L-2/T-1/EEE Date : 09/11/2019

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **EEE 201** (Electric Circuits I)

Full Marks : 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks. Symbols have their usual meaning.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) The transistors Q_1 and Q_2 shown in Fig. for Q. 1. a are identical, calculate the channel current of Q_1. Given: $V_t = 1.0$ V and $\mu_n C_{ox}\frac{W}{L} = 1000\mu A/V^2$. **(20)**

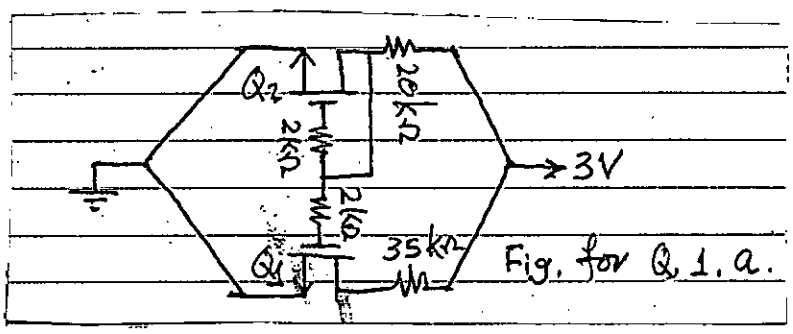

Fig. for Q. 1. a.

(b) Explain the operation of the electronic circuit shown in Fig. for Q. 1. b and sketch the output voltage (V_o) as a function of time (t) for the given sinusoid input voltage (V_i). **(15)**

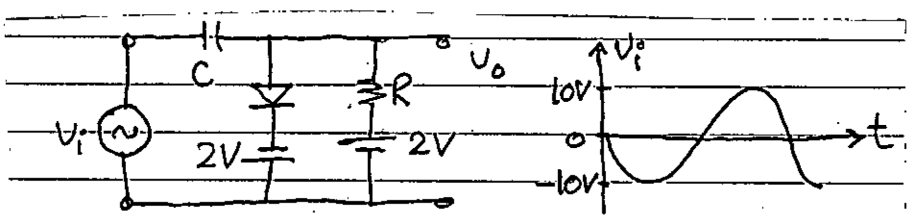

Fig. for Q. 1. b

2. (a) Determine currents through the diodes D_1, D_2 and D_3 in the circuit shown in Fig. for Q. 2. a Assume a pn junction built-in potential of 0.75 V and a reverse saturation current (scale current) of 1.0 fA at a temperature of 300 K, if necessary. **(18)**

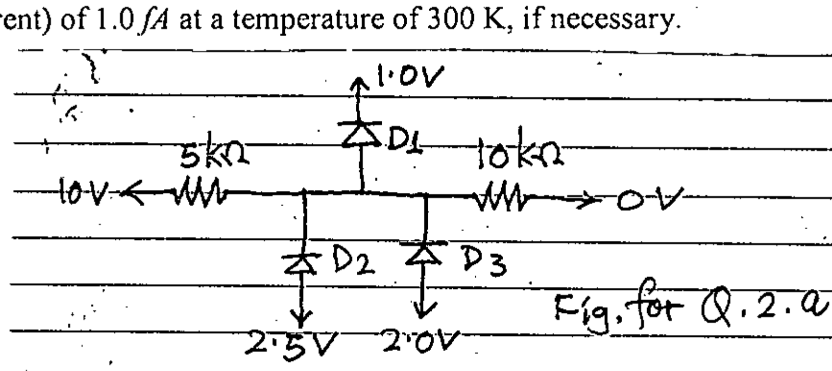

Fig. for Q. 2. a

(b) An MOS structure is fabricated with two different oxide layers as illustrated in Fig. for Q. 2. b Estimate the capacitance of the structure. Given: relative permittivities of HfO_2 and SiO_2 are 25 and 3.9, respectively. **(17)**

Contd P/2

EEE 201

Contd...Q. 2(b)

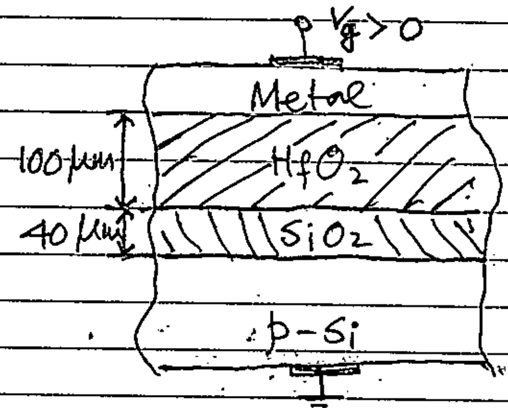

Fig. for Q.2.b

3. (a) Derive an expression for the drain current of a p-channel metal–oxide–semiconductor field effect transistor (p-ch MOSFET) with necessary illustrations. **(15)**

   (b) Explain operation of the circuit shown in Fig. for Q. 3. b and sketch time variation of the output voltage (V_o vs t) for the given V_s and V_k. Assume ideal diode. **(20)**

