AN APPROXIMATE METHOD FOR THE ANALYSIS OF MULTICELLULAR HIGH RISE TUBULAR STRUCTURES OF ARBITRARY PLAN SHAPE

A Thesis

by

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ABSTRACT

A method for approximate lateral load analysis of high rise tubular structures of any plan shape and having any number of interconnected cells is presented in this thesis.

In this method the structure is idealized by replacing the spandrel beams with a continuous medium of equivalent stiffness. The computational effort is greatly reduced by taking the shear force in the continuous medium as the primary unknown.

A computer program in BASIC has been developed on the basis of the approximate theory and it has been used to analyse two tubular structures, one with four rectangular cells and the other with two hexagonal cells, under two kinds of lateral loads viz. point load at the top and uniformly distributed load. The results of the approximate method have been compared with those obtained from a finite element analysis. The forces predicted by the approximate method, in general, agree with the finite element analysis values. However, it cannot predict the lateral deflection correctly.
### NOTATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A_i$</td>
<td>cross-sectional area of $i$th wall</td>
</tr>
<tr>
<td>$A_{p,i}$</td>
<td>cross-sectional area of spandrel beams in $i$th band of openings</td>
</tr>
<tr>
<td>$b_i$</td>
<td>clear span of spandrel beams in $i$th band of openings</td>
</tr>
<tr>
<td>$b_{x,i}$</td>
<td>$x$ projection of $b_i$</td>
</tr>
<tr>
<td>$b_{y,i}$</td>
<td>$y$ projection of $b_i$</td>
</tr>
<tr>
<td>$d_4$</td>
<td>depth of spandrel beams in $i$th band of openings</td>
</tr>
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<td>$d_1$</td>
<td>width of the stiffer wall</td>
</tr>
<tr>
<td>$E$</td>
<td>modulus of elasticity</td>
</tr>
<tr>
<td>$F_i$</td>
<td>axial force in $i$th wall</td>
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<tr>
<td>$G$</td>
<td>shear modulus</td>
</tr>
<tr>
<td>$H$</td>
<td>height of the building</td>
</tr>
<tr>
<td>$h$</td>
<td>storey height</td>
</tr>
<tr>
<td>$I_{p,i}$</td>
<td>moment of inertia of spandrel beams in $i$th band of openings</td>
</tr>
<tr>
<td>$I_{c,i}$</td>
<td>reduced moment of inertia spandrel beams in $i$th band of openings</td>
</tr>
<tr>
<td>$I_{p,i}$</td>
<td>moment of inertia of spandrel beams in $i$th band of openings</td>
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<tr>
<td>$I_{x,i}$</td>
<td>moment of inertia of $i$th wall about $x$-axis</td>
</tr>
<tr>
<td>$I_{y,i}$</td>
<td>moment of inertia of $i$th wall about $y$-axis</td>
</tr>
<tr>
<td>$i,j,k$</td>
<td>integer variables</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$L_x,i$</td>
<td>x projection of the distance between centroidal axes of two neighbouring walls</td>
</tr>
<tr>
<td>$L_y,i$</td>
<td>y projection of the distance between centroidal axes of two neighbouring walls</td>
</tr>
<tr>
<td>$m$</td>
<td>total number of walls/columns per storey</td>
</tr>
<tr>
<td>$M_x,i$</td>
<td>bending moment about x-axis in $i$th wall</td>
</tr>
<tr>
<td>$M_y,i$</td>
<td>bending moment about y-axis in $j$th wall</td>
</tr>
<tr>
<td>$n$</td>
<td>total number of lines of openings</td>
</tr>
<tr>
<td>$P_x$</td>
<td>$x$ component of the point load applied at top</td>
</tr>
<tr>
<td>$P_y$</td>
<td>$y$ component of the point load applied at top</td>
</tr>
<tr>
<td>$q_i$</td>
<td>intensity of shear force in the connecting medium in $i$th band of openings</td>
</tr>
<tr>
<td>$S_x,i$</td>
<td>flexibility in $x$ direction of $i$th column in a single storey high frame segment</td>
</tr>
<tr>
<td>$S_y,i$</td>
<td>flexibility in $y$ direction of $i$th column in a single storey high frame segment</td>
</tr>
<tr>
<td>$T$</td>
<td>integral of shear force</td>
</tr>
<tr>
<td>$T'$</td>
<td>$dT/dz$; $T''=d^2T/dz^2$</td>
</tr>
<tr>
<td>$U$</td>
<td>total strain energy of tube</td>
</tr>
<tr>
<td>$U_1$</td>
<td>strain energy due to bending and shear deformation of beams</td>
</tr>
<tr>
<td>$U_2$</td>
<td>strain energy due to axial deformation of walls</td>
</tr>
<tr>
<td>$U_3$</td>
<td>strain energy due to bending of walls</td>
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<td>$w_x$</td>
<td>intensity of uniformly distributed horizontal load applied in the $x$-direction</td>
</tr>
<tr>
<td>$w_y$</td>
<td>intensity of uniformly distributed horizontal load applied in the $y$-direction</td>
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</table>
$y_x$  \hspace{1cm} \text{horizontal deflection in } x\text{-direction}

$y_y$  \hspace{1cm} \text{horizontal deflection in } y\text{-direction}

$z$  \hspace{1cm} \text{distance of any horizontal section from top of the structure}

$\nu$  \hspace{1cm} \text{Poisson's ratio}
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## ACKNOWLEDGEMENT

## ABSTRACT

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1.1 General

Dense population concentration, scarcity and high cost of real estates have accelerated the transition from low rise construction to high-rise construction. The modern trend in urban planning is to build high-rise buildings in developing cities, particularly for office buildings. Low-rise buildings are often replaced by taller blocks in the more developed areas. The economy, beauty, efficiency and above all the prestige associated with tall buildings have, in recent years, increased their rate of construction all over the world. The massing of high-rise buildings evolves out of the designer's interpretation of the environmental context and his response to the purpose of building. However, the tall buildings of future may very well be an integral part of one large building organism, the city where the building or activity cells are interconnected by multi-level movement systems. High-rise buildings range in height from below 10 to more than 100 stories.

It is only in the last 30 years that reinforced concrete has found increasing use in the construction of tall buildings. In its initial development in the early parts of the twentieth century, reinforced concrete buildings were
limited to only a few stories in height. The structural type used was the traditional beam-column frame system which made the construction of taller buildings relatively expensive. In the early fifties the introduction of shear walls opened up the possibility of using concrete in apartment and office buildings as high as thirty stories. Taller buildings remained economically unattractive because the shear walls, which were mostly used in the core of the building, were relatively small in dimension compared to the height of the building, leading to insufficient stiffness to resist lateral loads. It was obvious that the overall dimensions of the interior cores were too small to economically provide the stability and stiffness for buildings over thirty to forty stories.

The natural tendency then was to find new systems of structures that would utilize the perimeter configurations of such buildings rather than to rely on the core configurations alone. The development the spatial wall frame i.e., perforated wall structure known as rigid tube was, therefore a logical outcome of this challenge. The modifications of the rigid tube system into a tube-in-tube, framed tube and other variations are indeed known to offer certain advantages in planning, design and construction.

The rigid tube system usually relies on "hull-core" i.e. tube-in-tube type configuration for its basic layout: this has formed the structural backbone of almost all the tallest buildings constructed in recent years. The exterior enclosure tube or "hull" usually consists of closely spaced columns connected together with deep spandrel beams at each floor level to form a multi-storey multi-bay box frame. For apartment buildings this tube alone or the hull with cross-walls provides the necessary stiffness against lateral
loads. For office buildings, the exterior hull is usually combined with an internal service "core" through the floor system. The resulting "hull-core" system is extremely efficient in resisting all kinds of horizontal loads viz. winds, earthquakes or blasts.

1.2 Lateral Load Resisting System

The need to resist large lateral forces is one of the major distinguishing characteristics of tall buildings. The normal lateral loads are those due to wind and earthquake. The lateral load resisting system of a tall building must be able to resist these loads and at the same time must prevent excessive deflections or accelerations and must help to provide stability. However, the lateral load resisting elements must not be too large, and must conform with the architectural, structural and mechanical schemes, or vice versa. A lateral system is generally considered to be efficient if the provision for lateral load resistance does not increase the floor and column sizes beyond those required for gravity loads.

Although there are as many concepts of structural systems as there are designers, it is possible to classify these systems' into categories. Each category tends to be most efficient for a certain height range or a certain type of occupancy. The most common structural systems for lateral load resistance are listed below:

1. Bearing wall
2. Shear core
3. Rigid frame
1.3 Tubular System

The concept of tubular behaviour introduced by F.R. Khan is one of the recent philosophies in the design of tall buildings. Four out of the five tallest buildings in the world now (1988) have been built using this concept. These buildings are the John Hancock Building, the Sears Tower and the Standard Oil Building in Chicago and the World Trade Center in New York. The system is so efficient that in most cases the amount of structural material used per square foot of floor space is comparable to that used in conventional framed buildings half the height.

Tubular design is based on the assumption that the facade structure responds to lateral loads as a closed hollow box beam cantilevering out of the ground. Since the exterior walls resist all or most of the lateral load, costly interior diagonal bracing or shear walls are eliminated.

There are several forms of tubular structures, which may be classified as shown in Table 1.1. A few examples are shown in Fig 1.1.
Table 1.1: Classification of tubular structures (22)

Tubular Structures

- Hollow Tube
  - Framed Tube
  - Trussed Tube
    - Column-diagonal Trussed Tube
    - Lattice Trussed Tube

- Interior Braced Tube
  - Tube with Parallel Shear Walls
    - Tube in Tube
  - Modified Tube
    - Framed Tube with Rigid Frames
      - Tube in Semitube
    - Modular Tube
Fig 1.1: Various types of tubular building structures (22)
1.2.1 Hollow Tube

(a) Framed Tube

The framed tube, the earliest among the various tubular systems, was first used in 1961 in the 43-storey Dewitt Chestnut Apartment Building in Chicago. In this Vierendeel tube system the exterior walls of the building, consisting of a closely spaced rectangular grid of beams and columns rigidly connected together, resist lateral loads through cantilever tube action without using interior bracing. The interior columns are assumed to carry gravity loads and do not contribute to the exterior tube's stiffness (Fig 1.2). The stiff floors act as diaphragms with respect to distributing the lateral forces to the perimeter walls.

Other examples of hollow framed tube buildings are the 83-storey Standard Oil Building in Chicago and the 110-storey World Trade Center in New York. Although these buildings have interior cores, they act as hollow tubes because the cores are not designed to resist lateral loads.

It would be ideal in the design of framed tube system if the exterior walls were to act as a unit, responding to lateral loads in pure cantilever bending. If this were the case, all columns that make up the tube, analogous to the fibers of a beam, would be either in axial tension or in compression. The linear stress distribution that would result is indicated by broken lines in Fig 1.3.

The true behaviour of the tube lies somewhere between that of a pure cantilever and a pure frame. The sides of the
Fig 1.2: Framed hollow tube

Fig 1.3: Stress distribution in a framed tube
tube parallel to the wind tend to act as independent multibay rigid frames, given the flexibility of the spandrel beams. This flexibility results in wracking of the frame due to shear, called shear lag. The effect of shear lag on the tube action results in nonlinear pressure distribution along the column envelope; the columns at the corners of the building are forced to take a higher share of the load than the columns in between. Furthermore, the deflected shape of the building no longer resembles that of a cantilever beam, as shear mode deformation becomes more significant.

The shear lag problem severely affects the efficiency of tubular systems, and all later developments of tubular design attempt to overcome it. The framed tube principle seems to be economical for steel buildings up to 80 stories and concrete buildings up to 60 stories.

(b) Trussed Tube

The inherent weakness of the framed tube lies in the flexibility of its spandrel beams. Its rigidity is greatly improved by adding diagonal members. The shear is now primarily absorbed by the diagonals and not by the spandrels. The diagonals carry the lateral forces directly in predominantly axial action. This reduction of shear lag provides for nearly pure cantilever behaviour.

Column Diagonal Trussed Tube

This system uses diagonals within the rectangular grid of beams and columns. The diagonals with the spandrel beams create a wall-like rigidity against lateral loads (Fig 1.4).
Fig 1.4: Column diagonal trussed tube
Not only do the diagonals carry the major portion of lateral loads, they act as inclined columns supporting gravity loads, as well.

Normally the compression induced by gravity loads is not overcome by the tension caused by lateral loads. This dual function of the diagonal members makes this system rather efficient for very tall buildings (up to about 100 stories in steel). It allows much larger spacing of columns than the framed tube.

An essential characteristic of the system is its capability to distribute a concentrated load evenly through the entire structure.

An interesting approach to achieving diagonals in an exterior concrete wall is to fill the window openings in a diagonal pattern.

Lattice Trussed Tube

In this system the tube is made up of closely spaced diagonals with no vertical columns. The diagonals act as inclined columns, carry all gravity loads and stiffen the structure against lateral loads. The diagonals may be tied together by horizontal beams.

The diagonals are extremely efficient in responding to lateral loads, but they are less efficient than vertical columns in transmitting gravity loads to the ground. Furthermore, the large number of joints required between diagonals and the problems related to window details make the lattice truss system generally impractical.
Interior Braced Tube

The framed exterior tube may be stiffened in plane by adding diagonals, or it may be stiffened from within the building by adding shear walls or interior cores. Several approaches to interior bracing are discussed in the following paragraphs.

Tube with Parallel Shear Walls

The exterior tubular wall can be stiffened by incorporating interior shear walls into the plan. One can visualize the exterior tube walls as the flanges of a huge built-up system in which the shear walls represent the webs. The stresses in the exterior tube walls are primarily axial, since shear lag is minimized.

The examples in Figs 1.5 (a)&(b), respectively, illustrate two approaches: wide spacing of facade columns, requiring a shear wall for every column, and close spacing of facade columns, requiring only two shear walls.

Tube-in-Tube

The stiffness of a hollow tube system is very much improved by using the core not only for gravity loads but to resist lateral loads, as well. The floor structure ties the exterior and interior tubes together, and they respond as a unit to lateral forces.

The reaction of a tube-in-tube system to lateral loads is similar to that of a frame and shear wall structure.
Fig 1.5: Tube with parallel shear walls
However the framed exterior tube is much stiffer than a rigid frame.

