APPLICATION OF DIFFERENT BOUNDARY CONDITIONS ON FLOW IN THE DRIVEN CAVITY PROBLEM

BY

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ABSTRACT

The integration of vorticity stream function formulation and control volume based element (CVFEM) has been applied to solve a fluid flow problem. The driven cavity problem is chosen for this purpose in which two walls of the cavity are moved with the same velocity instead of the conventional one (top) wall moving in horizontal direction. The resulting algorithm exhibits desirable qualities of consistency and stability in predicting the flow field.

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NOMENCLATURE

u,v	-	Horizontal and vertical velocities m/hr)
U,V	-	Dimensionless horizontal and vertical velocities
x,y	-	Cartesian co-ordinates horizontal and vertical(m)
X,Y	-	Dimensionless Cartesian co-ordinates horizontal and vertical
μ	-	Viscosity of fluid (kg/hr-m)
ν	-	Kinematic viscosity (m^2/hr)
ρ	-	Fluid density (kg/m^3)
ψ	-	Stream function (m^2/hr)
Ψ	-	Dimensionless stream function
ω	-	Vorticity (hr^{-1})
Ω	-	Dimensionless vorticity
τ	-	Shear stress (Kg/m^2)
τ^*	-	Dimensionless shear stress
Re	-	Reynolds number
δ	-	Dimensionless interior grid spaces
P	-	Pressure of fluid (kg/m^2)
L	-	Length of a wall of square box (m)

Subscripts -

w	-	At the wall
l	-	At the left wall
t	-	At the top wall.

CHAPTER I

INTRODUCTION

A. GENERAL

Flow with separation and reattachment is an old topic in the science of fluid mechanics. As an example, Leonardo da Vinci observed and sketched recirculating eddies formed in the flow over various configurations as early as in the fifteenth century[1,2]. Numerous researchers have worked on the said topic and many theoretical models have been proposed, but other than the complete Navier-stokes equations, none has been accepted widely.

The driven cavity problem is one of the standard examples of flow with separation and reattachment. In the original driven cavity problem, the top wall of a box (Figure-1) is moved with a particular velocity in the horizontal direction. But in this research work, two walls of a square box are moved with a particular velocity. This driven cavity problem will be referred to as the modified driven cavity problem in this thesis. Figure-1 is a cross-section of a parallelopiped with the top wall moving to the right at a speed u_w. The cavity is square with characteristic length L. The fluid contained within is assumed to be incompressible with constant properties. Further, the parallelopiped is assumed to be very long in the direction normal to its cross-section. These assumptions render the flow field 2-dimensional. In Figure-2, the modified driven cavity problem is shown. Here, two walls are moved at a time with the same magnitude of velocity. The top wall is always moved to the right at a velocity u_w. In conjunction with the top wall, either the left or the bottom wall is moved. Four separate types of movement of the left wall and the bottom wall are considered which are given below -

(a) Case-1 (LU) - Left wall moving up

(b) Case-2 (LD) - Left wall moving down

(c) Case-3 (BR) - Bottom wall moving right

(d) Case-4 (BL) - Bottom wall moving left

In subsequent description in this thesis, references will be made to case-1 through case-4 instead of illustrating the type of movement.

B. OBJECTIVES

The purpose of this study is to implement a numerical scheme to simulate Navier-Stokes equations for vorticity stream-function formulation for the modified driven cavity problem. The Control Volume based Finite Element Method (CVFEM) using the vorticity-stream function formulation of the Navier-Stokes equations will be used to predict the flow fields of the system. The details of the Control Volume based Finite Element Method (CVFEM) may be found in [8]. The main attention will be the observation of the following as predicted by the above mentioned numerical simulation.

(a) The variation of horizontal and vertical velocity profile at the midplane of the cavity with grid refinement.

(b) The variation of the shear stress along different walls with grid refinement.

(c) The variation of the horizontal velocity profile at the midplane with changes in Reynolds numbers.

(d) The variation of the shear stress along wall with changes in Reynold numbers.

(e) The shift of center of vortex of the main eddy with grid refinement

CHAPTER II

LITERATURE REVIEW

Several investigators have carried out extensive research on the original driven cavity problem. Simuni [3] published a paper in 1964 in Russia in which numerical solutions of the Navier-Stokes equations were presented both for rectangular cavities and for channel flow with forward and backward facing steps. The solutions were obtained by considering the large time limit of the unsteady equations of motion. Simuni [3] presented solutions for Reynolds numbers as high as 1000, showing isolated eddies within the flow.

Greenspan [4] published some numerical solutions for the recirculating flow in a square carried out at the University of Wisconsin in 1964. Solution were obtained for Reynolds number upto 256 with meshes as small as 1/16. Because of the coarse mesh, the vortex center tends erroneously toward the downstream wall.

An analytical study of the recirculating flow in a square based on Batchelor's [5] model of an inviscid core matched to a surrounding shear layer has been carried out by Mills [6]. Mills [6] also carried out experiments to verify his analysis. The measured flow properties were in qualitative agreement with the analysis, but the measured vorticity of the inviscid core was about 1/3 of the predicted value. Mills [6] attributed the deviation to the gap between the fixed and moving wall, to the secondary eddies at the bottom of cavity and to three dimensional effects.

Burgraf [7] performed an analytical and numerical study of the structure of steady separated flow. Analytical solutions, based on a linearized model, was obtained for an eddy bounded by a circular streamline. The validity of the linear analysis as a description of seperated eddies was confirmed by numerical solutions of the full Navier-Stokes equations for an eddy in a square cavity driven by a moving boundary at the top. These solutions were carried out by a relaxation procedure on a high speed digital computer. Results were presented for Reynolds numbers from 0 to 400 in the form of contour plots of stream function, vorticity and total pressure. At higher values of Reynolds number, an inviscid core was found to develop but secondary eddies were present in the bottom corners of the square at all Reynolds numbers.

The Control Volume based Finite Element Method (CVFEM) was first proposed by Baliga [8]. In Baliga's dissertation, the merits and demerits of the conventional Finite Element Method and Finite Difference Method are discussed in detail. Baliga used unequal order pressure-velocity interpolation in his CVFEM. In this unequal order method, pressure is stored at fewer locations than the velocities in the calculation domain which results in a coarse pressure resolution.

Later on Prakash and Patankar [9] improved the unequal order CVFEM which allows the storage of pressure at the same nodes as the velocities. This equal order CVFEM was found to produce more accurate pressures than those predicted by the unequal order method of Baliga [8]. Afterwards, Prakash [10] presented a new equal order method, the main idea of which is a modified exponential-linear shape function that directly takes into account the effects of sources such as pressure gradients in the momentum equations. The formulation performs even better in terms of accuracy than the previous ones at the cost of increased computer effort for each coefficient update cycle.

Recently, Husain [11] solved fluid flow problems using the Vorticity - Stream Function Formulation and the Control Volume based Finite Element Method (CVFEM). In [11], the CVFEM was successfully applied on the driven cavity problem. It was found to predict the overall flow field with greater accuracy than the procedure of Burgraf [7]. The main advantage of the vorticity stream function formulation is:- (a) The number of equations to be solved is reduced effectively by one. Instead of two momentum equations, a single transport equation for vorticity is encountered.(b) The pressure gradient terms and the continuity equation are conveniently absent. This eliminates the question of how the pressure is to be interpolated in order to represent the momentum equations in the continuity constraint.

The simulation of the modified driven cavity problem with the CVFEM and stream function formulation presnts an interesting prospects.Because it will reveal the suitability of the CVFEM and stream function formulation to predict the various types flow field.

CHAPTER III

METHOD OF SOLUTION

A. GOVERNING EQUATIONS

The Navier-Stokes equation in cartesian co-ordinates for two dimensional, incompressible flow may be written as follows:

x - momentum $$u\partial u/\partial x + v\partial u/\partial y = -1/\rho\partial p/\partial x + \nu(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2) + s^u \quad \ldots\ldots\ldots(1)$$

y - momentum $$u\partial v/\partial x + v\partial v/\partial y = -1/\rho\partial p/\partial y + \nu(\partial^2 v/\partial x^2 + \partial^2 v/\partial y^2) + s^v \quad \ldots\ldots\ldots(2)$$

Continuity $$\partial u/\partial x + \partial v/\partial y = 0 \quad \ldots\ldots\ldots(3)$$

In the above equation u and v represent velocity components in the x and y directions, respectively, and p is the pressure. The terms s^u and s^v are body forces in the x and y directions, respectively and ν is the Kinematic viscosity. Upon differentiating (1) with respect to y, (2) with respect to x and subtracting the former result from the later, the transport equation for vorticity is obtained:

$$u\partial\omega/\partial x + v\partial\omega/\partial y = \nu(\partial^2\omega/\partial x^2 + \partial^2\omega/\partial y^2) + (\partial s^v/\partial x - \partial s^u/\partial y) \quad \ldots\ldots\ldots\ldots(4)$$

where ω is the vorticity defined as

$$\omega \equiv \partial v/\partial x - \partial u/\partial y \quad \ldots\ldots\ldots(5)$$

Defining the the stream function ψ

$$u \equiv \partial\psi/\partial y \quad \ldots\ldots\ldots(6)$$

$$v \equiv -\partial\psi/\partial x \quad \ldots\ldots\ldots(7)$$

the relationship between ψ and ω is then expressed as follows

$$\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 + \omega = 0 \qquad \text{.........(8)}$$

Considering Figure 2 (modified driven cavity) and making following assumptions,

$$X \equiv x/L \qquad \text{.....} \qquad (9)$$

$$Y \equiv y/L \qquad \text{.....} \qquad (10)$$

$$\Psi \equiv \psi/u_w L \qquad \text{.....} \qquad (11)$$

$$\Omega \equiv \omega L/u_w \qquad \text{.....} \qquad (12)$$

$$U \equiv u/u_w \qquad \text{.....} \qquad (13)$$

$$V \equiv v/u_w \qquad \text{.....} \qquad (14)$$

equations (4) and through (8) may be written in dimensionless form as

$$U\partial\Omega/\partial X + V\partial\Omega/\partial Y = 1/Re(\partial^2\Omega/\partial X^2 + \partial^2\Omega/\partial Y^2) \qquad \text{.......} \qquad (15)$$

$$\Omega \equiv \partial V/\partial X - \partial U/\partial Y \qquad \text{.......} \qquad (16)$$

$$U \equiv \partial\Psi/\partial Y \qquad \text{........} \qquad (17)$$

$$V \equiv -\partial\Psi/\partial X \qquad \text{........} \qquad (18)$$

$$\partial^2\Psi/\partial X^2 + \partial^2\Psi/\partial Y^2 + \Omega = 0 \qquad \text{........} \qquad (19)$$

The main dimensionless group resulting is the Reynolds number, $Re = u_w L/\nu$.

B. <u>BOUNDARY CONDITIONS</u>

The boundary conditions used for different cases are summarized below:

$\Psi = 0$, at all walls and $U=1$, at the top wall in all cases (CASE-1 through CASE-4)

<u>CASE-1(LU):</u>

$U = V = 0$, at the bottom and right walls

V = 1, at the left wall.

CASE-2 (LD):

U = V = 0, at the bottom and right walls

V = -1, at the left wall

CASE-3(BR):

V = 0, at the left and right walls

U = 1, at the bottom wall

CASE-4(BL):

V = 0, at the left and right walls

U = -1, at the bottom wall

Whenever any wall is moved, the two extreme corners of the wall are kept fixed i.e. the velocities at these two points are set to zero.

C. METHOD OF SOLUTION

To obtain the nodal equations for the general transport equation, the calculation domain is subdivided into trangular elements. On each element, mid-points of its sides are extended to its centroid to form links. Control Volumes (Fig. 3b) are formed by adjoining these links to the others from the neighbour elements. The integral formulation of the general transport equation is then applied to each control volume in the calculation domain. An exponential-linear shape function is assigned within each element to the transported variable and based on this assigned shape function, the combined convective diffusive flux vector is integrated along the links of the element. Then the contributions of the links to the control volume portions that constitute it are assembled systematically. This procedure is repeated for all the elements in the calculation domain. This results a set of linear nodal equations for the general trasnport variable ϕ , which may be written as:

$$a_p^{\phi}\phi_p + \sum_{nb} a_{nb}^{\phi}\phi_{nb} = b_p^{\phi}$$

where nb indicates the neighbour nodes of the node p,a are coefficients of the nodal matrix equation (NEM) and b is the global load vector (GLV) component corresponding to node p.

The overall iterative procedure used in the calculation is as follows:-

(1) Compile the coefficients for the poisson equation for the stream function. This is done only once at the beginning of the calculation procedure.

(2) Guess the distribution of the stream function Ψ and compile the coefficients of the vorticity transport equation.

(3) Solve for the vorticity Ω and update wall values as proposed in [11].

(4) Using the values of Ω obtained in (3), compile the global load vector (GLV) for the poisson equation for Ψ .

(5) Solve for Ψ .

(6) Check for convergence. If not converged, go to step (2). If converged, print out results.

After obtaining the converged solution the shear stress are calculated according to the following formula:-

Shear stress along the top wall-

Along the top wall $\partial v/\partial x$ is equal to zero. So vorticity at the top wall becomes $\omega_{wt} \equiv -\partial u/\partial y$ . ... (20).

Now shear stress along the top wall is $\tau_{wt} \equiv \mu \frac{\partial u}{\partial y}$ ......(21).

$\equiv -\mu\omega_{wt}$ ....(22).

Non-dimensionalize shear stress τ_t^* is now calculated by dividing equation (22) by $1/2\,\rho u_w^2$.

i.e. $\tau_t^* \equiv -\mu\omega_{wt}/\frac{1}{2}\rho u_w^2$ .... (23)

The equation (22) and (23) can also be applied for the bottom wall.

Shear stress along the right wall -

The term $\partial u/\partial y$ is zero along the right wall. So the vorticity equation becomes $\omega_{wl} \equiv \partial v/\partial x$ ....(24)

So shear stress along the right wall is $\tau_w \equiv \mu\partial v/\partial x$ ... (25).

$\equiv \mu\omega_w$ ...... (26).

Again non-dimensionalize shear stress τ^* is given by

$$\tau^* \equiv \mu\omega/\tfrac{1}{2}\rho v_w^2 \qquad \text{..........(27).}$$

The equation (26) and (27) can also be applied for the left wall.

D. DOMAIN DISCRETIZATION

The discretized domain is shown in Figure 3 and is based on the dimensionless variables defined, by equations (9) and (10). An equal number of grid lines (LMAX) in the vertical and horizontal directions were used. The first grid space near the walls is kept half the distance of the interior grid space in order to resolve any boundary layer behaviour (sharp velocity gradients) that may arise. Therefore, the interior grid spacing, *k* , may be expresed in terms of number of grid lines as,

δ = 1/(LMAX - 2

CHAPTER IV

RESULTS AND DISCUSSIONS

Introduction

A detailed parametric study is conducted to assess the effects on the predicted flow field. In particular, the horizontal and vertical velocity profiles at midplanes of the cavity, the shear stress along the walls and movement of the primary vortex centers are examined with respect to the variation in Reynolds numbers and grid refinement. Results are obtained for Reynolds numbers of 10,100 and 400 and for each Reynolds number the grid sizes of 15x15, 21x21 and 27x27 are used. A summary of the runs is shown in TABLE-1. The solution converges quickly for every run and no spurious results are encountered. All results are presented in dimensionless form.

TABLE-1

CASE NO.	REYNOLDS NUMBER	GRID SIZES
CASE-1 (LU) Left wall up	10	15X15, 21X21, 27X27
	100	DO
	400	DO
CASE-2 (LD) Left wall down	10	DO
	100	DO
	400	DO
CASE-3 (BR) Bottom wall to the right	10	DO
	100	DO
	400	DO
CASE-4 (BL) Bottom wall to the left	10	DO
	100	DO
	400	DO

The computations were carried out on an IBM 4331 machine at BUET. A typical CPU time needed for case-1 is given in Table 2 and 3.

TABLE-2

Re-10 and Case-1 (LU)

Grid size	Converging Criteria	Time in Seconds
15 x 15	1E-4	223.4
21 x 21	1E-4	833.35
27 x 27	1E-4	-

TABLE 3

Grid size - 15 x 15 and Case-1 (LU)

Re. No.	Converging Criteria	Time in Seconds
10	1E-2	169.3
100	1E-2	202.9
400	1E-2	480.1

The converging criteria is calculated according to the formula,

$$\sum_{i=1}^{n} \left[\left(a_p^* \Omega_p + \sum a_n^* \Omega_n - R_\Omega \right)_i^2 \right] < \varepsilon$$

where a* are the new coefficients formed according to the step (2) of the 'Method of Solution' and R_Ω is the right hand side of the vorticity transport equation, Ω is the current value of the vorticity, n is the number of node. The value of ε is chosen arbitrarily and when the above equation satisfies the program than stops its iteration.

Effects of Grid Refinement on the Velocity Profile:

In observing the effects of grid refinements on flow fields, only the more demanding case for Reynolds number of 400 is discussed. Plots of the horizontal velocity profile (U) at the vertical center line of the cavity are shown for Reynolds numbers of 400 in Figure 4 for case-1 (LU). In this Figure, the result obtained for grid sizes of 15x15, 21x21 and 27x27 are presented in order to demonstrate the effects of grid refinement on velocity profile. It is clear upon examination of Figure 4 that even the prediction of horizontal velocity profile with coarse grid coincides with that of finer grids.

Plots of the horizontal velocity profile at the vertical center line of the cavity with grid refinement are shown for Reynolds numbers of 400 in Figures 5,6 and 7 for case-2 (LD), case-3 (BR) and case-4(BL) respectively. The vertical velocity profile at the horizontal midplane of the cavity with grid refinement for the same Reynolds number for case-2(LD) is shown in Figure 8. In all cases mentioned above the predictions of the velocity profiles with coarse grids coincide with those of finer grids. With regard to the effects of grid refinement, the velocity profiles generated by the present method attain insensitivity quickly for all cases as can be observed in Figures 4 through 8, demonstrating a healthy consistency.

Effects of Reynolds Number on the Velocity Profile:

Fig. 9 shows the distribution of the horizontal component of the velocity at the vertical centerline with variation in Reynolds numbers for case-2(LD). With the increasing Reynolds number, sharper changes in the profile near the top wall is evident which coroborates the idea that a boundary layer forms with higher Reynolds number. Two points of inflexion in the profile are observed. The lower inflexion point is at 0.18 from the bottom wall. This point is found not to shift with changes in Reynolds number. The upper inflexion point is found to occur above the horizontal midplane of the cavity. In contrast to the lower inflexion point, this upper inflexion point is found to vary with changes in Reynolds number. Similar plots are shown in Figure-10 for case-4 (BL). Here also the boundary layer effect is apparent near the moving walls for higher Reynolds number. In this case, two points of inflexion are found in the profiles and they are near the top and bottom walls. With the decreasing Reynolds number, the points of inflexion are found to shift interior of the wall. Between the points of inflexion, profile with almost constant slope ($\partial u/\partial y$) is found and the slope, $\partial u/\partial y$ is found to increase with higher Reynolds number. Nevertheless, an alomst constant vorticity is found near the centre of the cavity which dictates that an inviscid core develops around the centre. It is due to the reason that the slope $\partial v/\partial x$ is also found to increase correspondingly with higher Reynolds number. Relative strength of the vorticity at the centre of the cavity (ratio of an interior point vorticity to the corresponding point vorticity on the moving wall) is found to decrease with higher Reynolds number indicating that a larger inviscid core develops with lower Reynolds number which again reinforce that the boundary layer effect is more pronounced with higher Reynolds number.

Effects of Grid Refinement on the Shear Stress:

In all the Figures of the shear stress the corner points of the walls are excluded from the plots, because these are points of singularities. Plots of the shear stress along the top wall and right wall for Reynolds number of 400 with grid refinement are shown in Figure 11 and 12 for case-1 (LU). In Figure 12, it is interesting to observe that the shear stress is opposite in sign on the upper portion of the right wall as compared to the lower portion, indicating the existence of a small secondary vortex. Similar plots are shown in Figures 13 and 14 for case-2 (LD), in Figures 15 and 16 for case-3 (BR) and in Figures 17 and 18 for case-4(BL). From observing Figures 14 the existence of a secondary vortex near the upper portion of right wall is evident [case-2(LD)]. The shear stress changes its sign along the right wall for case-3 (BR) in Figure 16 indicating the existence of two primary vortices. Another important factor observed in the Figures 13, 15 and 17 is that the shear stress at the rear side is less than the front side of the top wall for case-2, case-3, case-4 respectively. It is clear upon observing the plots that the shear stress is more sensitive to grid refinement than those of the velocity profiles. But the sensitivity of the shear stress decreases with finer grids as compared to coarse grids in all the cases as shown in Figures 11 through 18.

Effects of Reynolds Number on the Shear Stress:

The variation of non-dimensional shear stress along the top wall with Reynolds number is shown in Figure 19 for case-1 (LU) using a grid size of 27 x 27. The same along the right wall is shown in Figure 20 for case-4 (BL). In both the cases, the non-dimensional shear stress is found to decrease with increasing Reynolds number. In Figure 19, the minimum non-dimensional shear stress is found to occur nearer to the beginning point of the top wall and in Figure 20, it shifts downward along the right wallwith decreasing Reynolds number. In Figure 20, a skewness near the minimum shear stress is observable for highest Reynolds number of 400 for case-4 (BL). With lower Reynolds number, the skewness is found to decrease and at lowest Reynolds number the shear stress becomes almost symmetric to the wall. It is due to the reason that with lower Reynolds number, the convective term of the governing equation becomes negligible and with symmetric boundary conditions applied on the simplified governing equation, the results become symmetric. A peculiarity is observed near the bottom in Figure 20. It is not clear whether any physical phenomena is causing this behaviour or any pitfall in the algorithm is responsible for it.