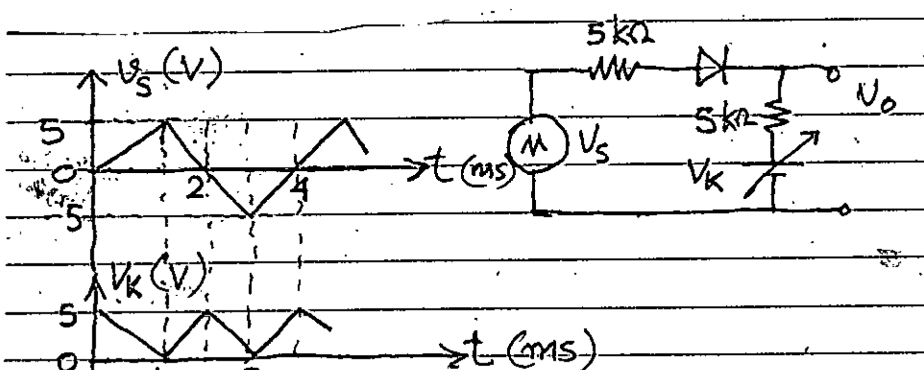

Fig. for Q. 3. b

4. (a) Calculate the current through the load, R_L in the circuit illustrated in Fig. for Q. 4. a. Transistor Q_1 and Q_2 are identical. Given: $V_t = 1.0$ V, $\mu_n C_{ox}\dfrac{W}{L} = 4mA/V^2$. **(18)**

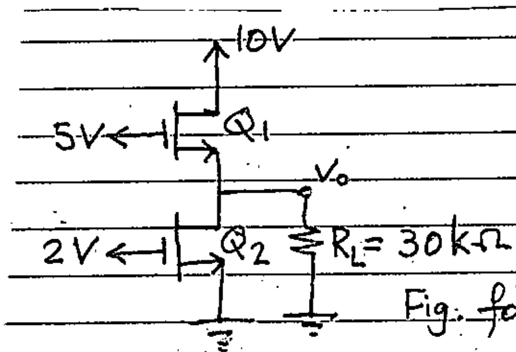

Fig. for Q. 4. a

   (b) Draw a single stage common gate amplifier circuit using n-channel MOSFET with appropriate biasing and small input signal. Derive expressions for input resistance, output resistance and voltage gain using small signal analysis. **(17)**

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SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Explain the requirements of biasing a transistor. Derive the design criteria for the biasing circuits of classical discrete-circuit bias arrangement and constant current source biasing. **(20)**

(b) The circuit shown in Fig. for Q. 5(b) provides a constant current I_0 as long as the circuit to which the collector is connected maintains the BJT in the active mode. Show that **(15)**

$$I_o = \frac{\alpha\left[\frac{V_{cc}R_2}{R_1+R_2} - V_{BE}\right]}{R_E + \frac{R_1 \| R_2}{(\beta+1)}}$$

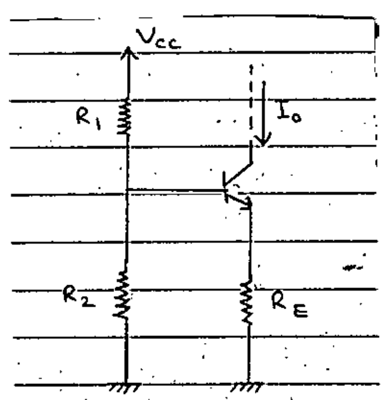

Fig. for Q. 5(b)

6. (a) A full-wave bridge rectifier with a filter capacitor C is to be designed to produce an output with a ripple of not more than 5 percent. An input line voltage of 120 V (rms), 50 Hz is available. Find the values of C, i_{Dmax}, $i_{D(avg)}$, PIV and turns ratio of the transformer. **(20)**

(b) Determine the maximum current flowing through the load resistance R_L in the circuit shown in Fig. for Q. 6(b). Plot load current i_L with respect to time (t) for the given sinusoid input voltage, v_i. **(15)**

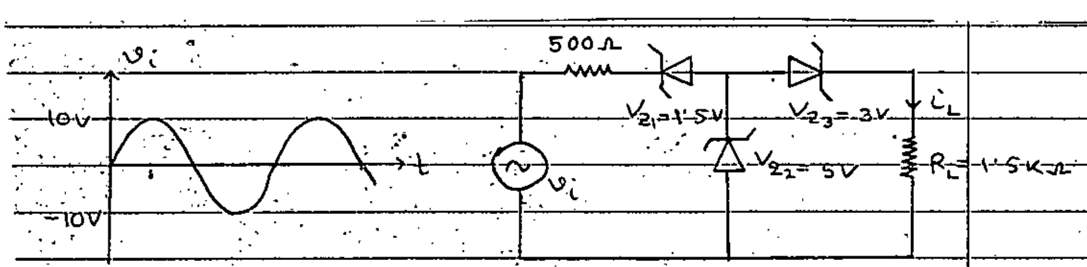

Fig. for Q. 6(b)

7. (a) Derive the small signal model of a BJT and calculate base resistance, emitter resistance and voltage gain. **(15)**

(b) Find the labeled node voltages in the circuit shown in Fig. for Q. 7(a). Assume $\beta = 100$. **(20)**

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Contd...Q.No. 7(b)

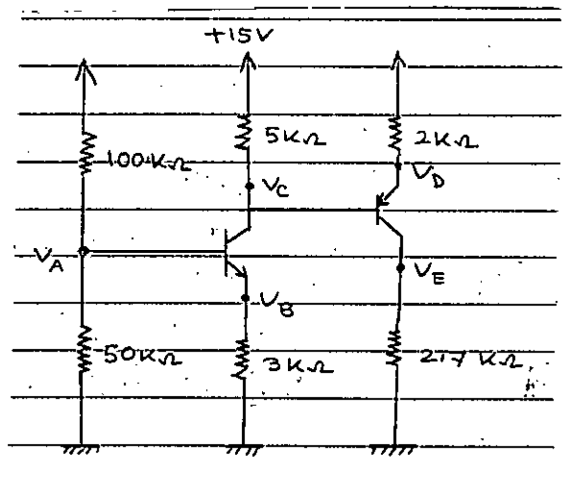

Fig. for Q.7(b)

