment

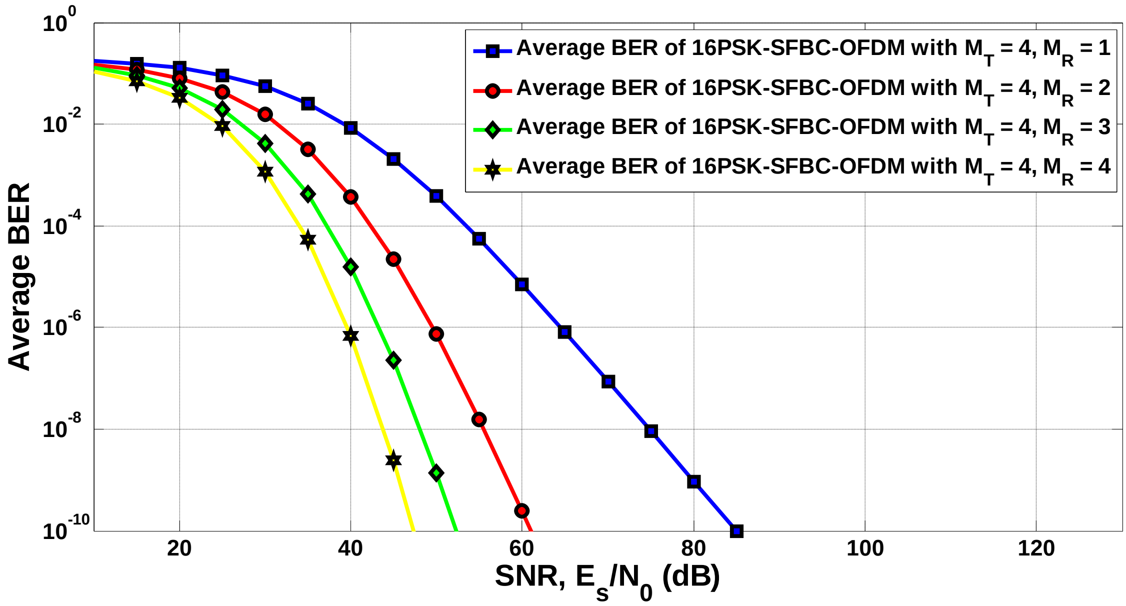

Fig. 3.10: BER performances of 16PSK-SFBC-OFDM with $M_T = 4$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 3/4$ in Rayleigh fading environment

The above two figures show the achieved results obtained by using 16PSK-OFDM system with SFBC instead of 16QAM. The performance of the 16QAM system is comparatively better than 16PSK system if we want to obtain the same BER.

Table 3.4: BER Performances MSK-SFBC-OFDM with $M_T = 2, 3, 4$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 1/2$ and $3/4$ in Rayleigh fading environment

Ser No	Code Rate, R_C	#Tx, M_T	$\overline{\text{BER}}$	SNR, $\gamma_S = E_S/N_0$(dB)			
				#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	2	10^{-6}	56.9774	28.1619	17.9624	12.3750
2	1/2	3	10^{-6}	41.4048	21.4842	13.8856	9.4761
3	1/2	4	10^{-6}	34.1825	18.3956	11.9749	8.1024
4	1/2	2	10^{-9}	86.9893	43.4366	28.6827	21.0064
5	1/2	3	10^{-9}	61.5040	32.2045	21.8601	16.2069
6	1/2	4	10^{-9}	49.4572	27.0270	18.7057	13.9687
7	3/4	2	10^{-6}	60.4992	31.6837	21.4842	15.8968
8	3/4	3	10^{-6}	44.9267	25.0061	17.4074	12.9980
9	3/4	4	10^{-6}	37.7043	21.9174	15.4967	11.6242
10	3/4	2	10^{-9}	90.5111	46.9584	32.2045	24.5282
11	3/4	3	10^{-9}	65.0258	35.7263	25.3819	19.7288
12	3/4	4	10^{-9}	52.9790	30.5488	22.2275	17.4905

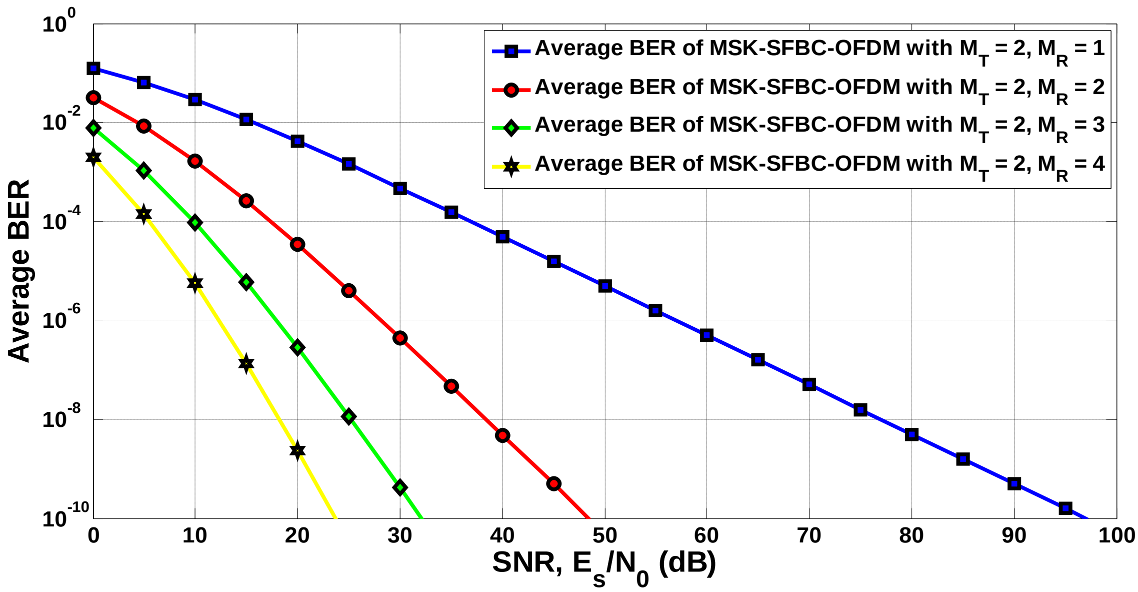

Fig. 3.11: BER performances of MSK-SFBC-OFDM with $M_T = 2$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 1/2$ in Rayleigh fading environment

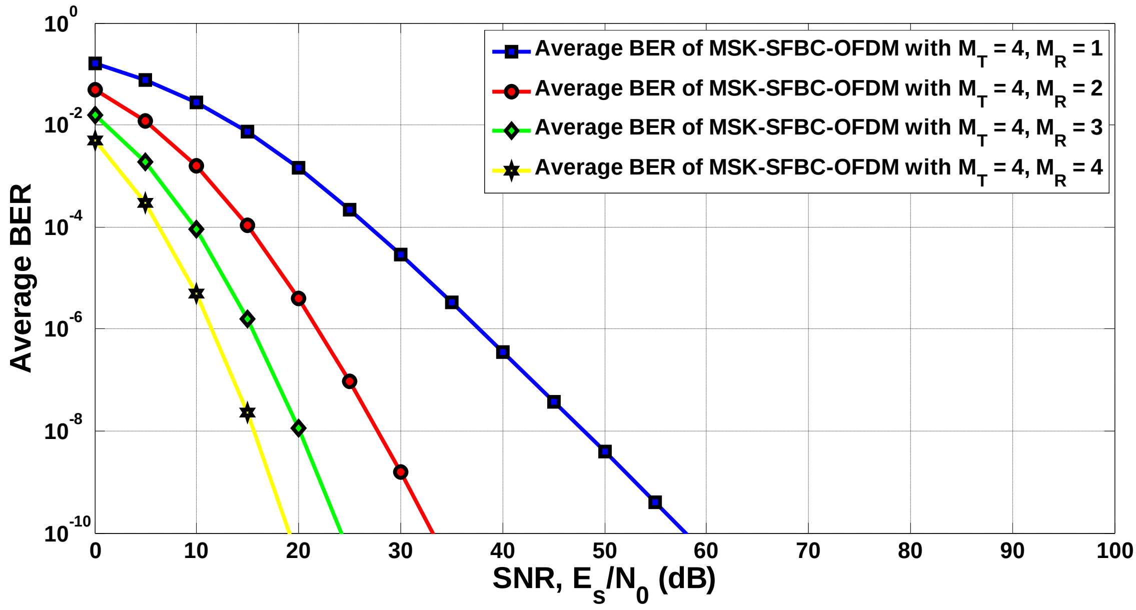

Fig. 3.12: BER performances of MSK-SFBC-OFDM with $M_T = 4$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 3/4$ in Rayleigh fading environment

From the Table 3.4 and Fig. 3.11 and Fig. 3.12 we can see that, the BER performances of MSK-SFBC-OFDM system resemble the same nature of changes with 16QAM and 16PSK in case of variation in number of transmitters, variation in number of receivers, variation in code rate and also variation in the desired average BER. But the ultimate diversity of the performance of MSK with the other two utilized modulation scheme is, it offers low SNR for same level of average BER considering all the same parameters. It has better performance than the other two modulation schemes used here. All the modulations are carried in Rayleigh environment with frequency selective fading channels.

Table 3.5 and Fig. 3.13, Fig. 3.14, Fig. 3.15 and Fig. 3.16 show the results with GMSK modulation scheme. GMSK has two values of α = 0.5 and 0.9. The results change significantly with the change of the values of α with the other previously used parameters. But there are similar changes in the performances with the variations in the parameters used. The performance of GMSK is also better than the 16QAM and 16PSK system, but it requires high SNR than MSK modulation schemes. Results are shown in the following tables and figures.

Table 3.5: BER Performances GMSK-SFBC-OFDM with α = 0.5, 0.9, M_T = 2, 3, 4 and M_R = 1, 2, 3, 4 with Code Rate, R_C = 1/2 and 3/4 in Rayleigh fading environment

Ser No	Code Rate, R_C	Gaussian Constant, α	#Tx, M_T	$\overline{\text{BER}}$	SNR, $\gamma_S = E_S/N_0$(dB)			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	69.0186	40.2031	30.0036	24.4162
2	1/2	0.5	3	10^{-6}	53.4460	33.5254	25.9268	21.5173
3	1/2	0.5	4	10^{-6}	46.2237	30.4368	24.0161	20.1436
4	1/2	0.5	2	10^{-9}	99.0305	55.4778	40.7239	33.0476
5	1/2	0.5	3	10^{-9}	73.5452	44.2457	33.9013	28.2481
6	1/2	0.5	4	10^{-9}	61.4984	39.0682	30.7469	26.0099
7	3/4	0.5	2	10^{-6}	72.5404	43.7249	33.5254	27.9380
8	3/4	0.5	3	10^{-6}	56.7679	37.0473	29.4486	25.0392
9	3/4	0.5	4	10^{-6}	49.7955	33.9586	27.5379	23.6654
10	3/4	0.5	2	10^{-9}	102.552	58.9996	44.2457	36.5694
11	3/4	0.5	3	10^{-9}	77.0670	47.7675	37.4231	31.7700
12	3/4	0.5	4	10^{-9}	65.0202	42.5900	34.2687	29.5317
13	1/2	0.9	2	10^{-6}	63.9132	35.0977	24.8982	19.3107
14	1/2	0.9	3	10^{-6}	48.3406	28.4200	20.8213	16.4119
15	1/2	0.9	4	10^{-6}	41.1183	25.3313	18.9107	15.0381
16	1/2	0.9	2	10^{-9}	93.9251	50.3723	35.6184	27.9421
17	1/2	0.9	3	10^{-9}	68.4398	39.1402	28.7959	23.1427
18	1/2	0.9	4	10^{-9}	56.3929	33.9627	25.6415	20.9044
19	3/4	0.9	2	10^{-6}	67.4350	38.6195	28.4200	22.8325
20	3/4	0.9	3	10^{-6}	51.8624	31.9418	24.3432	19.9337
21	3/4	0.9	4	10^{-6}	44.6401	28.8531	22.4325	18.5600
22	3/4	0.9	2	10^{-9}	97.4469	53.8941	39.1402	31.4640
23	3/4	0.9	3	10^{-9}	71.9616	42.6621	32.3177	26.6645
24	3/4	0.9	4	10^{-9}	59.9147	37.4846	29.1633	24.4263

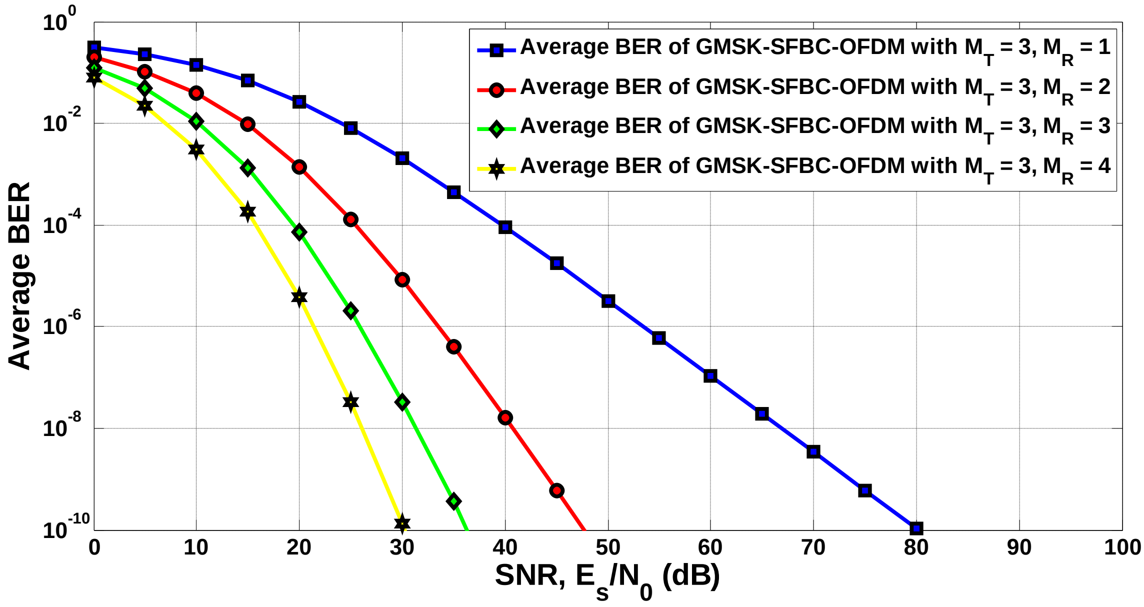

Fig. 3.13: BER performances of GMSK-SFBC-OFDM with $\alpha = 0.5$, $M_T = 3$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 3/4$ in Rayleigh fading environment

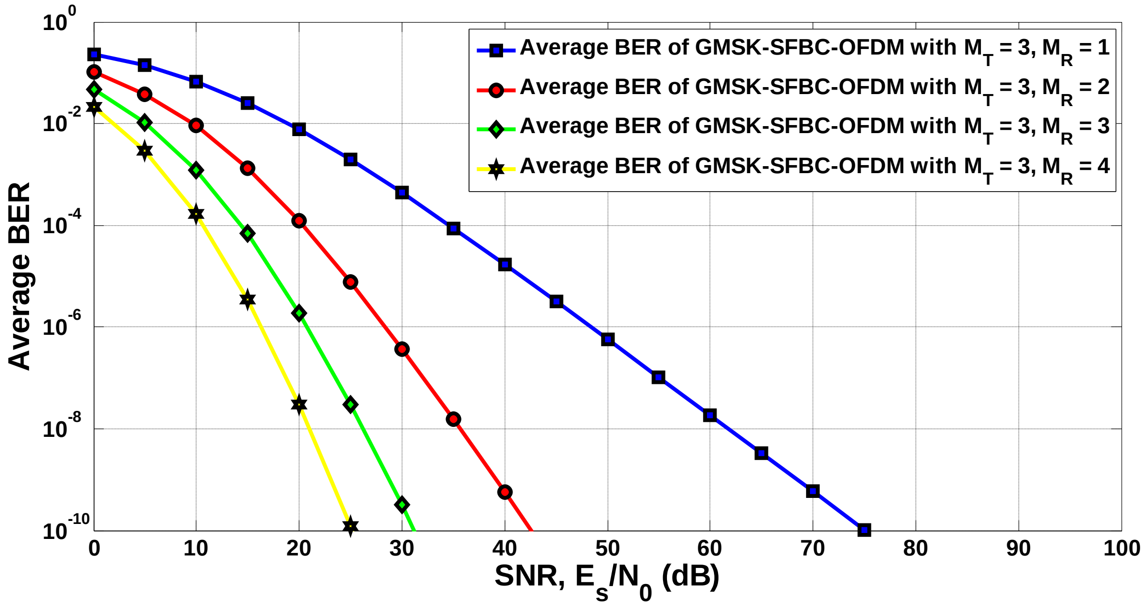

Fig. 3.14: BER performances of GMSK-SFBC-OFDM with $\alpha = 0.9$, $M_T = 3$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 1/2$ in Rayleigh fading environment

The Fig. 3.13 and Fig. 3.14 show that, with the variation of α, the SNR decreases for the same average BER. Here, the code rate remains same also. From the Table 3.5 we see that, if the code rate increases, the SNR will increase for the same average BER. Again the Gaussian constant, α, has the same effect on SNR with the variation of code rate. The following figures show the above results with the code rate, $R_C = 3/4$.

