AN INVENTORY ANALYSIS FOR DETERIORATING ITEMS DEPLOYING PRESERRVATION TECHNOLOGY WITH TIME DEPENDENT QUADRATIC DEMAND FUNCTION

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A part time student of M. Phil. (Mathematics) has been accepted as satisfactory in partial fulfilment of the degree of Master of Philosophy in Mathematics
On February 03, 2021

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DEDICATION

This work is dedicated to

Prof. Dr. Mohammed Forhad Uddin

For his advice, his patience, and his faith,

Because he always understood.

ABSTRACT

The effect of deterioration is very important in many inventory systems. Most of the physical goods undergo decay or deterioration over time. Preservation technology is a system which helps to reduce the rate of deterioration of the deteriorating items. The preservation technology has a significance and it can increase the life of some products.

In the present study, two individual model Inventory Model Without Preservation Technology (IMWOPT) and Inventory Model With Preservation Technology (IMWPT) have been formulated. The model IMWOPT consists of ordering cost, holding cost and purchase cost. On the other hand, the model IMWPT consists of ordering cost, holding cost, purchase cost, reduced deterioration rate and preservation technology cost. Both of these model has been developed by the sense of time dependent quadratic demand function and variable deterioration rate. The reason for considering the variable deterioration rate and time dependent demand rate is due to change in deterioration rate with respect to time. The development of these models is to minimize the total average cost per unit time. In order to validate the models, numerical examples have been considered. Then the sensitivity of several major parameters quadratic functions is analyzed using MATLAB.

The optimal solutions with the effects of major parameters have been represented graphically and in tabular form. From the numerical results of IMWOPT and IMWPT, it is evident that the values of parameters of quadratic demand function, ordering cost, holding cost, purchase cost and deterioration rate per unit time increase and decrease then the total cost per unit time increase and decrease respectively. Consequently, from the numerical results of IMWPT the values of reduced deterioration rate per unit time increase then the total cost per unit time decrease as well as the values of reduced deterioration rate per unit time decrease then the total cost per unit time increase and due to increase and decrease of preservation technology cost per unit time total cost per unit time increase and decrease respectively. Finally, comparison of both model has been discussed. It is observed that IMWPT is better than IMWOPT model.

Author's Declaration

I hereby announce that the work which is being presented in this thesis entitled

"AN INVENTORY ANALYSIS FOR DETERIORATING ITEMS DEPLOYING PRESERRVATION TECHNOLOGY WITH TIME DEPENDENT QUADRATIC DEMAND FUNCTION"

submitted in partial fulfillment of the requirements for the decoration of the degree of M.Phil., department of mathematics, BUET, Dhaka, is an authentic record of my own work.

The work is also original except where indicated by and attached with special references and figures in the context.

All views expressed in the dissertation are those of the author and in no or by no means represent those of Bangladesh University of Engineering and Technology (BUET), Dhaka. This dissertation has not been submitted to any other university for examination either in home or abroad.

(MD. ATIQUR RAHAMN)

Date: February 03, 2021

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(**MD. ATIQUR RAHMAN**)

Date: February 03, 2021

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T : length of replenishment cycle in traditional system (per year)

$I(t)$: inventory level at time t $(0 < t < T)$

A : ordering cost of inventory per order ($TK/order$)

h : unit holding cost per unit time

p : purchase cost per unit of item

$D(t)$: time dependent demand rate which is defined by

D (t) = $a + bt + ct^2$, $a > 0$, $b \neq 0$ & $c \neq 0$. Here a is the initial rate of demand, b is the rate with which the demand rate increases and the rate of change in the demand rate itself changes at a rate c.

$\theta\ (t)$: variable rate of deterioration of an item where $\theta(t) = \theta t^2$; $0 < \theta < 1$

IM : maximum inventory level during $[0, T]$

TC : average cost per unit time

M : reduced deterioration rate

τ_ρ : resultant deterioration rate which is considered by

$$\tau_\rho = (t) - m = \theta t^2 - m$$

PT : preservation technology

IMWOPT : Inventory model without preservation technology

IMWPT : Inventory model without preservation technology

Chapter 1

Introduction

1.1 Inventory

In present aggressive world, the capital problem and basic responsibility of an organization, whether it is a public sector, private sector or government section is to optimize the use of available resources. For the survival and magnification of an industrial enterprise, it is highly required that all the pervasive efforts made to minimize and dominance the total costs, to achieve operational efficiency and profitability of accompany. Inventory is a key resource of an enterprise. Inventory can be considered as a stock or store of goods or services, kept for use or sale in the future. The term 'inventory' refers to the stockpile of presentation a firm is offering for sale and the components that make up the production. Inventory investment is a portion of gross domestic product. The dissimilarity between goods produced and goods sold in a given year is to be known inventory investment. Inventory is always dynamic. A business can forward smoothly its operating activities only when suitable amount of inventory is controlled. Fundamentally, inventory dominates all operating functions such as sales, manufacturing, warehousing etc. Inventory takes a required action in operation management. Inventory management requires constant and careful evaluation of external and internal component and dominance through planning and review. Most of the organizations have an individual department or job function called inventory planners who continuously monitor, control and review inventory and interface with production, procurement and finance departments. In the supply chain one of the vital variables which has to be managed is inventory. The inventory involve a vast spectrum of materials that is being transferred, stored, consumed, produced, packaged, or sold in one way or another during a firm`s normal course of business. The planning, storing, moving and accounting for inventory is the mandatory for all logistics. Finally, principal purposes of inventory are maintain independence of separate operations, fulfill change of product demand, allowing the flexibility in production scheduling and considering the superiority of economic purchase order size.

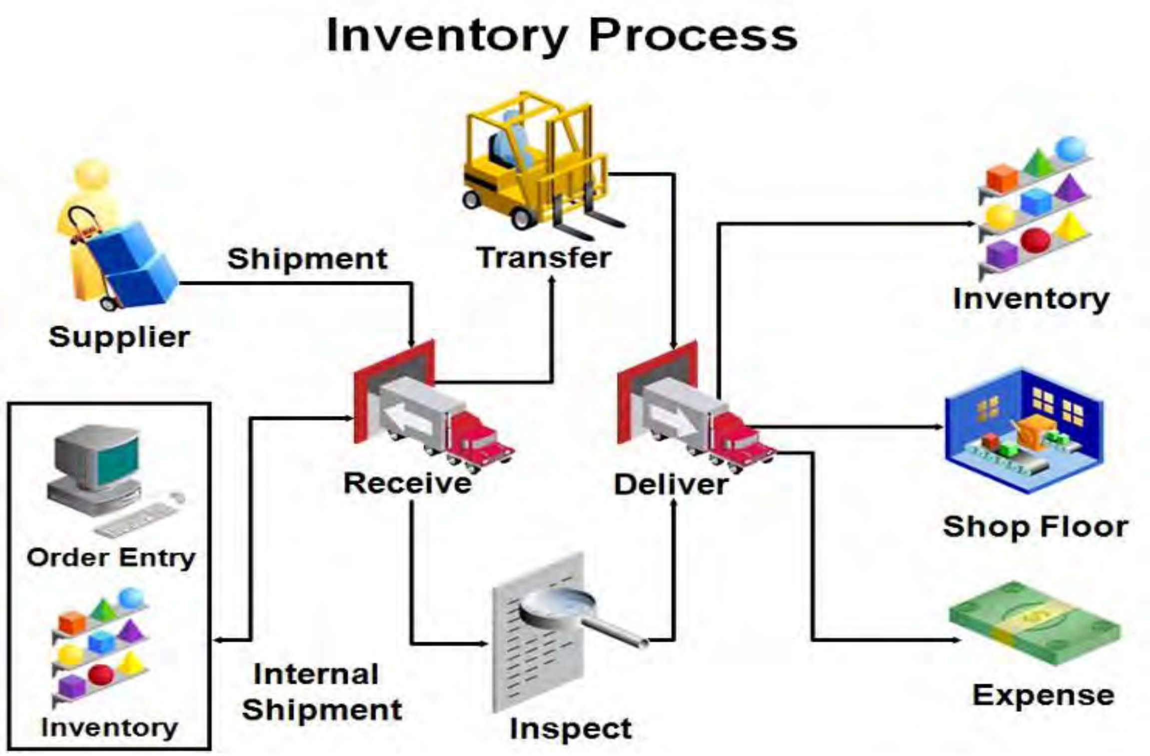

Figure 1.1 Inventory process from initial stage to last stage

1.2 Nature of Inventory

Inventory is an asset which is owned by a business that has the express purpose of being sold to a customer. Inventory of materials occurs at individual stages and departments of an organization. A manufacturing organization holds inventory of raw materials and consumables requisite for production. Further both row materials and finished goods those which are in transit in different area also form a section of inventory depending upon who own the inventory at the distinct juncture. Generally, inventory can be classified following four ways:

- Raw Materials and Supplies

Raw materials are unfinished inventory items which are used in the manufacturer's conversion process to produce components or finished products. These types of inventory items may be objects or elements that the farm has purchased from outside the organization. However, items such as nuts and bolts, ball bearing, casters wheels may be considered as raw materials.

- Work-in-Progress

Work-In-Progress is considered of all the materials, parts, semi-finished goods that are waiting to be prepared within the structure. Broadly, it contains each material from row material that has been released from starting processing up to the material that has been absolutely processed.

- Finished Goods

A finished good mention a completed portion that is prepared for a customer order. Therefore, it indicates finished goods inventory is the stock of completed goods. In essence, finished goods can be sold directly to their final user, sold to trailers, sold to wholesalers, sent to distribution centers.

- MRO Goods Inventory

MRO as an abbreviation Maintenance, repair and operating suppliers. MRO goods are items that are used to assist and preserve the production process i.e. its infrastructure. MRO goods are usually consumed as a result of the production process. As standard of MRO goods can be considered oils, lubricants, gloves, nuts, copier paper, toner etc.

1.3 Significance of Holding Inventory

Inventory is one of the important credits of a business. Inventory is mandatory for each business sector to ensure smooth running of the production system, to reduce the ordering cost of inventory, to take benefit of quantity discount, to utilize and optimize the plant capacity and to reduce the overall price. Finally, it can be said that inventory is invertible and has to be maintained in applicable quantity. The following is a list of reasons neither necessarily exhaustive nor mutually exclusive, through nearly related:

- Protection in opposition to uncertainties in Transit and handling
- To give customer guarantee of availability
- To hedge against expected surges in sales
- To await shipment to fill a definite order
- To handle production variations
- To make materials in economic lot sizes

1.4 Inventory Management

The idea of Inventory Management has been imagined differently by different authors, academicians and researchers on the subject. In an attempt to gain an insight in inventory management, it is found imperative to know different opinions on the subject. Inventory management is predominant scientific device for controlling inventory and eliminating wastage, is considered an integral part of Industrial management in present times. In other words, Inventory Management is the practice overseeing and controlling of the ordering, storage and importance of components that a company uses in the production of the goods which is to be sold. There are lot of possibilities to minimize inventory, both in terms of investment as well as quality, as it is a controllable decision variable. Inventory management is one of the major planning and control challenges facing managers today, especially in manufacturing facilities. The target of inventory management is to hold inventories at the minimum possible cost, given the objectives to make sure uninterrupted supplies for ongoing operations. When making decisions on inventory, management has to find a compromise between the individual cost components, such as the costs of supplying inventory, inventory holding costs and costs resulting from the short supply of inventories. Inventory investment should also be balanced with customer service which is related with changing production level, cost of placing orders and transportation cost. The benefit of carrying inventories should exceed the costs of carrying the inventories or else the purpose of carrying inventories is useless.

Every management problem can be defined as a decision problem. Decision is an essential task that all organizations have to take. The allocation of resource is a regular concern to all organizations. Organizations have to acquire, allocate and control the feature of production which is important for the attainment of the business's objectives. Operational research (OR) cover a wide range of problem-solving processes and methods applied in the pursuit of upgrade decision-making. Mathematical model can assist to make the unique decisions, among the possible alternatives. Operation research has been turned The Science of Batter. Individual methods of operations research are well adapted to such decision making in business sector. Operation research can be applied different sectors such as Management Science, Logistics Management, Supply Chain Management and Operations Management.

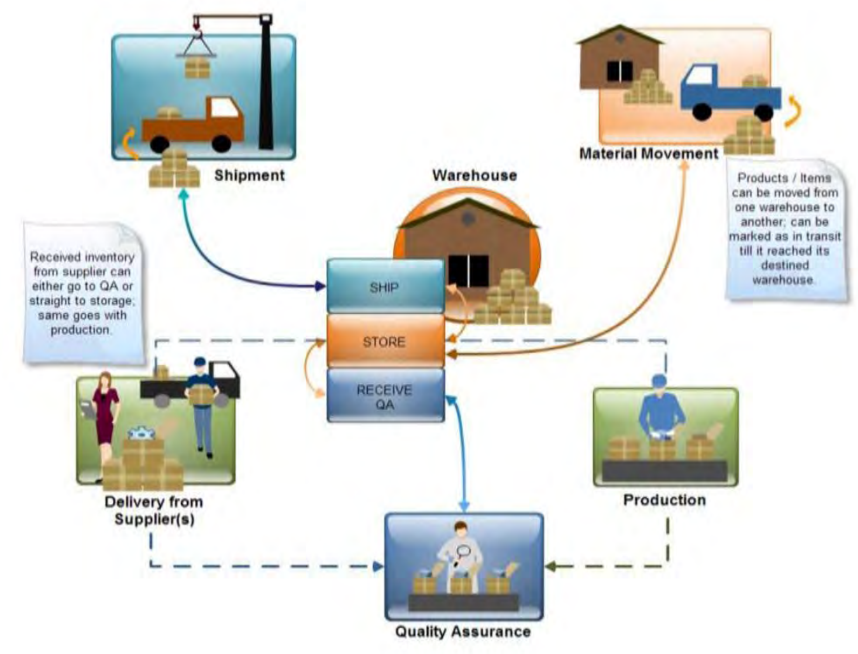

Figure 1.2 Inventory Management flow chart

1.4.1 Objectives of Inventory Management

The foremost objective of inventory management is to control at an appropriate stage to avoid excess or shortage of inventory. Inventory Management dominance system minimized inventory carrying cost. For the most part, it secure that the supply of raw material and finished goods remains continuous throughout the business operations. The purposes may be bisected into two categories such as operating objectives and financial objectives. Firstly, operating objectives are related to the operating activities of the business like purchase, production, sales etc. It also ensure continuous supply of material and ensure uninterrupted production system. Secondly, financial design helps to minimize the capital investment in the inventory and reduce inventory coats.

1.5 Economic Order Quantity (EOQ)

Economic order quantity is one of the essential course of function used to obtain the optimum quantity or number of orders to be placed from the suppliers. The Economic Order Quantity (EOQ) is the number of units that a firm should include to inventory with each order to minimize the total costs of inventory—such as holding costs, order costs, and shortage costs. The basic Economic Ordering Quantity (EOQ) model attempt to balance two basic costs of inventory, those being the cost of ordering and the cost of holding inventory. The EOQ is very useful tool for inventory control it may be applied to finished goods inventories, work-in-progress inventories and raw material inventories. Economic order quantity (EOQ) can be calculated by the following formula:

$$EOQ = \sqrt{\frac{2\,A\,B}{C\,S}}$$

where EOQ = Economic Order Quantity

A = Annual Consumption

B = Buying cost per order

C = Cost Per Unit

S= Storage and Carrying Cost per Annum

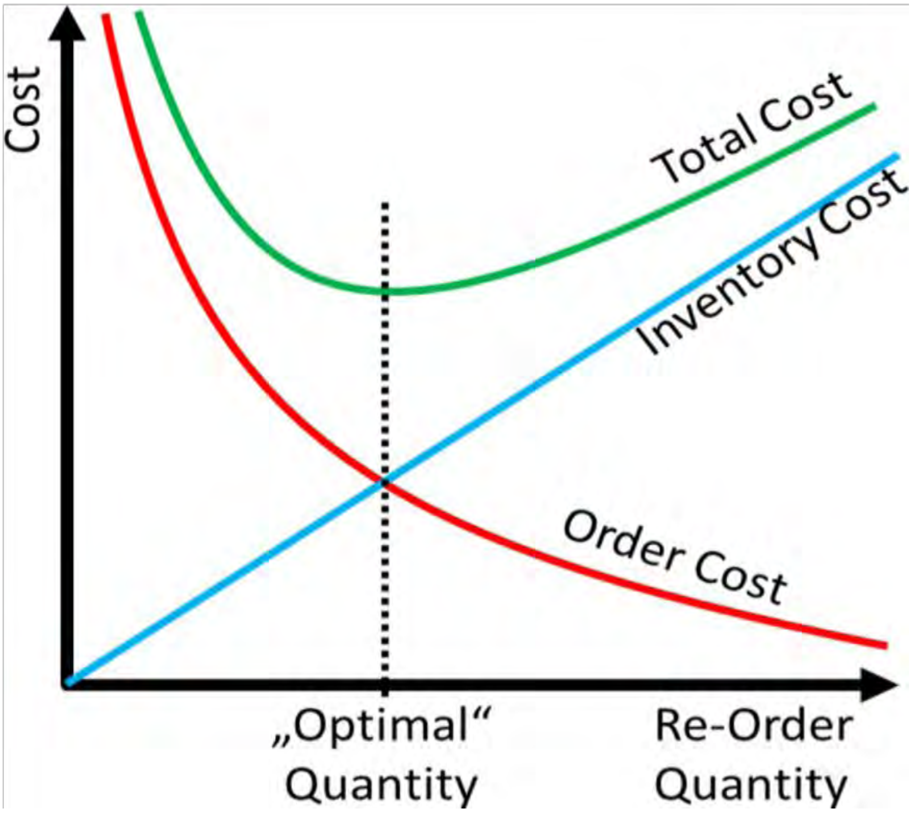

Figure 1.3 Economic Order Quantities

1.6 Economic Production Quantity (EPQ) Model

The economic production quantity (EPQ) model has been widely used in practice. Economic Production Quantity (EPQ) model is significantly used in industries and academics to study the optimal production lot size that minimizes overall production as well as inventory storage costs. This method is the extension of the Economic Order Quantity model. Mainly, the Economic Production Quantity (EPQ) model assumes the company will produce its own quantity or the parts are going to be shipped to the organization while they are being produced, therefore the orders are available or received in an incrementally manner while the products being produced. On the other hand, Economic Order Quantity model assumes the order quantity arrives complete and immediately after ordering, meaning that the parts are produced by another company and are ready to be shipped when the order is placed. Mathematically the EPQ model is represented as below:

$$EPQ = \sqrt{\frac{2 * D * C_0 * P}{(C_1 * (P - D))}}$$

Where D = Annual demand of the product

P = Annual rate of production

C_0 = Fixed cost per setup or the set up cost

C_1= Inventory holding cost per unit

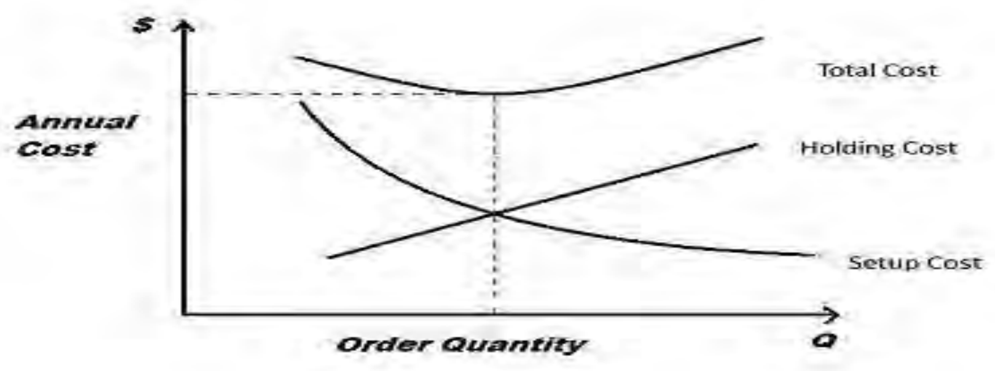

Figure 1.4 Economic Product Quantity model

1.7 Demand

A large number of consumers are willing to buy products based on a different factors which is known to be a variable. Among these variables own price is the key factor. Generally, economist observe that if the price of a good rises , buyers will like to buy less of these product as well as when its price falls the customer buy more. This observation can be called the law of demand. Also economist attempt to accumulate all of these variables for which demand influences and form a relationsonship among these variables can be defined as a demand function. In general, a function is a relationship that assigns a unique value to a dependent variable for any given set of values of a group of independent variables.

Demand is a relation of two different things which is showing the quantities of goods that consumers are willing and able to buy at various prices per period, other things constant. On the other hand, demand indicates rate at which consumers want to buy a product. Economic theory holds that demand consists of two factors: taste and ability to buy. Taste, which is the desire for a goods, determines the willingness to buy the goods at a specific price to fulfill the daily need. Ability to buy means that to buy a good at specific price, an individual must depend on sufficient wealth or income. Both factors of demand depend on the daily market price. When the market price for a product is high, the demand will be low. When price is low, demand is high. At very low prices, many consumers will be able to purchase a product. Acquiring additional increments of a good or service in some time period will yield less and less satisfaction. As a result, the demand for a product at low prices is limited by taste and is not infinite even when the price equals zero. As the price increases, the same amount of money will purchase fewer products. When the price for a product is very high, the demand will decrease because, while consumers may wish to purchase a product very much, they are limited by their ability to buy.

Any supply chain has only single point of independent demand or the amount of product demanded (by time and location) by the end-use customer of the supply chain. Whether this end-use customer is a consumer shopping in a retail establishment or online (B2C), or a business buying products for consumption in the process of conducting their business operations (B2B), these end-use customers determine the true demand for the product that will flow through the supply chain. The company in the supply chain that directly serves this end-use customer directly experiences this independent demand. All subsequent companies in the supply chain experience a demand that is tempered by the order fulfillment and purchasing policies of other companies in the supply chain. This second type of supply chain demand is called derived demand because it is not the independent demand of the end-use customer but rather a demand that is derived from what other companies in the supply chain do to meet their demand from their immediate customer (i.e., the company that orders from them). The derived demand for one company is often the dependent demand of their customers. Dependent demand is the demand for the component parts that go into a product.

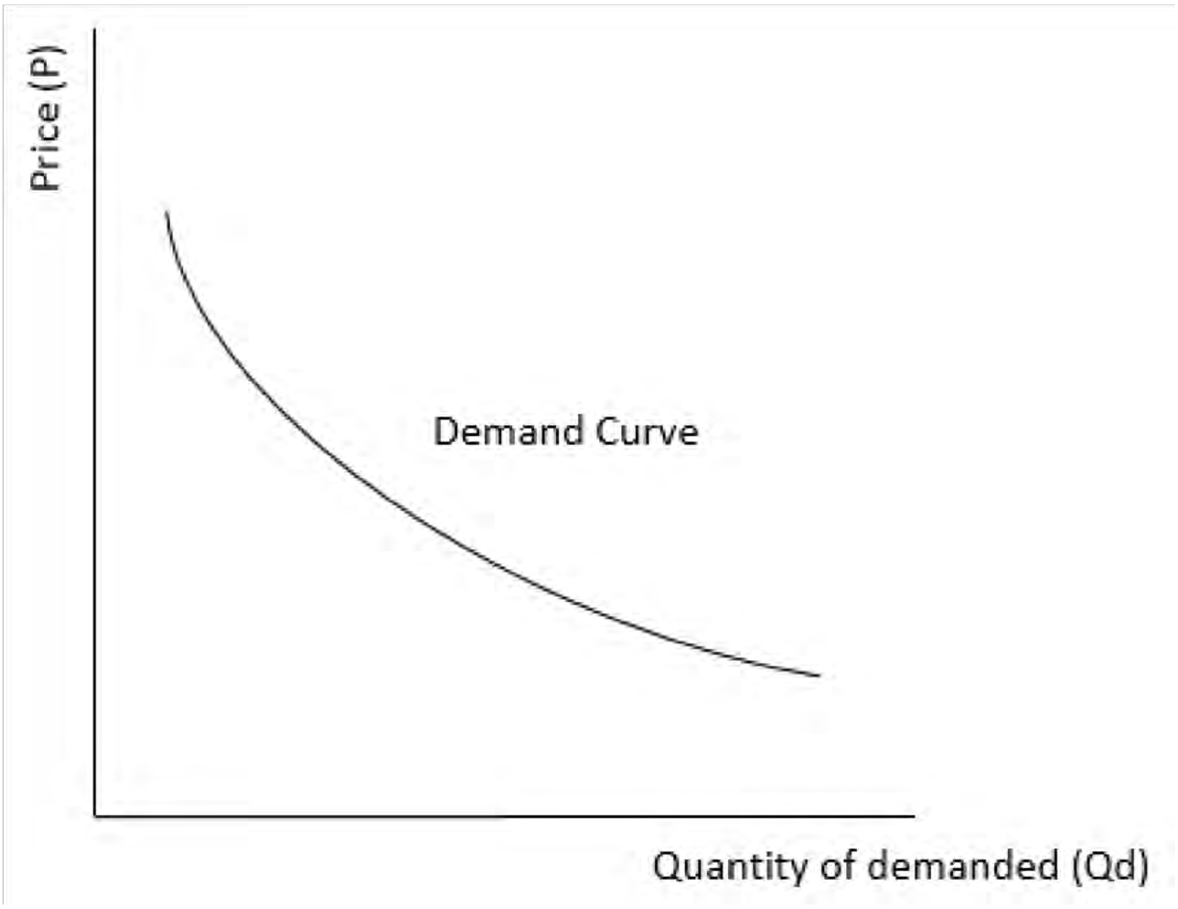

Figure 1.5 Demand curve

1.8 Types of Demand

Demand could be classified as below:

1.8.1 Deterministic Demand

Deterministic demand is formed such a way that decision variables related with demand functions are known and predictable i.e. conditions can be assumed. Deterministic demand can be divided into two parts such as static and dynamic. Firstly, static indicates which does not have any variation i.e. the amount of demand is predetermined. Secondly, dynamic which may be varied due to different factors.

