

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) An Armstrong number (after Michael F. Armstrong) is a number that is the sum of its own digits each raised to the power of the number of digits. E.g., 371 is an Armstrong number since $3^3 + 7^3 + 1^3 = 371$. Now write a C Program that takes an integer as input and determines whether the number is an Armstrong number or not. Note that you are not allowed to use array to solve this problem. However, you are allowed to use any library function from math.h. (16)

Sample Input	Output
371	371 is an Armstrong number
27	27 is not an Armstrong number

- (b) Write a C Program that takes an odd integer n as input and finds the sum of all odd numbers up to n using a recursive function *oddSum* that takes an odd integer m as parameter and returns the sum of all odd numbers up to m as an integer. (12)

Sample Input	Output
5	Sum is 9
8	Invalid input

- (c) Write an advantage and two disadvantages of using recursive functions. (2+2.5×2=7)

2. (a) Write a C Program that takes an odd integer (not greater than 5) as input and outputs a specific triangular number pattern as demonstrated in the Table of Sample Input and Output. Your program must use a function *numberPattern* that takes an odd integer parameter (not greater than 5) and outputs a specific triangular number pattern as demonstrated in the Table of Sample Input and Output. The function *numberPattern* returns void. (15)

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Contd... Q. No. 2(a)

Sample Input	Output
3	1 123 12345
5	1 123 12345 1234567 123456789
9	Invalid Input

(b) Write a C Program that takes the number of rows (not greater than 10) and columns (not greater than 10) of two matrices A and B, then the elements of the matrices A and B consecutively as input and outputs the elements of the matrix C, where $C = A - B$. (15)

Sample Input	Output
3 3 5 6 7 8 9 10 11 12 14 1 2 3 4 5 6 7 8 9	Results of subtraction: 4 4 4 4 4 4 4 4 5

(c) Explain the fall through property in switch-case construct with an appropriate example. (5)

3. (a) Write the output of the following C program. (10)

```
#include<stdio.h>
void fn()
{
    int *pa,i;
    int x[5]={5,15,25,35,45};
    pa=&x[0];
    for(i=0;i<5;i++)
    {
        *pa=*pa+5;
        printf("%d\t",*pa++);
    }
}

void main()
{
    fn();
}
```

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Contd... Q. No. 3

(b) Write the output of the following C program.

(10)

<pre>#include<stdio.h> void fn() { static int i=10; int j=10; printf("%d \t %d\n",i++,j++); }</pre>	<pre>void main() { int i; for(i=0;i<5;i++) fn(); }</pre>
---	---

(c) "Pointer subtraction can be used to find the length of a string"– Demonstrate this fact by writing a function ptrStrlen that takes a string as parameter and returns the length of that string as an integer.

(10)

(d) State two uses of enumerations.

(2.5×2=5)

4. (a) Write the output of the following C program.

(9)

<pre>#include<stdio.h> void fn(int x, int y, int *z) { x*=y; y*=*z; *z*=5; }</pre>	<pre>void main() { int a=100, b=50, c=5; fn(a,b,&c); printf("%d \t %d \t %d \n",a,b,c); }</pre>
--	---

(b) Sketch a diagram showing the overall structure of a computer system according to Von Neumann.

(10)

(c) Write the output of the following C program.

(7)

<pre>#include<stdio.h> #define mult1(x, y) x*y #define mult2(x,y) (x)*(y)</pre>	<pre>void main() { int a=2, b=3; a=mult1(a+1,b+1)*2; b=mult2(a+1,b+1)*2; printf("%d \t %d \n",a,b); }</pre>
---	---

(d) What is a bit-field? Why is it useful? Can the address of a bitfield variable be obtained? Justify your answer.

(3+2+4=9)

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SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) Derive the output of the following program. (10)

```
#include <stdio.h>

union r
{
    int a;
    char ch[4];
};

int main()
{
    union r c;
    c.a = 20;
    c.ch[0] = 'a';
    c.ch[1] = 'b';
    printf("%d", c.a);

    return 0;
}
```

- (b) Replace if-else with ternary operator from the following code snippet. (10)

```
int findmax(int a, int b)
{
    if(a>b)
        return a;
    else
        return b;
}
```

- (c) Draw a flowchart showing the procedure of computing the sum of the following arithmetic progression. (You can't apply direct summation formula here.) (15)

$$1 + 2 + 3 + \dots + n$$

6. Consider a scenario where a professor has a number of students doing thesis under him. He finds it difficult to memorize all of their information. So, he made a plan to store all of the students' information in a file. He will fetch information of a specific student whenever it is needed.

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Contd... Q. No. 6

(a) He wants to run a C code and give it the number of students, n, as input. Then he will give all the students' information sequentially. The information includes id, name & thesis topic of the student. The code will store all the information in a file named "students.txt". Write the code to accomplish the task. (You can assume name & thesis topic contains no spaces and length of each word is at most 20. You must use **structure** to store the information of students.)