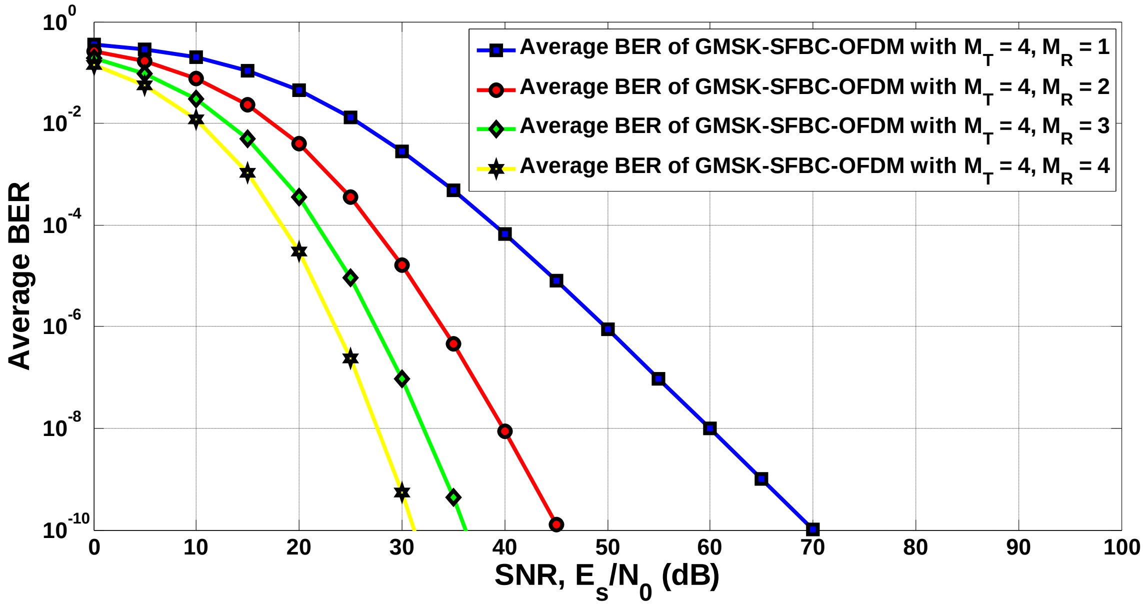

Fig. 3.15: BER performances of GMSK-SFBC-OFDM with $\alpha = 0.5$, $M_T = 4$ and M_R = 1, 2, 3, 4 with Code Rate, $R_C = 3/4$ in Rayleigh fading environment

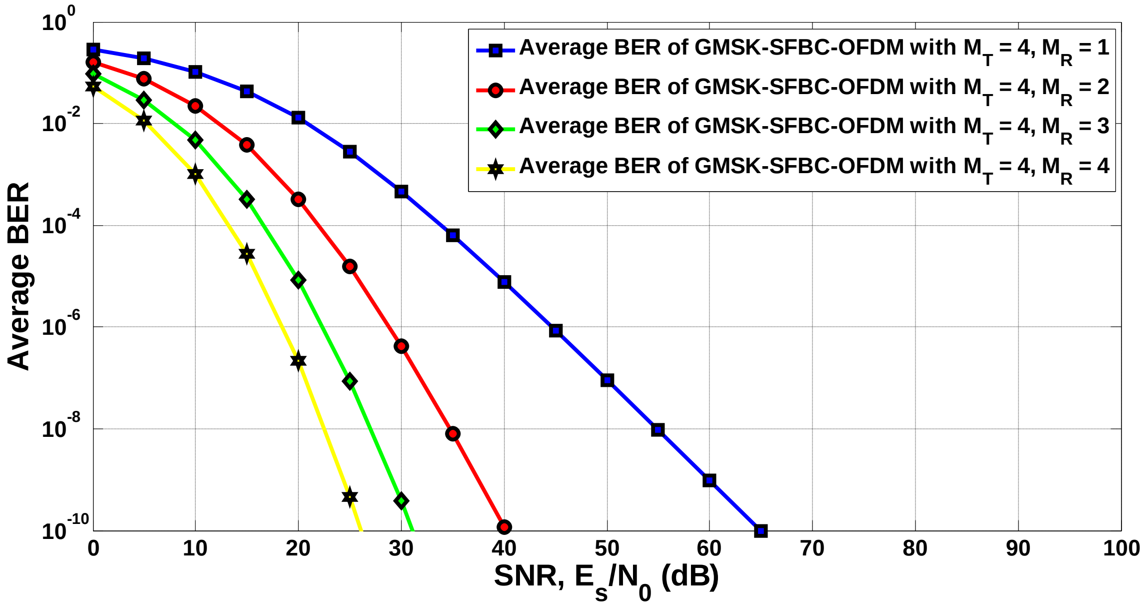

Fig. 3.16: BER performances of GMSK-SFBC-OFDM with $\alpha = 0.9$, $M_T = 4$ and M_R = 1, 2, 3, 4 with Code Rate, $R_C = 1/2$ in Rayleigh fading environment

All the above tables and figures represent the results by using SFBC-OFDM with four different modulation schemes under Rayleigh fading environment. The following figures and tables show the average BER performances of the same modulation schemes with Nakagami-m fading environment. The performances of the different modulation schemes will be nearly same in nature as in the Rayleigh fading environment.

Table 3.6: BER Performances 16QAM-SFBC-OFDM with m = 0.5, 2, 3, M_T = 2, 3, 4 and M_R = 1, 2, 3, 4 with Code Rate, R_C = 1/2, 3/4 in Nakagami-m fading environment

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	SNR, $\gamma_S = E_S/N_0$(dB)			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	122.450	70.9330	53.6924	44.9651
2	1/2	0.5	3	10^{-6}	91.6332	57.2143	45.5354	39.5061
3	1/2	0.5	4	10^{-6}	76.9536	50.9857	42.0049	37.2895
4	1/2	0.5	2	10^{-9}	182.450	100.944	73.7679	60.1521
5	1/2	0.5	3	10^{-9}	131.635	77.2897	59.1184	49.9373
6	1/2	0.5	4	10^{-9}	106.965	66.1727	52.4360	45.4101
7	3/4	0.5	2	10^{-6}	125.972	74.4548	57.2143	48.4869
8	3/4	0.5	3	10^{-6}	95.1550	60.7361	49.0572	43.0279
9	3/4	0.5	4	10^{-6}	80.4754	54.5075	45.5267	40.8113
10	3/4	0.5	2	10^{-9}	185.972	104.466	77.2897	63.6739
11	3/4	0.5	3	10^{-9}	135.157	80.8115	62.6403	53.4591
12	3/4	0.5	4	10^{-9}	110.487	69.6945	55.9579	48.9320
13	1/2	2.0	2	10^{-6}	48.3537	30.2428	20.2989	10.0314
14	1/2	2.0	3	10^{-6}	40.6173	23.8208	5.0981	10.499+ 27.288i
15	1/2	2.0	4	10^{-6}	36.2634	16.0520	12.998+ 27.288i	18.916+ 27.288i
16	1/2	2.0	2	10^{-9}	63.8510	40.2423	30.3698	23.5282
17	1/2	2.0	3	10^{-9}	52.0485	33.8916	23.9713	13.8284
18	1/2	2.0	4	10^{-9}	46.2629	29.5488	16.3272	5.660+ 27.288i
19	3/4	2.0	2	10^{-6}	51.8755	33.7647	23.8208	13.5532
20	3/4	2.0	3	10^{-6}	44.1392	27.3426	8.6199	14.021+ 27.288i
21	3/4	2.0	4	10^{-6}	39.7853	19.5738	16.520+ 27.288i	22.438+ 27.288i

22	3/4	2.0	2	10^{-9}	67.3728	43.7641	33.8916	27.0500
23	3/4	2.0	3	10^{-9}	55.5703	37.4135	27.4931	17.3502
24	3/4	2.0	4	10^{-9}	49.7847	33.0706	19.8490	9.182+ 27.288i
25	1/2	3.0	2	10^{-6}	39.6500	15.5552	15.481+ 27.288i	20.206+ 27.288i
26	1/2	3.0	3	10^{-6}	31.2545	19.003+ 27.288i	24.720+ 27.288i	26.259+ 27.288i
27	1/2	3.0	4	10^{-6}	21.5758	26.227+ 27.288i	28.758+ 27.288i	29.644+ 27.288i
28	1/2	3.0	2	10^{-9}	51.2352	30.2508	12.9343	12.619+ 27.288i
29	1/2	3.0	3	10^{-9}	42.1046	16.4561	19.946+ 27.288i	24.072+ 27.288i
30	1/2	3.0	4	10^{-9}	36.2714	18.639+ 27.288i	26.571+ 27.288i	28.414+ 27.288i
31	3/4	3.0	2	10^{-6}	43.1718	19.0770	19.003+ 27.288i	23.728+ 27.288i
32	3/4	3.0	3	10^{-6}	34.7763	22.524+ 27.288i	28.242+ 27.288i	29.781+ 27.288i
33	3/4	3.0	4	10^{-6}	25.0976	29.749+ 27.288i	32.280+ 27.288i	33.166+ 27.288i
34	3/4	3.0	2	10^{-9}	54.7570	33.7726	16.4561	16.141+ 27.288i
35	3/4	3.0	3	10^{-9}	45.6264	19.9779	23.468+ 27.288i	27.594+ 27.288i
36	3/4	3.0	4	10^{-9}	39.7932	22.161+ 27.288i	30.093+ 27.288i	31.935+ 27.288i

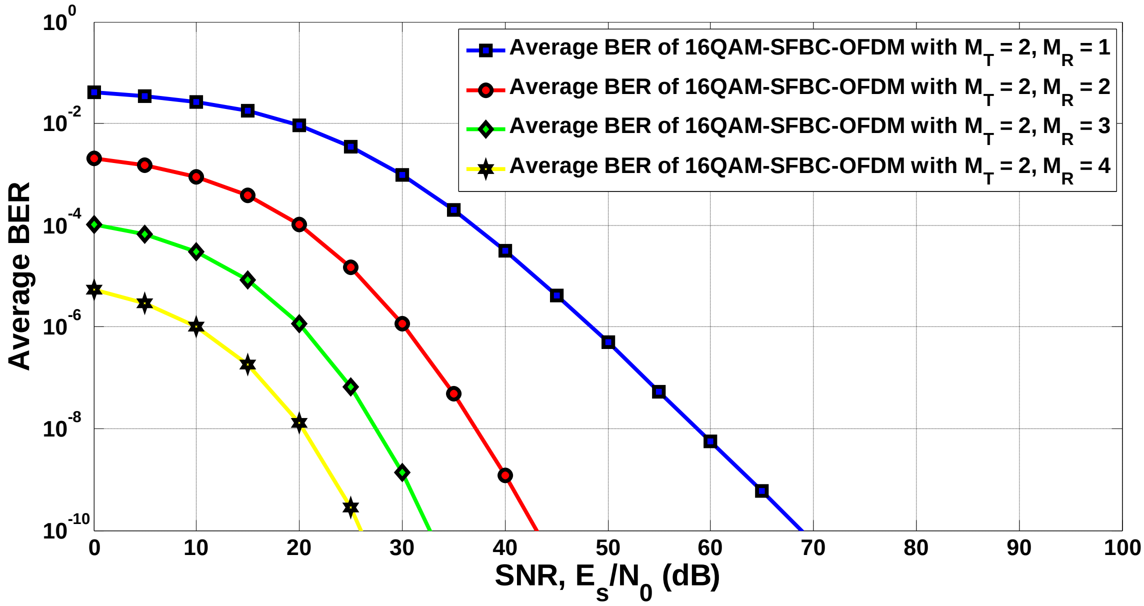

Fig. 3.17: BER performances of 16QAM-SFBC-OFDM, Nakagami constant, m=2, M_T = 2 and M_R = 1, 2, 3, 4 with Code Rate, R_C = 1/2 in Nakagami-m fading environment

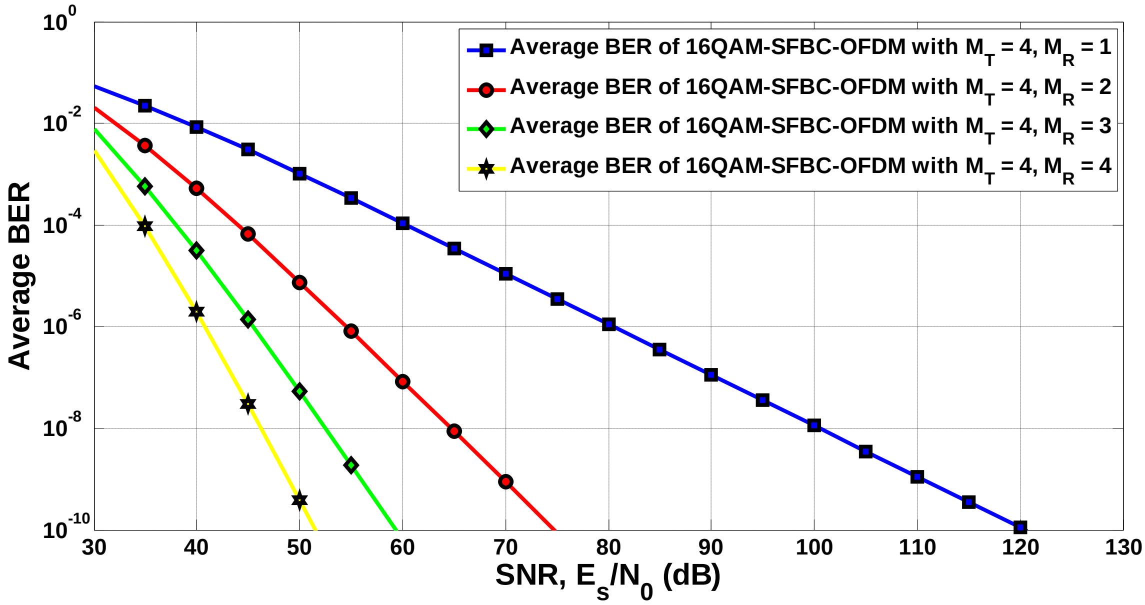

Fig. 3.18: BER performances of 16QAM-SFBC-OFDM, Nakagami constant, $m = 0.5$, $M_T = 4$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 3/4$ in Nakagami-m fading environment

From Table 3.6 it is observed that, keeping all others parameters fixed, with the increment in the number of transmitters, the BER performances improves, as the expected BER is obtained in lower SNR. Also if only the number of receiver increase then again the lower SNR provides the same BER with higher number of receiver. When the code rate changes, the BER is obtained for the higher SNR. The results have same nature of change with higher number of transmitters and receivers with this code rate. The Nakagami-m constant has significant effect on the results. If all the other parameters are fixed, then for the same code rate, when the value of increases, the result changes significantly as the expected value of BER is obtained for much lower SNR with same number of transmitters and receivers. Even when the code rate changes, for the same m value, the BER is obtained at lower SNR. As we know, when $m = 1$, Rayleigh = Nakagami, from the Table 3.2 and Table 3.6 it is clear that, Rayleigh fading influenced results have the values in the middle of $m = 0.5$ and $m = 2$. The values of SNR show that, for $m = 3$, there are complex valued SNRs as the average BER become much smaller when the value of m increases.

Table 3.7: BER Performances 16PSK-SFBC-OFDM with m = 0.5, 2, 3, M_T = 2, 3, 4 and M_R = 1, 2, 3, 4 with Code Rate R_C = 1/2, 3/4 in Nakagami-m fading environment

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	SNR, $\gamma_S = E_S/N_0$(dB)			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	131.918	80.4015	63.1609	54.4336
2	1/2	0.5	3	10^{-6}	101.102	66.6828	55.0039	48.9746
3	1/2	0.5	4	10^{-6}	86.4221	60.4542	51.4734	46.7580
4	1/2	0.5	2	10^{-9}	191.918	110.413	83.2364	69.6206
5	1/2	0.5	3	10^{-9}	141.103	86.7582	68.5869	59.4058
6	1/2	0.5	4	10^{-9}	116.433	75.6412	61.9045	54.8787
7	3/4	0.5	2	10^{-6}	135.440	83.9234	66.6828	57.9554
8	3/4	0.5	3	10^{-6}	104.624	70.2046	58.5267	52.4064
9	3/4	0.5	4	10^{-6}	89.944	63.976	54.9952	50.2798
10	3/4	0.5	2	10^{-9}	195.440	113.935	86.7582	73.1424
11	3/4	0.5	3	10^{-9}	144.625	90.280	72.1088	62.9276
12	3/4	0.5	4	10^{-9}	119.955	79.1630	65.4264	58.4005
13	1/2	2.0	2	10^{-6}	57.8222	39.7113	29.7675	19.4999
14	1/2	2.0	3	10^{-6}	50.0859	33.2893	14.5666	19.968+ 27.288i
15	1/2	2.0	4	10^{-6}	45.7319	25.5205	22.467+ 27.288i	28.385+ 27.288i
16	1/2	2.0	2	10^{-9}	73.3195	49.7108	39.8383	32.9967
17	1/2	2.0	3	10^{-9}	61.5170	43.3601	33.4398	23.2969
18	1/2	2.0	4	10^{-9}	55.7314	39.0173	25.7957	15.128+ 27.288i
19	3/4	2.0	2	10^{-6}	61.3440	43.2332	33.2893	23.0217
20	3/4	2.0	3	10^{-6}	53.6077	36.8111	18.0884	23.490+ 27.288i
21	3/4	2.0	4	10^{-6}	49.2538	29.0423	25.988+ 27.288i	31.907+ 27.288i
22	3/4	2.0	2	10^{-9}	76.8413	53.2326	43.3601	36.5185
23	3/4	2.0	3	10^{-9}	65.0388	46.8820	36.9616	26.8187

24	3/4	2.0	4	10^{-9}	59.2532	42.5391	29.3175	18.650+ 27.288i
25	1/2	3.0	2	10^{-6}	49.1185	25.0237	24.949+ 27.288i	29.675+ 27.288i
26	1/2	3.0	3	10^{-6}	40.7230	28.471+ 27.288i	34.188+ 27.288i	35.728+ 27.288i
27	1/2	3.0	4	10^{-6}	31.0443	35.695+ 27.288i	38.226+ 27.288i	39.112+ 27.288i
28	1/2	3.0	2	10^{-9}	60.7037	39.7193	22.4028	22.087+ 27.288i
29	1/2	3.0	3	10^{-9}	51.5731	25.9246	29.415+ 27.288i	33.541+ 27.288i
30	1/2	3.0	4	10^{-9}	45.7399	28.108+ 27.288i	36.039+ 27.288i	37.882+ 27.288i
31	3/4	3.0	2	10^{-6}	52.6404	28.5456	28.471+ 27.288i	33.196+ 17.288i
32	3/4	3.0	3	10^{-6}	44.2448	31.993+ 27.288i	37.71+ 27.288i	39.249+ 27.288i
33	3/4	3.0	4	10^{-6}	34.5662	39.217+ 27.288i	41.742+ 27.288i	42.634+ 27.288i
34	3/4	3.0	2	10^{-9}	64.2256	43.2411	25.9246	25.609+ 27.288i
35	3/4	3.0	3	10^{-9}	55.0949	29.4464	32.936+ 27.288i	37.063+ 27.288i
36	3/4	3.0	4	10^{-9}	49.2617	31.630+ 27.288i	39.561+ 27.288i	41.404+ 27.288i

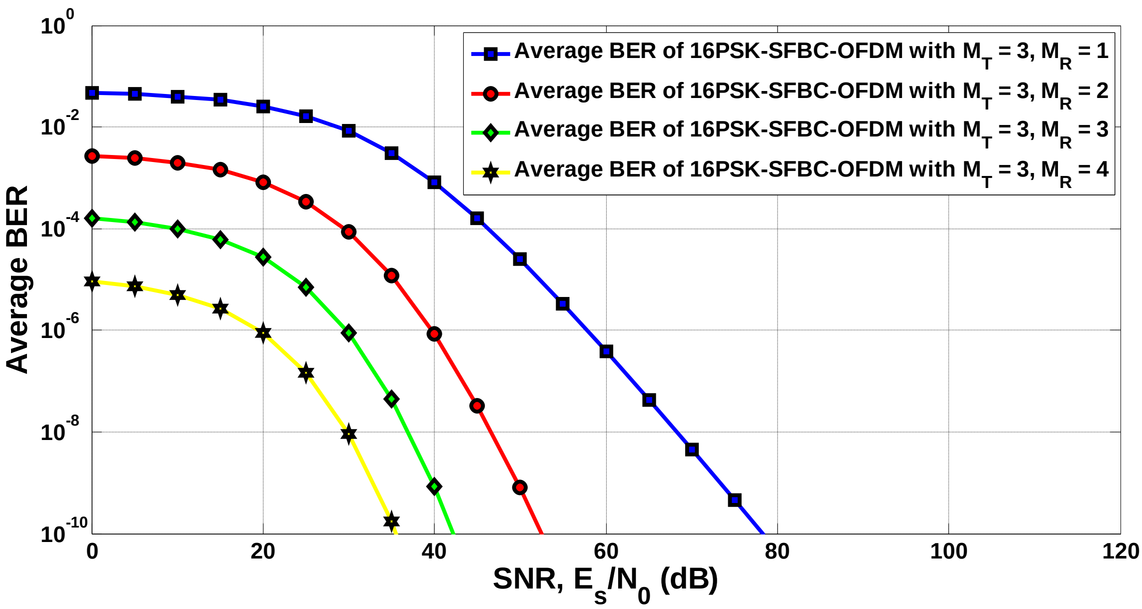

Fig. 3.19: BER performances of 16PSK-SFBC-OFDM, Nakagami constant, m = 2, M_T = 3 and M_R = 1, 2, 3, 4 with Code Rate R_C = 1/2 in Nakagami-m fading environment

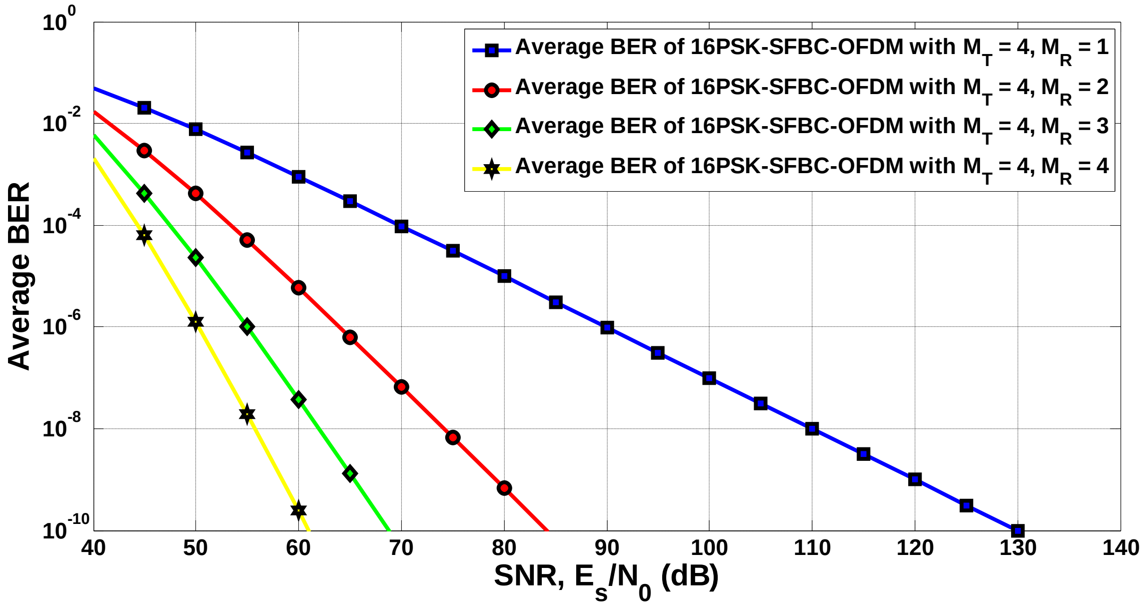

Fig. 3.20: BER performances of 16PSK-SFBC-OFDM, Nakagami constant, $m = 0.5$, $M_T = 4$ and $M_R = 1, 2, 3, 4$ with Code Rate, $R_C = 3/4$ in Nakagami-m fading environment

The Fig. 3.19 and Fig. 3.20 show the results obtained by using 16PSK with SFBC-OFDM in Nakagami-m fading environment. By observing the Table 3.7 it is also clear that, the 16PSK modulation schemes also has same nature of BER performance like 16QAM with the change of code rate, number of transmitters, number of receivers and the Nakagami-m constant, m. Here also the results of Table 3.3 and 3.7 show that, the results found in the Rayleigh fading environment are placed in between the results of Nakagami-m fading environment with $m = 0.5$ and 2.