Effects of Grid Refinement on Location of the Primary Vortex Centre:

The movement of the centres of the primary vortices with grid refinements are next discussed. In TABLE-4 below, the location of centers with grid refinement are shown for various cases.

In Figure-21, the location of center of primary vortices for different cases are shown.

TABLE-4

CASE	Numbers of primary vortices	Location of centers grid sizes 15x15	21x21	27x27
Case-1(LU)	1	(.5,.5767)	(.5,.5773)	(.5,.58)
Case-2(LD)	2	(.246,.309) (.691,.730)	(.25,.2895) (.698,.763)	(.26, .3) (.7, .74)
Case-3(BR)	2	(.576,.730) (.653,.230)	(.578,.720) (.616,.220)	(.58,.73) (.60,.22)
Case-4 (BL)	1	(.5,.5)	(.5,.5)	(.5,.5)

Case-1(LU):

One primary vortex is found in this case. The center is found to shift slightly upward with finer grids. The prediction is not well behaved because with finer grids, the movement of the centre is greater than that coarse grids.

Case-2(LD):

Two primary vortices are found in this case. Both the centers are found to shift a little with grid refinement. However, the shift is less with finer grids than with coarse grids.

Case-3(BR):

Two primary vortices are also found in this case as expected. Here also, the centers are found to move a little with grid refinement. Alike the earlier one, the shift in center is less with finer grids than with coarse grids.

Case-4(BL):

One primary vortex is found in this case. The center is found not to move its position with grid refinement.

The prediction of locations of the primary vortex centres is not very satisfactory. The main reasons of the above shortcomings may be the followings:

a) The computer program was written using single precision variable. This single precision might cause the above shortcoming. The program with double precision could yield more accurate solutions.

b) The converging criteria may be lowered to produce a better solution but of course, the time needed for getting a converged solution would be greater accordingly than that of with the higher converging criteria.

CHAPTER V

SUMMARY AND CONCLUSIONS

In this work, integration of vorticity stream-function formulation and (CVFEM) has been applied to solve fluid flow problem with separation and reattachment. The driven cavity problem with two walls moving tangentially with the same magnitude of velocity is chosen as a test problem instead of conventional one (top) wall moving in horizontal direction.

The combination of the vorticity-streamfunction equations with CVFEM leads to an algorithm that is free of complications of the pressure velocity coupling that plagues the primitive variable form of the Navier-Stokes equations. Consequently fewer equations have to be solved at each iteration step.The resulting algorithm shows favourable stability and consistency with grid refinement and with various Reynolds number in predicting the velocity profiles of the test problem. The algorithm is not as promising in predicting the shear stress with grid refinement as in the case of the velocity profile. It shows sensitivity in predicting the shear stress with grid refinement. But sensitivity decreases with finer grids i.e. the algorithm is more stable with finer grids in predicting the shear stress. The algorithm is found to predict the location of the primary vortex centres which vary slightly with grid refinement. But with double precision variable in the computer program and with lower converging criteria, the above short-coming is likely to improve. From observing such results, it can be concluded in overall sense that the combination of the vorticity stream function formulation and

CVFEM can be applied with some reservations in simulating various fluid flow problem of engineering interest such as rotating full or half annulas enclosure, rectangular cavities, channel flow with forward and backward facing steps etc.

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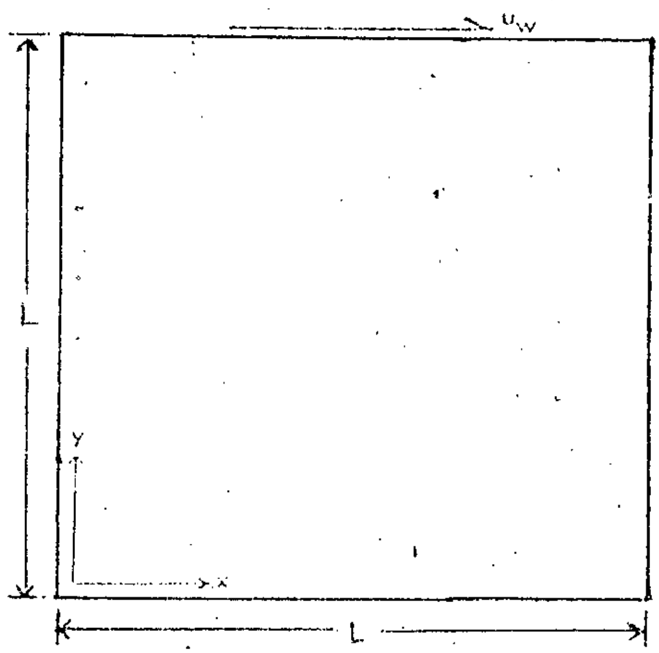

Fig. 1   Driven cavity problem schematic
(original)

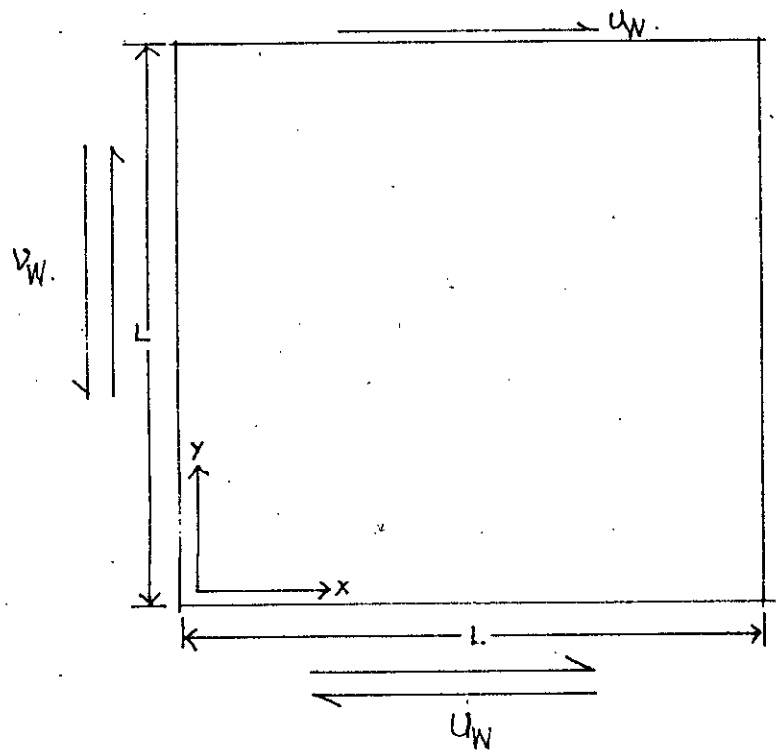

Fig.2 Modified driven cavity problem schematic.

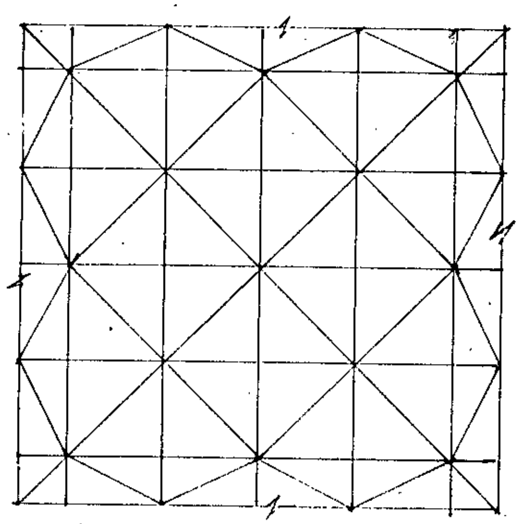

Fig. 3a Domain discretization for modified driven cavity problem.

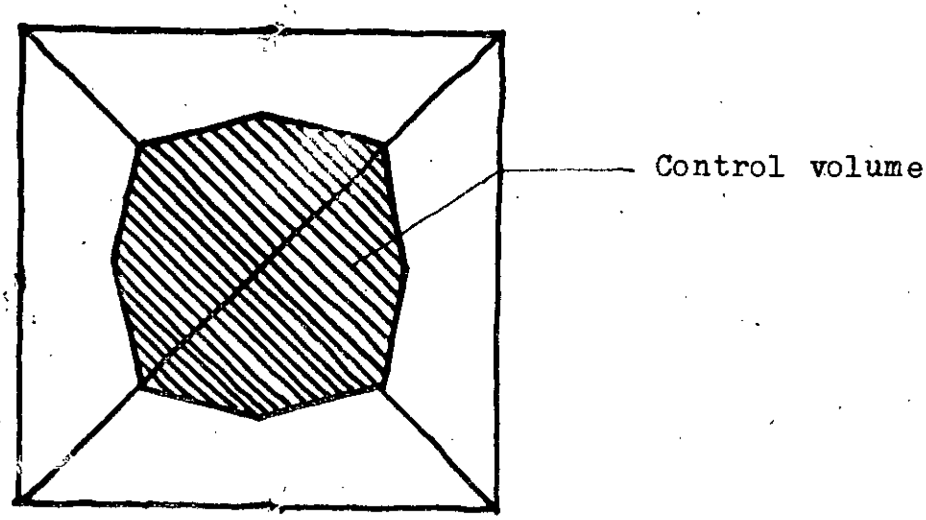

Fig. 3b A typical control volume.

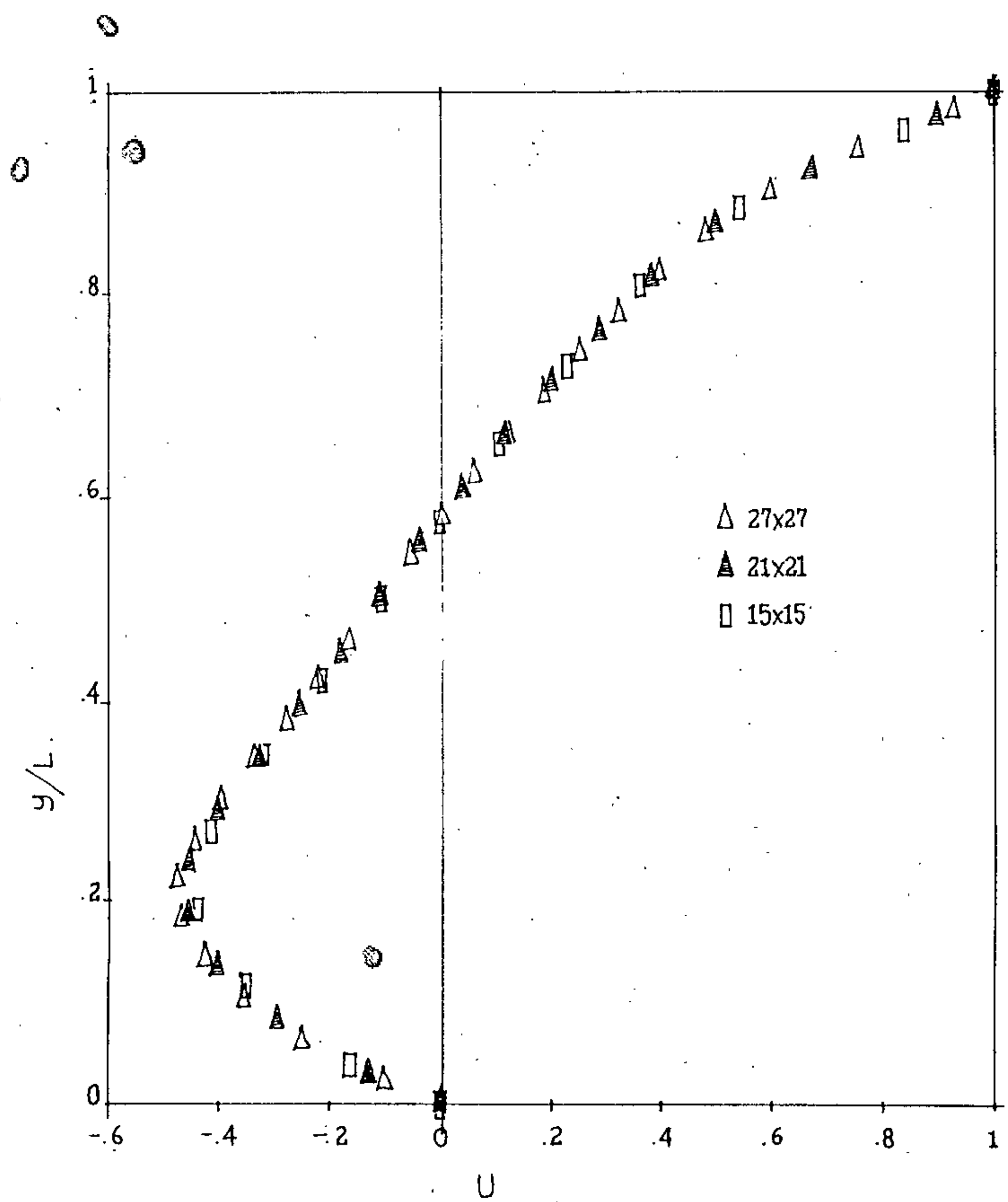

Fig.4 Horizontal velocity distribution at the vertical centerline: Re - 400, case - 1 (LU).

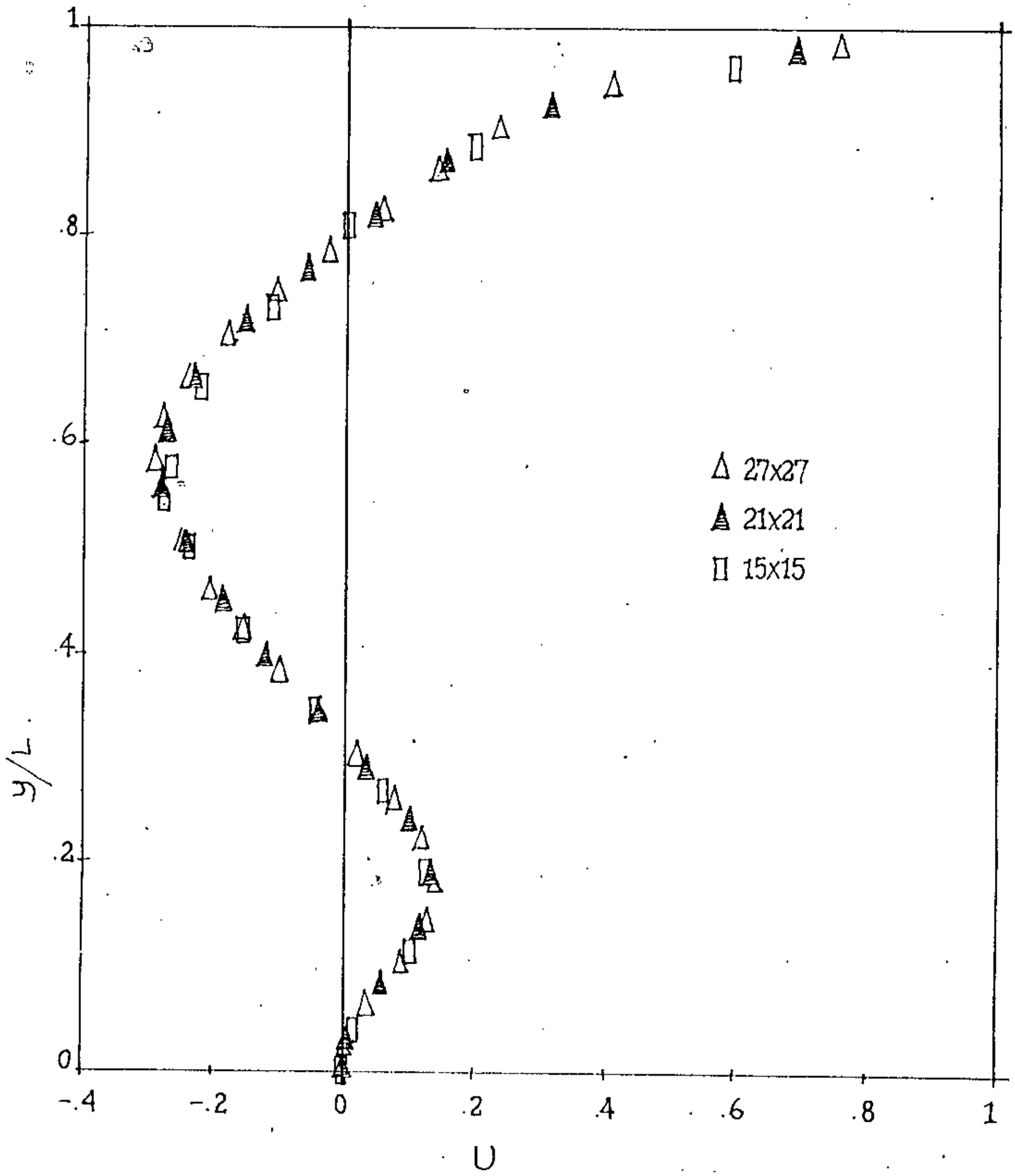

Fig.5 Horizontal velocity distribution at the vertical centerline: Re - 400, case -2 (LD).

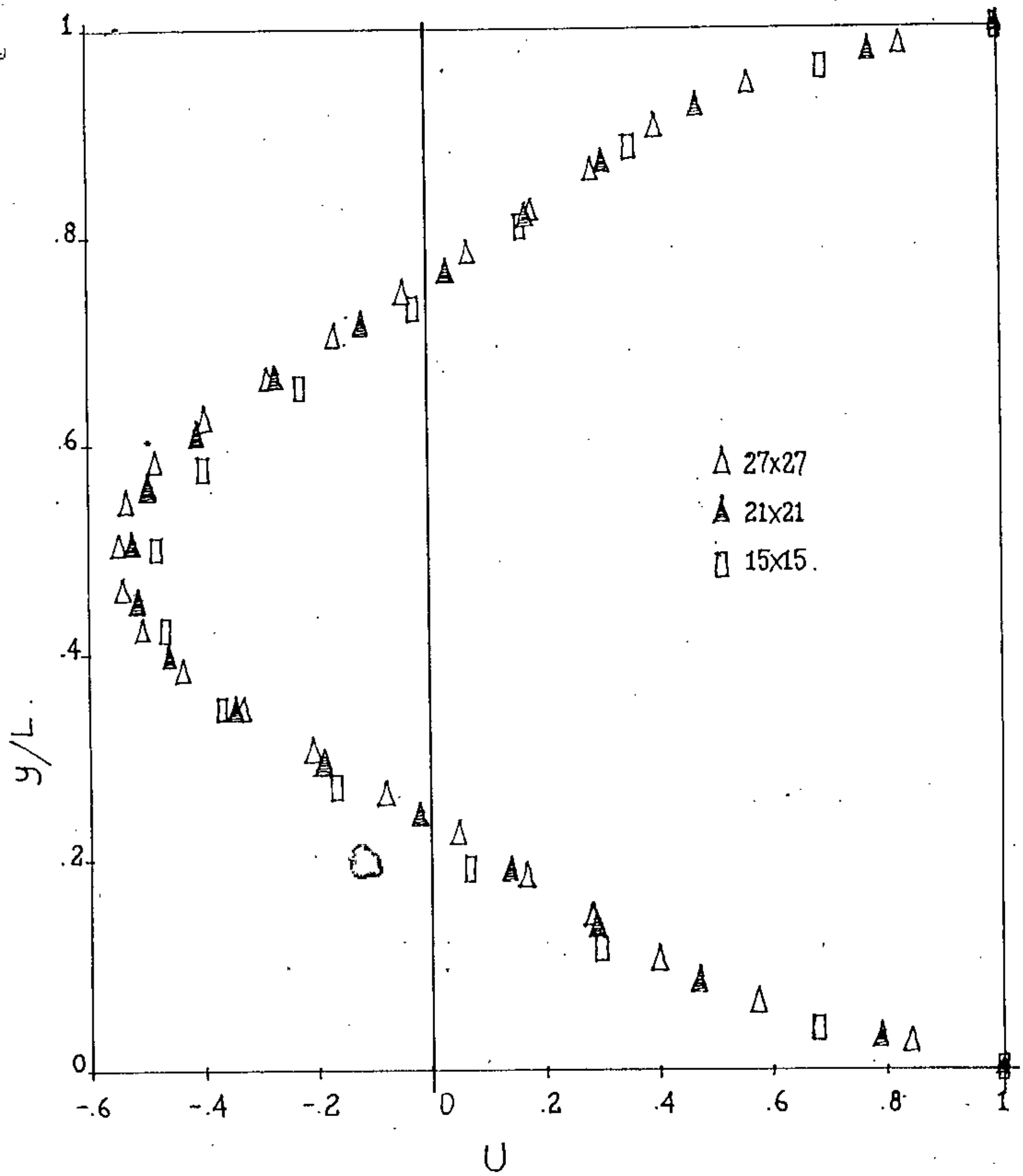

Fig.6 Horizontal velicity distribution at the vertical centerline: Re-400, case -3 (BR).

