PERFORMANCE ANALYSIS OF HYBRID NANOFLUID FLOWS IN LID-DRIVEN UNDULATED ENCLOSURE

by

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Candidate's Declaration

It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

Md. Saddam Hossain

This work is dedicated

to

My Mother

Acknowledgement

I would like to affirm the continual mercy, help and blessing showered by the Almighty without which it would have been impossible to accomplish the arduous job I was assigned to.

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Abstract

In this research, a numerical analysis has been carried out on combined convection in a lid driven trapezoidal enclosure with sinusoidal wavy bottom surface filled with hybrid nanofluid composed of equal quantities of Cu and Al_2O_3, Cu and CNT, Cu and CuO, and Cu and TiO_2 nanoparticles dispersed in water-based fluid. The left and right inclined walls of the trapezium have been considered as insulated while the top side of the cavity as cooled isothermally and the bottom sinusoidal wavy wall as hot. The two-dimensional governing partial differential equations of heat transfer and fluid flow with appropriate boundary conditions have been solved using finite element method of Galerkin's weighted residual approach built in COMSOL Multyphysics. The code validation has been shown. The implications of Richardson number, Prandtl number, number of waves, and solid volume fraction of nanoparticles of four types of hybrid nanofluids on the flow structure and heat transfer characteristics have been investigated in details. The comparison of heat transfer rate using hybrid nanofluids namely water-Cu-TiO_2, water-Cu-CuO and water-Cu-Al_2O_3, water-Cu-CNT and clear water has been also shown. Results have been presented in terms of streamlines, isothermal lines and average Nusselt number of the above-mentioned hybrid nanofluids for different values of governing parameters. The numerical results indicate that the Richardson number has significant effect on heat transfer performance from Ri = 0.1 (forced convection) to Ri = 10 (natural convection) about 15%. Moreover, it is noticed that combination of two different nanoparticles suspension has a better performance of heat transfer than single nanoparticle. Also, higher rate of heat transfer is found by 14.7% for wavy surface than flat surface of the enclosure. Higher heat transfer rate is found about 4.1, 3.1, and 2% using water-Cu-CNT hybrid nanofluid with compare to water-Cu-TiO_2, water-Cu-CuO and water-Cu-Al_2O_3, respectively. It is concluded that using hybrid nanofluid as heat transfer medium is more convenient than single nanofluid or base fluid.

Contents

Nomenclature

C_p	Specific heat at constant pressure ($Jkg^{-1}K^{-1}$)
g	Gravitational force (ms^{-2})
k	Thermal conductivity ($Wm^{-1}K^{-1}$)
Nu	Nusselt number
p	Pressure ($kgms^{-2}$)
Pr	Prandtl number
Ri	Richardson number
T	Dimensional temperature (K)
u, v	Dimensional x and y components of velocity (ms^{-1})
U, V	Dimensionless velocities
X, Y	Dimensionless coordinates
x, y	Cartesian coordinates (m)

Greek Symbols

β	Coefficient of thermal expansion (1/K)
ρ	Density (kgm^{-3})
μ	Dynamic viscosity (Nsm^{-2})
	Weight fraction (%)
θ	Dimensionless temperature
λ	Number of undulations

Subscripts

f	Fluid
hnf	Hybrid nanofluid
h	Hot
c	Cold
1	First nanoparticle
2	Second nanoparticle

Abbreviation

Al_2O_3	Aluminium tri-oxaide
CuO	Copper oxaide
CNT	Carbon nanotube
FEA	Finite element analysis

FEM	Finite element method
HTF	Heat transferring fluid
$Ti0_2$	Titanium oxaide

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Chapter 1

Introduction

1.1 Introduction

One of the proficient passive approaches is using nanofluid in heat transport improvement for enhancing the efficiency of thermal systems like heat exchangers, thermal storage, solar collectors, photovoltaic/thermal system, biomedical devices, nuclear reactors, cooling of electronic components etc. In recent years, there has been an increasing interest in merge of two or more nanoparticles in base fluid known as "hybrid nanofluid" due to improvement in cooling performance.

Researches on the nanofluids have been increased very rapidly over the past decade. In spite of some inconsistency in the reported results and insufficient understanding of the mechanism of the heat transfer in nanofluids, it has been emerged as a promising heat transfer fluid. In the continuation of nanofluids research, the researchers have also tried to use hybrid nanofluid recently, which is engineered by suspending dissimilar nanoparticles either in mixture or composite form. The idea of using hybrid nanofluids is to further improvement of heat transfer and pressure drop characteristics by trade-off between advantages and disadvantages of individual suspension, attributed to good aspect ratio, better thermal network and synergistic effect of nanomaterials.

1.1.1 Nanofluid

Nanofluids are treating as a two components mixture made of a base fluid and nano-particles (1-100 nm). The fundamental characteristics of the nanofluid are the raise of the thermal conductivity of the fount fluid, minimal impeding in flow passing, extensive stability and equity. In order to get better execution of heat generating, the nanofluid are utilized in several artificial applications such as chemical production, power generator in power plant, productions of micro-electronics, automotives, advance nuclear system, and nano-drug delivery. Sakiadis was the pioneer who established the concept of 2D boundary layer flow on continuous solid surface. The basic differential and integral momentum equations of boundary layer theory are governed and these equations are solved for moving continues flat surface and moving continuous cylindrical surface as well as for both laminar and turbulent flow in the boundary layer.

Nanofluids can be considered as the future of heat transfer fluids in various heat transfer applications. They are expected to give better thermal performance than conventional fluids due to the presence of suspended nanoparticles which have high thermal conductivity. Lately, there have been numerous investigations that have revealed the enhancement of thermal conductivity and higher heat transfer rate of nanofluids. Significant enhancement in the heat transfer rate with the use of various nanofluids in various application compared to conventional fluids have been reported by several researchers. Understanding the properties of nanofluids, such as thermal conductivity, viscosity and specific heat, is very important for the utilization of nanofluids in various applications. Further study of the fundamentals for heat transfer and friction factors in the case of nanofluids is considered to be very important in order to extend the applications of nanofluids.

Nanofluids are colloidal suspensions, made out of nanoparticles in some base fluid. These nanoparticles dictate the types of nanofluids and the distinction between them. Different properties and their role in green energy harvesting with an example where it has been used. Nanofluids are known for their thermal conductivity, which may be quite enhanced as compared with that of just the base fluid or that of the bulk colloidal suspensions. This property is useful in extracting more energy from nuclear and geothermal power plants, enhancing their efficiencies.

Nanofluids are fluids with different nanoparticles (particles smaller than 100 nm) and base fluids. Many types of nanoparticles such as metals (Cu, Ag, Au), oxide ceramics (Al_2O_3, CuO), carbon nanotubes and carbide ceramics (SiC, TiC) and various liquids such as water, oil, and ethylene glycol are used. Two general methods are used to produce nanofluids; namely two-step and one-step methods. In the two-step method, the nanoparticles are made separately and dispersed in base fluid by different method. Ultrasound and high-shear dispersion techniques are used to produce nanofluids with oxide nanoparticles by the two-step method. Nanoparticles are made and dispersed in a fluid in a single process in the one-step method. This method is used to produce nanofluids in small quantities for research purposes. For nanofluids with high conductivity metal nanoparticles, the one-step method is preferred. Figure 1.1 shows how nanofluid is made. According to figure the direct mixing of nanoparticle and base fluid forms nanofluid.

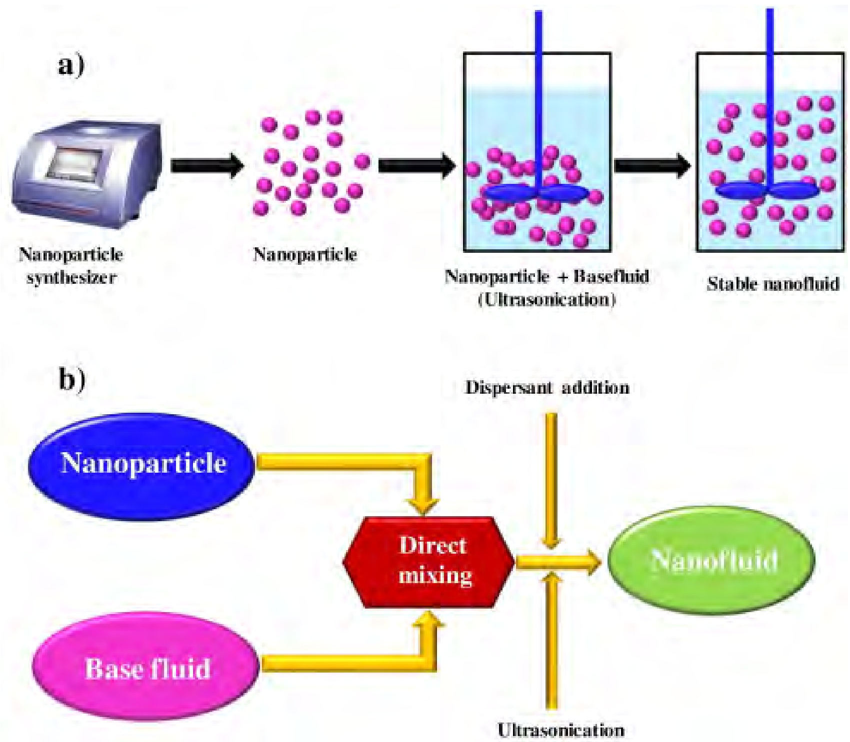

Figure 1.1: Formation of nanofluid

1.1.2 Hybrid nanofluid

"Hybrid" nanofluid can be obtained by suspending more than one type of nanoparticles in base fluid. Hybrid nanofluid is actually the mixture of two or more types of nanoparticles with a base fluid like water. Hybrid nanofluid is a new nanotechnology fluid that is synthesized by dispersing two different nanoparticles into conventional heat transfer fluid. Recently, researchers have indicated that hybrid nanofluids can effectively substitute the convectional coolant especially those working at very high temperatures. Hybrid nanofluids and hybrid nanolubricants are very new types of research which can be prepared by suspending two or more than two dissimilar nanoparticles either in a mixture or composite form in the base fluids. The term hybrid can be considered as different materials which are a combination of physical and chemical properties to form a homogeneous phase. The main objective of synthesizing hybrid nanofluids/nanolubricants is to improve the properties of single materials where it has great enhancement in thermal properties or rheological properties that are better than individually conventional nanofluids/nanolubricants. Figure 1.2 shows the preparation of hybrid nanofluid.

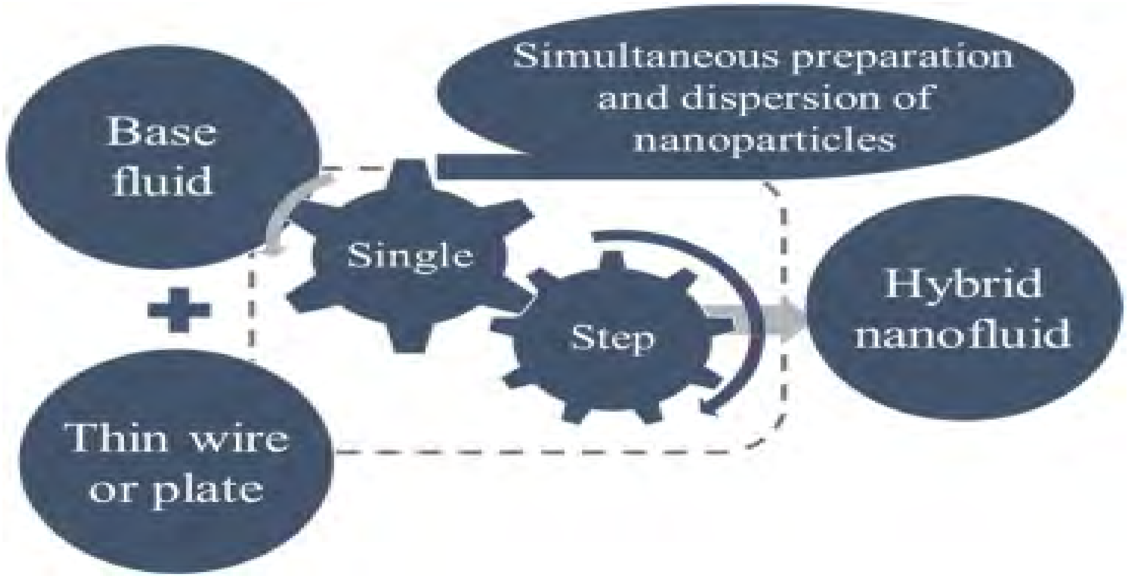

Figure 1.2: Preparation of hybrid nanofluid

1.1.3 Lid-driven cavity

The lid-driven cavity is an important fluid mechanical system serving as a benchmark for testing numerical methods and for studying fundamental aspects of incompressible flows in confined volumes which are driven by the tangential motion of a bounding wall. The lid-driven cavity is a well-known benchmark problem for viscous incompressible fluid flow. The geometry at stake has been shown in the Figure 1.3.

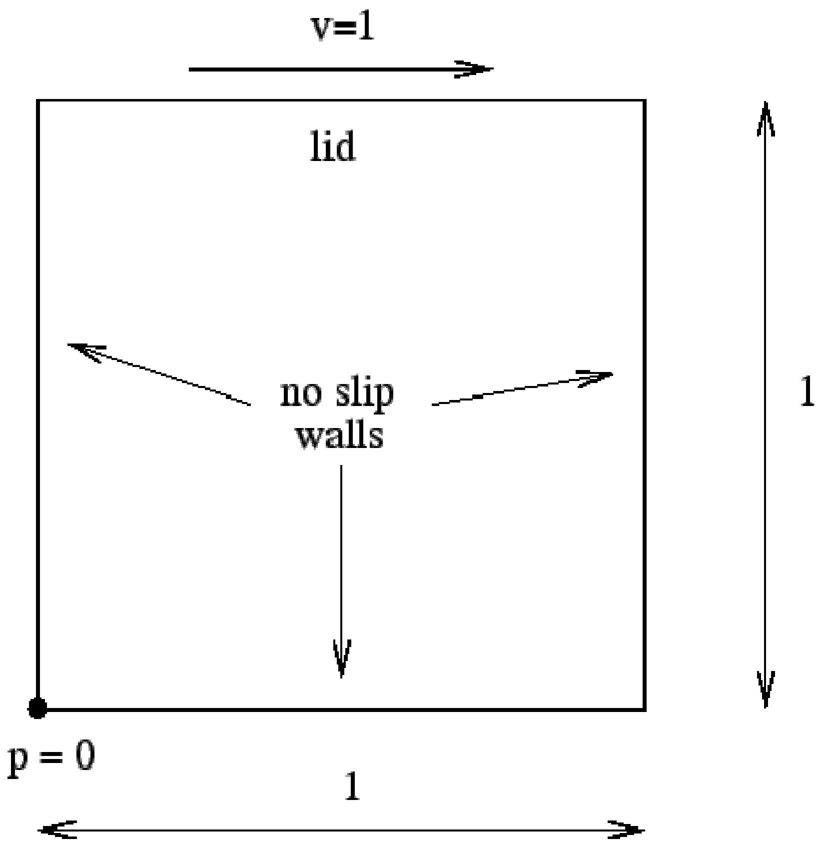

Figure 1.3: Geometry of the lid-driven cavity

1.1.4 Undulated enclosure

Undulation means having a wavy surface, edge, or markings. It also means to form or move in waves, to rise and fall in volume, pitch, or cadence, to present a wavy appearance. Undulation enclosure is to move with a sinuous or wavelike motion; display a smooth rising-and-falling or side-to-side alternation of movement. The geometry has been shown for undulated enclosure in the Figure 1.4.

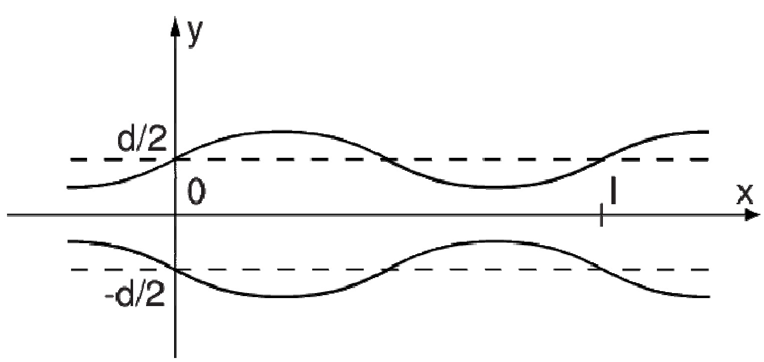

Figure 1.4: Undulated enclosure

1.1.5 Mixed convection

Mixed convection flows or combined free and forced convection flows occur in many technological and industrial applications in nature, e.g., solar receivers exposed to wind currents, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchangers placed in a low-velocity environment, flows in the ocean and in the atmosphere, and so on. Mixed (combined) convection is a combination of forced and free convections which is the general case of convection when a flow is determined simultaneously by both an outer forcing system (i.e., outer energy supply to the fluid-streamlined body system) and inner volumetric (mass) forces, viz., by the nonuniform density distribution of a fluid medium in a gravity field. The most vivid manifestation of mixed convection is the motion of the temperature stratified mass of air and water areas of the Earth that the traditionally studied in geophysics. However, mixed convection is found in the systems of much smaller scales, i.e., in many engineering devices. On heating or cooling of channel walls, and at the small velocities of a fluid flow that are characteristic of a laminar flow, mixed convection is almost always realized. Pure forced laminar convection may be obtained only in capillaries. Studies of turbulent channel flows with substantial gravity field effects have actively developed since the 1960s after their becoming important in engineering practice by virtue of the growth of heat loads and channel dimensions in modern technological applications (thermal and nuclear power engineering, pipeline transport).

1.1.6 Richardson number

Richardson number, parameter that can be used to predict the occurrence of fluid turbulence and, hence, the destruction of density currents in water or air. It was defined by the British meteorologist Lewis Fry Richardson, a pioneer in mathematical weather forecasting. Essentially the ratio of the density gradient (the change in density with depth) to the velocity gradient, the Richardson number is defined as

$$Ri = \frac{g}{\rho}\frac{\partial \rho}{\partial z} / (\frac{\partial u}{\partial z})^2$$

in which g is gravity, ρ is density, u is velocity, and z is depth. The Richardson number, or one of several variants, is of practical importance in weather forecasting and in investigating density and turbidity currents in oceans, lakes, and reservoirs.

1.1.7 Prandtl number

The relative thickness of the velocity and the thermal boundary layers are best described by the dimensionless parameter Prandtl number. Prandtl number is a dimensionless number, named after the German physicist Ludwig Prandtl, defined as the ratio of kinematic viscosity to thermal diffusivity and may be written as follows

$$Pr = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\upsilon}{D'_T}$$

The value of υ shows the effect of viscosity of the fluid. The smaller the value of υ is the narrower of the region which is affected by viscosity and which is known as the boundary layer region when υ is very small. The value of D'_T shows the thermal diffusivity due to heat conduction. The smaller the value of D'_T is the narrower of the region which is affected by the heat conduction and which is known as thermal boundary layer when D'_T is small. Thus, the Prandtl number shows the relative importance of heat conduction and viscosity of a fluid. For a gas the Prandtl number is of order of unity.

1.1.8 Nusselt number

From the temperature profile, the rate of heat transfer coefficient at the vertical plate can be obtained, which in non-dimensional form, in terms of Nusselt number, is defined by $Nu = \left(\frac{\partial T}{\partial y}\right)_{R=0}$.

1.1.9 Heat transfer

Heat transfer describes the exchange of thermal energy, between physical systems depending on the temperature and pressure, by dissipating heat. Figure 1.5 expresses general heat transfer procedure. The fundamental modes of heat transfer are conduction or diffusion, convection (free and forced) and radiation. Thermal equilibrium is reached when all involved bodies and the surroundings reach the same temperature.

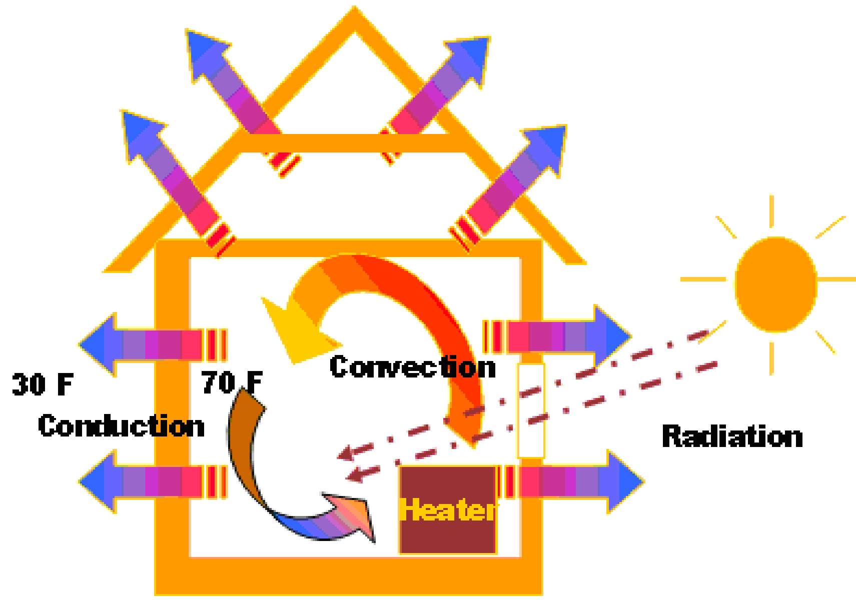

Figure 1.5: Heat transfer system

1.1.10 Viscosity

Viscosity is that property of real fluid as a result of which they offer some resistance to shearing, i.e., sliding movement of one particle past or near another particle. Viscosity is also known as internal friction of fluid. All known fluids have this property in varying degree. Viscosity of glycerin and oil is large in comparison to viscosity of water or gases.

1.1.11 Thermal conductivity

Thermal conductivity refers to the amount/speed of heat transmitted through a material. Heat transfer occurs at a higher rate across materials of high thermal conductivity than those of low thermal conductivity. Materials of high thermal conductivity are widely used in heat sink applications and materials of low thermal conductivity are used in thermal insulation. Thermal conductivity of materials is temperature dependent. The reciprocal of thermal conductivity is called thermal resistivity. Metals with high thermal conductivity, e.g. copper exhibits high electrical conductivity. The heat generated in high thermal conductivity materials is rapidly conducted away from the region of the weld. For metallic materials, the electrical and thermal conductivity correlate positively, i.e. materials with high electrical conductivity (low electrical resistance) exhibit high thermal conductivity. The proportionality constant k is called thermal conductivity of the material.

1.2 Literature Review

One of the proficient passive approaches is using nanofluid in heat transport improvement for enhancing the efficiency of thermal systems like heat exchangers, thermal storage, solar collectors, photovoltaic/thermal system, biomedical devices, nuclear reactors, cooling of electronic components etc. In recent years, there has been an increasing interest in merge of two or more nanoparticles in base fluid known as "hybrid nanofluid" due to improvement in cooling performance.

Madhesh and Kalaiselvam [1] conducted an experiment to analyze cooling performance of hybrid water-Cu-Ti nanofluid. Researchers [2-3] found enhanced thermal conductivity using hybrid nanoparticles such as CNT–Cu, CNT–Ag, CNT-Al_2O_3 and the increasing rate was about 21% at room temperature for hybrid CNT-Al_2O_3 particle volume fractions of 0.2%. Iqbal *et al.* [4] concluded that brick shaped hybrid Cu-CuO nanoparticles contributed to relative low temperature distribution while platelet shaped nanoparticles were efficient in a sense of rising fluid flow. Researchers [5-7] used hybrid nanofluids heat transfer medium by free convection inside different geometries like open wavy cavity, circular cylinder etc. as it has huge applications in environment and industry. In order to remain side by side developments in the extent applications and increasingly developing studies about nanofluids, review research has been conducted by Sarkar *et al*. [8].

