

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2020-2021

Sub: **CSE 203** (Data Structures and Algorithms I)

Full Marks: 210

Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

**SECTION – A**There are **FOUR** questions in this section. Answer any **THREE** questions.

In case of quotations and extracts, please give the full reference at the bottom of the paper.

1. (a) Deduce the time complexity of the following code fragments:
- (2×6=12)**

Code Fragment 1	Code Fragment 2
<pre> k = 1; while (n &gt; 1) {     n = n/2;     k++; } </pre>	<pre> sum = 0; for (k=1; k&lt;=n; k*=2)     for (j=1 ; j&lt;= k; j++)         sum++; </pre>

- (b) Using the substitution method to solve the following recurrence:
- (11)**

$$T(n) = 4T\left(\frac{n}{2}\right) + 100n$$

- (c) Identify whether the following statements are true or false clearly justifying your answer:
- (12)**

- If an algorithm has time complexity  $O(n^2)$  then it always makes  $n^2$  steps, where  $n$  is the size of the input.
- An algorithm with time complexity  $O(n)$  is always slower than an algorithm with time complexity  $O(\log_2 n)$ , for any input.
- An algorithm that makes  $c_1 \log_2 n$  steps and an algorithm that makes  $c_2 \log_4 n$  steps are in the same complexity class ( $c_1, c_2$  are constants).
- An  $O(2^n)$  algorithm can never be faster than an  $O(n)$  algorithm.

2. (a) While preparing for this exam, you have found an interesting question in a very old book with missing pages. It talks about a Binary Search Tree (BST) that stores integers in the range from 1 to 100 and proposes to search 22 and print the values encountered on the search path through the BST. Then it provides the following sequences of numbers (Sequences 1-3) and asks the following question: "Are these sequences possible search paths through the BST?"
- (9)**

Sequence 1 : 100, 40, 20, 30, 23, 35, 22

Sequence 2 : 100, 35, 15, 32, 20, 22

Sequence 3 : 50, 60, 90, 40, 75

Contd ..... P/2

**CSE 203**

**Contd... Q. No. 2(a)**

You have to answer the above question. If your answer is 'yes', please show the path (with branches), if 'no', explain why.

[Hint: you were thinking whether there was a missing page with a figure showing the BST, but your genius little brother smiled and hinted that no need to check for a figure as the answer lied already within the sequences provided.]

(b) In a Binary Search Tree (BST), successor of a node, N is defined as the node,  $N_{suc}$  with the smallest key greater than the key N. Similarly, the predecessor of N is defined as the node,  $N_{pre}$  with the largest key smaller than the key of N. Now answer the following questions ((i) ---(iii)).

(i) Write an efficient algorithm that will store the keys of the successor and predecessor of each node. Assume that you have two variables, suc and pre, respectively in the node class/data structure for that purpose. (7)

(ii) Discuss an efficient implementation strategy of `removeFullNode(N, Nsuc)` function (as defined below) with the help of appropriate illustrative examples. Analyze the time complexity of your strategy. (12)

<code>removeFullNode(N, N<sub>suc</sub>)</code>	N is a full node that has to be removed from the BST. N <sub>suc</sub> is the successor of N.
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(iii) Your genius little brother suggests that an equivalent strategy is available if we use  $N_{pre}$  instead of  $N_{suc}$ . Do you agree with him? Justify your answer using the same illustrative example you showed in 2.b(ii). (7)

3. (a) Assume that a binary heap is stored in an array called `treeNodes`, which has a capacity of 100 array elements, with array index 0 not being used. If the binary heap currently contains 86 keys, answer the following questions with proper explanation. (12)

- i) Is `treeNodes[44]` a leaf node?
- ii) How many children does `treeNodes[43]` have?
- iii) Is the subtree rooted at `treeNodes[8]` a full binary tree?
- iv) How many levels are there in the subtree rooted at `treeNodes[8]` including `treeNodes[8]`?
- v) Including the root node (`treeNodes[1]`) how many levels are there in the binary heap?
- vi) How many leaf nodes are there?

(b) Present and analyze the `BUILD_MAX_HEAP(array A)` function. Prove its correctness with the help of an appropriate loop invariant. (3+10+10=23)

**CSE 203**

4. (a) Suppose you are given an implementation of a list Abstract Data Type (ADT) which has the functions listed in Table A. Implement a stack which has the functionality shown in Table B. In your implementation, you can only use the list implementation of Table A. Assume that the elements are integers. (15)

Table A: List

insert(item)	Inserts an element at the current location.
append(item)	Appends an element at the end of the list.
remove()	Remove and return the current element.
moveToStart()	Set the current position to the start of the list.
moveToEnd()	Set the current position to the end of the list.
prev()	Move the current position one step left. No change if already at the beginning.
next()	Move the current position one step right. No change if already at the end.
length()	Return the number of elements in the list.
currPos()	Return the position (in the list) of the current element.
moveToPos(int pos)	Set current position.
getValue()	Return the current element.

Table B: Stack

clear()	Reinitialize the stack, i.e., make it (logically) empty stack.
push(item)	Pushes an element.
pop()	Pop an element.
length()	Return the number of elements in the stack.
max()	Return the value of the maximum element in the stack.

(b) Your friend uses your above stack implementation regularly. A bug has been reported for the max() function of your stack implementation. Now your friend needs to implement a separate independent max() function which uses the stack data structure (avoiding its own max() function). He does not have access to the list data structure of Table A. How will he implement the max() function? He cannot use any data structure other than your stack implementation. (10)

(c) We use a doubly linked list (DLL) to keep records of some books. In the list, all the books that belong to the same publisher are placed in consecutive nodes. Now, write a function addBook(title, publisher) that inserts a book into the DLL as the first book for its publisher. So, if the list already has three books for a particular publisher, the 4<sup>th</sup> book will be inserted in front of the three books already in the DLL. For a new publisher, the book should be inserted at the end of the DLL. Assume that each DLL node has 'prev' and 'next' pointers and stores the title and publisher of a book as Strings. (10)

**CSE 203**

**SECTION – B**

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Once an amateur thief entered into a museum full of tantalizing jewelry, geodes and rare gems. As he is new in this field, he brought just a single backpack with him that has a capacity of 6kg. The thief planned to get away with the most valuable objects without overloading the backpack. He found out that in a less-secured compartment, there were four (4) solid items, and their information are given in the following table.

