BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **NAME 117** (Hydrostatics and Stability)

Full Marks : 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

Assume reasonable value for missing data (if any).

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A ship of length 520 ft has a load displacement of 18,230 tons and LCB 9.20 ft abaft amidships. At a waterline tangential to the margin line, the areas of immersed sections are given in Table 1. Determine the length and position of the flooded compartment for this condition assuming permeability of 70%. **(17)**

Table 1 : Areas of immersed sections

Station	A.P.	½	1	1½	2	3	4	5	6	7	8	9	9½	FP
Area (ft^2)	380	855	1240	1615	1930	2290	2350	2320	2260	2040	1430	1015	600	245

Use added weight method for the calculation.

(b) A shop 150 m long has a displacement mass of 14,000 Ton and its center of buoyancy is 1 m forward of midship. When a forward compartment is flooded it sinks to a waterline which is tangential to the margin line. The areas of the immersed sections up to the flooded waterline at equally spaced intervals are as follows:

Station	A.P.	1	2	3	4	5	6	7	8	9	FP
Area Immersed section (m^2)	0	42	96	138	157	178	192	205	188	117	0

Find the volume of lost buoyancy due to flooding and the distance of its centroid from the original center of buoyancy. Also, obtain a first approximation to the floodable length assuming a permeability of 80%. **(18)**

2. (a) Find the relationship between the vertical prismatic coefficient, the water plane area coefficient and the block coefficient. **(10)**

(b) A container ship has the following water plane area coefficient values commencing at the base 0.427, 0.504, 0.577 and 0.647. At the summer load waterline, the water plane Area coefficient is 0.715. These water plane area coefficient values are equidistant apart up to the draft moulded. Calculate the block coefficient when this container ship is loaded up to her summer load waterline. **(10)**

(c) A ship 135 m long, 18 m beam and 7.6 m draught has a displacement of 14,000 Ton. The area of the load waterplane is 1,925 m^2 and the area of the immersed midship section is 130 m^2. Calculate the midship section area coefficient and the vertical prismatic coefficient. **(15)**

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3. (a) The lengths of the half-ordinates of an 80 m long waterplane commencing from forward as 0, 3.6, 5.0 5.3, 4.8 and 0.5. There is a 2.8 m long half-ordinate right midway between the last two half ordinates. Find the area of the waterplane. **(20)**

   (b) Define Bonjean Curves with schematic diagram. What are the uses of Beanjean Curves in ship design? **(10)**

   (c) Define the Angle of Loll. **(5)**

4. (a) Describe the requirements and procedure of conducting the inclining experiment of a ship. **(15)**

   (b) Derive the expression of transverse BM of a ship. **(12)**

   (c) Derive the wall-sided formula for righting lever. **(8)**

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) A box shaped vessel having length 110 m, breadth 12 m, depth 8 m is floating at draft 6 m. A midship compartment extending the full breadth of the vessel is having a length of 9 m. If the KG is 4.8 m and floating in salt water, Find **(20)**

   (i) the bilged draft;

   (ii) the initial GM of the vessel;

   (iii) the GM of the vessel in bilged condition;

   (iv) The GZ of the vessel at 1° heel for conditions (ii) and (iii).

   (b) A vessel having length 150 m, breadth 18 m, MCT 1 cm 150 tonnes-metres, TPC 25 is drawing 6.35 m F, 6.65 m A and loads the following: **(15)**

   230 tonnes in No. 1 hold 50 m fwd of C. F.

   800 tonnes in No. 3 hold 20 m fwd of C. F.

   500 tonnes in No. 4 hold 21 m abaft of C. F.

   She discharges 200 tonnes from No. 2 hold which is 36 m fwd of C. F. and discharges 105 tonnes from forepeak Tank which is 60 m fwd of C. F. The C. F. (centre of Flotation) is 5 m abaft the midship. Calculate the drafts at the perpendiculars.