1.8.2 Probabilistic Demand

A probabilistic approach is based on the theory of probability or the fact that randomness plays a vital role in predicting future steps. Probabilistic demand has attracted more attention due to its practical application. In modern business due to growing uncertainty the probabilistic demand is realistic. Deterministic demand is formed such a way that all variables related with demand functions are unpredictable.

1.8.3 Linear Demand

Linear demand is critical to learning the basics of how a market works and running a successful business. Being able to use a demand is almost like telling the future because it predicts consumer behavior. However, in real field, most curves are non-linear. That's why; constantly need to analyze demand for products. A linear demand is the graphical representation of the relationship between the price of a good and the quantity of those good consumers is willing to pay at a certain price at a point in time. The slope or rate that the line rises or falls is equal to the difference between two quantities of a product usually represented on the horizontal axis on the graph divided by the difference price of two points of the graph on the vertical axis.

1.8.4 Non Linear Demand

Non-linear functions are functions whose graphs are not straight lines. A nonlinear demand curve is one which is not straight. This means that decrease in price does not lead to an equal increase in the quantity demanded. The graph of a non-linear function indicates a curved line. A curved line is a line whose direction constantly changes. When the demand curve is non-linear then the slope moves with the price. It means that equal price changes do not lead to equal quantity changes. While there are several types of non-linear functions, this course will focus on three that are commonly used in business sector: parabolic functions, demand functions, and exponential functions. A basic graphic representation of each of these functions is shown below.

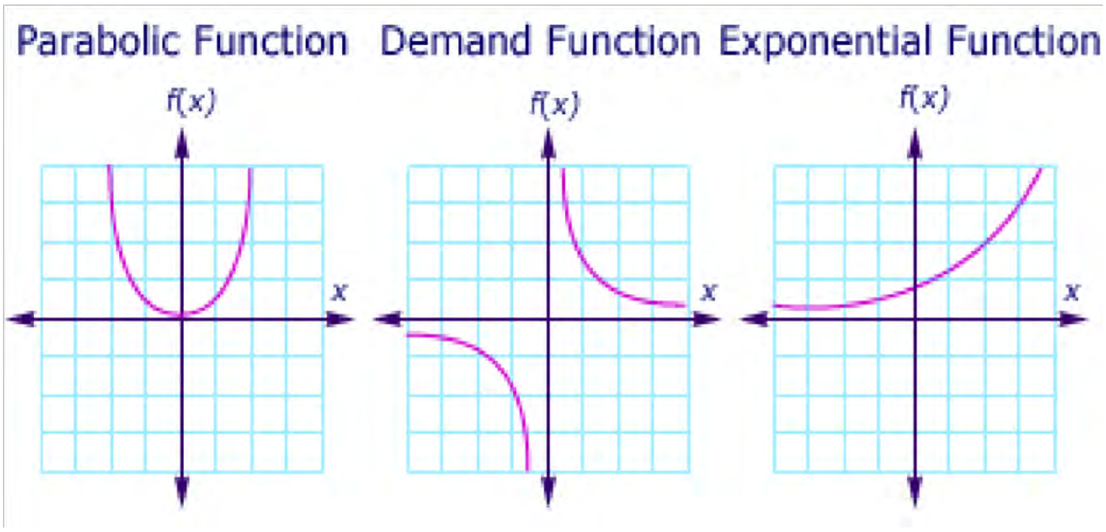

Figure 1.6 Different types of non-linear functions

1.8.4.1 Parabolic Functions

A parabolic function is defined as a symmetric which is a U-shaped function. This function has a term x^2 as its highest term. The standard form of this function is organized into an equation below on the right.

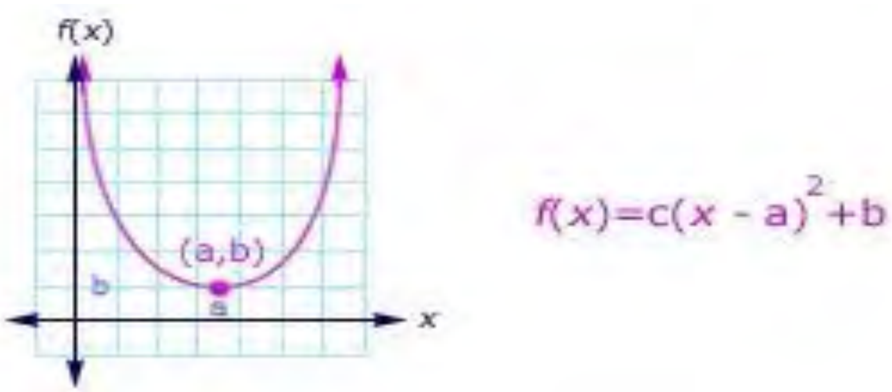

Figure 1.7 Upward shape of parabolic function

This form has basic importance because it indicates a number of things about the shape of the parabolic function which illustrated above on the left. This form allow pictorial representation of the function which indicates several key features. The direction of the parabola is determined by the constant c which can be described below:

- As in the above graph, the shape of parabola opens upward if the constant term c can be considered positive which allow the shape of a U i.e. upward open.
- As in the above graph, the shape of parabola opens downward if the constant term c can be considered positive which allow the shape of an inverted U i.e. downward open.

Let us consider the following production function, in which a company's marginal output in units per employee, $F(x)$, depends on how many workers are employed, x.

$$F(x) = -0.5\ (x - 5)^2 + 10$$

For the above function, we can define it as a parabola because its highest term, $(x - 5)^2$, is squared. A number of assumptions can be determined from the functions formula.

- For this function the value of the constant c is –0.5 which is negative. Hence, the parabola opens downward i.e. the shape of an inverted U.
- On the other hand, the absolute value of constant c, –0.5, is between 0 and 1 i.e. the parabola is relatively wide.
- The coordinates of vertex of the parabolas is (5, 10) i.e. it indicates the highest point. The vertex is the point at which marginal production can be maximized. It describes that when 5 people are employed then 10 units of output are produced per employee.

This function is graphed below.

Figure 1.8 Downward shape of parabolic function

1.8.4.2 Demand Functions

A demand function has a standard form $\frac{1}{x}$, which can also be written as x^{-1}. The demand function used often in business to describe demand; it is represented in equation form below on the right.

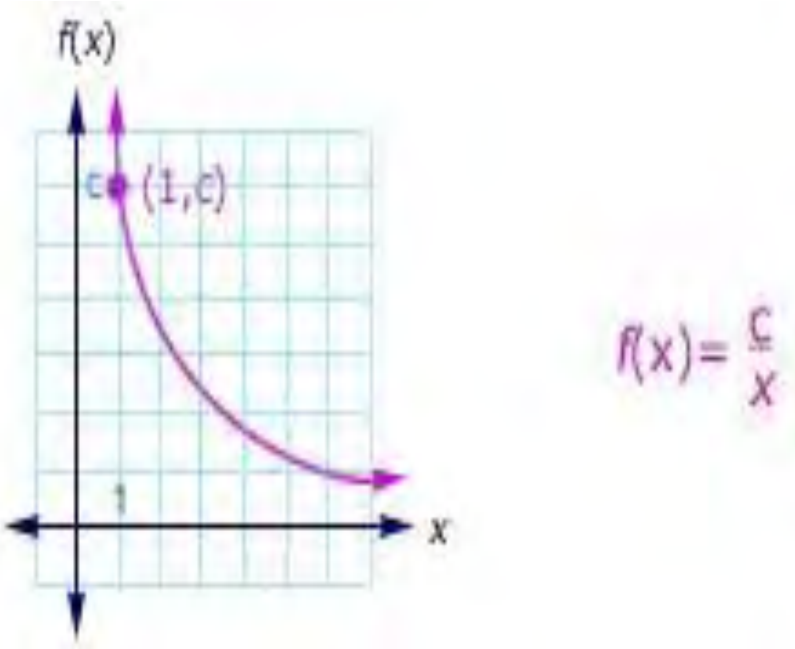

Figure 1.9 Shape of demand function $f(x) = \frac{c}{x}$

This form of the function describes few things about the shape and behavior of the demand function, which is illustrated above on the left. Graphically, the constant c tells how close the graph will be to the *x* and *y* axes. The smaller c is, the closer the graph is to the origin, the point (0, 0).

In general, demand functions will approach but will never cross the *x* or *y* axes.

Consider the following production function, in which the demand for a company's cookies. *f*(*x*), depends on the price of the cookies, *x*.

$$f(x) = \frac{2}{x}$$

When graphed on a coordinate plane, demand functions have two parts that are mirror images of each other. In business sector, only the section in the first quadrant is used because the values being examined are positive. For example, the demand for cookies and the price of cookies would not be negative. Therefore, the company's demand function for cookies would be illustrated in this way.

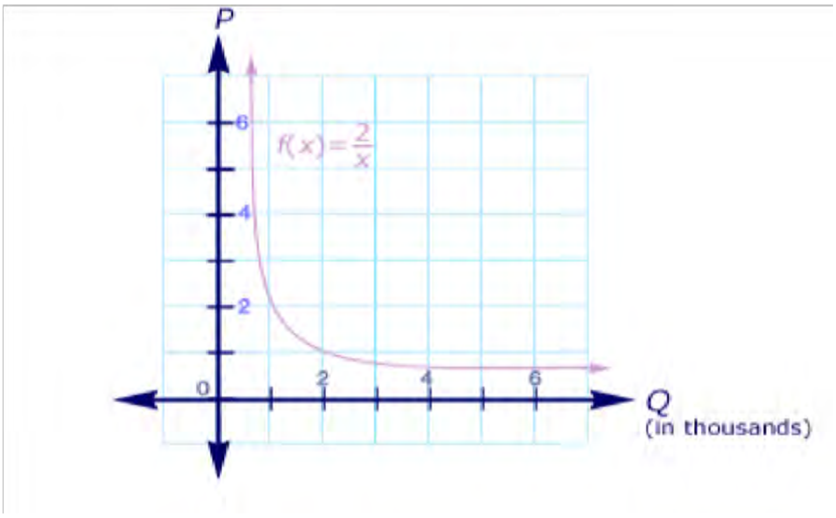

Figure 1.10 Shape of demand function $f(x) = \frac{2}{x}$

The constant, 2, determines how close the graph is to the *x* and *y* axes. If the demand for cookies increases, the constant in the demand function would increase. How would this change the shape of the demand function? Consider the following graph, which shows the original demand function, where c is 2, and the increased demand function, where c is 5.

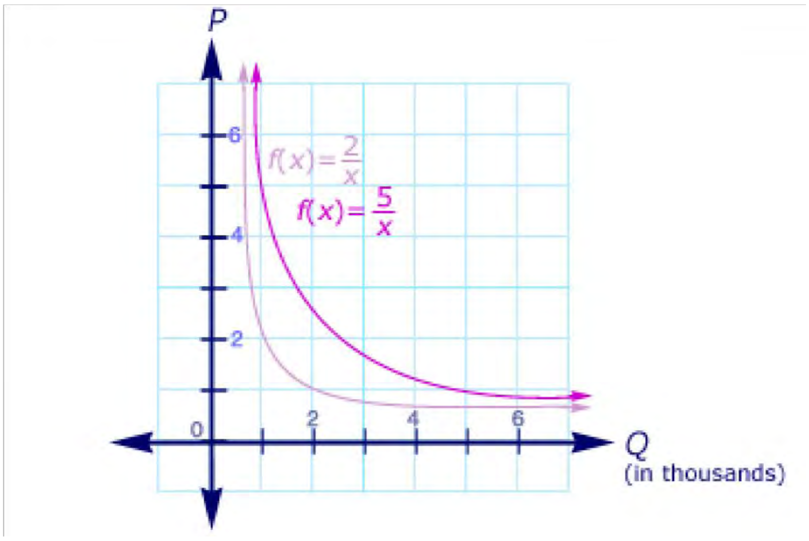

Figure 1.11 Shape of demand function under comparison $f(x) = \frac{2}{x}$ and $f(x) = \frac{5}{x}$

1.8.4.3 Exponential Functions

Any function where a constant is raised to a power of x is an exponential function. Exponential functions in business take the form of exponential growth and decay functions. In business scenarios, exponential functions often appear in the equation form that appears below on the right.

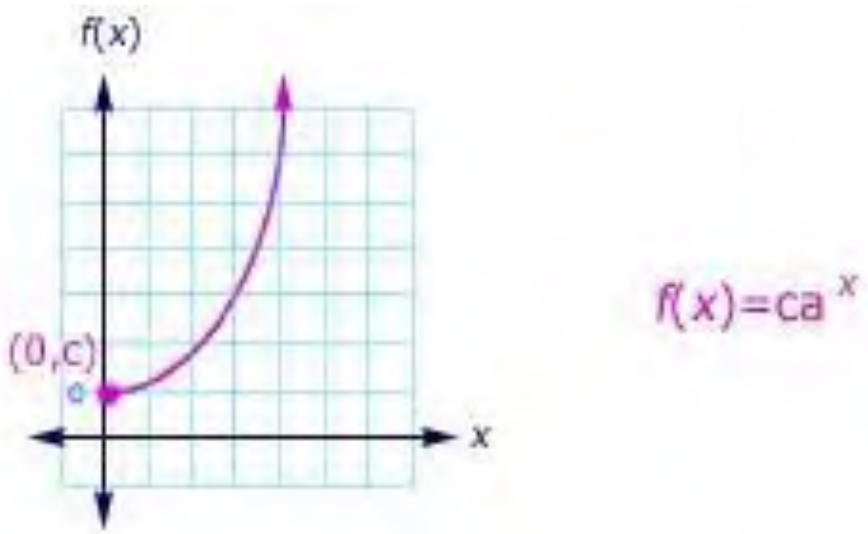

Figure 1.12 Shape of exponential function $f(x) = ca^x$

This form tells two important things about the function's shape, which is graphed above on the left.

- The point (0, c) is the function's *y*-intercept.
- The sign of the exponent *x* (positive or negative) indicates the direction of the function.
- If the exponent is positive, the function increases to the right. This is known as "exponential growth."
- If the exponent is negative, the function decreases to the right. This is known as "exponential decay."

The exponential function is commonly used for investments with compounded interest. For example, imagine that you put $10 into an account that compounds annually at a rate of seven percent. The function used to find the value of this investment at a future point in time is below.

$$P(x) = FV = 10(1.07)^N$$

FV = the future value, N= the number of years in future

Simply by looking at this function, it can be determined that it is an exponential function because it has a constant, 1.07, raised to a power *N*. Using this function, we can quickly think two points about its shape.

- Because the constant c is 10, the function's *y*-intercept is (0, 10).
- The exponent *N* is positive, which means the function is increasing as *N* increases to the right.

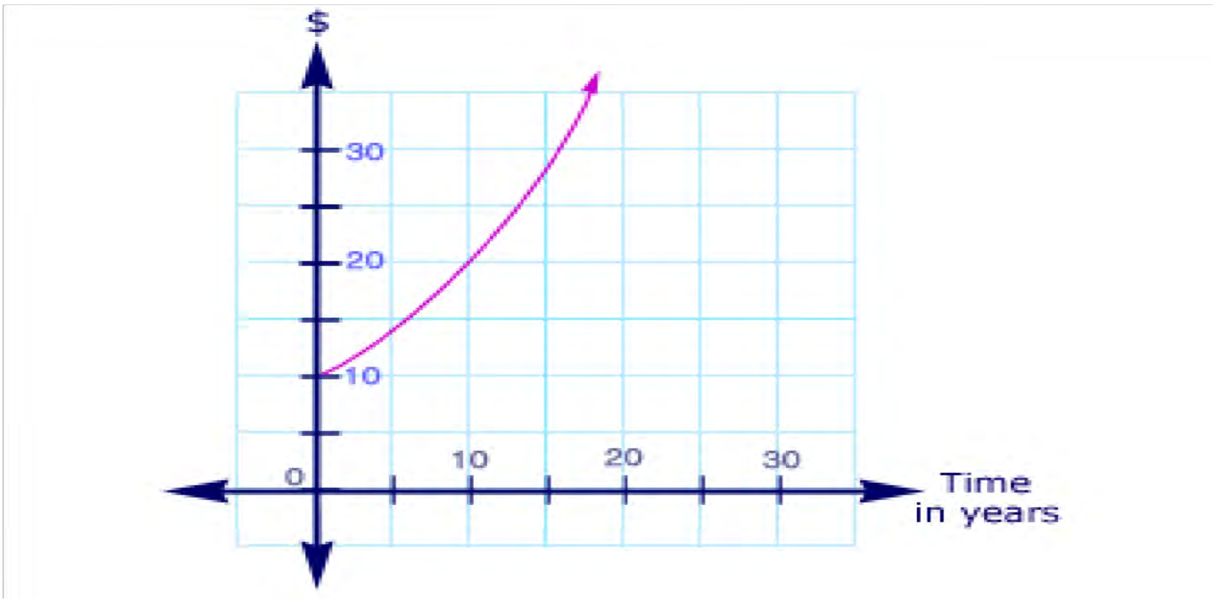

Figure 1.13 Shape of $P(x) = FV = 10(1.07)^N$ function

Consider the graph above with the following scenario the *y*-intercept tells that at the time of the initial investment, $10 was deposited. This investment will increase over the years, as indicated by the positive exponent. We can find the exact shape of the function by calculating the value of the investment for a number of years and then plotting the coordinates.

1.9 Elasticity of Demand

The Elasticity of Demand measures the percentage change in quantity demanded due to percentage change in the price. On the other hand, the relative change in demand for a commodity as a result of a relative change in its price is known as the elasticity of demand.

1.9.1 Types of Elasticity Demand

Elasticity of Demand can be classified four ways such as price elasticity of demand, income elasticity of demand, cross elasticity of demand and advertising elasticity of demand.

- **Price Elasticity of Demand**: The price elasticity of demand is the ratio of proportionate change in quantity demanded with proportionate change in price. Basically, it indicates to the responsiveness and sensitiveness of demand for product to the changes in its price.
- **Income Elasticity of Demand**: The income elasticity of demand is the ratio of percentage change in demand for a product with percentage change in income. Hence, it means the degree of responsiveness of a change in demand for a product due to the change in the income.
- **Cross Elasticity of Demand**: The cross elasticity of demand is the ratio proportionate change in purchase of commodity x with proportionate change in the price of commodity y. The individual commodities are said to be complementary if the prices of one commodity falls then the demand for other increases, on the other contrary, if the price of one commodity rises the demand for another commodity decreases. On the other hand, the individual commodities are said to be substitutes if the prices of one commodity falls then the demand for other decreases, on the other contrary, if the price of one commodity rises the demand for another commodity increases.
- **Advertising Elasticity of Demand**: The advertising elasticity of demand is the ratio of proportionate change in demand with proportionate change in advertising expenditure. In other words, the change in demand as a result of the change in advertisement and other promotional expenses.

1.10 Supply

The seller's activities are to be determined by willingness and ability to supply expected goods on time. At higher prices, more of the commodity will be available to the buyers. This is because the suppliers will be able to maintain a profit despite the higher costs of production that may result from short-term expansion of their capacity. In a real market, when the inventory is less than the desired inventory, manufacturers will raise both the supply of their product and its price. The short-term increase in supply causes manufacturing costs to rise, leading to a further increase in price. The price change in turn increases the desired rate of production.

1.11 Interaction between Supply and Demand

Demand is considered as the amount of a goods and service people are willing and able to buy at different prices. On the other hand, supply is defined as how much of a goods or service is offered at each price, so that the way to interact to control the market. In real situation, buyers and sellers react in opposite ways to a change in price. So when price of goods increases, the willingness and ability of sellers to offer goods will increase, while the willingness and ability of buyers to purchase goods will decrease.

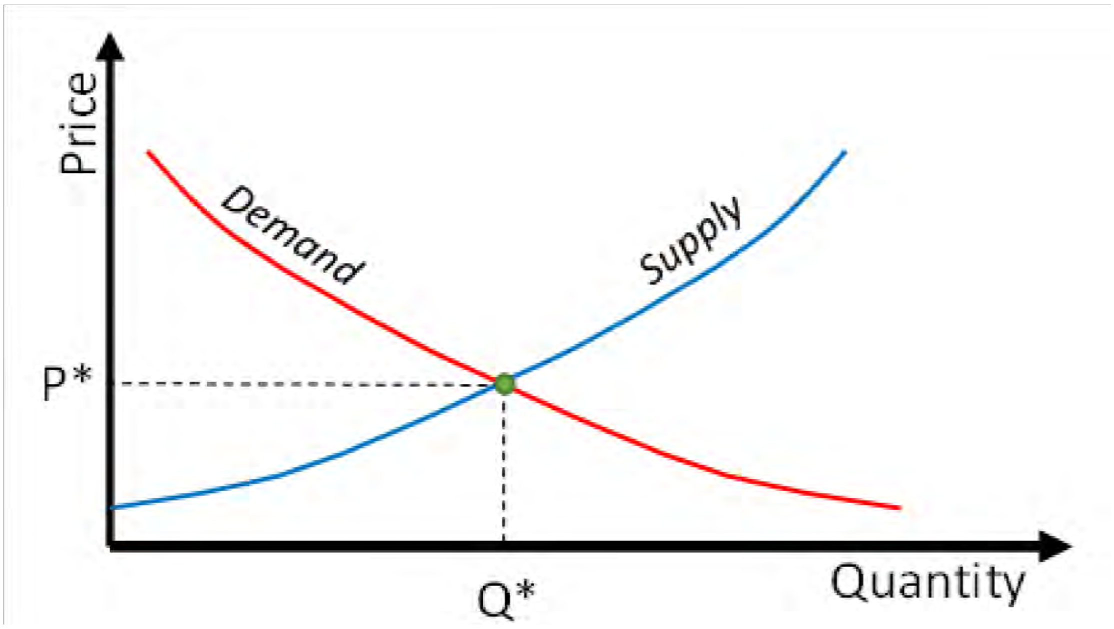

Figure 1.14 Interaction between supply and demand

1.12 Lead Time

The lead time is the delay applicable for inventory control purposes. This delay is typically the sum of the supply delay, that is, the time it takes a supplier to deliver the goods once an order is placed, and the reordering delay, which is the time until an ordering opportunity arises again. Lead Time is measured by elapsed time (minutes, hours, etc.), whereas Cycle Time is measured by the amount of time per unit (minutes/customer, hours/part, etc.). ... The Cycle Time above must be the process cycle time, which is determined by the bottleneck. From a sales perspective, lead and flow time reduction can allow for shorter lead times. From the production side of things, shorter lead times help to: Improve quality management by reducing the opportunity for work to be damaged, decreasing the time between manufacturer and defect detection. For manufacturing and assembly the concept of Lead Time is married to and has a direct relationship with the amount of inventory that exists at different points in the overall supply chain. If Customer Lead Time is less than: Material Lead Times, Production Lead Times, or Cumulative Lead Times it will result in the holding of inventory within the supply chain at some or all points. Variation and inconsistency will often compound this issue – it will cause the holding of stock or inventory to mitigate risks in the supply chain.

1.12.1 Types of Lead Time

There are several different types of Lead Time, but there are four primary types of Lead Time for our purposes in a manufacturing or assembly environment.

- **Customer Lead Time** – the amount of time taken between order confirmation and order fulfillment (either pick up or delivery depending on the agreement with the customer).
- **Material Lead Time** – the amount of time it takes to place an order with a supplier and receive it, from confirmed order to having it on hand.
- **Factory/Production Lead Time** – the amount of time it takes to build and ship a product if all the materials are available.
- **Cumulative Lead Time** – the total amount of time it would take from confirmed order to delivery of product if you had to order all the materials It is the summation of material lead time and factory lead time.

1.13 Cycle Time

Cycle Time is the amount of time it takes to complete a cycle of action. Completion of a specific task from start to finish. More specifically it is the measured time that explains how often a part is completed by a particular process.

1.14 Costs

The cost of inventory includes all costs associated with holding or storing inventory for sale. These costs include the opportunity cost of the money used to purchase the inventory, the space in which the inventory is stored, the cost of transportation or handling, and the cost of deterioration and obsolescence. This policy outlines guidelines and accounting policies to ensure that inventory is properly controlled and coasted, and losses or shortages are prevented. It applies to all inventory items, including raw materials/parts; work in progress, and finished goods and consigned inventory.

1.14.1 Types of Costs

The EOQ business accounts for many types of costs which are describing as below:

- **Holding Cost**- The holding cost (sometimes called the storage cost) represents all the costs associated with the storage of the inventory until it is sold or used. Included are the costs of capital tied up, space, insurance, protection, and taxes attributed to storage. The holding cost can be assessed either continuously or on a period-by-period basis. In the latter case, the cost may be a function of the maximum quantity held during a period, the average amount held, or the quantity in inventory at the end of the period. The last viewpoint is usually taken in this chapter. In the bicycle example, the holding cost is $1 per bicycle remaining at the end of the month. In the TV speaker's example, the holding cost is assessed continuously as $0.30 per speaker in inventory per month, so the average holding cost per month is $0.30 times the average number of speakers in inventory.

- **Shortage Cost**-The shortage cost (sometimes called the unsatisfied demand cost) is incurred when the amount of the commodity required (demand) exceeds the available stock. This cost depends upon which of the following two cases applies.

- **Ordering Costs**- Ordering costs are the costs associated with placing an order with the factory or a supplier. The ordering cost does not depend on the quantity ordered. It is a composite of all costs related to placing purchase orders or preparing shop orders, including Paperwork, Work station setups.

- **Carrying Cost**- Carrying cost is the total of costs related to maintaining the inventory, including Capital cost invested in inventory, or foregone earnings of alternate investment, Storage costs for space, equipment, and people, Taxes and insurance on inventory, Obsolescence caused by market, design, or competitors' product changes, Deterioration from long-term storage and handling, and Record keeping for inventory.

- **Risk Cost** – These are associated with the risk of obsolescence or shrinkage of inventory due to pilferage, spoilage, damage, disappearance (such as evaporation during storage), stock-dependent consumption, and devaluation of selling price.