Item	Weight (kg)	Value (USD)
Brooch of the queen	3	15
Crown of the king	2	20
Decorated vase	10	30
An ancient coin	2	14

Table for Ques No. 5(a) – Information of valuable items found in the museum

Since the items are unbreakable, for each item, the thief must pick the whole of it or not consider it at all. Now although our thief is amateur in this business, he possesses some algorithmic knowledge. Therefore, he applied a greedy technique and selected the items that gave him the maximum possible profit per weight, and then he left the museum with so much zeal! Now, using the information of the backpack of the thief and the item table given for this question, your first task is to show the steps of finding the maximum possible profit and the corresponding list of items using the technique that the thief adopted. You have also identified that the scenario at the museum was a special one and only dynamic programming is able to find the optimal solution in that case. Therefore, your second task is to find the optimal profit and the list of items using dynamic programming. **(10+10=20)**

(b) Once upon a time there was a city that had no proper roads. Getting around the city was particularly difficult after rainstorms because the ground used to become very muddy, cars got stuck in the mud, and people got their boots dirty. The mayor of the city decided that some of the streets must be paved, but he did not want to spend more money than necessary because the city also wanted to build a swimming pool. The mayor therefore specified two conditions:

- (i) Enough streets must be paved so that it is possible for everyone to travel from their house to anyone else's house only along the paved roads, and
- (ii) The paving should cost as little as possible.

The following figure shows the layout of the city. The number of paving stones in between two houses represents the cost of paving that route. Your task is to apply a greedy based algorithm that you have learned under this course to fulfill the

**CSE 203**

**Contd... Q. No. 5(b)**

requirement of the Mayor. Note that the bridge in the figure should not be considered as a paving stone. In your answer script, you must draw a simple graph representing the given layout of the muddy city. After that, you have to show the steps of applying your chosen greedy algorithm to find the connected subgraph which contains all the houses of the city (considering them as vertices of your drawn graph) and a subset of roads (considering them as edges) in a way that the total number of paving stones required is the minimum. You have to ensure that your connected subgraph does not contain any cycle, and you need to mention the algorithm that you are adopting to find your desired solution.

**(15)**

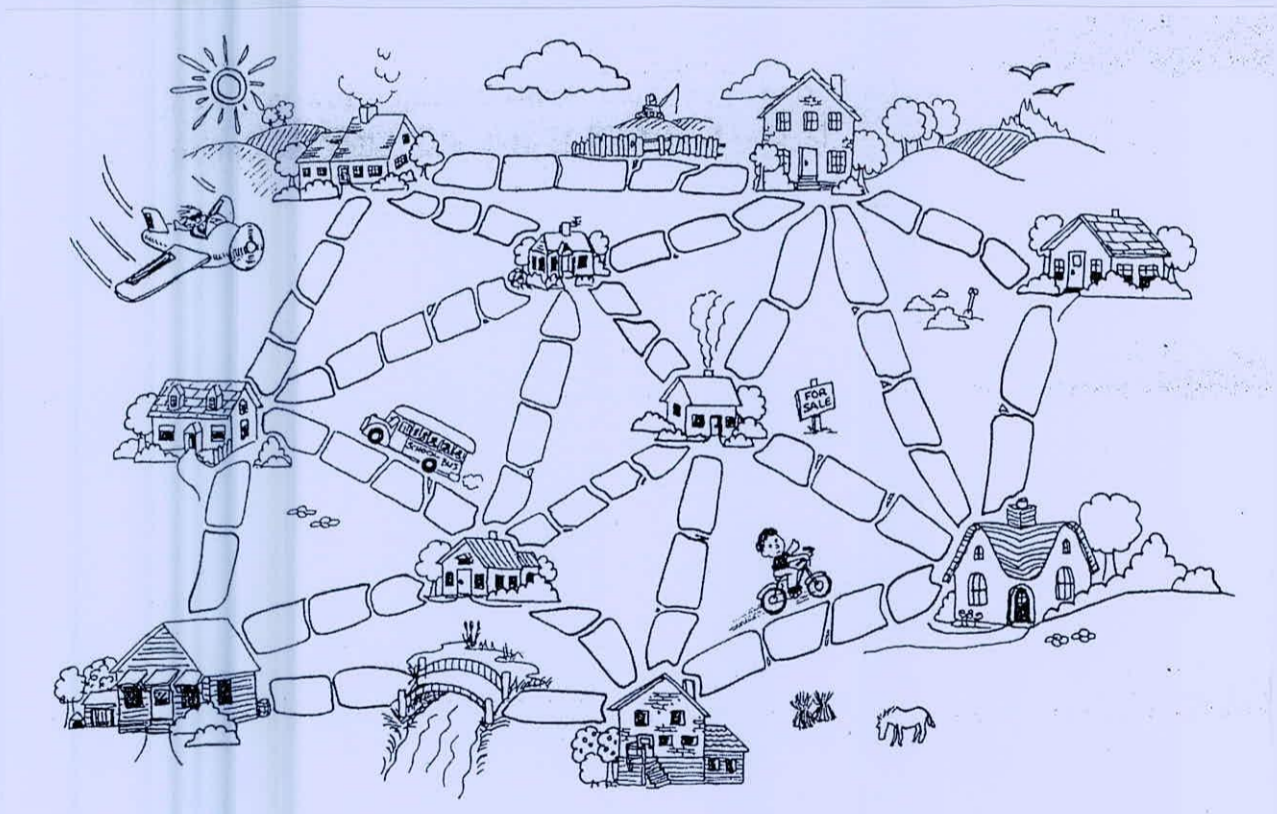


Figure for Ques No. 5(b) – The layout of the muddy city and the paving requirement

6. (a) Matrix Chain Multiplication (MCM) is an optimization problem that finds the most efficient way to multiply a given sequence of matrices. The problem is not actually to perform the multiplications but merely to decide the sequence of the matrix multiplications involved. The matrix multiplication is associative as no matter how the product is parenthesized, the result obtained will remain the same. However, the order in which the product is parenthesized affects the number of simple arithmetic