8. (a) The transistor amplifier shown in Fig. for Q. 8(a) is biased with a current source I and has a very high β. Find the dc voltage at the collector, V_c. Replace the transistor with the simplified hybrid-π model and hence find the voltage gain V_c/V_i. **(17)**

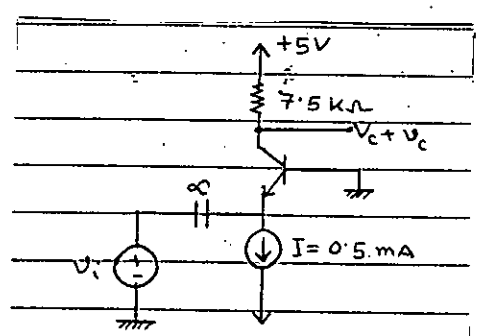

Fig. for Q. 8(a)

(b) For the emitter follower circuit shown in Fig. for Q. 8(b), the signal source is directly coupled to the transistor base. If the dc component of v_{sig} is zero, find the dc emitter current. Neglecting r_o, find R_{in}, the voltage gain v_o/v_{sig}, the current gain i_o/i_i and the output resistance R_{out}. Assume $\beta = 100$. **(18)**

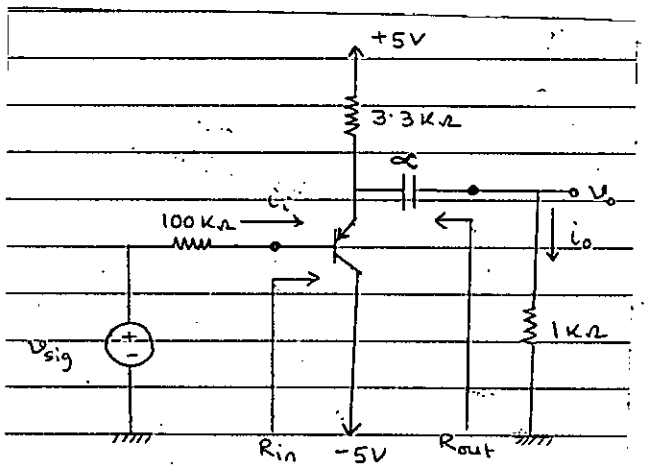

Fig. for Q. 8(b)

L-2/T-1/EEE **Date: 26/10/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2018-2019

Sub: **EEE 203** (Energy Conversion I)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

Symbols have their usual meanings.

1. (a) What are the assumptions made in an ideal transformer? Show that in a real transformer, **(12)**

   (i) $V_P/V_S \neq a$ (ii) $I_S/I_P \neq a$

   (b) What are the differences between core and shell type transformers? **(5)**

   (c) A single phase power source feeds a 180 kVA 20 kV/2.4 kV transformer through a feeder impedance of $(38.2 + j140)\Omega$. The transformer's equivalent impedance referred to L.T. side is $(0.25 + j1.0)\Omega$. The load on the transformer is 190 kW at 0.9 pf lagging and 2300 V. Calculate: **(18)**

   (i) The voltage at the power source.
   (ii) The voltage regulation of the transformer.
   (iii) The efficiency of the overall power system.

2. (a) Explain the magnetizing and core loss currents of a transformer. Explain with necessary formulae and diagrams why the magnetizing and core loss currents are non-sinusoidal even when the input voltage is sinusoidal. What are the power losses in a transformer? **(18)**

   (b) To determine the parameters of the equivalent circuit of a 15 kVA, 2400/240 V single phase transformer the open circuit and short circuit tests are performed. The recorded test data are: **(17)**

Open circuit test	Short circuit test
$V_{OC} = 230$ V	$V_{SC} = 47$ V
$I_{OC} = 2.1$ A	$I_{SC} = 6$ A
$P_{OC} = 50$ W	$P_{SC} = 160$ W

   Find–

   (i) The equivalent circuit referred to HT.
   (ii) The equivalent circuit referred to LT.
   (iii) The equivalent circuit in per unit.
   (iv) The core loss and copper loss at rated condition.

Contd P/2

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3. (a) Explain why Δ–Δ connected three-phase transformer is derated to 57.7% of its power rating when one of the phases is damaged and removed. What is the inrush of magnetizing current of a transformer? **(17)**

(b) A 100 VA, 120/12 V transformer is to be connected so as to form a step-up auto transformer. A primary voltage of 120 V is applied to the transformer (across the common winding). **(18)**

(i) What is the secondary voltage?

(ii) What is the maximum VA rating in this mode of operation?

(iii) Calculate the rating advantage of this auto transformer over the transformer's rating in conventional (two winding) 120/12 V operation.

4. (a) If two transformers are connected in parallel, show that the loads are divided between them in inverse proportion to their equivalent impedances when their turns ratios are equal. **(17)**

(b) Two transformers are operated in parallel to supply a load of 200 A at 0.8 pf lagging. The ratings of the two transformers are: **(18)**

T_1	T_2
41,000/2400 V	42,000/2400 V
200 kVA	400 kVA
$z_{eq} = (0.4 + j0.3)\Omega$ (ref. to LT)	$z_{eq} = (0.2 + j0.3)\Omega$ (ref. to LT)

Find–

(i) Load supplied by each transformer.

(ii) Circulating current if load current is zero.