The following Table 3.8 shows the results found by using MSK-SFBC-OFDM in Nakagami-m environment. Like the previous two modulation schemes, this modulation shows the same criterion in the performance. In all the above three sets of results it is clearly seen that, the BER performance improves with increase in the value of m. The Fig. 3.21 and Fig. 3.22 will also show the above findings. Among the above three modulation schemes, with same code rate, value of m, number of transmitters and number of receivers the MSK modulation scheme has better performance than the 16QAM and 16PSK schemes.

Table 3.8: BER Performances MSK-SFBC-OFDM with m = 0.5, 2, 3, M_T = 2, 3, 4 and M_R = 1, 2, 3, 4 with Code Rate R_C = 1/2, 3/4 in Nakagami-m fading environment

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	SNR, $\gamma_S = E_S/N_0$(dB)			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	110.969	55.4772	36.9281	27.5623
2	1/2	0.5	3	10^{-6}	77.5004	40.4499	27.9239	21.4842
3	1/2	0.5	4	10^{-6}	61.4978	33.5829	23.9830	18.9645
4	1/2	0.5	2	10^{-9}	170.969	85.4843	56.9836	42.7105
5	1/2	0.5	3	10^{-9}	117.501	60.5054	41.4601	31.8518
6	1/2	0.5	4	10^{-9}	91.5049	48.7311	34.3506	27.0127
7	3/4	0.5	2	10^{-6}	114.491	58.9991	40.4499	31.0841
8	3/4	0.5	3	10^{-6}	81.0222	43.9717	31.4457	25.0060
9	3/4	0.5	4	10^{-6}	65.0197	37.1047	27.5048	22.4863
10	3/4	0.5	2	10^{-9}	174.491	89.0061	60.5054	46.2324
11	3/4	0.5	3	10^{-9}	121.023	64.0272	44.9819	35.3736
12	3/4	0.5	4	10^{-9}	95.0267	52.2530	37.8724	30.5345
13	1/2	2.0	2	10^{-6}	31.0305	12.3240	2.6087	-6.4412
14	1/2	2.0	3	10^{-6}	22.8208	6.1305	-8.9096	-11.89+ 27.288i
15	1/2	2.0	4	10^{-6}	18.3446	-0.4206	-9.39+ 27.288i	-1.385+ 27.288i
16	1/2	2.0	2	10^{-9}	46.4226	21.9674	11.9491	5.1700
17	1/2	2.0	3	10^{-9}	34.0215	15.4709	5.7095	-3.5922
18	1/2	2.0	4	10^{-9}	27.9880	11.1906	-1.0934	-20.50+ 27.288i
19	3/4	2.0	2	10^{-6}	34.5523	15.8458	6.1305	-2.9194
20	3/4	2.0	3	10^{-6}	26.3426	9.6524	-5.3878	-8.367+ 27.288i
21	3/4	2.0	4	10^{-6}	21.8664	3.1012	-5.868+ 27.288i	2.1378+ 27.288i
22	3/4	2.0	2	10^{-9}	49.9444	25.4893	15.4709	8.6918
23	3/4	2.0	3	10^{-9}	37.5433	18.9927	9.2313	-0.0703

24	3/4	2.0	4	10^{-9}	31.5099	14.7124	2.4284	-16.98+ 27.288i
25	1/2	3.0	2	10^{-6}	21.8900	-0.6471	-5.842+ 27.288i	0.1552+ 27.288i
26	1/2	3.0	3	10^{-6}	13.6197	-2.32+ 27.288i	4.8341+ 27.288i	6.5808+ 27.288i
27	1/2	3.0	4	10^{-6}	5.3735	6.1758+ 27.288i	9.0795+ 27.288i	10.060+ 27.288i
28	1/2	3.0	2	10^{-9}	33.2167	12.0096	-3.6853	-8.947+ 27.288i
29	1/2	3.0	3	10^{-9}	23.8235	-0.1635	-0.525+ 27.288i	4.2576+ 27.288i
30	1/2	3.0	4	10^{-9}	18.0302	-2.926+ 27.288i	6.7563+ 27.288i	8.7820+ 27.288i
31	3/4	3.0	2	10^{-6}	25.4118	2.8747	-2.32+ 27.288i	3.677+ 27.288i
32	3/4	3.0	3	10^{-6}	17.1415	1.2019+ 27.288i	8.356+ 27.288i	10.103+ 27.288i
33	3/4	3.0	4	10^{-6}	8.8953	9.6976+ 27.288i	12.601+ 27.288i	13.582+ 27.288i
34	3/4	3.0	2	10^{-9}	36.7385	15.5314	-0.1635	-5.425+ 27.288i
35	3/4	3.0	3	10^{-9}	27.3453	3.3583	2.9966+ 27.288i	7.7794+ 27.288i
36	3/4	3.0	4	10^{-9}	21.552	0.5958+ 27.288i	10.278+ 27.288i	12.304+ 27.288i

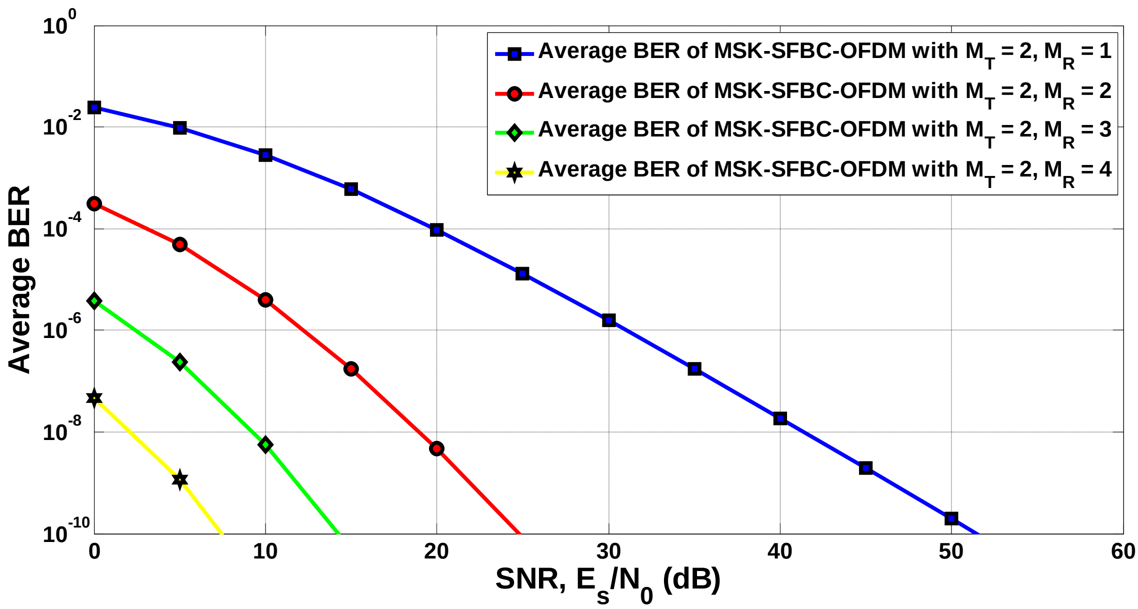

Fig. 3.21: BER performances of MSK-SFBC-OFDM, Nakagami constant, m = 2, M_T = 2 and M_R = 1, 2, 3, 4 with Code Rate, R_C = 1/2 in Nakagami-m fading environment

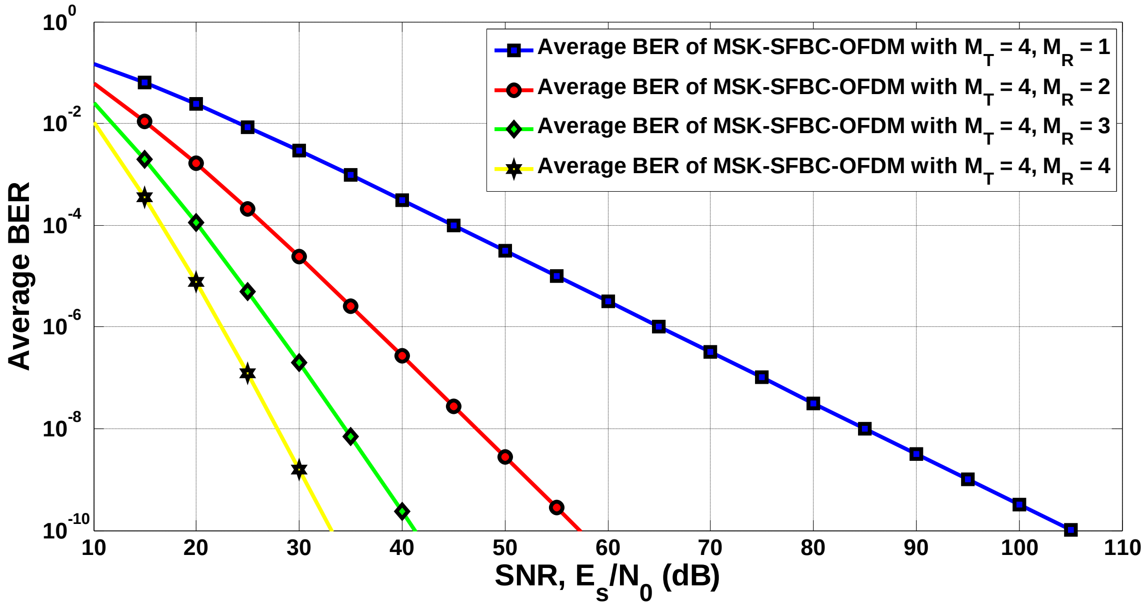

Fig. 3.22: BER performances of MSK-SFBC-OFDM, Nakagami constant, $m = 0.5$, $M_T = 4$ and $M_R = 1, 2, 3, 4$ with Code Rate $R_C = 3/4$ in Nakagami-m fading environment

Table 3.9: BER Performances GMSK-SFBC-OFDM with, Gaussian Constant, $\alpha = 0.5$, $m = 0.5, 2, 3$, $M_T = 2, 3, 4$ and $M_R = 1, 2, 3, 4$ with Code Rate $R_C = 1/2, 3/4$ in Nakagami-m fading environment

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	SNR, $\gamma_S = E_S/N_0$(dB)			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	123.010	67.5184	48.9693	39.6035
2	1/2	0.5	3	10^{-6}	89.5416	52.4911	39.9651	33.5254
3	1/2	0.5	4	10^{-6}	73.5390	45.6241	36.0242	31.0057
4	1/2	0.5	2	10^{-9}	183.010	97.5255	69.0248	54.7517
5	1/2	0.5	3	10^{-9}	129.542	72.5466	53.5013	43.8930
6	1/2	0.5	4	10^{-9}	103.546	60.7723	46.3918	39.0539
7	3/4	0.5	2	10^{-6}	126.532	71.0403	52.4911	43.1253
8	3/4	0.5	3	10^{-6}	93.0634	56.0129	43.4869	37.0472
9	3/4	0.5	4	10^{-6}	77.0609	49.1459	39.5460	34.5275
10	3/4	0.5	2	10^{-9}	186.532	101.047	72.5466	58.2736
11	3/4	0.5	3	10^{-9}	133.064	76.0684	57.0231	47.4148
12	3/4	0.5	4	10^{-9}	107.068	64.2942	49.9136	42.5757

13	1/2	2.0	2	10^{-6}	43.0717	24.3652	14.6499	5.6000
14	1/2	2.0	3	10^{-6}	34.8620	18.1717	3.1316	0.1522+ 27.288i
15	1/2	2.0	4	10^{-6}	30.3858	11.6206	2.6510+ 27.288i	10.657+ 27.288i
16	1/2	2.0	2	10^{-9}	58.4638	34.0086	23.9903	17.2112
17	1/2	2.0	3	10^{-9}	46.0627	27.5121	17.7507	8.4490
18	1/2	2.0	4	10^{-9}	40.0292	23.2318	10.9478	-8.459+ 27.288i
19	3/4	2.0	2	10^{-6}	46.5935	27.8870	18.1717	9.1218
20	3/4	2.0	3	10^{-6}	38.3838	21.6936	6.6534	3.6740+ 27.288i
21	3/4	2.0	4	10^{-6}	33.9076	15.1424	6.1728+ 27.288i	14.179+ 27.288i
22	3/4	2.0	2	10^{-9}	61.9856	37.5305	27.5121	20.7330
23	3/4	2.0	3	10^{-9}	49.5845	31.0339	21.2725	11.9709
24	3/4	2.0	4	10^{-9}	43.5511	26.7536	14.4696	-4.937+ 27.288i
25	1/2	3.0	2	10^{-6}	33.9312	11.3941	6.1994+ 27.288i	12.196+ 27.288i
26	1/2	3.0	3	10^{-6}	25.6609	9.7212+ 27.288i	16.875+ 27.288i	18.622+ 27.288i
27	1/2	3.0	4	10^{-6}	17.4147	18.217+ 27.288i	21.121+ 27.288i	22.102+ 27.288i
28	1/2	3.0	2	10^{-9}	45.2579	24.0508	8.3559	3.0946+ 27.288i
29	1/2	3.0	3	10^{-9}	35.8647	11.8777	11.516+ 27.288i	16.299+ 27.288i
30	1/2	3.0	4	10^{-9}	30.0714	9.1152+ 27.288i	18.798+ 27.288i	20.823+ 27.288i
31	3/4	3.0	2	10^{-6}	37.453	14.9159	9.7212+ 27.288i	15.718+ 27.288i
32	3/4	3.0	3	10^{-6}	29.1827	13.243+ 27.288i	20.397+ 27.288i	22.144+ 27.288i
33	3/4	3.0	4	10^{-6}	20.937	21.739+ 27.288i	24.643+ 27.288i	25.623+ 27.288i
34	3/4	3.0	2	10^{-9}	48.7797	27.5726	11.8777	6.6164+ 27.288i
35	3/4	3.0	3	10^{-9}	39.3865	15.3995	15.038+ 27.288i	19.821+ 27.288i
36	3/4	3.0	4	10^{-9}	33.5932	12.637+ 27.288i	22.319+ 27.288i	24.345+ 27.288i

The above Table 3.9 shows the BER performance of GMSK-SFBC-OFDM in Nakagami-m fading environment. Here, the results are shown with the value of Gaussian constant, $\alpha = 0.5$. The Fig. 3.23 and Fig. 3.24 will show the findings below.

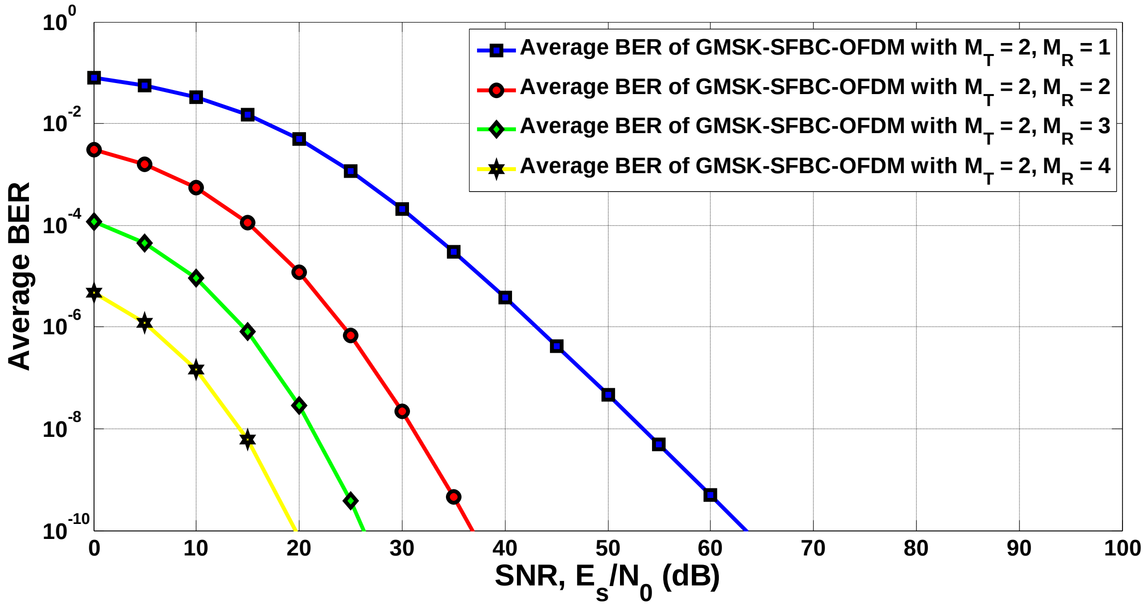

Fig. 3.23: BER performances of GMSK-SFBC-OFDM, Gaussian constant, $\alpha = 0.5$, Nakagami constant, $m = 2$, $M_T = 2$ and $M_R = 1, 2, 3, 4$ with Code Rate $R_C = 1/2$ in Nakagami-m fading environment

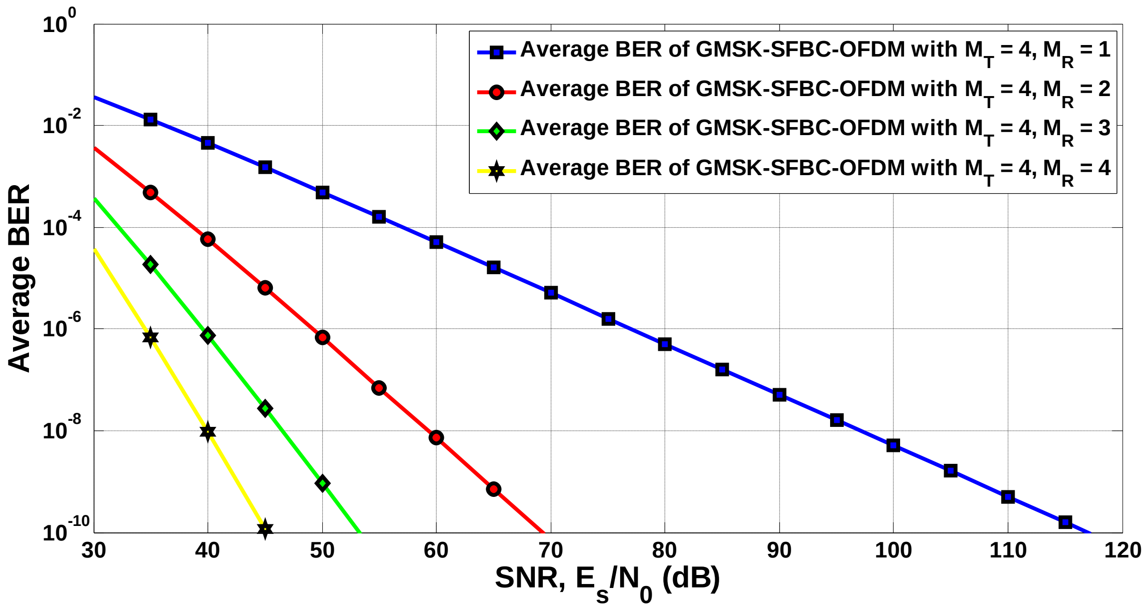

Fig. 3.24: BER performances of GMSK-SFBC-OFDM, Gaussian constant, $\alpha = 0.5$, Nakagami constant, $m = 0.5$, $M_T = 4$ and $M_R = 1, 2, 3, 4$ with Code Rate $R_C = 3/4$ in Nakagami-m fading environment

The Fig. 3.23 and Fig. 3.24 show that with the same Gaussian constant, the GMSK has same effects on the BER performance like the other three modulation schemes. Same kinds of results are obtained for the increment of the value of Nakagami constant, m.