**CSE 203**

**Contd... Q. No. 6(a)**

operations needed to compute the product. The popular MCM problem aims to find the optimal parenthesization so that the number of arithmetic operations is minimized. We have learned in the class that using dynamic programming, we can design an  $O(n^3)$  algorithm to find the optimal parenthesization and the required number of arithmetic operations. Now answer the following questions. **(7+6+10=23)**

- (i) Suppose  $m[i, j]$  represents the minimum number of multiplications needed to compute  $A_i \cdot A_{i+1} \dots A_j$ . If we are given  $n$  such matrices, then our goal is to compute  $A_1 \cdot A_2 \dots A_n$  or we can say to find the value of  $m[1, n]$ . Now, derive a recursive definition to find  $m[i, j]$ , which we can utilize to populate our DP table.
- (ii) With the help of recursion tree, describe the overlapping sub-problem property of the MCM problem.
- (iii) Suppose you have to multiply the following six matrices with the help of dynamic programming (DP). The number of rows and columns of each of the matrices are provided within the adjacent parenthesis.

$A_1 (30 \times 35)$

$A_2 (35 \times 15)$

$A_3 (15 \times 05)$

$A_4 (05 \times 10)$

$A_5 (10 \times 20)$

$A_6 (20 \times 25)$

You are also given with the following partially filled DP table which aims to solve the MCM problem. The unpopulated cells are marked by  $m[i, j]$ , where  $i$  and  $j$  represents their row and column indices, respectively. First, give an ordering how you will populate these marked cells. Second, using the values of the already populated cells, calculate the value of  $m[2, 5]$ .

	1	2	3	4	5	6
1	0	15750	7875	$m[1, 4]$	$m[1, 5]$	$m[1, 6]$
2		0	2625	4375	$m[2, 5]$	$m[2, 6]$
3			0	750	2500	$m[3, 6]$
4				0	1000	$m[4, 6]$
5					0	$m[5, 6]$
6						0

Table for Ques No. 6(a) – Partially filled DP table for MCM problem

**CSE 203**

**Contd... Q. No. 6**

(b) QUICK-SORT is a divide-and-conquer based algorithm. The traditional PARTITION function of QUICK-SORT works by selecting a *pivot* element from the input array and partitioning the other elements into two sub-arrays, according to whether they are less than or greater than the *pivot*. For this reason, it is sometimes called *partition-exchange* sort. The sub-arrays are then sorted recursively. This can be done *in-place*, requiring small additional amounts of memory to perform the sorting. Now answer the following questions. (5+7=12)

- (i) What is the worst case running time of QUICK-SORT algorithm if it is designed in a way that the last element of the input array is always selected as the *pivot*. Also give an example input scenario which will bring about the worst case running time.
- (ii) Suppose, you have been asked to evaluate the performance of QUICK-SORT algorithm. You are using a bad input array which results in a very unbalanced partition at each of the recursive steps. Consider that at each recursive call, the PARTITION function is producing an  $\alpha$ -to- $\beta$  proportional split. Now using a recursive tree, find the time complexity of the QUICK-SORT algorithm for this scenario. If the last three digits of your student ID represents a number  $x$ , then consider the following.

$$\alpha = x \text{ if } x \leq 100$$

$$\alpha = 200 - x \text{ if } x > 100$$

$$\beta = 100 - \alpha$$

Your answer must contain a well-drawn recursion tree where any kind of existing skewness of the tree is clearly illustrated.

7. (a) Suppose that we have a set of  $n$  activities,  $A = \{ a_1, a_2, \dots, a_n \}$ , where each activity  $a_i$  has a start time  $s_i$  and a finish time  $f_i$  ( $0 \leq s_i < f_i < \infty$ ). Two activities are compatible if and only if their intervals do not overlap. Our target is to generate the maximum-sized subset of mutually compatible activities. Now, prove that an optimal solution is always guaranteed if we design a greedy algorithm by considering the activity with the earliest finish time from the remaining activities. (10)

(b) At the beginning of the COVID-19 pandemic, two biologists Dr. Brown and Dr. William were trying to identify the similarity between COVID-19 and SARS viruses. To perform this task, they extracted the genome sequences of COVID-19 and SARS viruses and tagged them as  $S_1$  and  $S_2$ , respectively. Since they had a basic knowledge on algorithms, they planned to apply a dynamic programming based algorithm to guess an initial similarity between  $S_1$  and  $S_2$ . They found that finding the longest common subsequence of  $S_1$  and  $S_2$  or the edit distance between them can both be considered as good metrics of finding similarity between these two genome sequences. Therefore,

**CSE 203**

**Contd... Q. No. 7(b)**

they planned that Dr. Brown would find the longest common subsequence of  $S_1$  and  $S_2$ . In parallel, Dr. William would find the edit distance between  $S_1$  and  $S_2$ . Now as a student of algorithm course, your task is to carry out the role of Dr. Brown if your student ID is an odd number and the role of Dr. William if your student ID is an even number. To keep it simple, you have to consider only a substring of  $S_1$  and  $S_2$ , say  $S_1' = \langle \text{ATTGCAT} \rangle$  and  $S_2' = \langle \text{TAGCCT} \rangle$ , respectively. You have to write the required recursive definition of your assigned problem and populate the corresponding dynamic programming table. Your table should contain the arrows to illustrate how the cell values are propagating. In case of longest common subsequence problem, you have to find the length of the longest common subsequence and the subsequence itself. On the other hand, in case of the edit distance problem, you have to show the edit distance cost and the sequence of required operations. As the set of available operations, you have to use the set {Insert (cost 1), Delete (cost 1), Replacement (cost 1), Keep (cost 0)} where the terms carry their traditional meanings.