(20)

Sample Input	students.txt
2	1405003 Foo Bioinformatics
1405003 Foo Bioinformatics	1405020 Bar Data Mining
1405020 Bar Data Mining	

(b) He wants to run another C code and give it an id as input. It will show him all the information of the student with that id as output. If the student id doesn't exist in the file, it will print "Data not found!". Write a C code to accomplish this task.

(15)

Sample Input	Sample Output
1405003	1405003 Foo Bioinformatics
1405021	Data not found!

7. (a) Consider the following C code. It computes the sum of the elements of an array. The code assumes that user will **not** give more than 100 numbers as input. Modify the code to remove this assumption by dynamically allocating and deallocating the memory.

(12)

```
#include<stdio.h>

int main()
{
    int sum = 0, arr[100], n, i;
    printf("Enter number of integers: ");
    scanf("%d", &n);
    for(i=0; i<n; i++)scanf("%d", &arr[i]);
    for(i=0; i<n; i++)sum += arr[i];
    printf("Sum is %d", sum);
}
```

(b) What is the condition an array must fulfill so that binary search can be applied on it? Mention a scenario where you would prefer applying linear search on the array instead of binary search.

(2+6)

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Contd... Q. No. 7

(c) Consider the following code which has a function named **linear** which searches for a string in an array. Now write a function named **binary** to do the same task which will use binary search for this purpose. You can assume the array *arr* is sorted. (15)

```
#include<stdio.h>
#include<string.h>

void linear(char str[],char arr[][20],int length)
{
    int i;
    for(i=0;i<length;i++)
        if(!strcmp(str,arr[i]))
        {
            printf("Found %s in position %d",str,i);
            return;
        }
    printf("Not found");
}

int main()
{
    char names[][20] =
        {"Abir","Ashiq","Sifat","Snigdho","Tareq"};
    linear("Tareq",names,5);
}
```

8. (a) Assume the following code is saved in a file named "cmd.c". (7)

```
#include<stdio.h>

int main(int argc,char *argv[])
{
    int i;
    printf("%d\n",argc);
    for(i=0;i<argc;i++)
    {
        printf("%s\n",argv[i]);
    }
    return 0;
}
```

What is the output of this code if we run the following two commands in cmd?

```
gcc cmd.c
a a a a
```

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Contd... Q. No. 8

(b) Write a C code that will take *two* command line arguments from user, concatenate them and show the concatenated string as output. (Give warning if number of arguments is not 2.) (12)

(c) What are the scenarios when `fgetc()` returns `EOF`? How can you distinguish between the scenarios? (3+5)

(d) Explain why the following code segment will not produce expected output. (8)

```
#include<stdio.h>

int main()
{
    int celcius;
    float fahrenheit;
    scanf("%d",&celcius);
    fahrenheit = celcius/5 * 9 + 32;
    printf("%f", fahrenheit);

    return 0;
}
```

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) What are the three types of body materials required for traditional ceramic manufacturing? Give examples and describe their function as a raw material. (7)
- (b) What is meant by sintering process? Mention the problems associated with sintering process. (5 1/3)
- (c) The recipe of a glaze body is as follows: Albite ($\text{Na}_2\text{O} \cdot \text{Al}_2\text{O}_3 \cdot 6\text{SiO}_2$) = 35, Lead monosilicate ($\text{PbO} \cdot \text{SiO}_2$) = 57, Olastonite ($\text{CaO} \cdot \text{SiO}_2$) = 8 wt.%. Calculate the segar formula for this glaze. (11)

2. (a) With the help of a flowchart, discuss the typical steps encountered in the processing of traditional ceramics. (7)
- (b) Which of the forming methods would you prefer for ceramic teapots manufacturing? Briefly describe the manufacturing process. (9)
- (c) With a neat sketch, describe the Float or Flat glass manufacturing process. (7 1/3)

3. (a) How does the stress strain curve for polymers differ from that of metals? Discuss the factors that influence the mechanical properties of semi-crystalline polymers. (3+12=15)
- (b) With the help of a schematic diagram, describe the injection molding process for thermoplastic polymers. (8 1/3)

4. (a) What do you understand by glass transition temperature (T_g)? What are the factors that influence T_g ? (6 1/3)
- (b) A polyethylene sample contains 4000 chains with molecular weights between 0 and 5000 g/mol, 8000 chains with molecular weights between 5000 and 10,000 g/mol, 7000 chains with molecular weights between 10,000 and 15,000 g/mol, and 2000 chains with molecular weights between 15,000 and 20,000 g/mol. Determine both the number and weight average molecular weights. (8)
- (c) What is vulcanization? How can it be achieved? Write down the effect of vulcanization on properties of polymer. (2+2+5=9)