Researchers are interested in heat transfer by mixed convection inside lid driven cavities of different shapes as it has huge applications in environment and industry [9-10]. Flow and heat transfer from irregular surfaces are often encountered in many engineering applications to enhance heat transfer such as micro-electronic devices, flat-plate solar collectors and flat-plate condensers in refrigerators, flows in the earth's crust, underground cable systems, electric machinery, cooling system of micro-electronic devices, etc. In addition, roughened surfaces could be used in the cooling of electrical and nuclear components where the wall heat flux is known. Surfaces are intentionally roughened sometimes to enhance heat transfer [11-12].

One of the proficient passive approaches is using nanofluid in heat transport improvement for enhancing the efficiency of thermal systems like; heat exchangers, thermal storage, solar collectors, photovoltaic/thermal system, biomedical devices, nuclear reactors, engine/vehicle, cooling of electronic components, transformers etc. Several studies have been reported on an MHD convective lid-driven flow of considering different nanofluid under different physical situations. Record of such investigations can be found in the works of [13-19]. During the last 2 decades, the insistence of convective heat transfer enhancement inside enclosures have become an insistent demand due to their ever-increasing applications in lubrication technologies, electronic cooling, food processing and nuclear reactors. The basic study of this field was done by Choi and Eastman [20]. Researchers [21-22] are interested in conjugate heat transfer by free convection inside cavities as it has huge applications in environment and industry. In order to remain side by side developments in the extent applications and increasingly developing studies about nanofluids, review papers have been conducted.

“Hybrid” nanofluid can be obtained by suspending more than one type of nanoparticles in base fluid. Very recently, hybrid nanofluid, is a gradually mounting study field parallelly to the incessant developments of common nanofluids. Compromised properties between the advantages and disadvantages of the properties of individual nanoparticles are a declared task of hybridization. Moreover, nanoparticles suppliers exhibit noticeable differences in prices of different nanoparticles types. For example, the price of copper nanoparticles is about 10 times greater than that of alumina nanoparticles. Indeed, the “hybrid” nanoparticles should be limited to those prepared as a single composite substance in a base fluid for which their synthetization requires extra attention [23-26].

One can use the “hybrid” nanofluid issue to those prepared by mixing unlike nanoparticles with base fluid. The experiments of Ho *et al.* [27] conducted an experiment about the mixture of particles of micro-encapsulated phase-change material and alumina nanoparticles in base fluid water. The authors found good agreement between the experimental data of the density and mass fraction and those predicted from the mixture theory. Botha et al. [28] performed an experiment of hybrid nanofluid based on silver-silica-oil. The authors found more deviated value of the thermal conductivity with the Maxwell relation [29] at higher solid volume fractions.

In order to remain side by side developments in the extent applications and increasingly developing studies about nanofluids, review papers have been conducted. Hybrid nanoparticles mixture procedure has been reviewed comprehensively [30-33]. This review showed the arithmetical models of hybrid nanofluid properties.

From the above literature review it is observed that hybrid nanofluid has been used extensively to enhance heat transfer rate. Comparative study of different pair of mixing nanoparticles with base fluid has not been done yet. The present numerical research expects to investigate and compare the performance of hybrid nanofluids (water-Cu-CNT, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-TiO_2) on convective and conductive heat transfer inside a trapezoidal lid-driven wavy cavity.

1.3 Objectives

This research aims to simulate a two dimensional convective-conductive heat transfer and laminar flow physics using a trapezoidal wavy lid-driven enclosure. The specific aims of this research are as follows:

- To modify the mathematical model of heat and fluid flow applying hybrid nanofluid properties.
- To visualize the laminar flows of single nanofluid (Cu-water) and hybrid nanofluids (water-Cu-CNT, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-TiO_2).
- To find the effects of various pertinent parameters like wave number, Richardson number and Prandtl number on flow and temperature fields.
- To compare the heat transfer performance of the mentioned single and hybrid nanofluids using different solid volume fractions.

1.4 Scope of the Thesis

A brief description of the present numerical investigation of heat transfer inside a wavy trapezoidal enclosure using hybrid nanofluids have been presented in this thesis through four chapters as stated below:

Chapter 1 contains introduction with the aim and objectives of the present work. This chapter also includes a literature review of the past studies on heat transfer using different types of hybrid nanofluids which are relevant to the present work. Objectives of the present study has also been incorporated in this chapter.

Chapter 2 presents a detailed description for the numerical simulation of heat transfer and fluid flow characteristics inside a cavity taking heat transfer medium as water and water based single as well as hybrid nanofluids. Physical model of a undulated enclosure is described. Mathematical formulation has been given for numerical computation in this chapter.

In Chapter 3, the effects of wave number, Richardson number, Prandtl number and solid volume fraction of nanoparticles have been investigated. Results have been shown in isothermal lines, streamlines to better understand the heat transfer mechanism through undulated enclosure. Comparison has been also shown in terms of heat transfer rate among considered hybrid nanofluids.

Chapter 4 concludes remarks of the whole research and the recommendations for the future research have been presented systematically.

Chapter 2
Numerical Analysis

2.1. Introduction

Numerical analysis is the study of algorithms that use numerical approximation (as opposed to symbolic manipulations) for the problems of mathematical analysis (as distinguished from discrete mathematics). Numerical analysis naturally finds application in all fields of engineering and the physical sciences, but in the 21st century also the life sciences, social sciences, medicine, business and even the arts have adopted elements of scientific computations.

2.2. Finite Element Method

The finite element method (FEM) is a numerical method for solving problems of engineering and mathematical physics. Typical problem areas of interest include structural analysis, heat transfer, fluid flow, mass transport, and electromagnetic potential. The analytical solution of these problems generally requires the solution to boundary value problems for partial differential equations. The finite element method formulation of the problem results in a system of algebraic equations. The method approximates the unknown function over the domain. To solve the problem, it subdivides a large system into smaller, simpler parts that are called finite elements. The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem. FEM then uses variational methods from the calculus of variations to approximate a solution by minimizing an associated error function. Studying or analyzing a phenomenon with FEM is often referred to as finite element analysis (FEA).

A finite element method is characterized by a variational formulation, a discretization strategy, one or more solution algorithms and post-processing procedures. Examples of variational formulation are the Galerkin method, the discontinuous Galerkin method, mixed methods, etc. A discretization strategy is understood to mean a clearly defined set of procedures that cover (a) the creation of finite element meshes, (b) the definition of basis function on reference elements (also called shape functions) and (c) the mapping of reference elements onto the elements of the mesh. Examples of discretization strategies are the h-version, p-version, hp-version, x-FEM, iso-geometric analysis, etc. Each discretization strategy has certain advantages and disadvantages. A reasonable criterion in selecting a discretization strategy is to realize nearly optimal performance for the broadest set of mathematical models in a particular model class. There are various numerical solution algorithms that can be classified into two broad categories; direct and iterative solvers. These algorithms are designed to exploit the sparsity of matrices that depend on the choices of variational formulation and discretization strategy.

Post processing procedures are designed for the extraction of the data of interest from a finite element solution. In order to meet the requirements of solution verification, postprocessors need to provide for a posteriori error estimation in terms of the quantities of interest. When the errors of approximation are larger than what is considered acceptable then the discretization has to be changed either by an automated adaptive process or by action of the analyst. There are some very efficient postprocessors that provide for the realization of super convergence. FEM meshing has been displayed in the Figure 2.1.

This figure is an example of a two dimensional FEM (finite element method) mesh for a cylindrically shaped magnetic shield. The mesh is created by an analyst prior to solution by the FEM software. In this case "two dimensional" means that the picture shows a flat cross section of an assembly that extends to a large distance at right-angles to the paper (Cartesian coordinates). The rectangle outlined at the right of the picture has been designated the conducting component, which carries the electric current that creates the magnetic field. The cylindrical part has been designated to be a material of high magnetic permeability (for example iron). The gray areas have been designated air. The mesh is divided into smaller triangles inside the cylinder, which is an area where the lines of magnetic flux will be more concentrated.

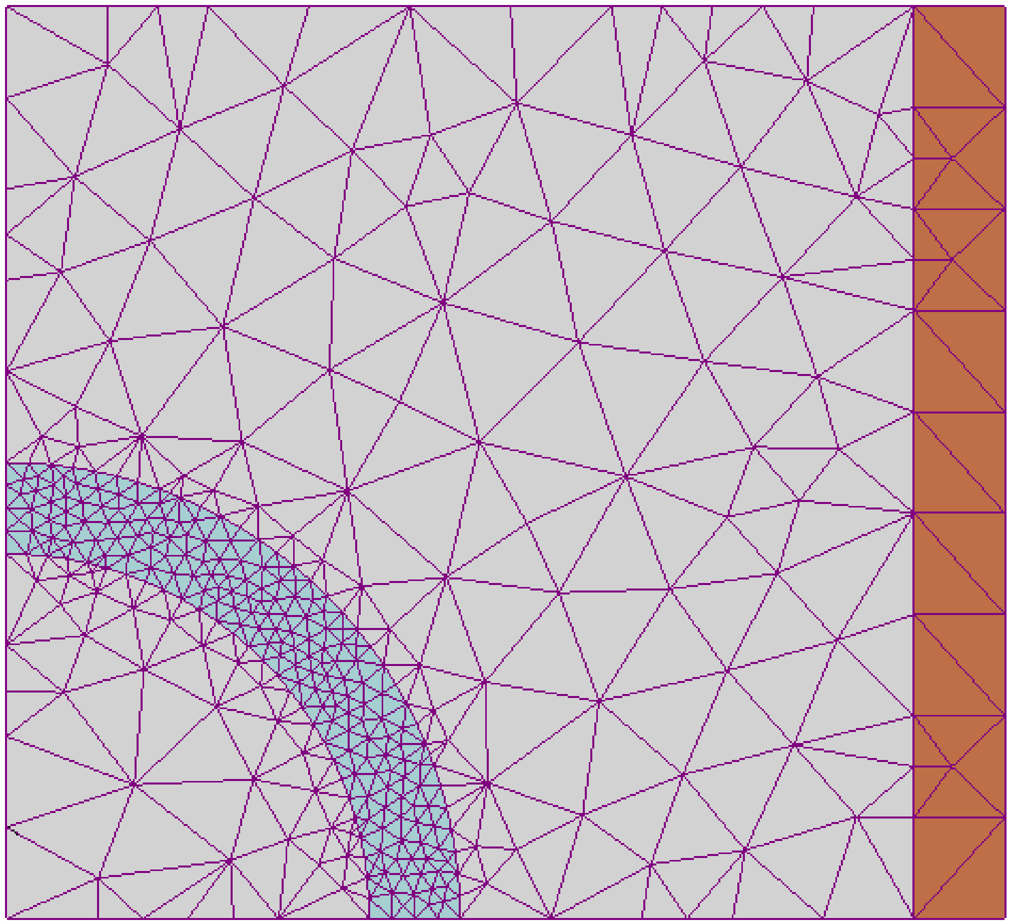

Figure 2.1: FEM Mesh

2.3 Problem Formulation

The schematic diagram of the studied configuration has been depicted in the Figure 2.2. It consists of a two dimensional lid-driven trapezoidal enclosure of side length H. The bottom surface has been assigned to temperature T_h while the sinusoidal wavy bottom surface of the enclosure has been cooled at a constant temperature T_c. Under all circumstance $T_h > T_c$ condition has been maintained. The inclined surface of the trapezoid cavity has been thermally insulated. The top surface has been considered as lid driven with uniform velocity. The heat transfer medium has been taken as four types of water-based hybrid nanofluids consisting equal solid volume fraction of each nanoparticle like (Cu and TiO_2), (Cu and CuO), (Cu and Al_2O_3) and (Cu and CNT). The diameter of the hybrid nanofluid is considered as 10 nm. Cu, TiO_2, CuO, and Al_2O_3 are spherical shaped and CNT is taken as cylindrical shaped. It has been also assumed that both the fluid and nanoparticles are in thermal equilibrium and there is no slip between them. The hybrid nanofluid used in the analysis has been considered as laminar and incompressible. The gravitational acceleration acts to the vertical downward surface.

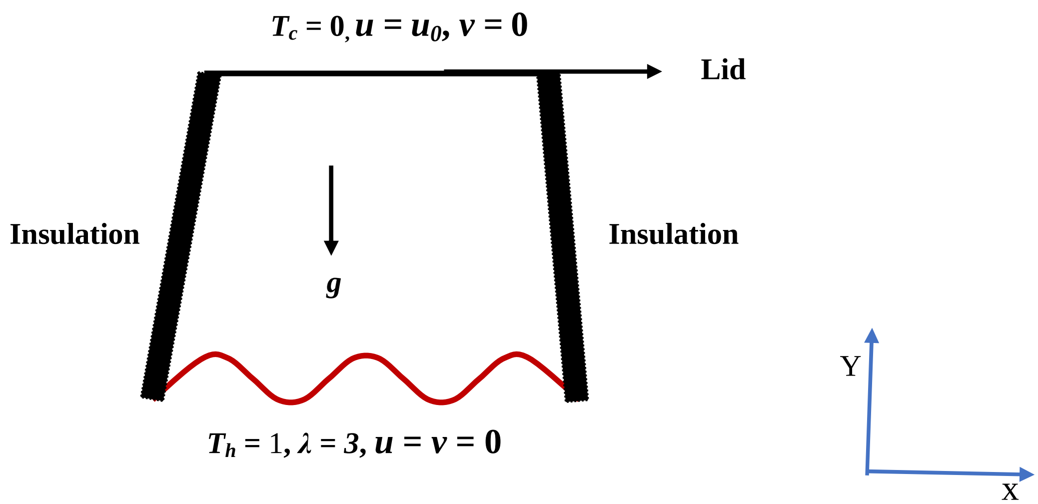

Figure 2.2: Physical model of computational domain

2.4 Mathematical Modeling

The 2D numerical simulation has been performed in steady state conditions. The governing partial differential equations according to [34-35], dimensional (2.1-2.4) and non-dimensional form (2.6-2.9) in terms of laminar flow and thermal energy physics using hybrid nanofluid are given below:

Dimensional form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x}+\upsilon_{hnf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2.2}$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial y}+\upsilon_{hnf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{(\rho\beta)_{hnf}}{(\rho\beta)_f}g(T-T_c) \tag{2.3}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha_{hnf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{2.4}$$

The above equations are non-dimensionalized by using the following dimensionless quantities

$$X=\frac{x}{H},\quad Y=\frac{y}{H},\ U=\frac{u_0 H}{\alpha_f},\quad V=\frac{u_0 H}{\alpha_f},\ \theta=\frac{T-T_c}{T_h-T_c},\ P=\frac{pH^2}{\rho_f \alpha_f^2} \tag{2.5}$$

Non-dimensional form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2.6}$$

$$\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P}{\partial X}+\frac{\rho_f}{\rho_{hnf}}\frac{\mu_{hnf}}{\mu_f}Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2.7}$$

$$\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+\frac{\rho_f}{\rho_{hnf}}\frac{\mu_{hnf}}{\mu_f}Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{(\rho\beta)_{hnf}}{(\rho\beta)_f}Ri\theta \tag{2.8}$$

$$\left(U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}\right)=\frac{\alpha_{hnf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{2.9}$$

where, $Pr=\dfrac{\mu_f C_{pf}}{k_f}$ and $Ri=\dfrac{g\beta_f \Delta TH}{v_0}$ be the Prandtl number and Richardson number, respectively.

The shape of the bottom wavy surface profile has been assumed to mimic the following pattern $Y=A[1-\cos(2\lambda\pi X)]$ (2.10)

where A is the dimensionless amplitude of the wavy surface and λ is the number of undulations.

The boundary conditions imposed on the flow are taken as:

On the left and right inclined wall: $U=0,\ V=0, \theta=0$

On the top wall: $U=1,\ V=0,\ \theta=0$

On the wavy bottom wall: $U=V=0, \theta=1$

The following models of effective properties of hybrid nanofluids have been chosen as:

thermal diffusivity $\alpha_{hnf}=k_{hnf}/\left(\rho C_p\right)_{hnf}$ (2.11)

density, $\rho_{hnf}=(1-\phi)\rho_f+\phi_1\rho_1+\phi_2\rho_2$ (2.12) heat capacitance, $\left(\rho C_p\right)_{hnf} = (1-\phi)\left(\rho C_p\right)_f + \phi_1\left(\rho C_p\right)_1 + \phi_2\left(\rho C_p\right)_2$ (2.13)

thermal expansion coefficient, $\left(\rho\beta\right)_{hnf} = (1-\phi)\left(\rho\beta\right)_f + \phi_1\left(\rho\beta\right)_1 + \phi_2\left(\rho\beta\right)_2$ (2.14)

specific heat at constant pressure, $C_{phnf} = \dfrac{(1-\phi)\left(\rho C_p\right)_f + \phi_1\left(\rho C_p\right)_1 + \phi_2\left(\rho C_p\right)_2}{(1-\phi)\rho_f + \phi_1\rho_1 + \phi_2\rho_2}$ (2.15)

viscosity of Brinkman model [36] $\mu_{hnf} = \dfrac{\mu_f}{\left(1-\phi_1-\phi_2\right)^{2.5}}$ (2.16)

thermal conductivity of Maxwell-Garnett model [29]

$$k_{hnf} = k_f \frac{\dfrac{\left(\phi_1 k_1 + \phi_2 k_2\right)}{\phi} + 2k_f + 2\left(\phi_1 k_1 + \phi_2 k_2\right) - 2\phi k_f}{\dfrac{\left(\phi_1 k_1 + \phi_2 k_2\right)}{\phi} + 2k_f - \left(\phi_1 k_1 + \phi_2 k_2\right) + \phi k_f} \tag{2.17}$$

and electrical conductivity of Maxwell-Garnett model [29]

$$\frac{\sigma_{hnf}}{\sigma_f} = 1 + \frac{3\left\{\dfrac{\left(\phi_1\sigma_1 + \phi_2\sigma_2\right)}{\sigma_f} - \left(\phi_1 + \phi_2\right)\right\}}{\left\{\dfrac{\left(\phi_1\sigma_1 + \phi_2\sigma_2\right)}{\phi\sigma_f} + 2\right\} - \left\{\dfrac{\left(\phi_1\sigma_1 + \phi_2\sigma_2\right)}{\sigma_f} - \left(\phi_1 + \phi_2\right)\right\}} \tag{2.18}$$

Here ϕ is the overall volume concentration of two different types of nanoparticles dispersed in hybrid nanofluid and is calculated as $\phi = \phi_1 + \phi_2$ (2.19)

The mean Nusselt number at the left heated wall of the cavity is

$$Nu = -\frac{k_{hnf}}{k_f}\int_0^L \frac{\partial\theta}{\partial N}\, dL \tag{2.20}$$

where L is the length of the undulated surface and N be the distances either along x or y directions acting normal to the inclined surface.

2.4.1 Thermo-physical properties

Thermo-physical properties [37-38] of base fluid and different nanoparticles have been shown in Table 2.1, assumed constant except for the density variation, which is maintained on Boussinesq approximation.

Table 2.1: Thermo- physical properties of base fluid and different nanoparticles

Physical Properties	Fluid Phase (Water)	Cu	TiO_2	CNT	CuO	Al_2O_3
C_p (J/kgK)	4179	385	686.2	796	535.6	765
ρ (kg/m^3)	997.1	8933	4250	1600	6500	3970
k (W/mK)	0.6	401	8.9538	3000	20	40
β (1/K)	21	1.67×10^{-5}	0.9×10^{-5}	------	5.1×10^{-5}	2.4×10^{-5}
σ (μS/cm)	0.05	5.96×10^{-7}	6.28×10^{-5}			

2.5 Computational Procedure

Using the Galerkin weighted residual finite element technique [39-40] the momentum and energy balance equations have been solved using COMSOL Multyphysics. In this method, the solution domain has been discretized into finite element meshes, which have been composed of non-uniform triangular elements. Then the nonlinear and non-dimensional governing partial differential equations have been transferred into a system of integral equations by applying Galerkin weighted residual method. The basic unknowns for the governing partial differential equations (2.6-2.9) are the velocity components U, V, the temperature θ and the pressure P. The six nodes with triangular element have been used in this numerical research. All six nodes have been associated with velocities as well as temperature while three corner nodes with pressure. The nonlinear algebraic equations so obtained have been modified by imposition of boundary conditions. These modified nonlinear equations have been transferred into linear algebraic equations by using Newton's method. Finally, these linear equations have been solved by using triangular factorization method. The convergence criterion for the solution procedure has been defined as $\left|\psi^{n+1}-\psi^{n}\right|\leq 10^{-6}$, where n is the number of iteration and ψ is a function of U, V and θ.

2.5.1 Code validation

In order to authenticate the exactness of present numerical technique, the obtained results in special cases have been compared with the results obtained by Rashad *et al.* [33]. These comparisons have been presented obviously in the Figure 2.3 in terms of the streamlines and Isotherms. The code validation has been conducted while employing the dimensionless parameters as Ha = 10, $\phi = 0.05$, D = 0.5, Ra = 10^4, Q = 1, B = 0.4. A very good agreement has been found between the present results and the results of Rashad *et al.*

[33]. These flattering comparisons provide confidence in the numerical results to be reported subsequently.

Rashad *et al.* [33] **Present Code**

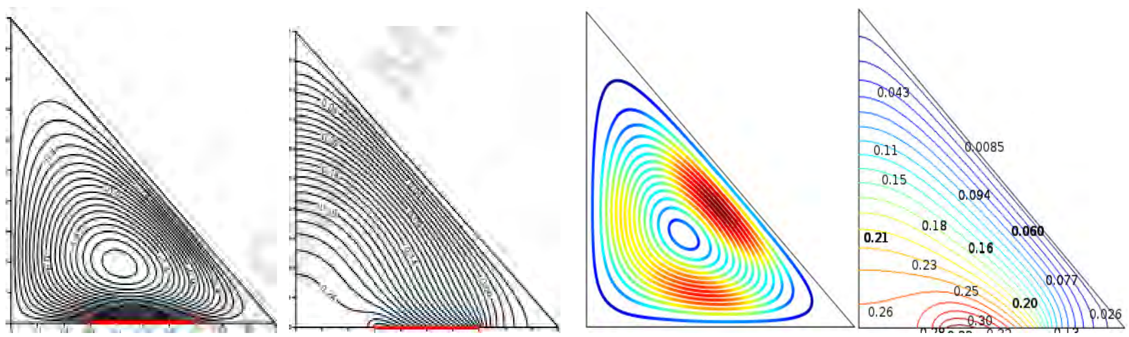

Figure 2.3: Code validation of the streamlines and isotherms between Rashad *et al.* [33] and that of present research at *Ha* = 10, $\phi = 0.05$, *D* = 0.5, *Ra* =10^4, *Q* = 1, *B* = 0.4

2.5.2 Mesh generation

The discrete locations at which the variables are to be calculated are defined by a mesh which covers the geometric domain on which the problem is to be solved. It divides the solution domain into a finite number of sub-domains called finite elements. The computational domains with irregular geometries by a collection of finite elements make the method a valuable practical tool for the solution of boundary value problems arising in various fields of engineering. Figure 2.4 displays the finite element mesh of the present physical domain. The meshing consists of tetrahedral element with ten nodes in subdomain and triangular element with six nodes in boundaries.