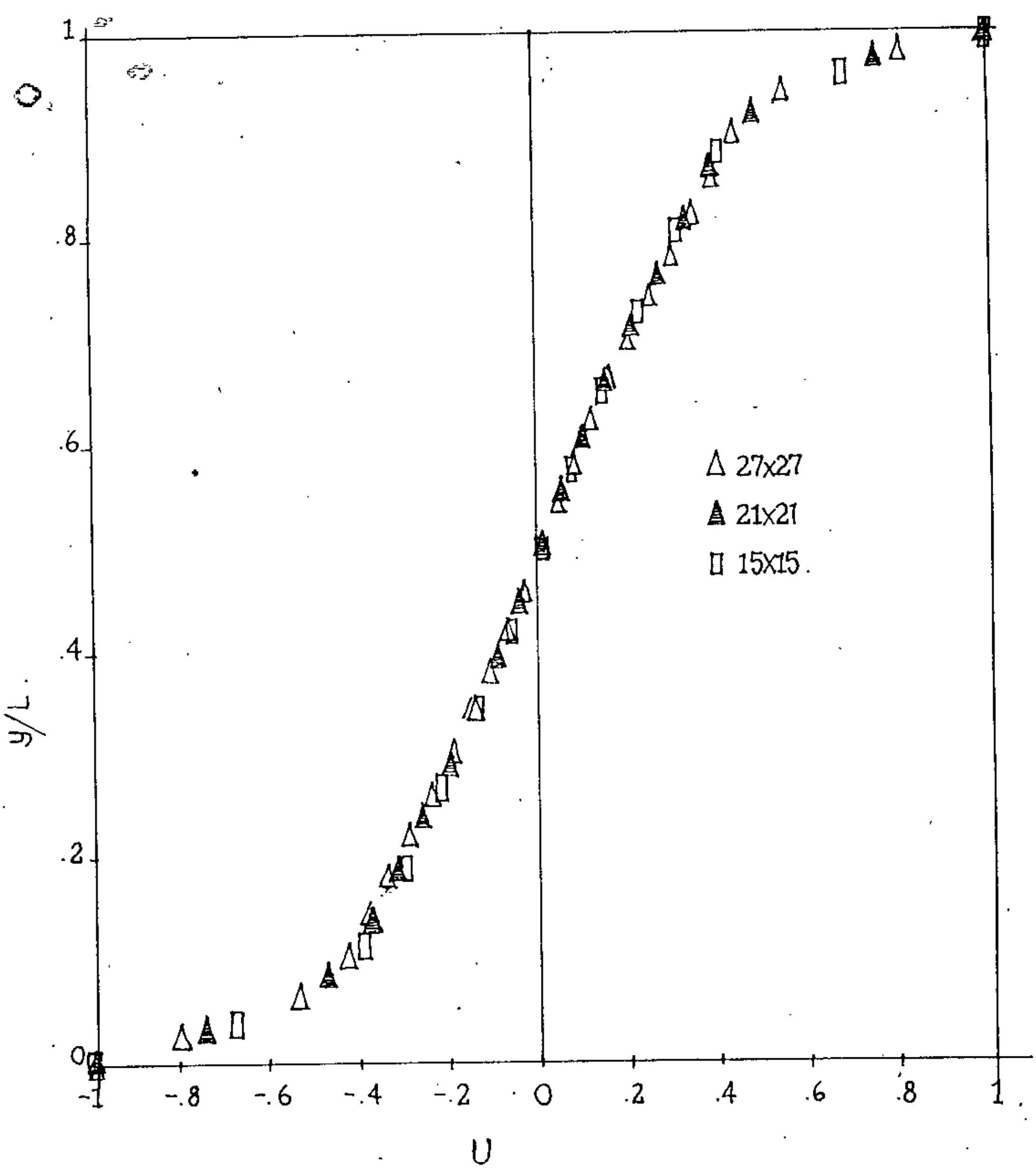

Fig.7 Horizontal velocity distribution at the vertical centerline: Re-400, case-4 (BL).

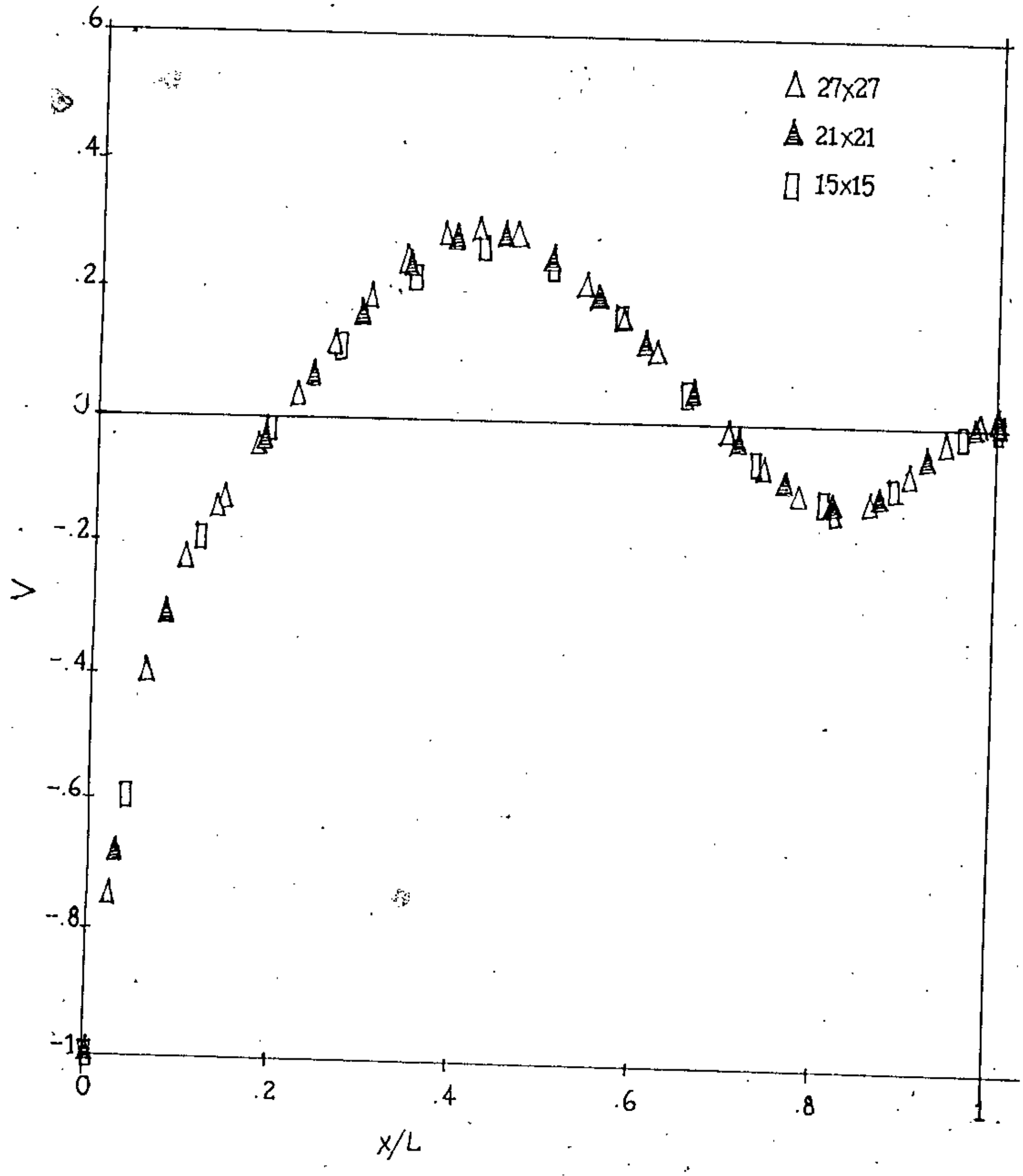

Fig.8 Vertical velocity distribution at the horizontal centerline: Re-400, case -2 (LD).

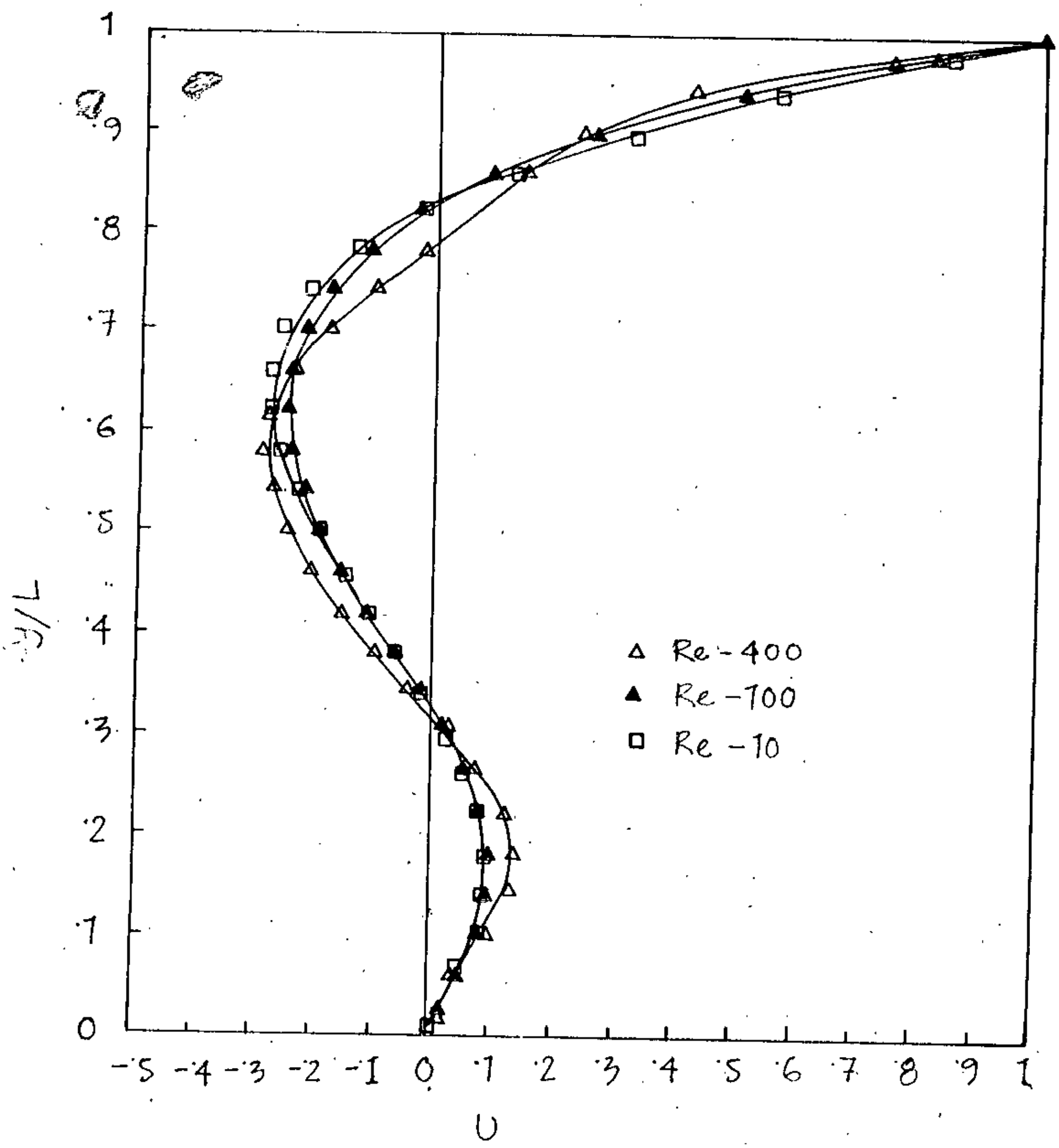

Fig.9 Horizontal velocity distribution with Reynolds number at the vertical centerline: Grid size - 27 x 27, case -2 (LD).

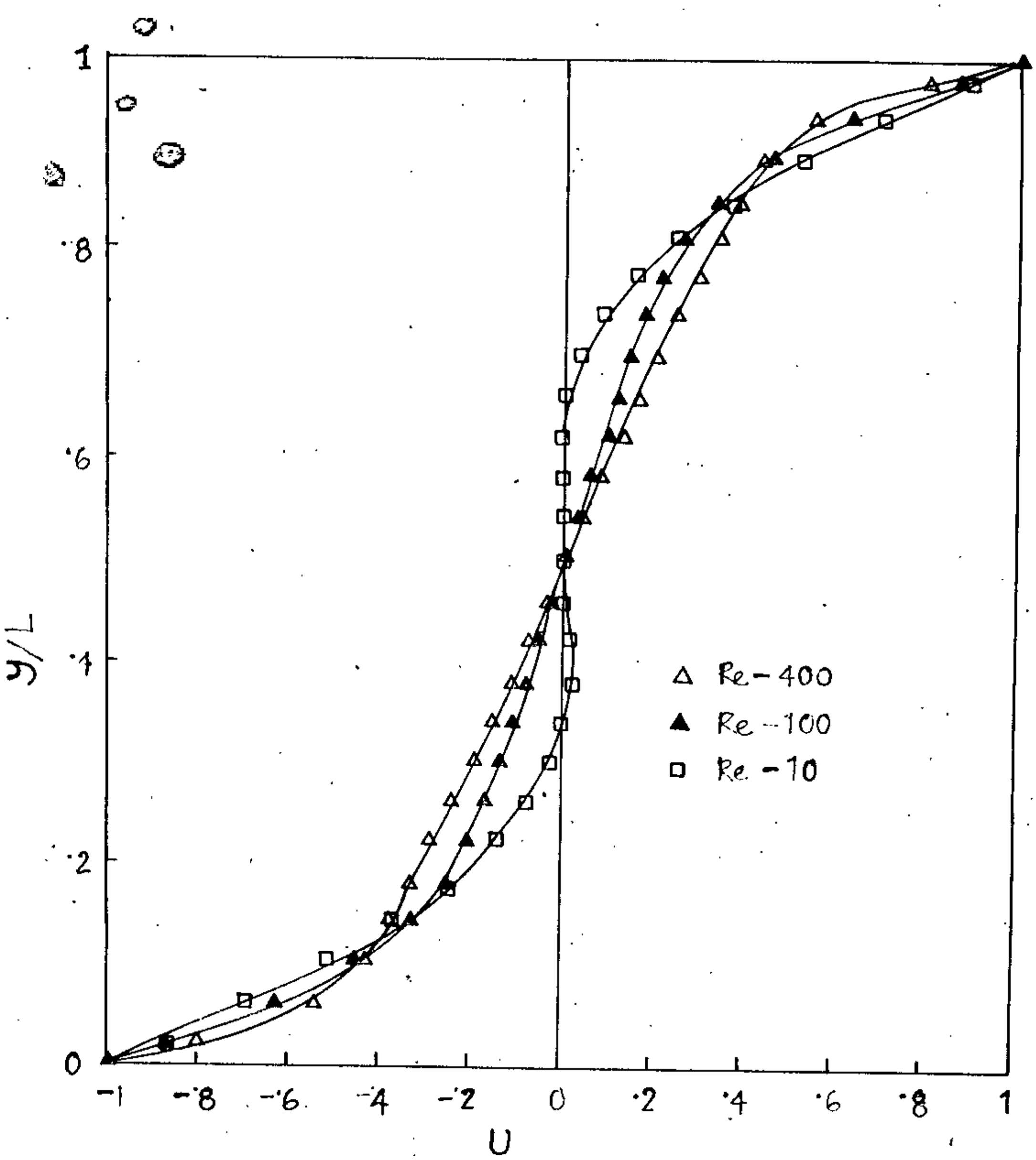

Fig.10 Horizontal velocity distribution with Reynolds number at the vertical centerline: Grid size- 27 x 27, case-4 (BL).

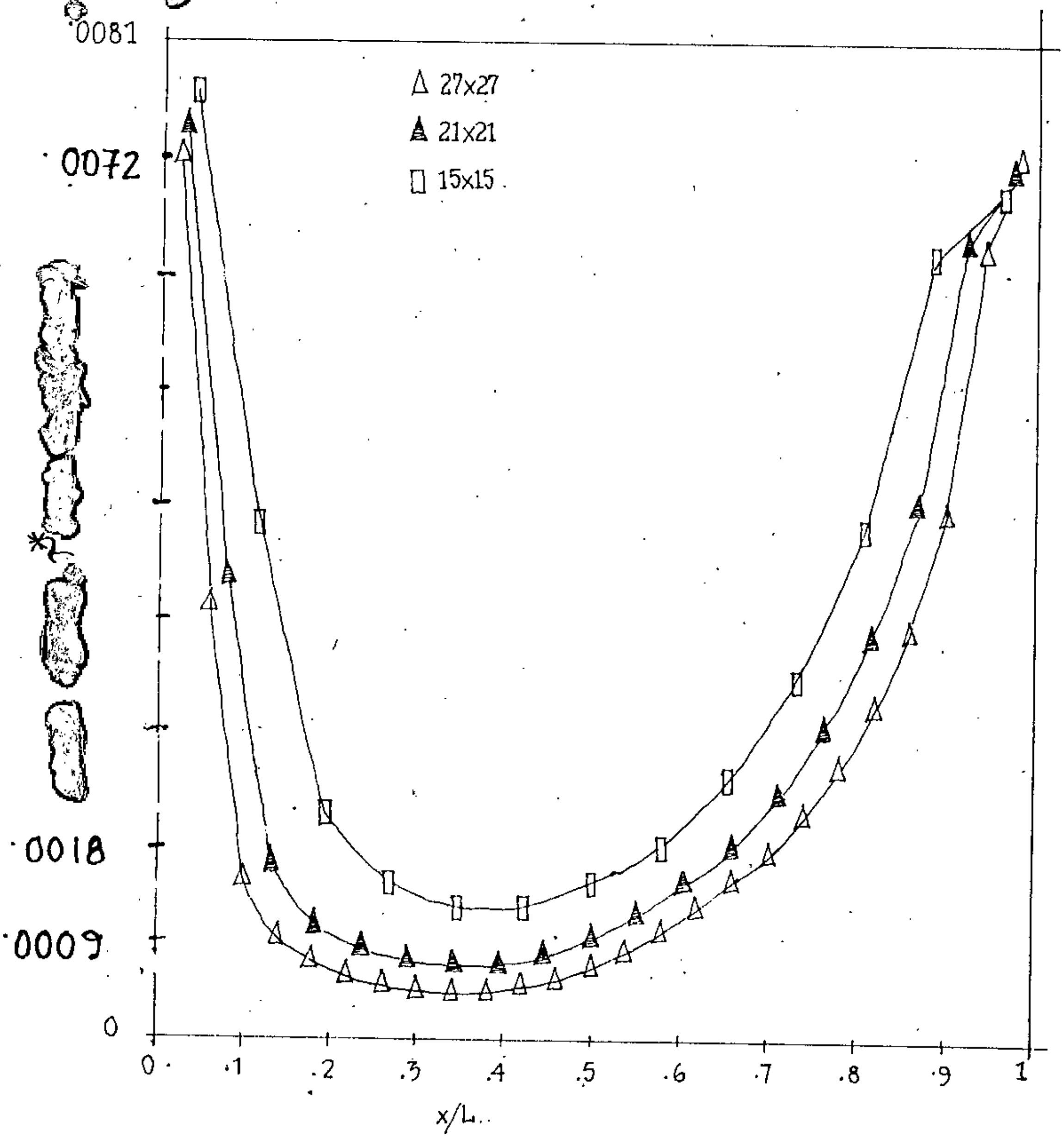

Fig.11 Distribution of the shear stress along the top wall: Re-400, case-1 (LU).

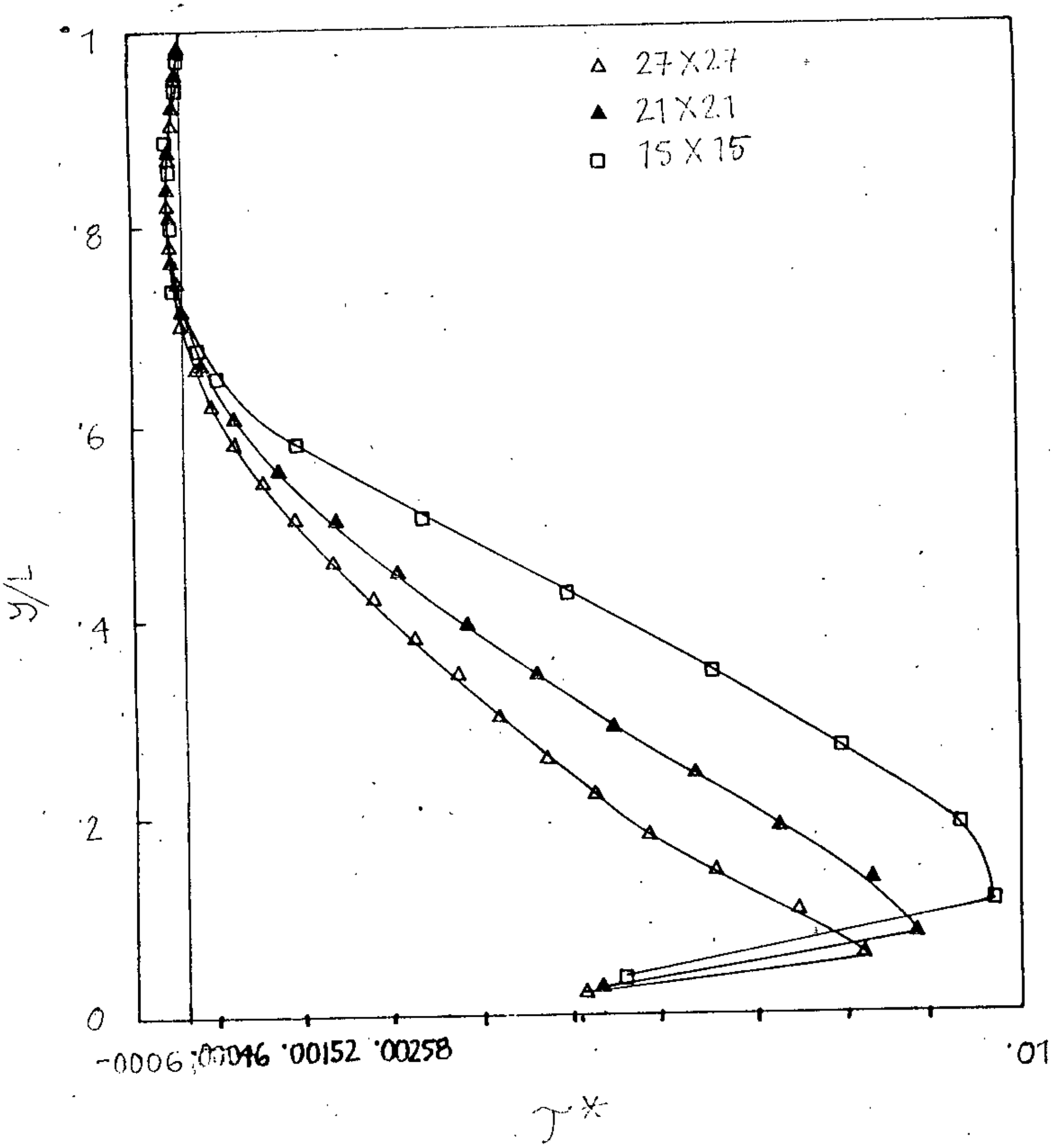

Fig.12 Distribution of the shear stress along the right wall : Re-400, case-1 (LU).

