Performance Analysis of a MIMO-OFDM System with Space Frequency Block Coding with MPSK and MQAM in Rician and Nakagami-m Fading Environments

by

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CERTIFICATION

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ABSTRACT

The approach here is to analyze the performance of SFBC-OFDM system in order to utilize the diversities in frequency and space with MPSK and MQAM modulation scheme over Rician and Nakagami-m fading environment. The analysis is done after deriving the closed-form expressions for MIMO-SFBC-OFDM systems over frequency-selective fading channels for different channel parameters, code rate and antenna configurations. These derived expressions are used also to analyse the difference in performance between coded and uncoded system and to find out the optimum number of antenna for a desired improvement using numerical analysis. And the final approach is to verify the analysis with the existing published literature. It can be seen that that the improvement in SNR(dB) in Nakagami-m fading is better than that of a Rician fading. The proposed expressions can be used to quantify the amount of degradation and improvement in the BER at the receiver.

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ABBREVIATIONS

AWGN	Additive White Gaussian Noise
4G	4th Generation
CP	Cyclic Prefix
CSI	Channel State Information
STBC	Space Time Block Coding
SFBC	Space Frequency Block Coding
PDF	Probability Density Function
FFT	Fast Fourier Transform
BER	Bit Error Rate
IEEE	Institute of Electrical and Electronic Engineers
IFFT	Inverse Fast Fourier Transform
ISI	Inter-Symbol Interference
MPSK	M-ary Phase Shift Keying
MQAM	M-ary Quadrature Amplitude Modulation
LOS	Line of Sight
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
MRC	Maximal Ratio Combining
BPSK	Bipolar Phase Shift Keying
QPSK	Quadrature Phase Shift Keying
OFDM	Orthogonal Frequency Division Multiplexing
STFBC	Space-Time-Frequency Block coding
QAM	Quadrature Amplitude Modulation
ADSL	Asymmetric Digital Subscriber Line
DFT	discrete Fourier Transform
IBI	Inter Block Interference
ICI	Inter Carrier Interference
DSP	Discrete Signal Processor
IDFT	Inverse discrete Fourier Transform
DSTBC	Differential Space Time Block Coding
FPGA	Field Programmable Gate Array
LOS	Line of Sight
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
MRC	Maximal Ratio Combining

CHAPTER 1

INTRODUCTION

This chapter provides a brief introduction of the thesis. This includes introduction to wireless communication, Multi-Carrier Modulation (MCM) and Multiple Access Technique in wireless communication, diversity in wireless communication, review of the research work in MIMO wireless communication system, objective of the thesis and outline of the thesis.

1.1 Introduction to Wireless Communication

The birth of wireless communications dates from the late 1800s, when M.G. Marconi did the pioneer work establishing the first successful radio link between a land-based station and a tugboat. Since then, wireless communication systems have been developing and evolving with a furious pace. Today, facilitated by RF circuit fabrication and digital switching techniques, affordable high speed telecommunication has been largely deployed across the world. The number of mobile subscribers has been growing tremendously in the past decades.

The early wireless systems consisted of a base station with a high-power transmitter and served a large geographic area. Each base station could serve only a small number of users and was costly as well. The systems were isolated from each other and only a few of them communicated with the public switched telephone networks. Today, the cellular systems consist of a cluster of base stations with low-power radio transmitters. Each base station serves a small cell within a large geographic area. The total number of users served is increased because of channel reuse and also larger frequency bandwidth. The cellular systems connect with each other via mobile switching and directly access the public switched telephone networks.

The most advertised advantage of wireless communication systems is that a mobile user can make a phone call anywhere and anytime. We know, communication means transfer of information from source to recipient. In traditional telephony, when source and recipient were located in long distance, this transfer used to happen by connecting source and recipient physically through conducting wires, which would carry information in the form of electrical signals. Any transfer of information between points that do not have a physical connection, like wire or cable connection, would be wireless communication. There are two types of wireless communication system- 1. Short Distance - TV controlled by remote 2. Long Distance - Space Radio Communication. The Components of a wireless communication system are: the source, input transducer, transmitter, channel, output transducer, receiver.

In a cellular network, an entire geographic area is divided into cells, with each cell being served by a base station. Because of the low transmission power at the base station, the same channels can be reused again in another cell without causing too much interference. The configuration and planning of the cell is chosen to minimize the interference from another cell and thus maximum capacity can be achieved. The cell is usually depicted as a hexagon, but in reality the actual shape varies according to the geographic environment and radio propagation.

Channel allocation is chosen based on the density of the users. If a cell has many users to serve, usually more channels are allocated. The channels are then reused in adjacent cells or cluster of cells. The spatial separation of the cells with the same radio channels, in conjunction with the low transmission power and antenna orientation, keeps the co-channel interference at an acceptable level. Mobility is one of the key features in wireless communication systems. There is a need to track the users moving into different cells and changing radio channels. A mobile switched to another channel in a different cell is called handoff. A signaling and call processing procedure is needed to support user mobility and handoff such that a mobile phone can be completed successfully. Paging is another key feature in cellular systems. It uses a common shared channel to locate the users within the service area and to broadcast some signaling messages.

1.2 Multi-Carrier Modulation (MCM) and Multiple Access Technique in Wireless Communication

1.2.1 Multi-Carrier Modulation (MCM)

Multicarrier modulation, MCM is a technique for transmitting data by sending the data over multiple carriers. Multicarrier modulation techniques are particularly beneficial because when the data rates increase, so wider bandwidths are needed. When this happens, different frequencies within the bandwidth are subject to different path lengths and different fading conditions. This can distort the transmission making it difficult to copy. MCM provides a way of increasing the bandwidth whilst still being able to tolerate the varying fading conditions present. A further advantage of multicarrier systems is that they are less susceptible to interference than single carrier system as interference may only affect a small number of the carriers.

Multi-carrier modulation (MCM) is a method of transmitting data by splitting it into several components, and sending each of these components over separate carrier signals. The individual carriers have narrow bandwidth , but the composite signal can have broad bandwidth.

The advantages of MCM include relative immunity to fading caused by transmission over more than one path at a time (multipath fading), less susceptibility than single-carrier systems to interference caused by impulse noise, and enhanced immunity to inter-symbol interference. Limitations include difficulty in synchronizing the carriers under marginal conditions, and a relatively strict requirement that amplification be linear. There are many forms of multicarrier modulation techniques that are in use of being investigated for future use.

Frequency-Division Multiplexing (FDM)

In frequency division multiplexing, the available bandwidth of a single physical medium is subdivided into several independent frequency channels. Independent message signals are translated into different frequency bands using modulation techniques, which are combined by a linear summing circuit in the multiplexer, to a composite signal. The resulting signal is then transmitted along the single channel by electromagnetic means. Basic approach is to divide the available bandwidth of a single physical medium into a number of smaller, independent frequency channels. Using modulation, independent message signals are translated into different frequency bands. All the modulated signals are combined in a linear summing circuit to form a composite signal for transmission. The carriers used to modulate the individual message signals are called sub-carriers. At the receiving end the signal is applied to a bank of band-pass filters, which separates individual frequency channels. The band pass filter outputs are then demodulated and distributed to different output channels. If the channels are very close to one other, it leads to inter-channel cross talk. Channels must be separated by strips of unused bandwidth to prevent inter-channel cross talk. These unused channels between each successive channel are known as guard bands. FDM are commonly used in radio broadcasts and TV networks.

Orthogonal Frequency Division Multiplexing

Frequency Division Multiplexing (FDM) is a technology that transmits multiple signals simultaneously over a single transmission path, such as a cable or wireless system. Each signal travels signal travels within its own unique frequency range (carrier), which is modulated by the data (text, voice, video, etc.).

Orthogonal FDM's (OFDM) spread spectrum technique distributes the data over a large number of carriers that are spaced apart at precise frequencies. This spacing provides the “orthogonality” in this technique, which prevents the demodulators from seeing frequencies other than their own.

The benefits of OFDM are high spectral efficiency, resiliency to RF interference, and lower multi-path distortion. This is useful because in a typical terrestrial broadcasting scenario there are multipath-channels (i.e. the transmitted signal arrives at the receiver using various paths of different length). Since multiple versions of the signal interfere with each other (inter symbol interference (ISI)), it becomes very hard to extract original information.

OFDM is a transmission technique that has been around for years, but only recently became popular due to the development of digital signal processors (DSPs) that can handle its heavy digital processing requirements. OFDM is being implemented in broadband wireless access systems as a way to overcome wireless transmission problems and to improve bandwidth. OFDM is sometimes called multi-carrier or discrete multi-tone modulation.

OFDM is similar to FDM but much more spectrally efficient by spacing the sub-channels much closer together (until they are actually overlapping). This is done by finding frequencies that are orthogonal, which means that they are perpendicular in a mathematical sense, allowing the spectrum of each sub-channel to overlap another without interfering.

1.2.2 Multiple Access Techniques

Multiple access is a technique to allow users to share a communication medium so that the overall capacity can be increased. There are three commonly used multiple access schemes: Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA). In FDMA, each call is assigned its own band of frequency for the duration of the call. The entire frequency band is divided into many small individual channels for users to access. In TDMA, users share the same band of frequencies. Each call is assigned a different time slot for its transmission. In CDMA, users share the same band of frequencies and time slots. Each call is assigned a unique code, which can spread the spectrum to the entire frequency band. The spectrum spread calls are sent on top of each other simultaneously, and are separated at the receiver by an inverse operation of the unique codes. A combination of the three multiple access schemes can also be applied.

FDMA (Frequency Division Multiple Access) each user is allocated a unique frequency band or channel, no other user can share the same frequency band.

TDMA (Time Division Multiple Access) divides the radio spectrum into time slots, and in each slot, only one user is allowed to either transmit or receive.

CDMA (Code Division Multiple Access) each user is assigned a special code sequence (signature) to modulate its message signal, all users are allowed to transmit over the same channel simultaneously and asynchronously.

SDMA (Space Division Multiple Access) controls the radiated energy for each user in space. SDMA serves different users by using spot beam antennas.

1.2.3 Coding Technique in MC-CDMA

The MC-CDMA scheme is a promising technology for future wireless communication systems. Future wireless communication requires a system supporting a large number of users, which can simultaneously provide high data rate. The MC-CDMA is a type of multiple access that utilize the benefits of both CDMA and OFDM schemes. The multi carrier part reduces the multipath fading and ISI, whereas the spread spectrum technology utilizes the limited spectrum in an efficient way. The high data rate transmission will make a resistive channel. The multi carrier part will overcome this problem by transmitting high data rate data into low rate parallel subcarriers. The overlapping of carriers provides high spectral efficiency. Compared to the other multicarrier technique, in a MC-CDMA scheme the original data symbols are first spread with a pseudorandom sequence, followed by the modulation on different carriers. That is, in a MC-CDMA system the chips of same symbol are modulated on different carriers. Hence the spreading is said to be done in frequency domain. Compared to DS-CDMA, in MC-CDMA the codes that is used to distinguish each user are modulated in frequency domain, hence the need of complex rake receiver is avoided.

1.3 Fading

The time variation of received signal power due to changes in transmission medium or paths is known as fading. Fading depends on various factors. In fixed scenario, fading depends on atmospheric conditions such as rainfall, lightening etc. In mobile scenario, fading depends on obstacles over the path which are varying with respect to time. These obstacles create complex transmission effects to the transmitted signal.

1.3.1 Types of Fading

Considering various channel related impairments and position of transmitter/receiver following are the types of fading in wireless communication system. As we know, fading signals occur due to reflections from ground and surrounding buildings as well as scattered signals from trees, people and towers present in the large area. There are two types of fading:

Large Scale Fading: Large scale fading occurs when an obstacle comes in between transmitter and receiver. It includes path loss and shadowing effects. This interference type causes significant amount of signal strength reduction. This is because EM wave is shadowed or blocked by the obstacle. It is related to large fluctuations of the signal over distance.

Small Scale Fading: Small scale fading is concerned with rapid fluctuations of received signal strength over very short distance and short time period. It is divided into two main categories i.e. multipath delay spread and doppler spread. The multipath delay spread is further divided into flat fading and frequency selective fading. Doppler spread is divided into fast fading and slow fading. Based on multipath delay spread there are two types of small scale fading e.g. flat fading and frequency selective fading. These multipath fading types depend on propagation environment.

Flat fading: The wireless channel is said to be flat fading if it has constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal. In this type of fading all the frequency components of the received signal fluctuate in same proportions simultaneously. It is also known as non-selective fading.

Signal BW<<Channel BW
Symbol period>> Delay Spread

The effect of flat fading is seen as decrease in SNR. These flat fading channels are known as amplitude varying channels or narrowband channels.

Frequency Selective fading: It affects different spectral components of a radio signal with different amplitude. Hence the name selective fading.

Signal BW>Channel BW
Symbol period< Delay Spread

Based on Doppler Spread there are two types of fading e.g. fast fading and slow fading. These Doppler spread fading types depend on mobile speed i.e. speed of receiver with respect to transmitter.

Fast Fading: The phenomenon of fast fading is represented by rapid fluctuation of signal over small areas (i.e. bandwidth). When the signals arrive from all the directions in the plane, fast fading will be observed for all directions of motion.

Fast fading occurs when channel impulse response changes very rapidly within the symbol duration.

- High Doppler spread
- Symbol period >Coherence time
- Signal variation < Channel variation

This parameters result into frequency dispersion or time selective fading due to doppler spreading. Fast fading is result of reflections of local objects and motion of objects relative to those objects.

In fast fading, receive signal is sum of numerous signals which are reflected from various surfaces. This signal is sum or difference of multiple signals which can be constructive or destructive based on relative phase shift between them. Phase relationships depend on speed of motion, frequency of transmission and relative path lengths.

Fast fading distorts the shape of the baseband pulse. This distortion is linear and creates ISI (Inter Symbol Interference). Adaptive equalization reduces ISI by removing linear distortion induced by channel.

Slow fading: Slow fading is result of shadowing by buildings, hills, mountains and other objects over the path.

- Low Doppler Spread
- Symbol period <<Coherence Time
- Signal Variation >> Channel Variation

Slow fading results in a loss of SNR. Error correction coding and receiver diversity techniques are used to overcome effects of slow fading.

1.3.2 Fading Models or Fading Distributions

Fading types are implemented in various models or distributions which include Rayleigh, Rician, Nakagami-m etc.

Implementations of fading models or fading distributions include Rayleigh fading, Rician fading, Nakagami-m fading. These channel distributions or models are designed to incorporate fading in the baseband data signal as per fading profile requirements.

Rayleigh fading

In Rayleigh model, only Non Line of Sight (NLOS) components are simulated between transmitter and receiver. It is assumed that there is no LOS path exists between transmitter and receiver. The power is exponentially distributed.The phase is uniformly distributed and independent from the amplitude. It is the most used types of Fading in wireless communication.

Rician fading

In rician model, both Line of Sight (LOS) and non Line of Sight (NLOS) components are simulated between transmitter and receiver.

Nakagami-m Fading

Nakagami-m channels are used when the received signal has contributions from both diffuse and specular scattering, i.e. the electric field is the sum of a strong component (which is not necessarily line of sight) and several contributions with less amplitude. Nakagami-m fading channels are generally used to model multiple paths fading.

The Nakagami distribution or the Nakagami-m distribution is a probability distribution related to the gamma distribution. It has two parameters: a shape parameter $m \geq 1/2$ and a second parameter controlling spread $\Omega > 0$.

1.4 Diversity in Wireless Communication

Diversity is a technique which is used to diminish the channel fading & is often implemented by using two or more receiving antennas. In 3G transmit diversity is used where base stations may transmit replicas of the signal on spatially alienated antennas or frequencies. With an equalizer, diversity improves the quality of a wireless communication link without alerting the common air interface & devoid of increasing the transmitted power or bandwidth. The difference in equalization & diversity is that equalizer technique is used to reduce ISI, whereas diversity technique is used to diminish the effect of fading on wireless communication. Diversity exploits the random nature of radio propagation by finding independent signal paths for communication. Diversity technique is mainly applied on the receiver, & unknown to the transmitter. By this technique the strongest or the best signal is received at the receiver. The following methods are used to obtain uncorrelated signals for combining:

Space diversity: Two antennas separated physically by a short distance can provide two signals with low correlation between their fades. The separation in general varies with antenna height and with frequency. The higher the frequency, the closer the two antennas can be to each other. Typically, a separation of a few wavelengths is enough to obtain uncorrelated signals. Taking into account the shadowing effect, usually a separation of at least 10 carrier wavelengths is required between two adjacent antennas. This diversity does not require extra system capacity; however, the cost is the extra antennas needed.

Frequency diversity: Signals received on two frequencies, separated by coherence bandwidth are uncorrelated. To use frequency diversity in an urban or suburban environment for cellular and personal communications services (PCS) frequencies, the frequency separation must be 300 kHz or more. This diversity improves link transmission quality at the cost of extra frequency bandwidths.

Time diversity: If the identical signals are transmitted in different time slots, the received signals will be uncorrelated, provided the time difference between time slots is more than the channel coherence time. This system will work for an environment where the fading occurs independent of the movement of the receiver. In a mobile radio environment, the mobile unit may be at a standstill at any location that has a weak local mean or is caught in a fade. Although fading still occurs even when the mobile is still, the time-delayed signals are correlated and time diversity will not reduce the fades. In addition to extra system capacity (in terms of transmission time) due to the redundant transmission, this diversity introduces a significant signal processing delay, especially when the channel coherence time is large. In practice, time diversity is more frequently used through bit interleaving, forward-error-correction, and automatic retransmission request (ARQ).

Polarization diversity: The horizontal and vertical polarization components transmitted by two polarized antennas at the base station and received by two polarized antennas at the mobile station can provide two uncorrelated fading signals. Polarization diversity results in 3 dB power reduction at the transmitting site since the power must be split into two different polarized antennas.

Angle diversity: When the operating frequency is 10 GHz, the scattering of signals from transmitter to receiver generates received signals from different directions that are uncorrelated with each other. Thus, two or more directional antennas can be pointed in different directions at the receiving site and provide signals for a combiner. This scheme is more effective at the mobile station than at the base station since the scattering is from local buildings and vegetation and is more pronounced at street level than at the height of base station antennas. Angle diversity can be viewed as a special case of space diversity since it also requires multiple antennas.

Path diversity: In code division multiple access (CDMA) systems, the use of direct sequence spread spectrum modulation allows the desired signal to be transmitted over a frequency bandwidth much larger than the channel coherence bandwidth. The spread spectrum signal can resolve in multipath signal components provided the path delays are separated by at least one chip period. A Rake receiver can separate the received signal components from different propagation paths by using code correlation and can then combine them constructively. In CDMA, exploiting the path diversity reduces the transmitted power needed and increases the system capacity by reducing interference

Types of Antennas

- SISO (Single Input Single Output)
- SIMO (Single Input Multiple Output)
- MISO (Multiple Input Single Output)
- MIMO (Multiple Input Multiple Output)

1.4.1 SISO (Single Input Single Output)

In SISO type of antenna, there is only one transmitting at the transmitter end and one receiving antenna at the receiver end. This makes SISO the simplest to implement and easiest to design amongst all the four types of antennas available. Following is the block diagram of SISO system.

1.4.2 SIMO (Single Input Multiple Output)

In SIMO technique, there is only one transmitting antenna and multiple receiving antennas at receiving end; this helps to increase the receiving diversity at the receiving end as compared with SISO. Following is the block diagram of SIMO system with one transmitting antenna and two receiving antenna at the receiving end for analysis (in this case only two, more than two also possible).

1.4.3 MISO (Multiple Input Single Output)

In MISO, there can be multiple transmitting antennas from which the signal can be sent, and there I only one receiving antenna to receive the signals coming from multiple transmitting antenna, which means there are different sources available but there is only one destination available. Following is the block diagram of MISO system with two (in this case only two, more than two also possible) transmitting antenna and one receiving antenna at the receiving end for analysis.

1.4.4 MIMO (Multiple Input Multiple Output)

In MIMO, there can be multiple transmitting antennas from which the signal can be sent, and also there are multiple receiving antennas through which the signal can be received. In MIMO, since there can be multiple transmitting antennas the signal can be transmitted by any antenna and therefore the signal can follow any path to reach to receiving end and this path followed by the signal depends on the position of the antenna i.e. if we move the antenna by small position the path will get change. The fading introduced in the signal from multiple paths can be termed as multipath fading. Following is the block diagram of MIMO system with N (in this case only two considered for practical analysis, more than two also possible) transmitting antenna and M (in this case only two considered for practical analysis, more than two also possible) receiving antenna at the receiving end for analysis.

1.4.5 Importance of STBC and SFBC

In the past years, Multiple-Input Multiple-Output (MIMO) wireless communications has received much interest. Multiple antennas are employed at both the receiver and the transmitter in a MIMO communication system to enhance channel capacity. Shannon Capacity Limit is difficult to be reached for Single Input Single Output (SISO) systems. It has been demonstrated in that further increases in channel capacity can be gained by the use of MIMO systems. Under ideal propagation conditions, the capacity limit grows linearly with the number of antennas. Space-time block coding (STBC) and Space Frequency block Code (SFBC) has emerged as major techniques to exploit the MIMO benefit. Both spatial and temporal diversity are achieved in STBC and SFBC. STBC and SFBC also offer simple decoding with the use of maximum likelihood detection algorithm at the receiver .

Other types of codes based on STBC and SFBC have then emerged and are of most interest as both full rate and full diversity can be achieved contrary to the STBC where full rate cannot be achieved for more than two transmit antennas. Therefore research is focusing on improving the complexity of the algorithm in order for the STBC and SFBC to be easier to implement.

These two codes are of much interest but require that channel parameters are known at the receiver to recover the transmitted data signal. Therefore, to counteract the need for channel estimation, codes based on differential modulation where the next transmitted symbol is phase shifted compared to the previously transmitted symbol. Detection and demodulation is based on the same principle. Complexity is of some concern to researchers. STBC and SFBC offer lower complexity at the transmitter and receiver for full diversity codes.

1.5 Review of Research Work in MIMO Wireless Communication System

Orthogonal Frequency Division Multiplexing (OFDM) is considered to be the most promising transmission technique to support future wireless multimedia communications because of its excellent performance in combating multipath fading as well as inter symbol interference (ISI). Multiple Input Multiple Output (MIMO)-OFDM system is an efficient approach to combat the adverse effects multipath spread in fading environment and to improve the bit error rate (BER) [1]-[3]. In diversity mechanisms, there are various ways of obtaining independently faded signals such as frequency, time, space, angle and path diversity [4].The diversity mechanism along with OFDM combined with channel coding such as Space Time Block Coding or Space Frequency Block Coding can be adopted to abate the fading effect [5]-[6].

Space Time Block Coding (STBC) has been developed to produce codes with different multiplexing and diversity gains for the MIMO antenna system [7]-[9]. Space Frequency Block Coding (SFBC) is also used to reduce the effect of destructive fading by passing the information symbols through multiple independently faded paths which benefits from the maximum coding advantage [10]-[15].

The conventional approach is to apply STBC in wireless communication system to reduce the fading effect and some researches has already been carried out on STBC-OFDM system [7]-[9]. SFBC-OFDM and also Space-Time-Frequency Block coding (STFBC) systems are efficient in handling frequency selective fading in wireless communication system. Some existing researches have already carried out considering only Rayleigh fading environment [10]-[15]. No analytical results are reported on SFBC-OFDM considering Rician and Nakagami-m fading environments.

In this work, analysis will be carried out for a wireless communication system with SFBC-OFDM for MPSK and MQAM over Rician and Nakagami-m fading channels. Based on Alamouti's encoding system we will consider Space Frequency Block Coded MIMO-OFDM system. SFBC-OFDM system model will be developed based on different antenna configurations. The BER performance with closed form expressions of MIMO-SFBC-OFDM system with MPSK and MQAM modulation schemes considering Rician and Nakagami-m fading channels will be evaluated conditioned on different channel parameters. The conditional BER will be derived and the overall BER expression will be analytically found by averaging the conditional BER over probability distribution function of the fading parameters. Analysis will be done based on different code rates as well as different antenna systems. Finally the results will be evaluated using numerical analysis. System performance results will be compared with different configurations for SFBC coded MIMO-OFDM system through numerical analysis. BER performance between uncoded and SFBC coded MIMO-OFDM system will be compared. System model with different antenna configurations will be analysed and the optimum number of antenna for desired improvement will be found out. Finally, to validate the analytical model, the analytical results will be compared to the simulation results published in the referred journals considering same system parameters.

1.6 Objective of the Thesis

The main objectives of this thesis are-

i. To find out the closed form expressions of BER to analyze the performance of the MIMO-OFDM wireless communication system with Space Frequency Block Coding using MPSK and MQAM modulation scheme over frequency selective Rician and Nakagami-m fading environment.

ii. To evaluate and compare analytically the BER performance results for Rician and Nakagami-m fading both in uncoded and coded system for different channel parameters, code rate, antenna configurations.

iii. To compare the analytical results with the published simulations results for same system parameters.

1.7 Outline of the thesis

The thesis is organised in the following way:

i. Chapter 2 reviews SFBC-OFDM system model with different number of antennas and analyzes the bit-error rate (BER) performance or average BER of SFBC-OFDM system with MPSK and MQAM modulation in Rician and Nakagami-m fading environment.

ii. Chapter 3 presents all the system performance results and also discusses and compares the results with others.

iii. Chapter 4 concludes the thesis and proposes the future work.

CHAPTER 2

ANALYSIS OF SFBC-MIMO-OFDM SYSTEM

2.1 SFBC-OFDM System Model

Figure 2.1 shows a block diagram of the SFBC-OFDM system with M_T transmit and M_R receive antennas.

A stream of information is first converted from serial to parallel. Then, all subbands are modulated using MQAM or MPSK and a signal vector S is provided as the input to the SFBC encoder. SFBC provides M_T blocks.

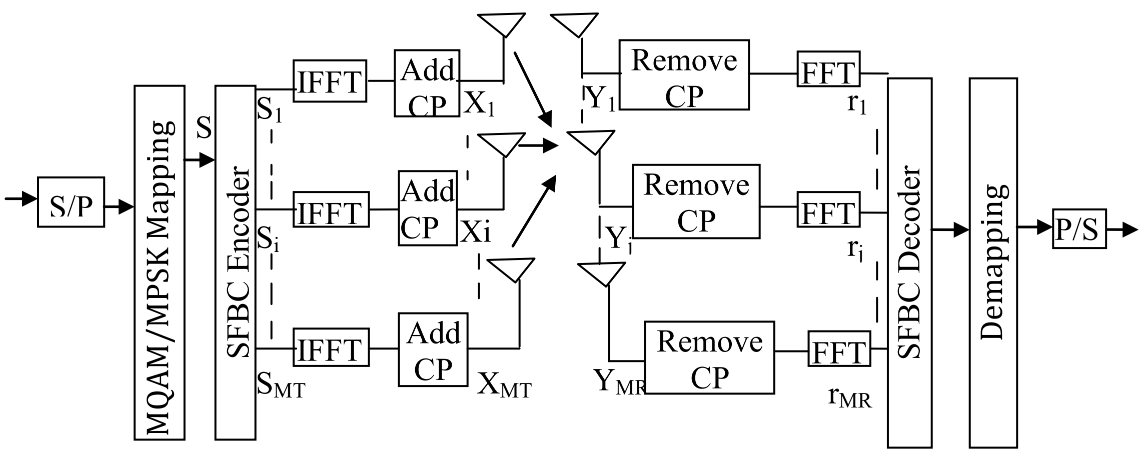

Fig. 2.1: SFBC-OFDM block diagram

Then, OFDM modulators generate blocks $\mathbf{X}_1$, $\mathbf{X}_2$,…$\mathbf{X}_{MT}$ to be transmitted by the first, second, ...,and M_T-th transmit antenna simultaneously. Then, after the cyclic prefix removal and FFT at the receiver side, the demodulated signal is received at the *j*-th receive antenna. Maximum Likelihood (ML) detection can be used for the SFBC decoding of the received signal, which is a simple linear process to extract the information.

Assuming OFDM with N subcarriers (or subchannels), let Ns be the number of subbands chosen to be Ns = N/q, i.e. each subband includes q adjacent subchannels where q is the symbol period of the SFBC system. All the subbands are modulated using MQAM or MPSK where M is determined by the number of allocated bits. A signal vector S ={s[0], s[1],….s[Nt − 1]}is provided as the input to the SFBC encoder, where N_t is equal to N multiplied by the SFBC code rate, R_c. A space-frequency block code is defined by a $\frac{N}{q}\times M_T$ transmission matrix G given by:

$$G = \begin{pmatrix} s_1[0] & s_2[0] & \dots & s_{MT}[0] \\ s_1[1] & s_2[1] & \dots & s_{MT}[1] \\ \vdots & \vdots & \vdots & \vdots \\ s_1[\frac{N}{q}-1] & s_2[\frac{N}{q}-1] & \dots & s_{MT}[\frac{N}{q}-1] \end{pmatrix} \quad \text{….. (2.1.1)}$$

where each element $s_{M_T}[\frac{N}{q}-1]$ is a linear combination of a subset of elements of S and their conjugates. The $\frac{N}{q}\times M_T$ transmission matrix **G** is based on a complex generalized orthogonal design. In order to utilize the space frequency diversity, the input blocks for OFDM at each transmit antenna should be of length N. SFBC provides M_T blocks, $\mathbf{S}_1$, $\mathbf{S}_2$,...$\mathbf{S}_{MT}$, each of length N and consisting of $\frac{N}{q}$ sub-blocks, i.e., $\mathbf{S}i = (\mathbf{s}_i[0]\ \mathbf{s}_i[1]\ ...\ \mathbf{s}_i[\frac{N}{q}-1])^T$ for i =1,2,..,M_T, where the superscript (.)T denotes the transpose operator. Then, OFDM modulators generate blocks $\mathbf{X}_1$, $\mathbf{X}_2$,...$\mathbf{X}_{MT}$ to be transmitted by the first, second, ...,and M_T-th transmit antenna simultaneously. Given that the guard time interval is longer than the delay spread of the multipath channel (to avoid inter symbol interference - ISI), there received signal will be the convolution of the channel and the transmitted signal. In typical indoor environments, the fading channel varies slowly and can be assumed to be static over an OFDM block. We assume that the fading process remains static during each OFDM block (one frequency slot) and that it varies from one block to another. The fading processes associated with different transmit-receive antenna pairs are considered to be uncorrelated and we assume perfect synchronization. The fading process impulse response of the link between the i-th transmit antenna and the j-th receive antenna can be expressed as:

$$h_{j,i}(t) = \sum_{m=0}^{L-1} \alpha_{m,j,i}(t)\delta(t-\tau_m(t)) \qquad \text{..... (2.1.2)}$$

where, (i =1,...,M_T ; j = 1,…,M_R); $h_{j,i}(t)$ is the tap weight, $\tau_m(t)$is the time delay of the m-th path and L is the total number of resolvable paths. The $\alpha_{m,j,i}(t)$'s are complex Gaussian random processes with zero mean and variance 1/L (equal power). With this model, we assume that the path delays, $\tau m(t)$, are multiples of the symbol duration Ts. Then, after the cyclic prefix removal and FFT at the receiver side, the demodulated received signal at the j-th receive antenna can be expressed as:

$$\mathrm{r}_j = \sum_{i=1}^{M_T} \mathrm{H}_{j,i}\mathrm{S}_i + \mathrm{W}_j \qquad \text{..... (2.1.3)}$$

where $\mathbf{r}_j = (r_j[0],..,r_j\ [N-1])^T$, $\mathbf{S}i = (\mathbf{s}_i[0],...,\mathbf{s}_i[\frac{N}{q}-1])^T$ is the transmitted signal at the i-th antenna, $\mathbf{W}j=(Wj[0],...Wj\ [N-1])^T$ denotes the AWGN, and $\mathbf{H}_{j,i} = diag\,\{\mathbf{H}_{j,i}[k]\}_{k=0}^{N-1}$ is an $N\times N$ diagonal matrix with elements corresponding to the DFT of the channel response between the i-th transmit and j-th receive antennas. Finally, under channel information knowledge at the receiver, Maximum Likelihood (ML) detection can be used for the SFBC decoding of the received signal, which is a simple linear process, and the elements of the block $S = \{s[m]\}_{m=0}^{Nt-1}$ are demodulated to extract the information.

2.1.1 SFBC-OFDM Systems for 2 Transmit Antennas

Consider an SFBC-OFDM system with two transmit antennas ($M_T = 2$).

The figure 2.2 shows the frequency vs time plots for 2 transmit antennas of SFBC-OFDM systems.

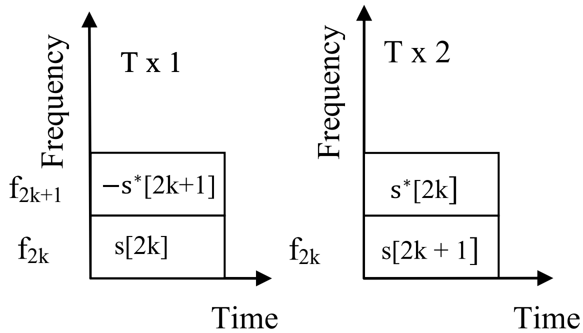

Fig. 2.2: Illustration of SFBC-OFDM with 2Tx-M_RRx antennas

Like in conventional OFDM, the sequence of input data is converted into parallel form, generating block $S = \{s[1], s[2], \ldots, s[N-1]\}$ where N is the block size chosen to be equal to the FFT size in the OFDM system[15].

Let's define sub-blocks $\mathbf{s}_1[k]$ and $\mathbf{s}_2[k]$ as

$\mathbf{s}_1[k] = (s[2k] \quad -s^*[2k+1])$ and
$\mathbf{s}_2[k] = (s[2k+1] \quad s^*[2k])$ respectively (Fig. 2.2). ….. (2.1.1.1)

Then, the orthogonal block code for two transmit antennas Using the code $\mathbf{G}_2$ can be written as

$$\mathbf{G}_2 = \begin{bmatrix} s[2k] & s[2k+1] \\ -s^*[2k+1] & s^*[2k] \end{bmatrix} \qquad k = 0, \ldots\ldots, \frac{N}{2} - 1$$

Or, $\mathbf{G}_2 = [(\boldsymbol{s_1}[k])^T \quad (\boldsymbol{s_2}[k])^T]$ ….. (2.1.1.2)

SFBC provides two blocks $\mathbf{S}_1$ and $\mathbf{S}_2$, each of the length N, for OFDM at the transmitter. In order to utilize the space frequency diversity, the input blocks are encoded as follows:

$$\mathbf{S}_1 = (s[0] \quad -s^*[1] \quad s[2] \quad -s^*[3] \quad \ldots\ldots\ldots \quad s[N-2] \quad -s^*[N-1])^T$$

$$\mathbf{S}_2 = (s[1] \quad +s^*[0] \quad s[3] \quad +s^*[2] \quad \ldots\ldots\ldots \quad s[N-1] \quad +s^*[N-2])^T.$$

….. (2.1.1.3)

OFDM modulators generate blocks $\mathbf{X}_1$ and $\mathbf{X}_2$, corresponding to $\mathbf{S}_1$ and $\mathbf{S}_2$, that are transmitted by the first and second transmit antenna respectively. Then given that the channel is assumed static during an OFDM block, after removing the cyclic prefix at the receiver side, the FFT output as the demodulated received signal at the j-th receive antenna can be expressed as:

$$\mathbf{r}_j = \mathbf{H}_{j,1}\,\mathbf{S}_1 + \mathbf{H}_{j,2}\,\mathbf{S}_2 + \mathbf{W}_j \qquad \text{..... (2.1.1.4)}$$

where $\mathbf{r}_j$, $\mathbf{H}_{j,1}$, $\mathbf{H}_{j,2}$ and $\mathbf{W}_j$ are as defined. ML detection can be used for decoding the received signal[16],

So, this detection scheme can be written as [Appendix-A]:

$$\tilde{s}[2k] = \sum_{j=1}^{M_R}\left(\mathrm{H}_{j,1}^{*}[2k]\mathrm{r}_j[2k] + \mathrm{H}_{j,2}[2k]\mathrm{r}_j^{*}[2k+1]\right),$$
$$\tilde{s}[2k+1] = \sum_{j=1}^{M_R}\left(\mathrm{H}_{j,2}^{*}[2k+1]\mathrm{r}_j[2k] - \mathrm{H}_{j,1}[2k+1]\mathrm{r}_j^{*}[2k+1]\right) \qquad \text{......(2.1.1.5)}$$

Then, assuming that the channel gains between the two adjacent subchannels are approximately equal, i.e., $H_{j,1}[2k] = H_{j,1}[2k+1]$ and $H_{j,2}[2k] = H_{j,2}[2k+1]$, and substituting (2.1.1.5) into (2.1.1.2), the decoded signal can be expressed as:

$$\tilde{s}[2k] = \sum_{j=1}^{M_R}\left(\left|\mathrm{H}_{j,1}^{*}[2k]\right|^2 + \left|\mathrm{H}_{j,2}[2k]\right|^2\right)\mathrm{s}[2k] + \eta[2k],$$
$$\tilde{s}[2k+1] = \sum_{j=1}^{M_R}\left(\left|\mathrm{H}_{j,1}^{*}[2k]\right|^2 + \left|\mathrm{H}_{j,2}[2k]\right|^2\right)\mathrm{s}[2k+1] + \eta[2k+1] \qquad \text{......(2.1.1.6)}$$

where η is the equivalent noise component given by

$$\eta[2k] = \sum_{j=1}^{M_R}\left(\mathrm{H}_{j,1}^{*}[2k]\mathrm{W}_j[2k] + \mathrm{H}_{j,2}[2k]\mathrm{W}_j^{*}[2k+1]\right),$$
$$\eta[2k+1] = \sum_{j=1}^{M_R}\left(\mathrm{H}_{j,2}^{*}[2k+1]\mathrm{W}_j[2k] - \mathrm{H}_{j,1}[2k+1]\mathrm{W}_j^{*}[2k+1]\right). \qquad \text{......(2.1.1.7)}$$

The above decision variables provide a diversity gain of order two for every $s[2k]$ and $s[2k+1]$. As can be seen, the total channel gain is the sum of squares of two channel gains. Therefore, the proposed scheme can provide significant performance gains over conventional OFDM.

2.1.2 SFBC-OFDM Systems for 3 Transmit Antennas

Consider an SFBC-OFDM system with three transmit antennas ($M_T = 3$).

The figure 2.3 shows the frequency vs time plots for 3 transmit antennas of SFBC-OFDM systems.

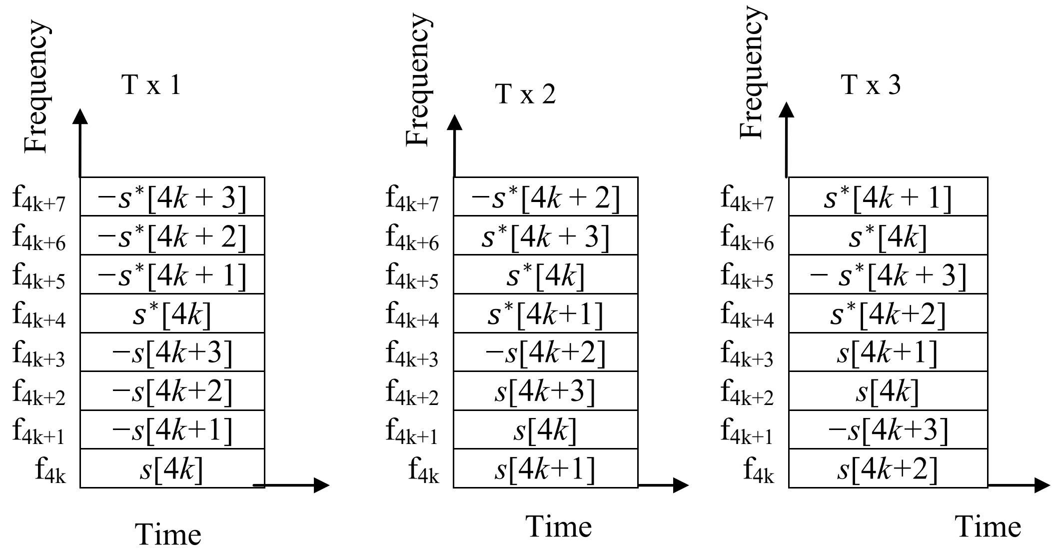

Fig. 2.3: Illustration of SFBC-OFDM with 3Tx-M_RRx antennas

Like in conventional OFDM, the sequence of input data is converted into parallel form, generating block $S = \{s[1], s[2], ..., s[N-1]\}$ where N is the block size chosen to be equal to the FFT size in the OFDM system.

Let's define sub-blocks $\mathbf{s}_1[k]$, $\mathbf{s}_2[k]$ and $\mathbf{s}_3[k]$ as

$$\mathbf{s}_1[k] = (s[4k] \quad -s[4k+1] \quad -s[4k+2] \quad -s[4k+3] \quad s^*[4k] \quad -s^*[4k+1] \quad -s^*[4k+2] \quad -s^*[4k+3])$$

$$\mathbf{s}_2[k] = (s[4k+1] \quad s[4k] \quad s[4k+3] \quad -s[4k+2] \quad s^*[4k+1] \quad s^*[4k] \quad s^*[4k+3] \quad -s^*[4k+2])$$

$$\mathbf{s}_3[k] = (s[4k+2] \quad -s[4k+3] \quad s[4k] \quad s[4k+1] \quad s^*[4k+2] \quad -s^*[4k+3] \quad s^*[4k] \quad s^*[4k+1])$$

respectively (Fig. 2.3).

……(2.1.2.1)

Then, the orthogonal block code for three transmit antennas Using the code $\mathbf{G}_3$ can be written as

$$\mathbf{G}_3 = \begin{bmatrix} s[4k] & (s[4k+1] & (s[4k+2] \\ -s[4k+1] & s[4k] & -s[4k+3] \\ -s[4k+2] & s[4k+3] & s[4k] \\ -s[4k+3] & -s[4k+2] & s[4k+1] \\ s^*[4k] & s^*[4k+1] & s^*[4k+2] \\ -s^*[4k+1] & s^*[4k] & -s^*[4k+3] \\ -s^*[4k+2] & s^*[4k+3] & s^*[4k] \\ -s^*[4k+3] & -s^*[4k+2] & s^*[4k+1] \end{bmatrix} \qquad k = 0, \ldots\ldots, \frac{N}{8} - 1$$

Or, $\mathbf{G}_3 = [(\boldsymbol{s_1}[k])^T \quad (\boldsymbol{s_2}[k])^T \quad (\boldsymbol{s_3}[k])^T]$ ……(2.1.2.1)

SFBC provides three blocks $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$ each of the length N, for OFDM at the transmitter.

