LINEAR PROGRAMMING PROBLEM FORMULATION AND SOLUTION USING BENDERS' DECOMPOSITION METHOD

By

SANJIDA AKTAR

Student ID: 1017092515F

Registration No: 1017092515,
Session: October 2017

MASTER OF SCIENCE

IN

MATHEMATICS

Department of Mathematics

Bangladesh University of Engineering and Technology (BUET),

Dhaka-1000, Bangladesh

The thesis entitled with

LINEAR PROGRAMMING PROBLEM FORMULATION AND SOLUTION USING BENDERS' DECOMPOSITION METHOD

Submitted by

SANJIDA AKTAR

Student ID: 1017092515F, Registration No. 1017092515, Session: Oct. 2017 a full time student of M.Sc. (Mathematics) has been accepted as satisfactory in partial fulfilment for the degree of Master of Science in Mathematics

On February 26, 2020.

BOARD OF EXAMINERS

1. ______________________________
Dr. Mohammed Forhad Uddin — **Chairman (Supervisor)**
Professor
Department of Mathematics, BUET, Dhaka-1000

2. ______________________________
Head — **Member (Ex-officio)**
Department of Mathematics, BUET, Dhaka-1000

3. ______________________________
Dr. Md. Manirul Alam Sarker — **Member**
Professor
Department of Mathematics, BUET, Dhaka-1000

4. ______________________________
Dr. Md. Zafar Iqbal Khan — **Member**
Professor
Department of Mathematics, BUET, Dhaka-1000

5. ______________________________
Dr. Mohammad Babul Hasan — **(External Member)**
Professor
Department of Mathematics,
Dhaka University, Dhaka-1000

DEDICATION

This work is dedicated

To

My beloved Parents and Teachers

AUTHOR'S DECLARATION

I, Sanjida Aktar, hereby announce that the work which is being presented in this thesis entitled **"LINEAR PROGRAMMING PROBLEM FORMULATION AND SOLUTION USING BENDERS' DECOMPOSITION METHOD"** is the outcome of the investigation carried out by the author under the supervision of Dr. Mohammed Forhad Uddin, Professor, Department Of Mathematics, Bangladesh University of Engineering and Technology (BUET), Dhaka-1000. This paper is submitted in partial fulfillment for the degree of M.Sc. This is an authentic record of my own work and has not been submitted to any other university for the award of any degree or diploma in home or abroad.

Signed: ___________________________________

Date: ___________________________________

ACKNOWLEDGEMENT

First and foremost, I would like to thank and praise the Almighty, most Merciful and most Gracious, for granting me the wisdom, the perseverance, and the necessary support and resources to navigate the M.Sc. study and finish the dissertation. It is my hope that this dissertation could glorify His name.

I am extremely blessed to have Dr. Mohammed Forhad Uddin as my M.Sc. supervisor. More importantly, he has spent significant effort on encouraging and facilitating my scholarly growth. I owe my sincere gratitude to him because this thesis would not be like this without his guidance, criticism, support, encouragement and motivation. I am very thankful to him for introducing me to this highly fascinating and applicable research area and for finishing this thesis successfully.

I am grateful to Dr. Md. Manirul Alam Sarker and Dr. Md. Zafar Iqbal Khan for being on my defense committee as well as reviewing and suggesting for the improvements of my dissertation.

Specially, I am extremely thankful and highly indebted to Dr. Mohammad Babul Hasan for his generous help, co-operation and valuable guidance during my research. I have learned a lot from him.

I am extremely thankful to Shah Abdullah Al Nahian for his help to go ahead in preparing my thesis.

Finally, I would like to express my deep regards to my parents and all other family members and friends for their constant cooperation and motivations. Their sincerest wishes for me have played a very important role in my study.

Signed: ________________________________

Date: ________________________________

ABSTRACT

In this thesis, a large scale linear programming problem consisting several parameters such as labor cost, raw material cost, machine and other cost have been formulated. Then the formulated problem has been solved by using Benders' Decomposition Method. In order to validate and calibrate the model, real data from a soap industry named MEGA SORNALI SOAP & COSMETICS INDUSTRY have been collected. Soap industry is one of the most feasible business options owing to the straightforward manufacturing process involved starting a soap and detergent manufacturing business in Bangladesh. With significant growth potential, this market is one segment of the Fast Moving Consumer Goods (FMCG) market in Bangladesh. People use it on daily basis for clothes, hand wash, and kitchen utensils and its demand is found in the market all through the year.

The formulated large model is divided into master and small sub problem. These models are solved by using a Mathematical Programming Language (AMPL). In order to validate the model, the sensitivity analysis of different cost parameters such as labor cost, raw material cost and machine cost will be considered. From the sensitivity analysis, the decision makers of the factory will be able to find out the ranges of cost coefficients and all the resources. As a result, they will be able to see how any change can affect the profit or loss of the factory.

From the numerical results, it is clear that Mega Sornali Sobi Marka Soap and Mega Washing Powder (25g) are not more profitable. The most profitable product of the company are found to be Sornali Soap (2015) and Mega Extra Powder (500g). Further, it is clear that raw material cost is the most sensitive cost. If the raw material cost can be decreased the profit will also increase. Finally, the result of the optimal solution will be represented in tabular form in addition to the graphs.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter One

Introduction

The development of linear programming is the most scientific advances in the mid-20th century. LP involves the planning of activities to obtain an optimal result which reaches the specialized goal best among all feasible alternatives. The best decision is found by solving a mathematical problem. Mathematical modeling plays an important role in many applications such as control theory, optimization, signal processing, large space flexible structures, game theory and design of physical system. Various complicated systems arise in many applications. They are described by very large mathematical models consisting of more and more mathematical systems with very large dimensions. Then it is very difficult to solve these problems. BDM is a popular technique for solving certain classes of difficult problems such as stochastic programming problems and mixed-integer LPP. It is a technique in mathematical programming that allows the solution of very large LPP that has special block structure. This structure often occurs in applications such as stochastic programming.

1.1 Literature Review

In the development of the subject LPP name G. B. Dantzig is the head. He first developed an LPP model although the similar problem was first formulated by the Russian economist-Mathematician L. V. Kantorovich as product allocation problem. Later the problem was formulated by G. B. Dantzig. He also formulated the method of solving such problem named simplex method.

Dantzig and Wolfe [1] established the decomposition algorithm for linear programming problem. Sweeny and Murphy [2] induced a method of decomposition for integer programs. It is based on the notion of searching for the optimal solution to an integer program among the near-optimal solutions to its Lagrangian relaxation. Benders' [3] showed partitioning procedures for solving mixed-variables programming problems. Laporte *et al.* [4] presented an integer programming algorithm for vehicle routing problem involving capacity and distance restrictions. They derived exact solutions for problems involving upto sixty cities. Hasan and Raffensperger [5] established decomposition based pricing model for solving a large-scale MILP for an integrated fishery. They integrated fishery planning problem (IFP). They described how a fishery manager can schedule fishing trawlers to determine when and where they should go and return their caught fish to the factory. Nonconvex nonlinear programming (NLP) problems arise frequently in water resources management, reservoir operations, groundwater remediation and integrated water quantity and quality management. Such problems are usually large and sparse. Cai *et al.* [6] presented technique for solving large nonconvex water resources management models using generalized BDM.

Andreas and Smith [7] developed a decomposition algorithm for the design of a nonsimultaneous capacitated evacuation tree network. They examined the design of an evacuation tree, in which evacuation is subject to capacity restrictions on arcs. The cost of evacuating people in the network is determined by the sum of penalties incurred on arcs on which they travel, where penalties are determined according to a nondecreasing function of time. Uddin *et al.* [8] analyzed Vendor-Bayer coordination and supply chain optimization with deterministic demand function. This research presents a model that deals with a vendor-buyer multi-product, multi-facility and multi-customer location selection problem, which subsume a set of manufacturer with limited production capacities situated within a geographical area. Georion [9] generalized Benders' decomposition algorithm. Eremin and Wallace [10] established hybrid Benders decomposition algorithms in constraint logic programming. They described an implementation of Benders Decomposition that enabled it to be used within a constraint programming framework. The programmer was spared from having to write down the dual form of any sub problem because it was derived by the system. Bazaraa *et al*. [11] established the nonlinear programming theory and algorithm. Costa [12] ran a survey on Benders decomposition applied to fixed-charge network design problems. Network design problems concern the selection of arcs in a graph in order to satisfy, at minimum cost, some flow requirements, usually expressed in the form of origin–destination pair demands. Benders decomposition methods, based on the idea of partition and delayed constraint generation, had been successfully applied to many of these problems. They presented a review of these applications. Nielsen and Zenios [13] founded the scalable parallel Benders decomposition for stochastic linear programming. They developed a scalable parallel implementation of the classical Benders decomposition algorithm for two-stage stochastic linear programs.

Taskin *et al.* [14] explained mixed-integer programming techniques for decomposing IMRT fluency maps using rectangular apertures. They studied the problem of minimizing the number of rectangles (and their associated intensities) necessary to decompose such a matrix. They proposed an integer programming-based methodology for providing lower and upper bounds on the optimal solution and demonstrate the efficacy of their approach on clinical data. Applegate *et al.* [15] implemented the Dantzig-Fulkerson-Johnson algorithm for large traveling salesman problems. An algorithm is described for solving large-scale instances of the Symmetric Traveling Salesman Problem (STSP) to optimality. Camargo *et al.* [16] showed a Benders' decomposition algorithm for the single allocation hub location problem under congestion. The single allocation hub location problem under congestion is addressed in this article. Then a very efficient and effective generalized Benders decomposition algorithm is deployed, enabling the solution of large scale instances in reasonable time. Cordeau *et al.* [17] showed an approach for the locomotive and car assignment problem using Benders' Decomposition. One of the problems faced by rail transportation companies is to optimize the utilization of the available stock of locomotives and cars. They described a decomposition method for the simultaneous assignment of locomotives and cars in the context of passenger transportation. Geffrion [18] generalized BDM. Magnanti and Wong [19] accelerated Benders' decomposition algorithom in enhancement and model selection criteria. They proposed a methodology for improving the performance of Benders decomposition when it was applied to mixed integer programs. They introduced a new technique for accelerating the convergence of the algorithm. Montemenni and Gambardella [20] solved the robust shortest path problem with interval data via BD. They investigated the well-known shortest path problem on directed acyclic graphs under arc length uncertainties. The data of the model was uncertainty by treating the arc lengths as interval ranges. Emeretlis *et al.* [21] mapped DAGs on heterogeneous platforms using logic-based BD. They presented a multiple cuts generation schemes that improved the performance of the solution process and extensive experimental results that showed significant speed ups compared to the pure ILP-based method. Rasmussen and Trick [22] described a BD to the constrained minimum break problem. It presents an algorithm for designing a double round robin schedule with a minimal number of breaks. Both mirrored and non-mirrored schedules with and without place constraints were considered. Ralphs *et al*. [23] solved the capacitated vehicle routing problem. Sherali and Fratichli [24] modified a BD algorithm for discrete sub-problems which was an approach for stochastic programs with integer resource. They modified Benders' decomposition method by using concepts from the Reformulation-Linearization Technique (RLT) and lift-and-project cuts in order to develop an approach for solving discrete optimization problems that yield integral sub problems such as those that arose in the case of two-stage stochastic programs with integer recourse. Xu *et al.* [25] induced a semi-smooth Newton’s method for traffic equilibrium problem with a general non-additive route cost. They presented a version of the (static) traffic equilibrium problem in which the cost incurred on each path was not simply the sum of the costs on the arcs that constituted that path. Santoso *et al.* [26] developed a stochastic programming approach for network design under uncertainty. Rahimi *et al*. [27] induced a new approach based on BD for unit commitment problem. They presented a hybrid model between Lagrange relaxation and Genetic algorithm to schedule generators economically based on forecasted information such as power prices and demand with an objective to maximize profit of Generation Company.

Salam [28] developed unit commitment solution methods. Osman and Demirli [29] developed a bilinear goal programming model and a modified Benders decomposition algorithm for supply chain reconfiguration and supplier selection. They solved the problem which was related to an aerospace company seeking to change its outsourcing strategies in order to meet the expected demand increase and customer satisfaction requirements regarding delivery dates and amounts. Lin *et al.* [30] proposed an efficient network-wide model-based predictive control for urban traffic networks. They developed a control system to deal with complex urban road networks more efficiently. Lu *et al.* [31] showed a new approach for combined freeway variable speed limits and coordinated ramp metering. Papamichail *et al.* [32] coordinated a ramp metering for freeway networks. Pisarski and Canudas-de-Wit [33] optimized balancing of road traffic density distributions for the cell transmission model. They studied the problem of optimal balancing of traffic density distributions. Wongpiromsarn *et al.* [34] distributed traffic signal control for maximum network throughput. They proposed a distributed algorithm for controlling traffic signals. Their algorithm was adapted from backpressure routing which had been mainly applied to communication and power networks. Chen *et al*. [35] developed a self-adaptive gradient projection algorithm for the nonadditive traffic equilibrium problem. Conejo *et al*. [36] showed decomposition technique in mathematical programming. Patriksson [37] formulated partial linearization methods in nonlinear programming. They characterized a class of feasible direction methods in nonlinear programming through the concept of partial linearization of the objective function. Barahona and Anbil [38] formulated primal solutions with a subgradient method. Shen and Smith [39] showed a decomposition approach for solving a broadcast domination network design problem. Lucena [40] developed non delayed relax-and-cut algorithms. Lysgaard *et al*. [41] developed a new branch-and-cut algorithm for the capacitated vehicle routing problem.

1.2 Chapter Outline

Chapter 01 provides introduction and the required literature review. It also contains some basic definitions and the object of the study and possible outcome. It also provides with different procedure of solving LPP. It also contains different kinds of decomposition, algorithm and block diagram so that anyone can easily solve any problem using these methods. In this chapter there is a discussion on BDM.

Chapter 02 is a main part of this thesis. At first there is a discussion about soap factories. Then it is taken real life data from a soap factory. Primary data is collected in this chapter.

In chapter 03, collected data are formulated into an LPP. After that it is solved using AMPL. Then it is solved by BDM using AMPL. Finally sensitivity analysis is also taken under consideration.

Chapter 04 shows conclusions and briefly discussion about the whole procedures and future research of the work.

1.3 Objectives with Specific Aim

The main objective of this research is to optimize the profit. The objectives of the proposed work are as follows:

➢ To formulate a linear programming model that would suggest a viable product-mix to ensure optimum profit for company.

➢ To minimize the production cost.
➢ To find out various types of effects of parameters in production period.
➢ To know about the constraints of the company regarding cost, resources.
➢ To highlight the peculiarities of using LP technique for the company.
➢ To maximize the production.