Table 3.10: BER Performances GMSK-SFBC-OFDM with, Gaussian Constant, α = 0.9, m = 0.5, 2, 3, M_T = 2, 3, 4 and M_R = 1, 2, 3, 4 with Code Rate R_C = 1/2, 3/4 in Nakagami-m fading environment

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	SNR, $\gamma_S = E_S/N_0$(dB)			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	117.905	62.4130	43.8638	34.4931
2	1/2	0.5	3	10^{-6}	84.4361	47.3856	34.8596	28.4200
3	1/2	0.5	4	10^{-6}	68.4336	40.5187	30.9187	25.9002
4	1/2	0.5	2	10^{-9}	177.905	92.4201	63.9193	49.6463
5	1/2	0.5	3	10^{-9}	124.437	67.4411	48.3958	38.7875
6	1/2	0.5	4	10^{-9}	98.4407	55.6669	41.2863	33.9485
7	3/4	0.5	2	10^{-6}	121.427	65.9348	47.3856	38.0199
8	3/4	0.5	3	10^{-6}	87.9579	50.9075	38.3814	31.9418
9	3/4	0.5	4	10^{-6}	71.9554	44.0405	34.4406	29.4220
10	3/4	0.5	2	10^{-9}	181.427	95.9419	67.4411	53.1681
11	3/4	0.5	3	10^{-9}	127.959	70.9630	51.9176	42.3094
12	3/4	0.5	4	10^{-9}	101.963	59.1887	44.8081	37.4703
13	1/2	2.0	2	10^{-6}	37.9662	19.2597	9.5445	0.4945
14	1/2	2.0	3	10^{-6}	29.7565	13.0663	1.9739	-4.953+ 27.288i
15	1/2	2.0	4	10^{-6}	25.2803	6.5151	-2.455+ 27.288i	5.5517+ 27.288i
16	1/2	2.0	2	10^{-9}	53.3548	28.9032	18.8848	12.1057
17	1/2	2.0	3	10^{-9}	40.9573	22.4066	12.6453	3.3436
18	1/2	2.0	4	10^{-9}	34.9238	18.1263	5.8424	-13.56+ 27.288i
19	3/4	2.0	2	10^{-6}	41.4880	22.7816	13.0663	4.0163

20	3/4	2.0	3	10^{-6}	33.2784	16.5881	1.5479	-1.431+ 27.288i
21	3/4	2.0	4	10^{-6}	28.8022	10.0369	1.0674+ 27.288i	9.0736+ 27.288i
22	3/4	2.0	2	10^{-9}	56.8802	32.4250	22.4066	15.6275
23	3/4	2.0	3	10^{-9}	44.4791	25.9285	16.1671	6.8654
24	3/4	2.0	4	10^{-9}	38.4456	21.6481	9.3642	-10.04+ 27.288i
25	1/2	3.0	2	10^{-6}	28.8257	6.2887	1.094+ 27.288i	7.091+ 27.288i
26	1/2	3.0	3	10^{-6}	20.5554	4.6158+ 27.288i	11.77+ 27.288i	13.517+ 27.288i
27	1/2	3.0	4	10^{-6}	12.3093	13.112+ 27.288i	16.015+ 27.288i	16.996+ 27.288i
28	1/2	3.0	2	10^{-9}	40.1524	18.9453	3.2504	-2.011+ 27.288i
29	1/2	3.0	3	10^{-9}	30.7593	6.7722	6.4105+ 27.288i	11.193+ 27.288i
30	1/2	3.0	4	10^{-9}	24.9659	4.0097+ 27.288i	13.692+ 27.288i	15.718+ 27.288i
31	3/4	3.0	2	10^{-6}	32.3476	9.8105	4.6158+ 27.288i	10.613+ 27.288i
32	3/4	3.0	3	10^{-6}	24.0773	8.1376+ 27.288i	15.292+ 27.288i	17.038+ 27.288i
33	3/4	3.0	4	10^{-6}	15.8311	16.633+ 27.288i	19.537+ 27.288i	20.518+ 27.288i
34	3/4	3.0	2	10^{-9}	43.6742	22.4671	6.7722	1.5109+ 27.288i
35	3/4	3.0	3	10^{-9}	34.2811	10.2941	9.9323+ 27.288i	14.715+ 27.288i
36	3/4	3.0	4	10^{-9}	28.4877	7.5315+ 27.288i	17.214+ 27.288i	19.24+ 27.288i

The above table show the results obtained for the Gaussian constant, $\alpha = 0.9$. With the comparison of the results obtained by the Gaussian constant, $\alpha = 0.5$, it is observed that, the increased value of the constant improves the performance for the same other parameters. Now by observing all the above sets of results for Nakagami-m fading it is seen that, the MSK modulation scheme has the best performance among the four modulation schemes. With the comparison of the results, GMSK with higher value of the Gaussian constant, has the next better result and 16QAM and finally 16PSK has the performances of results.

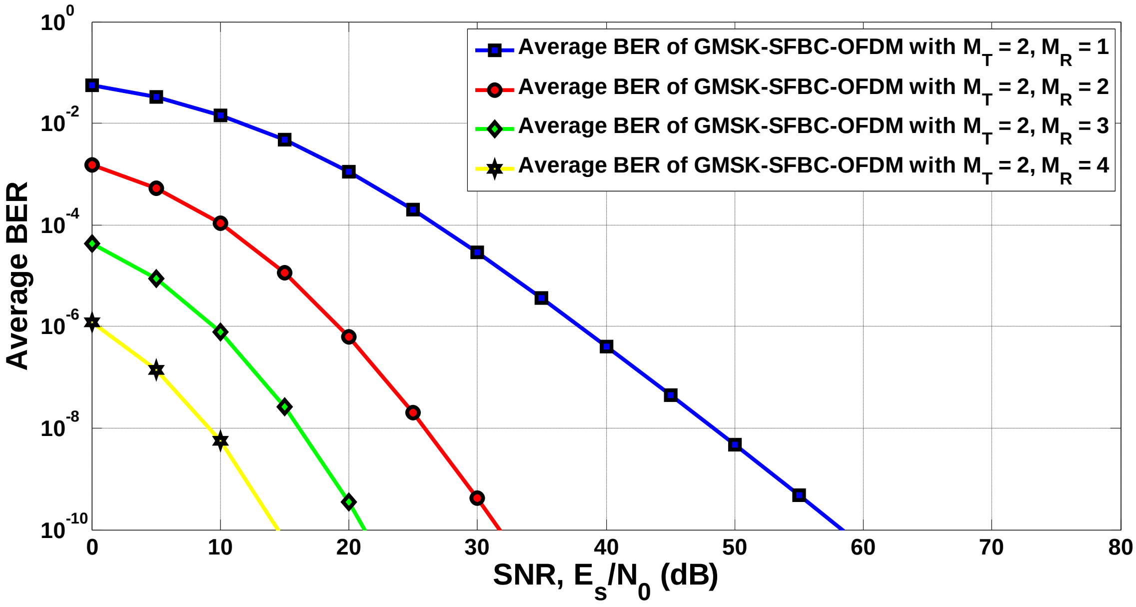

Fig. 3.25: BER performances of GMSK-SFBC-OFDM, Gaussian constant, $\alpha = 0.9$, Nakagami constant, $m = 2$, $M_T = 2$ and $M_R = 1, 2, 3, 4$ with Code Rate $R_C = 1/2$ in Nakagami-m fading environment

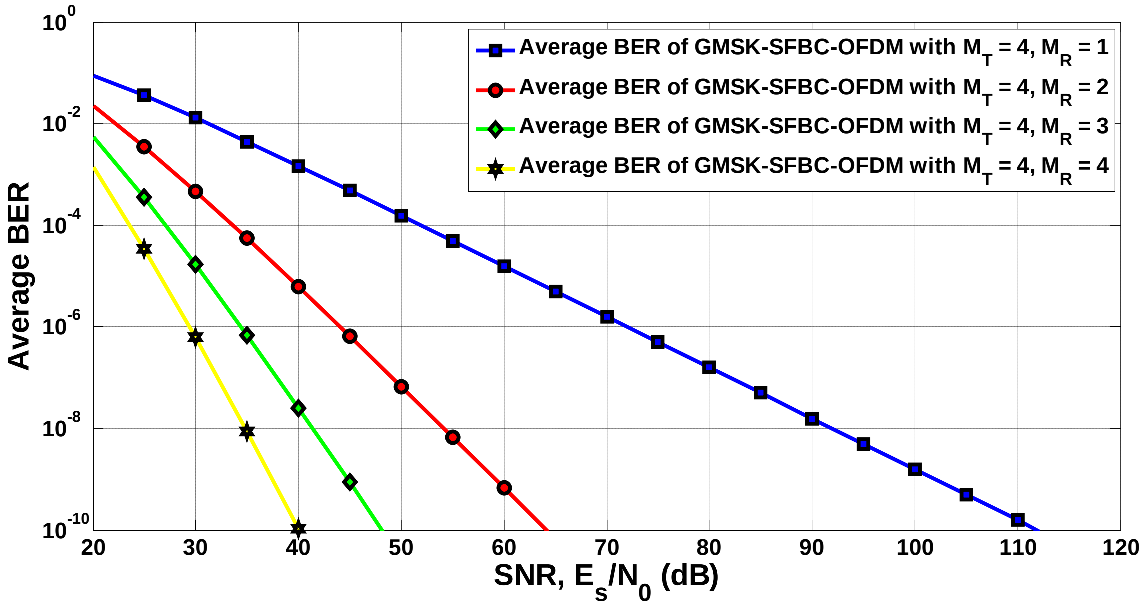

Fig. 3.26: BER performances of GMSK-SFBC-OFDM, Gaussian constant, $\alpha = 0.9$, Nakagami constant, $m = 0.5$, $M_T = 4$ and $M_R = 1, 2, 3, 4$ with Code Rate $R_C = 3/4$ in Nakagami-m fading environment

The above two figures support the findings of the Table 3.10. The higher value of the Gaussian constant improves the BER performances.

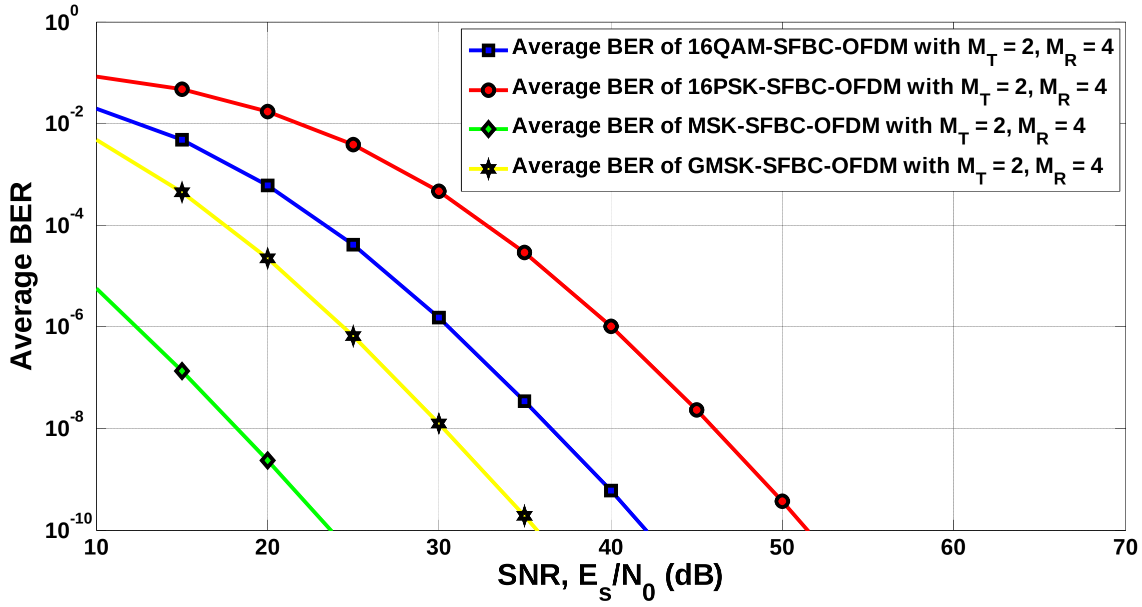

Fig. 3.27: BER performances of 16QAM, 16PSK, MSK and GMSK-SFBC-OFDM, Gaussian constant, $\alpha = 0.5$, $M_T = 2$ and $M_R = 4$ with Code Rate $R_C = 1/2$ in Rayleigh fading environment

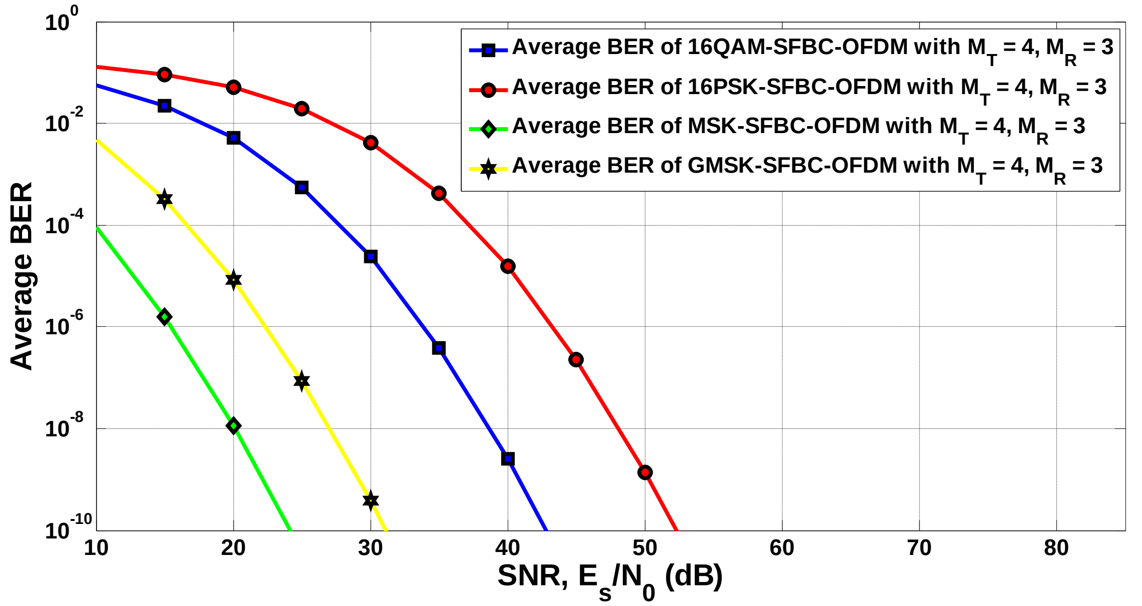

Fig. 3.28: BER performances of 16QAM, 16PSK, MSK and GMSK-SFBC-OFDM, Gaussian constant, $\alpha = 0.9$, $M_T = 4$ and $M_R = 3$ with Code Rate $R_C = 3/4$ in Rayleigh fading environment

The above two figures show that, with the same number of transmitters, receivers and code rate, the average BER performance of MSK-SFBC-OFDM system is the best among the four utilized schemes. GMSK modulation scheme provides the second better result. 16QAM-SFBC-OFDM has better result than 16PSK system. The results are obtained in Rayleigh fading environment.

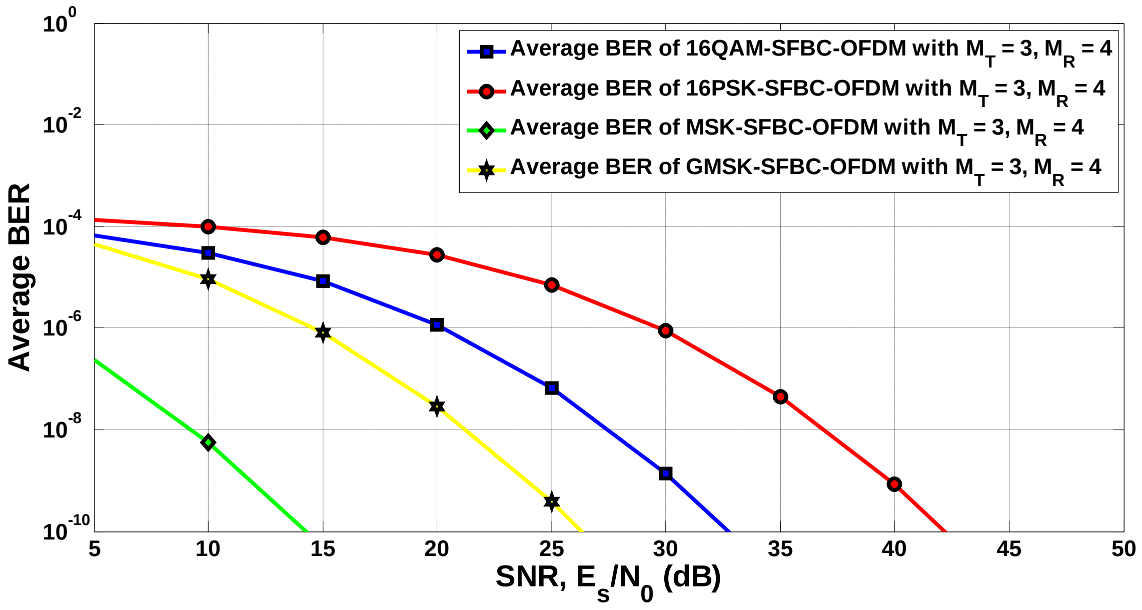

Fig. 3.29: BER performances of 16QAM, 16PSK, MSK and GMSK-SFBC-OFDM, Gaussian constant, $\alpha = 0.5$, Nakagami constant, $m = 2$, $M_T = 3$ and $M_R = 4$ with Code Rate $R_C = 1/2$ in Nakagami-m fading environment

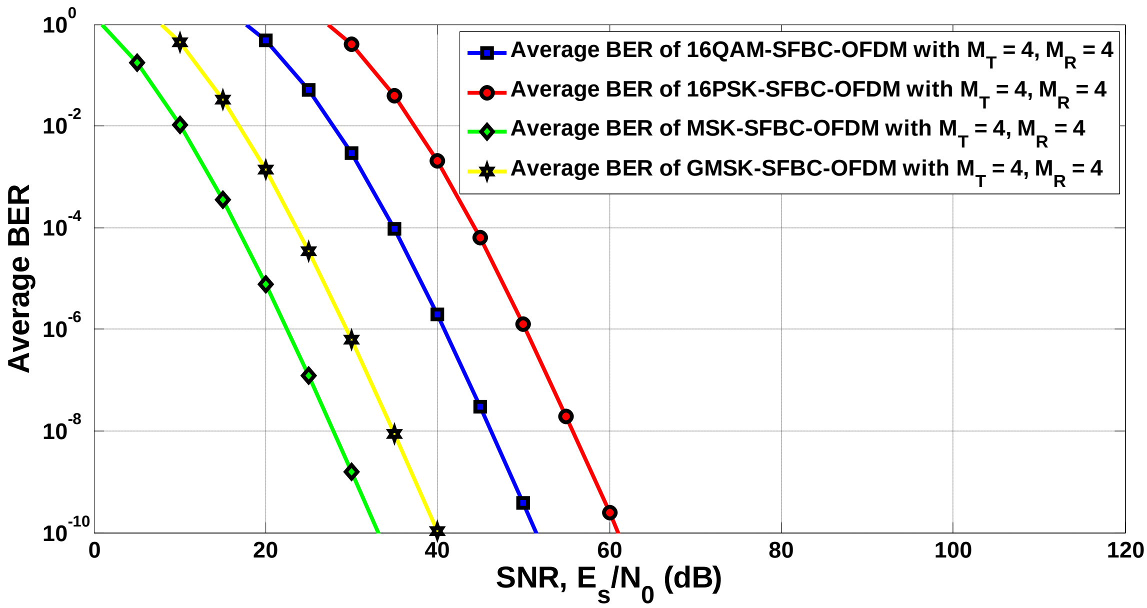

Fig. 3.30: BER performances of 16QAM, 16PSK, MSK and GMSK-SFBC-OFDM, Gaussian constant, $\alpha = 0.9$, Nakagami constant, $m = 0.5$, $M_T = 4$ and $M_R = 4$ with Code Rate $R_C = 3/4$ in Nakagami-m fading environment

The Fig. 3.29 and Fig. 3.30 demonstrate that, in Nakgami-m fading environment, the modulation schemes has same effect on the BER performances like in the Rayleigh fading environment.