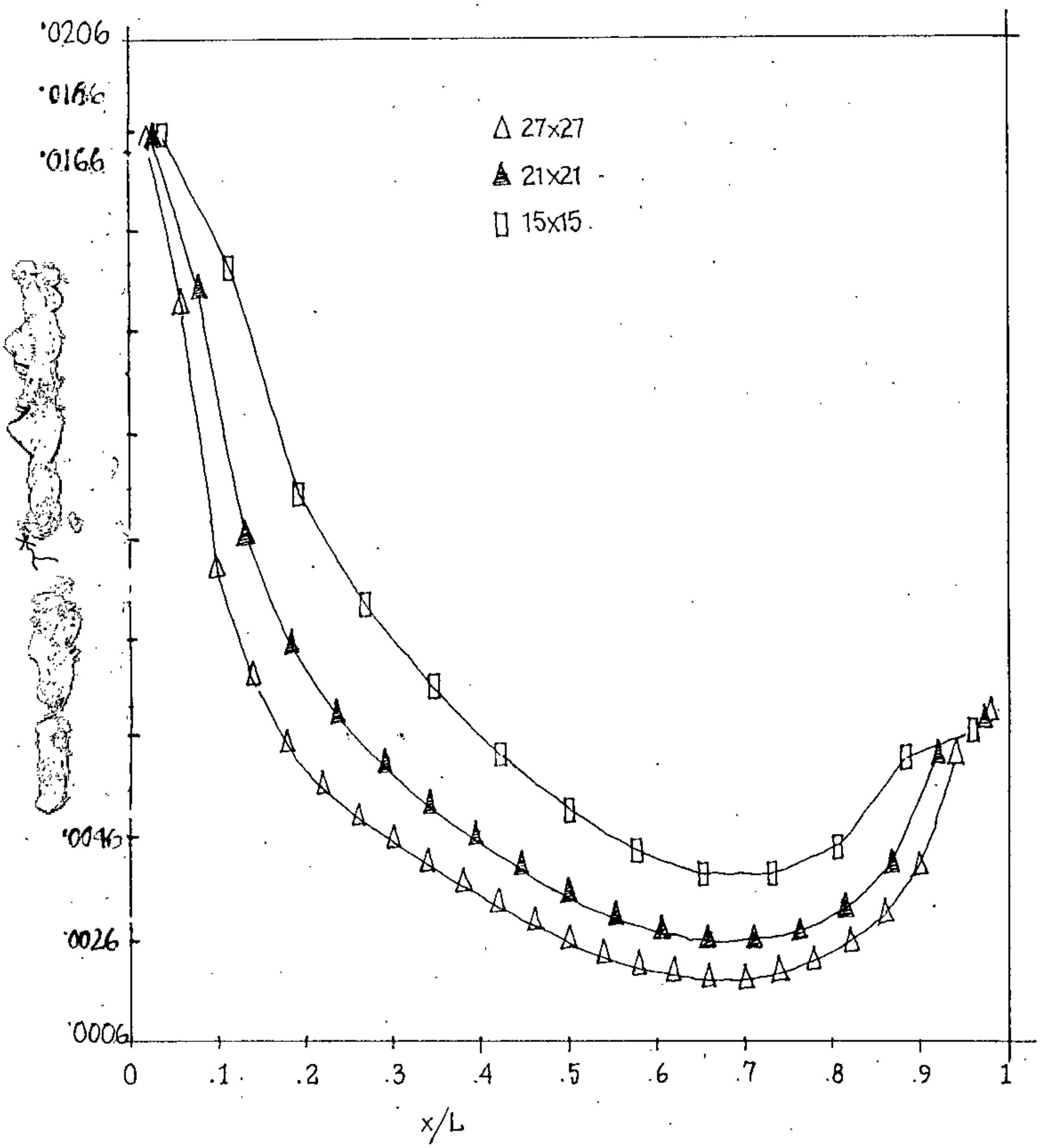

Fig.13 Distribution of the shear stress along the top wall : Re-400, case-2 (LD).

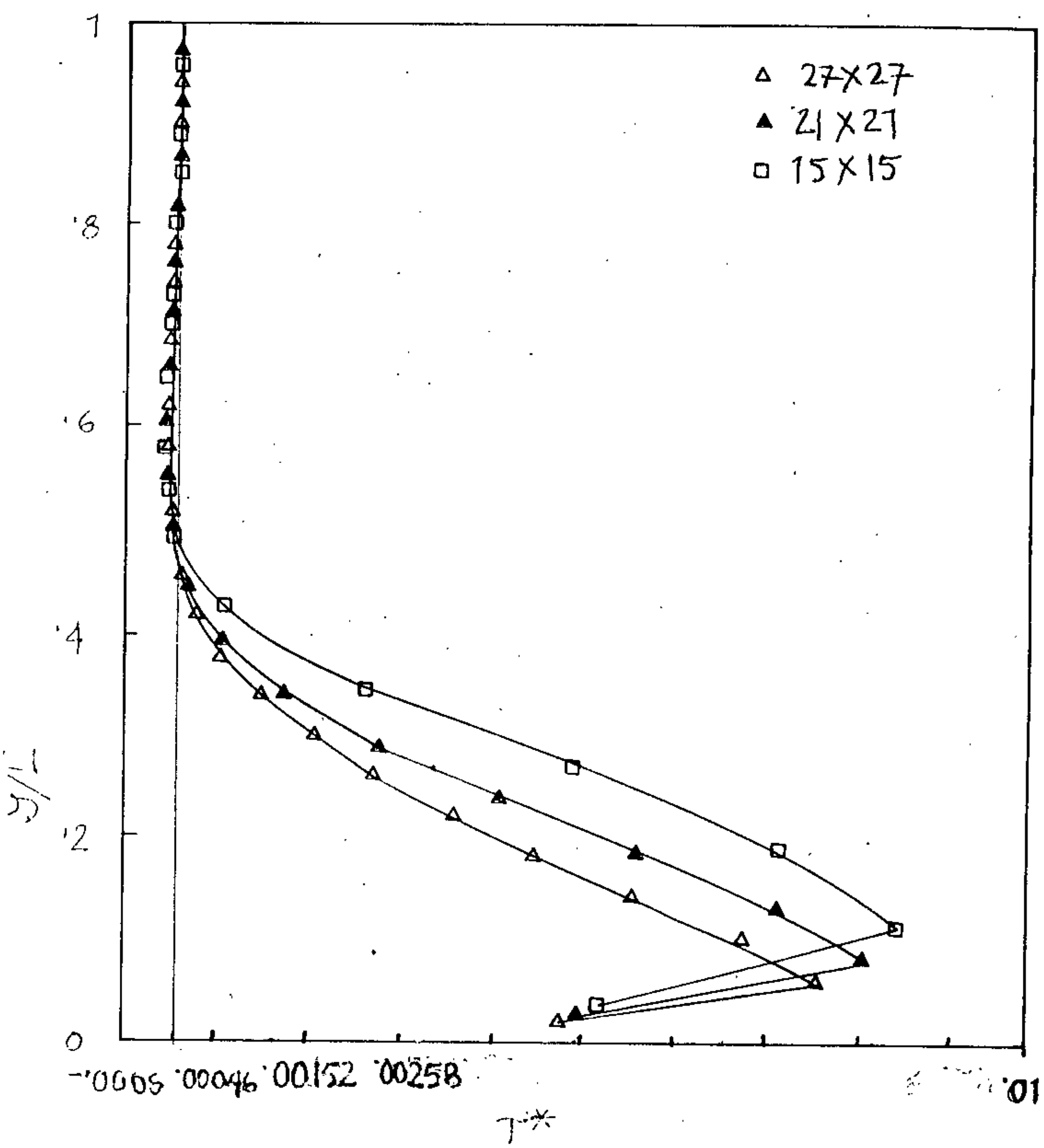

Fig.14 Distribution of the shear stress along the right wall: Re-400, case-2 (LD).

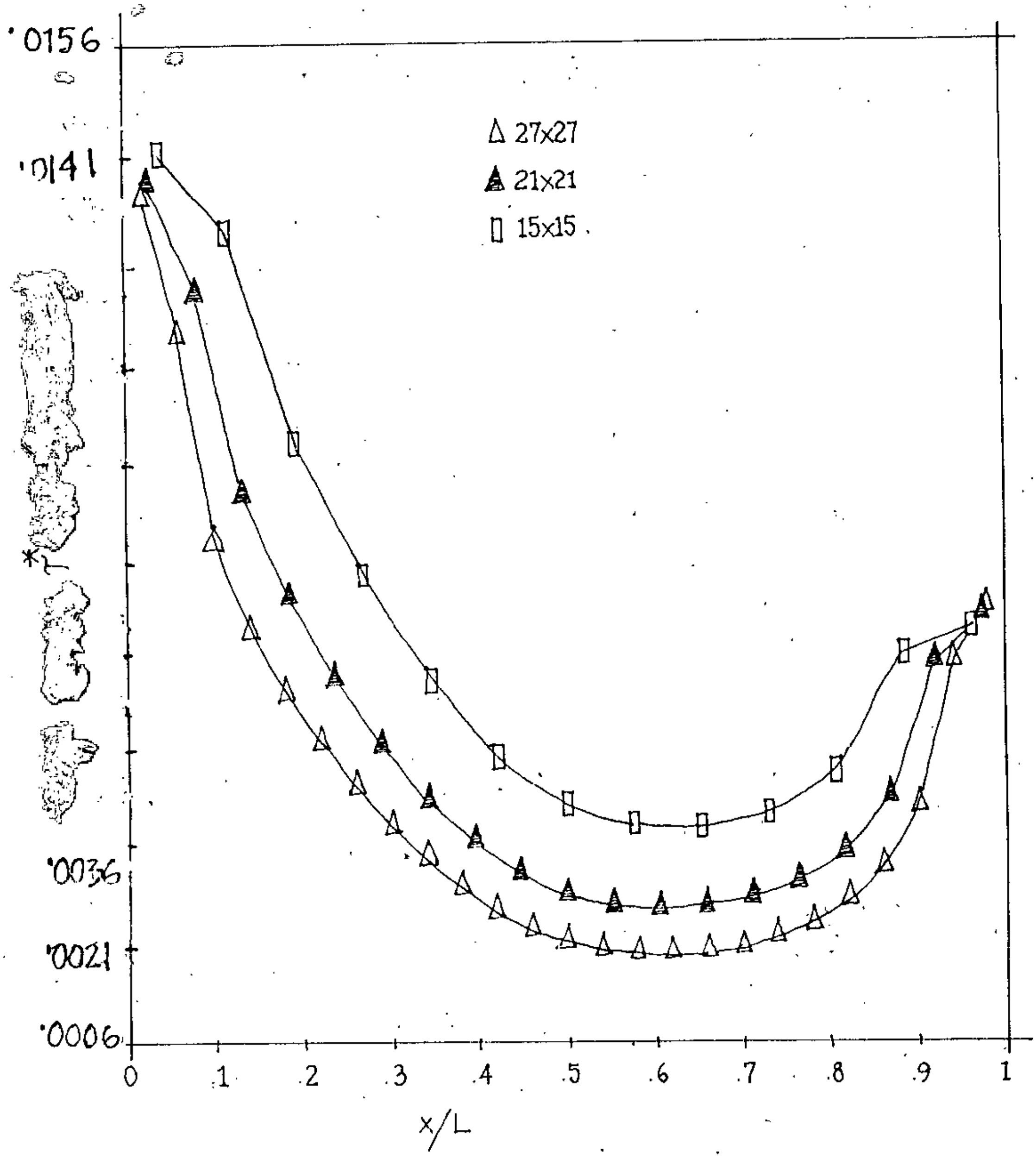

Fig.15 Distribution of the shear stress along the top wall : Re-400, case -3 (BR).

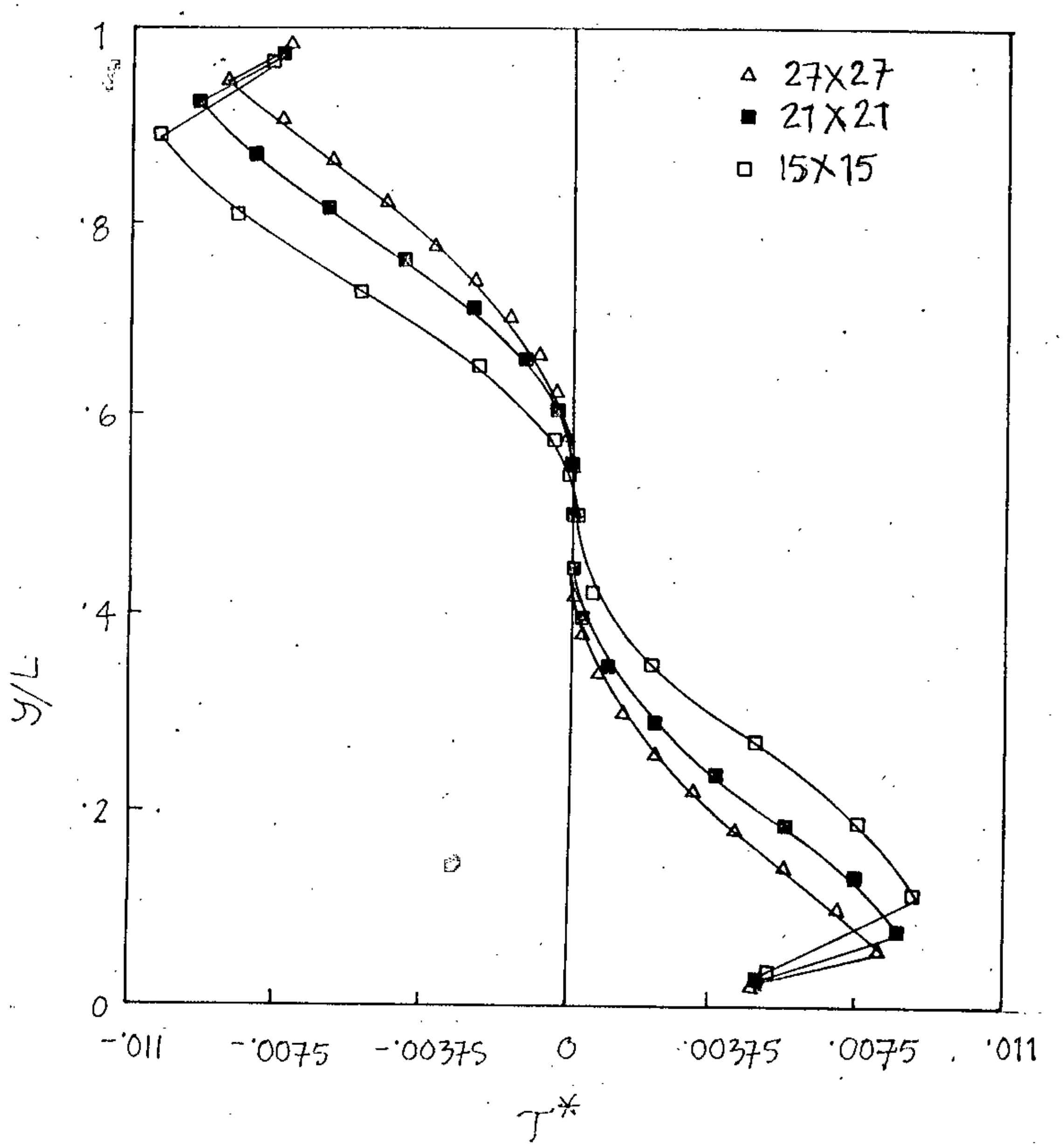

Fig.16 Distribution of the shear stress along the right wall: Re-400, case-3 (BR).

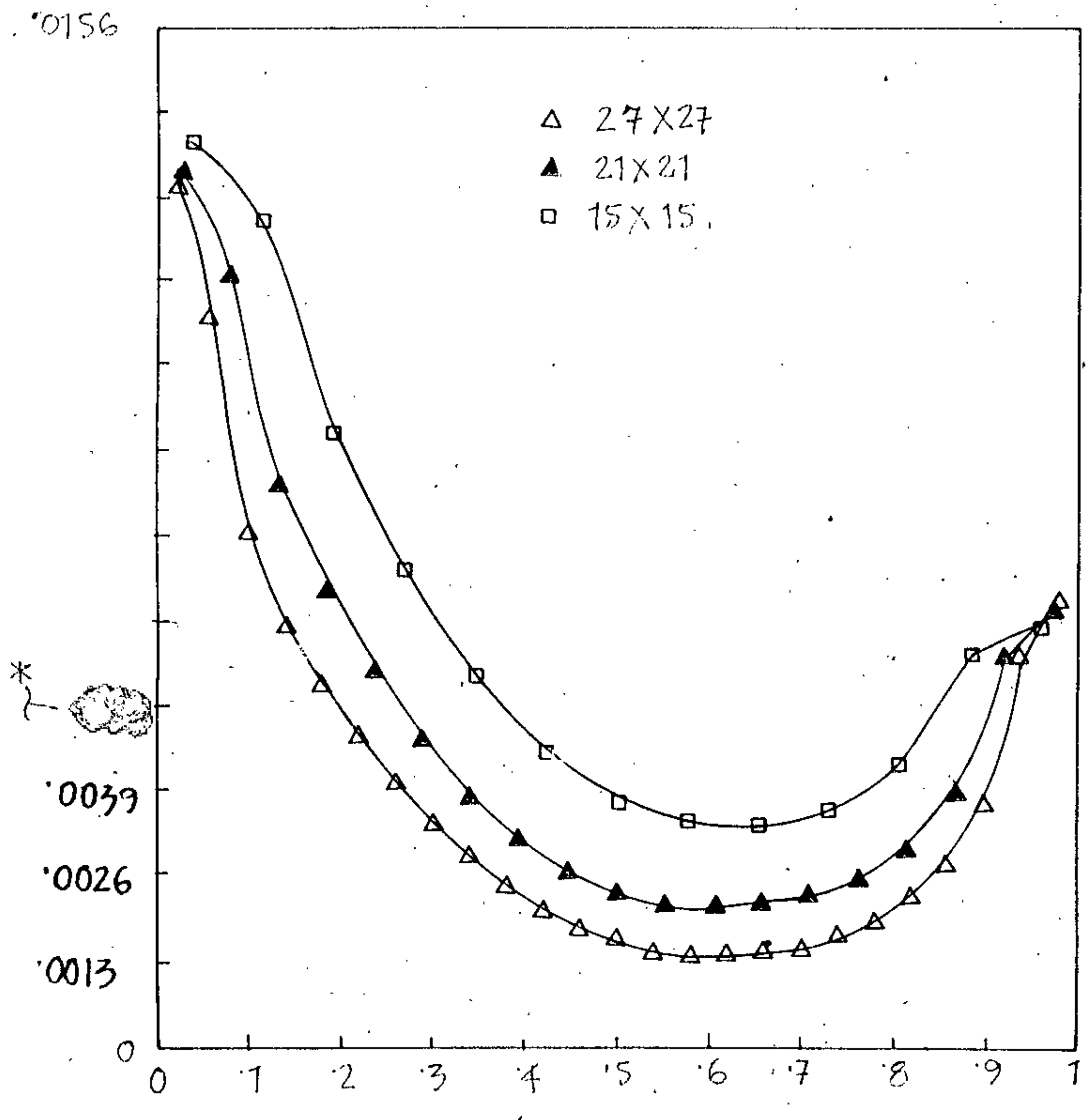

Fig.17 Distribution of the shear stress along the top wall: Re-400, case-4(BL).