Fig 1.6 indicates that the exterior tube resists most of the lateral load in the upper portion of the building, whereas the core carries most of the loads in the lower portion.

The tube-in-tube approach has been used in the 38-storey Brunswick Building in Chicago, and the 52-storey One Shell Plaza Building in Houston.

Taking the tube-in-tube concept one step further, the designers of a 60-storey office building in Tokyo used a triple tube (Fig. 1.7). In this system the exterior tube alone resists wind loads, but all three tubes, connected by the floor systems interact in resisting earthquake loads, a significant factor in Japan.

Modified Tube

Tubular action is most efficient in round and nearly square buildings. Buildings deviating from these forms present special structural considerations when tubular action is desired. The following two examples describe such conditions.

Framed Tube with Rigid Frames

The hexagonal shape of a 40-storey office building in Charlotte, North Carolina (Fig 1.8), forced the designers to modify the tubular principle. The pointed ends of this
Fig 1.6: Load distribution in a wall-frame structure

Fig 1.7: Triple tube (tube in tube)
hexagonal building exhibited excessive shear lag, making it impossible to get effective tubular response. Adding rigid frames in the transverse direction served to tie the exterior walls together. Thus the end walls in triangular arrangement were reinforced by rigid frames. By tying together the perimeter walls, effective tubular action was achieved.

Tube in Semitube

The irregular plan of the 32-storey Western Pennsylvania National Bank in Pittsburgh (Fig 1.9) gave rise to still another special solution of tubular design. In most tubular buildings, the tubular effect is generated by the exterior walls. In this building, however, the two intersecting octagons form a structural tube in the central part of the building.

The two end portions of the building are stiffened by channel-like wall frame systems. The lateral load is resisted by the combination of interior tube and the huge exterior end-wall channels.

Modular Tubes

One of the latest developments in tubular design is the modular or bundled tube principle. This system has been used for the Sears Tower in Chicago, currently the tallest building in the world.
Fig 1.8: Framed tube with rigid frames

Fig 1.9: Tube in semitube
The exterior framed tube is stiffened by interior cross diaphragms in both directions; an assemblage of cell tubes is formed. These individual tubes are independently strong, therefore may be bundled in any configuration and discontinued at any level. A further advantage of this bundled tube system lies in the extremely large floor areas that may be enclosed.

The interior diaphragms act as webs of a huge cantilever beam in resisting shear forces, thus minimizing shear lag. In addition, they contribute strength against bending.

The behaviour of this system is shown in the stress distribution diagram in Fig. 1.10. The diaphragms parallel to the wind (i.e. webs) absorb shear, thereby generating points of peak stress at points of intersection with perpendicular walls (i.e. flanges) indicating the individual action of each tube. The difference in axial stress distribution if there are no internal stiffeners that is, a single tube may be noted. The vertical diaphragms tend to distribute the axial stresses equally, although shear lag still occurs to some extent. However the deviation from ideal tubular behaviour, indicated by broken lines, does not seem to be very significant.

1.4 Objectives of the Present Work

The present study is aimed at
a) developing a method for approximate linear elastic analysis of multicellular high rise tubular structures of any plan shape under static lateral loads;
Fig 1.10: Behaviour of a modular tube
b) developing a computer programme on the basis of the method developed;

c) checking the accuracy of the approximate method by comparing the results of the approximate method with those obtained from an exact method of solution such as the finite element method; and

d) studying the shear lag effect in tubular structures of different aspect ratios and different stiffness factors.
CHAPTER TWO

AVAILABLE METHODS FOR
APPROXIMATE ANALYSIS OF TUBULAR STRUCTURES

2.1 Introduction

Structural engineers are always concerned with optimization to provide the most economic solution to any problem. But the process of optimization is an iterative one and for tall buildings which, in general, involve a large number of redundants requires considerable amount of time and money unless the number of iterative loops is kept to a minimum. The economy and in some cases, the true success of this iterative process depends to a considerable extent upon the correctness of the preliminary selection of member dimensions. The use of a dependable and informative approximate method of analysis is of great help in making the preliminary selection of members and this, in turn, enhances the possibility of rapid convergence.

Approximate methods have several uses in addition to providing a basis for preliminary selection of members. In the very early stage of design when the type of structure and basic dimensions are yet to be determined, approximate methods may prove useful in choosing the basic design that is best, or more likely to be best, from the standpoint of economy or other defined factors. Required sizes of members often may be calculated by approximate methods with
sufficient accuracy to provide relative figures of merit for the several designs which may be under consideration.

Finally, approximate methods provide values of forces, moments and deflections to serve as a check on the validity of more "exact" computer analyses which are in general so complex that error at any step is not unlikely.

2.2 Approximate Methods for Analysing Tubular Structures

A number of methods have been developed for approximate analysis of single celled tubular structures. Outlines of some of these methods are given in the following few paragraphs.

Recognizing the fact that under lateral loads the major interactions between the web frames and the flange frames of a rectangular tubular structure are the vertical shear forces at the corners, Coull and Subedi (9) have suggested that an approximate solution for forces in the various members of the tube may be obtained by analysing an equivalent plane frame. The equivalent plane frame is obtained by putting the orthogonal frames side by side and connecting them in series by fictitious linking member of such stiffnesses so as to allow only vertical forces to be transmitted between the frames. Since this method requires a two dimensional analysis the amount of computational effort is reduced considerably. Another advantage of this method is that a plane frame program with little modifications may be used to analyse tubular structures.

Rutenberg (21) has proposed a method quite similar to that of Coull and Subedi. In this method instead of using
fictitious links, fictitious beams of very short length with moment and thrust releases are used to transmit shear forces between orthogonal frames. This allows the use of standard plane frame programs without any modification to account for the shear transfer mechanisms.

Ast and Schwaighofer (1) have developed another plane frame approach which is quite efficient for large symmetric tube structures. It involves determining the interaction forces at a limited number of points on the junctions of the peripheral frames, and superimposing their effect on that of the horizontal external loading acting on the frames parallel to the external loading.

According to Khan (14) for a very preliminary analysis of the overall resistance, as well as the deflection of a tubular structure, the effective configuration of the tube may be reduced to two equivalent channels resisting the total overturning moments. Comparison of results of such an analysis with those obtained from exact analyses performed by a generalized computer program such as STRESS, STRUDL etc. has indicated that such an analysis generally gives conservative values of shear and moments.

Khan and Amin (18) have developed a semi-graphical semi-analytical solution method for analysing framed tubes of any dimension and of any height. On the basis of the results of computer analyses of a large number of framed tubes with various bending and shearing stiffness ratios of columns to beams they have developed a series of influence curves. From these curves the axial forces in the columns and shear forces in the spandrel beams of a tube of any height can be determined using a reduction modeling technique. The influence curves have been constructed taking the shear lag
effect into consideration and are quite useful for design engineers.

Ali Khan (17) has suggested a simplified method of analysis of tubular structures. The mathematical model for this method takes into account the effects of shear lag as well as of rigid joints. In this method the perforated tube is converted into an equivalent unperforated tube of undeformable cross sections with appropriate stiffness properties. The solid walled tube is analysed by the generalised energy principles and variational methods and then stresses and displacements of this equivalent tube are converted to the design stresses of the rigid tube structure using stress factors. This method is limited to structures having height to width ratio greater than two and with relatively deep members. Also the properties of the structure must be constant across its width and along its height. The accuracy of the method is lost appreciably when the ratios of beam depth to storey height and column depth to bay width are less than 0.25 (20).

Heiderbrecht and Stafford Smith (16) have, on the basis of the fact that the deformation of a tall wall-frame building structure is a combination of flexure and shear deformations, developed a few curves using non-dimensional parameters for evaluating stress resultants, interacting force and deflections. The mathematical model for these curves has been developed from the differential equations relating loads with deflections. For framed-tube structures these non-dimensional curves may be used assuming that the pair of outer walls parallel to the direction of lateral loading act as a frame whilst the outer walls normal to the loading act as flexural cantilevers.
A simple method for the analysis of large multistorey multibay frame work has been presented by Kinh et al (20). It is based on replacing the actual structure by an elastically equivalent orthotropic membrane which is then analysed by the finite element technique. The inflection points for the bottom storey column are assumed at 2/3 of the storey height from the base. The refined expressions for the equivalent elastic properties in combination with the versatility of the finite element technique make this method well adaptable to a wide range of tubular structures.

On the basis of the fact that in a rectangular tubular structure the lateral load is resisted primarily by (i) the rigidly jointed frame actions of the shear resisting panels parallel to the load, (ii) the axial deformations of the frame panels normal to the direction of the load and (iii) the axial forces in the discrete corner columns, Coull and Bose (7) have developed a simplified method for the analysis of tubular structures. This method consists of replacing the discrete structure by an equivalent orthotropic tube and obtaining stresses in the tube on the basis of a few simplifying assumptions. They have also developed a few curves which may be used in design offices for rapid assessment of stress and deflection.

Chang and Foutch (3) have presented an approximate method for the analysis of tube frames using an equivalent continuum. The idealized structure is first obtained by defining the material and geometric properties of the tube in terms of the properties of various substructures within the frame. Once the building is represented as a tube, it is analysed as a thin-walled tube. The tube model allows for shear lag in the flange as well as flexural and shear deformations. The governing differential equations are found
through the Minimum Potential Energy principle and from that approximate deflection of the model may be calculated. Although this method cannot be directly used for the calculation of forces and stresses for the final design of structural members, it does provide valuable information on the global behaviour of the structure.

2.3 Analysis of Bundled Tube Structures

The concept of bundled tube being a recent innovation, research on analysis of such structures is still at the initial stage. The problem with bundled tube structures is that a three dimensional analysis is a must, in no way it may be idealized as two dimensional structure.

A simplified method of obtaining closed form solutions for bundled tube structures has been developed by Coull et al (8). In this method the rigidly-jointed perimeter and interior web frame panels are replaced by equivalent orthotropic plates, whose properties are chosen to represent both the axial and shearing deformation characteristics of the frames. The force and stress distributions in the substitute panels are assumed to be represented with sufficient accuracy by polynomial series in the horizontal coordinates, the coefficients of the series being functions of the height only. The unknown functions are determined from the principle of least work. The influence of stiffer corner columns is included in the analysis. By incorporating simplifying assumptions regarding the form of stress distribution in the frame panels, the structural behaviour can be reduced to the solution of a single order differential equation.
2.4 Works on Shear Lag in Box Girders

Shear lag phenomenon, resulting in a nonuniform distribution of bending stresses across wide flanges of a beam cross-section has long been recognized. The analysis and design of box-beams with this special problem have also been investigated by aeronautical engineers. The pre and post World War II periods are especially marked for researches on box-like components of aircraft structure and so most of the significant papers on box beams were published during this time.

Ali Khan (17) has extensively studied the past research on shear-lag analysis of box beams. Many early works on shear-lag problem are referred to in this thesis. He used energy theorems and calculus of variation to present a general solution for bending and twisting of thin walled closed tubular structure. He assumed the spanwise displacements of a beam in the form of finite series incorporating the chordwise (transverse) displacements as some chosen and simple functions. A number of simultaneous differential equations are obtained which can be solved for stresses and displacements.

Foutch and Chang (15) have reported an interesting phenomenon associated with shear-lag in the flanges of box girders that is quite contrary to well established ideas concerning this subject. If a cantilever tube is loaded laterally under non-uniform shear, a reversal of the shear-lag distribution may occur at some point in the beam and the center-line stress may exceed the edge stress.
3.1 Introduction

In this chapter, a simplified method for analysis of tubular structures is presented. Choudhury (4) first developed the method for analysing plane shear walls with openings. Bari (2) extended the method to analyse single celled tubular structures having a rectangular plan shape. Later Zubair (24) modified the method slightly to use it for analysing single celled tubular structures of arbitrary plan shape. The method presented here is a further modification of the continuous medium method to analyse multicellular tubular structures having any plan shape. The method consists in replacing the spandrel beams by a continuous medium as shown in Figure 3.1, obtaining a set of simultaneous ordinary differential equations for the shear forces in the continuous medium by minimising the strain energy and solving the equations by using a weighted residual method. The primary advantage of this method is that the solution time is independent of the number of stories and depends only on the number of opening lines. This makes it possible to obtain approximate solutions very rapidly using a microcomputer.
Fig 3.1a: Idealization of a tubular structure - Actual structure.
Fig 3.1b: Idealization of a tubular structure — Beams replaced by continuous medium.
Fig 3.1c: Idealization of a tubular structure - Released structure.
3.2 Assumptions

Following are the assumptions made in developing the theory of the continuous medium method.

1) The connecting beams do not deform axially and hence the lateral deflection of the individual walls is the same at any level.

2) The ratios of the moment of inertia and cross-sectional area of the spandrel beam to the sum of half the heights of the stories on either side of the beam are constant throughout the height of the building.

3) The moment of inertia and cross-sectional area of the walls are constant throughout the height of the building.

4) The points of contraflexure in the connecting beams are at mid spans.

5) Engineer's Theory of Bending is valid i.e. plane sections of wall before bending remain plane after bending.

6) At any level the total moment to be carried by the walls is shared by the individual walls in proportion of their single storey stiffnesses as derived in Appendix A.

3.3 General Formulation

The general formulation technique follows that of Choudhury (4). Figure 3.2 shows a typical floor plan of a tubular structure, consisting of closely spaced walls/columns at arbitrary locations and interconnected by
Fig 3.2: Floor plan of a typical tubular structure
deep spandrel beams at floor levels. The discrete connecting beams of stiffness $EI_p$ are replaced by a continuous medium connecting the walls for the full height and having the same bending stiffness as the beams they replace, e.g. in an opening with storey height $h$ beams of stiffness $EI_p$ are replaced by continuous medium of stiffness $EI_p/h$. The structure is then released by introducing a cut along the line of contraflexure and the integral of distributed shear forces in the connecting medium,

$$ T = \int_{0}^{Z} q \, dz $$  \hspace{1cm} (3.1)$$

is taken as the redundant function (Fig.3.1c).

The governing equation for the function $T$ can now be derived from energy considerations (as originally suggested by Rosman).