The receiver sensitivity for the different modulation schemes with their code rate and various number of transmitters and receivers are calculated in this work. The considerations used for calculation of receiver sensitivity are listed in the following Table 3.11:

Table 3.11: Key Parameters for Evaluation of Receiver Sensitivity

Ser No	Parameters	Abbreviation and Value
1.	Reference Temperature	$T_0 = 290K$
2.	Boltzmann Constant	$k = 1.38 \times 10^{-23}$ J/K
3.	Receiver Bandwidth	B = 20 Mhz
4.	Noise Figure	NF = 10dB
5.	Implementation Margin	5 dB

The other parameters remain same for the calculation and evaluation of receiver sensitivities.

Table 3.12: Receiver sensitivity of 16QAM-SFBC-OFDM in Rayleigh fading environment with 20MHz bandwidth, 10dB noise figure, 5dB implementation margin

Ser No	Code Rate, R_C	#Tx, M_T	$\overline{\text{BER}}$	Rx Sensitivity = (-174+73+10+5)+γ_S dBm			
				#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	2	10^{-6}	-13.5697	-40.4762	-50.1077	-55.4378
2	1/2	3	10^{-6}	-27.8484	-46.5859	-53.8463	-58.1021
3	1/2	4	10^{-6}	-34.4556	-49.4172	-55.6033	-59.3681
4	1/2	2	10^{-9}	16.4491	-25.1288	-39.2573	-46.6434
5	1/2	3	10^{-9}	-7.7134	-35.7355	-45.6982	-51.1784
6	1/2	4	10^{-9}	-19.1082	-40.6228	-48.6796	-53.2968
7	3/4	2	10^{-6}	-10.0479	-36.9544	-46.5859	-51.9160

8	3/4	3	10^{-6}	-24.3265	-43.0641	-50.3245	-54.5802
9	3/4	4	10^{-6}	-30.9338	-45.8954	-52.0842	-55.8463
10	3/4	2	10^{-9}	19.9709	-21.6069	-35.7355	-43.1216
11	3/4	3	10^{-9}	-4.1916	-32.2136	-42.1764	-47.6565
12	3/4	4	10^{-9}	-15.5863	-37.1010	-45.1578	-49.7750

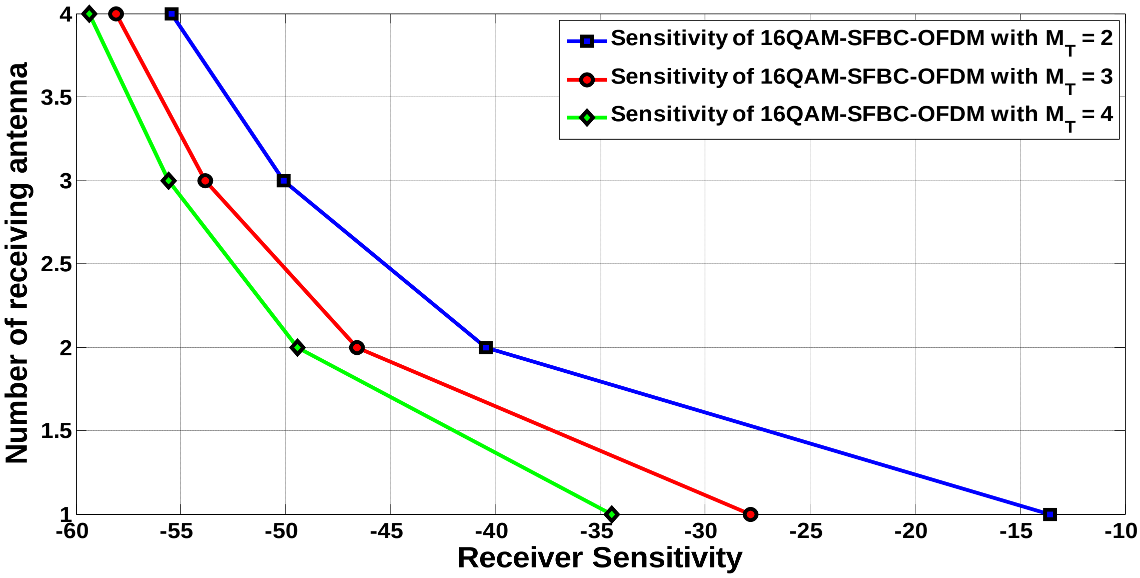

Fig. 3.31: Receiver Sensitivity of 16QAM-SFBC-OFDM under Rayleigh fading environment for code rate, R_C = 1/2 and average BER = 10^{-6}

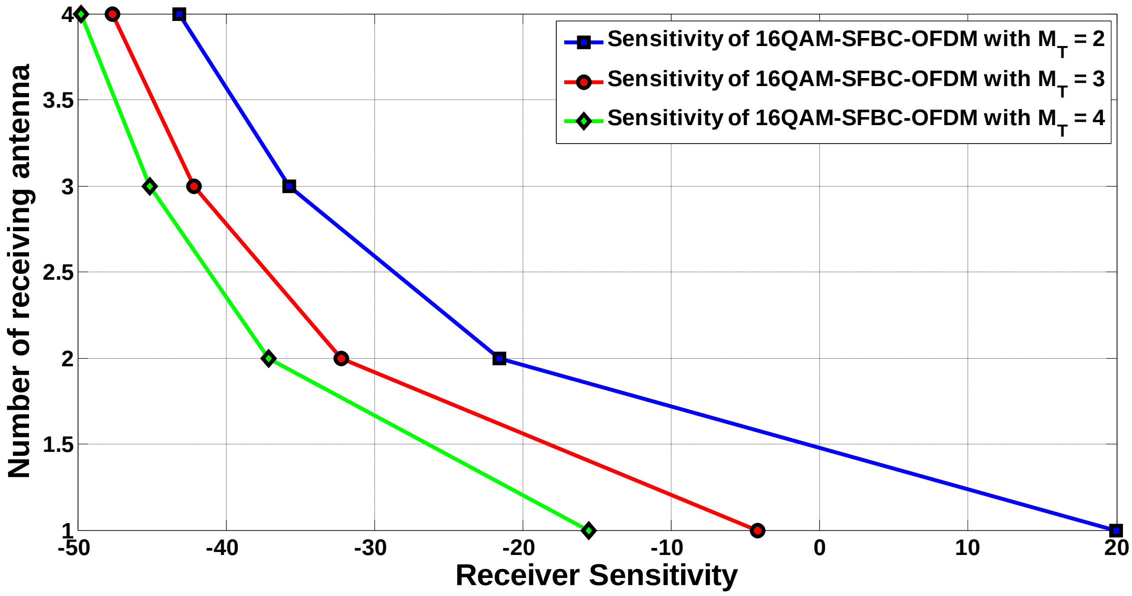

Fig. 3.32: Receiver Sensitivity of 16QAM-SFBC-OFDM under Rayleigh fading environment for code rate, R_C = 3/4 and average BER = 10^{-9}

The Table 3.12, Fig. 3.31 and Fig. 3.32 show the receiver sensitivities for 16QAM-SFBC-OFDM system. The results clearly demonstrate that, with the increment of receivers the sensitivity improves for the same number of transmitters. Sensitivity also improves if the number of transmitter is increased keeping the number of receivers fixed. But if the code rate increases, for the same condition of number of transmitters, receivers and average BER, the sensitivity performance diminishes slightly.

Table 3.13: Receiver sensitivity of 16PSK-SFBC-OFDM in Rayleigh fading environment with 20MHz bandwidth, 10dB noise figure, 5dB implementation margin

Ser No	Code Rate, R_C	#Tx, M_T	$\overline{\text{BER}}$	Rx Sensitivity = (-174+73+10+5)+γ_S dBm			
				#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	2	10^{-6}	-4.1012	-31.0077	-40.6392	-45.9693
2	1/2	3	10^{-6}	-18.3799	-37.1174	-44.3778	-48.6335
3	1/2	4	10^{-6}	-24.9871	-39.9487	-46.1348	-49.8996
4	1/2	2	10^{-9}	25.9176	-15.6603	-29.7888	-37.1749
5	1/2	3	10^{-9}	1.7551	-26.2670	-36.2297	-41.7099
6	1/2	4	10^{-9}	-9.6397	-31.1543	-39.2111	-43.8283
7	3/4	2	10^{-6}	-0.5794	-27.4858	-37.1174	-42.4475
8	3/4	3	10^{-6}	-14.8580	-33.9956	-40.8559	-45.1117
9	3/4	4	10^{-6}	-21.4652	-36.4269	-42.6129	-46.3778
10	3/4	2	10^{-9}	29.4394	-12.1384	-26.2670	-33.6531
11	3/4	3	10^{-9}	5.2769	-22.7451	-32.7079	-38.1880
12	3/4	4	10^{-9}	-6.1178	-27.6325	-35.6893	-40.3065

The above Table 3.13 demonstrates the sensitivity performances for 16PSK-SFBC-OFDM system. As seen above the 16QAM system, here the sensitivity also changes with the change in number of transmitters and receivers. Sensitivity improves in both cases for the increasing number of transmitters and receivers. The code rate increment diminishes the sensitivity performance. 16QAM has better sensitivity performance than 16PSK system. The following Fig. 3.33 and Fig.3.34 show the same performances.

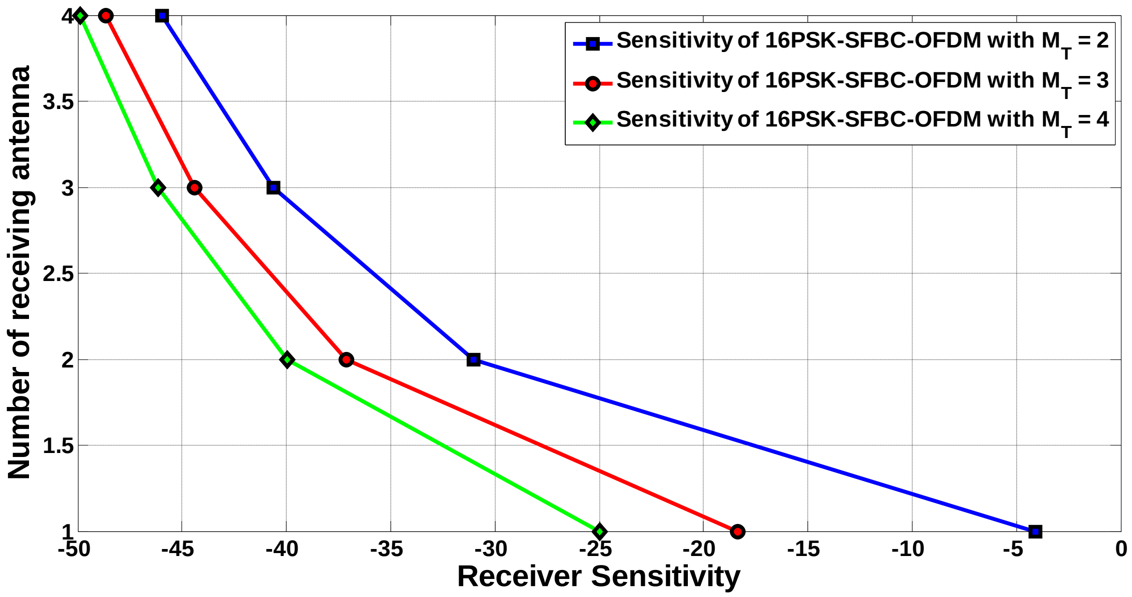

Fig. 3.33: Receiver Sensitivity of 16PSK-SFBC-OFDM under Rayleigh fading environment for code rate, R_C = 1/2 and average BER = 10^{-6}

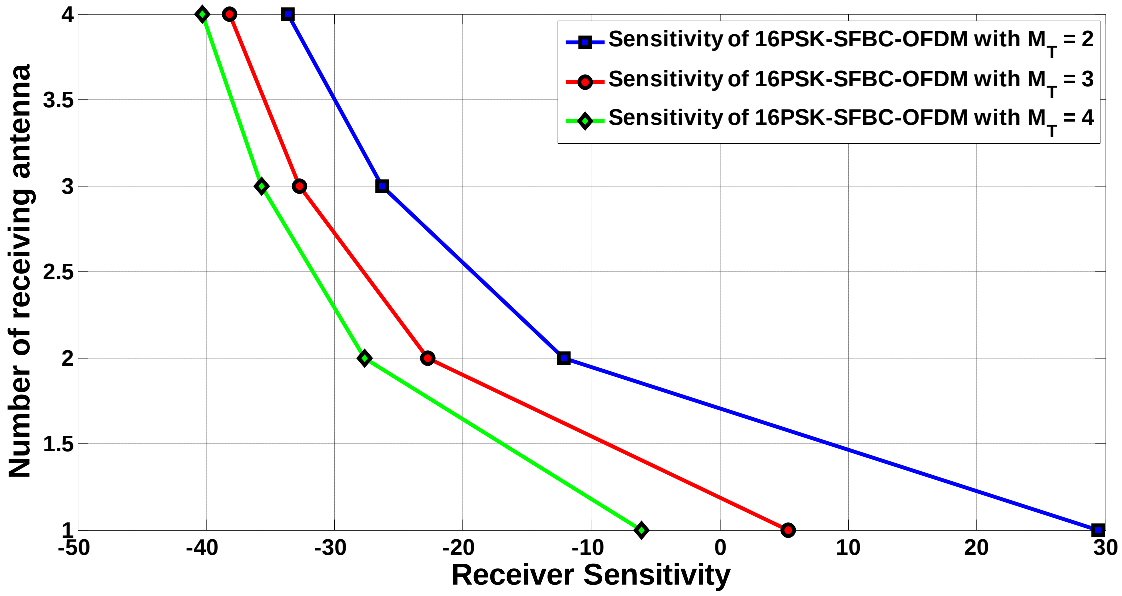

Fig. 3.34: Receiver Sensitivity of 16PSK-SFBC-OFDM under Rayleigh fading environment for code rate, R_C = 3/4 and average BER = 10^{-9}

The following Table 3.14, Fig. 3.35 and Fig. 3.36 show the sensitivity performances of MSK-SFBC-OFDM system. The MSK modulation schemes show better sensitivity than the other two modulation schemes.

Table 3.14: Receiver sensitivity of MSK-SFBC-OFDM in Rayleigh fading environment with 20MHz bandwidth, 10dB noise figure, 5dB implementation margin

Ser No	Code Rate, R_C	#Tx, M_T	$\overline{\text{BER}}$	Rx Sensitivity = (-174+73+10+5)+γ_S dBm			
				#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	2	10^{-6}	-29.0226	-57.8381	-68.0376	-73.6250
2	1/2	3	10^{-6}	-44.5952	-64.5158	-72.1144	-76.5239
3	1/2	4	10^{-6}	-51.8175	-67.6044	-74.0251	-77.8976
4	1/2	2	10^{-9}	0.9893	-42.5634	-57.3173	-64.9936
5	1/2	3	10^{-9}	-24.4960	-53.7955	-64.1399	-69.7931
6	1/2	4	10^{-9}	-36.5428	-58.9730	-67.2943	-72.0313
7	3/4	2	10^{-6}	-25.5008	-54.3163	-64.5158	-70.1032
8	3/4	3	10^{-6}	-41.0733	-60.9939	-68.5926	-73.0020
9	3/4	4	10^{-6}	-48.2957	-64.0826	-70.5033	-74.3758
10	3/4	2	10^{-9}	4.5111	-39.0416	-53.7955	-61.4718
11	3/4	3	10^{-9}	-20.9742	-50.2737	-60.6181	-66.2712
12	3/4	4	10^{-9}	-33.0210	-55.4512	-63.7725	-68.5095

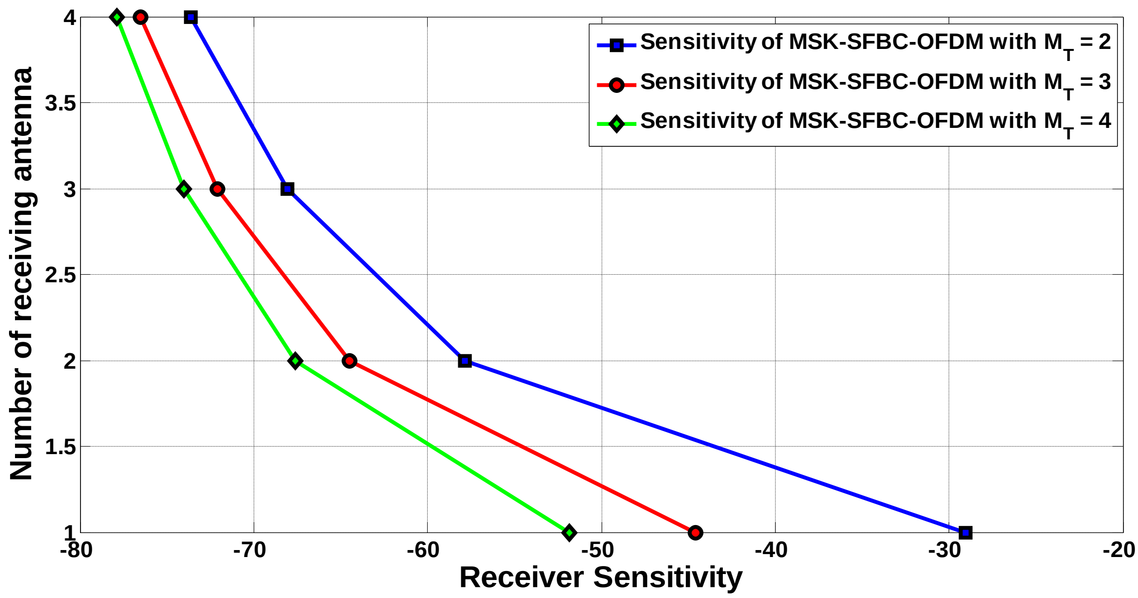

Fig. 3.35: Receiver Sensitivity of MSK-SFBC-OFDM under Rayleigh fading environment for code rate, R_C = 1/2 and average BER = 10^{-6}