1.4 Possible Outcomes

This research has the following possible outcomes. Here, BDM will be used for profit optimization of a soap factory. The LPP model will be capable to calculate how much of product should be produced to maximize profit. The model will be capable to help to minimize the production cost. The study would be able to identify the future production patterns. The study will be able to identify the limitations and indicate the effects of different parameters of real data.

1.5 Some definitions

1.5.1 Convex Set

A convex set is a set of points such that, given any two points A, B in that set, the line AB joining them lays entirely within that set. In other words, if all points of the line segment joining any two points of the set are in the set then the set is known as convex set.

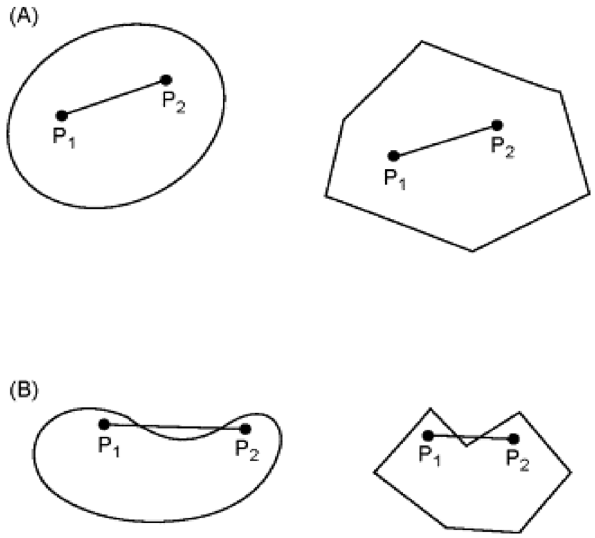

Figure 1.1: Convex Set

1.5.2 Hyper Plane

A hyper plane in $\mathbf{E}^{\mathbf{n}}$ is a set of X of points given by X = {**x: cx = k}** where **c** is a row vector, given by **c** = (c_1, c_2, ….., c_n) and not all $\mathbf{c_j}$ and **x** = (x_1, x_2,……, x_n) is an n component column vector.

1.5.3 Hyper Sphere

A hyper sphere is the set of points at a constant distance from a given point called its center. The hyper sphere in two dimensional is a circle and three dimensions is a sphere.

1.5.4 Convex Hull

The convex hull or convex envelope or convex closure of a set *X* of points in the Euclidean plane or in a Euclidean space is the smallest convex set that contains X.

1.5.5 Convex Polyhedron

If X is a set consist of finite number of points, then the set of all convex combination of sets of the points from X is called a convex polyhedron. A convex polyhedron is a convex set.

1.5.6 Linear Programming

A standard form of a Linear Program is

$$\text{Maximize } z = c^T x$$
$$\text{Subject to Constraints: } Ax \leq b$$
$$x \geq 0,$$

Where c $\in \mathbb{R}^n$, b $\in \mathbb{R}^m$ are given vectors and A $\in \mathbb{R}^{m\times n}$ is a matrix.

Or,

$$\text{Minimize } z = c^T x$$
$$\text{Subject to Constraints: } Ax \geq b$$
$$x \geq 0,$$

Where c $\in \mathbb{R}^n$, b $\in \mathbb{R}^m$ are given vectors and A $\in \mathbb{R}^{m\times n}$ is a matrix.

1.5.7 Mixed Integer Linear Program (MIP)

A Mixed Integer Linear Program (MIP) is given by vectors c $\in \mathbb{R}^n$, b $\in \mathbb{R}^m$, a matrix A $\in \mathbb{R}^{m\times n}$ and a number p $\in$ {0, n}. The goal of the problem is to find a vector x $\in \mathbb{R}^n$ solving the following optimization problem:

$$\text{Max } z = c^T x$$
$$\text{Subject to: } Ax \leq b.$$
$$x \geq 0$$
$$x \in \mathbb{Z}^p \times \mathbb{R}^{n-p}.$$

If p = 0, then there are no integrality constraints at all, so we obtain the Linear Program.

1.5.8 Integer Program (IP)

In the above equation, if p = n, then all variables are required to be integral. In this case, an Integer Linear Program (IP) is:

$$\text{Max } z = c^T x$$
$$\text{Subject to: } Ax \leq b$$
$$x \geq 0$$
$$x \in \mathbb{Z}^n.$$

1.5.9 Binary Integer Program (BIP)

If in an (IP) all variables are restricted to values from the set B = {0, 1}, then it is called a 0-1-Integer Linear Program or Binary Linear Integer Program:

$$\text{Max } z = c^T x$$
$$\text{Subject to constraint: } Ax \leq b$$
$$x \geq 0$$
$$x \in B^n.$$

1.5.10 Optimization in LPP

Optimization is the name given to computing the best solution to a problem modeled as a set of linear relationships. These problems arise in many scientific and engineering disciplines. A mathematical optimization problem is one in which some function is either maximized or minimized relative to a given set of alternatives.

Example 01:

$$\text{Maximize } z = 50x + 120y$$
$$\text{s. t. } x + 2y \leq 100;$$
$$x + 3y \leq 120;$$
$$x + y \leq 110;$$
$$x, y \geq 0;$$

1.5.11 Feasible Region

In mathematical optimization, a feasible region, feasible set, search space or solution space is the set of all possible points (sets of values of the choice variables) of an optimization problem that satisfy the problem's constraints, potentially including inequalities, equalities and integer constraints.

1.5.12 Feasible Solution

A feasible solution is a set of values for the decision variables that satisfies all of the constraints in an optimization problem.

1.5.13 Objective Function

The objective of linear programming is to maximize or to minimize some numerical value. This value may be the expected net present value of a project or a forest property; or it may be the cost of a project; it could also be the amount of wood produced, the expected number of visitor-days at a park, the number of endangered species that will be saved, or the amount of a particular type of habitat to be maintained. It is denoted by z.

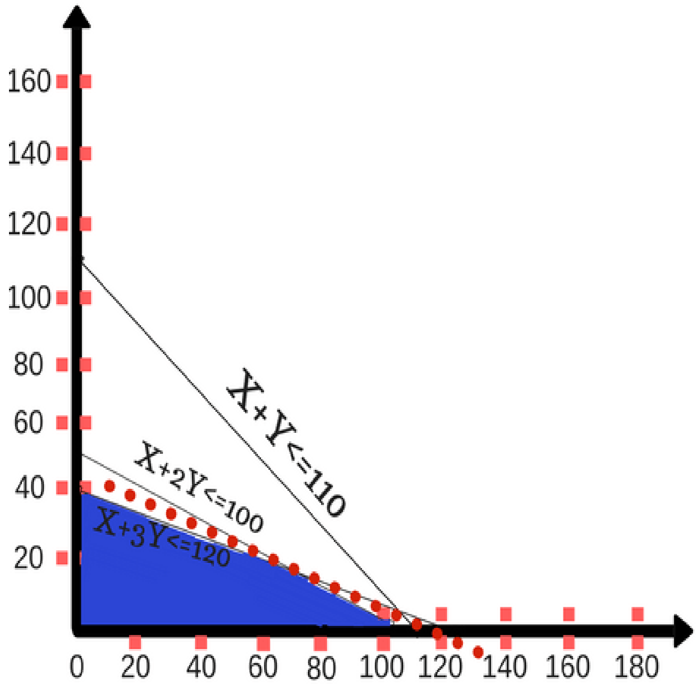

Figure 1.2: Feasible Solution of Example 01

The values for x and y which gives the optimal solution is at (60, 20).

Max z = 50 * (60) + 120 * (20)

=3000+2400

=5400

Here the value of objective function, z = 5400

1.5.14 Basic Solution

A basic solution is a solution that satisfies all the constraints.

1.5.15 Basic Feasible Solution

The solution set of an LPP which is feasible as well as basic is known as the basic feasible solution of the problem.

1.5.16 Degenerate Solution

A basic solution to the system **Ax=b** is called degenerate if one or more of the basic variables vanishes.

1.5.17 Non-degenerate Solution

If all component of a solution set corresponding to the basic variables are nonzero quantities then the basic solution is called non-degenerate basic solution.

1.5.18 Sensitivity Analysis

Sensitivity analysis is a financial model that determines how target variables are affected based on changes in other variables known as input variables. This model is also referred to as what-if or simulation analysis. It is a way to predict the outcome of a decision given a certain range of variables.

1.6 Process to Formulate a Linear Programming Problem

The steps are followed to solve a Linear Programming Problem generically:

1. Identify the decision variables
2. Write the objective function
3. Mention the constraints
4. Explicitly state the non-negativity restriction

For a problem to be a linear programming problem, the decision variables, objective function and constraints all have to be linear functions.

Table 1.1 Algorithm of Linear Programming Problem

Step 1. Study the given problem and find the key decision i. e. find out what will be determined. **Step 2.** Select variables for which problem will be determined. **Step 3.** Set all variables greater than or equal to zero for feasible solution. **Step 4.** Find total profit or total cost with the help of variables and declared as an objective function which will be maximized or minimized. **Step 5.** Express the constraints of the problem as linear equation. **Step 6.** Write the objective function and constraints as a linear programming problem.

Example:

Let's say a FedEx delivery man has 6 packages to deliver in a day. The warehouse is located at point A. The 6 delivery destinations are given by U, V, W, X, Y and Z. The numbers on the lines indicate the distance between the cities. To save on fuel and time the delivery person wants to take the shortest route.

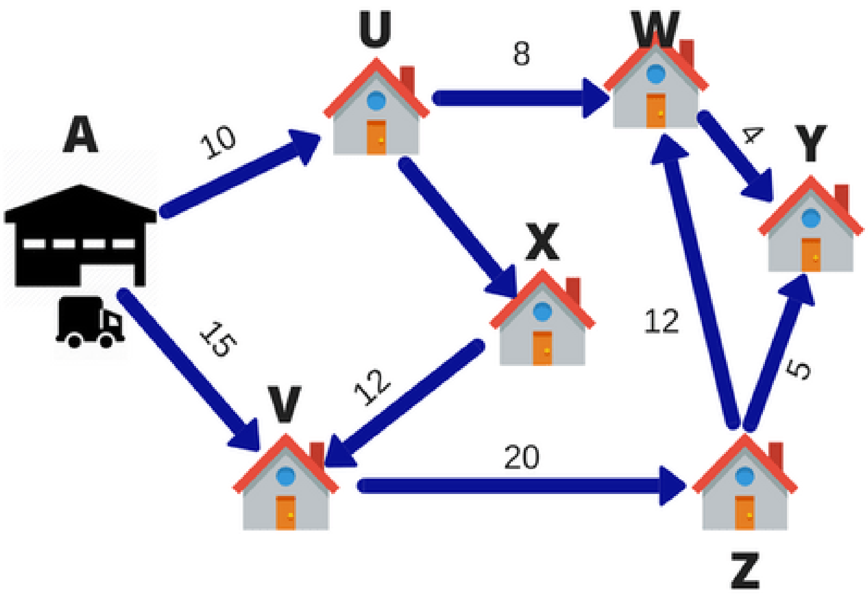

Figure 1.3: Shortest Route Problems

So, the delivery person will calculate different routes for going to all the 6 destinations and then come up with the shortest route. This technique of choosing the shortest route is called linear programming.

In this case, the objective of the delivery person is to deliver the parcel on time at all 6 destinations. The process of choosing the best route is called Operation Research. Operation research is an approach to decision-making, which involves a set of methods to operate a system. In the above example, my system was the Delivery model.

Linear programming is used for obtaining the most optimal solution for a problem with given constraints. In linear programming, we formulate our real life problem into a mathematical model. It involves an objective function, linear inequalities with subject to constraints.

1.7 Integer Programming (IP)

Integer programming expresses the optimization of a linear function subject to a set of linear constraints over integer variables. Integer programming is the class of problems that can be expressed as the optimization of a linear function subject to a set of linear constraints over integer variables.

Example:

$$\text{Max } 2x_1+5x_2$$
$$\text{s.t. } x_1 + x_2 \leq 6,$$
$$5x_1 + 9x_2 \leq 46,$$
$$x_1, x_2 \geq 0 \text{ and integer}$$

1.8 Pure Integer Problem (PIP)

An integer programming problem in which all variables are required to be integer is called a pure integer programming problem. If some variables are restricted to be integer and some are not then the problem is a mixed integer programming problem.

Example:

$$\text{Max } 2x_1+5x_2$$
$$\text{s.t. } x_1 + x_2 \leq 6,$$

$$5x_1 + 9x_2 \leq 46,$$

x_1, x_2 are all non-negative integers.

1.9 Fixed Charge Problem (FCP)

The fixed-charge problem deals with situations in which the economic activity incurs two types of costs: an initial "flat" fee that must be incurred to start the activity and a variable cost that is directly proportional to the level of the activity. For example, the initial tooling of a machine prior to starting production incurs a fixed setup cost regardless of how many units are manufactured. Once the setup is done, the cost of labor and material is proportional to the amount produced. Given that *F* is the fixed charge, e is the variable unit cost, and *x* is the level of production, the cost function is expressed as

$$C(x) = \begin{Bmatrix} F + cx, if\ x > 0 \\ 0, otherwise \end{Bmatrix}$$

The function *C(x)* is intractable analytically because it involves a discontinuity at $x = 0$.

1.10 Facility Location Problem (FLP)

Facility Location Problem deals with selecting the placement of a facility to best meet the demanded constraints. The problem often consists of selecting a factory location that minimizes total weighted distances from suppliers and customers, where weights are representative of the difficulty of transporting materials.

Consider a set of facilities (servers) I and a set of customers (clients) J. Let g_i(z) be a non-decreasing function for each facility i. The facility setup cost $g_i(z_i)$ occurs when facility i is opened with size z_i, such that z_i customers are served by it. The connection cost of assigning customer j to facility i is c_{ij} . z_i is a non-negative integer decision variable which denotes the number of customers of facility i. z_i> 0 if facility i is open, z_i =0 otherwise; $x_{ij}$is a binary decision variable which takes the value 1 if the customer j is served by facility i, 0 otherwise. The UFLP with general cost functions can be formulated as follows: Minimize z = i∈I $g_i(z_i$+,i ∈I, j∈J, c_{ij},

x_{ij}. subject to i∈I x_{ij} = 1, j ∈ J, (2) j∈J $x_{ij} = z_i$, i ∈ I, x_{ij}∈ {0, 1}, i ∈ I,j ∈ J, z_i = 0 integer i ∈ I.

There are many kinds of solving LPP. Simplex method is described below.

1.11 Algorithm of Simplex Method

To solve a linear programming problem in standard form, it has to use the following steps.