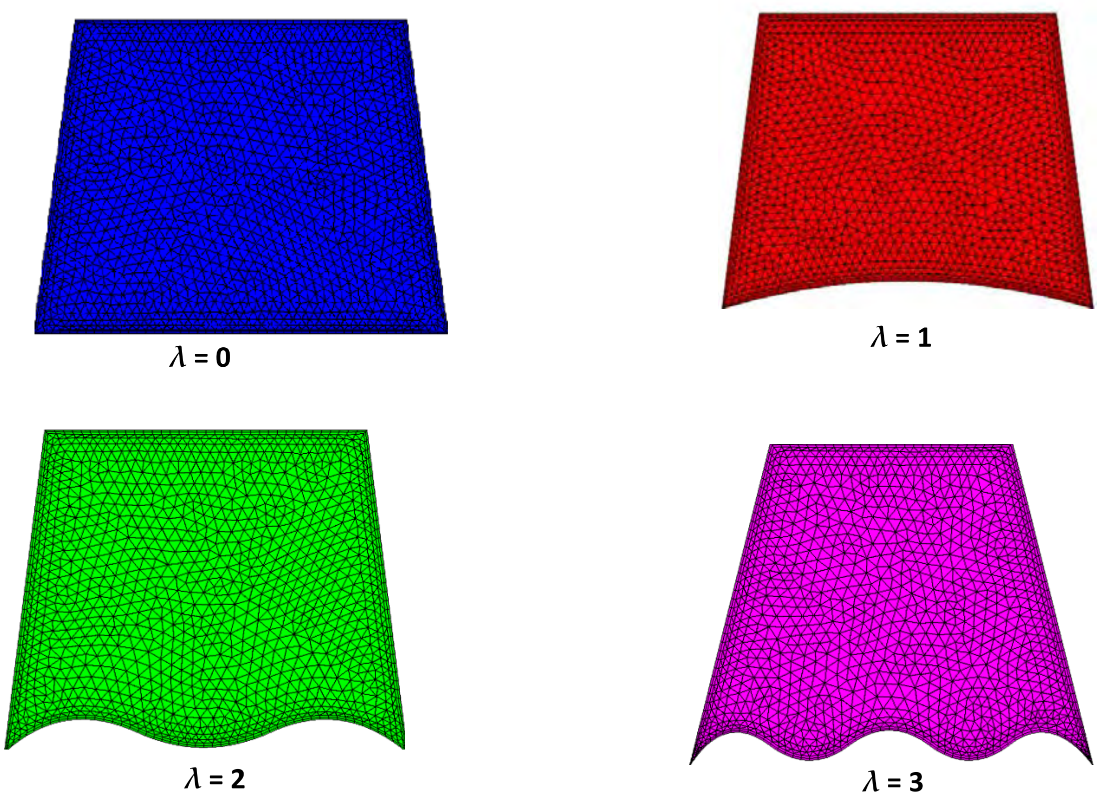

Figure 2.4: Mesh Generation of wavy enclosure

2.5.3 Grid sensitivity test

In order to determine the proper grid size for this study, a grid independence test is conducted with five types of mesh for *Pr* = 5.8, *Ri* =1.0, and λ = 3 which has been shown in Figure 2.5. Corresponding grid densities are 7182 nodes, 592 elements, time 29 s; 11062 nodes, 1133 elements, 42 s; 15074 nodes, 1744 elements, 64 s; 18082 nodes, 4484 elements, 105 s; and 29726 nodes, 12502 elements, 255 s. The extreme value of Nu is used as the monitoring variable for sensitivity measure of the accuracy of the solution. Taking into account both the precision of numerical values and computational time, the present calculations are performed with 18082 nodes and 4484 elements grid system. Figure 2.3, one can observe that no further improvement in accuracy occur using higher number of elements.

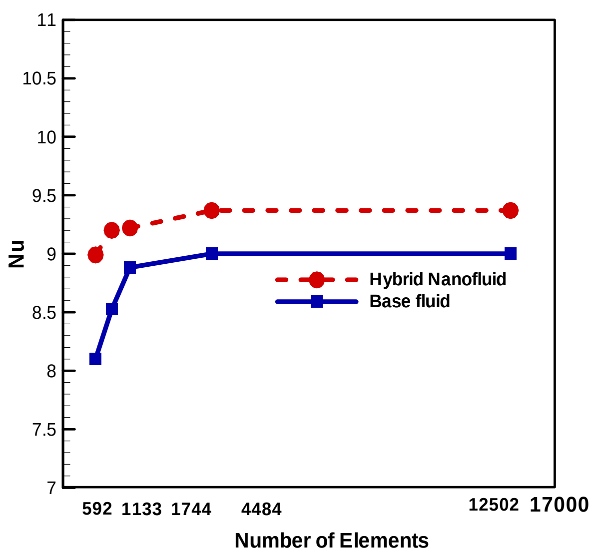

Figure 2.5: Grid refinement test

Chapter 3

Results and Discussions

3.1 Introduction

Heat transfer performance of different water-based hybrid nanofluids (water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT) flow in mixed convective lid-driven sinusoidal trapezoidal enclosure has been investigated numerically. The numerical results of the current study are expatiated graphically via stream function contours (streamline), dimensionless temperature contours (isotherms) and average Nusselt numbers. The relevant parameters those have a direct effect on the flow and thermal fields inside the considered cavity are; Richardson number Ri = 0.1 to10, nanoparticles volume fraction, φ = 0 to 3%, Prandtl number, Pr = 4.2 to 6.2 and undulation number, λ = 0 to 3. Also, comparison in heat transfer performance of four types of hybrid nanofluids have been displayed. Here total volume fraction is divided equally into two nanoparticles for each hybrid nanofluid. In order to display the results of these four independent parameters, three parameters are fixed (unless where stated) while the remainder single one is varied as gathered in the following categories:

3.2 Effect of Wave Numbers

Figure 3.1(a-b) provides the information about the sensitivity of the streamlines and isothermal lines pattern due to the variation of undulation number (λ) with Ri = 1, Pr = 5.8. HTF has been considered as water. From the streamlines contours it is found that for purely mixed convection (Ri = 1) and λ = 0 there exist two divided vortex and intertwining core near the left vertical line. When λ = 1, two divided vortices have been more sophisticated and intertwining core has not been complicated. When λ = 2, one oval shape core has been created and one vortex has been omitted. When λ = 3, one vortex has been small and intertwining core has not been complicated and core has been more vacuous.

The influence of Ri on temperature field has been plotted in the Figure 3.1(b). From this figure, it is noticed that for λ = 0, the isothermal line agglutinate to the left wall and undulated wall due to the influence of forced convection. Ditch inclined on the right of the enclosure. With further increase of λ = 1, the isotherms line increasing to the lid driven and ditch has been decreased. In this situation, it is also found that the isotherms depart to the right corner section of the enclosure and try to get crowded on the top wavy surface due to the influence of forced convection. Subsequently, the thermal current activity is influenced firmly by the conductivity properties of hybrid nanofluid. There is no major distinction in the isothermal lines for $\lambda = 1$ in compare to $\lambda = 2$ except the isothermal contours are clustered near the heated vertical wall of the cavity. For a fixed value of *Ri*, the thermal field has a little change with the variation in λ. Escalating the convective parameter $\lambda = 3$ steeper temperature gradients near the heated wall is evident and the isothermal lines get the parabolic shape near the inclined cold wall. In the considered regime, the buoyancy effect balances the flow patterns and a thin boundary layer is developed close to the bottom wavy surface. It is also captured that the mostly horizontal isothermal lines, occupy in the lower section of the cavity. Furthermore, the thermal boundary layer near the underside surface becomes slightly less parallel and concentrates when the number of waves increases.

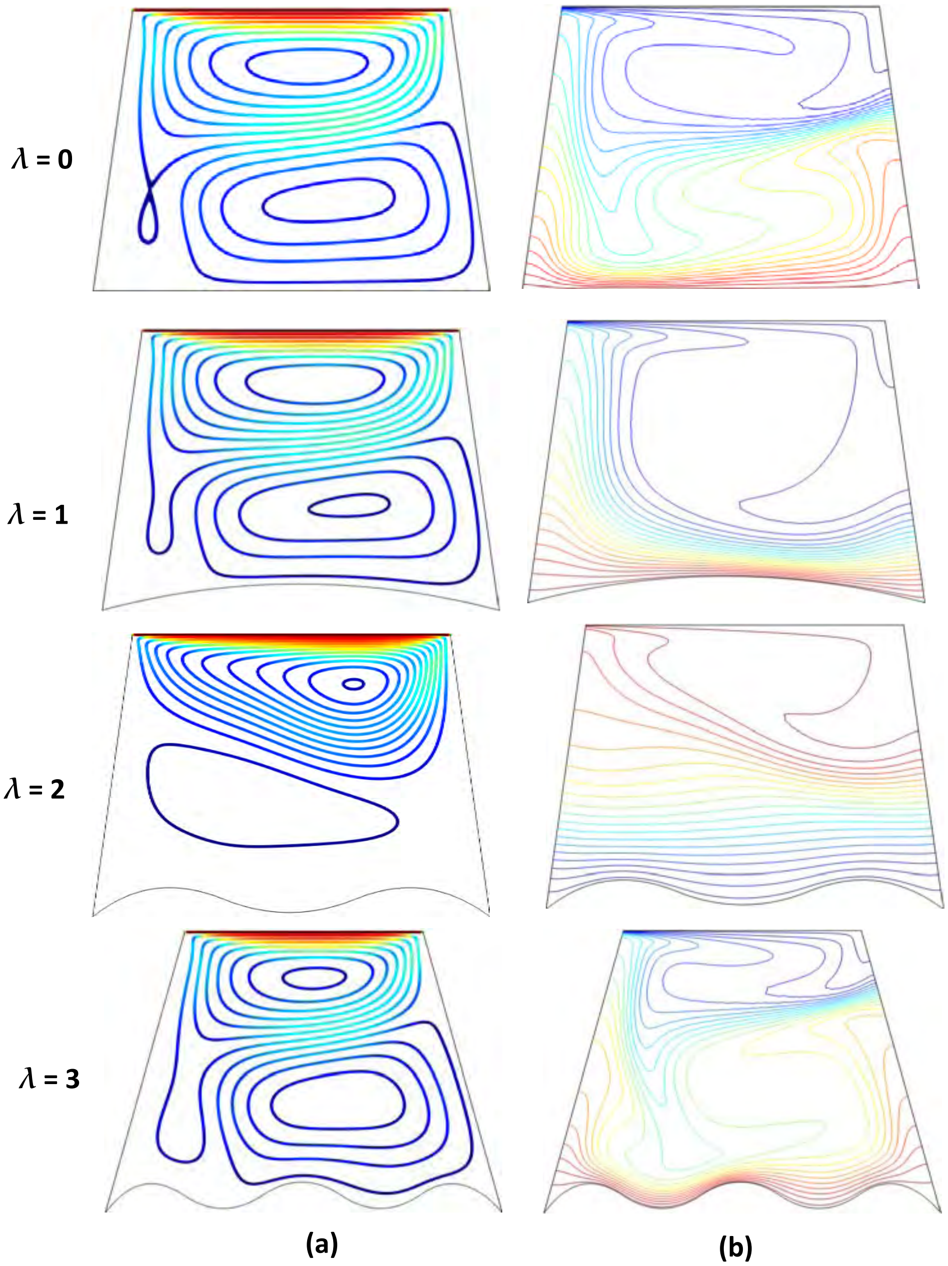

Figure 3.1: Effect of λ on (a) streamlines and (b) isotherms with *Ri* = 1, *Pr* = 5.8, ϕ = 1%

The distribution of Nusselt number for different wave number (undulation) has been presented in the figure 3.2. The augmentation in the Nusselt number has been recorded for a range of λ. It is also perceptible that rate of increasing heat transfer is much higher while the value of wave number (undulation) is high than for the small value of λ. The heat transfer rate raised by 14.7% compared to flat surface (λ = 0) to undulated surface (λ.= 3). So, it is very reasonable to conclude that the heat transfer rate becomes higher when the wave number (undulation) varies.

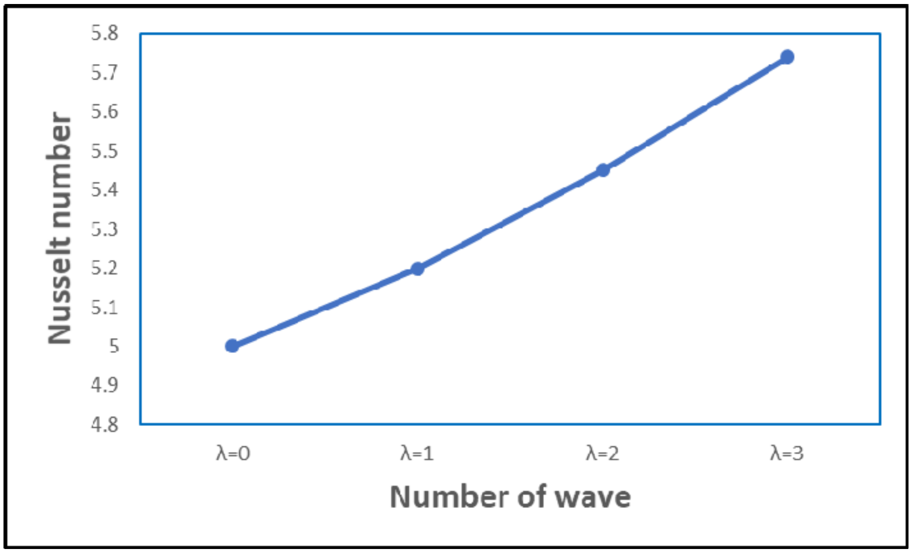

Figure 3.2: Average Nusselt number for various number of wave

3.3 Effect of Volume Fraction

Varying the parameter φ of water-Cu single nanofluid has led to few results and been shown in the Figure 3.3(a-b). From the Figure 3.3(a), When the nanoparticles volume fraction is increased from φ = 0.001 to φ = 0.03 for fixed λ = 3, Ri = 1, Pr = 5.8, an oval shape core inclined with the right vertical line and also near to the undulated line. When volume fraction is increased from φ = 0.01 to φ = 0.02, oval shape core becomes smaller a little bit. But remaining behavior of streamlines associate with pure water is transformed into behavior with the increase in the nanoparticle volume fraction and the intensification of the fluid flow behavior decrease when the fluid particle volume fraction drops off. This is due to the enhancement of viscosity of the hybrid nanofluid. Since with the augment of φ, the thermal conductivity of the nanofluid increases, hence the buoyancy flow, these interns improved the heat transfer rate.

However, it is also obvious from Figure 3.3(b), the isotherms are, generally, not affected by adding the nanoparticles with any fraction except that increasing φ the isothermes close to the heat source begin to be more scatterd. The enrichment of the thermal conductivity produces denser isotherms which is the indication of development nanofluid convection. We can conclude that the mixing advantage of Copper and Carbon nano tube nanoparticles may blow up the convective heat transfer. Hence, this result encourages us to examine other combination of different types nanoparticles, with different ratio of nanoparticles, and to work with innovative models on behalf of dissimilar thermo-physical properties.

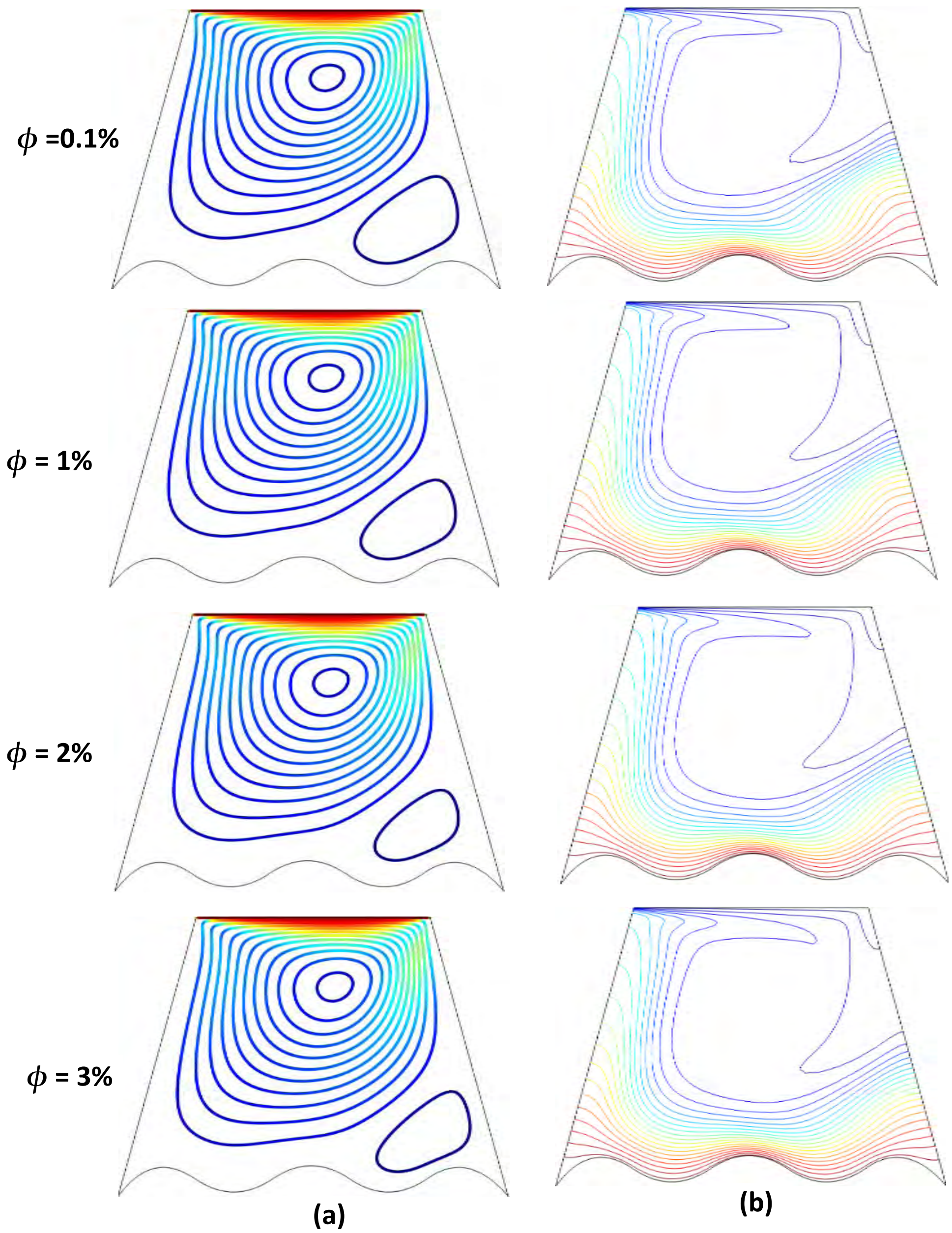

Figure 3.3: Effect of ϕ on (a) streamlines and (b) isotherms with *Ri* = 1, *Pr* = 5.8, λ=3

Figure 3.4 represents the average Nusselt number for various solid volume fraction of water-based Cu nanofluid. The value of volume fraction taken 0, 0.1, 1, 2, and 3%. The value of average Nusselt number of volume fraction 0, 0.001, 0.01, 0.02, 0.03 of water-Cu nanofluid are 4.4, 4.62, 4.76, 4.85 and 4.9 respectively. The heat transfer rate of volume fraction from 0 to 1% is greater than that of other volume fraction. The heat transfer rate is also increasing for other volume fraction (for 2 and 3%) but increasing rate is downcast. The rate of heat transfer is obtained 11.23% for increasing values of solid volume fraction from 0 to 3%.

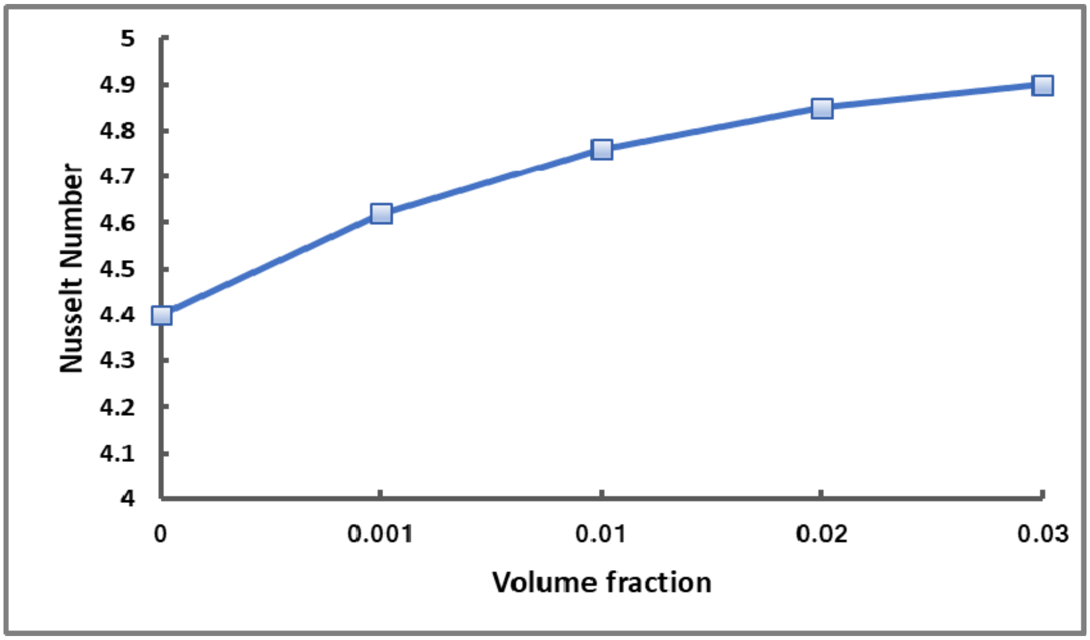

Figure 3.4: Average Nusselt number for various volume fraction of water-Cu nanofluid

3.4 Performance of Hybrid Nanofluids

The present numerical research expects to investigate and compare the performance of hybrid nanofluids (water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT) on convective and conductive heat transfer inside a trapezoidal lid-driven wavy cavity. Streamlines and (b) Isotherms for water-Cu-CNT hybrid nanofluid with fixed λ = 3, Ri = 1, Pr = 5.8, φ = 1% have shown in the figure It is obvious from Figure 3.5(a), the streamlines are, generally, less affected for using different hybrid nano-fluids. From the streamlines contours it is found that for Cu- TiO_2/water, there exist a primary elliptic circulation cell without any secondary core but one small oval shape core has been created. For water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT, it can be seen that shape of the primary cell has been bended a little bit and it is important to notice that oval shape core has been changed tiny bit than before. There less distinction in the streamlines.

However, it is also obvious from Figure 3.5(b), that the temperature gradients are also less affected on using the hybrid nanofluids (water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT). The enrichment of the thermal conductivity produces denser isotherms which is the indication of development nanofluid convection. It is concluded that the mixing advantage of copper and carbon nano tube nanoparticles may blow up the convective heat transfer. Hence, this result encourages us to examine other combination of different types nanoparticles, with different ratio of nanoparticles, and to work with innovative models on behalf of dissimilar thermo-physical properties.