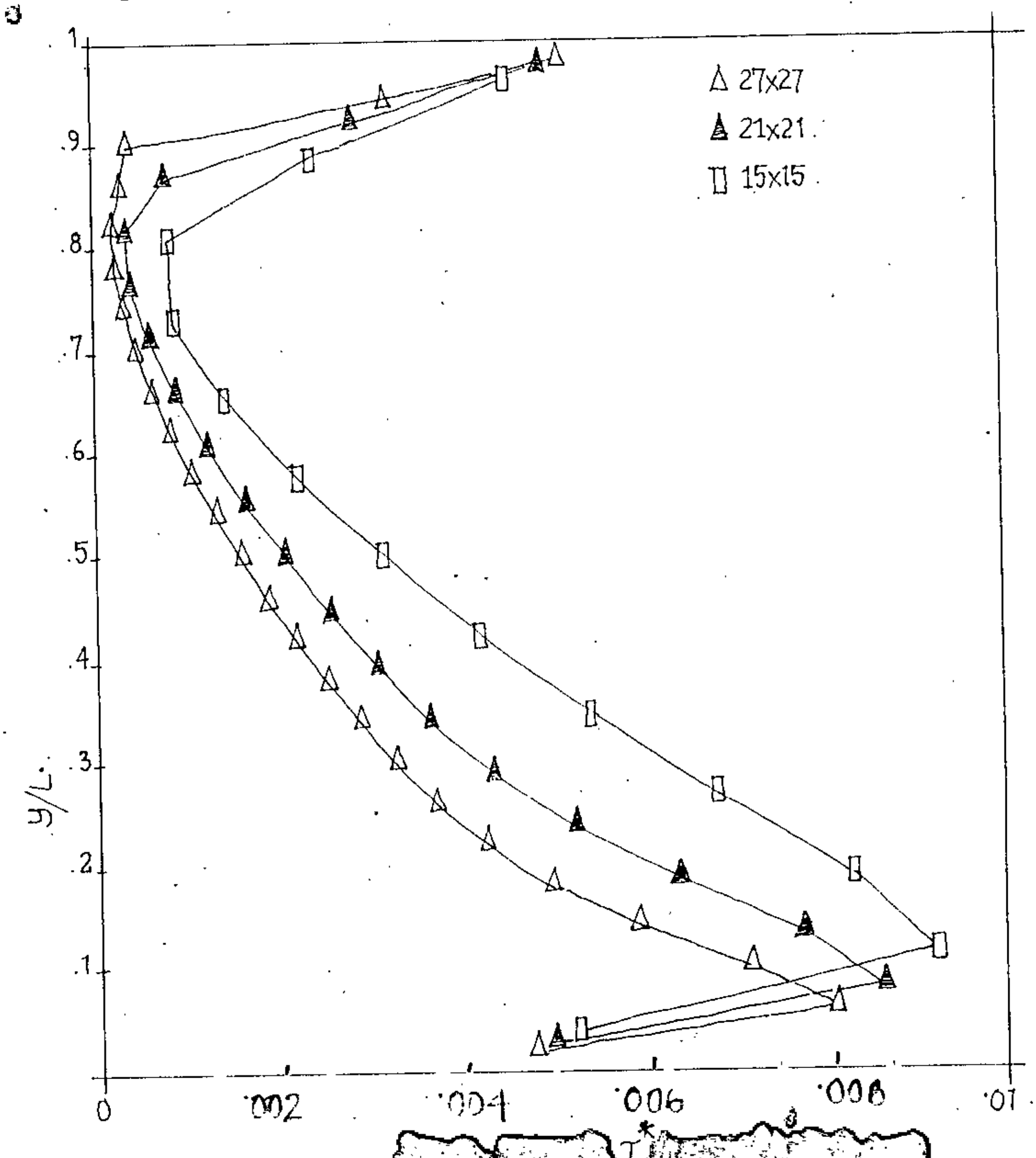

Fig.18 Distribution of the shear stress along the right wall; Re-400, case-4(BL).

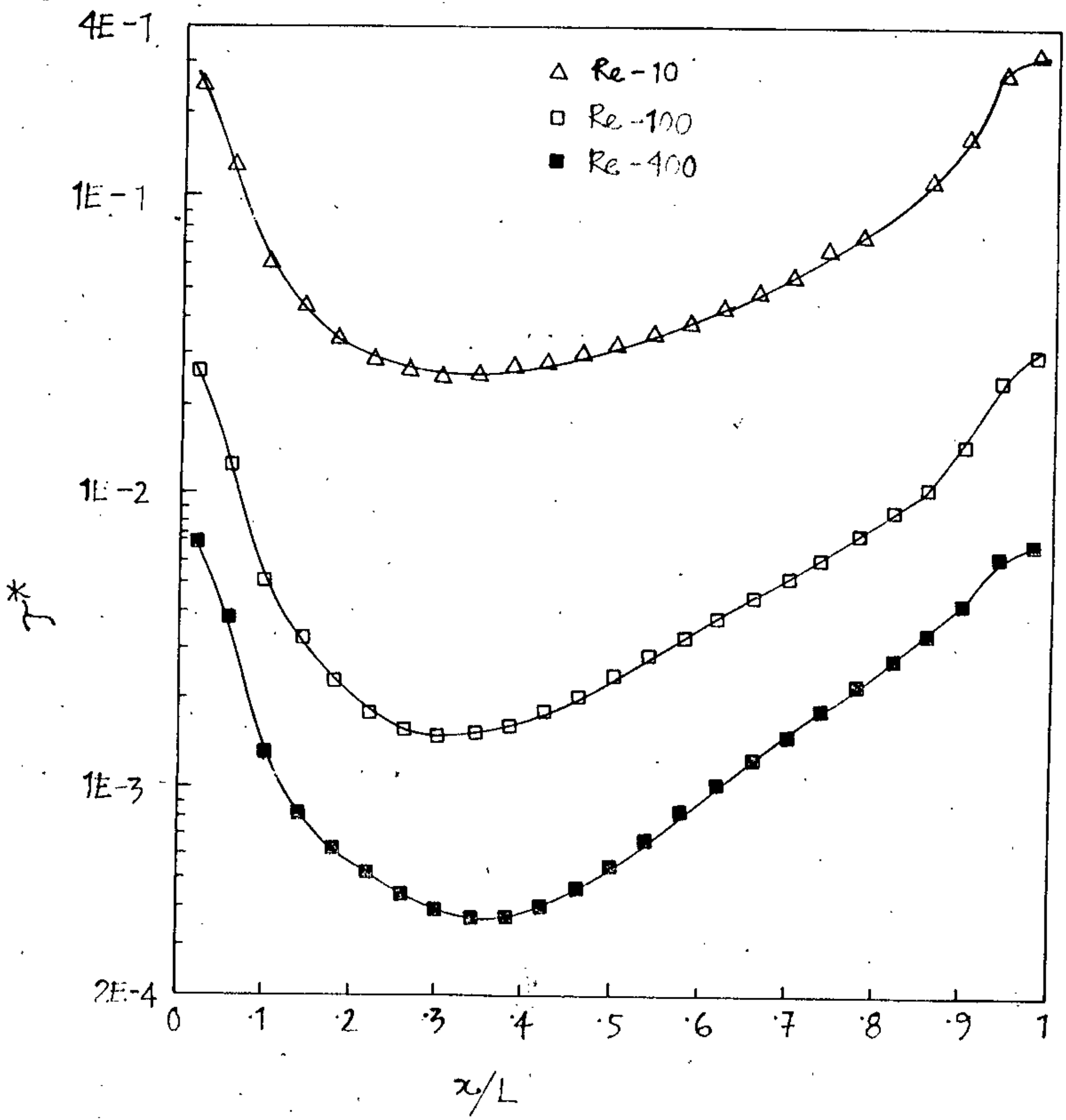

Fig.19 Variation of the shear stress along the top wall with Reynolds no. : Grid size-27x27, case-1(LU).

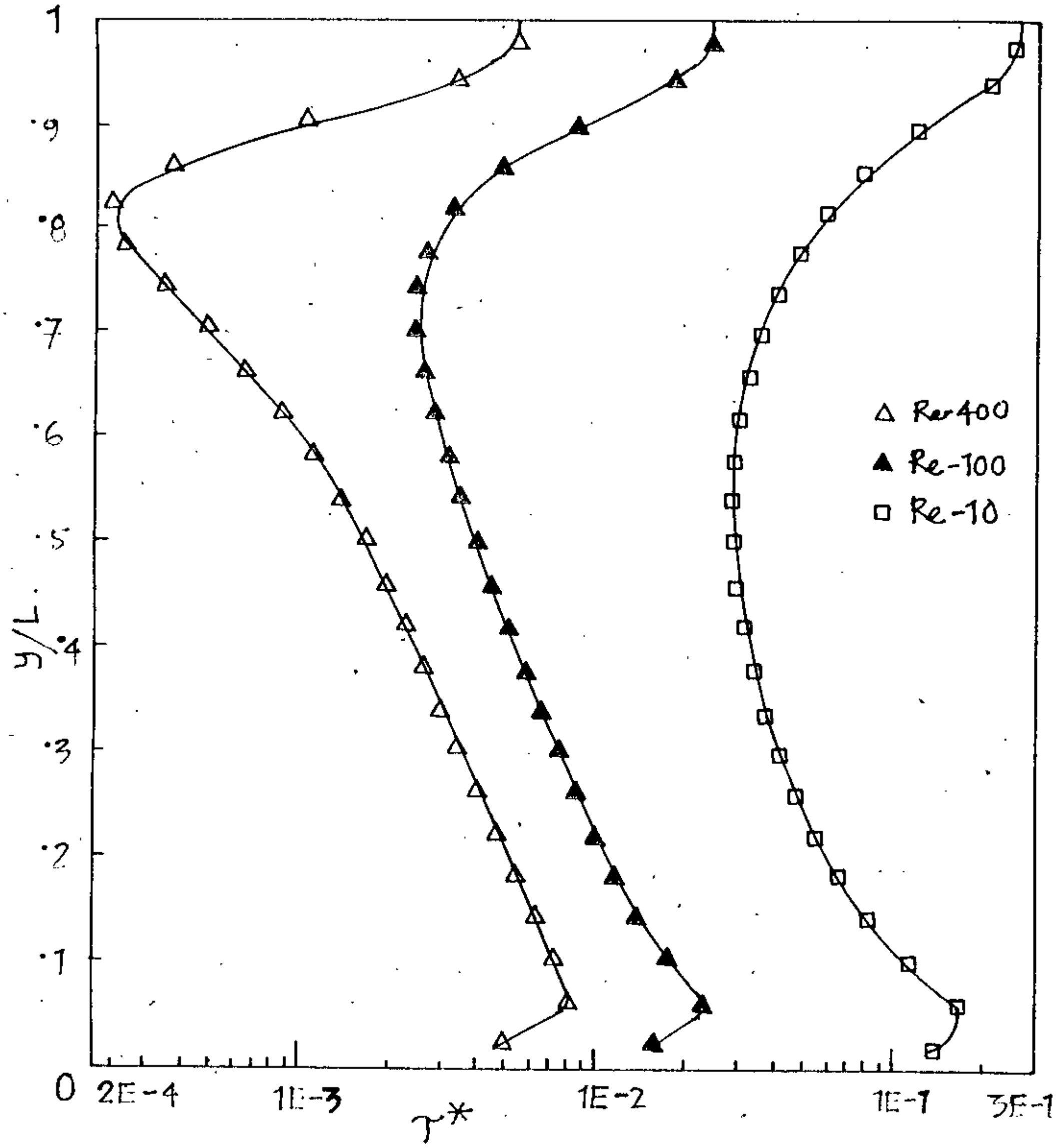

Fig.20 Variation of the shear stress along the right wall with Reynolds no. : Grid size- 27x27, case-4 (BL).

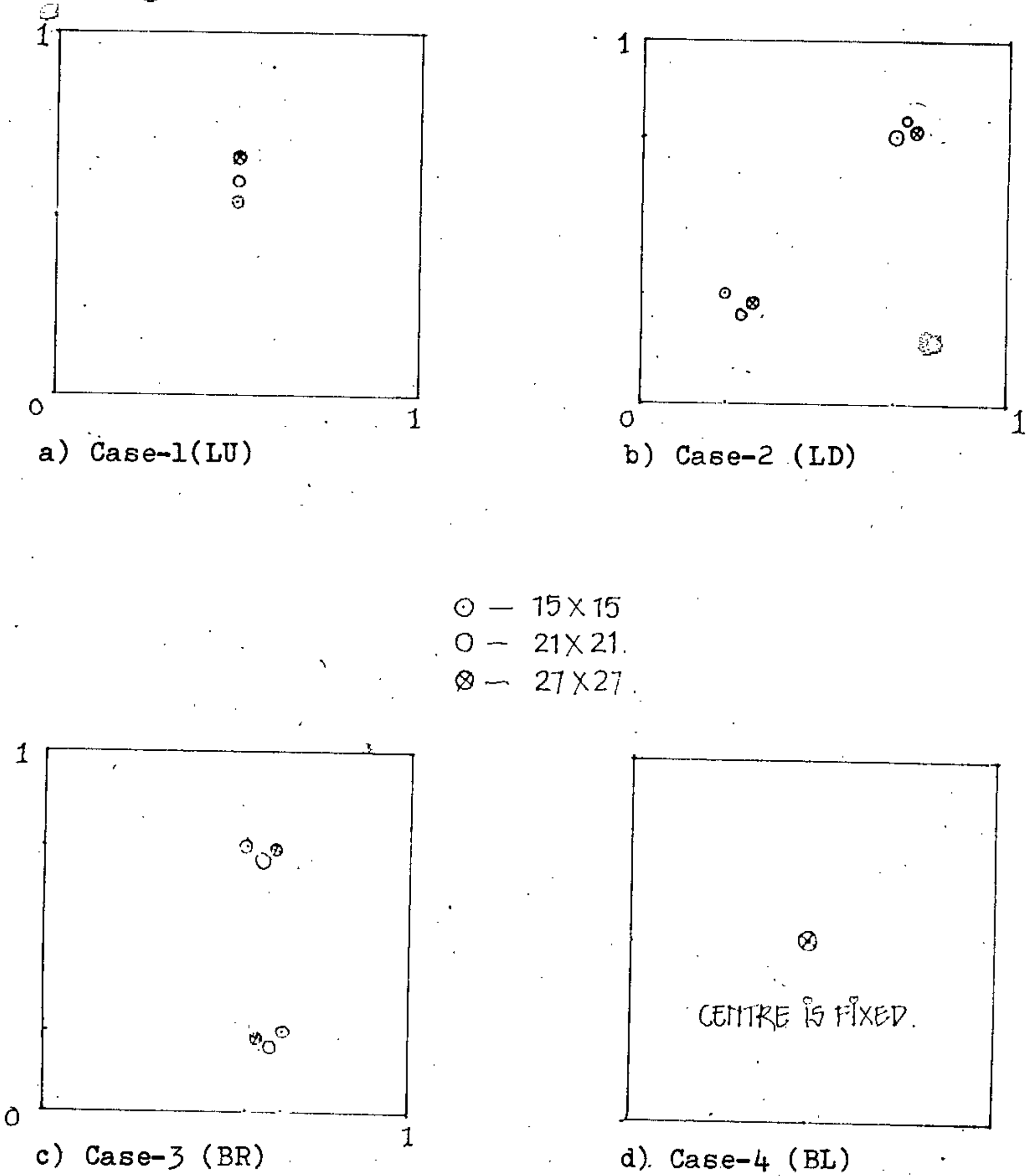

Fig.21 Effects of different grid spacing on primary vortex center: Re-400.

APPENDIX:A

(This section includes the computer program used in the analysis)