6. (a) A box shaped vessel 45 m × 10 m × 6 m is floating in salt water at a draft of 4 m Fwd and Aft. GM = 0.6 m. Calculate the dynamical stability to 20 degrees heel and also explain what it means. **(15)**

   (b) Describe the IMO stability requirements for intact stability applicable to all ships. **(12)**

   (c) When a vessel of 5300 tonnes displacement KM 7.7 m is inclined by shifting 10 tonnes 16 m, it is noted that the mean defection of a plumbline 12 m long is 33.25 cm. What is her KG and inclined angle θ? **(8)**

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NAME 117

7. (a) A ship of 9900 tonnes displacement has KM = 7.3 m, and KG = 6.4 m. She has yet to load two 50 tonne lifts with her own gear and the first lift is to be placed on deck on the inshore side (KG 9 m and C. G 6 m out from the centre line). When the derrick plumbs the quay its head is 15 m above the keel and 12 m out of the centre line. Calculate the maximum list during the operation. **(15)**

   (b) A ship has a displacement of 3000 tonnes. On the vessel, there is a rectangular double bottom tank having length 15 m long and 8 m wide. This tank is partially filled with ballast water of density 1025 kg/m^3. If the GM_T without free surface effect is 0.18 m, calculate the virtual loss in GM_T and the final GM_T when the double bottom has: **(20)**

   (i) No divisional bulkhead is fitted;

   (ii) One transverse bulkhead is fitted at midlength;

   (iii) One longitudinal bulkhead is fitted on the cenre line;

   (iv) Two longitudinal bulkheads fitted giving three equal divisions;

8. (a) With neat sketch, describe the three common methods of sideway launching. **(13)**

   (b) Explain a launching diagram, explain how do you get necessary information for plotting a launching diagram. **(12)**

   (c) Explain light weight deadweight, gross tonnage and net tonnage of a ship. **(10)**

L-1/T-1/NAME Date : 27/10/2019

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **PHY 113** (Structure of Matter, Electricity, Magnetism & Modern Physics)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define electric field and draw the field lines. **(5)**

   (b) Consider a circular plastic disk of radius R that has a positive surface charge of uniform density σ on its upper surface. Calculate the electric field at the point P which is at a distance z from the disk along its central axis. Explain the condition when (i) $R \rightarrow \infty$ and z is finite and (ii) $z \rightarrow 0$ and R is finite, respectively. **(20)**

   (c) At what distance along the central perpendicular axis of a uniformly charged plastic disk of radius 0.600 m is the magnitude of the electric field equal to one-half the magnitude of the field at the center of the surface of the disk? **(10)**

2. (a) Define drift speed. Deduce the current density equation in terms of drift speed. **(9)**

   (b) Consider an RC series circuit consisting of the capacitor C, an ideal battery of emf ξ, and a resistance R. Show that the potential difference $V_C(t)$ across the capacitor during the charging process is **(18)**

$$V_C(t) = \xi\left(1 - e^{-\frac{t}{RC}}\right)$$

   (c) What is the drift speed of the conduction electrons in a copper wire with radius r = 900 μm when it has a uniform current i = 17mA? Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section. Given that, copper's molar mass, $M = 63.54 \times 10^{-3}$ kg/mol and density, $\rho_{mass} = 8.96 \times 10^{3}$ kg/m^3, respectively. **(8)**

3. (a) Explain the mechanism of Hall effect. Derive an equation for the density of charge carries using Hall effect. **(10)**

   (b) State and explain Amperes Law. **(6)**

   (c) Briefly explain the Biot-Savart law. Show the magnitude of the magnetic field **B** at a perpendicular distance R from a long (infinite) straight wire carrying a current i is given by **(19)**

$$B = \frac{\mu_0 i}{4\pi R}$$

4. (a) Briefly describe the nuclear liquid drop model and shell model. **(14)**

   (b) Derive an expression for semi-empirical mass formula of a nucleus. **(21)**

Contd P/2

PHY 113

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What is Compton effect? Show that the expression for the change in wavelength of a photon undergoing Compton scattering is $\lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\varphi)$, where the symbols have their usual meaning. What is Compton wavelength? Calculate its value. **(25)**