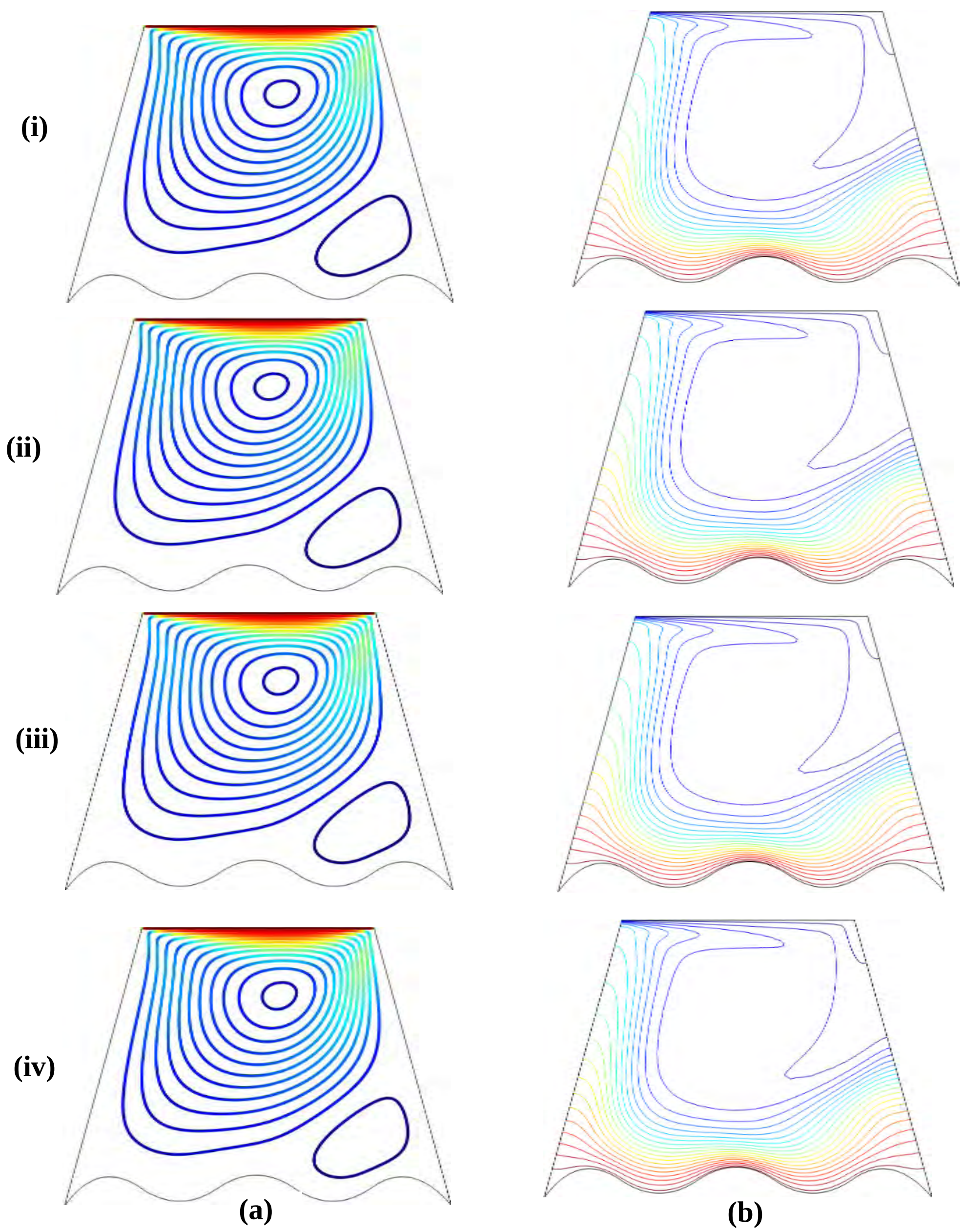

Figure 3.5: Effect of (i) water-Cu-TiO_2, (ii) water-Cu-CuO, (iii) water-Cu-Al_2O_3 and (iv) water-Cu-CNT hybrid nanofluids on (a) stream lines and (b) isotherms with λ = 3, *Ri* = 1, *Pr* = 5.8, ϕ = 1%

Figure 3.6 illustrates the Nusselt number of different hybrids nanofluid at λ = 3, Pr = 5.8 and ϕ = 1%. It is evident that with the increase of the value of thermal conductivity heat transfer rate increases. Nusselt number is found as increasing pattern in the sequence of water-based Cu-TiO_2< Cu-CuO<Cu-Al_2O_3 <Cu-CNT hybrid nanofluids. The heat transfer rate is increased about 6.3% using heat transfer medium as water-Cu-CNT hybrid nanofluid than water-Cu-TiO_2. nanofluid. Values of Nu are obtained as 4.4, 4.76, 4.80, 4.85, 4.90, and 5.0 for water, water-Cu, water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT, respectively. Thus, higher rate of heat transfer is found using water-Cu-CNT hybrid nanofluid than another considered hybrid nanofluids. Thus, heat transfer rate is higher for hybrid nanofluid compared to single nanofluid and base fluid. The enhanced rate of mean Nusselt number for water, water-Cu, water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT hybrid nanofluid with 1% solid volume fraction are found as 8.1, 9.1, 10.2, 11.4 and 13.6% respectively compared to base fluid.

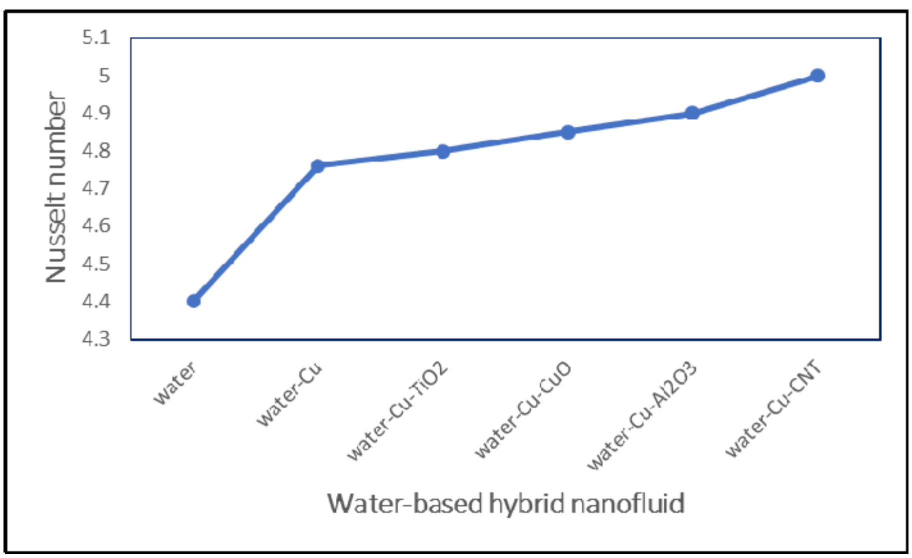

Figure 3.6: Average Nusselt number for various hybrid nanofluids

3.5 Effect of Richardson Number

In heat transfer problems, Richardson number represents the importance of the effect of forced to natural convection phenomena. Due to the influence of buoyancy force if the Richardson number is much less than unity, convection is unimportant in the flow. If it is much greater than unity, buoyancy is dominant the flow behavior. Figure 3.7(a-b) provides the information about the sensitivity of the streamlines and isothermal lines pattern due to the variation of Richardson number (Ri) and for $\lambda = 3$, $Pr = 5.8$ with the volume fraction of the suspended nanoparticles $\phi = 1\%$. Water-Cu-CNT hybrid nanofluid has been considered as heat transferring fluid (HTF). From the streamlines contours it is found that for purely forced convection ($Ri = 0.1$) there exist a primary elliptic circulation cell without any secondary core. This is due to the dominance of forced convection in the flow regime. At combined convection regime ($Ri = 1.0$) it can be seen that shape of the primary cell is bended a little bit and it is important to notice that one extra oval shape core is created. On the other hand, when $Ri = 5$ there creates a new shape core near the vertical wall and total contour separated by two parts. When $Ri = 10$, one oval shape core is created and two parts become larger. This is logical because escalating the parameter Ri assists buoyancy flow, hence the natural convection mode because buoyancy often plays a significant role in defining the mixed convection flow.

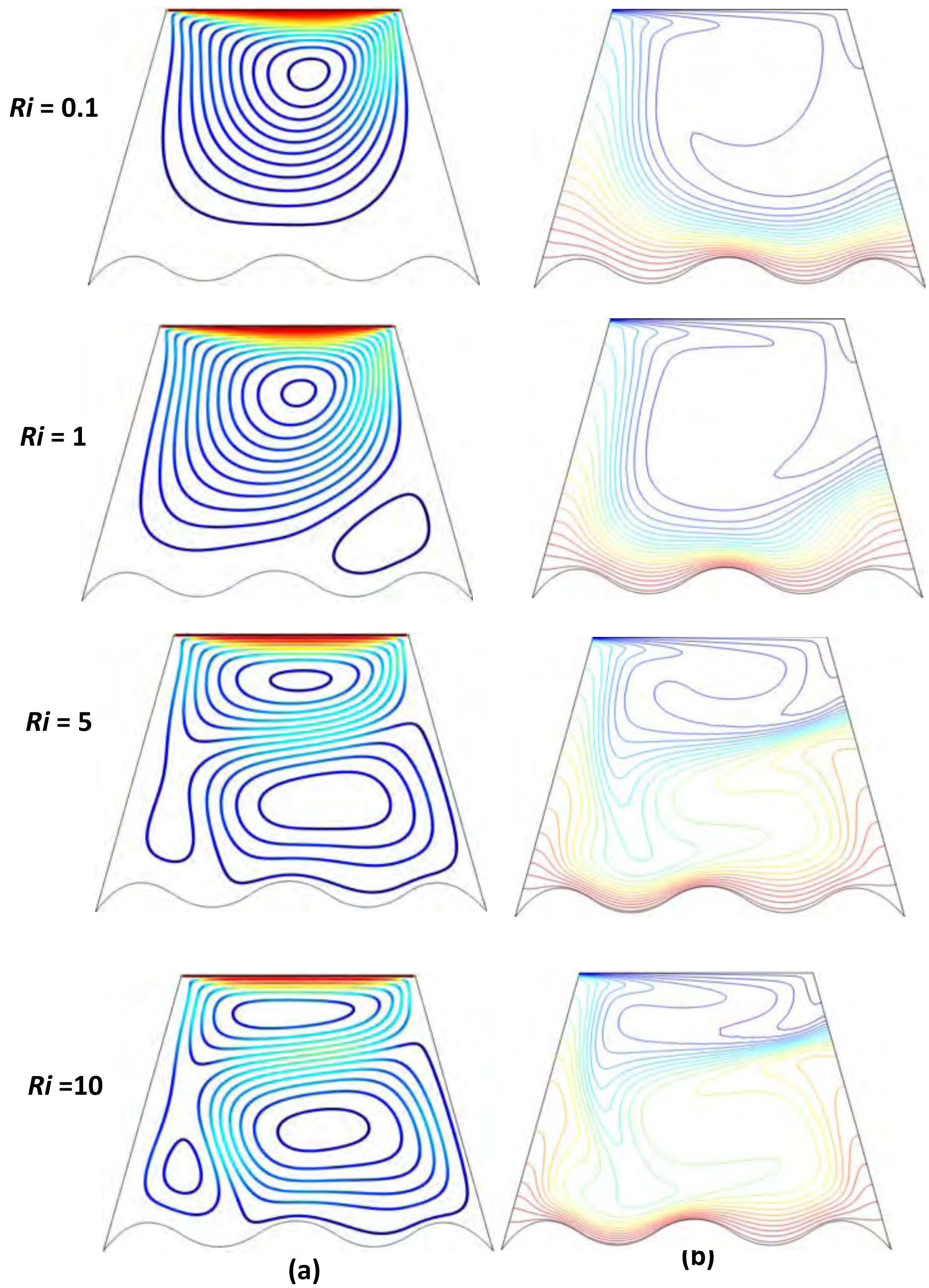

Figure 3.7: Effect of *Ri* on (a) streamlines and (b) isotherms with λ = 3, ϕ = 1%, *Pr* = 5.8

The influence of *Ri* on temperature field has been plotted in the Figure 3.7(b). From this figure, it is noticed that for *Ri* = 0.1, the isothermal line agglutinate to the left wall and undulated wall due to the influence of forced convection. Ditch inclined on the right of the enclosure. With further increase of *Ri* = 1, the isotherms line increasing to the lid driven and ditch has been decreased. In this situation, it is also found that the isotherms depart to the right corner section of the enclosure and try to get crowded on the top wavy surface due to the influence of forced convection. Subsequently, the thermal current activity is influenced firmly by the conductivity properties of hybrid nanofluid. There is no major distinction in the isothermal lines for *Ri* = 0.1 in compare to *Ri* = 1 except the isothermal contours are clustered near the heated vertical wall of the cavity. For a fixed value of λ, the thermal field has a little change with the variation in *Ri*. Escalating the convective parameter *Ri* = 10 steeper temperature gradients near the heated wall is evident and the isothermal lines get the parabolic shape near the inclined cold wall. In the considered regime, the buoyancy effect balances the flow patterns and a thin boundary layer is developed close to the bottom wavy surface. It is also captured that the mostly horizontal isothermal lines, occupy in the lower section of the cavity. Furthermore, the thermal boundary layer near the underside surface becomes slightly less parallel and concentrates when the number of waves increases.

Figure 3.8 represents the Nusselt number on the comparison of Richardson number. It is noticeable that the Nusselt number curve raised at the increment of Richardson number and it is also obvious that with the increase of the value of *Ri* heat transfer rate increase. Because with the augmentation of *Ri* buoyancy effect enhance, so large amount of heat is transfer from the heated wall to the enclosure. At *Ri* = 0.1, 1, 5 and *Ri* = 10 increasing rate are. 5.51, 4.54 and 5%.

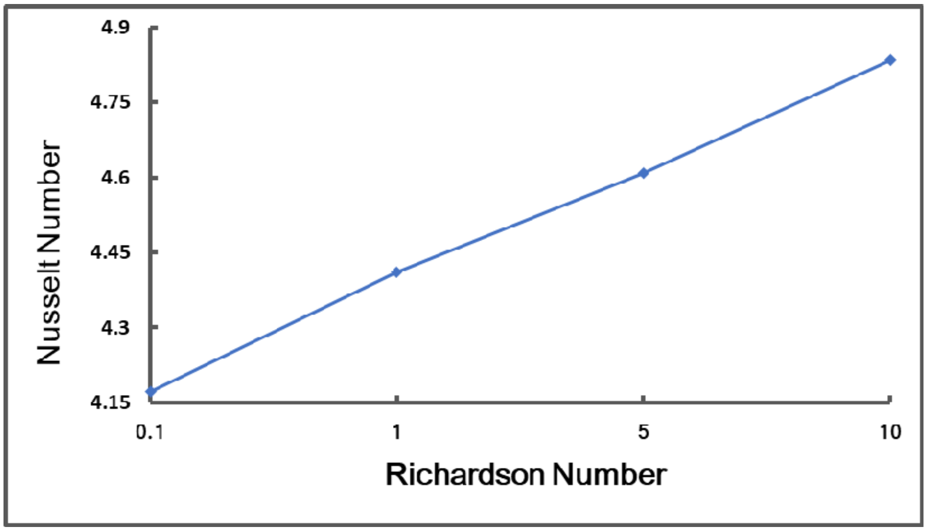

Figure 3.8: Average Nusselt number for various Richardson number

3.6 Effect of Prandtl Number

Figure 3.9-(a-b) displays he effect of Prandtl number (*Pr*) on water-Cu-CNT hybrid nanofluid flow and temperature fields with fixed $\lambda = 3$, $Ri = 1$, $\phi = 1\%$. Prandtl number is taken as 4.2, 5.2, 5.8 and 6.2 at temperature 308, 303, 300 and 298K, respectively. From the streamlines contours it is found that for $Pr = 4.2$, there exist a primary elliptic circulation cell without any secondary core but one small oval shape core has been created. At $Pr = 5.2$ it can be seen that shape of the primary cell has been bended a little bit and it is important to notice that oval shape core has been greater than before. On the other hand, when $Pr = 5.8$ and $Pr = 6.2$, there is no major distinction in the streamlines but there is only one change that gradually increment of oval shape core.

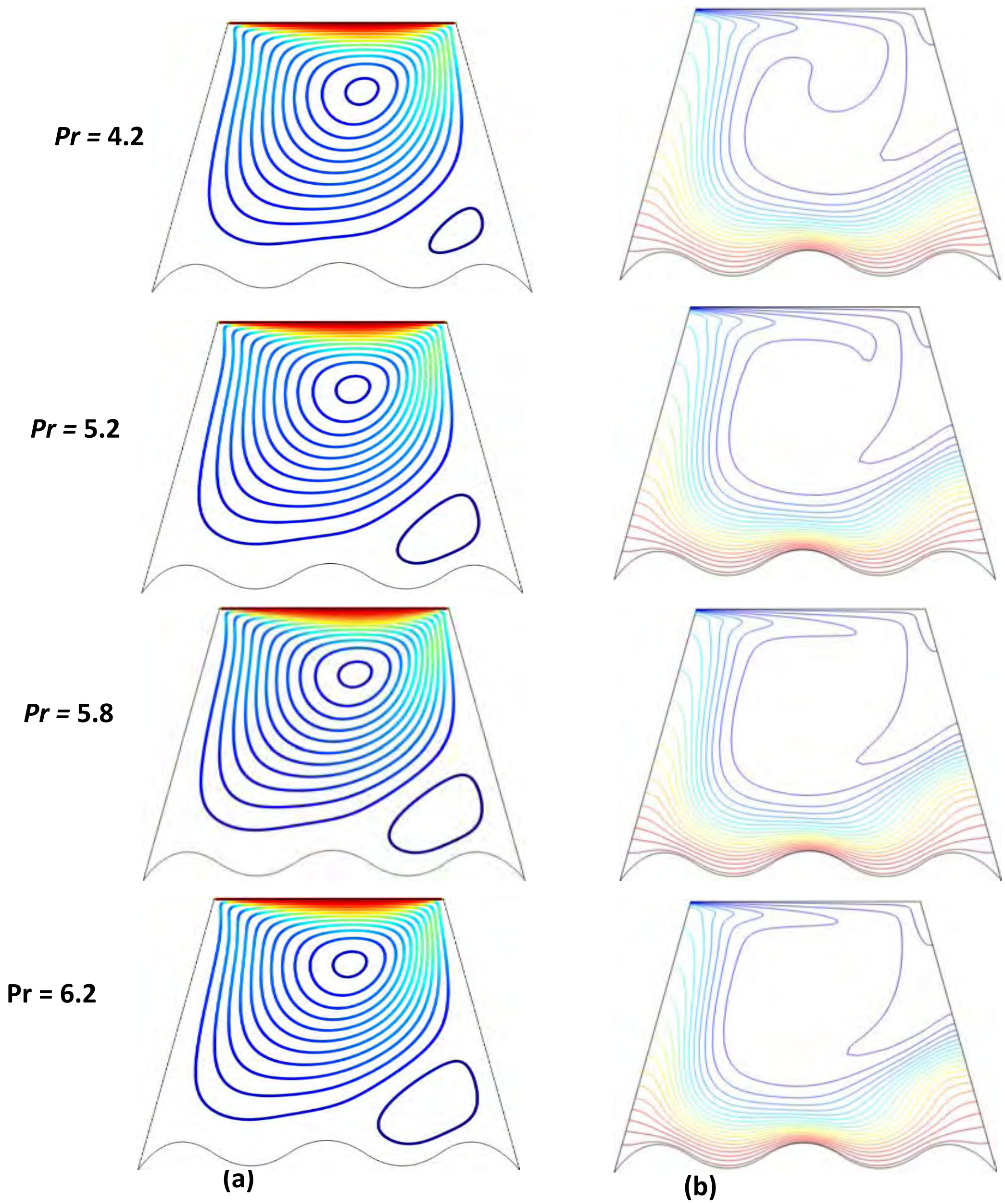

Figure 3.9: Effect of *Pr* on (a) streamlines and (b) isotherms with *Ri* =1, λ = 3, ϕ = 1%

However, it is also obvious from figure 3.9(b), that the temperature gradients are generally not affected on escalating the Prandtl number of nanoparticles with any fraction except with the increase of *Pr* the isotherms close to the heated wall begins to be more scattered. The enrichment of the thermal conductivity produces denser isotherms which is the indication of development nanofluid convection. We can conclude that the mixing advantage of Copper and Carbon nano tube nanoparticles may blow up the convective heat transfer. Hence, this result encourages us to examine other combination of different types nanoparticles, with different ratio of nanoparticles, and to work with innovative models on behalf of dissimilar thermo-physical properties.

Prandtl number λ = 3, *Ri* = 1, ϕ = 1%. From the figure 3.10, it is evident that with the increase of the value of Pr heat transfer rate increases. With the augmentation of *Pr* Nusselt number effect enhances, so large amount of heat is transferred from the heated wall to the enclosure. At *Pr* = 4.2, 5.2, 5.8 and 6.2 the Nusselt number are 4.29, 4.50, 4.60 and 4.68 respectively. Prandtl number varies from 4.2 to 6.2 whereas percentage of Nusselt number is 8.85%.

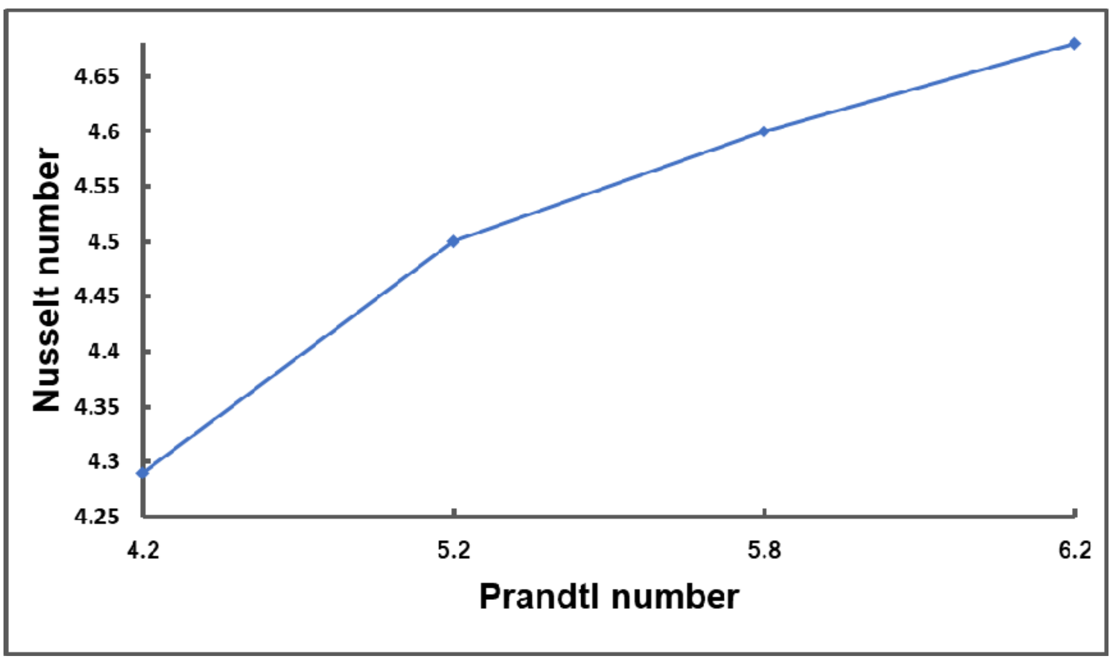

Figure 3.10: Average Nusselt number for various Prandlt number

Chapter 4

Conclusions and Recommendations

A numerical analysis has been conducted to show the performance of heat transfer phenomenon by using four types of hybrid nanofluids like water-Cu-CNT, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-TiO_2 along with single nanofluid water-Cu and base fluid water in a lid driven sinusoidal trapezium shaped cavity. The governing equations have been solved by using Galerkin's finite element method. The cavity is subject to conduction-convection heat transfer analysis. Varying the parameters Richardson number, Prandtl number, Volume fraction and Undulation number have led to the following conclusions:

4.1 Conclusions

- Ascending values of wave number (λ = 3) raises the heat transfer rate by 14.7% compared to flat surface (λ = 0). So, the corrugated lid driven cavity can be considered as an effective heat transfer system.
- The heat transfer rate increases according to increment of solid volume fraction of water-Cu nanofluid. The rate of heat transfer is obtained by 11.23% for increasing values of solid volume fraction from 0 to 3%.
- Nu increases about 8.1, 9.1, 10.2, 11.4 and 13.6% using heat transfer medium as water-Cu, water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT hybrid nanofluid with compared to base fluid.
- Higher heat transfer rate is found about 4.1, 3.1, and 2% using water-Cu-CNT hybrid nanofluid than water-Cu-TiO_2, water-Cu-CuO and water-Cu-Al_2O_3, respectively.
- The heat transfer rate is increased about 15% from *Ri* = 0.1 (forced convection) to *Ri* = 10 (natural convection).
- For the variation of Prandtl number (from 4.2 to 6.2), rising rate of heat transfer approximately 8.85% is found.