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SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) A non-stoichiometric oxide $Fe_{1-y}O$ has lattice parameter of 429 pm, density of 5.63 g/cm^3 , and has NaCl like structure. What is the composition of the oxide? **(8 1/3)**
- (b) Describe the perovskite structure of $BaTiO_3$ crystal. Explain ferro-electricity, pyroelectricity and piezoelectricity using the structure of $BaTiO_3$ and discuss how these properties are used in applications such as RAMs, ultrasound transducers and infrared detector. **(15)**
6. (a) Describe the working principle of semiconductor laser with necessary sketches. **(7)**
- (b) Explain superconductivity and discuss its application in the propulsion of trains by magnetic levitation. **(10)**
- (c) Why does magnetite (Fe_3O_4) exhibit ferrimagnetism? **(6 1/3)**
7. (a) Suppose you were given two Al_2O_3 (ceramic) samples. You tested one in tensile loading and the other in compressive loading. For which cases the obtained fracture load would be higher? With neat sketches explain your answer in terms of the mechanism of failure. **(10)**
- (b) Discuss some common methods of joining of high performance ceramics. **(5 1/3)**
- (c) A three-point bending test was performed on an aluminum oxide specimen having a circular cross section of radius 3.5 mm; the specimen fractured at a load of 950 N when the distance between the support points was 50 mm. Another test is to be performed on a specimen of this same material, but one that has a square cross section of 12 mm length on each edge. At what load would you expect this specimen to fracture if the support point separation is 40 mm. **(8)**
8. (a) What are the purposes of making composite material? **(3)**
- (b) Distinguish between particulate composite and dispersion strengthened materials. **(5 1/3)**
- (c) A continuous and aligned fiber-reinforced composite is to be produced consisting of 45 vol% aramid fibers and 55 vol% of a polycarbonate matrix. Mechanical characteristics of these two materials are as follows: **(15)**

Materials	Modulus Tensile of Elasticity (GPa)
Aramid Fiber	131
Polycarbonate	2.4

- (i) For this composite, compute the longitudinal modulus of elasticity.
- (ii) If the composite has a cross-sectional area of 480 mm^2 and is subjected to a longitudinal load of 53,400 N. Calculate the actual loads carried by both fiber and matrix phases.
- (iii) Determine the individual strain in each phase and comment on obtained values.
-

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols and abbreviations have their usual meaning.

1. (a) For an input signal shown in Fig. 1(a)(i), design a system using op-amps, which will give the output as shown in Fig. 1(a)(ii). Consider ideal cases. (25)

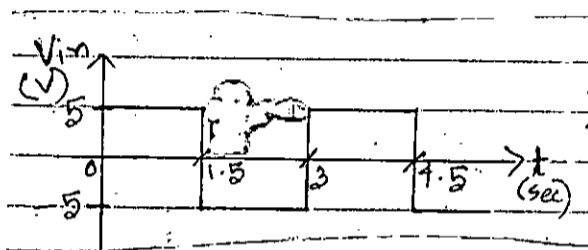


Fig. 1(a)(i)

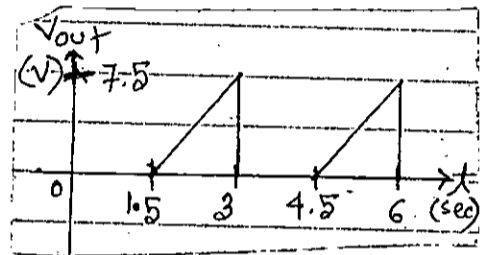


Fig. 1(a)(ii)

- (b) For any 3 input voltages: x, y, z ; design a circuit with op-amps to have an output voltage, V , where $V = k_1x + k_2y - k_3z$, where k_1, k_2 , and k_3 are constants. (10)
2. (a) What is 'breakover voltage' in an SCR? Briefly describe the working principle of an SCR with 'Diode Model' and 'Two Transistor Model'. (25)
- (b) Consider the circuit shown in Fig. 2(b). Here, $V_1 = 1V, V_2 = -2V$ and $V_3 = 3V$. Determine the output voltage, V_{out} and load current, I_L . (10)

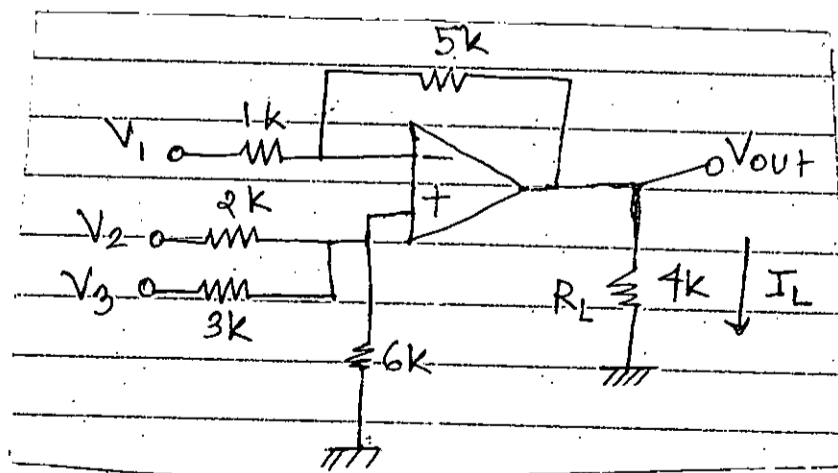


Fig. 2(b)

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3. (a) What is a 'transducer'? Briefly describe a pressure transducer and flow rate transducer with examples. (25)

(b) Explain with proper diagrams, how does the increase of field resistance for speed control in shunt dc motor affect its operation? (10)

4. (a) A 15 hp, 240V, 1200 rpm dc shunt motor has an armature resistance of 0.4Ω . Its field resistor has a resistance of 100Ω . A potentiometer is connected in series with the field resistor. The resistance of the potentiometer can be varied from 100Ω to 400Ω . There are 2700 turns per pole on the shunt field winding. The rated input current is 55 A. The magnetization curve of the motor is shown in Fig. 4(a). (25)

(i) Assuming no armature reaction, when the potentiometer is adjusted to 175Ω , what is the speed of the motor at full load?