(iii) Load supplied by each transformer if turns ratios are equal.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) A simple three-phase stator is shown in the figure 5(a). Currents in this stator are assumed positive if they flow into the unprimed end and out the primed end of the coils. The currents in the coil are expressed as: **(15)**

$$i_{aa'}(t) = I_M \sin \omega t$$
$$i_{bb'}(t) = I_M \sin(\omega t - 120^\circ)$$
$$i_{cc'}(t) = I_M \sin(\omega t - 240^\circ)$$

Find the expression of the resultant magnetic field, $\mathbf{B}_{net}$ **(t)**

Contd P/3

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Contd... Q. No. 5(a)

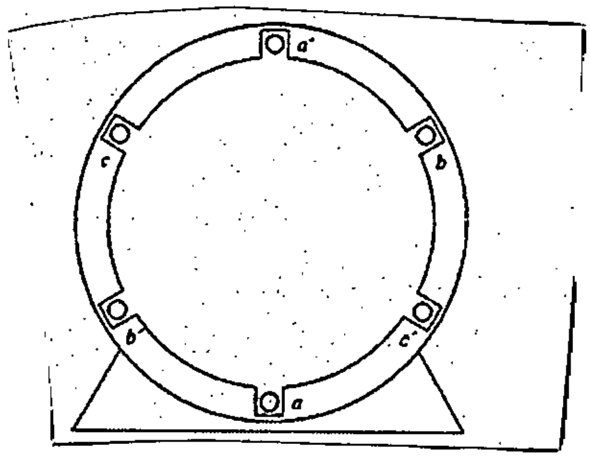

Fig. 5(a)

(b) A three-phase, design class C, Δ-connected, 25 hp, 600 V, 50 Hz induction motor operating at rated conditions, draws a line current of 27 A. Data from a locked rotor test (performed at 15 Hz), a no-load test (performed at 50 Hz), and a DC test are: **(20)**

Locked Rotor	No-Load	DC
$V_{line} = 54.7$ V	$V_{line} = 600$ V	$V_{DC} = 20$ V
$I_{line} = 27.0$ A	$I_{line} = 12.1$ A	$I_{DC} = 27.0$ A
$P_{3\text{-phase}} = 1650$ W	$P_{3\text{-phase}} = 1200$ W	

(i) Determine R_1, R_2, X_1, X_2, X_m.

(ii) Find the combined rotational losses.

(iii) Find stator and rotor power factor for slip = 0.02 at rated condition.

6. (a) Prove that, for a three phase induction motor, induced torque is given by **(17)**

$$T_{ind} = \frac{3\, V_{TH}^2\, {R_2}/{s}}{\omega_{sync}\left[\left(R_{TH} + {R_2}/{s}\right)^2 + \left(X_{TH} + X_2\right)^2\right]}$$

Induced torque becomes maximum if

$$s = \frac{R_2}{\sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}}$$

and maximum induced torque is independent of rotor resistance

$$T_{ind,\max} = \frac{3\, V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}\right]}$$

(b) A 460-V four-pole 75-hp 60-Hz Y-connected three-phase induction motor develops its full-load induced torque at 3.5 percent slip when operating at 60 Hz and 460 V. The per-phase circuit model impedances of the motor are **(18)**

$R_1 = 0.058\ \Omega$ $\quad X_M = 18\ \Omega$

$X_1 = 0.32\ \Omega$ $\quad X_2 = 0.386\ \Omega$

Mechanical, stray and core losses can be neglected in this problem

(i) Find the value of the rotor resistance R_2.

(ii) The rotor speed at maximum torque for this motor.

Contd P/4

7. (a) How a three-phase induction motor can run with single phase supply? **(5)**

(b) Describe with appropriate curves any three methods for speed control of a three phase induction motor. **(15)**

(c) The per-phase equivalent circuit of a 460-V, 50-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following parameters in per-unit values **(15)**

$R_1 = 0.02$ pu $\quad R_2 = 0.03$ pu

$X_1 = 0.1$ pu $\quad X_2 = 0.01$ pu

Determine the following:

(i) Slip at which the breakdown torque occurs.

(ii) Breakdown torque and starting torque.

(iii) New breakdown torque and starting torque when R_2 is 25% increased.

8. (a) (i) Explain how induction generators can be connected to a 3-phase bus. **(10)**

(ii) Explain the isolated operation of a 3-phase induction generator.

(b) What is "Dynamic braking" and "Plugging"? Briefly describe two methods of dynamic braking. **(15)**

(c) A 460-V 50-hp six-pole Δ-connected 60-Hz three-phase induction motor has a full-load slip of 4 percent, an efficiency of 91 percent, and a power factor of 0.87 lagging. At start up, the motor develops 1.75 times the full-load torque but draws 7 times the rated current at the rated voltage. This motor is to be started with reduced voltage. **(10)**

(i) What should be the input voltage of the induction motor so that the starting torque equals the rated torque of the motor?

(ii) What will be the motor starting current at this voltage?

L-2/T-1/EEE **Date: 20/10/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2018-2019

Sub: **EEE 211** (Continuous Signals and linear systems)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) State and explain the orthogonal condition of a set of signals over an interval. Are the exponential functions exp ($j2t$) and exp ($j4t$) orthogonal over the interval t = $[0,\pi]$? **(10)**

(b) Given the periodic signal

$$x(t) = 2 + \frac{1}{2}\cos(t + 45°) + 2\cos(3t) - \sin(4t + 30°),$$ **(10)**

(i) Find the exponential Fourier series of x(t)

(ii) Sketch the magnitude and phase spectra as a function of ω.