Neglecting the effect of axial forces and assuming that the wall cross sections are rectangular and doubly symmetric, the strain energy due to bending of the continuous medium in the $i$th opening and having a height of $dz$ is

$$ U_{1,i} = 2 \int_{0}^{b/2} \left\{ \frac{(q_1 \, dz \, x)^2}{2 \, E \, (I_{p,i} \, dz / h_i)} + \frac{1.2 \, (q_1 \, dz)^2}{2 \, G \, (A_{p,i} \, dz / h_i)} \right\} \, dx $$

$$ \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{0.5cm} \hspace{
Substituting the value of $G = \frac{E}{(1 + \nu)}$ and simplifying,

$$U_{1,i} = \frac{q_{1}^{2} b_{1}^{3} h_{1}}{24 E I_{p,i}} \left( 1 + 2.4 \frac{d_{1}}{b_{1}} \right)^{2} \frac{1}{(1 + \nu)} dz$$

$$= \left( T'_{1} \right)^{2} \frac{b_{1}^{3} h_{1}}{24 E I_{c,i}} dz \quad (3.3)$$

where $I_{c,i} = \frac{1 + 2.4 \frac{d_{1}}{b_{1}} (1 + \nu)}{1 + 2.4 \frac{d_{1}}{b_{1}} (1 + \nu)}$

is the reduced moment of inertia. If a non-rectangular section is used for the spandrel beams the general expression for the reduced moment of inertia given in section 3.5 should be used.

The total strain energy due to flexure and shear in the continuous medium in all the bands of openings is

$$U_{1} = \sum_{i=1}^{b} \int_{0}^{H} \left( T'_{1} \right)^{2} \frac{b_{1}^{3} h_{1}}{24 E I_{c,i}} dz \quad (3.5)$$

At this stage introducing a sign convention that in any band of openings of the released structure positive shear in the continuous medium adjacent to "End 1" wall of the beams (as defined in the beam connectivity matrix) is directed along decreasing $z$ and that in the continuous medium adjacent to "End 2" wall is directed along increasing $z$ and also that positive value of the center to center length of any band of openings is obtained by subtracting the
coordinates of "End 1" wall from those of "End 2" wall, we can write for the axial force in the ith wall

\[ F_i = \sum T_r - \sum T_s \]  \hspace{1cm} (3.6)

where \( r \) and \( s \) refer to the contents of the ith row of the wall connectivity matrix 1 ("WCM 1") and wall connectivity matrix 2 ("WCM 2") respectively. Each of these two matrices contain \( m \) rows and are generated by scanning the beam connectivity matrix. The ith row of wall connectivity matrix contains the numbers of the beam lines having wall i as their "End 1" and that of wall connectivity matrix 2 contains the numbers of the beam lines having wall i as their "End 2".

For the wall shown in Fig.3.2, \( r = i \) and \( l \) and \( s = t \) and \( p \), therefore, the axial force in the wall is given by

\[ F_i = T_r - T_l - T_t - T_p \]

Strain energy, due to axial deformation of the ith wall is

\[ U_{2,i} = \int_0^H \frac{(\sum T_r - \sum T_s)^2}{2EA_i} \, dz \]  \hspace{1cm} (3.7)

Therefore the total strain energy due to axial deformation of all the walls is

\[ U_2 = \sum_{i=1}^1 \int_0^H \frac{(\sum T_r - \sum T_s)^2}{2EA_i} \, dz \]  \hspace{1cm} (3.8)
Strain energy, due to flexural deformation of the \( i \)th wall is

\[
U_{3,i} = \int_{0}^{H} \left\{ \frac{(M_{x,i})^2}{2E I_{x,i}} + \frac{(M_{y,i})^2}{2E I_{y,i}} \right\} dz \quad (3.9)
\]

Now, on the basis of the sixth assumption mentioned in section 3.2, at any distance \( z \) from the top of the tube the bending moments \( M_{x,i} \) and \( M_{y,i} \) are given by

\[
M_{x,i} = \frac{w_{r} z^2}{2}\sum_{j=1}^{n} \frac{1}{1/S_{x,i}} \left[ (\Sigma T_{r} c_{r} + \Sigma T_{s} c')_{j} \right]
\]

\[
M_{y,i} = \frac{w_{r} z^2}{2}\sum_{j=1}^{n} \frac{1}{1/S_{y,i}} \left[ (\Sigma T_{r} a_{r} + \Sigma T_{s} a')_{j} \right]
\]

In any tubular structure because of continuity

\[
\sum_{j=1}^{n} (\Sigma T_{r} a_{r} + \Sigma T_{s} c')_{j} = \sum_{j=1}^{n} T_{j} L_{x,j} \quad (3.12)
\]

and

\[
\sum_{j=1}^{n} (\Sigma T_{r} c_{r} + \Sigma T_{s} c')_{j} = \sum_{j=1}^{n} T_{j} L_{y,j} \quad (3.13)
\]
Therefore,

\[ M_{x,i} = \frac{w_Y z^2}{2} \sum_{j=1}^{n} \frac{T_j L_y,j}{1/S_{x,j}} \]  
\[ M_{y,i} = \frac{w_X z^2}{2} \sum_{j=1}^{n} \frac{T_j L_x,j}{1/S_{y,j}} \]  

Substituting these values of \( M_{x,i} \) and \( M_{y,i} \) in Eqn. 3.9 and simplifying we get

\[ U_{x,i} = \int_{0}^{H} \left[ \frac{w_Y z^2}{2} \sum_{j=1}^{n} \frac{T_j L_y,j}{1/S_{x,j}} \right] ^{1} \]  
\[ + \frac{w_X z^2}{2} \sum_{j=1}^{n} \frac{T_j L_x,j}{1/S_{y,j}} \]  
\[ \int_{0}^{H} dz \]  
\[ (3.16) \]
Therefore the total strain energy due to flexural deformation of all the walls is

\[
U_3 = \begin{cases} 
\int_0^H & \frac{W_z z^2}{2} \left[ - \frac{1}{2} + \sum_{j=1}^n \frac{1}{E} \left( \frac{1}{S_x} \right)_j \right] \frac{1}{2} \sum_{i=1}^i \frac{1}{I_x, i S_x, i^2} \\
0 & + \frac{W_x z^2}{2} \left[ - \frac{1}{2} + \sum_{j=1}^n \frac{1}{E} \left( \frac{1}{S_y} \right)_j \right] \frac{1}{2} \sum_{i=1}^i \frac{1}{I_y, i S_y, i^2} 
\end{cases}
\] (3.17)

Total strain energy of the whole system, considering all kinds of deformation is

\[
U = U_1 + U_2 + U_3 = F(z,T,T') \, dz \quad (3.18)
\]

The Euler equation of the calculus of variations (23) states that if

\[
\phi = F(z,y,y') \, dz
\]

then for \( \phi \) to be a minimum

\[
\frac{\delta F}{\delta y} \frac{d}{dz} \left( \frac{\delta F}{\delta y'} \right) = 0
\]
Applying this to minimize the strain energy of the system we will have

\[
\frac{\delta U}{\delta T_i} \frac{d}{dz} \frac{\delta U}{\delta T_i'} = 0 \quad (3.19)
\]

Now

\[
\frac{\delta U}{\delta T_i} = \frac{\delta U_1}{\delta T_i} + \frac{\delta U_2}{\delta T_i} + \frac{\delta U_3}{\delta T_i} \quad (3.20)
\]

and

\[
\frac{d}{dz} \frac{\delta U}{\delta T_i'} = \frac{\delta U_1}{\delta T_i'} + \frac{\delta U_2}{\delta T_i'} + \frac{\delta U_3}{\delta T_i'} \quad (3.21)
\]

\[
\frac{\delta U_1}{\delta T_i} = 0 \quad (3.22)
\]

\[
\frac{\delta U_1}{\delta T_i'} = \frac{T_i' b_1^3 h_i}{12 E I_c, i} \quad (3.23)
\]

\[
\frac{d}{dz} \frac{\delta U_1}{\delta T_i'} = \frac{T_i' b_1^3 h_i}{12 E I_c, i} \quad (3.24)
\]

Similarly

\[
\frac{\delta U_2}{\delta T_i} = \frac{\Sigma T_r - \Sigma T_s}{A_u E} + \frac{\Sigma T_r - \Sigma T_s}{A_v E} \quad (3.25)
\]

given that for wall u one value of r is i and for wall v one value of s is i. It should be noted that u and v may have more than one value.
\[ \frac{\delta U_z}{\delta T'_1} = 0 \] (3.26)

and

\[ \frac{\delta U_x}{\delta T'_1} = \frac{w_x z^2}{2} \sum_{j=1}^{n} (T_j L_x, j) + \frac{w_x z^2}{2} L_x, i \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{n} \left( \frac{1}{S_{x,j} S_{x,k}} \right) \left( \frac{1}{I_{x,j} I_{x,k}} \right)^2 \]
\[ \sum_{j=1}^{n} \sum_{s=1}^{n} \left( \frac{1}{S_{x,j} S_{x,k}} \right) \left( \frac{1}{I_{x,j} I_{x,k}} \right)^2 = 0 \]

Substitution of the above expressions in Eqn.3.19 yields

\[ \frac{w_y z^2}{2} L_y, i \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{n} \left( \frac{1}{S_{x,j} S_{x,k}} \right) \left( \frac{1}{I_{x,j} I_{x,k}} \right)^2 \]
\[ \sum_{j=1}^{n} \sum_{s=1}^{n} \left( \frac{1}{S_{x,j} S_{x,k}} \right) \left( \frac{1}{I_{x,j} I_{x,k}} \right)^2 = 0 \] (3.29)
Rearranging the terms and simplifying

\[ T' = 12 I_c, i \quad b_1^3 h_1 \quad L_y, j \quad L_y, i \quad b_1^3 \quad \frac{1}{\Sigma T_j} \quad \left( \sum \frac{1}{S_x, k} \right)^2 \]

\[ + \quad \left( \sum \frac{1}{S_y, k} \right)^2 \quad \frac{1}{I_y, k \quad S_y, k^2} \quad \frac{1}{A_u \quad A_v} \quad \sum \quad k = 1 \]

\[ L_x, j \quad L_x, i \quad b_1^3 \quad h_1 \quad \frac{1}{\Sigma T_j} \quad \left( \sum \frac{1}{S_x, k} \right)^2 \quad \frac{1}{I_x, k \quad S_x, k^2} \quad \frac{1}{A_u \quad A_v} \quad \sum \quad k = 1 \]

\[ + \quad \left( \sum \frac{1}{S_y, k} \right)^2 \quad \frac{1}{I_y, k \quad S_y, k^2} \quad \frac{1}{A_u \quad A_v} \quad \sum \quad k = 1 \]

\[ \frac{12 I_c, i}{b_1^3 h_1} \quad \frac{1}{z^2} = 0 \quad \text{(3.30)} \]

where \( i = 1, 2, 3, \ldots \ldots \quad n \)

The above system of simultaneous second order differential equations may be written as

\[ T' = -\alpha_{1^1}^2 T_1 - \alpha_{1^2}^2 T_2 \quad \ldots \ldots \quad -\alpha_{1n}^2 T_n + \beta_1 z^2 = 0 \]

\[ T' = -\alpha_{2^1}^2 T_1 - \alpha_{2^2}^2 T_2 \quad \ldots \ldots \quad -\alpha_{2n}^2 T_n + \beta_2 z^2 = 0 \]

\[ \ldots \ldots \]

\[ T' = -\alpha_{n^1}^2 T_1 - \alpha_{n^2}^2 T_2 \quad \ldots \ldots \quad -\alpha_{nn}^2 T_n + \beta_n z^2 = 0 \]

\[ \ldots \ldots \quad \text{(3.31)} \]
where

\[
(a_1, j)^2 = \frac{12 E I_{c, i} L_y, j L_y, i}{b_i^2 h_i} \left\{ \sum_{k:1}^{\infty} \frac{1}{I_x, k S_x, k^2 (1/S_x, k)^2} \right\} \left( \sum_{k:1}^{\infty} \frac{1}{I_y, k S_y, k^2 (1/S_y, k)^2} \right) \]

when no entry in row numbers "End 1" and "End 2" (of the \(i\)th band of openings) of "WCM 1" or "WCM 2" is equal to \(j\).

\[
(a_1, j)^2 = \frac{12 E I_{c, i} L_y, j L_y, i}{b_i^2 h_i} \left\{ \sum_{k:1}^{\infty} \frac{1}{I_x, k S_x, k^2 (1/S_x, k)^2} \right\} \left( \sum_{k:1}^{\infty} \frac{1}{I_y, k S_y, k^2 (1/S_y, k)^2} \right) \]

when \(j\) is equal to one entry in row no "End 1" (of the \(i\)th band of openings), say row no \(u\), of "WCM 1".

\[
(a_1, j)^2 = \frac{12 E I_{c, i} L_y, j L_y, i}{b_i^2 h_i} \left\{ \sum_{k:1}^{\infty} \frac{1}{I_x, k S_x, k^2 (1/S_x, k)^2} \right\} \left( \sum_{k:1}^{\infty} \frac{1}{I_y, k S_y, k^2 (1/S_y, k)^2} \right) \]

when \(j\) is equal to one entry in row no "End 1" (of the \(i\)th band of openings), say row no \(u\), of "WCM 2".
\[(\alpha_{1,j})^2 = \frac{12 E I_c, i}{b_1^2 h_1} \left\{ \frac{L_y, j L_y, i}{\sum_{k=1}^{\infty} I_x, k S_x, k^2} \right\} \right\} \]

\[+ \left\{ \frac{L_x, j L_x, i}{\sum_{k=1}^{\infty} I_y, k S_y, k^2} \right\} \]

\[(\sum 1/S_x, k)^2 \]

when \( j \) is equal to one entry in row no "End 2" (of the \( i \)th band of openings), say row no \( v \), of "WCM 1".

\[(\alpha_{1,j})^2 = \frac{12 E I_c, i}{b_1^2 h_1} \left\{ \frac{L_y, j L_y, i}{\sum_{k=1}^{\infty} I_x, k S_x, k^2} \right\} \]

\[+ \left\{ \frac{L_x, j L_x, i}{\sum_{k=1}^{\infty} I_y, k S_y, k^2} \right\} \]

\[(\sum 1/S_y, k)^2 \]

when \( j \) is equal to one entry in row no "End 2" (of the \( i \)th band of openings), say row no \( v \), of "WCM 2".