(ii) If the motor is operating at full load, and if the resistance of the potentiometer is increased to 250Ω , what is the new speed of the motor?

(iii) Compare the speed of the motor in case (i) to the speed of the motor in case (ii).

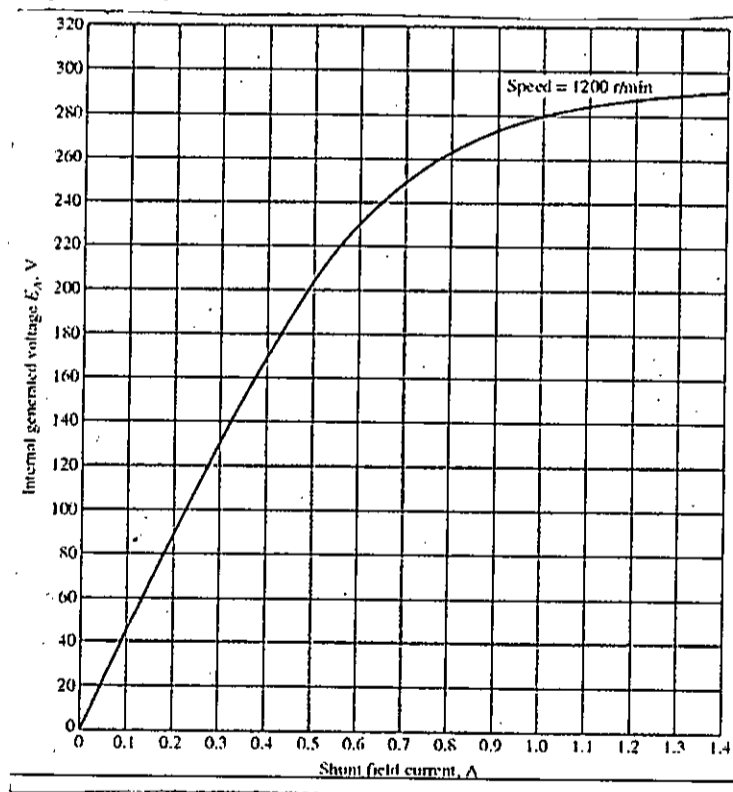


Fig. 4(a)

(b) Show that, the induced torque in a rotating loop is equal to $2rIlB$, under the pole faces and equal to zero, beyond the pole edges. (10)

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SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What are core loss current and excitation current? What is leakage flux? (5)
 (b) A 15-kVA, 2300/230-V transformer is to be tested to determine its excitation branch components; its series impedances, and its voltage regulation. The following test data have been taken from the primary side of the transformer: (30)

Open circuit test	Short circuit test
	$V_{sc} = 47 \text{ V}$
$I_{oc} = 0.21 \text{ A}$	$I_{sc} = 6 \text{ A}$
$P_{oc} = 50 \text{ W}$	$P_{sc} = 160 \text{ W}$

- (i) Find the equivalent circuit of this transformer referred to the high-voltage side.
 (ii) Find the equivalent circuit of this transformer referred to the low-voltage side.
 (iii) Calculate the full-load voltage regulation at 0.8 lagging power factor, 1.0 power factor, and at 0.8 leading power factor. Also, draw transformer phasor diagrams for these three cases.
 (iv) What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?
6. (a) Let 25V DC is given as input to the low voltage side of an ideal transformer having turns ratio 10:1. What will be the voltage at the high voltage side? (7)
 (b) On which side of a transformer, open circuit test and short circuit tests are carried out? Why? (8)
 (c) A 208-V, two-pole, 60-Hz Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are: (20)

$$R_1 = 0.2 \Omega \quad R_2 = 0.12 \Omega$$

$$X_1 = X_2 = 0.41 \Omega \quad X_M = 15 \Omega$$

$$P_{mech} = 250 \text{ W} \quad P_{misc} = 0 \text{ W} \quad P_{core} = 180 \text{ W}$$

For a slip of 0.05, find:–

- (i) line current,
 (ii) stator copper losses P_{SCL} ,
 (iii) air-gap power P_{AG} ,
 (iv) power converted from electrical to mechanical form P_{conv} , and
 (v) overall machine efficiency.