$S_i = [s_i[0] \quad s_i[1] \ldots\ldots\ldots\ldots s_i[\frac{N}{8}-1]]^T$ ……(2.1.2.2)

In order to utilize the space frequency diversity, the input blocks are encoded as follows:

$$\mathbf{S}_1 = (s[0] \quad -s[1] \quad -s[2] \quad -s[3] \quad s^*[0] \quad -s^*[1] \quad -s^*[2] \quad -s^*[3] \ldots$$
$$s[\tfrac{N}{2}-4] \; -s[\tfrac{N}{2}-3] \; -s[\tfrac{N}{2}-2] \; -s[\tfrac{N}{2}-1] \; s^*[\tfrac{N}{2}-4] \, -s^*[\tfrac{N}{2}-3] \; -s^*[\tfrac{N}{2}-2] \, -s^*[\tfrac{N}{2}-3])^T$$

$$\mathbf{S}_2 = (s[1] \quad s[0] \quad s[3] \quad -s[2] \quad s^*[1] \quad s^*[0] \quad s^*[3] \quad -s^*[2] \ldots$$
$$s[\tfrac{N}{2}-3] \; s[\tfrac{N}{2}-4] \; s[\tfrac{N}{2}-1] \; -s[\tfrac{N}{2}-2] \; s^*[\tfrac{N}{2}-3] \; s^*[\tfrac{N}{2}-4] \, s^*[\tfrac{N}{2}-1] \; -s^*[N-2])^T$$

$$\mathbf{S}_3 = (s[2] \quad -s[3] \quad s[0] \quad s[1] \quad s^*[2] \quad -s^*[3] \quad s^*[0] \quad s^*[1] \ldots$$
$$s[\tfrac{N}{2}-2] \; -s[\tfrac{N}{2}-1] \; s[\tfrac{N}{2}-4] \; s[\tfrac{N}{2}-3] \; s^*[\tfrac{N}{2}-2] \; -s^*[\tfrac{N}{2}-1] \; s^*[\tfrac{N}{2}-4] \; s^*[\tfrac{N}{2}-3])^T$$

…… (2.1.2.3)

OFDM modulators generate blocks $\mathbf{X}_1$, $\mathbf{X}_2$ and X_3 corresponding to $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$, that are transmitted by the first, second and third transmit antenna respectively. Then given that the channel is assumed static during an OFDM block, after removing the cyclic prefix at the receiver side, the FFT output as the demodulated received signal at the j-th receive antenna can be expressed as:

$$\mathbf{r}_j = \mathbf{H}_{j,1}\,\mathbf{S}_1 + \mathbf{H}_{j,2}\,\mathbf{S}_2 + \mathbf{H}_{j,3}\,\mathbf{S}_3 + \mathbf{W}_j \qquad \text{….. (2.1.2.4)}$$

where $\mathbf{r}_j$, $\mathbf{H}_{j,1}$, $\mathbf{H}_{j,2}$, $\mathbf{H}_{j,3}$ and $\mathbf{W}_j$ are as defined. ML detection can be used for decoding the received signal.

Assuming channel H is known at the receiver, the ML estimate is obtained by performing $\underset{G_2}{min}\left\|r_j - G_3 H\right\|_F^2$ which is the Frobenius form. The receiver decodes s[4k], s[4k+1], s[4k+2] and s[4k+3] by decomposing the measure $\left\|r_j - G_3 H\right\|_F^2$ into four parts, minimizes each separately over all possible values of s[4k], s[4k+1], s[4k+2] and s[4k+3] that belong to the constellation used [16]. Similarly, like the formula used in two transmit antennas, this detection scheme can be written as [Following Appendix-A]:

$$\tilde{s}[4k] = \sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,1}^*[8k]r_j[8k] & +H_{j,2}^*[8k]r_j[8k+1] & +H_{j,3}^*[8k]r_j[8k+2] \\ +H_{j,1}[8k]r_j^*[8k+4] & +H_{j,2}[8k]r_j^*[8k+5] & +H_{j,3}[8k]r_j^*[8k+6] \end{pmatrix},$$

$$\tilde{s}[4k+1] = \sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,2}^*[8k]r_j[8k] & -H_{j,1}^*[8k]r_j[8k+1] & +H_{j,3}^*[8k]r_j[8k+3] \\ +H_{j,2}[8k]r_j^*[8k+4] & -H_{j,1}[8k]r_j^*[8k+5] & +H_{j,3}[8k]r_j^*[8k+7] \end{pmatrix},$$

$$\tilde{s}[4k+2] = \sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,3}^*[8k]r_j[8k] & -H_{j,1}^*[8k]r_j[8k+2] & -H_{j,2}^*[8k]r_j[8k+3] \\ +H_{j,3}[8k]r_j^*[8k+4] & +H_{j,1}[8k]r_j^*[8k+6] & +H_{j,2}[8k]r_j^*[8k+7] \end{pmatrix},$$

$$\tilde{s}[4k+3] = \sum_{j=1}^{M_R} \begin{pmatrix} -H_{j,3}^*[8k]r_j[8k+1] & +H_{j,2}^*[8k]r_j[8k+2] & -H_{j,2}^*[8k]r_j[8k+3] \\ -H_{j,3}[8k]r_j^*[8k+5] & +H_{j,2}[8k]r_j^*[8k+6] & -H_{j,2}[8k]r_j^*[8k+7] \end{pmatrix},$$

…… (2.1.2.5)

Assuming that the channel gains between the two adjacent subchannels are approximately equal, i.e.

$H_{j,1}[8k] = H_{j,1}[8k+m]$, $H_{j,2}[8k] = H_{j,2}[8k+m]$ & $H_{j,3}[8k] = H_{j,3}[8k+m]$,

Then, the decoded signal can be expressed as:

$$\tilde{s}[4k] = 2\sum_{j=1}^{M_R} \left(|H_{j,1}[8k]|^2 + |H_{j,2}[8k]|^2 + |H_{j,3}[8k]|^2\right) s[4k] + \eta[4k],$$

$$\tilde{s}[4k+1] = 2\sum_{j=1}^{M_R} \left(|H_{j,1}[8k]|^2 + |H_{j,2}[8k]|^2 + |H_{j,3}[8k]|^2\right) s[4k+1] + \eta[4k+1]$$

$$\tilde{s}[4k+2] = 2\sum_{j=1}^{M_R} \left(|H_{j,1}[8k]|^2 + |H_{j,2}[8k]|^2 + |H_{j,3}[8k]|^2\right) s[4k+2] + \eta[4k+2]$$

$$\tilde{s}[4k+3] = 2\sum_{j=1}^{M_R} \left(|H_{j,1}[8k]|^2 + |H_{j,2}[8k]|^2 + |H_{j,3}[8k]|^2\right) s[4k+3] + \eta[4k+3]$$

…… (2.1.2.6)

where η is the equivalent noise component given by

$$\eta[4k] = \sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,1}^*[8k]W_j[8k] & +H_{j,2}^*[8k]W_j[8k+1] & +H_{j,3}^*[8k]W_j[8k+2] \\ +H_{j,1}[8k]W_j^*[8k+4] & +H_{j,2}[8k]W_j^*[8k+5] & +H_{j,3}[8k]W_j^*[8k+6] \end{pmatrix},$$

$$\eta[4k+1] = \sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,2}^*[8k]W_j[8k] & -H_{j,1}^*[8k]W_j[8k+1] & +H_{j,3}^*[8k]W_j[8k+3] \\ +H_{j,2}[8k]W_j^*[8k+4] & -H_{j,1}[8k]W_j^*[8k+5] & +H_{j,3}[8k]W_j^*[8k+6] \end{pmatrix},$$

$$\eta[4k+2] = \sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,3}^*[8k]W_j[8k] & -H_{j,1}^*[8k]W_j[8k+2] & -H_{j,2}^*[8k]W_j[8k+3] \\ +H_{j,3}[8k]W_j^*[8k+4] & +H_{j,1}[8k]W_j^*[8k+5] & +H_{j,2}[8k]W_j^*[8k+6] \end{pmatrix},$$

$$\eta[4k+3] = \sum_{j=1}^{M_R} \begin{pmatrix} -H_{j,3}^*[8k]W_j[8k+1] & +H_{j,2}^*[8k]W_j[8k+2] & -H_{j,2}^*[8k]W_j[8k+3] \\ -H_{j,3}[8k]W_j^*[8k+5] & +H_{j,2}[8k]W_j^*[8k+6] & +H_{j,2}[8k]W_j^*[8k+7] \end{pmatrix},$$

…… (2.1.2.7)

The above decision variables provide a diversity gain of order two for every s[4k], s[4k + 1], s[4k +2] and s[4k + 3]. As can be seen, the total channel gain is the sum of squares of three channel gains. Therefore, the proposed scheme can provide significant performance gains over conventional OFDM.

2.1.3 SFBC-OFDM Systems for 4 Transmit Antennas

Consider an SFBC-OFDM system with three transmit antennas (M_T = 4). The figure 2.4 shows the frequency vs time plots for 3 transmit antennas of SFBC-OFDM systems.

Like in conventional OFDM, the sequence of input data is converted into parallel form, generating block $S = \{s[1], s[2], ..., s[N-1]\}$ where N is the block size chosen to be equal to the FFT size in the OFDM system. Let's define sub-blocks $s_1[k]$, $s_2[k]$, $s_3[k]$ and $s_4[k]$ as

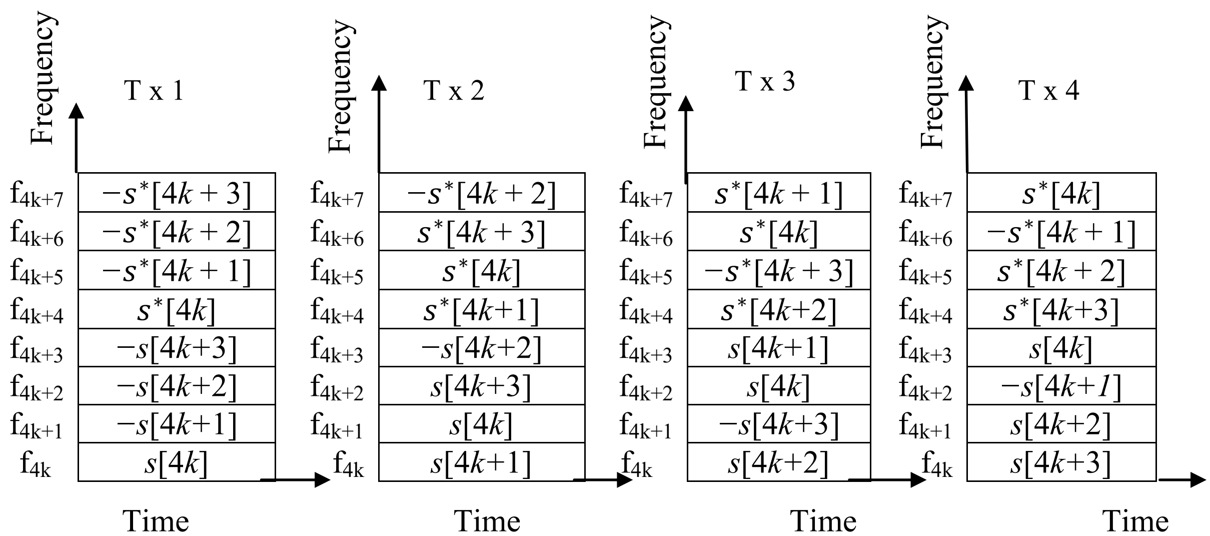

Fig. 2.4: Illustration of SFBC-OFDM with 4Tx-M_RRx antennas

$$s_1[k] = (s[4k] \quad -s[4k+1] \quad -s[4k+2] \quad -s[4k+3]$$
$$s^*[4k] \quad -s^*[4k+1] \quad -s^*[4k+2] \quad -s^*[4k+3])$$

$$s_2[k] = (s[4k+1] \quad s[4k] \quad s[4k+3] \quad -s[4k+2]$$
$$s^*[4k+1] \quad s^*[4k] \quad s^*[4k+3] \quad -s^*[4k+2])$$

$$s_3[k] = (s[4k+2] \quad -s[4k+3] \quad s[4k] \quad s[4k+1]$$
$$s^*[4k+2] \quad -s^*[4k+3] \quad s^*[4k] \quad s^*[4k+1])$$

$$s_4[k] = (s[4k+3] \quad s[4k+2] \quad -s[4k+1] \quad s[4k]$$
$$s^*[4k+3] \quad s^*[4k+2] \quad -s^*[4k+1] \quad s^*[4k])$$ respectively (Fig. 2.4)

…… (2.1.3.1)

Then, the orthogonal block code for three transmit antennas Using the code G_3 can be written as

$$G_4 = \begin{bmatrix} s[4k] & (s[4k+1] & (s[4k+2] & s[4k+3] \\ -s[4k+1] & s[4k] & -s[4k+3] & s[4k+2] \\ -s[4k+2] & s[4k+3] & s[4k] & -s[4k+1] \\ -s[4k+3] & -s[4k+2] & s[4k+1] & s[4k] \\ s^*[4k] & s^*[4k+1] & s^*[4k+2] & s^*[4k+3] \\ -s^*[4k+1] & s^*[4k] & -s^*[4k+3] & s^*[4k+2] \\ -s^*[4k+2] & s^*[4k+3] & s^*[4k] & -s^*[4k+1] \\ -s^*[4k+3] & -s^*[4k+2] & s^*[4k+1] & s^*[4k] \end{bmatrix}$$

Where, $k = 0, \ldots\ldots, \frac{N}{16} - 1$

$$G_4 = [(\boldsymbol{s_1}[k])^T \quad (\boldsymbol{s_2}[k])^T \quad (\boldsymbol{s_3}[k])^T \quad (\boldsymbol{s_4}[k])^T] \qquad \ldots\ldots (2.1.3.2)$$

SFBC provides four blocks S_1, S_2, S_3 and S_4 each of the length N, for OFDM at the transmitter.

$$S_i = [s_i[0] \quad s_i[1] \ldots\ldots\ldots\ldots s_i[\tfrac{N}{16} - 1]]^T \qquad \ldots\ldots (2.1.3.3)$$

In order to utilize the space frequency diversity, the input blocks are encoded as follows:

$$S_1 = (s[0] \quad -s[1] \quad -s[2] \quad -s[3] \quad s^*[0] \quad -s^*[1] \quad -s^*[2] \quad -s^*[3] \ldots$$
$$s[\tfrac{N}{4}-4] \quad -s[\tfrac{N}{4}-3] \; -s[\tfrac{N}{4}-2] \; -s[\tfrac{N}{4}-1] \quad s^*[\tfrac{N}{4}-4] \; -s^*[\tfrac{N}{4}-3] \; -s^*[\tfrac{N}{4}-2] \quad -s^*[\tfrac{N}{4}-3])^T$$
$$S_2 = (s[1] \quad s[0] \quad s[3] \quad -s[2] \quad s^*[1] \quad s^*[0] \quad s^*[3] \quad -s^*[2] \ldots$$
$$s[\tfrac{N}{4}-3] \; s[\tfrac{N}{4}-4] \quad s[\tfrac{N}{4}-1] \; -s[\tfrac{N}{4}-2] \quad s^*[\tfrac{N}{4}-3] \quad s^*[\tfrac{N}{4}-4] \quad s^*[\tfrac{N}{4}-1] \quad -s^*[N-2])^T$$
$$S_3 = (s[2] \quad -s[3] \quad s[0] \quad s[1] \quad s^*[2] \quad -s^*[3] \quad s^*[0] \quad s^*[1] \ldots$$
$$s[\tfrac{N}{4}-2] \quad -s[\tfrac{N}{4}-1] \quad s[\tfrac{N}{4}-4] \quad s[\tfrac{N}{4}-3] \quad s^*[\tfrac{N}{4}-2] \quad -s^*[\tfrac{N}{4}-1] \quad s^*[\tfrac{N}{4}-4] \quad s^*[\tfrac{N}{4}-3])^T$$
$$S_4 = (s[3] \quad s[2] \quad -s[1] \quad s[0] \quad s^*[3] \quad s^*[2] \quad -s^*[1] \quad s^*[0] \ldots$$
$$s[\tfrac{N}{4}-1] \quad s[\tfrac{N}{4}-2] \quad -s[\tfrac{N}{4}-3] \quad s[\tfrac{N}{4}-4] \quad s^*[\tfrac{N}{4}-1] \quad -s^*[\tfrac{N}{4}-2] \quad s^*[\tfrac{N}{4}-3] \quad s^*[\tfrac{N}{4}-4])^T$$

$$\ldots\ldots (2.1.3.4)$$

OFDM modulators generate blocks X_1, X_2, X_3 and X_4 corresponding to S_1, S_2, S_3 and S_4 that are transmitted by the first, second , third ad fourth transmit antenna respectively. Then given that the channel is assumed static during an OFDM block, after removing the cyclic prefix at the receiver side, the FFT output as the demodulated received signal at the j-th receive antenna can be expressed as:

$$r_j = H_{j,1} S_1 + H_{j,2} S_2 + H_{j,3} S_3 + H_{j,4} S_4 + W_j \qquad \ldots.. (2.1.3.5)$$

where r_j , $H_{j,1}$, $H_{j,2}$, $H_{j,3}$, $H_{j,4}$ and W_j are as defined. ML detection can be used for decoding the received signal.

Assuming channel H is known at the receiver, the ML estimate is obtained by performing $\min_{G_2} \left\| r_j - G_3 H \right\|_F^2$ which is the Frobenius form. The receiver decodes s[4k], s[4k+1], s[4k+2] and s[4k+3] by decomposing the measure $\left\| r_j - G_3 H \right\|_F^2$ into two parts, minimizes each separately over all possible values of s[4k], s[4k+1], s[4k+2] and s[4k+3] that belong to the constellation used. Similarly, like the formula used in two transmit antennas, this detection scheme can be written as [Following Appendix-A]:

$$\tilde{s}[4k] = 2\sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,1}^*[8k]r_j[8k] + H_{j,2}^*[8k]r_j[8k+1] + H_{j,3}^*[8k]r_j[8k+2] + H_{j,4}^*[8k]r_j[8k+3] \\ +H_{j,1}[8k]r_j^*[8k+4] + H_{j,2}[8k]r_j^*[8k+5] + H_{j,3}[8k]r_j^*[8k+6] + H_{j,4}[8k]r_j^*[8k+7] \end{pmatrix},$$

$$\tilde{s}[4k+1] = 2\sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,2}^*[8k]r_j[8k] - H_{j,1}^*[8k]r_j[8k+1] - H_{j,4}^*[8k]r_j[8k+2] + H_{j,3}^*[8k]r_j[8k+3] \\ +H_{j,2}[8k]r_j^*[8k+4] - H_{j,1}[8k]r_j^*[8k+5] - H_{j,4}[8k]r_j^*[8k+6] + H_{j,3}[8k]r_j^*[8k+7] \end{pmatrix}$$

$$\tilde{s}[4k+2] = 2\sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,3}^*[8k]r_j[8k] + H_{j,4}^*[8k]r_j[8k+1] - H_{j,1}^*[8k]r_j[8k+2] - H_{j,2}^*[8k]r_j[8k+3] \\ +H_{j,3}[8k]r_j^*[8k+4] + H_{j,4}[8k]r_j^*[8k+5] + H_{j,1}[8k]r_j^*[8k+6] + H_{j,2}[8k]r_j^*[8k+7] \end{pmatrix}$$

$$\tilde{s}[4k+3] = 2\sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,4}^*[8k]r_j[8k] - H_{j,3}^*[8k]r_j[8k+1] + H_{j,2}^*[8k]r_j[8k+2] - H_{j,2}^*[8k]r_j[8k+3] \\ +H_{j,4}[8k]r_j^*[8k+4] - H_{j,3}[8k]r_j^*[8k+5] + H_{j,2}[8k]r_j^*[8k+6] + H_{j,2}[8k]r_j^*[8k+7] \end{pmatrix}$$

…… (2.1.3.6)

Then, assuming that the channel gains between the two adjacent subchannels are approximately equal, i.e. $H_{j,1}[8k] = H_{j,1}[8k+m]$, $H_{j,2}[8k] = H_{j,2}[8k+m]$, $H_{j,3}[8k] = H_{j,3}[8k+m]$ and $H_{j,4}[8k] = H_{j,4}[8k+m]$ the decoded signal can be expressed as:

$$\tilde{s}[4k] = 2\sum_{j=1}^{M_R} \left(\left|H_{j,1}[8k]\right|^2 + \left|H_{j,2}[8k]\right|^2 + \left|H_{j,3}[8k]\right|^2 + \left|H_{j,4}[8k]\right|^2\right) s[4k] + \eta[4k],$$

$$\tilde{s}[4k+1] = 2\sum_{j=1}^{M_R} \left(\left|H_{j,1}[8k]\right|^2 + \left|H_{j,2}[8k]\right|^2 + \left|H_{j,3}[8k]\right|^2 + \left|H_{j,4}[8k]\right|^2\right) s[4k+1] + \eta[4k+1]$$

$$\tilde{s}[4k+2] = 2\sum_{j=1}^{M_R} \left(\left|H_{j,1}[8k]\right|^2 + \left|H_{j,2}[8k]\right|^2 + \left|H_{j,3}[8k]\right|^2 + \left|H_{j,4}[8k]\right|^2\right) s[4k+2] + \eta[4k+2]$$

$$\tilde{s}[4k+3] = 2\sum_{j=1}^{M_R} \left(\left|H_{j,1}[8k]\right|^2 + \left|H_{j,2}[8k]\right|^2 + \left|H_{j,3}[8k]\right|^2 + \left|H_{j,4}[8k]\right|^2\right) s[4k+3] + \eta[4k+3]$$

…… (2.1.3.7)

where η is the equivalent noise component given by

$$\eta[4k] = \sum_{j=1}^{M_R} \begin{pmatrix} +H_{j,1}^*[8k]W_j[8k] + H_{j,2}^*[8k]W_j[8k+1] + H_{j,3}^*[8k]W_j[8k+2] + H_{j,4}^*[8k]W_j[8k+3] \\ +H_{j,1}[8k]W_j^*[8k+4] + H_{j,2}[8k]W_j^*[8k+5] + H_{j,3}[8k]W_j^*[8k+6] + H_{j,4}[8k]W_j^*[8k+7] \end{pmatrix},$$

$\eta[4k+1] =$

$$\sum_{j=1}^{M_R}\begin{pmatrix}+H_{j,2}^{*}[8k]W_j[8k] & -H_{j,1}^{*}[8k]W_j[8k+1] & -H_{j,4}^{*}[8k]W_j[8k+2] & +H_{j,3}^{*}[8k]W_j[8k+3]\\ +H_{j,2}[8k]W_j^{*}[8k+4] & -H_{j,1}[8k]W_j^{*}[8k+5] & -H_{j,4}[8k]W_j^{*}[8k+6] & +H_{j,3}[8k]W_j^{*}[8k+6]\end{pmatrix}$$

$\eta[4k+2] =$

$$\sum_{j=1}^{M_R}\begin{pmatrix}+H_{j,3}^{*}[8k]W_j[8k] & +H_{j,4}^{*}[8k]W_j[8k+1] & -H_{j,1}^{*}[8k]W_j[8k+2] & -H_{j,2}^{*}[8k]W_j[8k+3]\\ +H_{j,3}[8k]W_j^{*}[8k+4] & +H_{j,4}[8k]W_j^{*}[8k+5] & +H_{j,1}[8k]W_j^{*}[8k+5] & +H_{j,2}[8k]W_j^{*}[8k+6]\end{pmatrix},$$

$\eta[4k+3] =$

$$\sum_{j=1}^{M_R}\begin{pmatrix}+H_{j,4}^{*}[8k]W_j[8k] & -H_{j,3}^{*}[8k]W_j[8k+1] & +H_{j,2}^{*}[8k]W_j[8k+2] & -H_{j,2}^{*}[8k]W_j[8k+3]\\ +H_{j,4}[8k]W_j^{*}[8k+4] & -H_{j,3}[8k]W_j^{*}[8k+5] & +H_{j,2}[8k]W_j^{*}[8k+6] & +H_{j,2}[8k]W_j^{*}[8k+7]\end{pmatrix},$$

…… (2.1.3.8)

The above decision variables provide a diversity gain of order two for every $s[4k]$, $s[4k+1]$, $s[4k+2]$ and $s[4k+3]$. As can be seen, the total channel gain is the sum of squares of three channel gains. Therefore, the proposed scheme can provide significant performance gains over conventional OFDM.

2.2 Analysis Of Uncoded Systems With MPSK And MQAM Modulation Scheme in Rician And Nakagami-m Fading Environment

In this section, the bit-error rate (BER) performance of conventional systems is reviewed. Let S be the MQAM/MPSK input signal and r be the signal after removing the cyclic prefix and performing FFT of the remaining signal. The received signal is given by

$$r = HS + W \quad \text{…… (2.2.1)}$$

Where,
S is the transmit signal,
W denotes the AWGN, *and*
H is a element corresponding to the DFT of the multipath channel response $h = [\alpha(0), \ldots\ldots\ldots\ldots, \alpha(L-1)]^T$ with L the total number of resolvable paths ($H = \sum_{l=0}^{L-1}\alpha(l)e^{-j\frac{2\pi}{N}l}$,
The $\alpha(l)$'s are complex Gaussian random variables with zero mean and variance $\frac{1}{L}$.
Therefore $|H|^2 = \alpha^2$
At the receiver side, the information data can be detected and extracted.

2.2.1 Average BER or BER Performance Of Uncoded MPSK System

Assume MPSK with Gray bit-mapping is employed. We can express the instantaneous BER as

$$BER_{MPSK} = \frac{1}{\beta}\, erfc\left(\sqrt{\gamma_s|H|^2}\, sin(\tfrac{\pi}{2^\beta})\right)$$

$$\text{Or, } BER_{MPSK} = \frac{1}{\beta}\, erfc\left(\sqrt{\gamma_s|H|^2}\, sin(\tfrac{\pi}{M})\right) \quad \text{…… (2.2.1.1)}$$

Therefore, the BER expression of the Uncoded system can be written as

$$BER_{MPSK} = \frac{1}{\beta}\, erfc\left(\sqrt{\gamma_s |H|^2}\, sin(\tfrac{\pi}{M})\right)$$
$$= \frac{1}{\beta}\, erfc\left(\sqrt{\gamma_s \alpha^2}\, sin(\tfrac{\pi}{M})\right) \qquad [\text{since, } |H|^2 = \alpha^2]$$

$$\text{Therefore, } BER_{MPSK}(\alpha) = \frac{1}{\beta}\, erfc\left(\sqrt{\gamma_s \alpha^2}\, sin(\tfrac{\pi}{M})\right) \qquad \text{...... (2.2.1.2)}$$

The average BER can then be evaluated using

$$\overline{BER}_{MPSK} = \int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)\, d\alpha \qquad \text{...... (2.2.1.3)}$$

$p(\alpha)$is the pdf of the amplitude fading α.

2.2.2 Average BER or BER Performance Of Uncoded MQAM System

Let us assume that MQAM with Gray bit-mapping is employed. We can express the instantaneous BER as

$$BER_{MQAM} = \frac{2(1-\frac{1}{\sqrt{2^\beta}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s |H|^2}{2^\beta - 1}}\right)$$

$$\text{Or, } BER_{MQAM} = \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s |H|^2}{M-1}}\right) \qquad \text{...... (2.2.2.1)}$$

Therefore, the BER expression of the Uncoded system can be written as

$$BER_{MQAM} = \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s |H|^2}{M-1}}\right)$$
$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s \alpha^2}{M-1}}\right) \qquad [\text{since, } |H|^2 = \alpha^2]$$
$$Therefore,\ BER_{MQAM}(\alpha) = \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s \alpha^2}{M-1}}\right) \qquad \text{...... (2.2.2.2)}$$

The average BER can then be evaluated using

$$\overline{BER}_{MQAM} = \int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)\, d\alpha \qquad \text{...... (2.2.2.3)}$$

$p(\alpha)$is the pdf of the amplitude fading α.

2.2.3 Average BER of Uncoded MPSK System in Rician fading Channel

For Uncoded MPSK, $BER_{MPSK}(\alpha) = \frac{1}{\beta} erfc\left(\sqrt{\gamma_s \alpha^2}\, sin(\frac{\pi}{M})\right)$

For a rician fading channel, α follows a rayleigh distribution with pdf

$$p(\alpha) = \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2}{2\sigma_\alpha^2}\right) \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) I_0\left(\frac{\alpha_0 \alpha}{\sigma_\alpha^2}\right),\ \ \alpha \geq 0$$

$$= \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2 + \alpha_0^2}{2\sigma_\alpha^2}\right) I_0\left(\frac{\alpha_0 \alpha}{\sigma_\alpha^2}\right),\ \ \alpha \geq 0 \qquad \text{...... (2.2.3.1)}$$

Here, $I_0(x)$ is the zero order modified Bessel function

$I_0(x) = \sum_{n=0}^{\infty} \frac{\left(\frac{x^2}{4}\right)^n}{(n!)^2}$ [Appendix C]

So, $p(\alpha) = \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2+\alpha_0^2}{2\sigma_\alpha^2}\right) \sum_{n=0}^{\infty} \frac{\left(\left(\frac{\alpha_0\alpha}{\sigma_\alpha^2}\right)^2/4\right)^n}{(n!)^2}, \ \alpha \geq 0$

$$= \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2+\alpha_0^2}{2\sigma_\alpha^2}\right) \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2\alpha^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}, \ \alpha \geq 0 \qquad \text{…….. (2.2.3.1)}$$

where the average received SNR/symbol with respect to α^2

$$\overline{\gamma_s} = \int_0^{\infty} \alpha^2 \gamma_s \ p(\alpha) d\alpha$$
$$= \gamma_s \int_0^{\infty} \alpha^2 \ p(\alpha) d\alpha \qquad \text{......(2.2.3.2)}$$
$$= \gamma_s \ E(\alpha^2)$$

From equation (2.2.3.1), we get,

$$\overline{\gamma_s} = \gamma_s \int_0^{\infty} \alpha^2 \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2+\alpha_0^2}{2\sigma_\alpha^2}\right) \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2\alpha^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2} \ d\alpha$$
$$= \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right)^n (n+1)}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \ 2\sigma_\alpha^2\gamma_s \ \text{[Appendix D1]}$$

Therefore, $\overline{\gamma_s} = \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right)^n (n+1)}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \ 2\sigma_\alpha^2\gamma_s$ ......(2.2.3.3)

and $E(\alpha^2) = \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right)^n (n+1)}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) 2\sigma_\alpha^2$ ......(2.2.3.4)

So, the average BER with uncoded MPSK in Rician fading channel

$$\overline{BER}_{MPSK-Rician} = \int_0^{\infty} BER_{MPSK}(\alpha)\, p(\alpha)_{Rician} \ d\alpha$$
$$= \int_0^{\infty} \frac{1}{\beta} \ erfc\left(\sqrt{\gamma_s\alpha^2} \sin\left(\frac{\pi}{M}\right)\right) \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2+\alpha_0^2}{2\sigma_\alpha^2}\right) I_0\left(\frac{\alpha_0\alpha}{\sigma_\alpha^2}\right) d\alpha \qquad \text{......(2.2.3.5)}$$

Therefore, $\overline{\boldsymbol{BER}}_{\boldsymbol{MPSK-Rician}}$

$$= \sum_{n=0}^{\infty} \frac{\overline{\gamma_s}}{2\sigma_\alpha^2\gamma_s(n+1)} \frac{1}{\beta} \left[1 - \sum_{i=0}^{n} \frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2} \left(\frac{1}{1+2\sigma_\alpha^2\gamma_s \sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i} \sqrt{\frac{2\sigma_\alpha^2\gamma_s \sin^2\left(\frac{\pi}{M}\right)}{1+2\sigma_\alpha^2\gamma_s \sin^2\left(\frac{\pi}{M}\right)}}\right] \ \text{[Appendix E1]}$$

......(2.2.3.6)

2.2.4 Average BER of Uncoded MQAM System in Rician Fading Channel

For Uncoded MQAM, $BER_{MQAM}(\alpha) = \frac{2\left(1-\frac{1}{\sqrt{M}}\right)}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s\alpha^2}{M-1}}\right)$ ....(2.2.4.1)

For a rician fading channel, the average received SNR/symbol with respect to α^2 is

$$\overline{\gamma_s} = \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right)^n (n+1)}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \ 2\sigma_\alpha^2\gamma_s \ \text{[Appendix D1]} \qquad \text{......(2.2.4.2)}$$

and $$\mathrm{E}(\alpha^2) = \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right)^n (n+1)!}{n!} (2\sigma_\alpha^2)^{n+1} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \quad ......(2.2.4.3)$$

So, the average BER with uncoded MQAM in Rician fading channel,

$$\overline{BER}_{MQAM-Rician} = \int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$$
$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s\alpha^2}{M-1}}\right) \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2+\alpha_0^2}{2\sigma_\alpha^2}\right) I_0\left(\frac{\alpha_0\alpha}{\sigma_\alpha^2}\right) d\alpha \quad ......(2.2.4.4)$$

Therefore, $\boldsymbol{\overline{BER}_{MQAM-Rician}}$

$$= \sum_{n=0}^{\infty} \frac{\overline{\gamma_s}}{2\sigma_\alpha^2\gamma_s(n+1)} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \left[1 - \sum_{i=0}^{n} \frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2} \left(\frac{M-1}{(M-1)+3\sigma_\alpha^2\gamma_s}\right)^{n-i} \sqrt{\frac{3\sigma_\alpha^2\gamma_s}{(M-1)+3\sigma_\alpha^2\gamma_s}}\right] \text{[Appendix E2]}$$

......(2.2.4.5)

2.2.5 Average BER of Uncoded MPSK System in Nakagami - m Fading Channel

For Uncoded MPSK, $BER_{MPSK}(\alpha) = \frac{1}{\beta} erfc\left(\sqrt{\gamma_s\alpha^2}\, sin(\frac{\pi}{M})\right)$ .... (2.2.5.1)

For a Nakagami - m fading channel, α follows a distribution with pdf

$$p(\alpha) = \frac{2m^m}{\Gamma(m)\Omega^m} \alpha^{2m-1} \exp\left(-\frac{m}{\Omega}\alpha^2\right) \quad .... (2.2.5.2)$$

where, $m = \frac{E^2(\alpha^2)}{VAR(\alpha^2)}$, $\Omega = E(\alpha^2)$ & $\sigma = \sqrt{VAR(\alpha^2)} = \sqrt{\frac{VAR(\alpha^2)}{E^2(\alpha^2)}} E(\alpha^2) = \frac{E(\alpha^2)}{\sqrt{\frac{E^2(\alpha^2)}{VAR(\alpha^2)}}} = \frac{\Omega}{m}$

where the average received SNR/bit with respect to α^2

$$\overline{\gamma_s} = \int_0^\infty \alpha^2\gamma_s\, p(\alpha)d\alpha$$
$$= \gamma_s \int_0^\infty \alpha^2\, p(\alpha)d\alpha \quad ......(2.2.5.3)$$
$$= \gamma_s\, E(\alpha^2)$$

From equation (2.2.5.3)

$$\overline{\gamma_s} = \gamma_s \int_0^\infty \alpha^2 \frac{2m^m}{\Gamma(m)\Omega^m} \alpha^{2m-1} \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha$$
$$= \gamma_s \int_0^\infty \alpha^{2m+1} \frac{2m^m}{\Gamma(m)\Omega^m} \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha$$
$$= \gamma_s\, \Omega \quad \text{[Appendix D2]}$$

......(2.2.5.4)

Therefore, $\mathbf{E(\alpha^2)=\Omega}$. ......(2.2.5.5)

So, the average BER with uncoded MPSK in Nakagami - m fading channel

$$\overline{BER}_{MPSK-Nakagami}$$
$$= \int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{Nakagami}\, d\alpha$$
$$= \int_0^\infty \frac{1}{\beta} erfc\left(\sqrt{\gamma_s\alpha^2}\, sin(\frac{\pi}{M})\right) \frac{2m^m}{\Gamma(m)\Omega^m} \alpha^{2m-1} \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha \quad ......(2.2.5.6)$$

Therefore, $\overline{BER}_{MPSK-Nakagami}$

$$= \frac{1}{\beta}\left[1 - \sum_{i=0}^{m-1} \frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2} \left(\frac{m}{\left(m+\overline{\gamma_s} sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i} \sqrt{\frac{\overline{\gamma_s} sin^2(\frac{\pi}{M})}{\left(m+\overline{\gamma_s} sin^2(\frac{\pi}{M})\right)}}\right] \text{[Appendix E3]}$$

......(2.2.5.7)

2.2.6 Average BER with Uncoded MQAM System in Nakagami - m Fading Channel

For Uncoded MPSK, $BER_{MQAM}(\alpha) = \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s \alpha^2}{M-1}}\right)$ ......(2.2.6.1)

For a Nakagami - m fading channel, the average received SNR/bit with respect to α^2

$\overline{\gamma_s} = \gamma_s \Omega$ [Appendix D2] ......(2.2.6.2)

and $E(\alpha^2)=\Omega$. ......(2.2.6.3)

So, the average BER with uncoded MQAM in Nakagami - m fading channel

$$\overline{BER}_{MQAM-Nakagami} = \int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Nakagami}\, d\alpha$$

$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s \alpha^2}{M-1}}\right) \frac{2m^m}{\Gamma(m)\Omega^m} \alpha^{2m-1} \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha$$

......(2.2.6.4)

Therefore, $\overline{BER}_{MQAM-Nakagami}$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\left[1 - \sum_{i=0}^{m-1} \frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2} \left(\frac{m(M-1)}{(m(M-1)+1.5\overline{\gamma_s})}\right)^{m-1-i} \sqrt{\frac{1.5\overline{\gamma_s}}{(m(M-1)+1.5\overline{\gamma_s})}}\right]$$

[Appendix E4]

......(2.2.6.5)

2.3 Analysis Of Uncoded OFDM Systems With MPSK And MQAM Modulation Scheme in Rician And Nakagami-m Fading Environment

In this section, the bit-error rate (BER) performance of conventional OFDM systems is reviewed. Let S be the MQAM/MPSK input signal and r be the signal after removing the cyclic prefix and performing FFT of the remaining signal. The received signal is given by

$$r[k] = H[k]\, s[k] + W[k], \qquad k = 0, \ldots\ldots\ldots\ldots N-1, \qquad ......(2.3.1)$$

which can be expressed as:

$$r = H\,S + W \qquad \text{......(2.3.1)}$$

where

$r = (r[0], \ldots\ldots\ldots, r[N-1])^T$,
$S = (s[0], \ldots\ldots\ldots, s[N-1])^T$ is the transmit signal,
$W = (W[0], \ldots\ldots\ldots, W[N-1])^T$ denotes the AWGN, *and*
H is a diagonal matrix of size $N \times N$ *given by* $H = diag\,\{H[k]\}_{k=0}^{N-1}$, with $H[k]$'s elements corresponding to the DFT of the multipath channel response $h = [\alpha(0), \ldots\ldots\ldots, \alpha(L-1)]^T$ with L the total number of resolvable paths $(H[k] = \sum_{l=0}^{L-1} \alpha(l) e^{-j\frac{2\pi}{N}kl}$, The $\alpha(l)$'s are complex Gaussian random variables with zero mean and variance $\frac{1}{L}$.
Therefore $|H[k]|^2 = \alpha^2$
At the receiver side, the information data can be detected and extracted. Finally, the BER expression of the OFDM system can be written as

$$BER = \frac{1}{N}\sum_{k=0}^{N-1} BER[k] \qquad \text{......(2.3.2)}$$

where $BER[k]$ is the instantaneous BER of the k-th subchannel in the OFDM block.

2.3.1 Average BER or BER Performance Of Uncoded MPSK-OFDM System

Assume MPSK with Gray bit-mapping is employed for each subchannel. We can express the instantaneous BER of the k-th subchannel as

$$BER_{MPSK}[k] = \frac{1}{\beta}\, erfc\left(\sqrt{\gamma_s |H[k]|^2}\, sin(\frac{\pi}{2^\beta})\right)$$

$$\text{Or, } BER_{MPSK}[k] = \frac{1}{\beta}\, erfc\left(\sqrt{\gamma_s |H[k]|^2}\, sin(\frac{\pi}{M})\right) \qquad \text{......(2.3.1.1)}$$

Therefore, the BER expression of the OFDM system can be written as

$$BER_{MPSK} = \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s |H[k]|^2}\, sin(\frac{\pi}{M})\right)$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s {\alpha_k}^2}\, sin(\frac{\pi}{M})\right) \; [|H[k]|^2 = {\alpha_k}^2]$$

$$Therefore, \quad BER_{MPSK}(\alpha) = \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s {\alpha_k}^2}\, sin(\frac{\pi}{M})\right) \qquad \text{......(2.3.1.2)}$$

The average BER can then be evaluated using

$$\overline{BER}_{MPSK} = \int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)\, d\alpha \qquad \text{......(2.3.1.3)}$$

$p(\alpha)$is the pdf of the amplitude fading α.

2.3.2 Average BER or BER Performance Of Uncoded MQAM-OFDM System

Assume MQAM with Gray bit-mapping is employed for each subchannel. We can express the instantaneous BER of the k-th subchannel as

$$BER_{MQAM}[k] = \frac{2(1 - \frac{1}{\sqrt{2^\beta}})}{\beta}\, erfc\left(\sqrt{\frac{1.5\gamma_s |H[k]|^2}{M-1}}\right)$$

Or, $BER_{MQAM}[k] = \frac{2(1-\frac{1}{\sqrt{k}})}{\beta}\ erfc\left(\sqrt{\frac{1.5\gamma_s|H[k]|^2}{2^{\beta}-1}}\right)$

Therefore, the BER expression of the OFDM system can be written as

$$BER_{MQAM} = \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s|H[k]|^2}{M-1}}\right)$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s{\alpha_k}^2}{M-1}}\right) \quad [|H[k]|^2 = {\alpha_k}^2]$$

Therefore, $$BER_{MQAM}(\alpha) = \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s{\alpha_k}^2}{M-1}}\right) \qquad ......(2.3.2.2)$$

The average BER can then be evaluated using,

$$\overline{BER}_{MQAM} = \int_0^{\infty} BER_{MQAM}(\alpha)\, p(\alpha)\, d\alpha \qquad ......(2.3.2.2)$$

$p(\alpha)$is the pdf of the amplitude fading α.