(b) If the maximum kinetic energy given to an electron in a Compton scattering experiment is 10KeV, what is the wavelength of the incident x-ray? **(10)**

6. (a) Explain inertial frame of reference and non-inertial frame of reference. **(6)**

(b) Describe the Michelson-Morley experiment. Explain the physical significance of the result of this experiment. **(20)**

(c) The rest mass and kinetic energy of a particle are m_0 and K, respectively. Show that the momentum of that particle is **(9)**

$$P = \sqrt{\frac{K^2}{c^2} + 2m_0 K}$$

where the symbols have their usual meaning.

7. (a) Establish a relation between Miller indices of plane (hkl) and interplanar distance (d_{hkl}) for a crystal system in which the crystallographic axes (X, Y and Z) are mutually perpendicular to each other. Show that the ratio interplanar distance for the low index planes d_{100}: d_{110}: d_{111} of an fcc crystal does not match to that od simple cubic crystal. **(15)**

(b) Write down the properties of X-rays that are useful for analyzing the structure of a solid. Deduce Bragg's law of X-ray diffraction. Write down the name of X-ray diffraction techniques used for structural analysis. **(15)**

(c) X-rays of wavelength 2.75 $\mathring{A}$ are incident on the first (220) plane of an fcc crystal at an angle 15.45°. Find the interplanar distance between two parallel (220) planes. **(5)**

8. (a) Describe cesium chloride (CsCl) structure with a schematic diagram. How does this structure differ from a bcc structure? Find the expression of packing factor of CsCl structure. Evaluate packing factor of CsCl structure considering the ionic radii of Cs^+ and Cl^- ions as 0.167 nm and 0.181 nm, respectively. How does packing factor of CsCl structure differ from a standard simple cubic structure? **(15)**

(b) Distinguish Metal, semiconductor and insulator in the light of band theory. What happens to the band structure of an intrinsic semiconductor when it is doped with n-type and p-type impurities? **(15)**

(c) Describe metallic bonds in solid. Why metallic bond is considered to an unsaturated bond? **(5)**

L-1/T-1/NAME **Date : 31/10/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **CHEM 117** (Chemistry-I)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Give two pieces of evidence for the wave model and two for the particle model of light. **(8)**

(b) What new idea about light did Einstein use to explain the photoelectric effect? Why does the photoelectric effect exhibit a threshold frequency but not a time lag? **(4+6)**

(c) (i) Why does the Bohr model of Hydrogen atom violate the uncertainty principle?

(ii) An electron (mass of electron is 9.11×10^{-31} kg) moving near an atomic nucleus has a speed of 6×10^{6} m/s ± 1%. Find the uncertainty in its position (Δx) and comment on the precision on locating the electron in the atom. **(5+5)**

(d) What physical meaning is attributed to Ψ^2? What feature of an orbital is related to each of the following? **(7)**

(i) Principle quantum number (n) (ii) Angular momentum quantum number (l) (iii) Magnetic quantum number (m_l).

2. (a) An electron in the hydrogen atom makes a transition from an energy state of principle quantum numbers n_i to the $n = 2$ state. If the photon emitted has a wavelength of 486 nm, what is the value of n_i ? The Rydberg constant for hydrogen is (2.18×10^{-18} J). **(8)**

(b) What is radial probability distribution curve? Consider both the filled and unfilled orbitals of element **X**. Determine the number of (i) radial nodes in a 3s (ii) angular nodes in the 4p_y orbital (iii) total nodes in a 4f orbital. **(4+6)**

(c) To form S^{2-} ions from gaseous sulfur atoms requires 214 kJ/mol, but these ions exist in solids such as K_2S; Explain. How does the lattice energy of an ionic compound depend on the charges and sizes of the ions? **(10)**

(d) What are main group elements? Sketch the outline of the periodic table and show group and period trends in the atomic size, first ionization energy and metallic property of the elements. **(7)**