- **Deterioration Cost**- the action or process of becoming impaired or inferior in quality, functioning, or condition: the state of having deteriorated rust deterioration the deterioration of academic standards. Deterioration cost is that cost which is incurred on any assets on any assets due to small wear and tear in asset due to use.

- **Opportunity Cost**- A benefit, profit, or value of something that must be given up to acquire or achieve something else. Since every resource (land, money, time, etc.) can be put to alternative uses, every action, choice, or decision has an associated opportunity cost. If, for example, you spend time and money going to a movie, you cannot spend that time at home reading a book, and you can't spend the money on something else.

1.15 Inventory Model

Inventory system involves with many reasons of variability and uncertainty. It means that the rate of withdrawal from the system may depend on customer demand which is variable in time and uncertain in amount. To avoid the risk of economic losses retailer should be determined the best model possible for nurturing inventory to get them just right. In inventory management two key models are considered such as deterministic model and stochastic model.

1.15.1 Deterministic Inventory Model

In inventory, a deterministic model is considered on the basis that all parameters related with inventory are known, predictable and can be predicted with a fair amount of certainty. Due to above reasons, inventory is counted, tracked, stocked and ordered according to a stable set of assumptions that largely remain the same.

1.15.2 Probabilistic Inventory Model

In Inventory Management situations uncertainty plays a vital rule in most of the cases. The retail merchant wants enough supply to satisfy customer demands, but ordering too much increases holding costs and the risk of losses through obsolescence. In probabilistic model, everything inventory control related in predicated on the assumption that demand may fluctuate and may not always be predictable. A probabilistic approach allows for fluctuations in demand and considers this when it comes to managing inventory. Hence, by using this model, inventory will be more adaptable.

1.16 Deteriorating Items

The effect of deterioration is very important in many inventory systems. Deterioration is defined as decay or damage such that the item cannot be used for its original purpose. Most of the physical goods undergo decay or deterioration over time.

Generally, deterioration is defined as decay, damage, evaporation, spoilage, obsolescence, loss of utility, pilferage, or loss of marginal values of a commodity and the item cannot be used for its original purpose. Deteriorating items are common in our daily life; however, academia has not reached a consensus on the definition of the deteriorating items. According to the study of Wee HM in 1993 [9], deteriorating items refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time. According to the definition, deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flowers, film and so on; the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods, and so on. Both of the two categories have the characteristic of short life cycle. For the first category, the items have a short natural life cycle. After a specific period (such as durability), the natural attributes of the items will change and then lose useable value and economic value; for the second category, the items have a short market life cycle. After a period of popularity in the market, the items lose the original economic value due to the changes in consumer preference, product upgrading and other reasons. The inventory problem of deteriorating items was first studied by Whitin T. M. [3], he studied fashion items deteriorating at the end of the storage period.

Figure 1.15 Different types of deteriorating items

1.17 Preservation Technology

Preservation technology is a system which helps to reduce the rate of deterioration of the deteriorating items. The consideration of preservation technology has a magical significance due to rapid social changes and the main fact that preservation technology can reduce the effect of deterioration of products significantly which help the retailer to reduce the economic losses. Most of the food products, for examples bakery items, fruits, and vegetables start deteriorating as soon as they are produced. Deterioration spoils the quality of the food for human use. Either the quality of the spoiled food is reduced or it becomes perishable after a certain period of time. Deterioration can occur due to attacks on harvested food items by enzymes, oxidation, and microorganisms, which include bacterial, mold, yeast, moisture, temperature, and chemical reactions. The deteriorated product can be analyzed on the basis of its appearance, which may not be fresh, change of color and texture, undesirable odor and taste. Such deteriorated food can cause forborne illness. Preservation technology refers to the methods, which can totally prevent, delay, or otherwise reduce the spoilage of deteriorating products. Several methods are being used to avoid food deterioration, purpose of which is to avoid exposure of the food items with bacteria, fungi, yeast, and other microorganism as well as to slow down the oxidation, which causes rancidity. Addition of preservatives can significantly increase the life of some products. Several juices and liquids are added with preservatives to increase their shelf life. Refrigeration is vastly used to preserve food items for longer time, though there is a limit of increasing shelf life of the products. Canning is used to preserve food items for very long time. This technique uses vacuum packing to keep oxygen out in order to avoid food oxidation and spoilage. Lactic acid fermentation is also adopted for food preservation. Other methods may include, drying, cooling, freezing, boiling, heating, salting, sugaring, smoking, pickling, jellying, jugging, pasteurization, vacuum packing, irradiation, pulsed electric field electro oration, atmospheric modification, non-thermal plasma, and high pressure food preservation.

1.18 Sensitivity Analysis

The technique used to determine how independent variable values will impact a particular dependent variable under a given set of assumptions is defined as sensitive analysis. Sensitivity Analysis is a tool used in financial modeling. More advanced types of financial models are built for valuation, planning, and to analyze how the different values of a set of independent variables affect a specific dependent variable under certain specific conditions. In a sensitivity analysis, only the unfavorable changes are accounted for to consider the impact of these changes on the profitability of the project. The sensitivity analysis serves following purposes: It helps in identifying the key variables that are major influence in the cost and benefits of the project. Sensitivity Analysis deals with finding out the amount by which we can change the input data for the output of our linear programming model to remain comparatively unchanged. This helps us in determining the sensitivity of the data we supply for the problem.

1.19 Supply Chain Management

Supply chain management (SCM) is the management of a network of interconnected businesses involved in the provision of product and service packages required by the end customers in a supply chain. [2] Supply chain management spans all movement and storage of raw materials, work-in-process inventory, and finished goods from point of origin to point of consumption. The term "supply chain management" entered the public domain when Keith Oliver, a consultant at Booz Allen Hamilton, used it in an interview for the Financial Times in 1982. The term was slow to take hold and the lexicon was slow to change. Supply chain management must address the following problems:

- **Distribution Network Configuration**: number, location and network missions of suppliers, production facilities, distribution centers, warehouses, cross-docks and customers.
- **Distribution Strategy**: questions of operating control (centralized, decentralized or shared); delivery scheme, e.g., direct shipment, pool point shipping, cross docking, direct store delivery (DSD), closed loop shipping; mode of transportation, e.g., motor carrier, including truckload, Less than truckload (LTL), parcel; railroad; intermodal transport, including trailer on flatcar (TOFC) and container on flatcar (COFC); ocean freight; airfreight; replenishment strategy (e.g., pull, push or hybrid); and transportation control (e.g., owner-operated, private carrier, common carrier, contract carrier, or third-party logistics (3PL)). • Trade-Offs in Logistical Activities: The above activities

must be well coordinated in order to achieve the lowest total logistics cost. Trade-offs may increase the total cost if only one of the activities is optimized. For example, full truckload (FTL) rates are more economical on a cost per pallet basis than LTL shipments. If, however, a full truckload of a product is ordered to reduce transportation costs, there will be an increase in inventory holding costs which may increase total logistics costs. It is therefore imperative to take a systems approach when planning logistical activities. This trade-offs are key to developing the most efficient and effective Logistics and SCM strategy.

- **Information**: Integration of processes through the supply chain to share valuable information, including demand signals, forecasts, inventory, transportation, potential collaboration, etc.
- **Inventory Management**: Quantity and location of inventory, including raw materials, work-in-process (WIP) and finished goods.
- **Cash-Flow**: Arranging the payment terms and methodologies for exchanging funds across entities within the supply chain.

Supply chain management is a cross-function approach including managing the movement of raw materials into an organization, certain aspects of the internal processing of materials into finished goods, and the movement of finished goods out of the organization and toward the end-consumer. As organizations strive to focus on core competencies and becoming more flexible, they reduce their ownership of raw materials sources and distribution channels. These functions are increasingly being outsourced to other entities that can perform the activities better or more cost effectively. The effect is to increase the number of organizations involved in satisfying customer demand, while reducing management control of daily logistics operations. Less control and more supply chain partners led to the creation of supply chain management concepts. The purpose of supply chain management is to improve trust and collaboration among supply chain partners, thus improving inventory visibility and the velocity of inventory movement.

Chapter 2

Literature Review

2.1 Introduction

Demand is the key factor in inventory management. Demand can be classified a few distinguish ways. Mainly, in inventory model, demand may be introduced four ways such as constant demand, time-dependent demand, probabilistic demand and stock-dependent demand. Also time-dependent demand can be classified based on two issues like linear and non-linear time-dependent demand function. Basically, constant demand is suitable only when the phase of product life cycle is matured for finite periods of time horizon. In real situation, inventory model which is formulated based on constant demand rate is not applicable for every types of items. In real life, demand may be changed due to different cases. For few inventory item constant demand should not consider such as fashionable clothes, electronic equipment's, different types of food etc. because of the reason of variation in demand rate. In reality, demand can be modified in competitive market due to consumer's preference on some eye catching product due to daily life style. Many researchers developed by assuming time-dependent demand as linear demand, quadratic demand and exponential demand rates require uniform change, steady increase or decrease and rapid change in demand rate respectively.

2.2 Comparative Studies of Inventory Model for Deteriorating Items and Preservation Technology

In recent years, most of the researchers have studied deteriorating items inventory model due to its economic impact. Basically, deterioration is a phase for which product will be damaged, decay, change or spoilage. It means item cannot be used for its actual purpose. In real situation, deterioration of any items is a natural phenomenon. So, it's mandatory to consider the factor deterioration rate while analyzing the inventory model. However, the consideration of preservation technology for reducing deterioration rate has received the attention of researcher in the past years. Due to rapid social changes, preservation technology should be considered because it helps reduce the deterioration rate of expected items significally. Hence, using preservation technology can be protected reduction of goods which may be beneficial for the commodities. By using the fact that the use of preservation technology which can reduce the deterioration rate and because of that the retailers to reduce their economic losses. The manufacturing process for an organization may produce defective items as a production. These types of items may be caused to reduce the maximum profit for the retailer. Thus, to sustain the supply of standard quality items, it is mandatory for the whole lot to be screened as soon as it comes into the inventory and defective items identified must be removed from the process. Because of this scenario, a number of researchers has been made in the direction of the development of Economic Order Quantity (EOQ) and Economic Product Quality (EPQ) models for defective items. The classical EOQ model inventory models were developed the assumptions of constant demand rate. Later, many researchers developed EOQ modes considering linearly increasing or decreasing demand and exponentially increasing or decreasing demand. Harris [1] was the first mathematician who studied inventory problems. He formulated famous EOQ model that was also derived independently by Wilson [2]. Inventory of deteriorating items first studied by Within [3], he considered the deterioration of fashionable goods at the end of prescribed storage period. After that Ghare and Schrader [4] extended the above ideas. They considered in their study that the consumption of the deteriorating items was closely relative to a negative exponential of time. Based on the criteria, they proposed and formulated an inventory model as stated below

$$\frac{dI(t)}{dt} + \theta I(t) = -f(t)$$

In that model, θ indicates the rate of deterioration of deteriorating items, (t) refers to the inventory level at time t and (t) means demand rate with respect to time. Mainly, this inventory model laid foundation for the follow-up study. Covert and Philip [5] derived an EOQ model for deteriorating items without shortages under the assumption of constant demand rate and two parameters Weiball distribution deterioration rate. Donaldson [6] examined the classical no shortage inventory model for deteriorating items with a linear trend in demand over a known and finite horizon. Another class of researchers on inventory model for deteriorating items was developed by considering the deteriorating rate as time proportional. Researchers like Dave and Patel [7], Goswami and Chaudhuri [8] etc. developed the inventory models for deteriorating items with trended demand. According to the study of Wee H.M. in 1993 [9], deteriorating items refers to the items that become decayed, damaged, expired, devaluation and so on through time. Hence, based on the above study, deteriorating items can be classified two categories. Firstly, it refers the items that become decayed, damaged, or expired through time period like vegetables, fruits, flowers etc. Secondly, it refers the items that lose part or total value through time due to introduce new technology in market. These types of items can be considered as computer chips, mobile phone, Televisions, Laptop and so on. Among these two categories have the same characteristic which is short life cycle? For first category, the items have a short life cycle due to natural cases. On the other hand, for second category, the items have a short life cycle because of market demand. K. J. Chung and P. S. Ting, in 1993[10] developed a heuristic for replenishment of deteriorating items with linear trend in demand. Sarker, Jamal and Wang [11] developed inventory model in which both the demand and deteriorating rate are constant. Kalpakam [12] and Shanthi [13] observed a modified perishable inventory policy. Chang [14] developed an EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. Sana, Goyal and Chaudhuri [15] considered a production inventory model for a deteriorating items with trended demand with shortages. WANG Sheng-dong and Wang jun-ping [16] formulated a model to determine optimal ordering policy for deteriorating items under inflation, partial backlogging and time-dependent demand. Yang [17] conducted research on deteriorating items inventory under the premise that the demand is time-dependent. In fact, due to uncertainty in the modern business environment the probabilistic demand has attracted more and more attentions. Chung and Liao [18] conducted research on the buyers allowed a delay period to pay for the items purchased. Although the constant demand assumption to simplify the problem but it's far away from real scenario. Dye, Chang, and Tang [19] discussed about a deteriorating inventory model with time-varying demand and shortage dependent partial backlogging. Panda et al. [20] discussed an inventory model for a seasonal product. In this inventory model, they considered ramp type tie –dependent demand function. Khanra, Ghosh and Chaudhuri [21] noticed an

EOQ model based on deteriorating items with time dependent demand. They also considered permissible delay in payment in their research. Mishra and Singh [22] proposed a model based on deteriorating items. In their research, they have considered time dependent demand and holding cost with partial backlogging. Bakker, Riezebos, and Teunter reviewed inventory systems with deterioration since 2001. Hsieh, and Dye observed a production-inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time.

Chapter 3

Inventory Analysis for Deteriorating Items with Quadratic Demand Functions without Preservation Technology

3.1 Introduction

Inventory is an accumulation of a product which may be used in future to fulfill the demand of this product for community. In a single world, Inventory is an assets of companies which tell when should be ordered, how much to be ordered and which product to be ordered. Managers have long recognized that good inventory control is crucial. On one hand, a firm can try to reduce costs by reducing on-hand inventory levels. On the other hand, customers become dissatisfied when frequent inventory outages, called stock outs, occur. Thus, companies must make the balance between low and high inventory levels. Cost minimization is the major factor in obtaining this delicate balance. Inventory is any stored resource that is used to satisfy a current or future need. Raw materials, work-in-process, and finished goods are examples of inventory. Inventory levels for finished goods, such as clothes dryers, are a direct function of market demand. By using this demand information, it is possible to determine how much raw materials and work-in-process are needed to produce the finished product.

The Inventory model without preservation technology (IMWOPT) has been developed with the help of time-dependent quadratic demand function as well as variable deteriorating rate i.e. deterioration may be moved with respect to time. Since the effect of deterioration cannot be ignored due to real world situation, we have taken variable deterioration. Also the model has been developed by the sense of time dependent quadratic demand function as quadratic demand rate requires steady increase or decrease. Demand increases or decreases due to the popularity of the product. Furthermore time is the vital factor which plays the key role in developing the inventory model. The reason for considering the variable deterioration rate and time dependent demand rate is due to change in deterioration rate with respect to time and the suitable demand for the present market situation. Shortages are not allowed for the model.

In this study, quadratic functions are assumed as demand functions. This model starts with a specific level of inventory and ends with zero-inventory after a certain time from where inventory starts accumulating to reach at the same specific level from which it started decreasing. Numerical example has been considered for illustrating the developed model.

3.2 Mathematical Formulation

This inventory model is developed by the consideration of the replenishment problem of a single non-instantaneous deteriorating item without preservation technology. The inventory model runs as follows:

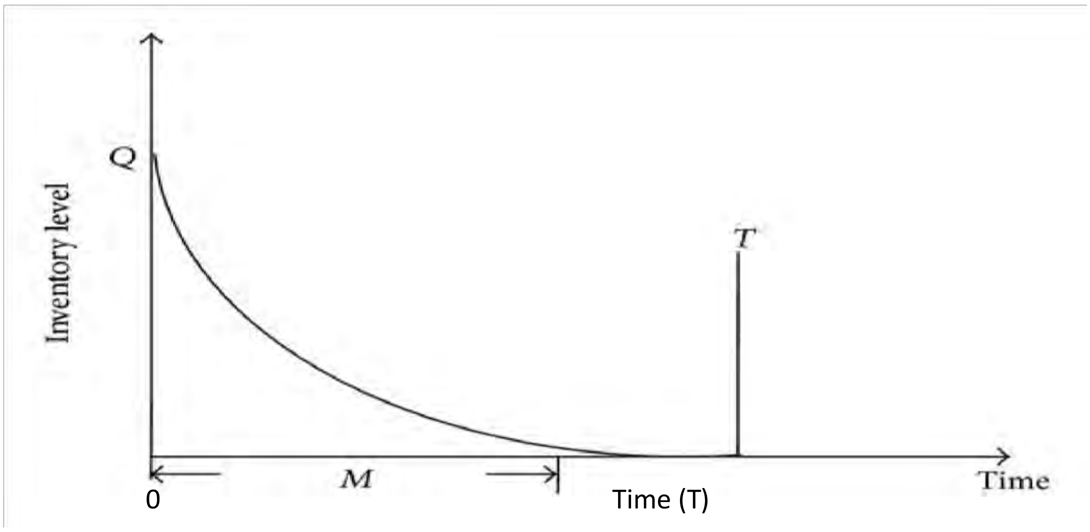

Figure 3.0 Inventory mode without preservation technology

- Quadratic demand function has been taken over the time interval $[0, T]$ with the initial inventory level $Q = IM$ at the beginning of time.
- Deteriorated items have been evaluated over the time interval $[0, T]$.
- Deterioration of the item has been considered as a variable time function.
- Negative sigh are given in front of quadratic demand function in the governed differential equation to indicate decrease in inventory.
- Shortage is not allowed throughout the time interval $[0, T]$.
- Holding cost has been considered.

3.2.1 Assumption and Notations

The following assumptions are made to develop this model:

- The inventory system involves single type of items.
- Replenishment rate is infinite, i.e. Replenishment rate is instantaneously.
- The demand rate of the item is considered by a quadratic and continuous function of time.
- The deterioration rate is variable rate of deterioration on the on-hand inventory per unit time and there is no repair or replenishment of the deteriorated items within the cycle.
- Shortages is not allowed.
- There is no provision for repair or replacement of deteriorated units.
- Time horizon is infinite.

Decision Variables

T Length of replenishment cycle in traditional system (per year)

Parameters

Symbols	Abbreviation
$I(t)$	Inventory level at time t $(0 < t < T)$
A	Ordering cost of inventory per order $(TK/order)$
h	Unit holding cost per unit time
p	Purchase cost per unit of item
$D(t)$	Time dependent demand rate which is defined by $D(t) = a + bt + ct^2, a > 0, b \neq 0 \,\&\, c \neq 0.$ Here a is the initial rate of demand, b is the rate with which the demand rate increases and the rate of change in the demand rate itself changes at a rate c.

$\theta\ (t)$	Variable rate of deterioration of an item where $\theta\ (t) = \theta t^2;\ 0 < \theta < 1$
IM	Maximum inventory level during $[0, T]$
TC	Average cost per unit time

3.2.2 Inventory Model Without Preservation Technology (IMWOPT)

The inventory level $I(t)$ at time t generally decreases from initial inventory to meet markets demand and products deterioration and reaches to zero at$T$. Hence, the variation of inventory with respect to time can be described by the governing differential equation:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -(a + bt + ct^2) \quad \text{……… (3.1)}$$

with conditions $0 \le t \le T, (0) = IM, (t) = 0\ if\ t = T.$ ……… (3.2)

I.F $= e^{\int \theta t^2\, dt} = e^{\frac{\theta t^3}{3}}$

Therefore, $I(t)e^{\frac{\theta t^3}{3}} = -\int (a + bt + ct^2)e^{\frac{\theta t^3}{3}} dt$

$= -\int (a + bt + ct^2)(1 + \frac{\theta t^3}{3})dt$ [Neglecting higher degree of theta]

$= -\int a\left(1 + \frac{\theta t^3}{3}\right) dt - bt\int \left(1 + \frac{\theta t^3}{3}\right) dt - ct^2 \int \left(1 + \frac{\theta t^3}{3}\right) dt$

$$= -a\left(t + \frac{\theta t^4}{12}\right) - b\left(\frac{t^2}{2} + \frac{\theta t^5}{15}\right) - c\left(\frac{t^3}{3} + \frac{\theta t^6}{18}\right) + W \quad \text{……… (3.3)}$$

$I(t) = 0\ if\ t = T$ in equation (3.3), we have

$$W = a\left(t + \frac{\theta t^4}{12}\right) + b\left(\frac{t^2}{2} + \frac{\theta t^5}{15}\right) + c\left(\frac{t^3}{3} + \frac{\theta t^6}{18}\right)$$

Therefore,

$$I(t) = a[(T-t) + \frac{\theta}{12}(T^4 - t^4)]e^{-\frac{\theta t^3}{3}}$$

$$+b\left[\frac{1}{2}(T^2 - t^2) + \frac{\theta}{15}(T^5 - t^5)\right]e^{-\frac{\theta t^3}{3}}$$

$$+c[\frac{1}{3}(T^3 - t^3) + \frac{\theta}{18}(T^6 - t^6)]e^{-\frac{\theta t^3}{3}}$$

$$= a\left[(T-t) + \frac{\theta}{12}(T^4 - t^4)\right]\left(1 - \frac{\theta t^3}{3}\right)$$

$$+b\left[\frac{1}{2}(T^2 - t^2) + \frac{\theta}{15}(T^5 - t^5)\right]\left(1 - \frac{\theta t^3}{3}\right)$$

$$+c[\frac{1}{3}(T^3 - t^3) + \frac{\theta}{18}(T^6 - t^6)](1 - \frac{\theta t^3}{3})$$

$$= a\left[(T-t) + \frac{\theta}{12}(T^4 - t^4)\right] - a\left[\frac{\theta}{3}(Tt^3 - t^4) + \frac{\theta^2}{36}(T^4t^3 - t^7)\right]$$

$$+b\left[\frac{1}{2}(T^2 - t^2) + \frac{\theta}{15}(T^5 - t^5)\right] - b\left[\frac{\theta}{6}(T^2t^3 - t^5) + \frac{\theta^2}{45}(T^5t^3 - t^8)\right]$$

$$+c\left[\frac{1}{3}(T^3 - t^3) + \frac{\theta}{18}(T^6 - t^6)\right] - c[\frac{\theta}{6}(T^3t^3 - t^6) + \frac{\theta^2}{54}(T^6t^3 - t^9)] \qquad \ldots\ldots\ldots (3.4)$$

We have, Inventory holding cost per cycle,

$$\text{IHC} = h\int_0^T I(t)dt$$

$$= h\int_0^T \{ a\left[(T-t) + \frac{\theta}{12}(T^4 - t^4)\right] - a\left[\frac{\theta}{3}(Tt^3 - t^4) + \frac{\theta^2}{36}(T^4t^3 - t^7)\right]$$

$$+b\left[\frac{1}{2}(T^2 - t^2) + \frac{\theta}{15}(T^5 - t^5)\right] - b\left[\frac{\theta}{6}(T^2t^3 - t^5) + \frac{\theta^2}{45}(T^5t^3 - t^8)\right]$$

$$+c\left[\frac{1}{3}(T^3 - t^3) + \frac{\theta}{18}(T^6 - t^6)\right]$$

$$-c[\frac{\theta}{6}(T^3t^3 - t^6)$$

$$+\frac{\theta^2}{54}(T^6t^3 - t^9)]\}\, dt$$

$$= h\left\{\left[a\left(Tt - \frac{t^2}{2}\right) + \frac{\theta}{12}\left(T^4t - \frac{t^5}{5}\right)\right] - a\left[\frac{\theta}{3}\left(\frac{Tt^4}{4} - \frac{t^5}{5}\right) + \frac{\theta^2}{36}\left(\frac{T^4t^4}{4} - \frac{t^8}{8}\right)\right]\right.$$

$$+ b\left[\frac{1}{2}\left(T^2t - \frac{t^3}{3}\right) + \frac{\theta}{15}\left(T^5t - \frac{t^6}{6}\right)] - b[\frac{\theta}{6}(\frac{T^2t^4}{4} - \frac{t^6}{6}) + \frac{\theta^2}{45}\left(\frac{T^5t^4}{4} - \frac{t^9}{9}\right)\right]$$

$$+ c\left[\frac{1}{3}\left(T^3t - \frac{t^4}{4}\right) + \frac{\theta}{18}\left(T^6t - \frac{t^7}{7}\right)\right] - c[\frac{\theta}{9}\left(\frac{T^3t^4}{4} - \frac{t^7}{7}\right) + \frac{\theta^2}{54}(\frac{T^6t^4}{4}$$

$$\left. - \frac{t^{10}}{10})]\right\}$$

$$= h\{a\left[\left(T^2 - \frac{T^2}{2}\right) + \frac{\theta}{12}\left(T^5 - \frac{T^5}{2}\right)\right] - a\left[\frac{\theta}{3}\left(\frac{T^5}{4} - \frac{T^5}{5}\right) + \frac{\theta^2}{36}\left(\frac{T^8}{4} - \frac{T^8}{8}\right)\right]$$

$$+ b\left[\frac{1}{2}\left(T^3 - \frac{T^3}{3}\right) + \frac{\theta}{15}\left(T^6 - \frac{T^6}{6}\right)] - b[\frac{\theta}{6}(\frac{T^6}{4} - \frac{T^6}{6}) + \frac{\theta^2}{45}\left(\frac{T^9}{4} - \frac{T^9}{9}\right)\right]$$

$$+ c\left[\frac{1}{3}\left(T^4 - \frac{T^4}{4}\right) + \frac{\theta}{18}\left(T^7 - \frac{T^7}{7}\right)\right] - c[\frac{\theta}{9}\left(\frac{T^7}{4} - \frac{T^7}{7}\right) + \frac{\theta^2}{54}(\frac{T^{10}}{4}$$

$$- \frac{T^{10}}{10})]\}$$

$$= h\left(\frac{aT^2}{2} + \frac{a\theta T^5}{20} - \frac{a\theta^2T^8}{288} + \frac{bT^3}{3} + \frac{b\theta T^6}{24} - \frac{b\theta^2T^9}{324} + \frac{cT^4}{4} + \frac{c\theta T^7}{28} - \frac{c\theta^2T^{10}}{360}\right)$$