2.3.3 Average BER of Uncoded MPSK-OFDM System in Rician Fading Channel

The BER expression for the MQAM-OFDM system can be written as

$$\frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s|H[k]|^2}\, sin(\tfrac{\pi}{M})\right) = \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s{\alpha_k}^2}\, sin(\tfrac{\pi}{M})\right) \qquad ......(2.3.3.1)$$

In case of OFDM, For a rician fading channel, α_k follows a rayleigh distribution with pdf

$$p(\alpha_k) = \frac{\alpha_k}{\sigma^2_{\alpha_k}}\exp\left(-\frac{{\alpha_k}^2}{2\sigma^2_{\alpha}}\right)\exp\left(-\frac{\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) I_0\left(\frac{\alpha_0\alpha_k}{\sigma^2_{\alpha_k}}\right), \quad \alpha_k \geq 0$$

$$= \frac{\alpha_k}{\sigma^2_{\alpha}}\exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) I_0\left(\frac{\alpha_0\alpha_k}{\sigma^2_{\alpha_k}}\right), \quad \alpha_k \geq 0 \qquad ......(2.3.3.2)$$

Here, $I_0(x)$ is the zero order modified Bessel function

$I_0(x) = \sum_{n=0}^{\infty}\frac{\left(\frac{x^2}{4}\right)^n}{(n!)^2}$ [Appendix C]

$$p(\alpha_k) = \sum_{k=0}^{N-1}\frac{\alpha_k}{\sigma^2_{\alpha_k}}\exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma^2_{\alpha_k}}\right)\sum_{n=0}^{\infty}\frac{\left(\left(\frac{\alpha_0\alpha_k}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^2\right)^n}{(n!)^2}, \quad \alpha_k \geq 0$$

$$= \sum_{k=0}^{N-1}\frac{\alpha_k}{\sigma^2_{\alpha_k}}\exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma^2_{\alpha_k}}\right)\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2{\alpha_k}^2}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2}, \quad \alpha_k \geq 0 \qquad .....(2.3.3.3)$$

For OFDM, where the average received SNR/bit with respect to ${\alpha_k}^2$

$$\overline{\gamma_s}_k = \int_0^{\infty}{\alpha_k}^2\gamma_s\ p(\alpha_k)d\alpha_k$$

$$= \gamma_s\int_0^{\infty}{\alpha_k}^2\, p(\alpha_k)d\alpha_k \qquad ......(2.3.3.4)$$

$$= \gamma_s\, E({\alpha_k}^2)$$

For OFDM, The total the average received SNR/bit

$$\overline{\gamma_s} = \sum_{k=0}^{N-1} \int_0^\infty {\alpha_k}^2 \gamma_s \; p(\alpha_k) d\alpha_k \; = \sum_{k=0}^{N-1} \gamma_s \, E({\alpha_k}^2) \qquad ......(2.3.3.5)$$

From equation (2.3.3.2)

$$\overline{\gamma}_{s\,k} = \gamma_s \int_0^\infty {\alpha_k}^2 \frac{\alpha_k}{\sigma_{\alpha_k}^2} \exp\left(-\frac{{\alpha_k}^2 + \alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \sum_{n=0}^\infty \frac{\left(\frac{{\alpha_0}^2 {\alpha_k}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2} \; d\alpha_k$$

$$= \sum_{n=0}^\infty \frac{\left(\frac{{\alpha_0}^2}{2\sigma_{\alpha_k}^2}\right)^n (n+1)}{n!} \; \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \; 2\sigma_{\alpha_k}^2 \gamma_s \quad \text{[Appendix D3]} \qquad ......(2.3.3.6)$$

and therefore, $E({\alpha_k}^2) = \sum_{n=0}^\infty \frac{\left(\frac{{\alpha_0}^2}{4\sigma_{\alpha_k}^4}\right)^n (n+1)!}{(n!)^2} \left(2\sigma_{\alpha_k}^2\right)^{n+1} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)$ ......(2.3.3.4)

So, The total the average received SNR/bit

$$\overline{\gamma_s} = \sum_{k=0}^{N-1} \sum_{n=0}^\infty \frac{\left(\frac{{\alpha_0}^2}{4\sigma_{\alpha_k}^4}\right)^n (n+1)!}{(n!)^2} \left(2\sigma_{\alpha_k}^2\right)^{n+1} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \; \gamma_s \qquad ......(2.3.3.7)$$

So, The average BER with uncoded MPSK-OFDM in Rician fading channel

$$\overline{BER}_{MPSK-OFDM-Rician}$$

$$= \int_0^\infty BER_{MPSK-OFDM}(\alpha) \, p(\alpha)_{Rician} \; d\alpha$$

$$= \int_0^\infty \frac{1}{N} \sum_{k=0}^{N-1} BER_{\text{MPSK}-\text{OFDM}}(\alpha_k) \, p(\alpha_k)_{Rician} \; d\alpha_k$$

$$= \int_0^\infty \frac{1}{N\beta} \; \sum_{k=0}^{N-1} \; erfc\left(\sqrt{\gamma_s {\alpha_k}^2} \, sin(\tfrac{\pi}{M})\right) \frac{\alpha_k}{\sigma_{\alpha_k}^2} \exp\left(-\frac{{\alpha_k}^2 + \alpha_0^2}{2\sigma_{\alpha_k}^2}\right) I_0\left(\frac{\alpha_0 \alpha_k}{\sigma_{\alpha_k}^2}\right) \; d\alpha_k$$

Therefore,

$$\boldsymbol{\overline{BER}_{MPSK-OFDM-Rician}} =$$

$$\boldsymbol{\frac{1}{N\beta} \; \sum_{k=0}^{N-1} \left[\sum_{n=0}^\infty \frac{\overline{\gamma_s}_k}{2\sigma_{\alpha_k}^2 \gamma_s (n+1)} \left[1 - \sum_{i=0}^{n} \frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2} \left(\frac{1}{1 + 2\sigma_{\alpha_k}^2 \gamma_s sin^2(\frac{\pi}{M})} \right)^{n-i} \sqrt{\frac{2\sigma_{\alpha_k}^2 \gamma_s sin^2(\frac{\pi}{M})}{1 + 2\sigma_{\alpha_k}^2 \gamma_s sin^2(\frac{\pi}{M})}} \right] \right]}$$

[Appendix F1]

......(2.3.3.8)

2.3.4 Average BER of Uncoded MQAM-OFDM System in Rician fading Channel

The BER expression for the MQAM-OFDM system can be written as

$$=\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s|H[k]|^2}{M-1}}\right)=\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s{\alpha_k}^2}{M-1}}\right) \quad ......(2.3.4.1)$$

In case of OFDM, For a rician fading channel,

The average received SNR/bit with respect to ${\alpha_k}^2$

$$\overline{\gamma_s}_k=\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{2\sigma_{\alpha_k}^2}\right)^n(n+1)}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\ 2\sigma_{\alpha_k}^2\gamma_s \ \text{[Appendix D3]} \quad ......(2.3.4.2)$$

and $E({\alpha_k}^2)=\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{2\sigma_{\alpha_k}^2}\right)^n(n+1)}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)2\sigma_{\alpha_k}^2$ ......(2.3.4.3)

So, The total the average received SNR/bit

$$\overline{\gamma_s}=\sum_{k=0}^{N-1}\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{2\sigma_{\alpha_k}^2}\right)^n(n+1)}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\ 2\sigma_{\alpha_k}^2\gamma_s \quad ......(2.3.4.4)$$

So, the average BER with uncoded MQAM-OFDM in Rician fading channel

$$\overline{BER}_{MQAM-OFDM-Rician}$$

$$=\int_0^{\infty}BER_{MQAM-OFDM}(\alpha)\,p(\alpha)_{Rician}\ d\alpha$$

$$=\int_0^{\infty}\frac{1}{N}\sum_{k=0}^{N-1}BER_{\text{MQAM}-\text{OFDM}}(\alpha_k)\,p(\alpha_k)_{Rician}\ d\alpha_k$$

$$=\int_0^{\infty}\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}erfc\left(\sqrt{\frac{1.5\gamma_s{\alpha_k}^2}{M-1}}\right)\frac{\alpha_k}{\sigma_{\alpha_k}^2}\exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)I_0\left(\frac{\alpha_0\alpha_k}{\sigma_{\alpha_k}^2}\right)d\alpha_k$$

Therefore,

$$\overline{\boldsymbol{BER}}_{\mathbf{MQAM-OFDM-Rician}}=$$

$$\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_s}_k}{2\sigma_{\alpha_k}^2\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2}\left(\frac{M-1}{(M-1)+3\sigma_{\alpha_k}^2\gamma_s}\right)^{n-i}\sqrt{\frac{3\sigma_{\alpha_k}^2\gamma_s}{(M-1)+3\sigma_{\alpha_k}^2\gamma_s}}\right]\right]$$

[Appendix F2]

......(2.3.4.5)

2.3.5 Average BER of Uncoded MPSK-OFDM System in Nakagami - m fading Channel

The BER expression for the MQAM-OFDM system can be written as

$$=\frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s|H[k]|^2}\, sin(\frac{\pi}{M})\right)=\frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s{\alpha_k}^2}\, sin(\frac{\pi}{M})\right) \quad ...... (2.3.5.1)$$

In case of OFDM system, for a Nakagami - m fading channel, α follows a rayleigh distribution with pdf

$$p(\alpha_k) = \sum_{k=0}^{N-1}\frac{2m^m}{\Gamma(m){\Omega_k}^m}{\alpha_k}^{2m-1}exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right) \quad ...... (2.3.5.2)$$

where, $m = \frac{E^2({\alpha_k}^2)}{VAR({\alpha_k}^2)}$, $\Omega_k = E({\alpha_k}^2)$ and $\sigma_{{\alpha_k}^2} = \frac{\Omega_k}{m}$

For OFDM, where the average received SNR/bit with respect to ${\alpha_k}^2$

$$\begin{aligned}\overline{\gamma_s}_k &= \int_0^\infty {\alpha_k}^2\gamma_s\, p(\alpha_k)d\alpha_k \\ &= \gamma_s \int_0^\infty {\alpha_k}^2\, p(\alpha_k)d\alpha_k \\ &= \gamma_s\, E({\alpha_k}^2)\end{aligned} \quad ...... (2.3.5.3)$$

For OFDM, The total the average received SNR/bit

$$\overline{\gamma_s} = \sum_{k=0}^{N-1}\int_0^\infty {\alpha_k}^2\gamma_s\, p(\alpha_k)d\alpha_k = \sum_{k=0}^{N-1}\gamma_s\, E({\alpha_k}^2) \quad ...... (2.3.5.4)$$

From equation (iv),

$$\overline{\gamma_s}_k = \gamma_s\int_0^\infty {\alpha_k}^2\frac{2m^m}{\Gamma(m){\Omega_k}^m}{\alpha_k}^{2m-1}\exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right)d\alpha_k$$

$$= \gamma_s\int_0^\infty {\alpha_k}^{2m+1}\frac{2m^m}{\Gamma(m){\Omega_k}^m}\exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right)d\alpha_k$$

$= \gamma_s\,\Omega_k$ [Appendix D4] ...... (2.3.5.5)

Therefore, $\mathbf{E}({\alpha_k}^2) = \boldsymbol{\Omega_k}$ ...... (2.3.5.6)

So, The total the average received SNR/bit

$$\boldsymbol{\overline{\gamma_s} = \sum_{k=0}^{N-1}\gamma_s\,\Omega_k} \quad ...... (2.3.5.7)$$

So, the average BER for MPSK-OFDM system in Nakagami - m fading Channel,

$$\overline{BER}_{MPSK-OFDM-Nakagami} = \int_0^\infty BER_{MPSK-OFDM}(\alpha)\, p(\alpha)_{Nakagami}\, d\alpha$$

$$= \int_0^\infty \frac{1}{N}\sum_{k=0}^{N-1} BER_{\mathrm{MPSK-OFDM}}(\alpha_k)\, p(\alpha_k)_{Nakagami}\, d\alpha_k$$

$$= \int_0^\infty \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[erfc\left(\sqrt{\gamma_s{\alpha_k}^2}\, sin(\frac{\pi}{M})\right)\frac{2m^m}{\Gamma(m){\Omega_k}^m}{\alpha_k}^{2m-1}\exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right)d\alpha_k\right]$$

Therefore, $\overline{\boldsymbol{BER}}_{\boldsymbol{MPSK-OFDM-Nakagami}}$

$$= \frac{\mathbf{1}}{\boldsymbol{N\beta}}\sum_{\boldsymbol{k=0}}^{\boldsymbol{N-1}}\left[\mathbf{1} - \sum_{\boldsymbol{i=0}}^{\boldsymbol{m-1}}\frac{(\mathbf{2}(\boldsymbol{m-1-i}))!}{\mathbf{2}^{\mathbf{2}(\boldsymbol{m-1-i})}\left((\boldsymbol{m-1-i})!\right)^{\mathbf{2}}}\left(\frac{\boldsymbol{m}}{\left(\boldsymbol{m}+\overline{\boldsymbol{\gamma_s}}_{\boldsymbol{k}}\boldsymbol{sin}^{\mathbf{2}}(\frac{\boldsymbol{\pi}}{\boldsymbol{M}})\right)}\right)^{\boldsymbol{m-1-i}}\sqrt{\frac{\overline{\boldsymbol{\gamma_s}}_{\boldsymbol{k}}\boldsymbol{sin}^{\mathbf{2}}(\frac{\boldsymbol{\pi}}{\boldsymbol{M}})}{\left(\boldsymbol{m}+\overline{\boldsymbol{\gamma_s}}_{\boldsymbol{k}}\boldsymbol{sin}^{\mathbf{2}}(\frac{\boldsymbol{\pi}}{\boldsymbol{M}})\right)}}\right]$$

[Appendix F3]

...... (2.3.5.8)

2.3.6 Average BER of Uncoded MQAM-OFDM System in Nakagami - m fading Channel

The BER expression for the MQAM-OFDM system can be written as

$$=\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s|H[k]|^2}{M-1}}\right) = \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s{\alpha_k}^2}{M-1}}\right) \quad \text{...... (2.3.6.1)}$$

In case of OFDM system, for a Nakagami - m fading channel, The average received SNR/bit with respect to ${\alpha_k}^2$

$\overline{\boldsymbol{\gamma_s}}_{\boldsymbol{k}} = \boldsymbol{\gamma_s\,\Omega_k}$ [Appendix D4] ...... (2.3.6.2)

and E$({\alpha_k}^2)$= $\boldsymbol{\Omega_k}$ ...... (2.3.6.3)

So, The total the average received SNR/bit is $\overline{\boldsymbol{\gamma_s}} = \sum_{\boldsymbol{k=0}}^{\boldsymbol{N-1}}\boldsymbol{\gamma_s\,\Omega_k}$ ...... (2.3.6.4)

So, the average BER for MQAM-OFDM system in Nakagami - m fading Channel,

$$\overline{BER}_{\text{MQAM -OFDM -Nakagami}} = \int_0^\infty BER_{\text{MQAM -OFDM}}(\alpha)\, p(\alpha)_{\,Nakagami}\, d\alpha$$

$$= \int_0^\infty \frac{1}{N}\sum_{k=0}^{N-1} BER_{\text{MQAM -OFDM}}(\alpha_k)\, p(\alpha_k)_{Nakagami}\, d\alpha_k$$

$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s{\alpha_k}^2}{M-1}}\right)\frac{2m^m}{\Gamma(m){\Omega_k}^m}{\alpha_k}^{2m-1}\exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right) d\alpha_k$$

Therefore,

$\overline{\boldsymbol{BER}}_{\textbf{MQAM}-\textbf{OFDM}-\textbf{Nakagami}} =$

$$\frac{\mathbf{2}(\mathbf{1}-\frac{\mathbf{1}}{\sqrt{\boldsymbol{M}}})}{\boldsymbol{N\beta}}\sum_{\boldsymbol{k=0}}^{\boldsymbol{N-1}}\left[\mathbf{1} - \sum_{\boldsymbol{i=0}}^{\boldsymbol{m-1}}\frac{(\mathbf{2}(\boldsymbol{m-1-i}))!}{\mathbf{2}^{\mathbf{2}(\boldsymbol{m-1-i})}\left((\boldsymbol{m-1-i})!\right)^{\mathbf{2}}}\left(\frac{\boldsymbol{m(M-1)}}{\left(\boldsymbol{m(M-1)}+\mathbf{1.5}\overline{\boldsymbol{\gamma_s}}_{\boldsymbol{k}}\right)}\right)^{\boldsymbol{m-1-i}}\sqrt{\frac{\mathbf{1.5}\overline{\boldsymbol{\gamma_s}}_{\boldsymbol{k}}}{\left(\boldsymbol{m(M-1)}+\mathbf{1.5}\overline{\boldsymbol{\gamma_s}}_{\boldsymbol{k}}\right)}}\right]$$

[Appendix F4]

...... (2.3.6.5)

2.4 Analysis Of SFBC-OFDM Systems With MPSK And MQAM Modulation Scheme in Rician And Nakagami-m Fading Environment

In this section, we derive the expressions for the BER of SFBC-OFDM systems employing M_T transmit and M_R receive antennas with MQAM or MPSK modulation.

Consider an MQAM-SFBC-OFDM system employing M_T transmit and M_R receive antennas. The decoder minimizes the decision metric

$$|\tilde{s}[k] - s[k]|^2 \qquad \text{where, } k = 0, \ldots\ldots, N-1. \qquad \ldots..\ (2.4.1)$$

From the previous analysis we can show that

$$\tilde{s}[k] = \frac{1}{R_c} \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \left|H_{j,i}[k]\right|^2 s[k] + \eta[k] \qquad \ldots...\ (2.4.2)$$

Where,

$H_{j,i}[k]$ is the k-th subchannel associated with the i-th transmit and j-th receive antennas,

R_c is the code rate of the SFBC system and $\eta[k]$ is the noise component.

$$\gamma = \frac{1}{R_c} \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \left|H_{j,i}[k]\right|^2 \gamma_s \qquad \ldots...\ (2.4.3)$$

we can express the normalized instantaneous SNR as

where γ_s is Similar to the case of uncoded OFDM.

2.4.1 Average BER or BER Performance Of SFBC-MPSK-OFDM System

The BER of MPSK-SFBC-OFDM can be written as

$$BER_{MPSK-SFBC-OFDM} = \frac{1}{N\beta} \sum_{k=0}^{N-1} erfc\left(\frac{\sqrt{\gamma_s \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \left|H_{j,i}[k]\right|^2} \sin\left(\frac{\pi}{2\beta}\right)}{R_c} \right)$$

$$= \frac{1}{N\beta} \sum_{k=0}^{N-1} erfc\left(\frac{\sqrt{\gamma_s \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \left|H_{j,i}[k]\right|^2} \sin\left(\frac{\pi}{M}\right)}{R_c} \right) \qquad \ldots...\ (2.4.1.1)$$

The average BER can then be evaluated using

$$\overline{BER}_{MPSK-SFBC-OFDM}$$

$$= \int_0^\infty \ldots \int_0^\infty BER_{MPSK-SFBC-OFDM}\, P(\alpha_{1,1}) \ldots P(\alpha_{M_R,M_T})\, d\alpha_{1,1} \ldots d\alpha_{M_R,M_T} \qquad \ldots...\ (2.4.1.2)$$

2.4.2 Average BER or BER Performance Of SFBC-MQAM-OFDM System

The BER of MPSK-SFBC-OFDM can be written as

$$\overline{BER}_{MQAM-SFBC-OFDM} = \frac{2\left(1-\frac{1}{\sqrt{2^{\beta}}}\right)}{N\beta}\sum_{k=0}^{N-1} erfc\left(\frac{\sqrt{1.5\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}}{R_c(2^{\beta}-1)}\right)$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\frac{\sqrt{1.5\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}}{R_c(M-1)}\right) \quad \text{…... (2.4.2.1)}$$

The average BER can then be evaluated using

$$\overline{BER}_{MQAM-SFBC-OFDM}$$

$$= \int_0^{\infty} \dots \int_0^{\infty} BER_{MQAM-SFBC-OFDM}\, P(\alpha_{1,1}) \dots P(\alpha_{\mathrm{M_R},M_T})\, d\alpha_{1,1} \dots d\alpha_{\mathrm{M_R},M_T} \quad \text{…... (2.4.2.2)}$$

2.4.3 Average BER Using SFBC with MPSK-OFDM system in Rician fading Channel

For SFBC system, The BER expression for the SFBC-MPSK-OFDM system can be written as

$$BER_{SFBC-MPSK-OFDM}\left(\alpha_{k_{i,j}}\right) = \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{i,j}[k]\right|^2}{R_c}}\, sin(\frac{\pi}{M})\right)$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\alpha_{k_{i,j}}^{\,2}}{R_c}}\, sin(\frac{\pi}{M})\right) \quad \text{…... (2.4.3.1)}$$

We assume that the fading process remains static during each OFDM block and that it varies from one block to another. Therefore,

$$\alpha_{k_{1,1}} = \cdots \dots \dots = \alpha_{k_{M_R,M_T}} = \alpha_k^{\,2} \quad \text{…... (2.4.3.2)}$$

So,In case of OFDM, For a rician fading channel, α_k follows a rayleigh distribution with pdf

$$p\left(\alpha_{k_{1,1}}\right) = \cdots = p\left(\alpha_{k_{M_R,M_T}}\right)$$

$$= \frac{\alpha_k}{\sigma_{\alpha_k}^2}\exp\left(-\frac{\alpha_k^{\,2}}{2\sigma_{\alpha}^2}\right)\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) I_0\left(\frac{\alpha_0\alpha_k}{\sigma_{\alpha_k}^2}\right), \quad \alpha_k \geq 0$$

$$= \frac{\alpha_k}{\sigma_{\alpha}^2}\exp\left(-\frac{\alpha_k^{\,2}+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) I_0\left(\frac{\alpha_0\alpha_k}{\sigma_{\alpha_k}^2}\right), \quad \alpha_k \geq 0 \quad \text{…... (2.4.3.3)}$$

For OFDM, the average received SNR/bit

$$\overline{\gamma_{s}}_k = \sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0^{\,2}}{2\sigma_{\alpha_k}^2}\right)^n (n+1)}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) 2\sigma_{\alpha_k}^2\gamma_s \text{ [Appendix D3]} \quad \text{…... (2.4.3.4)}$$

So, The total the average received SNR/bit

$$\overline{\gamma_s} = \sum_{k=0}^{N-1}\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{4\left(\sigma_{\alpha_k}^2\right)^2}\right)^n (n+1)!}{(n!)^2}\left(2\sigma_{\alpha_k}^2\right)^{n+1}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\ \gamma_s \qquad \text{…... (2.4.3.5)}$$

So, the average BER using SFBC with MQAM-OFDM in Rician fading Channel can be evaluated by,

$$\overline{BER}_{SFBC-MPSK-OFDM-Rician}$$

$$= \int_0^{\infty}\ldots\int_0^{\infty} BER_{SFBC-MPSK-OFDM}\left(\alpha_{M_R,M_T}\right) p\left(\alpha_{1,1}\right)_{Rician}\ldots p\left(\alpha_{M_R,M_T}\right)_{Rician}\, d\alpha_{1,1}\ldots d\alpha_{M_R,M_T}$$

…... (2.4.3.6)

Here, $\int_0^{\infty} BER_{SFBC-MPSK-OFDM}\, p\left(\alpha_{1,1}\right)_{Rician}\, d\alpha_{1,1}$

$$= \frac{1}{N\beta}\ \sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_s}_k}{2\sigma_{\alpha_k}^2\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2}\left(\frac{R_c}{R_c+2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}\right)^{n-i}\sqrt{\frac{2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}{R_c+2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}}\right]\text{erfc}\left(\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})\right)\right]$$

[Appendix G1]

…... (2.4.3.7)

Now, $\int_0^{\infty}\int_0^{\infty} BER_{SFBC-MPSK-OFDM}\, p\left(\alpha_{1,1}\right)_{Rician}\, d\alpha_{1,1}\, p\left(\alpha_{1,2}\right)_{Rician}\, d\alpha_{1,2}$

$$= \frac{1}{N\beta}\ \sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_s}_k}{2\sigma_{\alpha_k}^2\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2}\left(\frac{R_c}{R_c+2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}\right)^{n-i}\sqrt{\frac{2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}{R_c+2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}}\right]\right]\int_0^{\infty}\text{erfc}\left(\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})\right)p\left(\alpha_{1,2}\right)_{Rician}\, d\alpha_{1,2}$$

$$= \frac{1}{N\beta}\left[\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_s}_k}{2\sigma_{\alpha_k}^2\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2}\left(\frac{R_c}{R_c+2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}\right)^{n-i}\sqrt{\frac{2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}{R_c+2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}}\right]\right]\right]^2\text{erfc}\left(\sqrt{\frac{\gamma_s C_1}{R_c}}sin(\frac{\pi}{M})\right)$$

…... (2.4.3.8)

So, from equation (2.4.3.6), we get,

$$\boldsymbol{\overline{BER}_{SFBC-MPSK-OFDM-Rician}}$$

$$\boldsymbol{= \frac{1}{N\beta}\left[\ \sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_s}_k}{2\sigma_{\alpha_k}^2\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2}\left(\frac{R_c}{R_c+2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}\right)^{n-i}\sqrt{\frac{2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}{R_c+2\sigma_{\alpha_k}^2\gamma_s sin^2(\frac{\pi}{M})}}\right]\right]\right]^{M_R M_T}}$$

…... (2.4.3.9)

2.4.4 Average BER Using SFBC with MQAM-OFDM system in Rician fading Channel

For SFBC system, The BER expression for the SFBC-MQAM-OFDM system can be written as

$$BER_{SFBC-MQAM-OFDM} = \frac{2\left(1-\frac{1}{\sqrt{M}}\right)}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{i,j}[k]\right|^2}{R_c(M-1)}}\right)$$

$$= \frac{2\left(1-\frac{1}{\sqrt{M}}\right)}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}{\alpha_{k}}_{i,j}^{2}}{R_c(M-1)}}\right) \quad \text{…….. (2.4.4.1)}$$

We assume that the fading process remains static during each OFDM block and that it varies from one block to another. Therefore,

$$\alpha_{k_{1,1}} = \cdots \dots \dots = \alpha_{k_{M_R,M_T}} = {\alpha_k}^2 \quad \text{…….. (2.4.4.2)}$$

In case of OFDM, For a rician fading channel, the average received SNR/bit,

$$\overline{\boldsymbol{\gamma_{s_k}}} = \sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{4\left(\sigma_{\alpha_k}^2\right)^2}\right)^{n}(n+1)!}{(n!)^2}\left(2\sigma_{\alpha_k}^2\right)^{n+1}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\ \boldsymbol{\gamma_s} \quad \text{[Appendix D3]} \quad \text{…….. (2.4.4.3)}$$

So, The total the average received SNR/bit

$$\overline{\boldsymbol{\gamma_s}} = \sum_{k=0}^{N-1}\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{4\left(\sigma_{\alpha_k}^2\right)^2}\right)^{n}(n+1)!}{(n!)^2}\left(2\sigma_{\alpha_k}^2\right)^{n+1}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\ \boldsymbol{\gamma_s} \quad \text{…….. (2.4.4.4)}$$

So, the average BER using SFBC with MQAM-OFDM in Rician fading Channel can be evaluated by,

$$\overline{BER}_{SFBC-MQAM-OFDM-Rician} = \int_0^\infty \dots \int_0^\infty BER_{SFBC-MQAM-OFDM}\ p(\alpha_{1,1})_{Rician} \dots \dots p(\alpha_{M_R,M_T})_{Rician}\ d\alpha_{1,1} \dots \dots . . d\alpha_{M_R,M_T}$$

…….. (2.4.4.5)

Here,

$$\int_0^\infty BER_{SFBC-MQAM-OFDM}\ p(\alpha_{1,1})_{Rician}\ d\alpha_{1,1}$$

$$= \frac{2\left(1-\frac{1}{\sqrt{M}}\right)}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_s}_k}{2\sigma_{\alpha_k}^2\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2}\left(\frac{R_c(M-1)}{R_c(M-1)+3\sigma_{\alpha_k}^2\gamma_s}\right)^{n-i}\sqrt{\frac{3\sigma_{\alpha_k}^2\gamma_s}{R_c(M-1)+3\sigma_{\alpha_k}^2\gamma_s}}\right] erfc\left(\frac{1.5\gamma_s C}{R_c(M-1)}\right)\right]$$

[Appendix G2]

…….. (2.4.4.6)

Now, $\int_0^\infty \int_0^\infty BER_{SFBC-MQAM-OFDM}\; p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}\; p(\alpha_{1,2})_{Rician}\, d\alpha_{1,2}$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_s}_k}{2\sigma^2_{\alpha_k}\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2}\left(\frac{R_c(M-1)}{R_c(M-1)+3\sigma^2_{\alpha_k}\gamma_s}\right)^{n-i}\sqrt{\frac{3\sigma^2_{\alpha_k}\gamma_s}{R_c(M-1)+3\sigma^2_{\alpha_k}\gamma_s}}\right]\right]\int_0^\infty erfc\left(\frac{1.5\gamma_s C}{R_c(M-1)}\right)d\alpha_{1,1}\, p(\alpha_{1,2})_{Rician}\, d\alpha_{1,2}$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\left[\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_s}_k}{2\sigma^2_{\alpha_k}\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2}\left(\frac{R_c(M-1)}{R_c(M-1)+3\sigma^2_{\alpha_k}\gamma_s}\right)^{n-i}\sqrt{\frac{3\sigma^2_{\alpha_k}\gamma_s}{R_c(M-1)+3\sigma^2_{\alpha_k}\gamma_s}}\right]\right]\right]^2 erfc\left(\frac{1.5\gamma_s C_1}{R_c(M-1)}\right)$$

…….. (2.4.4.7)

So, from equation (2.4.4.5), we get,
,

$\boldsymbol{\overline{BER}_{SFBC-MQAM-OFDM-Rician}}$

$$= \frac{\mathbf{2(1-\frac{1}{\sqrt{M}})}}{\boldsymbol{N\beta}}\left[\sum_{\boldsymbol{k=0}}^{\boldsymbol{N-1}}\left[\sum_{\boldsymbol{n=0}}^{\infty}\frac{\boldsymbol{\overline{\gamma_s}_k}}{\boldsymbol{2\sigma^2_{\alpha_k}\gamma_s(n+1)}}\left[\mathbf{1}-\sum_{\boldsymbol{i=0}}^{\boldsymbol{n}}\frac{\boldsymbol{(2(n-i))!}}{\boldsymbol{2^{2(n-i)}((n-i)!)^2}}\left(\frac{\boldsymbol{R_c(M-1)}}{\boldsymbol{R_c(M-1)+3\sigma^2_{\alpha_k}\gamma_s}}\right)^{\boldsymbol{n-i}}\sqrt{\frac{\boldsymbol{3\sigma^2_{\alpha_k}\gamma_s}}{\boldsymbol{R_c(M-1)+3\sigma^2_{\alpha_k}\gamma_s}}}\right]\right]\right]^{\boldsymbol{M_R M_T}}$$

…….. (2.4.4.8)

2.4.5 Average BER Using SFBC with MPSK-OFDM System in Nakagami-m fading Channel

For SFBC system, The BER expression for the SFBC-MPSK-OFDM system can be written as

$$BER_{SFBC-MPSK-OFDM} = \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}|H_{i,j}[k]|^2}{R_c}}\, sin(\frac{\pi}{M})\right)$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}{\alpha_k}_{i,j}^2}{R_c}}\, sin(\frac{\pi}{M})\right) \quad …….. (2.4.5.1)$$

We assume that the fading process remains static during each OFDM block and that it varies from one block to another. Therefore,

$${\alpha_k}_{1,1} = \cdots \ldots \ldots = {\alpha_k}_{M_R,M_T} = {\alpha_k}^2 \quad …….. (2.4.5.2)$$

In case of OFDM system, for a Nakagami - m fading channel, α_k follows a rayleigh distribution with pdf

$$p(\alpha_k) = \sum_{k=0}^{N-1}\frac{2m^m}{\Gamma(m){\Omega_k}^m}{\alpha_k}^{2m-1} exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right) \quad …….. (2.4.5.3)$$

where, $m = \frac{E^2({\alpha_k}^2)}{VAR({\alpha_k}^2)}$, $\Omega_k = E({\alpha_k}^2)$ and $\sigma_{{\alpha_k}^2} = \frac{\Omega_k}{m}$

For OFDM, the average received SNR/bit

$$\overline{\gamma_s}_k = \gamma_s\, \Omega_k \text{ [Appendix D4]} \quad \text{........ (2.4.5.4)}$$

So, The total the average received SNR/bit

$$\overline{\gamma_s} = \sum_{k=0}^{N-1} \gamma_s\, \Omega_k \quad \text{........ (2.4.5.5)}$$

So, the average BER using SFBC with MQAM-OFDM in Rician fading Channel can be evaluated by,

$$\overline{BER}_{SFBC-MPSK-OFDM-Rician} = \int_0^{\infty} \dots \int_0^{\infty} BER_{SFBC-MPSK-OFDM} \dots . p(\alpha_{1,1})_{Nakagami} \dots p(\alpha_{M_R,M_T})_{Nakagami}\; d\alpha_{1,1} \dots d\alpha_{M_R,M_T}$$

.... (2.4.5.6)

Here,

$$\int_0^{\infty} BER_{SFBC-MPSK-OFDM}\, p(\alpha_{1,1})_{Nakagami}\, d\alpha_{1,1}$$

$$= \frac{1}{N\beta} \sum_{k=0}^{N-1} \left[1 - \sum_{i=0}^{m-1} \frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2} \left(\frac{mR_c}{\left(mR_c + \overline{\gamma_s}_k \sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i} \sqrt{\frac{\overline{\gamma_s}_k \sin^2(\frac{\pi}{M})}{\left(mR_c + \overline{\gamma_s}_k \sin^2(\frac{\pi}{M})\right)}}\right] erfc\left(\sqrt{\frac{\gamma_s C}{R_c}} \sin(\frac{\pi}{M})\right)$$

[Appendix G3]

.... (2.4.5.7)

Now, $\int_0^{\infty} \int_0^{\infty} BER_{SFBC-MQAM-OFDM}\, p(\alpha_{1,1})_{Nakagami}\, d\alpha_{1,1}\, p(\alpha_{1,2})_{Nakagami}\, d\alpha_{1,2}$

$$= \frac{1}{N\beta} \sum_{k=0}^{N-1} \left[1 - \sum_{i=0}^{m-1} \frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2} \left(\frac{mR_c}{\left(mR_c + \overline{\gamma_s}_k \sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i} \sqrt{\frac{\overline{\gamma_s}_k \sin^2(\frac{\pi}{M})}{\left(mR_c + \overline{\gamma_s}_k \sin^2(\frac{\pi}{M})\right)}}\right] \int_0^{\infty} erfc\left(\sqrt{\frac{\gamma_s C}{R_c}} \sin(\frac{\pi}{M})\right) p(\alpha_{1,2})_{Nakagami}\, d\alpha_{1,2}$$

$$= \frac{1}{N\beta} \sum_{k=0}^{N-1} \left[1 - \sum_{i=0}^{m-1} \frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2} \left(\frac{mR_c}{\left(mR_c + \overline{\gamma_s}_k \sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i} \sqrt{\frac{\overline{\gamma_s}_k \sin^2(\frac{\pi}{M})}{\left(mR_c + \overline{\gamma_s}_k \sin^2(\frac{\pi}{M})\right)}}\right]^2 erfc\left(\sqrt{\frac{\gamma_s C_1}{R_c}} \sin(\frac{\pi}{M})\right)$$

.... (2.4.5.8)

So, from equation (2.4.5.6), we get,

$$\overline{\boldsymbol{BER}}_{\boldsymbol{SFBC-MPSK-OFDM-Nakagami}}$$

$$= \frac{1}{N\beta} \left[\sum_{k=0}^{N-1} \left[1 - \sum_{i=0}^{m-1} \frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2} \left(\frac{mR_c}{\left(mR_c + \overline{\gamma_s}_k \sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i} \sqrt{\frac{\overline{\gamma_s}_k \sin^2(\frac{\pi}{M})}{\left(mR_c + \overline{\gamma_s}_k \sin^2(\frac{\pi}{M})\right)}}\right]\right]^{M_R M_T}$$

.... (2.4.5.9)

2.4.6 Average BER Using SFBC with MQAM-OFDM System in Nakagami-m fading Channel

For SFBC system, The BER expression for the SFBC-MQAM-OFDM system can be written as

$$BER_{SFBC-MQAM-OFDM} = \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{i,j}[k]\right|^2}{R_c(M-1)}}\right)$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\alpha_{k_{i,j}}^{2}}{R_c(M-1)}}\right) \quad \text{...... (2.4.6.1)}$$

We assume that the fading process remains static during each OFDM block and that it varies from one block to another. Therefore,

$$\alpha_{k_{1,1}} = \cdots = \alpha_{k_{M_R,M_T}} = \alpha_k^{\,2} \quad \text{........ (2.4.6.2)}$$

In case of OFDM system, for a Nakagami - m fading channel, α_k follows a distribution with pdf

$$p(\alpha_k) = \sum_{k=0}^{N-1}\frac{2m^m}{\Gamma(m)\Omega_k^{\,m}}\alpha_k^{\,2m-1} exp\left(-\frac{m}{\Omega_k}\alpha_k^{\,2}\right) \quad \text{........ (2.4.6.3)}$$

where, $m = \frac{E^2(\alpha_k^{\,2})}{VAR(\alpha_k^{\,2})}$, $\Omega_k = E(\alpha_k^{\,2})$ and $\sigma_{\alpha_k^{\,2}} = \frac{\Omega_k}{m}$

For OFDM, the average received SNR/bit is

$$\overline{\gamma_s}_k = \gamma_s\,\Omega_k \quad \text{[Appendix D4]} \quad \text{........ (2.4.6.4)}$$

So, The total the average received SNR/bit is

$$\overline{\gamma_s} = \sum_{k=0}^{N-1}\gamma_s\,\Omega_k \quad \text{........ (2.4.6.5)}$$

So, The average BER using SFBC with MQAM-OFDM in Nakagami-m fading Channel can be evaluated by,

$$\overline{BER}_{SFBC-MQAM-OFDM-Nakagami} = \int_0^\infty \ldots \int_0^\infty BER_{SFBC-MQAM-OFDM}\, p(\alpha_{1,1})_{Nakagami} \ldots p(\alpha_{M_R,M_T})_{Nakagami}\, d\alpha_{1,1} \ldots d\alpha_{M_R,M_T}$$

............(2.4.6.6)

Here, $\int_0^\infty BER_{SFBC-MQAM-OFDM}\, p(\alpha_{1,1})_{Nakagami}\, d\alpha_{1,1}$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}\frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2}\left(\frac{mR_c(M-1)}{(mR_c(M-1)+1.5\overline{\gamma_s}_k)}\right)^{m-1-i}\sqrt{\frac{1.5\overline{\gamma_s}_k}{(mR_c(M-1)+1.5\overline{\gamma_s}_k)}}\right] erfc\left(\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}\right)$$

[Appendix G4]

.... (2.4.6.7)

Now, $\int_0^\infty BER_{SFBC-MQAM-OFDM}\, p(\alpha_{1,1})_{Nakagami}\, d\alpha_{1,1} p(\alpha_{1,2})_{Nakagami}\, d\alpha_{1,2}$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta} \sum_{k=0}^{N-1} \left[1 - \sum_{i=0}^{m-1} \frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2} \left(\frac{mR_c(M-1)}{(mR_c(M-1)+1.5\overline{\gamma_s}_k)} \right)^{m-1-i} \sqrt{\frac{1.5\overline{\gamma_s}_k}{(mR_c(M-1)+1.5\overline{\gamma_s}_k)}} \right] \int_o^\infty erfc\left(\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}} \right) d\alpha_{1,1} p(\alpha_{1,2})_{Nakagami}\, d\alpha_{1,2}$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta} \left[\sum_{k=0}^{N-1} \left[1 - \sum_{i=0}^{m-1} \frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2} \left(\frac{mR_c(M-1)}{(mR_c(M-1)+1.5\overline{\gamma_s}_k)} \right)^{m-1-i} \sqrt{\frac{1.5\overline{\gamma_s}_k}{(mR_c(M-1)+1.5\overline{\gamma_s}_k)}} \right] \right]^2 erfc\left(\sqrt{\frac{1.5\gamma_s C_1}{R_c(M-1)}} \right)$$

So, from equation (2.4.6.6), we get,

$$\overline{\boldsymbol{BER}}_{\boldsymbol{SFBC-MQAM-OFDM-Nakagami}}$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta} \left[\sum_{k=0}^{N-1} \left[1 - \sum_{i=0}^{m-1} \frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2} \left(\frac{mR_c(M-1)}{(mR_c(M-1)+1.5\overline{\gamma_s}_k)} \right)^{m-1-i} \sqrt{\frac{1.5\overline{\gamma_s}_k}{(mR_c(M-1)+1.5\overline{\gamma_s}_k)}} \right] \right]^{M_R M_T}$$

…….. (2.4.6.8)

CHAPTER 3

RESULTS AND DISCUSSIONS

3.1 Introduction (Summary and Table of Parameters)

The BER performances of SFBC-OFDM with MPSK and MQAM (M=4) modulation schemes in Rician and Nakagami-m fading environment are evaluated. The considered channels are multipath fading channels with coherence bandwidth smaller than the total bandwidth of the multicarrier system and thus seen as frequency-selective channels. The fading process is assumed to be stationary and slow-varying compared to the symbol duration of the multicarrier signal and as such, it is approximately constant during one OFDM block length. The OFDM system includes N = 512 subcarriers and a cyclic prefix which is longer than the delay spread In the provided results, the term simulation in the figures refers to measurement of the average BER based on simulation of the proposed system which is obtained using the derived formulae.

M	The number of points in the signal constellation.
N	No of Subchannels
Rc	Code Rate
m	Shape Parameter
α	Fading parameter
σ_α^2	Variance of fading
α_o^2	Power of LOS component
β	No of bits
γ	SNR
M_R	No of receive antennas
M_T	No of Transmit antennas

3.2 Results and Discussions

3.2.1 Simulation Results of Average BER Performance of SFBC-OFDM system in Environment with Different Antenna Configurations

The average BER performance of the SFBC-OFDM system MPSK and MQAM modulation schemes in Rician fading environment are shown in Fig. 3.1 & Fig. 3.2 respectively.