4.2 Recommendations

There is a lot of scope for research in this area in future. Since study of hybrid nanofluids is under initial stages so there is a lot of scope in development of hybrid nanofluids. The size, shape, material and volume fraction of dispersed nanoparticles play a very important role in the absorption of heat. In consideration of the present investigation, the following recommendation for future works have been provided:

- Trapezium shaped cavity has been considered in the present study. So, this deliberation may be extended by considering other formations of enclosures to investigate the performance of hybrid nanofluids.
- Using hybrid nanofluids with single phase flow have been considered as heat transfer medium in this thesis work. It can be investigated for multiphase flow also.
- In this research, four types of hybrid nanofluids namely water-Cu-CuO, water-Cu-$Ti0_2$, water-Cu-CNT and water-Cu-Al_2O_3 have been used as HTF. Anyone can use other combinations of hybrid nanofluids to obtain better heat transfer rate.

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Chapter 1

Introduction

1.1 Introduction

One of the proficient passive approaches is using nanofluid in heat transport improvement for enhancing the efficiency of thermal systems like heat exchangers, thermal storage, solar collectors, photovoltaic/thermal system, biomedical devices, nuclear reactors, cooling of electronic components etc. In recent years, there has been an increasing interest in merge of two or more nanoparticles in base fluid known as "hybrid nanofluid" due to improvement in cooling performance.

Researches on the nanofluids have been increased very rapidly over the past decade. In spite of some inconsistency in the reported results and insufficient understanding of the mechanism of the heat transfer in nanofluids, it has been emerged as a promising heat transfer fluid. In the continuation of nanofluids research, the researchers have also tried to use hybrid nanofluid recently, which is engineered by suspending dissimilar nanoparticles either in mixture or composite form. The idea of using hybrid nanofluids is to further improvement of heat transfer and pressure drop characteristics by trade-off between advantages and disadvantages of individual suspension, attributed to good aspect ratio, better thermal network and synergistic effect of nanomaterials.

1.1.1 Nanofluid

Nanofluids are treating as a two components mixture made of a base fluid and nano-particles (1-100 nm). The fundamental characteristics of the nanofluid are the raise of the thermal conductivity of the fount fluid, minimal impeding in flow passing, extensive stability and equity. In order to get better execution of heat generating, the nanofluid are utilized in several artificial applications such as chemical production, power generator in power plant, productions of micro-electronics, automotives, advance nuclear system, and nano-drug delivery. Sakiadis was the pioneer who established the concept of 2D boundary layer flow on continuous solid surface. The basic differential and integral momentum equations of boundary layer theory are governed and these equations are solved for moving continues flat surface and moving continuous cylindrical surface as well as for both laminar and turbulent flow in the boundary layer.

Nanofluids can be considered as the future of heat transfer fluids in various heat transfer applications. They are expected to give better thermal performance than conventional fluids due to the presence of suspended nanoparticles which have high thermal conductivity. Lately, there have been numerous investigations that have revealed the enhancement of thermal conductivity and higher heat transfer rate of nanofluids. Significant enhancement in the heat transfer rate with the use of various nanofluids in various application compared to conventional fluids have been reported by several researchers. Understanding the properties of nanofluids, such as thermal conductivity, viscosity and specific heat, is very important for the utilization of nanofluids in various applications. Further study of the fundamentals for heat transfer and friction factors in the case of nanofluids is considered to be very important in order to extend the applications of nanofluids.

Nanofluids are colloidal suspensions, made out of nanoparticles in some base fluid. These nanoparticles dictate the types of nanofluids and the distinction between them. Different properties and their role in green energy harvesting with an example where it has been used. Nanofluids are known for their thermal conductivity, which may be quite enhanced as compared with that of just the base fluid or that of the bulk colloidal suspensions. This property is useful in extracting more energy from nuclear and geothermal power plants, enhancing their efficiencies.

Nanofluids are fluids with different nanoparticles (particles smaller than 100 nm) and base fluids. Many types of nanoparticles such as metals (Cu, Ag, Au), oxide ceramics (Al_2O_3, CuO), carbon nanotubes and carbide ceramics (SiC, TiC) and various liquids such as water, oil, and ethylene glycol are used. Two general methods are used to produce nanofluids; namely two-step and one-step methods. In the two-step method, the nanoparticles are made separately and dispersed in base fluid by different method. Ultrasound and high-shear dispersion techniques are used to produce nanofluids with oxide nanoparticles by the two-step method. Nanoparticles are made and dispersed in a fluid in a single process in the one-step method. This method is used to produce nanofluids in small quantities for research purposes. For nanofluids with high conductivity metal nanoparticles, the one-step method is preferred. Figure 1.1 shows how nanofluid is made. According to figure the direct mixing of nanoparticle and base fluid forms nanofluid.

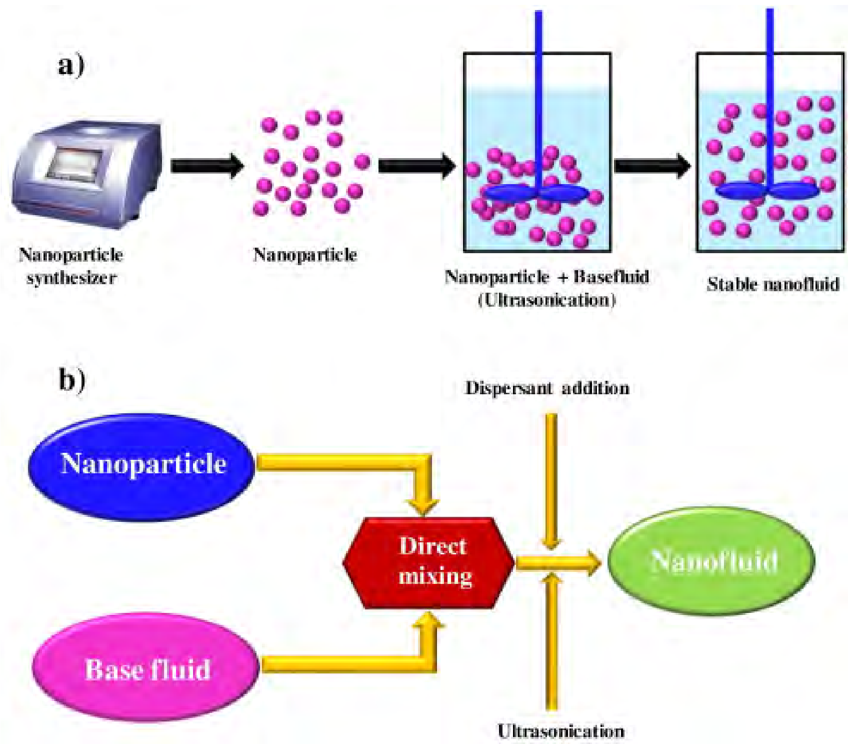

Figure 1.1: Formation of nanofluid

1.1.2 Hybrid nanofluid

"Hybrid" nanofluid can be obtained by suspending more than one type of nanoparticles in base fluid. Hybrid nanofluid is actually the mixture of two or more types of nanoparticles with a base fluid like water. Hybrid nanofluid is a new nanotechnology fluid that is synthesized by dispersing two different nanoparticles into conventional heat transfer fluid. Recently, researchers have indicated that hybrid nanofluids can effectively substitute the convectional coolant especially those working at very high temperatures. Hybrid nanofluids and hybrid nanolubricants are very new types of research which can be prepared by suspending two or more than two dissimilar nanoparticles either in a mixture or composite form in the base fluids. The term hybrid can be considered as different materials which are a combination of physical and chemical properties to form a homogeneous phase. The main objective of synthesizing hybrid nanofluids/nanolubricants is to improve the properties of single materials where it has great enhancement in thermal properties or rheological properties that are better than individually conventional nanofluids/nanolubricants. Figure 1.2 shows the preparation of hybrid nanofluid.

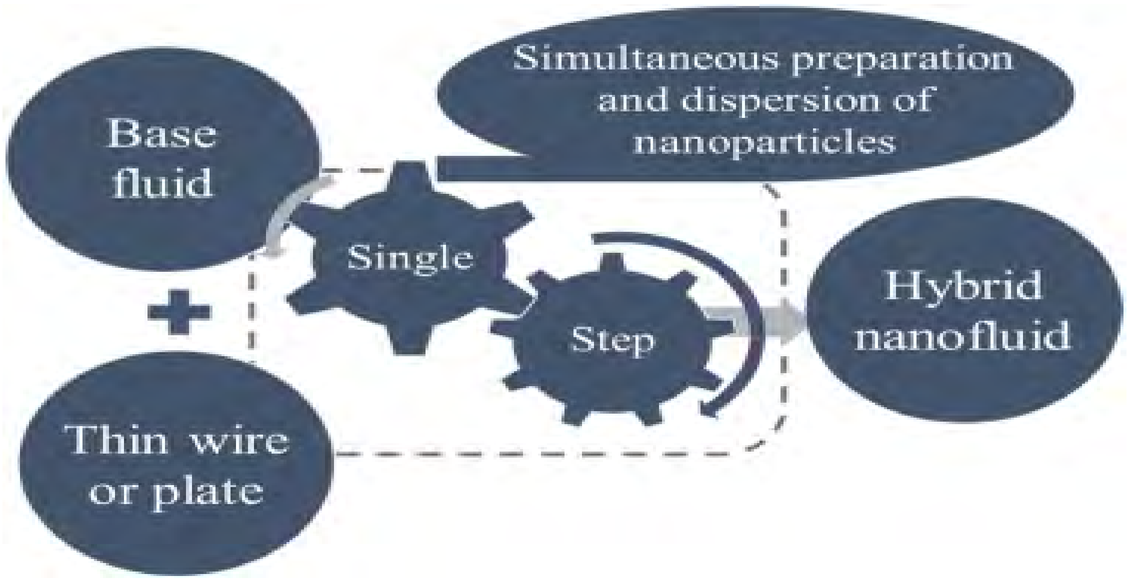

Figure 1.2: Preparation of hybrid nanofluid

1.1.3 Lid-driven cavity

The lid-driven cavity is an important fluid mechanical system serving as a benchmark for testing numerical methods and for studying fundamental aspects of incompressible flows in confined volumes which are driven by the tangential motion of a bounding wall. The lid-driven cavity is a well-known benchmark problem for viscous incompressible fluid flow. The geometry at stake has been shown in the Figure 1.3.

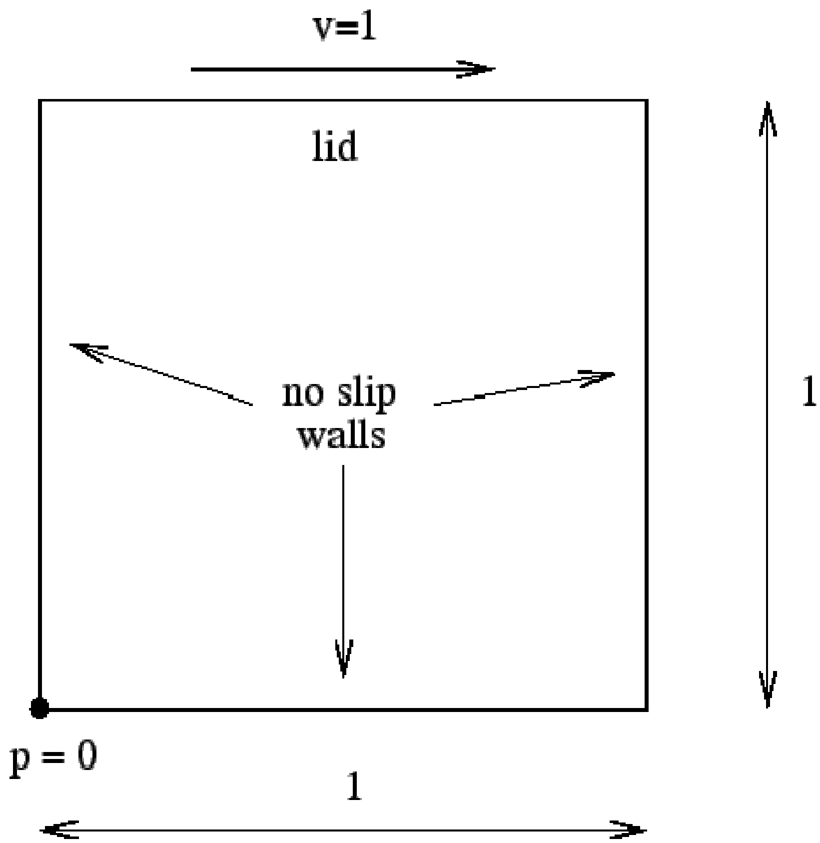

Figure 1.3: Geometry of the lid-driven cavity

1.1.4 Undulated enclosure

Undulation means having a wavy surface, edge, or markings. It also means to form or move in waves, to rise and fall in volume, pitch, or cadence, to present a wavy appearance. Undulation enclosure is to move with a sinuous or wavelike motion; display a smooth rising-and-falling or side-to-side alternation of movement. The geometry has been shown for undulated enclosure in the Figure 1.4.

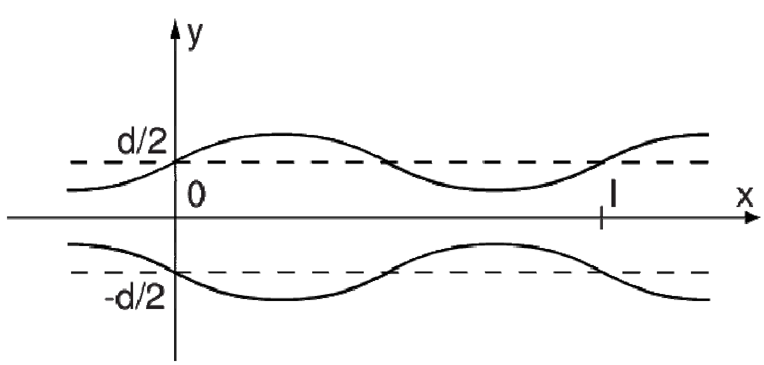

Figure 1.4: Undulated enclosure

1.1.5 Mixed convection

Mixed convection flows or combined free and forced convection flows occur in many technological and industrial applications in nature, e.g., solar receivers exposed to wind currents, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchangers placed in a low-velocity environment, flows in the ocean and in the atmosphere, and so on. Mixed (combined) convection is a combination of forced and free convections which is the general case of convection when a flow is determined simultaneously by both an outer forcing system (i.e., outer energy supply to the fluid-streamlined body system) and inner volumetric (mass) forces, viz., by the nonuniform density distribution of a fluid medium in a gravity field. The most vivid manifestation of mixed convection is the motion of the temperature stratified mass of air and water areas of the Earth that the traditionally studied in geophysics. However, mixed convection is found in the systems of much smaller scales, i.e., in many engineering devices. On heating or cooling of channel walls, and at the small velocities of a fluid flow that are characteristic of a laminar flow, mixed convection is almost always realized. Pure forced laminar convection may be obtained only in capillaries. Studies of turbulent channel flows with substantial gravity field effects have actively developed since the 1960s after their becoming important in engineering practice by virtue of the growth of heat loads and channel dimensions in modern technological applications (thermal and nuclear power engineering, pipeline transport).

1.1.6 Richardson number

Richardson number, parameter that can be used to predict the occurrence of fluid turbulence and, hence, the destruction of density currents in water or air. It was defined by the British meteorologist Lewis Fry Richardson, a pioneer in mathematical weather forecasting. Essentially the ratio of the density gradient (the change in density with depth) to the velocity gradient, the Richardson number is defined as

$$Ri = \frac{g}{\rho}\frac{\partial \rho}{\partial z} / (\frac{\partial u}{\partial z})^2$$

in which g is gravity, ρ is density, u is velocity, and z is depth. The Richardson number, or one of several variants, is of practical importance in weather forecasting and in investigating density and turbidity currents in oceans, lakes, and reservoirs.

1.1.7 Prandtl number

The relative thickness of the velocity and the thermal boundary layers are best described by the dimensionless parameter Prandtl number. Prandtl number is a dimensionless number, named after the German physicist Ludwig Prandtl, defined as the ratio of kinematic viscosity to thermal diffusivity and may be written as follows

$$Pr = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\upsilon}{D'_T}$$

The value of υ shows the effect of viscosity of the fluid. The smaller the value of υ is the narrower of the region which is affected by viscosity and which is known as the boundary layer region when υ is very small. The value of D'_T shows the thermal diffusivity due to heat conduction. The smaller the value of D'_T is the narrower of the region which is affected by the heat conduction and which is known as thermal boundary layer when D'_T is small. Thus, the Prandtl number shows the relative importance of heat conduction and viscosity of a fluid. For a gas the Prandtl number is of order of unity.

1.1.8 Nusselt number

From the temperature profile, the rate of heat transfer coefficient at the vertical plate can be obtained, which in non-dimensional form, in terms of Nusselt number, is defined by $Nu = \left(\frac{\partial T}{\partial y}\right)_{R=0}$.

1.1.9 Heat transfer

Heat transfer describes the exchange of thermal energy, between physical systems depending on the temperature and pressure, by dissipating heat. Figure 1.5 expresses general heat transfer procedure. The fundamental modes of heat transfer are conduction or diffusion, convection (free and forced) and radiation. Thermal equilibrium is reached when all involved bodies and the surroundings reach the same temperature.

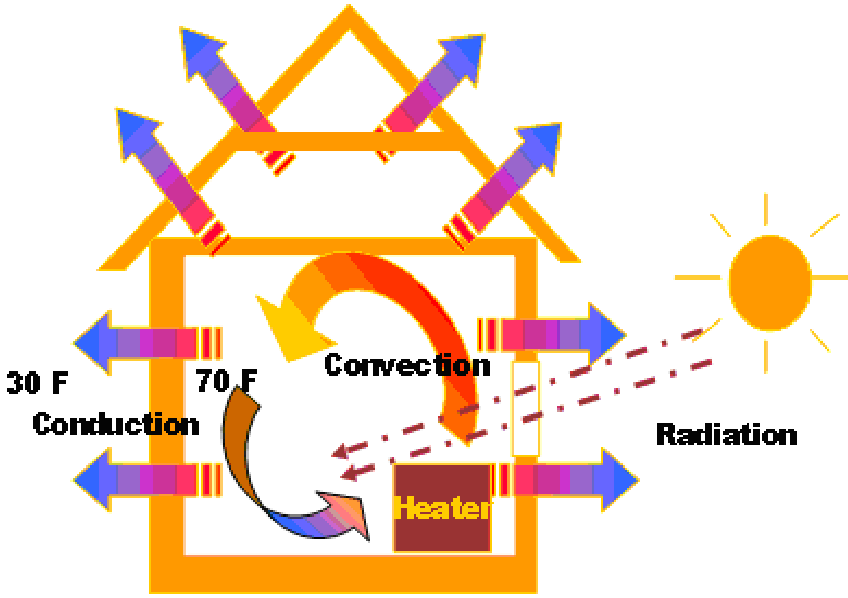

Figure 1.5: Heat transfer system

1.1.10 Viscosity

Viscosity is that property of real fluid as a result of which they offer some resistance to shearing, i.e., sliding movement of one particle past or near another particle. Viscosity is also known as internal friction of fluid. All known fluids have this property in varying degree. Viscosity of glycerin and oil is large in comparison to viscosity of water or gases.

1.1.11 Thermal conductivity

Thermal conductivity refers to the amount/speed of heat transmitted through a material. Heat transfer occurs at a higher rate across materials of high thermal conductivity than those of low thermal conductivity. Materials of high thermal conductivity are widely used in heat sink applications and materials of low thermal conductivity are used in thermal insulation. Thermal conductivity of materials is temperature dependent. The reciprocal of thermal conductivity is called thermal resistivity. Metals with high thermal conductivity, e.g. copper exhibits high electrical conductivity. The heat generated in high thermal conductivity materials is rapidly conducted away from the region of the weld. For metallic materials, the electrical and thermal conductivity correlate positively, i.e. materials with high electrical conductivity (low electrical resistance) exhibit high thermal conductivity. The proportionality constant k is called thermal conductivity of the material.

1.2 Literature Review

One of the proficient passive approaches is using nanofluid in heat transport improvement for enhancing the efficiency of thermal systems like heat exchangers, thermal storage, solar collectors, photovoltaic/thermal system, biomedical devices, nuclear reactors, cooling of electronic components etc. In recent years, there has been an increasing interest in merge of two or more nanoparticles in base fluid known as "hybrid nanofluid" due to improvement in cooling performance.

Madhesh and Kalaiselvam [1] conducted an experiment to analyze cooling performance of hybrid water-Cu-Ti nanofluid. Researchers [2-3] found enhanced thermal conductivity using hybrid nanoparticles such as CNT–Cu, CNT–Ag, CNT-Al_2O_3 and the increasing rate was about 21% at room temperature for hybrid CNT-Al_2O_3 particle volume fractions of 0.2%. Iqbal *et al.* [4] concluded that brick shaped hybrid Cu-CuO nanoparticles contributed to relative low temperature distribution while platelet shaped nanoparticles were efficient in a sense of rising fluid flow. Researchers [5-7] used hybrid nanofluids heat transfer medium by free convection inside different geometries like open wavy cavity, circular cylinder etc. as it has huge applications in environment and industry. In order to remain side by side developments in the extent applications and increasingly developing studies about nanofluids, review research has been conducted by Sarkar *et al*. [8].

Researchers are interested in heat transfer by mixed convection inside lid driven cavities of different shapes as it has huge applications in environment and industry [9-10]. Flow and heat transfer from irregular surfaces are often encountered in many engineering applications to enhance heat transfer such as micro-electronic devices, flat-plate solar collectors and flat-plate condensers in refrigerators, flows in the earth's crust, underground cable systems, electric machinery, cooling system of micro-electronic devices, etc. In addition, roughened surfaces could be used in the cooling of electrical and nuclear components where the wall heat flux is known. Surfaces are intentionally roughened sometimes to enhance heat transfer [11-12].

One of the proficient passive approaches is using nanofluid in heat transport improvement for enhancing the efficiency of thermal systems like; heat exchangers, thermal storage, solar collectors, photovoltaic/thermal system, biomedical devices, nuclear reactors, engine/vehicle, cooling of electronic components, transformers etc. Several studies have been reported on an MHD convective lid-driven flow of considering different nanofluid under different physical situations. Record of such investigations can be found in the works of [13-19]. During the last 2 decades, the insistence of convective heat transfer enhancement inside enclosures have become an insistent demand due to their ever-increasing applications in lubrication technologies, electronic cooling, food processing and nuclear reactors. The basic study of this field was done by Choi and Eastman [20]. Researchers [21-22] are interested in conjugate heat transfer by free convection inside cavities as it has huge applications in environment and industry. In order to remain side by side developments in the extent applications and increasingly developing studies about nanofluids, review papers have been conducted.