Table 1.2 Algorithm of Simplex Method

Step 1. Convert each inequality in the set of constraints to an equation by adding slack variables. **Step 2.** Create the initial simplex tableau. **Step 3.** Locate the most negative entry in the bottom row. The column for this entry is called the entering column. (If ties occur, any of the tied entries can be used to determine the entering column.) **Step 4.** Form the ratios of the entries in the “b-column” with their corresponding positive entries in the entering column. The departing row corresponds to the smallest nonnegative ratio (If all entries in the entering column are 0 or negative, then there is no maximum solution. For ties, choose either entry.) The entry in the departing row and the entering column is called the pivot. **Step 5**. Use elementary row operations so that the pivot is 1, and all other entries in the entering column are 0. This process is called pivoting. **Step 6.** If all entries in the bottom row are zero or positive, this is the final tableau. If not, go back to Step 3. **Step 7.** If it is obtained a final tableau, then the linear programming problem has a maximum solution, which is given by the entry in the lower-right corner of the tableau.

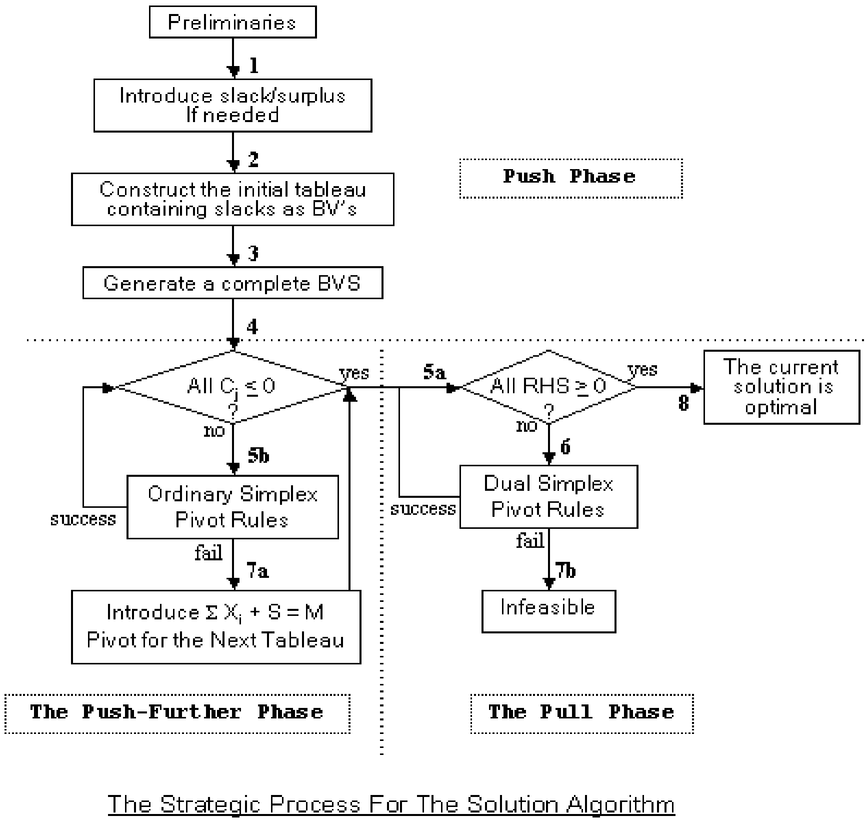

Figure 1.4: Diagram of Simplex Method

1.12 Algorithm of Graphical Method

The algorithm of solving an LPP in graphical method is given below:

Table 1.3 Algorithm of Graphical Method

Step 1. Formulate the mathematical model of the given linear programming problem (LPP).
Step 2. Treat inequalities as equalities and then draw the lines corresponding to each equation and non-negativity restrictions.
Step 3. Locate the end points (corner points) on the feasible region.
Step 4. Determine the value of the objective function corresponding to the end points determined in step 3.
Step 5. Find out the optimal value of the objective function.

Many linear programming problems of practical interest have the property that they may be described, in part, as composed of separate linear programming problems tied together by a number of constraints considerably smaller than the total number imposed on the problem. Now, it will be studied DWD, DBP, BD and ID method. Then it will be developed block diagram and made algorithm of these decomposition. The business organizations will be able to get best production rate and profit if they apply mathematical model in their business.

1.13 Different kinds of Decomposition

In this section, it will be discussed existing methods called DWD, DBP, BD and ID.

1.13.1 Dantzig-wolfe Decomposition (DWD) Method

Dantzig–Wolfe decomposition is an algorithm for solving linear programming problems with special structure. It was originally developed by George Dantzig and Philip Wolfe and initially published in 1960. Many texts on linear programming have sections dedicated to discussing this decomposition algorithm.

Dantzig–Wolfe decomposition relies on delayed column generation for improving the tractability of large-scale linear programs. For most linear programs solved via the revised simplex algorithm, at each step, most columns (variables) are not in the basis. In such a scheme, a master problem containing at least the currently active columns (the basis) uses a sub problem or to generate columns for entry into the basis such that their inclusion improves the objective function. In order to use Dantzig–Wolfe decomposition, the constraint matrix of the linear program must have a specific form. A set of constraints must be identified as "connecting", "coupling" or "complicating" constraints where in many of the variables contained in the constraints have non-zero coefficients. The remaining constraints need to be grouped into independent sub matrices such that if a variable has a non-zero coefficient within one sub matrix, it will not have a non-zero coefficient in another sub matrix.

While there are several variations regarding implementation, the Dantzig–Wolfe decomposition algorithm can be briefly described as follows:

Table 1.4 Algorithm of DWD

Step 1. Starting with a feasible solution to the reduced master program, formulate new objective functions for each sub problem such that the sub problems will offer solutions that improve the current objective of the master program. Sub problems are resolved given their new objective functions. An optimal value for each sub problem is offered to the master program. **Step 2.** The master program incorporates one or all of the new columns generated by the solutions to the sub problems based on those columns' respective ability to improve the original problem's objective. **Step 3.** Master program performs x iterations of the simplex algorithm, where x is the number of columns incorporated. **Step 4.** If objective is improved, go to step 1. Else, continue. **Step 5.** The master program cannot be further improved by any new columns from the sub problems, thus return.

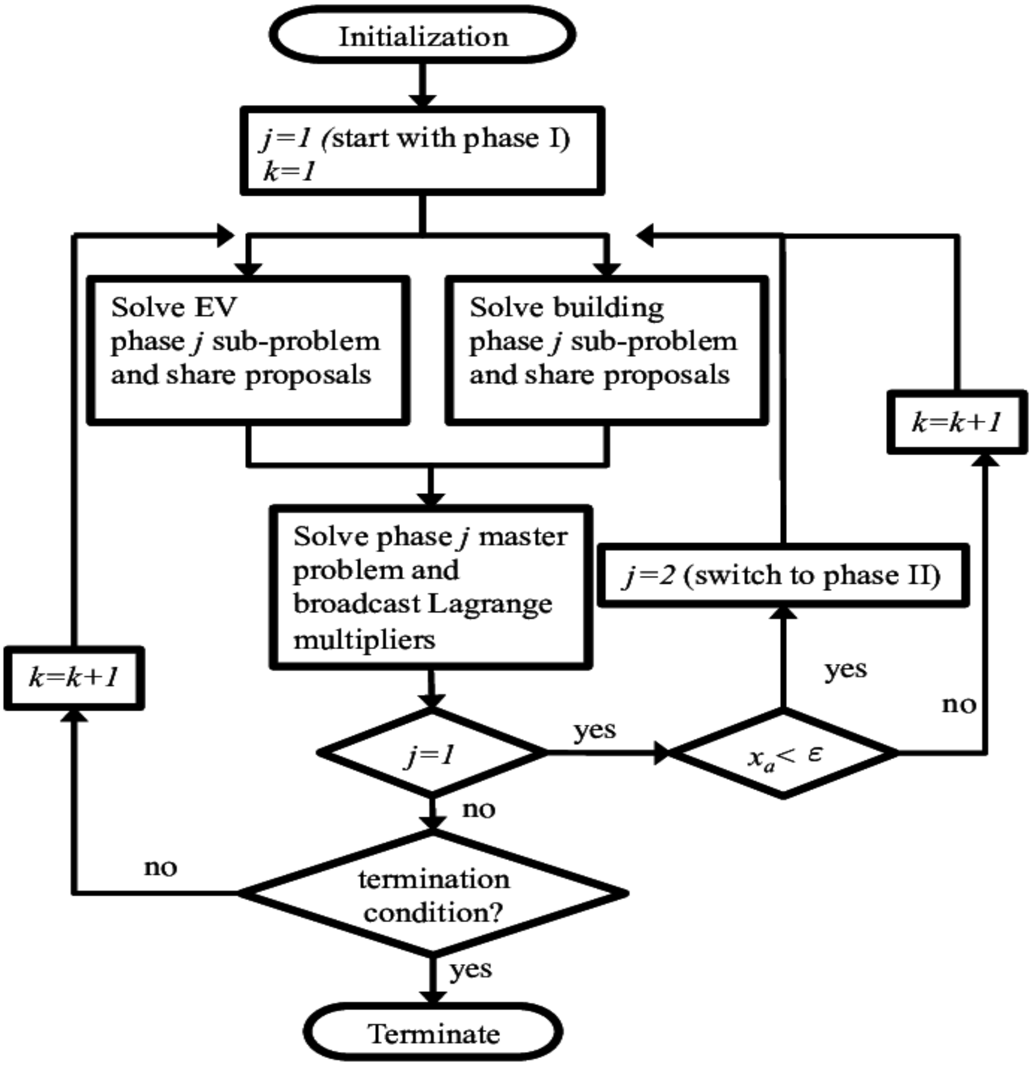

Figure 1.5: Diagram of DWD

1.13.2 Decomposition Based Pricing (DBP) Method

DBP is a procedure to filter the unnecessary decision ingredients from large scale mixed integer programming (MIP) problem, where the variables are in huge number will be abated and the complicacy of restrictions will be straightforward.

The idea of taking computational advantage of the special structure of a specific problem is to develop an efficient algorithm is not new. Certain structural forms of large-scale problems reappear frequently in applications, and large-scale systems.

The first step is to solve the problem by relaxing the integer restrictions. So it will be concentrated on solving the corresponding LP with continuous variable and then Then it is developed a real life model of DBP approach to solve.

This section improves a decomposition algorithm for the solution of two persons zero sum games using DBP method.

Table 1.5 Algorithm of DBP

Step 1. Search the minimum element from each row of the payoff matrix and then find the maximum element of these minimum elements.

Step 2. Search the maximum element from each column of the payoff matrix and then find the minimum element of these maximum elements.

Step 3. For the player I if the maximin less than zero then find k which is equal to addition of one and absolute value of maximization.

Step 4. For the player II if the minimax less than zero then find k which is equal to addition of one and absolute value of minimax.

Step 5. If maximin and minimax both are greater than zero then k=0.

Step 6. To construct the modified pay off matrix adding k with each payoff elements of the given payoff matrix.

Step 7. Then to find the mixed strategies with game value of the two players, formulate the game problem. Then follow the following Sub-steps.

Step 8. Subtract complicating constraint from objective function and generate sub-problems.

Sub-step 1. Solve sub-problem and determine the non-negative variables.

Sub-step 2. Delete all those variables which are not non-negative and generate the master problem.

Sub-step 3. Solve master problem.

Sub-step 4. If sub-problem value and master problem value become equal then stop the iterations. Otherwise repeat Sub-steps 1 to 3.

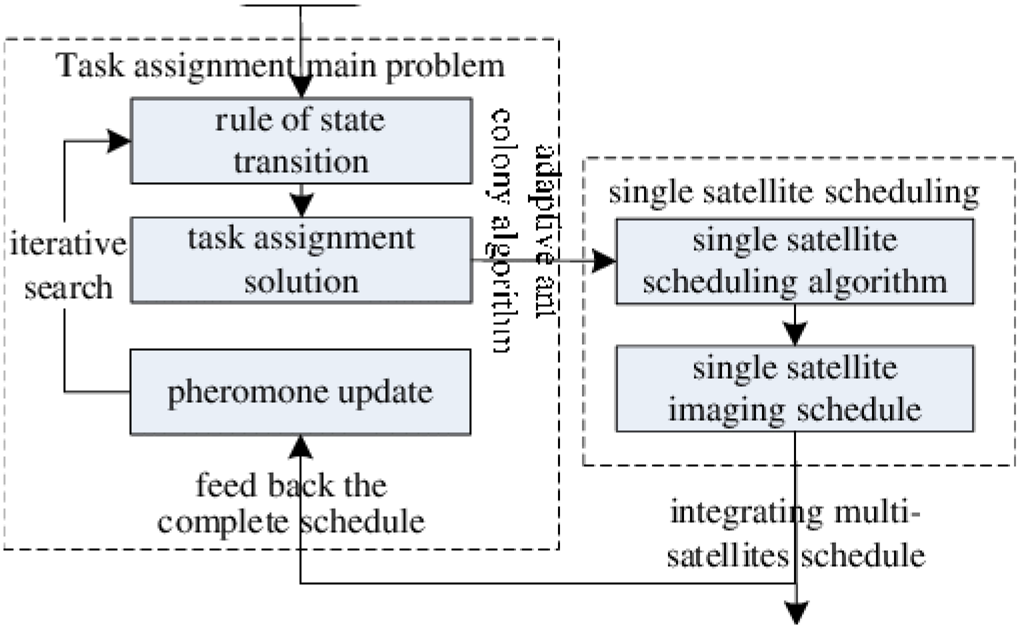

Figure 1.6: Diagram of DBP.

1.13.3 Benders Decomposition (BD) Method

Benders' decomposition is a classical solution approach for combinatorial optimization problems based on partition and delayed constraint generation. This method was originally purposed by J. F. Benders in 1962 for solving large scale combinatorial optimization problems and then several extensions were proposed. One of the most important ones was presented by Geoffrion who proposed a "generalized Benders' decomposition" approach. He used nonlinear duality theory and extended the Benders' method to the case where the sub-problem was convex. This development enabled the application of the Benders' decomposition to a whole new set of problems, particularly those in which a joint problem was generally nonconvex but could be made convex by fixing one set of variables. Examples of successful application of this methodology to mixed-integer problems are abundant. Also, there are a number of applications; for instance, the seminal paper by Geoffrion and Graves on multi commodity distribution network design and the extension presented by Cordea on the same problem can be mentioned.

Other applications include the locomotive and car assignment problems, large scale water resource management problem, two stage stochastic linear problems and robust shortest path problem. The method partitions the model to be solved into two simpler problems named master and sub problem.