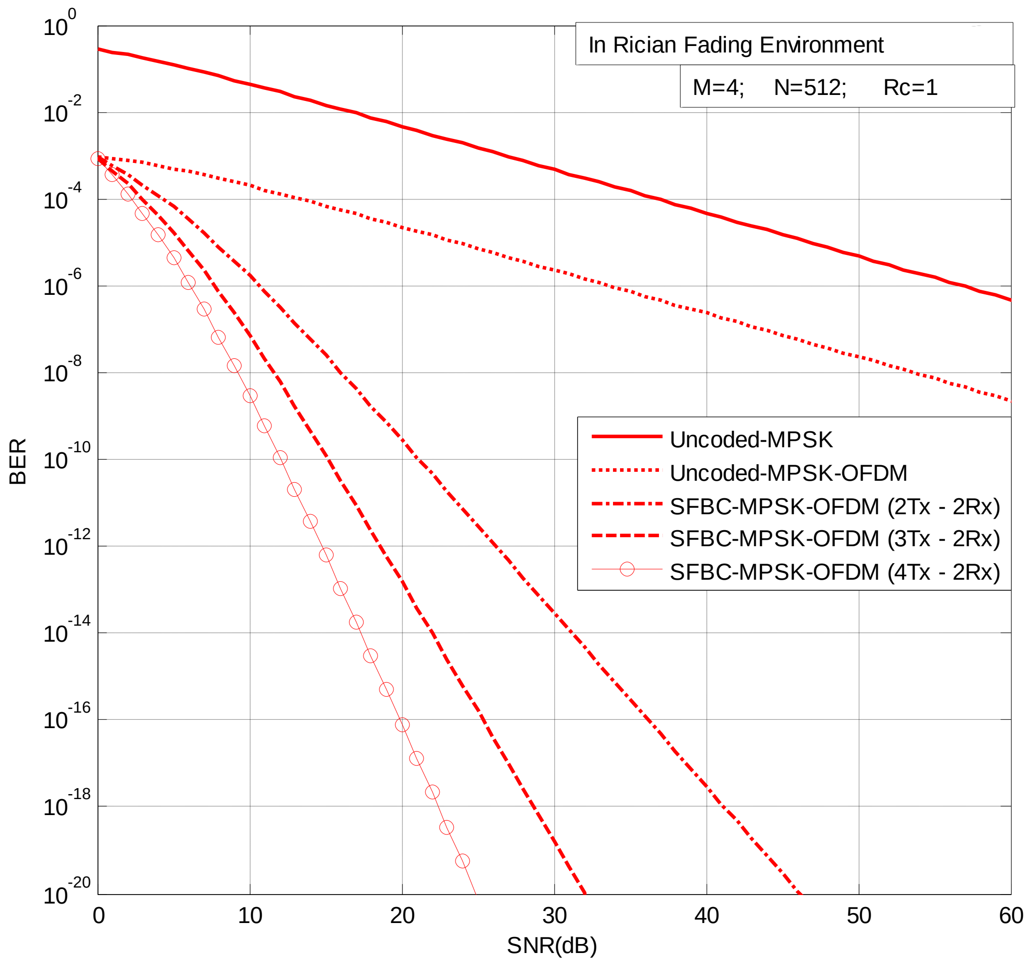

Fig 3.1: Average BER performance of MPSK-SFBC-OFDM in Rician fading environment with different antenna configurations

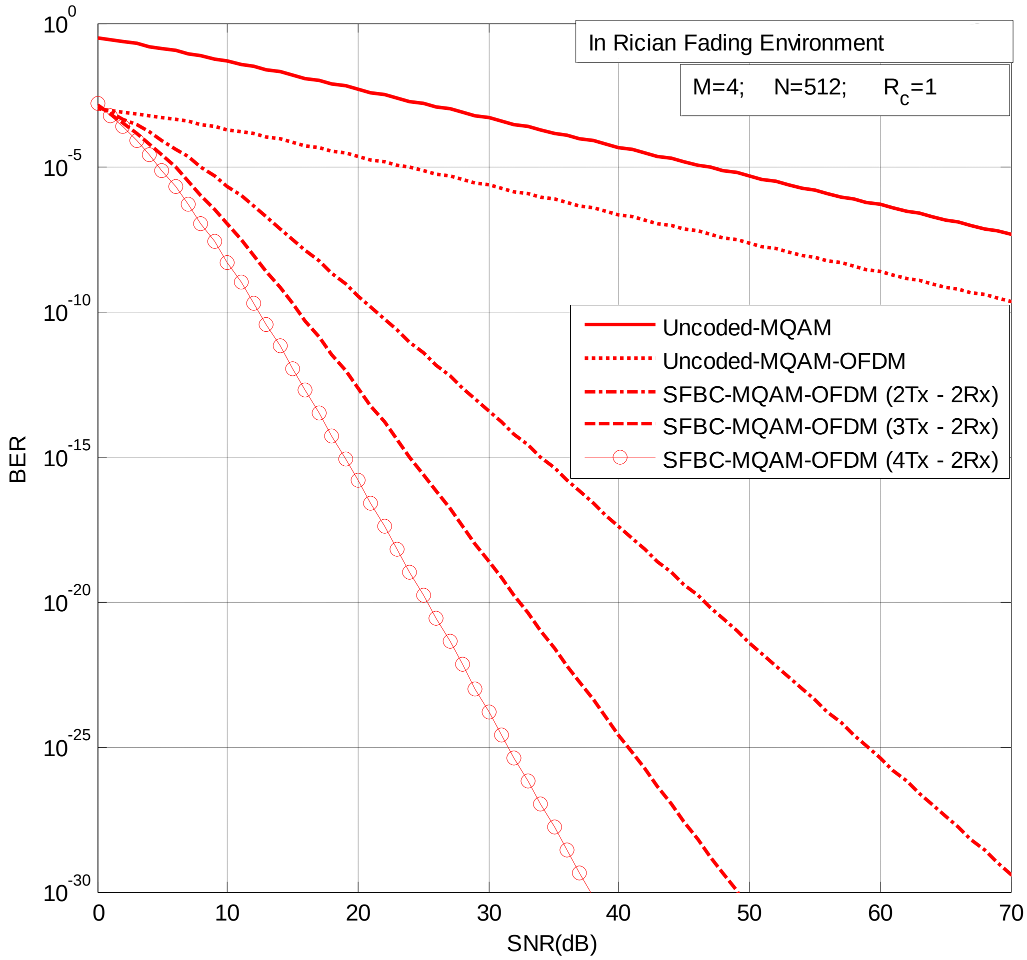

Fig 3.2: Average BER performance of MQAM-SFBC-OFDM in Rician fading environment with different antenna configurations

Results are presented in Rician fading environment for uncoded MPSK, uncoded MPSK OFDM, SFBC-MPSK-OFDM (Fig. 3.1) and also for uncoded MQAM, uncoded MQAM OFDM, SFBC-MQAM-OFDM (Fig. 3.2). SFBC with 2Tx-2Rx antennas, 3Tx-2Rx, 4Tx-2Rx are considered,. The code rate in each case is 1 bits/s/Hz. In the plot, the set of curves are obtained by calculating the average BER using derived formula. It can be seen that SFBC (2Tx-2Rx) provides a remarkable BER performance gain for OFDM and significantly outperforms uncoded OFDM; a gain of about 46 dB (Fig. 3.1) and 44 dB (Fig. 3.2) at a BER of 10^{-6} is obtained. Higher gains are achieved by using higher order SFBC with more antennas at the transmitter.

The average BER performance of the SFBC-OFDM system MPSK and MQAM modulation schemes in Nakagami-m fading environment are shown in Fig. 3.3 & Fig. 3.4 respectively.

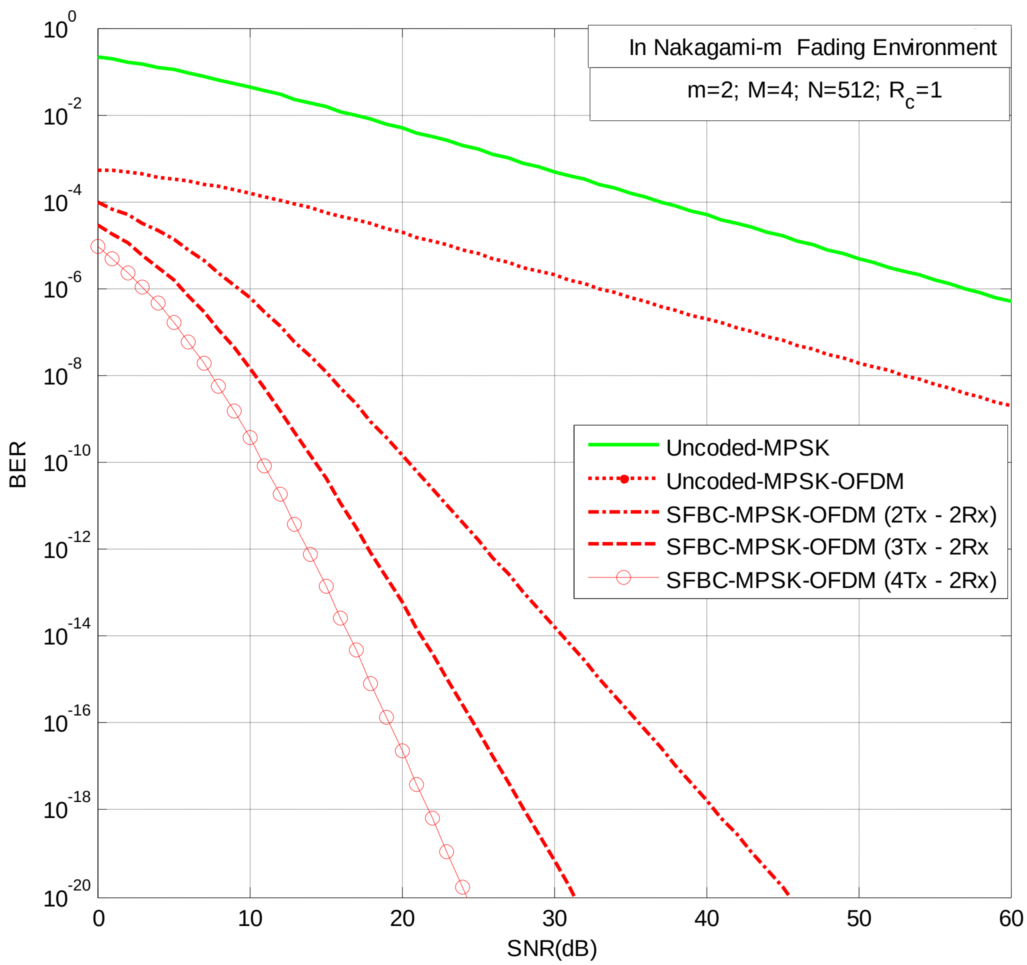

Fig 3.3: Average BER performance of MPSK-SFBC-OFDM system in Nakagami-m fading environment with different antenna configurations

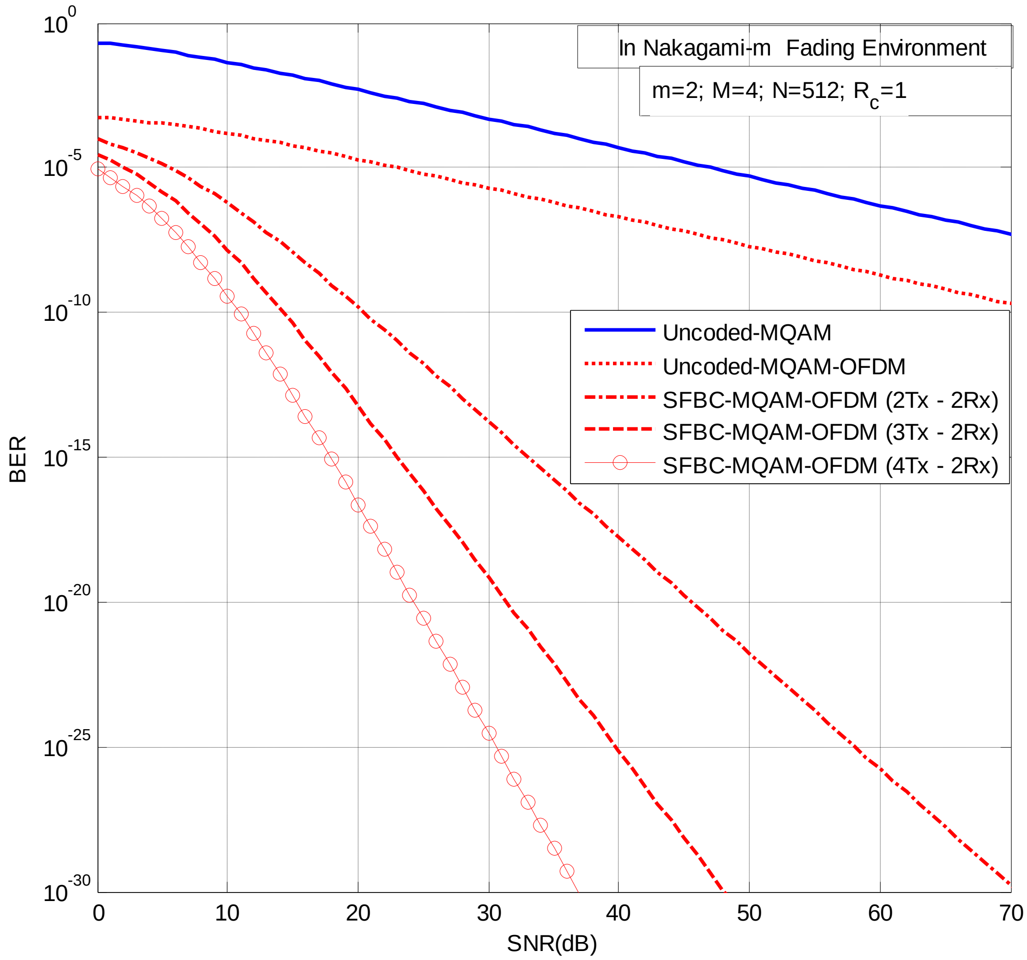

Fig 3.4: Average BER performance of MQAM-SFBC-OFDM system in Nakagami-m fading environment with different antenna configurations

Results are presented in Nakagami-m fading environment (m=2) for uncoded MPSK, uncoded MPSK OFDM, SFBC-MPSK-OFDM (Fig. 3.3) and also for uncoded MQAM, uncoded MQAM OFDM, SFBC-MQAM-OFDM (Fig. 3.4). SFBC with 2Tx-2Rx antennas, 3Tx-2Rx, 4Tx-2Rx are considered. The code rate in each case is 1 bits/s/Hz. In the plot, the set of curves are obtained by calculating the average BER using derived formula. It can be seen that SFBC (2Tx-2Rx) provides a remarkable BER performance gain for OFDM and significantly outperforms uncoded OFDM; a gain of about 48 dB (Fig. 3.3) and 46 dB (Fig. 3.4) at a BER of 10^{-6} is obtained. Higher gains are achieved by using higher order SFBC with more antennas at the transmitter.

3.2.2 Simulation Results of Average BER Performance of SFBC-OFDM System with Different Code Rate

The average BER performance of the SFBC-OFDM system MPSK and MQAM modulation schemes in Rician fading environment with different code rate are shown in Fig. 3.5 & Fig. 3.6 respectively.

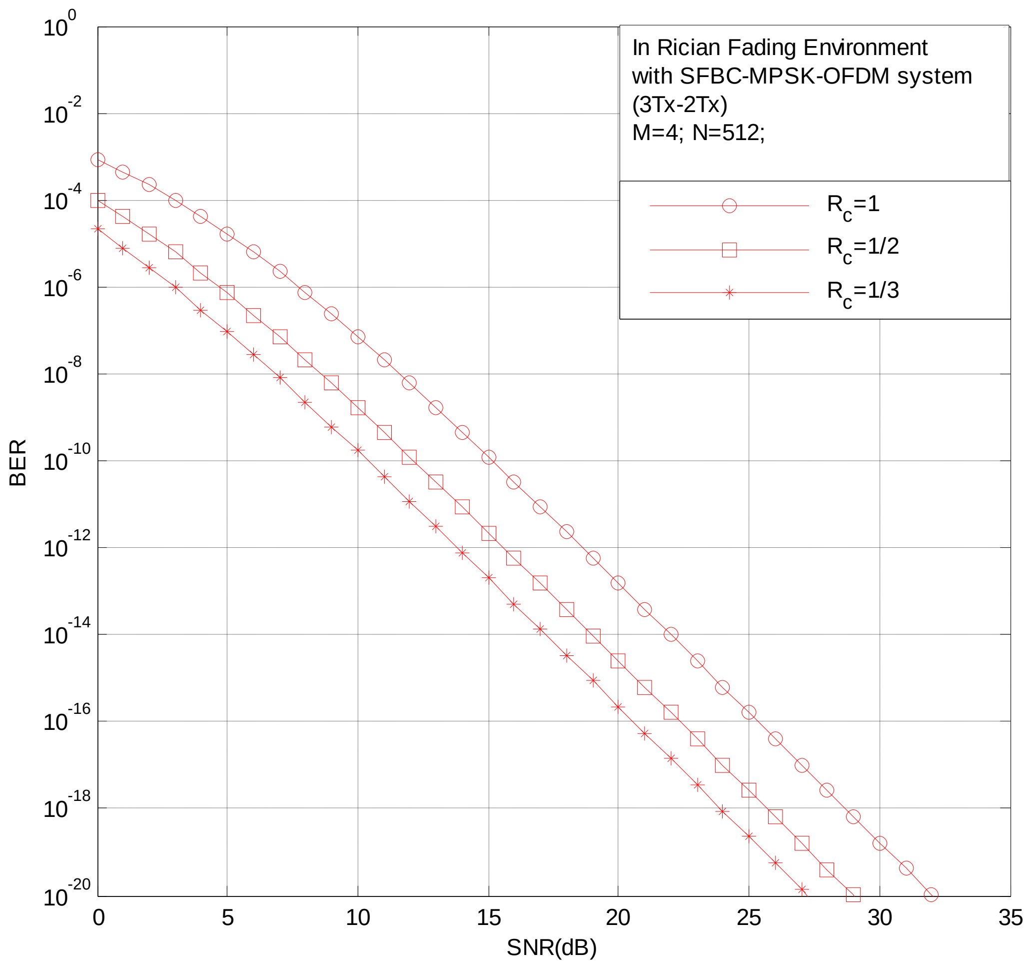

Fig. 3.5: Average BER performance of SFBC-MPSK-OFDM system with Rician fading with different code rate

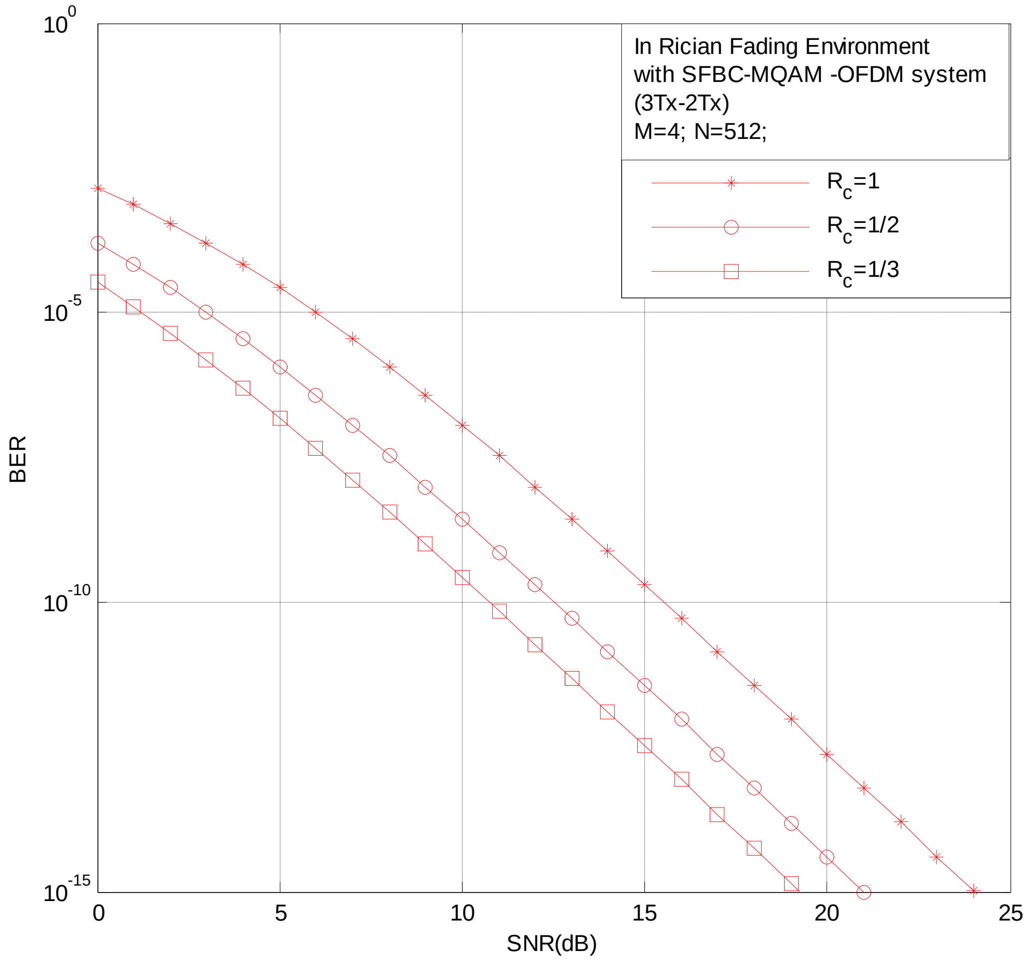

Fig. 3.6: Average BER performance of SFBC-MQAM-OFDM system with Rician fading with different code rate

Results are presented in Rician fading environment for SFBC-MPSK-OFDM (Fig. 3.5) and also for SFBC-MQAM-OFDM (Fig. 3.6). SFBC with 3Tx-2Rx are considered with different code rates. The code rates, Rc=1 bits/s/Hz, Rc=1/2 bits/s/Hz, Rc=1/3 bits/s/Hz are considered. In the plot, the set of curves are obtained by calculating the average BER using derived formula. It can be seen that SFBC (3Tx-2Rx) provides a small BER performance gain for OFDM and significantly a gain of about 3 dB (when shifted from Rc=1 to Rc=1/2)and 5 dB (when shifted from Rc=1 to Rc=1/3) at a BER of 10^{-6} is obtained.

The average BER performance of the SFBC-OFDM system MPSK and MQAM modulation schemes in Nakagami-m fading environment with different code rate are shown in Fig. 3.7 & Fig. 3.8 respectively.

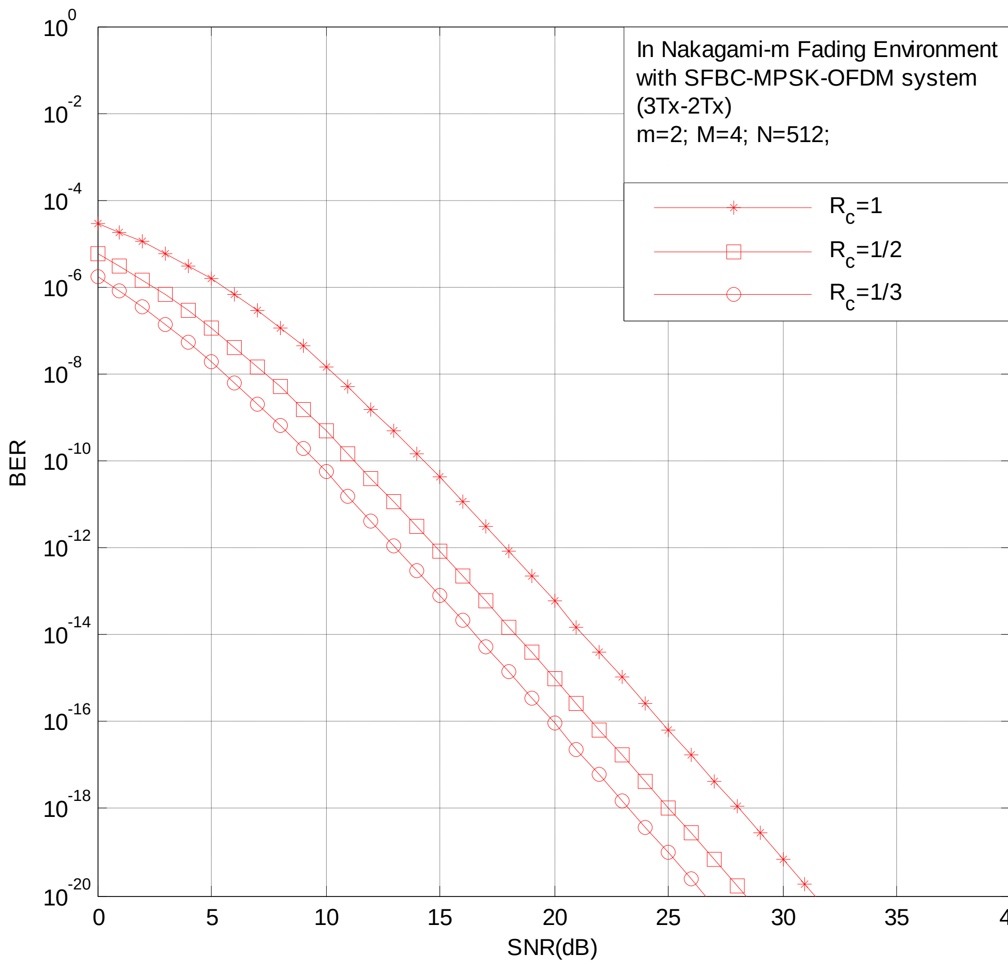

Fig. 3.7: Average BER performance of SFBC-MPSK-OFDM system with Nakagami-m fading with different code rate

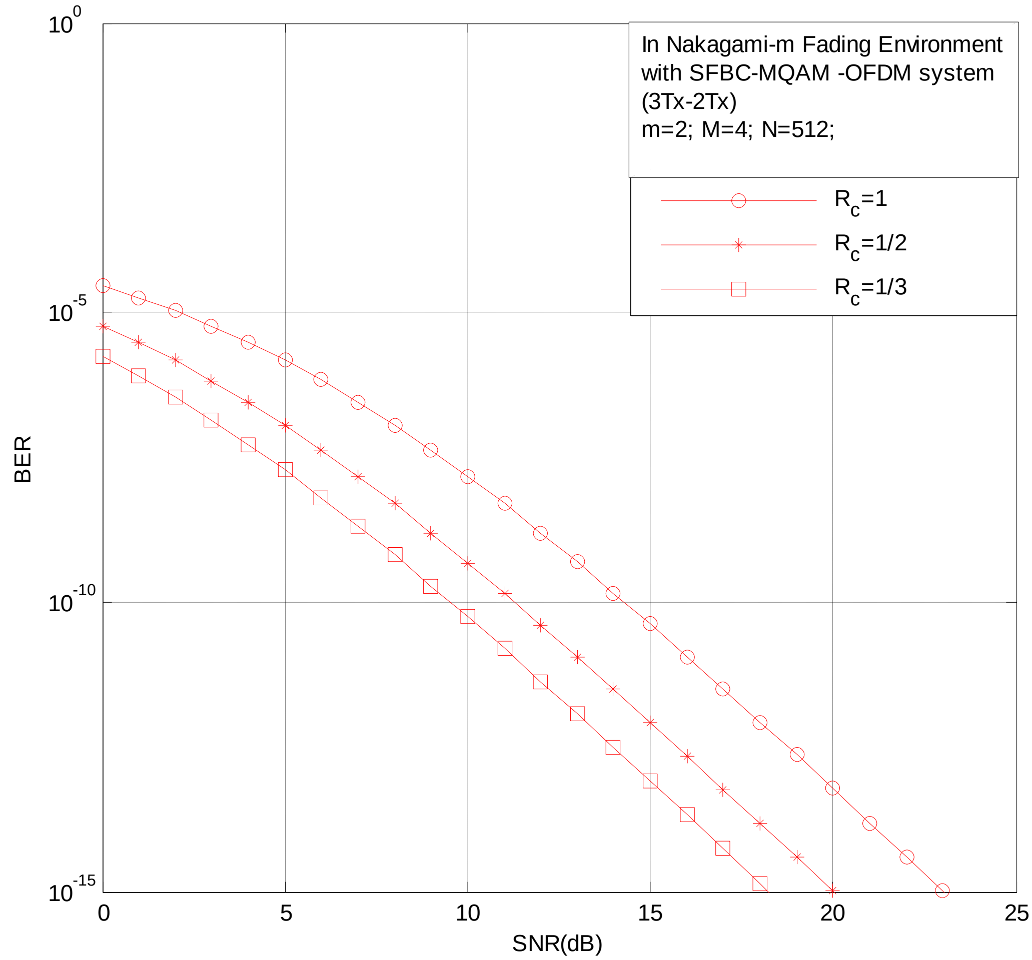

Fig. 3.8: Average BER performance of SFBC-MQAM-OFDM system with Nakagami-m fading with different code rate

Results are presented in Nakagami-m fading environment (m=2) for SFBC-MPSK-OFDM (Fig. 3.7) and also for, SFBC-MQAM-OFDM (Fig. 3.8). SFBC with 3Tx-2Rx are considered with different code rates. The code rates, Rc=1 bits/s/Hz, Rc=1/2 bits/s/Hz, Rc=1/3 bits/s/Hz are considered. In the plot, the set of curves are obtained by calculating the average BER using derived formula. It can be seen that SFBC (3Tx-2Rx) provides a small BER performance gain for OFDM and significantly a gain of about 3 dB (when shifted from Rc=1 to Rc=1/2)and 5 dB (when shifted from Rc=1 to Rc=1/3) at a BER of 10^{-6} is obtained.

3.2.3 Simulation Results of the Comparison of Average BER Performance of SFBC-OFDM System between Nakagami-m Fading and Rician Fading

The comparison of average BER performance of the SFBC-MPSK-OFDM system between Rician fading and Nakagami-m fading environment are shown in Fig. 3.9.

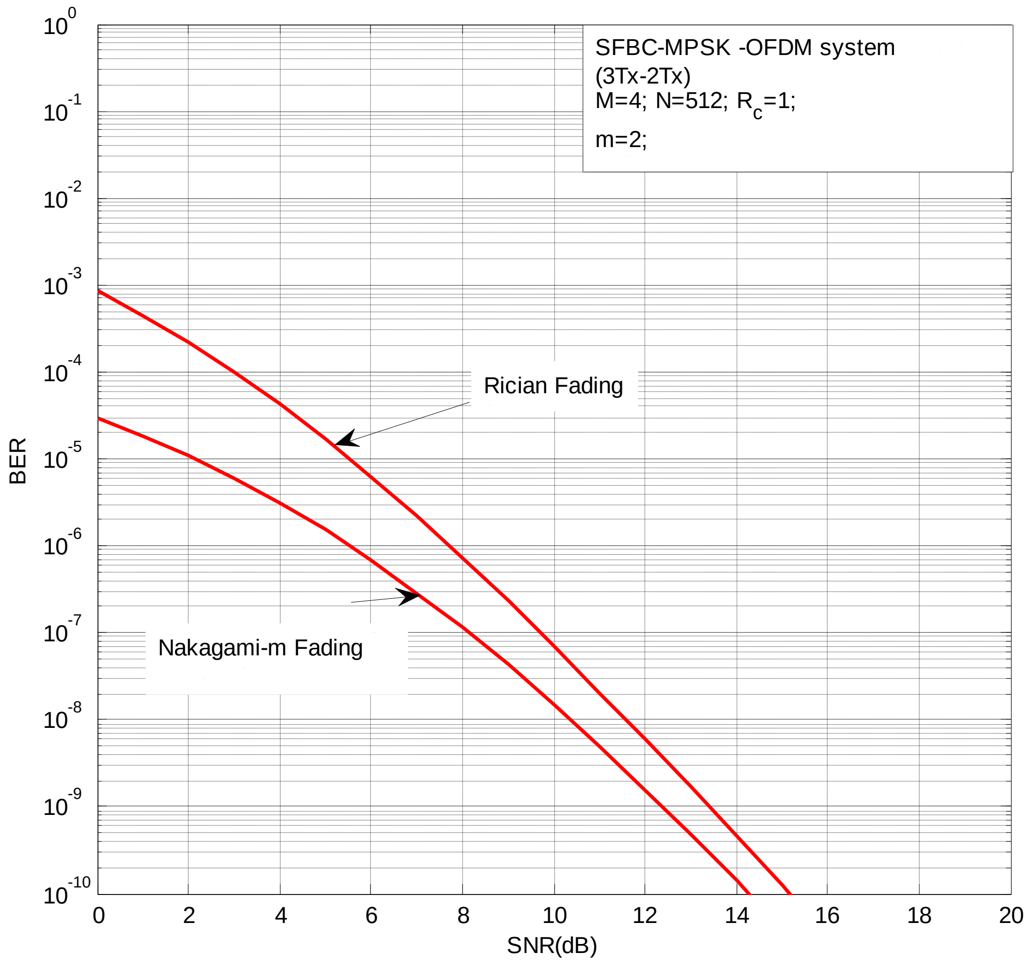

Fig. 3.9: Comparison of Average BER performance SFBC-MPSK-OFDM system between Nakagami-m Fading and Rician Fading

Results are presented for comparison of SFBC-MPSK-OFDM system between Nakagami-m Fading (m=2) and Rician Fading. SFBC with 3Tx-2Rx is considered. The code rate in this case is 1 bits/s/Hz. In the plot, the set of curves are obtained by calculating the average BER using derived formula. It can be seen that SFBC-MPSK-OFDM (3Tx-2Rx) with Nakagami-m fading performs better than SFBC-MPSK-OFDM (3Tx-2Rx) with Rician fading and improves to a remarkable BER performance gain for OFDM; a gain of about 2 dB at a BER of 10^{-6} is obtained. Higher gains are achieved by using higher order SFBC with more antennas at the transmitter.

The comparison of average BER performance of the SFBC-MQAM-OFDM system between Rician fading and Nakagami-m fading environment are shown in Fig. 3.10.

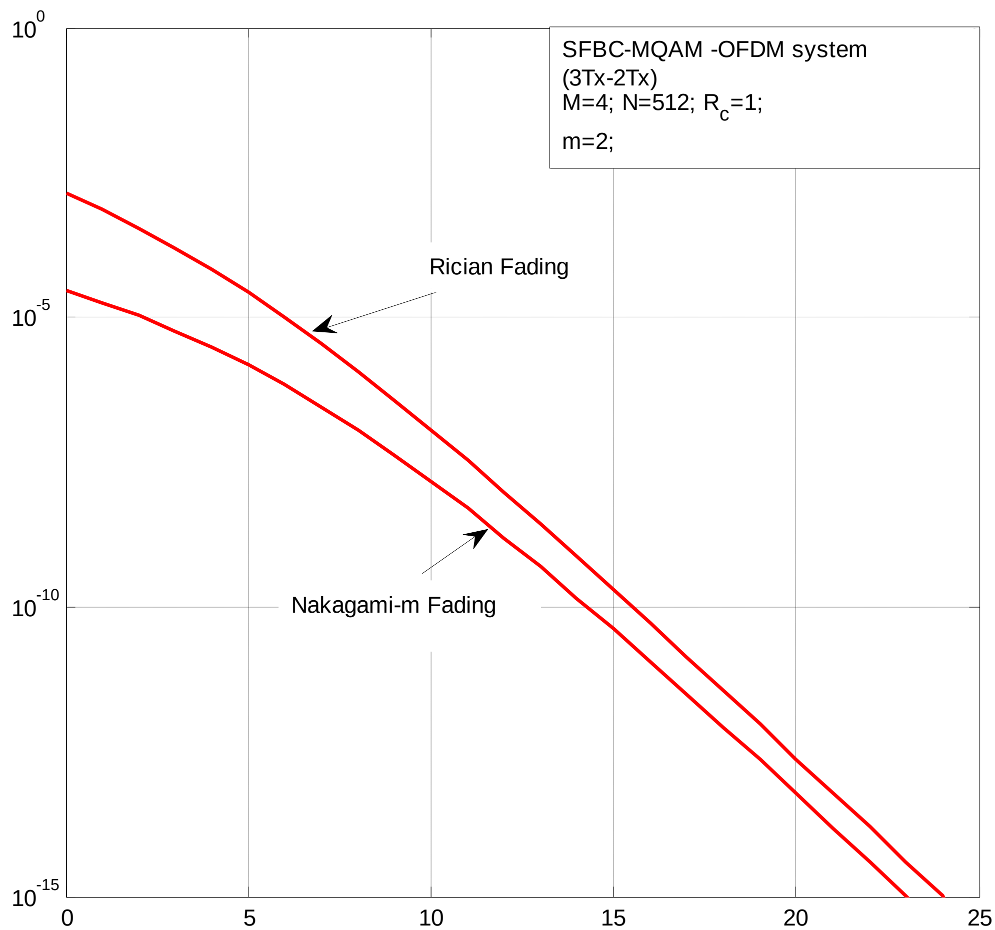

Fig. 3.10: Comparison of Average BER performance SFBC-MQAM-OFDM system between Nakagami-m Fading and Rician Fading

Results are presented for comparison of SFBC-MQAM-OFDM system between Nakagami-m Fading (m=2) and Rician Fading. SFBC with 3Tx-2Rx is considered. The code rate in this case is 1 bits/s/Hz. In the plot, the set of curves are obtained by calculating the average BER using derived formula. It can be seen that SFBC-MQAM-OFDM (3Tx-2Rx) with Nakagami-m fading performs better than SFBC-MQAM-OFDM (3Tx-2Rx) with Rician fading and improves to a remarkable BER performance gain for OFDM; a gain of about 2 dB at a BER of 10^{-6} is obtained. Higher gains are achieved by using higher order SFBC with more antennas at the transmitter.

3.2.4 Simulation Results of the Improvement in SNR(dB) from Uncoded to the SFBC-OFDM system at a fixed BER with the Increase of the No of Transmitting Antennas

The Improvement in SNR(dB) from Uncoded MPSK to the SFBC-MPSK-OFDM system & Uncoded MQAM to the SFBC-MQAM-OFDM system at a fixed BER with the increase of the no of transmitting antenna in Rician fading environment are shown in Fig. 3.11 & Fig. 3.12 respectively.

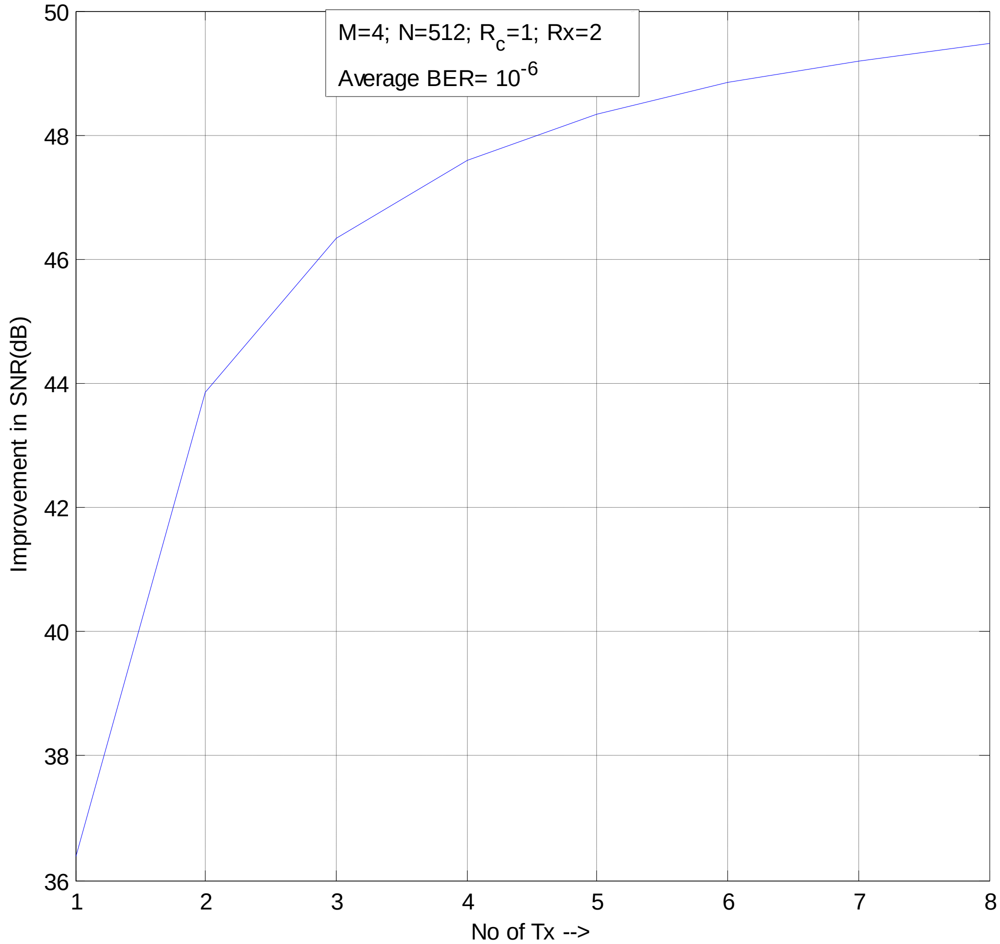

Fig. 3.11: The Improvement in SNR(dB) from uncoded to SFBC-MPSK-OFDM system in Rician Fading with the increase of the no of transmitting antenna at a BER of 10^{-6}

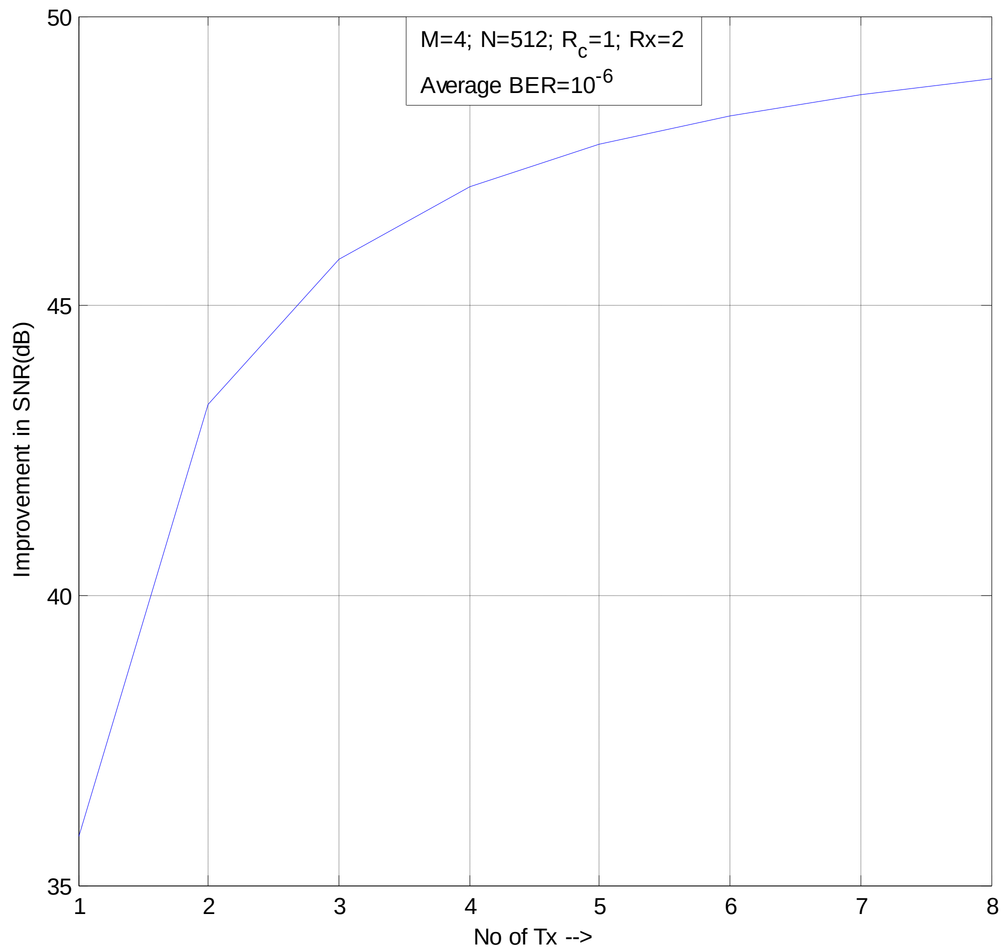

Fig. 3.12: The Improvement in SNR(dB) from uncoded to SFBC-MQAM-OFDM system in Rician Fading with the increase of the no of transmitting antenna at a BER of 10^{-6}

Results are presented in Rician fading environment at a average BER of 10^{-6} for SFBC-MPSK-OFDM (Fig. 3.11) and also for SFBC-MQAM-OFDM (Fig. 3.12). SFBC with 2Rx is considered. The code rate in this case is 1 bits/s/Hz. It can be seen that SFBC provides a significant improvement in SNR(dB) for OFDM with the increase of Tx (1-8) and significantly a gain of about 1.24 dB (when shifted from 3Tx to 4Tx) (Fig. 3.11) and 1.25 dB (when shifted from 3Tx to 4Tx) (Fig. 3.12). It is seen that the improvement is very poor after 5 Tx in both the cases. So, the optimum no of antenna is 5.

The Improvement in SNR(dB) from Uncoded MPSK to the SFBC-MPSK-OFDM system & Uncoded MQAM to the SFBC-MQAM-OFDM system at a fixed BER with the increase of the no of transmitting antenna in Nakagami-m fading environment are shown in Fig. 3.13 & Fig. 3.14 respectively.