(15)

(c)  $G = (V, E)$  is a connected graph with the vertex set  $V$  and edge set  $E$ , and  $G$  is represented using adjacency-list. A spanning tree of the graph  $G$  is a tree that spans  $G$ , that means, it includes every vertex of  $G$  and is a subgraph of  $G$  (every edge in the tree belongs to  $E$ ). The cost of the spanning tree is the sum of the weights of all the edges in the tree. There can be many spanning trees of the same graph. Minimum spanning tree is the spanning tree where the cost is minimum among all the spanning trees. There also can be many minimum spanning trees of the same graph. Now, in the following figure, you have been given with the Prim's algorithm to find the minimum spanning tree of any graph  $G$ . It uses the traditional Binary Min-Heap to maintain a minimum priority queue. Your task is to analyze the runtime complexity of this given algorithm. In your analysis, you have to mention the individual runtime of each of the significant operations. How this running time is improved if we decide to use a Fibonacci Heap instead?

(10)

**MST-PRIM( $G, w, r$ )**

```

1  for each  $u \in G.V$ 
2       $u.key = \infty$ 
3       $u.\pi = \text{NIL}$ 
4   $r.key = 0$ 
5   $Q = G.V$ 
6  while  $Q \neq \emptyset$ 
7       $u = \text{EXTRACT-MIN}(Q)$ 
8      for each  $v \in G.Adj[u]$ 
9          if  $v \in Q$  and  $w(u, v) < v.key$ 
10              $v.\pi = u$ 
11              $v.key = w(u, v)$ 

```

Figure for Ques No. 7(c) – Prim's Algorithm to find the MST



**CSE 203**

8. (a) Suppose, in a 2-dimensional co-ordinate system, you are given with  $n$  points, say  $P_1, P_2, \dots, P_n$ , where the  $x$ - $y$  co-ordinates of any point  $P_i$  is represented as  $(x_i, y_i)$ . You have been assigned to find the closest pair of points using divide and conquer method. **(5+10=15)**

(i) Write down the approach that you will follow to split these  $n$  points into two partitions, say *LEFT* and *RIGHT*, in such a way that each partition contains roughly  $\frac{n}{2}$  points.

Now suppose, the *LEFT* and *RIGHT* partitions, respectively calculate  $d_L$  and  $d_R$  as the distances between the closest pair of points within themselves. Now your task is to check whether there exist a pair of points  $P_{Li}$  and  $P_{Rj}$  such that  $P_{Li} \in$  *LEFT* and  $P_{Rj} \in$  *RIGHT*, and the distance between  $P_{Li}$  and  $P_{Rj}$  is less than  $\min(d_L, d_R)$ .

(ii) Write down the steps to perform this checking so that the minimum number of comparisons is ensured and the checking can be performed in  $O(n)$  time.

(b) In the problem of finding the number of inversions, we are given a sequence of  $n$  distinct numbers  $a_1, a_2, \dots, a_n$ . We define an inversion to be a pair  $i < j$  such that  $a_i > a_j$ .

Let's call a pair a significant inversion if  $i < j$  and  $a_i > 2a_j$ . Give an  $O(n \log n)$  algorithm to count the number of significant inversions of any given sequence of  $n$  numbers. You have to write the corresponding pseudo-code of your algorithm. **(10)**

(c) Discuss a problem scenario where adopting memoization over dynamic programming can result in a faster solution. Justify your answer. **(10)**

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**SECTION – A**

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Design a logic circuit which will take two 2 bit numbers  $X_1X_0$  and  $Y_1Y_0$  and finds the sum using 2-4 line decoder that has  $I_0, I_1$  as input lines,  $O_3, O_2, O_1, O_0$  active low output lines. The decoder is enabled by active low enables  $EN_1'$  and  $EN_2'$  (Use the minimum number of extra logic gates.) (15)
- (b) After school final result, Ishrat applied for college admission. Ishrat gave her choices from eight colleges which were indexed from  $A$  to  $H$ . She followed the priority order  $B, A, D, E, F, H, G, C$  ( $B$  has the highest priority and  $C$  has the lowest priority). Find the Boolean function of her choices and show the steps of designing the function. (15)
- (c) Can a decoder act like a demultiplexer? If the answer is *Yes*, then explain how it acts. (5)
2. (a) Design a digital security lock using PLA which will take BCD as input and will open after it converts the input to Excess-3 code. Consider the minimum number of product terms. (Show the fuse map.) (15)
- (b) Using  $K$ -Map, minimize  $f(v, w, x, y, z) = \Sigma(2, 7, 8, 15, 22, 23, 25, 29) + d(6, 9, 13, 18, 24, 31)$ . Find the *Prime Implicants* and *Essential Prime Implicants*, and the *Minimal Sums*. Write down the responsible *minterm* for each *Essential Prime Implicant*. (15)
- (c) In a network, for a 3 bit data transmission, an error will be generated if there is an odd parity. Design this logical circuit. (5)
3. (a) Math teacher Mina Rahman sent sixteen question choices  $Q_0, \dots, Q_{15}$  to her students with a 2 bit binary code  $C_1C_0$ . If  $C_1C_0 = 11$ ,  $Q_0$  is selected; if  $C_1C_0 = 10$ ,  $Q_1$  is selected; if  $C_1C_0 = 01$ ,  $Q_2$  is selected; if  $C_1C_0 = 00$ ,  $Q_3$  is selected and so on. The code is activated by active low  $EN_1'$  and active high  $EN_2$ . Design the multiplexing circuit to access the questions. (Design of the circuit will have the mentioned symbols.) (15)

**CSE 205****Contd... Q. No. 3**

- (b) There are 16 electronic turnstile gates in a cricket stadium. The gates are numbered from 0 to 15 denoted by four bits  $T_3T_2T_1T_0$ . You have to design a digital ticketing system which uses a switch, and the switch has four input lines  $I_0, I_1, I_2, I_3$ . These lines can be selected by  $S_0, S_1$ . The switch is activated by two active low enables  $EN_1'$  and  $EN_2'$ . (The selection policy of the switch is as follows. If  $S_1S_0 = 00$ ,  $I_0$  is selected; if  $S_1S_0 = 01$ ,  $I_1$  is selected; if  $S_1S_0 = 10$ ,  $I_2$  is selected; ... so on.). You can have entry to the stadium if you get the ticket for any of the following gate numbers (1, 3, 4, 6, 7, 9, 12, 15). Design the ticketing circuit using the switch. (15)
- (c) Express  $f(A, B, C) = \Pi(0, 2, 3, 5, 7)$  with NOR gates. (5)
4. (a) Design a logic circuit which finds the maximum value between two 3 bit numbers. Explain the operation using the two numbers 7 and 3. (10)
- (b) Design a BCD adder-subtractor which takes two 4 bit BCD numbers  $A$  and  $B$  as input. Addition and subtraction is selected by a selector bit  $S$ . If  $S$  is given 0, the circuit produces  $A + B$ , if  $S$  is given 1, the circuit produces  $A - B$ . (12)
- (c) Design a 4 bit Carry Look Ahead (CLA) adder which will take  $X_3X_2X_1X_0$  and  $Y_3Y_2Y_1Y_0$  as input, and produce their sum. Show the steps and the role of different carries of this CLA. (13)