3. (a) Justify why does hydrogen gas consist of H_2 molecules and not separate H atoms? **(5)**

(b) Write Lewis structures for the following species, including all resonance forms, show formal charges and name the geometry around the central atom: (i) OCN^- (ii) NNO (atom order as indicated). **(8)**

Contd P/2

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Contd... Q. No.3

(c) (i) Why is the study of molecular geometry important? (ii) Which of the following numbers of electron groups can give rise to a bent (V-shaped) molecule: two, three, four, five, six? Draw an example for each case showing the shape classification (AB_mE_n where A is the central atom, B and E represents terminal atoms and lone pairs respectively) and the ideal bond angle. **(4+8)**

(d) Draw molecular orbital diagram for N_2^+ molecule. Write the **valence** electron configuration of this molecule and compare it's bond energy and bond length with N_2 molecule. **(10)**

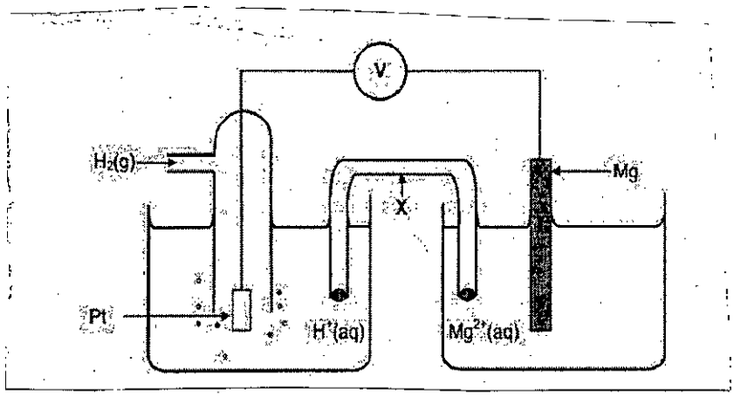
4. (a) Sketch the shapes of the following molecule orbitals: (i) $\sigma_{1s}\,\sigma_{1s}^*$ and (ii) π_{2p}, π_{2p}^*. Molecular oxygen shows paramagnetic property – explain. **(8)**

(b) Balance the following skeleton reactions and identify the oxidizing and reducing agents: **(6)**

(i) $ClO_3^-(aq) + I^-(aq) \rightarrow I_2(s) + Cl^-(aq)$[acidic]

(ii) $MnO_4^-(aq) + SO_3^{2-}(aq) \rightarrow MnO_2(s) + SO_4^{2-}(aq)$[basic]

(c) The galvanic cell represented below consists of a hydrogen half-cell and a magnesium half-cell at standard conditions. The reading on the voltmeter is **2.36v**. **(16)**



(i) Label all the components of this cell. What are the conditions needed for the hydrogen half-cell to function at standard conditions?

(ii) Write down the cell notation/diagram for this cell. Give the balanced NET (overall) cell reaction that takes place in this cell.

(iii) Calculate the standard reduction potential of the magnesium half-cell.

(iv) Comment on the spontaneity of the cell reaction based on the standard free energy change. (F= 96,484 C mol^{-1}).

(d) The concentration of K^+ ion in the interior and exterior of a nerve cell are 400 mM and 15 mM, respectively. Find the electrical potential across the membrane. $[K^+ + e^- = K; E^\circ = -2.92]$. Which direction the K^+ movement will be spontaneous? **(5)**

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SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Why is molecularity applicable only for elementary reactions and order is applicable for elementary as well as complex reactions? **(7)**

(b) Enumerate the different methods employed in the determination of the "order of a reaction". Give one method in detail. **(15)**

(c) Thermodynamic feasibility of the reaction alone cannot decide the rate of the reaction. Describe the factors that control the chemical reactivities. **(8)**

(d) The decomposition of dimethyl ether at 504°C is first order reaction with a half-life of 1570 seconds what fraction of an initial amount of dimethyl ether does remain after 4710 seconds? **(5)**