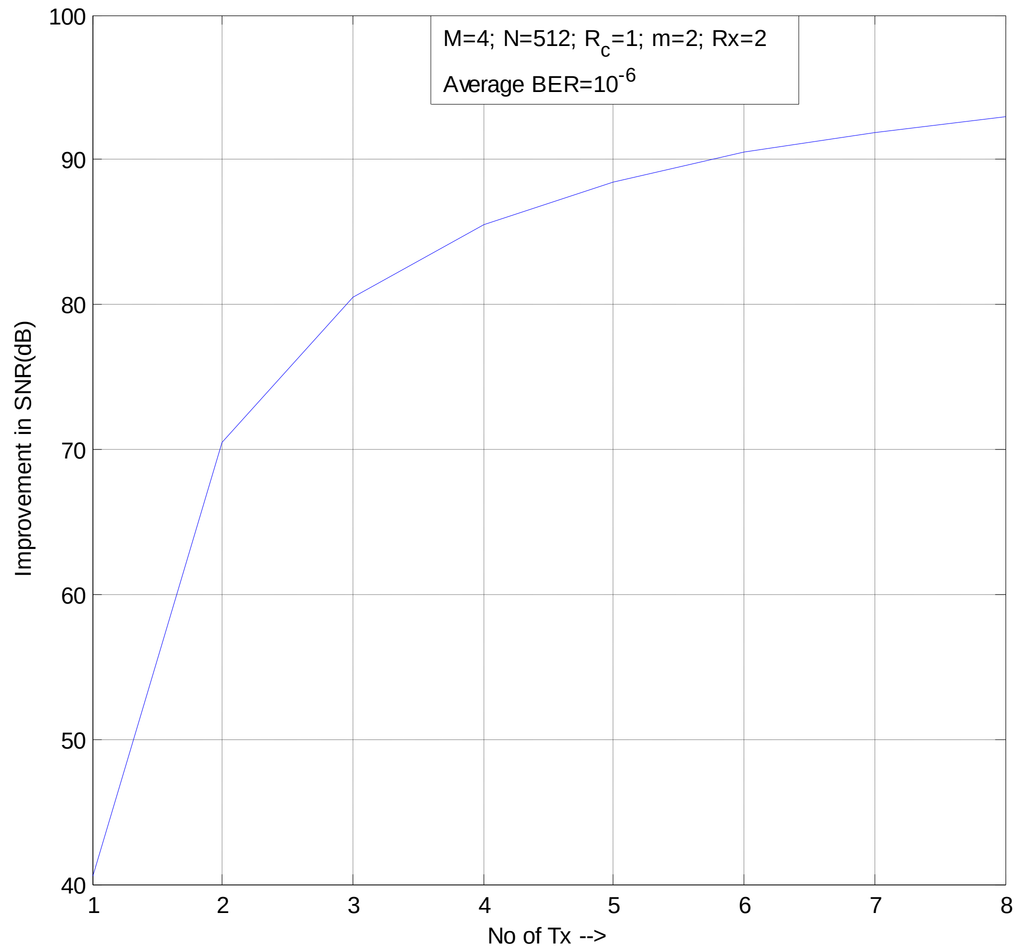

Fig. 3.13: The Improvement in SNR(dB) from uncoded to SFBC-MPSK-OFDM system in Nakagami-m Fading with the increase of the no of transmitting antenna at a BER of 10^{-6}

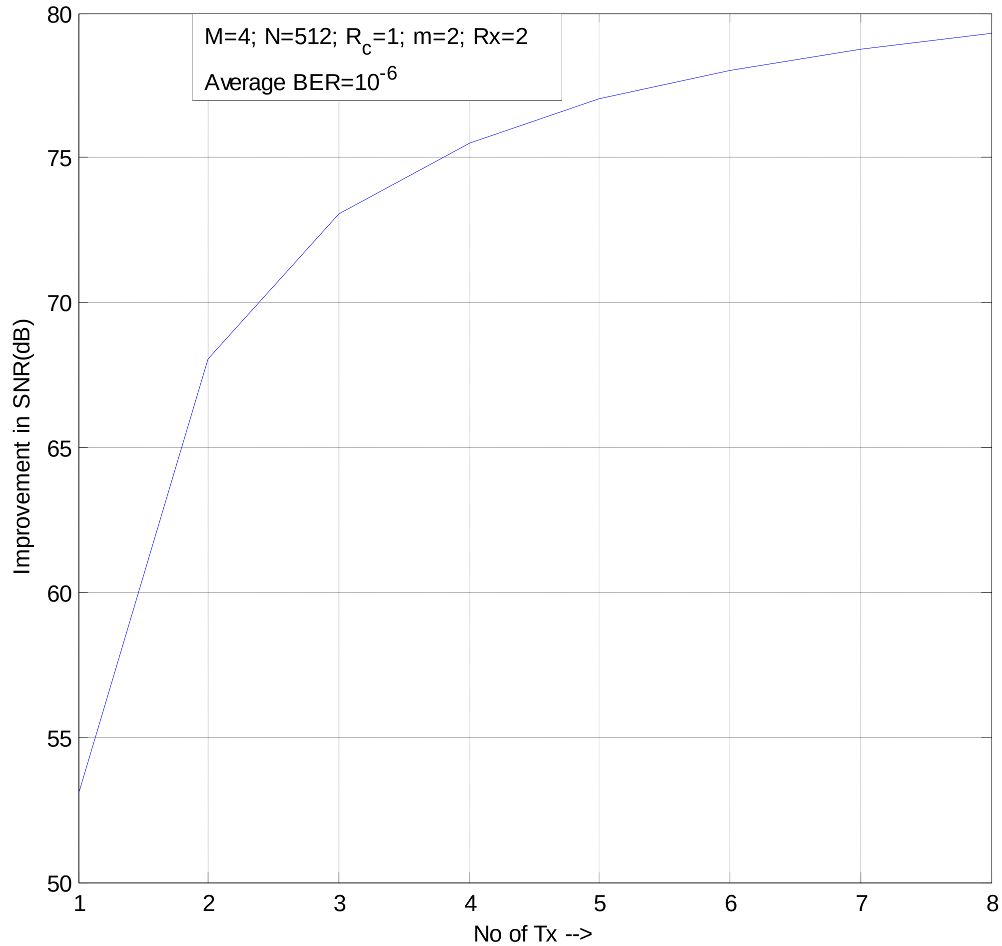

Fig. 3.14: The Improvement in SNR(dB) from uncoded to SFBC-MQAM-OFDM system in Nakagami-m Fading with the increase of the no of transmitting antenna at a BER of 10^{-6}

Results are presented in Nakagami-m fading environment at a average BER of 10^{-30} for SFBC-MPSK-OFDM (Fig. 3.13) and also for SFBC-MQAM-OFDM (Fig. 3.14). SFBC with 2Rx is considered. The code rate in this case is 1 bits/s/Hz. It can be seen that SFBC provides a significant improvement in SNR(dB) for OFDM with the increase of Tx (1-8) and significantly a gain of about 5 dB (when shifted from 3Tx to 4Tx) (Fig. 3.13) and 2.49 dB (when shifted from 3Tx to 4Tx) (Fig. 3.14). It is seen that the improvement is very poor after 5 Tx in both the cases. So, the optimum no of antenna is 5.

3.2.5 Simulation Results of the Comparison of the Improvement in SNR(dB) from Uncoded to the SFBC-OFDM system at a fixed BER with the Increase of the No of Transmitting Antennas between Rician Fading and Nakagami-m Fading Environment

The comparison of the improvement in SNR(dB) from Uncoded MPSK to the SFBC-MPSK-OFDM system at a fixed BER with the increase of the no of transmitting antenna between Rician fading and Nakagami-m fading environment are shown in Fig. 3.15.

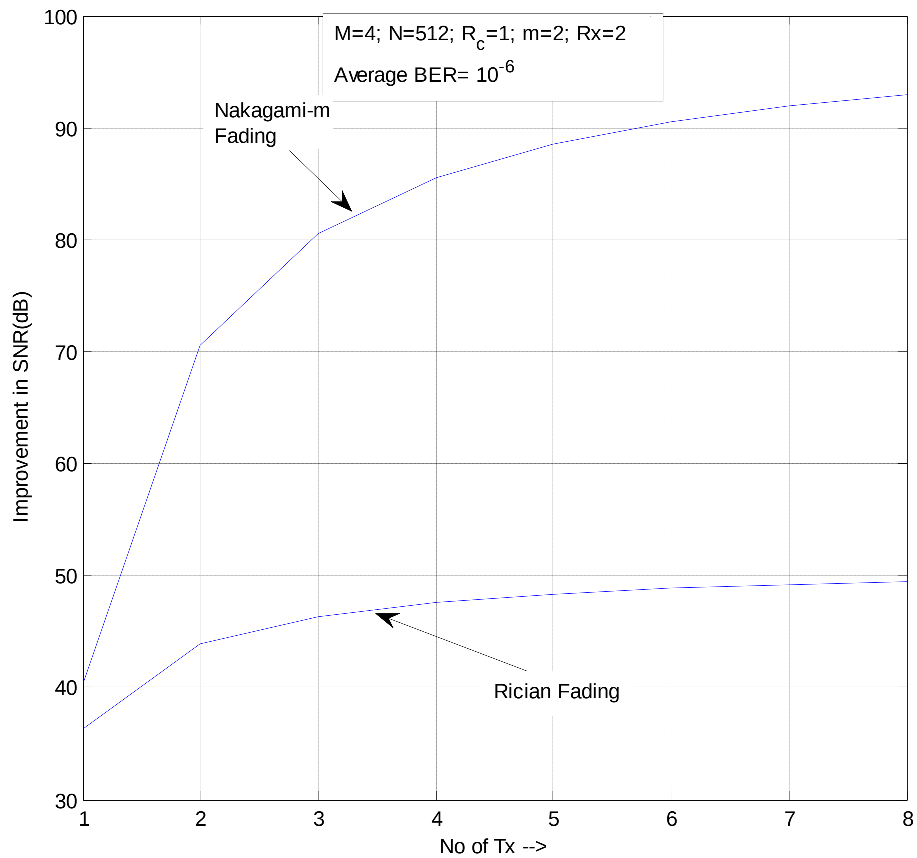

Fig. 3.15: The comparison of the improvement in SNR(dB) from uncoded to SFBC-MPSK-OFDM system between Rician Fading and Nakagami-m fading with the increase of the no of transmitting antenna at a BER of 10^{-6}

Results are presented in both Rician fading and nakagami-m fading environment at a average BER of 10^{-6} for SFBC-MPSK-OFDM. SFBC with 2Rx is considered. The code rate in each case is 1 bits/s/Hz. It can be seen that the improvement in SNR(dB) in Nakagami-m fading is better than that of a Rician fading and SFBC provides a significant improvement in SNR(dB) for OFDM with the increase of Tx (1-5) and significantly a gain of about 1.24 dB (when shifted from 3Tx to 4Tx) in Nakagami-m fading and 5 dB (when shifted from 3Tx to 4Tx) in Rician fading.

The comparison of the improvement in SNR(dB) from Uncoded MQAM to the SFBC-MQAM-OFDM at a fixed BER with the increase of the no of transmitting antenna between Rician fading and Nakagami-m fading environment are shown in Fig. 3.16.

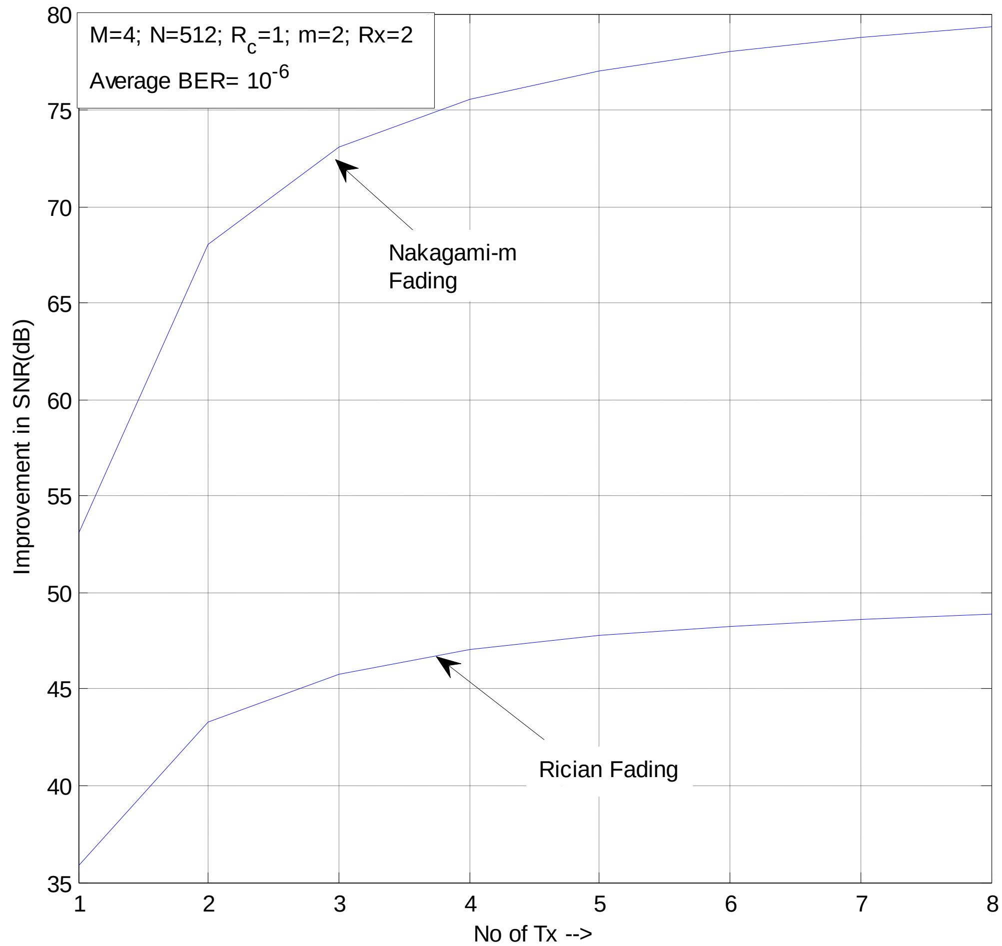

Fig. 3.16: The comparison of the improvement in SNR(dB) from uncoded to SFBC-MQAM-OFDM system between Rician Fading and Nakagami-m fading with the increase of the no of transmitting antenna at a BER of 10^{-6}

Results are presented in both Rician fading and nakagami-m fading environment at a average BER of 10^{-6} for SFBC-MQAM-OFDM. SFBC with 2Rx is considered. The code rate in each case is 1 bits/s/Hz. It can be seen that the improvement in SNR(dB) in Nakagami-m fading is better than that of a Rician fading and SFBC provides a significant improvement in SNR(dB) for OFDM with the increase of Tx (1-5) and significantly a gain of about 1.25 dB (when shifted from 3Tx to 4Tx) in Nakagami-m fading and 2.49 dB (when shifted from 3Tx to 4Tx) in Rician fading.

3.2.6 Simulation Results of the Improvement in SNR(dB) from Uncoded to the SFBC-OFDM system at a fixed BER with Different Code Rates

The Improvement in SNR(dB) from Uncoded MPSK to the SFBC-MPSK-OFDM system & Uncoded MQAM to the SFBC-MQAM-OFDM system at a fixed BER with different code rates in Rician fading environment are shown in Fig. 3.17 & Fig. 3.18 respectively.

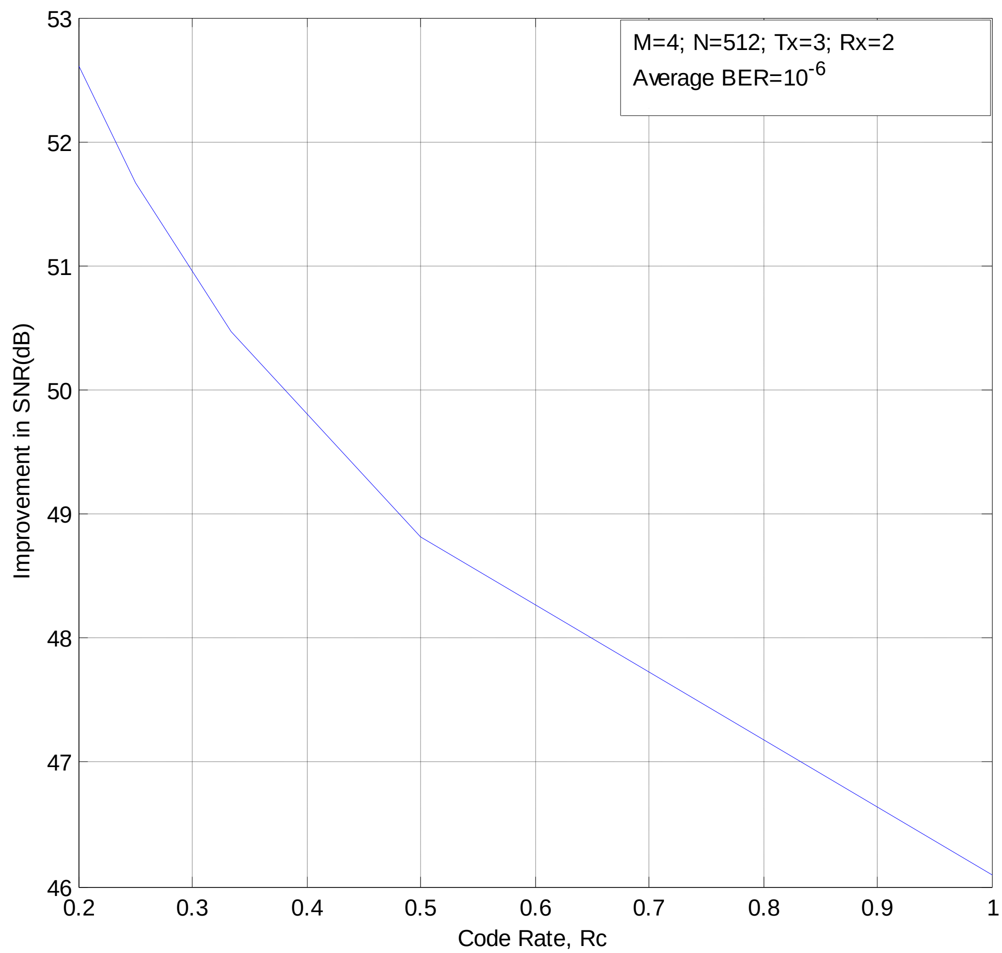

Fig. 3.17: The Improvement in SNR(dB) from uncoded to SFBC-MPSK-OFDM system in Rician fading with the different code rates at a BER of 10^{-6}

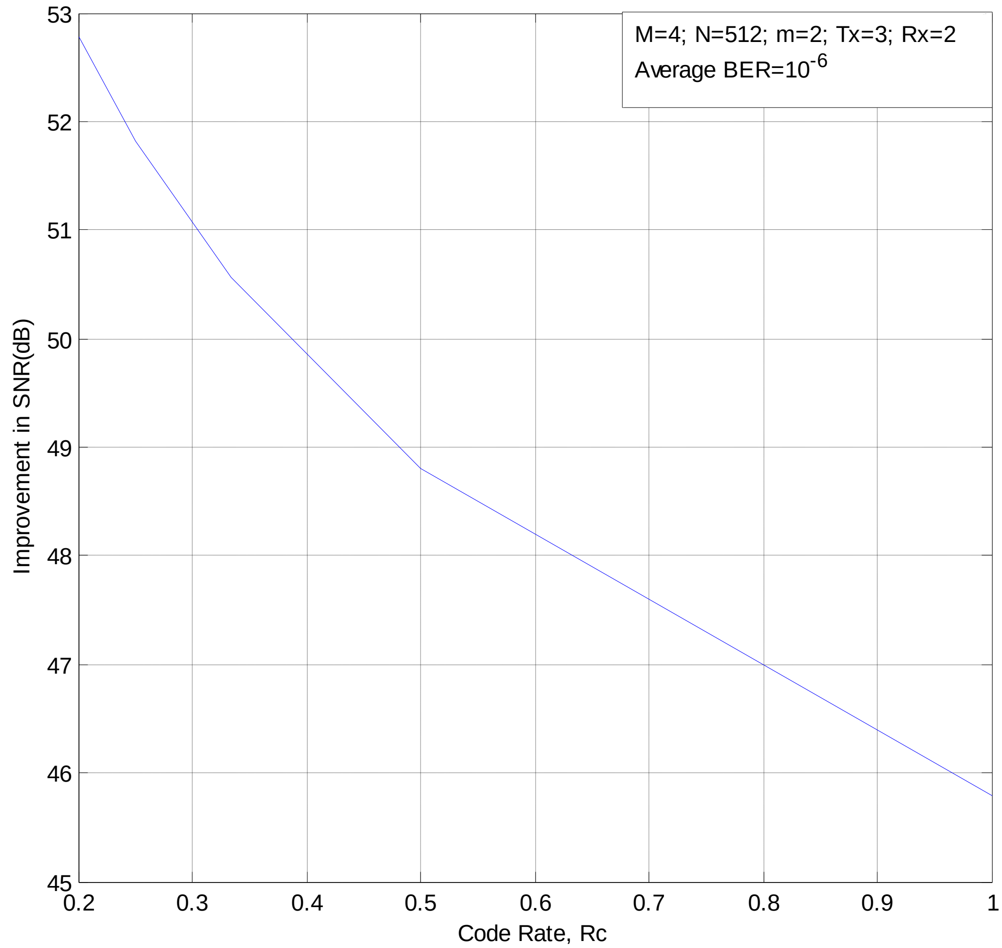

Fig. 3.18: The Improvement in SNR(dB) from uncoded to SFBC-MQAM-OFDM system in Rician Fading with the different code rates at a BER of 10^{-6}

Results are presented in Rician fading environment at a average BER of 10^{-6} for SFBC-MPSK-OFDM (Fig. 3.17) and also for SFBC-MQAM-OFDM (Fig. 3.18). SFBC with 2Rx is considered. The code rate in this case is 1 bits/s/Hz. It can be seen that SFBC provides a small improvement in SNR(dB) for OFDM with varying the code rates and significantly a gain of about 1.66 dB (when shifted from Rc=1/2 to Rc=1/3) (Fig. 3.17) and 1.76 dB (when shifted from Rc=1/2 to Rc=1/3) (Fig. 3.18).

The Improvement in SNR(dB) from Uncoded MPSK to the SFBC-MPSK-OFDM system & Uncoded MQAM to the SFBC-MQAM-OFDM system at a fixed BER with different code rates in Nakagami-m fading environment are shown in Fig. 3.19 & Fig. 3.20 respectively.

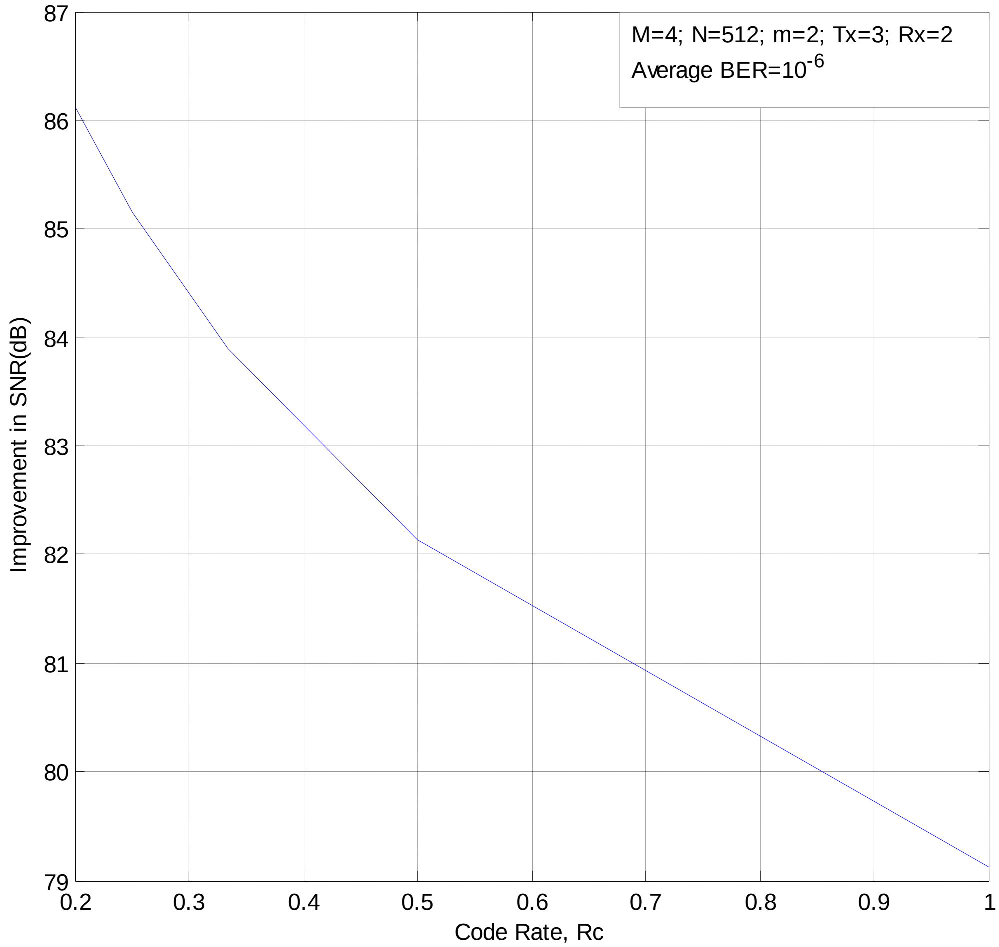

Fig. 3.19: The Improvement in SNR(dB) from uncoded to SFBC-MPSK-OFDM system in Nakagami-m Fading with the different code rates at a BER of 10^{-6}

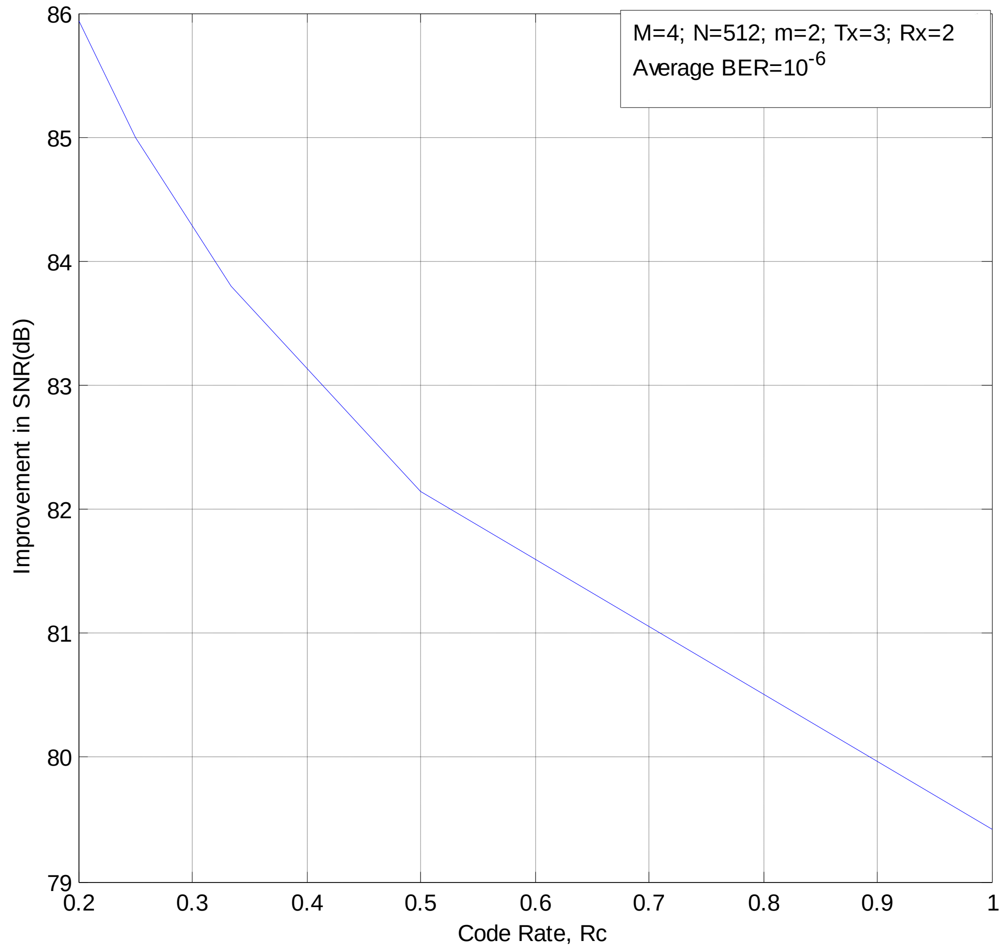

Fig. 3.20: The Improvement in SNR(dB) from uncoded to SFBC-MQAM-OFDM system in Nakagami-m Fading with the different code rates at a BER of 10^{-6}

Results are presented in Nakagami-m fading environment at a average BER of 10^{-6} for SFBC-MPSK-OFDM (Fig. 3.19) and also for SFBC-MQAM-OFDM (Fig. 3.20). SFBC with 2Rx is considered. The code rate in this case is 1 bits/s/Hz. It can be seen that SFBC provides a small improvement in SNR(dB) for OFDM with varying the code rates and significantly a gain of about 1.76 dB (when shifted from Rc=1/2 to Rc=1/3) (Fig. 3.19) and 1.66 dB (when shifted from Rc=1/2 to Rc=1/3) (Fig. 3.20).

3.2.7 Simulation Results of the Comparison of the Improvement in SNR(dB) from Uncoded to the SFBC-OFDM system at a fixed BER with Different Code Rates between Rician Fading and Nakagami-m Fading Environment

The comparison of the improvement in SNR(dB) from Uncoded MPSK to the SFBC-MPSK-OFDM at a fixed BER with different code rates between Rician fading and Nakagami-m fading environment are shown in Fig. 3.21.

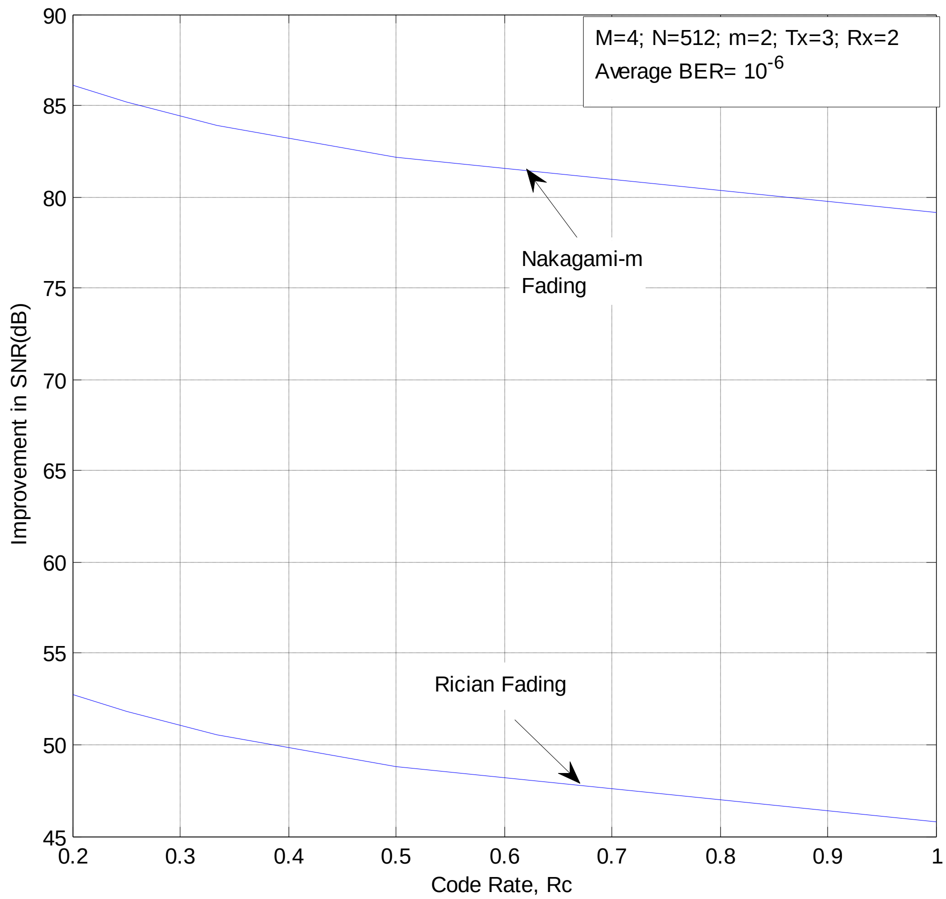

Fig. 3.21: The comparison of the improvement in SNR(dB) from uncoded to SFBC-MPSK-OFDM system between Rician fading and Nakagami-m Fading with the different code rates at a BER of 10^{-6}

Results are presented in both Rician fading and Nakagami-m fading environment at a average BER of 10^{-6} for SFBC-MPSK-OFDM, SFBC with 2Rx is considered. The code rate in this case is 1 bits/s/Hz. It can be seen that that the improvement in SNR(dB) in Nakagami-m fading is better than that of a Rician fading and SFBC provides a small improvement in SNR(dB) for OFDM with varying the code rates and significantly a gain of about 1.66 dB (when shifted from Rc=1/2 to Rc=1/3) in Nakagami-m fading and 1.76 dB (when shifted from Rc=1/2 to Rc=1/3) in Rician fading.

The comparison of the improvement in SNR(dB) from Uncoded MQAM to the SFBC-MQAM-OFDM at a fixed BER with different code rates between Rician fading and Nakagami-m fading environment are shown in Fig. 3.22.

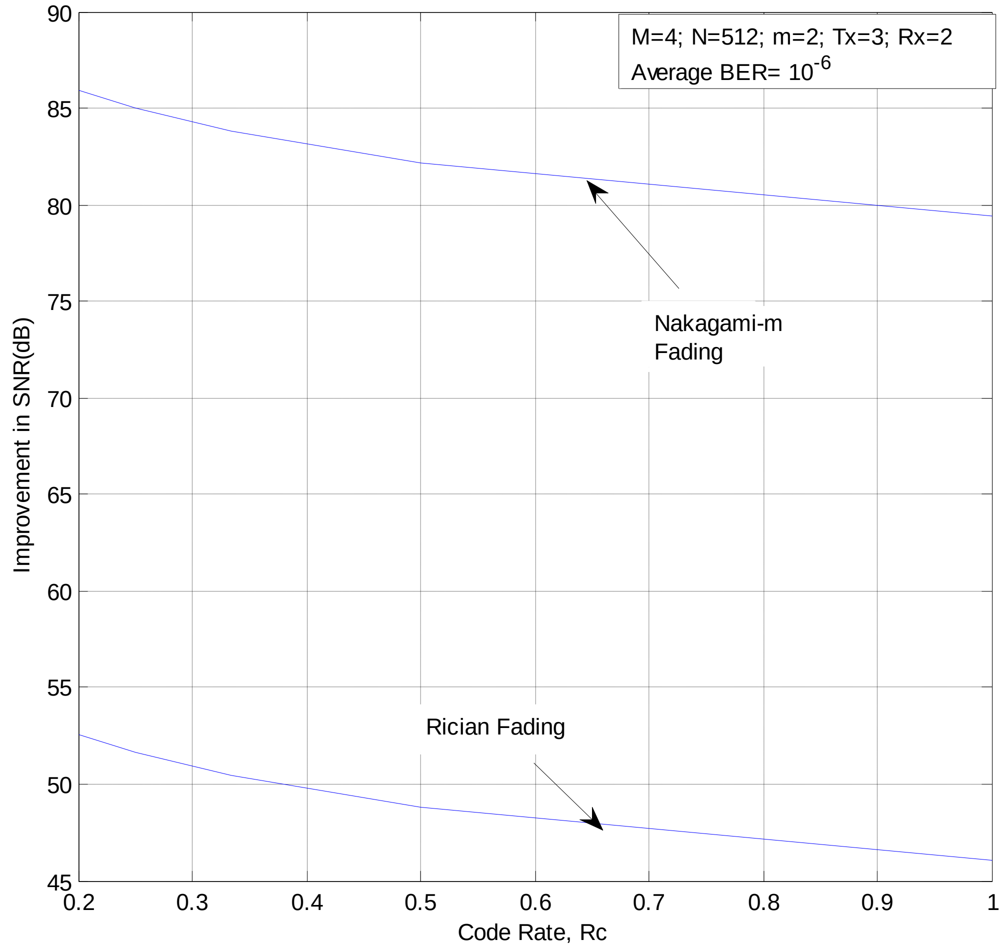

Fig. 3.22: The comparison of the improvement in SNR(dB) from uncoded to SFBC-MQAM-OFDM system between Rician fading and Nakagami-m Fading with the different code rates at a BER of 10^{-6}

Results are presented in both Rician fading and Nakagami-m fading environment at a average BER of 10^{-6} for SFBC-MQAM-OFDM, SFBC with 2Rx is considered. The code rate in this case is 1 bits/s/Hz. It can be seen that that the improvement in SNR(dB) in Nakagami-m fading is better than that of a Rician fading and SFBC provides a small improvement in SNR(dB) for OFDM with varying the code rates and significantly a gain of about 1.76 dB (when shifted from Rc=1/2 to Rc=1/3) in Nakagami-m fading and 1.66 dB (when shifted from Rc=1/2 to Rc=1/3) in Rician fading.

3.2.8 Comparing with the recently published Literature

No recent published literature is found which is similar to the analysis that is done here. But there is a published paper which I can refer is "Mohammad Torabi, Sonia Aissa, Senior Member, IEEE, and M. Reza Soleymani, Senior Member, IEEE, "On the BER Performance of Space-Frequency Block Coded OFDM Systems in Fading MIMO Channels", IEEE Transactions On Wireless Communications, Vol. 6, No. 4, April 2007."

In the above mentioned paper analysis is done with the MIMO-SFBC-OFDM system in Rayleigh fading environment using closed form (approximate) formula. If the proposed formula of this paper is analysed using Rayleigh fading and compared to the above mentioned oublished paper, the proposed analysis can be verified.

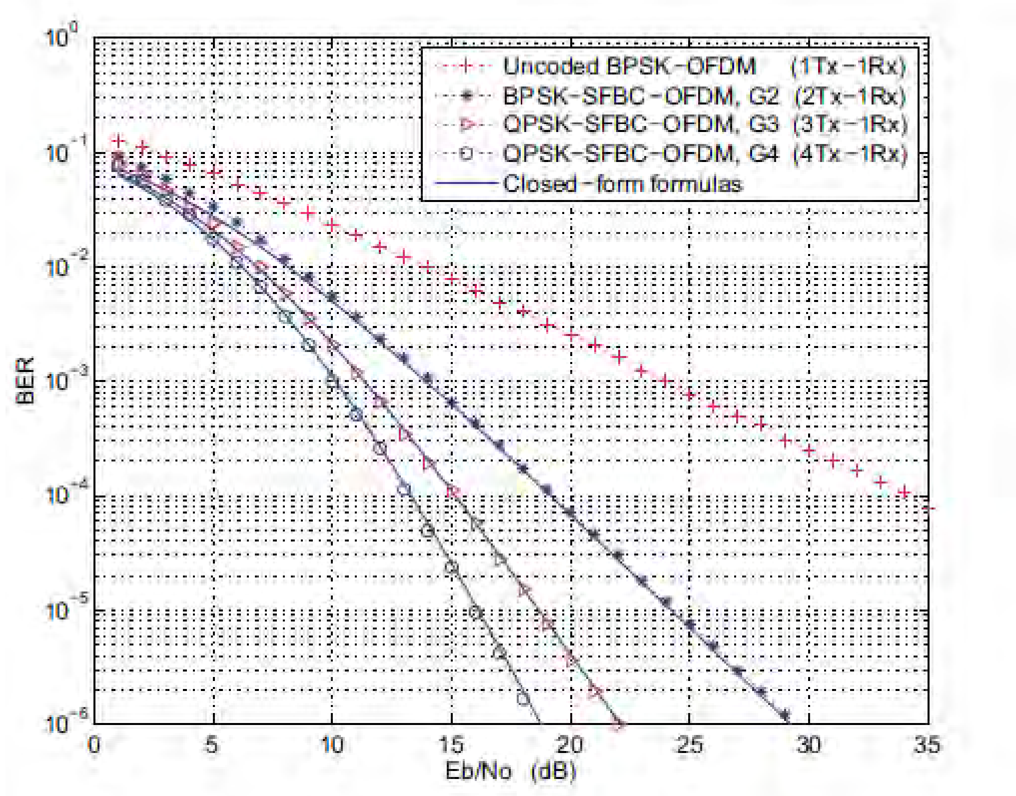

Fig. 3.23: Average BER performance analysis of MPSK-SFBC-OFDM with different antenna configurations according to the analysis done in the published paper.

The above figure 3.23 is the analysis done in the published paper with MPSK-SFBC-OFDM system using closed-form formula in Rayleigh fading environment with which we have to compare the proposed analysis.

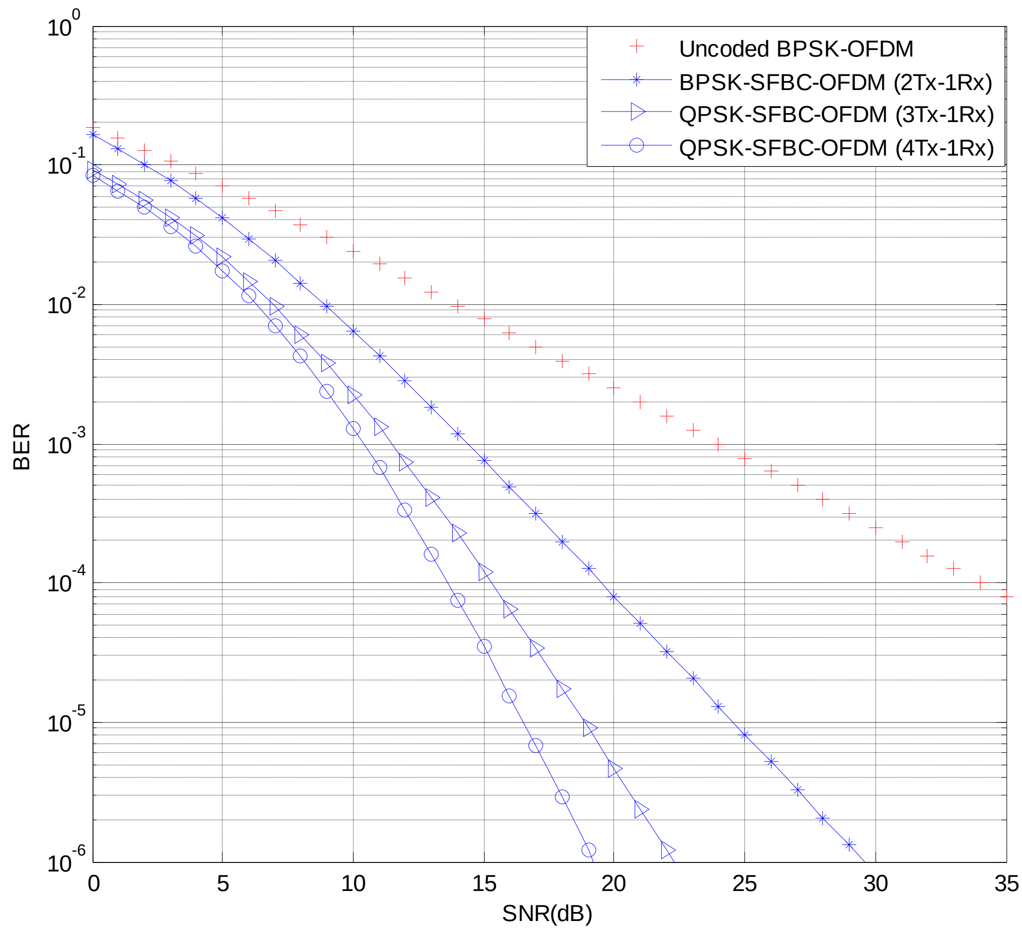

Fig. 3.24: Average BER performance analysis of MPSK-SFBC-OFDM with different antenna configurations according to the proposed analysis of this paper.

The above figure 3.24 is the analysis done in this paper with MPSK-SFBC-OFDM system using derived formula in Rayleigh fading environment which is approximately similar to the analysis done in the published paper with MPSK-SFBC-OFDM system using closed-form formula in Rayleigh fading environment.