\[\beta_i = \frac{12 E I_c, i}{b_1^2 h_1} \left\{ \frac{w_y L_y, i}{\sum_{k=1}^{\infty} I_x, k S_x, k^2} \right\} \]

\[+ \left\{ \frac{w_x L_x, i}{\sum_{k=1}^{\infty} I_y, k S_y, k^2} \right\} \]

\[(\sum 1/S_y, k)^2 \]

It should be noted that in each of the expressions above \( u \) and \( v \) may have multiple values.
When written in matrix notation, Eqn. 3.31 becomes

\[
\begin{bmatrix}
T_1 \\
d^2 \\
\vdots \\
T_n
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_{11}^2 & \alpha_{12}^2 & \cdots & \alpha_{1n}^2 \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{n1}^2 & \alpha_{n2}^2 & \cdots & \alpha_{nn}^2
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
\vdots \\
T_n
\end{bmatrix}
+ z^2 \begin{bmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_n
\end{bmatrix}
\]

or

\[
\frac{d^2}{dz^2} T = AT + z^2 B \tag{3.32}
\]

An analytical solution of the above system of equations is too tedious for more than two bands of openings. Assuming a family of solutions

\[
T_j = \sum_{i=1,3,5} a_{ij} \sin\left( \frac{i \pi z}{2H} \right) \tag{3.33}
\]

each term of which satisfies the boundary conditions

\[
\text{at } z = 0 \quad T_1 = 0 \quad \text{and} \quad T_1' = 0 \tag{3.34}
\]

and applying Galerkin's method to minimize residuals, the following system of simultaneous equations in terms of \(a_{ij}\)'s is obtained:


\[
\begin{bmatrix}
\frac{\pi}{2} & \alpha_{11}^2 & \alpha_{12}^2 & \ldots & \alpha_{1n}^2 \\
(-1)^2 + \alpha_{11}^2 & \alpha_{12}^2 & \ldots & \alpha_{1n}^2 \\
\frac{\pi}{2} & \alpha_{21}^2 & \ldots & \alpha_{2n}^2 \\
\vdots & \alpha_{31}^2 & \ldots & \alpha_{3n}^2 \\
\alpha_{n2}^2 & \ldots & (-1)^2 + \alpha_{nn}^2
\end{bmatrix}
\begin{bmatrix}
a_{11} \\
a_{12} \\
a_{13} \\
a_{1n}
\end{bmatrix}
= \frac{16}{\pi^3} \sin(\pi i/2) - 2 \begin{bmatrix}
\beta_1 \\
\beta_2 \\
\beta_3 \\
\beta_n
\end{bmatrix}
\]

where \( i = 1, 3, 5 \ldots \)

In matrix form,

\[
C D_1 = K \quad (3.36)
\]

the solution of which gives \( a_{ij} \)'s

\[
D_1 = C^{-1} K \quad (3.37)
\]

once the \( a_{ij} \)'s are determined, the values of the \( T_i \)'s at different heights may be evaluated by summing the series (Eqn. 3.33).

For wall \( i \), at height \( z \),

\[
M_{x,i} = \frac{W_y z^2}{2} \sum_{k=1}^{n} T_k L_{y,k} \left( \frac{1}{S_{x,i}} \right) \quad (3.38a)
\]

\[
\sum_{k=1}^{n} \left( \frac{1}{S_{x,k}} \right)
\]
The tube deflections can be determined by using the load-deflection equation as shown below:

\[
\frac{W_x z^2}{2} \sum_{k=1}^{n} T_k L_{x,k} - \sum_{k=1}^{n} \left( \frac{1}{S_{y,k}} \right) \frac{E I}{dz^2} = \frac{-E I}{dz^2} + \sum_{i=1}^{3} \left( T_i L_{y,i} \right) \quad (3.38b)
\]

where the \( T_k \)s are calculated at height \( z \).

Axial force in wall \( i \) at height \( z \) is

\[
F_i = \sum T_r - \sum T_z
\quad (3.39)

Deflections:

The tube deflections can be determined by using the load-deflection equation as shown below:

\[
\frac{E I}{dz^2} = \frac{W_x z^2}{2} \sum_{i=1}^{3} \left( T_i L_{x,i} \right) \quad (3.40a)
\]

\[
\frac{E I}{dz^2} = \frac{-W_y z^2}{2} \sum_{i=1}^{3} \left( T_i L_{y,i} \right) \quad (3.40b)
\]

Substituting the values of \( T_i \)s from Eqn. 3.33

\[
\frac{d^2 y_x}{dz^2} = -\frac{W_x z^2}{2} + \sum_{j=1}^{\infty} \sum_{i=1,3,5} \alpha_{i,j} \sin(i\pi z/2H) L_{x,i,j} \quad (3.41a)
\]
\[
\begin{align*}
\frac{d^3y_x}{dz^3} &= E I \sum_{j=1}^{\infty} a_{i,j} \sin(i\pi z/2H) L_{i,j} \\
\frac{d^3y_y}{dz^3} &= E I \sum_{j=1}^{\infty} a_{i,j} \sin(i\pi z/2H) L_{i,j} \\
\end{align*}
\]

Integrating twice and substituting the appropriate boundary conditions i.e.,
\[
\begin{align*}
\frac{dy_x}{dz} &= y_x = \frac{dy_y}{dz} = 0 \\
\end{align*}
\]
we get
\[
\begin{align*}
E I y_x &= \frac{w_x z^4}{2} \frac{H^3 z}{12} \frac{H^4}{4} \\
E I y_y &= \frac{w_y z^4}{2} \frac{H^3 z}{12} \frac{H^4}{4} \\
\end{align*}
\]
\[
\begin{align*}
&+ \frac{a_{i,j}}{\pi} \left( \sum_{j=1}^{\infty} \sum_{i=1,3,5} \{\sin(i\pi z/2H) - \sin(i\pi z/2H)\} L_{i,j} \right) \\
&+ \frac{a_{i,j}}{\pi^3} \left( \sum_{j=1}^{\infty} \sum_{i=1,3,5} \{\sin(i\pi z/2H) - \sin(i\pi z/2H)\} L_{i,j} \right) \\
\end{align*}
\]

For point load at top

The derivation follows exactly the same pattern as for the uniformly distributed load, the only difference is in the values of loading terms in Eqn. 3.32 expressed by \( z^4 \). The general expression for \( \beta_i \) now becomes
where $P_x$ and $P_y$ are the concentrated loads applied horizontally at top.

The equations for moments (Eqns. 3.38) are modified to

\[
\beta_i = \frac{12 E I_{c,i}}{b_i h_i} \left( \sum_{k=1}^{\infty} \frac{1}{I_{x,k} S_{x,k}^2} \right)^2 \frac{P_y L_y, i}{1} + \frac{P_x L_x, i}{1} \left( \sum_{k=1}^{\infty} \frac{1}{I_{y,k} S_{y,k}^2} \right)^2
\]

\[
(3.44)
\]

and the equations for deflections (Eqns. 3.43) become

\[
M_{x,i} = \frac{-P_y z + \sum_{j=1}^{n} T_j L_y, j}{\sum_{k=1}^{\infty} (1/S_{x,k})} \quad (1/S_{x, i}) \quad (3.45a)
\]

\[
M_{y,i} = \frac{-P_x z - \sum_{j=1}^{n} T_j L_x, j}{\sum_{k=1}^{\infty} (1/S_{y,k})} \quad (1/S_{y, i}) \quad (3.45b)
\]

\[
E I_y x = -\frac{z^2}{6} \frac{h^2 z}{2} \frac{H^3}{3} + \frac{4 h^2}{a_{1,j}} \sum_{i=1}^{n} \sum_{j=1}^{3} \sin(i \pi /2) - \sin(i \pi z /2H) \} L_{x,i} \quad (3.46a)
\]

\[
......
\]
3.4 Modification for Non-Rectangular Plan Shape

In buildings with rectangular plan shape the walls are always parallel to either of the global axes. Hence the moments of inertia of the walls about the global axes are the same as those about their local principal axes. But in buildings with non-rectangular plan shape the wall principal axes are, in general, not parallel to the global axes. Since evaluation of the $\alpha_{1,j}$ and $\beta_{1}$ requires wall moments of inertia about the global axes, the moments of inertia of the walls about their local axes must be transferred to the global axes.

\[
E I_{y} = P_{y} \left( \frac{z^{3}}{6} + \frac{H^{2} z}{2} + \frac{H^{3}}{3} \right)
\]

\[
4 \frac{H^{2}}{\pi^{2}} \sum_{1}^{\infty} \sum_{i=1,3,5}^{2} \left( \sin \left( \frac{i \pi \theta}{2} \right) - \sin \left( \frac{i \pi z}{2H} \right) \right) L_{x,j}
\]

\[
.. \quad (3.46b)
\]

**Fig 3.3: Axis transformation**
In Fig. 3.3 \(x-y\) are the global axes and \(u-v\) are the local axes of a wall making an angle \(\theta\) with the global \(x\) axis. The moments of inertia of the wall about the \(x\) and \(y\) axes i.e. \(I_x\) and \(I_y\) are given by

\[I_x = I_u \cos^2 \theta + I_v \sin^2 \theta\]  \hspace{1cm} (3.47a)

\[I_y = I_u \sin^2 \theta + I_v \cos^2 \theta\]  \hspace{1cm} (3.47b)

where \(I_u\) and \(I_v\) are the moments of inertia of the wall about its local axes.

3.5 Modification for Non-Rectangular Beams

The distribution of shear stress and consequently the shear deformation in any beam is closely related to the geometric shape of the beam. This fact must be taken into account while calculating the strain energy associated with shear deformation of beams. In section 3.3 the spandrel beams were assumed to be rectangular. If a non-rectangular section is used only the expression for the reduced moment of inertia \(I_c\) will be modified. The general expression for the reduced moment of inertia is

\[I_{c,1} = I_{p,1} \left/ \left\{ 1 + 24n \left(1 + \nu\right) I_{p,1} / b_1^2 A_{p,1} \right\} \right.\]  \hspace{1cm} (3.48)

where \(n\) is the shape factor which has a value of 1.2 for rectangular sections, 1.11 for solid circular sections, etc.
3.6 Modification for Shift of the Point of Contraflexure

In developing the theory in section 3.3 the points of contraflexure in the spandrel beams were assumed to lie at their mid-spans. This assumption is valid for tubular structures with flexible beams. In structures where the beams are stiff compared to the walls the point of contraflexure in a beam line may drift away from the mid-span if the supporting columns are of unequal stiffnesses. The two components of this drift of the point of contraflexure towards the less stiff wall may be approximated from the following formulae deduced from Beck's derivation (4)

\[
\begin{align*}
\Delta x,1 &= \frac{b_{x,1} d_1}{I_{y,1}} - \frac{I_{y,2}}{2 c} \quad (3.49a) \\
\Delta y,1 &= \frac{b_{y,1} d_1}{I_{x,1}} - \frac{I_{x,2}}{2 c} \quad (3.49b)
\end{align*}
\]

and the drift measured along the beam axis is given by

\[
\Delta 1 = (\Delta x,1^2 + \Delta y,1^2)^{1/2} \quad (3.50)
\]

When the drift is considered the strain energy of the structure due to flexure and shear in the continuous medium
is changed and consequently the expression for the reduced moment of inertia is modified to

\[ I_{c,i} = I_{p,i} / \left( 1 + 12(\Delta_i/b)^2 + 24n(1 + \nu) \right) \frac{I_{p,i}}{b l^2 A_{p,i}} \] ........ (3.51)

3.7 Computer Program

A computer program, based on the theory presented in section 3.3 and its modifications discussed in sections 3.4 to 3.6 has been written in BASIC. The program is completely general in nature and can be used for analysing tubular structures having any plan shape and any number of interconnected cells. It can also be used for analysing plane frames and tubular structures with one or more open cells by introducing fictitious beams of zero moment of inertia at appropriate locations.

The program can be run in any microcomputer having a BASIC compiler or a BASIC interpreter. Since the matrices used in the program are small they do not present any storage problem.

A block flow diagram of the program is given in Fig 3.4. A listing of the program with detailed instructions for preparing the input data file and a sample input data file are given in Appendix C.
Declare dimensions (1250 - 1320).

Define constants and functions (1340 - 1380).

Open input and output files (1400 - 1490).

Read in input data and print the same in the output file (1580 - 2530).

Generate wall connectivity matrix (2550 - 2710).

Calculate column stiffnesses (2720 (5550 - 5920)).

Calculate the shift of the point of contraflexure and the reduced moment of inertia for each band of openings (2970 - 3480).

Evaluate the matrices containing $a^2$ and $\beta$. (3500 - 3830).

Evaluate C and K matrices (3850 - 4000).

continued on page 55
Solve Eqn.3.35 using the Gaussian elimination technique to obtain the coefficients in the solution series for the T forces.
\[
\{ 4010 ( 5140 - 5540) \}
\]

Calculate q and T at various z/H levels
(4150 - 4260)

Calculate beam shears and end moments
(4280 - 4350)

Calculate wall axial forces and moments
(4460 - 4650)

Print shears, moments and axial forces
(4660 - 4800)

Calculate and print lateral deflections
(4820 - 5050)

Fig. 3.4: Block flow diagram for the computer program described in section 3.7
CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

The applicability of the continuous medium method developed in the previous chapter has been checked by analysing a twenty storied hypothetical structure. The computer program described in section 3.7 has been used to calculate column axial forces, beam shear forces, beam end moments and lateral deflections of the example problem and these results have been compared with those obtained from more exact finite element analysis using the general purpose finite element package ANSYS.

Fig 4.1 shows the plan of the twenty storied four-celled concrete modular tube which has been used to verify the applicability of the continuous medium method. As shown in the figure the tube, henceforth referred to as example problem 1, consists of 33 columns and 36 beams per storey. It is 200 ft high, each storey being 10 ft high. The columns are spaced at 8.33 ft c/c and each side of the tube has a length of 50 ft. The column spacing has been kept small in order to keep the base width to height ratio within a realistic range for high rise tubular structures. If a longer beam is used the height of the building should also be increased but that would make the finite element analysis
Fig 4.1: Floor plan of the example problem 1
much more time consuming because the number of unknowns increase as the number of stories increase.