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7. (a) Explain in brief the principle of operation of a 3 phase induction motor. How is rotating magnetic flux developed in an induction motor? (15)
- (b) What are slip and slip speed in an induction motor? Why is it impossible for an induction motor to operate at synchronous speed? (10)
- (c) Show that, $P_{\text{CONV}}: P_{\text{AG}} = (1-s):1$. (10)
8. (a) What is speed regulation of a synchronous motor? (3)
- (b) Explain the effect of load changes on a synchronous motor with phasor diagram and necessary equations. (15)
- (c) A 208-V, 45-kVA, 0.8 p.f leading, Δ -connected, 60-Hz synchronous machine has a synchronous reactance of 2.5Ω and a negligible armature resistance. Its friction and windage losses are 1.5 kW, and its core losses are 1.0 kW. The shaft is supplying a 15-hp load, and the motor's power factor is 0.80 leading. (17)
- Sketch the phasor diagram of this motor, and find the values of I_A , I_L and E_A .
-

The figures in the margin indicate full marks

Symbols indicate their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Determine the mass of each of the two cylinders as shown in the Figure for Question No. 1(a) if they cause a deformation of the springs for which $s = 0.5$ m when suspended from the rings at A and B . Note that $s = 0$ when the cylinders are removed. (23)

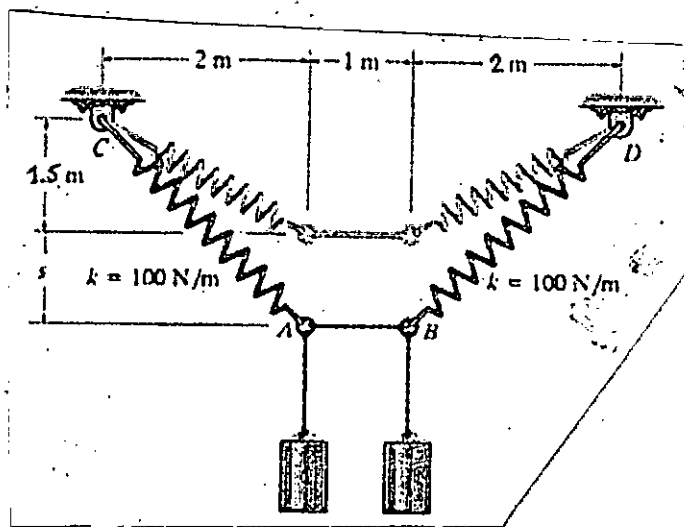


Figure for Question No. 1(a)

- (b) Determine the volume of the solid obtained by rotating the area as shown in Figure for Question No. 1(b) about y axis. Also determine the weight of the solid if the specific gravity of the constituent material is 7.85. (23 $\frac{2}{3}$)

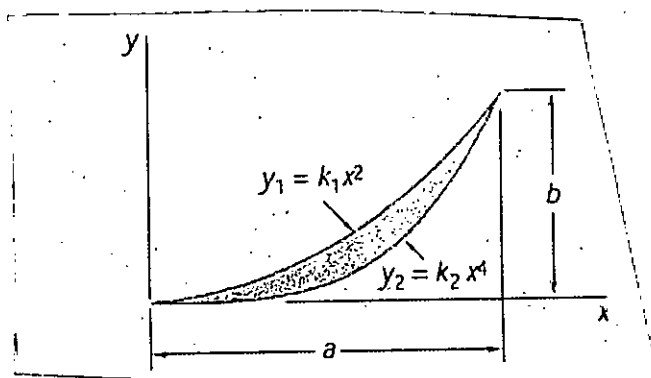


Figure for Question No. 1(b)

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2. (a) As shown in the Figure for Question No. 2(a), spring BC is unstretched with $\theta = 0^\circ$ and the bell crank achieves its equilibrium position when $\theta = 15^\circ$. Determine the force, F which is applied perpendicular to segment AD and the reaction forces acting at pin A . Spring BC remains in the horizontal position at all times due to the roller at C . (23)

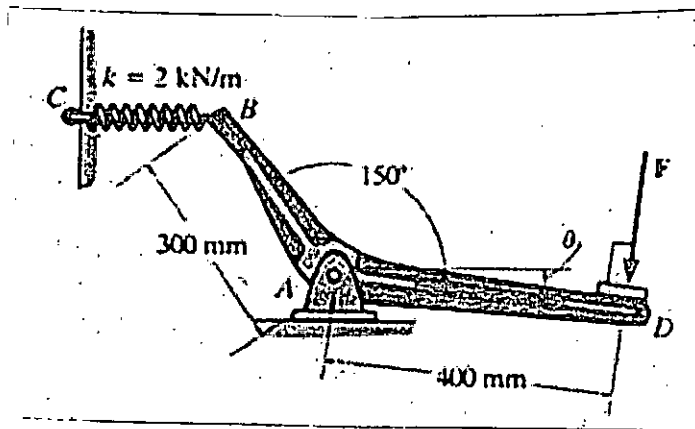


Figure for Question No. 2(a)

- (b) Knowing that the pulley as shown in the Figure for Question No. 2(b) has a radius of 0.5 m, determine the components of the reactions at A and E . (23 2/3)