FILE: DRIVCAV FORTRAN A1 BUET COMPUTER CENTRE, DHAKA VM/SP

```
      PROGRAM MAIN
C
C       THIS IS THE CVFEM CODE THAT USES THE STREAMFUNCTION-VORTICITY
C       FORMULATION OF THE N.S. EQUATIONS. THIS PROGRAM SOLVES THE TWO
C       STANDARD TEST PROBLEMS; 1) DRIVEN CAVITY, AND 2) NATURAL CON-
C       VECTION IN A SQUARE ENCLOSURE. RELAXATION FACTOR FOR WALL VOR-
C       TICITY OF 0.7 AND THAT FOR VORTICITY OF 0.7 ARE RECOMMENDED.
C       FOR TEMPERATURE, ALSO USE 0.7. HOWEVER, FOR HIGH REYNOLDS NUMBER
C       (RE=5000), USE VALUES FOR WALL VORTICITY AND VORTICITY ITSELF 0.
C       FOR GR=10**7, USE 0.5 FOR TEMPERATURE AS WELL AS OTHERS.
C
      INCLUDE(COMVORT)
      LOGICAL MCLRNC,UNIFRM,JV2,BV2,VCALC,PLOTY,BURG,READ,WRITE
      MCLRNC=.TRUE.
      OPEN(13,FILE='INVORT',STATUS='OLD')
      READ(13, ) UNIFRM,VCALC,JV2,BV2,BURG,GR,RELT,CRIT,DUDY,READ,WRITE
      READ(13, ) NGR,IPM,JPM,MAXIT,BVREL,RELV,RE,PLOTY,EXPX,EXPY
      CLOSE(13)
      OPEN(11,FILE= 'VORTOUT',STATUS='NEW')
        NODC=IPM*JPM
      NELTC=2 (IPM-1) (JPM-1)
        NUMELC=NELTC
      IPMP=IPM+1
      JPMP=JPM+1
      IHALF=IPMP/2
      JHALF=JPMP/2
      IMAXC=IPM+2
      JMAXC=JPM+2
      IPMT=5
      JPMT=5
      JTL=4
      JTR=8
      IPMM=IPM-1
      JPMM=JPM-1
      IPMTM=IPMT-1
      JPMTM=JPMT-1
      RHO=1.0
C
      IM1=IMAXC-1
      IM2=IMAXC-2
      JM1=JMAXC-1
      JM2=JMAXC-2
      IF(UNIFRM) GOTO 7002
C          SET DOUBLE DENSITY GRID AT THE BOUNDARIES
      IF(NGR.EQ.4) THEN
      DX(1)=0.05
      DX(2)=0.05
      DX(JPMM)=[illegible]
```

```
      DX(I)=DELTA
      DY(I)=DELTA
9311  CONTINUE
      GOTO 7003
      ENDIF
      IF(NGR.EQ.3) GOTO 7120
      IF(NGR.EQ.1) THEN
      DEL2=1./FLOAT(IPM-2)
      DEL=DEL2/2.
      DO 7301 I=1,IPMM
      DY(I)=DEL2
7301  DX(I)=DY(I)
      DX(1)=DEL
      DX(JPMM)=DEL
      DY(1)=DEL
      DY(IPMM)=DEL
      GOTO 7003
      ENDIF
      ADEL=FLOAT(IPMM)-4.0E0
      BDEL=2.*ADEL+4.0E0
      DEL=1.0/BDEL
      DEL2=2.*DEL
      DO 7000 N=1,IPMM
      DX(N)=DEL2
7000  DY(N)=DX(N)
      DX(1)=DEL
      DX(2)=DEL
      DX(JPMM)=DEL
      DX(JPMM-1)=DEL
      DY(1)=DEL
      DY(2)=DEL
      DY(IPMM)=DEL
      DY(IPMM-1)=DEL
      GOTO 7003
7002  DO 7201 I=1,IPMM
      DX(I)=1./FLOAT(IPMM)
7201  DY(I)=DX(I)
      GOTO 7003
7120  PRODX=EXPX
      JPEND=JHALF-3
      DO 7121 JP=1,JPEND
7121  PRODX=(PRODX+1.0)*EXPX
      DX(1)=0.5/(1.+PRODX)
      DX(JPMM)=DX(1)
      JE=JPMM
      DO 7122 JP=2,JHALF-1
      JE=JE-1
      DX(JP)=DX(JP-1)*EXPX
7122  DX(JE)=DX(JP)
      PRODY=EXPY
      IPEND=IHALF-3
      DO 7123 IP=1,IPEND
7123  PRODY=(PRODY+1.0)*EXPY
      DY(1)=0.5/(1.+PRODY)
      DY(I[illegible]Y(1)
```

```
      IE=IPMM
      DO 7124 IP=2,IHALF-1
      IE=IE-1
      DY(IP)=DY(IP-1)*EXPY
7124  DY(IE)=DY(IP)
7003  CONTINUE
C          INITIALIZE THE NODAL ARRAYS
      DO 1111 ND=1,NODC
      SU(ND)=0.0
      SV(ND)=0.0
      U(ND)=1.0
      V(ND)=0.0
      UH(ND)=0.0
      VH(ND)=0.0
      DU(ND)=0.0
      DV(ND)=0.0
      P(ND)=0.0
      PP(ND)=0.0
      SMASS(ND)=0.0
      UFLX(ND)=0.0
      VOR(ND)=1.0
      VOL(ND)=0.0
      GAM(1,ND)=1.0
      GAM(2,ND)=1.0
      DO 9313 I=1,3
      DO 9312 J=1,3
      A(I,J,ND)=1.0
9312  CONTINUE
9313  CONTINUE
1111  CONTINUE
C
      CALL GEOT
C        SET THE WALL VELOCITIES
      KASE=0
      WRITE(*,*)'ENTER CODE FOR WALL MOVEMENT'
      WRITE(*,*)'1= LEFT WALL MOVING UP,        -1=MOVING DOWN'
      WRITE(*,*)'2=BOTTOM WALL MOVING RIGHT,    -2=MOVING LEFT'
      READ(*,*)KASE
      IF(KASE.EQ.0)WRITE(*,*)'YOU HAVE SELECTED TOP WALL MOVING ONLY'
      VTOP=0.0
      VBOT=0.0
      VLEF=0.
      VRIT=0.
      ULEF=0.
      URIT=0.
      UTOP=RE
      UBOT=0.0
      IF(KASE.EQ.1)THEN
      VLEF=RE
      ELSE IF(KASE .EQ.-1)THEN
      VLEF=-RE
      ELSE IF(KASE.EQ.2)THEN
      UBOT=RE
      ELSE IF(KASE.EQ.-2)THEN
      UBOT=-RE
```

```
      ENDIF
      VOR(NP(1,1))=0.
      VOR(NP(1,JPM))=0.
      VOR(NP(IPM,1))=0.
      VOR(NP(IPM,JPM))=0.
      U(NP(1,1))=0.
      U(NP(1,JPM))=0.
      U(NP(IPM,1))=0.
      U(NP(IPM,JPM))=0.
      V(NP(1,1))=0.
      V(NP(1,JPM))=0.
      V(NP(IPM,1))=0.
      V(NP(IPM,JPM))=0.
      DO 1112 J=2,JPMM
      U(NP(1,J))=UBOT
      V(NP(1,J))=VBOT
      U(NP(IPM,J))=UTOP
1112  V(NP(IPM,J))=VTOP
      DO 1113 I=2,IPMM
      U(NP(I,1))=ULEF
      U(NP(I,JPM))=URIT
      V(NP(I,1))=VLEF
1113  V(NP(I,JPM))=VRIT
      IF(BURG) GOTO 1115
      DO 1114 I=1,IPM
      UFLX(NP(I,1))=1.0
1114  UFLX(NP(I,JPM))=0.0
      BLK=GR
1115  CONTINUE
      CALL ASMPSI
      CALL SVOR
      CALL LSOLVE(3,1.0E0,0,3,IM2,3,JM2,5,1,1)
C [illegible]
      COMP=1.0
      IF(READ) THEN
      OPEN(15,FILE='RAWDAT',STATUS='OLD')
      DO 2451 N=1,NODC
2451  READ(15,*) VOR(N),PP(N),UFLX(N)
      CLOSE(15)
      DO 9314 N=1,NODC
        VOR(N)=VOR(N) UTOP
        PP(N)=PP(N) UTOP
9314  CONTINUE
      ENDIF
      DO 9000 ITERG=1,MAXIT
      IF(.NOT.BV2)CALL BVT(BVREL)
      CALL COFSRT(1)
      IF(BURG) GOTO 1116
      CALL LSOLVE(5,PULT,1,2,IM1,3,JM2,2,1,1)
      CALL TSURC
1116  CONTINUE
      CALL LSOLVE(1,POLV,1,3,IM2,3,JM2,2,1,1)
C --------------------- CONVERGENCE CHECK ----------------------
      IF(MOD(ITERG,1).EQ.0) THEN
        SID=0.0
```

```
      DO 9316 I=2,IPM-1
      DO 9315 J=2,JPM-1
        N=NP(I,J)
        RES=A(1,1,N)*VOR(NP(I-1,J-1))+A(1,2,N)*VOR(NP(I-1,  J))+
     1      A(1,3,N)*VOR(NP(I-1,J+1))+A(2,1,N)*VOR(NP(I  ,J-1))+
     2      A(2,2,N)*VOR(NP(I  ,  J))+A(2,3,N)*VOR(NP(I  ,J+1))+
     3      A(3,1,N)*VOR(NP(I+1,J-1))+A(3,2,N)*VOR(NP(I+1,  J))+
     4      A(3,3,N)*VOR(NP(I+1,J+1))-RHP(NP(I,J))
        RESID=RESID+RES*RES
 9315 CONTINUE
 9316 CONTINUE
      WRITE(*,9210) ITERG,RESID,VOR(NP(IHALF,JHALF)),
     1PP(NP(IHALF,JHALF)),UFLX(NP(IHALF,JHALF))
      WRITE(11,9210) ITERG,RESID,VOR(NP(IHALF,JHALF)),
     1PP(NP(IHALF,JHALF)),UFLX(NP(IHALF,JHALF))
      ENDIF
        IF(RESID.LT.CRIT) GOTO 9001
      CALL SVOR
      CALL LSOLVE(3,1.0E0,0,3,IM2,3,JM2,2,1,1)
      DO 2033 I=1,IPM
      DO 2033 J=1,JPM
 2033 PSI(I,J)=PP(NP(I,J))
 9000 CONTINUE
 9001 CONTINUE
      CALL UVCALC
      WRITE(11,*) '          T E S T .  FOR ...
      WRITE(11,2440) NGR,IPM,JPM,BVREL,RELV,RELT
      IF(NGR.EQ.3) WRITE(11,2441) EXPX,EXPY
      WRITE(11,*) 'BURG IS',BURG
      WRITE(11,*) 'UNIFRM IS',UNIFRM
      IF(.NOT.BURG) WRITE(11,*) 'THE GRASHOF # IS',GR
      IF(BURG) THEN
      WRITE(11,*) 'REYNOLDS NUMBER=',RE
      CALL SHEAR
      DO 9317 N=1,NODE
      U(N)=U(N)/UTOP
      V(N)=V(N)/UTOP
      VOR(N)=VOR(N)/UTOP
      PP(N)=PP(N)/UTOP
 9317 CONTINUE
      ENDIF
      IF(.NOT.BURG) CALL FLUX
      WRITE(11,*) 'U-VELOCITY'
      CALL PRINT(U)
      WRITE(11,*) 'V-VELOCITY'
      CALL PRINT(V)
      WRITE(11,*) 'STREAM FUNCTION'
      CALL PRINT(PP)
      WRITE(11,*) 'VORTICITY'
      CALL PRINT(VOR)
      IF(.NOT.BURG) THEN
      WRITE(11,*) 'TEMPERATURE'
      CALL PRINT(UFLX)
      ENDIF
      CLOSE(11)
```

```
      IF(PLOTY) THEN
      OPEN(14,FILE='PLOTU',STATUS='NEW')
      OPEN(15,FILE='PLOTV',STATUS='NEW')
      OPEN(16,FILE='PLOTS',STATUS='NEW')
      OPEN(17,FILE='PLOTO',STATUS='NEW')
      DO 1351 I=IPM,1,-1
      WRITE(14,141) (U(NP(I,J)),J=1,JPM)
      WRITE(15,141) (V(NP(I,J)),J=1,JPM)
      WRITE(16,141) (PP(NP(I,J)),J=1,JPM)
      WRITE(17,141) (VOR(NP(I,J)),J=1,JPM)
 1351 CONTINUE
      CLOSE(14)
      CLOSE(15)
      CLOSE(16)
      CLOSE(17)
 141  FORMAT(1X,21(F10.3,1X))
      ENDIF
C
C          WRITE UNFORMATTED VALUES OF VOR,PSI,U,V
      IF(WRITE) THEN
      OPEN(15,FILE='RAWDAT',STATUS='NEW')
      DO 2452 N=1,NODC
 2452 WRITE(15, ) VOR(N),PP(N),UFLX(N)
      CLOSE(15)
      ENDIF
C
 2440 FORMAT(3X,'JOB',2X,'IPM',2X,'JPM',7X,'BVREL',
     12X,'      V',2X,'    F',/,3X,'---',2X,'---',2X,'---',7X,'-----',
     22X,'-----',2X,'-----',/,4X,I1,3X,I3,2X,I3,7X,F5.2,2X,F5.2,2X,
     3F5.2)
 2441 FORMAT(//,10X,'EXPANSION COEFFS. IN X AND Y',/,10X,28('-'),/,
     112X,F5.2,3X,F5.2)
 9210 FORMAT(1X,I4,4X,4(1PD11.4,2X))
      STOP
      END
      SUBROUTINE GEOT
      INCLUDE(COMVORT)
      NODC=IPM*JPM
      NUMELC=NELTC
      NELTC=NUMELC
C
C     NUMBER THE NODES FIRST
C
C     THE CLEARANCE
       NP(1,1)=1
       X(1)=0.0
       Y(1)=0.0
       I=1
       DO 100 J=2,JPM
       NP(I,J)=NP(I,J-1)+IPM
       X(NP(I,J))=X(NP(I,J-1))+DX(J-1)
 100   Y(NP(I,J))=0.0
C
       J=1
       DO 101 I=2,IPM
```

```
      NP(I,J)=I
      X(NP(I,J))=0.0
101   Y(NP(I,J))=Y(NP(I-1,J))+DY(I-1)
C
      DO 102 J=2,JPM
      DO 102 I=2,IPM
      NP(I,J)=NP(I,J-1)+IPM
      X(NP(I,J))=X(NP(I,J-1))+DX(J-1)
102   Y(NP(I,J))=Y(NP(I-1,J))+DY(I-1)
C
C     NOW SETUP THE NODAL CONNECTION ARRAY
      NB=0
      DO 2121 I=1,IPM
      NB=NB+1
2121  NBND(NB)=I
      NB1=NB
      DO 2122 J=2,JPM
      NB=NB+1
2122  NBND(NB)=NP(IPM,J)
      NB2=NB
      DO 2123 I=IPMM,1,-1
      NB=NB+1
2123  NBND(NB)=NP(I,JPM)
      NB3=NB
      DO 2124 J=JPMM,2,-1
      NB=NB+1
2124  NBND(NB)=NP(1,J)
      NBMAX=NB
      WRITE(11, ) NBMAX,NB1,NB2,NB3
C
      NEL=0
      DO 9319 IQ=1,IPMM
      DO 9320 JQ=1,JPMM
      NEL=NEL+1
      NCA(NEL,1)=NP(IQ,JQ)
      NCA(NEL,2)=NP(IQ,JQ+1)
      NCA(NEL,3)=NP(IQ+1,JQ+1)
      NEL=NEL+1
      NCA(NEL,1)=NP(IQ,JQ)
      NCA(NEL,2)=NP(IQ+1,JQ+1)
      NCA(NEL,3)=NP(IQ+1,JQ)
9320  CONTINUE
9319  CONTINUE
      NEL=2 IPMM
      NCA(NEL,1)=NP(IPM,1)
      NCA(NEL,2)=NP(IPMM,2)
      NCA(NEL,3)=NP(IPM,2)
      NEL=NEL-1
      NCA(NEL,1)=NP(IPM,1)
      NCA(NEL,2)=NP(IPMM,1)
      NCA(NEL,3)=NP(IPMM,2)
      NEL=NELTO-2 IPMM+1
      NCA(NEL,1)=NP(2,JPMM)
      NCA(NEL,2)=NP(1,JPMM)
      NCA(NEL,3)=NP(1,JPM)
```

```
      NEL=NEL+1
      NCA(NEL,1)=NP(2,JPMM)
      NCA(NEL,2)=NP(1,JPM)
      NCA(NEL,3)=NP(2,JPM)
C
      DO 2000 NEL=1,NELTC
      DO 2100 NODE=1,3
      XT(NODE)=X(NCA(NEL,NODE))
2100  YT(NODE)=Y(NCA(NEL,NODE))
C
      DET=XT(1)*YT(2)+XT(2)*YT(3)+XT(3)*YT(1)-
     $     YT(1)*XT(2)-YT(2)*XT(3)-YT(3)*XT(1)
C
      AREA3(NEL)=ABS(DET/2.0)/3.0
C
      ALPHA(NEL,1)=(YT(3)-YT(2))/DET
      ALPHA(NEL,2)=(YT(1)-YT(3))/DET
      ALPHA(NEL,3)=(YT(2)-YT(1))/DET
C
      BETA(NEL,1)= (XT(2)-XT(3))/DET
      BETA(NEL,2)= (XT(3)-XT(1))/DET
      BETA(NEL,3)= (XT(1)-XT(2))/DET
C
C
C
2000  CONTINUE
C        THE NODAL VOLUMES ARE ASSEMBLED HERE
C
      DO 2500 NEL=1,NELTC
      DO 2501 N=1,3
C        HERE DV IS USED TO ASSEMBLE INVERSE AREAS FOR EACH NODE
      DV(NCA(NEL,N))=DV(NCA(NEL,N))+1.0/AREA3(NEL)
2501  VOL(NCA(NEL,N))=VOL(NCA(NEL,N))+AREA3(NEL)
2500  CONTINUE
C        THE MAP FOR THE GENERAL PHI VARIABLE
      JM1=JMAXC-1
      IM1=IMAXC-1
      DO 9000 J=2,JM1
      DO 9000 I=2,IM1
9000  MAP(I,J)=NP(I-1,J-1)
C
      RETURN
      END
      SUBROUTINE UVCLC4
      INCLUDE(COMVORT)
      DIMENSION II(3),JJ(3)
C
      DO 2500 N=1,NODC
      UH(N)=U(N)
2500  VH(N)=V(N)
      DO 2000 N=1,NODC
      U(N)=0.0
2000  V(N)=0.0
C
      DO 1000 NEL=1,NELTC
```

```
      PS1=PP(NCA(NEL,1))
      PS2=PP(NCA(NEL,2))
      PS3=PP(NCA(NEL,3))
C
      VE=ALPHA(NEL,1)*PS1+ALPHA(NEL,2)*PS2+ALPHA(NEL,3)*PS3
      UE=(-1.)*(BETA(NEL,1)*PS1+BETA(NEL,2)*PS2+BETA(NEL,3)*PS3)
C
      DO 300 N=1,3
      U(NCA(NEL,N))=U(NCA(NEL,N))+UE/(AREA3(NEL)*DV(NCA(NEL,N)))
300   V(NCA(NEL,N))=V(NCA(NEL,N))+VE/(AREA3(NEL)*DV(NCA(NEL,N)))
C
1000  CONTINUE
C
C     REIMPOSE THE BOUNDARY VELOCITIES
      DO 3000 J=1,JPM
      U(NP(1,J))=UH(NP(1,J))
      V(NP(1,J))=VH(NP(1,J))
      U(NP(IPM,J))=UH(NP(IPM,J))
      V(NP(IPM,J))=VH(NP(IPM,J))
3000  CONTINUE
C
      DO 3001 I=2,IPM1
      U(NP(I,JPM))=UH(NP(I,JPM))
      U(NP(I,1))=UH(NP(I,1))
      V(NP(I,1))=VH(NP(I,1))
      V(NP(I,JPM))=VH(NP(I,JPM))
3001  CONTINUE
C
      RETURN
      END
      SUBROUTINE SVOR
      INCLUDE(COMVORT)
C
      DO 2000 N=1,NODC
2000  SU(N)=0.0E0
C
      DO 1000 NEL=1,NELTC
      N1=NCA(NEL,1)
      N2=NCA(NEL,2)
      N3=NCA(NEL,3)
      SU(N1)=SU(N1)+AREA3(NEL)*(22*VOR(N1)+7*VOR(N2)+7*VOR(N3))/36
      SU(N2)=SU(N2)+AREA3(NEL)*(7*VOR(N1)+22*VOR(N2)+7*VOR(N3))/36
      SU(N3)=SU(N3)+AREA3(NEL)*(7*VOR(N1)+7*VOR(N2)+22*VOR(N3))/36
1000  CONTINUE
C
      RETURN
      END
      SUBROUTINE ASMPSI
      INCLUDE(COMVORT)
      DIMENSION II(3),JJ(3)
      DO 2000 N=1,NODC
      DO 2000 I=1,3
      DO 2000 J=1,3
2000  APSI(I,J,N)=0.0E0
C
```

```
      DO 1000 NEL=1,NELTC
      DO 100 N=1,3
      XN(N)=X(NCA(NEL,N))
100   YN(N)=Y(NCA(NEL,N))
C
      DET=XN(1)*YN(2)+XN(2)*YN(3)+XN(3)*YN(1)-
     &     YN(1)*XN(2)-YN(2)*XN(3)-YN(3)*XN(1)
C
      Y23=YN(2)-YN(3)
      Y31=YN(3)-YN(1)
      Y12=YN(1)-YN(2)
C
      X32=XN(3)-XN(2)
      X13=XN(1)-XN(3)
      X21=XN(2)-XN(1)
C
      ESM(1,1)=Y23*Y23+X32*X32
      ESM(1,2)=Y23*Y31+X32*X13
      ESM(1,3)=Y23*Y12+X32*X21
      ESM(2,2)=Y31*Y31+X13*X13
      ESM(2,3)=Y31*Y12+X13*X21
      ESM(3,3)=Y12*Y12+X21*X21
C     STATEMENT OF SYMMETRY
      ESM(2,1)=ESM(1,2)
      ESM(3,1)=ESM(1,3)
      ESM(3,2)=ESM(2,3)
C
      DO 101 I=1,3
      DO 101 J=1,3
101   ESM(I,J)=0.5*ESM(I,J)/DET
C
C     THE ASSEMBLY PROCESS ONTO THE APSI MATRIX
      DO 120 NC=1,3
      JJ(NC)=(NCA(NEL,NC)-1)/IPM+1
120   II(NC)=NCA(NEL,NC)-IPM*(JJ(NC)-1)
      DO 121 I=1,3
      ND=NP(II(I),JJ(I))
      DO 122 J=1,3
      IG=2-(II(I)-II(J))
      JG=2-(JJ(I)-JJ(J))
      APSI(IG,JG,ND)=APSI(IG,JG,ND)+ESM(I,J)
122   CONTINUE
121   CONTINUE
C     END ASSEMBLY
1000  CONTINUE
      RETURN
      END
C
      SUBROUTINE TSORC
      INCLUDE(COMVORT)
      DIMENSION II(3),JJ(3)
C
      DO 2000 N=1,NODC
2000  RHP(N)=0.0
C
```

```
      DO 1000 NEL=1,NELTC
C
      IF(DUDY) GOTO 2100
      SORCT=(ALPHA(NEL,1)*UFLX(NCA(NEL,1))+ALPHA(NEL,2)*
     &UFLX(NCA(NEL,2))+ALPHA(NEL,3)*UFLX(NCA(NEL,3)))*AREA3(NEL)
      GOTO 2110
2100  SORCT=(BETA(NEL,1)*UFLX(NCA(NEL,1))+BETA(NEL,2)*UFLX(NCA(NEL,2))
     1+BETA(NEL,3)*UFLX(NCA(NEL,3)))*AREA3(NEL)
C
2110  SORCT=SORCT*BLK
      DO 300 N=1,3
300   RHP(NCA(NEL,N))=RHP(NCA(NEL,N))-SORCT
C
1000  CONTINUE
C
C
      RETURN
      END
      SUBROUTINE LSOLVE(NVAR,RL,NCOEF,ISTS,INDS,JSTS,JNDS,
     1NSWEEP,NSWPX,NSWPY)
      INCLUDE(COMVORT)
      DIMENSION AA(50),BB(50),CC(50),RHS(50)
      NEL=0
      IMX=INDS-ISTS+1
      JMX=JNDS-JSTS+1
      CREL=(1.0-RL)/RL
C        SET PHI TO 0.0
      DO 4999 I=1,IMAXC
      DO 4999 J=1,JMAXC
4999  PHI(I,J)=0.0
      IF(NVAR.EQ.3) GOTO 500
C        COPY A MATRIX ONTO AMT
      DO 5000 N=1,NODC
      DO 5000 I=1,3
      DO 5000 J=1,3
5000  AP(I,J,N)=A(I,J,N)
C
      DO 5001 N=1,NODC
      COEF=2.*(AMAX1(0.0E0,-A(2,2,N)))*FLOAT(NCOEF)
      DOEF=COEF+A(2,2,N)
      AP(2,2,N)=DOEF/RL
      IF(NVAR.EQ.1) RH(N)=RHP(N)+(DOEF*CREL+COEF)*VOR(N)
      IF(NVAR.EQ.5) RH(N)=(DOEF*CREL+COEF)*UFLX(N)+DU(N)
5001  CONTINUE
      GOTO 501
500   CONTINUE
C        COPY THE APSI MATRIX ONTO AMT
      DO 5002 N=1,NODC
      DO 5002 I=1,3
      DO 5002 J=1,3
      RH(N)=SU(N)
5002  AP(I,J,N)=APSI(I,J,N)
C
501   CONTINUE
      DO 5004 I=2,IM1
```

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FILE: DRIVCAV  FORTRAN  A1  BUET COMPUTER CENTRE, DHAKA
      DO 5004 J=2,JM1
      IF(NVAR.EQ.1) PHI(I,J)=VOR(MAP(I,J))
      IF(NVAR.EQ.5) PHI(I,J)=UFLX(MAP(I,J))
      IF(NVAR.EQ.3) PHI(I,J)=PP(MAP(I,J))
5004  CONTINUE
C
C
C           START THE SWEEPING
C
      DO 2000 NT=1,NSWEEP
      DO 1000 NS=1,NSWPX
      IFL=-1
      J=JSTS
      DO 100 NCT=1,2
      J=J+IFL
      IFL=-1*IFL
      DO 10 JC=1,JMX
      J=J+IFL
      I=ISTS-1
      DO 1 IC=1,IMX
      I=I+1
      N=MAP(I,J)
      AA(IC)=AP(1,2,N)
      BB(IC)=AP(2,2,N)
      CC(IC)=AP(3,2,N)
      RHS(IC)=RH(N)-AP(1,1,N)*PHI(I-1,J-1)-AP(1,3,N)*PHI(I-1,J+1)
     1-AP(2,1,N)*PHI(I,J-1)-AP(2,3,N)*PHI(I,J+1)
     2-AP(3,1,N)*PHI(I+1,J-1)-AP(3,3,N)*PHI(I+1,J+1)
1     CONTINUE
      RHS(1)=RHS(1)-AP(1,2,MAP(ISTS,J))*PHI(ISTS-1,J)
      RHS(IMX)=RHS(IMX)-AP(3,2,MAP(INDS,J))*PHI(INDS+1,J)
      CALL TRI(AA,BB,CC,RHS,1,IMX)
      I=ISTS-1
      DO 2 II=1,IMX
      I=I+1
2     PHI(I,J)=AA(II)
10    CONTINUE
100   CONTINUE
1000  CONTINUE
C
      DO 1001 NS=1,NSWPY
      IFL=-1
      I=ISTS
      DO 101 NCT=1,2
      I=I+IFL
      IFL=-1*IFL
      DO 11 IC=1,IMX
      I=I+IFL
      J=JSTS-1
      DO 3 JC=1,JMX
      J=J+1
      N=MAP(I,J)
      AA(JC)=AP(2,1,N)
      BB(JC)=AP(2,2,N)
      CC(JC)=AP(2,3,N)
```