... ... (3.5)

Now, the total demand of the cycle during $[0, T]$ is given by

$$\int_0^T (a + bt + ct^2)dt = T(a + \frac{bT}{2} + \frac{CT^2}{3})$$

Therefore, the number of deteriorating unit is

$$W - \int_0^T D(t)dt$$

$$= a\left(t + \frac{\theta t^4}{12}\right) + b\left(\frac{t^2}{2} + \frac{\theta t^5}{15}\right) + c\left(\frac{t^3}{3} + \frac{\theta t^6}{18}\right) - T\left(a + \frac{bT}{2} + \frac{CT^2}{3}\right)$$

$$= \frac{a\theta T^4}{12} + \frac{b\theta T^5}{15} + \frac{c\theta T^6}{18}$$

$$= \frac{\theta T^4}{3}(\frac{a}{4} + \frac{bT}{5} + \frac{CT^2}{6})$$

Hence, Deterioration rate per cycle

D_c = purchase cost per cycle×number of deteriorated units

$$= \frac{p\theta T^4}{3}(\frac{a}{4} + \frac{bT}{5} + \frac{CT^2}{6})$$

$$= \frac{p\theta T^4}{3}(\frac{a}{4} + \frac{bT}{5} + \frac{CT^2}{6})$$

At the initial stage i.e. at $t = 0$, $I(0) = IM$.

$$IM = a\left[(T-t)+\frac{\theta}{12}(T^4-t^4)\right]-a\left[\frac{\theta}{3}(Tt^3-t^4)+\frac{\theta^2}{36}(T^4t^3-t^7)\right]$$

$$+b\left[\frac{1}{2}(T^2-t^2)+\frac{\theta}{15}(T^5-t^5)\right]-b\left[\frac{\theta}{6}(T^2t^3-t^5)+\frac{\theta^2}{45}(T^5t^3-t^8)\right]$$

$$+c\left[\frac{1}{3}(T^3-t^3)+\frac{\theta}{18}(T^6-t^6)\right]-c[\frac{\theta}{6}(T^3t^3-t^6)+\frac{\theta^2}{54}(T^6t^3-t^9)]$$

$$= a\left[(T-0)+\frac{\theta}{12}(T^4-0)\right]-a\left[\frac{\theta}{3}(0-0)+\frac{\theta^2}{36}(0-0)\right]$$

$$+b\left[\frac{1}{2}(T^2-0)+\frac{\theta}{15}(T^5-0)\right]-b\left[\frac{\theta}{6}(0-0)+\frac{\theta^2}{45}(0-0)\right]$$

$$+c\left[\frac{1}{3}(T^3-0)+\frac{\theta}{18}(T^6-0)\right]-c[\frac{\theta}{6}(0-0)+\frac{\theta^2}{54}(0-0)]$$

$$= T\left(a+\frac{bT}{2}+\frac{CT^2}{3}\right)+\frac{\theta T^4}{3}(\frac{a}{4}+\frac{bT}{5}+\frac{CT^2}{6})$$

We have, ordering size $O_S = IM + IB$

$$= T\left(a+\frac{bT}{2}+\frac{CT^2}{3}\right)+\frac{\theta T^4}{3}\left(\frac{a}{4}+\frac{bT}{5}+\frac{CT^2}{6}\right)+0$$

$$= T\left(a+\frac{bT}{2}+\frac{CT^2}{3}\right)+\frac{\theta T^4}{3}(\frac{a}{4}+\frac{bT}{5}+\frac{CT^2}{6})$$

Also, Purchase cost $P_c = p \times O_S$

$$= p[T\left(a+\frac{bT}{2}+\frac{CT^2}{3}\right)+\frac{\theta T^4}{3}(\frac{a}{4}+\frac{bT}{5}+\frac{CT^2}{6})]$$

Therefore, total cost per unit time (TC) is given by

$$TC = \frac{Ordering\ cost + Holding\ cost + Deteriration\ cost + Purchase\ cost}{T}$$

$$= \frac{1}{T}[A + h\left(\frac{aT^2}{2} + \frac{a\theta T^5}{20} - \frac{a\theta^2 T^8}{288} + \frac{bT^3}{3} + \frac{b\theta T^6}{24} - \frac{b\theta^2 T^9}{324} + \frac{cT^4}{4} + \frac{c\theta T^7}{28} - \frac{c\theta^2 T^{10}}{360}\right)$$
$$+ \left[\frac{p\theta T^4}{3}\left(\frac{a}{4} + \frac{bT}{5} + \frac{CT^2}{6}\right)\right] + [p\{T\left(a + \frac{bT}{2} + \frac{CT^2}{3}\right) + \frac{\theta T^4}{3}(\frac{a}{4} + \frac{bT}{5}$$
$$+ \frac{CT^2}{6})\}]]$$

$$= \frac{1}{T}[A + \frac{haT^2}{2}\left(1 + \frac{\theta T^3}{10} - \frac{\theta^2 T^6}{144}\right) + \frac{hbT^3}{3}\left(1 + \frac{\theta T^3}{8} - \frac{\theta^2 T^6}{108}\right)$$
$$+ \frac{hcT^4}{4}\left(1 + \frac{\theta T^3}{7} - \frac{\theta^2 T^6}{90}\right) + pT\left(a + \frac{bT}{2} + \frac{cT^2}{3}\right)$$
$$+ \frac{2p\theta T^4}{3}(\frac{a}{4} + \frac{bT}{5} + \frac{CT^2}{6})] \qquad \dots\dots\dots\ \dots (3.6)$$

To get the optimal solution we need to solve the following equations

$$\frac{dTC}{dT} = 0\ and\ \frac{d^2TC}{dT} = 0.$$

Now $\frac{dT(C)}{dT} = \frac{1}{T}\left[\begin{matrix} h\left(aT + \frac{a\theta T^4}{4} - \frac{a\theta^2 T^7}{36} + bT^2 + \frac{b\theta T^5}{4} - \frac{b\theta^2 T^8}{36} + cT^3 + \frac{c\theta T^6}{4} - \frac{c\theta^2 T^9}{36}\right) \\ + \frac{2ap\theta T^3}{3} + \frac{2bp\theta T^4}{3} + \frac{2cp\theta T^5}{3} + ap + bpT + pcT^2 \end{matrix}\right]$

$$-\frac{1}{T^2}[A + \frac{haT^2}{2}\left(1 + \frac{\theta T^3}{10} - \frac{\theta^2 T^6}{144}\right) + \frac{hbT^3}{3}\left(1 + \frac{\theta T^3}{8} - \frac{\theta^2 T^6}{108}\right)$$
$$+ \frac{hcT^4}{4}\left(1 + \frac{\theta T^3}{7} - \frac{\theta^2 T^6}{90}\right) + pT\left(a + \frac{bT}{2} + \frac{cT^2}{3}\right)$$
$$+ \frac{2p\theta T^4}{3}(\frac{a}{4} + \frac{bT}{5} + \frac{CT^2}{6})]$$

$$= \frac{1}{T}\left[h\left(1+\frac{\theta T^3}{4}-\frac{\theta^2 T^6}{36}\right)(aT+bT^2+cT^3)+\left(\frac{2p\theta T^3}{3}+p\right)(a+bT+cT^2)\right]$$

$$-\frac{1}{T^2}[A+\frac{haT^2}{2}\left(1+\frac{\theta T^3}{10}-\frac{\theta^2 T^6}{144}\right)+\frac{hbT^3}{3}\left(1+\frac{\theta T^3}{8}-\frac{\theta^2 T^6}{108}\right)$$

$$+\frac{hcT^4}{4}\left(1+\frac{\theta T^3}{7}-\frac{\theta^2 T^6}{90}\right)$$

$$+pT\left(a+\frac{bT}{2}+\frac{cT^2}{3}\right)$$

$$+\frac{2p\theta T^4}{3}(\frac{a}{4}+\frac{bT}{5}+\frac{CT^2}{6})]$$

$$\frac{d^2T(C)}{dT^2}=\frac{1}{T}\left[h\left(1+\frac{\theta T^3}{4}-\frac{\theta^2 T^6}{36}\right)(a+2bT+3cT^2)\right.$$

$$+h(aT+bT^2+cT^3)\left(\frac{3\theta T^2}{4}-\frac{\theta^2 T^5}{6}\right)+\left(\frac{2p\theta T^3}{3}+p\right)(b+2cT)$$

$$\left.+2p\theta T^2(a+bT+cT^2)\right]$$

$$-\frac{1}{T^2}\left[h\left(1+\frac{\theta T^3}{4}-\frac{\theta^2 T^6}{36}\right)(aT+bT^2+cT^3)\right.$$

$$\left.+\left(\frac{2p\theta T^3}{3}+p\right)(a+bT+cT^2)\right]$$

$$-\frac{1}{T^2}\left[h\left(1+\frac{\theta T^3}{4}-\frac{\theta^2 T^6}{36}\right)(aT+2bT^2+cT^3)\right.$$

$$\left.+\frac{2p\theta T^3}{3}\left(a+\frac{b}{5}+\frac{4bT}{5}+\frac{cT}{3}+\frac{2cT^2}{3}\right)+p\left(bT+\frac{2cT^2}{3}+\frac{cT^3}{3}\right)\right]$$

$$+\frac{1}{T^3}[A+\frac{haT^2}{2}\left(1+\frac{\theta T^3}{10}-\frac{\theta^2 T^6}{144}\right)+\frac{hbT^3}{3}\left(1+\frac{\theta T^3}{8}-\frac{\theta^2 T^6}{108}\right)$$

$$+\frac{hcT^4}{4}\left(1+\frac{\theta T^3}{7}-\frac{\theta^2 T^6}{90}\right)+pT\left(a+\frac{bT}{2}+\frac{cT^2}{3}\right)+\frac{2p\theta T^4}{3}(\frac{a}{4}+\frac{bT}{5}$$

$$+\frac{CT^2}{6})]$$

3.3 Result Discussion and Computational Analysis

In this section, a numerical example is considered to illustrate this maintenance model. Parameter numerical example has been considered to check the validity inventory model without preservation technology (IMWOPT) in proper units:

$$A = 12, \quad a = 10, \quad b = 8, \quad c = 5, \quad h = 1, \quad p = 15, \quad \theta = 0.87$$

Solving the equation and it has been obtained optimum value of $= \mathbf{1.0028}$ and the minimum average cost per unit time is evaluated $TC = 46.0117$.

3.4 Sensitivity Analysis

Sensitivity analysis is the process on which the optimum solution of the model is affected by the changes in its input parameter values. In this analysis, the sensitivity analysis for total cost per unit time TC is carried out with respect to the changes in the values of the parameters of quadratic function, unit holding cost per un it time, ordering cost, purchase cost, deteriorating cost, parameters a, b, & c. These sensitivity analyses is performed by considering variation in each one of the above parameters by 5% change in stipulated standard value, keeping all of the remaining parameters as a fixed.

3.4.1 Sensitivity of different parameters with total cost per unit time for IMWOPT table:

Table 3.1 (Sensitivity of ordering cost A)

index	Parameter Value	TC
1	8.4	43.5777
2	9	43.9833
3	9.6	44.389
4	10.2	44.7947
5	10.8	45.2004
6	11.4	45.6061
7	**12.0000**	**46.0117**
8	12.6000	46.4174

9	13.2000	46.8231
10	13.8000	47.2288
11	14.4000	47.6345
12	15.0000	48.0401
13	15.6000	48.4458

Table 3.2 (Sensitivity of parameter a)

index	Parameter Value	TC
1	7	41.0872
2	7.5	41.9079
3	8	42.7287
4	8.5	43.5495
5	9	44.3702
6	9.5	45.191
7	**10.0000**	**46.0117**
8	10.5000	46.8325
9	11.0000	47.6532
10	11.5000	48.474
11	12.0000	49.2948
12	12.5000	50.1155
13	13.0000	50.9363

Table 3.3 (Sensitivity of parameter b)

index	Parameter Value	TC
1	5.6	42.221
2	6	42.8528
3	6.4	43.4846
4	6.8	44.1164
5	7.2	44.7482
6	7.6	45.3799

7	**8.0000**	**46.0117**
8	8.4000	46.6435
9	8.8000	47.2753
10	9.2000	47.9071
11	9.6000	48.5389
12	10.0000	49.1707
13	10.4000	49.8025

Table 3.4 (Sensitivity of parameter c)

index	Parameter Value	TC
1	3.5	43.3576
2	3.75	43.7999
3	4	44.2423
4	4.25	44.6846
5	4.5	45.127
6	4.75	45.5694
7	**5.0000**	**46.0117**
8	5.2500	46.4541
9	5.5000	46.8965
10	5.7500	47.3388
11	6.0000	47.7812
12	6.2500	48.2235
13	6.5000	48.6659

Table 3.5 (Sensitivity of holding cost h)

index	Parameter Value	TC
1	0.7	39.4593
2	0.75	40.5514
3	0.8	41.6435
4	0.85	42.7355
5	0.9	43.8276

6	0.95	44.9197
7	**1.0000**	**46.0117**
8	1.0500	47.1038
9	1.1000	48.1959
10	1.1500	49.2879
11	1.2000	50.38
12	1.2500	51.4721
13	1.3000	52.5641

Table 3.5 (Sensitivity of deterioration rate θ)

index	Parameter Value	TC
1	0.609	43.0083
2	0.6525	43.5237
3	0.696	44.0332
4	0.7395	44.5368
5	0.783	45.0344
6	0.8265	45.526
7	**0.8700**	**46.0117**
8	0.9135	46.4915
9	0.9570	46.9653
10	1.0005	47.4331
11	1.0440	47.895
12	1.0875	48.3509
13	1.1310	48.8009

Table 3.5 (Sensitivity of purchase cost p)

index	Parameter Value	TC
1	0.35	41.1947
2	0.375	41.9975
3	0.4	42.8004

4	0.425	43.6032
5	0.45	44.406
6	0.475	45.2089
7	**0.5000**	**46.0117**
8	0.5250	46.8146
9	0.5500	47.6174
10	0.5750	48.4203
11	0.6000	49.2231
12	0.6250	50.0259
13	0.6500	50.8288