**SECTION – B**

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) Design a synchronous counter that counts in the following sequence:  
 $0 \rightarrow 2 \rightarrow 4 \rightarrow 7 \rightarrow 5 \rightarrow 3 \rightarrow 0 \rightarrow 2 \rightarrow 4 \rightarrow \dots$   
 Design the counter using S-R flipflops and draw the circuit diagram. (20)
- (b) Distinguish between a ring counter and a Johnson counter. Design a 5-bit ring counter and a 5-bit Johnson Counter using shift registers and illustrate their operation using timing diagram. (10)
- (c) Design a BCD ripple down counter using D flip-flops. (5)
6. (a) Design a control unit with synchronous sequential circuit for a vending machine that accepts 5 Taka, 10 Taka and 20 Taka notes and releases a soft drink bottle worth of 20 Taka. The machine always releases a drink whenever the deposit amount equals or exceeds 20 Tk and keeps the change as a deposit for the next purchase. Hence the

**CSE 205**

**Contd... Q. No. 6(a)**

maximum deposit amount (saved in the vending machine) at any time is 15 Tk. The machine releases the deposit amount whenever the 'CHANGE' input is given. Hence, there are two outputs: Release Drink,  $Z_1$  and Release Change,  $Z_2$ . There are two bit inputs  $X_1X_2$  representing the input combinations as shown in the table for Question 6(a). Please use J-K flip-flops in your design. (20)

Input	$X_1X_2$
Tk 5	00
Tk 10	01
Tk 20	10
CHANGE	11

Table for Question 6(a)

(b) Design a serial parity generation circuit. The circuit receives a sequence of bits and determines whether the sequence contains an even or odd number of ones. The circuit output  $p$  should be 0 for even parity, that is, if the sequence contains an even number of ones, and 1 for odd parity. (10)

(c) Design a D-flipflop using a T-flipflop and gates. (5)

7. (a) For the state-table, in the figure for Question 7(a), of a completely specified circuit, find the equivalence partitions and write the state-table of the minimal machine. Name the states that are equivalent. (10)

PS	NS, Z	
	$x = 0$	$x = 1$
A	B, 1	H, 1
B	F, 1	D, 1
C	D, 0	E, 1
D	C, 0	F, 1
E	D, 1	C, 1
F	C, 1	C, 1
G	C, 1	D, 1
H	C, 0	A, 1

Figure for Question 7(a)

**CSE 205**

**Contd... Q. No. 7**

(b) For the incompletely specified machine shown in the figure for Question 7(b), complete the implication table and determine the maximal compatibles and the maximal incompatibles. Find the upper bound and the lower bound of the number of states of the minimal machine. Give the state table of the minimal machine. (25)

PS	NS, $Z_1Z_2$			
	00	01	11	10
A	A, 00	E, 01	-	A, 01
B	-	C, 10	B, 00	D, 11
C	A, 00	C, 10	-	-
D	A, 00	-	-	D, 11
E	-	E, 01	F, 00	-
F	-	G, 10	F, 00	G, 11
G	A, 00	-	-	G, 11

Figure for Question 7(b)

8. (a) Design a two input ( $x_1, x_2$ ), one output ( $z$ ) fundamental mode circuit that will operate as follows. The output changes from 0 to 1 only on the first  $x_1$  input change that follows an  $x_2$  input change. A 1 to 0 output change occurs only when  $x_1$  changes from 1 to 0 while  $x_2 = 1$ . Determine the primitive flow table, reduced flow table, race-free state assignment, K-maps and circuit diagram. (25)

(b) For the given reduced flow table in the figure for Question 8(b), find a valid assignment without any critical race and complete the modified flow table. (10)

$x_1x_2$	00	01	11	10
a	Ⓐ/0	b/-	Ⓐ/1	b/-
b	a/-	Ⓑ/0	c/-	Ⓑ/0
c	a/-	Ⓒ/1	Ⓒ/0	b/-

Figure for Question 8(b)

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The figures in the margin indicate full marks

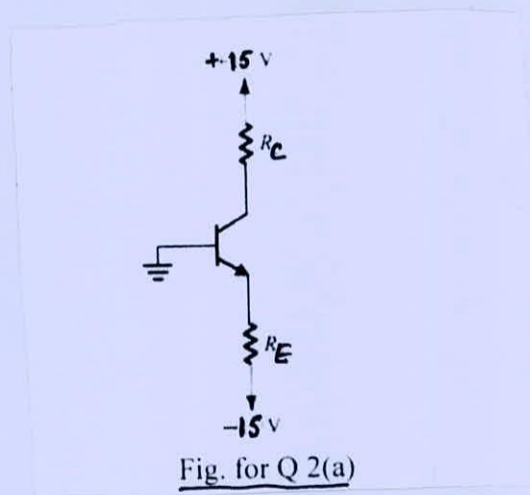
All the symbols have their usual meanings. Assume reasonable values for missing data.