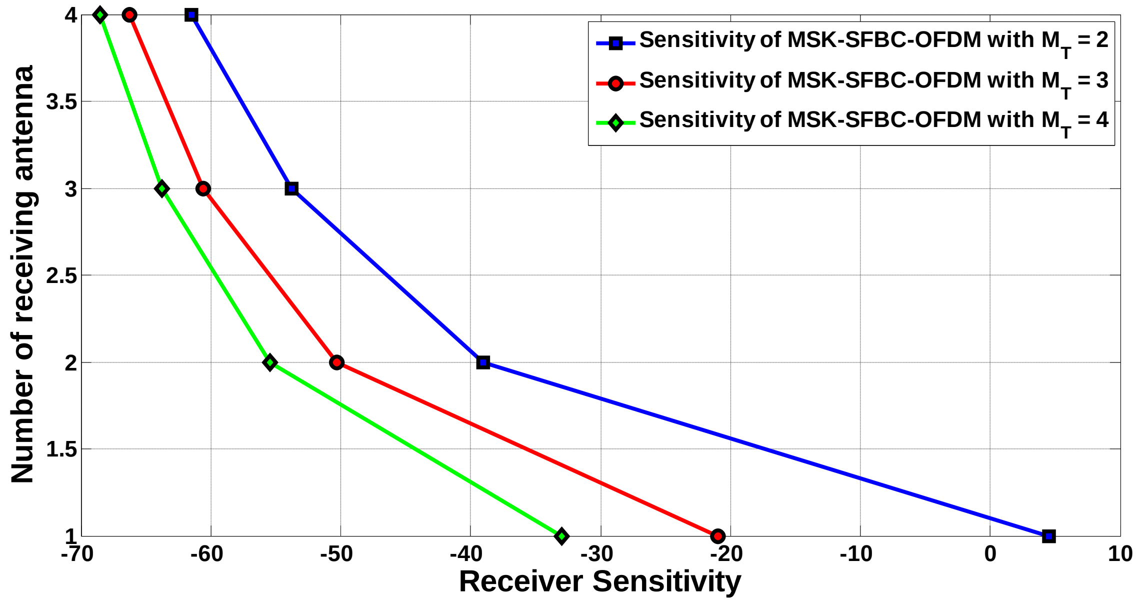

Fig. 3.36: Receiver Sensitivity of MSK-SFBC-OFDM under Rayleigh fading environment for code rate, R_C = 3/4 and average BER = 10^{-9}

Table 3.15, Fig. 3.37 and Fig. 3.38 show the receiver sensitivities of GMSK-SFBC-OFDM system. The results also follow the same nature of changes like the previous three schemes. GMSK has better result than 16QAM and 16PSK. But it has slightly lower result than MSK. With the increment of the value of the Gaussian constant, the performance improves in GMSK.

Table 3.15: Receiver sensitivity of GMSK-SFBC-OFDM in Rayleigh fading environment with 20MHz bandwidth, 10dB noise figure, 5dB implementation margin

Ser No	Code Rate, R_C	Gaussian Constant, α	#Tx, M_T	$\overline{\text{BER}}$	Rx Sensitivity = (-174+73+10+5)+γ_S dB			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	-16.981	-45.797	-55.996	-61.584
2	1/2	0.5	3	10^{-6}	-32.554	-52.475	-60.073	-64.483
3	1/2	0.5	4	10^{-6}	-39.776	-55.563	-61.984	-65.856
4	1/2	0.5	2	10^{-9}	13.031	-30.522	-45.276	-52.952
5	1/2	0.5	3	10^{-9}	-12.455	-41.754	-52.099	-57.752
6	1/2	0.5	4	10^{-9}	-24.502	-46.932	-55.253	-59.990
7	3/4	0.5	2	10^{-6}	-13.460	-42.275	-52.475	-58.062
8	3/4	0.5	3	10^{-6}	-29.232	-48.953	-56.551	-60.961

9	3/4	0.5	4	10^{-6}	-36.205	-52.041	-58.462	-62.335
10	3/4	0.5	2	10^{-9}	16.552	-27.001	-41.754	-49.431
11	3/4	0.5	3	10^{-9}	-8.933	-38.233	-48.577	-54.230
12	3/4	0.5	4	10^{-9}	-20.980	-43.410	-51.731	-56.468
13	1/2	0.9	2	10^{-6}	-22.087	-50.902	-61.102	-66.689
14	1/2	0.9	3	10^{-6}	-37.659	-57.580	-65.178	-69.588
15	1/2	0.9	4	10^{-6}	-44.882	-60.669	-67.089	-70.962
16	1/2	0.9	2	10^{-9}	7.9251	-35.628	-50.382	-58.058
17	1/2	0.9	3	10^{-9}	-17.560	-46.860	-57.204	-62.857
18	1/2	0.9	4	10^{-9}	-29.607	-52.037	-60.359	-65.096
19	3/4	0.9	2	10^{-6}	-18.565	-47.381	-57.580	-63.168
20	3/4	0.9	3	10^{-6}	-34.138	-54.058	-61.659	-66.066
21	3/4	0.9	4	10^{-6}	-41.360	-57.147	-63.568	-67.440
22	3/4	0.9	2	10^{-9}	11.4469	-32.106	-46.860	-54.536
23	3/4	0.9	3	10^{-9}	-14.038	-43.338	-53.682	-59.336
24	3/4	0.9	4	10^{-9}	-26.085	-48.515	-56.837	-61.574

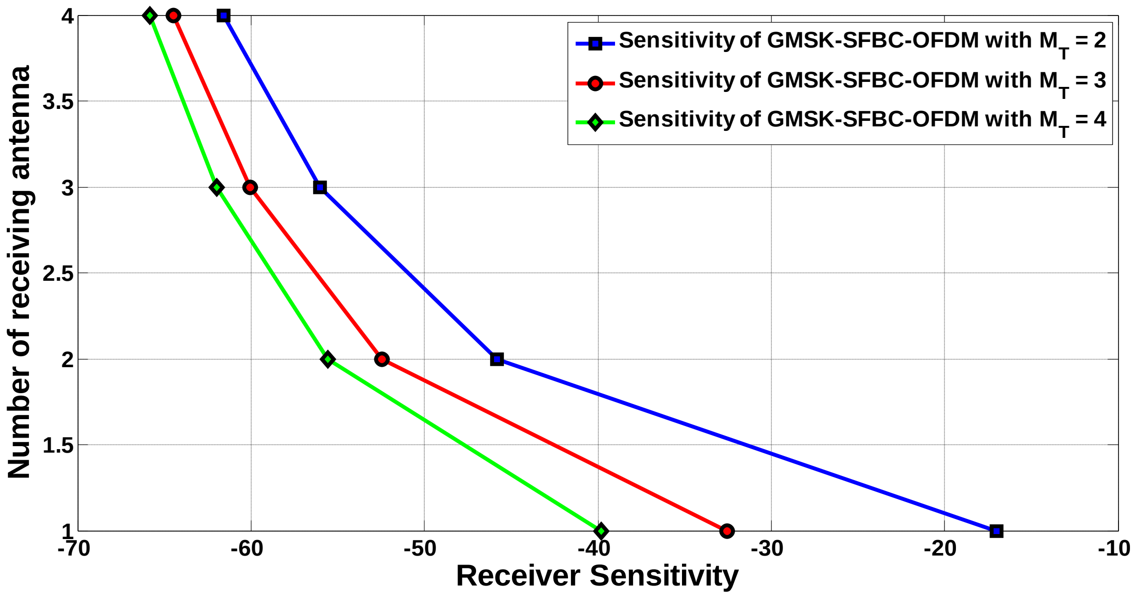

Fig. 3.37: Receiver Sensitivity of GMSK-SFBC-OFDM under Rayleigh fading environment for code rate, R_C =1/2, Gaussian constant, α = 0.5 and average BER = 10^{-6}

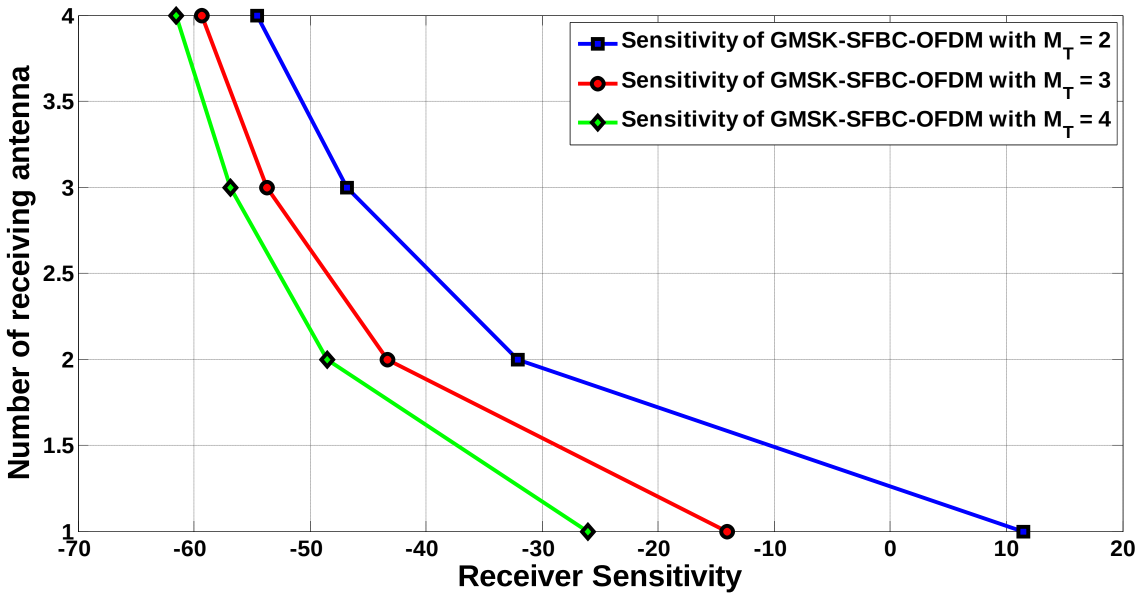

Fig. 3.38: Receiver Sensitivity of GMSK-SFBC-OFDM under Rayleigh fading environment for code rate, R_C =3/4, Gaussian constant, α = 0.9 and average BER = 10^{-9}

Table 3.16: Receiver sensitivity of 16QAM-SFBC-OFDM in Nakagami-m fading environment with 20MHz bandwidth, 10dB noise figure, 5dB implementation margin

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	Rx Sensitivity = (-174+73+10+5)+γ_S dB			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	36.450	-15.067	-32.308	-41.035
2	1/2	0.5	3	10^{-6}	5.6332	-28.786	-40.465	-46.494
3	1/2	0.5	4	10^{-6}	-9.0464	-35.014	-43.995	-48.711
4	1/2	0.5	2	10^{-9}	96.450	14.944	-12.232	-25.848
5	1/2	0.5	3	10^{-9}	45.635	-8.710	-26.882	-36.063
6	1/2	0.5	4	10^{-9}	20.965	-19.827	-33.564	-40.590
7	3/4	0.5	2	10^{-6}	39.972	-11.545	-28.786	-37.513
8	3/4	0.5	3	10^{-6}	9.155	-25.264	-36.943	-42.972
9	3/4	0.5	4	10^{-6}	-5.5246	-31.493	-40.473	-45.189
10	3/4	0.5	2	10^{-9}	99.9720	18.4660	-8.7103	-22.326
11	3/4	0.5	3	10^{-9}	49.1570	-5.1885	-23.360	-32.541
12	3/4	0.5	4	10^{-9}	24.487	-16.306	-30.042	-37.068

13	1/2	2.0	2	10^{-6}	-37.646	-55.757	-65.701	-75.969
14	1/2	2.0	3	10^{-6}	-45.383	-62.179	-80.902	-75.50+ 27.288i
15	1/2	2.0	4	10^{-6}	-49.737	-69.948	-73.0+ 27.288i	-67.08+ 27.288i
16	1/2	2.0	2	10^{-9}	-22.149	-45.758	-55.630	-62.472
17	1/2	2.0	3	10^{-9}	-33.952	-52.108	-62.029	-72.172
18	1/2	2.0	4	10^{-9}	-39.737	-56.451	-69.673	-80.34+ 27.288i
19	3/4	2.0	2	10^{-6}	-34.125	-52.235	-62.179	-72.447
20	3/4	2.0	3	10^{-6}	-41.861	-58.657	-77.38	-71.98+ 27.288i
21	3/4	2.0	4	10^{-6}	-46.215	-66.426	-69.48+ 27.288i	-63.56+ 27.288i
22	3/4	2.0	2	10^{-9}	-18.627	-42.236	-52.108	-58.95
23	3/4	2.0	3	10^{-9}	-30.43	-48.587	-58.507	-68.65
24	3/4	2.0	4	10^{-9}	-36.215	-52.929	-66.151	-76.82+ 27.288i
25	1/2	3.0	2	10^{-6}	-46.35	-70.445	-70.52+ 27.288i	-65.79+ 27.288i
26	1/2	3.0	3	10^{-6}	-54.745	-67.00+ 27.288i	-61.28+ 27.288i	-59.74+ 27.288i
27	1/2	3.0	4	10^{-6}	-64.424	-59.77+ 27.288i	-57.24+ 27.288i	-56.36+ 27.288i
28	1/2	3.0	2	10^{-9}	-34/765	-55.749	-73.066	-73.38+ 27.288i
29	1/2	3.0	3	10^{-9}	-43.895	-69.544	-66.05+ 27.288i	-61.93+ 27.288i
30	1/2	3.0	4	10^{-9}	-49.729	-67.36+ 27.288i	-59.43+ 27.288i	-57.59+ 27.288i
31	3/4	3.0	2	10^{-6}	-42.828	-66.923	-67.00+ 27.288i	-62.27+ 27.288i
32	3/4	3.0	3	10^{-6}	-51.224	-63.48+ 27.288i	-57.76+ 27.288i	-56.22+ 27.288i
33	3/4	3.0	4	10^{-6}	-60.902	-56.25+ 27.288i	-53.72+ 27.288i	-52.83+ 27.288i
34	3/4	3.0	2	10^{-9}	-31.243	-52.227	-69.544	-69.86+ 27.288i
35	3/4	3.0	3	10^{-9}	-40.374	-66.022	-62.53+ 27.288i	-58.41+ 27.288i
36	3/4	3.0	4	10^{-9}	-46.207	-63.84+ 27.288i	-55.91+ 27.288i	-54.07+ 27.288i

The sensitivity performances are also evaluated in Nakagami-m fading environment. The above Table 3.16 and the following Fig. 3.39 and Fig. 3.40 show the results for 16QAM scheme in this environment. The sensitivity performances for the increment of the number of transmitters, receivers and code rate are like the performances in Rayleigh fading environment. With the increment in the value of Nakagami constant, *m* the sensitivity performance improves.

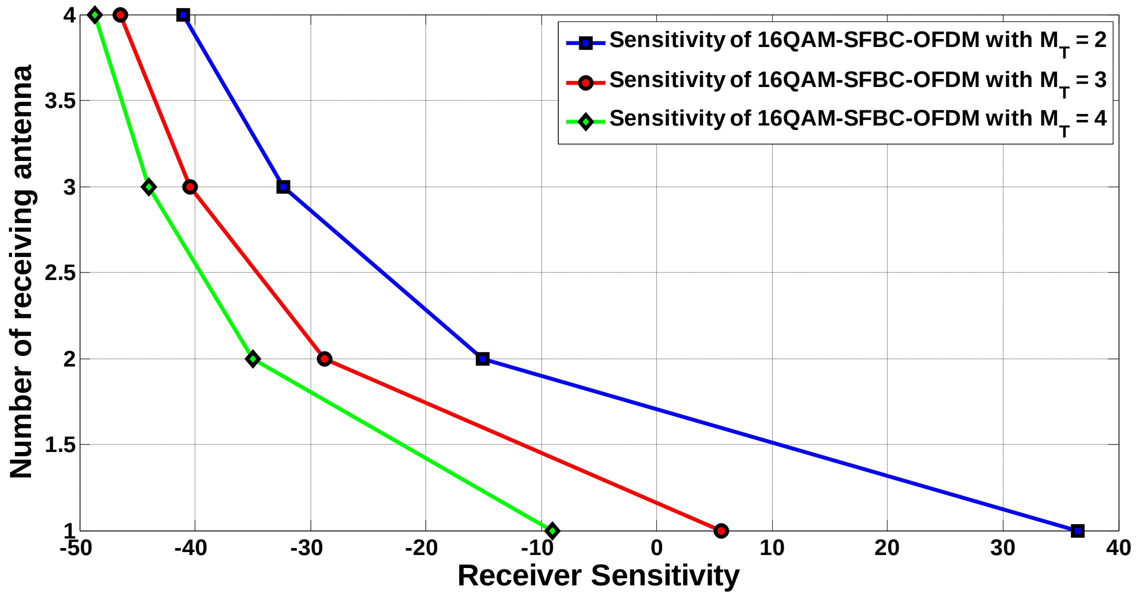

Fig. 3.39: Receiver Sensitivity of 16QAM-SFBC-OFDM under Nakagami-m fading environment for code rate, R_C=1/2, Nakagami constant, m = 0.5 and average BER=10^{-6}

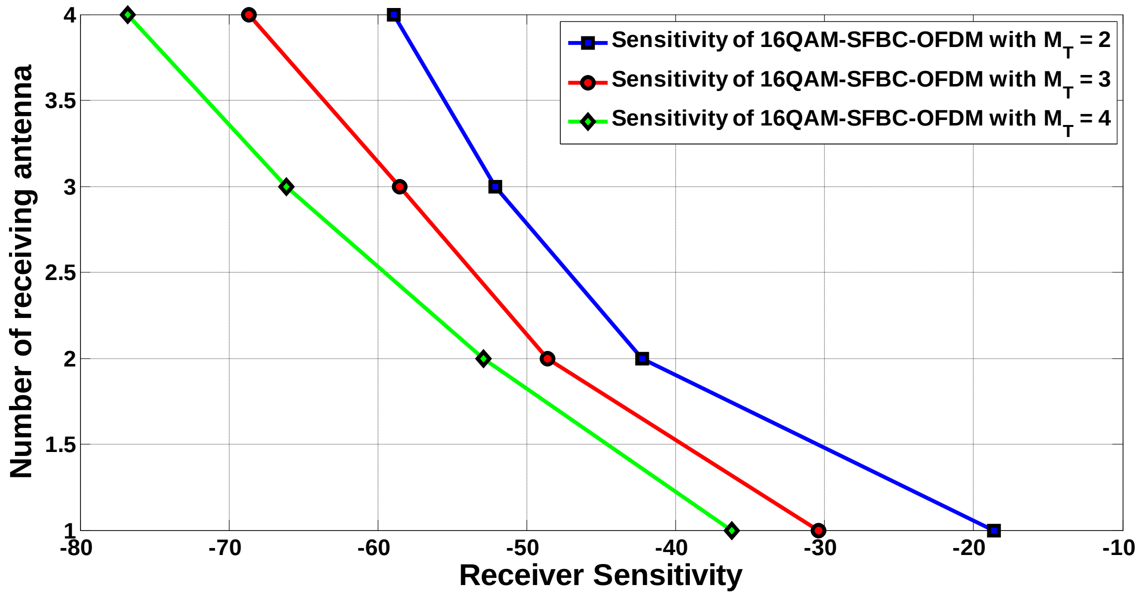

Fig. 3.40: Receiver Sensitivity of 16QAM-SFBC-OFDM under Nakagami-m fading environment for code rate, R_C=3/4, Nakagami constant, m=2 and average BER=10^{-9}