“Hybrid” nanofluid can be obtained by suspending more than one type of nanoparticles in base fluid. Very recently, hybrid nanofluid, is a gradually mounting study field parallelly to the incessant developments of common nanofluids. Compromised properties between the advantages and disadvantages of the properties of individual nanoparticles are a declared task of hybridization. Moreover, nanoparticles suppliers exhibit noticeable differences in prices of different nanoparticles types. For example, the price of copper nanoparticles is about 10 times greater than that of alumina nanoparticles. Indeed, the “hybrid” nanoparticles should be limited to those prepared as a single composite substance in a base fluid for which their synthetization requires extra attention [23-26].

One can use the “hybrid” nanofluid issue to those prepared by mixing unlike nanoparticles with base fluid. The experiments of Ho *et al.* [27] conducted an experiment about the mixture of particles of micro-encapsulated phase-change material and alumina nanoparticles in base fluid water. The authors found good agreement between the experimental data of the density and mass fraction and those predicted from the mixture theory. Botha et al. [28] performed an experiment of hybrid nanofluid based on silver-silica-oil. The authors found more deviated value of the thermal conductivity with the Maxwell relation [29] at higher solid volume fractions.

In order to remain side by side developments in the extent applications and increasingly developing studies about nanofluids, review papers have been conducted. Hybrid nanoparticles mixture procedure has been reviewed comprehensively [30-33]. This review showed the arithmetical models of hybrid nanofluid properties.

From the above literature review it is observed that hybrid nanofluid has been used extensively to enhance heat transfer rate. Comparative study of different pair of mixing nanoparticles with base fluid has not been done yet. The present numerical research expects to investigate and compare the performance of hybrid nanofluids (water-Cu-CNT, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-TiO_2) on convective and conductive heat transfer inside a trapezoidal lid-driven wavy cavity.

1.3 Objectives

This research aims to simulate a two dimensional convective-conductive heat transfer and laminar flow physics using a trapezoidal wavy lid-driven enclosure. The specific aims of this research are as follows:

- To modify the mathematical model of heat and fluid flow applying hybrid nanofluid properties.
- To visualize the laminar flows of single nanofluid (Cu-water) and hybrid nanofluids (water-Cu-CNT, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-TiO_2).
- To find the effects of various pertinent parameters like wave number, Richardson number and Prandtl number on flow and temperature fields.
- To compare the heat transfer performance of the mentioned single and hybrid nanofluids using different solid volume fractions.

1.4 Scope of the Thesis

A brief description of the present numerical investigation of heat transfer inside a wavy trapezoidal enclosure using hybrid nanofluids have been presented in this thesis through four chapters as stated below:

Chapter 1 contains introduction with the aim and objectives of the present work. This chapter also includes a literature review of the past studies on heat transfer using different types of hybrid nanofluids which are relevant to the present work. Objectives of the present study has also been incorporated in this chapter.

Chapter 2 presents a detailed description for the numerical simulation of heat transfer and fluid flow characteristics inside a cavity taking heat transfer medium as water and water based single as well as hybrid nanofluids. Physical model of a undulated enclosure is described. Mathematical formulation has been given for numerical computation in this chapter.

In Chapter 3, the effects of wave number, Richardson number, Prandtl number and solid volume fraction of nanoparticles have been investigated. Results have been shown in isothermal lines, streamlines to better understand the heat transfer mechanism through undulated enclosure. Comparison has been also shown in terms of heat transfer rate among considered hybrid nanofluids.

Chapter 4 concludes remarks of the whole research and the recommendations for the future research have been presented systematically.

Chapter 2
Numerical Analysis

2.1. Introduction

Numerical analysis is the study of algorithms that use numerical approximation (as opposed to symbolic manipulations) for the problems of mathematical analysis (as distinguished from discrete mathematics). Numerical analysis naturally finds application in all fields of engineering and the physical sciences, but in the 21st century also the life sciences, social sciences, medicine, business and even the arts have adopted elements of scientific computations.

2.2. Finite Element Method

The finite element method (FEM) is a numerical method for solving problems of engineering and mathematical physics. Typical problem areas of interest include structural analysis, heat transfer, fluid flow, mass transport, and electromagnetic potential. The analytical solution of these problems generally requires the solution to boundary value problems for partial differential equations. The finite element method formulation of the problem results in a system of algebraic equations. The method approximates the unknown function over the domain. To solve the problem, it subdivides a large system into smaller, simpler parts that are called finite elements. The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem. FEM then uses variational methods from the calculus of variations to approximate a solution by minimizing an associated error function. Studying or analyzing a phenomenon with FEM is often referred to as finite element analysis (FEA).

A finite element method is characterized by a variational formulation, a discretization strategy, one or more solution algorithms and post-processing procedures. Examples of variational formulation are the Galerkin method, the discontinuous Galerkin method, mixed methods, etc. A discretization strategy is understood to mean a clearly defined set of procedures that cover (a) the creation of finite element meshes, (b) the definition of basis function on reference elements (also called shape functions) and (c) the mapping of reference elements onto the elements of the mesh. Examples of discretization strategies are the h-version, p-version, hp-version, x-FEM, iso-geometric analysis, etc. Each discretization strategy has certain advantages and disadvantages. A reasonable criterion in selecting a discretization strategy is to realize nearly optimal performance for the broadest set of mathematical models in a particular model class. There are various numerical solution algorithms that can be classified into two broad categories; direct and iterative solvers. These algorithms are designed to exploit the sparsity of matrices that depend on the choices of variational formulation and discretization strategy.

Post processing procedures are designed for the extraction of the data of interest from a finite element solution. In order to meet the requirements of solution verification, postprocessors need to provide for a posteriori error estimation in terms of the quantities of interest. When the errors of approximation are larger than what is considered acceptable then the discretization has to be changed either by an automated adaptive process or by action of the analyst. There are some very efficient postprocessors that provide for the realization of super convergence. FEM meshing has been displayed in the Figure 2.1.

This figure is an example of a two dimensional FEM (finite element method) mesh for a cylindrically shaped magnetic shield. The mesh is created by an analyst prior to solution by the FEM software. In this case "two dimensional" means that the picture shows a flat cross section of an assembly that extends to a large distance at right-angles to the paper (Cartesian coordinates). The rectangle outlined at the right of the picture has been designated the conducting component, which carries the electric current that creates the magnetic field. The cylindrical part has been designated to be a material of high magnetic permeability (for example iron). The gray areas have been designated air. The mesh is divided into smaller triangles inside the cylinder, which is an area where the lines of magnetic flux will be more concentrated.

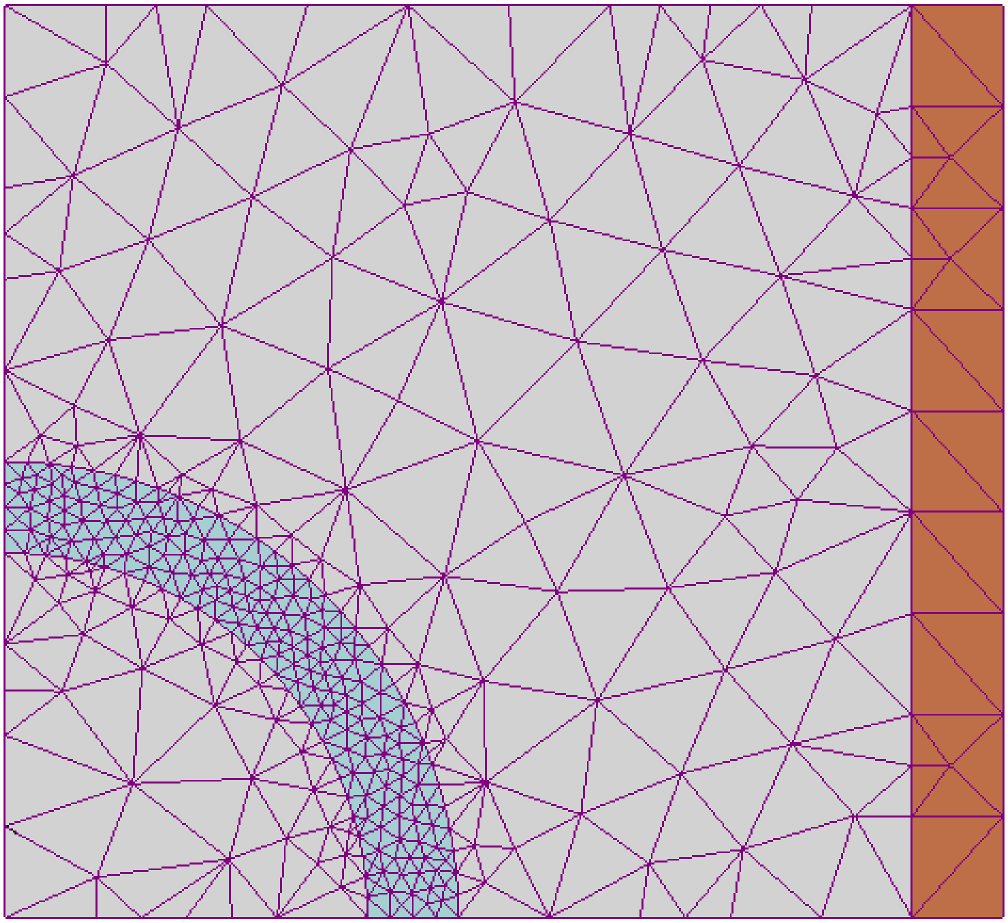

Figure 2.1: FEM Mesh

2.3 Problem Formulation

The schematic diagram of the studied configuration has been depicted in the Figure 2.2. It consists of a two dimensional lid-driven trapezoidal enclosure of side length H. The bottom surface has been assigned to temperature T_h while the sinusoidal wavy bottom surface of the enclosure has been cooled at a constant temperature T_c. Under all circumstance $T_h > T_c$ condition has been maintained. The inclined surface of the trapezoid cavity has been thermally insulated. The top surface has been considered as lid driven with uniform velocity. The heat transfer medium has been taken as four types of water-based hybrid nanofluids consisting equal solid volume fraction of each nanoparticle like (Cu and TiO_2), (Cu and CuO), (Cu and Al_2O_3) and (Cu and CNT). The diameter of the hybrid nanofluid is considered as 10 nm. Cu, TiO_2, CuO, and Al_2O_3 are spherical shaped and CNT is taken as cylindrical shaped. It has been also assumed that both the fluid and nanoparticles are in thermal equilibrium and there is no slip between them. The hybrid nanofluid used in the analysis has been considered as laminar and incompressible. The gravitational acceleration acts to the vertical downward surface.

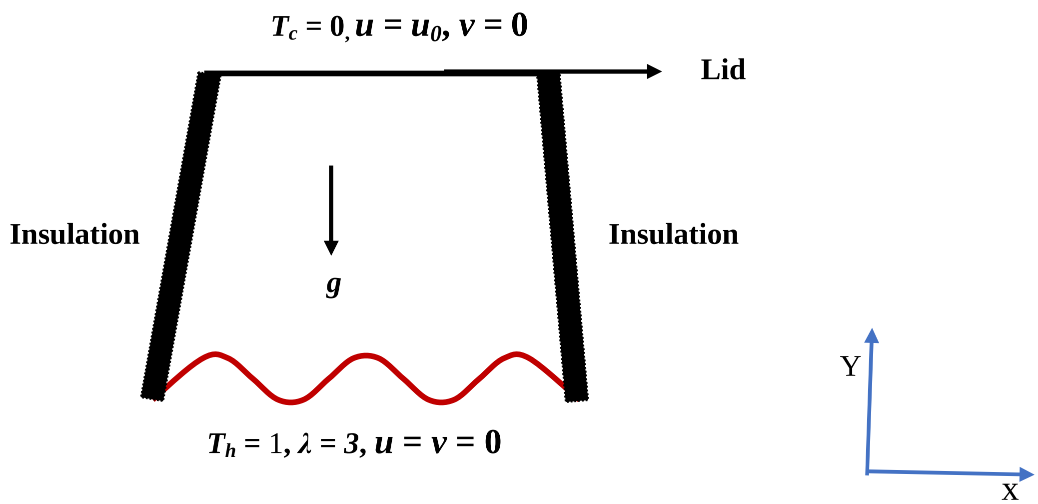

Figure 2.2: Physical model of computational domain

2.4 Mathematical Modeling

The 2D numerical simulation has been performed in steady state conditions. The governing partial differential equations according to [34-35], dimensional (2.1-2.4) and non-dimensional form (2.6-2.9) in terms of laminar flow and thermal energy physics using hybrid nanofluid are given below:

Dimensional form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x}+\upsilon_{hnf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2.2}$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial y}+\upsilon_{hnf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{(\rho\beta)_{hnf}}{(\rho\beta)_f}g(T-T_c) \tag{2.3}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha_{hnf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{2.4}$$

The above equations are non-dimensionalized by using the following dimensionless quantities

$$X=\frac{x}{H},\quad Y=\frac{y}{H},\; U=\frac{u_0 H}{\alpha_f},\quad V=\frac{u_0 H}{\alpha_f},\; \theta=\frac{T-T_c}{T_h-T_c},\; P=\frac{pH^2}{\rho_f\alpha_f^2} \tag{2.5}$$

Non-dimensional form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2.6}$$

$$\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P}{\partial X}+\frac{\rho_f}{\rho_{hnf}}\frac{\mu_{hnf}}{\mu_f}Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2.7}$$

$$\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+\frac{\rho_f}{\rho_{hnf}}\frac{\mu_{hnf}}{\mu_f}Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{(\rho\beta)_{hnf}}{(\rho\beta)_f}Ri\theta \tag{2.8}$$

$$\left(U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}\right)=\frac{\alpha_{hnf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{2.9}$$

where, $Pr=\frac{\mu_f C_{pf}}{k_f}$ and $Ri=\frac{g\beta_f\Delta TH}{v_0}$ be the Prandtl number and Richardson number, respectively.

The shape of the bottom wavy surface profile has been assumed to mimic the following pattern $Y=A[1-\cos(2\lambda\pi X)]$ (2.10)

where A is the dimensionless amplitude of the wavy surface and λ is the number of undulations.

The boundary conditions imposed on the flow are taken as:

On the left and right inclined wall: $U=0,\; V=0, \theta=0$

On the top wall: $U=1,\; V=0,\; \theta=0$

On the wavy bottom wall: $U=V=0, \theta=1$

The following models of effective properties of hybrid nanofluids have been chosen as:

thermal diffusivity $\alpha_{hnf}=k_{hnf}/\left(\rho C_p\right)_{hnf}$ (2.11)

density, $\rho_{hnf}=(1-\phi)\rho_f+\phi_1\rho_1+\phi_2\rho_2$ (2.12) heat capacitance, $\left(\rho C_p\right)_{hnf} = (1-\phi)\left(\rho C_p\right)_f + \phi_1\left(\rho C_p\right)_1 + \phi_2\left(\rho C_p\right)_2$ (2.13)

thermal expansion coefficient, $\left(\rho\beta\right)_{hnf} = (1-\phi)\left(\rho\beta\right)_f + \phi_1\left(\rho\beta\right)_1 + \phi_2\left(\rho\beta\right)_2$ (2.14)

specific heat at constant pressure, $C_{phnf} = \dfrac{(1-\phi)\left(\rho C_p\right)_f + \phi_1\left(\rho C_p\right)_1 + \phi_2\left(\rho C_p\right)_2}{(1-\phi)\rho_f + \phi_1\rho_1 + \phi_2\rho_2}$ (2.15)

viscosity of Brinkman model [36] $\mu_{hnf} = \dfrac{\mu_f}{\left(1-\phi_1-\phi_2\right)^{2.5}}$ (2.16)

thermal conductivity of Maxwell-Garnett model [29]

$$k_{hnf} = k_f \frac{\dfrac{\left(\phi_1 k_1 + \phi_2 k_2\right)}{\phi} + 2k_f + 2\left(\phi_1 k_1 + \phi_2 k_2\right) - 2\phi k_f}{\dfrac{\left(\phi_1 k_1 + \phi_2 k_2\right)}{\phi} + 2k_f - \left(\phi_1 k_1 + \phi_2 k_2\right) + \phi k_f} \tag{2.17}$$

and electrical conductivity of Maxwell-Garnett model [29]

$$\frac{\sigma_{hnf}}{\sigma_f} = 1 + \frac{3\left\{\dfrac{\left(\phi_1\sigma_1 + \phi_2\sigma_2\right)}{\sigma_f} - \left(\phi_1 + \phi_2\right)\right\}}{\left\{\dfrac{\left(\phi_1\sigma_1 + \phi_2\sigma_2\right)}{\phi\sigma_f} + 2\right\} - \left\{\dfrac{\left(\phi_1\sigma_1 + \phi_2\sigma_2\right)}{\sigma_f} - \left(\phi_1 + \phi_2\right)\right\}} \tag{2.18}$$

Here ϕ is the overall volume concentration of two different types of nanoparticles dispersed in hybrid nanofluid and is calculated as $\phi = \phi_1 + \phi_2$ (2.19)

The mean Nusselt number at the left heated wall of the cavity is

$$Nu = -\frac{k_{hnf}}{k_f}\int_0^L \frac{\partial\theta}{\partial N}\,dL \tag{2.20}$$

where L is the length of the undulated surface and N be the distances either along x or y directions acting normal to the inclined surface.

2.4.1 Thermo-physical properties

Thermo-physical properties [37-38] of base fluid and different nanoparticles have been shown in Table 2.1, assumed constant except for the density variation, which is maintained on Boussinesq approximation.

Table 2.1: Thermo- physical properties of base fluid and different nanoparticles

Physical Properties	Fluid Phase (Water)	Cu	TiO_2	CNT	CuO	Al_2O_3
C_p (J/kgK)	4179	385	686.2	796	535.6	765
ρ (kg/m^3)	997.1	8933	4250	1600	6500	3970
k (W/mK)	0.6	401	8.9538	3000	20	40
β (1/K)	21	1.67×10^{-5}	0.9×10^{-5}	------	5.1×10^{-5}	2.4×10^{-5}
σ (μS/cm)	0.05	5.96×10^{-7}	6.28×10^{-5}			

2.5 Computational Procedure

Using the Galerkin weighted residual finite element technique [39-40] the momentum and energy balance equations have been solved using COMSOL Multyphysics. In this method, the solution domain has been discretized into finite element meshes, which have been composed of non-uniform triangular elements. Then the nonlinear and non-dimensional governing partial differential equations have been transferred into a system of integral equations by applying Galerkin weighted residual method. The basic unknowns for the governing partial differential equations (2.6-2.9) are the velocity components *U*, *V*, the temperature θ and the pressure *P*. The six nodes with triangular element have been used in this numerical research. All six nodes have been associated with velocities as well as temperature while three corner nodes with pressure. The nonlinear algebraic equations so obtained have been modified by imposition of boundary conditions. These modified nonlinear equations have been transferred into linear algebraic equations by using Newton's method. Finally, these linear equations have been solved by using triangular factorization method. The convergence criterion for the solution procedure has been defined as $\left|\psi^{n+1}-\psi^{n}\right|\le10^{-6}$, where n is the number of iteration and ψ is a function of *U*, *V* and θ.

2.5.1 Code validation

In order to authenticate the exactness of present numerical technique, the obtained results in special cases have been compared with the results obtained by Rashad *et al*. [33]. These comparisons have been presented obviously in the Figure 2.3 in terms of the streamlines and Isotherms. The code validation has been conducted while employing the dimensionless parameters as Ha = 10, $\phi = 0.05$, D = 0.5, Ra = 10^4, Q = 1, B = 0.4. A very good agreement has been found between the present results and the results of Rashad *et al.*

[33]. These flattering comparisons provide confidence in the numerical results to be reported subsequently.

Rashad *et al.* [33] **Present Code**

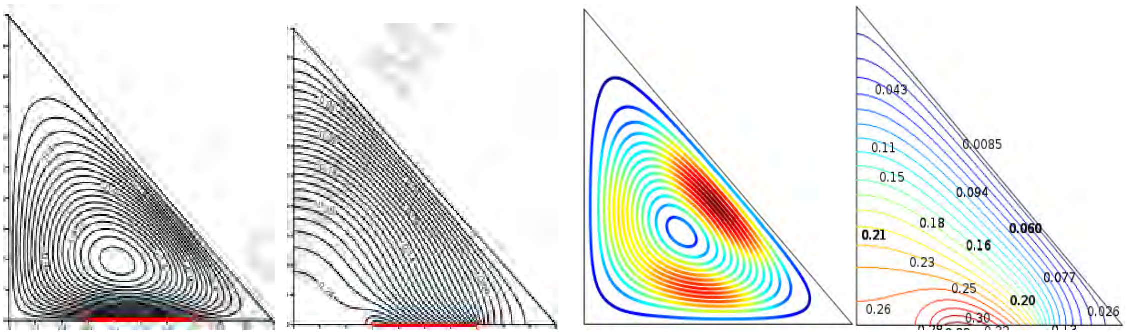

Figure 2.3: Code validation of the streamlines and isotherms between Rashad *et al.* [33] and that of present research at *Ha* = 10, $\phi = 0.05$, *D* = 0.5, *Ra* = 10^4, *Q* = 1, *B* = 0.4

2.5.2 Mesh generation

The discrete locations at which the variables are to be calculated are defined by a mesh which covers the geometric domain on which the problem is to be solved. It divides the solution domain into a finite number of sub-domains called finite elements. The computational domains with irregular geometries by a collection of finite elements make the method a valuable practical tool for the solution of boundary value problems arising in various fields of engineering. Figure 2.4 displays the finite element mesh of the present physical domain. The meshing consists of tetrahedral element with ten nodes in subdomain and triangular element with six nodes in boundaries.

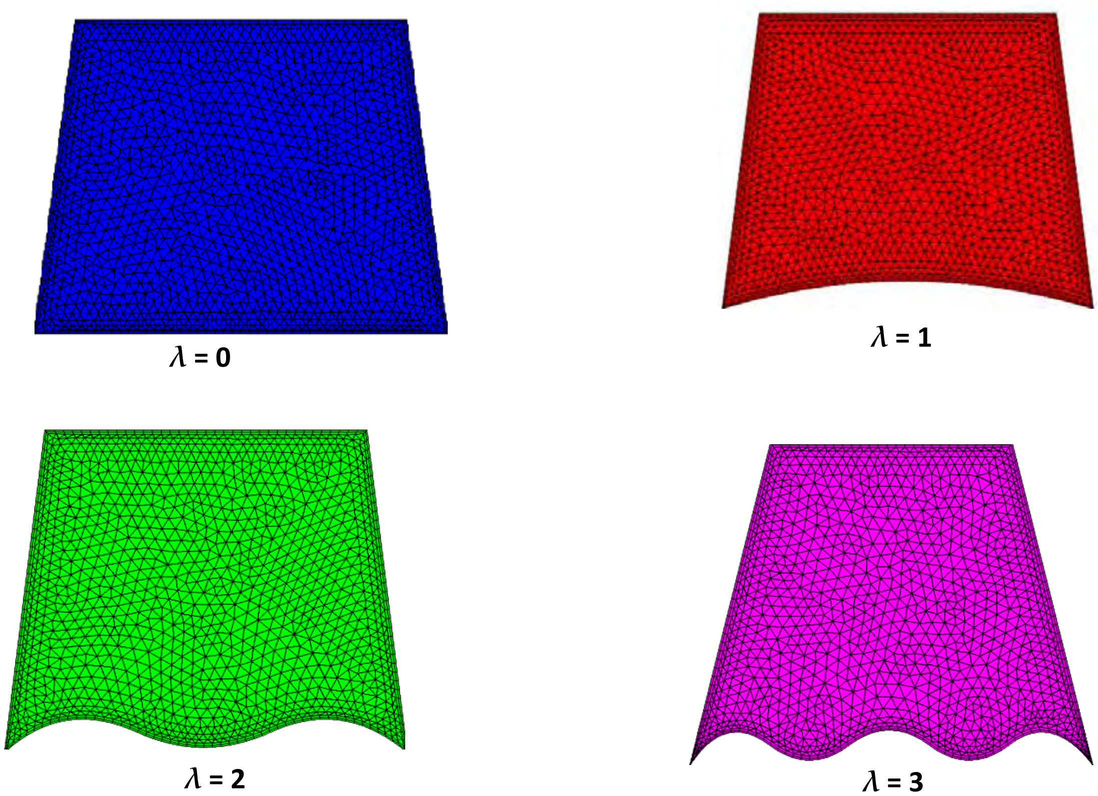

Figure 2.4: Mesh Generation of wavy enclosure

2.5.3 Grid sensitivity test

In order to determine the proper grid size for this study, a grid independence test is conducted with five types of mesh for *Pr* = 5.8, *Ri* =1.0, and λ = 3 which has been shown in Figure 2.5. Corresponding grid densities are 7182 nodes, 592 elements, time 29 s; 11062 nodes, 1133 elements, 42 s; 15074 nodes, 1744 elements, 64 s; 18082 nodes, 4484 elements, 105 s; and 29726 nodes, 12502 elements, 255 s. The extreme value of Nu is used as the monitoring variable for sensitivity measure of the accuracy of the solution. Taking into account both the precision of numerical values and computational time, the present calculations are performed with 18082 nodes and 4484 elements grid system. Figure 2.3, one can observe that no further improvement in accuracy occur using higher number of elements.