3.5 Graphical presentation for the effects of parameters on total cost of per unit Time

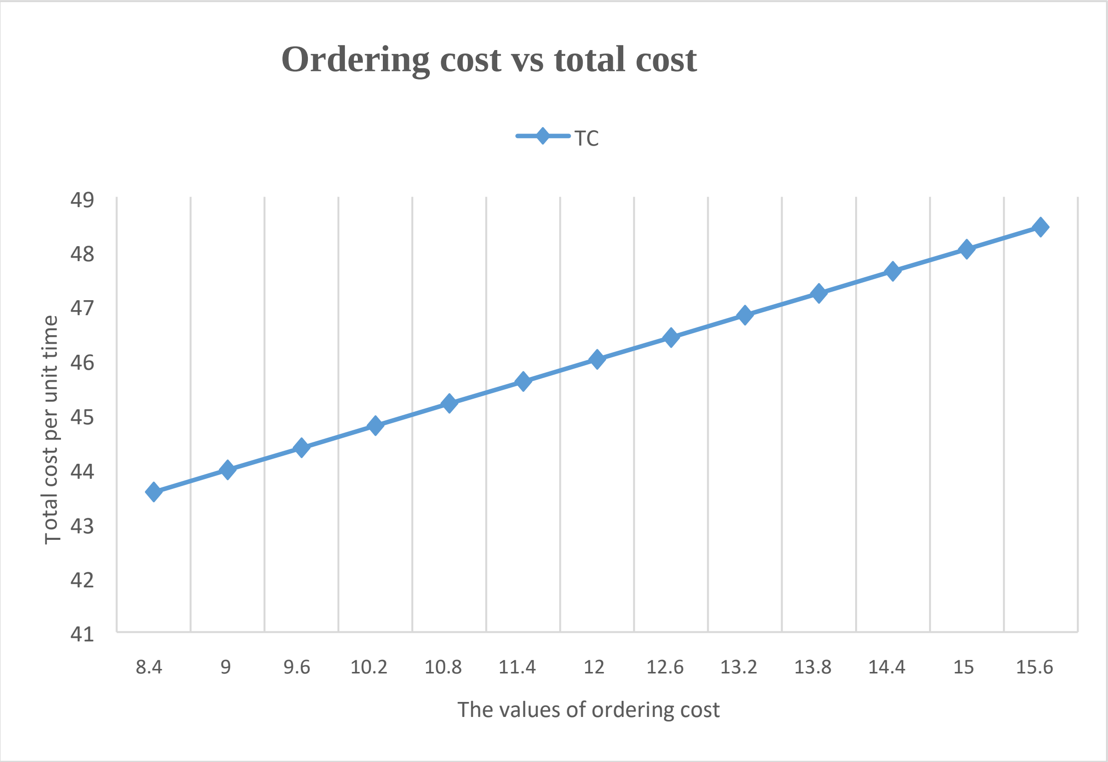

Figure 3.1 Effects of ordering cost on total cost per unit time

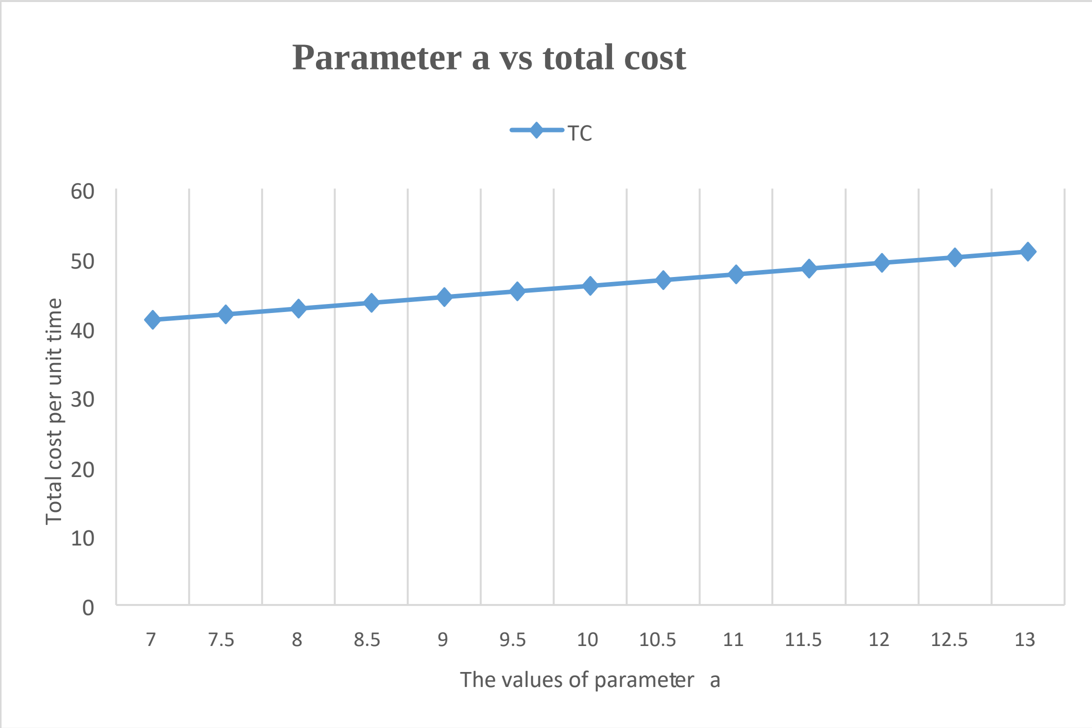

Figure 3.2 Effects of parameter a on total cost per unit time

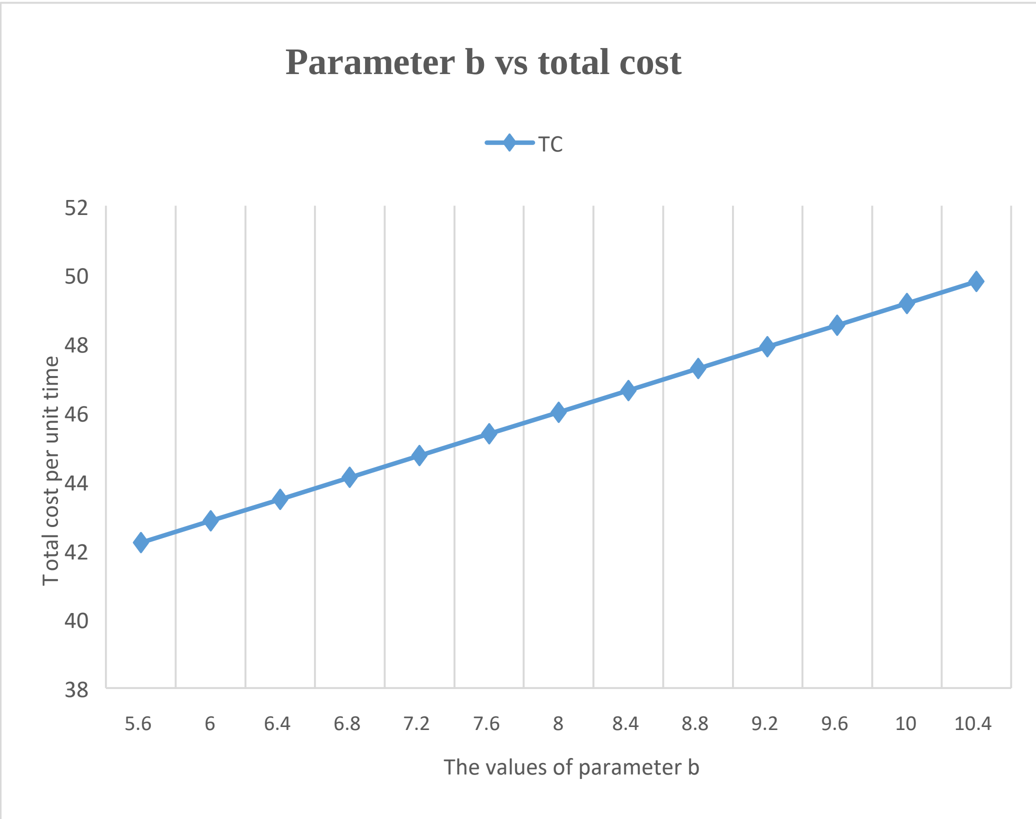

Figure 3.3 Effects of parameter b on total cost per unit time

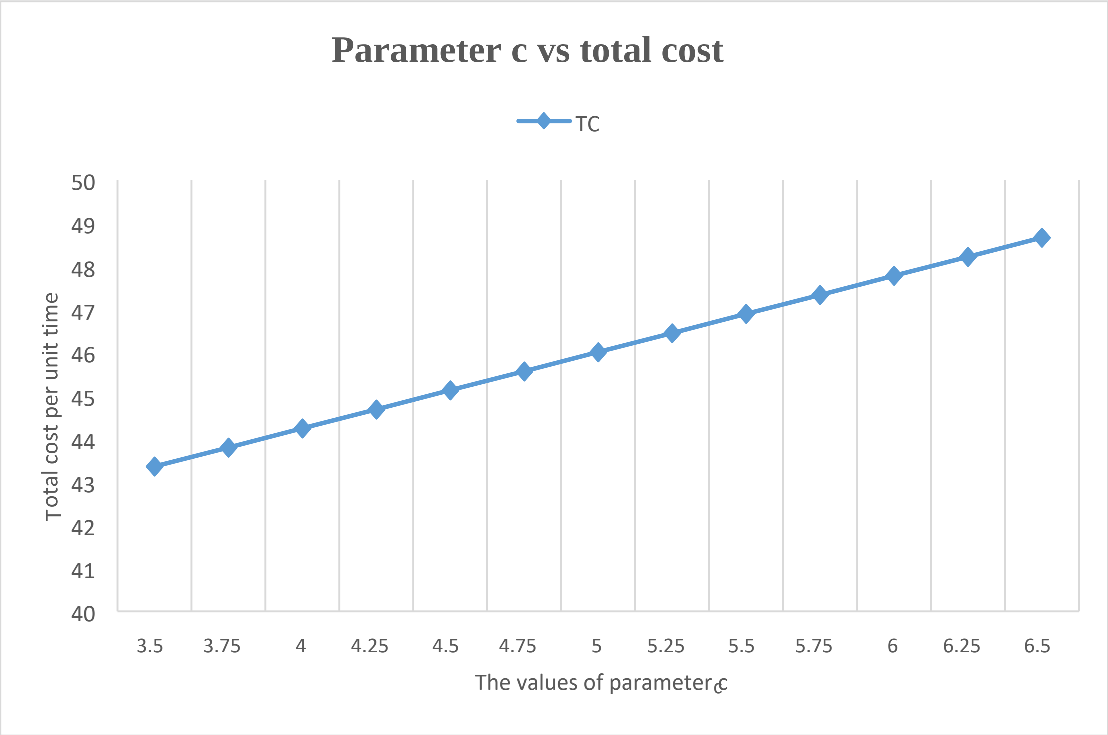

Figure 3.4 Effects of parameter c on total cost per unit time

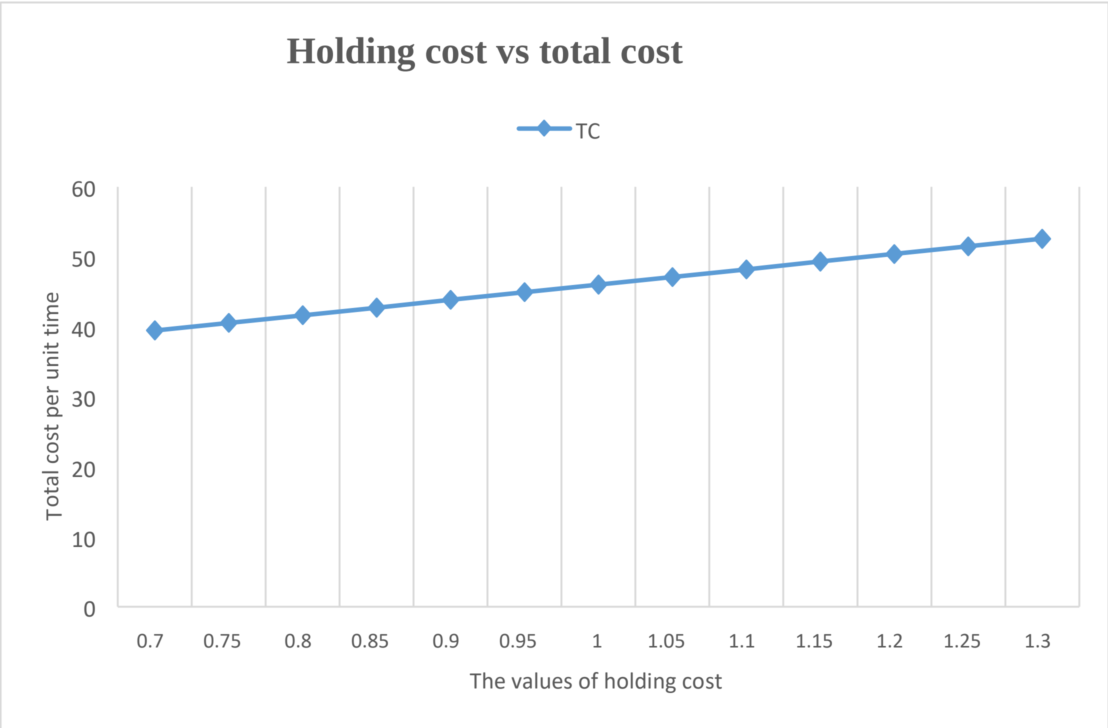

Figure 3.5 Effects of holding cost on total cost per unit time

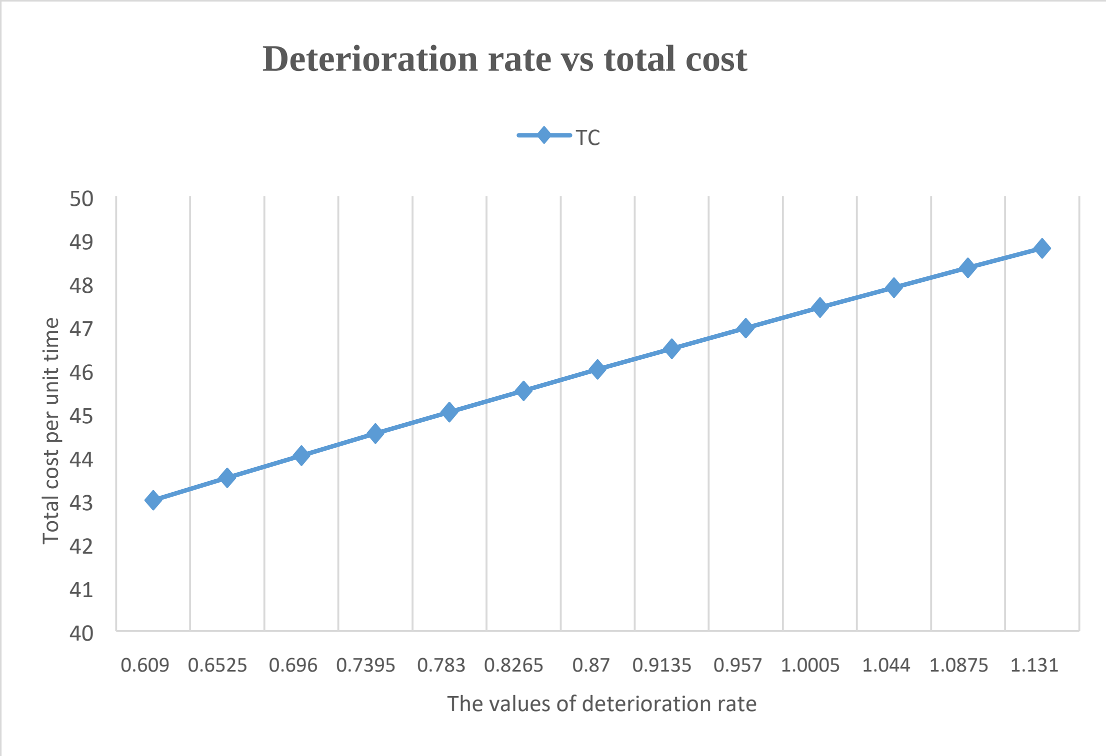

Figure 3.6 Effects of deterioration rate on total cost per unit time

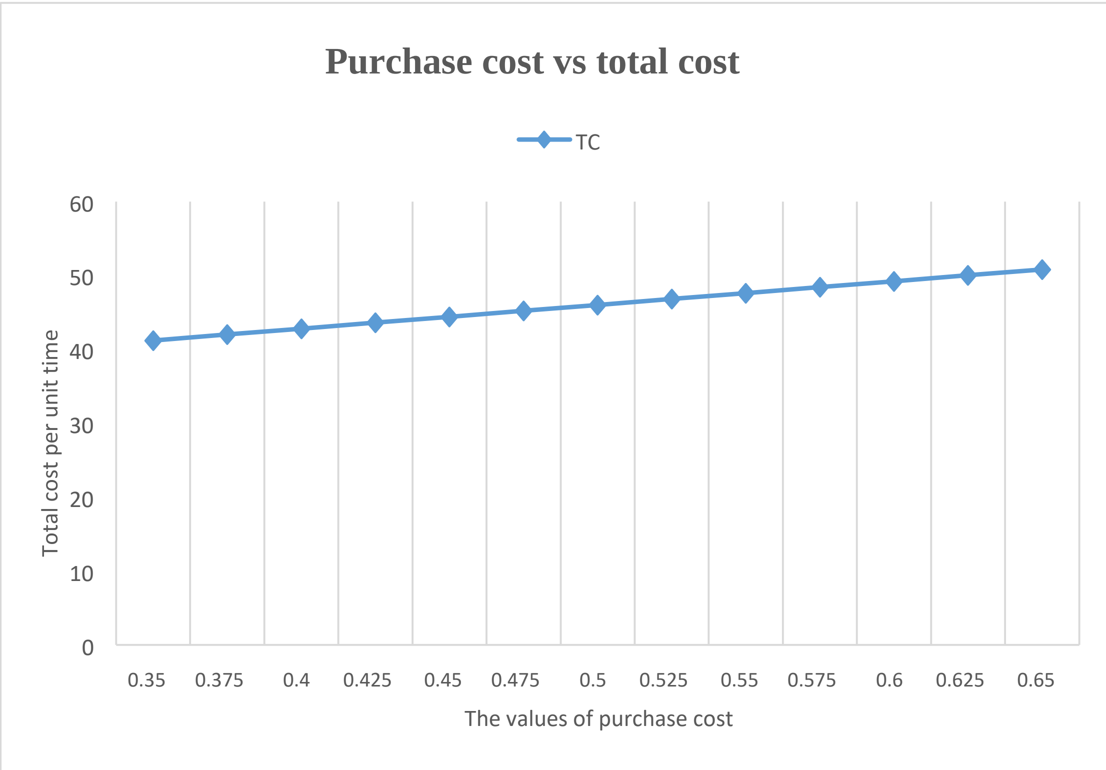

Figure 3.7 Effects of purchase cost on total cost per unit time

3.6 Conclusion

The inventory model has been upheld for IMWOPT with deterioration rate and without shortages. This model can apply in any industry to determine the effect of cost per unit time for variation of different parameters. It is evident that

- When the values of parameters of quadratic demand function increase then the total cost per unit time increase as well as when the values of parameters of quadratic demand function decrease then the total cost per unit time decrease.
- When the values of ordering cost, holding cost and purchase cost per unit time increase then the total cost per unit time increase as well as When the values of ordering cost, holding cost and purchase cost per unit time decrease then the total cost per unit time decrease.
- When the deterioration rate increase then the total cost per unit time increase rapidly whereas when the deterioration decrease then the total cost per unit time decrease rapidly.

Chapter 4

Inventory Analysis for Deteriorating Items With Time Dependent Quadratic Demand Functions with Preservation Technology

4.1 Introduction

In classical inventory models, it is assumed that the items can be preserved for an infinite time without any change of their physical status. However, in reality, many products become partially or totally unusable after a certain time period. Deterioration is regarded as a natural phenomenon for inventories which has been demonstrated very extensively for agricultural products, volatile liquids, food items, pharmaceutical products, perfumes, radioactive substances, gasoline, electronic components and photographic films. In general, it is found that items always deteriorate continuously with respect to time, but deterioration can be controlled by applying some suitable preservation technology. For example, the rate of deterioration of fish can be reduced by storing the fish in a deep fridge or by using ice. Applying cool supply-chain policy, the rate of deterioration for fruits becomes less. Though the cost of preservation technology may be high, it will be our attempt to reduce the total cost which is expended for preservation technology, and for calculating the amount of preservation technology investment to diminish the deterioration cost and to minimize the total cost.

In this study, we have developed an inventory model for instantaneous deteriorating items under the consideration of the facts: deterioration rate can be controlled by using the preservation technology (PT) during deteriorating period and deterioration rate is a function of time, which was treated as constant in most of the deteriorating inventory models. Since the effect of deterioration cannot be ignored, it has been taken variable deterioration. The consideration of preservation technology (PT) is important due to rapid social changes and it can reduce the deterioration rate significally. Also the model has been developed by the sense of time dependent quadratic demand function as quadratic demand rate requires steady increase or decrease. Demand increases or decreases due to the popularity of the product. Furthermore time is the vital factor which plays the key role in developing the inventory model. The reason for considering the variable deterioration rate and time dependent demand rate is due to change in deterioration rate with respect to time and the suitable demand for the present market situation. Shortages are not allowed for the model.

In practice, the deterioration rate of products can be controlled and reduced through various efforts such as procedural changes and specialized equipment acquisition. In this study, preservation technology is considered due to rapid social changes, and the fact that preservation technology can reduce the deterioration rate significantly. By the efforts of investing in preservation technology, we can reduce the deterioration rate. In this study, quadratic functions are assumed as demand functions. This model starts with a specific level of inventory and ends with zero-inventory after a certain time from where inventory starts accumulating to reach at the same specific level from which it started decreasing. Numerical example has been considered for illustrating the developed model. .

4.2 Mathematical Formulation

This inventory model is developed by the consideration of the replenishment problem of a single non-instantaneous deteriorating item with preservation technology. The inventory model runs as follows:

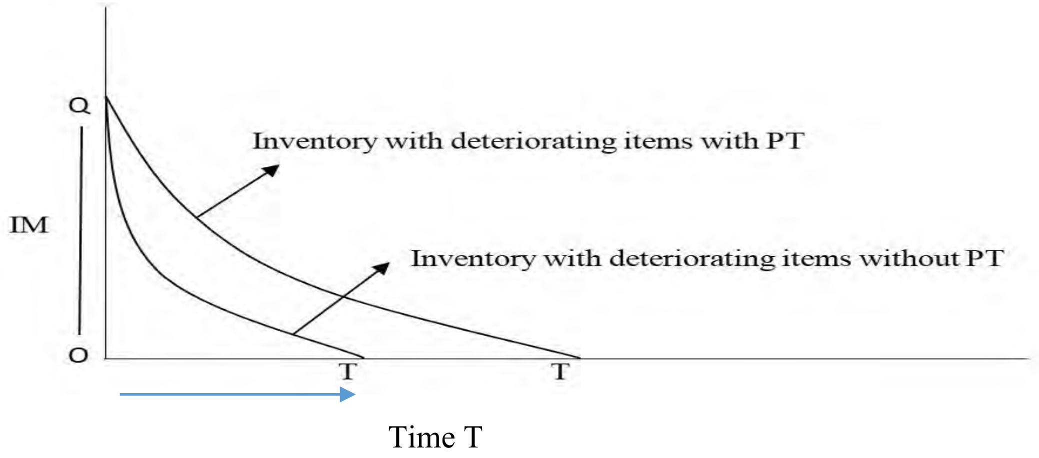

Figure 4.0 Inventory level with preservation technology

- Quadratic demand function has been taken over the time interval $[0, T]$ with the initial inventory level Q at the beginning of time.
- Deteriorated items have been evaluated over the time interval $[0, T]$.
- Deterioration of the item has been considered as a function of time i.e. $\theta = \theta t^2$.
- Resultant deterioration rate has been assumed as
  $\tau_\rho = \theta t^2 - m$, where m is a reduced deterioration rate.
- Negative sigh are given in front of quadratic demand function in the governed differential equation to indicate decrease in inventory.
- Shortage is not allowed throughout the time interval $[0, T]$.
- Holding cost has been considered which is fixed.

4.2.1 Assumption and Notations

The following assumptions are made to develop this model:

- The inventory system involves single type of items.
- Replenishment rate is infinite, i.e. Replenishment rate is instantaneously.
- The demand rate of the item is considered by a quadratic and continuous function of time.
- Preservation technology (PT) is used for reducing the deterioration rate.
- The deterioration rate is variable rate of deterioration on the on-hand inventory per unit time and there is no repair or replenishment of the deteriorated items within the cycle.
- Shortages are not allowed.
- There is no provision for repair or replacement of deteriorated units.
- Time horizon is infinite.

Decision Variables:

T Length of replenishment cycle in traditional system (per weak)

Parameters

Symbols	Abbreviation
$I(t)$	Inventory level at time t $(0 < t < T)$
A	Ordering cost of inventory per order $(TK/order)$
h	Unit holding cost per unit time
p	Purchase cost per unit of item
$D(t)$	Time dependent demand rate which is defined by $D(t) = a + bt + ct^2, a > 0, b \neq 0$ & $c \neq 0$. Here a is the initial rate of demand, b is the rate with which the demand rate increases and the rate of change in the demand rate itself changes at a rate c.
$\theta\ (t)$	Variable rate of deterioration of an item where $\theta\ (t) = \theta t^2;\ 0 < \theta < 1$
IM	Maximum inventory level during $[0, T]$
TC	Average cost per unit time
m	Reduced deterioration rate
τ_ρ	Resultant deterioration rate which is considered by $\tau_\rho = (t) - m = \theta t^2 - m$

4.2.2 Inventory model with preservation technology (IMWPT)

The inventory level $I(t)$ at time t generally decreases from initial inventory to meet markets demand and products deterioration and reaches to zero at T.

Hence, the variation of inventory with respect to time can be described by the governing differential equation:

$$\frac{dI(t)}{dt} + \tau_\rho I(t) = -(a + bt + ct^2)$$

$$\Rightarrow \frac{dI(t)}{dt} + (\theta(t) - m)I(t) = -(a + bt + ct^2)$$

$$\Rightarrow \frac{dI(t)}{dt} + (\theta t^2 - m)I(t) = -(a + bt + ct^2) \qquad \ldots\ldots\ldots (4.1)$$

with conditions $0 \le t \le T$, $(0) = IM$, $(t) = 0\ if\ t = T$

and resultant deterioration rate $\tau_\rho = \theta(t) - m$

Integrating factor I.F = $e^{\int (\theta t^2 - m)\, dt} = e^{\frac{\theta t}{3} - mt}$

Therefore the general solution is obtained by

$$I(t)e^{\frac{\theta t^3}{3} - mt} = -\int (a + bt + ct^2)e^{\frac{\theta t^3}{3} - mt} dt$$

$$= -\int (a + bt + ct^2)(1 + \frac{\theta t^3}{3} - mt)dt$$

[Neglecting higher degree of theta]

$$= -\int a\left(1 + \frac{\theta t^3}{3} - mt\right)dt - bt\int\left(1 + \frac{\theta t^3}{3} - mt\right)dt - ct^2\int\left(1 + \frac{\theta t^3}{3} - mt\right)dt$$

$$= -a\left(t + \frac{\theta t^4}{12} - \frac{mt^2}{2}\right) - b\left(\frac{t^2}{2} + \frac{\theta t^5}{15} - \frac{mt^3}{3}\right) - c\left(\frac{t^3}{3} + \frac{\theta t^6}{18} - \frac{mt^4}{4}\right) + W \qquad \ldots\ldots\ldots (4.2)$$

At the final stage, $t = 0\ if\ t = T$ in equation (4.2), we have

$$W = a\left(t + \frac{\theta t^4}{12} - \frac{mt^2}{2}\right) + b\left(\frac{t^2}{2} + \frac{\theta t^5}{15} - \frac{mt^3}{3}\right) + c\left(\frac{t^3}{3} + \frac{\theta t^6}{18} - \frac{mt^4}{4}\right)$$

Therefore,

$$I(t) = \ a\left[(T-t) + \frac{\theta}{12}(T^4 - t^4) - \frac{m}{2}(T^2 - t^2)\right]e^{-(\frac{\theta t}{3} - mt)}$$

$$+b\left[\frac{1}{2}(T^2 - t^2) + \frac{\theta}{15}(T^5 - t^5) - \frac{m}{3}(T^3 - t^3)\right]e^{-(\frac{\theta t^3}{3} - mt)}$$

$$+c[\tfrac{1}{3}(T^3 - t^3) + \tfrac{\theta}{18}(T^6 - t^6) - \tfrac{m}{4}(T^4 - t^4)]e^{-(\frac{\theta t}{3} - mt)^{3}}$$

$$= \ a\left[(T-t) + \frac{\theta}{12}(T^4 - t^4) - \frac{m}{2}(T^2 - t^2)\right](1 - \frac{\theta t^3}{3} + mt)$$

$$+b\left[\frac{1}{2}(T^2 - t^2) + \frac{\theta}{15}(T^5 - t^5) - \frac{m}{3}(T^3 - t^3)\right](1 - \frac{\theta t^3}{3} + mt)$$

$$+c[\tfrac{1}{3}(T^3 - t^3) + \tfrac{\theta}{18}(T^6 - t^6) - \tfrac{m}{4}(T^4 - t^4)](1 - \tfrac{\theta t^3}{3} + mt)$$

$$= a\{\left[(T-t) + \frac{\theta}{12}(T^4 - t^4) - \frac{m}{2}(T^2 - t^2)\right]$$
$$- \frac{\theta}{3}\left[(Tt^3 - t^4) + \frac{\theta}{12}(T^4t^3 - t^7) - \frac{m}{2}(T^2t^3 - t^5)\right]$$
$$+ [m\left\{(tT - t^2) + \frac{\theta}{12}(T^4t - t^5) - \frac{m}{2}(T^2t - t^3)\right]\}$$

$$+b\{\left[\frac{1}{2}(T^2 - t^2) + \frac{\theta}{15}(T^5 - t^5) - \frac{m}{3}(T^3 - t^3)\right]$$
$$- \frac{\theta}{3}\left[\frac{1}{2}(T^2t^3 - t^5) + \frac{\theta}{15}(T^5t^3 - t^8) - \frac{m}{3}(T^3t^3 - t^6)\right]$$
$$+ [m\left\{\frac{1}{2}(T^2t - t^3) + \frac{\theta}{15}(T^5t - t^6) - \frac{m}{3}(T^3t - t^4)\right]\}$$

$$+c\{[\tfrac{1}{3}(T^3 - t^3) + \tfrac{\theta}{18}(T^6 - t^6) - \tfrac{m}{4}(T^4 - t^4)] - \tfrac{\theta}{3}\left[\tfrac{1}{3}(T^3t^3 - t^6) + \tfrac{\theta}{18}(T^6t^3 - t^9) - \tfrac{m}{4}(T^4t^3 - t^7)\right] + [m\left\{\tfrac{1}{3}(T^3t - t^4) + \tfrac{\theta}{18}(T^6t - t^7) - \tfrac{m}{4}(T^4t - t^5)\right]\}$$

... … … (4.3)

We have, Inventory holding cost per cycle,

$$\text{IHC} = h\int_0^T I(t)dt = h\int_0^T \Big[a\{\Big[(T-t) + \frac{\theta}{12}(T^4-t^4) - \frac{m}{2}(T^2-t^2)\Big] - \frac{\theta}{3}\Big[(Tt^3-t^4) + \frac{\theta}{12}(T^4t^3-t^7) - \frac{m}{2}(T^2t^3-t^5)\Big] + \Big[m\Big\{(tT-t^2) + \frac{\theta}{12}(T^4t-t^5) - \frac{m}{2}(T^2t-t^3)\Big]\Big\} + b\{\Big[\frac{1}{2}(T^2-t^2) + \frac{\theta}{15}(T^5-t^5) - \frac{m}{3}(T^3-t^3)\Big] - \frac{\theta}{3}\Big[\frac{1}{2}(T^2t^3-t^5) + \frac{\theta}{15}(T^5t^3-t^8) - \frac{m}{3}(T^3t^3-t^6)\Big] + \Big[m\Big\{\frac{1}{2}(T^2t - t^3) + \frac{\theta}{15}(T^5t-t^6) - \frac{m}{3}(T^3t-t^4)\Big]\Big\} + c\{[\frac{1}{3}(T^3-t^3) + \frac{\theta}{18}(T^6-t^6) - \frac{m}{4}(T^4-t^4)] - \frac{\theta}{3}\Big[\frac{1}{3}(T^3t^3-t^6) + \frac{\theta}{18}(T^6t^3-t^9) - \frac{m}{4}(T^4t^3-t^7)\Big] + [m\Big\{\frac{1}{3}(T^3t-t^4) + \frac{\theta}{18}(T^6t - t^7) - \frac{m}{4}(T^4t-t^5)\Big]\Big\}\Big]dt$$

$$= h\Bigg\{\Bigg[a\left(Tt - \frac{t^2}{2}\right) + \frac{\theta}{12}\left(T^4t - \frac{t^5}{5}\right)\Bigg] - a\Bigg[\frac{\theta}{3}\left(\frac{Tt^4}{4} - \frac{t^5}{5}\right) + \frac{\theta^2}{36}\left(\frac{T^4t^4}{4} - \frac{t^8}{8}\right)\Bigg] + b\Bigg[\frac{1}{2}\left(T^2t - \frac{t^3}{3}\right) + \frac{\theta}{15}\left(T^5t - \frac{t^6}{6}\right)] - b[\frac{\theta}{6}(\frac{T^2t^4}{4} - \frac{t^6}{6}) + \frac{\theta^2}{45}\left(\frac{T^5t^4}{4} - \frac{t^9}{9}\right)\Bigg] + c\Bigg[\frac{1}{3}\left(T^3t - \frac{t^4}{4}\right) + \frac{\theta}{18}\left(T^6t - \frac{t^7}{7}\right)\Bigg] - c[\frac{\theta}{9}\left(\frac{T^3t^4}{4} - \frac{t^7}{7}\right) + \frac{\theta^2}{54}(\frac{T^6t^4}{4} - \frac{t^{10}}{10})]\Bigg\}$$

$$= h[\{a\left[\left(T^2 - \frac{T^2}{2}\right) + \frac{\theta}{12}\left(T^5 - \frac{T^5}{5}\right) - \frac{m}{2}\left(T^3 - \frac{T^3}{2}\right)\right]$$

$$- \frac{\theta}{3}\left[\left(\frac{T^5}{4} - \frac{T^5}{5}\right) + \frac{\theta}{12}\left(\frac{T^8}{4} - \frac{T^8}{8}\right) - \frac{m}{2}\left(\frac{T^6}{4} - \frac{T^6}{6}\right)\right.$$

$$\left. + m\left\{\left(\frac{T^3}{2} - \frac{T^3}{3}\right) + \frac{\theta}{12}\left(\frac{T^6}{2} - \frac{T^6}{6}\right) - \frac{m}{2}\left(\frac{T^4}{2} - \frac{T^4}{4}\right)\right\}\right]$$

$$+ b\{\left[\frac{1}{2}\left(T^3 - \frac{T^3}{2}\right) + \frac{\theta}{15}\left(T^6 - \frac{T^6}{6}\right) - \frac{m}{3}\left(T^4 - \frac{T^4}{4}\right)\right] - \frac{\theta}{3}[\frac{1}{2}\left(\frac{T^6}{4} - \frac{T^6}{6}\right)$$

$$+ \frac{\theta}{15}\left(\frac{T^9}{4} - \frac{T^9}{9}\right) - \frac{m}{3}\left(\frac{T^7}{4} - \frac{T^7}{7}\right)] + m[\frac{1}{2}\left(\frac{T^4}{2} - \frac{T^4}{4}\right) + \frac{\theta}{15}\left(\frac{T^7}{2} - \frac{T^7}{7}\right)$$

$$- \frac{m}{3}\left(\frac{T^5}{2} - \frac{T^5}{5}\right)]\} + c\{[\frac{1}{3}\left(T^4 - \frac{T^4}{2}\right) + \frac{\theta}{18}\left(T^7 - \frac{T^7}{7}\right) - \frac{m}{4}\left(T^5 - \frac{T^5}{5}\right)]$$

$$- \frac{\theta}{3}[\frac{1}{3}\left(\frac{T^7}{4} - \frac{T^7}{7}\right) + \frac{\theta}{18}\left(\frac{T^{10}}{4} - \frac{T^{10}}{10}\right) - \frac{m}{4}\left(\frac{T^8}{4} - \frac{T^8}{8}\right)] + m[\frac{1}{3}\left(\frac{T^5}{2} - \frac{T^5}{5}\right)$$

$$+ \frac{\theta}{18}\left(\frac{T^8}{2} - \frac{T^8}{8}\right) - \frac{m}{4}\left(\frac{T^6}{2} - \frac{T^6}{6}\right)]\Big]$$

$$= h\{\left[\left\{a\left(\frac{T^2}{2} + \frac{\theta T^5}{15} - \frac{mT^3}{3}\right) - \frac{a\theta}{3}\left(\frac{T^5}{20} + \frac{\theta T^8}{96} - \frac{mT^6}{24}\right) + ma\left(\frac{T^3}{6} + \frac{\theta T^6}{36} - \frac{mT^4}{8}\right)\right\}\right]$$

$$+ \left[\left\{b\left(\frac{T^3}{3} + \frac{\theta T^6}{18} - \frac{mT^4}{4}\right) - \frac{b\theta}{3}\left(\frac{T^6}{24} + \frac{\theta T^9}{108} - \frac{mT^7}{28}\right)\right.\right.$$

$$\left.\left. + mb\left(\frac{T^4}{8} + \frac{\theta T^7}{42} - \frac{mT^5}{10}\right)\right\}\right]$$

$$+ \left[\left\{c\left(\frac{T^4}{4} + \frac{\theta T^7}{21} - \frac{mT^5}{5}\right) - \frac{c\theta}{3}\left(\frac{T^7}{28} + \frac{\theta T^{10}}{120} - \frac{mT^8}{32}\right)\right.\right.$$

$$\left.\left. + mc\left(\frac{T^5}{10} + \frac{\theta T^8}{48} - \frac{mT^6}{12}\right)\right\}\right]\}$$