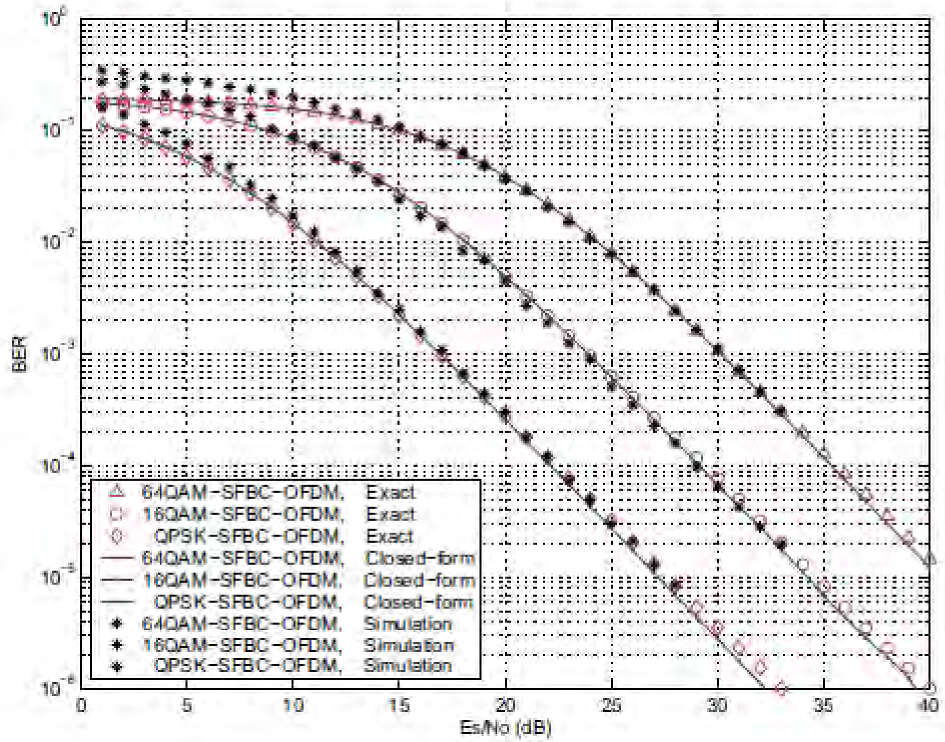

Fig. 3.25: Average BER performance analysis of MQAM-SFBC-OFDM with (2Tx-1Rx) antennas according to the analysis done in the published paper.

The above figure 3.25 is the analysis done in the published paper with MQAM-SFBC-OFDM system using closed-form formula in Rayleigh fading environment with which we have to compare the proposed analysis.

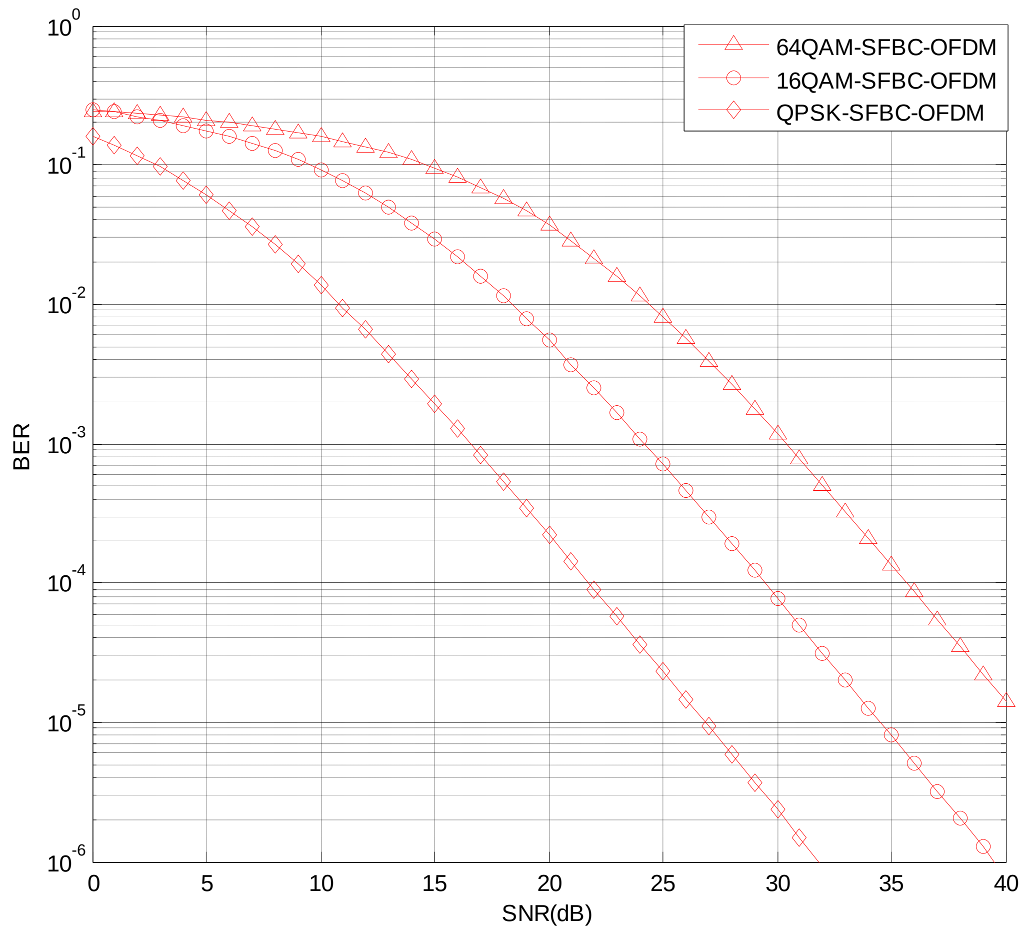

Fig. 3.26: Average BER performance analysis of MQAM-SFBC-OFDM with (2Tx-1Rx) antennas according to the proposed analysis of this paper.

The above figure 3.26 is the analysis done in this paper with MQAM-SFBC-OFDM system using derived formula in Rayleigh fading environment which is approximately similar to the the analysis done in the published paper with MQAM-SFBC-OFDM system using closed-form formula in Rayleigh fading environment.

So, it is verified that the proposed analysis is approximately correct.

CHAPTER 4

CONCLUSIONS AND FUTURE SCOPE OF WORK

4.1 Conclusion

This paper presented new expressions for the BER of space-frequency block coded OFDM (SFBC-OFDM) systems over Rician and Nakagami-m fading channels. In the performance evaluation, both M-ary phase shift keying (MPSK) and M-ary quadrature amplitude modulation (MQAM) were considered. Numerical results were provided for analysis and simulations and the performances of several forms of SFBC-OFDM system were evaluated and compared. The proposed expressions are used to quantify the amount of degradation and improvement in the BER at the receiver. Higher gains is achieved by using higher order SFBC with more antennas at the transmitter but the no of antenna must be optimum. SFBC-OFDM system with Nakagami-m fading performs better than that with Rician fading.

4.2 Future Work

Research within wireless communications is vast and endless in possibilities. This research has focused on the performance analysis for different MIMO and MIMO-OFDM systems with MPSK and MQAM using SFBC in Rician and Nakagami-m fading environment. While the research is comprehensive, there remains further work which could be done to further enhance system performance in simulation. I have a great intension to work further with this topic of multiple antenna wireless system so that further improvement can be achieved. For this reason if it is possible I have a idea of the following future works:

a) The proposed schemes can be also applied for the performance analysis for different MIMO and MIMO-OFDM systems with other modulation schemes such as GMSK, DMPSK etc using SFBC in Rician and Nakagami-m fading environment and can be compared to the performance of the proposed schemes
b) The proposed schemes can also be analyzed using STBC, STFBC, OSTFBC in Rician and Nakagami-m fading and the results can be compared to the proposed schemes.
c) The proposed schemes can be also applied to hybrid MIMO techniques that combine spatial diversity, spatial multiplexing and beamforming.
d) The proposed schemes can also be applied with suitable modifications for high mobility channel conditions.

REFERENCES

[1] A. Stamoulis, S.N. Diggavi, N. Al-Dhahir, "Intercarrier Interference in MIMO OFDM", IEEE Transactions on Signal Processing, Vol. 50, Issue. 10, October 2002.

[2] G. L. Stuber, J. R. Barry, S. W. McLaughlin, Ye Li, M. A. Ingram, T. G. Pratt, "Broadband MIMO-OFDM Wireless Communications", Proc. IEEE, Vol. 92, Issue. 2, February 2004.

[3] H. Bolcskei, "MIMO-OFDM Wireless Systems: Basics, Perspectives, and Challenges", IEEE Wireless Communications, Vol. 13, Issue. 4, August 2006.

[4] Junyoung Nam, Giuseppe Caire, Jeongseok Ha, "On the Role of Transmit Correlation Diversity in Multiuser MIMO Systems", IEEE Transactions on Information Theory, Vol. 63, Issue.1, January 2017.

[5] Junil Choi, David J. Love, T. Patrick Bidigare, "Coded Distributed Diversity: A Novel Distributed Reception Technique for Wireless Communication Systems", IEEE Transactions on Signal Processing", Vol. 63, Issue. 5, March 2015.

[6] Ui-Kun Kwon, Gi-Hong Im, Jong-Bu Lim, "MIMO Spatial Multiplexing Technique With Transmit Diversity", IEEE Signal Processing Letters, Vol. 16, Issue.7, July 2009.

[7] Tsung-Hsien Liu, "Analysis of the Alamouti STBC-MIMO System With Spatial Division Multiplexing Over the Rayleigh Fading Channel", IEEE Transactions on Wireless Communications, Vol. 14, Issue. 9, September 2015.

[8] Yoon Hyun Kim, Jin Young Kim, "Differential STBC-MIMO-OFDM System with Cyclic Delay Diversity for Mobile Multimedia Communication Systems" The 9th International Conference on Advanced Communication Technology, Vol. 3, February 2007.

[9] Samira Boualleg, Khalida Ghanem, Brahim Haraoubia, Mourad Nedil, "On the Performance Enhancement When Combining STBC and Channel Estimation Techniques in Underground MIMO Channels." IEEE Antennas and Wireless Propagation Letters, Vol. 15, July 2015.

[10] A. Morsali, S. Talebi, M. Shahabinejad, "On the Coding Advantages of the Quasi-Orthogonal Space-Frequency Block Codes .", IET Commun., Vol. 8, Issue. 4, March 2014.

[11] Z. Mohammadian, M. Shahabinejad, S. Talebi, "New full-diversity Space-Frequency Block Codes Based on the OSTBCs", IEEE Communication Letters, Vol. 16, Issue. 10, October 2012.

[12] Z. Rui, A. Linduska, J. Lindner, "Blockwise Space-Frequency Code Design for Non-coherent MIMO-OFDM Systems", Proc. IEEE Globecom, December 2008.

[13] Lei Shao, Sumit Roy, "Rate-One Space–Frequency Block Codes With Maximum Diversity for MIMO–OFDM." IEEE Transactions on Wireless Communications, Vol. 4, No. 4, July 2005.

[14] Zhiqiang Liu, Yan Xin, Georgios B. Giannakis, "Space-Time-Frequency Coded OFDM Over Frequency-Selective Fading Channels." IEEE Transactions On Signal Processing, Vol. 50, No. 10, October 2002.

[15] Mohammad Torabi, Sonia Aissa, M. Reza Soleymani, "On the BER Performance of Space-Frequency Block Coded OFDM Systems in Fading MIMO Channels", IEEE Transactions On Wireless Communications, Vol. 6, No. 4, April 2007.

[16] Luay Azzam, Ender Ayanoglu, "A Novel Maximum Likelihood Detection Algorithm for Orthogonal Space-Time Block Codes", IEEE Transactions On Communications, Vol. 57, Issue. 3, 2009.

[17] Lorenzo Lo Monte , Braham Himed , Thomas Corigliano , Christopher J. Baker , "Performance analysis of time division and code division waveforms in co-located MIMO", IEEE Radar Conference (RadarCon), 2015.

[18] S. Kuzhaloli , K. S. Shaji , "Comparison of equalization techniques for an MIMO MC-CDMA system", International Conference on Circuits, Power and Computing Technologies [ICCPCT], 2015.

[19] Li You, Xiqi Gao, A. Lee Swindlehurst, Wen Zhong, "Channel Acquisition for Massive MIMO-OFDM With Adjustable Phase Shift Pilots", IEEE Transactions on Signal Processing, Vol. 64 , Issue. 6 , March, 2016.

[20] Yang Hong, Tesi WU, Lian-Kuan Chen, "On the Performance of Adaptive MIMO-OFDM Indoor Visible Light Communications", IEEE Photonics Technology Letters, Vol. 28 , Issue. 8 , April, 2016.

APPENDIX

Appendix-A

Maximum Likelihood Detection

Assuming channel H is known at the receiver, the ML estimate is obtained by performing $\min_{G_2} \left\| r_j - G_2 H \right\|_F^2$ which is the Frobenius form. The receiver decodes s[2k] and s[2k+1] by decomposing the measure $\left\| r_j - G_2 H \right\|_F^2$ into two parts, minimizes each separately over all possible values of s[2k] and s[2k+1] that belong to the constellation used. Then ML is equivalent to [16],

$$\tilde{s}[2k] = \arg\min \begin{pmatrix} \left| \left[\sum_{j=1}^{M_R} \left(r_j[2k] H_{j,1}^*[2k] + r_j^*[2k+1] H_{j,2}[2k] \right) \right] - s[2k] \right|^2 \\ + \left(-1 + \sum_{j=1}^{M_R} \sum_{i=1}^{2} \left| H_{j,i}[2k] \right|^2 \right) |s[2k]|^2 \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A.1)}$$

$$\tilde{s}[2k+1] = \arg\min \begin{pmatrix} \left| \left[\sum_{j=1}^{M_R} \left(r_j[2k] H_{j,2}^*[2k+1] + r_j^*[2k+1] H_{j,1}[2k+1] \right) \right] - s[2k+1] \right|^2 \\ + \left(-1 + \sum_{j=1}^{M_R} \sum_{i=1}^{2} \left| H_{j,i}[2k+1] \right|^2 \right) |s[2k+1]|^2 \end{pmatrix} \ldots\ldots\ldots \text{(A.2)}$$

Now, equation (A.1) is equivalent to

$$\frac{\partial}{\partial s[2k]} \left(\left| \left[\sum_{j=1}^{M_R} \left(r_j[2k] H_{j,1}^*[2k] + r_j^*[2k+1] H_{j,2}[2k] \right) \right] - s[2k] \right|^2 + \left(-1 + \sum_{j=1}^{M_R} \sum_{i=1}^{2} \left| H_{j,i}[2k] \right|^2 \right) |s[2k]|^2 \right) = 0$$

$$\Rightarrow 2 \left| \left[\sum_{j=1}^{M_R} \left(r_j[2k] H_{j,1}^*[2k] + r_j^*[2k+1] H_{j,2}[2k] \right) \right] - s[2k] \right| = 0$$

So, $\tilde{s}[2k] = \sum_{j=1}^{M_R} \left(r_j[2k] H_{j,1}^*[2k] + r_j^*[2k+1] H_{j,2}[2k] \right)$ ………………………………..…………..…… (A.3)

Again, equation (A.2) is equivalent to

$$\frac{\partial}{\partial s[2k]} \begin{pmatrix} \left| \left[\sum_{j=1}^{M_R} \left(r_j[2k] H_{j,2}^*[2k+1] + r_j^*[2k+1] H_{j,1}[2k+1] \right) \right] - s[2k+1] \right|^2 \\ + \left(-1 + \sum_{j=1}^{M_R} \sum_{i=1}^{2} \left| H_{j,i}[2k+1] \right|^2 \right) |s[2k+1]|^2 \end{pmatrix} = 0$$

$$\Rightarrow 2 \left| \left[\sum_{j=1}^{M_R} \left(r_j[2k] H_{j,2}^*[2k+1] + r_j^*[2k+1] H_{j,1}[2k+1] \right) \right] - s[2k+1] \right| = 0$$

So, $\tilde{s}[2k+1] = \sum_{j=1}^{M_R} \left(r_j[2k] H_{j,2}^*[2k+1] + r_j^*[2k+1] H_{j,1}[2k+1] \right)$ …………………………...…..… (A.4)

Appendix-B

erf(x) plot

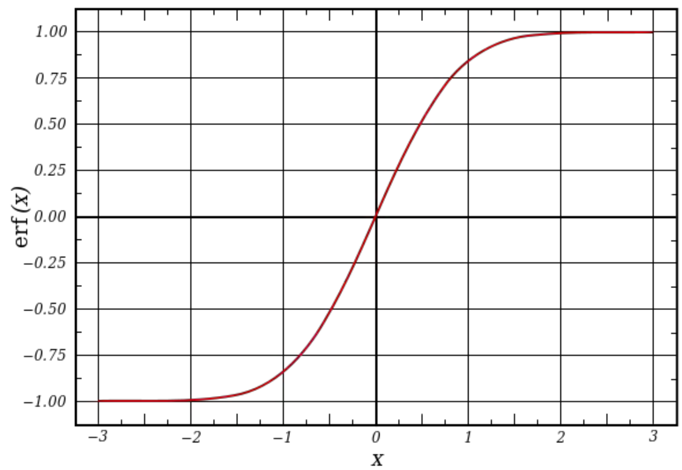

Figure B.1: Plot of the error function

Appendix-C

Zero Order Modified Bessel Function

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} exp(x\,cos\theta)\,d\theta$$

$$\Rightarrow I_0(x) = \frac{1}{2\pi}\int_0^{2\pi}\left(1+\frac{x\,cos\theta}{1!}+\frac{x^2\,cos^2\theta}{2!}+\frac{x^3\,cos^3\theta}{3!}+\frac{x^4\,cos^4\theta}{4!}+\cdots\right)d\theta \quad \text{[From exponential series]}$$

1^{st} *term*, $\frac{1}{2\pi}\int_0^{2\pi} d\theta = \frac{1}{2\pi}\,[\theta]_0^{2\pi} = 1$

2^{nd} *term*, $\frac{1}{2\pi}\int_0^{2\pi}\frac{x\,cos\theta}{1!}\,d\theta = \frac{x}{2\pi}\,[sin\theta]_0^{2\pi} = 0$

3^{rd} *term*, $\frac{1}{2\pi}\int_0^{2\pi}\frac{x^2\,cos^2\theta}{2!}\,d\theta = \frac{x^2}{8\pi}\int_0^{2\pi}(1+cos2\theta)\,d\theta = \frac{x^2}{8\pi}\,[\theta]_0^{2\pi} + \frac{x^2}{8\pi.2}[sin\,2\theta]_0^{2\pi} = \frac{x^2}{4}$

4^{th} *term*, $\frac{1}{2\pi}\int_0^{2\pi}\frac{x^3\,cos^3\theta}{3!}\,d\theta = \frac{x^3}{48\pi}\int_0^{2\pi}(3cos\theta+cos3\theta)d\theta = \frac{3x^3}{48\pi}\,[sin\theta]_0^{2\pi} + \frac{x^3}{48\pi.3}[sin\,3\theta]_0^{2\pi} = 0$

5^{th} *term*, $\frac{1}{2\pi}\int_0^{2\pi}\frac{x^4\,cos^4\theta}{4!}\,d\theta = \frac{x^4}{192\pi}\int_0^{2\pi}(1+cos2\theta)^2\,d\theta = \frac{x^4}{192\pi}\,[\theta]_0^{2\pi} + \frac{x^4}{384\pi}[\theta]_0^{2\pi} = \frac{x^4}{64}$

……………

$$\text{Therefore, } I_0(x) = 1+\frac{x^2}{4}+\frac{x^4}{64}+\frac{x^6}{2304}+\cdots = 1+\frac{(x^2/4)^1}{(1!)^2}+\frac{(x^2/4)^2}{(2!)^2}+\frac{(x^2/4)^3}{(3!)^2}+\ \ldots = \sum_{n=0}^{\infty}\frac{\left(\frac{x^2}{4}\right)^n}{(n!)^2} \qquad \text{……………… (C.1)}$$

Appendix-D1

Average Received SNR in Rician Fading Channel

$$\overline{\gamma_s} = \gamma_s \int_0^\infty \alpha^2 \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2 + \alpha_0^2}{2\sigma_\alpha^2}\right) \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2 \alpha^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\, d\alpha$$

Let, $y = \alpha^2$
$\Rightarrow dy = 2\alpha d\alpha$
$\Rightarrow \frac{dy}{2} = \alpha d\alpha$

$$= \frac{\gamma_s}{2\sigma_\alpha^2} \int_0^\infty y \exp\left(-\frac{y + \alpha_0^2}{2\sigma_\alpha^2}\right) \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2 y}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\, dy$$

$$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2} \frac{\gamma_s}{2\sigma_\alpha^2} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \int_0^\infty y^{n+1} \exp\left(-\frac{y}{2\sigma_\alpha^2}\right) dy \quad \text{(D1.1)}$$

We know, $\int_0^\infty \int x^n \exp(-ax)\, dx = \frac{n!}{a^{n+1}}$ (where n=0,1,2,……a>0)

So, from equation (D1.1), we get,

$$\overline{\gamma_s} = \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right)^n}{n!} \frac{1}{n!(2\sigma_\alpha^2)^n} \frac{\gamma_s}{2\sigma_\alpha^2} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \left[\frac{(n+1)!}{\left(\frac{1}{2\sigma_\alpha^2}\right)^{n+2}}\right]$$

$$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right)^n}{n!} (n+1) \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) 2\sigma_\alpha^2 \gamma_s \quad \text{(D1.2)}$$

Appendix-D2

Average Received SNR in Nakagami-m Fading Channel

$$\overline{\gamma_s} = \gamma_s \int_0^\infty \alpha^2 \frac{2m^m}{\Gamma(m)\Omega^m} \alpha^{2m-1} \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha$$

$$= \gamma_s \frac{2m^m}{\Gamma(m)\Omega^m} \int_0^\infty \alpha^{2m+1} \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha$$

$$= \gamma_s \frac{m^m}{\Gamma(m)\Omega^m} \int_0^\infty y^m \exp\left(-\frac{m}{\Omega}y\right) dy \quad \text{(D2.1)}$$

Let, $y = \alpha^2$
$\Rightarrow dy = 2\alpha d\alpha$
$\Rightarrow \frac{dy}{2} = \alpha d\alpha$

We know, $\int_0^\infty \int x^n \exp(-ax)\, dx = \frac{n!}{a^{n+1}}$ (where n=0,1,2,……a>0)

So, from equation (D2.1), we get,

$$\overline{\gamma_s} = \gamma_s \frac{m^m}{\Gamma(m)\Omega^m} \left[\frac{m!}{\left(\frac{m}{\Omega}\right)^{m+1}}\right] = \gamma_s \frac{m(m-1)!}{(m-1)!} \left(\frac{m}{\Omega}\right)^{-1} = \gamma_s \Omega \quad \text{(D2.2)}$$

Appendix-D3

Average Received SNR for OFDM in Rician Fading Channel

$$\overline{\gamma_s}_k = \gamma_s \int_0^\infty {\alpha_k}^2 \frac{\alpha_k}{\sigma^2_{\alpha_k}} \exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2{\alpha_k}^2}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2}\, d\alpha_k$$

Let, $y_k = {\alpha_k}^2$
$=> \mathrm{d}y_k = 2\alpha_k \mathrm{d}\alpha_k$

$$= \frac{\gamma_s}{2\sigma^2_{\alpha_k}} \int_0^\infty y_k \exp\left(-\frac{y_k+\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2 y_k}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2}\, dy_k$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2} \frac{\gamma_s}{2\sigma^2_{\alpha_k}} \exp\left(-\frac{\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \int_0^\infty {y_k}^{n+1} \exp\left(-\frac{y_k}{2\sigma^2_{\alpha_k}}\right)\, dy_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(D3.1)}$$

We know, $\int_0^\infty \int x^n \exp(-ax)\, dx = \frac{n!}{a^{n+1}}$ (where n=0,1,2,……a>0)

So, from equation (D3.1), we get,

$$\overline{\gamma_s}_k = \sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{2\sigma^2_{\alpha}}\right)^n}{n!} \frac{1}{n!\left(2\sigma^2_{\alpha_k}\right)^n} \frac{\gamma_s}{2\sigma^2_{\alpha_k}} \exp\left(-\frac{\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \left[\frac{(n+1)!}{\left(\frac{1}{2\sigma^2_{\alpha_k}}\right)^{n+2}}\right]$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{2\sigma^2_{\alpha_k}}\right)^n}{n!} (n+1) 2\sigma^2_{\alpha_k} \gamma_s \exp\left(-\frac{\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(D3.2)}$$

Appendix-D4

Average Received SNR for OFDM in Nakagami-m Fading Channel

$$\overline{\gamma_s}_k = \gamma_s \int_0^\infty {\alpha_k}^2 \frac{2m^m}{\Gamma(m){\Omega_k}^m} {\alpha_k}^{2m-1} \exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right) d\alpha_k$$

Let, $y_k = {\alpha_k}^2$
$=> \mathrm{d}y_k = 2\alpha_k \mathrm{d}\alpha_k$

$$= \gamma_s \int_0^\infty {\alpha_k}^{2m+1} \frac{2m^m}{\Gamma(m){\Omega_k}^m} \exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right) d\alpha_k$$

$$= \gamma_s \frac{m^m}{\Gamma(m){\Omega_k}^m} \int_0^\infty {y_k}^m \exp\left(-\frac{m}{\Omega_k} y_k\right) dy_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(D4.1)}$$

We know, $\int_0^\infty \int x^n \exp(-ax)\, dx = \frac{n!}{a^{n+1}}$ (where n=0,1,2,……a>0)

So, from equation (D4.1), we get,

$$\overline{\gamma_s}_k = \gamma_s \frac{m^m}{\Gamma(m){\Omega_k}^m} \left[\frac{m!}{\left(\frac{m}{\Omega_k}\right)^{m+1}}\right] = \gamma_s \frac{m(m-1)!}{(m-1)!} \left(\frac{m}{\Omega_k}\right)^{-1} = \gamma_s \Omega_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(D4.2)}$$

Appendix-E1

Average BER with Uncoded MPSK in Rician Fading Channel

Let $y = \alpha^2$
$\Rightarrow dy = 2\alpha d\alpha$
$\Rightarrow \frac{dy}{2} = \alpha d\alpha$

$\boldsymbol{erfc(x)} = \frac{\mathbf{2}}{\sqrt{\boldsymbol{\pi}}}\int_{\boldsymbol{x}}^{\infty} \mathbf{exp}(-\boldsymbol{u}^{\mathbf{2}})\,\boldsymbol{du}$

$$\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$$

$$= \int_0^\infty \frac{1}{\beta}\, erfc\left(\sqrt{\gamma_s \alpha^2}\, sin(\tfrac{\pi}{M})\right) \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2+\alpha_0^2}{2\sigma_\alpha^2}\right) I_0\left(\frac{\alpha_0 \alpha}{\sigma_\alpha^2}\right) d\alpha$$

$$= \frac{1}{\beta}\int_0^\infty erfc\left(\sqrt{\gamma_s \alpha^2}\, sin(\tfrac{\pi}{M})\right) \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2+\alpha_0^2}{2\sigma_\alpha^2}\right) \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2\alpha^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\, d\alpha$$

$$= \frac{1}{\beta}\frac{1}{2\sigma_\alpha^2}\int_0^\infty erfc\left(\sqrt{\gamma_s y}\, sin(\tfrac{\pi}{M})\right) \exp\left(-\frac{\alpha^2}{2\sigma_\alpha^2}\right) exp\left(-\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right) \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2 y}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\, dy$$

$$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2} \frac{1}{2\sigma_\alpha^2}\frac{1}{\beta}\frac{2}{\sqrt{\pi}} exp\left(-\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right) \int_0^\infty \left[\int_{\sqrt{\gamma_s y}\, sin(\frac{\pi}{M})}^\infty \exp(-u^2)\, du\right] \exp\left(-\frac{y}{2\sigma_\alpha^2}\right) y^n\, dy \quad \text{........... (E1.1)}$$

The region where $\sqrt{\gamma_s y}\, sin(\frac{\pi}{M}) < u < \infty$ for $0 < y < \infty$, fixing a value of u, the value of y varies from y=0 to y = $\frac{u^2}{\gamma_s sin^2(\frac{\pi}{M})}$; first slice is u= 0, last slice is u =∞.

So, from the equation (E1.1), we get,

$$\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$$

$$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2} \frac{1}{2\sigma_\alpha^2}\frac{1}{\beta}\frac{2}{\sqrt{\pi}} exp\left(-\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right) \int_0^\infty \exp(-u^2) \left[\int_0^{\frac{u^2}{\gamma_s sin^2(\frac{\pi}{M})}} y^n \exp\left(-\frac{y}{2\sigma_\alpha^2}\right) dy\right] du$$

$$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2} \frac{1}{2\sigma_\alpha^2}\frac{1}{\beta}\frac{2}{\sqrt{\pi}} exp\left(-\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right) \int_0^\infty \exp(-u^2) \left[\int_0^{\frac{u^2}{\gamma_s sin^2(\frac{\pi}{M})}} y^n \exp\left(-\frac{y}{2\sigma_\alpha^2}\right) dy\right] du \quad \text{.......... (E1.2)}$$

We know, $\int x^n \exp(cx)\, dx = \exp(cx) \sum_{i=0}^n (-1)^i \frac{n!}{(n-i)!\,c^{i+1}} x^{n-i}$. So, from equation (E1.2), we get,

$$\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$$

$$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2} \frac{1}{2\sigma_\alpha^2}\frac{1}{\beta}\frac{2}{\sqrt{\pi}} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \int_0^\infty \exp(-u^2) \left[\exp\left(-\frac{y}{2\sigma_\alpha^2}\right) \sum_{i=0}^n (-1)^i \frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_\alpha^2}\right)^{i+1}} y^{n-i}.\right]_0^{\frac{u^2}{\gamma_s sin^2(\frac{\pi}{M})}} du$$

$$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2} \frac{1}{2\sigma_\alpha^2}\frac{1}{\beta}\frac{2}{\sqrt{\pi}} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \int_0^\infty \exp(-u^2) \left[\begin{array}{c}\left(\exp\left(-\frac{u^2}{2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right) \sum_{i=0}^n (-1)^i \frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_\alpha^2}\right)^{i+1}} \left(\frac{u^2}{\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\right) \\ -\left(\exp(0)\,(-1)^n \frac{n!}{(n-n)!\left(-\frac{1}{2\sigma_\alpha^2}\right)^{n+1}} (0)^{n-n}\right)\end{array}\right] du$$

$$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{n!} \frac{1}{\beta}\frac{2}{\sqrt{\pi}} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \int_0^\infty \exp(-u^2) \left[-\exp\left(-\frac{u^2}{2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right) \sum_{i=0}^n (2\sigma_\alpha^2)^i \frac{1}{(n-i)!} \left(\frac{u^2}{\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i} + (2\sigma_\alpha^2)^n\right] du$$

$$= \sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{n!}\frac{1}{\beta}\frac{2}{\sqrt{\pi}}\exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right)\left[\begin{array}{c}-\sum_{i=0}^{n}\frac{(2\sigma_\alpha^2)^i}{(n-i)!}\left(\frac{1}{\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\int_0^\infty (u^2)^{n-i}\exp\left(-u^2\left(1+\frac{1}{2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)\right)du \\ +(2\sigma_\alpha^2)^n\int_0^\infty \exp(-u^2)\ du\end{array}\right]$$

……………..(E1.3)

We know, $\int \exp(-cx^2)\, dx = \sqrt{\frac{\pi}{4c}}\,\text{erf}(\sqrt{c}\,x)$ &

$\int_0^\infty x^n \exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}$ (where n=2k; k integer; a>0). So, from equation (E1.3), we get,

$$\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{Rician}\ d\alpha$$

$$= \sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{n!}\frac{1}{\beta}\frac{2}{\sqrt{\pi}}\exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right)\left[\begin{array}{c}-\sum_{i=0}^{n}\frac{(2\sigma_\alpha^2)^i}{(n-i)!}\left(\frac{1}{\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\left[\frac{(2(n-i)-1)!!}{2^{n-i+1}\left(1+\frac{1}{2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}}\sqrt{\frac{\pi}{\left(1+\frac{1}{2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)}}\right] \\ +(2\sigma_\alpha^2)^n\left[\sqrt{\frac{\pi}{4}}\,\text{erf}(u)\right]_0^\infty\end{array}\right] \quad \text{….. (E1.4)}$$

We know,

For an even positive integer $n = 2k$, $k \geq 0$, the double factorial is expressed as $n!! = 2^k k!$

For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!}{2^k k!} = \frac{n!}{(n-1)!!}$

So, from equation (E1.4), we get,

$$\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{Rician}\ d\alpha$$

$$= \sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{n!}\frac{1}{\beta}\exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right)\left[-\sum_{i=0}^{n}\left[\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2}(2\sigma_\alpha^2)^n\frac{1}{\left(1+2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)\right)^{n-i}}\sqrt{\frac{2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}{\left(1+2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)\right)}}\right]+(2\sigma_\alpha^2)^n[1-0]\right]$$

[From Appendix B]

$$= \frac{1}{\beta}\sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right)\left[1-\sum_{i=0}^{n}\left[\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2}\frac{1}{\left(1+2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)\right)^{n-i}}\sqrt{\frac{2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}{\left(1+2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)\right)}}\right]\right] \quad \text{……….…… (E1.5)}$$

Therefore, from equation (D1.2) & (E1.5), we get,

$$\boldsymbol{\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{Rician}\ d\alpha}$$

$$\boldsymbol{= \sum_{n=0}^{\infty}\frac{\overline{\gamma_s}}{2\sigma_\alpha^2\gamma_s(n+1)}\frac{1}{\beta}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2}\left(\frac{1}{1+2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\sqrt{\frac{2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}{1+2\sigma_\alpha^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}}\right]} \quad \text{…….... (E1.6)}$$

Appendix-E2

Average BER with Uncoded MQAM in Rician Fading Channel

$\int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Rician}\ d\alpha$

$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s\alpha^2}{M-1}}\right)\frac{\alpha}{\sigma_\alpha^2}\exp\left(-\frac{\alpha^2+\alpha_0^2}{2\sigma_\alpha^2}\right) I_0\left(\frac{\alpha_0\alpha}{\sigma_\alpha^2}\right)\ d\alpha$

$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s\alpha^2}{M-1}}\right)\frac{\alpha}{\sigma_\alpha^2}\exp\left(-\frac{\alpha^2+\alpha_0^2}{2\sigma_\alpha^2}\right) \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2\alpha^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\ d\alpha$

$= \frac{1}{2\sigma_\alpha^2}\int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s y}{M-1}}\right)\exp\left(-\frac{y+\alpha_0^2}{2\sigma_\alpha^2}\right)\sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2 y}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\ dy$

Let $y = \alpha^2$
$\Rightarrow dy = 2\alpha d\alpha$
$\Rightarrow \frac{dy}{2} = \alpha d\alpha$

$\boldsymbol{erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-u^2)\,du}$

$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\frac{1}{2\sigma_\alpha^2}\frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\frac{2}{\sqrt{\pi}}\exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right)\int_0^\infty\left[\int_{\sqrt{\frac{1.5\gamma_s y}{M-1}}}^\infty \exp(-u^2)\,du\right] y^n \exp\left(-\frac{y}{2\sigma_\alpha^2}\right)\ dy$ ... (E2.1)

The region where $\sqrt{\frac{1.5\gamma_s y}{M-1}} < u < \infty$ for $0 < y < \infty$, fixing a value of u, the value of y varies from y=0 to y = $\frac{u^2(M-1)}{1.5\gamma_s}$; first slice is u= 0, last slice is u =∞

So, from the equation (E2.1), we get,

$\int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Rician}\ d\alpha$

$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\frac{1}{2\sigma_\alpha^2}\frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right)\int_0^\infty \exp(-u^2)\left[\int_0^{\frac{u^2(M-1)}{1.5\gamma_s}} y^n\exp\left(-\frac{y}{2\sigma_\alpha^2}\right)\ dy\right] du$ ………… (E2.2)

We know, $\int x^n \exp(cx)\,dx = \exp(cx)\sum_{i=0}^n(-1)^i\frac{n!}{(n-i)!c^{i+1}}x^{n-i}$ So, from the equation (E2.2), we get,

$\int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Rician}\ d\alpha$

$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right)\frac{1}{2\sigma_\alpha^2}\frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\int_0^\infty \exp(-u^2)\left[\exp\left(-\frac{y}{2\sigma_\alpha^2}\right)\sum_{i=0}^n(-1)^i\frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_\alpha^2}\right)^{i+1}}y^{n-i}\right]_0^{\frac{u^2(M-1)}{1.5\gamma_s}} du$

$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{(n!)^2}\exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right)\frac{1}{2\sigma_\alpha^2}\frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\int_0^\infty \exp(-u^2)\left[\left(\exp\left(-\frac{u^2(M-1)}{2\sigma_\alpha^2 1.5\gamma_s}\right)\sum_{i=0}^n(-1)^i\frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_\alpha^2}\right)^{i+1}}\left(\frac{u^2(M-1)}{1.5\gamma_s}\right)^{n-i}\right) - \left(\exp(0)\,(-1)^n\frac{n!}{(n-n)!\left(-\frac{1}{2\sigma_\alpha^2}\right)^{n+1}}(0)^{n-n}\right)\right]du$

$= \sum_{n=0}^\infty \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right)\frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\int_0^\infty \exp(-u^2)\left[-\exp\left(-\frac{u^2(M-1)}{3\sigma_\alpha^2\gamma_s}\right)\sum_{i=0}^n(2\sigma_\alpha^2)^i\frac{1}{(n-i)!}\left(\frac{u^2(M-1)}{1.5\gamma_s}\right)^{n-i} + (2\sigma_\alpha^2)^n\right]$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \left[\begin{array}{c} -\sum_{i=0}^{n} \frac{(2\sigma_\alpha^2)^i}{(n-i)!} \left(\frac{(M-1)}{1.5\gamma_s}\right)^{n-i} \int_0^\infty (u^2)^{n-i} \exp\left(-u^2\left(1+\frac{M-1}{3\sigma_\alpha^2\gamma_s}\right)\right) du \\ +(2\sigma_\alpha^2)^n \int_0^\infty \exp(-u^2)\, du \end{array} \right]$$

……... (E2.3)

We know, $\int \exp(-cx^2)\,dx = \sqrt{\frac{\pi}{4c}}\,\mathrm{erf}(\sqrt{c}\,x)$ &

$\int_0^\infty x^n \exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}$ (where n=2k;k integer;a>0). So, from the equation (E2.3), we get,

$\int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \left[\begin{array}{c} -\sum_{i=0}^{n} \frac{(2\sigma_\alpha^2)^i}{(n-i)!} \left(\frac{M-1}{1.5\gamma_s}\right)^{n-i} \left[\frac{(2(n-i)-1)!!}{2^{n-i+1}\left(1+\frac{M-1}{3\sigma_\alpha^2\gamma_s}\right)^{n-i}} \sqrt{\frac{\pi}{\left(1+\frac{M-1}{3\sigma_\alpha^2\gamma_s}\right)}} \right] \\ +(2\sigma_\alpha^2)^n \left[\sqrt{\frac{\pi}{4}}\,\mathrm{erf}(u)\right]_0^\infty \end{array} \right] \text{.... (E2.4)}$$

We know,

For an even positive integer $n = 2k$, $k \geq 0$, the double factorial is expressed as $n!! = 2^k k!$

For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!}{2^k k!} = \frac{n!}{(n-1)!!}$,

So, from the equation (E2.4), we get,

$\int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{(2\sigma_\alpha^2)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \left[-\sum_{i=0}^{n} \left[\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2} (2\sigma_\alpha^2)^n \left(\frac{M-1}{((M-1)+3\sigma_\alpha^2\gamma_s)}\right)^{n-i} \sqrt{\frac{3\sigma_\alpha^2\gamma_s}{(M-1)+3\sigma_\alpha^2\gamma_s}} \right] + (2\sigma_\alpha^2)^n [1-0] \right]$$

[From Appendix B]

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{2\sigma_\alpha^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_\alpha^2}\right) \left[1 - -\sum_{i=0}^{n} \left[\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2} \left(\frac{M-1}{((M-1)+3\sigma_\alpha^2\gamma_s)}\right)^{n-i} \sqrt{\frac{3\sigma_\alpha^2\gamma_s}{(M-1)+3\sigma_\alpha^2\gamma_s}} \right] \right] \text{….. (E2.5)}$$

Therefore, from equation (D1.2) & (E2.5), we get,

$\boldsymbol{\int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha}$

$$= \boldsymbol{\frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{n=0}^{\infty} \frac{\overline{\gamma_s}}{2\sigma_\alpha^2\gamma_s(n+1)} \left[1 - \sum_{i=0}^{n} \frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2} \left(\frac{M-1}{(M-1)+3\sigma_\alpha^2\gamma_s}\right)^{n-i} \sqrt{\frac{3\sigma_\alpha^2\gamma_s}{(M-1)+3\sigma_\alpha^2\gamma_s}} \right]} \text{... (E2.6)}$$

Appendix-E3

Average BER with Uncoded MPSK in Nakagami-m Fading Channel

$\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{\ Nakagami}\ d\alpha$

$= \int_0^\infty \frac{1}{\beta} erfc\left(\sqrt{\gamma_s \alpha^2}\, sin(\frac{\pi}{M})\right) \frac{2m^m}{\Gamma(m)\Omega^m} \alpha^{2m-1} \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha$

$= \int_0^\infty \frac{1}{\beta} erfc\left(\sqrt{\gamma_s \alpha^2}\, sin(\frac{\pi}{M})\right) \frac{2m^m}{\Gamma(m)\Omega^m} \alpha^{2m-2}\alpha \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha$

$= \frac{2m^m}{2\Gamma(m)\Omega^m} \int_0^\infty \frac{1}{\beta} erfc\left(\sqrt{\gamma_s y}\, sin(\frac{\pi}{M})\right) y^{m-1} \exp\left(-\frac{m}{\Omega} y\right) dy$

$= \frac{1}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \int_0^\infty \left[\int_{\sqrt{\gamma_s y}\, sin(\frac{\pi}{M})}^\infty \exp(-u^2) du\right] y^{m-1}\alpha \exp\left(-\frac{m}{\Omega} y\right) dy$ …………………….. (E3.1)

Let $y = \alpha^2$
=> $dy = 2\alpha d\alpha$
=> $\frac{dy}{2} = \alpha d\alpha$

$\boldsymbol{erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-u^2)\, du}$

The region where $\sqrt{\gamma_s y}\, sin(\frac{\pi}{M}) < u < \infty$ for $0 < y < \infty$, fixing a value of u, the value of y varies from y=0 to y = $\frac{u^2}{\gamma_s sin^2(\frac{\pi}{M})}$; first slice is u= 0, last slice is u =∞ .