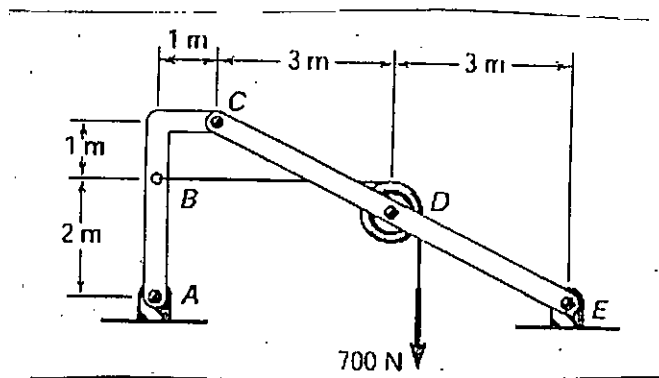


Figure for Question No. 2(b)

3. (a) Block C as shown in the Figure for Question No. 3(a) starts from rest at $t = 0$ and moves downward with a constant acceleration of 4 m/s^2 . Knowing that block B has a constant velocity of 3 m/s upward, determine (23)
- (i) the time when the velocity of block A is zero,
 - (ii) the time when the velocity of block A is equal to the velocity of block D ,
 - (iii) the change in position of block A after 5 s.

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Contd... Q. No. 3(a)

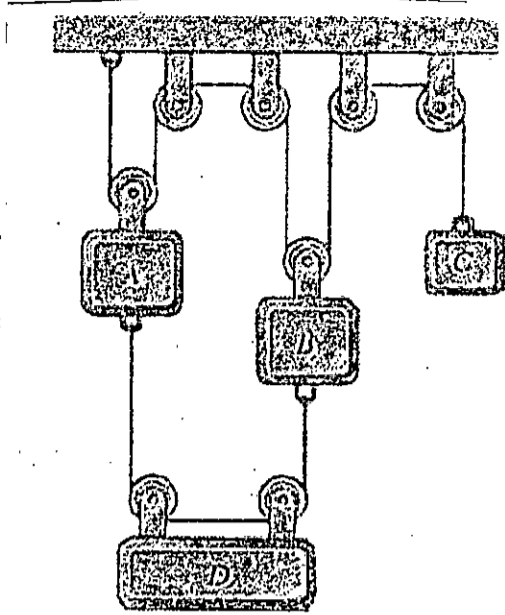


Figure for Question No. 3(a)

(b) Block *B* of mass 10 kg rests as shown in the Figure for Question No. 3(b) on the upper surface of a 22-kg wedge *A*. Knowing that the system is released from rest and neglecting friction, determine (i) the acceleration of *B*, (ii) the velocity of *B* relative to *A* at $t = 0.5$ s.

(23 $\frac{2}{3}$)

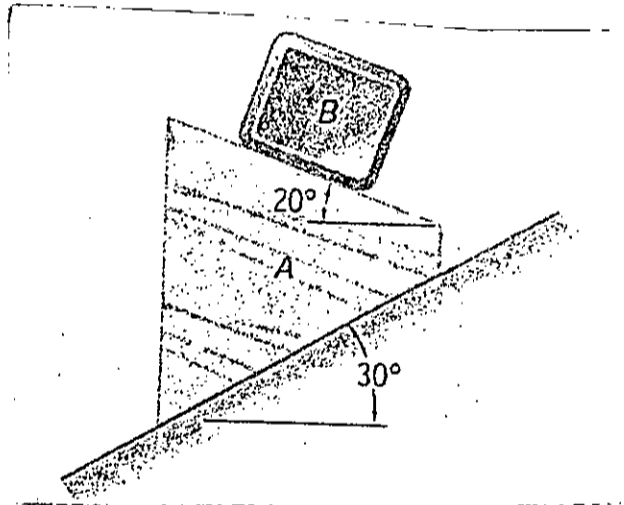


Figure for Question No. 3(b)

4. (a) Find the moment of inertia of the trapezoidal section with a semicircular slot as shown in Figure for Question No. 4(a) about the horizontal centroidal axis.

(20)

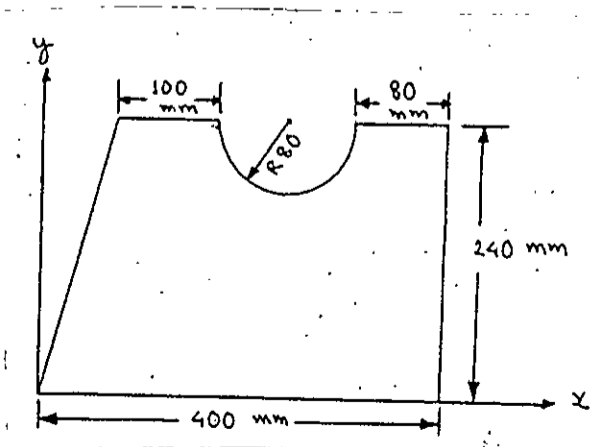


Figure for Question No. 4(a)

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Contd... Q. No. 4

(b) The roller-coaster track shown in the Figure for Question No. 4(b) is contained in a vertical plane. The portion of track between A and B is straight and horizontal, while the portions to the left of A and to the right of B have radii of curvature as indicated. A car is travelling at a speed of 72 km/h when the brakes are suddenly applied, causing the wheels of the car to slide on the track (coefficient of kinetic friction, $\mu_k = 0.20$).