Indeed, summarizing Benders' decomposition, first the relaxed master problem is solved to obtain a lower bound on the optimal values of the objective function of the initial problem, and then, the sub-problem uses inputs of the master problem to form an approximate cut and adds it to the master problem in the next iteration. Also, by solving the sub-problem, an upper bound is found for the initial problem. During the iterative process, by adding a new constraint to the master problem, the optimal value of its objective function can only increase or stay the same. On the other hand, in each iteration, by solving a sub-problem, the upper bound of objective function of the initial problem can only decrease or stay the same. As soon as the lower and upper bounds of the initial problem are sufficiently close, the iterative process can be terminated with a sufficiently small tolerance. Based on the convergence theorem of Benders' decomposition method, the algorithm achieves the optimal solution after a finite number of iterations. Benders decomposition can be used to solve:

- linear programming
- mixed-integer (non)linear programming
- two-stage stochastic programming (L-shaped algorithm)
- multistage stochastic programming (Nested Benders decomposition)

Table 1.6 Algorithm of BDM

min z = cx + f(y)

s.t. Ax + g(y) $\geq$ b

x, y $\geq$ 0;

Step 1. Choose y in original problem

Step 2. $\bar{z} \leftarrow -\infty$

Step 3. k $\leftarrow$ 0

Step 4. While (sub-problem dual has feasible solution $\geq \bar{z}$) do

Step 5. Derive lower bound function $\beta\bar{y}$(y) with $\beta\bar{y}$ ($\bar{y}$) = β

Step 6. k $\leftarrow$ k + 1

Step 7. $y^k \leftarrow \bar{y}$

Step 8. Add z $\geq$ $\beta\bar{y}$(y) to master problem,

Step 9. If (master problem is infeasible) then

Step 10. Stop. The original problem is infeasible.

Step 11. Else.

Step 12. Let ($\bar{y}$,$\bar{z}$) be the optimal value and solution to the master problem.

Step 13. Return (($\bar{y}$, $\bar{z}$).

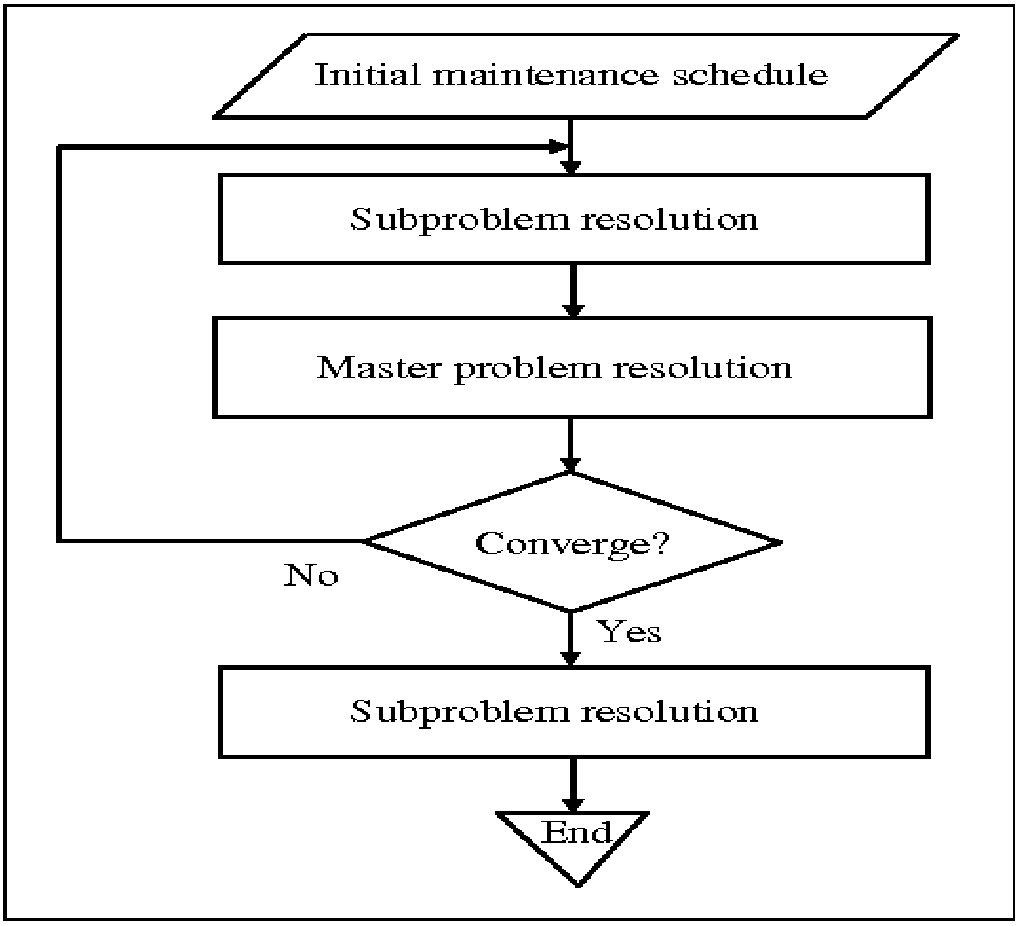

Figure 1.7: Diagram of BDM

1.13.4 Improved Decomposition (ID) Method

Due to the delayed column generation for solving large scale LPs by DWD principle, in 2011 Istiaq and Hasan presented an Improved Decomposition (ID) algorithm depending on DWD principle for solving LPs. This method is composed of three subproblems (which can be generalized for *n* sub problems) of an original problem and the master problem with the help of Lagrangian relaxation. Optimality holds when the value of the sum of the sub-problem will be equal to the master problem.

$$V(S1) + V(S2) + V(S3) = V(M)$$

Picking up an initial value of the dual variables randomly the sub problem(s) is solved from which current solution of the sub problem is imported to create the master problem. Then master problem is solved and tested the optimality condition. If the optimality condition does not hold, then the current dual value from the master problem is taken and imported this to update the sub problem(s) and continue the same process unless it meets the optimality condition. The method is so far the latest one to solve large-scale LPs which is relatively easier approach to carry on and has the simple algorithm and computational strategy to find the optimal solution. Although these methods are described to be successful in some special area but there are no mention about what will be the deportment of these method when solving an IP as well as a large-scale MIP. Also the optimality condition described by the equation does not hold always for IP. So in the next section, we developed a successful and relatively time consuming method to solve both large-scale LP and MIP.

Chapter Two

Data Collection

One of the most feasible business options owing to the straightforward manufacturing process involved starting a soap and detergent manufacturing business in Bangladesh. With significant growth potential, this market is one segment of the Fast Moving Consumer Goods (FMCG) market in Bangladesh. People use it on a daily basis for clothes, hand wash, and kitchen utensils and its demand is found in the market all through the year is a consumer good.

Moreover, with moderate capital investment, an entrepreneur can initiate a detergent manufacturing business. It's around 2.7kg per year the per capita detergent consumption in Bangladesh and its 3.7kg in Malaysia and Philippines and around 10kgs in the USA. On the other hand, the global liquid detergent market is expected to grow steadily over the next four years. So, this is a good business to start and more possibilities to be successful**.**

2.1 Market Potential of Soap and Detergent Manufacturing Business

From the last five years, the Bangladeshi soap and detergent industry is growing at 13.06%. There are three categories, lower, middle and higher-end markets while catering to the segment. And it's BDT. 500 crore detergents market is among the largest FMCG categories in Bangladesh and its next only to edible oils and biscuits. The demand for this product is flourishing due to rapid urbanization and the emergence of small pack size and sachets. Moreover, boost the purchasing capacity of the population while increasing per capita income.

In addition, other reasons for the growing demand for detergent powder are including a wide range of available choice, health awareness and hunger for good living. On the other hand, the rural population has replaced detergent cake with washing powder to wash their clothes in massive quantity.

2.2 Steps to Start a Soap and Detergent Manufacturing Business

It's a promising industry in Bangladesh of producing soap and detergent. As it requires a moderate capital investment, any individual can initiate a detergent manufacturing business in Bangladesh.

And it is intended to explore how to start a small-scale detergent powder manufacturing business in this article. Although it looks like an easy and simple to start the business, it's not so easy at all.

Not only some simple steps but many procedures are to follow if anyone wants success in it. There are some steps to start a soap and detergent manufacturing business in Bangladesh.

First Step: Business Plan

As an essential product used daily by billions of people, soap and detergent are a Fast Moving Consumer Good (FMCG) in Bangladesh. But, before starting a detergent manufacturing business, a great deal of market research and a well-framed business plan is needed. Also, a business plan should incorporate to this mission statement, budgeting, and target market.

Apart of these, there are some of the most important elements that should be included in a business plan for a detergent business:

- Target Market
- Cost of raw materials
- Source of raw materials
- Plant capacity cost
- Machinery cost
- Capital investment
- Marketing strategy
- Management structure
- Manufacturing process

Second Step: Required budget:

If it is chosen a medium sized detergent powder manufacturing unit then it is needed a 1000m sq. ft. area.

As there is the presence of a large number of competitors in the detergent manufacturing industry, initially, the struggle for selling would be too high. Also, there are required budgets for the following items:

- Manufacturing unit rent
- Raw Materials
- Employees
- Equipment
- Advertisement
- Insurance
- License and
- Registration

Third Step: Business Location

Keeping in mind that the location should have adequate availability of water, electric power, and transportation, the factory location should be chosen carefully. Also, the location should choose in a region having close proximity to the source of raw materials and somewhat nearby the target market.

Along with state and government zoning requirements, the factory should be in compliance. In addition, it should be selected the location that’s suitable for equipment and should have ample parking facility. Apart of these, the factory should be located in an industrial zone and there should be easy access to the factory through land transportation.

Fourth Step: Needed Equipment

A few modernized tools and equipment and ample space to work in the manufacturing facility to initiate the manufacturing process for an average detergent powder manufacturing plant are needed. Here is a complete list of the required equipment that is needed to start a soap and detergent manufacturing business in Bangladesh:

- Mixing vessels
- Reactors
- High-pressure tanks and reactors
- Neutralizer
- Pulverize
- Blender
- Cyclone
- Storage and raw materials tanks
- Weighing scale
- Blowers
- Furnace
- Spray dryer
- Conveyors sieve
- Perfumer
- Gas or electric stove
- Packaging machine
- Anti-pollution unit
- Waste disposal baggies and plastic bags
- Blenders, hand gloves and basins

Fifth Step: Required Raw Materials

Looking for the most ideal and cost-effective supplier of raw materials who can ship these to the business organizations at their manufacturing facility warehouse is the next important step. It consumes about 60% of the detergent business's working capital while purchasing raw materials.

- Although it can also be purchased these raw materials by the organizations from the wholesale market, doing this can be cost-effective and very time-consuming in the long run. Formulations essentially consist of active ingredients, STPP, Filler such as sodium sulfate and silicate of the detergent powder manufacturing. There is a list below that are the required raw materials to start a soap and detergent manufacturing business in Bangladesh:

- Soda ash light Surfactants
- Sodium sulfate
- Labsa
- Trisodium Phosphate
- Sodium Meta Silicate
- Sodium Tri Polyphosphate (STTP)
- Carboxy Methyl Cellouse
- Color
- Glauber's salt
- Fabric softeners
- Detergent builders
- Enzymes
- Bleaches and compounds
- Sodium silicate
- Caustic soda
- Synthetic perfumes and fragrances
- Polythene bags for packaging
- Alkyl benzene sulphonate

Sixth Step: Business Promotion

It plays an essential role in the success or failure of a business while promoting the product. As there are so many mediums through, companies can advertise or promote their soap or detergent powder and can reach a maximum number of people.

A huge amount of media promotions are required to establish the brand because detergent is a consumer goods business.

It can be thought of expanding operations to nearby areas after focus on targeting your local market. Also, it can be opened a detailed website of business describing all about organizations and product.

Moreover, traditional printing and television advertisement are to be used for different marketing strategies. Apart from these, many of the marketers utilize social media to boost the popularity of their company's laundry detergent brand.

Some Other Steps to Follow

- Business Branding
- Niche and Demographics
- Detail financial plan
- Manufacturing Process
- Decide Pricing
- Detergent Waste Disposal

2.3 Data Collection

For discussing a real life problem, it has chosen a Bangladeshi company named Mega Sornali Soap & Cosmetics Industries Ltd. It was established in 2015. This company produces five types of soap, three types of lemon powder and two types of mega extra powder.

Mega Sornali Soap & Cosmetics Industries Ltd

Established	2015
Employees	21+
Machine	5

Table 2.1 Measurement of Production: (Monthly)

Soap (10 base)	88000kg
Detergent Powder (1 base)	11200kg

Table 2.2 Roll Manpower

Number of managers	1
Number of mechanical engineer	1
Number of electrician	1
Number of machine operator	2
Fueling members	2
Gate keeper	1
Sweeper	1

Tool for Collection Data:

The collection of data is done through direct interview and telephonic conversation with the concerned people by visiting Soap & Cosmetics Industry.

Method of Collection Data:

Primary data is collected.

Primary Data:

During visit to Soap & Cosmetics industry by observation, the primary data like products process sequence, machine used for particular operation, no. of machines, no. of operator, skill matrix, learning performance are carried out by using through observation, recording and collections.

Duration of Work Shift:

8 workers work daily per shift. Sometimes two shifts are worked. These are day shift and night shift. Day shift continues from 8 a.m. to 5 p.m. and night shift continues from 6p.m. to 6 a.m. Friday is holiday.