USE SEPARATE SCRIPTS FOR EACH SECTION

**SECTION – A**

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Draw the energy band profile of a pnp transistor under zero bias and forward-active mode bias conditions. (16)
- (b) What is transistor biasing? Why do we need transistor biasing? How can you achieve biasing stability? (10<sup>2/3</sup>)
- (c) The transistor in the circuit of Fig. Q 2(a) has  $\beta = 100$  and exhibits a  $V_{BE}$  of 0.7 V at  $i_c = 1$  mA. Design the circuit so that a current of 2 mA flows through the collector and a voltage of +5 V appears at the collector. (20)



2. (a) For a common collector amplifier, derive the expression for  $R_i$ ,  $R_o$ ,  $A_v$  and  $A_i$  using T model with early effect for the transistor. Where is common collector amplifier used? (26)
- (b) For the circuit shown in Fig. for Q 2(b), find the voltages at all nodes and the currents through all branches. Assume  $\beta = 100$ . (20<sup>2/3</sup>)

Contd ..... P/2

**EEE 263/CSE**

**Contd... Q. No. 2(b)**

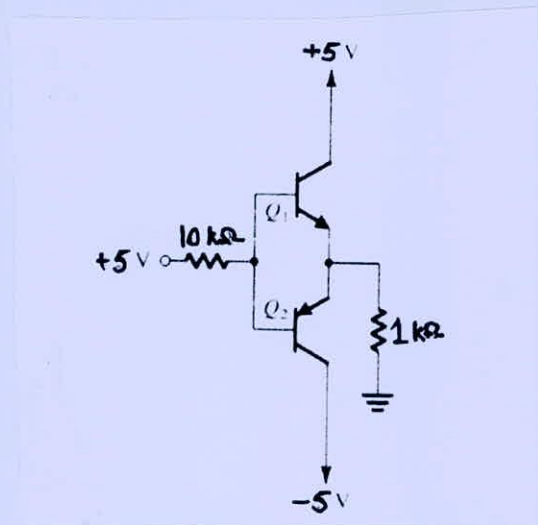


Fig. for Q. 2(b)

3. (a) Design an oscillator circuit using 555 Timer IC which has a square wave output with oscillation frequency of 20 KHz and duty cycle of 25%. Provide necessary illustration of the designed circuit and draw the wave shape. (20)

- (b) Draw the circuit of the Colpitts oscillator. Then using small signal model of the MOSFET, show that the frequency of oscillation, (26<sup>2</sup>/<sub>3</sub>)

$$\omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}} \quad \text{and} \quad \frac{C_2}{C_1} = g_m R$$

4. (a) Drawing appropriate circuits, show how you can generate triangular waves using a bistable multivibrator and appropriate Op-amp circuits. (26<sup>2</sup>/<sub>3</sub>)

- (b) By drawing appropriate circuits, explain the operation of a dual slope A/D converter. (20)

**SECTION – B**

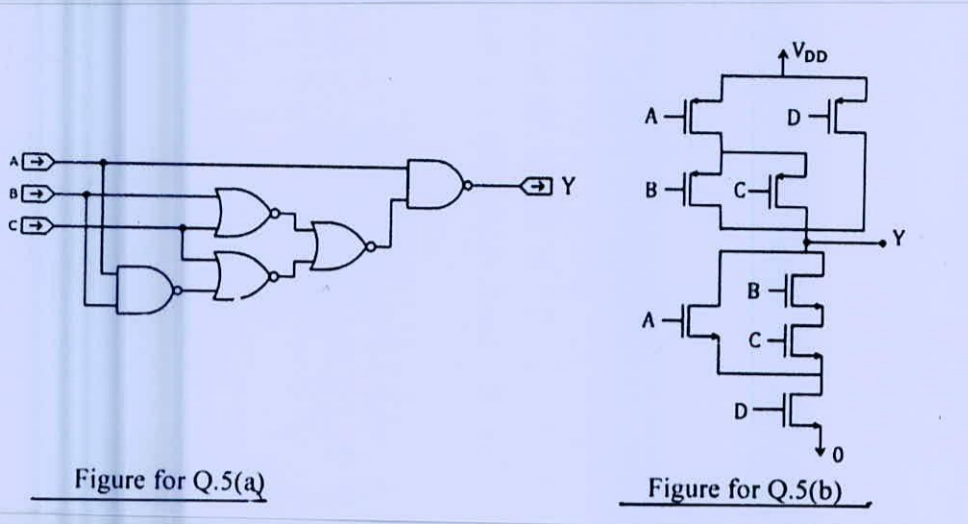
There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) The diagram in Figure for Q. 5(a) shows a Boolean function Y being implemented as a combinational circuit. Design a CMOS transistor level implementation of the Boolean function using minimum number of transistor. (20)

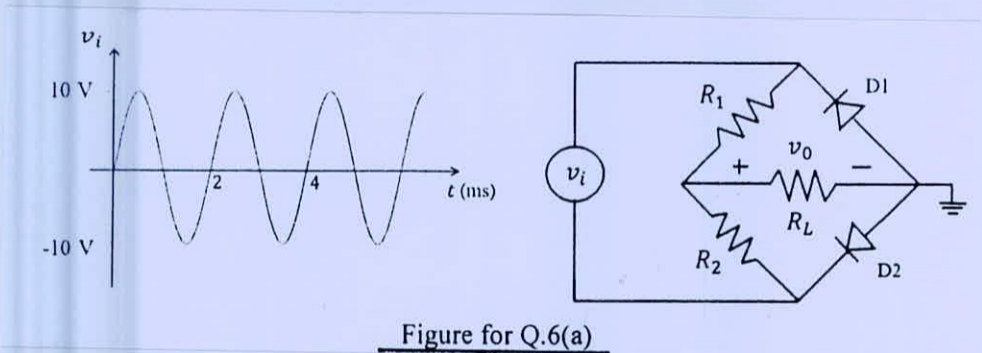
- (b) For the following logic circuit (shown in Figure for Q. 5(b) choose transistor widths to achieve equal rise and fall resistance as a unit inverter. Assume,  $\mu_n = 3\mu_p$ . What is the minimum and maximum propagation delay in the circuit? (26<sup>2</sup>/<sub>3</sub>)

**EEE 263/CSE**

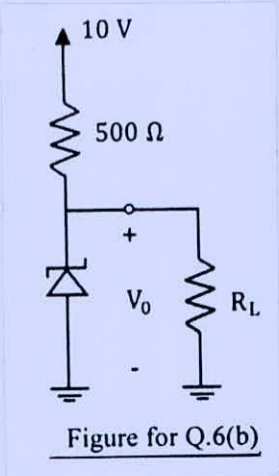
**Contd... Q. No. 5(b)**



6. (a) Draw the transfer curve of the circuit shown in Figure for Q.6(a) assuming all diodes are ideal,  $R_1 = R_2 = 20 \text{ k}\Omega$ , and  $R_L = 10 \text{ k}\Omega$ . (23 <sup>1</sup>/<sub>3</sub>)
- (i) What is the maximum peak voltage of the output signal?
  - (ii) What is the peak-inverse voltage?
  - (iii) What is the average current through the load?