So, from the equation (E3.1), we get,

$\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{\ Nakagami}\ d\alpha$

$= \frac{1}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \int_0^\infty \exp(-u^2) \left[\int_0^{\frac{u^2}{\gamma_s sin^2(\frac{\pi}{M})}} y^{m-1}\alpha \exp\left(-\frac{m}{\Omega} y\right) dy\right] du$ ………………..….. (E3.2)

We know, $\int x^n \exp(cx)\, dx = \exp(cx) \sum_{i=0}^{n} (-1)^i \frac{n!}{(n-i)!c^{i+1}} x^{n-i}$

So, from the equation (E3.2), we get,

$\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{\ Nakagami}\ d\alpha$

$= \frac{1}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \int_0^\infty \exp(-u^2) \left[\exp\left(-\frac{m}{\Omega} y\right) \sum_{i=0}^{m-1} (-1)^i \frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega}\right)^{i+1}} y^{m-1-i}\right]_0^{\frac{u^2}{\gamma_s sin^2(\frac{\pi}{M})}} du$

$= \frac{1}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \int_0^\infty \exp(-u^2) \left[\begin{array}{c} \exp\left(-\frac{m}{\Omega} \frac{u^2}{\gamma_s sin^2(\frac{\pi}{M})}\right) \sum_{i=0}^{m-1} (-1)^i \frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega}\right)^{i+1}} \left(\frac{u^2}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i} \\ -(-1)^{m-1-m} \frac{m-1!}{\left(\frac{m}{\Omega}\right)^m} \end{array}\right] du$ ……..….. (E3.3)

Now, from equation (D2.2) & (E3.3), we get

$$=\frac{1}{\beta}\frac{m^m}{\Gamma(m)\Omega^m}\frac{2}{\sqrt{\pi}}\int_0^\infty \exp(-u^2)\left[-\exp\left(-\frac{mu^2}{\overline{\gamma_s}sin^2(\frac{\pi}{M})}\right)\sum_{i=0}^{m-1}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega}\right)^{i+1}}(u^2)^{m-1-i}+\frac{m-1!}{\left(\frac{m}{\Omega}\right)^m}\right]du$$

$$=\frac{1}{\beta}\frac{m^m}{\Gamma(m)\Omega^m}\frac{2}{\sqrt{\pi}}\left[-\sum_{i=0}^{m-1}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega}\right)^{i+1}}\int_0^\infty \exp(-u^2)\left(1+\frac{m}{\overline{\gamma_s}sin^2(\frac{\pi}{M})}\right)(u^2)^{m-1-i}du+\int_0^\infty \exp(-u^2)\frac{m-1!}{\left(\frac{m}{\Omega}\right)^m}du\right]$$

…..……. (E3.4)

We know, $\int \exp(-cx^2)\,dx=\sqrt{\frac{\pi}{4c}}\,\text{erf}(\sqrt{c}\,x)$ &

$\int_0^\infty x^n \exp(-ax^2)=\frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}$(where n=2k;k integer;a>0). So, from the equation (E3.3), we get,

$\int_0^\infty BER_{MPSK}(\alpha)\,p(\alpha)_{\ Nakagami}\ d\alpha$

$$=\frac{1}{\beta}\frac{m^m}{\Gamma(m)\Omega^m}\frac{2}{\sqrt{\pi}}\left[\left[-\sum_{i=0}^{m-1}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega}\right)^{i+1}}\frac{(2(m-1-i)-1)!!}{2^{m-1-i+1}\left[\left(1+\frac{m}{\overline{\gamma_s}sin^2(\frac{\pi}{M})}\right)\right]^{m-1-i}}\sqrt{\frac{\pi}{\left(1+\frac{m}{\overline{\gamma_s}sin^2(\frac{\pi}{M})}\right)}}\right]+\left[\frac{m-1!}{\left(\frac{m}{\Omega}\right)^m}\left[\sqrt{\frac{\pi}{4}}\ \text{erf}(u)\right]_0^\infty\right]\right]$$

$$=\frac{1}{\beta}\frac{m^m}{\Gamma(m)\Omega^m}\left[\left[-\sum_{i=0}^{m-1}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega}\right)^{i+1}}\frac{(2(m-1-i)-1)!!}{2^{m-1-i}\left[\left(1+\frac{m}{\overline{\gamma_s}sin^2(\frac{\pi}{M})}\right)\right]^{m-1-i}}\sqrt{\frac{1}{\left(1+\frac{m}{\overline{\gamma_s}sin^2(\frac{\pi}{M})}\right)}}\right]+\frac{m-1!}{\left(\frac{m}{\Omega}\right)^m}[1-0]\right]$$ …..... (E3.5)

[From Appendix B]

We know,

For an even positive integer $n = 2k$, $k \geq 0$, the double factorial is expressed as $n!! = 2^k k!$

For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!}{2^k k!} = \frac{n!}{(n-1)!!}$

So, from the equation (E3.5), we get,

$\int_0^\infty BER_{MPSK}(\alpha)\,p(\alpha)_{\ Nakagami}\ d\alpha$

$$=\frac{1}{\beta}\left[1-\sum_{i=0}^{m-1}\left(\frac{m}{\Omega}\right)^{m-1-i}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2}\left(\frac{\overline{\gamma_s}sin^2(\frac{\pi}{M})}{\left(m+\overline{\gamma_s}sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i}\sqrt{\frac{\overline{\gamma_s}sin^2(\frac{\pi}{M})}{\left(m+\overline{\gamma_s}sin^2(\frac{\pi}{M})\right)}}\right]$$

$$=\frac{1}{\beta}\left[1-\sum_{i=0}^{m-1}(m)^{m-1-i}\left(\frac{1}{\overline{\gamma_s}sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2}\left(\frac{\overline{\gamma_s}sin^2(\frac{\pi}{M})}{\left(m+\overline{\gamma_s}sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i}\sqrt{\frac{\overline{\gamma_s}sin^2(\frac{\pi}{M})}{\left(m+\overline{\gamma_s}sin^2(\frac{\pi}{M})\right)}}\right]$$

Therefore,

$\boldsymbol{\int_0^\infty BER_{MPSK}(\alpha)\,p(\alpha)_{\ Nakagami}d\alpha}$

$$\boldsymbol{=\frac{1}{\beta}\left[1-\sum_{i=0}^{m-1}\frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2}\left(\frac{m}{\left(m+\overline{\gamma_s}sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i}\sqrt{\frac{\overline{\gamma_s}sin^2(\frac{\pi}{M})}{\left(m+\overline{\gamma_s}sin^2(\frac{\pi}{M})\right)}}\right]}$$ …………..… (E3.6)

Appendix-E4

Average BER with Uncoded MQAM in Nakagami-m Fading Channel

Let $y = \alpha^2$

$\Rightarrow dy = 2\alpha d\alpha$

$\Rightarrow \frac{dy}{2} = \alpha d\alpha$

$\boldsymbol{erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} \exp(-u^2)\, du}$

$$\int_0^{\infty} BER_{MQAM}(\alpha)\, p(\alpha)_{Nakagami}\; d\alpha$$

$$= \int_0^{\infty} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s\alpha^2}{M-1}}\right) \frac{2m^m}{\Gamma(m)\Omega^m} \alpha^{2m-1} \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha$$

$$= \int_0^{\infty} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s\alpha^2}{M-1}}\right) \frac{2m^m}{\Gamma(m)\Omega^m} \alpha^{2m-2}\alpha \exp\left(-\frac{m}{\Omega}\alpha^2\right) d\alpha$$

$$= \frac{2m^m}{2\Gamma(m)\Omega^m} \int_0^{\infty} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s y}{M-1}}\right) y^{m-1} \exp\left(-\frac{m}{\Omega}y\right) dy$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \int_0^{\infty} \left[\int_{\sqrt{\frac{1.5\gamma_s y}{M-1}}}^{\infty} \exp(-u^2)\, du\right] y^{m-1}\alpha \exp\left(-\frac{m}{\Omega}y\right) dy \qquad \text{(E4.1)}$$

The region where $\sqrt{\frac{1.5\gamma_s y}{M-1}} < u < \infty$ for $0 < y < \infty$, fixing a value of u, the value of y varies from y=0 to y = $\frac{u^2(M-1)}{1.5\gamma_s}$; first slice is u= 0, last slice is u =∞

So, from the equation (E4.1), we get,

$$\int_0^{\infty} BER_{MQAM}(\alpha)\, p(\alpha)_{Nakagami}\; d\alpha$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \int_0^{\infty} \exp(-u^2) \left[\int_0^{\frac{u^2(M-1)}{1.5\gamma_s}} y^{m-1}\alpha \exp\left(-\frac{m}{\Omega}y\right) dy\right] du \qquad \text{(E4.2)}$$

We know, $\int x^n \exp(cx)\, dx = \exp(cx) \sum_{i=0}^{n} (-1)^i \frac{n!}{(n-i)!c^{i+1}} x^{n-i}$

So, from the equation (E4.2), we get,

$$\int_0^{\infty} BER_{MQAM}(\alpha)\, p(\alpha)_{Nakagami}\; d\alpha$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \int_0^{\infty} \exp(-u^2) \left[\exp\left(-\frac{m}{\Omega}y\right) \sum_{i=0}^{m-1} (-1)^i \frac{(m-1)!}{(m-1-i)!\left(-\frac{m}{\Omega}\right)^{i+1}} y^{m-1-i}\right]_0^{\frac{u^2(M-1)}{1.5\gamma_s}} du$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \int_0^{\infty} \exp(-u^2) \left[\begin{array}{c} \exp\left(-\frac{m}{\Omega}\frac{u^2(M-1)}{1.5\gamma_s}\right) \sum_{i=0}^{m-1} (-1)^i \frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega}\right)^{i+1}} \left(\frac{u^2(M-1)}{1.5\gamma_s}\right)^{m-1-i} \\ -(-1)^{m-1-m} \frac{(m-1)!}{\left(\frac{m}{\Omega}\right)^m} \end{array}\right] du$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \int_0^{\infty} \exp(-u^2) \left[\begin{array}{c} \exp\left(-\frac{m}{\Omega}\frac{u^2(M-1)}{1.5\gamma_s}\right) \sum_{i=0}^{m-1} (-1)^i \frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega}\right)^{i+1}} \left(\frac{u^2(M-1)}{1.5\gamma_s}\right)^{m-1-i} \\ + \frac{m-1!}{\left(\frac{m}{\Omega}\right)^m} \end{array}\right] du$$

……… (E4.3)

Now, from equation (D2.2) & (E4.3), we get

$$\int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Nakagami}\ \ d\alpha$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \left[-\sum_{i=0}^{m-1} \left(\frac{(M-1)}{1.5\gamma_s}\right)^{m-1-i} \frac{(m-1)!}{(m-1-i)!\left(\frac{m}{\Omega}\right)^{i+1}} \int_0^\infty \exp(-u^2)\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}}\right)(u^2)^{m-1-i} du + \int_0^\infty \exp(-u^2)\frac{m-1!}{\left(\frac{m}{\Omega}\right)^m} du \right]$$

……… (E4.4)

Now, $\int \exp(-cx^2)\, dx = \sqrt{\frac{\pi}{4c}}\operatorname{erf}(\sqrt{c}\, x)$ &

$\int_0^\infty x^n \exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}$ (where n=2k;k integer;a>0). So, from the equation (E4.4), we get,

$$\int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Nakagami}\ \ d\alpha$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \frac{2}{\sqrt{\pi}} \left[\left[-\sum_{i=0}^{m-1} \left(\frac{(M-1)}{1.5\gamma_s}\right)^{m-1-i} \frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega}\right)^{i+1}} \frac{(2(m-1-i)-1)!!}{2^{m-1-i+1}\left[\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}}\right)\right]^{m-1-i}} \sqrt{\frac{\pi}{\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}}\right)}}\right] + \left[\frac{m-1!}{\left(\frac{m}{\Omega}\right)^m}\left[\sqrt{\frac{\pi}{4}}\operatorname{erf}(u)\right]_0^\infty\right]\right]$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \frac{m^m}{\Gamma(m)\Omega^m} \left[\left[-\sum_{i=0}^{m-1} \left(\frac{(M-1)}{1.5\gamma_s}\right)^{m-1-i} \frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega}\right)^{i+1}} \frac{(2(m-1-i)-1)!!}{2^{m-1-i}\left[\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}}\right)\right]^{m-1-i}} \sqrt{\frac{1}{\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}}\right)}}\right] + \frac{m-1!}{\left(\frac{m}{\Omega}\right)^m}[1-0]\right] \text{….. (E4.5)}$$

[From Appendix B]

We know,

For an even positive integer $n = 2k$, $k \geq 0$, the double factorial is expressed as $n!! = 2^k k!$

For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!}{2^k k!} = \frac{n!}{(n-1)!!}$

So, from the equation (E4.5), we get,

$$\int_0^\infty BER_{MQAM}(\alpha)\, p(\alpha)_{Nakagami}\ \ d\alpha$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\left[1-\sum_{i=0}^{m-1}(m)^{m-1-i}\left(\frac{M-1}{1.5\overline{\gamma_s}}\right)^{m-1-i} \frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2}\left(\frac{1.5\overline{\gamma_s}}{(m(M-1)+1.5\overline{\gamma_s})}\right)^{m-1-i}\sqrt{\frac{1.5\overline{\gamma_s}}{(m(M-1)+1.5\overline{\gamma_s})}}\right]$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\left[1-\sum_{i=0}^{m-1}(m)^{m-1-i} \frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2}\left(\frac{(M-1)}{(m(M-1)+1.5\overline{\gamma_s})}\right)^{m-1-i}\sqrt{\frac{1.5\overline{\gamma_s}}{(m(M-1)+1.5\overline{\gamma_s})}}\right]$$

Therefore,

$$\boldsymbol{\int_0^\infty BER_{MPSK}(\alpha)\, p(\alpha)_{\ Nakagami} d\alpha}$$

$$\boldsymbol{= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\left[1-\sum_{i=0}^{m-1}\frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2}\left(\frac{m(M-1)}{(m(M-1)+1.5\overline{\gamma_s})}\right)^{m-1-i}\sqrt{\frac{1.5\overline{\gamma_s}}{(m(M-1)+1.5\overline{\gamma_s})}}\right]} \text{ ………. (E4.6)}$$

Appendix-F1

Average BER with Uncoded MPSK-OFDM in Rician Fading Channel

Let, $y_k = \alpha_k^2$

$\Rightarrow 2\alpha_k d\alpha_k = dy_k$

$\Rightarrow \alpha_k d\alpha_k = \frac{dy_k}{2}$

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} \exp(-u^2)\,du$$

$$\int_0^{\infty} BER_{MPSK-OFDM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$$

$$= \int_0^{\infty} \frac{1}{N}\sum_{k=0}^{N-1} BER_{MPSK-OFDM}(\alpha_k)\, p(\alpha_k)_{Rician}\, d\alpha_k$$

$$= \frac{1}{N\beta}\int_0^{\infty} \sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s {\alpha_k}^2}\, sin(\tfrac{\pi}{M})\right) \frac{\alpha_k}{\sigma^2_{\alpha_k}} \exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) I_0\left(\frac{\alpha_0\alpha_k}{\sigma^2_{\alpha_k}}\right) d\alpha_k$$

$$= \frac{1}{N\beta}\int_0^{\infty} \sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s {\alpha_k}^2}\, sin(\tfrac{\pi}{M})\right) \frac{\alpha_k}{\sigma^2_{\alpha_k}} \exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2{\alpha_k}^2}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2}\, d\alpha_k$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{1}{2\sigma^2_{\alpha_k}}\int_0^{\infty} erfc\left(\sqrt{\gamma_s y_k}\, sin(\tfrac{\pi}{M})\right) \exp\left(-\frac{y_k+\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2 y_k}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2}\, dy_k\right]$$

$$= \frac{1}{N\beta}\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2} \frac{1}{2\sigma^2_{\alpha_k}} \exp\left(-\frac{\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \left[\int_0^{\infty}\left[\int_{\sqrt{\gamma_s y_k}\, sin(\frac{\pi}{M})}^{\infty} \exp(-{u_k}^2)\, du_k\right] {y_k}^n \exp\left(-\frac{y_k}{2\sigma^2_{\alpha_k}}\right) dy_k\right]\right]$$

………. (F1.1)

The region where $\sqrt{\gamma_s y}sin(\frac{\pi}{M}) < u_k < \infty$ for $0 < y_k < \infty$, fixing a value of u, the value of y varies from y_k=0 to $y_k = \frac{{u_k}^2}{\gamma_s sin^2(\frac{\pi}{M})}$; first slice is u_k= 0, last slice is u_k =∞, So, from equation (F1.1), we get,

$$\int_0^{\infty} BER_{MPSK-OFDM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$$

$$= \frac{1}{N\beta}\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2} \exp\left(-\frac{\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \frac{1}{2\sigma^2_{\alpha_k}}\int_0^{\infty} \exp(-{u_k}^2) \left[\int_0^{\frac{{u_k}^2}{\gamma_s sin^2(\frac{\pi}{M})}} \exp\left(-\frac{y_k}{2\sigma^2_{\alpha_k}}\right) {y_k}^n\, dy\right] du_k\right] \text{……..(F1.2)}$$

Now, $\int x^n \exp(cx)\, dx = \exp(cx)\sum_{i=0}^{n}(-1)^i \frac{n!}{(n-i)!c^{i+1}} x^{n-i}$. So, from equation (F1.2), we get,

$$\int_0^{\infty} BER_{MPSK-OFDM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$$

$$= \frac{1}{N\beta}\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2} \exp\left(-\frac{\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \frac{1}{2\sigma^2_{\alpha_k}}\left[\int_0^{\infty} \exp(-{u_k}^2) \left[\exp\left(-\frac{y_k}{2\sigma^2_{\alpha_k}}\right) \sum_{i=0}^{n}(-1)^i \frac{n!}{(n-i)!\left(-\frac{1}{2\sigma^2_{\alpha_k}}\right)^{i+1}} {y_k}^{n-i}\right]_0^{\frac{{u_k}^2}{\gamma_s sin^2(\frac{\pi}{M})}} du_k\right]\right]$$

$$= \frac{1}{N\beta}\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma^2_{\alpha_k}\right)^2}\right)^n}{(n!)^2} \exp\left(-\frac{\alpha_0^2}{2\sigma^2_{\alpha_k}}\right) \frac{1}{2\sigma^2_{\alpha_k}}\int_0^{\infty} \exp(-{u_k}^2) \left[\begin{array}{l}\exp\left(-\frac{{u_k}^2}{2\sigma^2_{\alpha_k}\gamma_s sin^2(\frac{\pi}{M})}\right) \sum_{i=0}^{n}(-1)^i \frac{n!}{(n-i)!\left(-\frac{1}{2\sigma^2_{\alpha_k}}\right)^{i+1}} \left(\frac{{u_k}^2}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{n-i} \\ -\exp(0).(-1)^n \frac{n!}{(n-n)!\left(-\frac{1}{2\sigma^2_{\alpha_k}}\right)^{n+1}}.(0)^{n-n}\end{array}\right] du_k\right]$$

$$= \frac{1}{N\beta}\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0{}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[\begin{array}{c}-\int_0^{\infty}\exp(-u_k{}^2)\exp\left(-\frac{u_k{}^2}{2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)\sum_{i=0}^{n}\frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i}(u_k{}^2)^{n-i}\left(\frac{1}{\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i} du_k \\ +\int_0^{\infty}\exp(-u_k{}^2)\left(2\sigma_{\alpha_k}^2\right)^n du_k\end{array}\right]\right]$$

$$= \frac{1}{N\beta}\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0{}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[\begin{array}{c}\sum_{i=0}^{n}-\frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i}\left(\frac{1}{\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\int_0^{\infty}\exp\left\{-u_k{}^2\left(1+\frac{1}{2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)\right\}(u_k{}^2)^{n-i}du_k \\ +\left(2\sigma_{\alpha_k}^2\right)^n\int_0^{\infty}\exp(-u_k{}^2)\,du_k\end{array}\right]\right]$$

…….. (F1.3)

We know, $\int \exp(-cx^2)\,dx = \sqrt{\frac{\pi}{4c}}\ \mathrm{erf}(\sqrt{c}\,x)$ &

$\int_0^{\infty} x^n \exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}$ (where n=2k; k integer; a>0). So, from equation (F1.3), we get,

$$= \frac{1}{N\beta}\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0{}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[\begin{array}{c}\sum_{i=0}^{n}-\frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i}\left(\frac{1}{\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\frac{(2(n-i)-1)!!}{2^{n-i+1}\left\{\left(1+\frac{1}{2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)\right\}^{n-i}}\sqrt{\frac{\pi}{\left\{\left(1+\frac{1}{2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)\right\}}} \\ +\left(2\sigma_{\alpha_k}^2\right)^n\left[\sqrt{\frac{\pi}{4.}}\mathrm{erf}\,(u_k)\right]_0^{\infty}\end{array}\right]\right]$$

$$= \frac{1}{N\beta}\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0{}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[\begin{array}{c}\sum_{i=0}^{n}-\frac{1}{2}\frac{n!\,(2(n-i)-1)!!}{2^{n-i}(n-i)!}\frac{\left(2\sigma_{\alpha_k}^2\right)^n}{\left(1+2\sigma_{\alpha_k}^2\gamma_s\right)^{n-i}}\sqrt{\frac{\pi.2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}{1+2\sigma_{\alpha_k}^2\gamma_b sin^2\left(\frac{\pi}{M}\right)}} \\ +n!\left(2\sigma_{\alpha_k}^2\right)^n\left[\sqrt{\frac{\pi}{4}}(1)-\sqrt{\frac{\pi}{4}}(0)\right]\end{array}\right]\right]$$ …….. (F1.4)

[From Appendix B]

We know,

For an even positive integer $n = 2k$, $k \geq 0$, the double factorial is expressed as $n!! = 2^k k!$

For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!}{2^k k!} = \frac{n!}{(n-1)!!}$

So, from the equation (F1.4), we get

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{\alpha_0{}^2}{2\sigma_{\alpha_k}^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2}\frac{1}{\left(1+2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)\right)^{n-i}}\sqrt{\frac{2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}{1+2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}}\right]\right]$$ ……... (F1.5)

Therefore, from equation (D3.2) & (F1.5), we get

$$\int_0^{\infty} \boldsymbol{BER_{MPSK-OFDM}(\alpha)\,p(\alpha)_{Rician}\,d\alpha}$$

$$\boldsymbol{= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_{s}}_k}{2\sigma_{\alpha_k}^2\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}((n-i)!)^2}\left(\frac{1}{1+2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\sqrt{\frac{2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}{1+2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}}\right]\right]}$$

……... (F1.6)

Appendix-F2

Average BER with Uncoded MQAM-OFDM in Rician fading Channel

$\int_0^\infty BER_{MQAM-OFDM}(\alpha)\, p(\alpha)_{Rician}\; d\alpha$

$= \int_0^\infty \frac{1}{N}\sum_{k=0}^{N-1} BER_{\mathrm{MQAM-OFDM}}(\alpha_k)\, p(\alpha_k)_{Rician}\; d\alpha_k$

$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta} \sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s {\alpha_k}^2}{M-1}}\right) \frac{\alpha_k}{\sigma_{\alpha_k}^2} \exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) I_0\left(\frac{\alpha_0\alpha_k}{\sigma_{\alpha_k}^2}\right) d\alpha_k$

$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta} \sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s {\alpha_k}^2}{M-1}}\right) \frac{\alpha_k}{\sigma_{\alpha_k}^2} \exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2{\alpha_k}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\; d\alpha_k$

$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta} \sum_{k=0}^{N-1} \frac{1}{2\sigma_{\alpha_k}^2} erfc\left(\sqrt{\frac{1.5\gamma_s\, y_k}{M-1}}\right) \exp\left(-\frac{y_k+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2\, y_k}{4\left(\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\; dy_k$

$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2} \frac{1}{2\sigma_{\alpha_k}^2} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \int_0^\infty \left[\int_{\sqrt{\frac{1.5\gamma_s\, y_k}{M-1}}}^{\infty} \exp(-{u_k}^2)\, du_k\right] \exp\left(-\frac{y_k}{2\sigma_{\alpha_k}^2}\right) {y_k}^n\; dy_k \right]$ ………. (F2.1)

Let, $y = {\alpha_k}^2$

$\Rightarrow 2\alpha_k d\alpha_k = dy_k$

$\Rightarrow \alpha_k d\alpha_k = \frac{dy_k}{2}$

$\boldsymbol{erfc(x)} = \frac{\mathbf{2}}{\sqrt{\boldsymbol{\pi}}}\int_x^\infty \mathbf{exp}(-\boldsymbol{u}^2)\, \boldsymbol{du}$

The region where $\sqrt{\frac{1.5\gamma_s y_k}{M-1}} < u_k < \infty$ for $0 < y_k < \infty$, fixing a value of u, the value of y varies from y_k=0 to $y_k = \frac{{u_k}^2(M-1)}{1.5\gamma_s}$; first slice is u_k= 0, last slice is u_k =∞. So, from the equation (F2.1), we get,

$\int_0^\infty BER_{MQAM-OFDM}(\alpha)\, p(\alpha)_{Rician}\; d\alpha$

$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \frac{1}{2\sigma_{\alpha_k}^2} \int_0^\infty \exp(-{u_k}^2) \left[\int_0^{\frac{{u_k}^2(M-1)}{1.5\gamma_s}} \exp\left(-\frac{y_k}{2\sigma_{\alpha_k}^2}\right) {y_k}^n\; dy_k\right] du_k \right]$

………. (F2.2)

We know, $\int x^n \exp(cx)\, dx = \exp(cx) \sum_{i=0}^{n} (-1)^i \frac{n!}{(n-i)!c^{i+1}} x^{n-i}$. So, from the equation (F2.2), we get,

$\int_0^\infty BER_{MQAM-OFDM}(\alpha)\, p(\alpha)_{Rician}\; d\alpha$

$$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \frac{1}{2\sigma_{\alpha_k}^2} \int_0^\infty \exp(-{u_k}^2) \left[\exp\left(-\frac{y}{2\sigma_{\alpha_k}^2}\right) \sum_{i=0}^{n} (-1)^i \frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_{\alpha_k}^2}\right)^{i+1}} {y_k}^{n-i}\right]_0^{\frac{{u_k}^2(M-1)}{1.5\gamma_s}} du_k \right]$$

$$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{{\alpha_0}^2}{4\left(\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \frac{1}{2\sigma_{\alpha_k}^2} \int_0^\infty \exp(-{u_k}^2) \left[\begin{array}{c} \exp\left(-\frac{{u_k}^2(M-1)}{2\sigma_{\alpha_k}^2 . 1.5\gamma_s}\right) \sum_{i=0}^{n} (-1)^i \frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_{\alpha_k}^2}\right)^{i+1}} \left(\frac{{u_k}^2(M-1)}{1.5\gamma_s}\right)^{n-i} \\ -\exp(0) . (-1)^n \frac{n!}{(n-n)!\left(-\frac{1}{2\sigma_{\alpha_k}^2}\right)^{n+1}} . (0)^{n-n} \end{array} \right] du_k \right]$$

$$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{4\left(\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \left[\begin{array}{c} -\int_0^{\infty} \exp(-\mathrm{u_k}^2) \exp\left(-\frac{u_k{}^2(M-1)}{3\sigma_{\alpha_k}^2\gamma_s}\right) \sum_{i=0}^{n} \frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i} (\mathrm{u_k}^2)^{n-i} \left(\frac{(M-1)}{1.5\gamma_s}\right)^{n-i} d\mathrm{u_k} \\ + \int_0^{\infty} \exp(-\mathrm{u_k}^2) \left(2\sigma_{\alpha_k}^2\right)^n d\mathrm{u_k} \end{array} \right]$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{4\left(\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \left[\begin{array}{c} \sum_{i=0}^{n} - \frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i} \left(\frac{(M-1)}{1.5\gamma_s}\right)^{n-i} \int_0^{\infty} \exp\left\{-\mathrm{u_k}^2\left(1+\frac{(M-1)}{3\sigma_{\alpha_k}^2\gamma_s}\right)\right\} (\mathrm{u_k}^2)^{n-i} d\mathrm{u_k} \\ + \left(2\sigma_{\alpha_k}^2\right)^n \int_0^{\infty} \exp(-\mathrm{u_k}^2) \, d\mathrm{u_k} \end{array} \right]$$

………. (F2.3)

Now, $\int \exp(-cx^2)\, dx = \frac{\pi}{4c} \operatorname{erf}(\sqrt{c}\, x)$

& $\int_0^{\infty} x^n \exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k} \sqrt{\frac{\pi}{a}}$ (where n=2k; k integer; a>0), So, from the equation (F2.3), we get,

$$\int_0^{\infty} BER_{MQAM-OFDM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$$

$$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \left[\begin{array}{c} \sum_{i=0}^{n} - \frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i} \left(\frac{(M-1)}{1.5\gamma_s}\right)^{n-i} \frac{(2(n-i)-1)!!}{2^{n-i+1}\left\{\left(1+\frac{(M-1)}{3\sigma_{\alpha_k}^2\gamma_s}\right)\right\}^{n-i}} \sqrt{\frac{\pi}{\left\{\left(1+\frac{M-1}{3\sigma_{\alpha_k}^2\gamma_s}\right)\right\}}} \\ + \left(2\sigma_{\alpha_k}^2\right)^n \left[\sqrt{\frac{\pi}{4}} \operatorname{erf}\left(\sqrt{\mathrm{u_k}}\right) \right]_0^{\infty} \end{array} \right] \right]$$

$$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{4\left(\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha}^2}\right) \left[\begin{array}{c} \sum_{i=0}^{n} - \frac{1}{2} \frac{(2(n-i)-1)!!}{(n-i)!\left(\frac{1}{3\sigma_{\alpha}^2\gamma_s}\right)^n} \frac{(M-1)^{n-i}\left(\frac{1}{1.5\gamma_s}\right)^n}{2^{n-i}\left((M-1)+3\sigma_{\alpha}^2\gamma_s\right)^{n-i}} \sqrt{\frac{\pi . 3\sigma_{\alpha}^2\gamma_s}{(M-1)+3\sigma_{\alpha}^2\gamma_s}} \\ + (2\sigma_{\alpha}^2)^n \left[\sqrt{\frac{\pi}{4}}(1) - \sqrt{\frac{\pi}{4}}(0) \right] \end{array} \right] \right] \quad \ldots \text{(F2.4)}$$

[From Appendix B]

We know,

For an even positive integer n = 2k, k ≥ 0, the double factorial is expressed as $n!! = 2^k k!$

For an odd positive integer n = 2k − 1, k ≥ 1, it has the expressions $n!! = \frac{(2k)!}{2^k k!} = \frac{n!}{(n-1)!!}$

So, from the equation (F2.4), we get,

$$\int_0^{\infty} BER_{MQAM-OFDM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha$$

$$= \frac{2\left(1-\frac{1}{\sqrt{M}}\right)}{N\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{2\sigma_{\alpha_k}^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \left[1 - \sum_{i=0}^{n} \frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2} \left(\frac{M-1}{(M-1)+3\sigma_{\alpha_k}^2\gamma_s}\right)^{n-i} \sqrt{\frac{3\sigma_{\alpha_k}^2\gamma_s}{(M-1)+3\sigma_{\alpha_k}^2\gamma_s}} \right] \right] \quad \ldots\ldots\text{. (F2.5)}$$

Therefore, from equation (D3.2) & (F2.5), we get

$$\boldsymbol{\int_0^{\infty} BER_{MQAM-OFDM}(\alpha)\, p(\alpha)_{Rician}\, d\alpha}$$

$$\boldsymbol{= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\overline{\gamma_{sk}}}{2\sigma_{\alpha_k}^2\gamma_s(n+1)} \left[1 - \sum_{i=0}^{n} \frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2} \left(\frac{M-1}{(M-1)+3\sigma_{\alpha_k}^2\gamma_s}\right)^{n-i} \sqrt{\frac{3\sigma_{\alpha_k}^2\gamma_s}{(M-1)+3\sigma_{\alpha_k}^2\gamma_s}} \right] \right]}$$

……. (F2.6)

Appendix-F3

Average BER with Uncoded MPSK-OFDM in Nakagami-m fading Channel

Let, $y = {\alpha_k}^2$

$=> \ 2\alpha_k \mathrm{d}\alpha_k = \mathrm{dy_k}$

$=> \ \alpha_k \mathrm{d}\alpha_k = \frac{\mathrm{dy_k}}{2}$

$\boldsymbol{erfc(x)} = \frac{\mathbf{2}}{\sqrt{\boldsymbol{\pi}}}\int_x^\infty \mathbf{exp}(-\boldsymbol{u}^2)\,\boldsymbol{du}$

$$\int_0^\infty BER_{MPSK-OFDM}(\alpha)\, p(\alpha)_{Nakagami}\ d\alpha$$

$$= \int_0^\infty \frac{1}{N}\sum_{k=0}^{N-1} BER_{\mathrm{MPSK-OFDM}}(\alpha_k)\, p(\alpha_k)_{Nakagami}\ d\alpha_k$$

$$= \int_0^\infty \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[erfc\left(\sqrt{\gamma_s{\alpha_k}^2}\, sin(\tfrac{\pi}{M})\right)\frac{2m^m}{\Gamma(m){\Omega_k}^m}{\alpha_k}^{2m-1}\exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right)d\alpha_k\right]$$

$$= \int_0^\infty \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[erfc\left(\sqrt{\gamma_s{\alpha_k}^2}\, sin(\tfrac{\pi}{M})\right)\frac{2m^m}{\Gamma(m){\Omega_k}^m}{\alpha_k}^{2m-2}\alpha_k\exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right)d\alpha_k\right]$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{2m^m}{2\Gamma(m){\Omega_k}^m}\int_0^\infty erfc\left(\sqrt{\gamma_s \mathrm{y_k}}\, sin(\tfrac{\pi}{M})\right){\mathrm{y_k}}^{m-1}\exp\left(-\frac{m}{\Omega_k}\mathrm{y_k}\right)d\mathrm{y_k}\right]$$

$$= \frac{1}{N\beta}\left[\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_0^\infty\left[\int_{\sqrt{\gamma_s \mathrm{y_k}}\, sin(\frac{\pi}{M})}^\infty \exp(-{u_k}^2)du_k\right]{\mathrm{y_k}}^{m-1}\exp\left(-\frac{m}{\Omega_k}\mathrm{y_k}\right)d\mathrm{y_k}\right] \quad \text{…………………. (F3.1)}$$

The region where $\sqrt{\gamma_s y}\,sin(\frac{\pi}{M}) < u_k < \infty$ for $0 < y_k < \infty$, fixing a value of u, the value of y varies from y_k=0 to $y_k = \frac{{u_k}^2}{\gamma_s sin^2(\frac{\pi}{M})}$; first slice is u_k= 0, last slice is u_k =∞, So, from the equation (F3.1), we get,

$$\int_0^\infty BER_{MPSK-OFDM}(\alpha)\, p(\alpha)_{Nakagami}\ d\alpha$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_0^\infty \exp(-{u_k}^2)\left[\int_0^{\frac{{u_k}^2}{\gamma_s sin^2(\frac{\pi}{M})}}{y_k}^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right)dy_k\right]du_k\right] \quad \text{………..…… (F3.2)}$$

We know, $\int x^n \exp(cx)\,dx = \exp(cx)\sum_{i=0}^{n}(-1)^i\ \frac{n!}{(n-i)!c^{i+1}}x^{n-i}$. So, from the equation (F3.2), we get,

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{2}{\sqrt{\pi}}\int_0^\infty \exp(-{u_k}^2)\left[\exp\left(-\frac{m}{\Omega_k}y_k\right)\sum_{i=0}^{m-1}(-1)^i\ \frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega_k}\right)^{i+1}}{y_k}^{m-1-i}\right]_0^{\frac{{u_k}^2}{\gamma_s sin^2(\frac{\pi}{M})}}du_k\right]$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_0^\infty \exp(-{u_k}^2)\left[\begin{array}{c}\exp\left(-\frac{m}{\Omega_k}\frac{{u_k}^2}{\gamma_s sin^2(\frac{\pi}{M})}\right)\sum_{i=0}^{m-1}(-1)^i\frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega_k}\right)^{i+1}}\left(\frac{{u_k}^2}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\\ -\exp(0)(-1)^{m-1}\frac{m-1!}{(0)!\left(-\frac{m}{\Omega_k}\right)^{m-1+1}}0^{m-1-m+1}\end{array}\right]du_k\right] \quad \text{…..…. (F3.3)}$$

Now, from equation (D4.2) & (F3.3), we get,

$$\int_0^\infty BER_{MPSK-OFDM}(\alpha)\, p(\alpha)_{Nakagami}\ d\alpha$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_0^\infty \exp(-{u_k}^2)\left[\begin{array}{c}-\exp\left(-\frac{m{u_k}^2}{\overline{\gamma_s} sin^2(\frac{\pi}{M})}\right)\sum_{i=0}^{m-1}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}({u_k}^2)^{m-1-i}\\ +\frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}\end{array}\right]du_k\right]$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{m^m}{\Gamma(m)\Omega_k{}^m}\frac{2}{\sqrt{\pi}}\left[-\sum_{i=0}^{m-1}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}\int_0^\infty \exp(-u_k{}^2)\left(1+\frac{m}{\bar{\gamma_s} sin^2\left(\frac{\pi}{M}\right)}\right)(u_k{}^2)^{m-1-i}du_k + \int_0^\infty \exp(-u_k{}^2)\frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}du_k\right]\right]$$

………..…… (F3.4)

We know, $\int \exp(-cx^2)\,dx = \sqrt{\frac{\pi}{4c}}\,\mathrm{erf}(\sqrt{c}\,x)$ &

$\int_0^\infty x^n \exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}$ (where n=2k; k integer; a>0). So, from the equation (F3.4), we get,

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{m^m}{\Gamma(m)\Omega_k{}^m}\frac{2}{\sqrt{\pi}}\left[\left[-\sum_{i=0}^{m-1}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}\frac{(2(m-1-i)-1)!!}{2^{m-1-i+1}\left[\left(1+\frac{m}{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}\right)\right]^{m-1-i}}\sqrt{\frac{\pi}{\left(1+\frac{m}{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}\right)}}\right]+\left[\frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}\left[\sqrt{\frac{\pi}{4}}\,\mathrm{erf}(u_k)\right]_0^\infty\right]\right]\right]$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{m^m}{\Gamma(m)\Omega_k{}^m}\frac{2}{\sqrt{\pi}}\left[-\sum_{i=0}^{m-1}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}\frac{1}{2}\frac{(2(m-1-i)-1)!!}{2^{m-1-i}\left[\left(1+\frac{m}{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}\right)\right]^{m-1-i}}\sqrt{\frac{\pi}{\left(1+\frac{m}{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}\right)}} + \frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}\left[\sqrt{\frac{\pi}{4}}.1-\sqrt{\frac{\pi}{4}}.0\right]\right]\right] \text{.. (F3.5)}$$

[From Appendix B]

We know,
For an even positive integer $n = 2k$, $k \geq 0$, the double factorial may be expressed as $n!! = 2^k k!$
For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!!}{2^k k!}$. So, from the equation (F3.5), we get,

$\int_0^\infty BER_{MPSK-OFDM}(\alpha)\,p(\alpha)_{Nakagami}\;d\alpha$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}\left(\frac{m}{\Omega_k}\right)^{m-1-i}\left(\frac{1}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2}\left(\frac{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}{\left(m+\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i}\sqrt{\frac{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}{\left(m+\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})\right)}}\right]$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}(m)^{m-1-i}\left(\frac{1}{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2}\left(\frac{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}{\left(m+\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i}\sqrt{\frac{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}{\left(m+\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})\right)}}\right]$$

Therefore,

$\boldsymbol{\int_0^\infty BER_{MPSK-OFDM}(\alpha)\,p(\alpha)_{Nakagami}\,d\alpha}$

$$\boldsymbol{= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}\frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2}\left(\frac{m}{\left(m+\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i}\sqrt{\frac{\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})}{\left(m+\bar{\gamma}_{s_k} sin^2(\frac{\pi}{M})\right)}}\right]}$$ .... (F3.6)

Appendix-F4

Average BER with Uncoded MQAM-OFDM in Nakagami-m fading Channel

Let , $y = \alpha_k^2$

$=> \; 2\alpha_k d\alpha_k = dy_k$

$=> \; \alpha_k d\alpha_k = \frac{dy_k}{2}$

$\boldsymbol{erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} \exp(-u^2)\, du}$

$$\int_0^{\infty} BER_{\text{MQAM}-\text{OFDM}}(\alpha)\, p(\alpha)_{Nakagami}\, d\alpha$$

$$= \int_0^{\infty} \frac{1}{N}\sum_{k=0}^{N-1} BER_{\text{MQAM}-\text{OFDM}}(\alpha_k)\, p(\alpha_k)_{Nakagami}\, d\alpha_k$$

$$= \int_0^{\infty} \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s \alpha_k^2}{M-1}}\right)\frac{2m^m}{\Gamma(m)\Omega_k^{\,m}}\alpha_k^{2m-1}\exp\left(-\frac{m}{\Omega_k}\alpha_k^2\right) d\alpha_k$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{2m^m}{2\Gamma(m)\Omega_k^{\,m}}\int_0^{\infty} erfc\left(\sqrt{\frac{1.5\gamma_s\alpha_k^2}{M-1}}\right)\alpha_k^{2m-2}\alpha_k \exp\left(-\frac{m}{\Omega_k}\alpha_k^2\right) 2\alpha_k d\alpha_k$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k^{\,m}}\int_0^{\infty} erfc\left(\sqrt{\frac{1.5\gamma_s y_k}{M-1}}\right) y_k^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right) dy_k$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k^{\,m}}\frac{2}{\sqrt{\pi}}\int_0^{\infty}\left[\int_{\sqrt{\frac{1.5\gamma_s y_k}{M-1}}}^{\infty}\exp(-u_k^2)du_k\right] y_k^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right) dy_k \quad \text{......................(F4.1)}$$

After Drawing the region where $\sqrt{\frac{1.5\gamma_s y_k}{M-1}} < u_k < \infty$ for $0 < y_k < \infty$, fixing a value of u, the value of y varies from y_k=0 to $y_k = \frac{u_k^2(M-1)}{1.5\gamma_s}$; first slice is u_k= 0, last slice is u_k =∞

So, from the equation (F4.1), we get,

$$\int_0^{\infty} BER_{\text{MQAM}-\text{OFDM}}(\alpha)\, p(\alpha)_{Nakagami}\, d\alpha$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k^{\,m}}\frac{2}{\sqrt{\pi}}\int_0^{\infty}\exp(-u_k^2)\left[\int_0^{\frac{u_k^2(M-1)}{1.5\gamma_s}}\sum_{k=0}^{N-1} y_k^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right) dy_k\right] du_k \quad \text{..............(F4.2)}$$

We know, $\int x^n \exp(cx)\, dx = \exp(cx)\sum_{i=0}^{n}(-1)^i \frac{n!}{(n-i)!c^{i+1}}x^{n-i}$. So, from the equation (F4.2), we get,

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k^{\,m}}\frac{2}{\sqrt{\pi}}\int_0^{\infty}\exp(-u_k^2)\left[\exp\left(-\frac{m}{\Omega_k}y_k\right)\sum_{i=0}^{m-1}(-1)^i\frac{(m-1)!}{(m-1-i)!\left(-\frac{m}{\Omega_k}\right)^{i+1}}y_k^{m-1-i}\right]_0^{\frac{u_k^2(M-1)}{1.5\gamma_s}} du_k$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k^{\,m}}\frac{2}{\sqrt{\pi}}\int_0^{\infty}\exp(-u_k^2)\begin{bmatrix}\exp\left(-\frac{m}{\Omega_k}\frac{u_k^2(M-1)}{1.5\gamma_s}\right)\sum_{i=0}^{m-1}(-1)^i\frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega_k}\right)^{i+1}}\left(\frac{u_k^2(M-1)}{1.5\gamma_s}\right)^{m-1-i} \\ -\exp(0)(-1)^{m-1}\frac{(m-1)!}{(0)!\left(-\frac{m}{\Omega_k}\right)^{m-1+1}}(0)^{m-1-m+1}\end{bmatrix} du_k$$

..............(F4.3)

Now, from equation (D4.2) & (F4.3), we get,

$$\int_0^{\infty} BER_{\text{MQAM}-\text{OFDM}}(\alpha)\, p(\alpha)_{Nakagami}\, d\alpha$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k^{\,m}}\frac{2}{\sqrt{\pi}}\int_0^{\infty}\exp(-u_k^2)\begin{bmatrix}-\exp\left(-\frac{m u_k^2(M-1)}{1.5\,\overline{\boldsymbol{\gamma_{sk}}}}\right)\sum_{i=0}^{m-1}\left(\frac{(M-1)}{1.5\gamma_s}\right)^{m-1-i}\frac{(m-1)!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}(u_k^2)^{m-1-i} \\ +\frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}\end{bmatrix} du_k$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k{}^m}\frac{2}{\sqrt{\pi}}\left[-\sum_{i=0}^{m-1}\left(\frac{(M-1)}{1.5\gamma_s}\right)^{m-1-i}\frac{(m-1)!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}\int_0^\infty \begin{matrix}\exp(-u_k{}^2)\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}_k}\right)(u_k{}^2)^{m-1-i}du_k \\ +\int_0^\infty \exp(-u_k{}^2)\frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}du_k\end{matrix}\right]$$

…………... (F4.4)

We know, $\int \exp(-cx^2)\,dx = \sqrt{\frac{\pi}{4c}}\,\mathrm{erf}(\sqrt{c}\,x)$ & $\int_0^\infty x^n \exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}$ (where n=2k; k integer; a>0).