Objectives:

Raw materials → Component→ Manufacturer → Retailer → Consumer

Types of Soap & Detergent Powder:

I. Soap
II. Lemon Detergent Powder
III. Extra Detergent Powder

Table 2.3 Raw Materials to Produce Soap

No.	Name	Cost (Tk)/Kg
01.	Silicate	14
02.	Palm Oil	76
03.	Palm Pati	80
04.	Rice Pati	54
05.	Palm Stearing	80.50
06.	Soybean	48.50
07.	Caustic Soda	32
08.	S.L.S.(Foam Powder)	290
09.	Perfumed	1000
10.	Colour	1000

Table 2.4 Raw Materials to Produce Lemon Detergent Powder

No.	Name	Cost (Tk)/Kg
01.	Dolomite	5
02.	Global Salt	12
03.	Calcium Carbonet	15
04.	Soda	32
05.	Lapsa (Foam)	125
06.	Colour	4000
07.	Perfume	1000

Table 2.5 Raw Materials to Produce Extra Detergent Powder

No.	Name	Cost(Tk)/Kg
01.	Limestone	7
02.	Soda	32
03.	Calcium Carbonet	15
04.	Global Salt	12
05.	Lapsa	125
06.	Sky White	300
07.	S. Perkel	55
08.	Perfume	1000

Table 2.6 Selling Price of Soap

No.	Name	Quantity(g)	Selling Prices Per Piece (Tk)
01.	Mega Sornali Sobi Marka Soap	250	11.66
02.	Sornali Bati Soap	175	6.50
03.	Sornali 2015	500	20
04.	Sornali Soap	250	10.41
05.	Mega Sornali Full Marka	250	8.33

Table 2.7 Selling Price of Lemon Powder

No.	Name	Quantity(g)	Selling Prices Per Piece (Tk)
01.	Mega Washing Powder	25	2.5
02.	Mega Washing Powder	200	6.86
03.	Mega Washing Powder	500	14

Table 2.8 Selling Price of Mega Extra Powder

No.	Name	Quantity(g)	Selling Prices Per Piece (Tk)
01.	Mega Extra Powder	200	10.32
02.	Mega Extra Powder	500	20

Selling prices are given in the following graph:

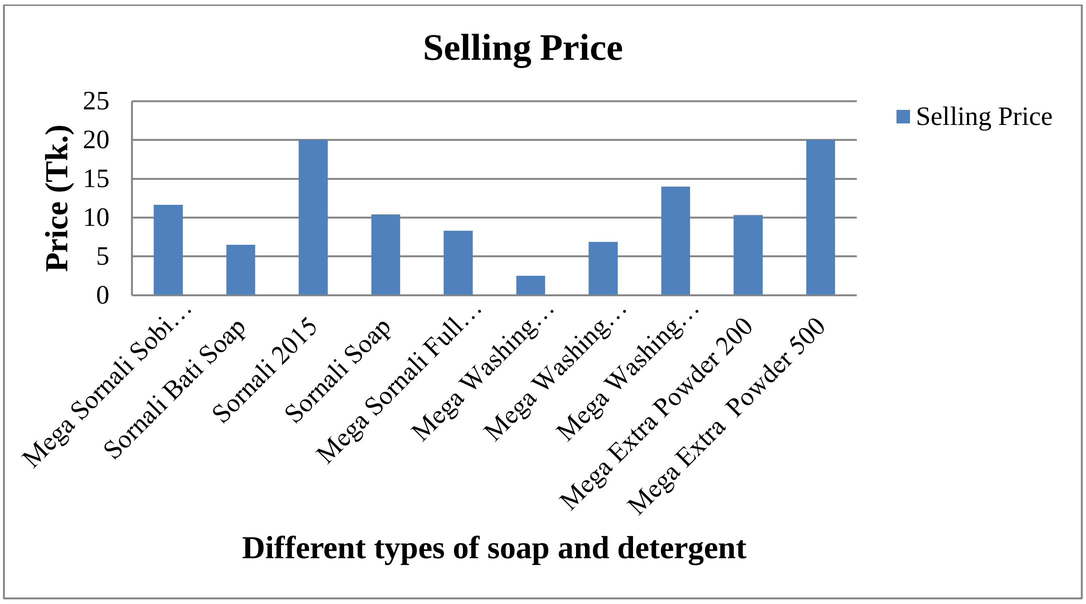

Figure 2.1: Selling Price of Soap & Detergent

Table 2.9 Price of Machine

No.	Name	Price(Tk)
01.	Mixer Machine	210000
02.	Sipter Machine	100000
03.	Packing Machine (Mini)	150000
04.	Packing Machine (250g, 500g)	300000

Table 2.10 Salary Structure

Post	Salary monthly(Tk)
Mechanical Engineer	15000
Manager	10000
Electrician	8000
Fueling	9500
Sweeper	5000
Machine Operator	5000

Table 2.11 Other Cost

Purpose	Cost(Tk)
Oil	2250
Tools	3000
Electric Motor (5 pieces)	9000
Total	**14250**

Order Duration:

Monthly

Festival Bonus:

Two Eid bonuses are given to every worker, manager and engineer. One Eid bonus is 50% of gross salary.

Electricity Bill:

Monthly electricity bill is 10,000 tk. Electricity bill per unit is tk. 8.

Local Order:

Maximum production goes to Chittagong, Sylhet, Cumilla, Noakhali market. Besides this, some orders come from Dhaka Market.

The company bears all expenses of workers accident.

Table 2.12 Some Brands of Foreign material

Country	Brand
Bhutan	Limestone, Dolomite
India	Lapsa
Taiwan	Foam Powder

Other raw materials come from Bangladesh.

Transportation System:

Covered van, Small truck

Rent Cost:

rent cost is 24000 tk.

Chapter Three

Mathematical Modeling

In recent years, Bangladesh has improved a lot in business sector. Though it is one of the most emerging sector and contributing a lot to our economy, still most of the business organizations don't use proper mathematical approaches to forecast their production rates, profits and losses. If they apply mathematical procedures in their business plan, they can get an exact idea about which product to produce at which rate and can also identify the ranges of costs of the products and the amount of required resources in which the profit will increase or remain the same.

In previous chapter, it is taken data from a soap factory of Bangladesh. In this chapter, it will be developed a mathematical model from this data which will be resulting into a large LPP and by applying the solving procedure of LPP and by applying the solving procedure of LPP in its production planning. It will be tried to identify its desired production rate and to answer some questions that may arise when thinking about the profit. It will be showed the impact of LPP in business planning.

3.1 Methodology

In this section, it will be discussed the steps of solving real life problem.

- Problem discussion
- Formulation of the Problem
- Solution of the Problem
- Result discussion
- Sensitivity Analysis

Problem Discussion

- To get an idea about the economic condition of the factory, at first it will be known the transaction data and other expenses. The demand and prices of the manufactured products of the factory will be obtained. Other expenses and inventories of the factory will also be taken into consideration.

- It will be used linear programming techniques to develop a mathematical model which will materialize the objectives of the study, based on the collection.

Solution of the Problem

- After formulating the mathematical model, the solution of the problem will be sought out with computational programming languages: AMPL.

Result Discussion

- The solution of the problem will be discussed briefly here. The interpretation of every value in the output will help to understand the problem.

Sensitivity Analysis

- It will be applied the technique of sensitivity analysis after obtaining the optimal result to see the effects of changes of costs or resources on the optimal solutions by using AMPL. It will be interpreted the sensitivity analysis results of the problem.

3.2 Formulation of the Problem

To solve the problem mathematically, first it is needed to formulate the problem as a mathematical model. To produce an LPP,

Let,

X_1 is the unit of Mega Sornali Sobi Marka Soap (250g).

X_2 is the unit of Sornali Bati Soap (175g).

X_3 is the unit of Sornali Soap 2015 (500g).

X_4 is the unit of Sornali Soap (250g).

X_5 is the unit of Mega Sornali Full Marka Soap (250g).

X_6 is the unit of Mega Washing Powder (25g).

X_7 is the unit of Mega Washing Powder (200g).

X_8 is the unit of Mega Washing Powder (500g).

X_9 is the unit of Mega Extra Powder (200g).

X_{10} is the unit of Mega Extra Powder (500g).

The objective function then becomes

Max z =

$2.725X_1+2.69X_2+7.649X_3+3.269X_4+2.663X_5+0.308X_6+2.717X_7+6.13X_8+3.62X_9+6.26X_{10}$

subject to

$$0.236X_1+0.295X_2+0.295X_3+0.268X_4+0.322X_5+0.163X_6+0.271X_7+0.295X_8+0.236X_9+0.3X_{10}\leq 60000 \qquad (3.1)$$

$$0.054X_1+0.068X_2+0.065X_3+0.055X_4+0.067X_5+0.029X_6+0.062X_7+0.075X_8+0.06X_9+0.1X_{10}\leq 850000 \qquad (3.2)$$

$$8.644X_1+3.447X_2+11.991X_3+6.818X_4+5.278X_5+2.0X_6+3.81X_7+7.5X_8+6.4X_9+13.33X_{10}\leq 1000000 \qquad (3.3)$$

$$0\leq X_1\leq 25000 \qquad (3.4)$$

$$0\leq X_2\leq 20000 \qquad (3.5)$$

$$0\leq X_3\leq 20000 \qquad (3.6)$$

$$0\leq X_4\leq 22000 \qquad (3.7)$$

$$0\leq X_5\leq 18000 \qquad (3.8)$$

$$0\leq X_6\leq 35000 \qquad (3.9)$$

$$0\leq X_7\leq 21000 \qquad (3.10)$$

$$0\leq X_8\leq 20000 \qquad (3.11)$$

$$0\leq X_9\leq 25000 \qquad (3.12)$$

$$0\leq X_{10}\leq 20000 \qquad (3.13)$$

Here, equation (3.1), (3.2), (3.3) mean labor cost, machine and other cost and raw material cost respectively.

Table 3.1 Showing data of the LPP

Variable	Labor cost	Machine and other cost	Raw material cost	Profit for each variable
X_1	0.236	0.054	8.644	2.725
X_2	0.295	0.068	3.447	2.69
X_3	0.295	0.065	11.991	7.649
X_4	0.268	0.055	6.818	3.269
X_5	0.322	0.067	5.278	2.663
X_6	0.163	0.029	2.0	0.308
X_7	0.271	0.062	3.81	2.717
X_8	0.295	0.075	7.5	6.13
X_9	0.236	0.06	6.4	3.62
X_{10}	0.3	0.1	13.33	6.26

3.3 Solution of the Problem

AMPL (A Mathematical Programming Language) is software to solve the LPP problem. LPP, Non-LPP, IP, stochastic programming, large LPP can be solved by AMPL. It consists of three parts. They are model file, data file and run file. After developing a model file, it has to arrange a data file according to the model file. Both the model and related data file must be called in command file with proper codes. Then to obtain the output of the problem it has to call command in AMPL. Then the solution can be found by run file using solver cplex.

Table 3.2 Model file of AMPL

```
set J;
set I;
param C {J} >=0;
param A {I,J} >=0;
param B {I} >=0;
var X{J} >=0;
maximize z: sum{j in J} C[j] * X[j];
s.t. Constraint {i in I}: sum {j in J} A[i,j] * X[j] <= B[i];
```

Data file: Value of different parameters

```
set J: = 1 2 3 4 5 6 7 8 9 10;
set I: = 1 2 3 4 5 6 7 8 9 10 11 12 13;
```

Table 3.3 Objective Function Coefficients

param C :=		
	1	2.725
	2	2.69
	3	7.649
	4	3.269
	5	2.663
	6	0.308
	7	2.717
	8	6.13
	9	3.62
	10	6.26;

Table 3.4 Cost Coefficients Matrix

param A :	1	2	3	4	5	6	7	8	9	10 :=
1	0.236	0.295	0.295	0.268	0.322	0.163	0.271	0.295	0.236	0.3
2	0.054	0.068	0.065	0.055	0.067	0.029	0.062	0.075	0.06	0.1
3	8.644	3.447	11.99	6.818	5.278	2.0	3.81	7.5	6.4	13.33
4	1	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	1

Table 3.5 Right Hand Side Constants

```
param   B:=
          1   60000
          2   850000
          3   1000000
          4   25000
          5   20000
          6   20000
          7   22000
          8   18000
          9   35000
         10   21000
         11   20000
         12   25000
         13   20000;
```

The optimal solution: maximum profit z = 623195.5866

Value of X_1=0,

X_2 = 20000,

X_3 = 20000,

X_4 = 22000,

X_5 = 18000,

X_6 = 0,

X_7 = 21000,

X_8 = 20000,

X_9 = 25000,

X_{10} = 4218.3;

3.4 Optimal Solution by BD

In this section it will be used BD to solve the problem.

Master Problem

max M = $2.725X_1+2.69X_2+7.649X_3+3.269X_4+2.663X_5$

subject to:

$0 \le X_1 \le 25000$ (3.14)

$0 \le X_2 \le 20000$ (3.15)

$0 \le X_3 \le 20000$ (3.16)

$0 \le X_4 \le 22000$ (3.17)

$0 \le X_5 \le 18000$ (3.18)

Solution: Iteration 01:

Master problem solution:

$X_1=25000$,

$X_2=20000$,

$X_3=20000$,

$X_4=22000$,

$X_5=18000$,

Master value 394667.

Primal Sub-Problem

max $P = 0.308X_6+2.717X_7+6.13X_8+3.62X_9+6.26X_{10}$

subject to:

$0.163X_6+0.271X_7+0.295X_8+0.236X_9+0.3X_{10} \le 60000-0.236X_1-0.295X_2-0.295X_3-0.268X_4+0.322X_5$ (3.19)

$0.029X_6+0.062X_7+0.075X_8+0.06X_9+0.1X_{10} \le 850000-0.054X_1-0.068X_2-0.065X_3-0.055X_4-0.067X_5$ (3.20)

$2.0X_6+3.81X_7+7.5X_8+6.4X_9+13.33X_{10} \le 1000000-8.644X_1-3.447X_2-11.991X_3-6.818X_4-5.278X_5$ (3.21)

$0 \le X_6 \le 35000$ (3.22)

$0 \le X_7 \le 21000$ (3.23)

$0 \le X_8 \le 20000$ (3.24)

$0 \le X_9 \le 25000$ (3.25)

$0 \le X_{10} \le 20000$ (3.26)

Dual Subproblem

Min D : λ_1(60000-0.236X_1-0.295X_2-0.295X_3-0.268X_4+0.322X_5)+λ_2(850000-0.054X_1-0.068X_2-0.065X_3-0.055X_4-0.067X_5)+λ_3(1000000-8.644X_1-3.447X_2-11.991X_3-6.818X_4-5.278X_5)+35000λ_4+21000λ_5+20000λ_6+25000λ_7+20000λ_8

= λ_1(60000-0.236*25000-0.295*20000-0.295*20000-0.268*22000-0.322*18000)+λ_2(850000-0.054*25000-0.068*20000-0.06520000-0.055*22000-0.067*18000)+λ_3(1000000-8.644*25000-3.447*20000-11.991*20000-6.818*22000-5.278*18000)+ 35000λ_4+21000λ_5+20000λ_6+25000λ_7+20000λ_8

=30600λ_1+843600λ_2+9230660λ_3+35000λ_4+21000λ_5+20000λ_6+25000λ_7+20000λ_8

Subject to:

$0.163\lambda_1+0.029\lambda_2+2.06\lambda_3+\lambda_4 \ge 0.308$ (3.27)

$0.271\lambda_1+0.062\lambda_2+3.81\lambda_3+\lambda_5 \ge 2.717$ (3.28)

$0.295\lambda_1+0.075\lambda_2+7.5\lambda_3+\lambda_6 \ge 6.13$ (3.29)

$0.236\lambda_1+0.06\lambda_2+6.49\lambda_3+\lambda_7 \ge 3.62$ (3.30)

$0.3\lambda_1+0.1\lambda_2+13.33\lambda_3+\lambda_8 \ge 6.26$ (3.31)

All $\lambda_i \ge 0$

Primal Subproblem Solution: **X_6=35000,**
X_7=21000,
X_8=20000,
X_9=25000,
X_{10}=20000;

Primal value 406137;

Dual problem solution: $\boldsymbol{\lambda_1 = 20.8667}$,
$\boldsymbol{\lambda_2=\lambda_3=\lambda_4=\lambda_5=\lambda_6=\lambda_7=\lambda_8=0}$**;**

Dual value 638520.