6. (a) Differentiate between internal energy and enthalpy of a chemical system. How are internal energies being changed during chemical reactions? **(8)**

(b) Explain the differences between heat and temperature. Under what condition is heat being transferred from one chemical system to another? How would you determine the enthalpy charges of chemical reaction systems? Describe the method in detail. **(15)**

(c) State and explain the third law of thermodynamics. How is Gibbs free energy related to reaction spontaneity? **(7)**

(d) Suppose you have 125 mL of coffee in a well-insulated cup, which is too hot to drink. What volume of cold milk at 12°C would you have to add to reduce the temperature of the coffee to 72°C? **(5)**

Assume coffee and milk have the same specific heat and density water: S_{H_2O} = 4.184 J/g°C, d_{H_2O} = 1 g/mL).

7. (a) Explain each of the following terms: **(8)**

(i) Ideal Solution (ii) Azeotropic mixture

(b) What is meant by reverse osmosis? How could the osmotic pressure be measured? Describe the laws of osmotic pressure. Explain how you would determine the molecular mass of a substance in solution from its osmotic pressure? **(15)**

(c) How would you illustrate the solubility of a gas into a liquid solvent using Henry's law? Explain the limitation of the laws. **(7)**

(d) Lysozyme extracted from chicken eggs white has a molar mass of 13930 $gmol^{-1}$. Exactly 0.1g of this protein is dissolved in 50 kg of water at 298K. Calculate the vapor pressure lowering, the depression in freezing point, the elevation of boiling point, and the osmotic pressure of this solution. (The vapor pressure of pure water at 298K is 23.76 mmHg). **(5)**

(K_b for water is 0.51 $Kmol^{-1}kg$ and K_f for H_2O is 1.86 $Kmol^{-1}kg$)

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8. (a) State the phase Rule. Illustrate the terms used in the Rule. Explain how the phase Rule could be applied to study the characteristics of heterogeneous equilibria systems. **(8)**

(b) Snow forms in the upper atmosphere in a cold air mass that is supersaturated with water vapor. When the snow later falls through a lower, warm air mass, rain forms. When this falls on a sunny spot, the drops evaporate. Describe all of the phase changes that have occurred in those observations. Draw a phase diagram and determine the number of degree of freedoms, phases, and components in each of the phase changes in the system. **(15)**

(c) Explain how the dynamic nature of chemical equilibria could be employed to determine the equilibrium constant of a reversible reaction. How is the equilibrium constant differed from the velocity constant of a chemical reaction? **(7)**

(d) Eggshells are composed mostly of calcium carbonate ($CaCO_3$) formed by the following reaction: **(5)**

$$Ca^{2+}_{(aq)} + CO^{2-}_{3(aq)} \rightleftharpoons CaCO_{3(s)}$$

The carbonate ions are supplied by carbon dioxide produced during metabolism. Explain why eggshells are thinner in the summer when the rate of panting by chickens is greater. Suggest a remedy for this situation.

L-1/T-1NAME **Date : 09/11/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **MATH 281** (Vector Analysis and Differential Equations (Special Types))

Full Marks : 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

Symbols have their usual meaning.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Solve the following differential equations: **(20)**

   (i) $\left(x^3D^3+6x^2D^2+8xD+2\right)y=x^2+3x-4$

   (ii) $\left[(3x+2)^2D^2+5(3x+2)D-3\right]y=x^2+x+1$

   (b) Solve the following differential equation by the method based on factorization of the operator: $\left[xD^2+(x-1)D-1\right]y=x^2$ **(15)**

2. Solve in series by the method of Fröbeneous of the following differential equation: **(35)**

   $\left(x-x^2\right)y''(x)+(1-x)y'(x)-y(x)=0$

3. Show that the following:

   (a) $xJ_n'(x)=nJ_n(x)-xJ_{n+1}(x)$ **(12)**

   (b) $\frac{d}{dx}\left[J_n^2(x)+J_{n+1}^2(x)\right]=2\left[\frac{n}{x}J_n^2(x)-\frac{n+1}{x}J_{n+1}^2(x)\right]$ **(12)**