(c) The triangular waveform x(t) of Fig. for Q. No. 1 (c) with period T = 4 and peak amplitude A = 4 is applied to a series combination of a resistor R = 100 Ω and an inductor L = 0.1 H. Determine the power dissipated in the resistor. **(15)**

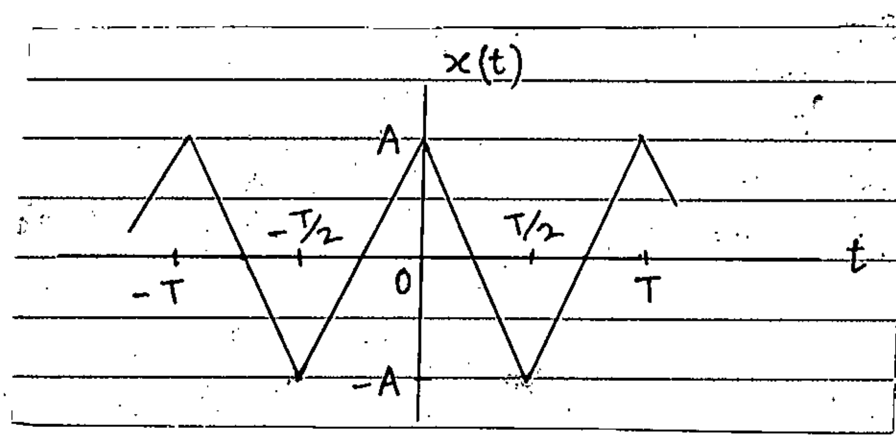

Fig. for Q. No. 1 (c)

2. (a) Prove that the energy of a signal x (t) is given by **(10)**

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |x(\omega)|^2 \, d\omega,$$

where X(ω) is the Fourier transform of x(t). Also, compute the energy of a sinc signal, sinc ($\frac{t}{2}$).

(b) The signal x(t) = exp [-αt] u(t) is input into a system with impulse response h(t) = sin (2t)/(πt). **(10)**

Contd P/2

EEE 211

Contd... Q. No. 2(b)

(i) Find the Fourier transform $Y(\omega)$ of the output.

(ii) For what value of α does the energy in the output signal equal one-half the input signal energy?

(c) A single-sideband, amplitude modulated signal is generated using the system shown in Fig. for Q. No. 2 (c). (i) Sketch the spectrum of y(t) for $\omega_f = \omega_m$. (ii) Find the mathematical expression for $h_f(t)$. Is it a realizable filter? **(15)**

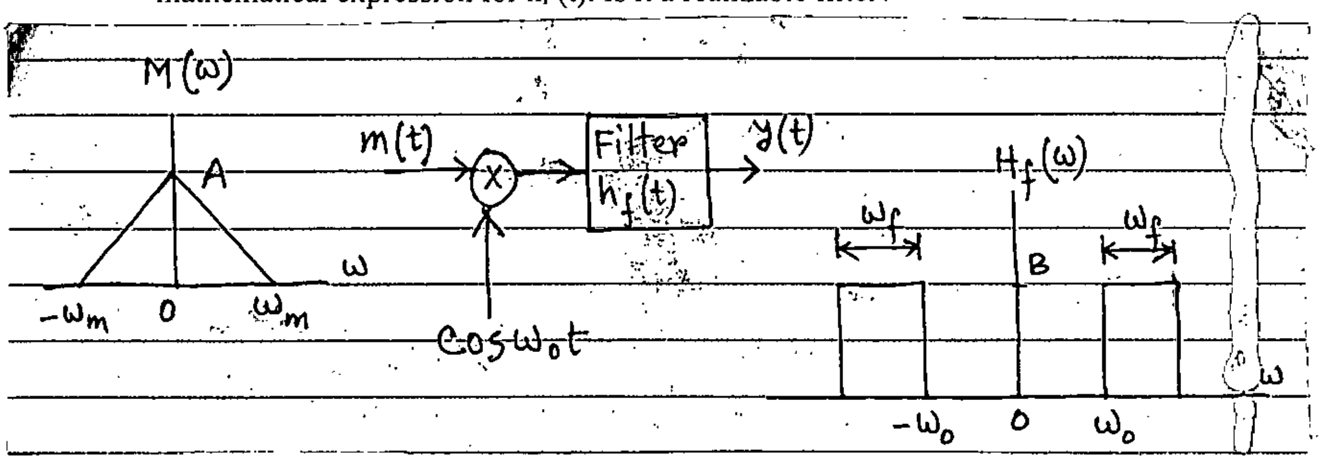

Fig. for Q. No. 2 (c)

3. (a) Show that the Fourier transform of a signal consisting of a train of impulses in time domain yields a train of impulses in frequency domain. Sketch both the signal and its spectrum. **(10)**

(b) Determine the Fourier transform of the signal $x(t) = \sin 1000\,\pi t + \cos 1200\,\pi t.$. Sketch the spectrum $X(\omega)$. What should be the sampling period that period that would ensure no aliasing? **(10)**

(c) A periodic signal x (t) with period T = 2 has the Fourier coefficients

$$C_n = \begin{cases} 0, & h = 0 \\ 0, n \text{ is even} \\ 1, n \text{ is odd} \end{cases}$$

The signal is applied to a linear time-invariant continuous-time system with the magnitude and phase spectra shown in Fig. for Q. No. 3 (c). Determine h(t) and the system output. (15)