```
      RHS(JC)=RH(N)-AP(1,1,N) PHI(I-1,J-1)-AP(3,1,N) PHI(I+1,J-1)
     1-AP(1,2,N) PHI(I-1,J)-AP(3,2,N) PHI(I+1,J)
     2-AP(1,3,N) PHI(I-1,J+1)-AP(3,3,N) PHI(I+1,J+1)
3,    CONTINUE
      RHS(1)=RHS(1)-AP(2,1,MAP(I,JSTS)) PHI(I,JSTS-1)
      RHS(JMX)=RHS(JMX)-AP(2,3,MAP(I,JNDS)) PHI(I,JNDS+1)
      CALL TRI(AA,BB,CC,RHS,1,JMX)
      J=JSTS-1
      DO 4 JJ=1,JMX
      J=J+1
4     PHI(I,J)=AA(JJ)
11    CONTINUE
101   CONTINUE
1001  CONTINUE
C
2000  CONTINUE
C
      IF(PHI(5,5).GE.1.0D10)  THEN
      WRITE(11, ) 'BOMBING OUT IN NVAR=',NVAR
      STOP
      ENDIF
C          RESUBSTITUTE PHI BACK INTO THE APPROPRIATE VARIABLE
      DO 3000 I=ISTS,INDS
      DO 3000 J=JSTS,JNDS
      IF(NVAR.EQ.1) VOR(MAP(I,J))=PHI(I,J)
      IF(NVAR.EQ.5) JFLX(MAP(I,J))=PHI(I,J)
      IF(NVAR.EQ.3) PP(MAP(I,J))=PHI(I,J)
3000  CONTINUE
C
C
      RETURN
      END
      SUBROUTINE TRI(A,B,C,D,M,N)
      DIMENSION A(50),B(50),C(50),D(50),E(50),F(50),G(50)
C     GAUSS ELIMINATION
      E(M)=B(M)
      F(M)=D(M)
      M1=M+1
      DO 10 I=M1,N
      G(I)=A(I)/E(I-1)
      E(I)=B(I)-G(I) C(I-1)
10    F(I)=D(I)-G(I) F(I-1)
C     BACK SUBSTITUTION. ANSWER STORED IN A(I)
      A(N)=F(N)/E(N)
      DO 20 J=M1,N
      I=N+M1-1-J
20    A(I)=(F(I)-C(I) A(I+1))/E(I)
      RETURN
      END
      SUBROUTINE SHEAR
      INCLUDE(COMVORT)
      DIMENSION SHRU(27),SHRB(27),SHRL(27),SHRR(27)
      DATA SHRU/27 0./SHRB/27 0./SHRL/27 0./SHRR/27 0./
C          TO DETERMINE THE VALUE OF SHEAR ALONG TOP WALL
      DO 10 J=1,JPMM
```

```
      NB=NP(IPM,J)
      NBP=NP(IPM,J+1)
      SHRU(J)=SHRU(J)+DX(J)*(VOR(NBP)+3.*VOR(NB))/8.
      SHRU(J+1)=SHRU(J+1)+DX(J)*(3.*VOR(NBP)+VOR(NB))/8.
10    CONTINUE
      SUM=0.
      DO 11 J=1,JPM
11    SUM=SUM+SHRU(J)
      SUM=ABS(2*SUM/(UTOP**2))
      WRITE(11,*) 'DRAG COEFFICIENT FOR THE TOP WALL =',SUM
      WRITE(11,*) 'SHEAR ARRAY FOR THE TOP WALL FOLLOWS'
      DO 12 J=1,JPM
12    WRITE(11,13) J,SHRU(J)
13    FORMAT(4X,I4,3X,D11.4)
C           TO DETERMINE THE VALUE OF SHEAR ALONG BOTTOM WALL
      DO 14 J=1,JPMM
      NB=NP(1,J)
      NBP=NP(1,J+1)
      SHRB(J)=SHRB(J)+DX(J)*(VOR(NBP)+3.*VOR(NB))/8.
      SHRB(J+1)=SHRB(J+1)+DX(J)*(3.*VOR(NBP)+VOR(NB))/8.
14    CONTINUE
      SUM=0.
      DO 15 J=1,JPM
15    SUM=SUM+SHRB(J)
      SUM=ABS(2*SUM/(UTOP**2))
      WRITE(11,*) 'DRAG COEFFICIENT FOR BOTTOM WALL=',SUM
      WRITE(11,*) 'SHEAR ARRAY FOR THE BOTTOM WALL FOLLOWS'
      DO 16 J=1,JPM
16    WRITE(11,13) J,SHRB(J)
C           TO DETERMINE THE VALUE OF SHEAR ALONG THE LEFT WALL
      DO 17 I=1,IPMM
      NB=NP(I,1)
      NBP=NP(I+1,1)
      SHRL(I)=SHRL(I)+DY(I)*(VOR(NBP)+3.*VOR(NB))/8.
      SHRL(I+1)=SHRL(I+1)+DY(I)*(3.*VOR(NBP)+VOR(NB))/8.
17    CONTINUE
      SUM=0.
      DO 18 I=1,IPM
18    SUM=SUM+SHRL(I)
      SUM=ABS(2*SUM/(UTOP**2))
      WRITE(11,*) 'DRAG COEFFICIENT FOR THE LEFT WALL=',SUM
      WRITE(11,*) 'SHEAR ARRAY FOR THE LEFT WALL FOLLOWS'
      DO 19 I=1,IPM
19    WRITE(11,13) I,SHRL(I)
C           TO DETERMINE THE VALUE OF SHEAR ALONG THE RIGHT WALL
      DO 20 I=1,IPMM
      NB=NP(I,JPM)
      NBP=NP(I+1,JPM)
      SHRR(I)=SHRR(I)+DY(I)*(VOR(NBP)+3.*VOR(NB))/8.
      SHRR(I+1)=SHRR(I+1)+DY(I)*(3.*VOR(NBP)+VOR(NB))/8.
20    CONTINUE
      SUM=0.
      DO 21 I=1,IPM
21    SUM=SUM+SHRR(I)
      SUM=ABS(2*SUM/(UTOP**2))
```

```
      WRITE(11,*) 'DRAG COEFFICIENT FOR THE RIGHT WALL=',SUM
      WRITE(11,*) 'SHEAR ARRAY FOR THE RIGHT WALL FOLLOWS'
      DO 22 I=1,IPM
22    WRITE(11,13) I,SHRR(I)
      RETURN
      END
      SUBROUTINE PRINT(PH)
      INCLUDE(COMVORT)
      DIMENSION PH(NODC)
      JST=-5
      KLIP=JPM/6+1
      JREM=MOD(JPM,6)
      DO 7800 K=1,KLIP
      JST=JST+6
      JND=JST+5
      IF(K.EQ.KLIP) THEN
      JND=JPM
      JST=JND-JREM+1
      ENDIF
      WRITE(11,7900) (X(NP(1,J)),J=JST,JND)
      WRITE(11,7903)
      DO 7801 I=IPM,1,-1
7801  WRITE(11,7901) Y(NP(I,1)),(PH(NP(I,J)),J=JST,JND)
7800  CONTINUE
7900  FORMAT(//,6X,'X=---->',6(1PE10.3,1X))
7901  FORMAT('Y=',1PE10.3,' ',6(1PE10.3,1X))
7903  FORMAT(79('-'))
      RETURN
      END
      SUBROUTINE FLUX
      INCLUDE(COMVORT)
      DO 1 I=1,IMAXC
      DO 1 J=1,JMAXC
1     PHI(I,J)=0.
      DO 2 I=2,IM1
      DO 2 J=2,JM1
      PHI(I,J)=UFLX(NP(I-1,J-1))
2     CONTINUE
      DO 4 N=1,NODC
4     RH(N)=DJ(N)
      SUM=0.0
      DO 3 N=1,IPM
      II=N
      JJ=1
      I=II+1
      J=JJ+1
      FLX=A(2,2,N)*PHI(I,J)+A(1,1,N)*PHI(I-1,J-1)+A(1,2,N)*
     1PHI(I-1,J)+A(1,3,N)*PHI(I-1,J+1)+A(2,1,N)*PHI(I,J-1)+A(2,3,N)*
     2PHI(I,J+1)+A(3,1,N)*PHI(I+1,J-1)+A(3,2,N)*PHI(I+1,J)+A(3,3,N)*
     3PHI(I+1,J+1)-RH(N)
      SUM=SUM+FLX
      WRITE(11,2000) N,FLX
      IF(N.GT.1 .AND. N.LT.IPM) EL=(DY(N-1)+DY(N))/2
      IF(N.EQ.1) EL=DY(1)/2
      IF(N.EQ.IPM) EL=DY(IPMM)/2
```

```
      JIN(N)=FLX/EL
 3    CONTINUE
      WRITE(11, ) 'NET FLUX=',SUM
2000  FORMAT(3X,I4,2X,D11.4)
      RETURN
      END
      SUBROUTINE BVT(BVREL)
      INCLUDE(COMVORT)
      DO 1 I=1,IMAXC
      DO 1 J=1,JMAXC
 1    PHI(I,J)=0.
      DO 2 I=2,IM1
      DO 2 J=2,JM1
      PHI(I,J)=PP(NP(I-1,J-1))
 2    CONTINUE
      DO 3 NB=1,NBMAX
      N=NBND(NB)
      JJ=(N-1)/IPM+1
      II=N-IPM*(JJ-1)
      I=II+1
      J=JJ+1
      VNS=APSI(2,2,N)*PHI(I,J)+APSI(1,1,N)*PHI(I-1,J-1)+
     1APSI(1,2,N)*PHI(I-1,J)+APSI(1,3,N)*PHI(I-1,J+1)+APSI(2,1,N)*
     2PHI(I,J-1)+APSI(2,3,N)*PHI(I,J+1)+APSI(3,1,N)*PHI(I+1,J-1)+
     3APSI(3,2,N)*PHI(I+1,J)+APSI(3,3,N)*PHI(I+1,J+1)
      IF((NB.GT.NB1).AND.(NB.LT.NB2)) THEN
      EL=(DX(JJ-1)+DX(JJ))/2.
      VNS=VNS-U(N)*EL
      ELSE IF((NB.GT.NB2).AND.(NB.LT.NB3))THEN
      EL=(DY(II-1)+DY(II))/2
      VNS=VNS+V(N)*EL
      ELSE IF((NB.GT.NB3).AND.(NB.LE.NBMAX))THEN
      EL=(DX(JJ-1)+DX(JJ))/2
      VNS=VNS+U(N)*EL
      ELSE IF((NB.GT.1).AND.(NB.LT.NB1))THEN
      EL=(DY(II-1)+DY(II))/2
      VNS=VNS-V(N)*EL
      ENDIF
      VN=VNS/VOL(N)
      VOR(N)=(VN-VOR(N))*BVREL+VOR(N)
 3    CONTINUE
      RETURN
      END
C
      SUBROUTINE COFSRT(KDF)
      INCLUDE(COMVORT)
      DIMENSION II(3),JJ(3),XU(3),YU(3),Z(10),XB(10),YB(10),
     #          F(10,3),G(10,3),YFXG(3,3),UF(3),VF(3),
     #          UT(10),VT(10)
      DO 2000 N=1,NODC
      DO 2000 I=1,3
      DO 2000 J=1,3
2000  A(I,J,N)=0.0D0
      DO 1000 NE=1,NTETC
      NI=ICA(NE,1)
```

```
      N2=NCA(NL,2)
      N3=NCA(NL,3)
      VE=ALPHA(NL,1)*PP(N1)+ALPHA(NL,2)*PP(N2)+ALPHA(NL,3)*PP(N3)
      UE=-1.*(BETA(NL,1)*PP(N1)+BETA(NL,2)*PP(N2)+BETA(NL,3)*PP(N3))
      U1=UE
      U2=UE
      U3=UE
      V1=VE
      V2=VE
      V3=VE
      XO=(X(N1)+X(N2)+X(N3))/3.0
      YO=(Y(N1)+Y(N2)+Y(N3))/3.0
      GAMMA=(GAM(1,N1)+GAM(1,N2)+GAM(1,N3))/3.0
      UAV=(U1+U2+U3)/3.0
      VAV=(V1+V2+V3)/3.0
      UBAV=SQRT(UAV*UAV+VAV*VAV)
      COST=UAV/UBAV
      SINT=VAV/UBAV
      DO 9300 I=1,3
      XB(I)=(X(NCA(NL,I))-XO)*COST+
     &       (Y(NCA(NL,I))-YO)*SINT
      YB(I)=(Y(NCA(NL,I))-YO)*COST-
     &       (X(NCA(NL,I))-XO)*SINT
9300  CONTINUE
      XB(10)=0.0
      YB(10)=0.0
      XB(4)=(XB(1)+XB(2))/2
      XB(5)=(XB(2)+XB(3))/2
      XB(6)=(XB(3)+XB(1))/2
      XB(7)=XB(4)/2
      XB(8)=XB(5)/2
      XB(9)=XB(6)/2
      YB(4)=(YB(1)+YB(2))/2
      YB(5)=(YB(2)+YB(3))/2
      YB(6)=(YB(3)+YB(1))/2
      YB(7)=YB(4)/2
      YB(8)=YB(5)/2
      YB(9)=YB(6)/2
      CST=RHO*UBAV/GAMMA
      XMX=AMAX1(XB(1),XB(2),XB(3))
      DO 9301 I=1,3
      PE=CST*(XMX-XB(I))
      BIG=AMAX1(0.,(1.-.1*PE)**5)
      Z(I)=BIG/(PE+BIG)
      Z(I)=(Z(I)-1)/CST
9301  CONTINUE
      PE=CST*XMX
      BIG=AMAX1(0.,(1.-.1*PE)**5)
      Z(10)=BIG/(PE+BIG)
      Z(10)=(Z(10)-1)/CST
      Y12=YB(1)-YB(2)
      Y31=YB(3)-YB(1)
      Y23=YB(2)-YB(3)
      Z32=Z(3)-Z(2)
      Z13=Z(1)-Z(3)
```

```
      Z21=Z(2)-Z(1)
      C1=Z(2)*YB(3)-Z(3)*YB(2)
      C2=Z(3)*YB(1)-Z(1)*YB(3)
      C3=Z(1)*YB(2)-Z(2)*YB(1)
      D=Z(1)*Y23+Z(2)*Y31+Z(3)*Y12
      XILI=(Y23*XB(1)+Y31*XB(2)+Y12*XB(3))/D
      XIMI=(Z32*XB(1)+Z13*XB(2)+Z21*XB(3))/D
      XINI=( C1*XB(1)+ C2*XB(2)+ C3*XB(3))/D
      RAV=RHO*UBAV
      DO 9302 I=4,10
      F(I,1)=(RAV*(Z32*YB(I)+C1)-GAMMA*Y23)/D
      F(I,2)=(RAV*(Z13*YB(I)+C2)-GAMMA*Y31)/D
      F(I,3)=(RAV*(Z21*YB(I)+C3)-GAMMA*Y12)/D
      G(I,1)=(-GAMMA*Z32)/D
      G(I,2)=(-GAMMA*Z13)/D
      G(I,3)=(-GAMMA*Z21)/D
 9302 CONTINUE
      DO 9310 J=1,3
      DO 9311 I=1,3
      YFXG(J,I)=((F(J+3,I)+4.*F(J+6,I)+F(10,I))*YB(J+3)
     #         -(G(J+3,I)+4.*G(J+6,I)+G(10,I))*XB(J+3))/6
 9311 CONTINUE
 9310 CONTINUE
      DO 9320 I=1,3
      ESM(1,I)=YFXG(3,I)-YFXG(1,I)
      ESM(2,I)=YFXG(1,I)-YFXG(2,I)
      ESM(3,I)=YFXG(2,I)-YFXG(3,I)
 9320 CONTINUE
      DO 120 NC=1,3
      JJ(NC)=(NCA(NL,NC)-1)/IPM+1
  120 II(NC)=NCA(NL,NC)-IPM*(JJ(NC)-1)
      DO 121 I=1,3
      ND=NP(II(I),JJ(I))
      DO 122 J=1,3
      IG=2-(II(I)-II(J))
      JG=2-(JJ(I)-JJ(J))
      A(IG,JG,ND)=A(IG,JG,ND)+ESM(I,J)
  122 CONTINUE
  121 CONTINUE
 1000 CONTINUE
      RETURN
      END
```

APPENDIX:B

(This section includes a typical output of the computer program for case-1)

	56	13	29	43
1	1.4949D+06	-2.0867D-01	5.3978D-02	0.0000D+00
2	1.3162D+06	-2.1513D+00	-3.2323D-01	0.0000D+00
3	4.0777D+05	-4.4899D+00	-5.9144D-01	0.0000D+00
4	1.9553D+05	-6.7570D+00	-7.7798D-01	0.0000D+00
5	6.4445D+04	-8.7654D+00	-9.7325D-01	0.0000D+00
6	2.9406D+04	-1.0415D+01	-9.6555D-01	0.0000D+00
7	7.5182D+03	-1.1725D+01	-1.0173D+00	0.0000D+00
8	4.1573D+03	-1.2734D+01	-1.0511D+00	0.0000D+00
9	1.3687D+03	-1.3500D+01	-1.0870D+00	0.0000D+00
10	6.1426D+02	-1.4074D+01	-1.1119D+00	0.0000D+00
11	1.9438D+02	-1.4592D+01	-1.1273D+00	0.0000D+00
12	7.2078D+01	-1.4819D+01	-1.1375D+00	0.0000D+00
13	2.7459D+01	-1.5054D+01	-1.1479D+00	0.0000D+00
14	1.4151D+01	-1.5229D+01	-1.1545D+00	0.0000D+00
15	3.9071D+00	-1.5357D+01	-1.1592D+00	0.0000D+00
16	2.2652D+00	-1.5451D+01	-1.1627D+00	0.0000D+00
17	5.7549D-01	-1.5522D+01	-1.1653D+00	0.0000D+00
18	3.5630D-01	-1.5573D+01	-1.1672D+00	0.0000D+00
19	9.2002D-02	-1.5611D+01	-1.1686D+00	0.0000D+00
20	7.1612D-02	-1.5640D+01	-1.1697D+00	0.0000D+00
21	1.6921D-02	-1.5660D+01	-1.1704D+00	0.0000D+00
22	1.4524D-02	-1.5676D+01	-1.1710D+00	0.0000D+00
23	3.7089D-03	-1.5687D+01	-1.1714D+00	0.0000D+00
24	3.2110D-03	-1.5696D+01	-1.1717D+00	0.0000D+00
25	9.6217D-04	-1.5702D+01	-1.1720D+00	0.0000D+00
26	7.7727D-04	-1.5706D+01	-1.1721D+00	0.0000D+00
27	2.9368D-04	-1.5710D+01	-1.1723D+00	0.0000D+00
28	2.3769D-04	-1.5712D+01	-1.1724D+00	0.0000D+00
29	8.4430D-05	-1.5714D+01	-1.1724D+00	0.0000D+00
30	5.7047D-05	-1.5715D+01	-1.1725D+00	0.0000D+00
31	2.8937D-05	-1.5716D+01	-1.1725D+00	0.0000D+00
32	2.0332D-05	-1.5717D+01	-1.1725D+00	0.0000D+00
33	1.2190D-05	-1.5719D+01	-1.1726D+00	0.0000D+00
34	8.7541D-06	-1.5718D+01	-1.1726D+00	0.0000D+00

T E S T . FOR

NGR	IPM	JPM	BVREL	RELV	RELT
1	15	15	0.70	0.30	0.00

BURG ISI
UNIFRM ISF
REYNOLDS NUMBER= 10.0000000
DRAG COEFFICIENT FOR THE TOP WALL = 1.75167965
SHEAR ARRAY FOR THE TOP WALL FOLLOWS

1	-0.1333D+01
2	-0.1270D+02
3	-0.5400D+01
4	-0.3050D+01
5	-0.2540D+01
6	-0.2503D+01
7	-0.2717D+01
8	-0.3170D+01
9	-0.3678D+01
10	-0.4429D+01
11	-0.5786D+01