… … … (4.4)

At the initial stage of inventory, $t = 0, I(0) = IM$

$$IM = a\{\left[(T-t)+\frac{\theta}{12}(T^4-t^4)-\frac{m}{2}(T^2-t^2)\right] - \frac{\theta}{3}\left[(Tt^3-t^4)+\frac{\theta}{12}(T^4t^3-t^7)-\frac{m}{2}(T^2t^3-t^5)\right] + [m\left\{(tT-t^2)+\frac{\theta}{12}(T^4t-t^5)-\frac{m}{2}(T^2t-t^3)\right]\}$$

$$+b\{\left[\frac{1}{2}(T^2-t^2)+\frac{\theta}{15}(T^5-t^5)-\frac{m}{3}(T^3-t^3)\right] - \frac{\theta}{3}\left[\frac{1}{2}(T^2t^3-t^5)+\frac{\theta}{15}(T^5t^3-t^8)-\frac{m}{3}(T^3t^3-t^6)\right] + [m\left\{\frac{1}{2}(T^2t-t^3)+\frac{\theta}{15}(T^5t-t^6)-\frac{m}{3}(T^3t-t^4)\right]\}$$

$$+c\{[\frac{1}{3}(T^3-t^3)+\frac{\theta}{18}(T^6-t^6)-\frac{m}{4}(T^4-t^4)] - \frac{\theta}{3}\left[\frac{1}{3}(T^3t^3-t^6)+\frac{\theta}{18}(T^6t^3-t^9)-\frac{m}{4}(T^4t^3-t^7)\right] + [m\left\{\frac{1}{3}(T^3t-t^4)+\frac{\theta}{18}(T^6t-t^7)-\frac{m}{4}(T^4t-t^5)\right]\}$$

$$= a\left(T+\frac{\theta}{12}T^4-\frac{m}{2}T^2\right)+b\left(\frac{1}{2}T^2+\frac{\theta}{15}T^5-\frac{m}{3}T^3\right)+c(\frac{1}{3}T^3+\frac{\theta}{18}T^6-\frac{m}{4}T^4)$$

We have, ordering size, $Q = IM + IB$

$$= a\left(T+\frac{\theta}{12}T^4-\frac{m}{2}T^2\right)+b\left(\frac{1}{2}T^2+\frac{\theta}{15}T^5-\frac{m}{3}T^3\right) +c(\frac{1}{3}T^3+\frac{\theta}{18}T^6-\frac{m}{4}T^4)+0$$

$$= a\left(T+\frac{\theta}{12}T^4-\frac{m}{2}T^2\right)+b\left(\frac{1}{2}T^2+\frac{\theta}{15}T^5-\frac{m}{3}T^3\right) +c(\frac{1}{3}T^3+\frac{\theta}{18}T^6-\frac{m}{4}T^4)$$

We have, Purchase cost $P_c = p \times Q$

$$= p[a\left(T+\frac{\theta}{12}T^4-\frac{m}{2}T^2\right)+b\left(\frac{1}{2}T^2+\frac{\theta}{15}T^5-\frac{m}{3}T^3\right) +c\left(\frac{1}{3}T^3+\frac{\theta}{18}T^6-\frac{m}{4}T^4\right)]$$

Therefore, total cost (TC) per unit time is given by

$$TC = \frac{Ordering\ cost + Holding\ cost + Purchase\ cost + Preservation\ cost}{T}$$

$$= \frac{1}{T}[A + h\{\left[\left\{a\left(\frac{T^2}{2} + \frac{\theta T^5}{15} - \frac{mT^3}{3}\right) - \frac{a\theta}{3}\left(\frac{T^5}{20} + \frac{\theta T^8}{96} - \frac{mT^6}{24}\right)\right.\right.$$
$$\left.\left. + ma\left(\frac{T^3}{6} + \frac{\theta T^6}{36} - \frac{mT^4}{8}\right)\right\}\right]$$
$$+ \left[\left\{b\left(\frac{T^3}{3} + \frac{\theta T^6}{18} - \frac{mT^4}{4}\right) - \frac{b\theta}{3}\left(\frac{T^6}{24} + \frac{\theta T^9}{108} - \frac{mT^7}{28}\right)\right.\right.$$
$$\left.\left. + mb\left(\frac{T^4}{8} + \frac{\theta T^7}{42} - \frac{mT^5}{10}\right)\right\}\right]$$
$$+ \left[\left\{c\left(\frac{T^4}{4} + \frac{\theta T^7}{21} - \frac{mT^5}{5}\right) - \frac{c\theta}{3}\left(\frac{T^7}{28} + \frac{\theta T^{10}}{120} - \frac{mT^8}{32}\right)\right.\right.$$
$$\left.\left. + mc\left(\frac{T^5}{10} + \frac{\theta T^8}{48} - \frac{mT^6}{12}\right)\right\}\right]\}$$
$$+p\left[a\left(T + \frac{\theta}{12}T^4 - \frac{m}{2}T^2\right) + b\left(\frac{1}{2}T^2 + \frac{\theta}{15}T^5 - \frac{m}{3}T^3\right)\right.$$
$$\left. + c\left(\frac{1}{3}T^3 + \frac{\theta}{18}T^6 - \frac{m}{4}T^4\right)\right] + \xi]$$

$$= \frac{1}{T}[A$$
$$+ h\left\{\left[\left\{\begin{matrix} a\left(\frac{T^2}{2} + \frac{\theta T^5}{15} - \frac{mT^3}{3}\right) - \frac{a\theta}{3}\left(\frac{T^5}{20} + \frac{\theta T^8}{96} - \frac{mT^6}{24}\right) + \\ \left(\frac{ma}{2} + b\right)\left(\frac{T^3}{3} + \frac{\theta T^6}{12} - \frac{mT^4}{4}\right) - \frac{b\theta}{3}\left(\frac{T^6}{24} + \frac{\theta T^9}{108} - \frac{mT^7}{28}\right) + \left(\frac{mb}{2} + c\right) \\ \left(\frac{T^4}{4} + \frac{\theta T^7}{21} - \frac{mT^5}{5}\right) - \frac{c\theta}{3}\left(\frac{T^7}{28} + \frac{\theta T^{10}}{120} - \frac{mT^8}{32}\right) \end{matrix}\right\}\right]\right\}$$
$$+p\left[a\left(T + \frac{\theta}{12}T^4 - \frac{m}{2}T^2\right) + b\left(\frac{1}{2}T^2 + \frac{\theta}{15}T^5 - \frac{m}{3}T^3\right)\right.$$
$$\left. + c\left(\frac{1}{3}T^3 + \frac{\theta}{18}T^6 - \frac{m}{4}T^4\right)\right] + \xi]$$

... ... (4.5)

To get the optimal solution we need to solve the following equations

$$\frac{dTC}{dT} = 0 \; and \; \frac{d^2TC}{dT} = 0$$

Therefore

$$\frac{dTC}{dT} = \frac{1}{T}\Bigg[h\Bigg\{\left[a + \left(\frac{ma}{2} + b\right)T + \left(\frac{mb}{2} + c\right)T^2\right]\left(T + \frac{\theta}{12}T^4 - mT^2\right)$$

$$-\frac{\theta}{3}\left(\frac{T^4}{4} + \frac{\theta T^7}{12} - \frac{mT^5}{4}\right)(a + bT + cT^2)\Bigg\}$$

$$+ p\left\{T(b - ma) + (c - mb)T^2 + \left(\frac{a\theta}{3} - cm\right)T^3 + \frac{\theta}{3}(bT^4 + cT^5)\right\}\Bigg]$$

$$-\frac{1}{T^2}[A$$

$$+ h\{\left[\left\{\begin{matrix} a\left(\frac{T^2}{2} + \frac{\theta T^5}{15} - \frac{mT^3}{3}\right) - \frac{a\theta}{3}\left(\frac{T^5}{20} + \frac{\theta T^8}{96} - \frac{mT^6}{24}\right) + \\ \left(\frac{ma}{2} + b\right)\left(\frac{T^3}{3} + \frac{\theta T^6}{12} - \frac{mT^4}{4}\right) - \frac{b\theta}{3}\left(\frac{T^6}{24} + \frac{\theta T^9}{108} - \frac{mT^7}{28}\right) + \left(\frac{mb}{2} + c\right) \\ \left(\frac{T^4}{4} + \frac{\theta T^7}{21} - \frac{mT^5}{5}\right) - \frac{c\theta}{3}\left(\frac{T^7}{28} + \frac{\theta T^{10}}{120} - \frac{mT^8}{32}\right) \end{matrix}\right\}\right]\}$$

$$+p\left[a\left(T + \frac{\theta}{12}T^4 - \frac{m}{2}T^2\right) + b\left(\frac{1}{2}T^2 + \frac{\theta}{15}T^5 - \frac{m}{3}T^3\right) + c\left(\frac{1}{3}T^3 + \frac{\theta}{18}T^6 -\right.\right.$$

$$\left.\left.\frac{m}{4}T^4\right)\right] + \xi] \qquad \ldots\ldots\ldots (4.6)$$

$$\frac{d^2TC}{dt^2} = \frac{1}{T}\left[h\left\{\left[\left(\frac{ma}{2}+b\right)+(mb+2c)T\right]\left(T+\frac{\theta}{12}T^4-mT^2\right)+\right.\right.$$

$$\left[a+\left(\frac{ma}{2}+b\right)T+\left(\frac{mb}{2}+c\right)T^2\left(1+\frac{4\theta}{12}T^3-2mT\right)\right]-\frac{\theta}{3}\left(\frac{T^4}{4}+\frac{\theta T^7}{12}-\frac{mT^5}{4}\right)(b+2cT)-$$

$$\frac{\theta}{3}\left(T^3+\frac{7\theta T^6}{12}-\frac{5mT^4}{4}\right)(a+bT+cT^2)\Big\}+p\Big\{(b-ma)+2T(c-mb)+$$

$$(a\theta-3cm)T^2+\frac{\theta}{3}(4bT^3+5cT^4)\Big\}\Big]-\frac{1}{T^2}\left[h\left\{\left[a+\left(\frac{ma}{2}+b\right)T+\left(\frac{mb}{2}+c\right)T^2\right]\left(T+\right.\right.\right.$$

$$\left.\frac{\theta}{12}T^4-mT^2\right)-\frac{\theta}{3}\left(\frac{T^4}{4}+\frac{\theta T^7}{12}-\frac{mT^5}{4}\right)(a+bT+cT^2)\Big\}+p\Big\{T(b-ma)+(c-mb)T^2+$$

$$\left(\frac{a\theta}{3}-cm\right)T^3+\frac{\theta}{3}(bT^4+cT^5)\Big\}\Big]-\frac{1}{T^2}\left[\left[h\left\{\left[a+\left(\frac{ma}{2}+b\right)T+\left(\frac{mb}{2}+c\right)T^2\right]\left(T+\right.\right.\right.\right.$$

$$\left.\frac{\theta}{12}T^4-mT^2\right)-\frac{\theta}{3}\left(\frac{T^4}{4}+\frac{\theta T^7}{12}-\frac{mT^5}{4}\right)(a+bT+cT^2)\Big\}+p\Big\{T(b-ma)+(c-mb)T^2+$$

$$\left(\frac{a\theta}{3}-cm\right)T^3+\frac{\theta}{3}(bT^4+cT^5)\Big\}\Big]\Big]+\frac{1}{T^3}[A+$$

$$h\left\{\left[\left\{\begin{matrix} a\left(\frac{T^2}{2}+\frac{\theta T^5}{15}-\frac{mT^3}{3}\right)-\frac{a\theta}{3}\left(\frac{T^5}{20}+\frac{\theta T^8}{96}-\frac{mT^6}{24}\right)+ \\ \left(\frac{ma}{2}+b\right)\left(\frac{T^3}{3}+\frac{\theta T^6}{12}-\frac{mT^4}{4}\right)-\frac{b\theta}{3}\left(\frac{T^6}{24}+\frac{\theta T^9}{108}-\frac{mT^7}{28}\right)+\left(\frac{mb}{2}+c\right) \\ \left(\frac{T^4}{4}+\frac{\theta T^7}{21}-\frac{mT^5}{5}\right)-\frac{c\theta}{3}\left(\frac{T^7}{28}+\frac{\theta T^{10}}{120}-\frac{mT^8}{32}\right)\end{matrix}\right\}\right]\right\}$$

$$+p\left[a\left(T+\frac{\theta}{12}T^4-\frac{m}{2}T^2\right)+b\left(\frac{1}{2}T^2+\frac{\theta}{15}T^5-\frac{m}{3}T^3\right)\right.$$

$$\left.+c\left(\frac{1}{3}T^3+\frac{\theta}{18}T^6-\frac{m}{4}T^4\right)\right]+\xi]$$

4.3 Result Discussion and Computational Analysis

In this section, a numerical example is considered to illustrate this maintenance model. Parameter numerical example has been considered to check the validity inventory model without preservation technology (IMWPT) in proper units:

$A = 12\frac{tk}{order},\ a = 10,\ b = 8,\ c = 5, h = 1tk/kg, p = 15, \theta = 87\%$,

$\xi = 2tk/kg, m = 0.0001$

Solving the equation and it has been obtained optimum value of $T = 1.4790$ and the minimum average cost per unit time is evaluated $TC = \mathbf{32.5139}$.

4.4 Sensitivity Analysis

Sensitivity analysis is the process on which the optimum solution of the model is affected by the changes in its input parameter values. In this analysis, the sensitivity analysis for total cost per unit time TC is carried out with respect to the changes in the values of the parameters of quadratic function, unit holding cost per un it time, ordering cost, purchase cost, deteriorating cost, parameters a, b, & c. These sensitivity analyses is performed by considering variation in each one of the above parameters by 5% change in stipulated standard value, keeping all of the remaining parameters as a fixed.

4.4.1 Sensitivity of Different Parameters with Total Cost Per Unit Time for IMWPT

Table 4.1 (Sensitivity of ordering cost A)

index	Parameter Value	TC
1	8.4	28.924
2	9	29.5223
3	9.6	30.1206
4	10.2	30.719
5	10.8	31.3173
6	11.4	31.9156
7	**12.0000**	**32.5139**
8	12.6000	33.1123
9	13.2000	33.7106
10	13.8000	34.3089
11	14.4000	34.9072
12	15.0000	35.5056
13	15.6000	36.1039

Table 4.2 (Sensitivity of parameter a)

index	Parameter Value	TC
1	7	29.2763
2	7.5	29.8159
3	8	30.3555
4	8.5	30.8951
5	9	31.4347

6	9.5	31.9743
7	**10.0000**	**32.5139**
8	10.5000	33.0536
9	11.0000	33.5932
10	11.5000	34.1328
11	12.0000	34.6724
12	12.5000	35.212
13	13.0000	35.7516

Table 4.3 (Sensitivity of parameter b)

index	Parameter Value	TC
1	5.6	30.8962
2	6	31.1658
3	6.4	31.4354
4	6.8	31.7051
5	7.2	31.9747
6	7.6	32.2443
7	**8.0000**	**32.5139**
8	8.4000	32.7836
9	8.8000	33.0532
10	9.2000	33.3228
11	9.6000	33.5925
12	10.0000	33.8621
13	10.4000	34.1317

Table 4.4 (Sensitivity of parameter c)

index	Parameter Value	TC
1	3.5	31.8035
2	3.75	31.9219
3	4	32.0403

4	4.25	32.1587
5	4.5	32.2771
6	4.75	32.3955
7	**5.0000**	**32.5139**
8	5.2500	32.6324
9	5.5000	32.7508
10	5.7500	32.8692
11	6.0000	32.9876
12	6.2500	33.106
13	6.5000	33.2244

Table 4.5 (Sensitivity of holding cost h)

index	Parameter Value	TC
1	0.7	29.5178
2	0.75	30.0172
3	0.8	30.5165
4	0.85	31.0159
5	0.9	31.5152
6	0.95	32.0146
7	**1.0000**	**32.5139**
8	1.0500	33.0133
9	1.1000	33.5127
10	1.1500	34.012
11	1.2000	34.5114
12	1.2500	35.0107
13	1.3000	35.5101

Table 4.6 (Sensitivity of deterioration cost)

index	Parameter Value	TC
1	0.609	31.9996
2	0.6525	32.0861
3	0.696	32.1722
4	0.7395	32.2581
5	0.783	32.3436
6	0.8265	32.4289
7	**0.8700**	**32.5139**
8	0.9135	32.5987
9	0.9570	32.6831
10	1.0005	32.7673
11	1.0440	32.8512
12	1.0875	32.9348
13	1.1310	33.0181

Table 4.7 (Sensitivity of purchase cost)

index	Parameter Value	TC
1	0.35	29.9442
2	0.375	30.3725
3	0.4	30.8008
4	0.425	31.2291
5	0.45	31.6574
6	0.475	32.0857
7	**0.5000**	**32.5139**
8	0.5250	32.9422
9	0.5500	33.3705
10	0.5750	33.7988
11	0.6000	34.2271

12	0.6250	34.6554
13	0.6500	35.0837

Table 4.8 (Sensitivity of reduced deterioration rate)

index	Parameter Value	TC
1	0.000070	32.5142
2	0.000075	32.5141
3	0.000080	32.5141
4	0.000085	32.5141
5	0.000090	32.514
6	0.000095	32.514
7	**0.000100**	**32.5139**
8	0.000105	32.5139
9	0.000110	32.5139
10	0.000115	32.5138
11	0.000120	32.5138
12	0.000125	32.5138
13	0.000130	32.5137

Table 4.9 (Sensitivity of preservation technology cost)

index	Parameter Value	TC
1	1.4	31.9156
2	1.5	32.0153
3	1.6	32.1151
4	1.7	32.2148
5	1.8	32.3145
6	1.9	32.4142
7	**2.0000**	**32.5139**
8	2.1000	32.6137
9	2.2000	32.7134