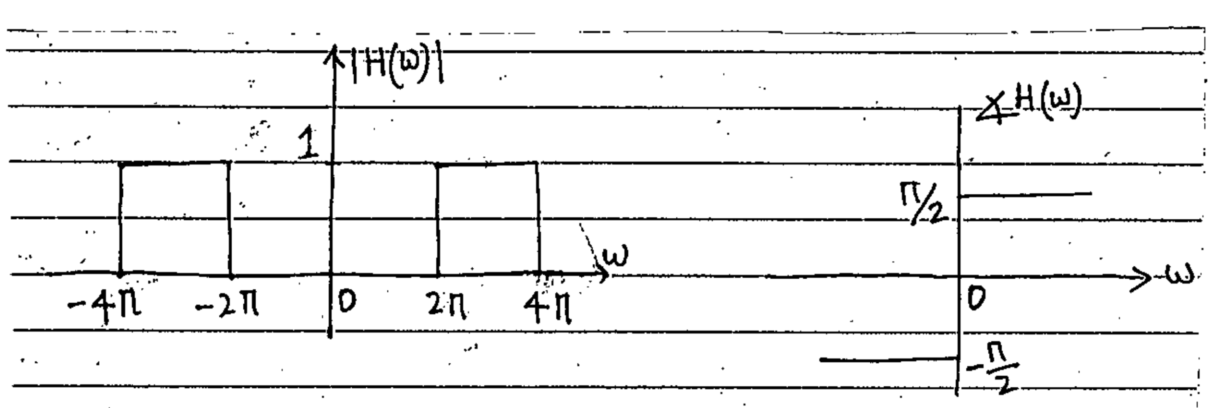

Fig. for Q. No. 3 (c)

Contd P/3

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4. (a) The input and the output of a causal LTI system are related by the differential equation $\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$. By using Fourier transformation,

(i) find the impulse response of the system,

(ii) find the output y(t) of the system if the input x (t) = $te^{-2t}u(t)$ (17)

b) For the mechanical system of Fig. for Q. No. 4(b), write the equation of motion in terms of the given mechanical quantities. Obtain the equivalent electrical equations and draw the f–i (force-current) and f-v (force–voltage) analogous electrical circuits. **(18)**

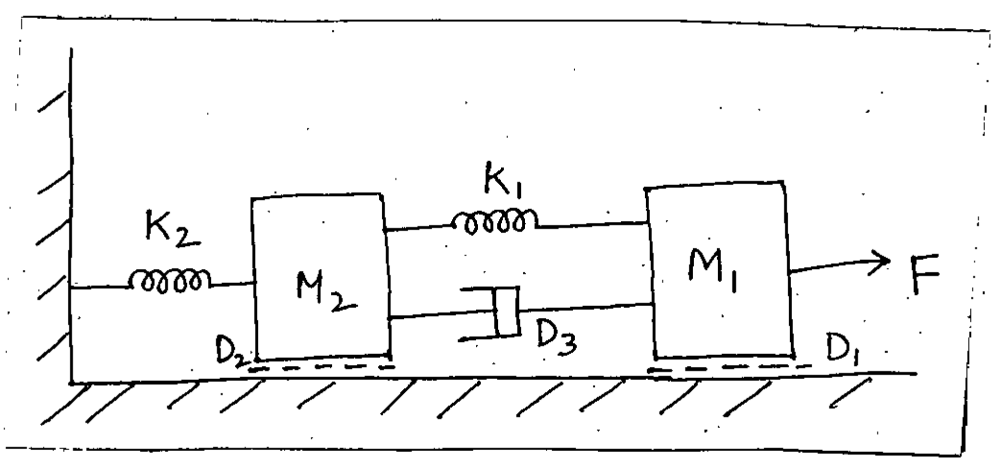

Fig. for Q. No. 4 (b)

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) The following information is provided about a real signal x(t) with Laplace transform X(s). **(15)**

(i) X(s) has exactly two poles.

(ii) X(s) has no zeros in the finite s-plane

(iii) X(s) has a pole at s = -1 + j

(iv) e^{2t} x(t) is not absolutely integrable.

(v) X(0) = 5

Determine X(s) and specify its region of convergence.

(b) Consider the circuit shown in Fig.for Q. No. 5 (b), where **(20)**

$$i_s(t) = \begin{cases} 2A, t \le 0 \\ 4e^{-2t}A, t \ge 0^+ \end{cases}$$

Determine $i_L(t)$ for $t \ge 0^+$ using Laplace transform method.

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Contd... Q. No. 5(b)

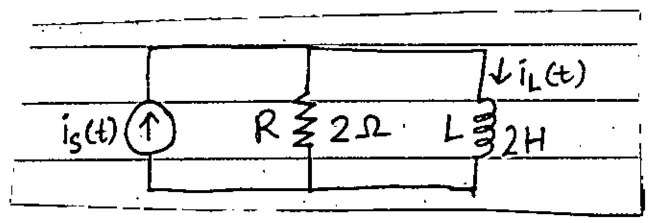

Fig. for Q. No. 5 (b)

6. (a) Determine whether the following signals are power or energy signals or neither. Justify the answer.

   (i) $x_1(t) = \exp\left[j\left(2\pi t + \frac{\pi}{4}\right)\right] + \exp[-2|t|]$,

   (ii) $x_2(t) = [u(\cos t) - \delta(t - k\pi)]^{0.5}$, where k = ±1, ± 3, ± 5, …….. ∞. **(15)**

   (b) Consider the voltage waveform v(t) shown in Fig. for Q. No. 6 (b). **(20)**

   (i) Express v(t) in terms of unit step and unit ramp functions.

   (ii) Compute the derivative of v(t) and sketch its waveform.