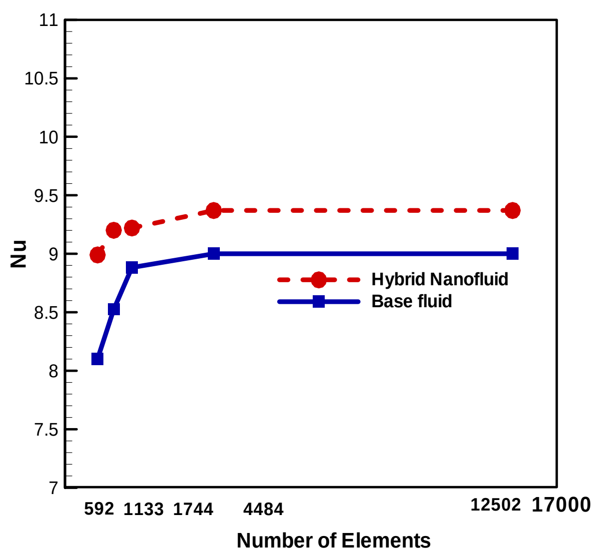

Figure 2.5: Grid refinement test

Chapter 3

Results and Discussions

3.1 Introduction

Heat transfer performance of different water-based hybrid nanofluids (water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT) flow in mixed convective lid-driven sinusoidal trapezoidal enclosure has been investigated numerically. The numerical results of the current study are expatiated graphically via stream function contours (streamline), dimensionless temperature contours (isotherms) and average Nusselt numbers. The relevant parameters those have a direct effect on the flow and thermal fields inside the considered cavity are; Richardson number Ri = 0.1 to10, nanoparticles volume fraction, φ = 0 to 3%, Prandtl number, Pr = 4.2 to 6.2 and undulation number, λ = 0 to 3. Also, comparison in heat transfer performance of four types of hybrid nanofluids have been displayed. Here total volume fraction is divided equally into two nanoparticles for each hybrid nanofluid. In order to display the results of these four independent parameters, three parameters are fixed (unless where stated) while the remainder single one is varied as gathered in the following categories:

3.2 Effect of Wave Numbers

Figure 3.1(a-b) provides the information about the sensitivity of the streamlines and isothermal lines pattern due to the variation of undulation number (λ) with Ri = 1, Pr = 5.8. HTF has been considered as water. From the streamlines contours it is found that for purely mixed convection (Ri = 1) and λ = 0 there exist two divided vortex and intertwining core near the left vertical line. When λ = 1, two divided vortices have been more sophisticated and intertwining core has not been complicated. When λ = 2, one oval shape core has been created and one vortex has been omitted. When λ = 3, one vortex has been small and intertwining core has not been complicated and core has been more vacuous.

The influence of Ri on temperature field has been plotted in the Figure 3.1(b). From this figure, it is noticed that for λ = 0, the isothermal line agglutinate to the left wall and undulated wall due to the influence of forced convection. Ditch inclined on the right of the enclosure. With further increase of λ = 1, the isotherms line increasing to the lid driven and ditch has been decreased. In this situation, it is also found that the isotherms depart to the right corner section of the enclosure and try to get crowded on the top wavy surface due to the influence of forced convection. Subsequently, the thermal current activity is influenced firmly by the conductivity properties of hybrid nanofluid. There is no major distinction in the isothermal lines for λ = 1 in compare to λ = 2 except the isothermal contours are clustered near the heated vertical wall of the cavity. For a fixed value of *Ri*, the thermal field has a little change with the variation in λ. Escalating the convective parameter λ = 3 steeper temperature gradients near the heated wall is evident and the isothermal lines get the parabolic shape near the inclined cold wall. In the considered regime, the buoyancy effect balances the flow patterns and a thin boundary layer is developed close to the bottom wavy surface. It is also captured that the mostly horizontal isothermal lines, occupy in the lower section of the cavity. Furthermore, the thermal boundary layer near the underside surface becomes slightly less parallel and concentrates when the number of waves increases.

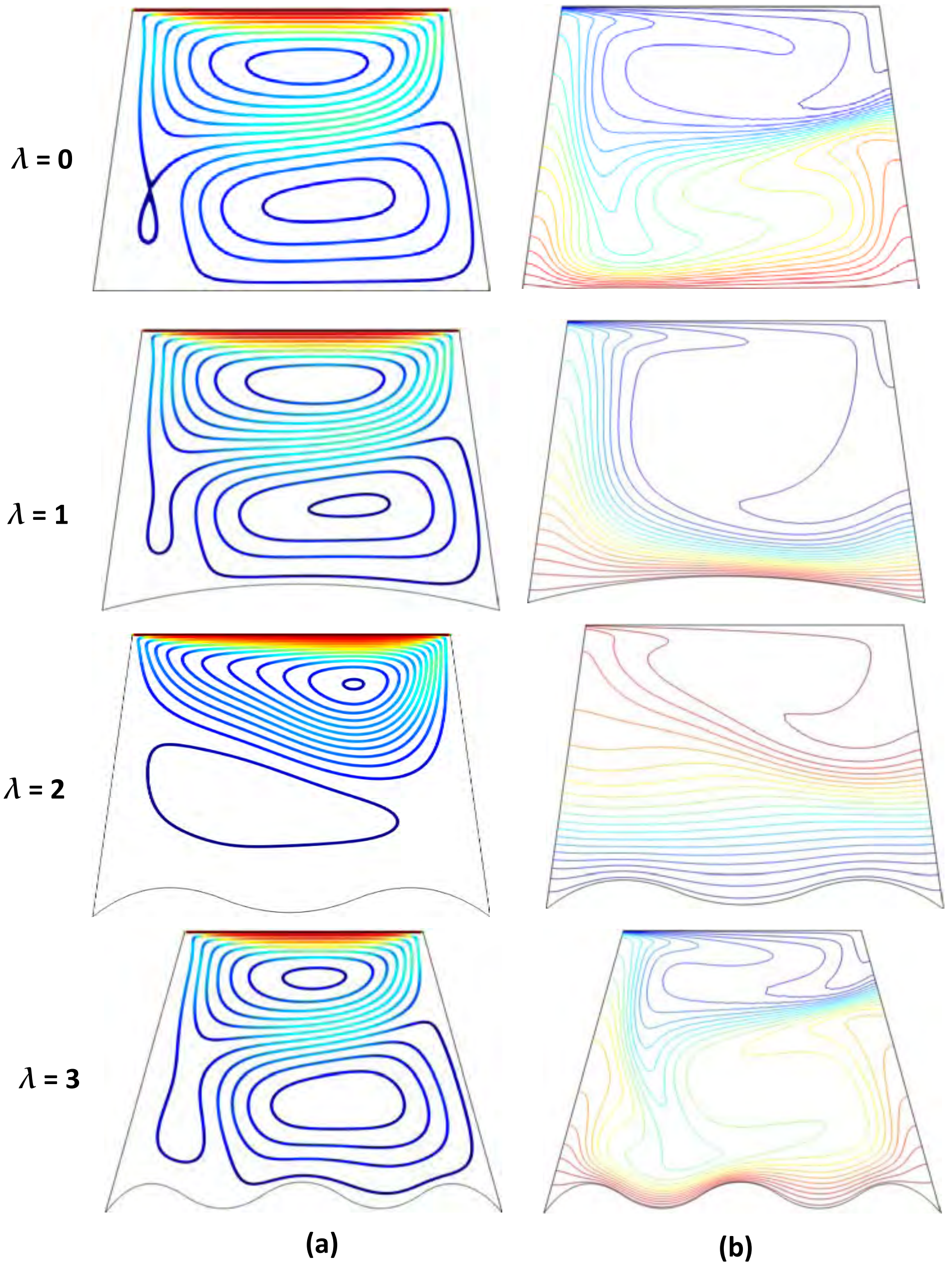

Figure 3.1: Effect of λ on (a) streamlines and (b) isotherms with *Ri* = 1, *Pr* = 5.8, ϕ = 1%

The distribution of Nusselt number for different wave number (undulation) has been presented in the figure 3.2. The augmentation in the Nusselt number has been recorded for a range of λ. It is also perceptible that rate of increasing heat transfer is much higher while the value of wave number (undulation) is high than for the small value of λ. The heat transfer rate raised by 14.7% compared to flat surface (λ = 0) to undulated surface (λ.= 3). So, it is very reasonable to conclude that the heat transfer rate becomes higher when the wave number (undulation) varies.

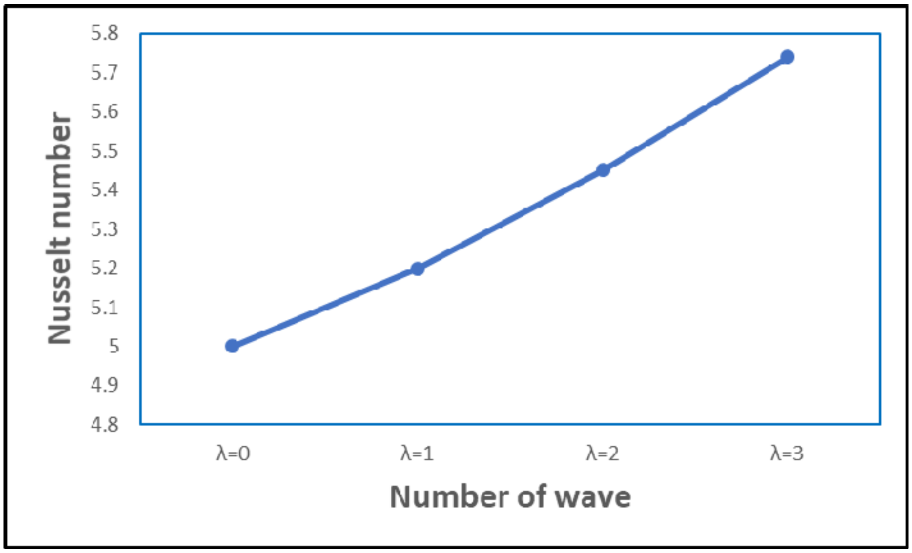

Figure 3.2: Average Nusselt number for various number of wave

3.3 Effect of Volume Fraction

Varying the parameter φ of water-Cu single nanofluid has led to few results and been shown in the Figure 3.3(a-b). From the Figure 3.3(a), When the nanoparticles volume fraction is increased from φ = 0.001 to φ = 0.03 for fixed λ = 3, *Ri* = 1, *Pr* = 5.8, an oval shape core inclined with the right vertical line and also near to the undulated line. When volume fraction is increased from φ = 0.01 to φ = 0.02, oval shape core becomes smaller a little bit. But remaining behavior of streamlines associate with pure water is transformed into behavior with the increase in the nanoparticle volume fraction and the intensification of the fluid flow behavior decrease when the fluid particle volume fraction drops off. This is due to the enhancement of viscosity of the hybrid nanofluid. Since with the augment of φ, the thermal conductivity of the nanofluid increases, hence the buoyancy flow, these interns improved the heat transfer rate.

However, it is also obvious from Figure 3.3(b), the isotherms are, generally, not affected by adding the nanoparticles with any fraction except that increasing φ the isothermes close to the heat source begin to be more scatterd. The enrichment of the thermal conductivity produces denser isotherms which is the indication of development nanofluid convection. We can conclude that the mixing advantage of Copper and Carbon nano tube nanoparticles may blow up the convective heat transfer. Hence, this result encourages us to examine other combination of different types nanoparticles, with different ratio of nanoparticles, and to work with innovative models on behalf of dissimilar thermo-physical properties.

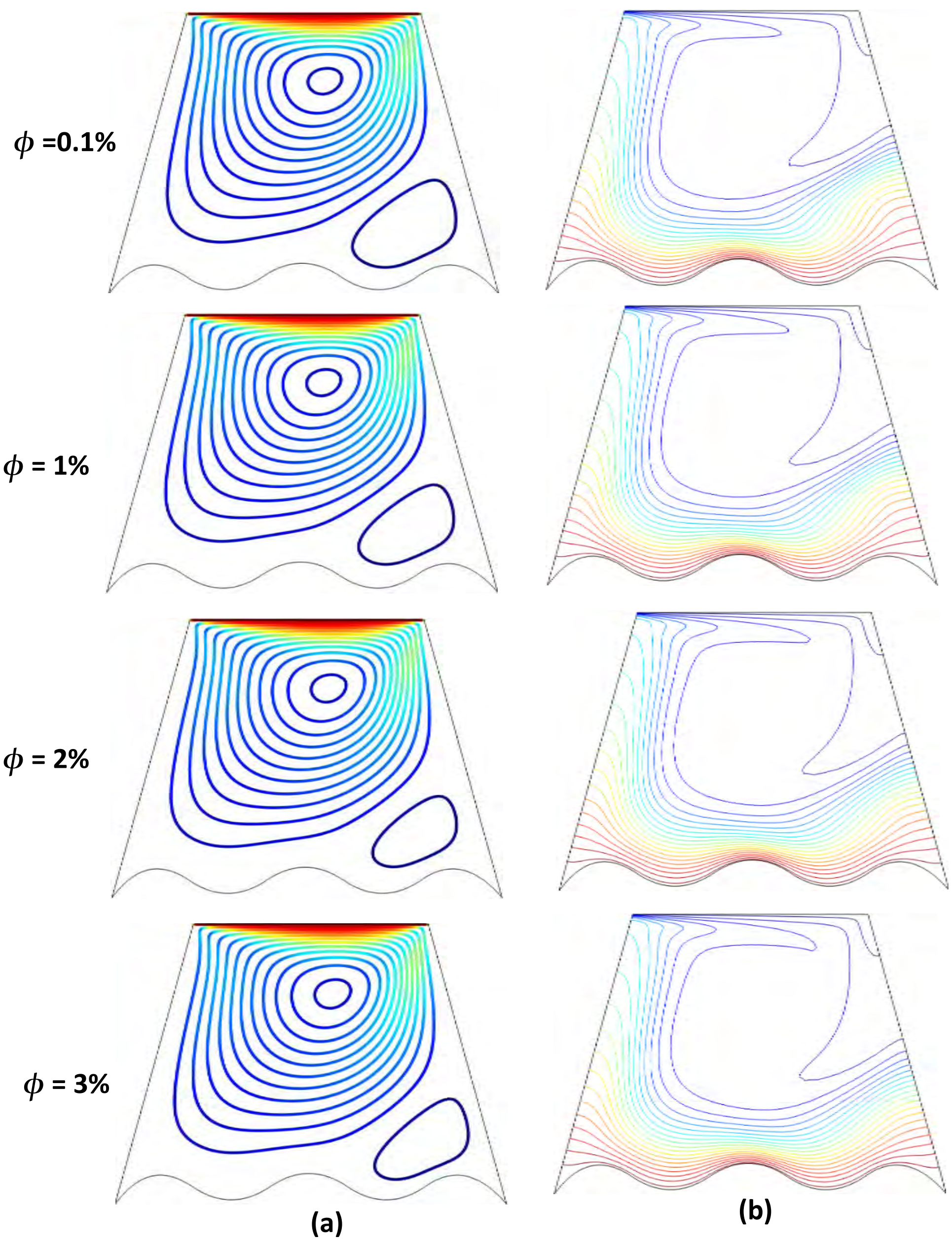

Figure 3.3: Effect of ϕ on (a) streamlines and (b) isotherms with *Ri* = 1, *Pr* = 5.8, λ=3

Figure 3.4 represents the average Nusselt number for various solid volume fraction of water-based Cu nanofluid. The value of volume fraction taken 0, 0.1, 1, 2, and 3%. The value of average Nusselt number of volume fraction 0, 0.001, 0.01, 0.02, 0.03 of water-Cu nanofluid are 4.4, 4.62, 4.76, 4.85 and 4.9 respectively. The heat transfer rate of volume fraction from 0 to 1% is greater than that of other volume fraction. The heat transfer rate is also increasing for other volume fraction (for 2 and 3%) but increasing rate is downcast. The rate of heat transfer is obtained 11.23% for increasing values of solid volume fraction from 0 to 3%.

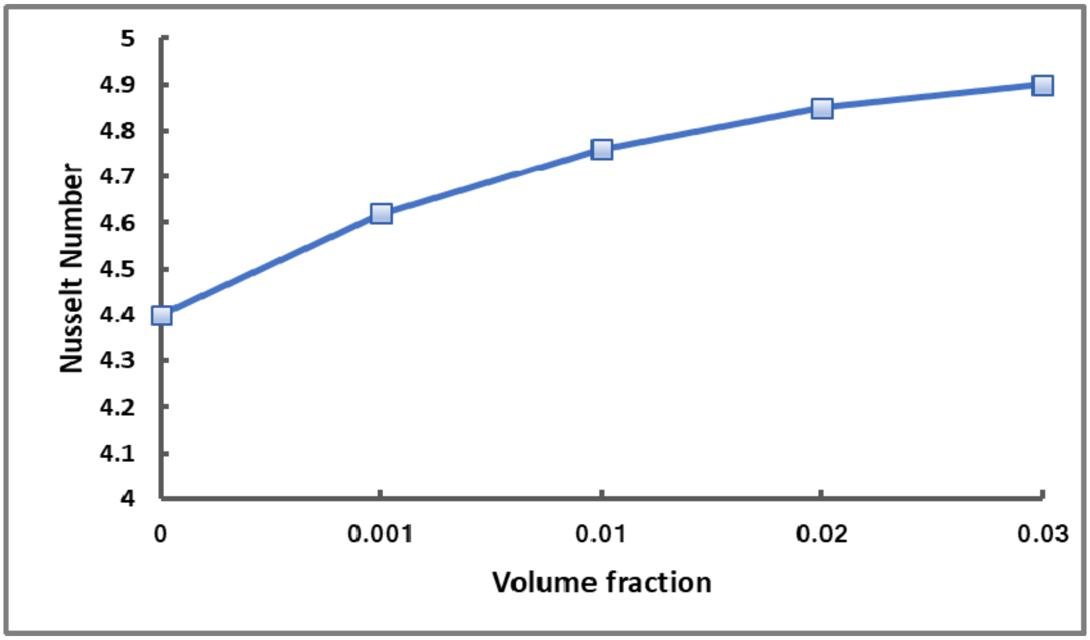

Figure 3.4: Average Nusselt number for various volume fraction of water-Cu nanofluid

3.4 Performance of Hybrid Nanofluids

The present numerical research expects to investigate and compare the performance of hybrid nanofluids (water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT) on convective and conductive heat transfer inside a trapezoidal lid-driven wavy cavity. Streamlines and (b) Isotherms for water-Cu-CNT hybrid nanofluid with fixed $\lambda = 3$, $Ri = 1$, $Pr = 5.8$, $\varphi = 1\%$ have shown in the figure It is obvious from Figure 3.5(a), the streamlines are, generally, less affected for using different hybrid nano-fluids. From the streamlines contours it is found that for Cu- TiO_2/water, there exist a primary elliptic circulation cell without any secondary core but one small oval shape core has been created. For water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT, it can be seen that shape of the primary cell has been bended a little bit and it is important to notice that oval shape core has been changed tiny bit than before. There less distinction in the streamlines.

However, it is also obvious from Figure 3.5(b), that the temperature gradients are also less affected on using the hybrid nanofluids (water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT). The enrichment of the thermal conductivity produces denser isotherms which is the indication of development nanofluid convection. It is concluded that the mixing advantage of copper and carbon nano tube nanoparticles may blow up the convective heat transfer. Hence, this result encourages us to examine other combination of different types nanoparticles, with different ratio of nanoparticles, and to work with innovative models on behalf of dissimilar thermo-physical properties.