The material properties used are:

Poisson's ratio = 0.18
Modulus of elasticity = 432000 ksf

Following Khan's (14) approach of reduction modelling three different 10 storey equivalent stiffness factors \( S_{10} \) have been considered. The stiffness factor has been changed by changing only the beam section, other properties of the structure were kept the same. The 10 storey equivalent stiffness factor \( S'_{10} \) is given by

\[
S'_{10} = S_t \times (N/10)^2
\]

(4.1)

where \( S_t \) is the actual stiffness factor for the tube and is given by

\[
S_t = S_b / S_c
\]

(4.2)

in which \( S_b \) = shear stiffness of spandrel beam

\[
= 12 \frac{E I_b}{L^3}
\]

(4.3)

and \( S_c \) = axial stiffness of the column

\[
= A_c E / H
\]

(4.4)

where \( I_b \) = moment of inertia of the spandrel beams

\( A_c \) = cross sectional area of the column

\( H \) = height of column

\( L \) = effective span of the spandrel beam

\( E \) = modulus of elasticity

\( N \) = number of stories
The properties of the different components of example problem 1 under different stiffness factors are given in Table 4.1. The tube has been analysed for two load conditions:

a) 100 kips point load applied at the top and
b) 1 kip/ft. uniformly distributed load.

4.2 Results Obtained by Continuous Medium Method

As already stated the program developed on the basis of the theory presented in Chapter Three of this thesis has been used to analyse the structure shown in Fig. 4.1 under two load conditions and three different 10-storey equivalent stiffness factors. The column axial forces at various levels are shown in Fig. 4.2 to Fig. 4.7. The deflected shapes of the tube under different load conditions and stiffness factors are shown in Fig. 4.8 to Fig. 4.13.

4.3 Results Obtained from Finite Element Analysis

The general purpose finite element package ANSYS (Version 4.3) available at the BUET Computer Centre was used to analyse all the tubes analysed by the continuous medium method. A three-dimensional beam element with two nodes and six degrees of freedom per node have been used to represent the spandrel beams and the columns. This element is a line element and uses linear displacement functions for axial displacement and torsional rotation and cubic polynomial displacement function for normal displacement. The effect of shear deformation in the beams has been included as an option. The coupling action of the floor slabs has been
Table 4.1: Properties of the tube shown in Fig. 4.1

<table>
<thead>
<tr>
<th>Property</th>
<th>1.0</th>
<th>1.5</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Width (ft)</td>
<td>1.0</td>
<td>1.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Beam depth (ft)</td>
<td>1.9420</td>
<td>3.6550</td>
<td>6.6416</td>
</tr>
<tr>
<td>Beam moment of inertia (ft^2)</td>
<td>0.6103</td>
<td>6.1034</td>
<td>61.0347</td>
</tr>
<tr>
<td>Beam length (ft)</td>
<td>8.3333</td>
<td>8.3333</td>
<td>8.3333</td>
</tr>
<tr>
<td>S_b/E (ft)</td>
<td>0.0127</td>
<td>0.1266</td>
<td>1.2656</td>
</tr>
<tr>
<td>Column size (ft x ft)</td>
<td>2.25 x 2.25</td>
<td>2.25 x 2.25</td>
<td>2.25 x 2.25</td>
</tr>
<tr>
<td>Column area (ft^2)</td>
<td>5.0625</td>
<td>5.0625</td>
<td>5.0625</td>
</tr>
<tr>
<td>Storey height (ft)</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>S_c/E (ft)</td>
<td>0.5063</td>
<td>0.5063</td>
<td>0.5063</td>
</tr>
<tr>
<td>S_f</td>
<td>0.0251</td>
<td>0.2510</td>
<td>2.5100</td>
</tr>
<tr>
<td>Number of stories</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>S_f10</td>
<td>0.10</td>
<td>1.00</td>
<td>10.00</td>
</tr>
</tbody>
</table>
simulated by coupling all lateral deflections at floor levels along the direction of the load.

The results of the finite element analysis are shown in the same diagrams on which the results of the Continuous Medium Method are plotted i.e. Figs 4.2 to 4.13.

4.4 Comparison between the Results of the Continuous Medium Method and those of Finite Element Method

The column axial forces obtained from the Continuous Medium analysis are compared with those obtained from Finite Element analysis in Fig. Nos. 4.2 to 4.7. In these diagrams the variation of column axial forces as given by the continuous medium method is shown by broken lines while that given by the finite element method is shown by solid lines.

When the structure is subjected to a 100k point load applied at the top, the continuous medium method gives stresses that are quite close to those obtained from finite element analysis. For all the three S_{fl} levels considered, at \( Z/H = 0.25 \) the continuous medium method slightly overestimates the axial forces in all the columns. The continuous medium method is seen to always overestimate the axial forces in the interior columns of the web frames. However, the degree of overestimation is quite low, in the order of 10%. In the flange frames the axial forces in the columns not common with the web frames are overestimated, the axial forces given by the approximate method being approximately 10% higher than those obtained from the finite element analysis. In the corner columns, at \( Z/H = 1.0 \) the finite element analysis is seen to give higher values.
for columns axial forces. In the corner column maximum difference between the two methods is about 11% for $S'_{110}=10.0$ and $Z/H=0.25$, the minimum being about 2% for $S'_{110}=1.0$ and $Z/H=0.75$. Both the methods show that the shear lag effect is pronounced only at $Z/H=1.0$ for $S'_{110}=0.10$ and 1.00. For $S'_{110}=10.0$ the shear lag effect is very small even at $Z/H=1.0$.

For an uniformly distributed load of 1 k/ft. of height applied along the global y axis an interesting reversal in the axial force distribution pattern upto $Z/H=0.50$ is observed for all the $S'_{110}$ levels considered. Both the methods show this reversal. Normally, one would expect a larger axial force to be produced in the corner columns than in the columns in between, but in these cases the corner columns are seen to carry significantly lower force than the columns adjacent to these corner columns. For $S'_{110}=0.10$ the finite element analysis shows a complete reversal of the direction of action of the axial forces in the corner columns at $Z/H=0.25$; the continuous medium method indicates a zero force in these columns.

The two methods give the same pattern of distribution of the axial forces in the columns in all cases although the magnitudes of the forces differ quite a lot. Both the methods show that a change in the axial force distribution occurs between $Z/H=0.50$ and $Z/H=0.75$.

The lateral deflections of the structure are shown in Fig. Nos. 4.8 to 4.13. These figures show that both the methods give the same kind of deflected shape for all stiffness factors. But the magnitudes of deflections predicted by the continuous medium method are much less than those given by the finite element method. The continuous
medium method deflections are almost one half of the corresponding finite element deflections.

At all $S'_{10}$ levels the beam shears and moments are predicted fairly accurately by the continuous medium method. The column moments given by the continuous medium method, however, vary considerably from their finite element analysis values. This is because, the small errors in axial forces are magnified when moments are calculated.

4.5 Discussion on Shear-Lag Effect

A study of the shear lag effect on multicellular tubular structures is made by using Fig. Nos. 4.2 to 4.7. It is observed that the shear lag effect, which results in nonlinear distribution of column axial forces, is almost negligible up to $Z/H=0.50$ from top for all the three conditions studied with a point load at the top. For $S'_{10}=0.10$ the distribution starts to become nonlinear quite pronouncedly at $Z/H=0.75$. On the other hand for $S'_{10}=10.0$ the shear lag effect is very small even at $Z/H=1.00$. For $S'_{10}=0.10$ and 1.0 at $Z/H=1.00$ the shear lag effect produces highly nonlinear axial force distribution pattern in the flange frames.

As reported earlier, an interesting phenomenon is observed when the structure is subjected to uniformly distributed load. Up to $Z/H=0.50$ from the top a reversal of the axial force distribution pattern due to shear lag effect is observed. Normally the corner columns carry higher load than those in between, but in this case the flange columns adjacent to the corner columns are seen to carry the maximum
force. For $S_{f10}=0.10$ at $Z/H=0.25$ a reversal of the direction of the force in the corner column is observed. As $Z/H$ increases the corner columns gradually begin to carry more load than the columns adjacent to them. At $Z/H=1.0$ the usual nonlinear axial force distribution pattern due to shear lag is obtained for all the three $S_{f10}$s considered. For uniformly distributed load also at $Z/H=1.00$ and $S_{f10}=10.0$ the effect of shear lag on the axial force distribution pattern is very weak.
Fig 4.2 a & b: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Fig 4.2 c & d: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Fig 4.3 a & b: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Fig 4.3 c & d: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Fig 4.4 a & b: Axial forces in the columns of the example problem 1.
(Values within parentheses are from continuous medium method)
Fig 4.4 c & d: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Fig 4.5 a & b: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Fig 4.5 c & d: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Fig 4.6 a & b: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Fig 4.6 c & d: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
\( w = 1 \text{k/ft} \)
\( S_{f1c} = 10.0 \)

(a) \( Z/H = 0.25 \)

(b) \( Z/H = 0.50 \)

Fig 4.7 a & b: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Fig 4.7 c & d: Axial forces in the columns of the example problem 1.

(Values within parentheses are from continuous medium method)
Continuous medium method

Finite element analysis

Fig 4.8: Lateral deflection of the example problem 1.
$P_x = 100k$

$S_{f1s} = 1.00$

- Continuous medium method
- Finite element analysis

Fig 4.9: Lateral deflection of the example problem 1.
Fig. 4.10: Lateral deflection of the example problem 1.
\[ w_r = 1 \text{ k/ft} \]
\[ S_{	ext{fle}} = 0.10 \]

Fig 4.11: Lateral deflection of the example problem 1.

$w_f = 1 \text{ k/ft}$

$S_{11e} = 1.00$

- Continuous medium method
- Finite element analysis

Fig 4.12: Lateral deflection of the example problem 1.
Fig 4.13: Lateral deflection of the example problem 1.
4.6 Structures of Arbitrary Plan Shape

In order to demonstrate the applicability of the continuous medium method to multicellular structures of arbitrary plan shape, example problem 2 whose plan shape is shown in Fig 4.14, has been analysed by the continuous medium method and the results have been checked against those obtained from a finite element analysis.

The column axial forces at various levels as obtained from the two analyses are compared in Fig 4.15. The column axial forces agree quite favourably. A maximum difference of 15% has been observed at Z/H=0.25. Here also the continuous medium method underestimates the axial force in the corner column and overestimates the same in the columns in between.

The shear lag effect is very weak at upper levels and is quite pronounced at the lower levels especially at Z/H=1.00.

As for example 1 here also the deflection predicted by the continuous medium method is in the order of 50% of that given by the finite element analysis.

4.7 Influence of the Number of Terms Considered in the Solution Series

The solution time in the continuous medium method depends not only on the number of bands of openings in the structure but also on the number of terms considered in the solution series for the T forces (Eqn. 3.33). Obviously the higher the number of terms the more is the number of times
Fig 4.14: Floor plan of the example problem 2.
$P_t = 100k$
$S_{f10} = 0.10$

(a) $Z/H = 0.25$

(b) $Z/H = 0.50$

Fig 4.15 a & b: Axial forces in the columns of the example problem 2.

(Values within parentheses are from continuous medium method)
Fig 4.15 c & d: Axial forces in the columns of the example problem 2.

(Values within parentheses are from continuous medium method)
$P_7 = 100k$
$S_{fr 10} = 0.10$

**Fig 4.16:** Lateral deflection of the example problem 2.
Eqn. 3.35 has to solved for the coefficients in the solution series which means more computer time. Fig 4.17 shows the variation of the primary unknown T in one of the openings of example 1 with the number of terms considered. It can easily be concluded from Fig. 4.17 that for all practical purposes only the first ten or so terms give results that are quite accurate. Use of more terms does not improve the force solution very much because the curve for T begins to become asymptotic to the horizontal axis at that point. Variation of Ts in other bands of openings has been found to be similar. On the basis of this information fifteen terms have been used in the analysis of both the example problems.

4.8 Shift of the Point of Contraflexure

As indicated in section 3.6 the point of contraflexure in the spandrel beams do not always lie at mid span and this introduces some error in the results obtained from the continuous medium method. A modified form of Beck's formula for estimating the shift of the point of contraflexure in the spandrel beams in a plane shear wall has been used in the continuous medium method to calculate the shift of the point of contraflexure but it is evident from Table 4.2 that it cannot predict the shift accurately. Finite element analysis shows that when the load is small or the beams are very stiff compared to the columns there is no point of contraflexure in the beams at the top floor. In other cases the shift is much higher than that estimated by Beck's formula. Also Beck's formula gives the same shift at all floor levels although finite element analysis shows that the drift varies from floor to floor and is maximum at the topmost floor.
Fig 4.17: Variation of T force with number of terms.
Table 4.2: Shift of the point of contraflexure in a corner panel of example 1

<table>
<thead>
<tr>
<th>Floor level</th>
<th>1st</th>
<th>2nd</th>
<th>19th</th>
<th>20th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beck's formula (Eqn 3.49)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S'_{110} = 0.10$</td>
<td>0.052</td>
<td>0.052</td>
<td>0.052</td>
<td>0.052</td>
</tr>
<tr>
<td>$S'_{110} = 1.00$</td>
<td>0.268</td>
<td>0.268</td>
<td>0.268</td>
<td>0.268</td>
</tr>
<tr>
<td>$S'_{110} = 10.0$</td>
<td>0.458</td>
<td>0.458</td>
<td>0.458</td>
<td>0.458</td>
</tr>
<tr>
<td>ANSYS, $P_y = 100k$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S'_{110} = 0.10$</td>
<td>0.135</td>
<td>0.124</td>
<td>0.133</td>
<td>0.356</td>
</tr>
<tr>
<td>$S'_{110} = 1.00$</td>
<td>0.930</td>
<td>0.957</td>
<td>1.188</td>
<td>2.270</td>
</tr>
<tr>
<td>$S'_{110} = 10.0$</td>
<td>3.557</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ANSYS, $w_y = 1 k/ft$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S'_{110} = 0.10$</td>
<td>0.134</td>
<td>0.122</td>
<td>0.311</td>
<td>*</td>
</tr>
<tr>
<td>$S'_{110} = 1.00$</td>
<td>0.918</td>
<td>0.935</td>
<td>2.108</td>
<td>*</td>
</tr>
<tr>
<td>$S'_{110} = 10.0$</td>
<td>3.500</td>
<td>4.150</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

* No point of contraflexure
CHAPTER FIVE

CONCLUSIONS

5.1 Conclusions

The following conclusions regarding the applicability of the proposed method may be drawn on the basis of the results presented in Chapter Four.