So, from the equation (F4.4), we get,

$$\int_0^\infty BER_{\text{MQAM}-\text{OFDM}}(\alpha)\,p(\alpha)_{Nakagami}\,d\alpha$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k{}^m}\frac{2}{\sqrt{\pi}}\left[\left[-\sum_{i=0}^{m-1}\left(\frac{(M-1)}{1.5\gamma_s}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}\frac{(2(m-1-i)-1)!!}{2^{m-1-i+1}\left[\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}_k}\right)\right]^{m-1-i}}\sqrt{\frac{\pi}{\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}}\right)}}\right] + \left[\frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}\left[\sqrt{\frac{\pi}{4}}\,\mathrm{erf}(u_k)\right]\right]_0^\infty\right]$$

$$= \frac{2\left(1-\frac{1}{\sqrt{M}}\right)}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k{}^m}\frac{2}{\sqrt{\pi}}\left[\left[-\sum_{i=0}^{m-1}\left(\frac{(M-1)}{1.5\gamma_s}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}\frac{(2(m-1-i)-1)!!}{2^{m-i}\left[\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}_k}\right)\right]^{m-1-i}}\sqrt{\frac{\pi}{\left(1+\frac{m(M-1)}{1.5\overline{\gamma_s}_k}\right)}} + \frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}\left[\sqrt{\frac{\pi}{4}}.1-\sqrt{\frac{\pi}{4}}.0\right]\right]\right] \quad \text{……... (F4.5)}$$

[From Appendix B]

We know,
For an even positive integer $n = 2k$, $k \geq 0$, the double factorial may be expressed as $n!! = 2^k k!$
For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!!}{2^k k!}$. So, from the equation (F4.5), we get,

$$\int_0^\infty BER_{\text{MQAM}-\text{OFDM}}(\alpha)\,p(\alpha)_{Nakagami}\,d\alpha$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}(m)^{m-1-i}\left(\frac{M-1}{1.5\overline{\gamma_s}_k}\right)^{m-1-i}\frac{(2(m-1-i))!}{((m-1-i)!)^2}\left(\frac{1.5\overline{\gamma_s}_k}{(m(M-1)+1.5\overline{\gamma_s}_k)}\right)^{m-1-i}\sqrt{\frac{1.5\overline{\boldsymbol{\gamma_s}}_{\boldsymbol{k}}}{(m(M-1)+1.5\overline{\boldsymbol{\gamma_s}}_{\boldsymbol{k}})}}\right]$$

Therefore,

$$\int_0^\infty \boldsymbol{BER_{\text{MQAM}-\text{OFDM}}(\alpha)\,p(\alpha)_{Nakagami}\,d\alpha}$$

$$= \boldsymbol{\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}\frac{(2(m-1-i))!}{2^{2(m-1-i)}((m-1-i)!)^2}\left(\frac{m(M-1)}{(m(M-1)+1.5\overline{\gamma_s}_k)}\right)^{m-1-i}\sqrt{\frac{1.5\overline{\gamma_s}_k}{(m(M-1)+1.5\overline{\gamma_s}_k)}}\right]} \quad \text{... (F4.6)}$$

Appendix-G1

Average BER using SFBC (j=1, i=1) with MPSK-OFDM in Rician fading Channel

$$\int_0^\infty BER_{SFBC-MPSK-OFDM}\; p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}$$

$$= \int_0^\infty \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}{\alpha_{k_{i,j}}}^2}{R_c}}\, sin(\frac{\pi}{M})\right) p\left(\alpha_{k_{1,1}}\right)_{Rician}\; d\alpha_k$$

$$= \int_0^\infty \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s\left({\alpha_{k_{1,1}}}^2+C\right)}{R_c(M-1)}}\, sin(\frac{\pi}{M})\right)\frac{\alpha_k}{\sigma_{\alpha_k}^2}\exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) I_0\left(\frac{\alpha_0\alpha_k}{\sigma_{\alpha_k}^2}\right)\, d\alpha_k$$

Let $y_k = {\alpha_k}^2$

$\Rightarrow 2\alpha_k d\alpha_k = dy_k$

$\Rightarrow \alpha_k d\alpha_k = \frac{dy_k}{2}$

$$\boldsymbol{erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-u^2)\, du}$$

$$= \int_0^\infty \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\gamma_s({\alpha_k}^2+C)}{R_c}}\, sin(\frac{\pi}{M})\right)\frac{\alpha_k}{\sigma_{\alpha_k}^2}\exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\sum_{n=0}^{\infty}\frac{\left(\left(\frac{\alpha_0\alpha_k}{2\sigma_{\alpha_k}^2}\right)^2\right)^n}{(n!)^2}\; d\alpha_k$$

$$= \sum_{k=0}^{N-1}\left[\frac{1}{2\sigma_{\alpha_k}^2}\int_0^\infty \frac{1}{N\beta}\; erfc\left(\sqrt{\frac{\gamma_s(y_k+C)}{R_c}}\, sin(\frac{\pi}{M})\right)\exp\left(-\frac{y_k+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2 y_k}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\; dy_k\right]$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\frac{1}{2\sigma_{\alpha_k}^2}\int_0^\infty\left[\int_{\sqrt{\frac{\gamma_s(y_k+C)}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\, du_k\right]{y_k}^n\exp\left(-\frac{y_k+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\; dy_k\right] \quad \text{... (G1.1)}$$

The region where $\sqrt{\frac{\gamma_s(y_k+C)}{R_c}}\, sin(\frac{\pi}{M}) < u_k < \infty$ for $0 < y_k < \infty$, fixing a value of u_k, the value of y_k varies from y_k=0 to $y_k = \frac{{u_k}^2 R_c}{\gamma_s sin^2(\frac{\pi}{M})}$; first slice is $u_k = \sqrt{\frac{\gamma_s C}{R_c}}\, sin(\frac{\pi}{M})$, last slice is u_k=∞. So, from equation (G1.1), we get,

$$\int_0^\infty BER_{SFBC-MPSK-OFDM}\; p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\frac{1}{2\sigma_{\alpha_k}^2}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\int_{\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\left[\int_0^{\frac{{u_k}^2 R_c}{\gamma_s sin^2(\frac{\pi}{M})}}{y_k}^n\exp\left(-\frac{y_k}{2\sigma_{\alpha_k}^2}\right)\, dy_k\right] du_k\right]$$

….. (G1.2)

We know, $\int x^n \exp(cx)\, dx = \exp(cx)\sum_{i=0}^{n}(-1)^i\;\frac{n!}{(n-i)!c^{i+1}}x^{n-i}$. So, from equation (G1.2), we get,

$$\int_0^\infty BER_{SFBC-MPSK-OFDM}\; p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\frac{1}{2\sigma_{\alpha_k}^2}\int_{\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\left[\exp\left(-\frac{y_k}{2\sigma_{\alpha_k}^2}\right)\sum_{i=0}^{n}(-1)^i\frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_{\alpha_k}^2}\right)^{i+1}}{y_k}^{n-i}\right]_0^{\frac{{u_k}^2 R_c}{\gamma_s sin^2(\frac{\pi}{M})}} du_k\right]$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\frac{1}{2\sigma_{\alpha_k}^2}\int_{\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\left[\begin{array}{c}\exp\left(-\frac{{u_k}^2 R_c}{2\sigma_{\alpha_k}^2\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)\sum_{i=0}^{n}(-1)^i\frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_{\alpha_k}^2}\right)^{i+1}}\left(\frac{{u_k}^2 R_c}{\gamma_s sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i} \\ -\exp(0)\,.\,(-1)^n\frac{n!}{(n-n)!\left(-\frac{1}{2\sigma_{\alpha_k}^2}\right)^{n+1}}\,.\,(0)^{n-n}\end{array}\right] du_k\right]$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[\begin{array}{c}-\int_{\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\exp\left(-\frac{{u_k}^2R_c}{2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}\right)\sum_{i=0}^{n}\frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i}({u_k}^2)^{n-i}\left(\frac{R_c}{\gamma_s\sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}du_k\\ +\int_{\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\left(2\sigma_{\alpha_k}^2\right)^n du_k\end{array}\right]\right]$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[\begin{array}{c}-\sum_{i=0}^{n}\frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i}\left(\frac{R_c}{\gamma_s\sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\int_{\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})}^{\infty}\exp\left\{-{u_k}^2\left(1+\frac{R_c}{2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}\right)\right\}({u_k}^2)^{n-i}du_k\\ +\left(2\sigma_{\alpha_k}^2\right)^n\int_{\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\,du_k\end{array}\right]\right]$$

….. (G1.3)

We know, $\int \exp(-cx^2)\,dx = \frac{\pi}{4c}\operatorname{erf}(\sqrt{c}\,x)$ & $\int x^n\exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}\operatorname{erf}(x)$ (where n=2k; k integer; a>0). So, from equation (G1.3), we get,

$$\int_0^{\infty} BER_{SFBC-MPSK-OFDM}\; p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[\begin{array}{c}-\sum_{i=0}^{n}\frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i}\left(\frac{R_c}{\gamma_s\sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\left[\frac{(2(n-i)-1)!!}{2^{n-i+1}\left\{\left(1+\frac{R_c}{2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}\right)\right\}^{n-i}}\sqrt{\frac{\pi}{\left\{\left(1+\frac{R_c}{2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}\right)\right\}}}\operatorname{erf}(u_k)\right]_{\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})}^{\infty}\\ +\left(2\sigma_{\alpha_k}^2\right)^n\left[\sqrt{\frac{\pi}{4.}}\operatorname{erf}(u_k)\right]_{\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})}^{\infty}\end{array}\right]\right]$$

$$= \frac{2}{\sqrt{\pi}}\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[\begin{array}{c}-\sum_{i=0}^{n}\frac{1}{2}\frac{(2(n-i)-1)!!}{2^{n-i}(n-i)!}\left(2\sigma_{\alpha_k}^2\right)^n\frac{(R_c)^{n-i}}{\left(R_c+2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)\right)^{n-i}}\sqrt{\frac{\pi.2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}{R_c+2\sigma_{\alpha_k}^2\gamma_b\sin^2\left(\frac{\pi}{M}\right)}}\left(1-\operatorname{erf}\left(\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})\right)\right)\\ +\left(2\sigma_{\alpha_k}^2\right)^n\left[\sqrt{\frac{\pi}{4}}(1)-\sqrt{\frac{\pi}{4}}\left(\operatorname{erf}\left(\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})\right)\right)\right]\end{array}\right]\right] \quad ..\text{(G1.4)}$$

[From Appendix B]

We know,

For an even positive integer $n = 2k$, $k \geq 0$, the double factorial may be expressed as $n!! = 2^k k!$

For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!!}{2^k k!}$. So, from equation (G1.4), we get,

$$\int_0^{\infty} BER_{SFBC-MPSK-OFDM}\; p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}$$

$$= \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\left(\frac{{\alpha_0}^2}{2\sigma_{\alpha_k}^2}\right)^n}{n!}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2}\frac{(R_c)^{n-i}}{\left(R_c+2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)\right)^{n-i}}\sqrt{\frac{2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}{R_c+2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}}\right]\left(1-\operatorname{erf}\left(\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})\right)\right)\right]$$

…... (G1.5)

Therefore, from equation (G1.5) & (D3.2), we get,

$$\boldsymbol{\int_0^{\infty} BER_{SFBC-MPSK-OFDM}\; p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}}$$

$$= \boldsymbol{\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{\infty}\frac{\overline{\gamma_{s_k}}}{2\sigma_{\alpha_k}^2\gamma_s(n+1)}\left[1-\sum_{i=0}^{n}\frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2}\left(\frac{R_c}{R_c+2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}\right)^{n-i}\sqrt{\frac{2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}{R_c+2\sigma_{\alpha_k}^2\gamma_s\sin^2\left(\frac{\pi}{M}\right)}}\right]\operatorname{erfc}\left(\sqrt{\frac{\gamma_s C}{R_c}}\sin(\frac{\pi}{M})\right)\right]}$$

…... (G1.6)

Appendix-G2

Average BER using SFBC (j=1, i=1) with MQAM-OFDM in Rician fading Channel

$$\int_0^\infty BER_{SFBC-MQAM-OFDM}\, p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}$$

Let, $y_k = {\alpha_k}^2$

$=> 2\alpha_k d\alpha_k = dy$

$=> \alpha_k d\alpha_k = \frac{dy}{2}$

$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-u^2)\, du$

$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\frac{\sqrt{1.5\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T} {\alpha_{k\,i,j}}^2}}{R_c(M-1)}\right) p\left(\alpha_{k_{1,1}}\right)_{Rician}\; d\alpha_k$$

$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\frac{\sqrt{1.5\gamma_s\left({\alpha_{k_{1,1}}}^2+C\right)}}{R_c(M-1)}\right)\frac{\alpha_k}{\sigma_{\alpha_k}^2}\exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) I_0\left(\frac{\alpha_0\alpha_k}{\sigma_{\alpha_k}^2}\right) d\alpha_k$$

$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s({\alpha_k}^2+C)}{R_c(M-1)}}\right)\frac{\alpha}{\sigma_{\alpha_k}^2}\exp\left(-\frac{{\alpha_k}^2+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\sum_{n=0}^\infty \frac{\left(\left(\frac{\alpha_0\alpha_k}{2\sigma_{\alpha_k}^2}\right)^2\right)^n}{(n!)^2}\, d\alpha_k$$

$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{1}{2\sigma_{\alpha_k}^2} erfc\left(\sqrt{\frac{1.5\gamma_s(y_k+C)}{R_c(M-1)}}\right)\exp\left(-\frac{y_k+\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\sum_{n=0}^\infty\frac{\left(\frac{{\alpha_0}^2 y_k}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\; dy_k$$

$$= \frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^\infty\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\frac{1}{2\sigma_{\alpha_k}^2}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\int_0^\infty\left[\int_{\sqrt{\frac{1.5\gamma_s(y_k+C)}{R_c(M-1)}}}^\infty \exp(-{u_k}^2)\, du_k\right]{y_k}^n\exp\left(-\frac{y_k}{2\sigma_{\alpha_k}^2}\right)\; dy_k\right] \quad ..(G2.1)$$

After Drawing the region where $\sqrt{\frac{1.5\gamma_s(y_k+C)}{R_c(M-1)}} < u_k < \infty$ for $0 < y_k < \infty$, fixing a value of u_k, the value of y varies from y_k=0 to $y_k = \frac{{u_k}^2 R_c(M-1)}{1.5\gamma_s}$; first slice is $u_k = \sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}$, last slice is u_k =∞.

So, from equation (G2.1), we get,

$$\int_0^\infty BER_{SFBC-MQAM-OFDM}\, p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}$$

$$= \frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^\infty\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\frac{1}{2\sigma_{\alpha_k}^2}\int_{\frac{1.5\gamma_s C}{R_c(M-1)}}^\infty \exp(-{u_k}^2)\left[\int_0^{\frac{{u_k}^2 R_c(M-1)}{1.5\gamma_s}}\exp\left(-\frac{y_k}{2\sigma_{\alpha_k}^2}\right){y_k}^n\; dy_k\right] du_k\right] \quad ..(G2.2)$$

We know, $\int x^n \exp(cx)\, dx = \exp(cx)\sum_{i=0}^n (-1)^i \frac{n!}{(n-i)!c^{i+1}}x^{n-i}$. So, from equation (G2.2), we get,

$$\int_0^\infty BER_{SFBC-MQAM-OFDM}\, p(\alpha_{1,1})_{Rician}\, d\alpha_{1,1}$$

$$= \frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^\infty\frac{\left(\frac{{\alpha_0}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\frac{1}{2\sigma_{\alpha_k}^2}\int_{\frac{1.5\gamma_s C}{R_c(M-1)}}^\infty \exp(-{u_k}^2)\left[\exp\left(-\frac{y}{2\sigma_{\alpha_k}^2}\right)\sum_{i=0}^n(-1)^i\frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_{\alpha_k}^2}\right)^{i+1}}{y_k}^{n-i}\right]_0^{\frac{{u_k}^2 R_c(M-1)}{1.5\gamma_s}} du_k\right]$$

$$= \frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{\beta}\sum_{k=0}^{N-1}\left[\sum_{n=0}^\infty\frac{\left(\frac{{\alpha_0}^2}{4\left(\sigma_{\alpha_k}^2\right)^2}\right)^n}{(n!)^2}\exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right)\frac{1}{2\sigma_{\alpha_k}^2}\int_{\frac{1.5\gamma_s C}{R_c(M-1)}}^\infty \exp(-{u_k}^2)\left[\begin{array}{c} exp\left(-\frac{{u_k}^2 R_c(M-1)}{2\sigma_{\alpha_k}^2 .1.5\gamma_s}\right)\sum_{i=0}^n(-1)^i\frac{n!}{(n-i)!\left(-\frac{1}{2\sigma_{\alpha_k}^2}\right)^{i+1}}\left(\frac{{u_k}^2(M-1)}{1.5\gamma_s}\right)^{n-i} \\ -\exp(0).(-1)^n\frac{n!}{(n-n)!\left(-\frac{1}{2\sigma_{\alpha_k}^2}\right)^{n+1}}.(0)^{n-n}\end{array}\right] du_k\right]$$

$$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \left[\begin{array}{c} -\int_{\frac{1.5\gamma_s C}{R_c(M-1)}}^{\infty} \exp(-\mathrm{u_k}^2) \exp\left(-\frac{u_k{}^2 R_c(M-1)}{3\sigma_{\alpha_k}^2 \gamma_s}\right) \sum_{i=0}^{n} \frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i} (\mathrm{u_k}^2)^{n-i} \left(\frac{(M-1)}{1.5\gamma_s}\right)^{n-i} d\mathrm{u_k} \\ + \int_{\frac{1.5\gamma_s C}{R_c(M-1)}}^{\infty} \exp(-\mathrm{u_k}^2) \ \left(2\sigma_{\alpha_k}^2\right)^n d\mathrm{u_k} \end{array} \right]$$

$$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \left[\begin{array}{c} \sum_{i=0}^{n} - \frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i} \left(\frac{R_c(M-1)}{1.5\gamma_s}\right)^{n-i} \int_{\frac{1.5\gamma_s C}{R_c(M-1)}}^{\infty} \exp\left\{-\mathrm{u_k}^2 \left(1+\frac{R_c(M-1)}{3\sigma_{\alpha_k}^2\gamma_s}\right)\right\} (\mathrm{u_k}^2)^{\,n-i} d\mathrm{u_k} \\ + \left(2\sigma_{\alpha_k}^2\right)^n \int_{\frac{1.5\gamma_s C}{R_c(M-1)}}^{\infty} \exp(-\mathrm{u_k}^2) \ \ d\mathrm{u_k} \end{array} \right]$$

……..(G2.3)

Now, $\int \exp(-cx^2)\,dx = \sqrt{\frac{\pi}{4c}}\ \mathrm{erf}(\sqrt{c}\,x)$ & $\int_0^{\infty} x^n \exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}\ \mathrm{erf(x)}$ (where n=2k; k integer; a>0). So, from equation (G2.3), we get,

$$\int_0^{\infty} BER_{SFBC-MQAM-OFDM}\ p\left(\alpha_{1,1}\right)_{Rician}\ d\alpha_{1,1}$$

$$= \frac{2}{\sqrt{\pi}} \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \left[\begin{array}{c} -\sum_{i=0}^{n} \frac{1}{(n-i)!\left(\frac{1}{2\sigma_{\alpha_k}^2}\right)^i} \left(\frac{(M-1)}{1.5\gamma_s}\right)^{n-i} \left[\frac{(2(n-i)-1)!!}{2^{n-i+1}\left\{\left(1+\frac{R_c(M-1)}{3\sigma_{\alpha_k}^2\gamma_s}\right)\right\}^{n-i}} \sqrt{\frac{\pi}{\left\{\left(1+\frac{R_c(M-1)}{3\sigma_{\alpha_k}^2\gamma_s}\right)\right\}}}\, erf(\mathrm{u_k}) \right]_{\frac{1.5\gamma_s C}{R_c(M-1)}}^{\infty} \\ + \left(2\sigma_{\alpha_k}^2\right)^n \left[\sqrt{\frac{\pi}{4}}\mathrm{erf}(\mathrm{u_k}) \right]_{\frac{1.5\gamma_s C}{R_c(M-1)}}^{\infty} \end{array} \right] \right]$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{\left(2\sigma_{\alpha_k}^2\right)^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha}^2}\right) \left[\begin{array}{c} -\sum_{i=0}^{n} \frac{(2(n-i)-1)!!}{(n-i)!} \frac{\left(2\sigma_\alpha^2\right)^n \left(R_c(M-1)\right)^{n-i}}{2^{n-i}\left(R_c(M-1)+3\sigma_\alpha^2\gamma_s\right)^{n-i}} \sqrt{\frac{3\sigma_\alpha^2\gamma_s}{R_c(M-1)+3\sigma_\alpha^2\gamma_s}} \left(1-\mathrm{erf}\left(\frac{1.5\gamma_s C}{R_c(M-1)}\right)\right) \\ + \left(2\sigma_\alpha^2\right)^n \left[1 - erf\left(\frac{1.5\gamma_s C}{R_c(M-1)}\right)\right] \end{array} \right] \right] \quad .. \text{(G2.4)}$$

[Appendix B]

We know,

For an even positive integer $n = 2k$, $k \geq 0$, the double factorial may be expressed as $n!! = 2^k k!$

For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!!}{2^k k!}$.

So, from equation (G2.4), we get,

$$\int_0^{\infty} BER_{SFBC-MQAM-OFDM}\ p\left(\alpha_{1,1}\right)_{Rician}\ d\alpha_{1,1}$$

$$= \frac{2\left(1-\frac{1}{\sqrt{M}}\right)}{N\beta} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_0{}^2}{2\sigma_{\alpha_k}^2{}^2}\right)^n}{n!} \exp\left(-\frac{\alpha_0^2}{2\sigma_{\alpha_k}^2}\right) \left[1 - \sum_{i=0}^{n} \frac{(2(n-i))!}{2^{2(n-i)}\left((n-i)!\right)^2} \left(\frac{R_c(M-1)}{R_c(M-1)+3\sigma_{\alpha_k}^2\gamma_s}\right)^{n-i} \sqrt{\frac{3\sigma_{\alpha_k}^2\gamma_s}{R_c(M-1)+3\sigma_{\alpha_k}^2\gamma_s}} \right] erfc\left(\frac{1.5\gamma_s C}{R_c(M-1)}\right) \right]$$

……. (G2.5)

Therefore, from equation (G2.5) & (D3.2), we get,

$$\int_0^{\infty} \boldsymbol{BER_{SFBC-MQAM-OFDM}\ p\left(\alpha_{1,1}\right)_{Rician} d\alpha_{1,1}}$$

$$= \frac{\mathbf{2(1-\frac{1}{\sqrt{M}})}}{\boldsymbol{N\beta}} \sum_{\boldsymbol{k=0}}^{\boldsymbol{N-1}} \left[\sum_{\boldsymbol{n=0}}^{\infty} \frac{\overline{\boldsymbol{\gamma_{s_k}}}}{\boldsymbol{2\sigma_{\alpha_k}^2\gamma_s(n+1)}} \left[\mathbf{1} - \sum_{\boldsymbol{i=0}}^{\boldsymbol{n}} \frac{\boldsymbol{(2(n-i))!}}{\boldsymbol{2^{2(n-i)}\left((n-i)!\right)^2}} \left(\frac{\boldsymbol{R_c(M-1)}}{\boldsymbol{R_c(M-1)+3\sigma_{\alpha_k}^2\gamma_s}}\right)^{\boldsymbol{n-i}} \sqrt{\frac{\boldsymbol{3\sigma_{\alpha_k}^2\gamma_s}}{\boldsymbol{R_c(M-1)+3\sigma_{\alpha_k}^2\gamma_s}}} \right] \boldsymbol{erfc}\left(\frac{\mathbf{1.5\gamma_s C}}{\boldsymbol{R_c(M-1)}}\right) \right]$$

……... (G2.6)

Appendix-G3

Average BER using SFBC (j=1, i=1)with MPSK-OFDM in Nakagami-m fading Channel

$$\int_0^\infty BER_{SFBC-MPSK-OFDM}\, p(\alpha_{1,1})_{Nakagami}\; d\alpha_{1,1}$$

$$=\int_0^\infty \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T}{\alpha_{k_{i,j}}}^2}{R_c}}\, sin(\frac{\pi}{M})\right) p(\alpha_{1,1})_{Nakagami}\; d\alpha_{1,1}$$

$$=\int_0^\infty \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s\left({\alpha_{k_{1,1}}}^2+C\right)}{R_c(M-1)}}\, sin(\frac{\pi}{M})\right) p\left(\alpha_{k_{1,1}}\right)_{Nakagami}\; d\alpha_{k_{1,1}}$$

$$=\int_0^\infty \frac{1}{N\beta}\sum_{k=0}^{N-1}\left[erfc\left(\sqrt{\frac{\gamma_s({\alpha_k}^2+C)}{R_c}}\, sin(\frac{\pi}{M})\right)\frac{2m^m}{\Gamma(m){\Omega_k}^m}{\alpha_k}^{2m-1}\exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right)d\alpha_k\right]$$

$$=\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{2m^m}{2\Gamma(m){\Omega_k}^m}\int_0^\infty erfc\left(\sqrt{\frac{\gamma_s(y_k+C)}{R_c}}\, sin(\frac{\pi}{M})\right){y_k}^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right)dy_k\right]$$

$$=\frac{1}{N\beta}\left[\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_0^\infty\left[\int_{\sqrt{\frac{\gamma_s(y_k+C)}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)du_k\right]{y_k}^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right)dy_k\right] \quad \text{... (G3.1)}$$

Let $y=\alpha_k{}^2$

$=> 2\alpha_k d\alpha_k = dy_k$

$=> \alpha_k d\alpha_k = \frac{dy_k}{2}$

$\boldsymbol{erfc(x)} = \frac{\mathbf{2}}{\sqrt{\boldsymbol{\pi}}}\int_x^\infty \mathbf{exp}(-\boldsymbol{u}^2)\,\boldsymbol{du}$

The region where $\sqrt{\frac{\gamma_s(y_k+C)}{R_c}}\, sin(\frac{\pi}{M}) < u_k < \infty$ for $0 < y_k < \infty$, fixing a value of uk, the value of y_k varies from y_k=0 to $y_k = \frac{{u_k}^2R_c}{\gamma_s sin^2(\frac{\pi}{M})}$; first slice is $u_k = \sqrt{\frac{\gamma_s C}{R_c}}\, sin(\frac{\pi}{M})$, last slice is u_k=∞. So, from equation (G3.1), we get,

$$\int_0^\infty BER_{SFBC-MPSK-OFDM}(\alpha_{1,1})\, p(\alpha_{1,1})_{Nakagami}\; d\alpha_{1,1}$$

$$=\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_{\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\left[\int_0^{\frac{{u_k}^2R_c}{\gamma_s sin^2(\frac{\pi}{M})}}{y_k}^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right)dy_k\right]du_k\right] \quad \text{.......... (G3.2)}$$

We know, $\int x^n \exp(cx)\,dx = \exp(cx)\sum_{i=0}^{n}(-1)^i\;\frac{n!}{(n-i)!c^{i+1}}x^{n-i}$. So, from equation (G3.2), we get,

$$\int_0^\infty BER_{SFBC-MPSK-OFDM}(\alpha_{1,1})\, p(\alpha_{1,1})_{Nakagami}\; d\alpha_{1,1}$$

$$=\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{2}{\sqrt{\pi}}\int_{\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\left[\exp\left(-\frac{m}{\Omega_k}y_k\right)\sum_{i=0}^{m-1}(-1)^i\;\frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega_k}\right)^{i+1}}{y_k}^{m-1-i}\right]_0^{\frac{{u_k}^2R_c}{\gamma_s sin^2(\frac{\pi}{M})}}du_k\right]$$

$$=\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_{\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-{u_k}^2)\left[\begin{array}{c}\exp\left(-\frac{m}{\Omega_k}\left(\frac{{u_k}^2R_c}{\gamma_s sin^2(\frac{\pi}{M})}\right)\right)\sum_{i=0}^{m-1}(-1)^i\frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega_k}\right)^{i+1}}\left(\frac{{u_k}^2R_c}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\\ -\exp(0)(-1)^{m-1}\frac{m-1!}{(0)!\left(-\frac{m}{\Omega_k}\right)^{m-1+1}}(0)^{m-1-m+1}\end{array}\right]du_k\right]$$

.... (G3.3)

Now, from the equation (G3.3) & (D4.2), we get,

$$=\frac{1}{N\beta}\frac{m^m}{\Gamma(m)\Omega_k{}^m}\frac{2}{\sqrt{\pi}}\left[\begin{array}{c}-\sum_{i=0}^{m-1}\left(\frac{R_c}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}\int_{\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-u_k{}^2)\left(1+\frac{mR_c}{\overline{\gamma_s}_k sin^2(\frac{\pi}{M})}\right)(u_k{}^2)^{m-1-i}du_k\\ +\int_{\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})}^{\infty}\exp(-u_k{}^2)\frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}du_k\end{array}\right] \text{..........} \quad (G3.4)$$

Now, $\int\exp(-cx^2)\,dx=\sqrt{\frac{\pi}{4c}}\,\mathrm{erf}(\sqrt{c}\,x)$ & $\int x^n\exp(-ax^2)dx=\frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}\,\mathrm{erf}(x)$ (where n=2k; k integer; a>0). So, from the equation (G3.4), we get,

$$\int_0^{\infty}BER_{SFBC-MPSK-OFDM}\;p(\alpha_{1,1})_{Nakagami}\;d\alpha_{1,1}$$

$$=\frac{1}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k{}^m}\frac{2}{\sqrt{\pi}}\left[\begin{array}{c}\left[-\sum_{i=0}^{m-1}\left(\frac{R_c}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}\frac{(2(m-1-i)-1)!!}{2^{m-1-i+1}\left[\left(1+\frac{mR_c}{\overline{\gamma_s}_k sin^2(\frac{\pi}{M})}\right)\right]^{m-1-i}}\sqrt{\frac{\pi}{\left(1+\frac{mR_c}{\overline{\gamma_s}_k sin^2(\frac{\pi}{M})}\right)}}\,\mathrm{erf}(u_k)\right]_{\left(\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})\right)}^{\infty}\\ +\left[\frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}\left[\sqrt{\frac{\pi}{4}}\,\mathrm{erf}(u_k)\right]\right]_{\left(\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})\right)}^{\infty}\end{array}\right]$$

$$=\frac{1}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m)\Omega_k{}^m}\left[\begin{array}{c}\left[-\sum_{i=0}^{m-1}\left(\frac{R_c}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{m-1!}{(m-1-i)!\left(\frac{m}{\Omega_k}\right)^{i+1}}\frac{(2(m-1-i)-1)!!}{2^{m-1-i}\left[\left(1+\frac{mR_c}{\overline{\gamma_s}_k sin^2(\frac{\pi}{M})}\right)\right]^{m-1-i}}\sqrt{\frac{1}{\left(1+\frac{mR_c}{\overline{\gamma_s}_k sin^2(\frac{\pi}{M})}\right)}}\left(1-erf\left(\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})\right)\right)\right]\\ +\frac{m-1!}{\left(\frac{m}{\Omega_k}\right)^m}\left[1-erf\left(\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})\right)\right]\end{array}\right] \text{...} \quad (G3.5)$$

[From Appendix B]

We know,
For an even positive integer $n = 2k$, $k \geq 0$, the double factorial is expressed as $n!! = 2^k k!$
For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!!}{2^k k!}$
So, from the equation (G3.5), we get,

$$\int_0^{\infty}BER_{SFBC-MPSK-OFDM}\;p(\alpha_{1,1})_{Nakagami}\;d\alpha_{1,1}$$

$$=\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}\left(\frac{m}{\Omega_k}\right)^{m-1-i}\left(\frac{R_c}{\gamma_s sin^2(\frac{\pi}{M})}\right)^{m-1-i}\frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2}\left(\frac{\overline{\gamma_s}_k sin^2(\frac{\pi}{M})}{\left(mR_c+\overline{\gamma_s}_k sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i}\sqrt{\frac{\overline{\gamma_s}_k sin^2(\frac{\pi}{M})}{\left(mR_c+\overline{\gamma_s}_k sin^2(\frac{\pi}{M})\right)}}\right]erfc\left(\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})\right)$$

Therefore,

$$\boldsymbol{\int_0^{\infty}BER_{SFBC-MPSK-OFDM}\;p(\alpha_{1,1})_{Nakagami}\,d\alpha_{1,1}}$$

$$\boldsymbol{=\frac{1}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}\frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2}\left(\frac{mR_c}{\left(mR_c+\overline{\gamma_s}_k sin^2(\frac{\pi}{M})\right)}\right)^{m-1-i}\sqrt{\frac{\overline{\gamma_s}_k sin^2(\frac{\pi}{M})}{\left(mR_c+\overline{\gamma_s}_k sin^2(\frac{\pi}{M})\right)}}\right]erfc\left(\sqrt{\frac{\gamma_s C}{R_c}}sin(\frac{\pi}{M})\right)} \text{......} \quad (G3.6)$$

Appendix-G4

Average BER using SFBC (j=1, i=1) with MQAM-OFDM in Nakagami-m fading Channel

Let $y = {\alpha_k}^2$

$=> \ 2\alpha_k \mathrm{d}\alpha_k = \mathrm{dy_k}$

$=> \ \alpha_k \mathrm{d}\alpha_k = \frac{\mathrm{dy_k}}{2}$

$\boldsymbol{erfc(x)} = \frac{\mathbf{2}}{\sqrt{\boldsymbol{\pi}}}\int_x^\infty \mathbf{exp}(-\boldsymbol{u}^2)\,\boldsymbol{du}$

$$\int_0^\infty BER_{SFBC-MQAM-OFDM}\; p(\alpha_{1,1})_{Nakagami}\; d\alpha_{1,1}$$

$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\frac{\sqrt{1.5\gamma_s \sum_{j=1}^{M_R}\sum_{i=1}^{M_T} {\alpha_{k_{i,j}}}^2}}{R_c(M-1)}\right) p\left(\alpha_{k_{1,1}}\right)_{Nakagami}\; d\alpha_{k_{1,1}}$$

$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\frac{\sqrt{1.5\gamma_s\left({\alpha_{k_{1,1}}}^2+C\right)}}{R_c(M-1)}\right) p\left(\alpha_{k_{1,1}}\right)_{Nakagami}\; d\alpha_{k_{1,1}}$$

$$= \int_0^\infty \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s({\alpha_k}^2+\mathrm{C})}{R_c(M-1)}}\right)\frac{2m^m}{\Gamma(m){\Omega_k}^m}{\alpha_k}^{2m-1}\exp\left(-\frac{m}{\Omega_k}{\alpha_k}^2\right) d\alpha_k$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m){\Omega_k}^m}\int_0^\infty erfc\left(\sqrt{\frac{1.5\gamma_s(y_k+C)}{R_c(M-1)}}\right){y_k}^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right) dy_k$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_0^\infty\left[\int_{\sqrt{\frac{1.5\gamma_s(y_k+C)}{R_c(M-1)}}}^\infty \exp(-{u_k}^2)du_k\right]{y_k}^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right) dy_k \quad \text{.............. (G4.1)}$$

After Drawing the region where $\sqrt{\frac{1.5\gamma_s(y_k+\mathrm{C})}{R_c(M-1)}} < u_k < \infty$ for $0 < y_k < \infty$, fixing a value of u, the value of y varies from y_k=0 to $y_k = \frac{{u_k}^2 R_c(M-1)}{1.5\gamma_s}$; first slice is $u_k = \sqrt{\frac{1.5\gamma_s\mathrm{C}}{R_c(M-1)}}$, last slice is u_k =∞. So, from equation (G4.1), we get,

$$\int_0^\infty BER_{SFBC-MQAM-OFDM}\; p(\alpha_{1,1})_{Nakagami}\; d\alpha_{1,1}$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_{\sqrt{\frac{1.5\gamma_s\mathrm{C}}{R_c(M-1)}}}^\infty \exp(-{u_k}^2)\left[\int_0^{\frac{{u_k}^2R_c(M-1)}{1.5\gamma_s}} {y_k}^{m-1}\exp\left(-\frac{m}{\Omega_k}y_k\right)dy_k\right]du_k$$

.... (G4.2)

We know, $\int x^n \exp(cx)\,dx = \exp(cx)\sum_{i=0}^{n}(-1)^i\frac{n!}{(n-i)!c^{i+1}}x^{n-i}$. So, from equation (G4.2), we get,

$$\int_0^\infty BER_{SFBC-MQAM-OFDM}\; p(\alpha_{1,1})_{Nakagami}\; d\alpha_{1,1}$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_{\sqrt{\frac{1.5\gamma_s\mathrm{C}}{R_c(M-1)}}}^\infty \exp(-{u_k}^2)\left[\exp\left(-\frac{m}{\Omega_k}y_k\right)\sum_{i=0}^{m-1}(-1)^i\frac{(m-1)!}{(m-1-i)!\left(-\frac{m}{\Omega_k}\right)^{i+1}}{y_k}^{m-1-i}\right]_0^{\frac{{u_k}^2R_c(M-1)}{1.5\gamma_s}} du_k$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{m^m}{\Gamma(m){\Omega_k}^m}\frac{2}{\sqrt{\pi}}\int_{\sqrt{\frac{1.5\gamma_s\mathrm{C}}{R_c(M-1)}}}^\infty \exp(-{u_k}^2)\left[\begin{array}{c}\exp\left(-\frac{m}{\Omega_k}\left(\frac{{u_k}^2R_c(M-1)}{1.5\gamma_s}\right)\right)\sum_{i=0}^{m-1}(-1)^i\frac{m-1!}{(m-1-i)!\left(-\frac{m}{\Omega_k}\right)^{i+1}}\left(\frac{{u_k}^2R_c(M-1)}{1.5\gamma_s}\right)^{m-1-i}\\ -\exp(0)(-1)^{m-1}\frac{(m-1)!}{(0)!\left(-\frac{m}{\Omega_k}\right)^{m-1+1}}(0)^{m-1-m+1}\end{array}\right]du_k \quad \text{............ (G4.3)}$$

Now, from equation (G4.3) & (D4.2), we get,

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\frac{2}{\sqrt{\pi}}\int_{\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}}^{\infty}\exp(-u_k{}^2)\left[\begin{matrix}-\exp\left(-u_k{}^2\left(1+\frac{mR_c(M-1)}{1.5\,\overline{\gamma_{s_k}}}\right)\right)\sum_{i=0}^{m-1}\frac{1}{(m-1-i)!}\left(\frac{R_c(M-1)}{1.5\gamma_s}\right)^{m-1-i}\left(\frac{m}{\Omega_k}\right)^{m-1-i}(u_k{}^2)^{m-1-i}\\+1\end{matrix}\right]du_k$$

$$= \frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[\begin{matrix}-\sum_{i=0}^{m-1}\left(\frac{mR_c(M-1)}{1.5\overline{\gamma_{s_k}}}\right)^{m-1-i}\frac{1}{(m-1-i)!}\int_{\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}}^{\infty}(u_k{}^2)^{m-1-i}\exp(-u_k{}^2)\left(1+\frac{mR_c(M-1)}{1.5\overline{\gamma_{s_k}}}\right)du_k\\+\int_{\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}}^{\infty}\exp(-u_k{}^2)\,du_k\end{matrix}\right] \quad \text{........ (G4.4)}$$

Now, $\int \exp(-cx^2)\,dx = \sqrt{\frac{\pi}{4c}}\,\mathrm{erf}(\sqrt{c}\,x)$ & $\int x^n \exp(-ax^2) = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}}\,\mathrm{erf}(x)$ (where n=2k; k integer; a>0).

$$= \frac{2}{\sqrt{\pi}}\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[\begin{matrix}\left[-\sum_{i=0}^{m-1}\left(\frac{mR_c(M-1)}{1.5\overline{\gamma_{s_k}}}\right)^{m-1-i}\frac{1}{(m-1-i)!}\frac{(2(m-1-i)-1)!!}{2^{m-1-i+1}\left[\left(1+\frac{mR_c(M-1)}{1.5\overline{\gamma_{s_k}}}\right)\right]^{m-1-i}}\sqrt{\frac{\pi}{\left(1+\frac{m(M-1)}{1.5\overline{\gamma_{s_k}}}\right)}}\,\mathrm{erf}(u_k)\right]_{\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}}^{\infty}\\+\left[\left[\sqrt{\frac{\pi}{4}}\,\mathrm{erf}(u_k)\right]_{\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}}^{\infty}\right.\end{matrix}\right]$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[\begin{matrix}\left[-\sum_{i=0}^{m-1}\left(\frac{mR_c(M-1)}{1.5\overline{\gamma_{s_k}}}\right)^{m-1-i}\frac{1}{(m-1-i)!}\frac{(2(m-1-i)-1)!!}{2^{m-i-1}\left[\left(1+\frac{mR_c(M-1))}{1.5\overline{\gamma_{s_k}}}\right)\right]^{m-1-i}}\sqrt{\frac{1}{\left(1+\frac{mR_c(M-1)}{1.5\overline{\gamma_{s_k}}}\right)}}\left[1-\mathrm{erf}\left(\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}\right)\right]\right]\\+\left[1-\mathrm{erf}\left(\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}\right)\right]\end{matrix}\right] \quad \text{.......... (G4.5)}$$

[Appendix B**]**

We know,
For an even positive integer n = 2k, k ≥ 0, the double factorial may be expressed as $n!! = 2^k k!$
For an odd positive integer $n = 2k - 1$, $k \geq 1$, it has the expressions $n!! = \frac{(2k)!!}{2^k k!}$. So, from equation (G4.5), we get,

$$\int_0^{\infty} BER_{SFBC-MQAM-OFDM}\; p(\alpha_{1,1})_{Nakagami}\; d\alpha_{1,1}$$

$$= \frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}\frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2}\left(\frac{mR_c(M-1)}{(mR_c(M-1)+1.5\overline{\gamma_{s_k}})}\right)^{m-1-i}\sqrt{\frac{1.5\overline{\gamma_{s_k}}}{(mR_c(M-1)+1.5\overline{\gamma_{s_k}})}}\right]erfc\left(\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}\right)$$

Therefore,

$$\int_0^{\infty} \boldsymbol{BER_{SFBC-MQAM-OFDM}\, p(\alpha_{1,1})_{Nakagami} d\alpha_{1,1}}$$

$$= \boldsymbol{\frac{2(1-\frac{1}{\sqrt{M}})}{N\beta}\sum_{k=0}^{N-1}\left[1-\sum_{i=0}^{m-1}\frac{(2(m-1-i))!}{2^{2(m-1-i)}\left((m-1-i)!\right)^2}\left(\frac{mR_c(M-1)}{(mR_c(M-1)+1.5\overline{\gamma_{s_k}})}\right)^{m-1-i}\sqrt{\frac{1.5\overline{\gamma_{s_k}}}{(mR_c(M-1)+1.5\overline{\gamma_{s_k}})}}\right]erfc\left(\sqrt{\frac{1.5\gamma_s C}{R_c(M-1)}}\right)}$$

.... (G4.6)