10	2.3000	32.8131
11	2.4000	32.9128
12	2.5000	33.0126
13	2.6000	33.1123

4.5 Graphical presentation for the effects of parameters on total cost of per unit time:

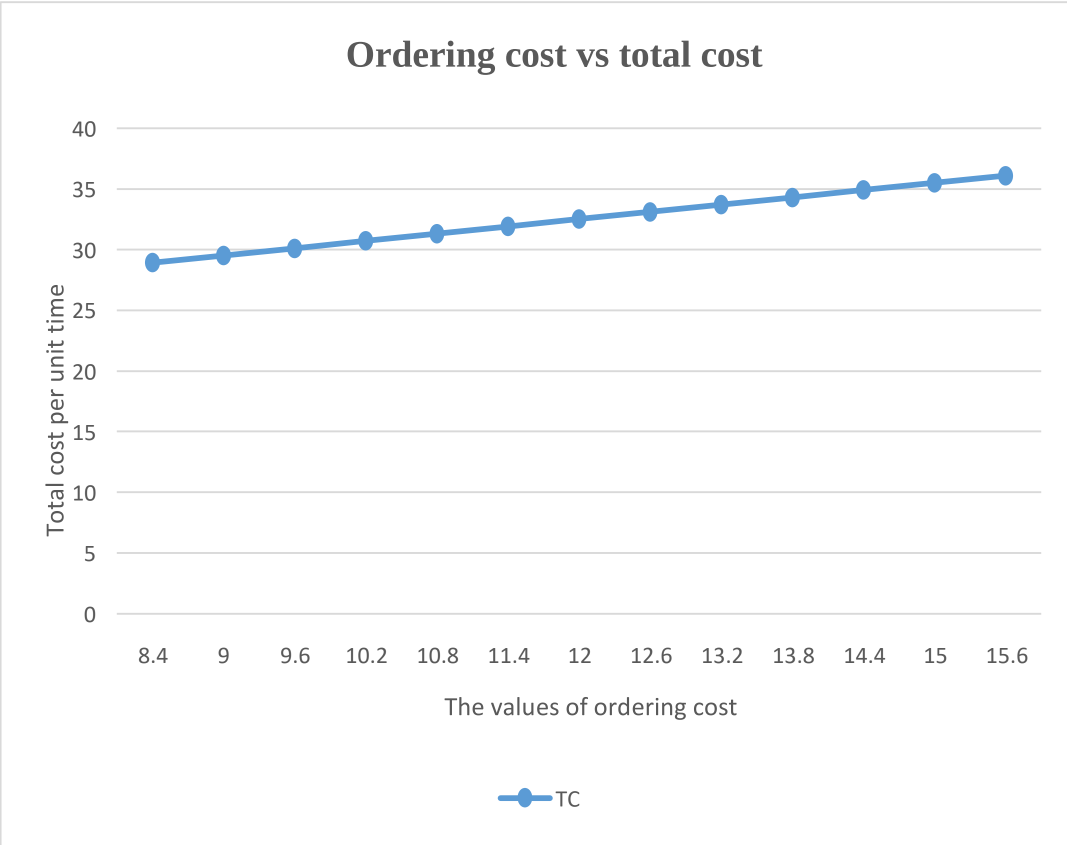

Figure 4.1 Effects of ordering cost on total cost per unit time

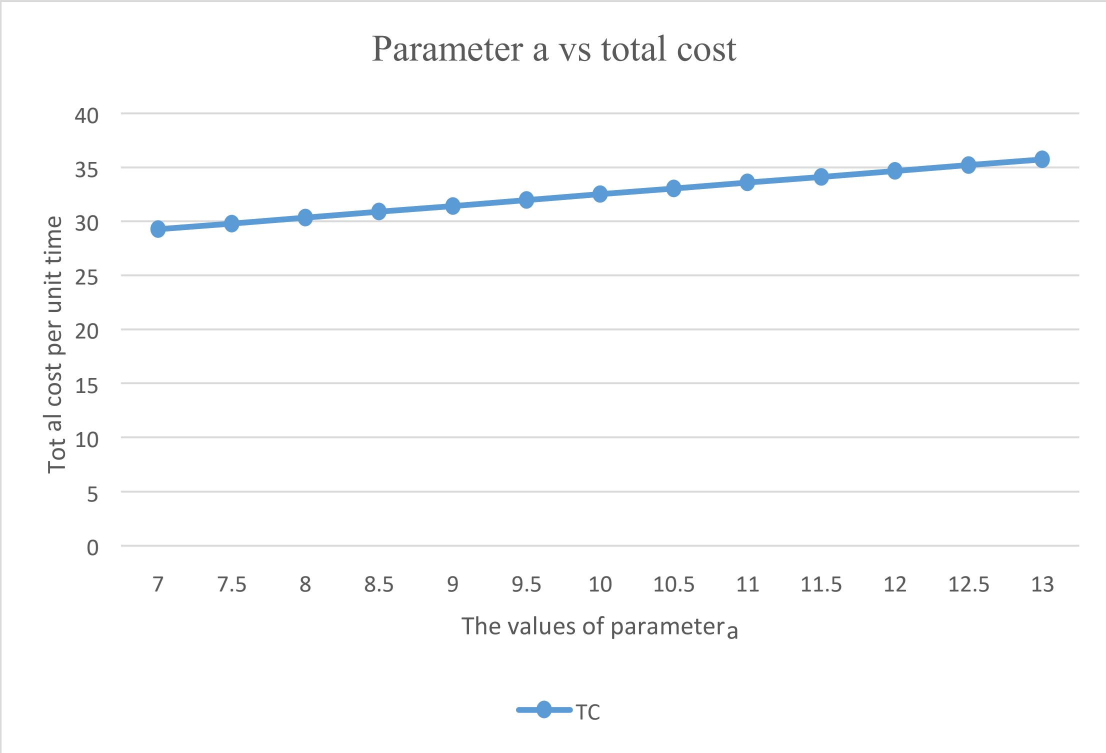

Figure 4.2 Effects of parameter a on total cost per unit time

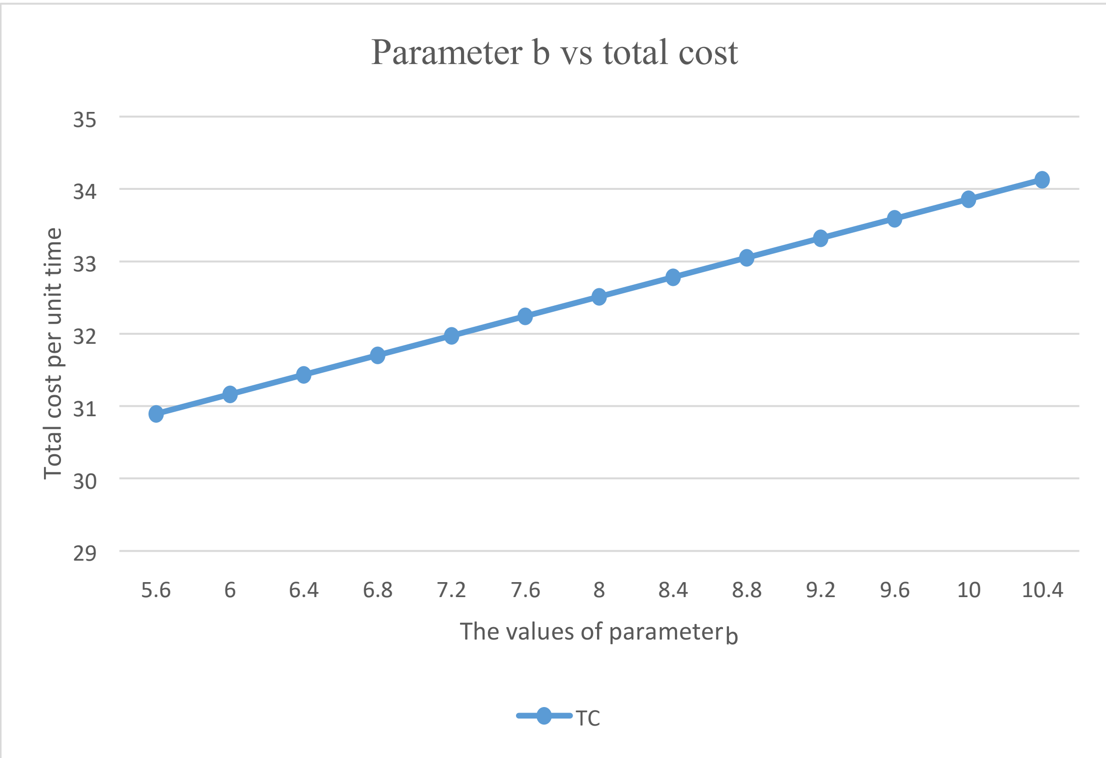

Figure 4.3 Effects of parameter b on total cost per unit time

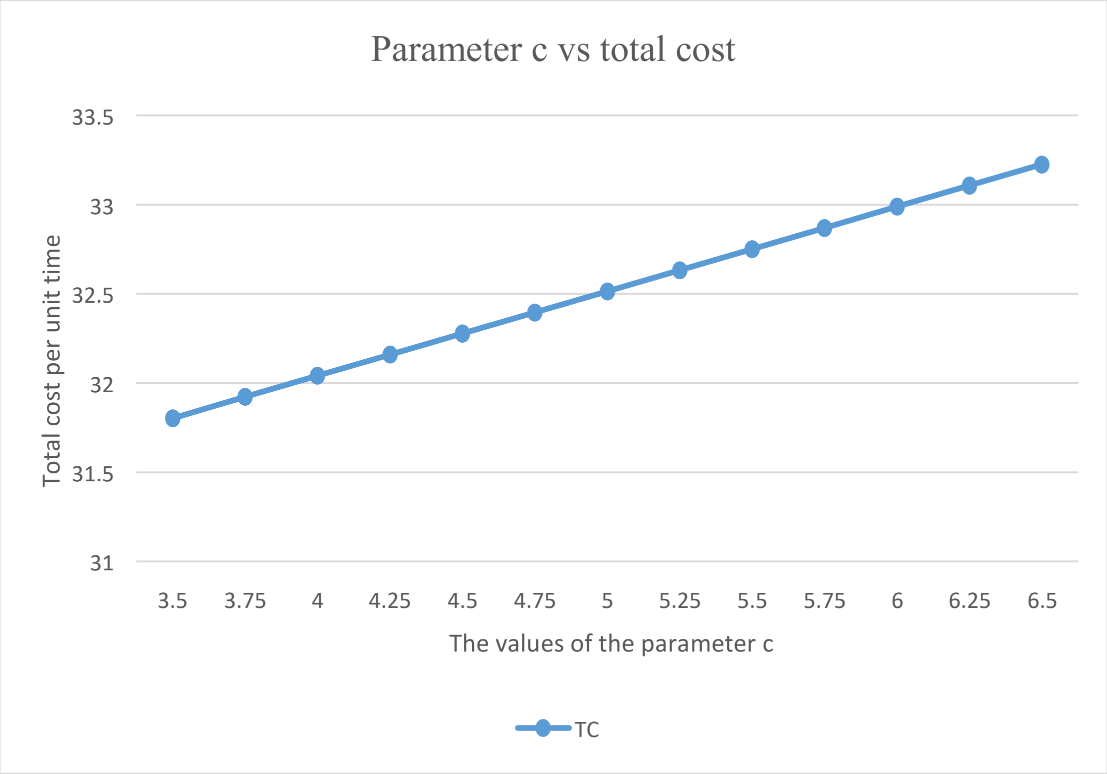

Figure 4.4 Effects of parameter c on total cost per unit time

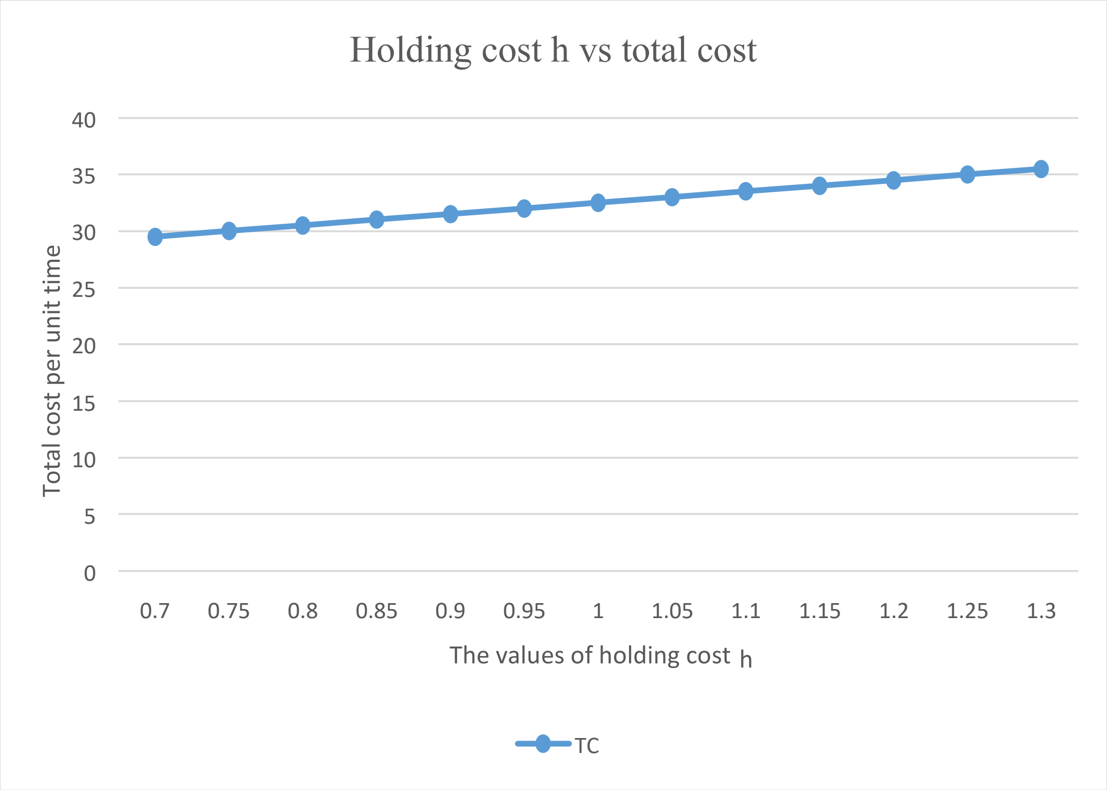

Figure 4.5 Effects of holding cost h on total cost per unit time

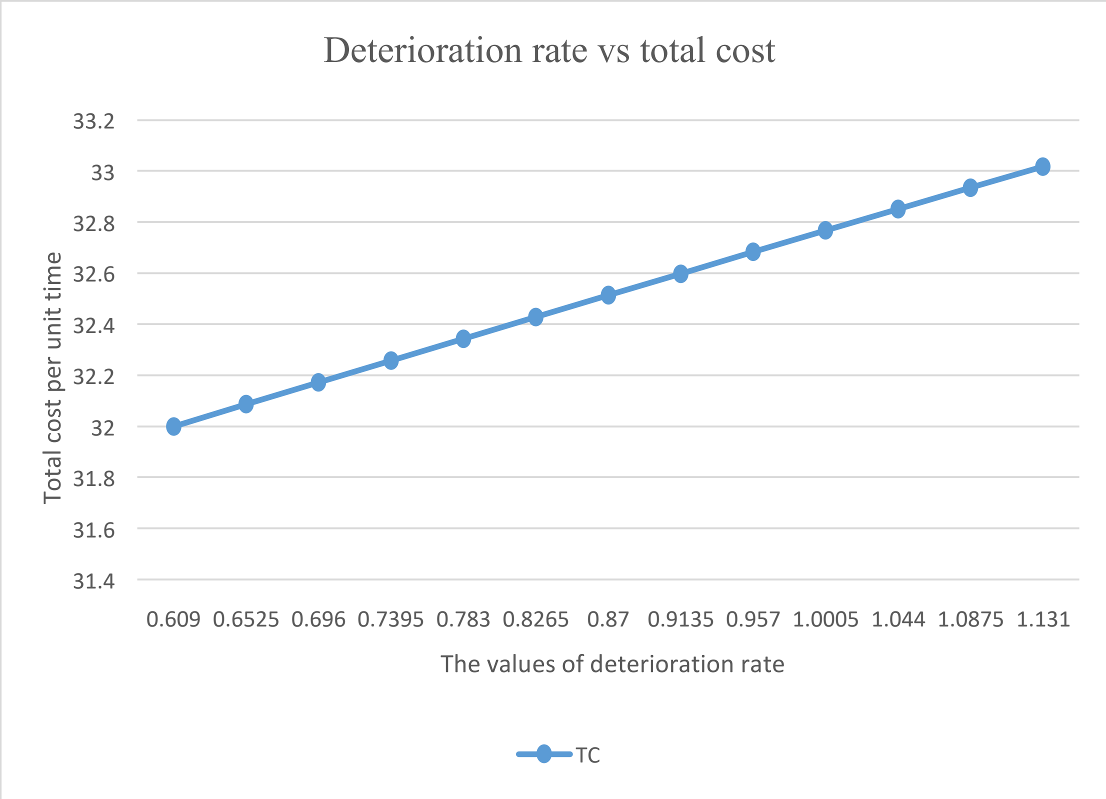

Figure 4.6 Effects of deterioration rate on total cost per unit time

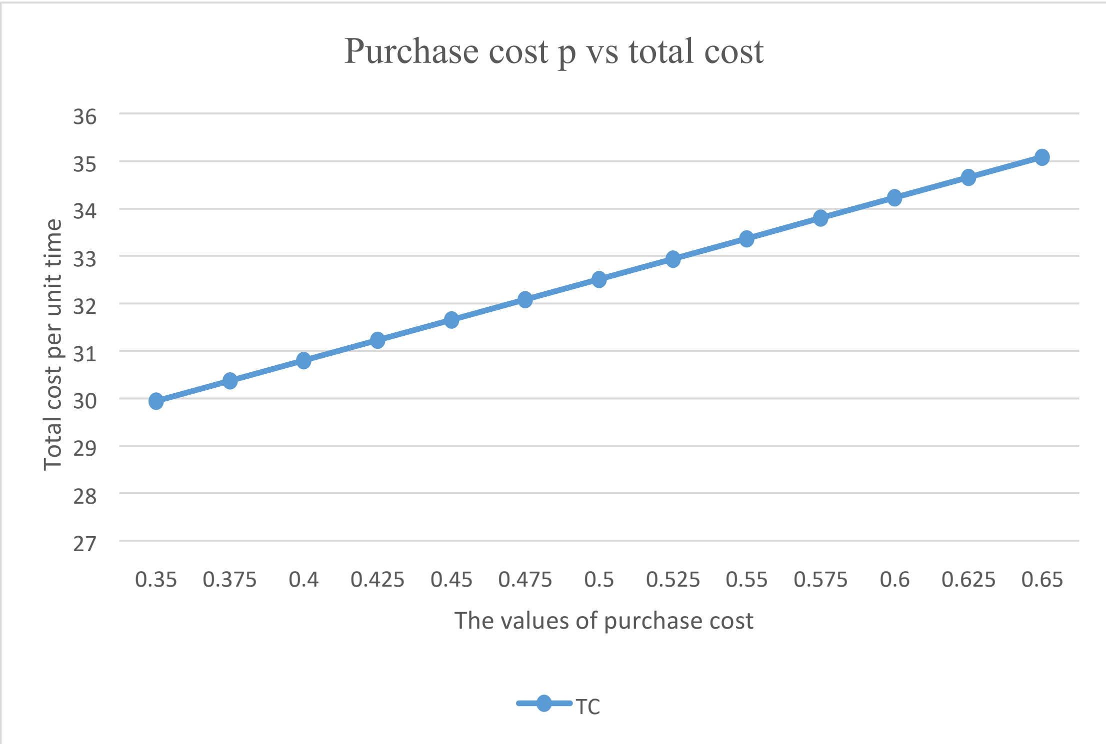

Figure 4.7 Effects of purchase cost p on total cost per unit time

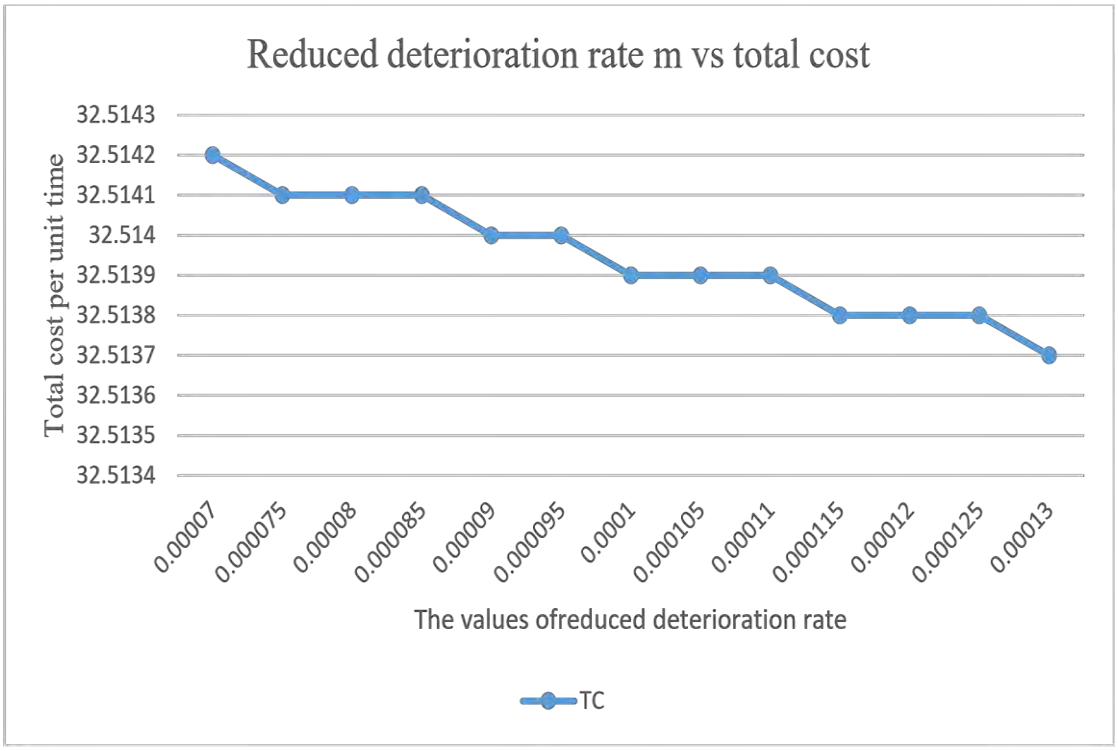

Figure 4.8 Effects of reduced deterioration rate on total cost per unit time

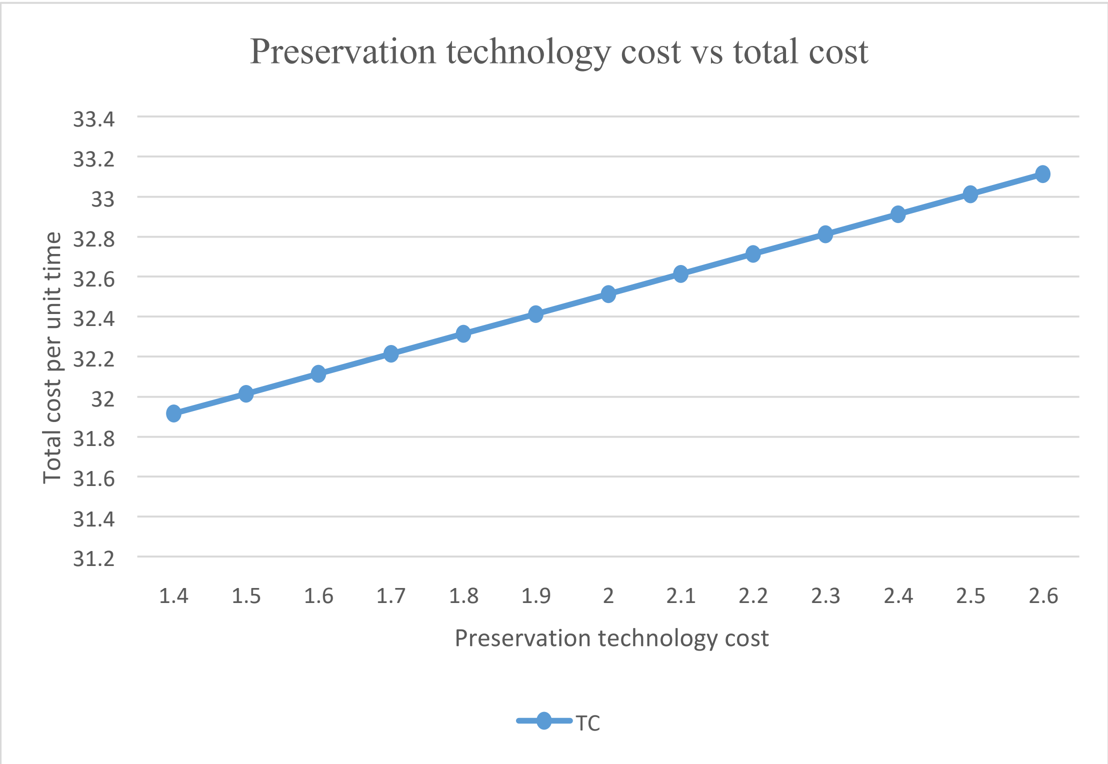

Figure 4.9 Effects of preservation technology cost on total cost per unit time

4.6 Conclusion

The inventory model has been upheld for IMWPT with deterioration rate and without shortages. This model can apply in any industry to determine the effect of cost per unit time for variation of different parameters. It is evident that

- When the values of parameters of quadratic demand function increase then the total cost per unit time increase as well as when the values of parameters of quadratic demand function decrease then the total cost per unit time decrease.
- When the values of ordering cost, holding cost and purchase cost per unit time increase then the total cost per unit time increase as well as When the values of ordering cost, holding cost and purchase cost per unit time decrease then the total cost per unit time decrease.
- When the reduced deterioration rate increase then the total cost per unit time decrease whereas when the deterioration decrease then the total cost per unit time increase.
- When the preservation technology cost increase then the total cost per unit time increase whereas when the deterioration decrease then the total cost per unit time decrease.

Chapter 5

Comparison between Inventory Model without Preservation Technology and Inventory Model without Preservation Technology

5.1 Introduction

A comparative study has been established between the inventory model without preservation technology (IMWOPT) and the inventory model with preservation technology (IMWPT) with the help of some major parameters A, h, p, a, b, c, θ respectively. Each of the parameters has been changed by 5% and effect of them has been noted on total cost per unit time. Then graphically represented the effects of individual parameters on total coat per unit time for without preservation technology (IMWOPT) and the inventory model with preservation technology (IMWPT).

5.2 Comparison table between IMWOPT Model and IMWPT Model

Ordering Cost A	**IMWOPT**		**IMWPT**	
	PARAMETER VALUE	TOTAL COST	PARAMETER VALUE	TOTAL COST
A	8.4	43.5777	8.4	28.924
	9	43.9833	9	29.5223
	9.6	44.3890	9.6	30.1206
	10.2	44.7947	10.2	30.719
	10.8	45.2004	10.8	31.3173
	11.4	45.6061	11.4	31.9156
	12.0000	**46.0117**	**12.0000**	**32.5139**

	12.6000	46.4174	12.6000	33.1123
	13.2000	46.8231	13.2000	33.7106
A	13.8000	47.2288	13.8000	34.3089
	14.4000	47.6345	14.4000	34.9072
	15.0000	48.0401	15.0000	35.5056
	15.6000	48.4458	15.6000	36.1039
Parameter a	**IMWOPT**		**IMWPT**	
	PARAMETER VALUE	TOTAL COST	PARAMETER VALUE	TOTAL COST
	7	41.0872	7	29.2763
	7.5	41.9079	7.5	29.8159
	8	42.7287	8	30.3555
	8.5	43.5495	8.5	30.8951
a	9	44.3702	9	31.4347
	9.5	45.191	9.5	31.9743
	10.0000	**46.0117**	10.0000	**32.5139**
	10.5000	46.8325	10.5000	33.0536
	11.0000	47.6532	11.0000	33.5932
	11.5000	48.474	11.5000	34.1328
	12.0000	49.2948	12.0000	34.6724
	12.5000	50.1155	12.5000	35.212
	13.0000	50.9363	13.0000	35.7516
Parameter b	**IMWOPT**		**IMWPT**	
	PARAMETER VALUE	TOTAL COST	PARAMETER VALUE	TOTAL COST
	5.6	42.221	5.6	30.8962
	6	42.8528	6	31.1658
b	6.4	43.4846	6.4	31.4354
	6.8	44.1164	6.8	31.7051
	7.2	44.7482	7.2	31.9747
	7.6	45.3799	7.6	32.2443

b	**8.0000**	**46.0117**	**8.0000**	**32.5139**
	8.4000	46.6435	8.4000	32.7836
	8.8000	47.2753	8.8000	33.0532
	9.2000	47.9071	9.2000	33.3228
	9.6000	48.5389	9.6000	33.5925
	10.0000	49.1707	10.0000	33.8621
	10.4000	49.8025	10.4000	34.1317
Parameter c	**IMWOPT**		**IMWPT**	
c	PARAMETER VALUE	TOTAL COST	PARAMETER VALUE	TOTAL COST
	3.5	43.3576	3.5	31.8035
	3.75	43.7999	3.75	31.9219
	4	44.2423	4	32.0403
	4.25	44.6846	4.25	32.1587
	4.5	45.127	4.5	32.2771
	4.75	45.5694	4.75	32.3955
	5.0000	**46.0117**	5.0000	**32.5139**
	5.2500	46.4541	5.2500	32.6324
	5.5000	46.8965	5.5000	32.7508
	5.7500	47.3388	5.7500	32.8692
	6.0000	47.7812	6.0000	32.9876
	6.2500	48.2235	6.2500	33.106
	6.5000	48.6659	6.5000	33.2244
Holding cost	**IMWOPT**		**IMWPT**	
h	PARAMETER VALUE	TOTAL COST	PARAMETER VALUE	TOTAL COST
	0.7	39.4593	0.7	29.5178
	0.75	40.5514	0.75	30.0172
	0.80	41.6435	0.80	30.5165
	0.85	42.7355	0.85	31.0159
	0.90	43.8276	0.90	31.5152

h	0.95	44.9197	0.95	32.0146
	1.0000	**46.0117**	1.0000	**32.5139**
	1.0500	47.1038	1.0500	33.0133
	1.1000	48.1959	1.1000	33.5127
	1.1500	49.2879	1.1500	34.012
	1.2000	50.3800	1.2000	34.5114
	1.2500	51.4721	1.2500	35.0107
	1.3000	52.5641	1.3000	35.5101
Deterioration rate	**IMWOPT**		**IMWPT**	
$\boldsymbol{\theta}$	PARAMETER VALUE	TOTAL COST	PARAMETER VALUE	TOTAL COST
	0.609	43.0083	0.609	31.9996
	0.6525	43.5237	0.6525	32.0861
	0.696	44.0332	0.696	32.1722
	0.7395	44.5368	0.7395	32.2581
	0.783	45.0344	0.783	32.3436
	0.8265	45.5260	0.8265	32.4289
	0.8700	**46.0117**	**0.8700**	**32.5139**
	0.9135	46.4915	0.9135	32.5987
	0.9570	46.9653	0.9570	32.6831
	1.0005	47.4331	1.0005	32.7673
	1.0440	47.895	1.0440	32.8512
	1.0875	48.3509	1.0875	32.9348
	1.1310	48.8009	1.1310	33.0181
Purchase cost	**IMWOPT**		**IMWPT**	
p	PARAMETER VALUE	TOTAL COST	PARAMETER VALUE	TOTAL COST
	0.35	41.1947	0.35	29.9442
	0.375	41.9975	0.375	30.3725
	0.4	42.8004	0.4	30.8008
	0.425	43.6032	0.425	31.2291
	0.45	44.406	0.45	31.6574

	0.475	45.2089	0.475	32.0857
	0.5000	**46.0117**	**0.5000**	**32.5139**
	0.5250	46.8146	0.5250	32.9422
	0.5500	47.6174	0.5500	33.3705
	0.5750	48.4203	0.5750	33.7988
	0.6000	49.2231	0.6000	34.2271
	0.6250	50.0259	0.6250	34.6554
	0.6500	50.8288	0.6500	35.0837

5.3 Graphical Comparison between Inventory Model without Preservation Technology and Inventory Model with Preservation:

Figure 5.1 Comparison of Ordering Cost between IMWOPT & IMWPT

From the above figure, we have observed that the cost per unit time increases with the increase of values of parameters A of the quadratic function for the inventory model without preservation technology (IMWOPT) and the inventory model with preservation technology (IMWPT). For the inventory model with preservation technology cost is minimized more than the inventory model without preservation technology which is determined using numerical results for these individual model. Comparison between IMWOPT model and IMWPT model on optimum total cost per unit time exposes that IMWPT model is better than IMWOPT model.