- (b) The zener diode in the circuit shown in Figure for Q.6(b) is specified to have  $V_Z = 6.8 \text{ V}$  at  $I_Z = 5 \text{ mA}$ ,  $r_z = 20 \Omega$  and  $I_{ZK} = 0.2 \text{ mA}$ . (23 <sup>1</sup>/<sub>3</sub>)
- (i) Find  $V_0$  at no load conditions.
  - (ii) What is the minimum value of  $R_L$  for which the diode will still operate in the breakdown region?





**EEE 263/CSE**

7. (a) Design a circuit using ideal operational amplifiers that would implement the following equation:  $v_o \propto \int (v_1 + v_2) dt$ . Write the final output expression in-terms of the circuit components used in your design. (20)

(b) Determine the overall small-signal voltage gain of the common source amplifier shown in Figure for Q. 7(b). The transistor parameters are: (26  $\frac{2}{3}$ )

$$V_t = 0.4 \text{ V}, k_n = 0.5 \text{ mA/V}^2, \lambda = 0.02 \text{ V}^{-1}$$

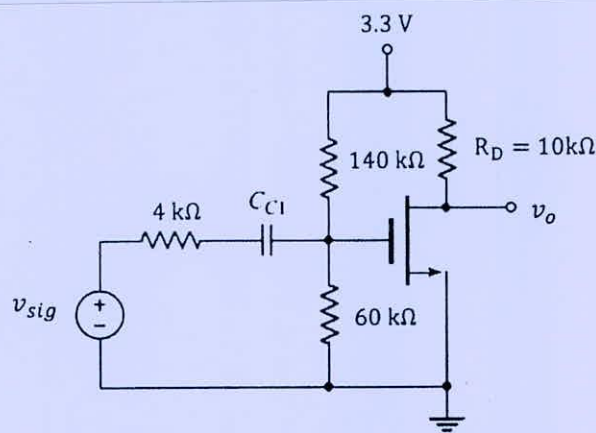


Figure for Q.7(b)

8. (a) Determine the closed-loop voltage gain and current-gain of the ideal op-amp network shown in Figure for Q. 8(a). (23  $\frac{1}{3}$ )

(b) Design the circuit shown in Figure for Q. 8(b) so that the transistor operated at  $I_D = 0.3 \text{ mA}$  and  $V_D = 0.4 \text{ V}$ . The NMOS transistor has a threshold voltage of 1 V, aspect ratio of 40 and a process transconductance parameter of  $60 \mu\text{A/V}^2$ . Neglect channel-length modulation effect. (23  $\frac{1}{3}$ )

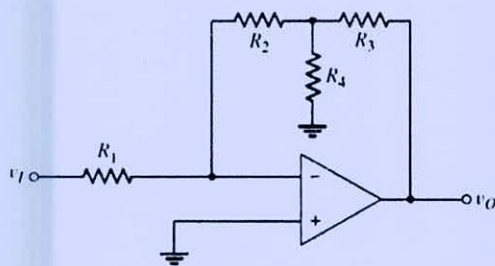


Figure for Q.8(a)

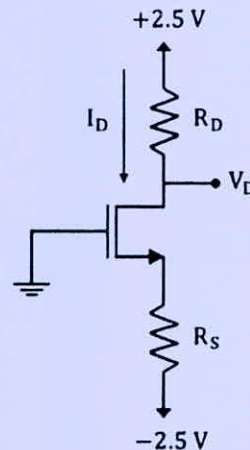


Figure for Q.8(b)

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2020-2021

Sub: **MATH 245** (Complex Variable and Statistics)

Full Marks: 210

Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

**SECTION – A**There are **FOUR** questions in this section. Answer any **THREE** questions.

Symbols used have their usual meaning.

1. (a) Test the differentiability of the function  $f(z) = \frac{64}{z^4}$  in its natural domain. (11)
- (b) Transform the circle  $x^2 + y^2 - 25x = 0$  into a straight line applying the  $w = \frac{5z+4}{z+3}$  transformation. (12)
- (c) Find all roots of the equation  $\cos z = 4$  by equating the real and imaginary parts in the equation. (12)
2. (a) Prove that  $u(x, y) = \sinh x \sin y + \frac{y}{x^2 + y^2}$  is harmonic. Find a harmonic conjugate of  $u(x, y)$ . (15)
- (b) Let  $C$  be the arc of the circle  $|z| = 2$  from  $z = 2$  to  $z = 2i$  that lies in the first quadrant, then show that  $\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}$ . (10)
- (c) Show that  $\text{Log}(-1+i)^2 \neq 2 \log(-1+i)$ . (10)
3. (a) Use Cauchy's integral formula to evaluate the integral  $\frac{1}{2\pi i} \oint_C \frac{e^{-3z}}{(z^2+4)} dz$ , where  $C$  is the circle  $|z| = 3$ , taken in the positive sense. (10)
- (b) Expand  $f(z) = \frac{5}{(z+1)(z+3)}$  in a Laurent's series valid for (i)  $1 < |z| < 3$  and (ii)  $|z| < 1$ . (10)
- (c) Evaluate  $\oint_C \frac{\sin \pi z}{(z-3)^3} dz$ , where  $C$  is the circle  $|z| = 4$ , by Cauchy's residue theorem. (15)
4. Evaluate the following integral using the method of contour integration:
- (i)  $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)^2}$  (17)
- (ii)  $\int_0^{\infty} \frac{x^2 dx}{(1+x^6)}$  (18)

**MATH 245/CSE**

**SECTION – B**

There are **FOUR** questions in this section. Answer any **THREE** questions.