Table 3.6 AMPL Model File for BDM

```
param k>=1 default 1;
set I;
set J;
set K;
set L;
set M;
set N;

param nv :=5;
param nr :=10;
param vs :=8;

param c {I} >=0;
param d {J,I} >=0;
param b {J} >=0;
var xm {I} >=0;

param a {K} >=0;
param f {L,K} >=0;
param e {L} >=0;
var xs {K} >=0;

param g {M} >=0;
param h {N,M} >=0;
param r {N} >=0;
var xp {M} >=0;

maximize M1: sum {j in I} c[j]*xm[j];
subject to m1 {i in J}: sum {j in I} d[i,j]*xm[j]<= b[i];

var z;
maximize M2: sum {j in 1..nv-1} c[j]*xm[j] +c[nv]*z;
subject to m2 {i in J}: sum {j in 1..nv-1} d[i,j]*xm[j]+d[i,nv]*z<= b[i];

minimize D: sum {j in K} a[j]*xs[j];
subject to s {i in L}: sum {j in K} f[i,j]*xs[j]>= e[i];

maximize P: sum {j in M} g[j]*xp[j];
subject to w {i in N}: sum {j in M} h[i,j]*xp[j]<= r[i];
```

Data file: Value of different parameters

set I:= 1 2 3 4 5;

set J:= 1 2 3 4 5;

set K:= 1 2 3 4 5 6 7 8;

set L:= 1 2 3 4 5;

set M:= 1 2 3 4 5 6 7 8 9 10;

set N:= 1 2 3 4 5 6 7 8;

Table 3.7 Coefficient of Objective Function of Master Problem

```
param   c  :=
        1    2.725
        2    2.69
        3    7.649
        4    3.269
        5    2.663;
```

Table 3.8 Cost Coefficient Matrix of Master Problem

param d :	1	2	3	4	5	:=
1	1	0	0	0	0	
2	0	1	0	0	0	
3	0	0	1	0	0	
4	0	0	0	1	0	
5	0	0	0	0	1	

Table 3.9 Right Hand Constraints of Master Problem

```
param   b  :=
        1    25000
        2    20000
        3    20000
        4    22000
        5    18000;
```

Table 3.10 Coefficients of Objective Function of Dual Problem

param a :=	
1	30600
2	843600
3	9230660
4	35000
5	21000
6	20000
7	25000
8	20000;

Table 3.11 Coefficients of Variables in Constraints of Dual Problem

param f :	1	2	3	4	5	6	7	8 :=
1	0.163	0.029	2.06	1	0	0	0	0
2	0.271	0.062	3.81	0	1	0	0	0
3	0.295	0.075	7.5	0	0	1	0	0
4	0.236	0.06	6.4	0	0	0	1	0
5	0.3	0.1	13.33	0	0	0	0	1

Table 3.12 Right Hand Constraints of Dual Problem

param e :=	
1	0.308
2	2.717
3	6.13
4	3.62
5	6.26;

Table 3.13 Coefficients of Objective Function of Primal Problem

```
param  g  :=
        1    0
        2    0
        3    0
        4    0
        5    0
        6    0.308
        7    2.717
        8    6.13
       10    6.26;
```

Table 3.14 Coefficients of Variable in Constraints of Primal Problem

param h :	1	2	3	4	5	6	7	8	9	10 :=
1	0.236	0.295	0.295	0.268	0.322	0.163	0.271	0.295	0.236	0.3
2	0.054	0.068	0.065	0.055	0.067	0.029	0.062	0.075	0.06	0.1
3	8.644	3.447	11.99	6.818	5.278	2.0	3.81	7.5	6.4	13.33
4	1	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	1

Table 3.15 Right Hand Constants of Primal Problem

```
param  r  :=
          1     60000
          2     850000
          3     10000000
          4     35000
          5     21000
          6     20000
          7     25000
          8     20000;
```

Solution:

Iteration 02:

Master solution:

X_1=0,

X_2=16000,

X_3=20000,

X_4=22000;

Master value: 354757;

Primal sub problem solution

X_5=18000,

X_6=0,

X_7=21000,

X_8=20000,

X_9=25000,

X_{10}=11000;

Primal value 386951;

Dual solution:

$\lambda_1=14.327$,

$\lambda_2=6.27$,

$\lambda_3=\lambda_4=\lambda_5=\lambda_6=\lambda_7=\lambda_8=0$;

Dual value 563806.2;

Iteration 03:

Master solution:

$X_1=0,$
$X_2=20000,$
$X_3=20000;$

Master value: 306075.265;

Primal sub problem solution:

$X_4=22000,$
$X_5=18000,$
$X_6=0,$
$X_7=21000,$
$X_8=20000,$
$X_9=25000,$
$X_{10}=4218.3;$

Primal value: 306075.558;

Dual solution:

$\lambda_1=14.003,$
$\lambda_2=6.27,$
$\lambda_3=\lambda_4=\lambda_5=\lambda_6=\lambda_7=\lambda_8=0;$

Dual value 553891.8;

Since at iteration 3 the value of master problem and primal problem are same so optimal solution is obtained.

Optimal solution:

$X_1=0,$
$X_2 = 20000,$
$X_3 =20000,$
$X_4 = 22000,$
$X_5 = 18000,$
$X_6 = 0,$
$X_7 = 21000,$

X_8 = 20000,

X_9 = 25000,

X_{10} = 4218.5;

Objective z = 623194.859.

Table 3.16 Showing Result in BDM Using AMPL

Iteration number	Master solution	Primal solution	Dual solution
01.	X_1=25000, X_2=20000, X_3=20000, X_4=22000, X_5=18000, Master value: 394667.	X_6=35000, X_7=21000, X_8=20000, X_9=25000, X_{10}=20000; Primal value: 406137	λ_1 = 20.8667, λ_2=λ_3= λ_4=λ_5= λ_6= λ_7=λ_8=0; Dual value 638520
02.	X_1=0, X_2=16000, X_3=20000, X_4=22000; Master value: 354757	X_5=18000, X_6=0, X_7=21000, X_8=20000, X_9=25000, X_{10}=11000; Primal value 386951	λ_1=14.327, λ_2=6.27, λ_3= λ_4= λ_5= λ_6= λ_7= λ_8=0; Dual value 563806.2
03.	X_1=0, X_2=20000, X_3=20000; Master value: 306075.265	X_4=22000, X_5=18000, X_6=0, X_7=21000, X_8=20000, X_9=25000, X_{10}=4218.3; Primal value 306075.558	λ_1=14.003, λ_2=0, λ_3= λ_4= λ_5= λ_6= λ_7= λ_8=0; Dual value 553891.8

Dual variables indicate the shadow price of the problem.

Table 3.17 Comparison of Manually Result and BDM Result

Solution of main problem	Solution of BDM
X_1=0, X_2 = 20000, X_3 =20000, X_4 = 22000, X_5 = 18000, X_6 = 0, X_7 = 21000, X_8 = 20000, X_9 = 25000, X_{10} = 4218.3; objective z = 623195.5866	X_1=0, X_2 = 20000, X_3 =20000, X_4 = 22000, X_5 = 18000, X_6 = 0, X_7 = 21000, X_8 = 20000, X_9 = 25000, X_{10} = 4218.1; objective z = 623194.859

There are two graphs about selling price, profit and cost, selling price.

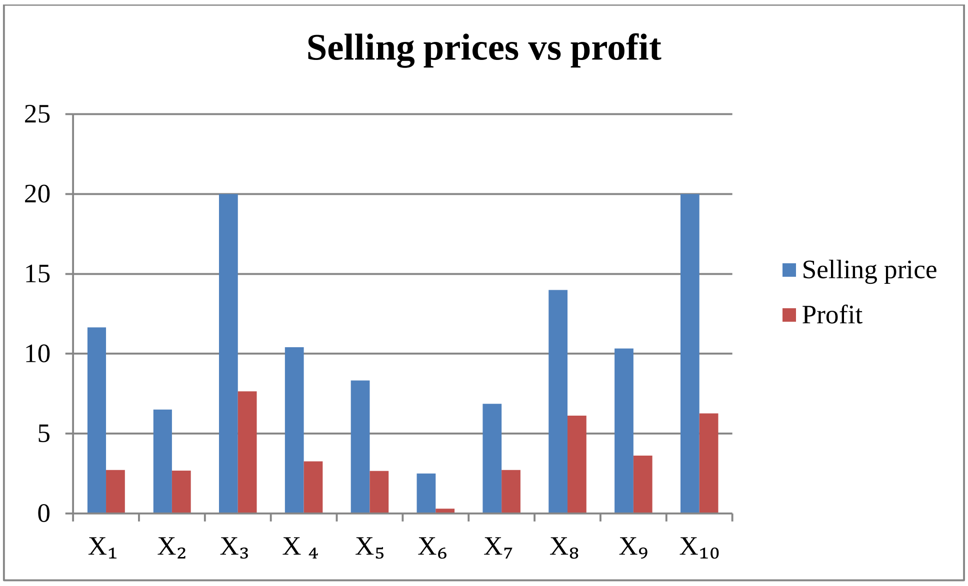

Figure 3.1: Selling Price and Profit

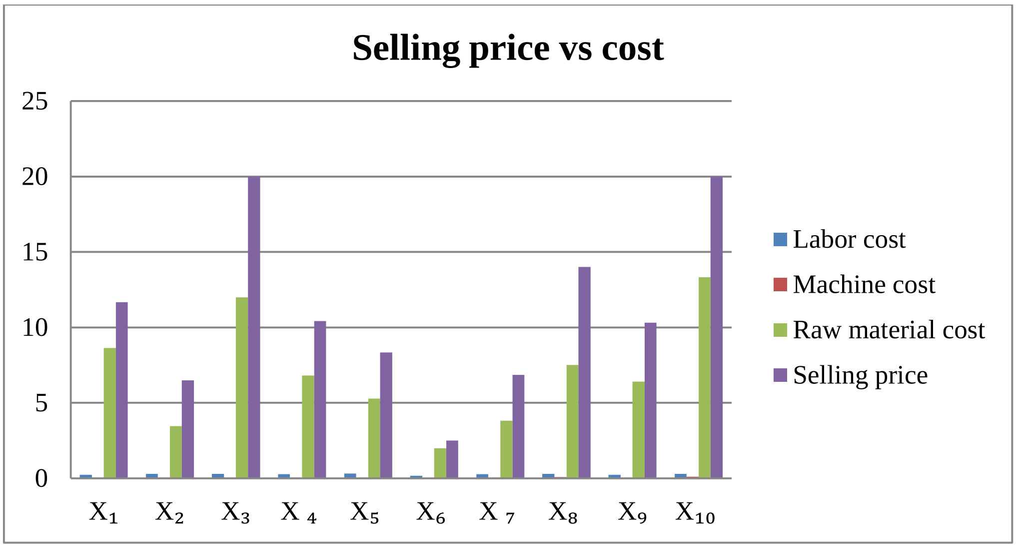

Figure 3.2: Selling Price and Cost

3.5 Sensitivity Analysis

Now, it will be discussed sensitivity analysis increasing and decreasing cost parameters by 5%, 10%, 15%.

Table 3.18 Showing Data of Increasing Cost Parameters by 5%, 10%, 15%

Variable	5% increase			10% increase			15% increase		
	Labor cost	Mach. and other cost	Raw mat. Cost	Labor cost	Mach. and other cost	Raw mat. Cost	Labor cost	Mach. and other cost	Raw mat. Cost
X_1	0.248	0.057	9.077	0.26	0.059	9.51	0.271	0.062	9.942
X_2	0.31	0.071	3.619	0.325	0.074	3.79	0.339	0.078	3.964
X_3	0.31	0.068	12.562	0.325	0.074	13.19	0.339	0.075	13.79
X_4	0.282	0.057	7.159	0.295	0.06	7.5	0.308	0.063	7.841
X_5	0.338	0.07	5.542	0.354	0.073	5.81	0.371	0.077	6.069
X_6	0.171	0.03	2.1	0.179	0.031	2.2	0.187	0.033	2.3
X_7	0.285	0.065	4	0.299	0.069	4.19	0.312	0.071	4.381
X_8	0.31	0.079	7.875	0.324	0.083	8.25	0.339	0.868	8.625
X_9	0.248	0.063	6.72	0.26	0.066	7.04	0.271	0.069	7.36
X_{10}	0.315	0.105	14.075	0.33	0.11	14.69	0.345	0.115	15.35

Data file for increasing cost by 5%

set J: = 1 2 3 4 5 6 7 8 9 10;

set I: = 1 2 3 4 5 6 7 8 9 10 11 12 13;

Table 3.19 Objective Function Coefficients Increasing Cost by 5%

```
param  C :=
        1   2.6
        2   2.5
        3   7.06
        4   2.912
        5   2.38
        6   0.20
        7   2.51
        8   5.736
        9   3.289
        10  5.51;
```

Table 3.20 Coefficients Matrix of Cost Increasing by 5%

param A :	1	2	3	4	5	6	7	8	9	10 :=
1	0.248	0.31	0.31	0.282	0.338	0.171	0.285	0.31	0.248	0.315
2	0.057	0.071	0.068	0.057	0.07	0.03	0.065	0.079	0.063	0.105
3	9.077	3.619	12.56	7.159	5.542	2.1	4.0	7.875	6.72	14.08
4	1	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	1

Table 3.21 Right Hand Side Constants Increasing Cost by 5%

param B:=	
1	60000
2	850000
3	1000000
4	25000
5	20000
6	20000
7	22000
8	18000
9	35000
10	21000
11	20000
12	25000
13	20000

Solution: optimal solution is z= 551372.7964;

Data file for increasing cost by 10%

set J: = 1 2 3 4 5 6 7 8 9 10;

set I: = 1 2 3 4 5 6 7 8 9 10 11 12 13;

Table 3.22 Objective Function Coefficients Increasing Cost by 10%

param	C :=	
	1	1.83
	2	2.31
	3	6.41
	4	2.56
	5	2.093
	6	0.09
	7	2.30
	8	5.34
	9	2.95
	10	4.87;