Determine the initial deceleration of the car if the brakes are applied as the car (20)

- (i) has almost reached A ,
- (ii) is traveling between A and B ,
- (iii) has just passed B .

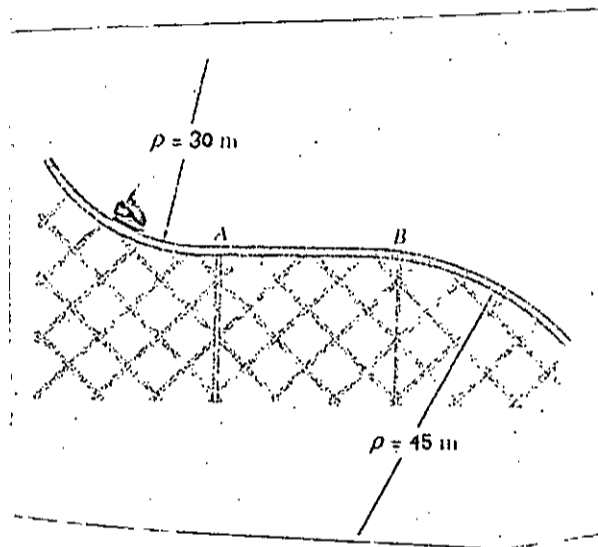


Figure for Question No. 4(b)

(c) Why motion of particles are different from motion of rigid bodies? Explain briefly with necessary example. (6 2/3)

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. A cam, with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to give a knife edge follower the motion as described below: (46 2/3)

- (i) To move outwards through 40 mm during 100° rotation of the cam;
- (ii) To dwell for the next 80°;
- (iii) To return to its starting position during the next 90°, and
- (iv) To dwell for the rest period of a revolution i.e. 90°.

The displacement of the follower is to take place with uniform acceleration and uniform retardation. Draw the displacement diagram and profile of the cam very neatly showing all construction details when the line of stroke of the follower passes through the center of the camshaft. Find the maximum velocity and acceleration of the follower when the cam rotates at 300 rpm.

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6. (a) Four masses A, B, C and D as shown below are to be completely balanced. (23)

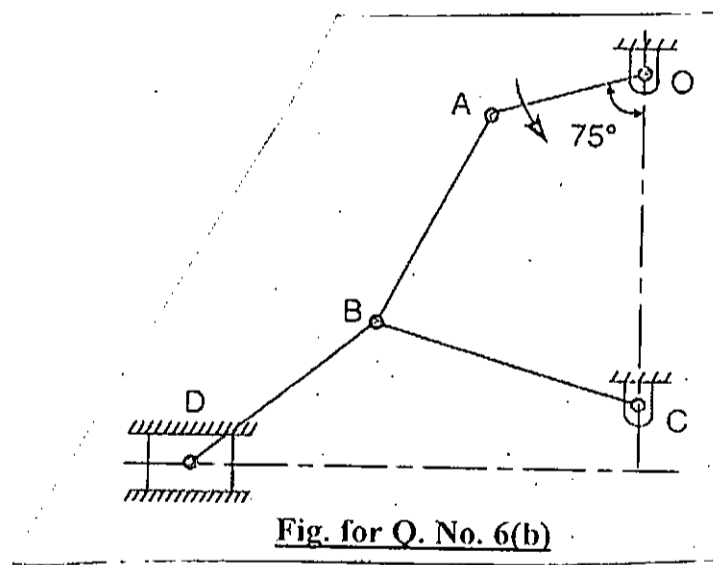
	A	B	C	D
Masses (Kg)	m_A	30	50	40
Radius (mm)	180	240	120	150

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90° . B and C make angles of 210° and 120° respectively with D in the same sense. Find:

- (i) The magnitude and the angular position of mass A; and
- (ii) The position of planes A and D.

(b) In Fig. for Q. No. 6(b), the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are: OA = 28 mm; AB = 44 mm; BC = 49 mm; and BD = 46 mm. The center distance between the centers of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below point C. The slider moves along a horizontal path and OC is vertical.

(23 $\frac{2}{3}$)



7. (a) In the epicyclic gear shown in Fig. for Q. No. 7(a), the wheels B and C are attached to the driving shaft X and gear with annular wheels D and E through the pinions F and H. The pinion F is free to revolve on a pin carried on the arm A which is keyed to the driven shaft Y. The pinion H revolves on a pin carried by the annular wheel D. The numbers of teeth on certain wheels and pinions are as follows: B, 28; D, 78; C, 24; E, 80. The wheel E is fixed by means of a band brake. If the shaft X rotates at a speed of 2000 rev/min, find the speed of Y.