The proposed approximate method gives column axial forces that are quite close to those given by the more exact finite element method. For the three cases with a point load at the top a maximum difference of 11% between the two methods is observed at \( Z/H = 0.25 \) and \( S_{f10} = 10.0 \). The continuous medium method slightly underestimates the axial forces in the corner columns and overestimates the same in the columns in between. But in most of the columns the approximate method predicts a value which is within 90-95% of the exact value given by the finite element method.

For uniformly distributed load the continuous medium method works quite well at lower levels \((Z/H > 0.50)\). At these levels the columns axial forces given by the continuous medium method are within 85-90% of their finite element values. At upper levels, where a reversal of the axial force distribution is observed the difference between the values of the axial forces given by the two methods is high in terms of percentage.
The proposed continuous medium method does not work well for deflections. Although it predicts the same deflected shape as the finite element method, it cannot predict the magnitude of the deflection correctly. The continuous medium method gives deflections that are in the order of 50% of those given by the finite element method.

An analysis of example problem 2 shows that the continuous medium method gives similar results for tubular structures of arbitrary plan shape.

For the example problem 1 the continuous medium method requires less than 580 kilobytes of RAM on a microcomputer while ANSYS, the general purpose finite element package requires 10 megabytes of virtual storage on a mainframe computer. With regard to time, with a BASIC compiler the continuous medium method requires approximately 30 minutes on an IBM PC/AT with 80287 math coprocessor whereas ANSYS requires 3700 CPU seconds on a IBM 4331 mainframe. Although the continuous medium method cannot predict deflections accurately it gives forces that are close to their exact values. Since it requires much less computer time and storage it may be used for preliminary analysis of tubular structures.

5.2 Recommendations for Further Study

The present study has yielded an approximate method that can fairly accurately predict the forces in a tubular structure of any plan shape and having any number of interconnected cells; but it cannot predict deflections
correctly. A different approach e.g. the moment area method may be developed to be used in conjunction with the continuous medium method.

As discussed in section 4.8, the formula used for calculating the drift of the point of contraflexure does not give satisfactory results, a more refined formula may be developed.

In developing the theory the torsional stiffness of the flange beams has been neglected. A more general method may be developed taking care of the torsional stiffness of the flange beams.

Finally, a series of influence curves may be drawn for different stiffness factors using the shear lag analyses made in Fig. Nos. 4.2 to 4.7.
REFERENCES


APPENDIX A

STIFFNESS OF AN ONE-STOREY FRAME SEGMENT

The stiffness of any column in the single storey high frame shown in Fig. A.1 under lateral load can easily be derived from equilibrium considerations, assuming that points of contraflexure occur at mid column height and at mid span of beams.

The load deflection relationship for the isolated column shown in Fig. A.2 is given by

\[
P = \frac{12EI_h}{h^3} \left( \frac{2I_h b^3}{b^3} + \frac{1}{h^3} \left( \frac{1}{I_{b1}(b_1+b_2)^2} + \frac{1}{I_{b2}(b_1+b_2)^2} \right) \right) \]

\[\ldots\ldots\ldots (A.1)\]
Fig A.2: Horizontal deflection in a column in a multibay frame

Now in a multibay frame for the same floor level lateral deflection $\Delta$, the load on the $i$th column which supports beams $l$ and $m$ is given by,

$$P_i = \frac{\Delta}{S_i}$$  \hspace{1cm} (A.2)

where

$$\frac{1}{S_i} = \frac{12EI_{b,1}}{h_1^3} \frac{1}{h_1} + \frac{2I_{b,1}}{h_1} \frac{b_1^3}{b_1^3} \frac{b_m}{b_m^3} \frac{1}{b_m} \left( \frac{I_{b,1}(b_1+b_m)^2}{h_1} + \frac{I_{b,m}(b_1+b_m)^2}{h_1} \right)$$  \hspace{1cm} (A.3)
The lateral load $P$ on the frame is therefore given by

$$ P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} (1/S_i) \quad (A.4) $$

where $n$ is the total number of columns in the frame.

From Eqn. A.4 we get

$$ \Delta = P/\sum (1/S_i) \quad (A.5) $$

Substituting this in Eqn. A.2 we get,

$$ P_i = \frac{P}{\sum_{i=1}^{n} (1/S_i)} \quad (A.6) $$

Assuming a similar distribution of moments among the columns we have

$$ M_i = \frac{M}{\sum_{i=1}^{n} (1/S_i)} \quad (A.7) $$

where $M$ is the total moment to be carried by the columns at the level under consideration.
APPENDIX B

EFFECT OF WIDTH TO HEIGHT RATIO

The accuracy of the continuous medium method has been found to be dependent upon the width to height ratio of the structure. For low values of the ratio the continuous medium method gives deflections that are within 90% of the deflections predicted by the finite element method. The difference between the predictions of the two methods increases as the width to height ratio increases. This has been tested by analysing a ten storied 100 ft high two bay plane frame. The variation of deflections predicted by the two methods are shown in Fig.B.1. The column axial forces are, however, predicted fairly accurately by the continuous medium method for all ratios.

Fig B.1: Variation of top deflection in a two bay frame with width to height ratio.
C.1 Listing of the Computer Program

1010 REM
1020 REM ***********************************************************************
1030 REM *** APPROXIMATE ANALYSIS OF A TUBULAR STRUCTURE USING ***
1040 REM *** THE CONTINUOUS MEDIUM METHOD ***
1050 REM ***********************************************************************
1060 REM
1070 REM
1080 REM *** PRINT INFORMATION REGARDING THE PROGRAM ********************************
1090 REM
1100 CLS: COLOR 4,3,7
1110 PRINT SPC(11); "APPROXIMATE ANALYSIS OF A TUBULAR STRUCTURE"
1120 PRINT SPC(11); " USING THE CONTINUOUS MEDIUM METHOD": PRINT
1130 PRINT: PRINT
1140 PRINT SPC(17); "Program developed by Asif Ahmed"
1150 PRINT: PRINT
1160 PRINT: PRINT: PRINT "STRIKE ANY KEY TO CONTINUE"
1170 AA$=INKEY$: IF AA$="" THEN GOTO 1170
1180 BEEP: BEEP
1190 PRINT: PRINT "Execution begins........"
1200 FOR I=1 TO 3: PRINT: NEXT I
1210 KEY OFF: LOCATE 25,1: COLOR 4,7,7
1220 PRINT SPC(20); "PROGRAM IN EXECUTION. DO NOT INTERFERE."; STRINGS(21," ")
1230 COLOR 4,3,7
1240 REM
1250 REM *** DECLARE DIMENSIONS ***********************************************
1260 REM
1270 DIM A2(50,50),AA(50,50),ANO(50),ANW(50),AP(50),AR(50),B(50),B1(50),B2(50)
1280 DIM BCON(50,2),BS(50),BT(50),C(50,50),D(50),DELT(50),F(50),H(50),IC(50)
1290 DIM IP(50),IU(50),IV(50),IX(50),LY(50),K(50),L(50),LOCX(50),LOCY(50)
1300 DIM LX(50),LY(50),MX(50),MY(50),NDN(50),NUP(50),Q(50),RCON(10,5),T(50)
1310 DIM WCONDN(50,5),WCONUP(50,5),WIDX(50),WIDY(50),WXY(50,2)
1320 DIM BM1(50),BM2(50),ENX(2),ENY(2),SX(50),SY(50),TOT(50),WCON(50,5)
1330 REM
1340 REM *** DEFINE CONSTANTS AND FUNCTIONS **************************************
1350 REM
1360 PI=3.141592654#
1370 DEF FNC(ANGLE)=COS(ANGLE*PI/180)
1380 DEF FNS(ANGLE)=SIN(ANGLE*PI/180)
1390 REM
1400 REM *** OPEN INPUT AND OUTPUT FILES ******************************************
1410 REM
1420 LOCATE 17,3
1430 INPUT "ENTER INPUT FILE NAME":IN$ 1440 PRINT
1450 LOCATE 19,3
1460 INPUT "ENTER OUTPUT FILE NAME";OT$
1470 OPEN IN$ FOR INPUT AS #1
1480 OPEN OT$ FOR OUTPUT AS #3
1490 WIDTH #3, 132
1500 REM
1510 REM *** PRINT TITLE *****************************************************************************
1520 REM
1530 PRINT #3,STRING$(130," "); PRINT #3,
1540 PRINT #3,CHR$(14);SPC(11); "APPROXIMATE ANALYSIS OF A TUBULAR STRUCTURE"
1550 PRINT #3,CHR$(14);SPC(11); " USING THE CONTINUOUS MEDIUM METHOD"
1560 PRINT #3,STRING$(130," "); PRINT #3, : PRINT #3,
1570 REM
1580 REM *** DATA INPUT *****************************************************************************
1590 REM
1600 INPUT #1,A$,TITL$
1610 PRINT #3,"PROBLEM TITLE ":";TITL$ : PRINT #3,
1620 INPUT #1,A$,E
1630 INPUT #1,A$,NU
1640 INPUT #1,A$,NW
1650 INPUT #1,A$,NB
1660 INPUT #1,A$,TH
1670 PRINT #3,"TOTAL HEIGHT OF THE STRUCTURE ":";TH;" ft"
1680 PRINT #3,
1690 PRINT #3,"TOTAL NUMBER OF BANDS OF OPENINGS ":";NB
1700 PRINT #3,"TOTAL NUMBER OF COLUMNS ":";NW
1710 INPUT #1, A$,NRCON
1720 FOR 1=1 TO NRCON
1730 INPUT #1, A$,SETNO
1740 INPUT #1, A$,RCON(SETNO,1)
1750 INPUT #1, A$,RCON(SETNO,2)
1760 INPUT #1, A$,RCON(SETNO,3)
1770 INPUT #1, A$,RCON(SETNO,4)
1780 INPUT #1, A$,RCON(SETNO,5)
1790 NEXT I
1800 PRINT #3, : PRINT #3, "WALL DATA"
1810 PRINT #3,STRING$(130," "); PRINT #3,
1820 PRINT #3," SL NO WALL NO COORDINATES in ft AREA in ft^2 IU in";
1830 PRINT #3,STRING$(130," "); PRINT #3,
1840 FOR 1=1 TO NW
1850 INPUT #1,A$,WN
1860 INPUT #1,A$,WXY(WN,1),WXY(WN,2)
1870 INPUT #1, A$,LOCX(WN),LOCY(WN)
1880 INPUT #1,A$,RSETNO
1890 AR(WN)=RCON(RSETNO,1)
1900 IU(WN)=RCON(RSETNO,2)
1910 IV(WN)=RCON(RSETNO,3)
1920 WIDX(WN)=RCON(RSETNO,4)
1930 WIDY(WN)=RCON(RSETNO,5)
1940 INPUT #1,A$,ANGW
1950 IX(WN)=IU(WN)*FNC(ANGW)^2+IV(WN)*FNS(ANGW)^2
1960 IY(WN)=IU(WN)*FNS(ANGW)^2+IV(WN)*FNC(ANGW)^2
1970 PRINT #3, USING "####"; I,
1980 PRINT #3, SPC(4); USING "####"; WN,
1990 PRINT #3, SPC(5); USING "####."; WXY(WN, 1),
2000 PRINT #3, SPC(1); USING "####."; WXY(WN, 2),
2010 PRINT #3, USING "################"; AR(WN), IU(WN), IV(WN),
2020 PRINT #3, SPC(3); USING "############"; ANGW;
2030 PRINT #3, USING "############"; IX(WN), IY(WN)
2040 NEXT I
2050 PRINT #3, STRING$(130, "-") : PRINT #3, : PRINT #3, : PRINT #3,
2060 PRINT #3, "OPENING DATA" : PRINT #3, STRING$(130, "-")
2070 PRINT #3, " SL NO OPN NO CONNECTIVITY(I,J) CLEAR SPAN WALL C/C"
2080 PRINT #3, STRING$(130, "-")
2090 FOR I=1 TO NB
2100 INPUT #1, AS, OPN
2110 INPUT #1, AS, BCON(OPN, 1), BCON(OPN, 2)
2120 INPUT #1, AS, B(OPN)
2130 INPUT #1, AS, H(OPN)
2140 INPUT #1, AS, RSETNO
2150 AP(OPN) = RCON(RSETNO, 1)
2160 IP(OPN) = RCON(RSETNO, 2)
2170 LX(OPN) = WXY(BCON(OPN, 2), 1) - WXY(BCON(OPN, 1), 1)
2180 LY(OPN) = WXY(BCON(OPN, 2), 2) - WXY(BCON(OPN, 1), 2)
2190 L(OPN) = ((LX(OPN))^2 + (LY(OPN))^2)^.5
2200 PRINT #3, USING "####"; I,
2210 PRINT #3, SPC(2); USING "####"; OPN,
2220 PRINT #3, SPC(4); USING "############"; BCON(OPN, 1), BCON(OPN, 2),
2230 PRINT #3, SPC(2); USING "############"; B(OPN), L(OPN), H(OPN);
2240 PRINT #3, USING "############"; AP(OPN), IP(OPN), LX(OPN), LY(OPN)
2250 NEXT I
2260 PRINT #3, STRING$(130, "-")
2270 PRINT #3, : PRINT #3, : PRINT #3,
2280 PRINT #3, : PRINT #3, "MATERIAL SPECIFICATIONS;"
2290 PRINT #3, : PRINT #3, " MODULUS OF ELASTICITY : "; E; "ksf"
2300 PRINT #3, : PRINT #3, " POISSON'S RATIO : "; NU
2310 INPUT #1, BS, LDTYP$,
2320 IF LDTYP$ = "POINT" OR LDTYP$ = "point" THEN LDTYP$ = "P" : GOTO 2390
2330 IF LDTYP$ = "UDL" OR LDTYP$ = "udl" THEN LDTYP$ = "U" : GOTO 2450
2340 PRINT : PRINT
2350 PRINT "Error in load type definition card. Program terminated."
2360 PRINT #3, "ERROR **** Error in load definition card. Program terminated."
2370 GOTO 5110
2380 INPUT #1, BS, P
2390 INPUT #1, BS, ANGLD
2400 PX = P*FNC(ANGLD)
2410 PY = P*FNS(ANGLD)
2420 PRINT #3, : PRINT #3, "LOAD SPECIFICATIONS;"
2430 PRINT #3, : PRINT #3, " POINT LOAD AT TOP : "; P; "k at"; ANGLD; "degrees with the ";
2440 GOTO 2510
2450 GOTO 2510
2460 INPUT #1, BS, W
2470 WX = W*FNC(ANGLD)
WY = W \times \text{FNS(ANGLD)}

PRINT #3, : PRINT #3, "LOAD SPECIFICATIONS:"
PRINT #3, " UNIFORMLY DISTRIBUTED LOAD : "; W; "\text{k/ft at}"; \text{ANGLD}; "degrees"; with the global x axis.