   (c) $\int_0^x x^2J_o(x)J_1(x)dx=\frac{x^2J_1^2(x)}{2}$ **(11)**

4. (a) Show that $nP_n(x)=(2n-1)xP_{n-1}(x)-(n-1)P_{n-2}(x)$. **(13)**

   (b) Show that $P_n(-x)=(-1)^nP_n(x)$ and hence deduce that $P_n(-1)=(-1)^n$. **(11)**

   (c) Prove that $\int_{-1}^{1}P_m(x)P_n(x)dx=0,(m\neq n)$. **(11)**

Contd P/2

MATH 281

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Two adjacent sides of a parallelogram are the vectors $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $-2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$. Find the angle between the two diagonals of the parallelogram and the angles which the shorter diagonal makes with each side. Find also a unit vector perpendicular to the plane containing the parallelogram. **(12)**

(b) Find the volume of the tetrahedron having vertices $A(0, -1, -1)$, $B(4, 5, q)$, $C(3, 9, 4)$ and $D(-1, 1, 1)$. Also find the value of q for which these four points are coplanar. **(12)**

(c) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{p}, \mathbf{q}, \mathbf{r}$ are such that

$\mathbf{p} \cdot \mathbf{a} = \mathbf{q} \cdot \mathbf{b} = \mathbf{r} \cdot \mathbf{c} = 1$

$\mathbf{p} \cdot \mathbf{b} = \mathbf{p} \cdot \mathbf{c} = \mathbf{q} \cdot \mathbf{a} = \mathbf{q} \cdot \mathbf{c} = \mathbf{r} \cdot \mathbf{a} = \mathbf{r} \cdot \mathbf{b} = 0$

Prove that $\mathbf{p} = \dfrac{b \times c}{a \cdot (b \times c)}$, $\mathbf{q} = \dfrac{c \times a}{a \cdot (b \times c)}$, $\mathbf{r} = \dfrac{a \times b}{a \cdot b \times c}$. **(11)**

6. (a) A particle moves so that its position vector is given by $\mathbf{r} = \cos \omega t + \sin \omega t\, \mathbf{j}$ where ω is a constant. Show that **(12)**

(i) the velocity $\mathbf{v}$ of the particle is perpendicular to $\mathbf{r}$,

(ii) the acceleration $\mathbf{a}$ is directed toward the origin and has magnitude proportional to the distance from the origin,

(iii) $\mathbf{r} \times \mathbf{v}$ = a constant vector.

(b) Differentiate twice with respect to t

$$V \bullet \left(\frac{dV}{dt} \times \frac{d^2V}{dt^2} \right)$$ where $\mathbf{V}$ is a vector function of t. **(12)**

(c) Find the curvature and torsion at any point '$\mathbf{t}$' of the curve $\mathbf{x} = \mathbf{t}$, $\mathbf{y} = \mathbf{t}^2$, $\mathbf{z} = \mathbf{t}^3$. **(11)**

7. (a) Find the equation of tangent line and normal plane to the curve of intersection of

$x^2 + y^2 + z^2 = 1, x + y + z = 1$ at $(1,0,0)$. **(12)**

(b) Evaluate $\int_C \mathbf{F}.dr$ where $\mathbf{F} = \left(xz^2 + y, z - y, xy - z\right)$ and C is the curve $x^2 + y^2 = a^2, z = h$ from (0, a, h) to $(a, 0, h)$. **(12)**

(c) Using Green's theorem in plane, evaluate $\oint_C [(y - \sin x)dx + \cos x dy]$ where C is the triangle whose vertices are (0,0), $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 1\right)$. **(11)**

8. (a) State Divergence theorem and verify it for $\mathbf{F} = \left(x^2 - yz\right)\mathbf{i} + \left(y^2 - zx\right)\mathbf{j} + \left(z^2 - xy\right)\mathbf{k}$ taken over the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. **(20)**