Table 3.23 Coefficients Matrix of Cost Increasing by 10%

param A :	1	2	3	4	5	6	7	8	9	10	:=
1	0.26	0.325	0.325	0.295	0.354	0.179	0.299	0.324	0.26	0.33	
2	0.059	0.074	0.074	0.06	0.073	0.031	0.069	0.083	0.066	0.11	
3	9.51	3.79	13.19	7.5	5.81	2.2	4.19	8.25	7.04	14.69	
4	1	0	0	0	0	0	0	0	0	0	
5	0	1	0	0	0	0	0	0	0	0	
6	0	0	1	0	0	0	0	0	0	0	
7	0	0	0	1	0	0	0	0	0	0	
8	0	0	0	0	1	0	0	0	0	0	
9	0	0	0	0	0	1	0	0	0	0	
10	0	0	0	0	0	0	1	0	0	0	
11	0	0	0	0	0	0	0	1	0	0	
12	0	0	0	0	0	0	0	0	1	0	
13	0	0	0	0	0	0	0	0	0	1	

Table 3.24 Right Hand Side Constants Increasing Cost by 10%

param	B:=	
	1	60000
	2	850000
	3	1000000
	4	25000
	5	20000
	6	20000
	7	22000
	8	18000
	9	35000
	10	21000
	11	20000
	12	25000
	13	20000;

Solution: Optimal solution, z = 488032.0267;

Data file for increasing cost by 15%

set J: = 1 2 3 4 5 6 7 8 9 10;

set I: = 1 2 3 4 5 6 7 8 9 10 11 12 13;

Table 3.25 Objective Function Coefficients Increasing Cost by 15%

param	C :=	
	1	1.385
	2	2.119
	3	5.8
	4	2.2
	5	1.813
	6	-0.02
	7	2.09
	8	4.95
	9	2.62
	10	4.19;

Table 3.26 Coefficients Matrix of Cost Increasing by 15%

param A	: 1	2	3	4	5	6	7	8	9	10	:=
1	0.271	0.339	0.339	0.308	0.371	0.187	0.312	0.339	0.271	0.345	
2	0.062	0.078	0.075	0.063	0.077	0.033	0.071	0.868	0.069	0.115	
3	9.94	3.964	13.79	7.841	6.069	2.3	4.381	8.625	7.36	15.35	
4	1	0	0	0	0	0	0	0	0	0	
5	0	1	0	0	0	0	0	0	0	0	
6	0	0	1	0	0	0	0	0	0	0	
7	0	0	0	1	0	0	0	0	0	0	
8	0	0	0	0	1	0	0	0	0	0	
9	0	0	0	0	0	1	0	0	0	0	
10	0	0	0	0	0	0	1	0	0	0	
11	0	0	0	0	0	0	0	1	0	0	
12	0	0	0	0	0	0	0	0	1	0	
13	0	0	0	0	0	0	0	0	0	1	

Table 3.27 Right Hand Side Constants Increasing Cost by 15%

param	B:=
1	60000
2	850000
3	1000000
4	25000
5	20000
6	20000
7	22000
8	18000
9	35000
10	21000
11	20000
12	25000
13	20000;

Solution: Optimal solution can not be found. Because the param C [6] = -0.02 is less than zero which is contradictory to the condition.

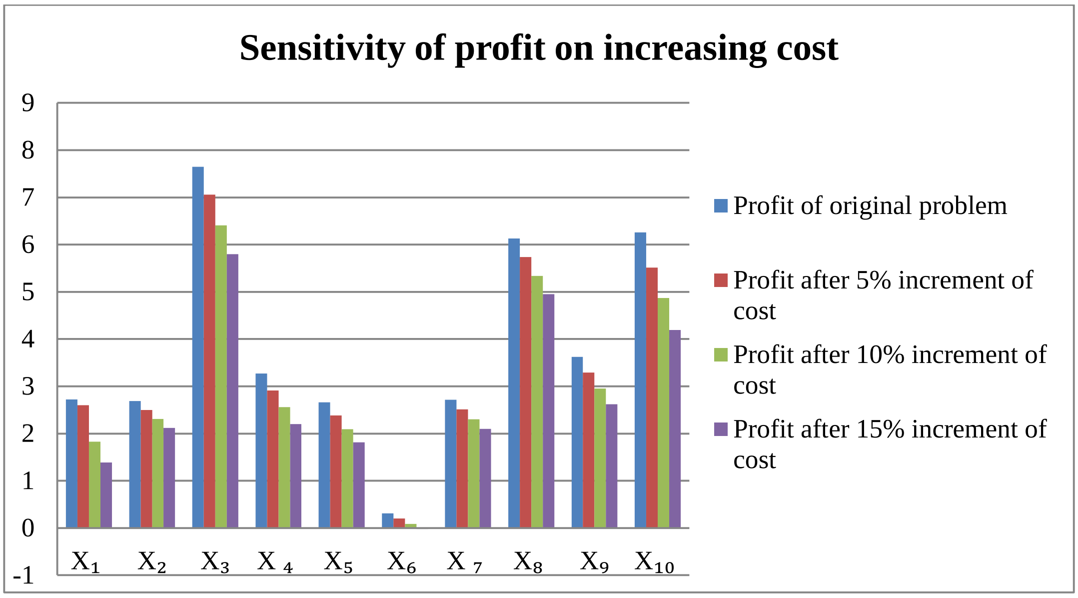

Figure 3.3: Decreasing of Profit by Increasing Cost Parameters

In the above graph, it is shown that how profit change if the cost parameters increase. It is clear from the graph that, if cost parameters increase, then profit decrease.

Table 3.28 Showing Data of Decreasing Cost Parameters by 5%, 10%, 15%

Variable	**5% decrease**			**10% decrease**			**15% decrease**		
	Labor cost	**Mach. and other cost**	**Raw mat. Cost**	**Labor cost**	**Mach. and other cost**	**Raw mat. cost**	**Labor cost**	**Mach. and other cost**	**Raw mat. Cost**
X_1	0.224	0.051	8.213	0.212	0.049	7.78	0.201	0.046	7.349
X_2	0.28	0.064	3.275	0.266	0.061	3.102	0.251	0.057	2.93
X_3	0.28	0.062	11.366	0.266	0.059	10.768	0.251	0.055	10.17
X_4	0.254	0.052	6.477	0.241	0.049	6.136	0.228	0.046	5.795
X_5	0.306	0.063	5.014	0.29	0.06	4.75	0.274	0.057	4.486
X_6	0.155	0.027	1.9	0.147	0.026	1.8	0.138	0.024	1.7
X_7	0.258	0.059	3.62	0.244	0.056	4.19	0.231	0.053	3.238
X_8	0.28	0.071	7.125	0.266	0.068	8.25	0.251	0.064	6.375
X_9	0.224	0.057	6.08	0.212	0.054	7.04	0.201	0.051	5.44
X_{10}	0.285	0.095	12.68	0.27	0.09	14.69	0.255	0.085	11.348

Data file for decreasing cost by 5%

set J: = 1 2 3 4 5 6 7 8 9 10;

set I: = 1 2 3 4 5 6 7 8 9 10 11 12 13;

Table 3.29 Objective Function Coefficients Decreasing Cost by 5%

param	C :=
1	3.172
2	2.881
3	8.292
4	3.627
5	2.947
6	0.418
7	2.923
8	6.524
9	3.959
10	6.937;

Table 3.30 Coefficients Matrix of Cost Decreasing by 5%

param A :	1	2	3	4	5	6	7	8	9	10 :=
1	0.224	0.28	0.28	0.254	0.306	0.155	0.258	0.28	0.224	0.285
2	0.051	0.064	0.062	0.052	0.063	0.027	0.059	0.071	0.057	0.95
3	8.213	3.275	11.36	6.477	5.014	1.9	3.62	7.125	6.08	12.68
4	1	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	1

Table 3.31 Right Hand Side Constants Decreasing Cost by 5%

```
param  B:=
    1  60000
    2  850000
    3  1000000
    4  25000
    5  20000
    6  20000
    7  22000
    8  18000
    9  35000
   10  21000
   11  20000
   12  25000
   13  20000;
```

Solution: Optimal solution is 703974.0339;

Data file for decreasing cost by 10%

set J: = 1 2 3 4 5 6 7 8 9 10;

set I: = 1 2 3 4 5 6 7 8 9 10 11 12 13;

Table 3.32 Objective Function Coefficients Decreasing Cost by 10%

```
param  C :=
    1  3.619
    2  3.071
    3  8.907
    4  3.984
    5  3.23
    6  0.527
    7  3.131
    8  6.916
    9  4.294
   10  7.625;
```

Table 3.33 Coefficients Matrix of Cost Decreasing by 10%

param A :	1	2	3	4	5	6	7	8	9	10 :=
1	0.212	0.266	0.266	0.268	0.241	0.29	0.147	0.244	0.266	0.212
2	0.049	0.061	0.059	0.049	0.06	0.026	0.056	0.068	0.054	0.09
3	7.78	3.102	10.76	6.136	4.75	1.8	3.429	6.75	5.76	12.015
4	1	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	1

Table 3.34 Right Hand Side Constants Decreasing Cost by 10%

param B:=	
1	60000
2	850000
3	1000000
4	25000
5	20000
6	20000
7	22000
8	18000
9	35000
10	21000
11	20000
12	25000
13	20000;

Solution: optimal solution is z=792659.9592

Data file for decreasing by15%

set J: = 1 2 3 4 5 6 7 8 9 10;

set I:= 1 2 3 4 5 6 7 8 9 10 11 12 13;

Table 3.35 Objective Function Coefficients Decreasing Cost by 15%

param	C :=	
	1	4.064
	2	3.262
	3	9.524
	4	4.341
	5	3.513
	6	0.638
	7	3.338
	8	7.31
	9	4.628
	10	8.312;

Table 3.36 Coefficients Matrix of Cost Decreasing by 15%

param A	: 1	2	3	4	5	6	7	8	9	10	:=
1	0.201	0.251	0.251	0.228	0.274	0.138	0.231	0.251	0.201	0.255	
2	0.046	0.057	0.055	0.046	0.057	0.024	0.053	0.064	0.051	0.085	
3	7.349	2.93	10.17	5.795	4.486	1.7	3.238	6.375	5.44	11.348	
4	1	0	0	0	0	0	0	0	0	0	
5	0	1	0	0	0	0	0	0	0	0	
6	0	0	1	0	0	0	0	0	0	0	
7	0	0	0	1	0	0	0	0	0	0	
8	0	0	0	0	1	0	0	0	0	0	
9	0	0	0	0	0	1	0	0	0	0	
10	0	0	0	0	0	0	1	0	0	0	
11	0	0	0	0	0	0	0	1	0	0	
12	0	0	0	0	0	0	0	0	1	0	
13	0	0	0	0	0	0	0	0	0	1	

Table 3.37 Right Hand Side Constants Decreasing Cost by 15%

param	B:=	
	1	60000
	2	850000
	3	1000000
	4	25000
	5	20000
	6	20000
	7	22000
	8	18000
	9	35000
	10	21000
	11	20000
	12	25000
	13	20000;

Solution: optimal solution is z = 891675.2168

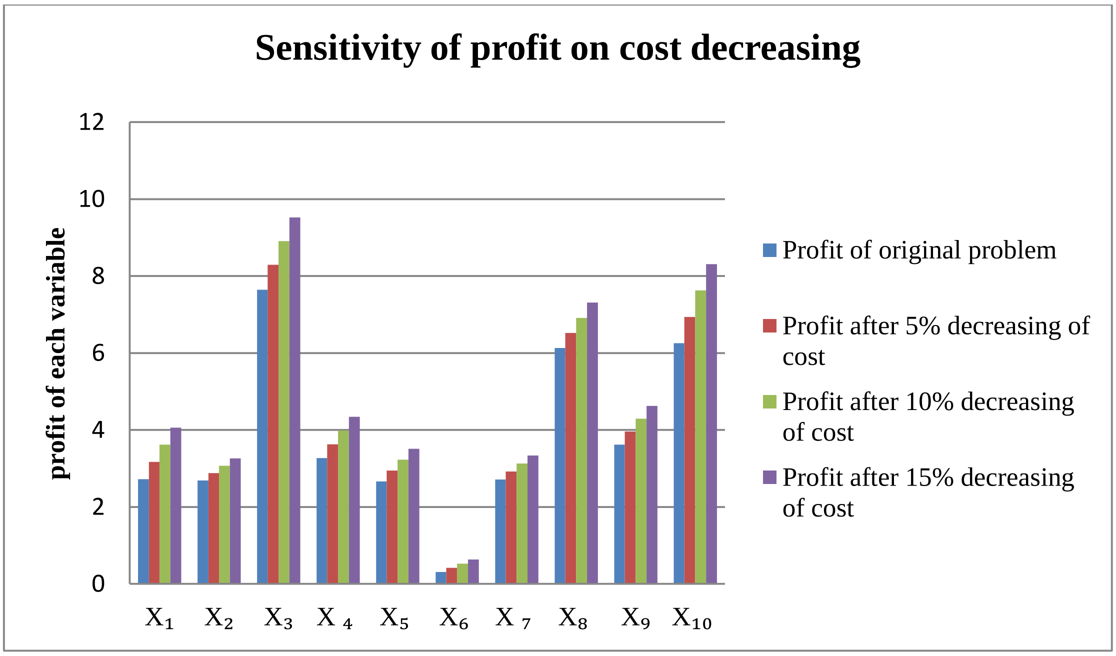

Figure 3.4: Increasing of Profit by Decreasing Cost Parameters

In the above graph, it is shown that how profit change if the cost parameters decrease. It is clear from the graph that, if cost parameters decrease, then profit increase.

It is clear that labor cost and machine cost have not much effect on profit. Now it will be shown the effect of raw material cost on profit.

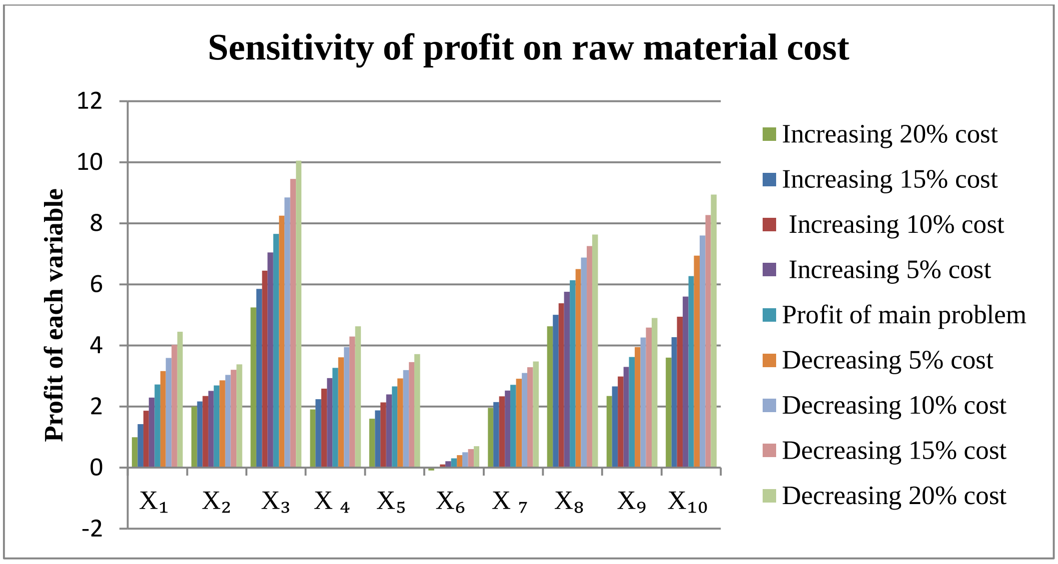

Figure 3.5: Profit Analysis on Raw Material Cost

In the above graph, it is shown that how profit changes if raw material cost changes. From the above graph, it is clear that if raw material cost increases profit decreases. Again, if the raw material cost decreases profit increases.