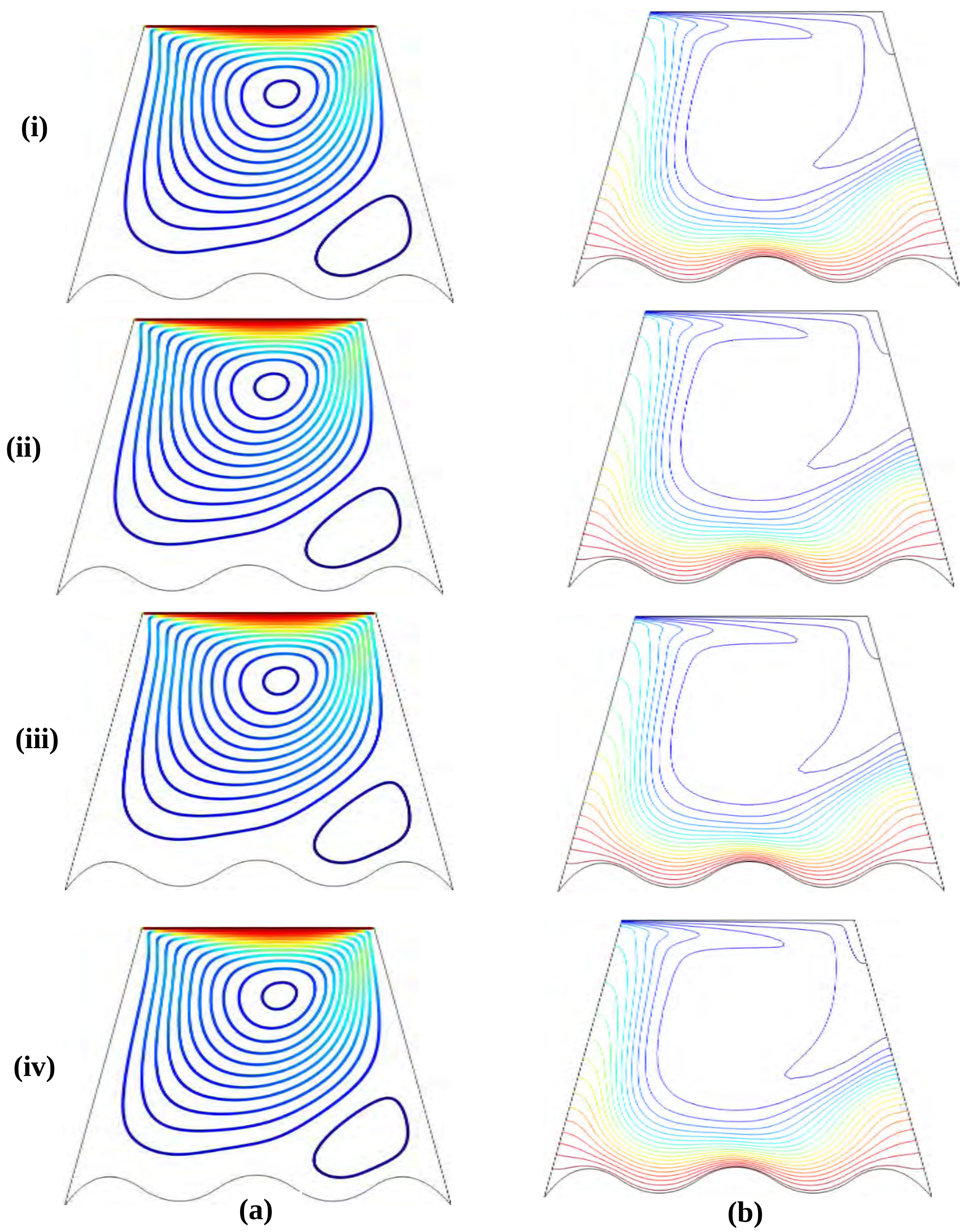

Figure 3.5: Effect of (i) water-Cu-TiO_2, (ii) water-Cu-CuO, (iii) water-Cu-Al_2O_3 and (iv) water-Cu-CNT hybrid nanofluids on (a) stream lines and (b) isotherms with λ = 3, Ri = 1, Pr = 5.8, ϕ = 1%

Figure 3.6 illustrates the Nusselt number of different hybrids nanofluid at $\lambda = 3$, $Pr = 5.8$ and $\phi = 1\%$. It is evident that with the increase of the value of thermal conductivity heat transfer rate increases. Nusselt number is found as increasing pattern in the sequence of water-based Cu-TiO_2< Cu-CuO<Cu-Al_2O_3 <Cu-CNT hybrid nanofluids. The heat transfer rate is increased about 6.3% using heat transfer medium as water-Cu-CNT hybrid nanofluid than water-Cu-TiO_2. nanofluid. Values of Nu are obtained as 4.4, 4.76, 4.80, 4.85, 4.90, and 5.0 for water, water-Cu, water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT, respectively. Thus, higher rate of heat transfer is found using water-Cu-CNT hybrid nanofluid than another considered hybrid nanofluids. Thus, heat transfer rate is higher for hybrid nanofluid compared to single nanofluid and base fluid. The enhanced rate of mean Nusselt number for water, water-Cu, water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT hybrid nanofluid with 1% solid volume fraction are found as 8.1, 9.1, 10.2, 11.4 and 13.6% respectively compared to base fluid.

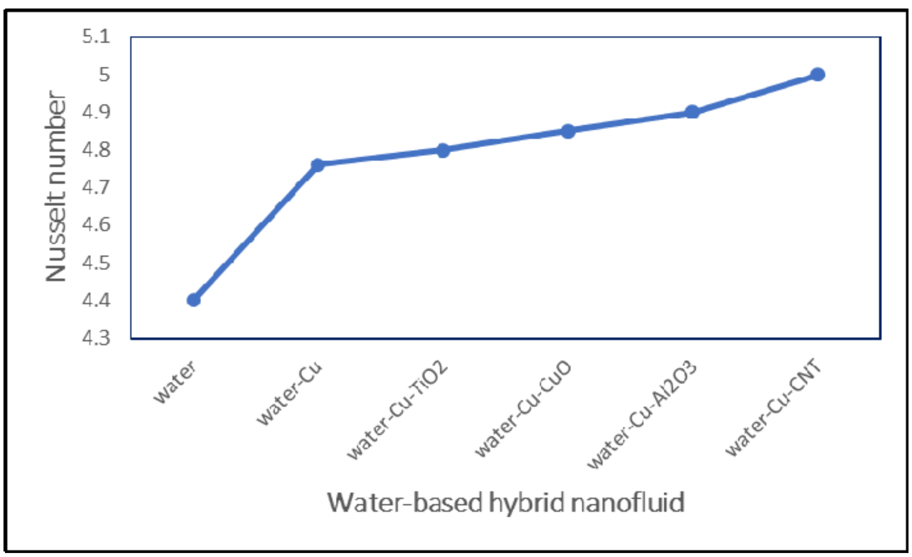

Figure 3.6: Average Nusselt number for various hybrid nanofluids

3.5 Effect of Richardson Number

In heat transfer problems, Richardson number represents the importance of the effect of forced to natural convection phenomena. Due to the influence of buoyancy force if the Richardson number is much less than unity, convection is unimportant in the flow. If it is much greater than unity, buoyancy is dominant the flow behavior. Figure 3.7(a-b) provides the information about the sensitivity of the streamlines and isothermal lines pattern due to the variation of Richardson number (Ri) and for $\lambda = 3$, $Pr = 5.8$ with the volume fraction of the suspended nanoparticles $\phi = 1\%$. Water-Cu-CNT hybrid nanofluid has been considered as heat transferring fluid (HTF). From the streamlines contours it is found that for purely forced convection ($Ri = 0.1$) there exist a primary elliptic circulation cell without any secondary core. This is due to the dominance of forced convection in the flow regime. At combined convection regime (Ri =1.0) it can be seen that shape of the primary cell is bended a little bit and it is important to notice that one extra oval shape core is created. On the other hand, when $Ri = 5$ there creates a new shape core near the vertical wall and total contour separated by two parts. When $Ri = 10$, one oval shape core is created and two parts become larger. This is logical because escalating the parameter Ri assists buoyancy flow, hence the natural convection mode because buoyancy often plays a significant role in defining the mixed convection flow.

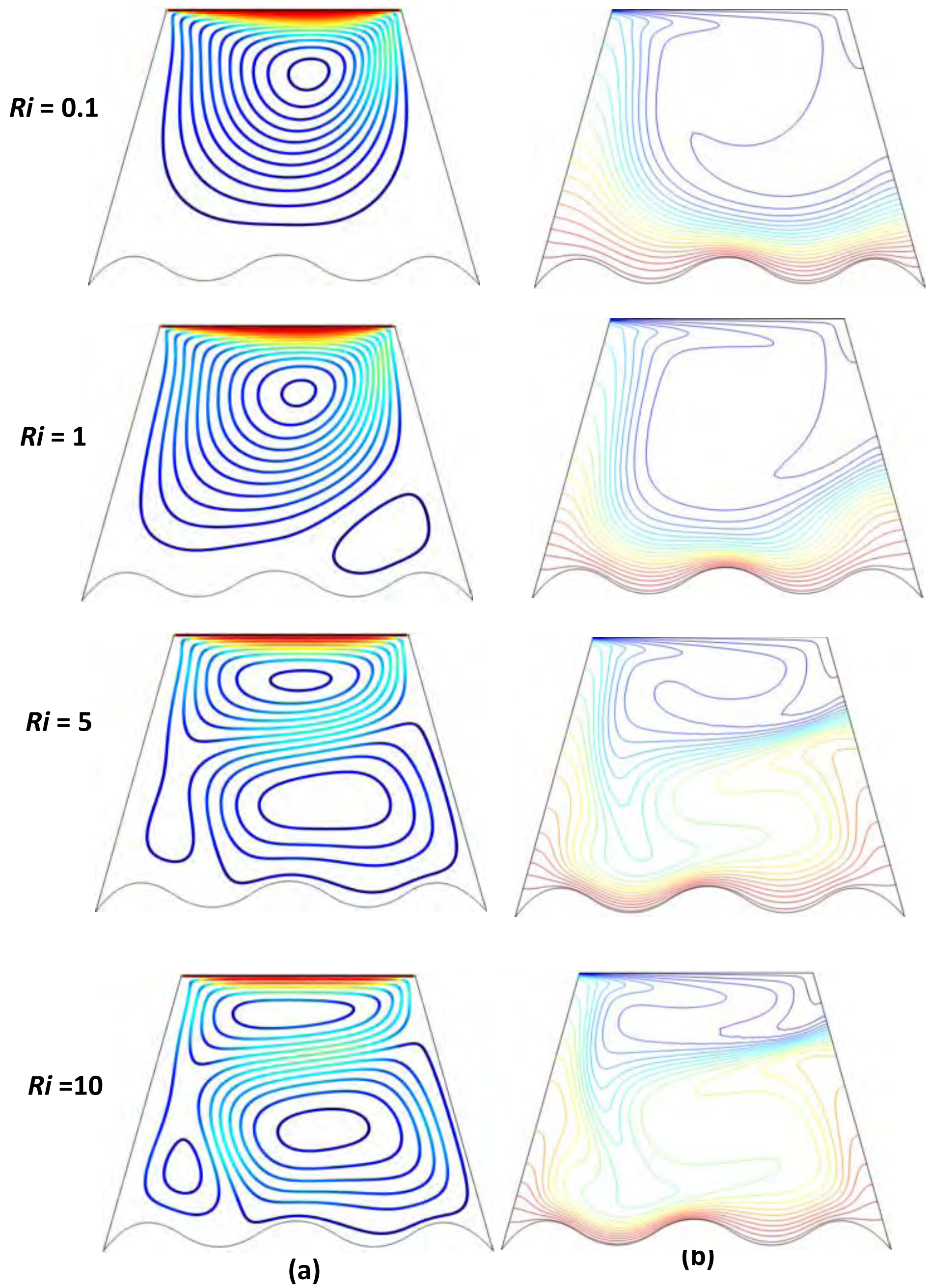

Figure 3.7: Effect of *Ri* on (a) streamlines and (b) isotherms with λ = 3, ϕ = 1%, *Pr* = 5.8

The influence of *Ri* on temperature field has been plotted in the Figure 3.7(b). From this figure, it is noticed that for *Ri* = 0.1, the isothermal line agglutinate to the left wall and undulated wall due to the influence of forced convection. Ditch inclined on the right of the enclosure. With further increase of *Ri* = 1, the isotherms line increasing to the lid driven and ditch has been decreased. In this situation, it is also found that the isotherms depart to the right corner section of the enclosure and try to get crowded on the top wavy surface due to the influence of forced convection. Subsequently, the thermal current activity is influenced firmly by the conductivity properties of hybrid nanofluid. There is no major distinction in the isothermal lines for *Ri* = 0.1 in compare to *Ri* = 1 except the isothermal contours are clustered near the heated vertical wall of the cavity. For a fixed value of λ, the thermal field has a little change with the variation in *Ri*. Escalating the convective parameter *Ri* = 10 steeper temperature gradients near the heated wall is evident and the isothermal lines get the parabolic shape near the inclined cold wall. In the considered regime, the buoyancy effect balances the flow patterns and a thin boundary layer is developed close to the bottom wavy surface. It is also captured that the mostly horizontal isothermal lines, occupy in the lower section of the cavity. Furthermore, the thermal boundary layer near the underside surface becomes slightly less parallel and concentrates when the number of waves increases.

Figure 3.8 represents the Nusselt number on the comparison of Richardson number. It is noticeable that the Nusselt number curve raised at the increment of Richardson number and it is also obvious that with the increase of the value of *Ri* heat transfer rate increase. Because with the augmentation of *Ri* buoyancy effect enhance, so large amount of heat is transfer from the heated wall to the enclosure. At *Ri* = 0.1, 1, 5 and *Ri* = 10 increasing rate are. 5.51, 4.54 and 5%.

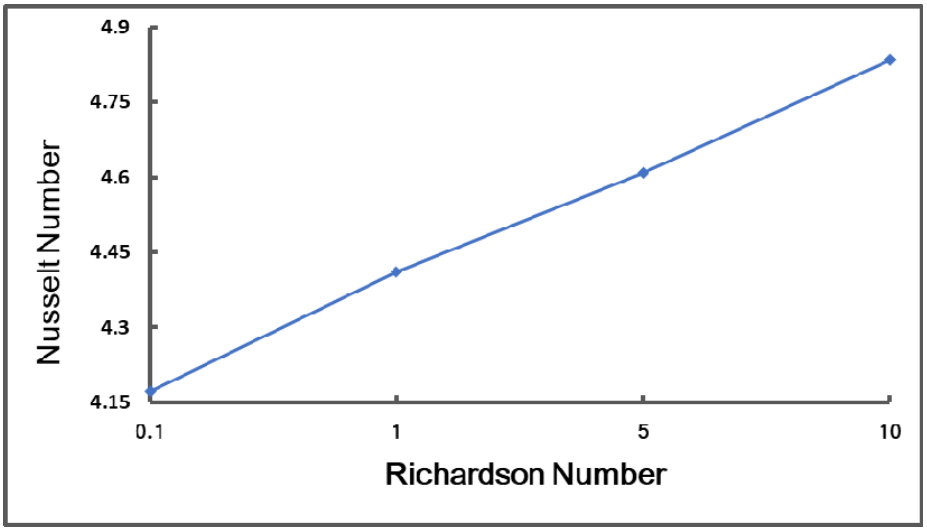

Figure 3.8: Average Nusselt number for various Richardson number

3.6 Effect of Prandtl Number

Figure 3.9-(a-b) displays he effect of Prandtl number (*Pr*) on water-Cu-CNT hybrid nanofluid flow and temperature fields with fixed $\lambda = 3$, $Ri = 1$, $\phi = 1\%$. Prandtl number is taken as 4.2, 5.2, 5.8 and 6.2 at temperature 308, 303, 300 and 298K, respectively. From the streamlines contours it is found that for $Pr = 4.2$, there exist a primary elliptic circulation cell without any secondary core but one small oval shape core has been created. At $Pr = 5.2$ it can be seen that shape of the primary cell has been bended a little bit and it is important to notice that oval shape core has been greater than before. On the other hand, when $Pr = 5.8$ and $Pr = 6.2$, there is no major distinction in the streamlines but there is only one change that gradually increment of oval shape core.

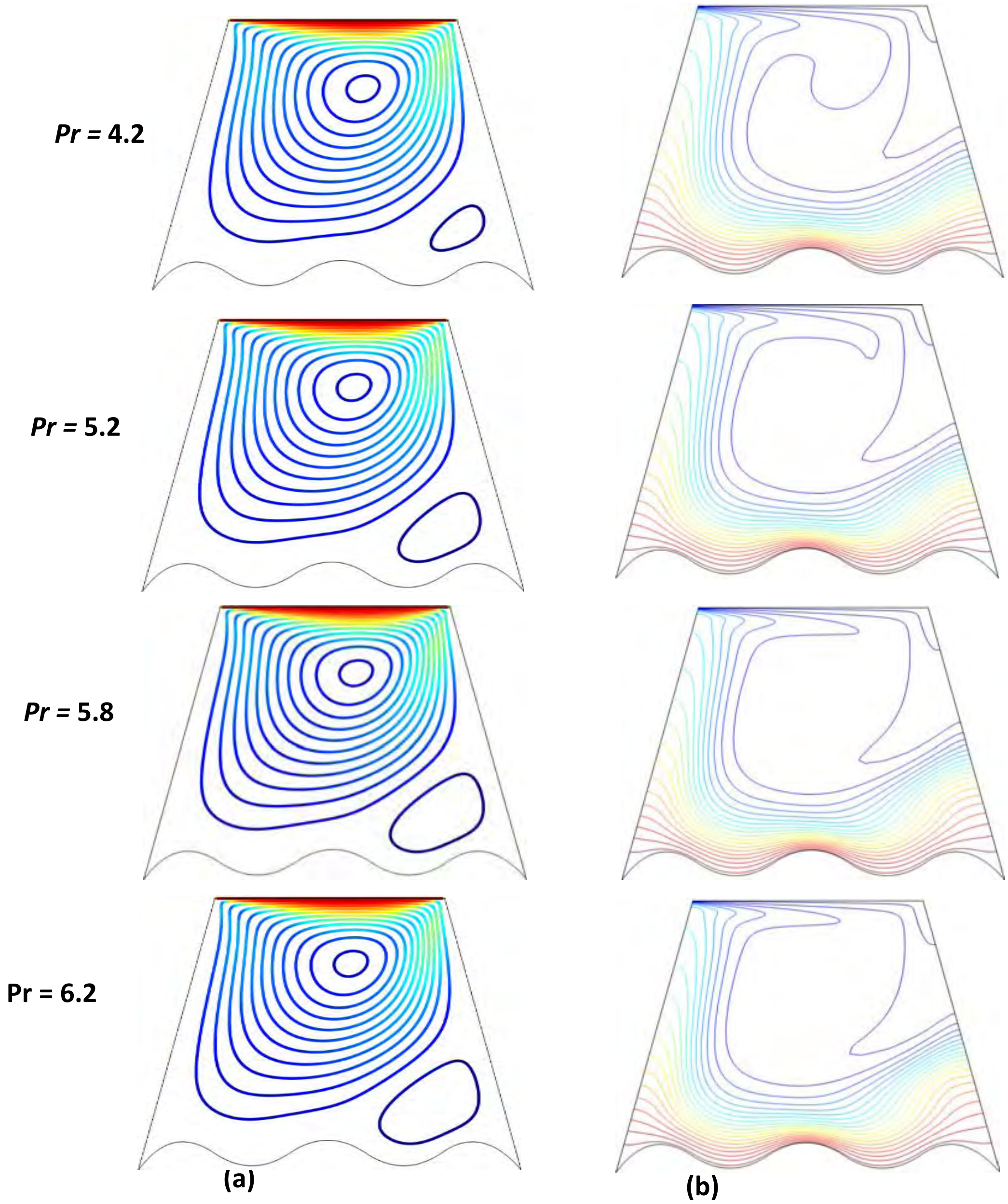

Figure 3.9: Effect of *Pr* on (a) streamlines and (b) isotherms with *Ri* =1, λ = 3, ϕ = 1%

However, it is also obvious from figure 3.9(b), that the temperature gradients are generally not affected on escalating the Prandtl number of nanoparticles with any fraction except with the increase of *Pr* the isotherms close to the heated wall begins to be more scattered. The enrichment of the thermal conductivity produces denser isotherms which is the indication of development nanofluid convection. We can conclude that the mixing advantage of Copper and Carbon nano tube nanoparticles may blow up the convective heat transfer. Hence, this result encourages us to examine other combination of different types nanoparticles, with different ratio of nanoparticles, and to work with innovative models on behalf of dissimilar thermo-physical properties.

Prandtl number λ = 3, *Ri* = 1, ϕ = 1%. From the figure 3.10, it is evident that with the increase of the value of Pr heat transfer rate increases. With the augmentation of *Pr* Nusselt number effect enhances, so large amount of heat is transferred from the heated wall to the enclosure. At *Pr* = 4.2, 5.2, 5.8 and 6.2 the Nusselt number are 4.29, 4.50, 4.60 and 4.68 respectively. Prandtl number varies from 4.2 to 6.2 whereas percentage of Nusselt number is 8.85%.

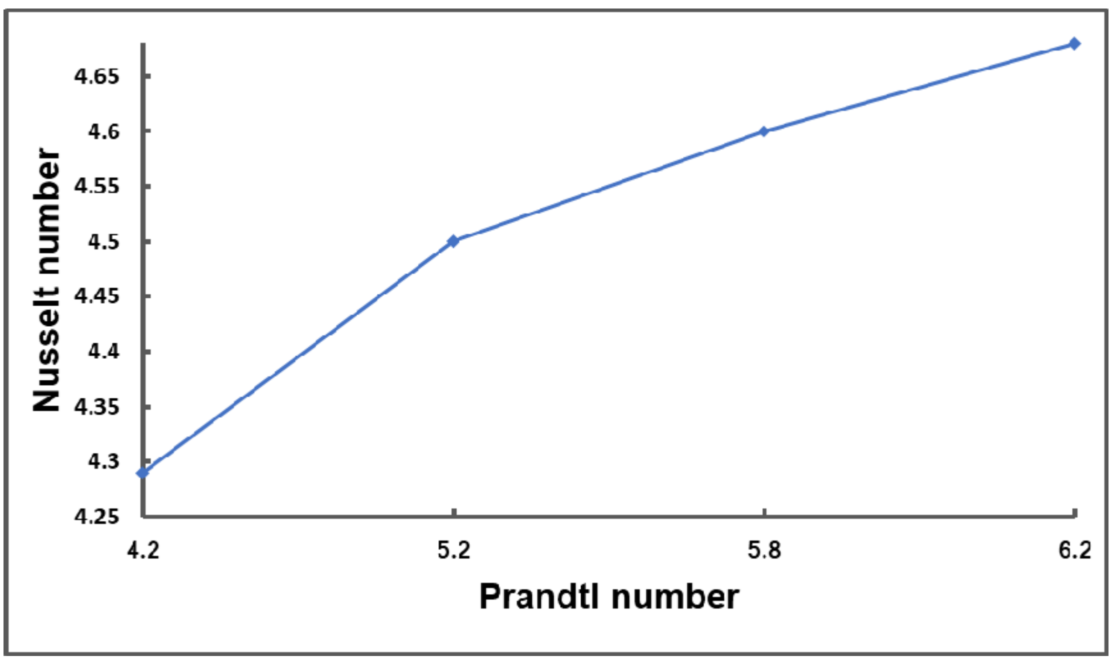

Figure 3.10: Average Nusselt number for various Prandlt number

Chapter 4

Conclusions and Recommendations

A numerical analysis has been conducted to show the performance of heat transfer phenomenon by using four types of hybrid nanofluids like water-Cu-CNT, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-TiO_2 along with single nanofluid water-Cu and base fluid water in a lid driven sinusoidal trapezium shaped cavity. The governing equations have been solved by using Galerkin's finite element method. The cavity is subject to conduction-convection heat transfer analysis. Varying the parameters Richardson number, Prandtl number, Volume fraction and Undulation number have led to the following conclusions:

4.1 Conclusions

- Ascending values of wave number (λ = 3) raises the heat transfer rate by 14.7% compared to flat surface (λ = 0). So, the corrugated lid driven cavity can be considered as an effective heat transfer system.
- The heat transfer rate increases according to increment of solid volume fraction of water-Cu nanofluid. The rate of heat transfer is obtained by 11.23% for increasing values of solid volume fraction from 0 to 3%.
- Nu increases about 8.1, 9.1, 10.2, 11.4 and 13.6% using heat transfer medium as water-Cu, water-Cu-TiO_2, water-Cu-CuO, water-Cu-Al_2O_3 and water-Cu-CNT hybrid nanofluid with compared to base fluid.
- Higher heat transfer rate is found about 4.1, 3.1, and 2% using water-Cu-CNT hybrid nanofluid than water-Cu-TiO_2, water-Cu-CuO and water-Cu-Al_2O_3, respectively.
- The heat transfer rate is increased about 15% from *Ri* = 0.1 (forced convection) to *Ri* = 10 (natural convection).
- For the variation of Prandtl number (from 4.2 to 6.2), rising rate of heat transfer approximately 8.85% is found.

4.2 Recommendations

There is a lot of scope for research in this area in future. Since study of hybrid nanofluids is under initial stages so there is a lot of scope in development of hybrid nanofluids. The size, shape, material and volume fraction of dispersed nanoparticles play a very important role in the absorption of heat. In consideration of the present investigation, the following recommendation for future works have been provided:

- Trapezium shaped cavity has been considered in the present study. So, this deliberation may be extended by considering other formations of enclosures to investigate the performance of hybrid nanofluids.
- Using hybrid nanofluids with single phase flow have been considered as heat transfer medium in this thesis work. It can be investigated for multiphase flow also.
- In this research, four types of hybrid nanofluids namely water-Cu-CuO, water-Cu-$Ti0_2$, water-Cu-CNT and water-Cu-Al_2O_3 have been used as HTF. Anyone can use other combinations of hybrid nanofluids to obtain better heat transfer rate.

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