(b) Evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot n dS$, where $\mathbf{F} = y\mathbf{i} + (x - 2xz)\mathbf{j} - xy\mathbf{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. **(15)**

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L-1/T-1/NAME Date : 21/10/2019

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **MATH 181** (Differential Calculus and Integral Calculus)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A function $f(x)$ is defined as follows: **(15)**

$$f(x)=\begin{cases} x^2 & \text{when} \quad 0<x<1 \\ 3+x & \text{when} \quad 1\le x\le 2 \\ 3x-x^2 & \text{when} \quad x>2 \end{cases}$$

Discuss the continuity and differentiability of $f(x)$ at $x = 2$. Also sketch the graph of $f(x)$.

(b) Evaluate: $\lim\limits_{x\to 0}(\cos x)^{\cos ec^2 x}$ **(10)**

(c) Find the n-th derivative of the function $y=\dfrac{1}{x^2+16}$. **(10)**

2. (a) If $x=\sin\left(\dfrac{\ln y}{a}\right)$ prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$ and also find the value of y_n when $x=0$. **(13)**

(b) If $u=r^3$ and $x^2+y^2+z^2=r^2$ then prove that $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}=12r$. **(12)**

(c) Expand $y=e^x \sec x$ in a series of ascending powers of x. **(10)**

3. (a) Find the maximum and minimum values of u where **(12)**

$$u=\frac{4}{x}+\frac{36}{y} \text{ and } x+y=2.$$

(b) If the straight line $x\cos\alpha+y\sin\alpha=p$ touches the curve $\left(\dfrac{x}{a}\right)^{\frac{n}{n-1}}+\left(\dfrac{y}{b}\right)^{\frac{n}{n-1}}=1$, then show that $(a\cos\alpha)^n+(b\sin\alpha)^n=p^n$. **(13)**

(c) Find the value of c in the Mean value theorem for the function $f(x)=x^3-5x^2+12x-20$ in the interval (0, 4). **(10)**

4. (a) Find the pedal equation of the cardioid $r=a(1-\cos\theta)$. **(10)**

(b) Find the radius of curvature at the point θ on the cycloid $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$. **(10)**

(c) Find all the asymptotes of the curve

$$4x^3-x^2y-4xy^2+y^3+3x^2+2xy-y^2-7x+5=0.$$ **(15)**

Contd P/2

MATH 181

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Evaluate the following integrals:

(i) $\int \frac{(\sin\theta+\cos\theta)}{5-\sin 2\theta}d\theta$ **(10)**

(ii) $\int \frac{x+3}{\sqrt{3-4x+4x^2}}dx$ **(10)**

(b) Derive reduction formula for $\int e^{az}\sin^n z\,dz$ and then evaluate $\int e^{2z}\sin^9 z\,dz$ **(15)**

6. (a) Evaluate $\underset{n\to\infty}{Lim}\left[\frac{1}{n^2}\sec^2\left(\frac{1}{n^2}\right)+\frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right)+\frac{3}{n^2}\sec^2\left(\frac{9}{n^2}\right)+....\frac{1}{n}\sec^2(1)\right]$. **(10)**

(b) Show that $\int_0^\infty \frac{\ln x}{1+x^2}dx = 0$. **(10)**

(c) Using process of summation find the integral $\int_0^1 \left(\sqrt{x}\right)^3 dx$. **(15)**

7. (a) Define Gamma and Beta functions. Using Gamma and Beta function evaluate $\int_0^\infty x^m(\ln x)^n dx$, where $m>-1$ and, n is a positive integer. **(15)**

(b) Write down the formula to calculate arc length and area of a curve in Cartesian and polar coordinate systems. Also, find the perimeter of the four cusped hypocycloid $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$. **(20)**

8. (a) Find the area of the limacon $r = a\cos\theta \pm b$, where a and b are positive. **(15)**

(b) Find the volume and surface area of the solid generated by revolving the leminscate $r^2 = a^2\cos 2\theta$ about the initial line. **(20)**