3.6 Result and Discussion

For the considered problem, the objective function is of maximization type and the objective function value gives the maximum profit. Here, the objective function value is 623195.5866. That means the maximum profit of the company is Tk 623195.

At first it is developed an LPP model using different kind of cost parameters. Then this problem is solved by AMPL. After that this problem is solved by BDM. To validate the problem, the LPP model is divided into master problem and primal sub problem. After three times iteration, the values of master and primal remain same. Then iteration is stopped. It is clear that manually and BDM achieve same result.

From the result it is found that, 20000 unit of Sornali Bati Soap, 20000 unit of Sornali Soap (2015), 22000 unit of Sornali Soap, 18000 unit of Mega Sornali Full Marka Soap, 0 unit of Mega Washing Powder (25g), 21000 unit of Mega Washing Powder (200g), 20000 unit of Mega Washing Powder (500g), 25000 unit of Mega Extra Washing Powder (200g), 4218 unit of Mega Extra Washing Powder (500g) are produced.

Table 3.38 Showing Profit for Per Unit Production

Name of variable	Profit for per unit production
X_1	2.725
X_2	2.69
X_3	7.649
X_4	3.269
X_5	2.663
X_6	0.308
X_7	2.717
X_8	6.13
X9	3.62
X_{10}	6.26

It is noticed from the result that, the production of product type one and six are zero. They are not so profitable. So, the company can stop to produce these two types of products. It is also noticed that production types three and ten are more profitable than other types of production. From sensitivity analysis, it has been found that if cost parameters increase by 5%, 10% and 15%, profit decrease. If cost parameters are decreased by 5%, 10%, 15% profit increase. It is also clear that labor cost and machine cost have not much effect on profit in the soap industry because labor cost and machine cost are very low in compare to other costs. Further, raw material cost is very much effective on profit. From the sensitivity analysis, it is also clear that if raw material cost can be reduced profit will be increased. It is shown that a small change can affect the profit a lot.

So, if the government can reduce tax and vat on raw materials of soap industry that is imported from abroad, this sector will become more profitable for the businessmen. If these raw materials can be produced in our country, soap industry will be more profitable in future than before. This sector can increase our GDP. It will also be able to contribute a lot to our economy.

From this data, the company can easily get a clear idea about their profit, production rate and selling procedures. The main aim of any company is to maximize their gain with minimum resources. In the case of this company, they can get best profit with minimum cost. Dual variable that is shadow prices help the company to assume their profit. Now it can be said that, if the company uses mathematical modeling technique and plans about its production according to the optimal solution, obtained by computer programming, they will get an accurate idea about the cost, production rate and profit.

3.7 Conclusion

In this thesis, a new technique for solving large LPP is presented. This problem is solved by BDM. To solve the problem easily, a computer code namely AMPL is used.

Chapter Four

Conclusion and Future Study

4.1 Conclusion

In this thesis, for the maximization of profit of Mega Sornali Soap & Cosmetic Industries Ltd., Benders' Decomposition Method (BDM) is used. After obtaining the optimal result, sensitivity analysis is also used to see the changes of optimal result after changing cost parameters.

In this thesis, ten types of production from the selected company have been taken into consideration. Labor cost, machine cost and raw material cost have also been taken into consideration. Then using these data, a Linear Programming Problem (LPP) is formulated. In this LPP, objective function is to maximize profit. Labor cost, machine cost, raw material cost and other cost are considered as subject to constraints. Maximum production rate that the company provided are also taken into consideration as subject to constraints. After that this LPP is solved in A Mathematical Programming Language (AMPL). Then this problem is solved by BDM using AMPL. Both the solutions gave the same result. The maximum profit of the company is Tk 623195. After that sensitivity analysis is discussed.

Sensitivity analysis helps the company to improve their business policy. In the sensitivity analysis, cost parameters are increased by 5%, 10% and 15%. Then it is found that profits decreased. Again, cost parameters are decreased by same percentages. In this case, it is found that profits increase. Both cases have been shown graphically. It is known that labor cost is very low in this country. Machine cost is also very low here. But raw material cost is very high. If the raw material cost can be reduced, this sector will become more profitable.

Like this company, applying of mathematical programming can help the owners of business organization to take correct decisions. This can identify the future production patterns and outlook resulting in the establishment of new production units, while thinking for maximizing profit and minimizing the cost of the company.

4.2 Future Study

In consideration of the present research the following can be put forward for the further works as follow-ups of the present research as. The recommendations for future works are as under:

- When we want to collect data the industry owners did not want to disclose their real data.
- In this study, we have collected data from a single industry. In future, it can be done by collecting data from more industries to get better result.
- In future some other cost parameters such as transportation cost can be included.
- In this paper, there is no discussion on shadow price. In future, anyone can work on it.
- In future this model can be used in other industries.

REFERENCES

[1] Dantzig, G. B. and Wolfe, P., “The Decomposition Algorithm for Linear Programming”, Econometrica, vol. 29, no. 4, pp. 767-778, 1961.

[2] Sweeny, D. J. and Murphy, R. A., “A Method of Decomposition for Integer Programs”, Operation Research, vol. 27, no. 6, pp. 1128-1141, 1979.

[3] Benders, J. F., “Partitioning Procedures for Solving Mixed-variables Programming Problems”, Numerische Mathematics, vol. 4, no. 1, pp. 238–252, 1962.

[4] Laporte, G., Nobert, Y. and Desrochers, M., “Optimal Routing with Capacity and Distance Restrictions”, Operations Research, vol. 33, pp. 1050–1073, 1985.

[5] Hasan, M. B. and Raffensperger, J. F., “A Decomposition Based Pricing Model for Solving a Large-scale MILP Model for an Integrated Fishery”, Hindawi Publishing Corporation, Journal of Applied Mathematics and Decision Sciences, vol. 2007, article id-56404, 10 pages, doi-10.1155/2007/56404, 2007.

[6] Cai, X., McKinney, D. C., Lasdon, L. S. and Watkins, Jr. D.W., “ Solving Large Non Convex Water Resources Management Models Using Generalized Benders Decomposition”, Operations Research, vol. 49, no. 2, pp. 235–245, 2001.

[7] Andreas, A. K. and Smith, J. C., “Decomposition Algorithms for The Design of A Nonsimultaneous Capacitated Evacuation Tree Network”, networks, vol. 53, no.2, pp. 91–103, 2009.

[8] Uddin, M. F., Mondal, M. and Kazi, A. H., “Vendor-Bayer coordination and supplychain optimization with Deterministic Demand Function”, Yugoslav Journal of Operation Research, Vol. 26, pp. 361-379, 2016.

[9] Georion, A. M., “Generalized Benders decomposition”, Journal of Optimization Theory and Applications, vol. 10, no. 4, pp. 237-260, 1972.

[10] Eremin A. and Wallace M., ‘‘Hybrid Benders decomposition algorithms in constraint logic programming”, In T. Walsh, editor, Principles and Practice of Constraint Programming of Lecture Notes in Computer Science,

Springer, vol. 2239, pp. 1-15, 2001.

[11] Bazaraa, M. S., Sherali, H. D. and Shetty, C. M., “Nonlinear Programming: Theory and Algorithms”, John Wiley & Sons, Inc., Hoboken, NJ, 2nd edition, 1993.

[12] Costa, A. M., “A survey on Benders decomposition applied to fixed-charge network design problems. Computers & Operations Research”, vol. 32, no.6, pp-1429-1450, 2005.

[13] Nielsen, S. S. and Zenios, S. A., “Scalable parallel Benders decomposition for stochastic linear programming”, Parallel Computing, vol. 23, no.8, pp. 1069-1088, 1997.

[14] Taskin, Z. C., Smith, J. C. and Romeijn, H. E., “Mixed-integer programming techniques for decomposing IMRT fluence maps using rectangular apertures”, Technical report, Department of Industrial and Systems Engineering, University of Florida, Gainesville, FL, 2009.

[15] Applegate, D., Bixby, R., Chavatal, V. and Cook W., “Implementing the Dantzig-Fulkerson-Johnson algorithm for large traveling salesman problems”, Mathematical Programming, vol. 97, no. (1-2), pp. 91-153, 2003.

[16] De Camargo, R.S., de Miranda, Jr. G. and Ferreira, R.P. M., “A hybrid outer approximation/benders decomposition algorithm for the single allocation hub location problem under congestion”, Operations Research Letters, vol. 39, no. 5, pp. 329-337, 2011.

[17] Cordeau, J. F., Soumis F. and Desrosiers, J., “A Benders’ decomposition approach for the locomotive and car assignment problem”, Transportation Science, vol. 34, no. 2, pp. 133-149, 2000.

[18] Geffrion, A.M., “Generalized Benders’ decomposition”, Journal of Optimization Theory and Applications, vol. 10, no.4, pp. 237-260, 1972.

[19] Magnanti, T. L. and Wong, R.T., “Accelerated Benders’ decomposition: algorithmic enhancement and model selection criteria”, Operations Research, vol. 29, no.3, pp. 464-484, 1981.

[20] Montemenni, R. and Gambardella, L.M., “The robust shortest path problem with interval data via Benders’ Decomposition”, Quantity Journal

of Operations Research, vol. 3, no. 4, pp. 315-328, 2005.

[21] Emeretlis, A., Theodoridis, G., Alefragis, P. and Voros, N., “Mapping DAGs on heterogeneous platforms using logic-based Benders decomposition”, In IEEE computer Society Annual Symposium on VLSI (ISVLSI), pp. 119–124, 2015.

[22] Rasmussen, R. V. and Trick, M. A., “A Benders approach to the constrained minimum break problem”, European Journal of Operational Research, vol. 177, pp. 198–213, 2007.

[23] Ralphs, T. K., Kopman, L., Pulleyblank, W. R. and Trotter Jr., L. E., “On the capacitated vehicle routing problem”, Mathematical Programming, vol. 94, pp. 343-359, 2003.

[24] Sherali, H. D. and Fratichli, B.R., “A modification of Benders’ decomposition algorithm for discrete sub-problems: an approach for stochastic programs with integer resource”, Journal of Global Optimization, vol. 22, no. 1, pp. 319-342, 2002.

[25] Xu, M., Chen, A., Qu, Y. and Gao, Z., “A semi-smooth Newton’s method for traffic equilibrium problem with a general non-additive route cost, Applied Mathematical Modelling”, vol. 35, no. 6, pp. 3048–3062, 2011.

[26] Santoso, T., Ahmed, S., Geeetschalckx, M. and Shapiro, A., “A stochastic programming approach for supply chain network design under uncertainty”, European Journal of Operational Research, vol. 167, pp. 96-115, 2005.

[27] Rahimi, S., Niknam, T. and Fallahi, F., “A new approach based on Benders’ decomposition for unit commitment problem”, Word Applied Science Journal, vol. 6, no. 12, pp. 1665-1672, 2009.

[28] Salam, S., “Unit commitment solution methods”, Word Academy of Science, Engineering and Technology, vol. 1, pp. 11-22, 2007.

[29] Osman, H. and Demirli, K., “A bilinear goal programming model and a modified Benders decomposition algorithm for supply chain reconfiguration and supplier selection”, Int. J. Production Economics, vol. 124, no. 1, 97-105, 2010.

[30] Lin, S., De Schutter, B., Xi, Y. and H. Hellendoorn, H., “Efficient network-

wide model-based predictive control for urban traffic networks", Transportation Research Part C, vol. 24, pp. 122-140, 2012.

[31] Lu, X. Y., Varaiya, P., Horowitz, R., Su, D. and Shladover, S., "A new approach for combined freeway variable speed limits and coordinated ramp metering", In 13th IEEE International Conference on Intelligent Transportation Systems, pp. 491-498, 2010.

[32] Papamichail, I., Kotsialos, A., Margonis, I. and Papageorgiou, M., "Coordinated ramp metering for freeway networks: a model-predictive hierarchical control approach", Transportation Research Part C: Emerging Technologies, vol. 18, no. 3, pp.311-331, 2010.

[33] Pisarski D. and Canudas-de-Wit, C., "Optimal balancing of road traffic density distributions for the cell transmission model", In 51st IEEE Conference on Decision and Controls, 2012, Maui, Hawaii, pp. 6969-6974, 2012.

[34] Wongpiromsarn, T. Uthaicharoenpong, T., Wang, Y., Frazzoli, E. and Wang, D., "Distributed traffic signal control for maximum network throughput", in 15th Inter-national IEEE Conference on Intelligent Transportation Systems, Anchorage, Alaska, pp. 588-595, 2012.

[35] Chen, A., Zhou, Z. and Xu, X., "A self-adaptive gradient projection algorithm for the nonadditive traffic equilibrium problem", Computers & Operations Research, vol. 39, no. 2, pp. 127–138, 2012.

[36] Conejo, R., Minguez, E., Castillo, R. and Bertrand, G., "Decomposition Technique in Mathematical Programming", Engineering and Science Applications, Springer, 2005.

[37] Patriksson, M., "Partial linearization methods in nonlinear programming", Journal of Optimization Theory and Application, vol.78, no. 2, pp. 227-246, 1993.

[38] Barahona, F. and Anbil, R., "The volume algorithm: Producing primal solutions with a sub gradient method", Mathematical Programming, vol. 87, pp.385-399, 2000.

[39] Shen, S. and Smith, J. C., "A decomposition approach for solving a broadcast domination network design problem", Annals of Operations

Research, 210:333–360, 2011.

[40] Lucena, A., “Non delayed relax-and-cut algorithms”, Annals of Operations Research, vol. 140, no. 1, pp. 375-410, 2005.

[41] Lysgaard, J., Letchford, A. N. and Eglese, R. W., “A new branch-and-cut algorithm for the capacitated vehicle routing problem”, Mathematical Programming, vol. 100, pp. 423-445, 2004.