```
     12   -0.2217D+01
     13   -0.1398D+02
     14   -0.1606D+02
     15   -0.1623D+01
DRAG COEFFICIENT FOR BOTTOM WALL= 0.667816463
SHEAR ARRAY FOR THE BOTTOM WALL FOLLOWS
      1    0.6183D+00
      2    0.4527D+01
      3    0.7634D+01
      4    0.4952D+01
      5    0.3538D+01
      6    0.2780D+01
      7    0.2253D+01
      8    0.1812D+01
      9    0.1395D+01
     10    0.9901D+00
     11    0.6046D+00
     12    0.2758D+00
     13    0.5675D-01
     14   -0.1406D-01
     15   -0.2084D-02
DRAG COEFFICIENT FOR THE LEFT WALL=  1.85959171
SHEAR ARRAY FOR THE LEFT WALL FOLLOWS
      1   -0.1632D+01
      2   -0.1644D+02
      3   -0.1485D+02
      4   -0.9000D+01
      5   -0.6374D+01
      6   -0.4919D+01
      7   -0.3975D+01
      8   -0.3343D+01
      9   -0.2954D+01
     10   -0.2803D+01
     11   -0.2937D+01
     12   -0.3508D+01
     13   -0.5756D+01
     14   -0.1265D+02
     15   -0.1338D+01
DRAG COEFFICIENT FOR THE RIGHT WALL= 0.745080829
SHEAR ARRAY FOR THE RIGHT WALL FOLLOWS
      1   -0.2084D-02
      2   -0.1416D-01
      3    0.5512D-01
      4    0.2672D+00
      5    0.5985D+00
      6    0.9816D+00
      7    0.1430D+01
      8    0.1935D+01
      9    0.2524D+01
     10    0.3244D+01
     11    0.4274D+01
     12    0.5661D+01
     13    0.8582D+01
     14    0.4933D+01
     15    0.6481D+02
```

U-VELOCITY

X=---->	0.000E+00	3.846E-02	1.154E-01	1.923E-01	2.692E-01	3.462E-01
Y= 1.000E+00	0.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
Y= 9.615E-01	0.000E+00	3.522E-01	7.134E-01	8.095E-01	8.510E-01	8.638E-01
Y= 8.846E-01	0.000E+00	9.915E-02	3.196E-01	4.691E-01	5.521E-01	5.852E-01
Y= 8.077E-01	0.000E+00	2.918E-02	1.452E-01	2.471E-01	3.135E-01	3.415E-01
Y= 7.308E-01	0.000E+00	1.105E-02	5.899E-02	1.066E-01	1.386E-01	1.488E-01
Y= 6.538E-01	0.000E+00	2.256E-03	1.092E-02	1.598E-02	1.433E-02	4.499E-03
Y= 5.769E-01	0.000E+00	-3.523E-03	-2.059E-02	-4.717E-02	-7.590E-02	-1.021E-01
Y= 5.000E-01	0.000E+00	-8.172E-03	-4.551E-02	-9.696E-02	-1.459E-01	-1.825E-01
Y= 4.231E-01	0.000E+00	-1.288E-02	-6.994E-02	-1.429E-01	-2.057E-01	-2.459E-01
Y= 3.462E-01	0.000E+00	-1.896E-02	-9.943E-02	-1.923E-01	-2.613E-01	-2.966E-01
Y= 2.692E-01	0.000E+00	-2.900E-02	-1.411E-01	-2.496E-01	-3.122E-01	-3.315E-01
Y= 1.923E-01	0.000E+00	-5.188E-02	-2.054E-01	-3.109E-01	-3.439E-01	-3.352E-01
Y= 1.154E-01	0.000E+00	-1.141E-01	-2.877E-01	-3.327E-01	-3.106E-01	-2.742E-01
Y= 3.846E-02	0.000E+00	-1.819E-01	-2.558E-01	-1.942E-01	-1.483E-01	-1.199E-01
Y= 0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

X=---->	4.231E-01	5.000E-01	5.769E-01	6.539E-01	7.308E-01	8.077E-01
Y= 1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
Y= 9.615E-01	8.605E-01	8.470E-01	8.242E-01	7.900E-01	7.365E-01	6.408E-01
Y= 8.846E-01	5.840E-01	5.578E-01	5.095E-01	4.368E-01	3.325E-01	1.899E-01
Y= 8.077E-01	3.371E-01	3.063E-01	2.524E-01	1.773E-01	8.513E-02	-6.985E-03
Y= 7.308E-01	1.378E-01	1.083E-01	6.393E-02	9.985E-03	-4.285E-02	-7.489E-02
Y= 6.538E-01	-1.339E-02	-3.800E-02	-6.637E-02	-9.268E-02	-1.072E-01	-9.763E-02
Y= 5.769E-01	-1.241E-01	-1.415E-01	-1.525E-01	-1.534E-01	-1.385E-01	-1.039E-01
Y= 5.000E-01	-2.045E-01	-2.126E-01	-2.072E-01	-1.874E-01	-1.523E-01	-1.036E-01
Y= 4.231E-01	-2.628E-01	-2.596E-01	-2.393E-01	-2.039E-01	-1.558E-01	-9.995E-02
Y= 3.462E-01	-3.027E-01	-2.865E-01	-2.532E-01	-2.068E-01	-1.515E-01	-9.342E-02
Y= 2.692E-01	-3.212E-01	-2.914E-01	-2.481E-01	-1.956E-01	-1.386E-01	-8.281E-02
Y= 1.923E-01	-3.061E-01	-2.658E-01	-2.185E-01	-1.669E-01	-1.145E-01	-6.610E-02
Y= 1.154E-01	-2.358E-01	-1.956E-01	-1.564E-01	-1.157E-01	-7.633E-02	-4.171E-02
Y= 3.846E-02	-9.851E-02	-7.977E-02	-6.189E-02	-4.441E-02	-2.793E-02	-1.388E-02
Y= 0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

X=---->	8.846E-01	9.615E-01	1.000E+00
Y= 1.000E+00	1.000E+00	1.000E+00	0.000E+00
Y= 9.615E-01	4.344E-01	1.263E-01	0.000E+00
Y= 8.846E-01	3.504E-02	-2.977E-02	0.000E+00
Y= 8.077E-01	-5.515E-02	-2.700E-02	0.000E+00
Y= 7.308E-01	-6.344E-02	-1.736E-02	0.000E+00
Y= 6.538E-01	-5.977E-02	-1.318E-02	0.000E+00
Y= 5.769E-01	-5.495E-02	-1.102E-02	0.000E+00
Y= 5.000E-01	-5.050E-02	-9.646E-03	0.000E+00
Y= 4.231E-01	-4.655E-02	-8.622E-03	0.000E+00
Y= 3.462E-01	-4.215E-02	-7.658E-03	0.000E+00
Y= 2.692E-01	-3.630E-02	-6.464E-03	0.000E+00
Y= 1.923E-01	-2.781E-02	-4.711E-03	0.000E+00

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Y= 1.154E-01 -1.531E-02 -2.252E-03  0.000E+00
Y= 3.846E-02 -4.160E-03 -4.466E-05  0.000E+00
Y= 0.000E+00  0.000E+00  0.000E+00  0.000E+00
```

V-VELOCITY

X=---->	0.000E+00	3.846E-02	1.154E-01	1.923E-01	2.692E-01	3.462E-01
Y= 1.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
Y= 9.615E-01	1.000E+00	3.519E-01	9.769E-02	3.027E-02	1.087E-02	1.779E-03
Y= 8.846E-01	1.000E+00	7.080E-01	3.240E-01	1.516E-01	6.341E-02	1.454E-02
Y= 8.077E-01	1.000E+00	8.017E-01	4.710E-01	2.558E-01	1.171E-01	2.808E-02
Y= 7.308E-01	1.000E+00	8.423E-01	5.490E-01	3.194E-01	1.517E-01	3.363E-02
Y= 6.538E-01	1.000E+00	8.551E-01	5.759E-01	3.403E-01	1.592E-01	2.618E-02
Y= 5.769E-01	1.000E+00	8.513E-01	5.673E-01	3.261E-01	1.418E-01	6.684E-03
Y= 5.000E-01	1.000E+00	8.357E-01	5.322E-01	2.947E-01	1.054E-01	-2.059E-02
Y= 4.231E-01	1.000E+00	8.091E-01	4.741E-01	2.221E-01	5.679E-02	-4.988E-02
Y= 3.462E-01	1.000E+00	7.694E-01	3.926E-01	1.431E-01	3.792E-03	-7.436E-02
Y= 2.692E-01	1.000E+00	7.120E-01	2.842E-01	5.528E-02	-4.274E-02	-8.597E-02
Y= 1.923E-01	1.000E+00	6.098E-01	1.479E-01	-2.374E-02	-6.694E-02	-7.693E-02
Y= 1.154E-01	1.000E+00	4.070E-01	1.221E-02	-5.713E-02	-5.355E-02	-4.595E-02
Y= 3.846E-02	1.000E+00	1.166E-01	-3.273E-02	-2.470E-02	-1.396E-02	-9.837E-03
Y= 0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

X=---->	4.231E-01	5.000E-01	5.769E-01	6.538E-01	7.308E-01	8.077E-01
Y= 1.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
Y= 9.615E-01	-3.330E-03	-7.321E-03	-1.112E-02	-1.650E-02	-2.627E-02	-4.986E-02
Y= 8.846E-01	-1.549E-02	-3.772E-02	-5.979E-02	-9.883E-02	-1.341E-01	-2.072E-01
Y= 8.077E-01	-3.210E-02	-7.883E-02	-1.241E-01	-1.778E-01	-2.478E-01	-3.308E-01
Y= 7.308E-01	-5.159E-02	-1.195E-01	-1.825E-01	-2.492E-01	-3.176E-01	-3.753E-01
Y= 6.538E-01	-7.295E-02	-1.520E-01	-2.222E-01	-2.881E-01	-3.442E-01	-3.676E-01
Y= 5.769E-01	-9.380E-02	-1.730E-01	-2.397E-01	-2.957E-01	-3.331E-01	-3.320E-01
Y= 5.000E-01	-1.113E-01	-1.811E-01	-2.367E-01	-2.784E-01	-2.987E-01	-2.827E-01
Y= 4.231E-01	-1.224E-01	-1.757E-01	-2.152E-01	-2.431E-01	-2.502E-01	-2.271E-01
Y= 3.462E-01	-1.228E-01	-1.568E-01	-1.816E-01	-1.957E-01	-1.941E-01	-1.701E-01
Y= 2.692E-01	-1.092E-01	-1.252E-01	-1.367E-01	-1.416E-01	-1.359E-01	-1.151E-01
Y= 1.923E-01	-8.037E-02	-8.372E-02	-8.575E-02	-8.695E-02	-8.089E-02	-6.616E-02
Y= 1.154E-01	-4.184E-02	-4.059E-02	-4.047E-02	-3.940E-02	-3.571E-02	-2.798E-02
Y= 3.846E-02	-8.262E-03	-7.744E-03	-7.589E-03	-7.307E-03	-6.487E-03	-4.814E-03
Y= 0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

X=---->	8.846E-01	9.615E-01	1.000E+00
Y= 1.000E+00	0.000E+00	0.000E+00	0.000E+00
Y= 9.615E-01	-1.165E-01	-1.912E-01	0.000E+00
Y= 8.846E-01	-3.103E-01	-2.837E-01	0.000E+00
Y= 8.077E-01	-3.737E-01	-2.261E-01	0.000E+00
Y= 7.308E-01	-3.532E-01	-1.749E-01	0.000E+00
Y= 6.538E-01	-3.118E-01	-1.320E-01	0.000E+00
Y= 5.769E-01	-2.622E-01	-1.124E-01	0.000E+00
Y= 5.000E-01	-2.123E-01	[illegible]	0.000E+00
Y= 4.231E-01	-1.632E-01	-6.415E-02	0.000E+00

Y=	3.462E-01	-1.183E-01	-4.470E-02	0.000E+00
Y=	2.692E-01	-7.697E-02	-2.762E-02	0.000E+00
Y=	1.923E-01	-4.173E-02	-1.354E-02	0.000E+00
Y=	1.154E-01	-1.509E-02	-4.108E-03	0.000E+00
Y=	3.846E-02	-2.288E-03	-4.407E-05	0.000E+00
Y=	0.000E+00	0.000E+00	0.000E+00	0.000E+00

STREAM FUNCTION

X=---->	0.000E+00	3.846E-02	1.154E-01	1.923E-01	2.692E-01	3.462E-01
Y= 1.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
Y= 9.615E-01	0.000E+00	-2.474E-02	-3.415E-02	-3.567E-02	-3.607E-02	-3.608E-02
Y= 8.846E-01	0.000E+00	-3.377E-02	-6.852E-02	-8.384E-02	-9.013E-02	-9.198E-02
Y= 8.077E-01	0.000E+00	-3.521E-02	-8.309E-02	-1.097E-01	-1.226E-01	-1.271E-01
Y= 7.308E-01	0.000E+00	-3.558E-02	-8.898E-02	-1.217E-01	-1.389E-01	-1.452E-01
Y= 6.538E-01	0.000E+00	-3.580E-02	-9.054E-02	-1.253E-01	-1.438E-01	-1.502E-01
Y= 5.769E-01	0.000E+00	-3.565E-02	-8.977E-02	-1.234E-01	-1.406E-01	-1.458E-01
Y= 5.000E-01	0.000E+00	-3.528E-02	-8.704E-02	-1.173E-01	-1.315E-01	-1.342E-01
Y= 4.231E-01	0.000E+00	-3.467E-02	-8.258E-02	-1.078E-01	-1.175E-01	-1.172E-01
Y= 3.462E-01	0.000E+00	-3.379E-02	-7.612E-02	-9.458E-02	-9.912E-02	-9.591E-02
Y= 2.692E-01	0.000E+00	-3.245E-02	-6.690E-02	-7.721E-02	-7.656E-02	-7.119E-02
Y= 1.923E-01	0.000E+00	-3.017E-02	-5.338E-02	-5.501E-02	-5.054E-02	-4.470E-02
Y= 1.154E-01	0.000E+00	-2.499E-02	-3.320E-02	-2.857E-02	-2.367E-02	-1.974E-02
Y= 3.846E-02	0.000E+00	-1.057E-02	-7.403E-03	-4.559E-03	-3.334E-03	-2.649E-03
Y= 0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

X=---->	4.231E-01	5.000E-01	5.769E-01	6.538E-01	7.308E-01	8.077E-01
Y= 1.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
Y= 9.615E-01	-3.586E-02	-3.548E-02	-3.494E-02	-3.417E-02	-3.300E-02	-3.092E-02
Y= 8.846E-01	-9.133E-02	-8.803E-02	-8.525E-02	-7.965E-02	-7.124E-02	-5.805E-02
Y= 8.077E-01	-1.263E-01	-1.216E-01	-1.135E-01	-1.017E-01	-8.512E-02	-6.229E-02
Y= 7.308E-01	-1.438E-01	-1.358E-01	-1.248E-01	-1.079E-01	-8.564E-02	-5.804E-02
Y= 6.538E-01	-1.479E-01	-1.388E-01	-1.241E-01	-1.041E-01	-7.927E-02	-5.089E-02
Y= 5.769E-01	-1.420E-01	-1.314E-01	-1.152E-01	-9.421E-02	-6.945E-02	-4.290E-02
Y= 5.000E-01	-1.283E-01	-1.173E-01	-1.009E-01	-8.076E-02	-5.803E-02	-3.482E-02
Y= 4.231E-01	-1.103E-01	-9.866E-02	-8.339E-02	-6.543E-02	-4.601E-02	-2.697E-02
Y= 3.462E-01	-8.911E-02	-7.723E-02	-6.403E-02	-4.936E-02	-3.403E-02	-1.952E-02
Y= 2.692E-01	-6.354E-02	-5.448E-02	-4.436E-02	-3.355E-02	-2.268E-02	-1.270E-02
Y= 1.923E-01	-3.860E-02	-3.232E-02	-2.582E-02	-1.916E-02	-1.266E-02	-6.860E-03
Y= 1.154E-01	-1.645E-02	-1.347E-02	-1.055E-02	-7.571E-03	-4.913E-03	-2.516E-03
Y= 3.846E-02	-2.153E-03	-1.739E-03	-1.342E-03	-9.477E-04	-5.748E-04	-2.518E-04
Y= 0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

X=---->	8.846E-01	9.615E-01	1.000E+00
Y= 1.000E+00	0.000E+00	0.000E+00	0.000E+00
Y= 9.615E-01	-2.589E-02	-1.107E-02	0.000E+00
Y= 8.846E-01	-3.691E-02	-8.425E-03	0.000E+00
Y= 8.077E-01	-3.318E-02	-5.439E-03	0.000E+00
Y= 7.308E-01	-2.790E-02	-3.999E-03	0.000E+00
Y= 6.538E-01	-2.284E-02	-3.089E-03	0.000E+00

FILE: FIL- RESULTS AT BUET COMPUTER CENTRE, DHAKA VM/SP (4331-L02)

Y=	5.769E-01	-1.993E-02	-2.410E-03	0.000E+00
Y=	5.000E-01	-1.435E-02	-1.851E-03	0.000E+00
Y=	4.231E-01	-1.099E-02	-1.357E-03	0.000E+00
Y=	3.462E-01	-7.762E-03	-9.373E-04	0.000E+00
Y=	2.692E-01	-4.889E-03	-5.576E-04	0.000E+00
Y=	1.923E-01	-2.694E-03	-2.438E-04	0.000E+00
Y=	1.154E-01	-7.995E-04	-3.539E-05	0.000E+00
Y=	3.846E-02	-3.614E-05	2.137E-05	0.000E+00
Y=	0.000E+00	0.000E+00	0.000E+00	0.000E+00

VORTICITY

X=---->	0.000E+00	3.846E-02	1.154E-01	1.923E-01	2.692E-01	3.462E-01
Y= 1.000E+00	0.000E+00	-2.782E+01	-5.827E+00	-3.776E+00	-3.238E+00	-3.215E+00
Y= 9.615E-01	-2.783E+01	-2.136E+01	-8.992E+00	-5.235E+00	-3.977E+00	-3.591E+00
Y= 8.846E-01	-6.340E+00	-8.370E+00	-6.221E+00	-5.429E+00	-4.461E+00	-3.939E+00
Y= 8.077E-01	-4.393E+00	-4.991E+00	-5.073E+00	-4.569E+00	-4.201E+00	-3.821E+00
Y= 7.308E-01	-3.759E+00	-4.094E+00	-4.175E+00	-4.037E+00	-3.770E+00	-3.498E+00
Y= 6.538E-01	-3.601E+00	-3.751E+00	-3.817E+00	-3.644E+00	-3.376E+00	-3.092E+00
Y= 5.769E-01	-3.902E+00	-3.896E+00	-3.759E+00	-3.418E+00	-3.038E+00	-2.658E+00
Y= 5.000E-01	-4.398E+00	-4.304E+00	-3.883E+00	-3.293E+00	-2.728E+00	-2.280E+00
Y= 4.231E-01	-5.120E+00	-4.953E+00	-4.118E+00	-3.154E+00	-2.398E+00	-1.843E+00
Y= 3.462E-01	-6.313E+00	-5.911E+00	-4.398E+00	-2.974E+00	-1.974E+00	-1.334E+00
Y= 2.692E-01	-8.125E+00	-7.253E+00	-4.500E+00	-2.568E+00	-1.354E+00	-6.947E-01
Y= 1.923E-01	-1.121E+01	-9.162E+00	-4.390E+00	-1.672E+00	-3.798E-01	1.885E-01
Y= 1.154E-01	-1.922E+01	-1.143E+01	-2.777E+00	2.245E-01	1.152E+00	1.357E+00
Y= 3.846E-02	-3.394E+01	-7.984E+00	3.319E+00	3.391E+00	3.366E+00	2.342E+00
Y= 0.000E+00	0.000E+00	1.286E+01	1.001E+01	6.154E+00	4.508E+00	3.591E+00

X=---->	4.231E-01	5.000E-01	5.769E-01	6.538E-01	7.308E-01	8.077E-01
Y= 1.000E+00	-3.504E+00	-4.015E+00	-4.744E+00	-5.779E+00	-7.369E+00	-1.018E+01
Y= 9.615E-01	-3.627E+00	-3.983E+00	-4.469E+00	-5.290E+00	-6.509E+00	-8.446E+00
Y= 8.846E-01	-3.738E+00	-3.747E+00	-3.984E+00	-4.349E+00	-4.778E+00	-4.951E+00
Y= 8.077E-01	-3.590E+00	-3.471E+00	-3.439E+00	-3.443E+00	-3.265E+00	-2.486E+00
Y= 7.308E-01	-3.246E+00	-3.052E+00	-2.858E+00	-2.583E+00	-2.066E+00	-9.828E-01
Y= 6.538E-01	-2.823E+00	-2.565E+00	-2.279E+00	-1.856E+00	-1.195E+00	-1.352E-01
Y= 5.769E-01	-2.372E+00	-2.063E+00	-1.710E+00	-1.247E+00	-5.962E-01	3.038E-01
Y= 5.000E-01	-1.911E+00	-1.572E+00	-1.209E+00	-7.661E-01	-2.018E-01	5.016E-01
Y= 4.231E-01	-1.434E+00	-1.025E+00	-7.614E-01	-3.876E-01	5.315E-02	5.628E-01
Y= 3.462E-01	-9.174E-01	-6.141E-01	-3.489E-01	-7.609E-02	2.242E-01	5.486E-01
Y= 2.692E-01	-3.170E-01	-9.736E-02	6.158E-02	2.053E-01	3.564E-01	5.105E-01
Y= 1.923E-01	4.173E-01	4.220E-01	5.021E-01	4.918E-01	4.813E-01	4.746E-01
Y= 1.154E-01	1.324E+00	1.179E+00	9.844E-01	7.986E-01	6.114E-01	4.467E-01
Y= 3.846E-02	2.382E+00	1.954E+00	1.535E+00	1.123E+00	7.332E-01	3.988E-01
Y= 0.000E+00	2.917E+00	2.331E+00	1.814E+00	1.284E+00	7.772E-01	3.404E-01

X=---->	8.846E-01	9.615E-01	1.000E+00
Y= 1.000E+00	-1.698E+01	-3.335E+01	0.000E+00
Y= 9.615E-01	-1.127E+01	-6.346E+00	1.743E+01
Y= 8.846E-01	-3.575E+00	3.393E+00	1.140E+01

FILE: FILE, RESULTS AT BUET COMPUTER CENTRE, DHAKA VM/SP (4381-[illegible])

Y=	8.077E-01	-2.693E-01	4.439E+00	7.352E+00
Y=	7.308E-01	1.002E+00	3.931E+00	5.392E+00
Y=	6.538E-01	1.432E+00	3.290E+00	4.179E+00
Y=	5.769E-01	1.453E+00	2.695E+00	3.261E+00
Y=	5.000E-01	1.303E+00	2.130E+00	2.504E+00
Y=	4.231E-01	1.107E+00	1.620E+00	1.849E+00
Y=	3.462E-01	8.710E-01	1.154E+00	1.257E+00
Y=	2.692E-01	5.990E-01	7.398E-01	7.539E-01
Y=	1.923E-01	4.5[illegible]E-01	3.926E-01	3.296E-01
Y=	1.154E-01	3.054E-01	1.501E-01	4.734E-02
Y=	3.846E-02	1.600E-01	2.448E-02	-4.335E-02
Y=	0.000E+00	4.885E-02	-4.334E-02	0.000E+00