L-3/T-2/NAME

Date: 10/01/2021

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2018-2019

Sub: **MATH 381** (Fourier Analysis, Harmonic Function, Complex Variable and Laplace Transforms)

Full Marks: 240 Time: 2 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

SECTION-A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Prove that, $\sqrt{2}\,|z| \geq |\mathrm{Re}(z)| + |\mathrm{Im}(z)|$, where, z is a non-zero complex number **(10)**

(b) Describe locus of points z that satisfy $\mathrm{Re}\left(\frac{1}{z}\right) \leq \frac{1}{2}$. **(10)**

(c) Find the bilinear transformation that maps distinct points $z_1 = 1,\ z_2 = 0,\ z_3 = -1$ onto the points $w_1 = i,\ w_2 = \infty,\ w_3 = 1$. **(20)**

2. (a) Show that $u(x, y) = \frac{y}{x^2+y^2}$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$ such that $f(z) = u + iv$. Also express $f(z)$ in terms of z. **(20)**

(b) Evaluate $\int_C (x^2 - iy^2)dz$ along the straight lines from $z = 1 + i$ to $z = 1 + 8i$ and then along a line parallel to y-axis from $z = 1 + 8i$ to $z = 2 + 8i$. **(20)**

3. (a) Evaluate $\oint_C \frac{\cos z}{z(z^2+8)}dz$, where, C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. **(20)**

(b) Give two Laurent series expansions in powers of z for the function $f(z) = \frac{1}{z^2(1-z)}$ and specify the regions in which those expansions are valid. **(20)**

4. Evaluate the following integral by contour integration: **(40)**

$$\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}dx .$$

<u>**SECTION-B**</u>

There are **FOUR** questions in this section. Answer any **THREE**.

5. **(a)** Find the Fourier sine series expansion of the function $f(x) = x(x-2)$ defined over the interval $0 < x < 2$. Also, sketch the graph of the function. **(20)**

   (b) Find the Fourier cosine integral formula of $f(x) = (1-x)e^{-2x}$ for $x \geq 0$. **(20)**

6. **(a)** Find the Fourier sine transform of $f(x) = \begin{cases} 2x, & 0 < x < 1/2 \\ x-1, & 1/2 < x < 1 \\ 0, & x > 1. \end{cases}$ **(15)**

   (b) If the values of a potential function on the boundary of a circle of radius 4 cm are given by $v(\theta) = \sin\theta$, find the potential at any interior point of the circle. **(25)**

7. **(a)** Find the steady temperature inside a solid sphere of unit radius if one hemisphere of its surface is kept at temperature zero and the other at temperature $F(\theta) = \cos\theta$. **(20)**

   (b) Use convolution theorem to evaluate $L^{-1}\left\{\dfrac{1}{s^2(s+1)^2}\right\}$. **(20)**

8. **(a)** Use Laplace transform to evaluate: $\int_0^x x^2 J_0(x) J_1(x)\, dx$. **(20)**

   (b) Solve the following differential equation by using Laplace transform: **(20)**

   $$tX''(t) + X'(t) + 4tX(t) = 0$$

   where $X(0) = 3,\ X'(0) = 0$.