(30)

ME 245/IPE

Contd... Q. No. 7(a)

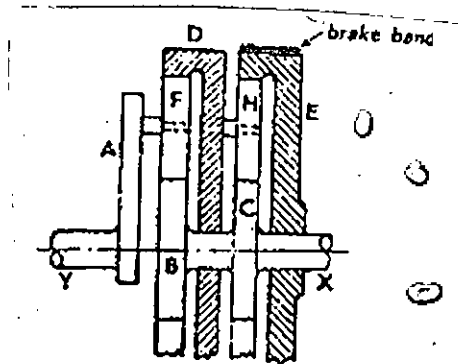


Fig. for Q. No. 7(a)

(b) The mass of a single degree damped vibrating system is 7.5 kg and makes 24 free oscillations in 14 seconds when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.25 of its initial value after five oscillations. Determine:
 (i) stiffness of the spring, (ii) Logarithmic decrement, and (iii) damping factor. (16 $\frac{2}{3}$)

8. (a) The uniform bar of mass 12 kg is supported by a spring as shown in Fig. for Q. No. 8(a). A disc of mass 4 kg and radius 0.085 m is fixed to the bar at its free end. Calculate the natural frequency of free vibration of the system. (23)

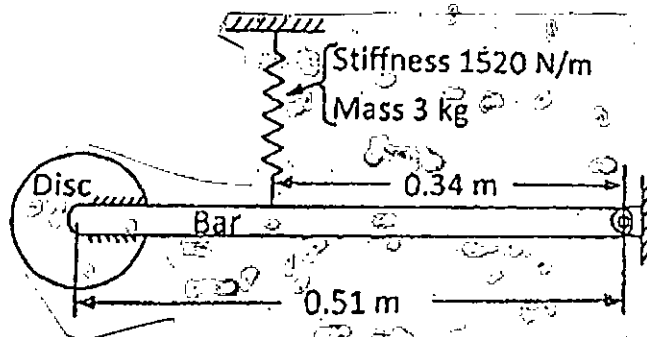


Fig. for Q. No. 8(a)

(b) A shaft 1.5 m long, supported in flexible bearings at the ends carries two wheels each of 50 kg mass. One wheel is situated at the center of the shaft and the other at a distance of 375 mm from the center towards left. The shaft is hollow of external diameter 75 mm and internal diameter 40 mm. The density of the shaft material is 7700 kg/m³ and its modulus of elasticity is 200 GN/m². Find the lowest whirling speed of the shaft, taking into account the mass of the shaft. (23 $\frac{2}{3}$)

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols used have their usual meaning.

1. (a) Find the differential equation by eliminating arbitrary constants a and b from the equation (10)

$$y = ae^{3x} + be^{-2x}$$

- (b) Solve the following differential equations:

(i) $(2x + y + 3) dy = (x + 2y + 3) dx$ (13)

(ii) $(2\sqrt{xy} - y)dx = xdy$ (12)

2. (a) Find the integrating factor and then solve (11)

$$x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$$

- (b) Solve the following higher order differential equations with $D \equiv \frac{d}{dx}$: (12)

$$(D^2 + 2D + 1)y = x \cos x$$

- (c) Solve the Cauchy-Euler differential equation: $x^2y'' - 3xy' + 4y = x + x^2 \ln x$. (12)

3. Find the series solution of the differential equation $2x^2y'' + xy' - (x + 1)y = 0$ by the method of Frobenius. (35)

4. (a) Show that the Legendre polynomial $P_n(x)$ is the coefficient of h^n in the expansion of $(1 - 2xh + h^2)^{-1/2}$ (15)

- (b) Show that

(i) $xJ'_n(x) = xJ_{n+1}(x) - nJ_n(x)$ (10)

(ii) $nP'_n(x) = xP'_n(x) - P'_{n-1}(x)$ (10)

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SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols used have their usual meaning.

5. (a) Define Sine Integral function and find its Laplace Transform. Also show that (12)

$$\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}.$$

- (b) Find (i) $L\left\{\frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-u^2} du\right\}$ (ii) $L\left\{\int_0^t \frac{1-e^{-u}}{u} du\right\}$ (14)

- (c) Find the Laplace Transform of $J_0(t)$, where $J_0(t)$ is the Bessel's function of order zero. (9)

6. (a) State Convolution theorem and using this theorem, find $L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}$. (10)

- (b) Find $L^{-1}\left\{\frac{s}{(s^2-2s+2)(s^2+2s+2)}\right\}$. (14)

- (c) Using Laplace Transform to evaluate $\int_0^{\infty} \cos x^2 dx$. (11)

7. (a) Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (11)

- (b) Evaluate $\int_C \underline{F} \cdot d\underline{r}$ where $\underline{F} = (x^2+y^2)\underline{i} - 2sy\underline{j}$ and curve C is the rectangle in xy -plane

bounded by $y = 0, x = a, y = b, x = 0$. (12)

- (c) Determine the constants a, b, c that the vector

$\underline{F} = (x+2y+az)\underline{i} + (bx-3y-z)\underline{j} + (4x+cy+2z)\underline{k}$ is irrotational.

8. (a) Verify Stoke's theorem for the function $\underline{F} = x\underline{i} + z^2\underline{j} + y^2\underline{k}$ over the plane surface $x + y + z = 1$ lying in the first octant. (12)

- (b) Evaluate $\iint_S (x^3 dydz + y^3 dzdx + z^3 dxdy)$ over the surface S of a cube bounded by the coordinate planes and the planes $x = y = z = a$. (12)

- (c) Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2 dy]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. (11)