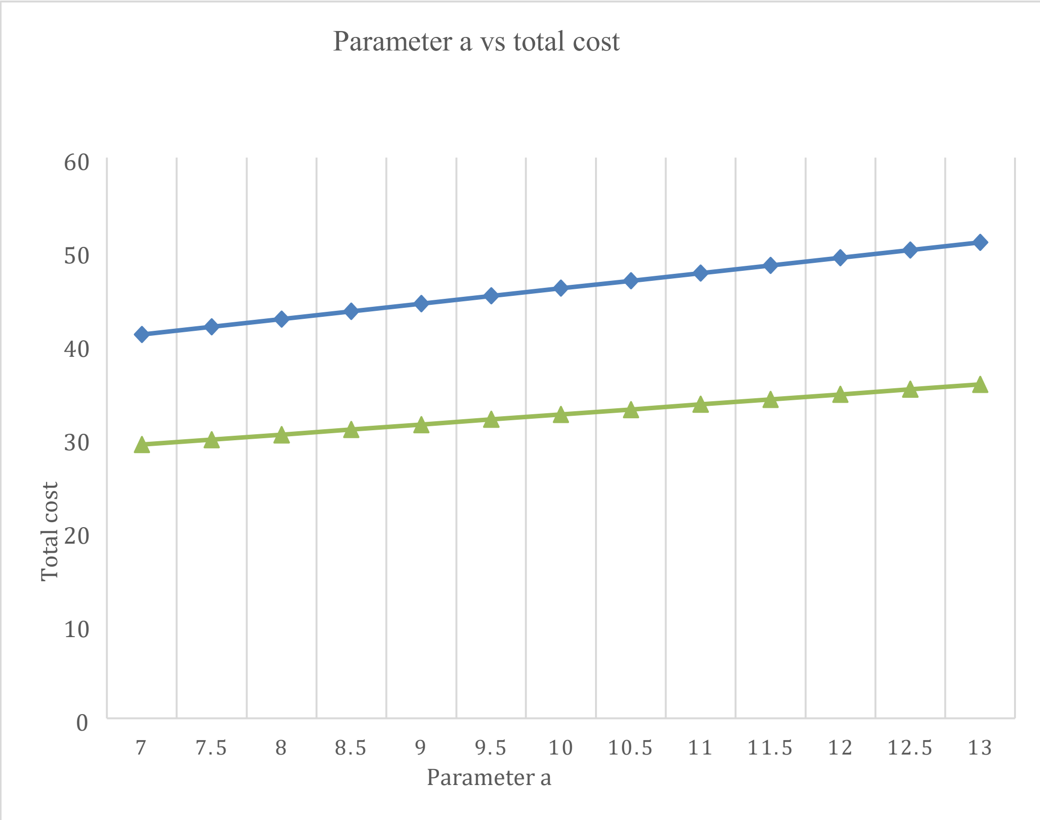

Figure 5.2 Comparison of Parameters a between IMWOPT & IMWPT

From the above figure, we have observed that the cost per unit time increases with the increase of values of parameters a of the quadratic function for the inventory model without preservation technology (IMWOPT) and the inventory model with preservation technology (IMWPT). For the inventory model with preservation technology cost is minimized more than the inventory model without preservation technology which is determined using numerical results for these individual model. Comparison between IMWOPT model and IMWPT model on optimum total cost per unit time exposes that IMWPT model is better than IMWOPT model.

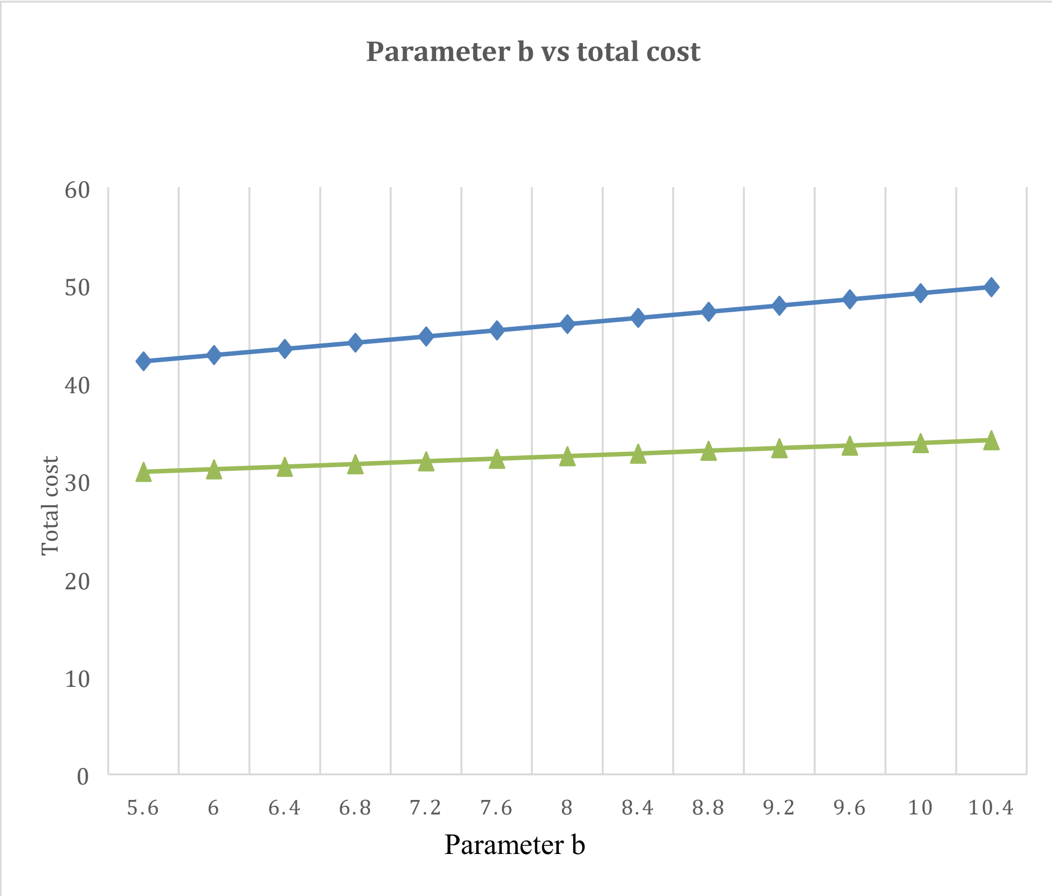

Figure 5.3 Comparison of Parameters b between IMWOPT & IMWPT

From the above figure, we have observed that the cost per unit time increases with the increase of values of parameters A of the quadratic function for the inventory model without preservation technology (IMWOPT) and the inventory model with preservation technology (IMWPT). For the inventory model with preservation technology cost is minimized more than the inventory model without preservation technology which is determined using numerical results for these individual model. Comparison between IMWOPT model and IMWPT model on optimum total cost per unit time exposes that IMWPT model is better than IMWOPT model.

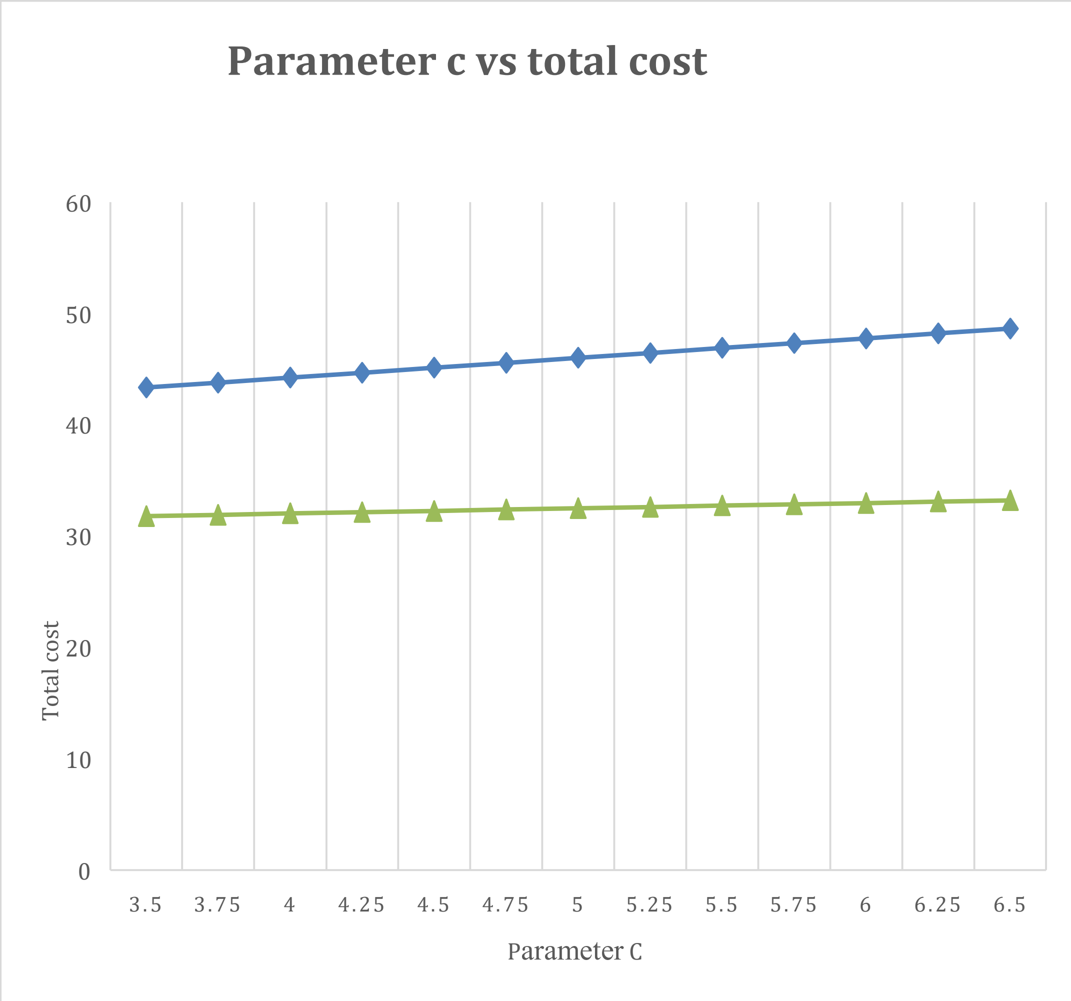

Figure 5.4 Comparison of Parameters c between IMWOPT & IMWPT

From the above figure, we have observed that the cost per unit time increases with the increase of values of parameters A of the quadratic function for the inventory model without preservation technology (IMWOPT) and the inventory model with preservation technology (IMWPT). For the inventory model with preservation technology cost is minimized more than the inventory model without preservation technology which is determined using numerical results for these individual model. Comparison between IMWOPT model and IMWPT model on optimum total cost per unit time exposes that IMWPT model is better than IMWOPT model.

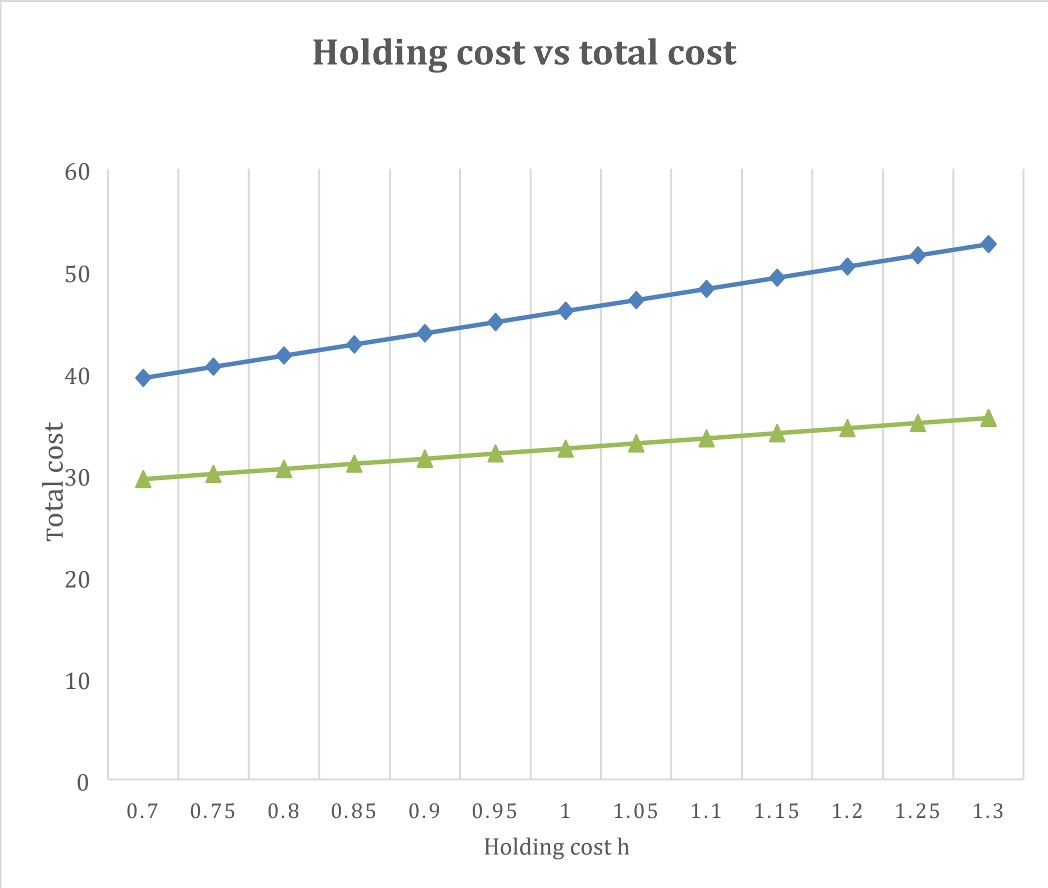

Figure 5.5 Comparison of Parameters h between IMWOPT & IMWPT

From the above figure, we have observed that the cost per unit time increases with the increase of values of parameters A of the quadratic function for the inventory model without preservation technology (IMWOPT) and the inventory model with preservation technology (IMWPT). For the inventory model with preservation technology cost is minimized more than the inventory model without preservation technology which is determined using numerical results for these individual model. Comparison between IMWOPT model and IMWPT model on optimum total cost per unit time exposes that IMWPT model is better than IMWOPT model.

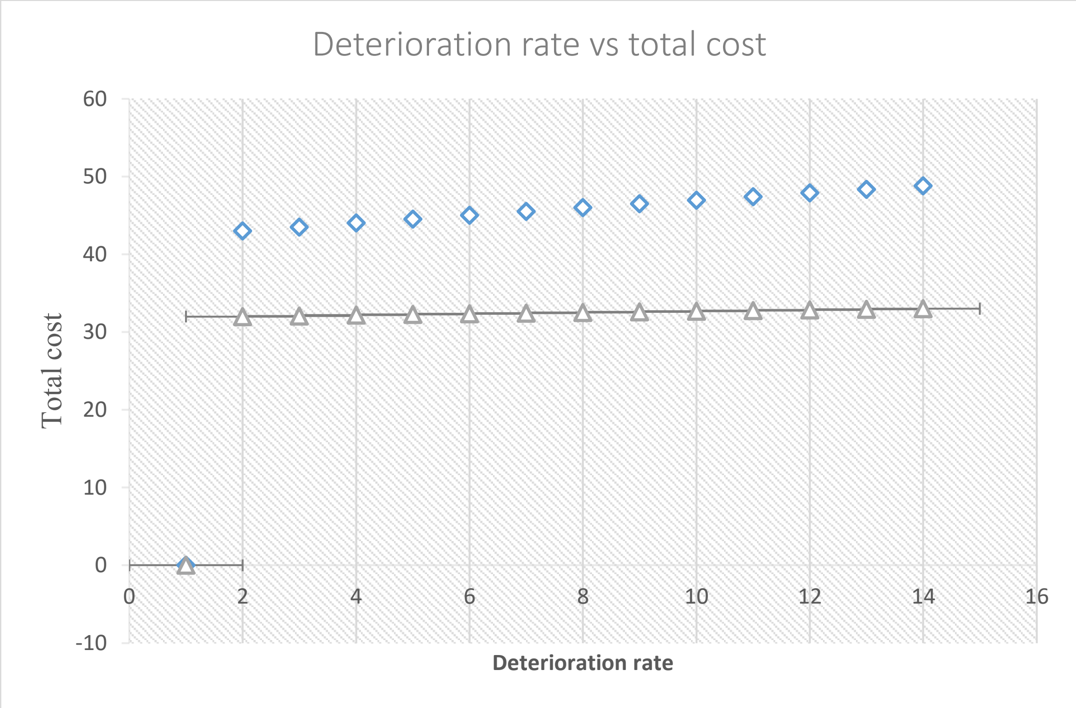

Figure 5.6 Comparison of Parameters θ between IMWOPT & IMWPT

From the above figure, we have observed that the cost per unit time increases with the increase of values of parameters A of the quadratic function for the inventory model without preservation technology (IMWOPT) and the inventory model with preservation technology (IMWPT). For the inventory model with preservation technology cost is minimized more than the inventory model without preservation technology which is determined using numerical results for these individual model. Comparison between IMWOPT model and IMWPT model on optimum total cost per unit time exposes that IMWPT model is better than IMWOPT model.

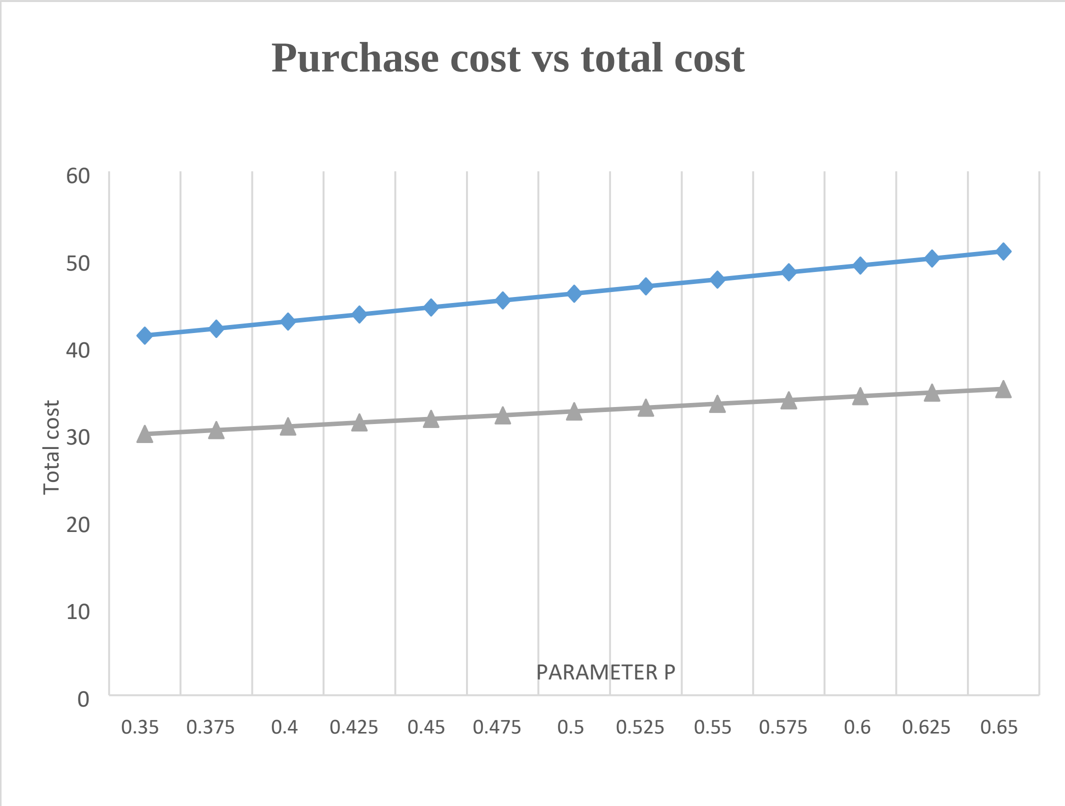

Figure 5.7 Comparison of Parameters p between IMWOPT & IMWPT

From the above figure, we have observed that the cost per unit time increases with the increase of values of parameters A of the quadratic function for the inventory model without preservation technology (IMWOPT) and the inventory model with preservation technology (IMWPT). For the inventory model with preservation technology cost is minimized more than the inventory model without preservation technology which is determined using numerical results for these individual model. Comparison between IMWOPT model and IMWPT model on optimum total cost per unit time exposes that IMWPT model is better than IMWOPT model.

Chapter 6

Conclusion and Future Recommendations

6.1 Introduction

In this chapter the effects of parameter of quadratic demand function have been depicted for models IMWOPT and IMWPT with a target of optimum total cost per unit time Afterwards, preferences of models indifferent levels have been manifested for optimum total cost per unit time. Finally, several ways have been upheld to extend the present working days ahead.

In daily life, the deterioration of goods is a common phenomenon. Recently, deteriorating items in inventory system have become an interesting feature for its practical importance. Deterioration is defined as decay, spoilage, damage, evaporation, obsolescence, pilferage, loss of utility or marginal value of a commodity that results in decreasing usefulness from the original one. Food items, photographic films, drugs, chemicals, pharmaceuticals, electronic components and radioactive substances are some examples of items. The effect of deterioration of physical goods cannot be disregarded in many inventory systems

Two inventory models IMWOPT and IMWPT with time dependent demand function have been formulated where it has been assumed that deterioration rate is variable with respect to time. Shortage is not allowed for the both models IMWOPT and IMWPT. Quadratic demand function has been introduced to formulate both the models. Also preservation technology has been introduced to formulate the model IMWPT. To validate the two models, numerical examples with the sensitivity of the several parameters have been considered.

From the table 5.1 different phenomena are observed. Optimum total cost per unit time increase and decreases for IMWOPT model, where as it increases and decrease of values of parameters of quadratic function. Optimum total cost per unit time increase and decreases for IMWPT model, where as it increases and decrease of values of parameters of quadratic function.

6.2 Studies in Days Ahead

In consideration of the present research “an inventory analysis for deteriorating items deploying preservation technology of time dependent quadratic demand function” the recommendations for future works are as under:

- To consider probabilistic demand function for IMWOPT model and IMWPT model respectively.
- To consider exponential demand function for IMWOPT model and IMWPT model respectively.
- To consider variable holding cost for the entire time cycle to evaluate the total cost for IMWOPT model and IMWPT model respectively.
- To validate the model collecting real data.

REFERENCES

[1] Harris, F. W., (1915), “Operations and cost (factory management series)”, A. W. Shaw Co.

[2] Wilson, R. H., (1934), “A scientific routine for stock control, Harvard Business Review”, Vol. 13, pp. 116-128.

[3] Whitin, T. M., (1957), “The theory of inventory management”, Princeton University Press, Princeton, NJ.

[4] Ghare, P. M., and Schrader, G. F., (1963), “A model for an exponentially decaying inventory”, Journal of Industrial Engineering, Vol. 14, pp. 238-243.

[5] Covert RP, and Philip GC, (1973), “An EOQ model for items with Weibull distribution deterioration”, AIIE Trans. Vol. 5, pp. 323-326.

[6] Donaldson WA, (1977), “ Inventory replenishment policy for a linear trend in demandan analytic solution”, Journal of Operational Research Society, Vol. 28, pp. 663-670.

[7] Dave, U., and Patel, L. K., (1981), “Policy inventory model for deteriorating items with time proportional demand”, Journal of Operational Research Society, Vol. 32, pp. 137-142.

[8] Goswami, A., and Chaudhuri, K. S., (1991), “An EOQ model for deteriorating items with linear trend in demand”, Journal of Operational Research Society”, Vol. 42, pp. 1105-1110.

[9] Wee, H. M., (1993) “Economic production lot size model for deteriorating items with partial back ordering [J]”, Compute~& Industrial Engineering, Vol. 24, No. 3, pp. 449– 458.

[10] Chung, K. J., and Ting, P. S., (1993), “A heuristic for replenishment of deteriorating items with linear trend in demand”, Journal of Operational Research Society, Vol. 44(12), pp. 1235-1241.

[11] Sarker, B. R., Jamal, A. M. M., and Wang, S. J., (2000), “Supply chain models for perishable products under inflation and permissible delay in payment [J]”,Vol. 27, pp. 59–75.

[12] Kalpakam, S. and Shanthi, S., (2000) “A perishable system with modified base stock policy and random supply quantity [J]”,Vol. 39, pp. 79–89.

[13] Kalpakam, S. and Shanthi, S., (2001) “A perishable inventory system with modified(S-1, S) policy and arbitrary processing times [J]”, Computers & Operations Research, Vol. 28, pp. 453–471.

[14] Chang, C.-T., (2004), “An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity [J]”, International Journal of Production Economics, Vol. 88, pp. 307–316.

[15] Sana, S., Goyal, S. K., and Chaudhuri, K. S.(2004), “A production-inventory model for a deteriorating item with trended demand and shortages [J]”, European Journal of Operational Research, Vol. 157, pp. 357–371, 2004.

[16] Wang S.-D., and Wang, J.-P., (2005) “A multi-stage optimal inventory model for deteriorating items by considering time value and inflation rate [J]”, Operations Research and Management Science, Vol. 14(6), pp. 142–148.

[17] YUNG, C., (2005)“A comparison among various partial backlogging inventory lotsize models for deteriorating items on the basis of maximum profit ”, International Journal of Production Economics, Vol. 96, pp. 119–128.

[18] Chung, K.-J., and Liao, J.-J., (2006), “The optimal ordering policy in a DCF analysis for deteriorating items when trade credit depends on the order quantity [J]”, International Journal of Production Economics, Vol. 100, pp. 116–130.

[19] Dye, C.-Y., Chang, H.-J., and Teng, J.-T., (2006) “A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging [J]”, European Journal of Operational Research, Vol. 172, pp. 417–429.

[20] Panda, S., Senapati, S., and Basu, M. (2008) “Optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand [J]”, Computers & Industrial Engineering, Vol. 54, pp. 301–314.

[21] Khanra, S., Ghosh S. K., and Chaudhuri, S. K. (2011), “An EOQ model for a deteriorating items with time dependent quadratic demand under permissible delay in payment”, Applied Mathematics and Computation, Vol. 218, pp. 1-9.

[22] Mishra V. K. and Singh, L. S., (2011), “ Deteriorating inventory model for time dependent demand and holding cost with partial backlogging”, International Journal of Management Science and Engineering Management, Vol. 6, pp. No.4, pp. 267-271.

[23]Bakker, M. Riezebos, J., and Teunter, R.H., (2012), “Review of inventory systems with deterioration”, European Journal of Operations Research, Vol. 221, pp. 275–284.

[24] Hsieh, T.P., Dye, C.Y. (2013), “A production-inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time”, Journal of Computational and Applied Mathematics, Vol. 239, pp. 25–36.