INPUT #1, C$, NT
PRINT #3, : PRINT #3,"NUMBER OF TERMS CONSIDERED IN THE SOLUTION SERIES"; NT
INPUT #1, C$, NLEVEL
REM
REM *** GENERATE THE WALL CONNECTIVITY MATRIX **************************
FOR I = 1 TO NW
  NUP(I) = 0 NDN(I) = 0 TOT(I) = 0
  FOR J = 1 TO NB
    IF BCON(J, 1) <> I THEN GOTO 2650
    NUP(I) = NUP(I) + 1
    WCONUP(I, NUP(I)) = J
    TOT(I) = TOT(I) + 1
    WCON(I, TOT(I)) = J
  NEXT J
NEXT I
GOSUB 5550
REM
REM *** EVALUATE SUMS ********************************************************
IXEQV = 0! IYEQV = 0! SUMAR = 0! SUMARX = 0! SUMARY = 0!
SUMIX = 0! SUMIY = 0! SUM1SX = 0! SUM1SY = 0!
SUM1IXSX2 = 0! SUM1IYSY2 = 0!
FOR I = 1 TO NW
  SUMAR = SUMAR + AR(I)
  SUMARX = SUMARX + AR(I) * WXY(I, 1)
  SUMIX = SUMIX + IX(I)
  SUMIY = SUMIY + IY(I)
  SUM1SX = SUM1SX + 1! / SX(I)
  SUM1SY = SUM1SY + 1! / SY(I)
  SUM1IXSX2 = SUM1IXSX2 + 1! / (IX(I) * SX(I)^2)
  SUM1IYSY2 = SUM1IYSY2 + 1! / (IY(I) * SY(I)^2)
NEXT I
XBAR = SUMARX / SUMAR
YBAR = SUMARY / SUMAR
FOR I = 1 TO NW
  IXEQV = IXEQV + IX(I) + AR(I) * (WXY(I, 1) - XBAR)^2
  IYEQV = IYEQV + IY(I) + AR(I) * (WXY(I, 2) - YBAR)^2
NEXT I
REM
REM *** CALCULATE THE REDUCED MOMENTS OF INERTIA **************************
FOR OPN = 1 TO NB
IF \( \text{ABS}(LY(OPN)) < .00001 \) THEN \( \Delta L = 0! \) : GOTO 3170

**CALCULATE \( \Delta L \)**

\( I_1 = IX(BCON(OPN,1)) \)
\( I_2 = IX(BCON(OPN,2)) \)
IF \( \text{LOC}(BCON(OPN,1)) = 0 \) THEN \( I_1 = I_1/2! \)
IF \( \text{LOC}(BCON(OPN,2)) = 0 \) THEN \( I_2 = I_2/2! \)
IF \( I_1 = I_2 \) THEN \( \Delta L = 0! \) : GOTO 3170
IF \( I_2 > I_1 \) THEN GOTO 3080
\( \text{END1} = BCON(OPN,1) ; \text{END2} = BCON(OPN,2) \) : GOTO 3120
\( \text{END1} = BCON(OPN,2) ; \text{END2} = BCON(OPN,1) \)
\( \text{TEMP} = I_1 \)
\( I_1 = I_2 \)
\( I_2 = \text{TEMP} \)
\( B = \text{ABS}(B(OPN) \times LY(OPN)/L(OPN)) \)
\( C = \text{ABS}(LY(OPN)/2! \)
\( D = \text{WIDY}(	ext{END1}) \)
\( SCRTCH = I_1 + I_2 + I_2 \times B \times I_1 \times I_2/(H(OPN) \times IP(OPN)) \)
\( \Delta L = (I_1 - (B + D) \times I_2/(2 \times C)) \times C/SCRTCH \)
IF \( \text{ABS}(LX(OPN)) < .00001 \) THEN \( \Delta X = 0! \) : GOTO 3350
**CALCULATE \( \Delta X \)**
\( I_1 = IY(BCON(OPN,1)) \)
\( I_2 = IY(BCON(OPN,2)) \)
IF \( \text{LOC}(BCON(OPN,1)) = 0 \) THEN \( I_1 = I_1/2! \)
IF \( \text{LOC}(BCON(OPN,2)) = 0 \) THEN \( I_2 = I_2/2! \)
IF \( I_1 = I_2 \) THEN \( \Delta X = 0! \) : GOTO 3350
IF \( I_2 > I_1 \) THEN GOTO 3260
\( \text{END1} = BCON(OPN,1) ; \text{END2} = BCON(OPN,2) \) : GOTO 3300
\( \text{END1} = BCON(OPN,2) ; \text{END2} = BCON(OPN,1) \)
\( \text{TEMP} = I_1 \)
\( I_1 = I_2 \)
\( I_2 = \text{TEMP} \)
\( B = \text{ABS}(B(OPN) \times LX(OPN)/L(OPN)) \)
\( C = \text{ABS}(LX(OPN)/2! \)
\( D = \text{WIDX}(	ext{END1}) \)
\( SCRTCH = I_1 + I_2 + I_2 \times B \times I_1 \times I_2/(H(OPN) \times IP(OPN)) \)
\( \Delta L = (I_1 - (B + D) \times I_2/(2 \times C)) \times C/SCRTCH \)
**CALCULATE \( \Delta \)**
\( \text{IF LDTYP} = \text{"P"} \) GOTO 3400
\( \text{IF} WX < .00001 \) THEN \( \Delta X = 0! \)
\( \text{IF} WY < .00001 \) THEN \( \Delta Y = 0! \)
\( \text{GOTO} 3420 \)
\( \text{IF} PX < .00001 \) THEN \( \Delta X = 0! \)
\( \text{IF} PY < .00001 \) THEN \( \Delta Y = 0! \)
\( \text{DELTA}(OPN) = (\Delta L \times 2 + \Delta X \times 2)^.5 \)
\( IC(OPN) = IP(OPN)/(1+28.8\times(1+NU)\times IP(OPN)/(AP(OPN)\times B(OPN)^2) + 12\times(DELTA(OPN)/B(OPN))^2) \)
\( \text{FACT} = -1 \)
\( \text{IF END1} = BCON(OPN,2) \) THEN \( \text{FACT} = 1 \)
\( B1(OPN) = B(OPN)/2! - \text{DELTA}(OPN) \times \text{FACT} \)
\( B2(OPN) = B(OPN)/2! + \text{DELTA}(OPN) \times \text{FACT} \)
**NEXT OPN**
3500 REM *** EVALUATE MATRICES CONTAINING ALPHA SQUARE AND BETA ***************
3510 REM
3520 FOR I = 1 TO NB
3530 MF = 12 * IC(I) / (B(I)^3 * H(I))
3540 ENDI = BCON(I, 1)
3550 RNDI = BCON(I, 2)
3560 Upi = NUP(ENDI)
3570 DNI = NDN(ENDI)
3580 Upj = NUP(ENDJ)
3590 DNJ = NDN(ENDJ)
3600 FOR J = 1 TO NB
3610 AAA = LY(J)^2 / (SUM1SX^2 / SUM1IXSX2) + LX(I) * LX(I) / (SUM1SY^2 / SUM1IYSY2)
3620 A2(I, J) = AAA * MF
3630 NEXT J
3640 FOR K = 1 TO UPI
3650 KK = WCONUP(ENDI, K)
3660 A2(I, KK) = A2(I, KK) + (MF / AR(ENDI))
3670 NEXT K
3680 FOR K = 1 TO DNI
3690 KK = WCONDN(ENDI, K)
3700 A2(I, KK) = A2(I, KK) - (MF / AR(ENDI))
3710 NEXT K
3720 FOR K = 1 TO UPJ
3730 KK = WCONUP(ENDJ, K)
3740 A2(I, KK) = A2(I, KK) - (MF / AR(ENDJ))
3750 NEXT K
3760 FOR K = 1 TO DNJ
3770 KK = WCONDN(ENDJ, K)
3780 A2(I, KK) = A2(I, KK) + (MF / AR(ENDJ))
3790 NEXT K
3800 IF LDTYP$ = "U" GOTO 3820
3810 BT(I) = (PX * LX(I) / (SUM1SX^2 / SUM1IXSX2) + PY * LY(I) / (SUM1SY^2 / SUM1IYSY2)) * MF : GOTO 3830
3820 BT(I) = (WX * LX(I) / (SUM1SX^2 / SUM1IYSY2) + WY * LY(I) / (SUM1SX^2 / SUM1IXSX2)) * MF * .5
3830 NEXT I
3840 REM
3850 REM *** EVALUATE THE C AND K MATRICES AND ************************************************
3860 REM *** THE COEFFICIENTS IN THE SOLUTION SERIES ****************************************
3870 REM
3880 II = -1
3890 FOR III = 1 TO NT
3900 II = II + 2
3910 AF = ((II * PI / (2 * TH))^2
3920 FOR I = 1 TO NB
3930 FOR J = 1 TO NB
3940 C(I, J) = A2(I, J)
3950 IF J = I THEN C(I, J) = C(I, J) + AF
3960 NEXT J
3970 IF LDTYP$ = "U" GOTO 3990
3980 K(I) = (8 * TH / (II * PI)^2) * SIN((II * PI / 2) * BT(I)) : GOTO 4000
3990 K(I) = (16 * TH^2 / (II * PI)^3) * (II * PI * SIN((II * PI / 2) - 2) * BT(I)
4000 NEXT I
U*U*U*U*U*T

4010 GOSUB 5140
4020 NEXT III
4030 REM
4040 REM *** EVALUATE STRESSES AND DEFLECTIONS AT VARIOUS HEIGHTS **************
4050 REM
4060 Z=-TH/NLEVEL
4070 FOR DIST=1 TO (NLEVEL+1)
4080 Z=Z+TH/NLEVEL
4090 ZH=Z/TH
4100 PRINT #3, : PRINT #3, : PRINT #3, : PRINT #3,
4110 PRINT #3, : PRINT #3, : PRINT #3,
4120 PRINT #3, SPC(47);"FORCES AND DEFLECTIONS AT Z/H = ";USING ";.##";ZH
4130 PRINT #3, : PRINT #3, : PRINT #3,
4140 REM
4150 REM *** EVALUATE Q AND T ***********************************************
4160 REM
4170 FOR J=1 TO NB
4180 Q(J)=0!
4190 T(J)=0!
4200 II=-1
4210 FOR III=1 TO NT
4220 II=II+2
4230 Q(J)=Q(J)+AA(III,J)*(II*PI/(2*TH»)*COS(II*PI*Z/(2*TH))
4240 T(J)=T(J)+AA(III,J)*SIN(II*PI*Z/(2*TH))
4250 NEXT III
4260 NEXT J
4270 REM
4280 REM *** EVALUATE BEAM SHEARS AND MOMENTS ********************************
4290 REM
4300 FOR I=1 TO NB
4310 BS(I)=Q(I)*H(I)
4320 IF Z=0! THEN BS(I)=BS(I)/2!
4330 BM1(I)=BS(I)*B1(I)
4340 BM2(I)=BS(I)*B2(I)
4350 NEXT I
4360 REM
4370 REM *** EVALUATE THE SUMS OF T*LX AND OF T*LY **************************
4380 REM
4390 SUMTLX=0!
4400 SUMTLY=0!
4410 FOR I=1 TO NB
4420 SUMTLX=SUMTLX+T(I)*LX(I)
4430 SUMTLY=SUMTLY+T(I)*LY(I)
4440 NEXT I
4450 REM
4460 REM *** EVALUATE WALL AXIAL FORCES AND BENDING MOMENTS ******************
4470 REM
4480 PRINT #3,STRING$(130,"-")
4490 PRINT #3," WALL AXIAL FORCE in k B.M. @ X in k-ft B.M. @ Y in";
4500 PRINT #3, STRING$(130,"")
4510 FOR I=1 TO NW
109