Table 3.17: Receiver sensitivity of 16PSK-SFBC-OFDM in Nakagami-m fading environment with 20MHz bandwidth, 10dB noise figure, 5dB implementation margin

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	Rx Sensitivity = (-174+73+10+5)+γ_S dBm			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	45.918	-5.5985	-22.839	-31.566
2	1/2	0.5	3	10^{-6}	15.102	-19.317	-30.996	-37.025
3	1/2	0.5	4	10^{-6}	0.4221	-25.546	-34.527	-39.242
4	1/2	0.5	2	10^{-9}	105.92	24.413	-2.7636	-16.379
5	1/2	0.5	3	10^{-9}	55.103	0.7582	-17.413	-26.594
6	1/2	0.5	4	10^{-9}	30.433	-10.359	-24.096	-31.121
7	3/4	0.5	2	10^{-6}	49.440	-2.0766	-19.317	-28.045
8	3/4	0.5	3	10^{-6}	18.624	-15.795	-27.473	-33.594
9	3/4	0.5	4	10^{-6}	3.944	-22.024	-31.005	-35.720
10	3/4	0.5	2	10^{-9}	109.44	27.935	0.7582	-12.858
11	3/4	0.5	3	10^{-9}	58.625	4.280	-13.891	-23.072
12	3/4	0.5	4	10^{-9}	33.955	-6.837	-20.574	-27.60
13	1/2	2.0	2	10^{-6}	-28.178	-46.289	-56.233	-66.500
14	1/2	2.0	3	10^{-6}	-35.914	-52.712	-71.433	-66.03+ 27.288i
15	1/2	2.0	4	10^{-6}	-40.268	-60.480	-63.53+ 27.288i	-57.62+ 27.288i
16	1/2	2.0	2	10^{-9}	-12.681	-36.289	-46.162	-53.003
17	1/2	2.0	3	10^{-9}	-24.483	-42.640	-52.560	-62.703
18	1/2	2.0	4	10^{-9}	-30.269	-46.983	-60.204	-70.87+ 27.288i
19	3/4	2.0	2	10^{-6}	-24.656	-42.767	-52.711	-62.978
20	3/4	2.0	3	10^{-6}	-32.392	-49.189	-67.912	-62.51+ 27.288i
21	3/4	2.0	4	10^{-6}	-36.746	-56.958	-60.01+ 27.288i	-54.09+ 27.288i
22	3/4	2.0	2	10^{-9}	-9.1587	-32.767	-42.640	-49.482

23	3/4	2.0	3	10^{-9}	-20.961	-39.118	-49.038	-59.181
24	3/4	2.0	4	10^{-9}	-26.747	-43.461	-56.683	-67.35+ 27.288i
25	1/2	3.0	2	10^{-6}	-36.882	-60.976	-61.05+ 27.288i	-56.33+ 27.288i
26	1/2	3.0	3	10^{-6}	-45.277	-57.53+ 27.288i	-51.81+ 27.288i	-50.27+ 27.288i
27	1/2	3.0	4	10^{-6}	-54.956	-50.31+ 27.288i	-47.77+ 27.288i	-46.89+ 27.288i
28	1/2	3.0	2	10^{-9}	-25.296	-46.281	-63.597	-63.91+ 27.288i
29	1/2	3.0	3	10^{-9}	-34.427	-60.075	-56.59+ 27.288i	-52.46+ 27.288i
30	1/2	3.0	4	10^{-9}	-40.260	-57.89+ 27.288i	-49.96+ 27.288i	-48.12+ 27.288i
31	3/4	3.0	2	10^{-6}	-33.360	-57.454	-57.53+ 27.288i	-52.80+ 27.288i
32	3/4	3.0	3	10^{-6}	-41.755	-54.01+ 27.288i	-48.29+ 27.288i	-46.75+ 27.288i
33	3/4	3.0	4	10^{-6}	-51.434	-46.78+ 27.288i	-44.26+ 27.288i	-43.37+ 27.288i
34	3/4	3.0	2	10^{-9}	-21.774	-42.759	-60.075	-60.39+ 27.288i
35	3/4	3.0	3	10^{-9}	-30.905	-56.554	-53.06+ 27.288i	-48.94+ 27.288i
36	3/4	3.0	4	10^{-9}	-36.738	-54.37+ 27.288i	-46.44+ 27.288i	-44.60+ 27.288i

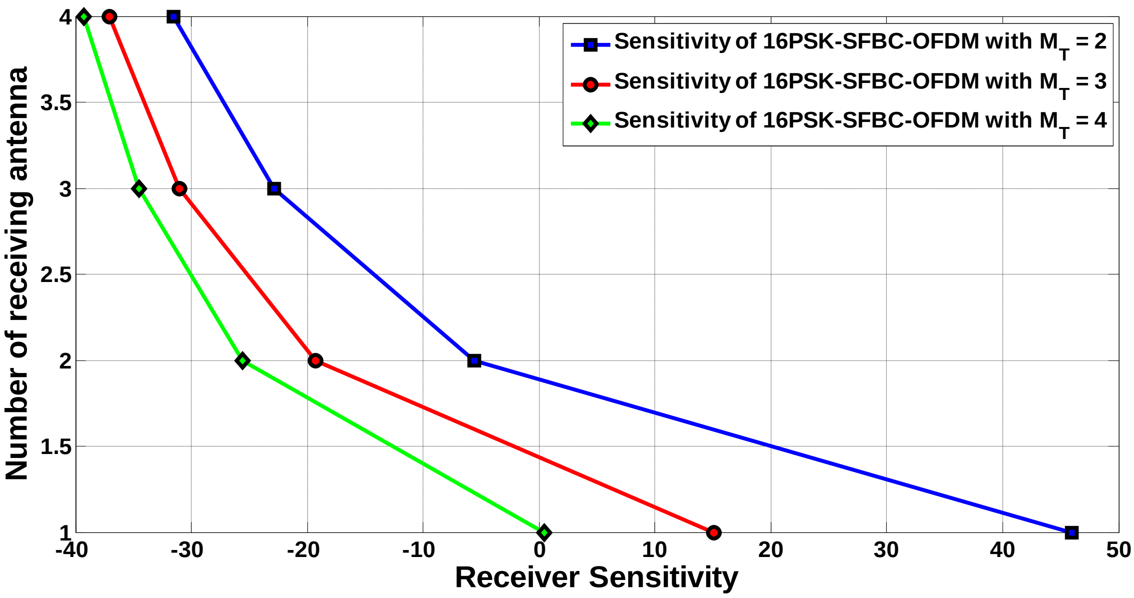

Fig. 3.41: Receiver Sensitivity of 16PSK-SFBC-OFDM under Nakagami-m fading environment for code rate, R_C=1/2, Nakagami constant, m=0.5 and average BER=10^{-6}

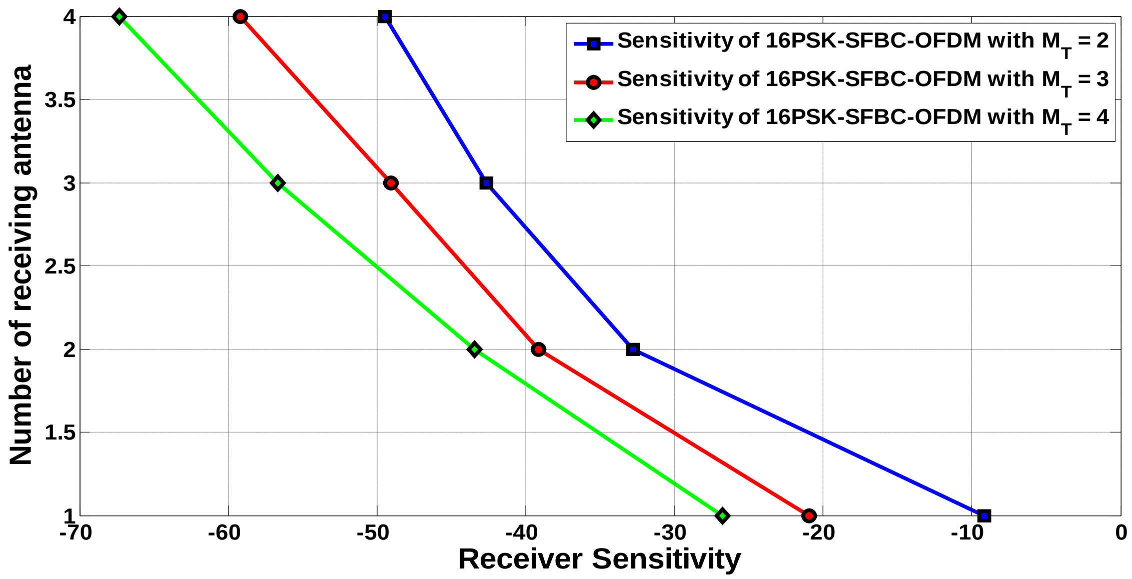

Fig. 3.42: Receiver Sensitivity of 16PSK-SFBC-OFDM under Nakagami-m fading environment for code rate, R_C=3/4, Nakagami constant, m=2 and average BER=10^{-9}

The Table 3.17, Fig. 3.41 and Fig. 3.42 show that, like Rayleigh environment, 16QAM has better performance than 16PSK also in Nakagami-m environment.

Table 3.18: Receiver sensitivity of MSK-SFBC-OFDM in Nakagami-m fading environment with 20MHz bandwidth, 10dB noise figure, 5dB implementation margin

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	Rx Sensitivity = (-174+73+10+5)+γ_S dBm			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	24.969	-30.523	-49.072	-58.438
2	1/2	0.5	3	10^{-6}	-8.4996	-45.550	-58.076	-64.516
3	1/2	0.5	4	10^{-6}	-24.502	-52.417	-62.017	-67.036
4	1/2	0.5	2	10^{-9}	84.969	-0.5157	-29.016	-43.290
5	1/2	0.5	3	10^{-9}	31.501	-25.495	-44.540	-54.148
6	1/2	0.5	4	10^{-9}	5.5049	-37.269	-51.649	-58.987
7	3/4	0.5	2	10^{-6}	28.491	-27.000	-45.550	-54.916
8	3/4	0.5	3	10^{-6}	-4.9778	-42.028	-54.554	-60.994
9	3/4	0.5	4	10^{-6}	-20.980	-48.895	-58.495	-63.514
10	3/4	0.5	2	10^{-9}	88.491	3.0061	-25.495	-39.768
11	3/4	0.5	3	10^{-9}	35.023	-21.973	-41.018	-50.626

12	3/4	0.5	4	10^{-9}	9.0267	-33.747	-48.128	-55.466
13	1/2	2.0	2	10^{-6}	-54.970	-73.676	-83.391	-92.441
14	1/2	2.0	3	10^{-6}	-63.179	-79.870	-94.910	-97.89+ 27.288i
15	½	2.0	4	10^{-6}	-67.655	-86.421	-95.39+ 27.288i	-87.39+ 27.288i
16	1/2	2.0	2	10^{-9}	-39.577	-64.033	-74.051	-80.83
17	1/2	2.0	3	10^{-9}	-51.979	-70.529	-80.291	-89.592
18	1/2	2.0	4	10^{-9}	-58.01	-74.81	-87.09	-106.5+ 27.288i
19	3/4	2.0	2	10^{-6}	-51.448	-70.154	-79.870	-88.920
20	3/4	2.0	3	10^{-6}	-59.657	-76.348	-91.388	-94.37+ 27.288i
21	3/4	2.0	4	10^{-6}	-64.134	-82.90	-91.87+ 27.288i	-83.86+ 27.288i
22	3/4	2.0	2	10^{-9}	-36.056	-60.511	-70.529	-77.308
23	3/4	2.0	3	10^{-9}	-48.457	-67.007	-76.769	-86.070
24	3/4	2.0	4	10^{-9}	-54.49	-71.29	-83.57	-103.0+ 27.288i
25	1/2	3.0	2	10^{-6}	-64.110	-86.647	-91.84+ 27.288i	-85.85+ 27.288i
26	1/2	3.0	3	10^{-6}	-72.380	-88.32+ 27.288i	-81.17+ 27.288i	-79.42+ 27.288i
27	1/2	3.0	4	10^{-6}	-80.627	-79.82+ 27.288i	-76.92+ 27.288i	-75.94+ 27.288i
28	1/2	3.0	2	10^{-9}	-52.783	-73.990	-89.685	-94.95+ 27.288i
29	1/2	3.0	3	10^{-9}	-62.177	-86.163	-86.52+ 27.288i	-81.74+ 27.288i
30	1/2	3.0	4	10^{-9}	-67.970	-88.93+ 27.288i	-79.24+ 27.288i	-77.22+ 27.288i
31	3/4	3.0	2	10^{-6}	-60.588	-83.125	-88.32+ 27.288i	-82.32+ 27.288i
32	3/4	3.0	3	10^{-6}	-68.859	-84.80+ 27.288i	-77.64+ 27.288i	-75.90+ 27.288i
33	3/4	3.0	4	10^{-6}	-77.105	-76.30+ 27.288i	-73.40+ 27.288i	-72.42+ 27.288i
34	3/4	3.0	2	10^{-9}	-49.262	-70.469	-86.163	-91.43+ 27.288i
35	3/4	3.0	3	10^{-9}	-58.655	-82.642	-83.00+ 27.288i	-78.22+ 27.288i
36	3/4	3.0	4	10^{-9}	-64.448	-85.40+ 27.288i	-75.72+ 27.288i	-73.70+ 27.288i

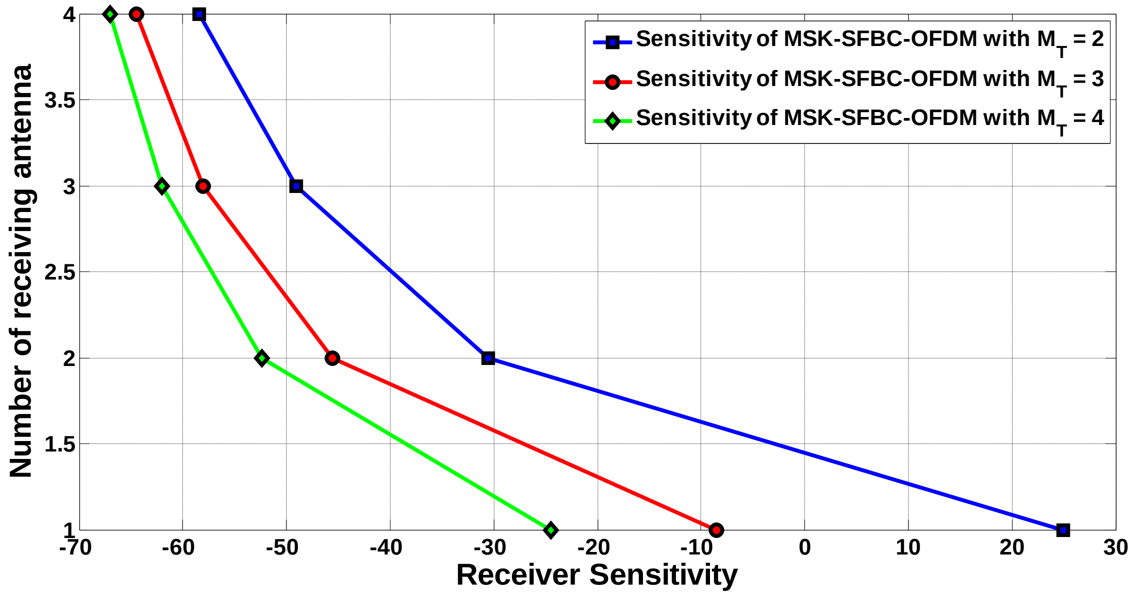

Fig. 3.43: Receiver Sensitivity of MSK-SFBC-OFDM under Nakagami-m fading environment for code rate, R_C=1/2, Nakagami constant, m=0.5 and average BER=10^{-6}

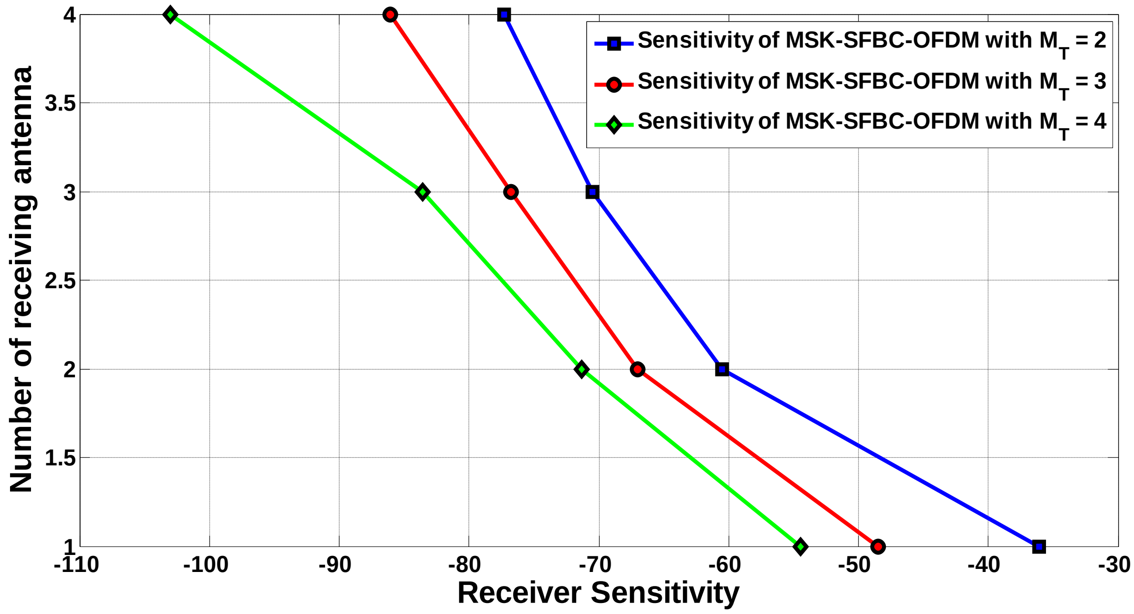

Fig. 3.44: Receiver Sensitivity of MSK-SFBC-OFDM under Nakagami-m fading environment for code rate, R_C=3/4, Nakagami constant, m=2 and average BER=10^{-9}

Table 3.18, Fig. 3.43 and Fig. 3.44 demonstrate the results obtained by MSK-SFBC-OFDM in Nakagami-m fading environment. Among the three utilized modulation schemes so far, MSK has the best performances according to sensitivity. The following Table 3.19 and 3.20 show the results for GMSK modulation with two different α.