   (ii) Evaluate and sketch the even and odd components of the derivative of v(t)

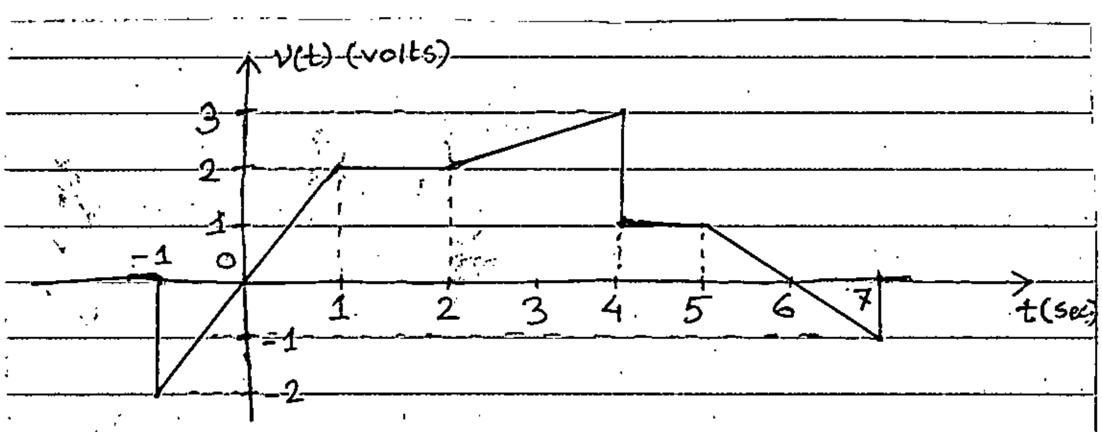

Fig. for Q. No. 6 (b)

7. (a) A linear system h(t) has the input-output as shown in Fig. for Q. No. 7(a). Determine whether the system is causal, time-invariant, and memory less. Also, find the output of the system for the input $x_1(t) = u(t) + u(t-1) - 2u(t-2)$ (20)

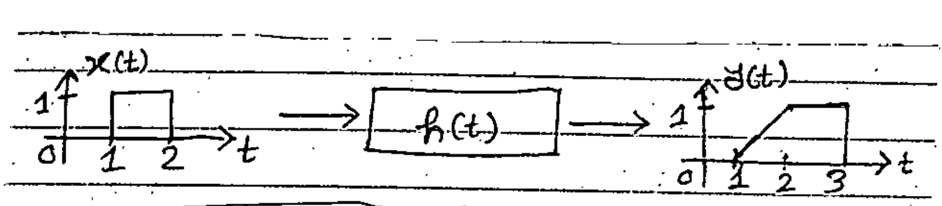

Fig. for Q. No. 7 (a)

EEE 211

Contd... Q. No. 7

(b) Consider an LTI system which has the impulse response of h(t) = u(t) –u(t-1). If $x(t) = t^2\,[u(t) - u(t-3)]$ is applied at the input of the system, obtain the output y(t) of the system using the graphical interpretation of convolution. Also, write the analytical expression of y(t) **(15)**

8. (a) Consider an LTI system with input x(t) and output y(t) governed by the differential equation $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + ky(t) = \frac{d^2x(t)}{dt^2}$, for $t \geq 0^+$

(i) Assume K is a real-valued constant and all the initial conditions to be zero. For what values of K is the system bounded-input bounded-output stable?

(ii) Find the zero-input response for the initial conditions, $y(0^+) = 0$ and $y'(0^+) = 1$. Assume K =2 for this case. (15)

(b) Consider the RLC circuit shown in Fig. for Q. No. 8 (b). This circuit is excited with a voltage source of x(t). Assuming the current in the circuit as the output and the current through the inductor, $v_1(t)$, and the voltage across the capacitor, $v_2(t)$, as the state variables, find the state space model of the circuit. Also, find a closed-form expression for the state transition matrix of the system. (20)

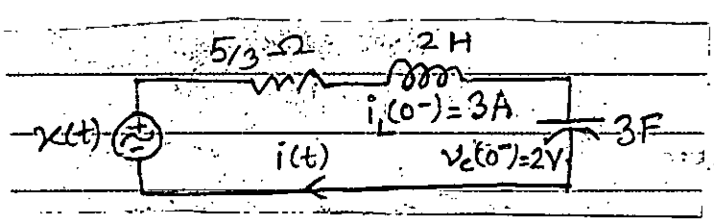

Fig. for Q. No. 8 (b)

L-2/T-1/EEE **Date: 03/11/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2018-2019

Sub: **MATH 259** (Linear Algebra)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Prove that every square matrix A can be expressed as the sum of a symmetric and skew-symmetric matrix. **(11)**

(b) Determine the values of a for which the system has no solutions, exactly one solution, or infinitely many solutions. **(12)**

$$\begin{aligned} x+y+\quad 7z &= -7 \\ 2x+3y+\quad 17z &= -16 \\ x+2y+(a^2+1)z &= 3a \end{aligned}$$

(c) For the matrix $A=\begin{bmatrix}1 & -2 & 1 & 3\\ 4 & -1 & 5 & 8\\ 2 & 3 & 3 & 2\end{bmatrix}$, find non-singular matrices P and Q such that PAQ is in the normal form B. **(12)**

2. (a) Find an upper triangular matrix that satisfies $A^3=\begin{bmatrix}1 & 30\\ 0 & -8\end{bmatrix}$, if possible. **(11)**

(b) Define elementary matrix. Express $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$ as a product of elementary matrices. **(12)**