Symbols used have their usual meaning.

5. (a) A survey, data on daily wages paid to workers of two factories A and B are as follows: **(10)**

Daily Wages	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Factory A	15	30	44	60	30	14	7
Factory B	25	40	60	35	20	15	5

Find out:

- (i) Which factory pays higher average wages?
- (ii) Which factory has greater variability about paying wages?

- (b) An analysis of weight distribution resulted in the following table: **(15)**

Weight	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	18	37	45	27	15	8

Calculate Bowley’s coefficient of skewness and Kelly’s coefficient of skewness for the above data. Hence, interpret your results by discussing the advantages and drawbacks of the methods.

- (c) Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pounds per square inch (psi). Let X denote the actual air pressure for the right tire and Y denote the actual air pressure for the left tire. Suppose that X and Y are random variables with the joint density function: **(10)**

$$f(x, y) = \begin{cases} k(x^2 + y^2), & 30 \leq x < 50, 30 \leq y < 50, \\ 0, & \text{elsewhere.} \end{cases}$$

- (i) Find the value of k.
- (ii) Find  $P(30 \leq X \leq 40 \text{ and } 40 \leq Y < 50)$ .
- (iii) Find the probability that both tires are underfilled.

6. (a) The following table shows the experience (X) and the performance rating (Y) of 5 persons: **(15)**

X	16	12	18	4	3
Y	87	88	89	68	58

- (i) Fit a linear regression model for Y on X.
- (ii) Find sum of squares for regression (SSR) and total sum of squares (SST).

**MATH 245/CSE**

**Contd... Q. No. 6**

(b) Measurement of scientific system are always subject to variation, some more than others. There are many structures for measurement error, and statisticians spend a great deal of time modeling these errors. Suppose the measurement error  $X$  of a certain physical quantity is decided by the density function:

(10)

$$f(x) = \begin{cases} k(3-x)^2, & -1 \leq x \leq 1, \\ 0 & \text{elsewhere.} \end{cases}$$

- (i) Determine  $k$  that renders  $f(x)$  a valid density function.
- (ii) Find the probability that a random error in measurement is less than  $\frac{1}{2}$ .
- (iii) For this particular measurement, it is undesirable if the magnitude of the error exceeds 0.8. What is the probability that this occurs?

(c) According to a study published by a group of University of Massachusetts sociologists, about two-thirds of the 20 million persons in this country who take Valium are women. Assuming this figure to be a valid estimate, find the probability that on a given day the fifth prescription written by a doctor for Valium is

(10)

- (i) the first prescribing Valium for a women
- (ii) the third prescribing Valium for a women.

7. (a) Imperfections in computer circuit boards and computer chips lend themselves to statistical treatment. For a particular type of board, the probability of a diode failure is 0.03 and the board contains 200 diodes.

(17)

- (i) What is the mean number of failures among the diodes?
- (ii) What is the variance?
- (iii) The board will work if there are no defective diodes. What is probability that a board will work?

(b) A research scientist reports that mice will live an average of 40 months when their diets are sharply restricted and then enriched with vitamins and proteins. Assuming that the lifetimes of such mice are normally distributed with a standard deviation of 6.3 months, find the probability that a given mouse will live

(18)

- (i) more than 32 months
  - (ii) between 37 and 49 months.
- (Necessary Table 1 and 2 are attached)

8. (a) The hospitalization period, in days, for patients following treatment for a certain type of kidney disorder is a random variable  $Y = X + 4$ , where  $X$  has the density function:

(10)

$$f(x) = \begin{cases} \frac{32}{(x+4)^3}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the average number of days that a person is hospitalized following treatment for this disorder.

**MATH 245/CSE**

**Contd... Q. No. 8**

(b) According to a dietary study, high sodium intake may be related to ulcers, stomach cancer, and migraine headaches. The human requirement for salt is only 220 milligrams per day, which is surpassed in most single servings of ready-to-eat cereals. If a random sample of 20 similar servings of a certain cereal has a mean sodium content of 244 milligrams and a standard deviation of 24.5 milligrams, does this suggest at the 0.05 level of significance that the average sodium content for a single serving of such cereal is greater than 220 milligrams?

**(15)**

(Necessary Table 3 is attached)

(c) In a scientific paper, one researcher described the reliability of DNA paternity testing as follows: "To get a completely accurate result, you would have to be tested, and so would (the man) and your mother. The test is 100% accurate if the man is not the father and 99.9% accurate if he is." Consider using the results of DNA paternity testing to decide between the following two hypotheses:

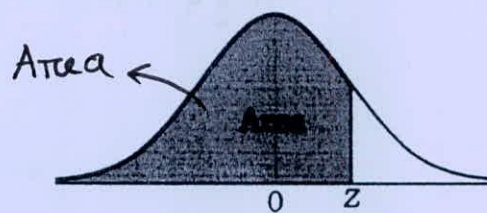
**(10)**

$H_0$  : a particular man is the father

$H_a$  : a particular man is not the father

In the context of this problem, describe Type I and Type II errors.

= 5 =

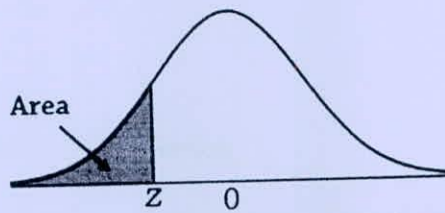


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table 1: Necessary Table for Question No. 7(b)

Contd. .... P/6

= 6 =



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

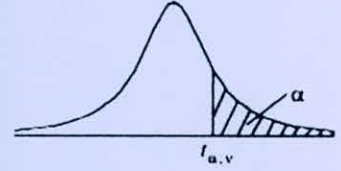
Table 2: Necessary Table for Question No. 7(b)

Contd - - - - P/7

$$= 7 =$$

### Table of the Student's $t$ -distribution

The table gives the values of  $t_{\alpha, v}$  where  
 $\Pr(T_v > t_{\alpha, v}) = \alpha$ , with  $v$  degrees of freedom



$\alpha \backslash v$	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
$\infty$	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Table: 3 Necessary Table for Question No. 8(b)