4520 \( F(I) = 0 \)
4530 FOR \( K = 1 \) TO \( NUP(I) \)
4540 \( KK = \text{WCONUP}(I, K) \)
4550 \( F(I) = F(I) + T(KK) \)
4560 NEXT \( K \)
4570 FOR \( K = 1 \) TO \( NDN(I) \)
4580 \( KK = \text{WCONDN}(I, K) \)
4590 \( F(I) = F(I) - T(KK) \)
4600 NEXT \( K \)
4610 \( MX(I) = \frac{(-PY*Z+SUMTLY)*(1/3X(I))}{SUM1SX} \)
4620 \( MY(I) = \frac{PF*Z-SUMTLX)*(1/3Y(I))}{SUM1SY} \)
4630 PRINT \#3, TAB(5); USING "###"; I;
4640 IF \( I < NB \) THEN GOTO 4670
4650 IF \( I = NB \) THEN GOTO 4670
4660 PRINT \#3, USING "#######.###"; \( F(I), MX(I), MY(I) \); GOTO 4710
4670 PRINT \#3, USING "#######.###"; \( F(I), MX(I), MY(I) \);
4680 PRINT \#3, USING "###"; I;
4690 PRINT \#3, USING "#######.###"; BS(I),
4700 PRINT \#3, USING "#######.###"; BM1(I), BM2(I)
4710 NEXT \( I \)
4720 EXT = NB-NW
4730 IEXT=NW
4740 FOR \( I = 1 \) TO \( EXT \)
4750 IEXT=IEXT+1
4760 PRINT \#3, USING "#######.###"; \( BS(I), BM1(I), BM2(I) \)
4770 NEXT \( I \)
4780 PRINT \#3, STRING$(130,"-")
4800 REM
4810 REM *** CALCULATE THE DEFLECTIONS
4820 REM ****************************************
4830 REM
4840 G=E/(2*(1+NU))
4850 SMASINLX=0!
4860 SMASINY=0!
4870 FOR \( J = 1 \) TO \( NB \)
4880 \( II = 1 \)
4890 FOR \( III = 1 \) TO \( NT \)
4900 \( II = II+2 \)
4910 SMASINLX=SMASINLX+AA(III,J)*(SIN(II*PI*Z/(2*TH))-SIN(II*PI/2))*LX(I)/II^2
4920 SMASINY=SMASINY+AA(III,J)*(SIN(II*PI*Z/(2*TH»-SIN(II*PI/2»)*LY(J)/II^2
4930 NEXT III
4940 NEXT \( J \)
4950 IF LDTYPE$="U" GOTO 4980
4960 EIVX=-PX*(Z^3/6-TH^2*Z/2+TH^3/3)-4*TH^2*SMASINLX/PI^2
4970 EIVY=-PY*(Z^3/6-TH^2*Z/2+TH^3/3)+4*TH^2*SMASINLX/PI^2 : GOTO 5000
4980 EIVX=-.5*WX*(Z^4/12-TH^3*Z/3+TH^4/4)-4*TH^2*SMASINLX/PI^2
4990 EIVY=-.5*WY*(Z^4/12-TH^3*Z/3+TH^4/4)+4*TH^2*SMASINLX/PI^2
5000 YXFLEX=EIVX/(E*SUMIY)
5010 YYFLEX=EIVY/(E*SUMIX)
5020 PRINT \#3, ; PRINT \#3, SPC(5);"DEFLECTION IN X DIRECTION ";
5030 USING "#####"; YXFLEX ;
5030 PRINT #3, " ft"; SPC(9); "DEFLECTION IN Y DIRECTION ";
    USING "###.###";YYFLEX;
5040 PRINT #3, " ft"
5050 NEXT DIST
5060 CLOSE #1
5070 CLOSE #2
5080 CLOSE #3
5090 BEEP: BEEP: BEEP
5100 PRINT: PRINT "Execution complete." : PRINT: PRINT
5110 KEY ON
5120 STOP
5130 REM
5140 REM ***** SUBROUTINE GAUSS **************************************************
5150 REM
5160 X1=NB
5170 XX=NB-1
5180 FOR KS=1 TO XX
5190 KP=KS+1
5200 LS=KS
5210 FOR IS=KS TO X1
5220 IF ABS(C(IS,KS)) <= ABS(C(LS,KS)) GOTO 5240
5230 LS=IS
5240 NEXT IS
5250 IF LS=KS GOTO 5340
5260 FOR JS=KS TO X1
5270 AJ=C(KS,JS)
5280 C(KS,JS)=C(LS,JS)
5290 C(LS,JS)=AJ
5300 NEXT JS
5310 AB=K(KS)
5320 K(KS)=K(LS)
5330 K(LS)=AB
5340 FOR IS=KP TO X1
5350 FAC=C(IS,KS)/C(KS,KS)
5360 FOR JS=KS TO X1
5370 C(IS,JS)=C(IS,JS)-FAC*C(KS,JS)
5380 NEXT JS
5390 K(IS)=K(IS)-FAC*K(KS)
5400 NEXT IS
5410 NEXT KS
5420 AA(III,X1)=K(X1)/C(X1,X1)
5430 IS=X1-1
5440 I2=IS+1
5450 SUM=0.0
5460 FOR JS=I2 TO X1
5470 SUM=SUM+C(IS,JS)*AA(III,JS)
5480 NEXT JS
5490 AA(III,IS)=(K(IS)-SUM)/C(IS,IS)
5500 IS=IS-1
5510 IF IS=0 GOTO 5530
5520 GOTO 5440
5530 RETURN
SUBROUTINE SX & SY

FOR WN = 1 TO NW
  IF TOT(WN) < 2 THEN PRINT "Error in connectivity definition."
      " Program terminated." ; PRINT #3, "ERROR *** Error in connectivity"
      " definition. Program terminated." : GOTO 5110
  XXX=0 : YYY=0
  FOR I=1 TO TOT(WN)
    IF ABS(LY(WCON(WN,I)) < .0001 GOTO 5650
    XXX= XXX+1
  IF XXX>2 THEN PRINT "More than two beams contribute to SX. Only the first"
      " two have been considered." : GOTO 5650

  ENX(1)=0 : ENX(2)=0 : ENY(1)=0 : ENY(2)=0
  FOR I=1 TO TOT(WN)
    IF ABS(LX(WCON(WN,I)) < .0001 GOTO 5690
    YYY = YYY +1
  IF YYY>2 THEN PRINT "More than two beams contribute to SY. Only the first"
      " two have been considered." : GOTO 5690
  ENY(YYY)=WCON(WN,I)

  NEXT I

  REM ******* CALCULATE SX *******

  IH=IX(WN)
  IF ENX(1)=0 AND ENX(2)=0 THEN HH=H(ENX(1)) : GOTO 5800
  HH=H(ENX(1)) : IB1=IP(ENX(1)) : B1=ABS(LY(ENX(1)))
  IF ENX(2)=0 THEN IB2=1 ! : B2=0 ! : GOTO 5780
  IB2=IP(ENX(2)) : B2=ABS(LY(ENX(2)))
  IF IB1=0 ! THEN IB1=1 ! : B1=0 !
  IF IB2=0 ! THEN IB2=1 ! : B2=0 !
  SX(WN)=HH^3*(1+2*IH*(B1^3/IB1+B2^3/IB2)/(HH*(B1+B2)^2))/(12*IH)
  GOTO 5810

  SX(WN)=HH^3/(3*IH)

  REM ******* CALCULATE SY *******

  IH=IFY(WN)
  IF ENY(1)=0 AND ENY(2)=0 THEN HH=H(ENY(1)) : GOTO 5910
  HH=H(ENY(1)) : IB1=IP(ENY(1)) : B1=ABS(LX(ENY(1)))
  IF ENY(2)=0 THEN IB2=1 ! : B2=0 ! : GOTO 5890
  IB2=IP(ENY(2)) : B2=ABS(LX(ENY(2)))
  IF IB1=0 ! THEN IB1=1 ! : B1=0 !
  IF IB2=0 ! THEN IB2=1 ! : B2=0 !
  SY(WN)=HH^3*(1+2*IH*(B1^3/IB1+B2^3/IB2)/(HH*(B1+B2)^2))/(12*IH)
  GOTO 5920

  SY(WN)=HH^3/(3*IH)

  NEXT WN
  RETURN

END
C.2 Input Instructions for Continuous Medium Method

A separate input data file should be created for the program using any editor e.g. PPAS, SPFPC etc. The data card images should be 80 columns wide and each data item should be preceded by a string identifier and a comma. The file should contain appropriate number of the following types of card images in the same sequence as they are described.

Card No. 1 (1600)
This is the problem title card. The problem title is read from the card as a string and is presented at the beginning of the output file.

Card No. 2 (1620)
This is the first material property card. The value of the modulus of elasticity E is read in from this card.

Card No. 3 (1630)
This second material property card is used to define the value of the Poisson's ratio NU.

Card No. 4 (1640)
This data card should contain the value of NW, number of column lines.

Card No. 5 (1650)
The value of NB, number of beam lines is read from this card.

Card No. 6 (1660)
This data card is used to define the total height of the structure TH.
Card No. 7 (1710)
The total number of real constant sets i.e NRCON is to be read in from this card.

Card No. 8a to 8f (1730-1780)
These six cards together define a real constant set. These should be NRCON of these cards in the input file. The first card should contain the real constant set number. The second card should contain the cross-sectional area and the next two cards are to contain the two moments of inertia about the two local axes. The last two cards of the set are used to define the dimensions along the local axes. For a beam real constant set the last three cards in the set may contain zeros.

Card No. 9a to 9e (1850-1880 & 1940)
This set of cards defines the columns. The first entry in the card is the set defines the column number. This entry must be an integer. The second card is used to define the coordinates of the column center. The third card in the set is used to define whether the wall is an external or an internal wall while scanning along the y axis. The integer number 1 indicates an external column and 0 indicates an internal column. Card 9c is used to select the real constant set associated with the current column; it should contain the appropriate real constant set no. The last card of this set is used to indicate the angle of rotation of the local u axis with respect to the global X axis. This item must be in degrees.

Cards 9a to 9e must be repeated exactly NW times.
Card No. 10a to 10e (2100-2140)
These five cards comprise a set which must be repeated a total of NB times. The first card defines the current beam line number. The beam connectivity i.e the numbers of column lines connected by the current current beam is read in from this card. The next two cards are used to define clear opening B and c/c height H respectively. The last card in the set defines the real constant number associated with the current beam line.

Card No. 11 (2310)
This is the first load definition card and should indicate whether the load is a point load or an UDL. For point load the input should be "UDL" or "udl" and for point load at the top it should be "POINT" or "point".

Card No. 12 (2380/2450)
The magnitude of the load is defined through this card.

Card No. 13 (2390/2460)
This card is used to define the direction of action of the load.

Card No. 14 (2510)
The number of terms NT, to be considered in the series solution.

Card No. 15 (2530)
This last data card should contain the value of the variable i.e the number of levels excluding the base at which stresses and deflections are to be calculated.
C.3 Sample Input Data File for Continuous Medium Method

The following is an image of the input data file used for the analysis of example problem 1 using the continuous medium method.

Title of the problem ===>, UDL. ......... Sf10 = 1.0
Modulus of Elasticity ================>, 432000.0
Poisson's Ratio ================>, 0.18
Number of Column lines ================>, 33
Number of Beam Lines ================>, 36
Total Height of the Structure ================>, 200.0
Number of Real Constant Sets ================>, 2
Real Constant Set Number ================>, 1
Cross Sectional Area ================>, 5.0625
Moment of Inertia @ u Axis ================>, 2.1357
Moment of Inertia @ v Axis ================>, 2.1357
Dimension along u ================>, 2.25
Dimension along v ================>, 2.25
Real Constant Set Number ================>, 2
Cross Sectional Area ================>, 5.4825
Moment of Inertia @ u Axis ================>, 6.1034
Moment of Inertia @ v Axis ================>, 0.0
Dimension along u ================>, 0.0
Dimension along v ================>, 0.0
Column Number ================>, 1
Coordinates of Column Center ================>, 0.0, 0.0, 0.0
External/Internal ================>, 1, 1
Real Constant Set Number ================>, 1
Angle of Rotation of u wrt X ================>, 0.0
Column Number ================>, 2
Coordinates of Column Center ================>, 0.0, 8.3333
External/Internal ================>, 1, 0
Real Constant Set Number ================>, 1
Angle of Rotation of u wrt X ================>, 0.0
Column Number ================>, 3
Coordinates of Column Center ================>, 0.0, 16.6667
External/Internal ================>, 1, 0
Real Constant Set Number ================>, 1
Angle of Rotation of u wrt X ================>, 0.0
<table>
<thead>
<tr>
<th>Column Number</th>
<th>Coordinates of Column Center</th>
<th>External/Internal</th>
<th>Real Constant Set Number</th>
<th>Angle of Rotation of u wrt X</th>
</tr>
</thead>
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<td>1</td>
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</tr>
<tr>
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<td>1</td>
<td>0.0</td>
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<td>0.0</td>
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<tr>
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<td>1, 0</td>
<td>1</td>
<td>0.0</td>
</tr>
<tr>
<td>Column Number</td>
<td>Coordinates of Column Center</td>
<td>External/Internal</td>
<td>Real Constant Set Number</td>
<td>Angle of Rotation of ( u ) wrt X</td>
</tr>
<tr>
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<td>----------------------------</td>
<td>------------------</td>
<td>--------------------------</td>
<td>----------------------------------</td>
</tr>
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<td></td>
</tr>
<tr>
<td>Column Number</td>
<td>Coordinates of Column Center</td>
<td>External/Internal</td>
<td>Real Constant Set Number</td>
<td>Angle of Rotation of u wrt X</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------------------------</td>
<td>------------------</td>
<td>-------------------------</td>
<td>-----------------------------</td>
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External/Internal: 0, 0
Real Constant Set Number: 1
Angle of Rotation of u wrt X: 0.0

Column Number: 32
Coordinates of Column Center: (-33.3333, 25.0)
External/Internal: 0, 0
Real Constant Set Number: 1
Angle of Rotation of u wrt X: 0.0

Column Number: 33
Coordinates of Column Center: (-41.6667, 25.0)
External/Internal: 0, 0
Real Constant Set Number: 1
Angle of Rotation of u wrt X: 0.0

Beam Connectivity:
1, 2, CI 0, >60833
2, 3, C/C Height: 10.0
3, 4, Clear Opening: 6.0833
4, 5, C/C Height: 10.0
5, 6, Beam Connectivity: 1, 2
6, 7, Clear Opening: 6.0833
7, Beam Connectivity: 2, 2
<table>
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<th>Real Constant Set Number</th>
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C/C Height = 10.0
Real Constant Set Number = 2
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C/C Height = 10.0
Real Constant Set Number = 2
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Beam Connectivity = 20, 21
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C/C Height = 10.0
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Beam Connectivity: 27, 32
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C/C Height: 10.0
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Beam Line Number: 35
Beam Connectivity: 32, 33
Clear Opening: 6.0833
C/C Height: 10.0
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Beam Connectivity: 33, 16
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C/C Height: 10.0
Real Constant Set Number: 2
Load Type (Point/UDL): UDL
Magnitude of the Load: 1.0
Direction of the Load: 90.0
Number of Terms to be Considered: 15
Number of Levels to be Considered: 4
APPENDIX D

ANSYS INPUT DATA FILE

The following is a sample of the input data file for finite analysis of the example problem no 1 using the general purpose finite element package ANSYS.

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