Table 3.19: Receiver sensitivity of GMSK-SFBC-OFDM with Gaussian constant, α =0.5 in Nakagami-m fading environment with 20MHz bandwidth, 10dB noise figure, 5dB implementation margin

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	Rx Sensitivity = (-174+73+10+5)+γ_S dBm			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	37.010	-18.482	-37.031	-46.397
2	1/2	0.5	3	10^{-6}	3.5416	-33.509	-46.035	-52.475
3	1/2	0.5	4	10^{-6}	-12.461	-40.376	-49.976	-54.994
4	1/2	0.5	2	10^{-9}	97.010	11.5255	-16.975	-31.248
5	1/2	0.5	3	10^{-9}	43.542	-13.453	-32.499	-42.107
6	1/2	0.5	4	10^{-9}	17.546	-25.228	-39.608	-46.946
7	3/4	0.5	2	10^{-6}	40.532	-14.960	-33.509	-42.875
8	3/4	0.5	3	10^{-6}	7.0634	-29.987	-42.513	-48.953
9	3/4	0.5	4	10^{-6}	-8.9391	-36.854	-46.454	-51.473
10	3/4	0.5	2	10^{-9}	100.532	15.047	-13.453	-27.726
11	3/4	0.5	3	10^{-9}	47.064	-9.9316	-28.977	-38.585
12	3/4	0.5	4	10^{-9}	21.068	-21.706	-36.086	-43.424
13	1/2	2.0	2	10^{-6}	-42.928	-61.635	-71.350	-80.400
14	1/2	2.0	3	10^{-6}	-51.138	-67.828	-82.868	-85.85+ 27.288i
15	1/2	2.0	4	10^{-6}	-55.614	-74.380	-83.35+ 27.288i	-75.34+ 27.288i
16	1/2	2.0	2	10^{-9}	-27.536	-51.991	-62.01	-68.789
17	1/2	2.0	3	10^{-9}	-39.937	-58.488	-68.249	-77.551
18	1/2	2.0	4	10^{-9}	-45.971	-62.768	-75.052	-94.46+ 27.288i
19	3/4	2.0	2	10^{-6}	-39.407	-58.113	-67.828	-76.878
20	3/4	2.0	3	10^{-6}	-47.616	-64.306	-79.347	-82.33+ 27.288i
21	3/4	2.0	4	10^{-6}	-52.092	-70.858	-79.83+ 27.288i	-71.82+ 27.288i
22	3/4	2.0	2	10^{-9}	-24.014	-48.470	-58.490	-65.267

23	3/4	2.0	3	10^{-9}	-36.416	-54.966	-64.728	-74.029
24	3/4	2.0	4	10^{-9}	-42.449	-59.246	-71.530	-90.94+ 27.288i
25	1/2	3.0	2	10^{-6}	-52.069	-74.606	-79.80+ 27.288i	-73.80+ 27.288i
26	1/2	3.0	3	10^{-6}	-60.339	-76.28+ 27.288i	-69.13+ 27.288i	-67.38+ 27.288i
27	1/2	3.0	4	10^{-6}	-68.585	-67.78+ 27.288i	-64.88+ 27.288i	-63.90+ 27.288i
28	1/2	3.0	2	10^{-9}	-40.742	-61.949	-77.644	-82.91+ 27.288i
29	1/2	3.0	3	10^{-9}	-50.135	-74.122	-74.49+ 27.288i	-69.70+ 27.288i
30	1/2	3.0	4	10^{-9}	-55.929	-76.89+ 27.288i	-67.20+ 27.288i	-65.18+ 27.288i
31	3/4	3.0	2	10^{-6}	-48.547	-71.084	-76.28+ 27.288i	-70.28+ 27.288i
32	3/4	3.0	3	10^{-6}	-56.817	-72.76+ 27.288i	-65.60+ 27.288i	-63.86+ 27.288i
33	3/4	3.0	4	10^{-6}	-65.063	-64.26+ 27.288i	-61.36+ 27.288i	-60.38+ 27.288i
34	3/4	3.0	2	10^{-9}	-37.220	-58.428	-74.122	-79.39+ 27.288i
35	3/4	3.0	3	10^{-9}	-46.614	-70.601	-70.96+ 27.288i	-66.18+ 27.288i
36	3/4	3.0	4	10^{-9}	-52.407	-73.36+ 27.288i	-63.68+ 27.288i	-61.66+ 27.288i

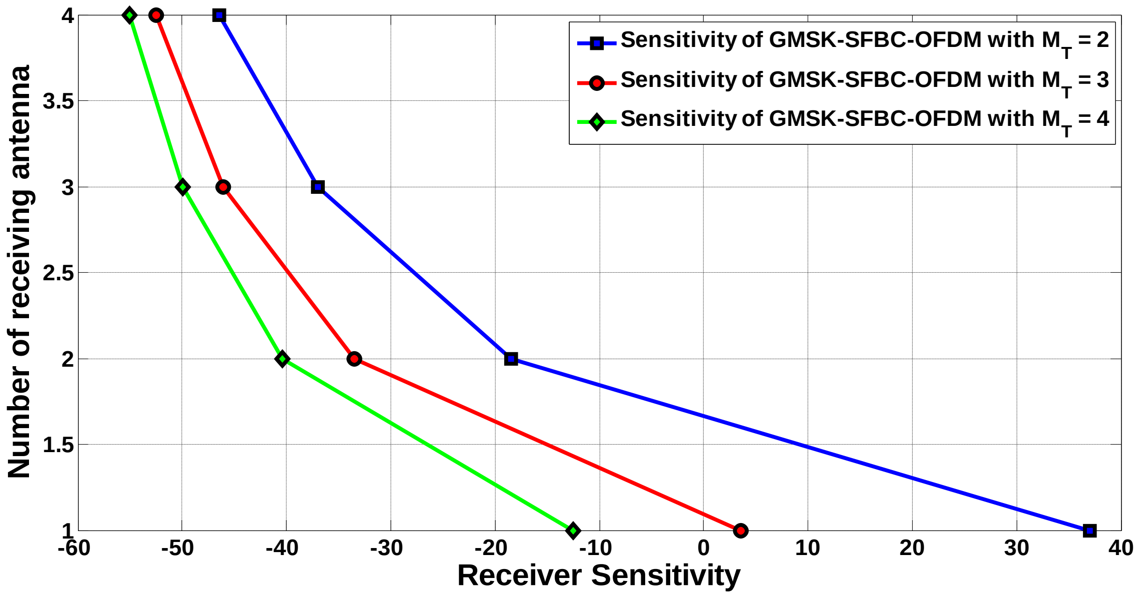

Fig. 3.45: Receiver Sensitivity of GMSK-SFBC-OFDM under Nakagami-m fading environment for code rate, R_C=1/2, Gaussian constant, α = 0.5, Nakagami constant, m=0.5 and average BER=10^{-6}

Table 3.20: Receiver sensitivity of GMSK-SFBC-OFDM with Gaussian constant, α =0.9 in Nakagami-m fading environment with 20MHz bandwidth, 10dB noise figure, 5dB implementation margin

Ser No	Code Rate, R_C	Nakagami Constant, m	#Tx, M_T	$\overline{\text{BER}}$	Rx Sensitivity = (-174+73+10+5)+γ_S dBm			
					#Rx, M_R = 1	#Rx, M_R = 2	#Rx, M_R = 3	#Rx, M_R = 4
1	1/2	0.5	2	10^{-6}	31.905	-23.587	-42.136	-51.507
2	1/2	0.5	3	10^{-6}	-1.5639	-38.614	-51.140	-57.580
3	1/2	0.5	4	10^{-6}	-17.566	-45.481	-55.081	-60.100
4	1/2	0.5	2	10^{-9}	91.905	6.4201	-22.081	-36.354
5	1/2	0.5	3	10^{-9}	38.437	-18.559	-37.604	-47.213
6	1/2	0.5	4	10^{-9}	12.441	-30.333	-44.714	-52.052
7	3/4	0.5	2	10^{-6}	35.427	-20.065	-38.614	-47.980
8	3/4	0.5	3	10^{-6}	1.9579	-35.093	-47.619	-54.058
9	3/4	0.5	4	10^{-6}	-14.045	-41.960	-51.560	-56.578
10	3/4	0.5	2	10^{-9}	95.427	9.9419	-18.559	-32.832
11	3/4	0.5	3	10^{-9}	41.959	-15.037	-34.082	-43.691
12	3/4	0.5	4	10^{-9}	15.963	-26.811	-41.192	-48.530
13	1/2	2.0	2	10^{-6}	-48.034	-66.740	-76.456	-85.506
14	1/2	2.0	3	10^{-6}	-56.244	-72.934	-84.026	-90.95+ 27.288i
15	1/2	2.0	4	10^{-6}	-60.720	-79.485	-88.46+ 27.288i	-80.45+ 27.288i
16	1/2	2.0	2	10^{-9}	-32.645	-57.097	-67.115	-73.894
17	1/2	2.0	3	10^{-9}	-45.043	-63.593	-73.355	-82.656
18	1/2	2.0	4	10^{-9}	-51.076	-67.874	-80.158	-99.56+ 27.288i
19	3/4	2.0	2	10^{-6}	-44.512	-63.218	-72.934	-81.984
20	3/4	2.0	3	10^{-6}	-52.722	-69.412	-84.452	-87.43+ 27.288i
21	3/4	2.0	4	10^{-6}	-57.198	-75.963	-84.93+ 27.288i	-76.93+ 27.288i
22	3/4	2.0	2	10^{-9}	-29.120	-53.575	-63.593	-70.373

23	3/4	2.0	3	10^{-9}	-41.521	-60.072	-69.833	-79.135
24	3/4	2.0	4	10^{-9}	-47.554	-64.352	-76.636	-96.04+ 27.288i
25	1/2	3.0	2	10^{-6}	-57.174	-79.711	-84.91+ 27.288i	-78.91+ 27.288i
26	1/2	3.0	3	10^{-6}	-65.445	-81.38+ 27.288i	-74.23+ 27.288i	-72.48+ 27.288i
27	1/2	3.0	4	10^{-6}	-73.691	-72.89+ 27.288i	-69.99+ 27.288i	-69.00+ 27.288i
28	1/2	3.0	2	10^{-9}	-45.848	-67.055	-82.750	-88.01+ 27.288i
29	1/2	3.0	3	10^{-9}	-55.241	-79.228	-79.59+ 27.288i	-74.81+ 27.288i
30	1/2	3.0	4	10^{-9}	-61.034	-81.99+ 27.288i	-72.31+ 27.288i	-70.28+ 27.288i
31	3/4	3.0	2	10^{-6}	-53.652	-76.190	-81.38+ 27.288i	-75.39+ 27.288i
32	3/4	3.0	3	10^{-6}	-61.923	-77.86+ 27.288i	-70.71+ 27.288i	-68.96+ 27.288i
33	3/4	3.0	4	10^{-6}	-70.169	-69.37+ 27.288i	-66.46+ 27.288i	-65.48+ 27.288i
34	3/4	3.0	2	10^{-9}	-42.326	-63.533	-79.228	-84.49+ 27.288i
35	3/4	3.0	3	10^{-9}	-51.719	-75.706	-76.07+ 27.288i	-71.29+ 27.288i
36	3/4	3.0	4	10^{-9}	-57.512	-78.47+ 27.288i	-68.77+ 27.288i	-66.76+ 27.288i

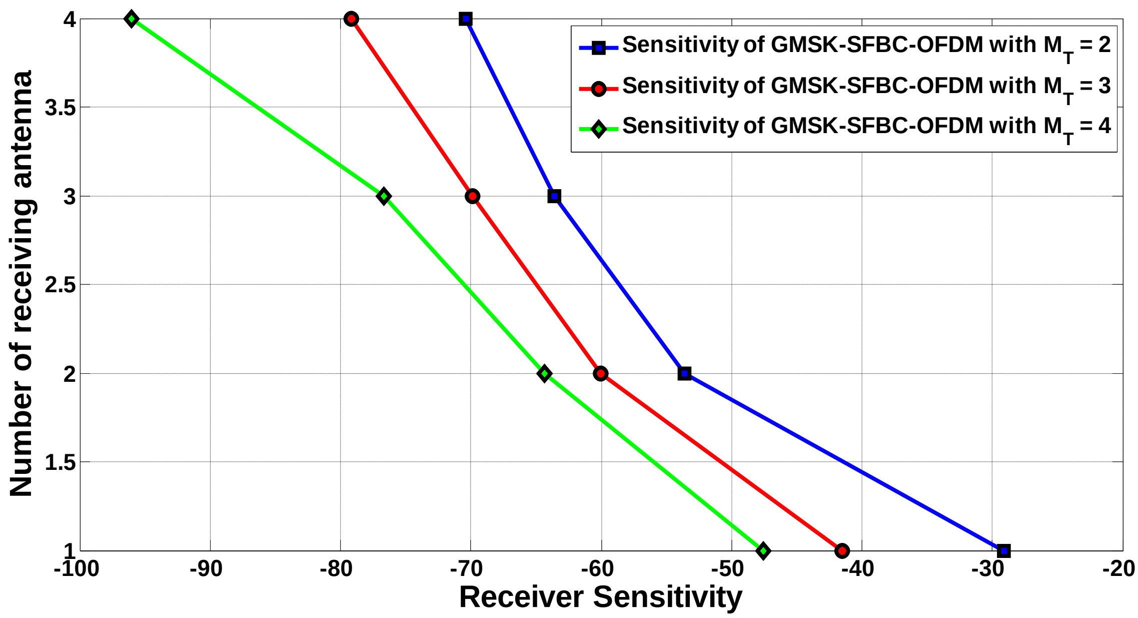

Fig. 3.46: Receiver Sensitivity of GMSK-SFBC-OFDM under Nakagami-m fading environment for code rate, R_C=3/4, Gaussian constant, α = 0.9, Nakagami constant, m=2 and average BER=10^{-9}

The Fig. 3.45 and Fig. 3.46 support the results shown in the Table 3.19 and Table 3.20. With the increment of the value of α, the performance improves for GMSK.

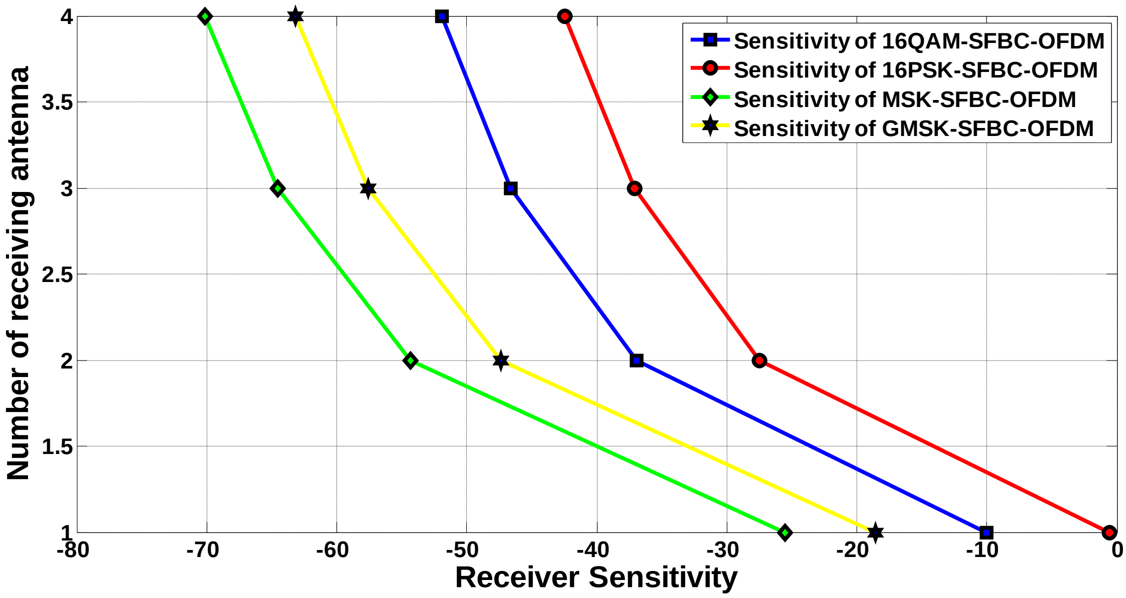

Fig. 3.47: Receiver Sensitivity of 16QAM, 16PSK, MSK and GMSK-SFBC-OFDM in Rayleigh fading environment with $M_T = 4$ and $M_R = 1, 2, 3, 4$, code rate $R_C = 3/4$ and average BER = 10^{-6}

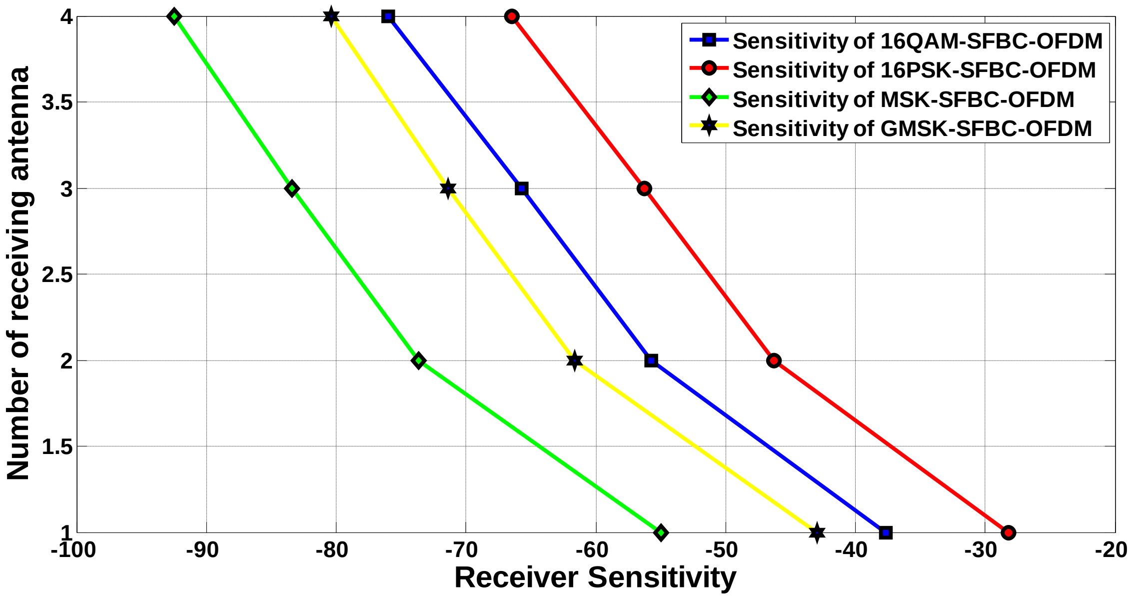

Fig. 3.48: Receiver Sensitivity of 16QAM, 16PSK, MSK and GMSK-SFBC-OFDM in Nakagami-m fading environment with $M_T = 2$ and $M_R = 1, 2, 3, 4$, nakagami constant, $m = 2$, code rate $R_C = 1/2$ and average BER = 10^{-6}

Finally, the above two figures, Fig. 3.47 and Fig. 3.48 show the performance comparison among the four modulation scheme. It is clear that, according to superior performance, results can be stated as: MSK>GMSK>MQAM>MPSK.

CHAPTER 4
CONCLUSIONS

4.1 Conclusions

The outcomes which are obtained from this research are as follows:

a. Analytical expressions of average BER for the four modulation schemes MQAM, MPSK, MSK and GMSK are obtained under both Rayleigh and Nakagami-m fading.

b. Receiver sensitivities are obtained for all the modulation schemes.

c. Comparisons among the results for BER performances and receiver sensitivities for the four modulation schemes are shown in the research.

d. The tables and figures of BER performances have shown that, same average BER has been obtained for lower value of SNR when the number of transmitters or number of receivers are increased with the same code rate. All the modulation schemes have demonstrated the same nature of results in this respect.

e. It is seen from the results that, with the increment of code rate the BER performances have been improved for all the modulations, that is increased code rate has better BER for lower SNR values.

f. The tables and figures for the receiver sensitivities have also shown the same criterion of results as the BER performances. When the number of transmitters or number of receivers has increased the receiver sensitivity have also improved. Increment of code rate also has improved performances.

g. MSK modulation has the best BER performance among the four modulation schemes. GMSK modulation has the next better result. Finally, 16QAM has better performance than 16PSK. So it can be said that, MSK has the most impressive and 16PSK has the least impressive BER performance.

h. In case of receiver sensitivity again the MSK modulation has best performance. The 16PSK modulation has the worst receiver sensitivity among the four modulations.

i. Again GMSK has the better receiver sensitivity than 16QAM.

j. Finally, it is also seen from the tables and figures, Rayleigh distribution has performance in between the Nakagami constant $m = 0.5$ and 2 as for $m = 1$ Rayleigh = Nakagami.

4.2 Future Works

The presented research work is carried out for a fixed value of number of subchannels, number of symbol bits and two different code rates. Future work can be carried out by varying the above parameters. Here, Rayleigh and Nakagami-m distributions are applied, other fading distributions can also be used to evaluate the BER performances and receiver sensitivity. Receiver sensitivity can also be found with 40MHz bandwidth of receiver. Rather than the proposed four modulation schemes, other modulation schemes can also be evaluated. Different detection schemes can be applied to finally detect the decoded results.

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