(c) Use the inversion algorithm to find the inverse of $A=\begin{bmatrix}2 & -4 & 0 & 0\\ 1 & 2 & 12 & 0\\ 0 & 0 & 2 & 0\\ 0 & -1 & -4 & -5\end{bmatrix}$, if the inverse exists. **(12)**

3. (a) State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}2 & 2 & -2\\ 2 & 3 & -1\\ -2 & -1 & 3\end{bmatrix}$ and hence find A^{-1}. **(20)**

(b) Reduce the quadratic form $q=x_1^2+2x_2^2-3x_3^2+8x_1x_2+10x_1x_3-16x_2x_3$ to the canonical form and hence find the corresponding linear transformation, rank, index and signature. **(15)**

Contd P/2

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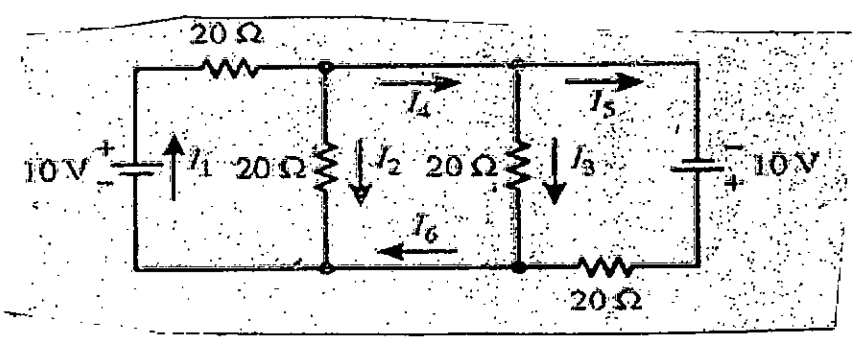
4. (a) Prove that if λ is an eigenvalue of an invertible matrix A, and $\mathbf{x}$ is a corresponding eigenvector, then $1/\lambda$ is an eigenvalue of A^{-1}, an $\mathbf{x}$ is a corresponding eigenvector. **(11)**

(b) Determine whether the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 5 & 6 \\ 4 & 6 & 7 \end{bmatrix}$ is diagonalizable. If so, find a matrix P such that $P^{-1}AP = D$. Also compute A^{17}. **(12)**

(c) Find the unknown currents in the circuit shown in the following figure: **(12)**



SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) Determine whether the following subsets are subspaces of R^4. If so, then find a basis in each case and their dimensions. **(15)**

(i) Set of vectors of the form (a, b, c, d), where $b + c + d = 0$.

(ii) Set of vectors of the form (a, b, c, d), where $d = 2a + 7c$ and $3c = 2a - 5b$.

(b) Let $T: R^4 \to R^3$ be the linear transformation defined by **(20)**

$$T(x, y, s, t) = (4x + y - 2s - 3t,\ 2x + y + s - 4t,\ 6x - 9s + 9t)$$

Find a basis and the dimension of (i) Kernel of T and (ii) Range of T.

6. (a) Find the standard matrix for the transformation T on R^3 where T is the composition of a reflection about the xy-plane, followed by a reflection about the xz-plane, followed by an orthogonal projection on the yz-plane. **(17)**

(b) Determine whether the linear operator $T: R^3 \to R^3$ defined by the equations **(18)**

$$w_1 = x_1 + 4x_2 - x_3$$
$$w_2 = 2x_1 + 7x_2 + x_3$$
$$w_3 = x_1 + 3x_2$$

is one-to-one; if so, find the standard matrix for the inverse operator and find T^{-1}.

7. (a) Let $\underline{u} = (2,1,0,3)$, $\underline{v} = (3,-1,5,2)$, $\underline{w} = (-1,0,2,1)$. Examine whether $\underline{x} = (-4,6,-13,4)$ is in span $\{\underline{u}, \underline{v}, \underline{w}\}$. **(10)**

(b) Consider the set P_2 of all polynomials of degree 2 and the set $S = \{p_1, p_2, p_3\}$ where $p_1 = 1 + x$, $p_2 = 1 + x^2$, $p_3 = x + x^2$. Is the S a basis for P_2? If so, then find the coordinate vector of the polynomial $p(x) = 2 - x + x^2$ relative to the basis S. **(15)**

(c) Let W be the subspace of R^4 spanned by the vectors $\underline{v_1} = (1,4,5,2)$, $\underline{v_2} = (2,1,3,0)$, and $\underline{v_3} = (-1,3,2,2)$. Find a basis for the orthogonal complement of W. **(10)**

8. (a) Consider the vector space R^3. Apply Gram-Schmidt process to transform the basis vectors $\underline{u_1} = (1,-1,0)$, $\underline{u_2} = (2,0,-2)$, and $\underline{u_3} = (3,-3,3)$ into an orthogonal basis $\{\underline{v_1}, \underline{v_2}, \underline{v_3}\}$; then obtain the orthonormal basis $\{\underline{q_1}, \underline{q_2}, \underline{q_3}\}$. Finally, write down the QR-decomposition of $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -3 \\ 0 & -2 & 3 \end{bmatrix}$. **(20)**

(b) Let $T : P_2 \to P_3$ be the linear transformation defined by $T(p(x)) = xp(3x-5)$, that is, $T(c_0 + c_1 x + c_2 x^2) = x(c_0 + c_1(3x-5) + c_2(3x-5)^2)$. **(15)**

Find the matrix for T with respect to the standard bases $B = \{1, x, x^2\}$ and $B' = \{1, x, x^2, x^3\}$. Finally, for any $\underline{x} \in P_2$, verify the relation $[T]_{B',B}[\underline{x}]_B = [T(\underline{x})]_{B'}$.
